Characterization and Prediction of Reactive Forces for Compressed 3D Printed Hyperelastic Lattice Structures

by
David So

A THESIS

submitted to

Oregon State University

Honors College

in partial fulfillment of
the requirements for the
degree of

Honors Baccalaureate of Science in Mechanical Engineering
(Honors Associate)

Presented May 22, 2020
Commencement June 2020

AN ABSTRACT OF THE THESIS OF

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Abstract approved:____

Brian Paul

In order to generate practical ways to easily design functional flexible lattice structures, this research seeks to find a way to predict reactive forces of compressed hyperelastic cubic lattice structures. Using 3D printed test coupons, material data was collected for use in finite element analysis. Cubic lattice structures with varying densities and member thicknesses were analyzed via FEA methods and the resultant relation between compressive and reactive forces was curve fitted using the closest fitting function, in this case a quadratic function. Using triangular interpolation of the physical parameters of the lattices and the quadratic equation coefficients, new predictive equations were generated for two new cubic lattice structures of untested parameters. The untested lattices were then analyzed using FEA methods and compared against their predicted interpolated equations. The average percent errors between the predictive equations and the FEA methods for the two models were 1.20% and 4.17%. This shows that for this material and lattice structure, using triangular interpolation to predict reactive forces is sufficient for use in a design cycle.

Key Words: 3D Printing, Additive Manufacturing, Finite Element Analysis, Lattice Structures, Hyperelastic, Nonlinear Elastic, Interpolation

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Chris Hoyle, Committee Member, representing School of Mechanical, Industrial, and Manufacturing Engineering

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I understand that my project will become part of the permanent collection of Oregon State University, Honors College. My signature below authorizes release of my project to any reader upon request.

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David So, Author

Introduction

Additive manufacturing is making possible the cost-effective manufacturing of custom mechanical and electromechanical apparatuses designed to meet the unique needs of the individual. This is in large part due to the lack of product-specific tooling, enabling the manufacture of different designs without the need for tooling changeover and the cost-effective production of one-off designs made for specific people. This technology has been exploited in this manner in industry applications for dental implants, prosthetic limbs [1], and shoe insoles [2].

Besides the ability to easily produce custom designs, the lack of product-specific tooling enables additive manufacturing to produce complex geometries without significant cost penalty, making possible the fabrication of geometries which would be impossible to produce using traditional manufacturing methods. Al-Ketan et al. [3], in looking to understand the topology-property relationship of several types of periodic cellular materials, found that Diamond Triply Periodic Minimal Surface (TPMS) structures achieved the best mechanical properties among the structures evaluated, properties which seemed to be nearly independent of relative density.

The ability to print different types of unit cell lattice structures, combined with the ability to print hyperelastic polymers, have opened new possibilities for compliant design. There have been several studies evaluating different types of lattice structures. Habib et al. [4] looked at the impact of lattice shape in energy absorption and was able to find differences in performance of bending-dominated deformation versus stretch and buckling dominated structures. Yuan et al. [5] analyzed different thermoplastic polyurethane (TPU) powders and their response to being laser sintered and cyclically loaded using Bucklicrystals, showing that these structures could withstand repeated compression cycles over large deformations. Abueidda et al. [6] compared experimental and finite element analysis (FEA) results for gyroidal lattice structures, finding that they perform similarly in comparison with other TPMS structures. Importantly, they also found that their FEA methods correlated well with the experimental results.

When properly validated, FEA is a powerful tool for being able to calculate the response of designs and materials without the need for costly fabrication. For simulating with hyperelastic materials, or materials which do not follow the traditional linear elastic deformation pattern, the proper model must be used. The Neo-Hookean model, developed in 1948 by Dr. Ronald Rivlin, is a method used to calculate the elastic strain energy potential energy [7]. It is suitable for accurately modeling up to 40% strain in uniaxial tension [8].

While there seems to be an ever-expanding amount of research in understanding the mechanics and optimal structure of different lattices and materials, there is a gap between this advanced knowledge and implementing it in practical designs. For industry engineers to be able to more effectively leverage these potential benefits of additive manufacturing, simple and straightforward predictive tools need to be developed. This paper focuses on analyzing rectangular lattice structures under compression with different relative densities, ultimately examining if simple two-dimensional curve fitting interpolation can be used to predict the reactive force response to compression. The aim is to produce a simple method that could easily be integrated in the beginning of design cycles to allow for faster selection of appropriate functional hyperelastic lattice structures. An Objet additive manufacturing system was used to produce coupons used to characterize the properties of the material as well as for comparison with FEA results. Nonlinear FEA methods were loaded with material data and used to simulate behavior of various lattice structures.

Methods and Materials

The general approach was to determine the compressive response of the material, use the interpreted compressive properties in simulating the response of a computer-aided design (CAD) test article, print a test article, run compression experiments and compare finite element analysis (FEA) and experimental results. Based on acceptable percent deviations, the compressive properties were used to explore a new set of material lattices to understand the effect of the lattice on the bulk response of the structure. The bulk response was modeled as a quadratic curve and interpolated to predict the response of new structures. The response of the quadratic curve was compared to FEA results for the same structure to determine whether this method was more computationally efficient method of determining the structure needed to achieve a particular bulk response.

Material

The polymer used was Stratasys' TangoBlackPlus FLX980, a resin meant to simulate rubber thermoplastic elastomers. Material properties of TangoBlackPlus are listed in Table 1.

Tensile Strength	0.8-1.5 MPa
Elongation at Break	170-220%
Compressive Set	4-5%
Shore Hardness (A)	26-28 Scale A
Tensile Tear Resistance	2-4 Kg/cm
Polymerized Density	1.12-1.13 g/cm^3

Table 1. Physical Properties of TangoBlackPlus, provided by Stratasys

Material Property Validation

A physical test article was produced in order to measure the compressive mechanical response of the material (**Fig. 1**). This test article was a cube, 38.1 mm in each dimension. This model was used for its simple cross-sectional area and geometries, helpful for easy calculation of the nominal stress. As shown in **Fig. 2 (right)**, the test cube was 3D printed using the polymer described above. The length, width, and height were recorded and were used to convert the force-displacement data into nominal stress and strain.

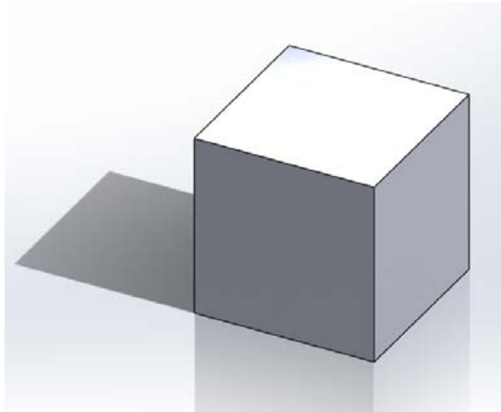


Figure 1. CAD model of the material property measurement test article.

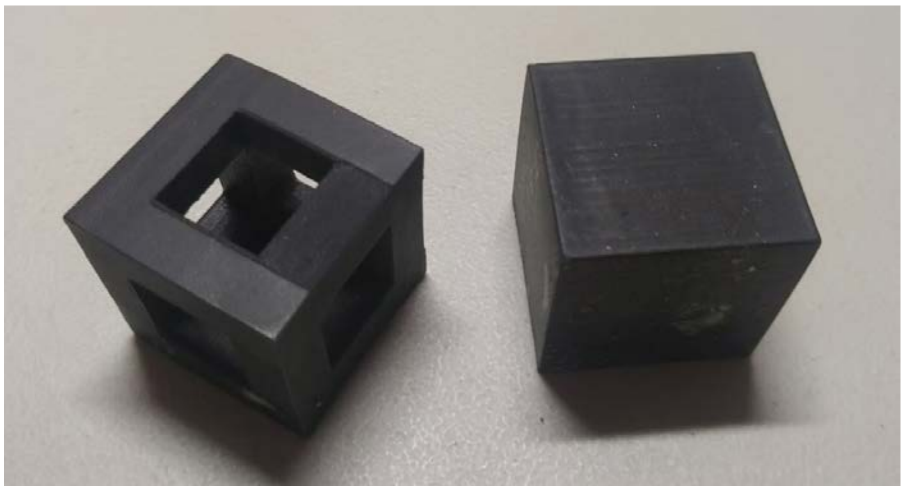


Figure 2. 3D printed test articles for (left) material property validation; and (right) material property measurement.

In addition, a cored cube (**Fig. 2 [left]**) was printed for use in validating the material properties derived from the first test article. The CAD model (**Fig. 3**) was a cube 38.1 mm on a side with square profile cutouts 19.05 mm on a side, centered in each face of the cube.

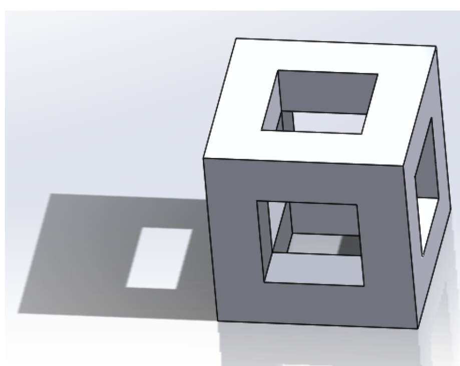


Figure 3. CAD model of the material property validation test article

The cubes were compressed within an Instron universal testing machine equipped with compressive platens, which generated compressive force-displacement data for the two physical test articles. Prior to testing, the actual dimensions of the cubes were recorded in order to calculate the apparent engineering stress-engineering strain of the structures. Each sample was put between the compressive plates and compressed at a rate of 5 mm/min up to 15 mm of displacement. The universal tester recorded data every 0.02 seconds. The resultant force-displacement data and measured dimensions of the cubes were used to calculate the nominal stress-strain curve of the bulk structures material. The behavior of the solid cube was used to inform a hyperelastic material within Abaqus 6.14 CAE. In Abaqus, the cored cube model was assigned the properties of the hyperelastic material described above using the nominal stress and strain. The top face of the cube was then negatively displaced, and the reactive force recorded.

Finite Element Analysis

Abaqus 6.14 CAE was used to analyze all geometries. The material property assigned to each model was a Neo-Hookean hyperelastic material, using nominal stress and strain data obtained from the experimental data of the solid cube. The element type was an 8-node linear brick, with reduced integration and hourglass control. The bottom face of all geometries was encastered, while the top face was displaced from 1 to 10 mm, in increments of 1 mm. The total reactive force was recorded from the top face. Each model's mesh was refined using a top face displacement of 5 mm until the results stabilized, with a minimum of 20,320 nodes used.

To equate force outputs and displacements between cubes, the reactive forces were divided by the overall square cross-sectional area taken up by the cube.

$$\sigma' = \frac{F_{reaction}}{A_{cross}} \text{ where } A_{cross} = a_{side}{}^2 \text{ and } a_{side} \text{ is the overall length of the side of the cube}$$

Displacements were divided by the length of the side of the cube.

$$\varepsilon' = \frac{\Delta L}{a_{height}} \text{ where } \Delta L \text{ is the displacement of the top face and } a_{height} \text{ is the original height of the cube}$$

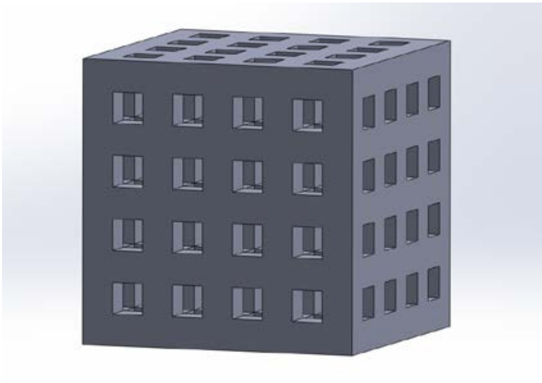
Below, to create curve fit data, normalized reactive forces and strain data was plotted and then curve fit in Excel using a quadratic model. Using two-dimensional interpolation and the constants from the quadratic equations, new equations were generated for two new dimensions of lattices. These two cubes were tested with FEA methods, one with a member thickness of 3.5 mm and a spacing of 3 mm, and one with a member thickness of 2.5 mm and a spacing of 3.5 mm. The data from the FEA methods was then compared against the interpolated quadratic equations.

Model Exploration

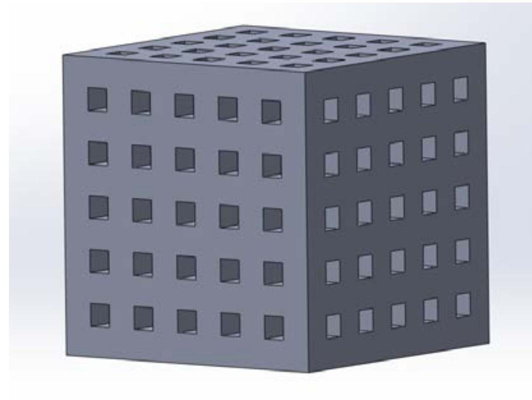
To explore the relationship between the morphology of the lattice and the bulk response of the lattice, multiple 3D lattice models were created using square members, with the parameters being the side length of the square member and the spacing between the members. The models were assigned the same material properties as above, the top face displaced and the reaction force recorded. The reactive forces were then divided by the outer cross-sectional area footprint of each cube to provide an apparent engineering stress, and the displacement divided by the outside length of the cube to provide an apparent engineering strain for the structures. The resultant 'stress' and 'strain' were then curve fit with a quadratic curve.

Afterwards, efforts were made to determine if a new more computationally efficient method could be used to predict the response of new lattice structures. The prior quadratic curves were used to interpolate a new predictive quadratic equation for predicting the response of a new set of lattices. The new lattices were then modeled and evaluated in Abaqus 6.14 CAE, and the apparent engineering stress and strain compared against the predictive interpolated equation.

All lattice structures were comprised of square members. Nine models were generated, a combination of 2, 3, and 4 mm thick members and spacings. These geometries were chosen due to the geometric similarities to the cored cube, both in structure and in size. The models were referred to by the size of the member struts and spacing as #S#M, where a cube with 4 mm member struts with 4 mm wide spacing was named 4S4M. All models used are shown in **Fig 4**.



(a)



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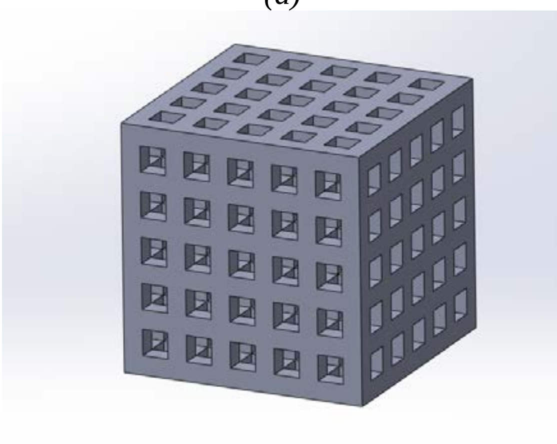

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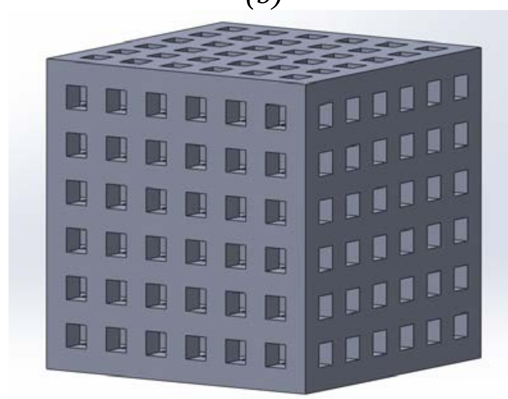

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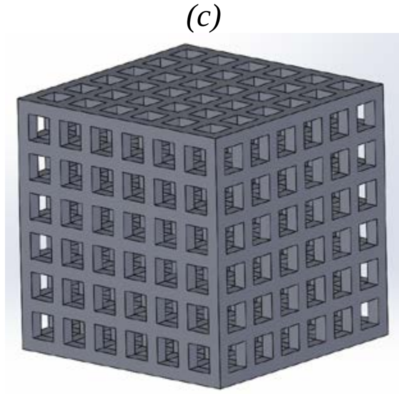

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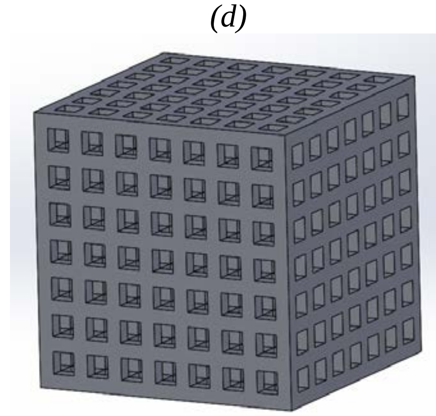

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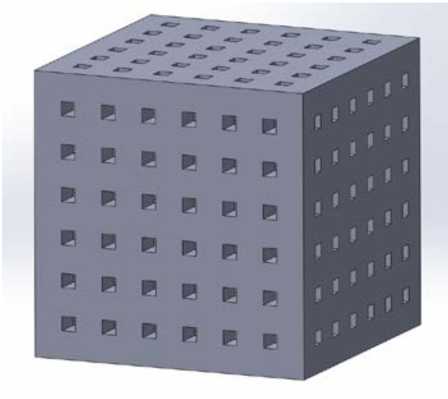

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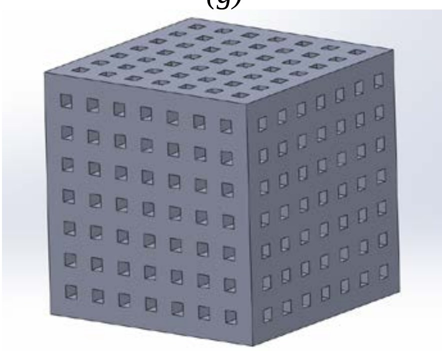

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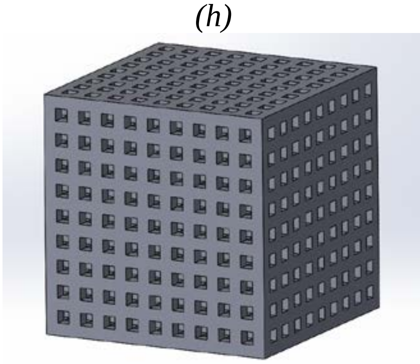
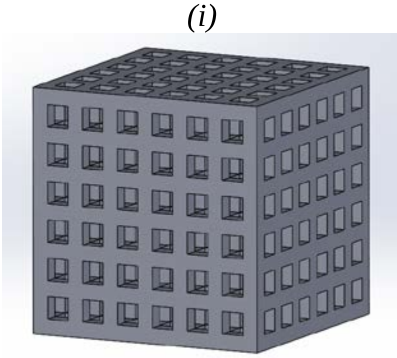


(i)



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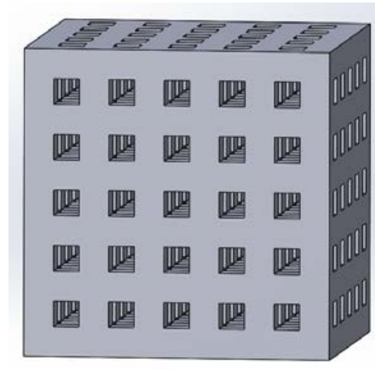


(k)

Fig 4. The different lattice cubes used. *(a)* 4 mm members with 4 mm spacing (4S4M), overall side width: 36 mm. *(b)* 4 mm members with 3 mm spacing (3S4M), overall side width: 39 mm. *(c)* 3 mm members with 4 mm spacing (4S3M), overall side width: 38 mm. *(d)* 3mm members with 3 mm spacing (3S3M), overall side width: 39 mm. *(e)* 2 mm members with 4 mm spacing (4S2M), overall side width: 38 mm. *(f)* 2 mm members with 3 mm spacing (3S2M), overall side width: 37 mm. *(g)* 4 mm members with 2 mm spacing (2S4M), overall side width: 40 mm. *(h)* 3 mm members with 2 mm spacing (2S3M), overall side width: 38 mm. *(i)* 2 mm members with 2 mm spacing (2S2M), overall side width: 38 mm. *(j)* Test Cube: 2.5 mm members with 3.5 mm spacing (3.5S2.5M), overall side width: 38.5 mm. *(k)* Test Cube: 3.5 mm members with 3 mm spacing (3S3.5M), overall side width: 36 mm

Results and Discussion

Material Property Validation

For the solid cube, the cross-sectional area calculated based on measured dimensions was 1440.96 mm^2 and the vertical height for strain calculations was found to be 37.96 mm. Force-displacement and engineering stress-strain plots are shown in Fig. 5 up to a maximum compressive force of 506.5 N. The nominal stress and strain data was entered into Abaqus CAE as a hyperelastic Neo-Hookean material and a compressive simulation of the solid cube was run. The resultant force-displacement is plotted in **Fig. 6**.

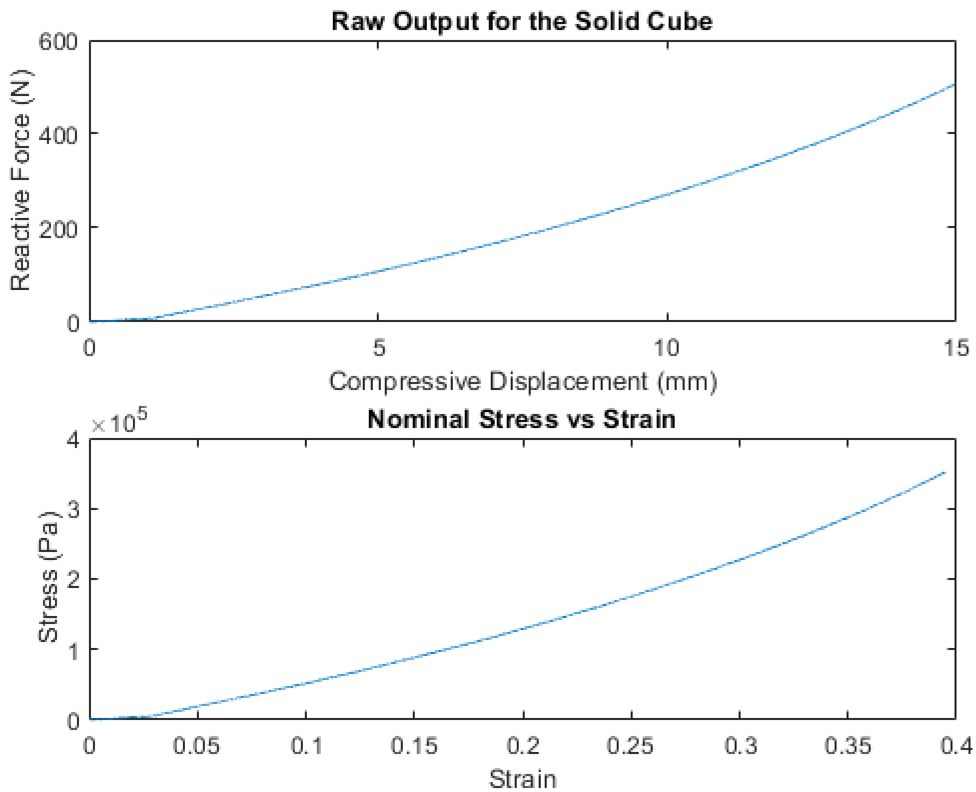
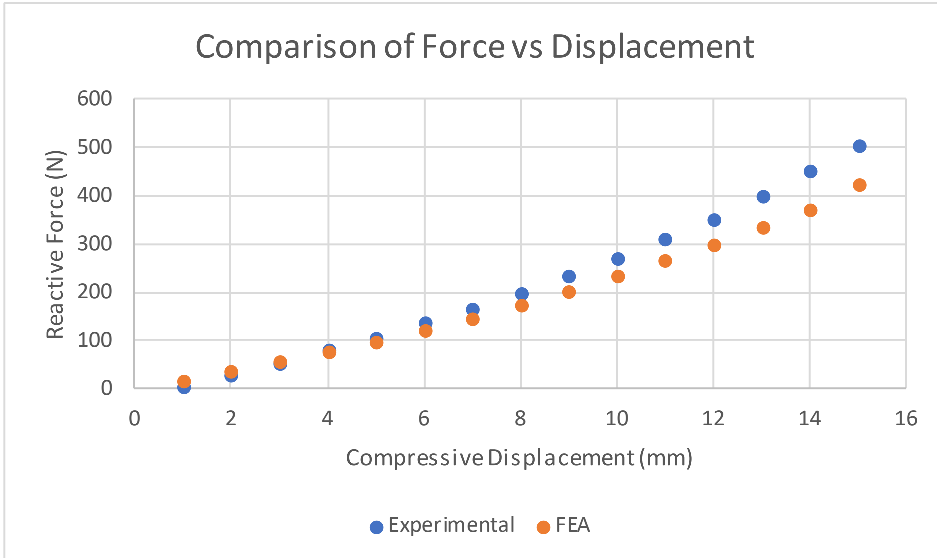


Fig 5. Plots showing the experimental data of reactive force versus compressive displacement of the solid cube as well as the calculated nominal stress and strain



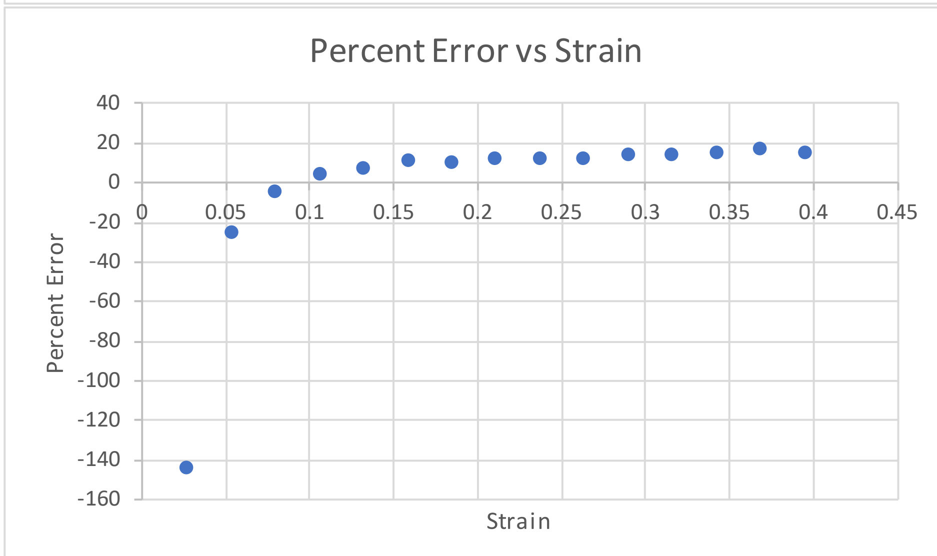


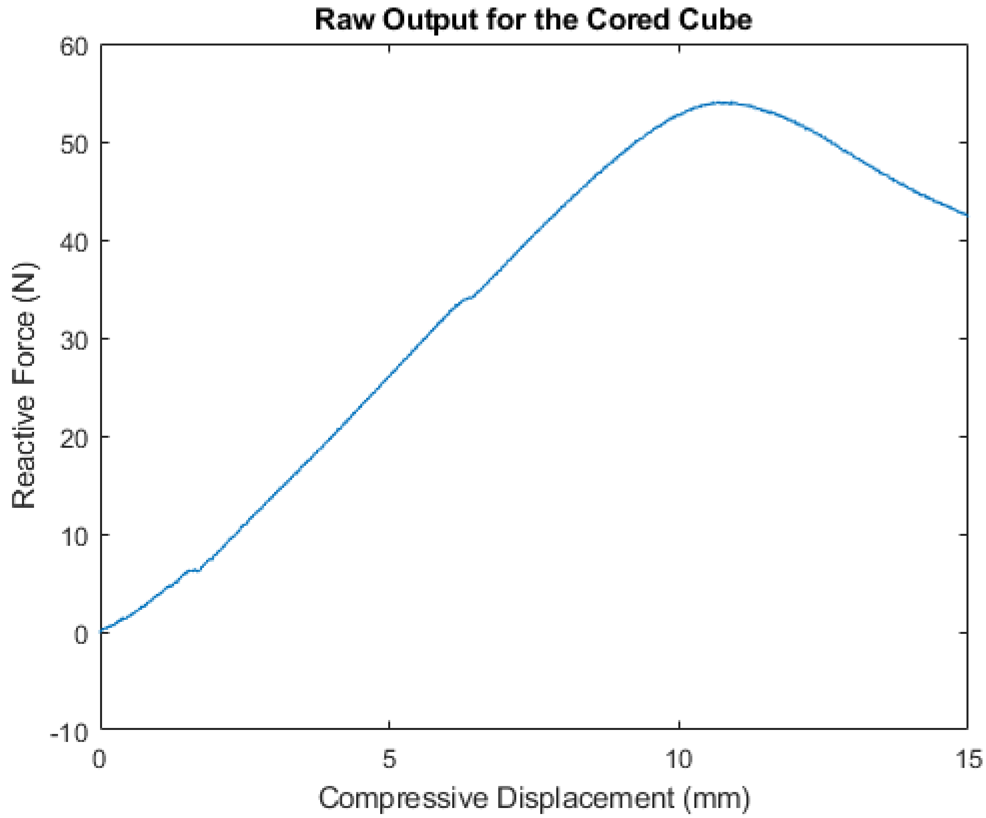
Fig 6. Plots comparing the solid cube experimental results and FEA results. As can be seen, after about 0.1 strain, the percent error is approximately 15%.

The percent error is initially very high, which can be attributed to the imprecision of setting a perfect zero point on the Instron. The top face of the printed model is not perfectly flat, thus making it hard to set the physical point where the 3D printed model is at the point just before deformation.

The percent error stabilizes around 17.5%, which is by itself unacceptably high. If this is the only data used to benchmark the FEA method results, the FEA method would not be reliable enough.

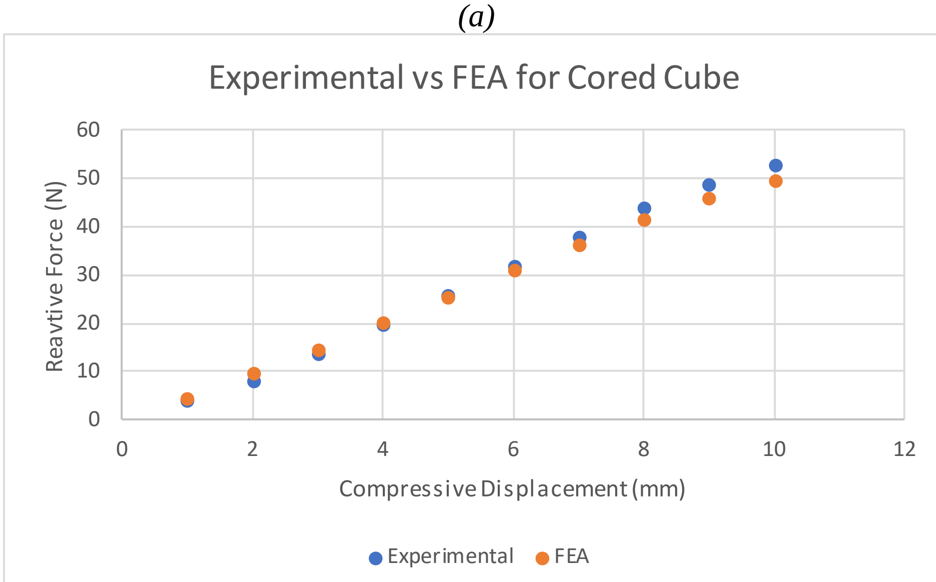
Cored Cube Analysis

Using the nominal data from the solid cube, the cored cube model was imported into FEA and force displacements outputted and compared against experimental data.



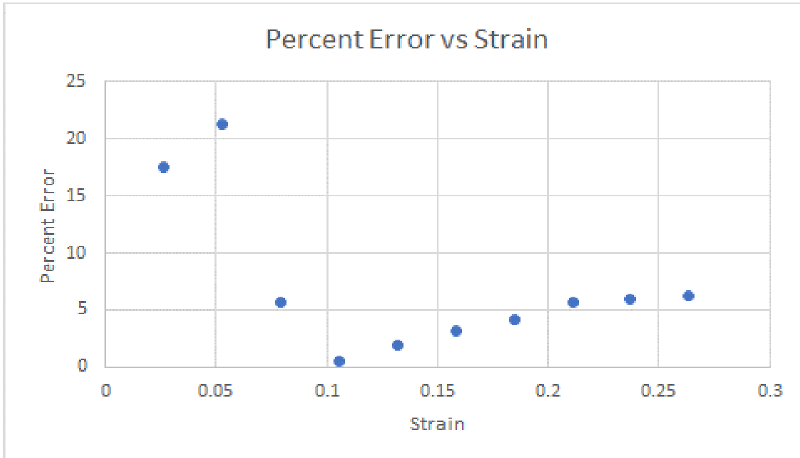
Raw Output for the Cored Cube
Reactive Force (N)
Compressive Displacement (mm)
60
50
40
30
20
10
0
-10
0
5
10
15

(a)



Experimental vs FEA for Cored Cube
Reavtive Force (N)
Compressive Displacement (mm)
60
50
40
30
20
10
0
0
2
4
6
8
10
12
Experimental
FEA

(b)



(c)

Fig 7. Plots looking at the cored cube data. *(a)* Data from compressive test. *(b)* Comparison between FEA and experimental results. *(c)* Percent error between FEA and experimental results

In **Fig 7a**, the data shows that after 11 mm of displacement, the reactive force drops as compression continues. This most likely is due to buckling from the side members of the cored cube. This is consistent with efforts in FEA, shown in **Fig 7b**, which failed to converge on a solution at 11 mm, suggesting a complex behavior like buckling. As shown in **Fig 7c**, the final error stabilizes to just over 6%. Like the solid cube, the large percent error at the start of the compression cycle is likely due to imprecise setting of the zero point in the experiment.

Comparing the two percent differences in correlation with strain, the solid cube's percent error is about twice that of the cored cube. The reason for this difference could be attributed by how the two cubes deform. The cored cube's hollow center allows the cube to stretch and deform inwardly, while the solid cube can only deform outwardly. The boundary condition from the bottom was set as encastered, but with large amounts of deformation, the theoretical model may behave differently from the physical model. Employing two compressive plate bodies for the top and bottom face with frictional forces between the plates and the cube may be a more precise model. This method would require measuring the frictional coefficients between the polymer and the metal plates used on the Instron, something not possible to measure due to the constraints from COVID19.

One way that the experimental protocol could be improved upon is through the application of a standard, like ASTM D575. Further, efforts to test multiple cubes would reduce measurement error, improving the quality of stress and strain data. Given time and monetary constraints, only one sample cube was made in this project. It is expected that tests would be destructive as the dimensions of the cube changed after compression suggesting that the material may be elastoviscoplastic, meaning the cube simultaneously strains both elastically and plastically.

FEA Results of Varying Lattice Structures

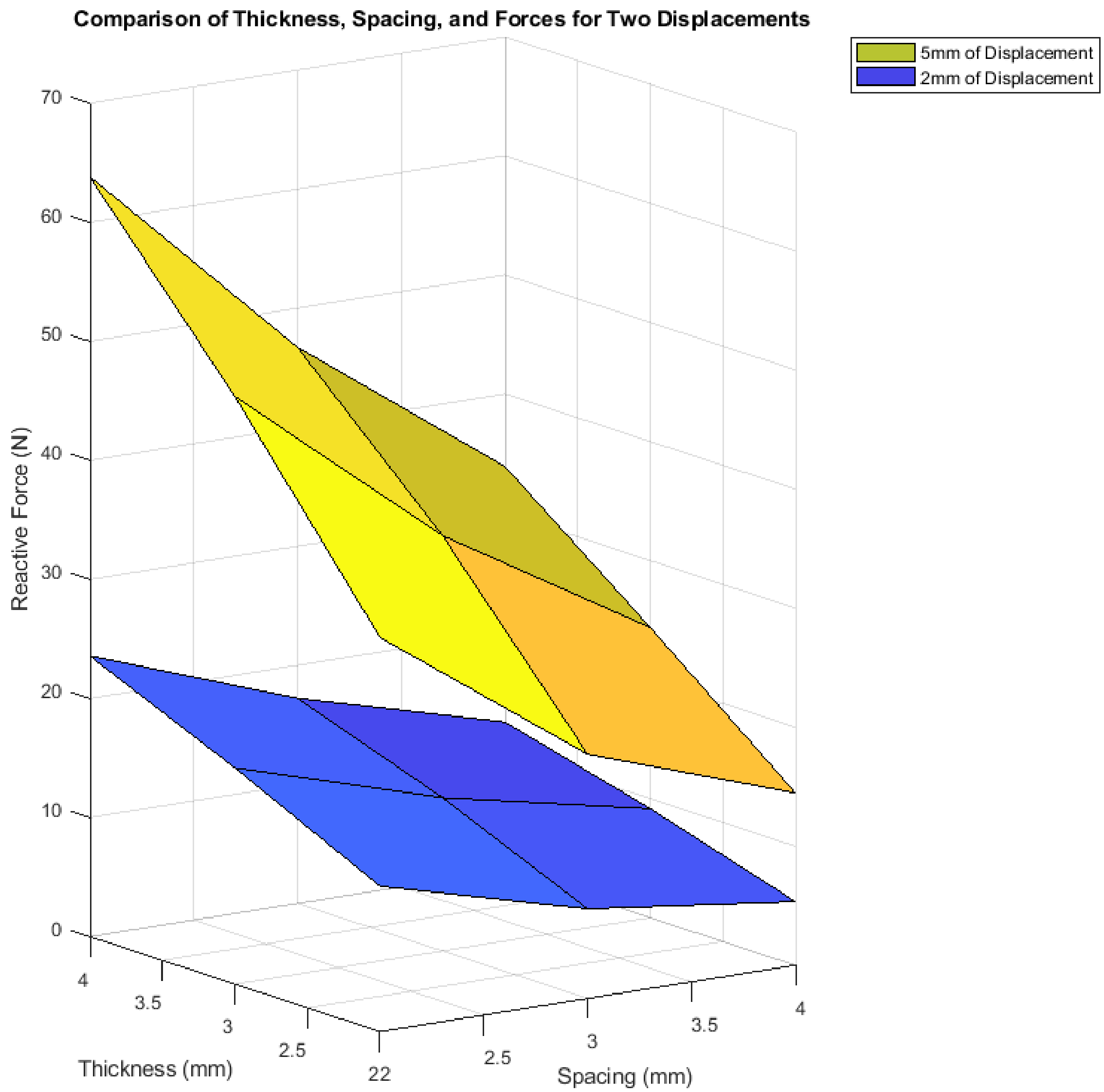


Fig 8. Surface plots of thickness and spacing with reactive force. Color gradient is based on the surface gradient

Fig 8 illustrates the effect of the reactive force with respect to different member thickness and spacings. Between the two surfaces, there is a greater change in reactive forces across the different thicknesses and spacings with more compression. The difference in material between 2S4M and 2S3M is nonlinear due to the cross-sectional area changing. Thus, the higher change in reactive force reflects the nonlinearity change in the amount of material used in each lattice. It can also be seen in the variance of the surface shade that the change within each displacement is also nonlinear. Once again, given that change in spacing and in thickness refers in effect to change is cross sectional area, it would make sense that it is nonlinear.

As shown below in **Fig 9**, there is a general trend that thinner members with greater spacing results in less reactive forces and thicker members with less spacing results in larger reactive forces. It should be noted that for 4S2M, there was excessive deformation for the model at a displacement of 6 mm, thus there were only five data points, from 1 to 5 mm. If there is too much bending from the members, then the FEA software is not able to converge on a solution. A similar error occurred when modeling the higher displacement levels for the cored cube. This happened when the top face displaced by 11 mm, which, when compared against the experimental compressive data, is after the maximum reactive force. Most likely, the physical specimen buckles, which is hard for FEA solutions to predict and model accurately.

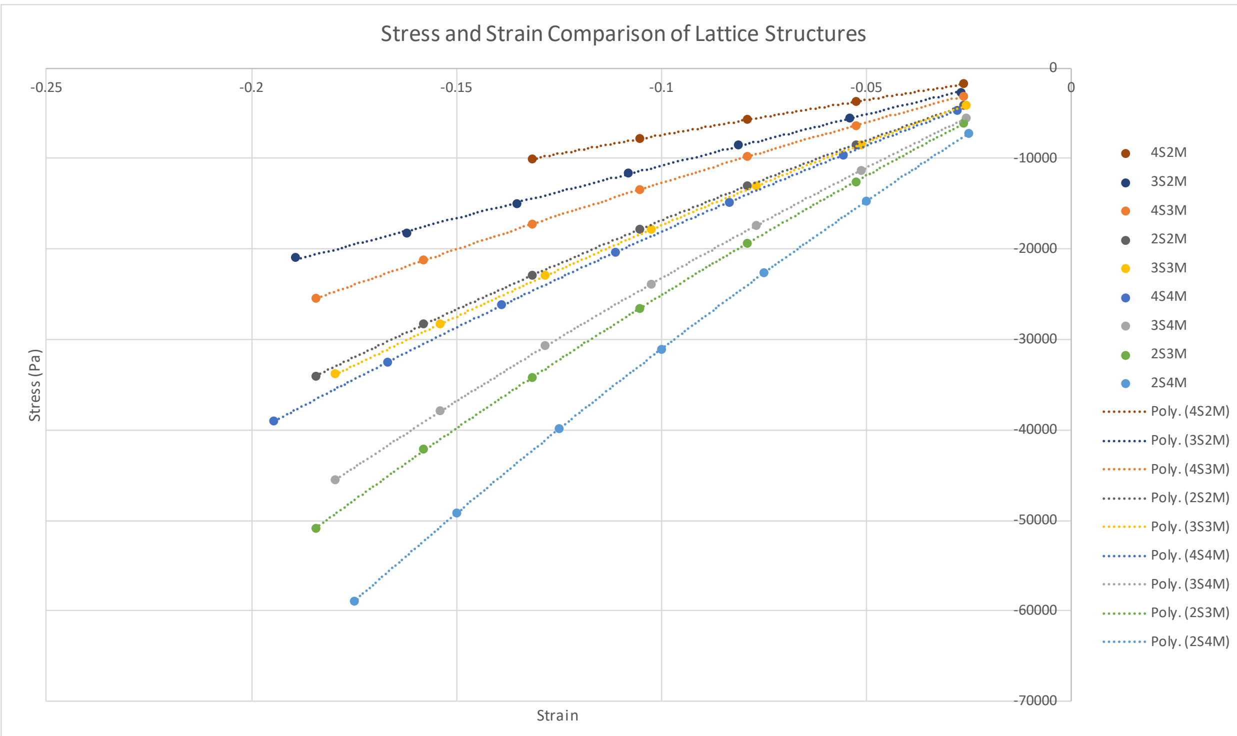


Fig 9. Comparison of the nine different lattice cubes with respect to strain and stress

<u>Equations for lines of best fit:</u>

<u>4S4M:</u> $F(x) = -218095x^2 + 157546x - 149.91$

<u>4S3M:</u> $F(x) = -134881x^2 + 113211x - 9.8932$

<u>4S2M:</u> $F(x) = -71794x^2 + 66853x + 6.91$

<u>3S4M:</u> $F(x) = -278690x^2 + 202640x - 149.91$

<u>3S3M:</u> $F(x) = -192857x^2 + 153938x - 49.779$

<u>3S2M:</u> $F(x) = -22714x^2 + 110070x + 440.88$

<u>2S4M:</u> $F(x) = -362619x^2 + 272637x - 164.29$

<u>2S3M:</u> $F(x) = -309643x^2 + 217472x - 236.45$

<u>2S2M:</u> $F(x) = -199570x^2 + 146952x - 150.12$

When comparing the changes in data, from 2S4M to 3S4M and 2S3M, the increase in spacing caused a bigger reduction in the reactive forces compared to a decrease in member thickness. From 3S4M to 4S4M and 3S3M, however, the decrease in member size caused a bigger reduction compared to increase spacing. From 3S3M to 4S3M and 3S2M, the reduction in member size once again caused a greater reduction than the increasing of the spacing. One factor which may be causing this discrepancy is that the percent changes are not the same. Going from a 4 mm to a 3 mm member thickness is a 25% reduction, while going from a 2 mm to a 3 mm spacing is a 50% increase.

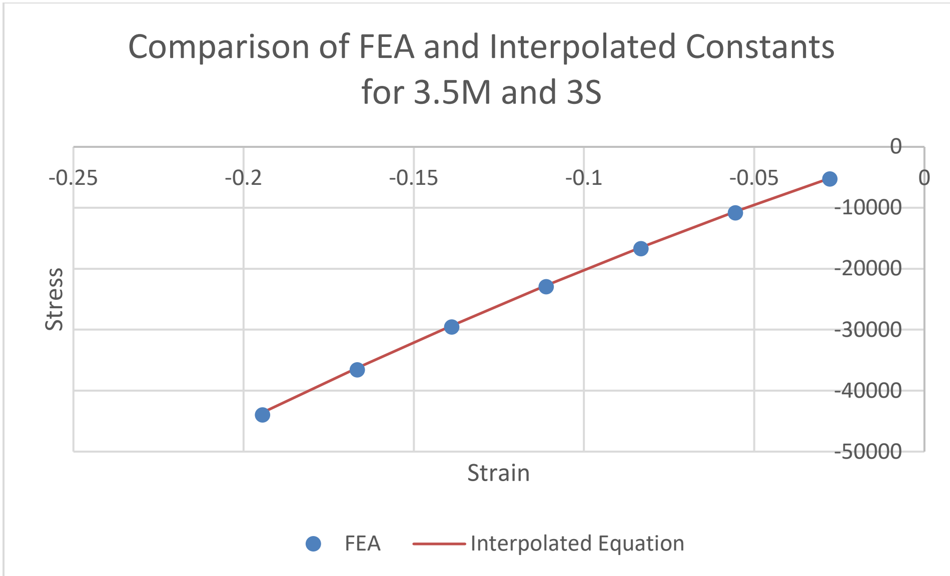
Practically, these different lattice structures' responses point to different use cases. For applications where a higher level of compression is desirable without large increases in forces, a structure like 4S2M is preferred. For a use case like padding, a structure like this would be more comfortable to a user, having a softer feel than the stiffness of 2S4M. However, for applications where stiffness is desirable, such as only marginally compliant gripping mechanisms, structures like 2S4M would allow for more force to be exerted on the gripped object than 4S2M, which would have large deformations instead of transferring the forces.

Fig 10. Comparison of reactive force per mm³ and strain

Comparing **Fig 9** and **Fig 10** when accounting for volume, the trend remains the same, but with less overall variance. This makes sense, as the material is the main driving factor. It seems then therefore that these structures are not as volume and thus mass reliant compared to the cross-sectional footprint shown in **Fig 9**.

Interpolation Results

Using a 3.5 mm member with 3 mm of spacing, the interpolated equation output was $F(x) = -235770x^2 + 178289x - 13.4$. The stress-strain responses from the equation were compared against the FEA result of the physical model.



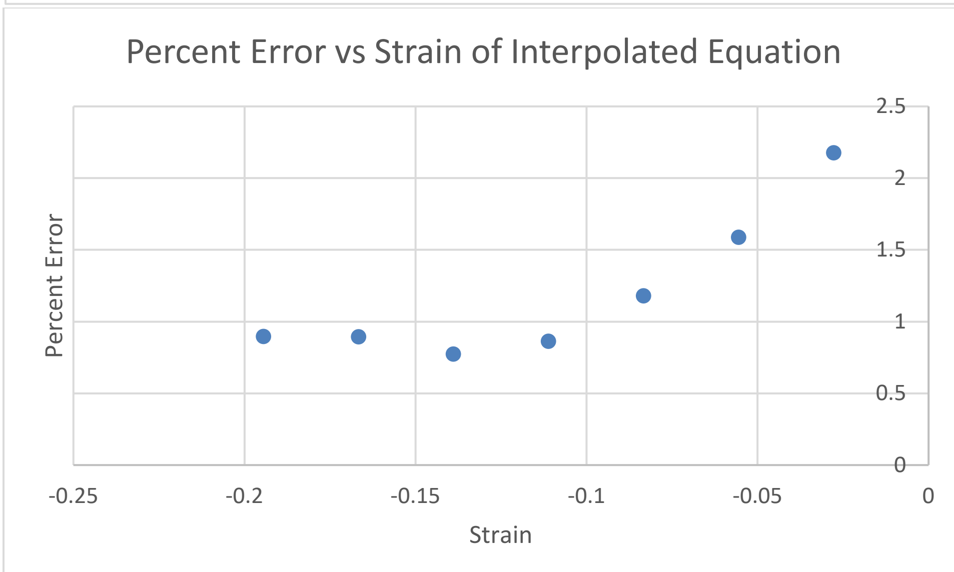
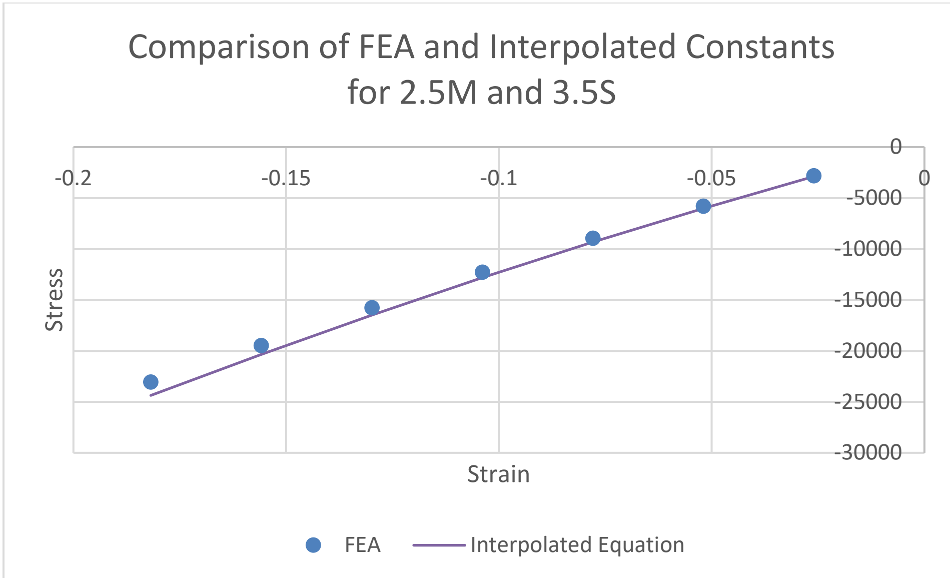


Fig 11. Plots comparing the interpolated equation and the FEA results of the 3S3.5M lattice cube

Using a 2.5 mm member with 3.5 mm of spacing, the interpolated equation outputted was $F(x) = -132330x^2 + 110400x - 78.3445$.



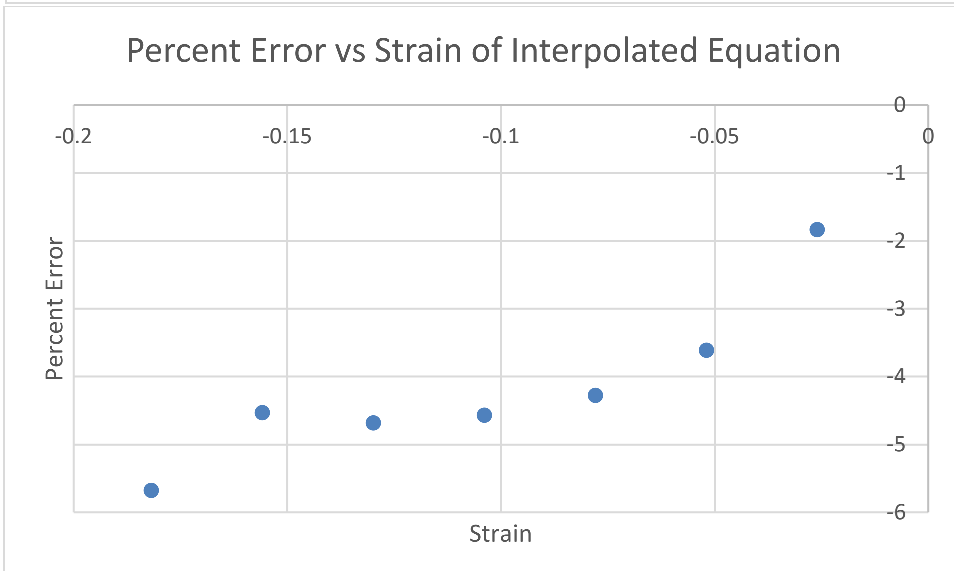


Fig 12. Plots comparing the interpolated equation and the FEA results of the 3.5S2.5M lattice cube

Overall, the interpolated predictions were accurate. With the 3 mm spacing and 3.5 mm members, the maximum error was 2.17%, which occurs at the first point. The error stabilizes out at around 0.89%. The average was 1.20%. However, for the 3.5S2.5M cube, the error is seen to be increasing, starting off at 1.8% and increasing by the end to 5.7%, with an overall average of 4.17%. The 3.5S2.5M interpolated prediction also overpredicts the reactionary force, while the 3S3.5M prediction underpredicted.

One reason why the 3S3.5M predicted model was closer to the FEA result than the 3.5S2.5M prediction was the proximity of the interpolated points to the known points.

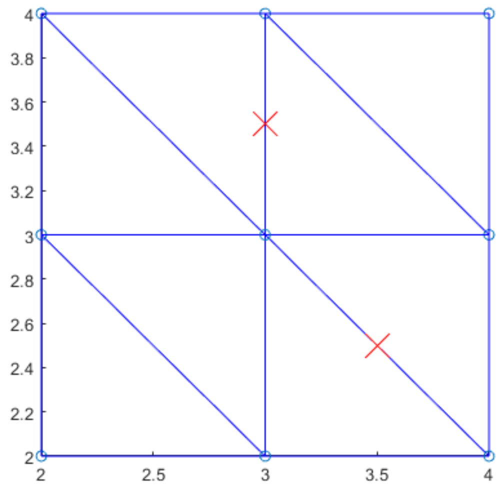


Fig 13. Triangulation of the points. The light blue circles are known nodes. The top-most 'x' is 3S3.5M and the lower 'x' is 3.5S2.5M

As can be seen in **Fig 13**, 3S3.5M is closer to known nodes, while 3.5S2.5M is farther from known values. Thus, the values generated by 3S3.5M are more accurate, as the interpolated data is better defined.

Conclusion

This data shows that it is possible to experimentally derive systems and methods to be able to predict reactive forces from compressive displacements of 3D printed hyperelastic lattice structured cubes. The predictive equations deviate from the FEA results by a maximum of 5.7%, with one model only being off by 1.2%. This level of accuracy is sufficient that a designer could generate the 3D lattice model with iterations to achieve the desired results. For comparison, the Indentation Load Deflection (ILD) values for latex sheets can vary by over 15% [9]. Importantly, this method would save a designer significant time when designing, as opposed to a guess and check method which relies of chance to get the desired performance.

Beyond validating the FEA results from testing physical models, further testing would be needed to validate this method with other materials and lattice structures. While this method works with this range of lattice parameters, lattice structure, and part material, it is unknown how well the triangulation method works with structures like TPMS and Bucklicrystals. Another improvement would be able to prescribe an ideal displacement and reactive force, and then be able to generate from those inputs the proper geometry of the lattice, as opposed to the current method of prescribing input geometries and generating displacements and reactive forces.

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