Order Scheduling is proposed as a new class of scheduling problems combining features that are now attributed exclusively to either a production scheduling problem or a project scheduling problem. Orders are defined as non-recurrent jobs with a sequence-dependent set-up time and a time-dependent priority. The objective is to minimize the total project cost subject to technological constraints for individual orders. Each order must start after its permitted starting date and completed before its due date. The float between the completion of an order and its due date is known as the dynamic slack.

A business firm specializing in spraying chemicals for agricultural and residential clients is studied. The major objective of the study is to reveal the characteristics of an industrial order scheduling problem as well as to seek and design, if necessary, a scheduling technique appropriate for industrial uses.

Though no single existing technique could be directly
modified travelling salesman principle provided a viable structure. Each entry in the distance matrix is continually updated by its "reduction factor", a function of the individual order's dynamic slack, and can be interpreted as the discounted cost of completing the order. In the case of the spray service firm, the "desirability factor" was chosen to be a combination of a linear and an exponential function of the dynamic slack (x).

\[
\text{Desirability Factor} = [1 + B - BKx - Be^{-Kx}] \text{ where } 0 < B \leq 1 \text{ and } 0 \leq K \leq 1.
\]

The format for this factor was chosen upon consultation with the management of the firm and to reflect the consideration that the first derivative of the Desirability Factor (DF) with respect to the Dynamic Slack (x) should yield a negative-exponential "reduction factor" which is 0 when x is 0 and which approaches a constant value asymptotically when the dynamic slack becomes increasingly large.

\[
\text{Reduction Factor} = + G[1 - e^{-Kx}] \text{ where } G = BK.
\]

Various combinations of numerical values for the parameters B and K were found to adequately approximate all dynamic characteristics of order execution costs encountered during the study.

Branch-and-Bound algorithm and Modified Next Best Heuristic Rule were applied to data generated by Monte Carlo simulation. B and K were uniformly samples between 0 and 1. The distance matrix was computed from orders that were randomly chosen from a trapezoidal region approximating the firm's customer distribution. The starting
distributions (i.e., Poisson intervals) and also by using normal distributions (i.e., Normally distributed intervals). Computational results and a plan for further studies are included.
HEURISTIC SCHEDULING OF ORDERS WITH DUE DATES AND
SEQUENCE DEPENDENT SET-UP TIMES

by

SUDHIR MADHUKAR JOSHI

A THESIS
submitted to
Oregon State University

in partial fulfillment of
the requirements for the
degree of

MASTER OF SCIENCE

June 1975
APPROVAL:

Redacted for privacy

Professor of Industrial and General Engineering

in charge of major

Redacted for privacy

Dean of School of Engineering

Redacted for privacy

Dean of Graduate School

Date thesis is presented May 28, 1974

Typed by Mary Syhlman for SUDHIR MADHUKAR JOSHI
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II THE PROBLEM</td>
<td>6</td>
</tr>
<tr>
<td>2.1 Structure of Organization, BSS</td>
<td>6</td>
</tr>
<tr>
<td>2.2 Time Limitations</td>
<td>9</td>
</tr>
<tr>
<td>2.3 Goodwill Consideration of BSS</td>
<td>14</td>
</tr>
<tr>
<td>2.4 Order Scheduling for BSS</td>
<td>16</td>
</tr>
<tr>
<td>III CLASSICAL METHODS</td>
<td>18</td>
</tr>
<tr>
<td>3.1 Solution Approach</td>
<td>18</td>
</tr>
<tr>
<td>3.2 Queueing Theory Approach</td>
<td>20</td>
</tr>
<tr>
<td>3.3 Mathematical Programming Approach</td>
<td>23</td>
</tr>
<tr>
<td>3.4 Heuristic Approach</td>
<td>26</td>
</tr>
<tr>
<td>IV HEURISTIC METHOD FOR ORDER SCHEDULING</td>
<td>29</td>
</tr>
<tr>
<td>4.1 Computational Scheme</td>
<td>29</td>
</tr>
<tr>
<td>4.2 Solution Methods for TSM</td>
<td>34</td>
</tr>
<tr>
<td>4.3 Selection of Desirability and Reduction Factors</td>
<td>42</td>
</tr>
<tr>
<td>V EVALUATION OF TECHNIQUE</td>
<td>49</td>
</tr>
<tr>
<td>5.1 Scheme of Evaluation</td>
<td>49</td>
</tr>
<tr>
<td>5.2 Economic Evaluation of Technique</td>
<td>53</td>
</tr>
<tr>
<td>5.3 Flexibility Evaluation of Technique</td>
<td>59</td>
</tr>
<tr>
<td>5.4 Numerical Examples</td>
<td>60</td>
</tr>
<tr>
<td>VI DISCUSSION AND CONCLUSION</td>
<td>68</td>
</tr>
<tr>
<td>6.1 Complexity of Order Scheduling Problems</td>
<td>68</td>
</tr>
<tr>
<td>6.2 Approach Used in This Study</td>
<td>69</td>
</tr>
<tr>
<td>6.3 The Proposed Order Scheduling Procedure</td>
<td>71</td>
</tr>
<tr>
<td>6.4 Evaluation of the Study</td>
<td>73</td>
</tr>
<tr>
<td>6.5 Future Research Efforts</td>
<td>74</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>77</td>
</tr>
<tr>
<td>APPENDIX A</td>
<td>79</td>
</tr>
</tbody>
</table>
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>10</td>
</tr>
<tr>
<td>2.2</td>
<td>12</td>
</tr>
<tr>
<td>3.1</td>
<td>19</td>
</tr>
<tr>
<td>4.1</td>
<td>31</td>
</tr>
<tr>
<td>4.2</td>
<td>32</td>
</tr>
<tr>
<td>4.3</td>
<td>32</td>
</tr>
<tr>
<td>4.4</td>
<td>33</td>
</tr>
<tr>
<td>4.5</td>
<td>33</td>
</tr>
<tr>
<td>4.6</td>
<td>33</td>
</tr>
<tr>
<td>4.7</td>
<td>34</td>
</tr>
<tr>
<td>4.8</td>
<td>35</td>
</tr>
<tr>
<td>4.9</td>
<td>36</td>
</tr>
<tr>
<td>4.10</td>
<td>36</td>
</tr>
<tr>
<td>4.11</td>
<td>37</td>
</tr>
<tr>
<td>4.12</td>
<td>37</td>
</tr>
<tr>
<td>4.13</td>
<td>38</td>
</tr>
<tr>
<td>4.14</td>
<td>38</td>
</tr>
<tr>
<td>4.15</td>
<td>39</td>
</tr>
<tr>
<td>5.1</td>
<td>54</td>
</tr>
<tr>
<td>5.2</td>
<td>57</td>
</tr>
<tr>
<td>5.3</td>
<td>61</td>
</tr>
<tr>
<td>5.4</td>
<td>63</td>
</tr>
<tr>
<td>5.5</td>
<td>67</td>
</tr>
<tr>
<td>6.1</td>
<td>70</td>
</tr>
<tr>
<td>Figures</td>
<td>PAGE</td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
</tr>
<tr>
<td>1.1 Order Scheduling</td>
<td>2</td>
</tr>
<tr>
<td>2.1 Operating Domain of BSS</td>
<td>7</td>
</tr>
<tr>
<td>3.1 Queueing Theory Model</td>
<td>21</td>
</tr>
<tr>
<td>4.1 Computing Time as a Function of Number of Jobs</td>
<td>41</td>
</tr>
<tr>
<td>4.2 Computing Time as a Function of Number of Jobs and Data Variability</td>
<td>41</td>
</tr>
<tr>
<td>4.3 Computing Time Variation for NB and MNB Rules</td>
<td>43</td>
</tr>
<tr>
<td>4.4 Variation of NDS with Time</td>
<td>45</td>
</tr>
<tr>
<td>4.5 Desirability Factor</td>
<td>47</td>
</tr>
<tr>
<td>4.6 Reduction Factor</td>
<td>47</td>
</tr>
<tr>
<td>5.1 Distribution of Simulated Data</td>
<td>51</td>
</tr>
<tr>
<td>5.2 Distribution of Orders - Uniform Distribution</td>
<td>52</td>
</tr>
<tr>
<td>5.3 Computing Time as a Function of Number of Jobs</td>
<td>55</td>
</tr>
<tr>
<td>5.4 Variation of Reduction Factor</td>
<td>58</td>
</tr>
<tr>
<td>5.5 Variation of Reduction Factor - Example 1</td>
<td>62</td>
</tr>
<tr>
<td>5.6 Block Diagram for FORTRAN Program</td>
<td>66</td>
</tr>
</tbody>
</table>
# Table of Contents

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>I</strong> INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td><strong>II</strong> THE PROBLEM</td>
<td>6</td>
</tr>
<tr>
<td>2.1 Structure of Organization, BSS</td>
<td>6</td>
</tr>
<tr>
<td>2.2 Time Limitations</td>
<td>9</td>
</tr>
<tr>
<td>2.3 Goodwill Consideration of BSS</td>
<td>14</td>
</tr>
<tr>
<td>2.4 Order Scheduling for BSS</td>
<td>16</td>
</tr>
<tr>
<td><strong>III</strong> CLASSICAL METHODS</td>
<td>18</td>
</tr>
<tr>
<td>3.1 Solution Approach</td>
<td>18</td>
</tr>
<tr>
<td>3.2 Queueing Theory Approach</td>
<td>20</td>
</tr>
<tr>
<td>3.3 Mathematical Programming Approach</td>
<td>23</td>
</tr>
<tr>
<td>3.4 Heuristic Approach</td>
<td>26</td>
</tr>
<tr>
<td><strong>IV</strong> HEURISTIC METHOD FOR ORDER SCHEDULING</td>
<td>29</td>
</tr>
<tr>
<td>4.1 Computational Scheme</td>
<td>29</td>
</tr>
<tr>
<td>4.2 Solution Methods for TSM</td>
<td>34</td>
</tr>
<tr>
<td>4.3 Selection of Desirability and Reduction Factors</td>
<td>42</td>
</tr>
<tr>
<td><strong>V</strong> EVALUATION OF TECHNIQUE</td>
<td>49</td>
</tr>
<tr>
<td>5.1 Scheme of Evaluation</td>
<td>49</td>
</tr>
<tr>
<td>5.2 Economic Evaluation of Technique</td>
<td>53</td>
</tr>
<tr>
<td>5.3 Flexibility Evaluation of Technique</td>
<td>59</td>
</tr>
<tr>
<td>5.4 Numerical Examples</td>
<td>60</td>
</tr>
<tr>
<td><strong>VI</strong> DISCUSSION AND CONCLUSION</td>
<td>68</td>
</tr>
<tr>
<td>6.1 Complexity of Order Scheduling Problems</td>
<td>68</td>
</tr>
<tr>
<td>6.2 Approach Used in This Study</td>
<td>69</td>
</tr>
<tr>
<td>6.3 The Proposed Order Scheduling Procedure</td>
<td>71</td>
</tr>
<tr>
<td>6.4 Evaluation of the Study</td>
<td>73</td>
</tr>
<tr>
<td>6.5 Future Research Efforts</td>
<td>74</td>
</tr>
</tbody>
</table>

BIBLIOGRAPHY 77

APPENDIX A 79
HEURISTIC SCHEDULING OF ORDERS WITH DUE DATES AND SEQUENCE DEPENDENT SET-UP TIMES

CHAPTER I

INTRODUCTION

Every business firm must operate under constraints of limited resources, and every firm has a scheduling problem of its own. Ever since the inception of Operations Research and Management Science, scheduling problems have provided many challenges to their researchers and practitioners. Numerous prototypes were proposed, and an even larger number of variations were studied. Some are highly successful in solving a wide variety of problems, and others, applicable only to specific situations, have quietly become forgotten. Most of those models, however, can today be classified into either production scheduling or project scheduling techniques. This is reminiscent of the origin of Industrial Engineering in production systems, and of governmental efforts to adopt similar techniques to the scheduling of large-scale projects in the 1960's. Figure 1.1 illustrates the two areas of traditional scheduling studies.

Job shop scheduling is an example of production scheduling problem that focuses its primary objective on programming of tasks on various machines.

The problem of job shop scheduling can be characterized as a continuous inflow of jobs into a plant where each order requires one or more operations to be performed on one or more machines. The problem is to schedule the
Figure 1.1. Order Scheduling
operations on the machines in order to minimize idle machine time, total time on the jobs in the system, or some other acceptable basis. (Thierauf and Gresse, 1970)

A project scheduling program, on the other hand, can be considered a process of programming activities and resources subject to technological constraints. Its objective usually is to minimize the time or cost of completing the given project. These projects have several characteristics:

1. The project consists of a well-defined collection of jobs, or activities, which when completed mark the end of the project.
2. The jobs may be started and stopped independently of each other, within a given sequence. (...)
3. The jobs are ordered, that is, they must be performed in technological sequence. (...) (Wiest and Levy, 1969)

In comparing job shop scheduling problems to project scheduling problems, some definite and distinct features are noted. For example, the process time is usually longer in the latter than in a comparable job shop scheduling situation. Several scheduling methods are available in each field, but very few methods are applicable to both. But most importantly, there seems to be a gap between the two fields.

Order Scheduling (OS) is the name given to types of problems that are becoming increasingly important to Industrial Engineers. This class of problems will have the following characteristics:

1. Each order (job) is undertaken only once during the planning horizon. This is similar to most project scheduling problems.
2. Separate set-up times are found for each order dependent on what order was last processed. This is similar to a job shop scheduling problem.

3. There is a penalty associated with scheduling jobs either too early or too late, and this will dynamically affect the priority for the order to be processed. This is a unique feature of Order Scheduling.

4. Depending on the type of resources required to perform a job, the crew may have to return to their home base. This is similar to a complete overhaul or clean-up required prior to undertaking a special job in a job shop.

An order scheduling problem is most easily found among service industries and primary industries, the two areas that are now opening up to Industrial Engineers.

This thesis is an attempt to study the characteristics of Order Scheduling (OS) problems and solution methods available to find optimal schedules. Chapter II analyzes the OS problem characteristics. An agricultural spray service firm is studied as a specific example, and its operations are described in terms of an OS problem structure and assumptions. Chapter III surveys the analytical tools used in both production and project scheduling, and their applicability to OS problems. Queueing models, mathematical programming models, and
heuristic models are investigated. Chapter IV develops the heuristic method for Order Scheduling. The computational scheme is described and various algorithms for solving Travelling Salesman problem are compared. A heuristic procedure is adopted incorporating the desirability and reduction factors to account for the OS constraints and priorities. Chapter V presents the computational procedure used to solve the OS problems. Data to simulate actual OS situations were used to evaluate the computational efficiency of the algorithm. Finally, Chapter VI discusses other applications of OS scheduling and suggests future research activities that will enhance the value of OS models to solve the types of problems that future Industrial Engineers are likely to encounter.
CHAPTER II

THE PROBLEM

2.1 Structure of Organization, BSS

Beaver Spraying Service, Inc., Albany, Oregon, undertakes services of spraying chemicals on weeds, orchards, and lawns, and spreading fertilizers on agricultural lands. Beaver Spraying Service, Inc., (BSS) operates its business within a radius of one hundred miles of its main office. It serves customers who are located in and near Albany and in the neighboring communities of Corvallis, Lebanon, and Salem (Figure 2.1). The warehouse of BSS is situated near Corvallis, approximately eleven miles west of Albany.

Beaver Spraying Service undertakes between 90-200 orders per month with its permanent staff of 6 people. The number of orders in a particular month fluctuates widely according to seasonal and climatic conditions. For example, the number of orders executed in summer months is as much as three times that during the winter months. These wide fluctuations in monthly orders pose many problems. First, the fluctuations create a variation of the workforce and since the amount of training involved is significant, a large turnover is undesirable. Secondly, the spraying equipment remains idle for a long time when orders are relatively few in number.
Figure 2.1. Operating Domain of BSS.
Since each order is executed at the customer's farm, lawn or orchard, the spraying equipment and chemicals have to be transported to the site. The travel includes transportation of equipment and chemicals from the warehouse to the customer, from one customer to another customer, and from the last customer back to the warehouse.

The management of Beaver Spraying Service is therefore concerned with the following problems:

1. How to spread the workload evenly between
   and within seasons; and,
2. How to minimize the idle travel time involved
   between the execution of orders.

In order to appreciate Beaver Spraying Service's problem, it is necessary to understand and define certain common terminology associated with the business.

An order may be defined as a single application of a particular chemical at a given site. For example, spraying a chemical $x$ at a grass seed farm $y$ located at $z$ will constitute an order. However, if the chemical $x$ is spread at another grass seed farm $w$ located at $z$, then it is considered a separate order. Each order is identified by the following characteristics:

1. Physical site where the order is to be executed.
2. Size of the farm, lawn or orchard to be serviced.
3. Earliest date at which the order can be executed.
4. Latest date before which the order has to be
executed, and

5. Approximate time required to execute the order.

Most of the orders are received over the telephone by Beaver Spraying Service, and fall under the category of weed control, Insect control, Disease control or Fertilizing. These may be further divided into subcategories (Table 2.1). For example, Weed Control may be divided into Residential Weed Control, Industrial Weed Control and Agricultural Weed Control.

2.2 Time Limitation on the Execution of an Order

The starting date and the due date constitute the important factors in the problem. The starting date (SD) is the earliest day at which a particular order can be undertaken. Due date (DD) is the latest day before which the order has to be completed.

The starting and due dates depend upon the type of chemical used and the type of application. A particular chemical is chosen on the basis of the type and the extent of the needs. Table 2.2 illustrates some of the starting and due dates for the orders executed by Beaver Spraying Service. For example, October 1 and December 1 are the starting and due dates for the application of Atrazine on Perenial Ryegrass.

Proper execution of an order in the specified period is a very important aspect of spraying business. For example, the effectiveness of the chemical sprayed on weeds is closely related to the time of application. In most cases the effectiveness of a
TABLE 2.1. Type of activities undertaken by BSS.

Spraying

A. Weed Control

1. Residential-Recreational-Commercial
   a. Shrub Beds
   b. Lawn
   C. Brush - Blackberry

2. Industrial
   a. Soil Sterilization
   b. Brush - Blackberry

3. Agricultural
   a. Grass Seed Fields
      i. Fine Fescue
      ii. Perennial Ryegrass
      iii. Bentgrass
      iv. Bluegrass
      v. Tall Fescue
   b. Legume Seed Fields
      i. Red Clover, Lading and other White Clover
      ii. Alfalfa
   c. Cereal Crops
      i. Winter Wheat
         - Annual Broadleaf Weeds
         - Annual Grass Weeds
   d. Sterilization of Fence-Rows

B. Insect Control

1. Residential-Recreational-Commercial
   a. Shrubs and Trees
   b. Lawns

2. Fruit Trees - Nut Trees
   a. Cherry
   b. Apple
   c. Peach
   d. Filbert
   e. Walnut

C. Disease Control

1. Residential-Recreational-Commercial
   a. Shrubs and Trees
   b. Lawns
TABLE 2.1. Continued

2. Fruit Trees - Nut Trees
   a. Cherry
   b. Apple
   c. Peach
   d. Filbert
   e. Walnut

D. Fertilizing

1. Residential-Recreational-Commercial
   a. Shrubs and Trees
   b. Lawns

2. Agricultural
   a. Grass Seed Fields
   b. Cereals
### TABLE 2.2. Data for Execution of Some Orders

<table>
<thead>
<tr>
<th>Crop</th>
<th>Chemical</th>
<th>Rate</th>
<th>SD</th>
<th>DD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perennial Ryegrass</td>
<td>IPC</td>
<td>3</td>
<td>Sept. 15</td>
<td>Nov. 15</td>
</tr>
<tr>
<td></td>
<td>CIPC</td>
<td>2</td>
<td>Sept. 15</td>
<td>Nov. 15</td>
</tr>
<tr>
<td></td>
<td>Atrazine</td>
<td>1.6</td>
<td>Oct. 1</td>
<td>Nov. 15</td>
</tr>
<tr>
<td>Bentgrass</td>
<td>IPC</td>
<td>4</td>
<td>Sept. 15</td>
<td>Nov. 15</td>
</tr>
<tr>
<td></td>
<td>CIPC</td>
<td>3</td>
<td>Sept. 15</td>
<td>Nov. 15</td>
</tr>
<tr>
<td></td>
<td>Karmex</td>
<td>3</td>
<td>Sept. 15</td>
<td>Nov. 15</td>
</tr>
<tr>
<td></td>
<td>Atrazine</td>
<td>2</td>
<td>Sept. 15</td>
<td>Nov. 15</td>
</tr>
<tr>
<td>Bluegrass</td>
<td>Karmex</td>
<td>4</td>
<td>Sept. 15</td>
<td>Nov. 15</td>
</tr>
<tr>
<td></td>
<td>Karmex</td>
<td>1.6</td>
<td>Dec. 1</td>
<td>Feb. 15</td>
</tr>
<tr>
<td>Tall Fescue</td>
<td>Karmex</td>
<td>3.2</td>
<td>Sept. 15</td>
<td>Nov. 15</td>
</tr>
<tr>
<td></td>
<td>Karmex</td>
<td>1.6</td>
<td>Dec. 1</td>
<td>Feb. 15</td>
</tr>
<tr>
<td></td>
<td>Atrazine</td>
<td>2.4</td>
<td>Sept. 15</td>
<td>Nov. 15</td>
</tr>
<tr>
<td>Fine Fescue</td>
<td>IPC</td>
<td>3</td>
<td>Sept. 15</td>
<td>Oct. 20</td>
</tr>
<tr>
<td></td>
<td>CIPC</td>
<td>2</td>
<td>Sept. 15</td>
<td>Oct. 20</td>
</tr>
<tr>
<td>Alfalfa</td>
<td>Karmex</td>
<td>2.4</td>
<td>Oct. 15</td>
<td>Feb. 1</td>
</tr>
<tr>
<td></td>
<td>IPC</td>
<td>4</td>
<td>Nov. 1</td>
<td>Feb. 15</td>
</tr>
</tbody>
</table>

**Note:**

Rate - Pounds of active ingredient per acre.

Karmex -
IPC -
Atrazine -
CIPC - Registered Trade Names
single application of a chemical diminishes drastically if the order is executed earlier than the starting date. If the chemicals were sprayed at a later date, it would allow the weeds to grow further in the meantime, either additional doses of the same chemical or an application of a different chemical becomes necessary to control the growth of the weeds.

To illustrate the absolute necessity of executing an order at the proper time, the case of controlling weeds in Bentgrass, may be considered. An appropriate timing is very important in the effectiveness of Atrazine in killing weeds in Bentgrass. Bentgrass seeds are generally planted in the last week of September or the first week of October. Weeds sprout along the seed plants. Atrazine, when used to kill weeds in Bentgrass, must therefore be sprayed in the first week of October. If the chemical is sprayed earlier, there is a likelihood that it may be wasted and additional spraying may be necessary at a proper time. On the other hand, if the spraying is delayed until the third or fourth week of October, larger doses of chemicals may be needed to effectively stop the growth of weeds.

An inappropriate timing in the use of chemicals results in the necessity of usage of chemicals in larger quantities. Consequently, spraying costs rise. Beaver Spraying Service bills the customer for the spraying services and the quantity of chemicals used. The cost of spraying service is primarily based on the number of acres sprayed, the larger the acreage the higher the costs. The cost of
a chemical is determined on the basis of the quantity and the concentration of the chemical sprayed. Certain chemicals cost more than the others.

2.3 Goodwill Considerations of BSS

Goodwill is an important aspect of the firm's business. Goodwill depends upon many factors including reputation of the firm, number of years in the business, social services performed to the community, working efficiently and, most important, consumer trust and satisfaction. The management of Beaver Spraying Service is always concerned about the firm's goodwill. Customers' trust and satisfaction are certainly important to them. However, there are many instances in which customers' trust is easily lost. For example, the terms of acceptance of an order are generally such that the extra cost over and above the normal cost incurred due to inappropriate timing of an order, are borne by the customers. Although the customers finally end up paying these extra costs, often they remain unsatisfied. If the extra costs involved are relatively high, the customer's dissatisfaction also runs high. On the other hand, relatively low extra costs develop low dissatisfaction. This eventually, in the long run, leads to loss of trust on the part of the customer and corresponding loss of business to Beaver Spraying Service.

The loss or gain in goodwill is often difficult to measure in terms of dollars since it is subjective in nature. Nonetheless,
the factors that cause goodwill to rise and fall can be quantified.

a. Size of order - The most important factor involved in actual comparison of two different customers is the size of their order. The larger the size of an order, the smaller the total overhead involved in the undertaking of that order. A single large order reduces the travel involved, and the corresponding cost, by an appreciable amount. Also, a small dissatisfaction on part of a customer with a large order gets mapped into a magnified loss of goodwill. The size of order can be measured in terms of the amount of orders received from the customer in a given year or the total acreage sprayed for that customer. Beaver Spraying Service prefers to keep this account in terms of number of acres sprayed in a given year.

b. Percentage extra cost incurred by the customer - This is the extra cost incurred by the customer over the regular cost, represented as a fraction of the regular cost. The dissatisfaction on the part of the customer can be assumed to be directly proportional to the extra cost borne by the customer.

c. Type of customer - Beaver Spraying Service's customers are of two types: steady and incidental. Steady customers are those who place their order every season. The size of their orders is usually large. Also, their orders are relatively easy to handle since their needs are known to the firm. Beaver Spraying Service pays special attention to execute orders of steady customers. Incidental customers are those who place their orders intermittently.
Some of the incidental customers become steady in long run. Orders such as spraying of chemicals on house lawns are generally incidental in nature.

2.4 Order Scheduling for BSS

In summarizing the operation of BSS the features of Order Scheduling can be easily noted. The case is by no means general, but an attempt to tackle it will lead to the better understanding of Order Scheduling.

As noted in the previous chapter, in OS the orders are undertaken only once. The same fact is true in the case of BSS. Since every working unit has to visit the actual location and undertake the orders, the facility has to be mobile. But the organization has an option of increasing or decreasing the number and efficiency of the working units by varying the crew size. Though the organization tends to favor the orders with longer processing times, a broad range of processing times is observed. In case of BSS each order is independent in the sense that the actual execution of the order does not depend upon any of the remaining orders, but the total travel involved in undertaking of the orders is greatly dependent upon the sequence in which the orders are undertaken. Last, the cost of execution of certain orders is greatly dependent upon the time at which it is undertaken.

Keeping the case of BSS in mind, the method for OS has to be studied. This guideline will serve in next chapter to explore
the project scheduling (PS) and job shop scheduling (JSS) methods.
CHAPTER III

CLASSICAL METHODS

3.1 Solution Approach

The problem of scheduling faced by Beaver Spraying Service, Inc. (BSS), can be identified as a typical Order Scheduling (OS) problem. In this chapter the problem will be compared with typical project scheduling and job shop scheduling problems.

The problem has some of the basic characteristics observed in project scheduling. Each order, which can be considered as a project, consists of a single activity. The cost and duration required for execution of each of the activities are functions of time at which they are performed and the labor force used. It can also be observed that each of these activities can be performed independently of each other and have a distinct identity. If this interpretation of an order as a project is accepted, the scheduling problem of BSS can belong to the category of "Multi-project Scheduling," i.e., scheduling of two or more simultaneous projects. As multi-project scheduling is a special case of project scheduling (PS), it may be expected that the methods available for PS can be extended to multi-project scheduling. On the contrary, Fendley (1967) argued that:

Many attempts have been made to use the principles behind the PERT technique to develop a method for multi-project scheduling. Many attempts met limited success because of assumptions describing multiproject scheduling; that is, the model used was not entirely realistic.
The other approach that is also pertinent is the job shop scheduling method (JSS). The various characteristics observed in JSS can also be identified and observed in the present problem. The concept of an order can be compared to the concept of a job, and the travel involved between carrying out the orders can be identified as the "set-up" time required to prepare the equipment to carry out the jobs (Table 3.1).

TABLE 3.1. Order-job Comparison

<table>
<thead>
<tr>
<th>Order</th>
<th>Job</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working unit</td>
<td>Machine capable of carrying out any kind of job.</td>
</tr>
<tr>
<td>Travelling time</td>
<td>Set-up time.</td>
</tr>
<tr>
<td>Due date</td>
<td>Due date</td>
</tr>
<tr>
<td>Starting date</td>
<td>Date of undertaking a job</td>
</tr>
</tbody>
</table>

The resemblance noted above makes the further research in the field of JSS justifiable. There is, however, a major difference between the two models. The efficiency of each working unit, machine, which is usually not a variable in JSS, is a major factor in an order scheduling problem. It varies widely according to the weather conditions, number of people working as a unit, capabilities of available equipment, etc. As the available resources are limited and the set-up times are not uniform in the present problem, the efficiencies are in turn dependent upon the schedule followed.

An attempt of scheduling jobs against due dates by McNaughton (1959) was the first in this field. Eastman, et al. (1964) further
attempted to establish bounds on the costs of operations by scheduling \( n \) jobs on \( m \) processors. Their major contribution was to tie the time of undertaking to the cost of operation. They did not consider the effect of due dates. Lawler (1964) pursued McNaughton (1959) approach with the help of linear programming.

Root (1965) made the first attempt to schedule jobs, on more than one machine, with due dates and corresponding loss functions. It should be noted that none of the above researchers did incorporate the variable set-up time in their approach. The algorithms presented by these authors cannot easily be modified to deal with a sequence dependent set-up time. The first attempt to schedule jobs with due dates and sequence dependent set-up times was made by Marsh and Montgomery (1973). The results presented by them were especially useful to the research for this thesis.

In studying the JSS techniques three different approaches have been investigated.

a. Queueing theory approach
b. Mathematical programming approach
c. Heuristic approach

3.2 Queueing Theory Approach

The situation of Queue arises in a job shop when the number of jobs arriving to the shop at a particular time is more than the number of jobs that the shop can service at that time. The process
of serving is called "service mechanism". The jobs which are in the shop, but not yet processed, constitute a "waiting line" or a "queue". The rule by which the jobs are selected for loading a service mechanism is called the "priority rule". Figure 3.1 illustrates the waiting line concepts as applied to the BSS problem. According to Kendall-Lee-Taha (Taha, 1971, p. 505), this model is classified as a (GI/G/k):(NPNR/N/n) queue.

![Queueing Theory Model](image)

Figure 3.1. Queueing Theory Model.

The pool of customers who have or who could have placed an order with BSS service is the Source Population, from which the waiting line arises. The orders which can be undertaken at a given date can be considered as the jobs which are waiting in line to be processed.

An extensive research on the properties of queues is reported in literature under the common heading of Queueing Theory (QT). The situations are analyzed when any two of the above parameters are known.

However, the emphasis thus far has been on developing a mathematical descriptive theory. Little attention has yet been given to the
practical application of this theory to achieving the goal of operations research, optimal decision making, (Hillier and Lieberman, 1967)

The queueing theory is based upon probabilistic models. The service time in the BSS problem has a small variance and can be considered a deterministic parameter. Moreover, BSS is a relatively young firm and historical data are not available. Also, the real life competition makes it impossible to gather data from another firm, or the customers. Consequently, it appears difficult to apply queueing theory directly to the present problem.

"Priority Dispatching," is the study of various dispatch rules. First come, first served (FCFS), shortest operation time (SOT), last come, first served (LCFS) are some of the commonly used priority rules. The first large scale research of these dispatching rules was done by Rowe (1958). Baker and Dzielsinki (1960), Conway and Maxwell (1962), Nanot (1963), Carroll (1965) followed his work further. Harris (1965) made an extensive investigation to compare these models.

Since the priority dispatching deals with due dates and loss functions, this concept has been found useful in modelling the BSS problem.
3.3 **Mathematical Programming Approach**

A large variety of different analytical techniques are encountered in scheduling literature. Among those applied to JSS, are linear programming, dynamic programming and integer programming models.

An integer programming analytical model that fits the BSS problem with justifiable closeness is the "Travelling Salesman Model" (TSM). The mathematical form originally given by Taha (1971) is described as follows:

... there are n towns with known distances between any two of them. A salesman wants to start from a given town, visit each town once, and then return to his starting point. The objective is to minimize his total travelling time.

Let

\[ x_{ijk} = \begin{cases} 
1, & \text{if the kth directed arc is from town } i \text{ to town } j, \\
0, & \text{if otherwise,} 
\end{cases} \]

where \( i, j, \) and \( k \) are integers that vary between 1 and \( n \). The constraints of the problem can be classified under the four types:

1. Only one directed arc may be assigned to specific \( k \), thus,

\[
\sum_{i} \sum_{j} x_{ijk} = 1, \ k = 1,2,\ldots n. \quad \text{if} \ j
\]

2. Only one other town may be reached from a specific town \( i \), thus

\[
\sum_{j} \sum_{k} x_{ijk} = 1, \ i = 1,2,\ldots n.
\]
3. Only one other town can initiate a directed arc to a specific town, thus:

\[ \sum_i \sum_k x_{ijk} = 1, \quad j = 1, 2, \ldots, n. \]

4. Given the kth directed arc ends at some specific town j, the (k+1)th directed arc must start at the same town j, thus

\[ \sum_i x_{ijk} = \sum_r x_{rjk}, \quad \text{for all } j \text{ and } k. \]

This constraint ensures that the round trip will consist of connected segments (directed arcs).

The objective function is thus:

\[
\min x_0 = \sum_i \sum_j \sum_k d_{ij} x_{ijk}, \quad i \neq j
\]

where \( d_{ij} \) is the distance from town i to town j.

This model can now be compared with the BSS problem. It may be observed that in the absence of starting date and due date the problem exactly fits this model, and the problem can be solved by the methods available in the literature.

Even in the presence of time limitations imposed by SD and DD, the original Taha constraints remain unchanged. Constraint 1 remains the same because a particular working unit can only be operating at a single place. The constraints 2 and 3 remain the same because each order can be executed once only (Chapter III).

To get a better understanding of the adaptability of the above model to the present problem, a slight modification of the objective function will be made to replace \( d_{ij} \) by \( d_{ijk} \). In the
above model \( d_{ij} \) represents the distance between town \( i \) and town \( j \). The presence of \( k \) relates this distance to time. This variable parameter, \( k \), for example, can be interpreted as the chronological order of execution, larger \( k \) means that a job is undertaken at a relatively later time in the \( n \)-job chain of travel by a particular working unit, while a smaller \( k \) means that the job is started earlier: \( 1 \leq k \leq n \).

The time at which the job is undertaken, \( T_k \), can be represented as

\[
T_k = \sum_{i=1}^{k-1} t_i
\]

where \( t_i \) is the duration of service for the \( i \)th job + time to travel from the \( i \)th to \( (i+1) \)th job.

Hence, \( T_k \) has to be adjusted such that

\[
SS_k \leq T_k \leq DD_k
\]

or

\[
SS_k \leq \sum_{i=1}^{k-1} t_i \leq DD_k
\]

where

\( SD_k = \) starting date for job \( k \)

\( DD_k = \) due date for job \( k \)

Another interpretation of \( d_{ijk} \) can be made as the quantification of "how a particular job \( j \) looks from the \( k \)th job \( i \). This means that having completed a job \( i \), there is a choice of choosing the next job from any of the remaining jobs. If the factor of time
was absent, the best choice will be that given by the solution of TSM. But in the present problem it may happen that some job, other than the one suggested by the solution of TSM, may be approaching its due date and it may be necessary to pickup that job immediately. And the final optimal schedule may not match the optimal solution of the equivalent TSM.

3.4 Heuristic Approach

At this point it may be clear that no single approach is uniquely applicable to an order scheduling problem such as the one faced by BSS. The weakness of the above two approaches has led researchers to look for heuristic models.

Both job shop and the project scheduling problems have been discussed frequently in the literature. Analytic approaches have been unsuccessful in solving the scheduling problems of practical size, and little practical success has been achieved by iterative approaches. The most successful approach to the scheduling problem has been the heuristic approach ... (Fendley, 1967)

In the case of BSS problem a combination of TSM algorithm and Priority Dispatching rules have been adopted. In this model \(d_{ijk}\) is considered as having two components. One component that varies with \(k\) and the other that varies with the duplet \(ij\). The component varying with \(ij\) is a factor related to the distance. The \(k\) component is affected by the time of execution of order.

The actual computation of \(d_{ijk}\) makes use of two factors called the desirability factor and the reduction factor. The
desirability factor is defined as the measure of "how a particular job is important to the organization at a particular time." This numerical factor is put on the scale of zero to one. The desirability factor of zero means that there is no immediate urgency of undertaking the job and the factor of one means that the job must be undertaken immediately.

An important aspect of the present problem is that the desirability factor changes with time. At the starting date, the selection of an order essentially depends upon the respective distances of orders from each other and the desirability factor is zero. The actual shape of the graph depends upon the type of order (see, for example, Figure 4-5 in Chapter IV). There is a different graph for each order. The major factors which govern the shape of the graph are growth of weeds and insects; loss of goodwill, etc. It should be noted that the desirability factor is just a tool to compare various orders. It does not give an absolute measure of any parameter.

The complimentary factor that arises out of the desirability factor is the reduction factor. The reduction factor is defined here as the ratio of the fictitious distance used in computation to its physical distance. The idea here is to reduce the value of each actual physical distance by the desirability of undertaking that particular order.

\[ D = D_{\text{physical}} \times \alpha \]
where

\[ D \text{ is the fictitious distance} \]

\[ \alpha \text{ is the reduction factor } 0 < \alpha \leq 1 \]

\[ D_{\text{physical}} \text{ is the physical distance between location of any two orders.} \]

The mechanism of this heuristic method will now be described in Chapter VI.
CHAPTER IV

HEURISTIC METHOD FOR ORDER SCHEDULING

4.1 Computational Scheme

The desirability factor, used in the heuristic approach to solve the BSS problem enables the incorporation of Priority Dispatching Rule within the framework of Travelling Salesman Model (TSM).

The general outline of the approach consists of reducing the physical distances with the help of desirability factor and then using TSM approach to select the schedule.

The resulting technique can be stated step-wise as follows:

Step 1. For each of the orders received, identify the SD and DD. Identify the order R from which the schedule must start.

Step 2. At any given point T in time, select from all the orders received, those m orders which can be undertaken at that point.

Step 3. Calculate the physical distance ($d_{ij}$ with $i=1,\ldots,m+1; j=1,\ldots,m$) for each order i from all remaining orders, $j\neq i$.

Step 4. Form $((m+1) \times m)$ "distance matrix" of the distances calculated in Step 3.
Step 5. Calculate the reduction factor for each of the \( m \) orders, according to the rules discussed in Section 4.3.

Step 6. Multiply each column of the distance matrix by corresponding reduction factor. The resulting matrix is denoted as "discounted distance matrix."

Step 7. Find out the minimum element from each column and subtract it from each element of that column. The resulting matrix is called the "reduced distance matrix."

Step 8. Starting in line \( R \), find out the minimum element in that row. Note the corresponding column \( C \).

Step 9. Cross out the column \( C \) and the row \( R \).

Step 10. Replace \( R \) by \( C \).

Step 11. Check if all the elements of matrix are crossed out. If not, go to 8.

Step 12. The series of \( R \)'s noted is the resulting schedule.

Example. BSS has received five different orders and wants to decide the sequence in which these are to be undertaken. The schedule is to be decided starting at time \( T = 5 \) and starting at order 1. The starting dates and the due dates for each order are shown in Table 4.1. The table also shows the reduction
factors and the distances of each order from all of the remaining orders.

TABLE 4.1. Data Matrix

<table>
<thead>
<tr>
<th>Reduction factor (k_R)</th>
<th>.5</th>
<th>.9</th>
<th>.7</th>
<th>.5</th>
<th>.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order number (R)</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Starting date (SD_R)</td>
<td>0</td>
<td>5</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Due date (DD_R)</td>
<td>14</td>
<td>9</td>
<td>15</td>
<td>18</td>
<td>20</td>
</tr>
<tr>
<td>Duration of execution (D_R)</td>
<td>1</td>
<td>.5</td>
<td>1.5</td>
<td>.2</td>
<td>3</td>
</tr>
</tbody>
</table>

The orders which can be undertaken at time \(T = 5\) are 2 through 6. Hence, all these orders have to be accommodated in the computations. Corresponding multiplication of distances with their reduction factors leads to the discounted distance matrix shown in Table 4.2.
TABLE 4.2. Discounted distance matrix.

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15.0</td>
<td>17.1</td>
<td>11.9</td>
<td>7.5</td>
<td>16.8</td>
</tr>
<tr>
<td>2</td>
<td>∞</td>
<td>13.5</td>
<td>12.6</td>
<td>9.0</td>
<td>10.4</td>
</tr>
<tr>
<td>3</td>
<td>3.5</td>
<td>∞</td>
<td>7.7</td>
<td>1.0</td>
<td>10.4</td>
</tr>
<tr>
<td>4</td>
<td>5.5</td>
<td>8.1</td>
<td>∞</td>
<td>2.5</td>
<td>18.4</td>
</tr>
<tr>
<td>5</td>
<td>3.0</td>
<td>7.2</td>
<td>3.5</td>
<td>∞</td>
<td>8.8</td>
</tr>
<tr>
<td>6</td>
<td>1.0</td>
<td>.9</td>
<td>0.0</td>
<td>9.0</td>
<td>∞</td>
</tr>
</tbody>
</table>

After corresponding subtraction the reduced distance matrix is obtained. (Table 4.3)

TABLE 4.3. Reduced distance matrix No. 1.

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14</td>
<td>16.2</td>
<td>11.9</td>
<td>6.5</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>∞</td>
<td>12.6</td>
<td>12.6</td>
<td>8</td>
<td>1.6</td>
</tr>
<tr>
<td>3</td>
<td>2.5</td>
<td>8</td>
<td>7.7</td>
<td>0</td>
<td>1.6</td>
</tr>
<tr>
<td>4</td>
<td>4.5</td>
<td>7.2</td>
<td>∞</td>
<td>1.5</td>
<td>9.6</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>6.3</td>
<td>3.5</td>
<td>∞</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>8</td>
<td>∞</td>
</tr>
</tbody>
</table>

In row 1 the minimum element is located in column 5. Hence, the order that is undertaken after 1 is 5. After line 1 and column 5 have been discarded the matrix becomes (Table 4.4).
TABLE 4.4. Reduced distance matrix No. 2.

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>∞</td>
<td>12.6</td>
<td>12.6</td>
<td>1.6</td>
</tr>
<tr>
<td>3</td>
<td>2.5</td>
<td>8</td>
<td>7.7</td>
<td>1.6</td>
</tr>
<tr>
<td>4</td>
<td>4.5</td>
<td>7.2</td>
<td>∞</td>
<td>9.6</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>6.3</td>
<td>3.5</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>∞</td>
</tr>
</tbody>
</table>

Following the same procedure, Table 4.5 through 4.7 are obtained.

TABLE 4.5. Reduced distance matrix No. 3.

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>∞</td>
<td>12.6</td>
<td>12.6</td>
</tr>
<tr>
<td>3</td>
<td>2.5</td>
<td>∞</td>
<td>7.7</td>
</tr>
<tr>
<td>4</td>
<td>4.5</td>
<td>7.2</td>
<td>∞</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

TABLE 4.6. Reduced distance matrix No. 4.

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>∞</td>
<td>12.6</td>
</tr>
<tr>
<td>3</td>
<td>2.5</td>
<td>∞</td>
</tr>
<tr>
<td>4</td>
<td>4.5</td>
<td>7.2</td>
</tr>
</tbody>
</table>
TABLE 4.7. Reduced distance matrix No. 5.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>12.6</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Hence, the schedule at T = 5 is established as

<table>
<thead>
<tr>
<th>Order No.</th>
<th>Starting time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>5.2</td>
</tr>
<tr>
<td>4</td>
<td>8.2</td>
</tr>
<tr>
<td>2</td>
<td>9.7</td>
</tr>
<tr>
<td>3</td>
<td>10.7</td>
</tr>
</tbody>
</table>

4.2 Solution Methods for Travelling Salesman Model

Steps 7 through 12, in the above technique are the same as those observed in the "Modified Next Best Rule," a heuristic approach used for solving TSM.

Various solution approaches for TSM have been attempted and reported in literature. Some of the methods commonly used are

1. Branch-and-Bound Technique
2. Heuristic Rules
   a. Next Best Rule (NB)
   b. Modified Next Best Rule (MNB)
4.2.1. Branch-and-Bound Technique - This is a modification of Branch-and-Bound algorithm developed by Little, Murty, Sweeney, and Karel (1963). The algorithm was originally developed for closed path scheduling, scheduling where the starting point of a schedule is same as the end point of the same. Since the schedule required in the present case is not a closed path schedule, the distance matrix needs to be modified. An additional column has to be added to the original matrix. In the example above, the distance matrix, after modification, results in the matrix shown in Table 4.8.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>∞</td>
<td>30</td>
<td>19</td>
<td>17</td>
<td>15</td>
<td>21</td>
</tr>
<tr>
<td>2</td>
<td>∞</td>
<td>∞</td>
<td>15</td>
<td>18</td>
<td>18</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>∞</td>
<td>7</td>
<td>∞</td>
<td>11</td>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>∞</td>
<td>11</td>
<td>9</td>
<td>∞</td>
<td>5</td>
<td>23</td>
</tr>
<tr>
<td>5</td>
<td>∞</td>
<td>6</td>
<td>8</td>
<td>5</td>
<td>∞</td>
<td>11</td>
</tr>
<tr>
<td>6</td>
<td>∞</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>18</td>
<td>∞</td>
</tr>
</tbody>
</table>

Various different modifications of the distance matrix have been attempted. These modifications facilitate the use of some basic algorithm to more complex cases. Marsh and Montgomery (1973) attempted the use of this technique for scheduling jobs with sequence dependent change over times on parallel processes.
Now, the Branch-and-Bound technique can be applied to the above example. Tables 4.9 through 4.14 show the detail steps involved in arriving at the optimal schedule by this technique. The general procedure for application of this technique is described by Ramalingam (1969).

Continuing further with Table 4.8, the row reduction of matrix results in Table 4.9.

**TABLE 4.9. Reduced distance matrix No. 1.**

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>\infty</td>
<td>15</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>\infty</td>
<td>\infty</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>\infty</td>
<td>5</td>
<td>\infty</td>
<td>9</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>\infty</td>
<td>6</td>
<td>4</td>
<td>\infty</td>
<td>0</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>\infty</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>\infty</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>\infty</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>18</td>
<td>\infty</td>
</tr>
</tbody>
</table>

Table 4.10 shows the column reduction of the matrix in Table 4.9.

**TABLE 4.10. Reduced distance matrix No. 2.**

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>\infty</td>
<td>14</td>
<td>3</td>
<td>2</td>
<td>0^2</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>\infty</td>
<td>\infty</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>0^7</td>
</tr>
<tr>
<td>3</td>
<td>\infty</td>
<td>4</td>
<td>\infty</td>
<td>9</td>
<td>0^4</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>\infty</td>
<td>5</td>
<td>3</td>
<td>\infty</td>
<td>0^3</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>\infty</td>
<td>0^1</td>
<td>2</td>
<td>0^0</td>
<td>\infty</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>\infty</td>
<td>1</td>
<td>0^1</td>
<td>0^0</td>
<td>18</td>
<td>\infty</td>
</tr>
</tbody>
</table>
The penalty costs also shown in the above table, lead to the choice of (2-6) as one of the links in the schedule.

TABLE 4.11. Reduced distance matrix No. 3.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>∞</td>
<td>14</td>
<td>3</td>
<td>2</td>
<td>0²</td>
</tr>
<tr>
<td>3</td>
<td>∞</td>
<td>4</td>
<td>∞</td>
<td>9</td>
<td>0⁴</td>
</tr>
<tr>
<td>4</td>
<td>∞</td>
<td>5</td>
<td>3</td>
<td>∞</td>
<td>0³</td>
</tr>
<tr>
<td>5</td>
<td>∞</td>
<td>0³</td>
<td>2</td>
<td>0⁰</td>
<td>∞</td>
</tr>
<tr>
<td>6</td>
<td>∞</td>
<td>1</td>
<td>0²</td>
<td>0⁰</td>
<td>18</td>
</tr>
</tbody>
</table>

Here the choice is 3 to 5.

TABLE 4.12. Reduced distance matrix No. 4.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>∞</td>
<td>12</td>
<td>1</td>
<td>0¹</td>
</tr>
<tr>
<td>4</td>
<td>∞</td>
<td>2</td>
<td>0²</td>
<td>∞</td>
</tr>
<tr>
<td>5</td>
<td>∞</td>
<td>0¹</td>
<td>2</td>
<td>0⁰</td>
</tr>
<tr>
<td>6</td>
<td>∞</td>
<td>1</td>
<td>0</td>
<td>0⁰</td>
</tr>
</tbody>
</table>

The choice here is 4-3.
The choice here is 1-4.

The obvious choice here is 5-2.

The above disjoint links may be combined together to give a complete schedule as

1 - 4 - 3 - 5 - 2 - 6

The total distance that has to be travelled in this case is 47 miles.

4.2.2. Heuristic Rules - These rules were developed by Gavett (1965).

a) Next Best Rule (NB): closest unvisited city method; the method here is to choose the unassigned order which is the nearest to the order that has just been completed. In the example this
rule might be applied starting at order 1. Order 5 has to be undertaken after order 1, since it is the nearest to the order 1. Now, the next order to be chosen has to be the closest to order 5. Proceeding in this fashion following schedule can be obtained.

1 - 5 - 4 - 3 - 2 - 6

Total distance to be travelled to execute this schedule is 49 miles.

b) Modified Next Best Rule (MNB): In this method the NB rule is applied after subtracting the minimum value in each column of the distance matrix from each value in that column. The distance matrix in the example after column reduction is shown in Table 4.15.

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>28</td>
<td>18</td>
<td>17</td>
<td>13</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>∞</td>
<td>15</td>
<td>18</td>
<td>16</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>∞</td>
<td>11</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>8</td>
<td>∞</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>7</td>
<td>5</td>
<td>∞</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>16</td>
<td>∞</td>
</tr>
</tbody>
</table>

Now the NB Rule may be applied to the above matrix to obtain the schedule.

1 - 6 - 3 - 5 - 2 - 4
with the travel involved = 48 miles.

At this point a choice has to be made between the techniques discussed above. The first argument that may be made in favor of Branch-and-Bound technique is that it yields an optimum schedule, as against near-optimum schedules in the heuristic rules. This optimality of the schedule is attained at the cost of increasing the complexity of the technique, and the inherent problem of computational efforts required. Here a balance has to be struck between the deviation from the optimality and the ease of computations.

For problems with uniformly distributed distances, Ramalingam (1969) reported that the average percentage increase in total distance travelled over optimum value is 57% for NB Rule and 25% for MNB Rule. For the uniformly distributed distances the average increase for NB Rule is 30%. The extra travel involved has to be compared with the computational savings achieved.

In the present problem, the interrelationships are very direct. The marginal costs for an extra mile of travel of about 7¢ per mile is weighed against the marginal time required for the computation. In the present problem the number of orders that have to be handled simultaneously is so large that the computational efforts increase rapidly with every additional order received. A graph of computational efforts against the number of orders may be plotted as by Marsh and Montgomery (1973) Figure 4.1 and Figure 4.2. The rate of increase in computation
Figure 4.1. Computing Time as a Function of M and N'.

Figure 4.2. Computing Time as a Function of M, N', and Data Variability.

(Marsh and Montgomery 1973)
time for every extra order can be easily noted. And there should be no problem in concluding that, in the present case, where orders of the order of 100's are to be scheduled, the application of the Branch-and-Bound techniques is almost impossible. In cases of NB Rule and MNB Rule this relationship between computational time and number of orders is linear (Figure 4.3).

It may be also noted that Branch-and-Bound techniques at each step schedules a disjoint link. The algorithm has to be continued till the entire schedule is generated. But in most of the cases, as in the present problem, only the first few elements of the schedule are used. Consequently, a large computational effort is wasted. But the NB Rule and MNB Rule select the orders in the sequence in which they are to be executed and it is possible to terminate computation as soon as enough orders to cover the desired period are scheduled.

The combined effect of these two factors is a large saving in computational time by the use of heuristic rules. A crude estimate is that only 1/20 of cpu time is needed by the heuristic methods as compared to the Branch-and-Bound method applied to a 6 x 6 problem.

4.3 Selection of Desirability and Reduction Factors

In the discussion of solution methods, thus far, no attempt has been made to modify the algorithms to account for the time
Figure 4.3. Computation Time Variation for NB and MNB Rules. (Schematic only)
limitations present on the execution of orders. The steps 2 through 6 in the method outlined at the beginning of this chapter accounts for the time restrictions.

The idea of desirability factor is based upon the "Dynamic Slack" concept of Priority Dispatching. Dynamic Slack can be defined as the time gap available for sliding an activity in the schedule. As in the present problem, it can be represented as

\[ S = DD - D - T, \]

where \( D \) = duration of execution and \( T \) = time at which the slack is calculated. 

\( S \) can also be interpreted as the time difference between the latest possible and the earliest possible starts of an activity. Dynamic Slack (\( S \)) can be normalized by dividing it by maximum possible slack (\( S_{\text{max}} \)).

\[ x = \frac{S}{S_{\text{max}}} = \frac{DD - D - T}{DD - D - SD} \tag{4.1} \]

This non-dimensioned parameter \( x \) is called a "Normalized Dynamic Slack" (NDS). The variation of NDS with time is shown in Figure 4.4. Some properties of this factor have been summarized by Buffa (1972).

**Example.** An order A has a start date of 2 and a due date of 15: \( SD = 2; DD = 15 \). It takes 3 days to execute the order: \( D = 3 \). At the start date this particular order has \((15 - 3 - 2 = 10)\) days of slack period. But as the time passes, the slack goes on reducing. At time \( T = 5 \), the order has only \((15 - 3 - 5 = 7)\) days of slack period and \( x = 7/10 = .7 \).
Figure 4.4. Time Variation of Normalized Dynamic Slack
It can be easily seen that the NDS decreases as the necessity of undertaking that particular order goes on increasing. If the desirability factor was assumed to be varying linearly with respect to NDS, the graph shown by dotted line in Figure 4.5, is obtained. This direct relationship accounts only for the time variation of the Desirability Factor (DF). As indicated in Chapter II, other factors such as type of the application, the size of an order, the percentage extra cost to the customer, and the type of customer also influence the desirability factor. The additional variation in DF may be shown as in Figure 4.5. These two factors can be combined as, for example:

\[ D = 1 + B - BKx - Be^{-Kx} \]  \hspace{1cm} (4.2)

Here \((1 + B - BKx)\) represents the variation of desirability factor with time. The factor \(Be^{-Kx}\) represents the variation of desirability factor with respect to the other factors. \(K\) can be associated with the exponential factors which are dependent upon the type of order, factors such as type of application, percentage extra cost to the customer, etc. The value of \(B\), on the other hand is a multiplier associated with the customer, by the size of the order, type of customer, etc. \(B = 0\) represents that the other factors do not contribute much to the desirability and the solid line in Figure 4.5 coincides with the dotted line. If the conditions are present where \(K = 0\), it represents a case where the type of order does not influence the desirability. The spraying of a lawn may be cited as an example of the \(K = 0\) class.
Figure 4.5. Variation of Desirability Factor

Figure 4.6. Variation of Reduction Factor.
where a low value of $K$ is appropriate.

The effect of $B$ is best described as a "good-will" factor. When $B = 0$, $D$ also becomes a constant ($D = 1$). Smaller $B$ values indicate relatively unimportant customers, while larger values of $B$ are given to customers whose good-will is valued highly by the management. The size and frequency of orders received from the customer, the length of business association with the customer, and the type of services rendered by the customer are all important factors to be considered in determining the value of $B$.

In deciding the schedule, or in comparing two different orders, the rate of increase of desirability as time advances ($x$ approaches 0) is an important criterion. Hence, the reduction factor ($\alpha$) has been defined as:

$$\alpha = - \frac{dD}{dx} = - (- BK + KBe^{-Kx})$$

$$= BK - KBe^{-Kx}$$

$$= BK (1 - e^{-Kx})$$

$$= G (1 - e^{-Kx}), \text{ where } G = BK$$

Figure 4.6 pictorially depicts this relationship. We can now use the reduction factor to discount the physical distances in the determination of the proper schedule for executing orders according to the physical distance and the desirability of effecting each order.
CHAPTER V

EVALUATION OF TECHNIQUE

5.1. Scheme of Evaluation

The approach presented in the previous chapter, though feasible, has to be evaluated. A flexible tool displays different characteristics under different operating conditions. But a broad evaluation will be attempted in this chapter in light of BSS problem. The basic premise for this evaluation is that the efficiency of a scheduling technique can be determined by using three major criteria.

a. Economy of computation.

b. Flexibility of application

c. Optimality of results.

Throughout the development of this thesis care has been taken that the resulting technique will have these three properties. The flexibility is achieved by the inclusion of two subjective parameters, G and K. The factor of economy depends upon the cost of operating and maintaining the scheduling system. This can be easily quantified. Finally, there is the saving made by the use of the technique. This criteria is subjective and difficult to quantify. The mileage saved in case of BSS can be estimated. But, other costs such as goodwill and damage to the crop are difficult to quantify.
The evaluation will be carried out with the help of two examples. The first example has fourteen orders and will indicate the manner in which the technique operates. The second example will be based upon the data from selected statistical distributions and will simulate the problem faced by BSS and the other Order Scheduling systems.

As indicated before in Chapter III, a sufficient amount of historical data to properly identify the statistical parameters has not been available in the case of BSS. But, from the information available, statistical distributions have been anticipated. It is assumed in the second example that all the parameters, except Starting Date and Due Date, are distributed uniformly throughout their ranges (0 to 1 for G and K). For example, it is assumed that all the orders received by BSS are to be executed at the locations randomly distributed throughout the region indicated in Figure 2.1. Other factors such as duration of execution of order (D_K) do not affect the schedule and any distribution can be assumed for these factors. The values of G and K can be distributed in any fashion as these values for all the orders will be kept constant, except for one order.

The distributions for Starting Date (SD) and Due Date (DD) affect the schedule to the largest extent. The values for SD and DD are generated to have a range of 300 days. But, only the time range of 100 to 200 will be observed, avoiding the transient effects (Figure 5.1). In this second example, two different
Figure 5.1. Distribution of Simulated Data.
FIGURE 5-2. Distribution of Orders - Uniform Distribution.
distributions are considered (Figure 5.1). The first distribution considered is a Poisson Input Process. Orders were generated from uniform distribution in the given range to simulate the arrival pattern. The second distribution is generated from two normal distributions. This is assumed to simulate a common situation faced by BSS. The time zone of interest is the one between the mean values of these distributions. The location of the two peaks (modes), affects the mean length of time available to execute orders.

Though the results obtained by the use of the heuristic technique will be used throughout this chapter, the details of the example will be presented at the end of this chapter.

5.2 Economic Evaluation of Technique

As indicated before, only quantitative evaluation based on an objective criterion will be treated in this section. The major assumption has to be made that G and K remain constant.

As experimental determination of computing time was made on CDC 3300. The parameters involved in the technique were varied to observe their effect on computing time. The major factor that governs the computing time is the number of orders to be scheduled. The results of such an investigation in case of second example can be plotted as in Figure 5.3. The Due Dates and Starting Dates were generated from a Normal Distribution Data. This plot can be compared with Figure 4.1 and Figure 4.2. The comparison clearly shows the savings in computational time
TABLE 5.1. Variation of Computing Time.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Normal</th>
<th>Uniform (Poisson Arrivals)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Order</td>
<td>5 10 20 40 80 100</td>
<td>100</td>
</tr>
<tr>
<td>G (for order 86)</td>
<td>.98</td>
<td>0</td>
</tr>
<tr>
<td>K (for order 86)</td>
<td>.49</td>
<td>0</td>
</tr>
<tr>
<td>Computational time (cpu)</td>
<td>6.1 7.0 9.1 19.9 74.5 111.3</td>
<td>52.9 49.8 50.1</td>
</tr>
</tbody>
</table>
Figure 5.3. Computing Time as a Function of Number of Jobs.
due to the use of MNB Rule. It may also be observed that the discrepancy between these two methods increases rapidly as the number of orders becomes large. From this analysis the dollar-value of computational time can be estimated easily.

A large number of additional runs were made after varying parameters like G and K. The computational times required to arrive at the schedule in these cases are summarized in Table 5.1. It is observed that for a single distribution the variation of computational times is not significant. But, there is a large difference in the results obtained from the Uniform Distribution case and the Normal Distribution case. In Normal Distribution, since about 25% of the orders have to be undertaken between dates 60 and 100, large matrices need be handled in this period and a large computational time results.

The data, gathered in the above form and transformed in dollar-value, has to be compared with the results obtained by the use of other techniques. Two extreme cases can be visualized. In the first case TSM is used, i.e., the time restrictions are ignored. The schedule indicated in the first example shows the deviation from optimality. The schedule given by TSM, though not feasible, required 101 miles of travel, as against the new technique giving 338 miles of travel. The other extreme case may be the use of first come, first served (FCFS) rule of Priority Dispatching. In the first example, use of FCFS gives a schedule with a total travel of 351. In the second example, the schedule can be obtained by FCFS using Figure 5.2.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th>.47</th>
<th>.5</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>K</strong></td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>SD for 86 = 115 + .28</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.5</td>
<td>SD for 86 = 115 + .28</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>SD for 68 = 115 + .28</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.98</td>
<td>SD for 86 = 141.26</td>
<td>SD for 68 = 115.28</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>SD for 86 = 115.28</td>
<td>SD for 68 = 115 + .28</td>
<td>SD for 68 = (1-e^{-x})</td>
<td></td>
</tr>
</tbody>
</table>
Figure 5.4. Variation of Reduction Factor.
These savings in the total travel must be compared with the amount of computational time required.

5.3 Flexibility Evaluation of Technique

The type of comparison indicated at the end of previous section becomes difficult as a large number of different values can be assigned to the parameters G and K. Each value represents the weight of an order in the schedule and is subject to consideration of intangible factors.

To find the actual values of these factors an extensive investigation should be made. In the present investigation a few different values were assigned to the parameters G and K. The effect of parameter variation on Order Number 86 from the Uniform Distribution case was closely studied through the investigation. The variation of G and K causes the variation of Desirability Factor (DF) and Reduction Factor ($\alpha$). Figure 5.4 portrays $\alpha$ variations for the Order 86. Using these values, their effect on a schedule can be observed (Table 5.2). For $G = 0.98$ and $K = 0.47$ the Order 86 is undertaken in the week of 140. But assigning value of reduction factor $= 0$ for Order 86, its desirability increased rapidly above the desirability of Order 68, which was undertaken in the week of 115 before. Consequently, the higher priority order 86 pre-empted the lower priority Order 86 completely. On the other hand, any other degenerate values of G and K caused the deletion of Order 86 from the schedule completely.
The nature of the variation for a large number of different values of G and K and their effects on the schedule were observed. It is expected that after some experience with varying values of G and K, the assignment of these values to a particular customer and a particular type of order can be made intuitively by the management of BSS.

5.4 Numerical Examples

To illustrate the procedures already discussed in Chapter IV, following examples will be considered.

Example 1

Table 5.3 indicates the data that has to be compiled or calculated before the actual process of scheduling is started. Columns 6 to 8 of Table 5.3 list the information received by BSS along with the order. The location where order has to be executed is indicated by the Area Code. Area Codes are the dummy numbers used to indicate geographical regions and simplify calculations of distances between any two orders. From the type of order and the number of acres to be sprayed, the data in the remaining columns can be computed. Table 5.4 shows the distance matrix calculated with the help of Area Codes.

The use of Branch-and-Bound technique yielded the following schedule with total distance to be travelled = 101 miles.

\[1 \rightarrow 4 \rightarrow 3 \rightarrow 5 \rightarrow 2 \rightarrow 6 \rightarrow 7 \rightarrow 8 \rightarrow 13 \rightarrow 12 \rightarrow 14 \rightarrow 9 \rightarrow 10 \rightarrow 11\]

It can be noted that this schedule is not feasible because the
<table>
<thead>
<tr>
<th>Order Number</th>
<th>Area Code</th>
<th>Start Date</th>
<th>Duration</th>
<th>Due Date</th>
<th>Acres</th>
<th>Type of Customer</th>
<th>Order Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>101</td>
<td>0</td>
<td>5</td>
<td>6</td>
<td>1000</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>106</td>
<td>13</td>
<td>4</td>
<td>51</td>
<td>500</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>104</td>
<td>9</td>
<td>8</td>
<td>76</td>
<td>50</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>102</td>
<td>41</td>
<td>5</td>
<td>63</td>
<td>100</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>104</td>
<td>1</td>
<td>3</td>
<td>17</td>
<td>200</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>207</td>
<td>10</td>
<td>7</td>
<td>22</td>
<td>700</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>208</td>
<td>0</td>
<td>2</td>
<td>15</td>
<td>1300</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>203</td>
<td>9</td>
<td>9</td>
<td>51</td>
<td>1600</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>301</td>
<td>6</td>
<td>4</td>
<td>49</td>
<td>160</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>311</td>
<td>16</td>
<td>7</td>
<td>76</td>
<td>250</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>309</td>
<td>0</td>
<td>1</td>
<td>77</td>
<td>100</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>401</td>
<td>4</td>
<td>4</td>
<td>39</td>
<td>10</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>13</td>
<td>405</td>
<td>24</td>
<td>1</td>
<td>39</td>
<td>600</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>14</td>
<td>402</td>
<td>0</td>
<td>5</td>
<td>70</td>
<td>4000</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5.3:

Area Code - First digit denotes zone.
Second and third digits denotes exact location
Customer type - Annual or Non-Annual
Figure 5-5. Variation of Reduction Factor for Example 1.
TABLE 5-4. Distance Matrix (Rounded to nearest mile).

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>∞</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>14</td>
<td>14</td>
<td>12</td>
<td>36</td>
<td>38</td>
<td>38</td>
<td>21</td>
<td>21</td>
<td>21</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>∞</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>17</td>
<td>18</td>
<td>15</td>
<td>39</td>
<td>41</td>
<td>41</td>
<td>24</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>5</td>
<td>∞</td>
<td>3</td>
<td>0</td>
<td>17</td>
<td>18</td>
<td>15</td>
<td>39</td>
<td>41</td>
<td>41</td>
<td>24</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>∞</td>
<td>3</td>
<td>16</td>
<td>17</td>
<td>14</td>
<td>38</td>
<td>40</td>
<td>40</td>
<td>23</td>
<td>23</td>
<td>23</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>5</td>
<td>0</td>
<td>3</td>
<td>∞</td>
<td>17</td>
<td>18</td>
<td>15</td>
<td>39</td>
<td>41</td>
<td>41</td>
<td>24</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>6</td>
<td>14</td>
<td>17</td>
<td>17</td>
<td>16</td>
<td>17</td>
<td>∞</td>
<td>2</td>
<td>4</td>
<td>28</td>
<td>30</td>
<td>30</td>
<td>19</td>
<td>19</td>
<td>19</td>
</tr>
<tr>
<td>7</td>
<td>15</td>
<td>18</td>
<td>18</td>
<td>17</td>
<td>18</td>
<td>2</td>
<td>∞</td>
<td>5</td>
<td>29</td>
<td>31</td>
<td>20</td>
<td>31</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
<td>15</td>
<td>15</td>
<td>14</td>
<td>15</td>
<td>4</td>
<td>5</td>
<td>∞</td>
<td>26</td>
<td>29</td>
<td>28</td>
<td>17</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>9</td>
<td>36</td>
<td>39</td>
<td>39</td>
<td>38</td>
<td>39</td>
<td>28</td>
<td>29</td>
<td>26</td>
<td>∞</td>
<td>2</td>
<td>4</td>
<td>38</td>
<td>38</td>
<td>38</td>
</tr>
<tr>
<td>10</td>
<td>38</td>
<td>41</td>
<td>41</td>
<td>40</td>
<td>41</td>
<td>30</td>
<td>31</td>
<td>29</td>
<td>2</td>
<td>∞</td>
<td>6</td>
<td>40</td>
<td>40</td>
<td>39</td>
</tr>
<tr>
<td>11</td>
<td>38</td>
<td>41</td>
<td>41</td>
<td>40</td>
<td>41</td>
<td>30</td>
<td>31</td>
<td>28</td>
<td>4</td>
<td>6</td>
<td>∞</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>12</td>
<td>21</td>
<td>24</td>
<td>24</td>
<td>23</td>
<td>24</td>
<td>19</td>
<td>20</td>
<td>17</td>
<td>38</td>
<td>40</td>
<td>40</td>
<td>∞</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>13</td>
<td>21</td>
<td>24</td>
<td>24</td>
<td>23</td>
<td>24</td>
<td>19</td>
<td>20</td>
<td>17</td>
<td>38</td>
<td>40</td>
<td>40</td>
<td>2</td>
<td>∞</td>
<td>3</td>
</tr>
<tr>
<td>14</td>
<td>21</td>
<td>24</td>
<td>24</td>
<td>23</td>
<td>23</td>
<td>19</td>
<td>20</td>
<td>17</td>
<td>38</td>
<td>39</td>
<td>40</td>
<td>2</td>
<td>3</td>
<td>∞</td>
</tr>
</tbody>
</table>
orders 4, 6, 7, 12, 13, 9 cannot be undertaken in the specified time period. When NB and MNB Rules were used the schedules with distances varying between 101 and 116 were obtained. On the other hand the use of First come, first served (FCFS) rule of priority dispatching generated the schedule.

1 - 7 - 11 - 14 - 5 - 3 - 12 - 9 - 8 - 6 - 2 - 10 - 13 - 4.

With total distance to be travelled = 351 miles. Here all the orders are undertaken in the desired periods.

To illustrate the effects of reduction factor on the schedule a simpler definition of $\alpha$, $\alpha = x$ (Figure 5.5) was used. The following schedule with a total distance of 338 miles was obtained.

1 - 7 - 5 - 12 - 3 - 6 - 9 - 14 - 2 - 13 - 8 - 10 - 4 - 11.

Example 2

This example is an attempt to handle a large and more realistic case. The computations in this example are performed with the help of FORTRAN IV program written for CDC 3300 system. The parameters used in the program are

$\text{ALB}(J, J) = \text{Distance between point } I \text{ and point } J, \text{ both points being in the Albany region.}$

$\text{CIT} = \text{Running index of time used up in performing the orders.}$

$\text{COR}(I, J) = \text{Distance between point } I \text{ and point } J, \text{ both points being in the Corvallis region.}$
COS = Running cumulative of time durations of executions.
CT(I,1) = Completion time for order I.
DINT(I,J) = Distance between town I and Town J.
IAT(I,1) = Row matrix of order numbers.
IBT(I,1) = Column matrix of order numbers.
IP = Starting Date of a schedule.
IPT(I,1) = Schedule in each period.
IPT(I,3) = Rush state of an order, I.
IPT(I,1) = Starting Date of an order, I.
ITP(I,2) = Due Date of an order, I.
ITP(I,3) = Tag to indicate unfinished orders.
ITP(I,4) = Area Code.
KKK = Starting order of a schedule for a period.
LAB(I,J) = Distance between point I and point J, both points being in the Lebanon region.
M = Number of active orders
N = M + 1
PN(I,1) = Duration required for completion of an order, I = D_i.
PN(I,2) = G
PN(I,3) = K
PN(I,4) = Duration corrected for rush state.

The flowchart in Figure 5.6 shows the scheme of the program detailed in the Appendix. It can be noted from the program that
REARRANGE AND INITIATE PARAMETERS

CHECK AND MODIFY DURATIONS, IF THERE ARE MORE ORDERS THAN CAN BE HANDLED

MAKE A LIST OF UNFINISHED ACTIVE ORDERS

GENERATE THE REDUCED DISTANCE MATRIX

APPLY MNB RULE

OUTPUT THE SCHEDULE
the different parameters shown in Table 5.5 can take different values depending upon the type of application. The values in this case were generated from the statistical distributions indicated in Table 5.5.

TABLE 5.5. Parameters of Simulated Data

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location of Order</td>
<td>Uniform, *a = 1 b = 35</td>
</tr>
<tr>
<td>Duration of excretion $D_i$</td>
<td>Uniform, a = 0 b = 27.00</td>
</tr>
<tr>
<td>Starting Date</td>
<td>Uniform, a = 0 b = 300</td>
</tr>
<tr>
<td></td>
<td>Normal, mean = 60 variance = 20</td>
</tr>
<tr>
<td>Due Date</td>
<td>Uniform, a = 0 b = 300</td>
</tr>
<tr>
<td></td>
<td>Normal, mean = 100 variance = 20</td>
</tr>
<tr>
<td>G</td>
<td>Uniform, a = 0 b = 1</td>
</tr>
<tr>
<td>K</td>
<td>Uniform, a = 0 b = 1</td>
</tr>
</tbody>
</table>

*b-a is the range of the distribution*
CHAPTER VI

DISCUSSION AND CONCLUSION

Order Scheduling represents a class of industrial application problems where no satisfactory algorithm has been found for use by managers. As defined in Chapter I, it incorporates both the slack scheduling feature of non-recurrent projects and the sequence dependent feature of job-shop order schedules.

6.1 Complexity of Order Scheduling Problems

The lack of literature on this class of problems is attributable to several factors. First, this class of problems is most frequently encountered in areas where industrial engineers have not previously been very active, namely the service-oriented industries.

Second, the interactions between the sequence dependency of the set-up time and the time dependency of priority for each individual job create problems that are almost insurmountable. A simple job-shop scheduling model utilizing a Branch-and-Bound algorithm to optimize the sequence-dependent set-up times is already plagued by the curse of dimensionality (Ramalingam, 1969). Similarly, a project cost optimization model with non-linear, and especially non-convex, discontinuous cost functions are painful to solve (Wiest and Levy, 1969). An order scheduling problem that combines the two features would seem to defy almost any logical attempt at solving a practical problem.
Third, the types and the variety of data that need be collected to create a viable mathematical model are beyond what an ordinary small firm is willing to expend for the sake of advancing the art of management.

6.2 Approach Used in this Study

The first difficulty was overcome by selecting a small service firm that willingly cooperated in this study of order scheduling. The management of the firm provided the data needed to construct and test the models, and acted in an advisory capacity to evaluate the effectiveness of the resulting scheduling procedures.

The second difficulty was overcome by adopting a heuristic approach to the solution of the problem. By sacrificing the guarantee for optimality, the model was simplified so that the computations can be performed practically.

The last difficulty was solved by utilizing existing data as much as possible and limiting additional data requirement to a minimum. Table 6-1 compares the list of existing data with the additional data collected experimentally for use in this study. A rough estimate of 15% additional reporting effort (35 minutes per day instead of 30 minutes per day) was observed during a month period when the additional data collection procedure was in effect.
TABLE 6-1. Data Required Before Scheduling.

<table>
<thead>
<tr>
<th>Presently Available Data</th>
<th>Required Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name of the customer</td>
<td>Number of customer</td>
</tr>
<tr>
<td></td>
<td>Number of years the customer is on file</td>
</tr>
<tr>
<td>Address</td>
<td>Area Code</td>
</tr>
<tr>
<td>Type of order</td>
<td>Number indicating type of order</td>
</tr>
<tr>
<td>Acres serviced</td>
<td>Acres serviced</td>
</tr>
<tr>
<td>Amount of chemical used</td>
<td>Time of execution of the order</td>
</tr>
<tr>
<td></td>
<td>Time required to complete the order</td>
</tr>
<tr>
<td></td>
<td>Extent of disease</td>
</tr>
<tr>
<td></td>
<td>Total miles travelled</td>
</tr>
<tr>
<td></td>
<td>Equipment used</td>
</tr>
<tr>
<td></td>
<td>Amount of chemical used</td>
</tr>
</tbody>
</table>
6.3 The Proposed Order Scheduling Procedure

From the experimental work carried out mathematically and evaluated by Beaver Spraying Service (BSS) the following Order Scheduling procedure is recommended.

a) Company Information File: The geographical and/or technological areas served by the organization must be identified. In the case of BSS, Figure 2.1 was formed as a grid system to contain the geographical distribution of present and prospective customers. Table 2.2 is a partial list of technological constraints imposed by the type of service provided by BSS. Data from Figure 2.1 were used to compute entries in the distance matrix, while Table 2.2 provided the starting and due dates for the orders received. A catalogue file of the firm's personnel and equipment must also be available.

A general desirability factor curve (Figure 4.5) should be plotted by the management. The desirability factor equation (4.2) will be fitted to the graph to determine the G and K factors.

b) Customer File: A data file must be set up for each customer containing permanent data on the customer (e.g., name, address, acreage, etc.) and augmented by data pertaining to his orders. Each order will be cross-referenced to the company's information file (e.g., area code, type of job, etc.) and other operational information (Table 6.1).

c) Order Scheduling: At a predetermined interval, orders will be searched to create a list of active orders. Active orders
are defined as uncompleted jobs with starting dates that have already past. In the case of BSS, a scheduling interval of one week is contemplated in the computerized version.

A distance matrix must then be compiled to indicate all possible set-up times \([d_{ij}]\) from one active order \((i)\) to another \((j)\).

The distance matrix, in case of BSS, contained actual mileage needed to go from one active job to another. In another industry, this may simply be time required to dismantle one job to set-up the system ready for the next job.

The distance matrix is then discounted by the reduction factor to obtain the discounted distance matrix \([d_{ijk}]\). The \(G\) and \(K\) values estimated in step (a) are used as their first approximations.

The Modified Next Best Rule can then be applied to the reduced distance matrix to produce a tentative schedule (Figure 5.6).

d) Implementation: The tentative order schedule must be modified according to other factors such as weather, holidays, machine failures, etc. In specific, the active order file should be reviewed to check if all orders will be executed before their due dates are past. It may then be necessary to schedule overtime, multiple shifts, and week-end works to satisfy order requirements.
e) **Evaluation and Adaptation:** The performance of the above order scheduling procedure should be reviewed from time to time to update company files and to expand the service areas, if necessary. In particular, the values of G and K should be modified to improve the schedule effectiveness.

It is not necessary that this scheduling procedure should always be implemented on a computer system. BSS appears to have an effective manual filing system which can utilize the above procedure with a limited additional data processing capability.

6.4 **Evaluation of the Study**

The need for Order Scheduling is evident even from a limited experiment conducted on fourteen uniformly distributed orders. The direct application of the classical Branch-and-Bound job-shop scheduling procedure resulted in six orders missing their due dates (Section 5.4). With the Order Scheduling procedure, all of the fourteen jobs were completed on time. More generally, it was found that only 15% of uniformly distributed jobs would miss their due dates when scheduled with the Next Best Rule as proposed in this study (Section 5.4).

The reduction factor that reflects the subjective judgment of BSS appeared to have a concave form. The proposed reduction factor formula (equation 5.3) approximated all conceivable forms of concave reduction factors as proposed by the BSS management.
The effectiveness of an order scheduling model depends greatly on the availability of historical data. In the case of BSS, 300 orders are processed per year and at least 100% improvement in accuracy is hoped by the management. It is estimated that about four times as much data as presently available on BSS's file is needed. Thus, the desired accuracy may not be reached until three or four more years of continued business operation and data collection is completed.

The use of the heuristic Next Best Rule over the more exacting Branch-and-Bound procedure seems justifiable in consideration of the computational efforts. The Branch-and-Bound computational time, reported by Marsh and Montgomery (1973), suggested that about 20% more CPU time is needed for that technique than that required to run the ten order normally distributed model of Example 2 in Section 5.4 on CDC-3300 computer system. This computational advantage of MNB Rule increased rapidly to twenty times for fifteen orders. The advantage of computational time is offset by the loss of optimality, estimated by Ramalingam (1969) to be 10-20%.

6.5 Future Research Efforts

One major problem faced during this investigation was the lack of a measure to judge the actual effectiveness of a scheduling approach. Factors involved in judging the commercial effectiveness of a service schedule are mostly subjective and seem to depend
generally on the philosophy of management. A method for incorporating these subjective factors directly in the evaluation of mathematical models will greatly increase the usefulness of any order scheduling algorithm.

To fully evaluate the utility of order scheduling models, a large number of industrial applications need be investigated. A promising case is a food processing plant that combines the canning of vegetables with canning of sea-food products to fully utilize their equipment all year around. A considerable amount of set-up time is required in switching from one product to another. The reduction in set-up times and the reduction in the number of changeovers, mean longer storage of the fresh produces and sea-foods. This may result in lower quality of product and increased wasteage. In this case, the heuristic approach can be easily implemented by assigning different desirability functions to approximate the spoilage of different products.

More generally, the order scheduling approach should find application in any industry where the product or the raw material deterioriates with time and at the same time set-up costs are involved in changing the batches. Similarly, order scheduling may find applications among governmental agencies which cater to the welfare of the public in general.

Industrial Engineering is rapidly expanding into hospital Engineering and service industry as well as into agriculture.
It is hoped that order scheduling provides a new model that will aid Industrial Engineers of the future to carry out their function more effectively.
BIBLIOGRAPHY


SUHAR

**PROGRAM EXAMPLE**

DINENSION ITB(101,1), IPT(100,3), ITP(100,4), PN(100,4),
1 IAT(100,1), TDIS(100,100), CT(100,1)
2, COK(7,7), ALB(8,8), SAL(12,12), DINT (4,4), LAB(8,8)

C IN THIS SECTION INFORMATION AND THE DATA ABOUT THE ORDER ARE READ.

READ(1,95) ((PN(I,J), J=1,4), I=1,100)
READ(1,12) ((ITP(I,J), J=1,4), I=1,100)
12 FORMAT(4F14.4)
READ(1,51) ((COR(I,J), J=1,7), I=1,7)
51 FORMAT(7F6.8)
READ(1,52) ((LAB(I,J), J=1,8), I=1,8)
52 FORMAT(6F6.2)
READ(1,53) ((SAL(I,J), J=1,12), I=1,12)
53 FORMAT(6F6.2)
READ(1,54) ((DINT(I,J), J=1,4), I=1,4)
54 FORMAT(4F6.2)
READ(1,55) ((LAB(I,J), J=1,8), I=1,8)
55 FORMAT(3F6.2)

C IN THIS SECTION INFORMATION IS SORTED AND THE PARAMETERS ARE INITIATED.

IPT(1,1) = 1
00 11 I = 1,100
IF (ITP(I,1)-ITP(I,2) + 30) 101,103,103
103 IK = ITP(I,1) + 30
IPT(I,1) = IK
105 CONTINUE

CONTINUE
95 FORMAT(4F6.2)
IP = 0
181 CONTINUE
KK = IPT(I,1)
IC = 1
00 11 I = 1,100
131 IPT(I,1) = KK
COS = J
DO 1 K = 1,100
1 IF (ITP(K,1) .LE. IP .AND. ITP(K,2) .LE. (IP + 30) .AND. ITP(K,3) .EQ
1.0) GO TO 2
GO TO 1
2 COS = COS + PN(K,1)
1 CONTINUE
IF (COS .LE. 56.70) GO TO 94
IF (COS .LE. 76.00) GO TO 11
IF (COS .LE. 72.00) GO TO 13
IF (COS .LE. 112.00) GO TO 14
94 OUR = 1.00
IPF = 1
GO TO 4
11 CONTINUE
OUR = 5.00/6.00
IPF = 1
GO TO 4
13 OUR = 5.00/7.00
IPF = 2
GO TO 4
14 OUR = 3.5
IN THIS SECTION LIST OF ACTIVE ORDERS IS COMPILED.

M = 0
N = 1
C IT(1, 1) = KKK
DO 15 I = 2, 100
IF (I TP (I, 3) .GE. IP 1 0) GO TO 15
GO TO 16
15 M = M + 1
N = N + 1
C IT(1, 1) = I
C IT(1, 1) = I
16 CONTINUE.
C IN THIS SECTION THE REDUCED DISTANCE MATRIX IS
C CALCULATED.
DO 17 I = 1, N
DO 18 J = 1, I
K = I TP(I, 1)
L = I TP(J, 1)
IF (IP(1, 4) .GT. 27) GO TO 69
IF (IP(K, 4) .GT. 15) GO TO 74
IF (ITP(K, 4) .GT. 6) GO TO 75
IIPL1 = 1
1PS1 = ITP(K, 4)
GO TO 86
86 IIPL1 = IIPL1 + 1
1PS1 = ITP(K, 4) - 27
GO TO 89
87 IIPL1 = IIPL1 + 1
1PS1 = ITP(K, 4) - 15
GO TO 89
88 IIPL1 = IIPL1 + 1
1PS1 = ITP(K, 4) - 7
89 CONTINUE.
C IF (I - J) 77, 76, 77
77 IF (ITP(L, 4) .GT. 27) GO TO 73
IF (ITP(L, 4) .GT. 15) GO TO 74
IF (ITP(L, 4) .GT. 6) GO TO 75
IIPL2 = 1
1PS2 = ITP(L, 4)
GO TO 86
73 IIPL2 = IIPL2 + 1
1PS2 = ITP(L, 4) - 27
GO TO 73
74 IIPL2 = IIPL2 + 1
1PS2 = ITP(L, 4) - 15
GO TO 73
75 IIPL2 = IIPL2 + 1
1PS2 = ITP(L, 4) - 7
GO TO 73
76 CONTINUE.
IF (IPL1=IPL2) 69, 81, 69
60 IF (IPL1 < 20, 1) 02 = CCP (IPS1, IPS2)
IF (IPL1 > F0, 2) 02 = ALU (IPS1, IPS2)
IF (IPL1 < 7, 3) 02 = SL (IPS1, IPS2)
IF (IPL1 < 9, 4) 02 = LA (IPS1, IPS2)
DIST = 32
GO TO 82
69. QI = INIT (IPL1, IPL2)
    IF (IPL1 < 20, 1) QI1 = COR (1, IPS1)
    IF (IPL1 > 20, 1) QI1 = ALB (1, IPS1)
    IF (IPL1 < 0, 3) QI1 = SAL (4, IPS1)
    IF (IPL1 > 0, 3) QI1 = LAB (4, IPS1)
DIST = QI1 + QI2 + 012
GO TO 76
76 DIST = 39, 39
82 CONTINUE
83 X = PN (L, 1)
   C = (TIP (L, 1) - IP (3, 025) * PN (L, 1))
   C = (TIP (L, 1) - ITF (L, 1) - (0, 025) * PN (L, 4))
   E = C /
   b = 1.0 - X * a
   A = FN (L, 2) * b
   TJIS (I, J) = DIST * A
18 CONTINUE
19 CONTINUE
C
IN THIS SECTION MNB RULE IS APPLIED TO OBTAIN THE
10 SCHEDULE.
DO 24 J = 1, M
   CON = 39, 39
DO 24 I = 1, N
   IF (TIS (I, J) - CON) 26, 26, 25
26 CON = TIS (I, J)
25 CONTINUE
DO 43 K = 1, N
   TJIS (K, J) = TOIS (K, J) - CON
43 CONTINUE
24 CONTINUE
114 CON = 39, 39
114 DO 19 J = 1, M
   K = IAT (I, J, 1)
   IF (TIS (<K, J) - CON) 21, 19, 19
21 CON = TIS (<K, J)
   KK = IAT (J, 1)
19 CONTINUE
114 DO 131 I = 1, IC
   IF (IIP (I, 1, 1) = 0, KK) GO TO 5
151 CONTINUE
10 J = J + 1
151 IIP (I, J, 1) = KK
   ITP (<KK, 7) = 1
   CIT = 1 + PI (KKK, 4)
   GT (I, 1) = CIT
DO 111 I = 1, M
   IF (K < EQ. IAT (I, 1)) GO TO 112
GO TO 111
112 DO 113 J = 1, M
   TJIS (J, I) = 999, 99
113 CONTINUE
CONTINUE
GO TO 127 I=1,N
GO TO 127 J=1,M
IF(T(IS(I,J)),G.E.99.99) GO TO 122
GO TO 123
122 CONTINUE
121 CONTINUE
120 CONTINUE
GO TO 5
123 IF(CIF(GE.43)) GO TO 5
GO TO 114
114 IN THIS SECTION RESULTS ARE OUTPUT.
5 WRITE(2,116)
116 FORMAT(FCC # ORDER NO.1 RUSH STATE : COMPLETION DATE*)
115 CONTINUE
DO 115 I=1,N
WRITE(2,7) IPT(1,1),IPT(I,3),CT(I,1)
115 CONTINUE
7 FORMAT(5X,I3,4X,3X,I3,5X,6X,2)
96 WRITE(2,6) IP
6 FORMAT(* START DATE*I3)
IF(IP=300) 181,181,99
99 CONTINUE
END