A comparative study was done on data collected by Weyerhaeuser Company on 1,644 second-growth Douglas-fir, Pseudotsuga menziesii, trees. The variable used in the study was the percent difference, positive or negative, between the volumes given by each of three different log rules and the standard tree volumes determined by Weyerhaeuser Company.

The data were analyzed in a two-way analysis of variance and factors, log rules and DBH classes were highly significant (99% level).

Comparative tests (LSD for unequal sample size) were done on pairs of log rules means as well as on pairs of DBH classes for each log rule. All the differences between means are presented and significances are summarized and discussed.

A regression analysis was performed between percent difference volume as the dependent variable and DBH as the independent for each log rule. Results showed no significant differences and the coefficients of determination were very low.
A COMPARATIVE STUDY OF THREE LOG RULES COMMONLY USED IN THE PACIFIC NORTHWEST, U.S.A.

by

Julio De Castro Paixao

A THESIS submitted to Oregon State University

in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Completed April 1980

Commencement June 1981
APPROVED:

Professor of Forest Management
in charge of major

Head of Department of Forest Management

Dean of Graduate School

Date thesis is presented April 24, 1980

Typed by CAMPUS PRINTING & COPY CENTER (Mary Syhlman)

for Julio De Castro Paixao
ACKNOWLEDGEMENTS

I wish to express my gratitude to my advisor, Dr. John F. Bell, for all his guidance and patience throughout my stay at Oregon State University.

I would also like to thank the rest of my committee: Dr. David P. Paine, Dr. David Faulkenberry, Dr. James W. Funck, and especially Dr. Paula Kanarek for her suggestions and criticism in the statistical project.

The financial assistance of the Food and Agriculture Organization of United Nations (FAO), Empresa Brasileira de Pesquisa Agropecuaria (EMBRAPA) and Instituto Brasileiro de Desenvolvimento Florestal are acknowledged.

For the very helpful correction of this paper writing, my special appreciation to Dick Monsen.

Finally, as before, it is with pleasure and much gratitude for all her support, I would like to dedicate this work to my wife, Maria Lidia.
<table>
<thead>
<tr>
<th>TABLE OF CONTENTS</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>LITERATURE REVIEW</td>
<td>4</td>
</tr>
<tr>
<td>Cubic-foot log rules - A review</td>
<td>10</td>
</tr>
<tr>
<td>Cubic volume tariff system</td>
<td>16</td>
</tr>
<tr>
<td>Constructing a tariff table - An example</td>
<td>28</td>
</tr>
<tr>
<td>STUDY MATERIAL</td>
<td>38</td>
</tr>
<tr>
<td>METHODS</td>
<td>46</td>
</tr>
<tr>
<td>RESULTS</td>
<td>51</td>
</tr>
<tr>
<td>Analysis of Variance</td>
<td>51</td>
</tr>
<tr>
<td>Test of Homogeneity of Variance: Bartlett's test</td>
<td>51</td>
</tr>
<tr>
<td>Test for comparing means: LSD-Test</td>
<td>54</td>
</tr>
<tr>
<td>Correlation between Average Percent Difference Volume and DBH class midpoint</td>
<td>60</td>
</tr>
<tr>
<td>Correlation between Percent Difference Volume and DBH</td>
<td>65</td>
</tr>
<tr>
<td>DISCUSSION AND CONCLUSIONS</td>
<td>66</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>70</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>18</td>
</tr>
<tr>
<td>6</td>
<td>19</td>
</tr>
<tr>
<td>7</td>
<td>19</td>
</tr>
<tr>
<td>8</td>
<td>24</td>
</tr>
<tr>
<td>9</td>
<td>24</td>
</tr>
<tr>
<td>10</td>
<td>26</td>
</tr>
<tr>
<td>11</td>
<td>31</td>
</tr>
<tr>
<td>12</td>
<td>34</td>
</tr>
<tr>
<td>13</td>
<td>37</td>
</tr>
<tr>
<td>14</td>
<td>39</td>
</tr>
<tr>
<td>15</td>
<td>42</td>
</tr>
<tr>
<td>16</td>
<td>43</td>
</tr>
<tr>
<td>17</td>
<td>44</td>
</tr>
</tbody>
</table>
LIST OF FIGURES (Continued)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>Regression of total height on DBH Equation.</td>
<td>45</td>
</tr>
<tr>
<td>19</td>
<td>Regression of average PDV on DBH class midpoint.</td>
<td>62</td>
</tr>
<tr>
<td>20</td>
<td>Regression of average PDV of DBH class on DBH class midpoint, for Log Rule B.</td>
<td>63</td>
</tr>
<tr>
<td>21</td>
<td>Regression of Mean Percent Volume (PDV) on DBH class midpoint.</td>
<td>64</td>
</tr>
<tr>
<td>Table</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>-------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>1</td>
<td>Quantities needed to construct a tariff table.</td>
<td>29</td>
</tr>
<tr>
<td>2</td>
<td>Measures of sample tree #1611.</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>Measures of sample tree #43.</td>
<td>32</td>
</tr>
<tr>
<td>4</td>
<td>Analysis of variance table for two factors: log rules and DBH classes.</td>
<td>51</td>
</tr>
<tr>
<td>5</td>
<td>Quantities needed for Bartlett's test on the homogeneity of the variance of the three log rules.</td>
<td>51</td>
</tr>
<tr>
<td>6</td>
<td>Quantities needed for Bartlett's test in log rule A.</td>
<td>52</td>
</tr>
<tr>
<td>7</td>
<td>Differences between two log rules.</td>
<td>54</td>
</tr>
<tr>
<td>8</td>
<td>Table of important statistics for log rules.</td>
<td>56</td>
</tr>
<tr>
<td>9</td>
<td>Difference between two DBH class means for log rule A.</td>
<td>57</td>
</tr>
<tr>
<td>10</td>
<td>Differences between two DBH class means for log rule B.</td>
<td>57</td>
</tr>
<tr>
<td>11</td>
<td>Table of important statistics for DBH classes.</td>
<td>59</td>
</tr>
<tr>
<td>12</td>
<td>Differences between two DBH class means for log rule C.</td>
<td>59</td>
</tr>
<tr>
<td>13</td>
<td>Regression analysis for log rule A.</td>
<td>61</td>
</tr>
<tr>
<td>14</td>
<td>Regression analysis for log rule B.</td>
<td>61</td>
</tr>
<tr>
<td>15</td>
<td>Regression analysis for log rule C.</td>
<td>61</td>
</tr>
</tbody>
</table>
COMPARATIVE STUDY OF THREE LOG RULES COMMONLY USED IN THE PACIFIC NORTHWEST, U.S.A.

INTRODUCTION

The basic reference quantity in timber measurement may be expressed in such terms as dry weight, length, number of pieces, or green weight. However, with good reason, the tradition has been to use solid volume to express timber measurement. Often when other parameters, such as stacked volume or weight are used these are changed to solid volume estimates for purposes of management. The use of solid volume is good for other reasons, too. Many timber products are sold by solid volume. Also it is easy to measure solid volume for both felled and standing timber. It is impossible to use weight measurements of any kind in standing timber. So, solid volume is generally "the expression of quantity".

The nature of forest products has led to the use of volume estimates, since very precise measurement in forestry is seldom practical. There are several ways of estimating timber volume even in the smallest logs. Consequently the resulting estimates are inevitably different depending on the method used. This creates a necessity of measurement conventions and a description of the material used if full understanding of timber volume estimates is to be obtained.

When making an estimate of quantity the degree of precision desired must be established. Different degrees of precision may be obtained for any forest product. Costs and benefits that
correspond to these different levels of precision must be considered before the measurements.

First of all, greater precision almost always costs more per unit volume because it requires more measurements and this involves more work. On the other hand, almost always, greater precision produces greater benefits. G. J. Hamilton, 1975, gives very clear considerations about the relationship of price to precision and of cost of measurement to precision.

Log measurement has been one of the most important and controversial topics in the field of forest management and wood technology. In the past, many different log rules have appeared in the United States with each generally being used in a particular area as a consequence of variations in acceptable utilization standards, slab allowance, taper, shrinkage, sawkerf and method of construction. Buyer-seller relationships remain normal because each accepts volumes and values involved in log transactions although they are aware of the inaccuracy of the log rules. For research purposes serious difficulties can appear when comparisons of volumes by two or more different log rules are tried. Log rule characteristics are such that only with difficulty can two measurement systems be successfully compared.

It is known that Tarif Tables and Bruce's Cubic Volume Table for Immature Douglas-fir are very practical log rules, but it is easy to perceive how important it is to know the accuracy of each one in a specific situation (as species, class of DBH, and site)
to help in selecting the one which gives the best results. Weyer-
haeuser Douglas-fir Table, British Columbia Coast Immature Douglas-
fir and Bruce's Table for Immature Douglas-fir are specific for the
material of study (Young growth Douglas-fir from Western Washington).
The objective of this study is to compare the volumes estimated by
those three volume tables with the actual volume of second growth
Douglas-fir calculated by standard measurement done by James E. King
of Weyerhaeuser Company.

Knowing that the results of a such investigation are precise
only for that specific population, we could have a reasonable idea
how these volumes compare with actual volumes.

The objective of this study is the investigation of a methodo-
logy which could serve as a base to begin a study to find out the
most accurate log rule for each species and/or site anywhere but
mainly in countries like Brazil where Forestry is really a very new
activity and practical log rules are seldom used in the many species
of Eucalyptus and Pinus for pulpwood and paper production and sawmill
purposes.

This will be accomplished by an analysis of variance on the
per cent difference between the total tree volume given by each of
three log rules and the standard volume calculated by Weyerhaeuser
Company. Average per cent volume difference by each of 8 diameter
classes will be compared for each log rule.
LITERATURE REVIEW

Since trees vary in geometric form from the stump to the top, no single mathematical formula can express the exact cubic volume of each log.

Measuring the volume contents of individual trees or sections by assimilating them to geometric solids is the most accurate means of determining volume. This is usually called "standard scaling" when assuming the stem as a sequence of super-imposed geometric solids. The interpretation and use of formulas varies with the mensurationist but the geometric solids more commonly used are the cone, the quadratic paraboloid, the neiloid, their frustrums and the cylinder.

It is conventional to calculate the volume of the stump as a cylinder and the volume of the first butt log as a frustrum of a neiloid. The subsequent logs may be assumed as frustrums of neiloids, cones or paraboloids, depending on the species and the mensurationist's interpretation.

A method that provides very accurate estimates is the one used by Weyerhaeuser Company which assumes the several logs of the stem are composed as follows:

2. Stump to 4.5 feet - neiloid frustrum.
3. Logs above 4.5 feet - paraboloid frustrums.
4. Top - cone.
Its accuracy was proved in the past for many species by the immersion method, the only perfectly accurate method for determining log volumes.

The formulas for these geometric forms give very different volumes. If the volume of a cylinder of given diameter and height is expressed as 100%, the volumes of the paraboloid, cone and neiloid of the same basal diameter and height are respectively 50, 33 1/3 and 25 per cent of the cylinder volume. Fig. 1.

Figure 1. The paraboloids which logs are more frequently assumed to resemble.
Actually the paraboloid mentioned here is the quadratic paraboloid. In general that is the only paraboloid used in tree measurement but the use of cubic and semi-cubic paraboloids should improve the accuracy in the determination of standard volume of a tree. In some European countries, like France and Portugal, researchers frequently use both cubic and semi-cubic paraboloid frustrums for certain species; the cubic for the lower logs (immediately after the neiloid frustrum) and the semi-cubic for the upper one (before the top). The intermediate logs are assumed as quadratic paraboloid (or simply paraboloid as it is commonly known) frustrums. Fig. 2.

Figure 2. Cubic, quadratic and semi-cubic paraboloids.
Since the cited geometric figures are solids of revolution, the volume formula for each one of them is obtained by integration rotating the graph of the general equation \( Y = K \sqrt[3]{X} \) around the \( X \) axis:

\[
V = \pi \int_{b}^{a} Y^2 \, dX
\]  

The paraboloid (more precisely: quadratic paraboloid) is generated when \( r = 1 \) (so \( Y = K X^{1/2} \)); the cone when \( r = 2 \) (\( Y = K X \)); the neiloid when \( r = 3 \) (\( Y = K X^{3/2} \)); and the cylinder when \( r = 0 \) (\( Y = K \)). The cubic and semi-cubic paraboloids are generated respectively from the formulas \( Y = K X^{1/3} \) and \( Y = K X^{2/3} \).

For any of the geometric curves, the constant \( K \) changes in accord with the ratio \( Y/X \). For example, the curve which generates the paraboloid of basal radius \( Y = 2 \) and height \( X = 7 \) has the constant \( K = .755929 \) while the curve which generates a paraboloid of same height \( X = 7 \) but with basal radius \( Y = 1 \) requires a constant \( K = .377964 \). This is a crucial point in comparative study of volumes of different paraboloids of the same height and basal radius. Fig. 3.

![Figure 3. Generating two quadratic paraboloids with different ratio radius/height (Y/X).](image)
In figure 3 if \( Y \) is the radius of the basal circle of either paraboloid the volume will be:

\[
V = \pi \int_a^b Y^2 \, dX \tag{2}
\]

Since \( Y \) is function of \( X \),

\[
V = \pi \int_a^b \left( K x^{r/2} \right)^2 \, dX \tag{3}
\]

Or:

\[
V = \pi K^2 \int_a^b X \, dX \tag{4}
\]

As \( r \) for the quadratic paraboloid is equal to 1, the volumes for both paraboloids in figure 3 can be calculated as follows:

\[
V = \pi (0.755929)^2 \left[ \frac{x^2}{2} \right]_0^7 = 43.982303
\]

for the thickest one, and

\[
V = \pi (0.377964)^2 \left[ \frac{x^2}{2} \right]_0^7 = 10.9955...
\]

for the thinnest one.

A paraboloid frustrum has a perfect formula for its volume as the product of an average cross-sectional area of its basal area \( (A_b) \) and top cross-sectional area \( (A_u) \) by its height. This is the Smalian's formula, often used in log scaling:

\[
V = \frac{H}{2} (A_b + A_u) \tag{5}
\]
where

\[ V = \text{volume of the log} \]
\[ H = \text{length of the log} \]
\[ A_b = \text{basal area at the base} \]
\[ A_u = \text{basal area at the top} \]

A different formula is often used for paraboloid frustrums. It is Huber's formula, which is considered more accurate because the volume of a log is more dependent on the middle diameter than on the end diameter, and other practical formulas utilize end diameters. Huber's formula is expressed as follows:

\[ V = A_m H \] (6)

where

\[ A_m = \text{cross-sectional area at midpoint} \]

The mathematical formula for a cone frustrum is:

\[ V = \frac{H}{3} \left( A_b + \sqrt{A_b A_u} + A_u \right) \] (7)

Smalian's formula overscales a truncated cone and Huber's formula also slightly overscales the frustrum.

Neiloid frustrums have a more complicated geometrical formula for volume:

\[ V = \frac{H}{4} \left( A_b + \frac{3}{\sqrt{A_b}} \frac{2}{A_u} + \frac{3}{\sqrt{A_b A_u}} + A_u \right) \] (8)
A very accurate practical formula for all frustums we are concerned with in the present study is Newton's formula, which is inconvenient because of the necessity of taking three diameter measurements: at both extremeties and at the middle of the log:

\[ V = \frac{H}{6} (A_b + 4 A_m + A_u) \]  

(9)

Most of the practical log rules determine board-foot volume. This work however will be limited to a comparison of the accuracy of three cubic-foot log rules when applied to young Douglas-fir, *Pseudotsuga menziesii*.

CUBIC-FOOT LOG RULES - A REVIEW

1. Newton's Formula

It fits almost any geometric figure and gives the cubic contents of frustums of paraboloids, cones and neiloids accurately. However, it requires the measurement of diameters inside the bark at the base, middle and top of the log. Obviously, it is inefficient for practical use. The formula is:

\[ V = \frac{H}{6} \frac{\pi}{4} \frac{1}{144} (D_b^2 + 4 D_m^2 + D_u^2) \]  

(10)

where

\[ V = \text{volume of log in cubic feet} \]
\[ D_b = \text{diameter inside bark in inches at large end} \]
\[ D_u = \text{diameter inside the bark in inches at small end} \]
\[ D_m = \text{diameter inside bark in inches at the middle point of the log} \]

\[ H = \text{height} \]

In Western Oregon and Western Washington the more common length of log to be measured is 32 feet and so the formula can appear in the following expression:

\[
V = 0.0291 \left( D_b^2 + 4D_m^2 + D_u^2 \right) \quad (11)
\]

2. Smalian's Formula

It's perfect for a frustum of paraboloid but has the inconvenience of considering only the extreme end diameters, excluding the middle one. It overscales logs which have a truncated cone form and especially logs with neiloid frustum form:

\[
V = \frac{H \pi}{2} \frac{1}{144} \left( D_b^2 + 2D_u^2 \right) \quad (12)
\]

For a 32-foot log:

\[
V = 0.087264 \left( D_b^2 + D_u^2 \right) \quad (13)
\]

3. Huber's Formula

This formula is considered the most accurate of the more practical log rules but as in Newton's the inconvenience is in taking diameter measurement at the middle of the log:

\[
V = \frac{\pi H}{4} \frac{1}{144} D_m^2 \quad (14)
\]
and for 32-foot log:

\[ V = 0.174528 \frac{D^2}{m} \]  

(15)

4. Rapraeger's Formula

It is a modification of Huber's rule which attempts to combine the supposed accuracy of Huber's with a practical method of scaling decked, rafted and loaded logs to determine the diameter of the middle of the log. Rapraeger proposed an arbitrary taper allowance of one inch for every 8 feet of length from the small end of the log, with the resulting formula being:

\[ V = 0.005454 H \left( D + \frac{H}{16} \right)^2 \]  

(16)

where

\[ D = \text{Diameter inside bark at small end of the log in inches} \]
\[ H = \text{Length of the log in feet} \]

5. Sorensen's Formula

This rule is based on the cone frustum formula. It is not considered very satisfactory unless the logs scaled closely approach a taper of 1 inch in 10 feet of length. Sorensen advocates measuring only the small end diameter and using 1 inch of taper for every 10 feet of length to get the diameter at the middle of the log.

It saves time since the only measurements are length and diameter at the small end. This rule underscales drastically neiloidal
and paraboloidal logs. The formula can be expressed as follows:

\[
V = 0.005454154 \left( D + \frac{L}{20} \right) L \quad (17)
\]

where

\[D = \text{Diameter inside bark at small end}\]
\[\text{in inches}\]
\[L = \text{Length of the log in feet}\]

The \(\frac{L}{20}\) term is, in effect, a conversion from small end to midpoint diameter based on the assumed taper of 1 inch in 10 feet.


Although very specific, this rule is listed here because it is a major part of this study.

David Bruce and Donald J. DeMars Volume Equations were first published in November 1974 by Pacific Northwest Forest and Range Experimental Station as a USDA Forest Service Research Note. They were constructed as a result of a request for a reasonable table for small Douglas-Fir.

The tables were based on a sample of 1,127 trees which ranged from 0.4 inch DBH and 6 feet in height to 32 inches DBH and 167 feet in height. The independent variables were DBH outside bark (o.b.) and inside bark (i.b.); the dependent variable was form factor based on total volume inside bark, including the stump calculated as a
cylinder. The form factors are used in the equation and so are calculated before.

Outside bark form factor equations for small sample trees (FOS) and for large trees (FOL) are presented below. Trees considered small here are those of total height equal or smaller than 18 feet and consequently large trees are those of total height greater than 18 feet.

\[
FOS = 0.406098(H-0.9)^2/(H-4.5)^2 - 0.0762998 \frac{D(H-0.9)^3}{(H-4.5)^3}
+ 0.00262715 \frac{D(H-0.9)^3}{(H-4.5)^3}
\]  
(18)
(based on 59 trees for young stands in Oregon, Washington and B.C.)

\[
FOL = 0.480961 + 42.46542/H^2 - 10.99643 \frac{D}{H^2} - 0.107809 \frac{D}{H}
- 0.00409083 D
\]  
(19)
(based on 1,068 trees from young stands in Oregon, Washington and B.C.)

Volumes are simply calculated through one of the following formulas:

\[
VS = 0.005454154 \frac{FOS}{(D^2H)}
\]  
(20)
when \(H \leq 18\) feet, and

\[
VL = 0.005454154 \frac{FOL}{(D^2H)}
\]  
(21)
when \(H \geq 18\) feet.
Equation (21) can not be applied for very small trees (trees with DBH smaller than 1.4 inches and height less than 13 feet) as volume decreases when height increases, holding the same diameter. This frequently happens in different rules because of the use of DBH that is located at 4.5 feet above the ground level.

The test criterium for significance of the regression, for both form factor and volume was the root mean square error: 12.2 and 8.0% for FOS and FOL, respectively; and 12.7 and 16.8% for VS and VL.

7. Tarif Tables

Tarif Tables are constructed for different species and sites. They are considered very simple and more accurate than conventional tables especially in young growth stands. The application of the tarif system is very easy both for field use (tables) and computers (formulas). It provides easy conversion between units of measure and its authors claim sufficient accuracy for volume and growth in research application.

"A tarif table is a local volume table that gives tree volume by diameters for trees of the same general height class. They are particularly suited, but not limited, to even-aged stands" (Hoyer, 1971).

The tarif table system is a group of "preconstructed" local volume tables applicable to the specific stand, each one having its tarif access number. The tarif number is the total cubic foot volume from the stump to a 4-inch top for a tree of 1.0 square foot of basal area.
The tarif number of a stand is found through "Access Tables", which provide a number for each sample tree, given its DBH and height. In finding the tarif number, DBH's have to be measured to the nearest 1/10th inch and total height to the nearest foot for each sample tree. The tarif numbers found are averaged and this average is the tarif number. In general only 20 representative trees are necessary to determine the average tarif number for a stand. The tarif number is the index-number for the table to be used.

Tarif Tables being a major subject in the present work, a detailed discussion is required.

Cubic Volume Tarif System

Here are the main considerations about cubic volume tarif systems:

CV4 is the volume of a tree above stump height to a top diameter of 4.0 inches. CV4 curves are the basis for the Tarif System since total cubic volume curves (TCV) had been shown to be an inadequate basis for some important reasons:

1. The total volume/basal area straight line is satisfactory for the larger trees but the actual trend in lower range values is curvilinear (Fig.4).

2. The intercept in the horizontal axis is not stationary, so a moving intercept has to be adopted if volume/basal area trend is employed for the total cubic volume curves. The authors primarily investigated the moving intercept by fitting the trend of a coefficient in relation to b coefficient for all individual CVTS/basal
area lines. Then a system of harmonized CVTS/basal area lines was constructed by using the smoothed values of $a$ for a given $b$ but due to movement of the intercept along the horizontal axis because of change in the slope of the line (trees of small DBH showing higher volume for smaller slope and lower volume for greater slope, as shown in figure 5), and considering that the slope increases with increasing age of the stand, "this system of lines would cause the smaller trees to have less volume with advancing age". (Turbull and Hoyer, 1965).

In CV4 curves no curvilinearity of trend was found even in the lower range of basal area because the 4-inch top limit excluded the small trees which are responsible for the curvature. Furthermore the volume/basal area line is equally satisfactory for all plots in the sample and the horizontal intercept is stationary.

Subsequent tests showed important points:

1. No trend of the basal area intercept value in relation to steepness of the line.

2. None of the intercept values differed significantly from 0.0873 sq. ft. of basal area which is equivalent to a 4-inch DBH.

3. Trees with a 4-inch DBH outside bark have stump diameter inside bark consistently within 3.9 to 4.0 inches and hence they have zero volume to 4-inch top above stump. Fig. 6.
Total Volume

Basal Area

Figure 4. Curvilinearity of the basal area/total volume trend for the smaller trees.

Total Volume

Basal Area

Figure 5. Harmonized volume trends with "moving intercept."

Let's call the regression coefficient \( b_4 \), Fig. 7.

\[
CV4 = a + (b_4) (B)
\]  \hspace{1cm} (22)

where \( B = \text{basal area} \).
But when $B = 0.0873$ square feet, $CV4 = 0.0$

Figure 6. Relation between 4-inch diameter outside bark at breast height and the diameter inside bark at stump height.

Figure 7. When basal area is 0.0873 square foot, $CV4$ is always 0.0.
So:

\[ a = b y (0.0873) \]  \hspace{1cm} (23)

Then:

\[ CV4 = b y (0.0873) + b_4 \cdot B \]  \hspace{1cm} (24)

Or:

\[ CV4 = b y (basal \ area - 0.0873) \]  \hspace{1cm} (25)

which represents the fixed intercept CV4/basal area line.

This shows that equations for different stands only differ by the slope of the regression line and each different equation is designated by the volume for a tree of 1 square foot of basal area. This is the tariff number (T), the CV4 for a tree of 1 square foot of basal area, that is, the average CV4 for the tree of 1 square foot of basal area.

If

\[ CV4 = b_4 \cdot (1.0 - 0.0873) \]  \hspace{1cm} (26)

we have

\[ T = 0.9127 \cdot b_4 \]  \hspace{1cm} (27)

and

\[ b_4 = \frac{7}{0.9127} \]  \hspace{1cm} (28)

If tariff number and DBH are known, the CV4 can be computed as:

\[ CV4 = \frac{T}{0.9127} \cdot (B - 0.0873) \]  \hspace{1cm} (29)
If on the other hand, CV4 and DBH are known, the tarif number can be computed as:

\[ T = \frac{CV4 \cdot (0.9127)}{B - (0.0873)} \]  

(30)

The ratio \( \frac{0.9127}{B - 0.0873} \) is called Tarif Access Constant for CV4, represented by TA4.

So, the equation (30) can be written as:

\[ T = (CV4) \cdot (TA4) \]  

(31)

In using tarif system in terms of CV4 one should follow these four steps as recommended by Turbull and Hoyer:

1. Measure sample trees in a stand and estimate CV4 for each tree.
2. Compute T for each sample tree by using equation (30).
3. Average the sample tree T values to obtain T mean, \( \bar{T} \).
4. Use this sample mean, \( \bar{T} \), in equation (29) to compute the CV4 estimate for each DBH class midpoint. This will yield the equivalent of a Local Volume Table.

In choosing equation (30) in step 2, use tarif access constant tables of CV4.

Notice that

\[ CV4 = \frac{T}{0.9127} \cdot (B-0.0873) \]  

(32)

and

\[ \frac{B - 0.0873}{0.9127} = TV4 \]  

(33)
which is called Tarif Volume Constant for CV4 that is the inverse of TA4:

\[ TV4 = \frac{1}{TA4} \] (34)

To adopt this system of fixed-intercept CV4/basal area as the basis for the comprehensive tarif system the authors constructed weighted CV4/basal area lines for each sample plot by a special method conditioning the regression and compared them with the original CV4/basal area local volume tables, only weighted, following 2 criteria:

1. The volume for trees with sample mean basal area estimated by both equations should not differ by more than 2 standard errors of the mean.

2. The tarif volume line should lie within the confidence interval of the local volume table regression.

If no significant difference or consistent bias are shown in either test, the difference between the two lines is considered as sampling error.

The estimated Tarif has a statistical error that is Student's t times the standard error of the mean tarif and so its confident limits are easily determined.

As trees vary in form from species to species, from age to age and from site to site for the same species, tarif tables have been constructed for a combination of different species, ages and sites. As these tables are limited to those factors combinations, another
variable, Tree Total Height, may be used to make a more convenient
determination of individual sample tree volume, through a double
entry table in place of using equation (30). This avoids the use
of inadequate standard volume tables to get the volume in function
of DBH only and utilizes sample trees total height, a very strong
covariable when species, age and site are controlled.

Tarif tables offer not only DV4 but other merchantable volumes
like CVTS (cubic foot volume including top and stump), CVT (cubic ft. volume including only top), CV6 (cubic foot volume to 6-inch
top), CV8 (cubic foot volume to 8-inch top), IV6 (International 1/4-
inch volume to 6-inch top), IV8 (International 1/4 inch to 8-inch
top), SV6 (Scribner to 6-inch top) and SV8 (Scribner to 8-inch top).

Since the relationship CV4/basal area is the basis of the system, the conversions to other volume units were made through the
study of the trend of the ratio "volume in study"/CV4 over DBH. In
this paper will be shown the conversion of CV4 to CVTS, the unit used
in the future research.

When a trend of CVTS/CV4 (=RTS4) over DBH of a sample is traced,
it is easy to see that this trend is asymptotic (Fig. 8) "but since
there is actual volume to 4-inch trees of less than 0.0873 sq. ft.
of basal area, it is impossible to derive an actual ratio of CVTS/CV4", (Turbull, 1965). The problem can be overcome by measuring all volumes
from a "zero" level which is located 2 times the volume intercept
distance below the origin (Fig. 9).
Figure 8. Asymptotic curve of CVTS/CV4 ratio on DBH.

Figure 9. The artificial base for deriving an actual CVTS/CV4 ratio.
Let's call CVTS' and CV4' respectively CVTS and CV4 measured from the artificial base located two times the Volume-intercept below the origin. And let's all RTS4' the ratio CVTS'/CV4'.

\[
\text{CVTS}' = \text{CVTS} + 2a \\
\text{CV4}' = \text{CV4} + 2a
\]

So, if \( \frac{\text{CVTS}'}{\text{CV4}'} = \text{RTS4}' \),

\[
\text{CVTS} = \text{CVTS}' - 2a
\]

and

\[
\text{CVTS} = \text{RTS4}' \left( \text{CV4}' - 2a \right)
\]

\[
= \text{RTS4}' \left( \text{CV4} + 2a - 2a \right)
\]

\[
= \text{RTS4}' \left( b_4B - a + 2a \right) - 2a
\]

\[
= \text{RTS4}' \left( b_4B + a \right) - 2a
\]

Note that if \( B = 0 \), DBH = 0 and CVTS = 0 and note that

\[
a = -b_4 \left( 0.0873 \right)
\]

So, for CVTS = 0:

\[
0 = \text{RTS4}' \left[ b_4 \left( 0.0 \right) - b_4 \left( 0.0873 \right) \right] + 2 \left[ b_4 \left( 0.0873 \right) \right]
\]

\[
0 = \text{RTS4}' \left[ -b_4 \left( 0.0873 \right) \right] + 2 b_4 \left( 0.0873 \right)
\]

\[
\text{RTS4}' \left( b_4 \left( 0.0873 \right) \right) = 2 b_4 \left( 0.0873 \right)
\]

\[
\text{RTS4}' = 2
\]

when DBH = 0.

Now, the function RTS4' was found to be a sample asymptotic regression with formula:
\[ \text{RTS}_{i} = A + B e^{-kx} \]  

and when \( DBH = 0 \):

\[ \text{RTS}_{i} = A + B e^{-k(0)} \]

\[ \text{RTS}_{i} = A + B \]  

So:

\[ 2 = A + B \]  

or:

\[ B = 2 - A \]  

where \( A \) is the estimate asymptote.

The trend of \( \text{CVTS}'/\text{CV4}' \) ratio over \( DBH \) is shown in figure (10).

![Figure 10. Trend of CVTS'/CV4' ratio.](image)
As we have the value for CVTS as shown in formula (41), and we know that $b_4$, the slope of the volume line regression for CV4, is $rac{T}{0.9127}$, and more, that $a = b_4 (0.0873)$, we find the general formula for CVTS:

$$CVTS = T \left[ \frac{RTS4' (a) + RTS4' (0.0873) - 2 (0.0873)}{0.9127} \right]$$  \hspace{1cm} (51)

or:

$$CVTS = T (TVTS)$$  \hspace{1cm} (52)

where TVTS is total volume top and stump.

If, for example, a tree is a 15-inch DBH,

$$T = \frac{52.3 (0.9127)}{1.2272 - 0.0873} = 41.9$$  \hspace{1cm} (53)

$$TVTS = \frac{RTS4' (1.2272 + 0.0873) - 0.1746}{0.9127}$$  \hspace{1cm} (54)

In our case, DBH = 15:

$$RTS4' = 1.0385704$$  \hspace{1cm} (55)

$$TVTS = \frac{1.0385704 (1.2272 - 0.0873) - 0.1746}{0.9127}$$

$$= 1.3045$$

$$CVTS = 41.9 (1.3045) = 54.7$$  \hspace{1cm} (57)
Constructing a Tarif Table - An Example

As a demonstration of tarif table construction, a sample of 20 trees was taken, randomized within each diameter class. The randomization was done using the HP-97 "Random Number Generator" program. On the next page is presented a list of the sample trees with their DBH, basal area, total volume and CV4. The regression volume by three different equations - simple regression, weighted regression and tarif - is also presented, as well as values 1/B, 1/B^2, V/B and V/B^2, necessary to compute the coefficients a and b of the weighted regression.

CV4 was calculated by subtracting the volume of the stump and the volume above the 4 inches top diameter from the total volume. When the data did not show a top volume exactly above 4 inches diameter, the proportion method was used to find this volume considering either a frustrum of paraboloid or a cone according to the method Weyerhaeuser Company adopted to calculate the standard volume of the trees. Two examples were chosen to demonstrate this method.

1. The frustrum of paraboloid: Table (2) shows the values for sample tree #1611.

   The intermediary sections are assumed to be frustrums of paraboloids which volumes are determined by Smalian's formula. Thus the volume of section #5 is calculated as follows:

   \[ V = \frac{(4.4)^2 + (2.8)^2 (0.7854)}{(144)(12)} (10) = 0.74 \text{ cu. ft.} \]  

(58)
Table 1. Quantities needed to construct a tariff table.
It can easily be seen that DIB 4.0 inches falls in section #5 since 4.4 inches and 2.8 inches are the top limits of sections #4 and #5, respectively.

To find the length of the sub-section with extremities of 4.0 and 2.8 inches it is easier to use the proportion method. Since the method works only if the section is a frustrum of cone, there will be some error here as the section is assumed to be a frustrum of paraboloid. However, the error is insignificant.

So, if the diameter decreases from 4.4 to 2.8 (1.6 inches) at length of 10 feet (from 31.5' to 21.5') then at length of \( x \) feet the diameter will decrease from 4.0 to 2.8 (1.2 inches). Fig. (8-a).

\[
x = \frac{(10)(1.2)}{(1.6)} = 7.5 \text{ ft.}
\]

And the volume of this sub-section is:

\[
V = \frac{(4.0)^2 + (2.8)^2 (0.7854)}{(144)(2)} (7.5) = 0.488 \text{ cu. ft.} \quad (60)
\]
The stump volume is found by dividing the volume of the first section by twice its own height which gives the volume of a cylinder of 1/2 foot high and diameter equal to the top diameter of the first section. In this case there is a major mathematical negative error since the actual diameter of the stump is bigger than the top diameter of the first section, but, from the practical point of view, this error can be neglected given the volume of the stump is already very small in comparison to the total volume of the tree. In the computation of the stump volume of the biggest sample tree for example, the theoretical error is no more than 0.4 cubic foot or 0.1% of the CV4, while the error for the smallest sample tree, in this case, is no more than 0.02 cubic foot or 0.4% of the CV4.

![Diagram](image.png)

Figure II. Necessary measures to calculate the volume above 4.0 inches diameter in a paraboloid frurtrum (a) and in a cone (b).
In this example the volume of the stump is:

\[
\text{Stump vol.} = \frac{0.5}{(2)(1.5)} = 0.167 \text{ cu. ft.} \quad (61)
\]

DV4 of the sample tree #1611 is calculated by subtracting from CVTS (4.8 cu. ft.) the stump volume (0.167 cu. ft.), the calculated volume of the upper sub-section of section #5 (0.488 cu. ft.) and the volume of the top section #6 (0.2 cu. ft.):

\[
CV4 = 4.8 - (0.167 + 0.488 + 0.2) = 3.945 = 3.9 \text{ cu. ft.} \quad (62)
\]

2. The cone section: Sample tree #43 has the following values:

<table>
<thead>
<tr>
<th>DBH</th>
<th>HEIGHT</th>
<th>CVTS</th>
<th>SECT.</th>
<th>CUM. HT.</th>
<th>DIB</th>
<th>CUB. VOL.</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.0</td>
<td>95.5</td>
<td>37.7</td>
<td>1</td>
<td>1.0</td>
<td>13.3</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>4.5</td>
<td>12.0</td>
<td>12.0</td>
<td>1.5</td>
<td>11.3</td>
<td>9.6</td>
</tr>
<tr>
<td>3</td>
<td>17.5</td>
<td>27.5</td>
<td>45.5</td>
<td>11.3</td>
<td>11.3</td>
<td>9.6</td>
</tr>
<tr>
<td>4</td>
<td>34.0</td>
<td>76.5</td>
<td>76.5</td>
<td>17.5</td>
<td>11.3</td>
<td>9.6</td>
</tr>
<tr>
<td>5</td>
<td>49.5</td>
<td>126.0</td>
<td>126.0</td>
<td>34.0</td>
<td>8.0</td>
<td>6.8</td>
</tr>
<tr>
<td>6</td>
<td>50.5</td>
<td>176.5</td>
<td>176.5</td>
<td>49.5</td>
<td>8.0</td>
<td>6.8</td>
</tr>
<tr>
<td>7</td>
<td>59.5</td>
<td>236.0</td>
<td>236.0</td>
<td>50.5</td>
<td>7.8</td>
<td>0.3</td>
</tr>
<tr>
<td>8</td>
<td>95.5</td>
<td>321.5</td>
<td>321.5</td>
<td>59.5</td>
<td>6.0</td>
<td>2.4</td>
</tr>
</tbody>
</table>

Table 3: Measures of sample tree #43.

In this case the computation of the length of the 4-inch diameter base cone has theoretically no error (Fig. 8-b). The length of this top conical section is computed as follows:
Length \( x = \frac{(95.5 - 59.5)(4.0)}{(6.0)} = 24.0 \) inches \( (63) \)

Its volume is computed as follows:

\[
V = \frac{(4.0)^2(0.7854)}{(144)(3)}(24.0) = 0.698 \text{ cu. ft.} \quad (64)
\]

The volume of the stump is given by:

\[
\text{Stump vol.} = \frac{(1.0)}{(2)(1.0)} = 0.5 \text{ cu. ft.} \quad (65)
\]

And the \( CV_4 \) of the sample tree is:

\[
CV_4 = 37.5 - (0.698 + 0.5) = 34.5 \text{ cu. ft.} \quad (66)
\]

The graph on the next page (Fig. 12) shows the three regression links. Simple regression line has the following equation, with \( r^2 = 0.9638 \):

\[
CV_4_{SR} = -16.3609 + 56.1069 \text{ } B \quad (67)
\]

where \( B \) is basal area.

The values of the coefficients \( a \) and \( b \) for the weighted regression were computed through equations (68) and (69) below:

\[
b = \frac{\Sigma(V/B)}{N} \quad (68)
\]

\[
b = \frac{\Sigma(V/B^2) - b_2 \Sigma(1/B)}{\Sigma(1/B^2)} \quad (69)
\]
Figure 12: Comparing SR, WR and T regression lines.
The weighted regression equation is:

\[
CV_{WR}^4 = -3.5583 + 40.5389 B
\]  

(70)

The tariff number, \( T \), for each sample tree was computed through equation 71 below and the sample mean \( T = 41.8701 \) was used in the tariff volume equation 72.

\[
T = \frac{CV_4 (0.9127)}{(B - 0.0873)} \]  

(71)

\[
CV_4 T = \frac{41.8701}{0.9127} (b - 0.0873) \]  

(72)

As it was seen before, there are two conditions for accepting the results of a tariff table, although neither of those constitutes a valid significant test but "they do demonstrate the order of magnitude of difference between the two regressions in relation to estimate sampling error" (Turbull, 1965).

The first is that "the volume estimated for trees with sample mean basal area should not differ by more than 2 (Sym) standard errors between tariff and local volume table regressions" (Turbull, 1965).

These values in the present example are:

<table>
<thead>
<tr>
<th>Tarif volume estimated for trees with sample mean basal area</th>
<th>104.5 cu. ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Similar volume given by local volume regression</td>
<td>92.3 cu. ft.</td>
</tr>
<tr>
<td>Difference</td>
<td>12.2 cu. ft.</td>
</tr>
<tr>
<td>2 (Sym)</td>
<td>42.6 cu. ft.</td>
</tr>
</tbody>
</table>

Since 2 (Sym) is bigger than the difference between the two mean volumes, the condition is satisfied.
The second is that "the tariff volume line should lie within the confidence bands of the local volume table regression" (Turbull, 1965).

This condition is also perfectly satisfied as shown in the graph of figure 12.

Figure 13 compares CVTS and CV4 lines.
Figure 13. Comparing CVTS line with CV4 line.
STUDY MATERIAL

The data were provided by the Department of Natural Resources, State of Washington and consist of a computer output of the volume of 1,765 trees used in the construction of a standard volume table for a young growth of Douglas-fir, *Pseudotsuga menziesii* in Western Washington.

Total tree volume is the summation of the volumes of sections of the entire stem from the base to the top. The volume of the first section, which is called "stump", is calculated on the basis of it being considered a cylinder equal to its top cross-sectional area and length ranging between 0.5 and 4.5 feet. The next section from the stump to 4.5 feet is assumed to be a neiloid frustrum and its volume is calculated by formula (8). Sections above 4.5 feet to the top section were assumed to be paraboloid frustrums, their volumes are calculated by Smalian's formula (5). Top section was assumed to be a cone, its volume calculated by the formula for a cone.

A problem that could be considered of major importance is that the intermediary segments are not consistently divided into sections of equal length. As a result the total volume varies depending on the length of the sections since they are seldom perfect paraboloid frustrums. A good example can be seen in sample tree #240 which has the intermediary segment divided in sections of 6 feet except sections #13 and #14 of 4.5 and 1.5 feet, respectively. (Fig. 14). The summation of their volumes is 1.24 cubic feet while the volume of a whole 6-foot section (4.5 + 1.5) is 1.22 cubic feet. This
0.02-foot difference was not shown in the computer output. Many other trials were made in different trees with different sections lengths and the difference always had been shown to be minimum and so, insignificant, except for the largest trees that are out of the sample used in the present work.

Figure 14. In sample tree #240 the difference between the calculated volume of a 6-foot log and the summation of the calculated volumes of a 4.5-foot log and a 1.5-foot log is insignificant.

From a total of 1,765 trees, 1,644 were used for the study, since these had DBH between 4.0 and 35.0 inches, which are the limits for the tariff table used.

Trees were grouped in 8 diameter classes with width of 3.9 inches except 7 with width 3.8 inches:
The distribution of the frequency in the DBH classes is shown in figure (15). The regression equation for frequency and DBH mean for the 8 DBH classes is:

\[ F = 610.7028 - 29.6370 \text{ (DBH)} + 0.3734 \text{ (DBH)}^2 \]  

(73)

Second degree polynomial was the one which fits the best with \( R^2 = 0.9852 \) compared to \( r^2 = 0.9522 \) for linear regression. The regression curve is shown in figure (15).

The regression that fits the best Volume X DBH is:

\[ TV = -10.5723 + 0.1026 \text{ (DBH)} + 0.3162 \text{ (DBH)}^2 \]  

(74)

with

\[ R^2 = 0.9995 \]  

(75)

and where \( TV = \text{total volume of the tree. Figure (16).} \)

As it was expected, the best regression for Volume X Basal Area (B) was the simple linear regression. Figure (17). The equation is

\[ TV = -9.7436 + 58.5016 \text{ (B)} \]  

(76)
with

$$r^2 = 0.9984$$  \hspace{1cm} (77)

In search for the hypsometric relationships, a second degree convex polynomial (Figure 18) was found to show total height (TH) in function of DBH:

$$TH = 7.4317 + 9.0597 \text{ (DBH)} - 0.1119 \text{ (DBH)}^2$$  \hspace{1cm} (78)

with

$$R^2 = 0.9995$$

After all these investigations and considerations, Weyerhaeuser tree total volumes were considered to be a very good standard basis for this study.
Figure 15. Regression of frequency on DBH.

\[ F = 610.7028 - 29.6370 \text{ (DBH)} + 0.3734 \text{ (DBH)}^2 \]

\( Q \) : Average number of trees by average diameter for each DBH class
Figure 16. Regression of Total Volume on DBH.
TV = 10.5723 + 0.1026 (DBH) + 0.3165 (DBH)^2
θ: Average volume by average diameter for each DBH class.
R^2 = 0.9984
Figure 17. Regression of Total Volume over Basal Area.
Regression: $TV = -9.7436 + 58.5016 \cdot B$

$\Theta$: Average volume by average basal area for each diameter class.

$r^2 = 0.9984$
Figure 18. Regression of total height on DBH Equation:
\[ TH = 7.4317 + 9.0597 \text{ (DBH)} - 0.1119 \text{ (DBH)}^2 \]

\( 0 = \text{Average total height by average diameter for each DBH class} \)

\( R^2 = 0.9995 \)
METHODS

The variable used to compare the three log rules was percent difference between the volume given by each of the three log rules and the standard volume calculated by Weyerhaeuser Company:

\[
\text{Percent Difference Volume} = \frac{X - S}{S} \times 100 \quad (100)
\]

where \( S \) is the standard volume and \( X \) is either A, the volume given by the Weyerhaeuser Douglas-Fir Cubic Volume Equation, B, the volume given by the British Columbia Coast Immature Douglas-Fir Volume Equation or C, the volume given by David Bruce's Equations for Second-Growth Douglas-Fir.

Log rule A equation is the original tariff equation modified by Turnbull and King:

\[
\log \text{CVTS} = 0.321809 + 0.04948 \log \text{TH} \log \text{DBH} - 0.15664 (\log \text{DBH})^2 + 2.02132 \log \text{DBH} + 1.63408 \log \text{TH} - 0.16185 (\log \text{TH})^2 \quad (81)
\]

which has been changed for the computer to:

\[
\text{CVTS} = 10^{(3.21809 - 0.04948 \log \text{DBH} + 0.15664 (\log \text{DBH})^2 + 2.02132 \log \text{DBH} + 1.63408 \log \text{TH} - 0.16185 (\log \text{TH})^2)} \quad (82)
\]
Log rule B equation is presented in the form:
\[
\log \text{CVTS} = -2.658025 + 1.739925 \log \text{DBH} + 1.133187 \log \text{TH}
\]
which has been changed for the computer to
\[
\text{CVTS} = 10^{-2.658025} \cdot \text{DBH}^{1.739925} \cdot \text{TH}^{1.133187}
\]

Log rule C equations are VS and VL:
\[
\text{VS} = 0.005454154 \cdot \text{FOS} \cdot \text{DBH}^2 \cdot \text{TH}
\]
where VS is the volume of trees with total height (TH) equal or smaller than 18 feet and FOS is a form factor for trees of TH = 18 feet:
\[
\text{VL} = 0.005454154 \cdot \text{FOL} \cdot \text{DBH}^2 \cdot \text{TH}
\]
where VL is the volume of trees with TH = 18 feet and FOL is the form factor for those trees.

As there is no definite breaking point between these two categories, VS and VL give virtually the same volume for an 18-foot tree.
FOS and FOL formulas are given below:

\[
FOS = 0.406098 (TH - 0.9)^2 / (TH - 4.5)^2 - 0.0762998 (DBH) (TH - 0.9)^3 / (TH - 4.5)^3 + 0.00262615 (DBH) (TH) (TH - 0.9)^3 / (TH - 4.5)^3
\]

\[
FOL = 0.480961 + 42.46542 / (TH)^2 - 10.99643 (DBH) / (TH)^2 - 0.107809 (DBH) / (TH) - 0.00409083 (DBH)
\]

Since 1644 trees were used in the present work and 3 log rules were compared, a total of 4832 data were processed for the comparative study. The data were split into 8 DBH classes as shown below:

<table>
<thead>
<tr>
<th>Class</th>
<th>Interval in inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.0 - 7.9</td>
</tr>
<tr>
<td>2</td>
<td>8.0 - 11.8</td>
</tr>
<tr>
<td>3</td>
<td>11.9 - 15.6</td>
</tr>
<tr>
<td>4</td>
<td>15.7 - 19.5</td>
</tr>
<tr>
<td>5</td>
<td>19.6 - 23.4</td>
</tr>
<tr>
<td>6</td>
<td>23.5 - 27.3</td>
</tr>
<tr>
<td>7</td>
<td>27.4 - 31.1</td>
</tr>
<tr>
<td>8</td>
<td>31.2 - 35.0</td>
</tr>
</tbody>
</table>

A two-way analysis of variance was performed on the data to see if there were any significant differences among the percent difference volumes given by the 3 log rules as well as to see if the percent difference volumes were significantly different among the 8 DBH classes.
The purpose of comparing percent difference volumes among the DBH classes was to investigate the possibility of a log rule being a better estimator than another for a certain size of tree.

In case there is a difference among log rules and/or DBH classes, a test was planned for inspection of all differences between pairs of means.

Since "with unequal sample size the F- and t-test are more affected by non-normality and heterogeneity of the variance than with equal sample size" (Snedecor, 1973), before using any test for comparing pairs of means, a $\chi^2$-test - the Bartlett's Test of Homogeneity of Variance - was performed to see if there was homogeneity among the variances of the 8 DBH classes. Although the 3 log rules samples have the same size, the Bartlett's test was applied among their variance because, based on a preliminary test, they showed not being homogenous.

The Bartlett's test has the following equation:

$$\chi^2_{(K-1)} = 2.3026 \left( \log S^2 \right) \left( \Sigma (n_i - 1) \right) - \Sigma (n_i - 1) \left( \log S_i^2 \right).$$

where $S^2$ is the pooled within class variance and $S_i^2$ is the variance of class i. The figure 2.3026 is a constant of approximate transformation of common logarithm to natural logarithm which is used in the original formula. K is the number of variances to be tested.

The formula for the pooled within DBH class variance, $S^2$ is:

$$S^2 = \frac{\Sigma s_{ij}}{\Sigma (n-1)}.$$
When $\chi^2$ is significant, a correction should be applied. Its formula is:

$$C = \frac{3(K-1) + \left[ \sum_{i=1}^{K} \left( \frac{1}{n_i-1} \right) - \frac{1}{n_i-1} \right]}{3(K-1)}$$  \hfill (93)

And the corrected value of $\chi^2$ is then:

$$\text{Corrected } \chi^2 = \frac{\text{uncorrected } \chi^2}{C}$$  \hfill (94)

As the homogeneity of variance was expected to be rejected, the test chosen for inspection of difference between pairs of means was LSD-test because it deals with the standard error of the difference, $S_d$, which can be calculated separately for each pair of means, and in case of heterogeneity of variance, it seems to be more prudent to calculate $S_d$ for each pair of means, as shown below, rather than use a $S_d$ from a pooled variance of the analysis of variance:

$$S_d = \sqrt{\frac{s_i^2}{n_i} + \frac{s_k^2}{n_k}}$$  \hfill (95)

$$\text{LSD} = t' \ (S_d)$$

where

$$t' = \frac{w_1 t_1 + w_2 t_2}{w_1 + w_2}$$  \hfill (96)

and

$$w_i = \frac{s_i^2}{n_i}$$

$t_i$ is Student's $t$ for $n_i - 1$ degrees of freedom.
RESULTS

ANALYSIS OF VARIANCE

The two-way analysis of variance with highly ($\alpha = 0.01$) significant value of F for both log rules and DBH classes is shown below.

<table>
<thead>
<tr>
<th>SOURCES</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOG RULES</td>
<td>3074.994</td>
<td>2</td>
<td>1537.497</td>
<td>14.825**</td>
</tr>
<tr>
<td>DBH CLASSES</td>
<td>15414.585</td>
<td>7</td>
<td>2202.084</td>
<td>21.234**</td>
</tr>
<tr>
<td>LOG RULES X DBFH CLASSES</td>
<td>6504.032</td>
<td>14</td>
<td>464.574</td>
<td>4.480**</td>
</tr>
<tr>
<td>RESIDUAL</td>
<td>508996.153</td>
<td>4098</td>
<td>103.707</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>540959.128</td>
<td>4931</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE 4. Analysis of Variance table for two factors; log rules and DBH classes. ** Significantly different at 0.01.

TEST OF HOMOGENEITY OF VARIANCE

1. FOR LOG RULES:

The quantities needed for the Bartlett's Test of Homogeneity of Variance are tabulated here:

<table>
<thead>
<tr>
<th>RULE</th>
<th>$S_i^2$</th>
<th>n-1</th>
<th>CORRECTED SS</th>
<th>$\frac{1}{n-1}$</th>
<th>LOG $S_i^2$</th>
<th>$\frac{(n-1)(\log S_i^2)}{n-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>88.5204</td>
<td>1643</td>
<td>145,539.01</td>
<td>0.0006086</td>
<td>1.9470433</td>
<td>3,198.9921</td>
</tr>
<tr>
<td>2</td>
<td>97.6866</td>
<td>1643</td>
<td>160,499.08</td>
<td>0.0006086</td>
<td>1.9898348</td>
<td>3,269.2987</td>
</tr>
<tr>
<td>3</td>
<td>136.9304</td>
<td>1643</td>
<td>224,976.64</td>
<td>0.0006086</td>
<td>2.1364998</td>
<td>3,510.2692</td>
</tr>
<tr>
<td>TOTAL</td>
<td>929</td>
<td>530,914.73</td>
<td>0.0018258</td>
<td>9,667.2829</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE 5. Quantities needed for Bartlett's test on the homogeneity of the variance of the three log rules.
The pooled within variance is computed through the corrected Sum of Squares (SS):

\[ \bar{S}^2 = \frac{\sum SS}{\sum (n-1)} = 107.7125 \]

The calculated value of \( \chi^2 \) was significant in contrast with the tabulated value:

Calculated \( \chi^2 = 805.566 \)

Tabulated \( \chi^2 \) (2 d.f., = 0.01) = 0.020

2. FOR BDH CLASSES

Similar table, as for log rules, is presented for BDH classes in log rule A. For log rules B and C the results are very similar.

<table>
<thead>
<tr>
<th>CLASS</th>
<th>( S^2 )</th>
<th>n-1</th>
<th>CORRECTED SS</th>
<th>( \frac{1}{n-1} )</th>
<th>Log ( Si^2 )</th>
<th>(n-1)(log ( Si^2 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>83.1167</td>
<td>420</td>
<td>34,909.014</td>
<td>0.00238</td>
<td>1.9196882</td>
<td>806.26904</td>
</tr>
<tr>
<td>2</td>
<td>69.9372</td>
<td>399</td>
<td>27,865.042</td>
<td>0.00251</td>
<td>1.8440868</td>
<td>735.79063</td>
</tr>
<tr>
<td>3</td>
<td>79.6907</td>
<td>263</td>
<td>20,958.654</td>
<td>0.00380</td>
<td>1.9014076</td>
<td>500.07019</td>
</tr>
<tr>
<td>4</td>
<td>88.5585</td>
<td>216</td>
<td>19,128.636</td>
<td>0.00463</td>
<td>1.9472302</td>
<td>420.60172</td>
</tr>
<tr>
<td>5</td>
<td>112.1073</td>
<td>137</td>
<td>15,358.700</td>
<td>0.00730</td>
<td>2.0496338</td>
<td>280.79984</td>
</tr>
<tr>
<td>6</td>
<td>113.9831</td>
<td>97</td>
<td>11,056.360</td>
<td>0.01031</td>
<td>2.0568404</td>
<td>199.51352</td>
</tr>
<tr>
<td>7</td>
<td>110.9611</td>
<td>57</td>
<td>6,324.783</td>
<td>0.01754</td>
<td>2.0451707</td>
<td>116.57472</td>
</tr>
<tr>
<td>8</td>
<td>179.1625</td>
<td>47</td>
<td>8,420.638</td>
<td>0.02128</td>
<td>2.2532471</td>
<td>105.90261</td>
</tr>
<tr>
<td>TOTAL</td>
<td>1636</td>
<td>144</td>
<td>144,021.827</td>
<td>0.06975</td>
<td>3,165.52227</td>
<td></td>
</tr>
</tbody>
</table>

TABLE 6. Quantities needed for Bartlett's test in log rule A.
The pooled within variance for DBH classes is:

\[ S^2 = \frac{144.021.827}{1636} = 88.0329 \]

Calculated \( \chi^2 = 36.65 \)

Tabulated \( \chi^2 (7 \text{ d.f.}, \alpha = 0.01) = 1.24 \)

The homogeneity assumption was rejected for both log rules and DBH classes. That means that the standard error of the difference, \( S_q \), should be calculated for each pair of means, to compare them through LSD-test.
TEST FOR COMPARING MEANS: LSD-TEST

1. COMPARING LOG RULES MEANS:

Table 8 shows the means and variances of the 3 log rules, as well as the means and variances for the 8 DBH classes within each log rule.

The table below (Table 7) shows the absolute differences between the log rules means.

<table>
<thead>
<tr>
<th>Log Rule</th>
<th>A</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>2.62**</td>
<td>3.31**</td>
</tr>
<tr>
<td>A</td>
<td>--</td>
<td>0.69</td>
</tr>
</tbody>
</table>

** = significantly different at 0.01.

TABLE 7. Differences between two log rules.

The results of the test can be summarized as follows:

<table>
<thead>
<tr>
<th>Log Rule</th>
<th>B</th>
<th>A</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-1.33</td>
<td>1.29</td>
<td>1.98</td>
</tr>
</tbody>
</table>

Any two means not underscored by the same line are significantly different.

2. COMPARING DBH CLASSES MEANS FOR EACH LOG RULE:

Table 8 also shows means and variances of the 8 DBH classes for each log rule. As in table 7 for log rules means differences,
the differences between two means are presented in tables 9, 10 and 11.

Tables 9, 10 and 12 show LSD for each pair of means.
<table>
<thead>
<tr>
<th>Code</th>
<th>Sum</th>
<th>Mean</th>
<th>STD DEV</th>
<th>Variance</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>For entire population</td>
<td>3203.0000</td>
<td>.6494</td>
<td>10.4741</td>
<td>107.7058</td>
<td>4932</td>
</tr>
<tr>
<td>Log Rule</td>
<td>2125.0000</td>
<td>1.2926</td>
<td>9.4085</td>
<td>88.5204</td>
<td>1644</td>
</tr>
<tr>
<td>Class</td>
<td>898.3000</td>
<td>2.1337</td>
<td>9.1168</td>
<td>83.1166</td>
<td>421</td>
</tr>
<tr>
<td>Class</td>
<td>473.7000</td>
<td>1.1842</td>
<td>8.3569</td>
<td>69.8372</td>
<td>400</td>
</tr>
<tr>
<td>Class</td>
<td>119.6000</td>
<td>.4530</td>
<td>8.9270</td>
<td>79.6907</td>
<td>264</td>
</tr>
<tr>
<td>Class</td>
<td>8.9000</td>
<td>.0410</td>
<td>9.4106</td>
<td>88.5585</td>
<td>217</td>
</tr>
<tr>
<td>Class</td>
<td>204.2000</td>
<td>1.4797</td>
<td>10.5881</td>
<td>112.1073</td>
<td>138</td>
</tr>
<tr>
<td>Class</td>
<td>56.7000</td>
<td>.5786</td>
<td>10.6763</td>
<td>113.9831</td>
<td>98</td>
</tr>
<tr>
<td>Class</td>
<td>235.0000</td>
<td>4.0517</td>
<td>10.5338</td>
<td>110.9611</td>
<td>58</td>
</tr>
<tr>
<td>Class</td>
<td>128.6000</td>
<td>2.6792</td>
<td>13.3852</td>
<td>179.1625</td>
<td>48</td>
</tr>
<tr>
<td>Log Rule</td>
<td>-2184.6000</td>
<td>-1.3286</td>
<td>9.8837</td>
<td>97.6866</td>
<td>1644</td>
</tr>
<tr>
<td>Class</td>
<td>75.3000</td>
<td>.1789</td>
<td>9.2030</td>
<td>84.6959</td>
<td>421</td>
</tr>
<tr>
<td>Class</td>
<td>-1573.6000</td>
<td>-3.9340</td>
<td>8.0915</td>
<td>65.4730</td>
<td>400</td>
</tr>
<tr>
<td>Class</td>
<td>-1045.8000</td>
<td>-4.0030</td>
<td>8.9129</td>
<td>79.4392</td>
<td>264</td>
</tr>
<tr>
<td>Class</td>
<td>-624.8000</td>
<td>-2.8793</td>
<td>9.4342</td>
<td>89.0050</td>
<td>217</td>
</tr>
<tr>
<td>Class</td>
<td>73.4000</td>
<td>.5319</td>
<td>10.6003</td>
<td>112.3660</td>
<td>138</td>
</tr>
<tr>
<td>Class</td>
<td>149.1000</td>
<td>1.5214</td>
<td>10.9346</td>
<td>119.5660</td>
<td>98</td>
</tr>
<tr>
<td>Class</td>
<td>411.9000</td>
<td>7.1017</td>
<td>11.2172</td>
<td>125.8265</td>
<td>58</td>
</tr>
<tr>
<td>Class</td>
<td>360.9000</td>
<td>7.5187</td>
<td>14.0564</td>
<td>197.5820</td>
<td>48</td>
</tr>
<tr>
<td>Log Rule</td>
<td>3262.6000</td>
<td>1.9845</td>
<td>11.7017</td>
<td>136.9304</td>
<td>1644</td>
</tr>
<tr>
<td>Class</td>
<td>772.3000</td>
<td>1.8344</td>
<td>15.5563</td>
<td>241.9982</td>
<td>421</td>
</tr>
<tr>
<td>Class</td>
<td>-8.2000</td>
<td>-0.0205</td>
<td>8.5122</td>
<td>72.4578</td>
<td>400</td>
</tr>
<tr>
<td>Class</td>
<td>313.4000</td>
<td>1.1871</td>
<td>9.4757</td>
<td>89.7884</td>
<td>264</td>
</tr>
<tr>
<td>Class</td>
<td>469.7000</td>
<td>2.1645</td>
<td>9.8224</td>
<td>96.4797</td>
<td>217</td>
</tr>
<tr>
<td>Class</td>
<td>665.9000</td>
<td>4.8254</td>
<td>11.0295</td>
<td>121.6488</td>
<td>138</td>
</tr>
<tr>
<td>Class</td>
<td>394.7000</td>
<td>4.0276</td>
<td>11.0916</td>
<td>123.0245</td>
<td>98</td>
</tr>
<tr>
<td>Class</td>
<td>406.3000</td>
<td>7.0052</td>
<td>11.2968</td>
<td>127.6170</td>
<td>58</td>
</tr>
<tr>
<td>Class</td>
<td>248.5000</td>
<td>5.1771</td>
<td>13.7089</td>
<td>187.9346</td>
<td>48</td>
</tr>
</tbody>
</table>

Table 8. Table of important statistics for log rules.
2a. FOR LOG RULE A:

<table>
<thead>
<tr>
<th>DBH CLASS</th>
<th>3</th>
<th>6</th>
<th>2</th>
<th>5</th>
<th>1</th>
<th>8</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.41</td>
<td>0.54</td>
<td>1.14</td>
<td>1.44</td>
<td>2.09**</td>
<td>2.64**</td>
<td>4.01**</td>
</tr>
<tr>
<td>3</td>
<td>0.13</td>
<td>0.73</td>
<td>1.03</td>
<td>1.68</td>
<td>2.23</td>
<td>3.60</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.60</td>
<td>0.90</td>
<td>1.55</td>
<td>2.10</td>
<td>3.47</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.30</td>
<td>0.95</td>
<td>1.50</td>
<td>2.87</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.65</td>
<td>1.20</td>
<td>2.57</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.55</td>
<td>1.90</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td>1.37</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Significantly different at 0.01.

Table 9. Differences between two DBH class means for log rule A.

TEST SUMMARY:

<table>
<thead>
<tr>
<th>DBH Class:</th>
<th>2</th>
<th>4</th>
<th>1</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>DBH CLASS</td>
<td>3</td>
<td>4.18**</td>
<td>4.53**</td>
<td>5.52**</td>
<td>11.10**</td>
<td>11.52**</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4.11**</td>
<td>4.46**</td>
<td>5.45**</td>
<td>11.03**</td>
<td>11.45**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3.06**</td>
<td>3.42**</td>
<td>4.40**</td>
<td>9.98**</td>
<td>10.40**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.35</td>
<td>1.34</td>
<td>6.92**</td>
<td>7.34**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.99</td>
<td>6.57**</td>
<td>6.99**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>5.58**</td>
<td>6.00**</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>0.42</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Significantly different at 0.01.

Table 10. Differences between two DBH class means for log rule B.
<table>
<thead>
<tr>
<th>Class</th>
<th>SUM</th>
<th>MEAN</th>
<th>STD DEV</th>
<th>VARIANCE</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>For entire population</td>
<td>3203.0000</td>
<td>.6494</td>
<td>10.4741</td>
<td>109.7058</td>
<td>4932</td>
</tr>
<tr>
<td>Class</td>
<td>1</td>
<td>1745.9000</td>
<td>1.3823</td>
<td>11.7101</td>
<td>137.1268</td>
</tr>
<tr>
<td>Log Rule A</td>
<td>898.3000</td>
<td>2.1337</td>
<td>9.1168</td>
<td>83.1167</td>
<td>421</td>
</tr>
<tr>
<td>Log Rule B</td>
<td>75.3000</td>
<td>.1789</td>
<td>9.2030</td>
<td>84.6959</td>
<td>421</td>
</tr>
<tr>
<td>Log Rule C</td>
<td>772.3000</td>
<td>1.8344</td>
<td>15.5563</td>
<td>241.9982</td>
<td>421</td>
</tr>
<tr>
<td>Class</td>
<td>2</td>
<td>-1108.1000</td>
<td>-.9234</td>
<td>8.5976</td>
<td>73.9182</td>
</tr>
<tr>
<td>Log Rule A</td>
<td>473.7000</td>
<td>1.1842</td>
<td>8.3569</td>
<td>69.8372</td>
<td>400</td>
</tr>
<tr>
<td>Log Rule B</td>
<td>-1571.6000</td>
<td>-3.9530</td>
<td>8.0915</td>
<td>65.4730</td>
<td>400</td>
</tr>
<tr>
<td>Log Rule C</td>
<td>-8.2000</td>
<td>-1.2050</td>
<td>8.5122</td>
<td>72.4578</td>
<td>400</td>
</tr>
<tr>
<td>Class</td>
<td>3</td>
<td>-621.8000</td>
<td>-.7876</td>
<td>9.3824</td>
<td>88.0288</td>
</tr>
<tr>
<td>Log Rule A</td>
<td>119.6000</td>
<td>.6530</td>
<td>8.9720</td>
<td>79.6907</td>
<td>264</td>
</tr>
<tr>
<td>Log Rule B</td>
<td>75.5000</td>
<td>.9030</td>
<td>9.1293</td>
<td>79.4392</td>
<td>264</td>
</tr>
<tr>
<td>Log Rule C</td>
<td>313.4000</td>
<td>1.1871</td>
<td>9.4757</td>
<td>89.7884</td>
<td>264</td>
</tr>
<tr>
<td>Class</td>
<td>4</td>
<td>-146.2000</td>
<td>-.2246</td>
<td>9.7647</td>
<td>95.3845</td>
</tr>
<tr>
<td>Log Rule A</td>
<td>8.9000</td>
<td>.0410</td>
<td>9.4108</td>
<td>88.4484</td>
<td>217</td>
</tr>
<tr>
<td>Log Rule B</td>
<td>-624.8000</td>
<td>-2.8733</td>
<td>9.4342</td>
<td>89.0050</td>
<td>217</td>
</tr>
<tr>
<td>Log Rule C</td>
<td>462.7000</td>
<td>2.1645</td>
<td>9.8224</td>
<td>96.4797</td>
<td>217</td>
</tr>
<tr>
<td>Class</td>
<td>5</td>
<td>943.5000</td>
<td>2.2790</td>
<td>10.6727</td>
<td>118.2153</td>
</tr>
<tr>
<td>Log Rule A</td>
<td>204.2000</td>
<td>1.4797</td>
<td>10.5881</td>
<td>112.1073</td>
<td>138</td>
</tr>
<tr>
<td>Log Rule B</td>
<td>73.4000</td>
<td>.5319</td>
<td>10.6003</td>
<td>112.3660</td>
<td>138</td>
</tr>
<tr>
<td>Log Rule C</td>
<td>665.3000</td>
<td>4.8524</td>
<td>11.0295</td>
<td>121.6488</td>
<td>138</td>
</tr>
<tr>
<td>Class</td>
<td>6</td>
<td>600.5000</td>
<td>2.0425</td>
<td>10.9623</td>
<td>120.1221</td>
</tr>
<tr>
<td>Log Rule A</td>
<td>56.7000</td>
<td>.5786</td>
<td>10.6763</td>
<td>113.9831</td>
<td>98</td>
</tr>
<tr>
<td>Log Rule B</td>
<td>149.1000</td>
<td>1.5214</td>
<td>10.9346</td>
<td>119.5660</td>
<td>98</td>
</tr>
<tr>
<td>Log Rule C</td>
<td>394.7000</td>
<td>4.0276</td>
<td>11.0916</td>
<td>123.0245</td>
<td>98</td>
</tr>
<tr>
<td>Class</td>
<td>7</td>
<td>1053.2000</td>
<td>5.0529</td>
<td>11.0490</td>
<td>122.0794</td>
</tr>
<tr>
<td>Log Rule A</td>
<td>235.0000</td>
<td>1.0517</td>
<td>10.5338</td>
<td>110.9611</td>
<td>58</td>
</tr>
<tr>
<td>Log Rule B</td>
<td>411.9000</td>
<td>7.1017</td>
<td>11.2172</td>
<td>125.8265</td>
<td>58</td>
</tr>
<tr>
<td>Log Rule C</td>
<td>406.3000</td>
<td>7.0052</td>
<td>11.2968</td>
<td>127.6170</td>
<td>58</td>
</tr>
<tr>
<td>Class</td>
<td>8</td>
<td>738.0000</td>
<td>5.1250</td>
<td>13.7668</td>
<td>189.5261</td>
</tr>
<tr>
<td>Log Rule A</td>
<td>128.6000</td>
<td>2.6792</td>
<td>13.3852</td>
<td>179.1625</td>
<td>48</td>
</tr>
<tr>
<td>Log Rule B</td>
<td>360.9000</td>
<td>7.5187</td>
<td>14.0564</td>
<td>197.5820</td>
<td>48</td>
</tr>
<tr>
<td>Log Rule C</td>
<td>248.5000</td>
<td>5.1771</td>
<td>13.7089</td>
<td>187.9346</td>
<td>48</td>
</tr>
</tbody>
</table>

Table 11. Table of important statistics for DBH classes
TABLE 11.

TEST SUMMARY:

<table>
<thead>
<tr>
<th>DBH Class</th>
<th>3</th>
<th>2</th>
<th>4</th>
<th>1</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean:</td>
<td>-4.00</td>
<td>-3.93</td>
<td>-2.88</td>
<td>0.18</td>
<td>0.53</td>
<td>1.52</td>
<td>7.10</td>
<td>7.52</td>
</tr>
</tbody>
</table>

**DBH CLASS: 3**

<table>
<thead>
<tr>
<th>DBH CLASS</th>
<th>Mean:</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.21</td>
</tr>
<tr>
<td>3</td>
<td>0.64</td>
</tr>
<tr>
<td>4</td>
<td>0.33</td>
</tr>
<tr>
<td>5</td>
<td>1.87</td>
</tr>
<tr>
<td>6</td>
<td>0.80</td>
</tr>
<tr>
<td>7</td>
<td>0.35</td>
</tr>
<tr>
<td>8</td>
<td>1.83</td>
</tr>
</tbody>
</table>

**Significantly different at 0.01.**

Table 12. Differences between two DBH class means for log rule C.

TEST SUMMARY

<table>
<thead>
<tr>
<th>DBH CLASS:</th>
<th>2</th>
<th>3</th>
<th>1</th>
<th>4</th>
<th>6</th>
<th>5</th>
<th>8</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean:</td>
<td>-0.02</td>
<td>1.19</td>
<td>1.83</td>
<td>2.16</td>
<td>4.03</td>
<td>4.83</td>
<td>5.18</td>
<td>7.01</td>
</tr>
</tbody>
</table>
CORRELATION BETWEEN AVERAGE PERCENT DIFFERENCE VOLUME AND MIDPOINT DBH CLASS

The regression equations that show the best coefficient of determination are the following:

LOG RULE A:
\[ PDV = 4.6429 - 0.5066 \text{ (DBHmidpoint)} + 0.0151 \text{ (DBHmid)}^2 \]
with
\[ R^2 = 0.6858 \]

LOG RULE B:
\[ PDV = 4.2106 - 1.0798 \text{ (DBH mid)} + 0.0390 \text{ (DBH mid)}^2 \]
with
\[ R^2 = 0.9402 \]

LOG RULE C:
\[ PDV = 1.0941 + 0.1894 \text{ (DBH mid)} + 0.0015 \text{ (DBH mid)}^2 \]
with
\[ R^2 = 0.9765 \]

Figures 19, 20 and 21, illustrate those three regression equations in contrast with the points representing the DBH classes midpoints and the average percent difference volume for each class.

The regression analysis for the 3 log rules are shown in tables 13, 14, and 15.
Log Rule A:

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>8.8336</td>
<td>2</td>
<td>4.4418</td>
<td>5.7715*</td>
</tr>
<tr>
<td>Residual</td>
<td>3.8480</td>
<td>5</td>
<td>0.7696</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>12.4316</td>
<td>7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Regression significant at 0.05.

TABLE 13. Regression analysis for log rule A.

Log Rule B:

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>136.1409</td>
<td>2</td>
<td>68.0704</td>
<td>39.4735**</td>
</tr>
<tr>
<td>Residual</td>
<td></td>
<td>5</td>
<td>1.7246</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Regression significant at 0.01.

TABLE 14. Regression analysis for log rule B.

Log Rule C:

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>38.1752</td>
<td>2</td>
<td>19.0876</td>
<td>102.9203**</td>
</tr>
<tr>
<td>Residual</td>
<td>0.9272</td>
<td>5</td>
<td>0.1855</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>39.1028</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Regression significant at 0.01.

TABLE 15. Regression analysis for log rule C.
Figure 19. Regression of average PDV on DBH class midpoint.
Figure 20. Regression of average PDV of DBH class on DBH class midpoint, for Log Rule B.
Figure 21. Regression of Mean Percent Volume (PDV) on DBH class midpoint.
CORRELATION BETWEEN PERCENT DIFFERENCE VOLUME (PDV) AND DBH

For the three different log rules this correlation was unsatisfactory, with very low coefficient of determination, $R^2$, respectively:

$$R^2 = 0.0077 \text{ for rule A}$$
$$R^2 = 0.0933 \text{ for rule B}$$
$$R^2 = 0.0190 \text{ for rule C}.$$
DISCUSSIONS AND CONCLUSIONS

Although the percent "error", more precisely, the percent difference between the volumes given by the three studied log rules and the "actual" volume computed by the standard method, were very small (no more than 2% of the actual volume), they proved to be significantly different.

The large F value is due in part to the different number of trees in the 8 DBH classes, the second factor in this study. From a statistical point of view, it would be more convenient to deal with the same number of trees for each DBH class. However, the smallest size class (class number 8) had only 48 trees and a maximum of 384 (48 trees times 8 log rules) measurements was not sufficient for the experiment because the high variance of the variable "percent difference volume". The variance had shown, in a general way, to be still bigger in the larger classes which were exactly those of small size.

When the log rules means were compared through LSD-test, log rule B was significantly different from log rule A as well as from log rule C, at 99% level of probability (which is the probability level used throughout this study). Log rules A and C didn't prove to be significantly different.

Of all the log rules Bruce's Equation was the one which presented the highest volume difference, averaging 1.98%, even though very low. Log rule A, Weyerhaeuser Douglas-Fir Cubic, showed the smallest absolute mean difference, 1.29%, but very close, in absolute
value, to rule B, British Columbia Immature, with a volume difference averaging -1.33%.

All the log rules presented very high variance: Coefficient of variation equal to 59.0% for C, 72.9% for B and 74.3% for A. Only this high variance could explain the significance of the difference between so close averages. The greater difference was 3.31%.

The means of the DBH classes in the same log rule showed more variation. Log rule A, for example, showed a range from -4.00 in class 3 to 7.25 in class 8. To arrange them in an increasing or decreasing order with DBH increment was impossible, meanwhile almost always the 4 highest DBH classes had presented higher averages. The only exception was class 6 in rule A with a very small mean, 0.58%, while class 1 presented an average of 2.13%, the 3rd highest.

The tables of the differences between means for log rules A and C (tables 9 and 12) showed a peculiar situation related to class 8. For log rule A, class 4 was significantly different from class 6 with a difference of 2.09 but was not significantly different from class 8 which had a bigger difference (2.64). Log rule C had the same situation: classes 2 and 3 showed significantly different from class 5 but not from class 8 with larger mean. A reason for this apparent contradiction could be the high value of the variance of class 8 in both rules associated with its very small sample size. This seems to reinforce the evidence discussed above: DBH class 8, with very high variation and only 48 trees, did not have a sample size big enough for a statistical analysis.
It seems that the same peculiarity did not happen in log rule B because of its smaller relative variance and so for this rule the sample size of class 8 would have been sufficient.

With a look at the trend of the DBH class means of PDV on the midpoint of DBH classes (Figures 15, 16 and 17), it is reasonable to suspect that there exists a curvilinear correlation between these two variables. To test whether there is a relation between the variables an analysis of regression was performed. For log rule B and C the correlation was significant at 0.01 level. Values of coefficient of determination, $R^2$, equal to respectively, 0.94, 0.97 showed a very strong correlation between the variables. For log rule A, with $R^2 = 0.67$, the significance is shown only at 95% level.

A regression analysis of the correlation between PDV and DBH, using all data, (1644 trees) for each log rule, showed very low values of $R^2$ although the values of F had shown significant relation between the variables. The reason for those high values of F is justified by two major reasons: The large sample size and the high variance of the data.

In the conclusion about this topic, the major comment is that even though DBH classes midpoint seems to be very highly correlated with the mean of PDV for the DBH classes, DBH by itself does not seem to be a good predictor of PDV, the use of PDV mean and DBH midpoint should be discouraged and even not acceptable for DBH classes sample sizes smaller than those used in this study. Class 8 anyway should have a bigger sample size.
As it was explained in the Introduction, the objective of this paper was the investigation on a methodology of comparing log rules in a statistical basis. No attempt was made to ascertain the relative performance of the three log rules as an estimator of the total volume of a young second-growth Douglas-fir stand. The good performance of all of them for young Douglas-fir stands was already known. The resulting figures of this study prove this with very small average percent difference between the volume given by each of them and the standard volume although one of them, the British Columbia Coast Immature Douglas-fir Tarif Table showed to be statistically different of another tarif table, the Weyerhaeuser Douglas-fir Tarif Table and of Bruce's Table for Immature Douglas-fir.
BIBLIOGRAPHY


