AN ABSTRACT OF THE THESIS OF

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TITLE: A COMPARATIVE STUDY OF THREE LOG RULES COMMONLY USED IN THE PACIFIC NORTHWEST, U.S.A.

Abstract Approved: ______________________________

A comparative study was done on data collected by Weyerhaeuser Company on 1,644 second-growth Douglas-fir, Pseudotsuga menziesii, trees. The variable used in the study was the percent difference, positive or negative, between the volumes given by each of three different log rules and the standard tree volumes determined by Weyerhaeuser Company.

The data were analyzed in a two-way analysis of variance and factors, log rules and DBH classes were highly significant (99% level).

Comparative tests (LSD for unequal sample size) were done on pairs of log rules means as well as on pairs of DBH classes for each log rule. All the differences between means are presented and significances are summarized and discussed.

A regression analysis was performed between percent difference volume as the dependent variable and DBH as the independent for each log rule. Results showed no significant differences and the coefficients of determination were very low.

A COMPARATIVE STUDY OF THREE LOG RULES COMMONLY USED IN THE PACIFIC NORTHWEST, U.S.A.

by

Julio De Castro Paixao

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COMPARATIVE STUDY OF THREE LOG RULES COMMONLY USED IN THE PACIFIC NORTHWEST, U.S.A.

INTRODUCTION

The basic reference quantity in timber measurement may be expressed in such terms as dry weight, length, number of pieces, or green weight. However, with good reason, the tradition has been to use solid volume to express timber measurement. Often when other parameters, such as stacked volume or weight are used these are changed to solid volume estimates for purposes of management. The use of solid volume is good for other reasons, too. Many timber products are sold by solid volume. Also it is easy to measure solid volume for both felled and standing timber. It is impossible to use weight measurements of any kind in standing timber. So, solid volume is generally "the expression of quantity".

The nature of forest products has led to the use of volume estimates, since very precise measurement in forestry is seldom practical. There are several ways of estimating timber volume even in the smallest logs. Consequently the resulting estimates are inevitably different depending on the method used. This creates a necessity of measurement conventions and a description of the material used if full understanding of timber volume estimates is to be obtained.

When making an estimate of quantity the degree of precision desired must be established. Different degrees of precision may be obtained for any forest product. Costs and benefits that correspond to these different levels of precision must be considered before the measurements.

First of all, greater precision almost always costs more per unit volume because it requires more measurements and this involves more work. On the other hand, almost always, greater precision produces greater benefits. G. J. Hamilton, 1975, gives very clear considerations about the relationship of price to precision and of cost of measurement to precision.

Log measurement has been one of the most important and controversial topics in the field of forest management and wood technology. In the past, many different log rules have appeared in the United States with each generally being used in a particular area as a consequence of variations in acceptable utilization standards, slab allowance, taper, shrinkage, sawkerf and method of construction. Buyer-seller relationships remain normal because each accepts volumes and values involved in log transactions although they are aware of the inaccuracy of the log rules. For research purposes serious difficulties can appear when comparisons of volumes by two or more different log rules are tried. Log rule characteristics are such that only with difficulty can two measurement systems be successfully compared.

It is known that Tarif Tables and Bruce's Cubic Volume Table for Immature Douglas-fir are very practical log rules, but it is easy to perceive how important it is to know the accuracy of each one in a specific situation (as species, class of DBH, and site) to help in selecting the one which gives the best results. Weyerhaeuser Douglas-fir Table, British Columbia Coast Immature Douglas-fir and Bruce's Table for Immature Douglas-fir are specific for the material of study (Young growth Douglas-fir from Western Washington). The objective of this study is to compare the volumes estimated by those three volume tables with the actual volume of second growth Douglas-fir calculated by standard measurement done by James E. King of Weyerhaeuser Company.

Knowing that the results of a such investigation are precise only for that specific population, we could have a reasonable idea how these volumes compare with actual volumes.

The objective of this study is the investigation of a methodology which could serve as a base to begin a study to find out the most accurate log rule for each species and/or site anywhere but mainly in countries like Brazil where Forestry is really a very new activity and practical log rules are seldom used in the many species of *Eucalyptus* and *Pinus* for pulpwood and paper production and sawmill purposes.

This will be accomplished by an analysis of variance on the per cent difference between the total tree volume given by each of three log rules and the standard volume calculated by Weyerhaeuser Company. Average per cent volume difference by each of 8 diameter classes will be compared for each log rule.

LITERATURE REVIEW

Since trees vary in geometric form from the stump to the top, no single mathematical formula can express the exact cubic volume of each log.

Measuring the volume contents of individual trees or sections by assimilating them to geometric solids is the most accurate means of determining volume. This is usually called "standard scaling" when assuming the stem as a sequence of super-imposed geometric solids. The interpretation and use o formulas varies with the mensurationist but the geometric solids more commonly used are the cone, the quadratic paraboloid, the neiloid, their frustrums and the cylinder.

It is conventional to calculate the volume of the stump as a cylinder and the volume of the first butt log as a frustrum of a neiloid. The subsequent logs may be assumed as frustrums of neiloids, cones or paraboloids, depending on the species and the mensurationist's interpretation.

A method that provides very accurate estimates is the one used by Weyerhaeuser Company which assumes the several logs of the stem are composed as follows:

1. Stump - cylinder.
2. Stump to 4.5 feet - neiloid frustrum.
3. Logs above 4.5 feet - paraboloid frustrums.
4. Top - cone.

Its accuracy was proved in the past for many species by the immersion method, the only perfectly accurate method for determining log volumes.

The formulas for these geometric forms give very different volumes. If the volume of a cylinder of given diameter and height is expressed as 100%, the volumes of the paraboloid, cone and neiloid of the same basal diameter and height are respectively 50, 33 1/3 and 25 per cent of the cylinder volume. Fig. 1.

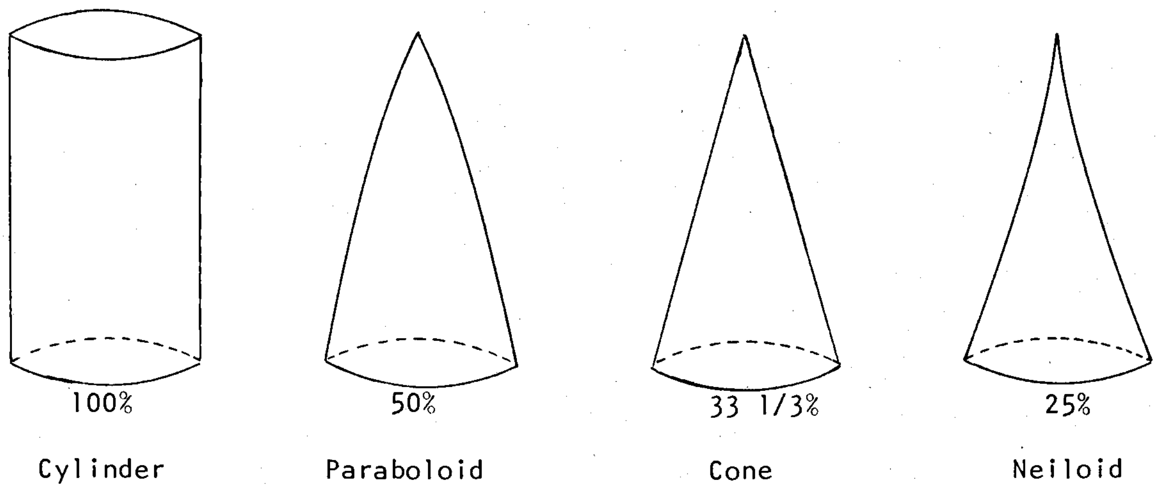

Figure 1. The paraboloids which logs are more frequently assumed to resemble.

Actually the paraboloid mentioned here is the quadratic paraboloid. In general that is the only paraboloid used in tree measurement but the use of cubic and semi-cubic paraboloids should improve the accuracy in the determination of standard volume of a tree. In some European countries, like France and Portugal, researchers frequently use both cubic and semi-cubic paraboloid frustrums for certain species; the cubic for the lower logs (immediately after the neiloid frustrum) and the semi-cubic for the upper one (before the top). The intermediate logs are assumed as quadratic paraboloid (or simply paraboloid as it is commonly known) frustrums. Fig. 2.

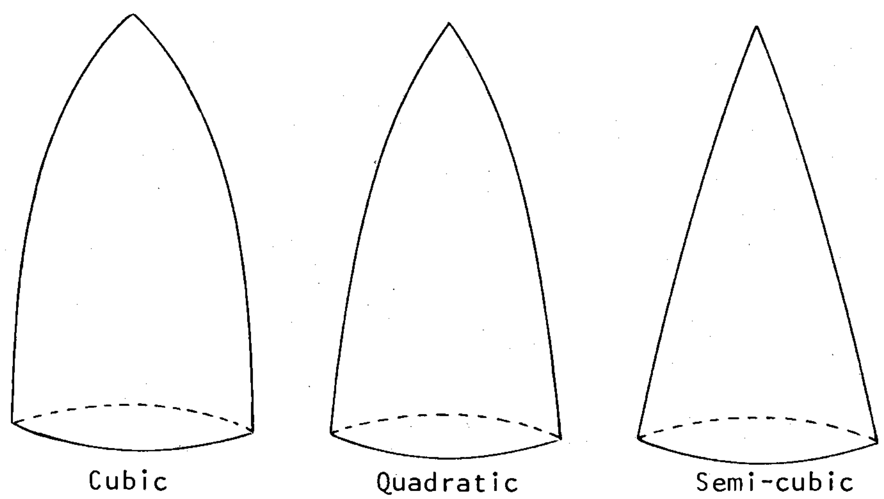

Figure 2. Cubic, quadratic and semi-cubic paraboloids.

Since the cited geometric figures are solids of revolution, the volume formula for each one of them is obtained by integration rotating the graph of the general equation $Y = K \sqrt{X^r}$ around the X axis:

$$V = \pi \int_b^a Y^2 \, dX \qquad (1)$$

The paraboloid (more precisely: quadratic paraboloid) is generated when r = 1 (so $Y = K X^{1/2}$); the cone when r = 2 (Y = K X); the neiloid when r = 3 ($Y = K X^{3/2}$); and the cylinder when r = 0 (Y = K). The cubic and semi-cubic paraboloids are generated respectively from the formulas $Y = K X^{1/3}$ and $Y = K X^{2/3}$.

For any of the geometric curves, the constant K changes in accord with the ratio Y/X. For example, the curve which generates the paraboloid of basal radius Y = 2 and height X = 7 has the constant K = .755929 while the curve which generates a paraboloid of same height X = 7 but with basal radius Y = 1 requires a constant K = .377964. This is a crucial point in comparative study of volumes of different paraboloids of the same height and basal radius. Fig. 3.

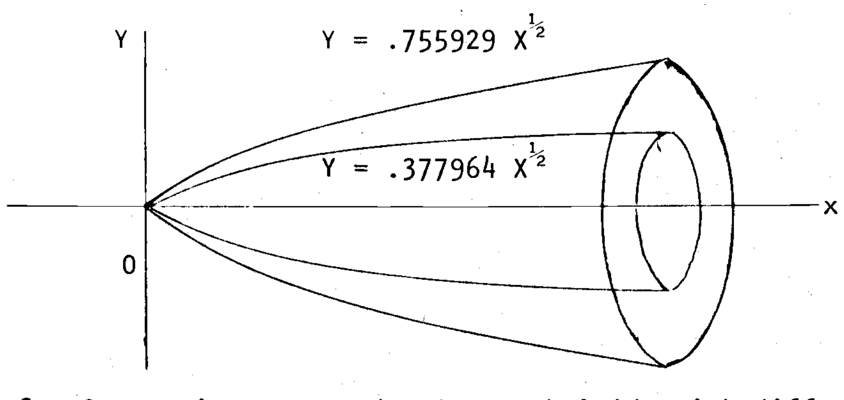

Figure 3. Generating two quadratic paraboloids with different ratio radius/height (Y/X).

In figure 3 if Y is the radius of the basal circle of either paraboloid the volume will be:

$$V = \pi \int_a^b Y^2 \, dX \tag{2}$$

Since Y is function of X,

$$V = \pi \int_a^b (K \, X^{r/2})^2 \, dX \tag{3}$$

Or:

$$V = \pi \; K^2 \int_a^b X \, dX \tag{4}$$

As r for the quadratic paraboloid is equal to 1, the volumes for both paraboloids in figure 3 can be calculated as follows:

$$V = \pi \, (.755929)^2 \left[X^2/2 \right]_0^7 = 43.982303$$

for the thickest one, and

$$V = \pi \, (.377964)^2 \left[X^2/2 \right]_0^7 = 10.9955...$$

for the thinnest one.

A paraboloid frustrum has a perfect formula for its volume as the product of an average cross-sectional area of its basal area (A_b) and top cross-sectional area (A_u) by its height. This is the Smalian's formula, often used in log scaling:

$$V = \frac{H}{2} \, (A_b + A_u) \tag{5}$$

where

$$V = \text{volume of the log}$$

$$H = \text{length of the log}$$

$$A_b = \text{basal area at the base}$$

$$A_u = \text{basal area at the top}$$

A different formula is often used for paraboloid frustrums. It is Huber's formula, which is considered more accurate because the volume of a log is more dependent on the middle diameter than on the end diameter, and other practical formulas utilize end diameters. Huber's formula is expressed as follows:

$$V = A_m H \tag{6}$$

where

$$A_m = \text{cross-sectional area at midpoint}$$

The mathematical formula for a cone frustrum is:

$$V = \frac{H}{3} \left(A_b + \sqrt{A_b A_u} + A_u\right) \tag{7}$$

Smalian's formula overscales a truncated cone and Huber's formula also slightly overscales the frustrum.

Neiloid frustrums have a more complicated geometrical formula for volume:

$$V = \frac{H}{4} \left(A_b + \sqrt[3]{A_b^2 A_u} + \sqrt[3]{A_b A_u^2} + A_u\right) \tag{8}$$

A very accurate practical formula for all frustrums we are concerned with in the present study is Newton's formula, which is inconvenient because of the necessity of taking three diameter measurements: at both extremeties and at the middle of the log:

$$V = \frac{H}{6} (A_b + 4\ A_m + A_u) \tag{9}$$

Most of the practical log rules determine board-foot volume. This work however will be limited to a comparison of the accuracy of three cubic-foot log rules when applied to young Douglas-fir, _Pseudotsuga menziesii_.

CUBIC-FOOT LOG RULES - A REVIEW

1. Newton's Formula

It fits almost any geometric figure and gives the cubic contents of frustrums of paraboloids, cones and neiloids accurately. However, it requires the measurement of diameters inside the bark at the base, middle and top of the log. Obviously, it is inefficient for practical use. The formula is:

$$V = \frac{H}{6}\ \frac{\pi}{4}\ \frac{1}{144} (D_b^2 + 4\ D_m^2 + D_u^2) \tag{10}$$

where

V = volume of log in cubic feet

D_b = diameter inside bark in inches at large end

D_u = diameter inside the bark in inches at small end

D_m = diameter inside bark in inches at the middle point of the log

H = height

In Western Oregon and Western Washington the more common length of log to be measured is 32 feet and so the formula can appear in the following expression:

$$V = .0291 \ (D_b^2 + 4 \ D_m^2 + D_u^2) \tag{11}$$

2. Smalian's Formula

It's perfect for a frustrum of paraboloid but has the inconvenience of considering only the extreme end diameters, excluding the middle one. It overscales logs which have a truncated cone form and especially logs with neiloid frustrum form:

$$V = \frac{H}{2} \frac{\pi}{4} \frac{1}{144} \ (D_b^2 + D_u^2) \tag{12}$$

For a 32-foot log:

$$V = .087264 \ (D_b^2 + D_u^2) \tag{13}$$

3. Huber's Formula

This formula is considered the most accurate of the more practical log rules but as in Newton's the inconvenience is in taking diameter measurement at the middle of the log:

$$V = \frac{\pi H}{4} \frac{1}{144} \ D_m^2 \tag{14}$$

and for 32-foot log:

$$V = .174528\ D_m^2 \qquad (15)$$

4. Rapraeger's Formula

It is a modification of Huber's rule which attempts to combine the supposed accuracy of Huber's with a practical method of scaling decked, rafted and loaded logs to determine the diameter of the middle of the log. Rapraeger proposed an arbitrary taper allowance of one inch for every 8 feet of length from the small end of the log, with the resulting formula being:

$$V = .005454\ H\ (D + \frac{H}{16})^2 \qquad (16)$$

where

D = Diameter inside bark at small end of the log in inches

H = Length of the log in feet

5. Sorensen's Formula

This rule is based on the cone frustrum formula. It is not considered very satisfactory unless the logs scaled closely approach a taper of 1 inch in 10 feet of length. Sorensen advocates measuring only the small end diameter and using 1 inch of taper for every 10 feet of length to get the diameter at the middle of the log.

It saves time since the only measurements are length and diameter at the small end. This rule underscales drastically neiloidal and paraboloidal logs. The formula can be expressed as follows:

$$V = .005454154\ (D + \frac{L}{20})\ L \qquad (17)$$

where

D = Diameter inside bark at small end in inches

L = Length of the log in feet

The $\frac{L}{20}$ term is, in effect, a conversion from small end to mid-point diameter based on the assumed taper of 1 inch in 10 feet.

6. Bruce and DeMars' Volume Equations for Second-Growth Douglas-Fir.

Although very specific, this rule is listed here because it is a major part of this study.

David Bruce and Donald J. DeMars Volume Equations were first published in November 1974 by Pacific Northwest Forest and Range Experimental Station as a USDA Forest Service Research Note. They were constructed as a result of a request for a reasonable table for small Douglas-Fir.

The tables were based on a sample of 1,127 trees which ranged from 0.4 inch DBH and 6 feet in height to 32 inches DBH and 167 feet in height. The independent variables were DBH outside bark (o.b.) and inside bark (i.b.); the dependent variable was form factor based on total volume inside bark, including the stump calculated as a cylinder. The form factors are used in the equation and so are calculated before.

Outside bark form factor equations for small sample trees (FOS) and for large trees (FOL) are presented below. Trees considered small here are those of total height equal or smaller than 18 feet and consequently large trees are those of total height greater than 18 feet.

$$FOS = 0.406098(H-0.9)^2/(H-4.5)^2 - 0.0762998\ D(H-0.9)^3/(H-4.5)^3 + 0.00262715\ DH(H-0.9)^3/(H-4.5)^3 \quad (18)$$

(based on 59 trees for young stands in Oregon, Washington and B.C.)

$$FOL = 0.480961 + 42.46542/H^2 - 10.99643\ D/H^2 - 0.107809\ D/H - 0.00409083\ D \quad (19)$$

(based on 1,068 trees from young stands in Oregon, Washington and B.C.)

Volumes are simply calculated through one of the following formulas:

$$VS = 0.005454154\ FOS\ (D^2H) \quad (20)$$

when $H \leq 18$ feet, and

$$VL = 0.005454154\ FOL\ (D^2H) \quad (21)$$

when $H \geq 18$ feet.

Equation (21) can not be applied for very small trees (trees with DBH smaller than 1.4 inches and height less than 13 feet) as volume decreases when height increases, holding the same diameter. This frequently happens in different rules because of the use of DBH that is located at 4.5 feet above the ground level.

The test criterium for significance of the regression, for both form factor and volume was the root mean square error: 12.2 and 8.0% for FOS and FOL, respectively; and 12.7 and 16.8% for VS and VL.

7. Tarif Tables

Tarif Tables are constructed for different species and sites. They are considered very simple and more accurate than conventional tables especially in young growth stands. The application of the tarif system is very easy both for field use (tables) and computers (formulas). It provides easy conversion between units of measure and its authors claim sufficient accuracy for volume and growth in research application.

"A tarif table is a local volume table that gives tree volume by diameters for trees of the same general height class. They are particularly suited, but not limited, to even-aged stands" (Hoyer, 1971).

The tarif table system is a group of "preconstructed" local volume tables applicable to the specific stand, each one having its tarif access number. The tarif number is the total cubic foot volume from the stump to a 4-inch top for a tree of 1.0 square foot of basal area.

The tarif number of a stand is found through "Access Tables", which provide a number for each sample tree, given its DBH and height. In finding the tarif number, DBH's have to be measured to the nearest 1/10th inch and total height to the nearest foot for each sample tree. The tarif numbers found are averaged and this average is the tarif number. In general only 20 representative trees are necessary to determine the average tarif number for a stand. The tarif number is the index-number for the table to be used.

Tarif Tables being a major subject in the present work, a detailed discussion is required.

Cubic Volume Tarif System

Here are the main considerations about cubic volume tarif systems:

CV4 is the volume of a tree above stump height to a top diameter of 4.0 inches. CV4 curves are the basis for the Tarif System since total cubic volume curves (TCV) had been shown to be an inadequate basis for some important reasons:

1. The total volume/basal area straight line is satisfactory for the larger trees but the actual trend in lower range values is curvilinear (Fig.4).

2. The intercept in the horizontal axis is not stationary, so a moving intercept has to be adopted if volume/basal area trend is employed for the total cubic volume curves. The authors primarily investigated the moving intercept by fitting the trend of _a_ coefficient in relation to _b_ coefficient for all individual CVTS/basal

area lines. Then a system of harmonized CVTS/basal area lines was constructed by using the smoothed values of a for a given b but due to movement of the intercept along the horizontal axis because of change in the slope of the line (trees of small DBH showing higher volume for smaller slope and lower volume for greater slope, as shown in figure 5), and considering that the slope increases with increasing age of the stand, "this system of lines would cause the smaller trees to have less volume with advancing age". (Turbull and Hoyer, 1965).

In CV4 curves no curvilinearity of trend was found even in the lower range of basal area because the 4-inch top limit excluded the small trees which are responsible for the curvature. Furthermore the volume/basal area line is equally satisfactory for all plots in the sample and the horizontal intercept is stationary.

Subsequent tests showed important points:

1. No trend of the basal area intercept value in relation to steepness of the line.

2. None of the intercept values differed significantly from 0.0873 sq. ft. of basal area which is equivalent to a 4-inch DBH.

3. Trees with a 4-inch DBH outside bark have stump diameter inside bark consistently within 3.9 to 4.0 inches and hence they have zero volume to 4-inch top above stump. Fig. 6.

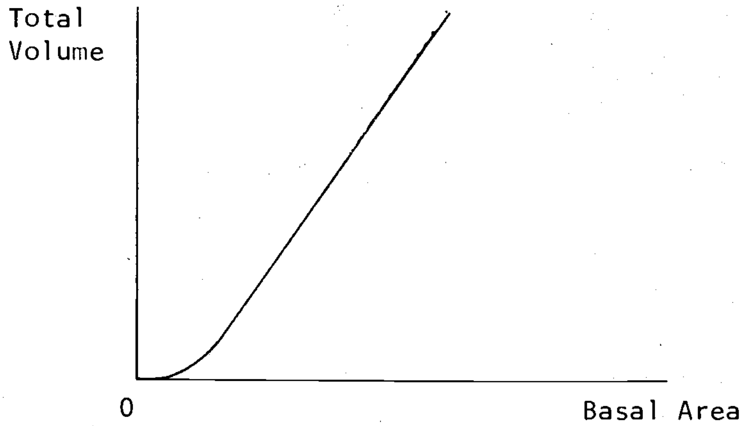

Figure 4. Curvilinearity of the basal area/total volume trend for the smaller trees.

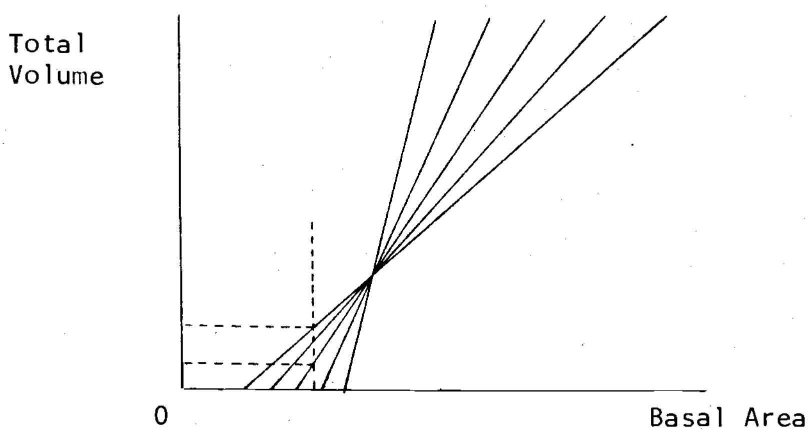

Figure 5. Harmonized volume trends with "moving intercept."

Let's call the regression coefficient <u>b</u>, b_4, Fig. 7.

$$CV4 = a + (b_4)\ (B) \qquad (22)$$

where B = basal area.

But when B = 0.0873 square feet, CV4 - 0.0

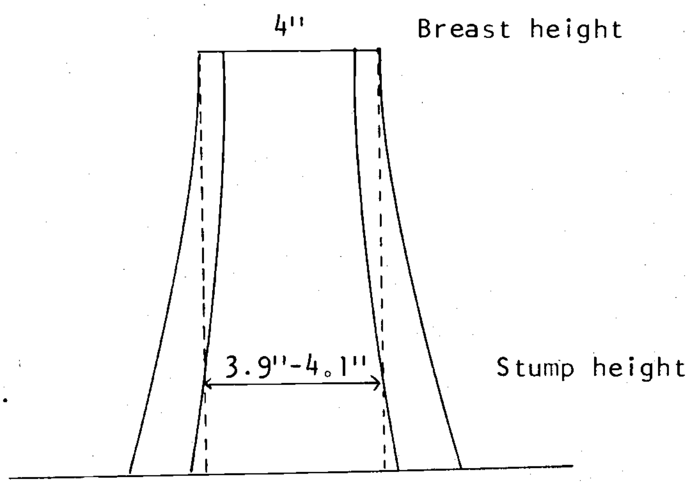

Figure 6. Relation between 4-inch diameter outside bark at breast height and the diameter inside bark at stump height.

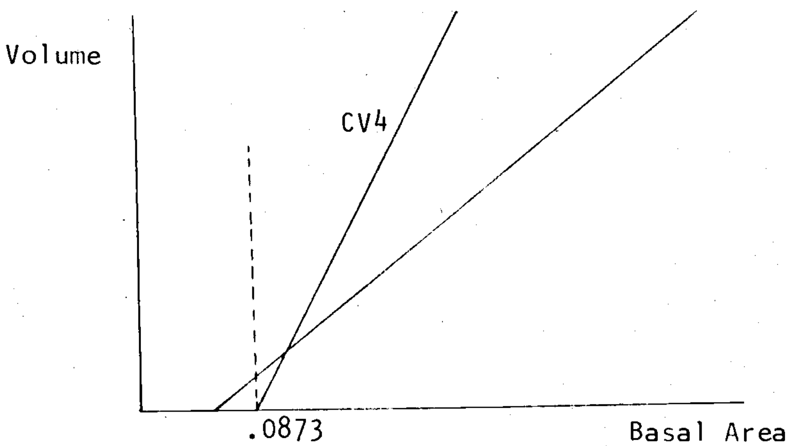

Figure 7. When basal area is 0.0873 square foot, CV4 is always 0.0.

So:

$$a = by\ (0.0873) \tag{23}$$

Then:

$$CV4 = by\ (0.0873) + b_4\ B \tag{24}$$

Or:

$$CV4 = by\ (\text{basal area} - 0.0873) \tag{25}$$

which represents the fixed intercept CV4/basal area line.

This shows that equations for different stands only differ by the slope of the regression line and each different equation is designated by the volume for a tree of 1 square foot of basal area. This is the tarif number (T), the CV4 for a tree of 1 square foot of basal area, that is, the average CV4 for the tree of 1 square foot of basal area.

If

$$CV4 = b_4\ (1.0 - 0.0873) \tag{26}$$

we have

$$T = 0.9127\ b_4 \tag{27}$$

and

$$b_4 = \frac{7}{0.9127} \tag{28}$$

If tarif number and DBH are known, the CV4 can be computed as:

$$CV4 = \frac{\overline{T}}{0.9127}\ (B - 0.0873) \tag{29}$$

If on the other hand, CV4 and DBH are known, the tarif number can be computed as:

$$T = \frac{CV4\ (0.9127)}{B - (0.0873)} \tag{30}$$

The ratio $\frac{0.9127}{B - 0.0873}$ is called <u>Tarif Access Constant for CV4</u>, represented by <u>TA4</u>.

So, the equation (30) can be written as:

$$T = (CV4).(TA4) \tag{31}$$

In using tarif system in terms of CV4 one should follow these four steps as recommended by Turbull and Hoyer:

1. Measure sample trees in a stand and estimate CV4 for each tree.

2. Compute T for each sample tree by using equation (30).

3. Average the sample tree T values to obtain T mean, $\overline{T}$.

4. Use this sample mean, $\overline{T}$, in equation (29) to compute the CV4 estimate for each DBH class midpoint. This will yield the equivalent of a Local Volume Table.

In choosing equation (30) in step 2, use tarif access constant tables of CV4.

Notice that

$$CV4 = \frac{T}{0.9127}\ (B-0.0873) \tag{32}$$

and

$$\frac{B - 0.0873}{0.9127} = TV4 \tag{33}$$

which is called Tarif Volume Constant for CV4 that is the inverse of TA4:

$$TV4 = 1/TA4 \tag{34}$$

To adopt this system of fixed-intercept CV4/basal area as the basis for the comprehensive tarif system the authors constructed weighted CV4/basal area lines for each sample plot by a special method conditioning the regression and compared them with the original CV4/basal area local volume tables, only weighted, following 2 criteria:

1. The volume for trees with sample mean basal area estimated by both equations should not differ by more than 2 standard errors of the mean.

2. The tarif volume line should lie within the confidence interval of the local volume table regression.

If no significant difference or consistent bias are shown in either test, the difference between the two lines is considered as sampling error.

The estimated Tarif has a statistical error that is Student's t times the standard error of the mean tarif and so its confident limits are easily determined.

As trees vary in form from species to species, from age to age and from site to site for the same species, tarif tables have been constructed for a combination of different species, ages and sites. As these tables are limited to those factors combinations, another variable, Tree Total Height, may be used to make a more convenient determination of individual sample tree volume, through a double entry table in place of using equation (30). This avoids the use of inadequate standard volume tables to get the volume in function of DBH only and utilizes sample trees total height, a very strong covariable when species, age and site are controlled.

Tarif tables offer not only DV4 but other merchantable volumes like CVTS (cubic foot volume including top and stump), CVT (cubic ft. volume including only top), CV6 (cubic foot volume to 6-inch top), CV8 (cubic foot volume to 8-inch top), IV6 (International 1/4-inch volume to 6-inch top), IV8 (International 1/4 inch to 8-inch top), SV6 (Scribner to 6-inch top) and SV8 (Scribner to 8-inch top).

Since the relationship CV4/basal area is the basis of the system, the conversions to other volume units were made through the study of the trend of the ratio "volume in study"/CV4 over DBH. In this paper will be shown the conversion of CV4 to CVTS, the unit used in the future research.

When a trend of CVTS/CV4 (=RTS4) over DBH of a sample is traced, it is easy to see that this trend is asymptotic (Fig. 8) "but since there is actual volume to 4-inch trees of less than 0.0873 sq. ft. of basal area, it is impossible to derive an actual ratio of CVTS/CV4", (Turbull, 1965). The problem can be overcome by measuring all volumes from a "zero" level which is located 2 times the volume intercept distance below the origin (Fig. 9).

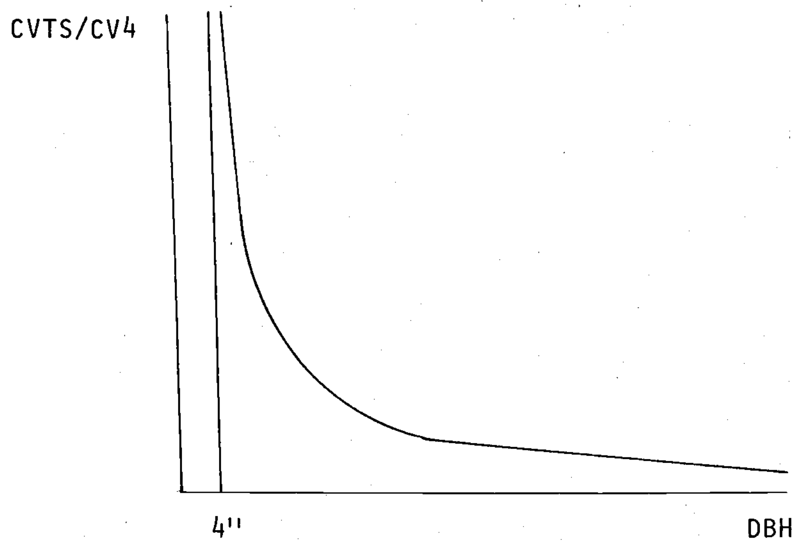

Figure 8. Asymptotic curve of CVTS/CV4 ratio on DBH.

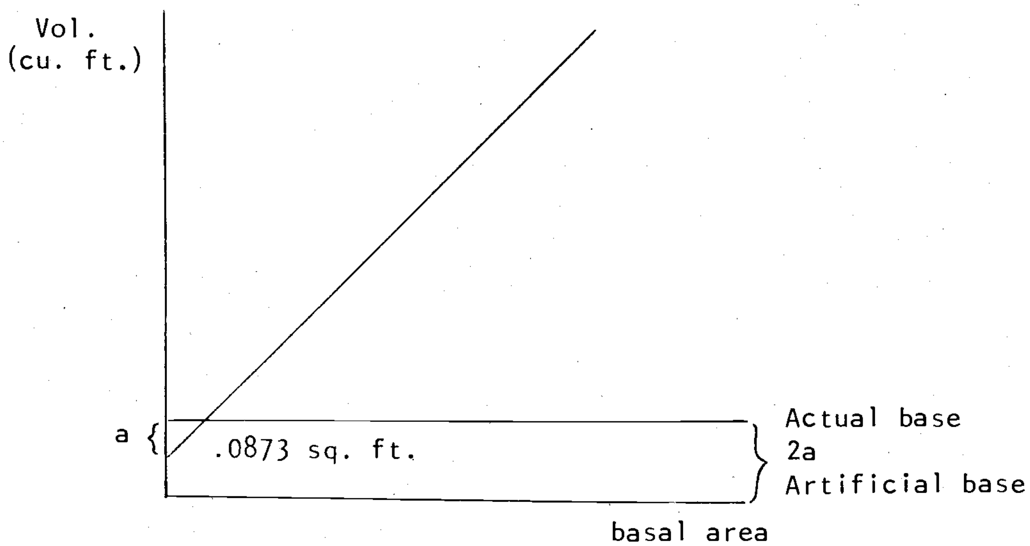

Figure 9. The artificial base for deriving an actual CVTS/CV4 ratio.

Let's call CVTS' and CV4' respectively CVTS and CV4 measured from the artificial base located two times the Volume-intercept below the origin. And let's all RTS4' the ratio CVTS'/CV4'.

$$CVTS' = CVTS + 2a \tag{35}$$

$$CV4' = CV4 + 2a \tag{36}$$

So, if $\dfrac{CVTS'}{CV4'} = RTS4'$,

$$CVTS = CVTS' - 2a \tag{37}$$

and

$$CVTS = RTS4' \; (CV4') - 2a \qquad (38)\,(38)$$

$$= RTS4' \; (CV4 + 2a) - 2a \tag{39}$$

$$= RTS4' \; (b_4 B - a + 2a) - 2a \tag{40}$$

$$= RTS4' \; (b_4 B + a) - 2a \tag{41}$$

Note that if B = 0,DBH = 0 and CVTS = 0 and note that

$$a = -b_4 \; (0.0873) \tag{42}$$

So, for CVTS = 0:

$$0 = RTS4' \; [b_4 \; (0.0) - b_4 \; (0.0873)] + 2 \; [b_4 \; (0.0873)] \tag{43}$$

$$0 = RTS4' \; [-b_4 \; (0.0873)] + 2 \; b_4 \; (0.0873) \tag{44}$$

$$RTS4' \; b_4 \; (0.0873) = 2 \; b_4 \; (0.0873) \tag{45}$$

$$RTS4' = 2$$

when DBH = 0 .

Now, the function RTS4' was found to be a sample asymptotic regression with formula:

$$RTS4' = A + B\ e^{-kX} \tag{47}$$

and when DBH = 0:

$$RTS4' = A + Be^{-k(o)}$$

$$RTS4' = A + B \tag{48}$$

So:

$$2 = A + B \tag{49}$$

or:

$$B = 2 - A \tag{50}$$

where A is the estimate asymptote.

The trend of CVTS'/CV4' ratio over DBH is shown in figure (10).

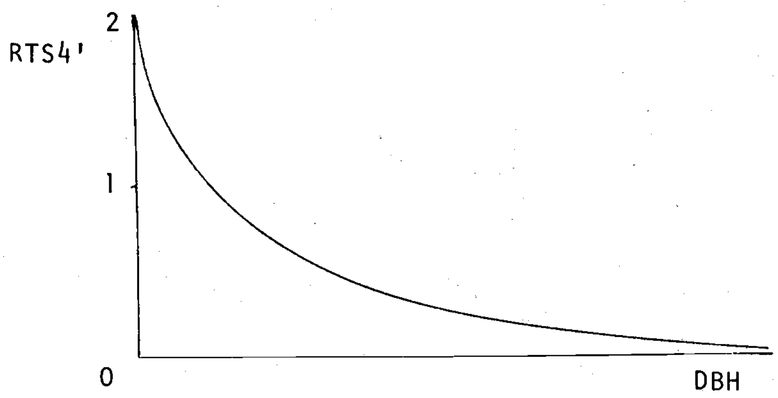

Figure 10. Trend of CVTS'/CV4' ratio.

As we have the value for CVTS as shown in formula (41), and we know that b_4, the slope of the volume line regression for CV4, is $\frac{T}{0.9127}$, and more, that $a = b_4$ (0.0873), we find the general formula for CVTS:

$$CVTS = T\ [\frac{RTS4'\ (B) + RTS4'\ (0.0873) - 2\ (0.0873)}{0.9127}] \qquad (51)$$

or:

$$CVTS = T\ (TVTS) \qquad (52)$$

where TVTS is total volume top and stump.

If, for example, a tree is a 15-inch DBH,

$$T = \frac{52.3\ (0.9127)}{1.2272 - 0.0873} = 41.9 \qquad (53)$$

$$TVTS = \frac{RTS4'\ (1.2272 + 0.0873) - 0.1746}{0.9127} \qquad (54)$$

In our case, DBH = 15:

$$RTS4' = 1.0385704 \qquad (55)$$

$$TVTS = \frac{1.0385704\ (1.2272 - 0.0873) - 0.1746}{0.9127} \qquad (56)$$

$$= 1.3045$$

$$CVTS = 41.9\ (1.3045) = 54.7 \qquad (57)$$

Constructing a Tarif Table - An Example

As a demonstration of tarif table construction, a sample of 20 trees was taken, randomized within each diameter class. The randomization was done using the HP-97 "Random Number Generator" program. On the next page is presented a list of the sample trees with their DBH, basal area, total volume and CV4. The regression volume by three different equations - simple regression, weighted regression and tarif - is also presented, as well as values $1/B$, $1/B^2$, V/B and V/B^2, necessary to compute the coefficients $\underline{a}$ and $\underline{b}$ of the weighted regression.

CV4 was calculated by subtracting the volume of the stump and the volume above the 4 inches top diameter from the total volume. When the data did not show a top volume exactly above 4 inches diameter, the proportion method was used to find this volume considering either a frustrum of paraboloid or a cone according to the method Weyerhaeuser Company adopted to calculate the standard volume of the trees. Two examples were chosen to demonstrate this method.

1. The frustrum of paraboloid: Table (2) shows the values for sample tree #1611.

The intermediary sections are assumed to be frustrums of paraboloids which volumes are determined by Smalian's formula. Thus the volume of section #5 is calculated as follows:

$$V = \frac{(4.4)^2 + (2.8)^2 \; (0.7854)}{(144) \; (12)} \; (10) = 0.74 \text{ cu. ft.} \qquad (58)$$

							REGRESSIONS						
1	2	3	4	5	6	7	8	9	10	11	12	13	14
#	DBH CLASS	SAMPLE NUMBER	DBH	BASAL AREA	CVTS	CV4	CV4 (S.R.)	CV4 (W.R.)	TARIF VOL.	1/B	$1/B^2$	V/B	V/B^2
1.	1.	1669.	4.1	.0917	2.1	0.2	-11.2	0.2	0.2	10.9051	118.9218	2.1810	23.7443
2.	1.	175.	6.6	.2376	8.6	7.3	- 3.6	6.1	6.9	4.2088	17.7136	30.7239	129.3094
3.	1.	1611.	7.3	.2907	4.8	3.9	- 0.1	8.2	9.3	3.4400	11.8334	13.4159	46.1503
4.	2.	980.	10.2	.5675	19.4	18.4	15.5	19.4	22.0	1.7621	3.1050	32.4229	57.1329
5.	2.	1221.	11.2	.6842	26.8	25.9	22.0	24.2	27.4	1.4616	2.1362	37.8544	55.3266
6.	3.	821.	11.7	.7466	19.7	18.8	25.5	26.7	30.2	1.3394	1.7940	25.1800	33.7273
7.	3.	43.	13.0	.9218	35.7	34.5	35.4	33.8	38.3	1.0848	1.1769	37.4268	40.6013
8.	3.	1640.	15.4	1.2935	50.8	49.7	56.2	48.9	55.1	0.7731	0.5977	38.4229	29.7046
9.	4.	132.	15.5	1.3104	74.6	73.6	57.2	49.6	56.1	0.7631	0.5824	56.1661	42.8618
10.	4.	796.	16.6	1.5030	44.4	43.6	68.0	57.4	64.9	0.6653	0.4427	29.0086	19.3005
11.	5.	512.	20.6	2.3145	124.0	122.4	113.5	90.3	102.2	0.4321	0.1867	52.8840	22.8490
12.	5.	799.	21.3	2.4745	83.6	82.0	122.5	96.8	109.5	0.4021	0.1633	33.1380	13.3918
13.	5.	1765.	23.3	2.9610	116.9	115.2	149.8	116.5	131.8	0.3377	0.1141	38.9058	13.1394
14.	6.	1081.	23.7	3.0636	160.9	159.1	155.5	120.6	136.5	0.3264	0.1065	51.9324	16.9514
15.	6.	526.	23.9	3.1155	215.7	213.8	158.4	122.7	138.9	0.3210	0.1030	68.6246	22.0268
16.	7.	488.	28.0	4.2761	223.4	217.3	223.6	169.8	192.2	0.2339	0.0547	50.8173	11.8840
17.	7.	229.	29.3	4.6823	234.9	231.1	246.4	186.3	210.8	0.2136	0.0456	49.3561	10.5410
18.	7.	1063.	29.7	4.8111	263.0	260.0	253.6	191.5	216.7	0.2079	0.0432	54.0417	11.2327
19.	8.	1160.	32.0	5.5851	286.6	283.6	297.0	222.9	252.2	0.1790	0.0321	50.7780	9.0917
20.	8.	1090.	34.2	6.3794	370.0	366.8	341.6	255.1	288.7	0.1568	0.0246	57.4976	9.0130
TOTAL			377.6	47.3101	2365.9	2327.2	2327.4	1857.0	2089.9	29.2156	159.1773	810.7788	617.9763
MEAN			18.9	2.3655	118.3	116.4	116.4	92.3	104.5				

Table 1. Quantities needed to construct a tarif table.

TABLE 2: Measures of sample tree #1611.

DBH	HEIGHT	CVTS	SECT. #	CUM HT.	DIB	CUB. VOL.
7.3	48.8	4.8	1	1.5	7.8	0.5
			2	4.5	5.8	0.8
			3	11.5	5.4	1.2
			4	21.5	4.4	1.3
			5	31.5	2.8	0.7
			6	48.5	0.0	0.2

It can easily be seen that DIB 4.0 inches falls in section #5 since 4.4 inches and 2.8 inches are the top limits of sections #4 and #5, respectively.

To find the length of the sub-section with extremities of 4.0 and 2.8 inches it is easier to use the proportion method. Since the method works only if the section is a frustrum of cone, there will be some error here as the section is assumed to be a frustrum of paraboloid. However, the error is insignificant.

So, if the diameter decreases from 4.4 to 2.8 (1.6 inches) at length of 10 feet (from 31.5'' to 21.5'') then at length of <u>x</u> feet the diameter will decrease from 4.0 to 2.8 (1.2 inches). Fig. (8-a).

$$X = \frac{(10)\ (1.2)}{(1.6)} = 7.5 \text{ ft.}$$

And the volume of this sub-section is:

$$V = \frac{(4.0)^2 + (2.8)^2\ (0.7854)}{(144)\ (2)}\ (7.5) = 0.488 \text{ cu. ft.} \qquad (60)$$

The stump volume is found by dividing the volume of the first section by twice its own height which gives the volume of a cylinder of 1/2 foot high and diameter equal to the top diameter of the first section. In this case there is a major mathematical negative error since the actual diameter of the stump is bigger than the top diameter of the first section, but, from the practical point of view, this error can be neglected given the volume of the stump is already very small in comparison to the total volume of the tree. In the computation of the stump volume of the biggest sample tree for example, the theoretical error is no more than 0.4 cubic foot or 0.1% of the CV4, while the error for the smallest sample tree, in this case, is no more than 0.02 cubic foot or 0.4% of the CV4.

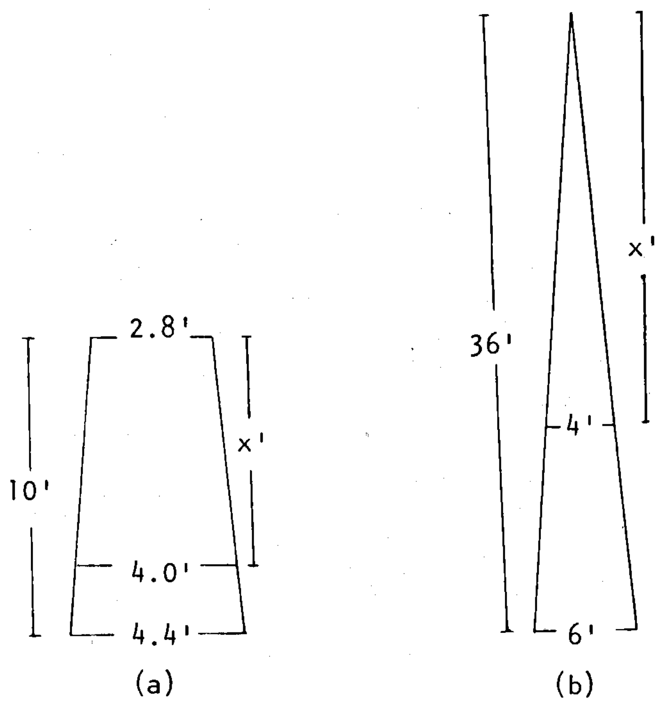

Figure 11. Necessary measures to calculate the volume above 4.0 inches diameter in a paraboloid frurtrum (a) and in a cone (b).

In this example the volume of the stump is:

$$\text{Stump vol.} = \frac{0.5}{(2)\ (1.5)} = 0.167 \text{ cu. ft.} \tag{61}$$

DV4 of the sample tree #1611 is calculated by subtracting from CVTS (4.8 cu. ft.) the stump volume (0.167 cu. ft.), the calculated volume of the upper sub-section of section #5 (0.488 cu. ft.) and the volume of the top section #6 (0.2 cu. ft.):

$$CV4 = 4.8 - (0.167 + 0.488 + 0.2) = 3.945 = 3.9 \text{ cu. ft.} \tag{62}$$

2. <u>The cone section</u>: Sample tree #43 has the following values

DBH	HEIGHT	CVTS	SECT. #	CUM. HT.	DIB	CUB. VOL.
13.0	95.5	37.7	1	1.0	13.3	1.0
			2	4.5	12.0	3.1
			3	17.5	11.3	9.6
			4	34.0	9.9	10.2
			5	49.5	8.0	6.8
			6	50.5	7.8	0.3
			7	59.5	6.0	2.4
			8	95.5	0.0	2.4

Table 3: Measures of sample tree #43.

In this case the computation of the length of the 4-inch diameter base cone has theoretically no error (Fig. 8-b). The length of this top conical section is computed as follows:

$$\text{Length } x = \frac{(95.5 - 59.5)\ (4.0)}{(6.0)} = 24.0 \text{ inches} \tag{63}$$

Its volume is computed as follows:

$$V = \frac{(4.0)^2\ (0.7854)}{(144)\ (3)}\ (24.0) = 0.698 \text{ cu. ft.} \tag{64}$$

The volume of the stump is given by:

$$\text{Stump vol.} = \frac{(1.0)}{(2)\ (1.0)} = 0.5 \text{ cu. ft.} \tag{65}$$

And the CV4 of the sample tree is:

$$CV4 = 37.5 - (0.698 + 0.5) = 34.5 \text{ cu. ft.} \tag{66}$$

The graph on the next page (Fig. 12) shows the three regression links. Simple regression line has the following equation, with r^2 = 0.9638:

$$CV4_{SR} = -16.3609 + 56.1069\ B \tag{67}$$

where B is basal area.

The values of the coefficients a and b for the weighted regression were computed through equations (68) and (69) below:

$$b = \frac{\Sigma(V/B)}{N} \tag{68}$$

$$b = \frac{\Sigma(V/B^2) - b_2\Sigma(1/B)}{\Sigma(1/B^2)} \tag{69}$$

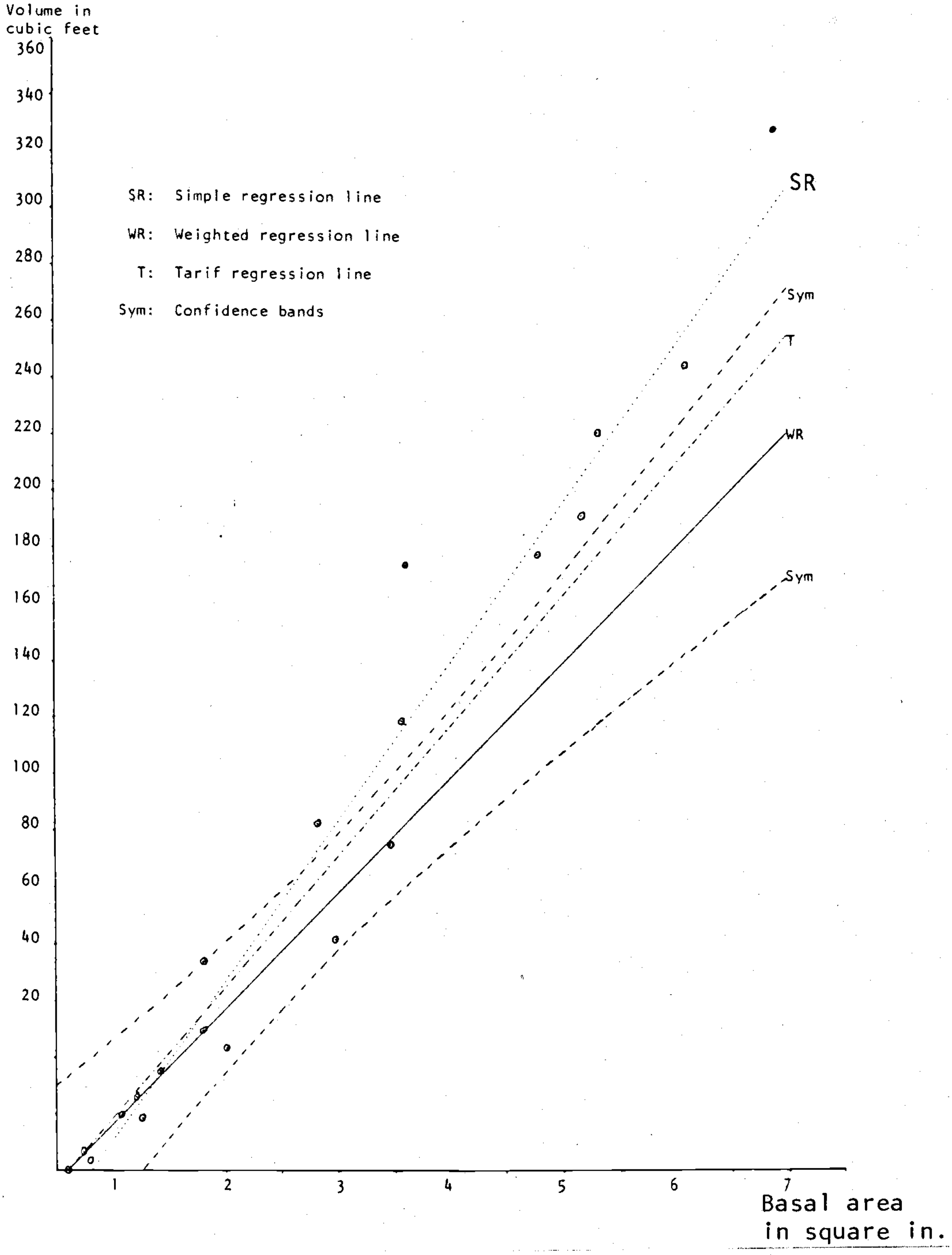

Figure 12: Comparing SR, WR and T regression lines.

The weighted regression equation is:

$$CV4_{WR} = -3.5583 + 40.5389\ B \tag{70}$$

The tarif number, T, for each sample tree was computed through equation 71 below and the sample mean T = 41.8701 was used in the tarif volume equation 72.

$$T = \frac{CV4\ (0.9127)}{(B - 0.0873)} \tag{71}$$

$$CV4_T = \frac{41.8701}{0.9127}\ (b - 0.0873) \tag{72}$$

As it was seen before, there are two conditions for accepting the results of a tarif table, although neither of those constitutes a valid significant test but "they do demonstrate the order of magnitude of difference between the two regressions in relation to estimate sampling error" (Turbull, 1965).

The first is that "the volume estimated for trees with sample mean basal area should not differ by more than 2 (Sym) standard errors between tarif and local volume table regressions" (Turbull, 1965).

These values in the present example are:

Tarif volume estimated for trees with

Sample mean basal area ....104.5 cu. ft.

Similar volume given by local volume regression.. 92.3 cu. ft.

Difference .... 12.2 cu. ft.

2 (Sym) .... 42.6 cu. ft.

Since 2 (Sym) is bigger than the difference between the two mean volumes, the condition is satisfied.

The second is that "the tarif volume line should lie within the confidence bands of the local volume table regression" (Turbull, 1965).

This condition is also perfectly satisfied as shown in the graph of figure 12.

Figure 13 compares CVTS and CV4 lines.

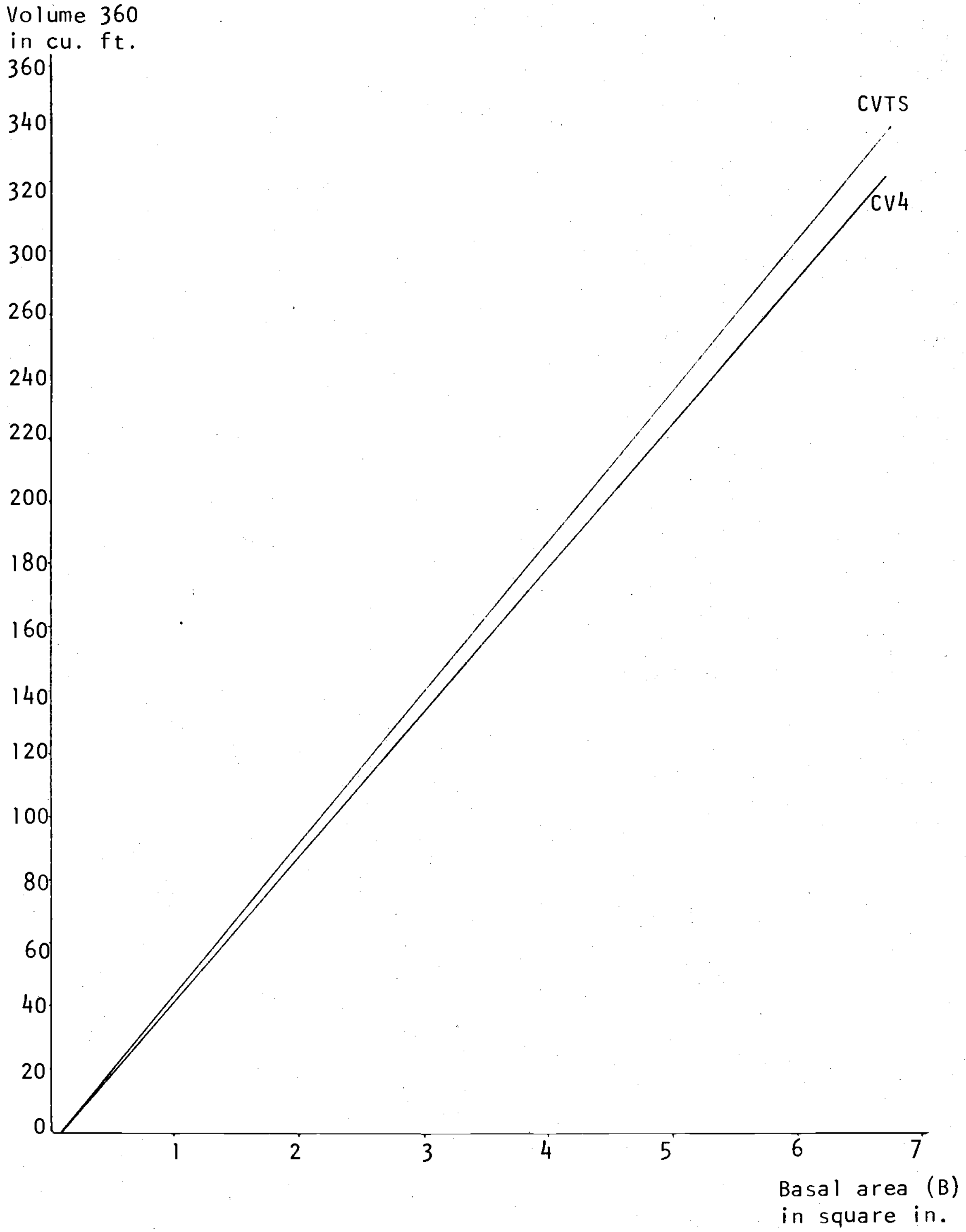

Figure 13. Comparing CVTS line with CV4 line.

STUDY MATERIAL

The data were provided by the Department of Natural Resources, State of Washington and consist of a computer output of the volume of 1,765 trees used in the construction of a standard volume table for a young growth of Douglas-fir, *Pseudotsuga menziesii* in Western Washington.

Total tree volume is the summation of the volumes of sections of the entire stem from the base to the top. The volume of the first section, which is called "stump", is calculated on the basis of it being considered a cylinder equal to its top cross-sectional area and length ranging between 0.5 and 4.5 feet. The next section from the stump to 4.5 feet is assumed to be a neiloid frustrum and its volume is calculated by formula (8). Sections above 4.5 feet to the top section were assumed to be paraboloid frustrums, their volumes are calculated by Smalian's formula (5). Top section was assumed to be a cone, its volume calculated by the formula for a cone.

A problem that could be considered of major importance is that the intermediary segments are not consistently divided into sections of equal length. As a result the total volume varies depending on the length of the sections since they are seldom perfect paraboloid frustrums. A good example can be seen in sample tree #240 which has the intermediary segment divided in sections of 6 feet except sections #13 and #14 of 4.5 and 1.5 feet, respectively. (Fig. 14). The summation of their volumes is 1.24 cubic feet while the volume of a whole 6-foot section (4.5 + 1.5) is 1.22 cubic feet. This

0.02-foot difference was not shown in the computer output. Many other trials were made in different trees with different sections lengths and the difference always had been shown to be minimum and so, insignificant, except for the largest trees that are out of the sample used in the present work.

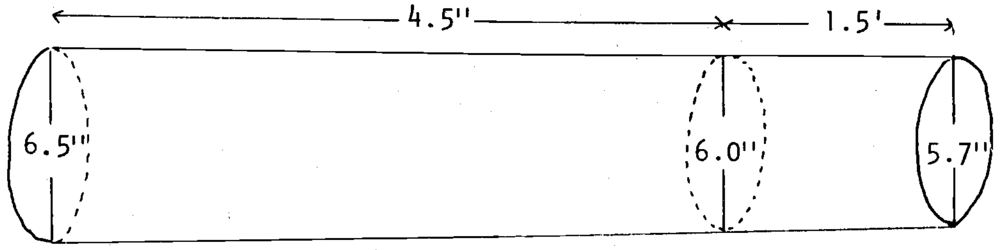

Figure 14. In sample tree #240 the difference between the calculated volume of a 6-foot log and the summation of the calculated volumes of a 4.5-foot log and a 1.5-foot log is insignificant.

From a total of 1,765 trees, 1,644 were used for the study, since these had DBH between 4.0 and 35.0 inches, which are the limits for the tarif table used.

Trees were grouped in 8 diameter classes with width of 3.9 inches except 7 with width 3.8 inches:

Class	Interval	Class	Interval
1	4.0" - 7.8"	5	19.6" - 23.4"
2	7.9" - 11.7"	6	23.5" - 27.3"
3	11.8" - 15.6"	7	27.8" - 31.1"
4	15.7" - 19.5"	8	31.2" - 35.0"

The distribution of the frequency in the DBH classes is shown in figure (15). The regression equation for frequency and DBH mean for the 8 DBH classes is:

$$F = 610.7028 - 29.6370\ (DBH) + 0.3734\ (DBH)^2 \tag{73}$$

Second degree polynomial was the one which fits the best with $R^2 = 0.9852$ compared to $r^2 = 0.9522$ for linear regression. The regression curve is shown in figure (15).

The regression that fits the best Volume X DBH is:

$$TV = -\ 10.5723 + 0.1026\ (DBH) + 0.3162\ (DBH)^2 \tag{74}$$

with

$$R^2 = 0.9995 \tag{75}$$

and where TV = total volume of the tree. Figure (16).

As it was expected, the best regression for Volume X Basal Area (B) was the simple linear regression. Figure (17). The equation is

$$TV = -9.7436 + 58.5016\ (B) \tag{76}$$

with

$$r^2 = 0.9984 \tag{77}$$

In search for the hypsometric relationships, a second degree convex polynomial (Figure 18) was found to show total height (TH) in function of DBH:

$$TH = 7.4317 + 9.0597\ (DBH) - 0.1119\ (DBH)^2 \tag{78}$$

with

$$R^2 = 0.9995$$

After all these investigations and considerations, Weyerhaeuser tree total volumes were considered to be a very good standard basis for this study.

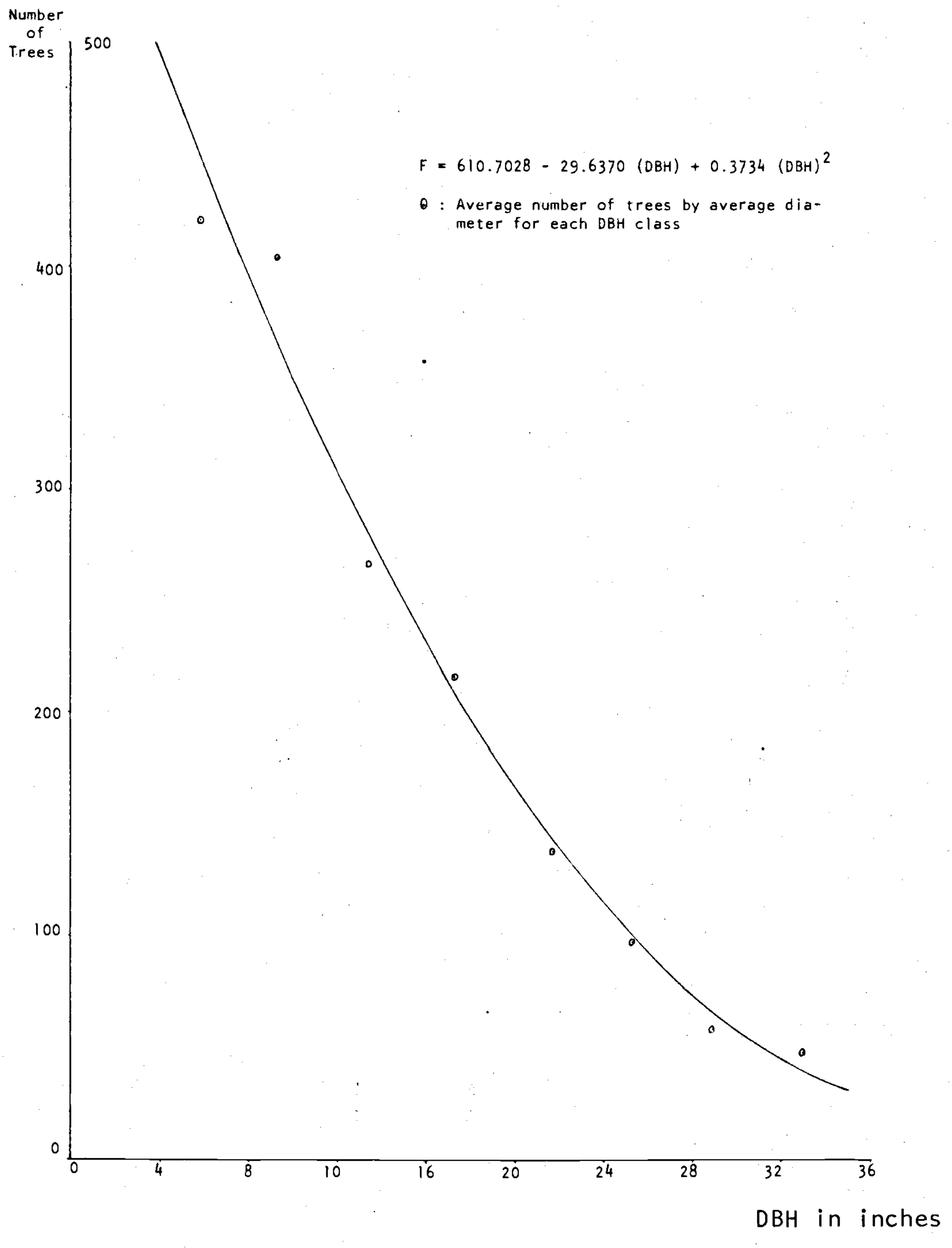

Figure 15. Regression of frequency on DBH.

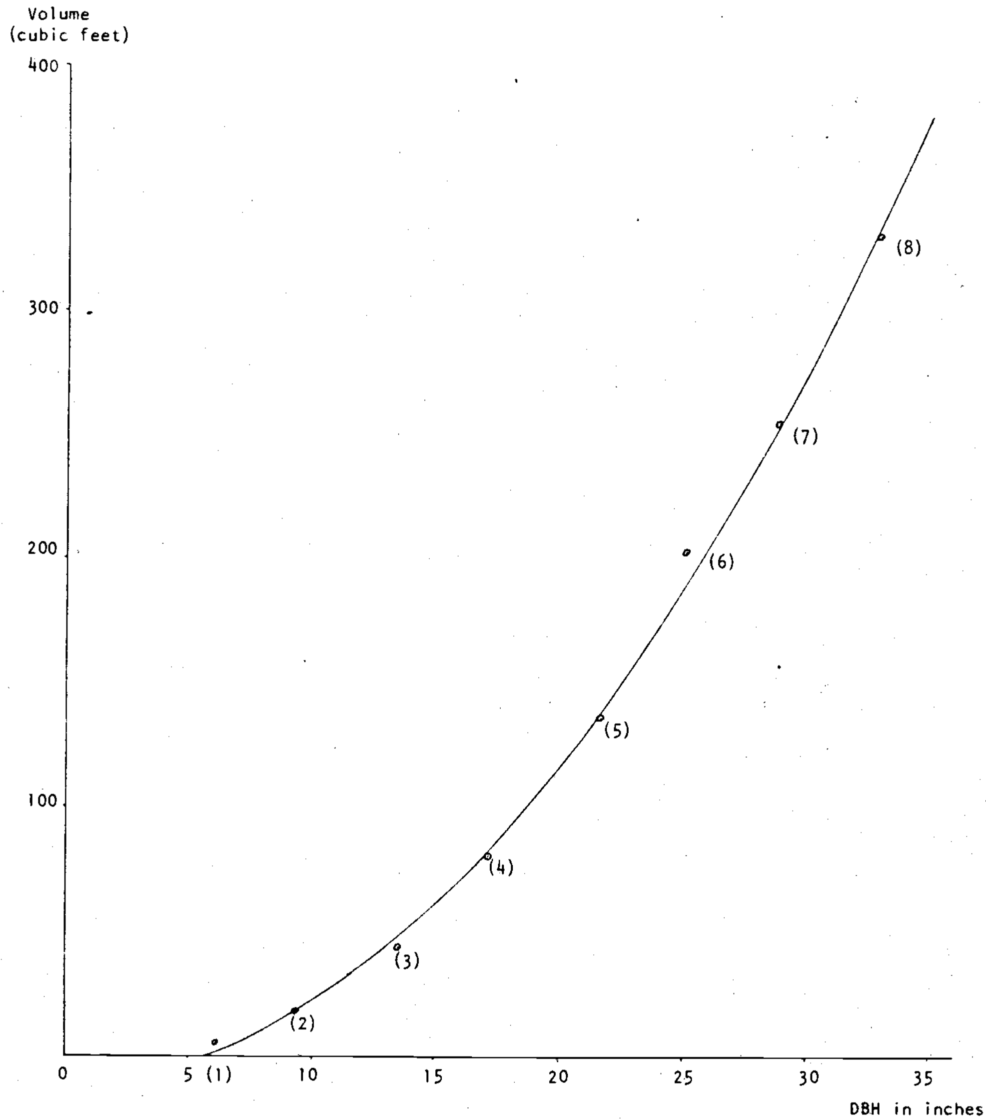

Figure 16. Regression of Total Volume on DBH.
$TV = 10.5723 + 0.1026\ (DBH) + 0.3165\ (DBH)^2$
Θ: Average volume by average diameter for each DBH class.
$R^2 = 0.9984$

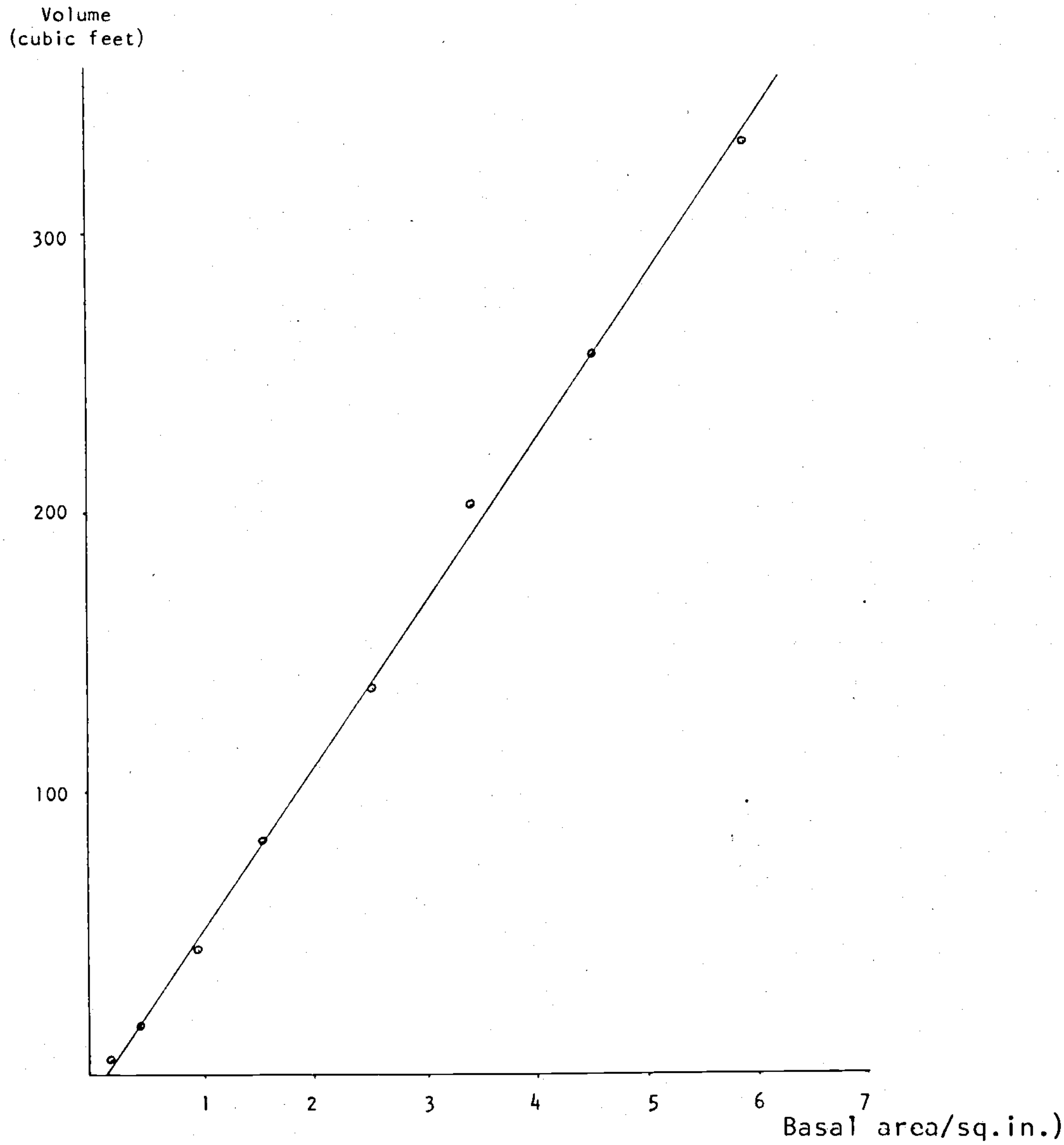

Figure 17. Regression of Total Volume over Basal Area.
Regression: TV = -9.7436 + 58.5016 (B)

θ: Average volume by average basal area for each diameter class.

r^2 = 9984

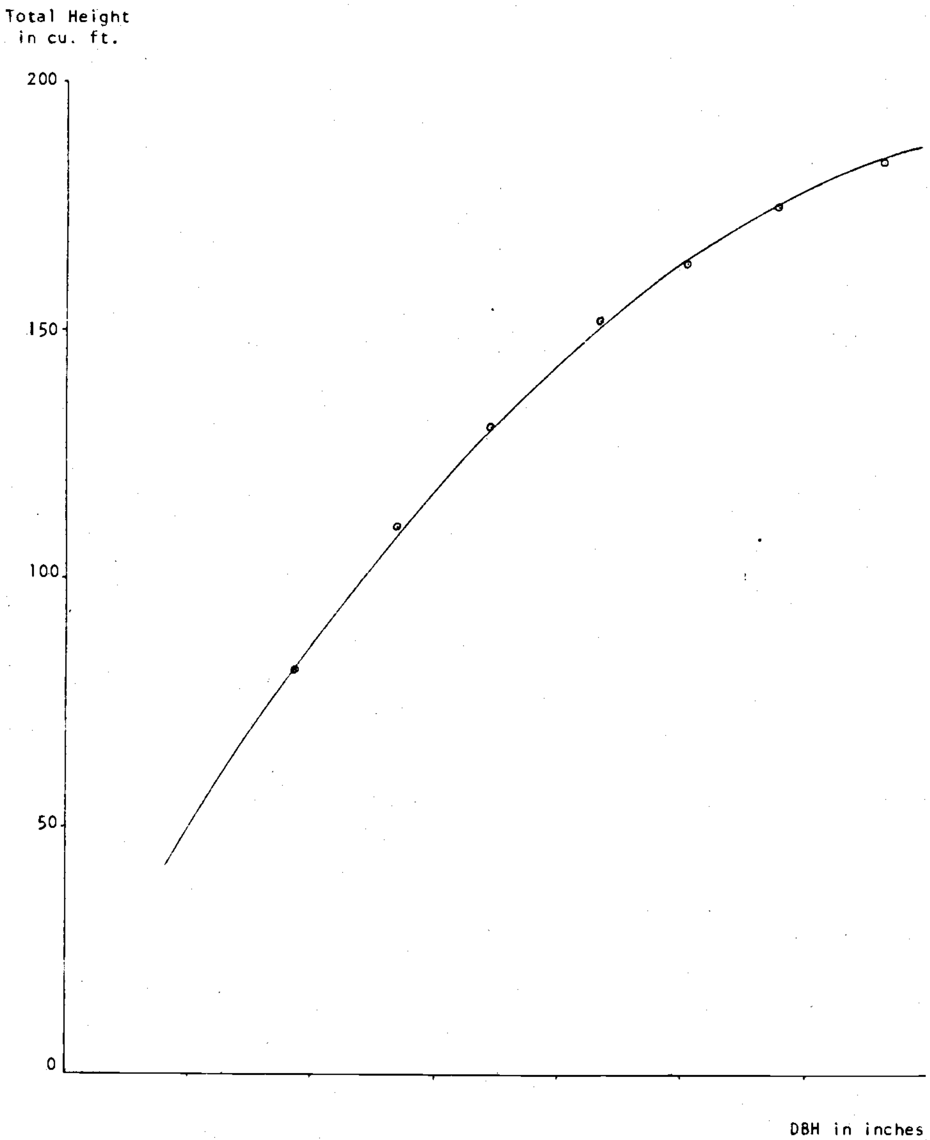

Figure 18. Regression of total height on DBH Equation:
$TH = 7.4317 + 9.0597\ (DBH) - 0.1119\ (DBH)^2$

Θ = Average total height by average diameter for each DBH class

$R^2 = 0.9995$

METHODS

The variable used to compare the three log rules was percent difference between the volume given by each of the three log rules and the standard volume calculated by Weyerhaeuser Company:

$$\text{Percent Difference Volume} = \frac{X - S}{S} (100) \qquad (80)$$

where S is the standard volume and X is either A, the volume given by the Weyerhaeuser Douglas-Fir Cubic Volume Equation, B, the volume given by the British Columbia Coast Immature Douglas-Fir Volume Equation or C, the volume given by David Bruce's Equations for Second-Growth Douglas-Fir.

Log rule A equation is the original tarif equation modified by Turnbull and King:

$$\begin{aligned} \text{Log CVTS} = \; & 0.321809 \qquad (81) \\ & + 0.04948 \log TH \log DBH \\ & - 0.15664 (\log DBH)^2 \\ & + 2.02132 \log DBH \\ & + 1.63408 \log TH \\ & - 0.16185 (\log TH)^2 \end{aligned}$$

which has been changed for the computer to:

```
CVTS  =  10** (-3.21809)                                        (82)
         *TH** (LOG(DBH)* 0.04948)
         *DBH** (LOG(DBH)*(-0.15664)
         *DBH**2.02132
         *TH**1.63408
         *TH**(LOG(TH)*(-0.16185)
```

$$CVTS = (10^{-3.21809}) (TH^{0.04948 \log DBH}) (DBH^{-0.15664 \log DBH}) (DBH^{2.02132}) (TH^{1.63408}) (TH^{-0.16185}) \quad (83)$$

Log rule B equation is presented in the form:

$$\text{Log } CVTS = -2.658025 + 1.739925 (\log DBH) + 1.133187 (\log TH) \quad (84)$$

which has been changed for the computer to

CVTS = 10** - 2.658025* DBH** 1.739925* TH** 1.133187 (85)

$$CVTS = (10^{-2.658025})(DBH^{1.739925})(TH^{1.133187}) \quad (86)$$

Log rule C equations are VS and VL:

$$VS = 0.005454154 \text{ FOS } (DBH^2 \text{ TH}) \quad (87)$$

where VS is the volume of trees with total height (TH) equal or smaller than 18 feet and FOS is a form factor for trees of TH = 18 feet:

$$VL = 0.005454154 \text{ FOL } (DBH^2 \text{ TH}) \quad (88)$$

where VL is the volume of trees with TH = 18 feet and FOL is the form factor for those trees.

As there is no definite breaking point between these two categories, VS and VL give virtually the same volume for an 18-foot tree.

FOS and FOL formulas are given below:

$$\begin{aligned} FOS = {} & 0.406098\ (TH - 0.9)^2 / (TH - 4.5)^2 && (89) \\ & - 0.0762998\ (DBH)\ (TH - 0.9)^3 / (TH - 4.5)^3 \\ & + 0.00262615\ (DBH)\ (TH)\ (TH - 0.9)^3 / (TH - 4.5)^3 \end{aligned}$$

$$\begin{aligned} FOL = {} & 0.480961 && (90) \\ & + 42.46542 / (TH)^2 \\ & - 10.99643\ (DBH) / (TH)^2 \\ & - 0.107809\ (DBH) / (TH) \\ & - 0.00409083\ (DBH) \end{aligned}$$

Since 1644 trees were used in the present work and 3 log rules were compared, a total of 4832 data were processed for the comparative study. The data were split into 8 DBH classes as shown below:

Class	Interval in inches
1	4.0 - 7.9
2	8.0 - 11.8
3	11.9 - 15.6
4	15.7 - 19.5
5	19.6 - 23.4
6	23.5 - 27.3
7	27.4 - 31.1
8	31.2 - 35.0

A two-way analysis of variance was performed on the data to see if there were any significant differences among the percent difference volumes given by the 3 log rules as well as to see if the percent difference volumes were significantly different among the 8 DBH classes.

The purpose of comparing percent difference volumes among the DBH classes was to investigate the possibility of a log rule being a better estimator than another for a certain size of tree.

In case there is a difference among log rules and/or DBH classes, a test was planned for inspection of all differences between pairs of means.

Since "with unequal sample size the F- and t-test are more affected by non-normality and heterogeneity of the variance than with equal sample size" (Snedecor, 1973), before using any test for comparing pairs of means, a χ^2-test - the Bartlett's Test of Homogeneity of Variance - was performed to see if there was homogeneity among the variances of the 8 DBH classes. Although the 3 log rules samples have the same size, the Bartlett's test was applied among their variance because, based on a preliminary test, they showed not being homogenous.

The Bartlett's test has the following equation:

$$\chi^2_{(K-1)} = 2.3026 \ (\log S^2)(\Sigma(n_i - 1)) - \Sigma \ (n_i - 1)(\log S_1^2) \quad (91)$$

where S^2 is the pooled within class variance and S^2 is the variance of class i. The figure 2.3026 is a constant of approximate transformation of common logarithm to natural logarithm which is used in the original formula. K is the number of variances to be tested.

The formula for the pooled within DBH class variance, S^2 is:

$$S^2 = \frac{\Sigma ss_i}{\Sigma \ (n-1)} \quad (92)$$

When χ^2 is significant, a correction should be applied. Its formula is:

$$C = \frac{3\ (K - 1) + [\ \Sigma(\frac{1}{n_i - 1}) - \frac{1}{(n_i - 1)}\ ]}{3(K-1)} \qquad (93)$$

And the corrected value of χ^2 is then:

$$\text{Corrected } \chi^2 = \frac{\text{uncorrected } x^2}{C} \qquad (94)$$

As the homogeneity of variance was expected to be rejected, the test chosen for inspection of difference between pairs of means was LSD-test because it deals with the standard error of the difference, $S_{\bar{d}}$, which can be calculated separately for each pair of means, and in case of heterogeneity of variance, it seems to be more prudent to calculate $S_{\bar{d}}$ for each pair of means, as shown below, rather than use a $S_{\bar{d}}$ from a pooled variance of the analysis of variance:

$$S_{\bar{d}} = \sqrt{\frac{S_i^2}{n_i} + \frac{S_k^2}{n_k}} \qquad (95)$$

$$LSD = t'\ (S_{\bar{d}})$$

where

$$t' = \frac{w_1 t_1 + w_2 t_2}{w_1 + w_2} \qquad (96)$$

and

$$w_i = \frac{S_i^2}{ni}$$

t_i is Student's $\underline{t}$ for $n_i - 1$ degrees of freedom.

RESULTS

ANALYSIS OF VARIANCE

The two-way analysis of variance with highly ($\alpha = 0.01$) significant value of F for both log rules and DBH classes is shown below.

SOURCES	SS	DF	MS	F
LOG RULES	3074.994	2	1537.497	14.825**
DBH CLASSES	15414.585	7	2202.084	21.234**
LOG RULES X DBH CLASSES	6504.032	14	464.574	4.480**
RESIDUAL	508996.153	4098	103.707	
TOTAL	540959.128	4931		

TABLE 4. Analysis of Variance table for two factors; log rules and DBH classes. ** Significantly different at 0.01.

TEST OF HOMOGENEITY OF VARIANCE

1. FOR LOG RULES:

The quantities needed for the Barlett's Test of Homogeneity of Variance are tabulated here:

RULE	Si^2	n-1	CORRECTED SS	$\frac{1}{n-1}$	LOG Si^2	(n-1)(log Si^2)
1	88.5204	1643	145,539.01	0.0006086	1.9470433	3,198,9921
2	97.6866	1643	160,499.08	0.0006086	1.9898348	3,269.2987
3	136.9304	1643	224,976.64	0.0006086	2.1364998	3,510.2692
TOTAL		4929	530,914.73	0.0018258		9,667.2829

TABLE 5. Quantities needed for Bartlett's test on the homogeneity of the variance of the three log rules.

The pooled within variance is computed through the corrected Sum of Squares (SS):

$$\bar{S}^2 = \frac{\Sigma\ SS_1}{\Sigma\ (n_1 - 1)} = 107.7125$$

The calculated value of χ^2 was significant in contrast with the tabulated value:

Calculated χ^2 = 805.566

Tabulated χ^2 (2 d.f., = 0.01) = 0.020

2. FOR BDH CLASSES

Similar tabke, as for log rules, is presented for DBH classes in log rule A. For log rules B and C the results are very similar.

CLASS	S^2	n-1	CORRECTED SS	$\frac{1}{n-1}$	Log Si^2	(n-1)(log Si^2)
1	83.1167	420	34,909.014	0.00238	1.9196882	806.26904
2	69.9372	399	27,865.042	0.00251	1.8440868	735.79063
3	79.6907	263	20,958.654	0.00380	1.9014076	500.07019
4	88.5585	216	19,128.636	0.00463	1.9472302	420.60172
5	112.1073	137	15,358.700	0.00730	2.0496338	280.79984
6	113.9831	97	11,056.360	0.01031	2.0568404	199.51352
7	110.9611	57	6,324.783	0.01754	2.0451707	116.57472
8	179.1625	47	8,420.638	0.02128	2.2532471	105.90261
TOTAL		1636	144,021.827	0.06975		3,165.52227

TABLE 6. Quantities needed for Bartlett's test in log rule A.

The pooled within variance for DBH classes is:

$$\bar{S}^2 = \frac{144.021.827}{1636} = 88.0329$$

Calculated $\chi^2 = 36.65$

Tabulated $\chi^2_{(7\text{ d.f.},\ \alpha = 0.01)} = 1.24$

The homogeneity assumption was rejected for both log rules and DBH classes. That means that the standard error of the difference, $S_{\bar{d}}$, should be calculated for each pair of means, to compare then through LSD-test.

TEST FOR COMPARING MEANS: LSD-TEST

1. COMPARING LOG RULES MEANS:

Table 8 shows the means and variances of the 3 log rules, as well as the means and variances for the 8 DBH classes within each log rule.

The table below (Table 7) shows the absolute differences between the log rules means.

Log Rule	Log Rule A	Log Rule C
B	2.62**	3.31**
A	--	0.69

** = significantly different at 0.01.

TABLE 7. Differences between two log rules.

The results of the test can be summarized as follows:

Log Rule:	B	A	C
Mean:	-1.33	1.29	1.98

Any two means not underscored by the same line are significantly different.

2. COMPARING DBH CLASSES MEANS FOR EACH LOG RULE:

Table 8 also shows means and variances of the 8 DBH classes for each log rule. As in table 7 for log rules means differences, the differences between two means are presented in tables 9, 10 and 11.

Tables 9, 10 and 12 show LSD for each pair of means.

	Code	Sum	Mean	STD DEV	Variance	N
For entire population		3203.0000	.6494	10.4741	107.7058	4932
Log Rule	A.	2125.0000	1.2926	9.4085	88.5204	1644
Class	1.	898.3000	2.1337	9.1168	83.1166	421
Class	2.	473.7000	1.1842	8.3569	69.8372	400
Class	3.	119.6000	.4530	8.9270	79.6907	264
Class	4.	8.9000	.0410	9.4106	88.5585	217
Class	5.	204.2000	1.4797	10.5881	112.1073	138
Class	6.	56.7000	.5786	10.6763	113.9831	98
Class	7.	235.0000	4.0517	10.5338	110.9611	58
Class	8.	128.6000	2.6792	13.3852	179.1625	48
Log Rule	B.	-2184.6000	-1.3286	9.8837	97.6866	1644
Class	1.	75.3000	.1789	9.2030	84.6959	421
Class	2.	-1573.6000	-3.9340	8.0915	65.4730	400
Class	3.	-1045.8000	-4.0030	8.9129	79.4392	264
Class	4.	-624.8000	-2.8793	9.4342	89.0050	217
Class	5.	73.4000	.5319	10.6003	112.3660	138
Class	6.	149.1000	1.5214	10.9346	119.5660	98
Class	7.	411.9000	7.1017	11.2172	125.8265	58
Class	8.	360.9000	7.5187	14.0564	197.5820	48
Log Rule	C.	3262.6000	1.9845	11.7017	136.9304	1644
Class	1.	772.3000	1.8344	15.5563	241.9982	421
Class	2.	-8.2000	-.0205	8.5122	72.4578	400
Class	3.	313.4000	1.1871	9.4757	89.7884	264
Class	4.	469.7000	2.1645	9.8224	96.4797	217
Class	5.	665.9000	4.8254	11.0295	121.6488	138
Class	6.	394.7000	4.0276	11.0916	123.0245	98
Class	7.	406.3000	7.0052	11.2968	127.6170	58
Class	8.	248.5000	5.1771	13.7089	187.9346	48

Table 8. Table of important statistics for log rules.

2a. FOR LOG RULE A:

DBH CLASS \ DBH CLASS	3	6	2	5	1	8	7
4	0.41	0.54	1.14	1.44	2.09**	2.64**	4.01**
3		0.13	0.73	1.03	1.68	2.23	3.60
6			0.60	0.90	1.55	2.10	3.47
2				0.30	0.95	1.50	2.87
5					0.65	1.20	2.57
1						0.55	1.90
8							1.37

**Significantly different at 0.01.

Table 9. Differences between two DBH class means for log rule A.

TEST SUMMARY:

DBH Class:

DBH CLASS \ DBH CLASS	2	4	1	5	6	7	8
3	0.07	1.12	4.18**	4.53**	5.52**	11.10**	11.52**
2		1.05	4.11**	4.46**	5.45**	11.03**	11.45*8
4			3.06**	3.42**	4.40**	9.98**	10.40**
1				0.35	1.34	6.92**	7.34**
5					0.99	6.57**	6.99**
6						5.58**	6.00**
7							0.42

**Significantly different at 0.01.

Table 10. Differences between two DBH class means for log rule B.

		SUM	MEAN	STD DEV	VARIANCE	N
For entire population		3203.0000	.6494	10.4741	109.7058	4932
Class	1	1745.9000	1.3823	11.7101	137.1268	1263
Log Rule	A	898.3000	2.1337	9.1168	83.1167	421
Log Rule	B	75.3000	.1789	9.2030	84.6959	421
Log Rule	C	772.3000	1.8344	15.5563	241.9982	421
Class	2	-1108.1000	-.9234	8.5976	73.9182	1200
Log Rule	A	473.7000	1.1842	8.3569	69.8372	400
Log Rule	B	-1573.6000	-3.9340	8.0915	65.4730	400
Log Rule	C	-8.2000	-.0205	8.5122	72.4578	400
Class	3	-623.8000	-.7876	9.3824	88.0288	782
Log Rule	A	119.6000	.4530	8.9270	79.6907	264
Log Rule	B	-1056.8000	-4.0030	8.9129	79.4392	264
Log Rule	C	313.4000	1.1871	9.4757	89.7884	264
Class	4	-146.2000	-.2246	9.7647	95.3485	651
Log Rule	A	8.9000	.0410	9.4108	88.4484	217
Log Rule	B	-624.8000	-2.8793	9.4342	89.0050	217
Log Rule	C	469.7000	2.1645	9.8224	96.4797	217
Class	5	943.5000	2.2790	10.6727	118.2153	414
Log Rule	A	204.2000	1.4797	10.5881	112.1073	138
Log Rule	B	73.4000	.5319	10.6003	112.3660	138
Log Rule	C	665.9000	4.8254	11.0295	121.6488	138
Class	6	600.5000	2.0425	10.9623	120.1721	294
Log Rule	A	56.7000	.5786	10.6763	113.9831	98
Log Rule	B	149.1000	1.5214	10.9346	119.5660	98
Log Rule	C	394.7000	4.0276	11.0916	123.0245	98
Class	7	1053.2000	6.0529	11.0490	122.0794	174
Log Rule	A	235.0000	4.0517	10.5338	110.9611	58
Log Rule	B	411.9000	7.1017	11.2172	125.8265	58
Log Rule	C	406.3000	7.0052	11.2968	127.6170	58
Class	8	738.0000	5.1250	13.7668	189.5261	144
Log Rule	A	128.6000	2.6792	13.3852	179.1625	48
Log Rule	B	360.9000	7.5187	14.0564	197.5820	48
Log Rule	C	248.5000	5.1771	13.7089	187.9346	48

Table 11. Table of important statistics for DBH classes

TABLE 11.

TEST SUMMARY:

DBH Class:	3	2	4	1	5	6	7	8
Mean:	-4.00	-3.93	-2.88	0.18	0.53	1.52	7.10	7.52

2 c: FOR LOG RULE C:

DBH CLASS (rows) \ DBH CLASS	3						
2	0.21	1.85	2.18**	4.05**	4.85**	5.20	7.03**
3		0.64	0.97	2.84	3.64**	3.99	5.82**
4			0.33	2.22	3.00	3.35	5.18**
1				1.87	2.67	3.02	4.85**
6					0.80	1.15	2.93
5						0.35	2.18
8							1.83

**Significantly different at 0.01.

Table 12. Differences between two DBH class means for log rule C.

TEST SUMMARY

DBH CLASS:	2	3	1	4	6	5	8	7
Mean:	-0.02	1.19	1.83	2.16	4.03	4.83	5.18	7.01

CORRELATION BETWEEN AVERAGE PERCENT DIFFERENCE VOLUME AND MIDPOINT DBH CLASS

The regression equations that show the best coefficient of determination are the following:

LOG RULE A:

$$\overline{PDV} = 4.6429 - 0.5066\ (\text{DBHmidpoint}) + 0.0151\ (\text{DBHmid})^2$$

with

$$R^2 = 0.6858$$

LOG RULE B:

$$\overline{PDV} = 4.2106 - 1.0798\ (\text{DBH mid}) + 0.0390\ (\text{DBH mid})^2$$

with

$$R^2 = 0.9402$$

LOG RULE C:

$$\overline{PDV} = 1.0941 + 0.1894\ (\text{DBH mid}) + 0.0015\ (\text{DBH mid})^2$$

with

$$R^2 = 0.9765$$

Figures 19, 20 and 21, illustrate those three regression equations in contrast with the points representing the DBH classes midpoints and the average percent difference volume for each class.

The regression analysis for the 3 log rules are shown in tables 13, 14, and 15.

Log Rule A:

Source	SS	DF	MS	F
Regression	8.8336	2	4.4418	5.7715*
Residual	3.8480	5	0.7696	
Total	12.4316	7		

* Regression significant at 0.05.

TABLE 13. Regression analysis for log rule A.

Log Rule B:

Source	SS	DF	MS	F
Regression	136.1409	2	68.0704	39.4735**
Residual		5	1.7246	
Total		7		

* Regression significant at 0.01.

TABLE 14. Regression analysis for log rule B.

Log Rule C:

Source	SS	DF	MS	F
Regression	38.1752	2	19.0876	102.9203**
Residual	0.9272	5	0.1855	
Total	39.1028			

*Regression significant at 0.01.

TABLE 15. Regression analysis for log rule C.

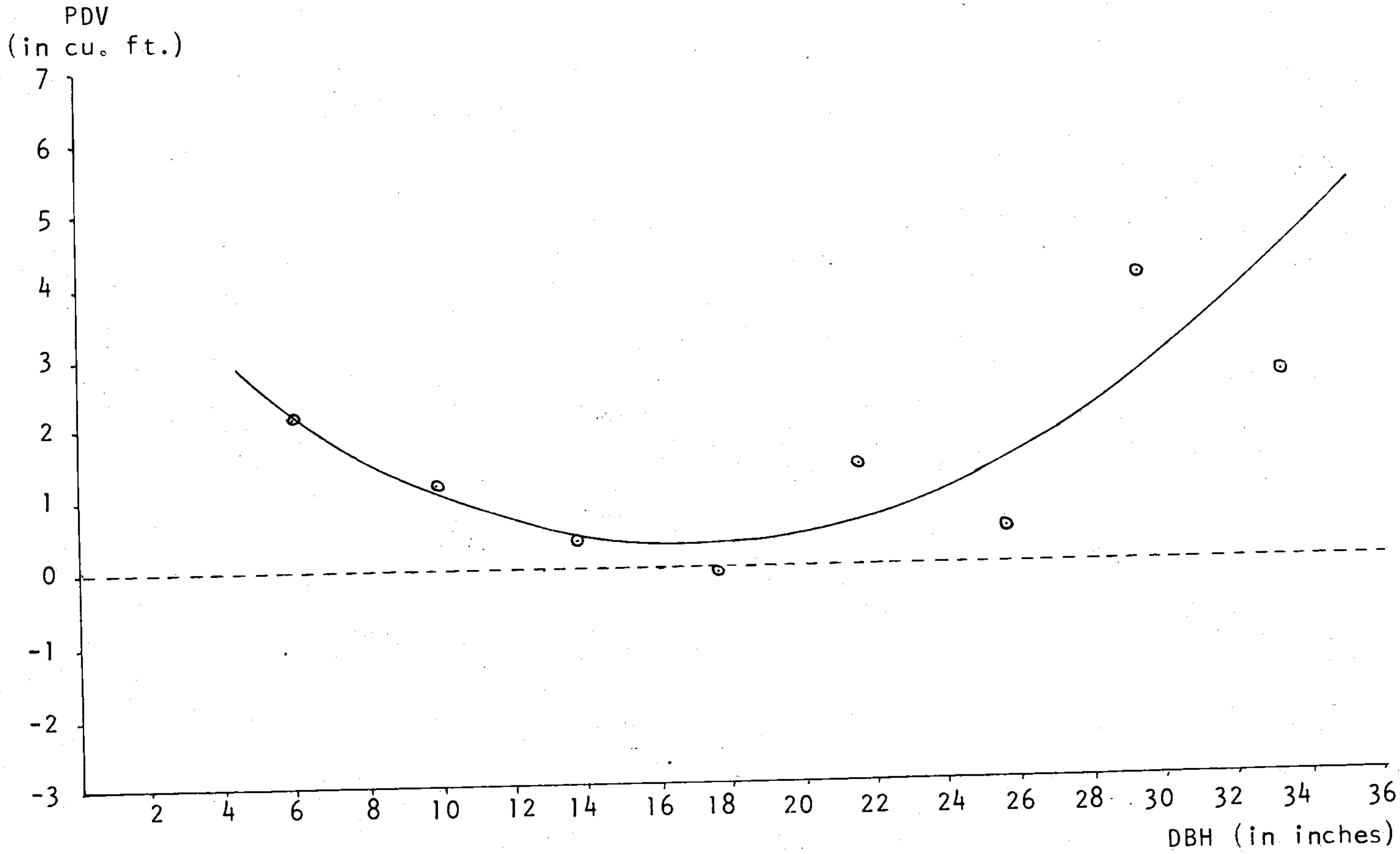

Figure 19. Regression of average PDV on DBH class midpoint.

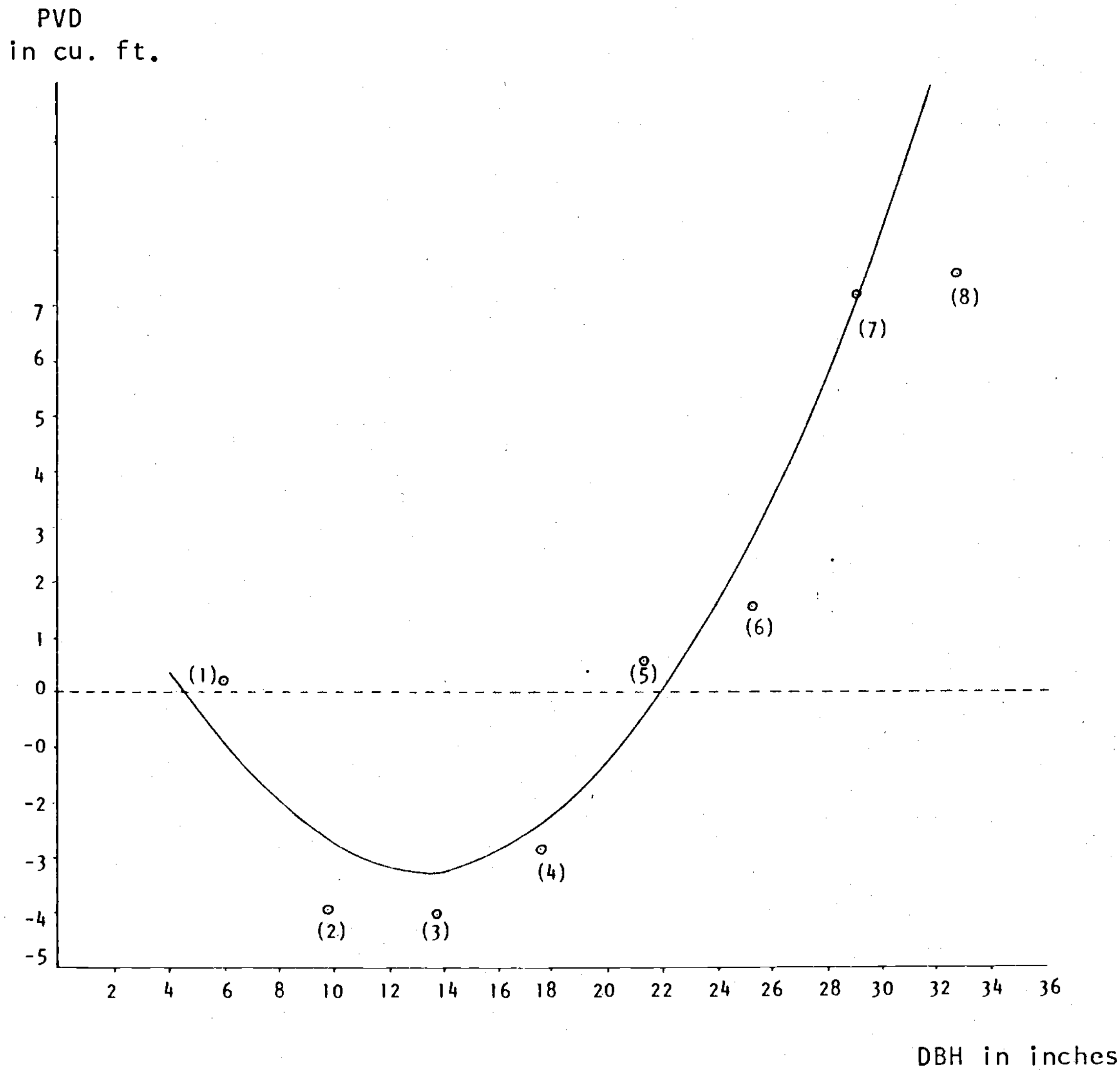

Figure 20. Regression of average PDV of DBH class on DBH class mid-point, for Log Rule B.

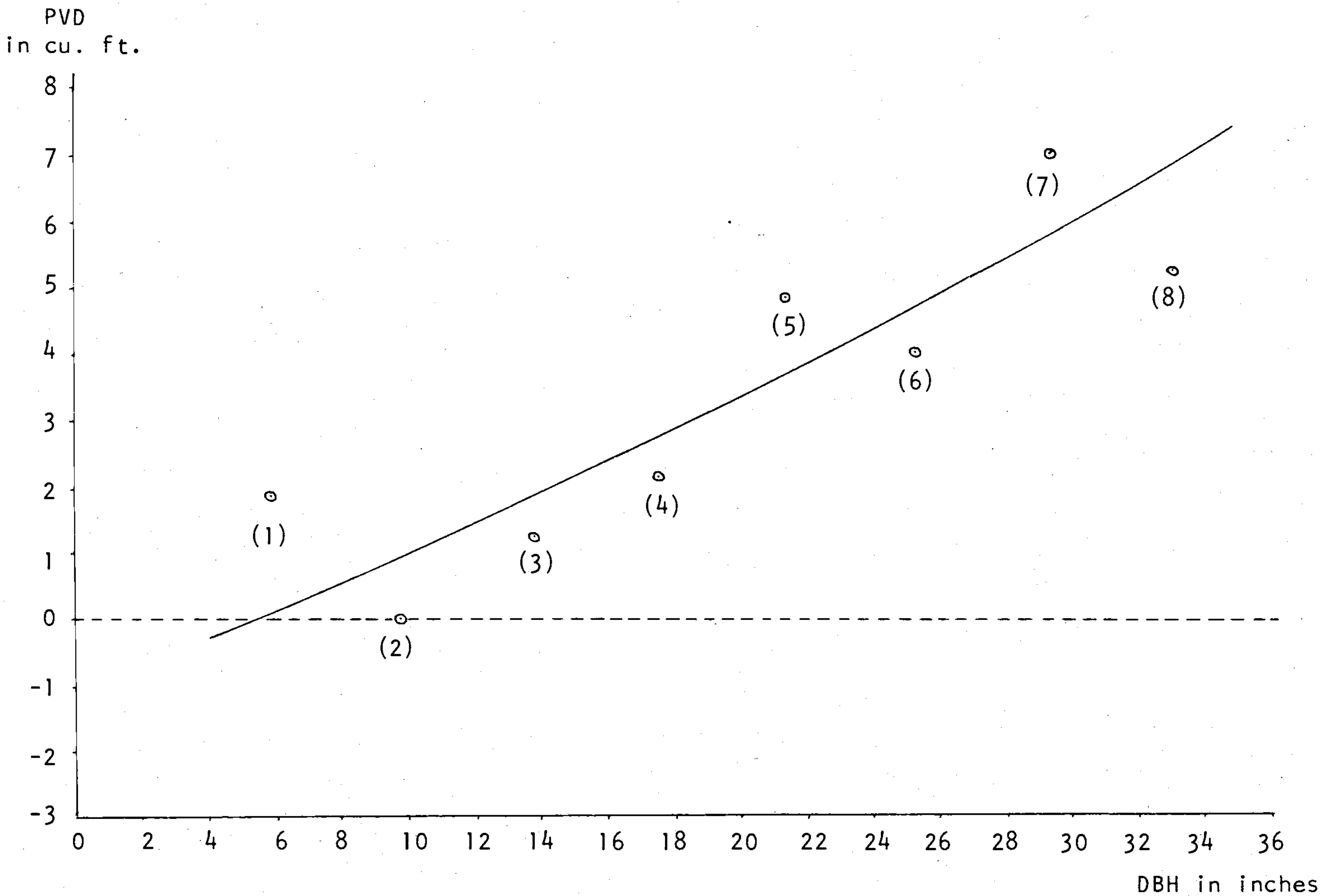

Figure 21. Regression of Mean Percent Volume (PDV) on DBH class midpoint.

CORRELATION BETWEEN PERCENT DIFFERENCE VOLUME (PDV) AND DBH

For the three different log rules this correlation was unsatisfactory, with very low coefficient of determination, R^2, respectively:

$R^2 = 0.0077$ for rule A

$R^2 = 0.0933$ for rule B

$R^2 = 0.0190$ for rule C.

DISCUSSIONS AND CONCLUSIONS

Although the percent "error", more precisely, the percent difference between the volumes given by the three studied log rules and the "actual" volume computed by the standard method, were very small (no more than 2% of the actual volume), they proved to be significantly different.

The large F value is due in part to the different number of trees in the 8 DBH classes, the second factor in this study. From a statistical point of view, it would be more convenient to deal with the same number of trees for each DBH class. However, the smallest size class (class number 8) had only 48 trees and a maximum of 384 (48 trees times 8 log rules) measurements was not sufficient for the experiment because the high variance of the variable "percent difference volume". The variance had shown, in a general way, to be still bigger in the larger classes which were exactly those of small size.

When the log rules means were compared through LSD-test, log rule B was significantly different from log rule A as well as from log rule C, at 99% level of probability (which is the probability level used throughout this study). Log rules A and C didn't prove to be significantly different.

Of all the log rules Bruce's Equation was the one which presented the highest volume difference, averaging 1.98%, even though very low. Log rule A, Weyerhaeuser Douglas-Fir Cubic, showed the smallest absolute mean difference, 1.29%, but very close, in absolute value, to rule B, British Columbia Immature, with a volume difference averaging -1.33%.

All the log rules presented very high variance: Coefficient of variation equal to 590% for C, 729% for B and 743% for A. Only this high variance could explain the significance of the difference between so close averages. The greater difference was 3.31%.

The means of the DBH classes in the same log rule showed more variation. Log rule A, for example, showed a range from -4.00 in class 3 to 7.25 in class 8. To arrange them in an increasing or decreasing order with DBH increment was impossible, meanwhile almost always the 4 highest DBH classes had presented higher averages. The only exception was class 6 in rule A with a very small mean, 0.58%, while class 1 presented an average of 2.13%, the 3rd highest.

The tables of the differences between means for log rules A and C (tables 9 and 12) showed a peculiar situation related to class 8. For log rule A, class 4 was significantly different from class 6 with a difference of 2.09 but was not significantly different from class 8 which had a bigger difference (2.64). Log rule C had the same situation: classes 2 and 3 showed significantly different from class 5 but not from class 8 with larger mean. A reason for this apparent contradiction could be the high value of the variance of class 8 in both rules associated with its very small sample size. This seems to reinforce the evidence discussed above: DBH class 8, with very high variation and only 48 trees, did not have a sample size big enough for a statistical analysis.

It seems that the same peculiarity did not happen in log rule B because of its smaller relative variance and so for this rule the sample size of class 8 would have been sufficient.

With a look at the trend of the DBH class means of PDV on the midpoint of DBH classes (Figures 15, 16 and 17), it is reasonable to suspect that there exists a curlinear correlation between these two variables. To test whether there is a relation between the variables an analysis of regression was performed. For log rule B and C the correlation was significant at 0.01 level. Values of coefficient of determination, R^2, equal to , respectively, 0.94, 0.97 showed a very strong correlation between the variables. For log rule A, with $R^2 = 0.67$, the significance is shown only at 95% level.

A regression analysis of the correlation between PDV and DBH, using all data, (1644 trees) for each log rule, showed very low values of R^2 although the values of F had shown significant relation between the variables. The reason for those high values of F is justified by two major reasons: The large sample size and the high variance of the data.

In the conclusion about this topic, the major comment is that even though DBH classes midpoint seems to be very highly correlated with the mean of PDV for the DBH classes, DBH by itself does not seem to be a good predictor of PDV, the use of PDV mean and DBH midpoint should be discouraged and even not acceptable for DBH classes sample sizes smaller than those used in this study. Class 8 anyway should have a bigger sample size.

As it was explained in the Introduction, the objective of this paper was the investigation on a methodology of comparing log rules in a statistical basis. No attempt was made to ascertain the relative performance of the three log rules as an estimator of the total volume of a young second-growth Douglas-fir stand. The good performance of all of them for young Douglas-fir stands was already known. The resulting figures of this study prove this with very small average percent difference between the volume given by each of them and the standard volume although one of them, the British Columbia Coast Immature Douglas-fir Tarif Table showed to be statistically different of another tarif table, the Weyerhaeuser Douglas-fir Tarif Table and of Bruce's Table for Immature Douglas-fir.

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