Relationship of Moist Convection to Boundary-Layer Properties: Application to a Semiarid Region

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ABSTRACT

A simple analytical expression is developed to relate the energy required to initiate moist convection to boundary-layer properties. This expression and exploratory regression are applied to data from the National Hail Research Experiment to discriminate between environments leading to cumulus congestus and well-developed hail-producing thunderstorms in northeast Colorado.

In this semiarid region, the parcel stability below the lifted condensation level is greater in environments leading to hail-producing thunderstorms compared to environments producing only cumulus congestus, as has been found in previous studies. As a result, boundary-layer properties have a multiplicity of contrasting influences on the severity of moist convection. For example, convection severity in this region generally increases with increasing low-level moisture. However, for a fixed mixed-layer depth and temperature, convection severity increases with decreasing low-level moisture because such a decrease increases the parcel stability below the condensation level.

1. Introduction

Boundary-layer properties exert a variety of competing and sometimes very nonlinear influences on the initiation and development of moist convection. For example, the development of moist convection is usually enhanced by the destablilization of low-level flow. However, in dry regions where the supply of atmospheric moisture is marginal and limited primarily to the boundary layer, strong moist convection is most likely to occur in the presence of significant low-level stratification in which case sufficient forcing is required to start cloud development (Fulks, 1951; Browning and Foote, 1976; Mahrt, 1977). This low-level stable stratification is usually concentrated in an inversion capping the mixed layer. If the energy required to initiate moist convection vanishes, then moist convection will be widespread but less likely to be severe due to competition for the limited moisture supply. In other terms, the widespread developing moist convection dries out the boundary layer before severe convection can develop. This drying is due to cloud-enhanced mass exchange between the boundary layer and the drier overlying free flow. On the other hand, if low-level stratification is too strong then moist convection will be suppressed altogether.

In the present study we examine the influence of boundary-layer properties, and the related energy required to initiate moist convection, on the severity of convective development occurring during the National Hail Research Experiment conducted in northeast Colorado. Before analyzing these data, we develop an analytical expression for the energy required to initiate moist convection.

2. Required initiation energy

Since the occurrence of some low-level stability below the condensation level seems to be conducive to severe storm development, we now develop a relationship for the energy required to lift a boundary-layer parcel to its condensation level.

As a preliminary step, we relate the parcel relative humidity at the mixed-layer top to initial parcel conditions by using the Clausius-Clapyron equation in the form

$$q_{\text{sat}} \simeq \frac{0.622}{p} e_0 \exp\left[\frac{L/R_v}{T_0^2}(T - T_0)\right], \qquad (1)$$

which is accurate to $O[(T - T_0)/T_0]^2$, where p is pressure of the parcel, T the parcel temperature, e_0 the reference saturation vapor pressure, T_0 the

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[1] This work was completed while this author was a visiting scientist at the Risø National Laboratory, Denmark.

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reference temperature, and $L/R_v \approx 5.414 \times 10^3$ K, where L is the latent heat of condensation and R_v the gas constant for water vapor. Neglecting entrainment so that, prior to condensation, the parcel conserves potential temperature θ and specific humidity q, the parcel relative humidity at the mixed layer top RH(h) is

$$\mathrm{RH}(h) = q\left[\frac{0.622}{p} e_0 \exp\left\{\frac{L/R_v}{T_0^2}\left[\theta(p/p_0)^\kappa - T_0\right]\right\}\right]^{-1}.$$

Assuming that the mixed-layer depth is thin compared to the scale height of the atmosphere, the hypsometric equation is approximately

$$\frac{P_h}{p_s} = \exp\left[-\frac{h}{H}\right], \quad H \equiv \frac{RT_0}{g}, \tag{2}$$

where p_s is the surface pressure. Using (2), the expression for RH(h) becomes

$$\mathrm{RH}(h) = qe^{h/H}\left[\frac{0.622}{p_s} e_0 \times \exp\left\{\frac{L/R_v}{T_0^2}\left[\theta(p_s/p_0)^\kappa - T_0\right]\right\}\right]^{-1}. \tag{3}$$

We do not consider energy involved in the movement of the representative parcel to the mixed-layer top.

The energy required to lift the parcel from the top of the mixed layer to the condensation level, hereafter referred to as initiation energy, is

$$E \equiv -R\int_{P_h}^{P_{\mathrm{LCL}}} (T - \bar{T})d(\ln p), \tag{4}$$

where P_{LCL} is the pressure of the lifted condensation level, P_h the pressure at the mixed-layer top and $\bar{T}$ the ambient temperature. The pressures of the parcel and environment are assumed to be the same.

If the influence of moisture on buoyancy is significant, T in Eq. (4) must be replaced by the virtual temperature. For the data in this study, we can neglect such effects. The potential temperature of the environment above the mixed layer is approximated as

$$\bar{\theta} = \theta_h + \overline{\Delta\theta} + \gamma(p - P_h), \quad \gamma \equiv \frac{\partial\theta}{\partial p}, \tag{5}$$

where $\overline{\Delta\theta}$ is the strength of the inversion capping the mixed layer and γ the free-flow stratification parameter, both assumed to be constant. The thickness between the top of the mixed layer and the lifted condensation level will be small compared to the thickness of the troposphere so that we can assume

$$\frac{P_h - P_{\mathrm{LCL}}}{P_h} \equiv \epsilon \ll 1. \tag{6}$$

Substituting (5) into (4) and using the definition of potential temperature, integrating and then simplifying by using (6), we obtain to O(ϵ) after considerable algebra and rearranging

$$E = R\gamma\left(\frac{P_h}{P_{\mathrm{LCL}}}\right)^\kappa \frac{(P_h - P_{\mathrm{LCL}})^2}{2P_h} + \frac{R\overline{\Delta\theta}}{P_{\mathrm{LCL}}^\kappa} P_h^{\kappa-1}(P_h - P_{\mathrm{LCL}}). \tag{7}$$

The pressure thickness between the condensation level and mixed-layer top can be related to the relative humidity RH of the representative mixed-layer parcel by expressing RH in the logarithmic form

$$\ln(\mathrm{RH}) = \ln(e) - \ln(e_s),$$

where e and e_s are the actual and saturation vapor pressures, respectively. Since specific humidity and therefore e/p are approximately conserved,

$$d(\ln \mathrm{RH}) = d(\ln p) - d(\ln e_s). \tag{8}$$

Using the Clausius-Clapyron equation, integrating from P_h to P_{LCL} and noting by definition

$$\ln[\mathrm{RH}(P_{\mathrm{LCL}})] = 0,$$

Eq. (8) becomes

$$\ln[\mathrm{RH}(P_h)] = \ln[P_h/P_{\mathrm{LCL}}] - \frac{L}{R_v}\left[\frac{1}{T(P_{\mathrm{LCL}})} - \frac{1}{T(P_h)}\right]. \tag{9}$$

Using the definition of potential temperature to relate the last term to pressure, using approximation (6) and solving for $(P_h - P_{\mathrm{LCL}})$, we obtain from (7) with an error of O(ϵ):

$$\left.\begin{aligned} P_h - P_{\mathrm{LCL}} &= \ln[\mathrm{RH}(P_h)]P_h/(1 - A) \\ A &\equiv \kappa L/[R_v T(P_{\mathrm{LCL}})] \end{aligned}\right\},$$

in which case

$$E = -\frac{R\gamma P_h^{\kappa+1}}{2P_{\mathrm{LCL}}^\kappa}\{\ln[\mathrm{RH}(P_h)]\}^2(1 - A)^{-2} + \frac{R\overline{\Delta\theta}}{P_0^\kappa} P_h^\kappa \ln[\mathrm{RH}(P_h)](1 - A)^{-1}. \tag{10}$$

Since A varies by less than 10% for the data set of this study, Eqs. (3) and (10) then indicate that the energy required to lift a parcel to its condensation level depends primarily on the inversion strength, stratification of overlying fluid, the mixed-layer depth and the initial parcel specific humidity and potential temperature. In the absence of significant variations in surface heat flux and advection, the mixed-layer depth should be highly correlated to the overlying stratification and inversion strength (Lilly, 1968; Tennekes, 1973).

Relationship (10) could be developed into a predictive model for boundary-layer initiation of moist convection by differentiating with respect to time and combining the result with a mixed-layer growth model and conservation equations for moisture and heat. Such a predictive model could statistically include the actual initiation process by parameterizing the penetration of boundary-layer thermals into the overlying free flow (Mahrt, 1979).

3. Data analysis

We will now carry out statistical procedures for discrimination between environments leading to only weak cumulus development and environments leading to hail-producing thunderstorms. The purpose of this analysis is primarily to help identify physical influences on convective severity, such as discussed in the Introduction, rather than to decide upon a particular predictive model.

Toward this goal, we will analyze data from radiosondes released at Sterling, Colorado in early afternoon (1400 MDT) as a part of the National Hail Research Program during the summers of 1972–74. Modahl (1975,[2] 1979) has classified the days during this experiment according to the maximum development of convective activity as observed from visual observations, cloud photography and a surface hail observing network. Of the various classes designated by Modahl, we will attempt to discriminate between environmental conditions occurring on "cumulus-congestus days" and "hail days". This differentiation should be least vulnerable to classification difficulties. We further require that the cloud development occurred within 75 km of Sterling within 6 h after the sounding release and that no hail was reported 6 h prior to the release. After imposing these restrictions, there remain 20 cumulus congestus days and 38 hail days.

In particular, we will apply a two-group discriminant analysis on the low-level environment as motivated by the discussion in the Introduction. To select descriptive variables, we vertically partition the atmosphere using the top of the mixed layer and the lifted condensation level. To describe the mixed-layer environment we use the specific humidity (q) and potential temperature (θ) at the first radiosonde contact level above ground ($\sim$100 m) and the mixed-layer depth (h). We also compute the energy required to lift a mixed-layer averaged parcel to its condensation level (E), hereafter referred to as initiation energy, the gradient of such energy (E_z) and the energy required to lift a mixed-layer averaged parcel from its condensation level to 300 mb above this level (EC), hereafter referred to as cloud energy. Finally, we use eastward and northward wind components vertically averaged over the lowest kilometer (u and v) and the magnitude of the shear vector in the lowest 5 km (V_z) [see Mahrt (1977) for further discussion of these variables]. We increase the normality of the distributions of h, E, u and v by transforming to a log scale and for the same reason transform E_z to a square-root scale, $\mathrm{sign}(x)|x|^{1/2}$.

The following statistical analyses will focus on the discriminatory power of individual variables which is *in addition* to that provided by other specified subsets of variables. This is called the "test for additional information" (Rao, 1973, Section 8c,4). This procedure is essentially the same as that discussed by Miller (1962, Chap. 1, Section 3c).

For the case of discriminating between just two groups, there is a particularly simple computational device for these calculations, as pointed out in Snedecor and Cochran (1967, Section 13.15). For example, one may obtain t-values describing the importance of individual environmental parameters by regressing on them a variable which is zero or one, depending on whether the result was cumulus congestus or hail, respectively.

The t-values corresponding to regressing this index variable on the environmental parameters are shown in Table 1 for data studied here. Consider, for example, the "model" represented by line 7. Those variables which are in the model are noted by an asterisk, *viz.*, q, h and EC. The ordinary t-statistic for testing the significance of the three predictor variables in the model are respectively -1.8, -3.8 and -3.7. These t-values measure the information carried by a given variable which is not carried jointly by the remaining variables in the model. The remaining t-statistics, corresponding to variables not in the current model, are computed by transforming partial correlation coefficients to t-statistics, in the standard manner of multiple-regression analysis. These t-values measure the discriminatory power of a given variable not in the model which is not carried jointly by those variables which are in the model.

The analysis in Table 1 is on roughly 50–60 degrees of freedom; thus, values near ± 2 are marginally significant and values greater than about ± 3 are highly significant. For example, line 9 indicates that there is substantial information carried by $u(t = -2.8)$ which is not carried by EC, whereas line 8 indicates that there is little information carried by $u(t = -1.4)$ which is not carried jointly by EC and h. Substantial conservatism should be adopted in using these statistics for formal significance

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[2] Modahl, A. C., 1975: Weather and hail event stratification of NHRE operational days. NHRE Tech. Rep. NCAR/71000-75/4, National Center for Atmospheric Research.

TABLE 1. t-values representing additional information in any single variable, adjusted for those variables marked by an asterisk. See text for definition of variables.

	q	θ	h	E	E_z	EC	u	v	V_z
1.	−1.6*	1.1*	−2.0*	0.3*	−0.5*	−1.9*	−1.5*	0.7*	−0.0*
2.	−1.6*	1.2*	−2.1*	0.3*	−0.5*	−1.9*	−1.5*	0.8*	−0.0
3.	−2.0*	1.9*	−3.1*	0.3	−0.4*	−2.0*	−1.7*	0.8*	−0.0
4.	−2.1*	1.9*	−3.1*	0.1	−0.4	−2.2*	−1.7*	0.8*	−0.1
5.	−2.3*	1.8*	−3.3*	0.2	−0.3	−2.5*	−1.7*	0.8	−0.4
6.	−2.1*	1.8*	−4.2*	0.7	−0.3	−3.0*	−1.7	0.7	−0.4
7.	−1.8*	1.8	−3.8*	1.7	−0.1	−3.7*	−1.7	0.4	−0.5
8.	−1.8	1.3	−3.3*	2.0	−0.1	−3.3*	−1.4	0.8	−0.6
9.	−0.1	0.0	−3.3	3.1	−0.0	−4.2*	−2.8	1.1	−0.3
10.	+2.7	1.8	−4.1	2.5	−0.6	−4.2	−4.4	1.7	−0.6
11.	−0.6*	2.3*	−3.1*	−0.3*	−1.0	−3.0	−2.4	1.4	−0.7
12.	−0.6	2.5*	−4.1*	0.1*	−1.1	−2.5	−2.0	1.5	−0.7
13.	2.4*	0.2*	−3.2	2.4*	−1.7	−3.7	−2.7	1.8	−0.5
14.	2.7*	0.2	−2.1	2.6*	−1.7	−3.5	−2.6	1.7	−0.5
15.	1.2*	2.2	−2.1*	1.4*	−1.3	−3.8	−2.0	1.4	−0.8
16.	1.2	2.5	−3.3*	1.2*	−1.1	−3.7	−2.4	1.3	−0.9
17.	2.5	1.1*	−4.1	2.1*	−1.7	−4.6	−3.7	1.9	−0.4

tests due to the multiplicity of statistics presented. Our aim is more toward exploratory analysis rather than formal significance testing.

Lines 1–10 in Table 1 result from the backward elimination procedure. Forward regression changed the ordering of importance of individual variables but offered no new physical insight relevant to this study. Lines 11–17 result from testing various combinations of variables affecting the influence of required initiation energy on the convective severity.

Table 1 indicates that hail-producing thunderstorms are more likely to occur with shallow, moist, mixed layers with large energy required to initiate moist convection (large E) but significant parcel instability aloft (negative EC). Severe storm development is also more probable with significant low-level easterly flow which normally leads to positive moisture advection over the High Plains. The importance of instability aloft and moisture advection for development of severe convection in this region is well documented (e.g., Fankhauser, 1976; Mahrt, 1977). However, the relationship between low-level parcel stability and mixed-layer moisture, temperature and depth, and the influence of this relationship on the severity of moist convection needs further clarification.

Toward this need, we now examine Table 1 in more detail and compare with Eq. (10) in Section 2. We first note that the potential importance of required initiation energy (E) increases considerably as mixed-layer temperature, moisture and depth are dropped from the model (lines 6–9) as might be expected from Eq. (10). That is, the discriminating power of required initiation energy is largely explained by these mixed-layer properties. The additional potential importance of variations of low-level inversion strength suggested by Eq. (10) is apparently small or more likely is significantly correlated with mixed layer depth as discussed in Section 2.

The discriminating importance of required initiation energy is most closely related to mixed-layer depth as is evident by examining lines 6–9 and comparing lines 11, 12, 15 and 16 with 13, 14 and 17. The strong relationship between the required initiation energy and mixed-layer properties, especially mixed-layer depth, is also evident by the increased discriminating importance of mixed-layer properties when initiation energy is dropped from the model (cf. lines 2 and 3). These comparisons verify that severe hail-producing convection, compared to weaker moist convection, is most likely to occur when the mixed layer depth is relatively thin. Then, with everything else fixed, the required initiation energy is relatively large. However, the roles of mixed-layer moisture and temperature are more complicated and need further discussion.

As the potential temperature of the mixed layer increases, for a given moisture value, the lifted condensation level rises and the required initiation energy increases. The importance of this thermal effect on initiation energy is suggested by the increased importance of initiation energy when temperature is dropped from the model (lines 6 and 7) and suggested by the increased importance of temperature when initiation energy is dropped from the model (lines 2 and 3). This discriminating importance of temperature requires knowledge of the mixed layer depth as can be seen by comparing lines 8 and 9 and also by comparing lines 11, 12, 15 and 16 with 13, 14 and 17. Apparently, temperature, together with mixed-layer depth, can account for much of the discriminating importance of initiation energy. However, without information on mixed layer depth, the role of temperature is less clear due to statistical relationships with other variables such as moisture and cloud energy.

With no information about other variables (line 10), severe convection is most likely to occur when the mixed layer is moist. This positive influence of moisture on convection severity is mainly through cloud parcel energy, since moisture is unimportant when information on cloud energy is included (cf. lines 9 and 10). That is, one of the roles of low-level moisture is to provide fuel for latent heat release which drives the convective storm. However, together with knowledge of mixed-layer depth (line 8), and to a lesser extent mixed layer temperature (line 6), the role of moisture reverses sign; i.e., severe convection becomes inversely related to low-level moisture through the influence of moisture on initiation energy. With the further addition of initiation energy to the model (line 2), the role of moisture becomes less important again. Contrasting lines 6, 7 and 8 with their counterparts 11, 15 and 16 further indicates that convective severity is negatively correlated with moisture if information on cloud energy is included with no other information on required initiation energy, but is positively correlated with moisture if information on required initiation energy is included with no other information on cloud energy.

4. Conclusions and further discussion

The unique influence of properties of the mixed layer in semiarid regions on the severity of moist convection is elucidated by the comparison of the physical relationship developed in Section 2 and the exploratory regression conducted in Section 3. When the initiation of moist convection from boundary-layer air requires little local forcing, the moist convection will be weak presumably due to the widespread development of many cumulus clouds which compete for the limited supply of moisture. When this moisture supply is confined primarily to the boundary layer, cumulus clouds transport relatively moist boundary-layer air upward, while between clouds, drier free-flow air is entrained downward into the boundary layer. On the other hand, significant low-level stratification will slow mixed-layer growth and cloud development and thus retard the drying of the mixed layer. Then with sufficient local forcing, a few well-developed convective storms may develop. However, if low-level stability is further strengthened, cloud development may be suppressed altogether. The amount of energy or local forcing, which is needed to start cloud development, can be estimated from Eq. (10).

The above conclusions for a moisture-deficient environment are quite different from the usual usage of stability indices where destabilization of low-level flow categorically implies increased probability of thunderstorm development. The above conclusions also illustrate the complex dependence of storm severity on low-level temperature and moisture. For example, the main fuel supply for the convective storms in the data examined here is moisture in the boundary layer. However, increasing low-level moisture will most effectively increase the probability of strong moist convection only if the low-level stratification is sufficient to inhibit widespread cumulus development and associated boundary-layer drying. That is, increased moisture also decreases the amount of energy needed to generate cumulus clouds. When variables, such as moisture, have a multiplicity of important roles, exploratory regression motivated by physical analysis is more illuminating than use of forward and backward regression alone. Further understanding might be obtained by including nonlinear relationships between variables although initial attempts to do this did not seem promising.

The above interrelationships also suggest that lack of precipitation, in those semi-arid regions where cumulus clouds frequently occur, cannot necessarily be attributed to the strengthening of the low-level inversion by, for example, radiative cooling at the top of a dust layer. That is, if the inversion became weaker, enhanced mixed-layer growth could result in drying of the mixed layer, perhaps terminating cloud initiation altogether depending on the vertical structure of moisture. The influence of changes in boundary-layer and inversion properties on the development of cumulus can be constructed from Eq. (10).

Finally, we must qualify the above conclusions because they represent only statistical tendencies and have excluded the role of larger scale kinematics. In general, the wind field exerts an important influence on thunderstorm development (e.g., Charba, 1979). However, at this point the statistical role of the larger scale flow on the development of hail storms over the high plains remains uncertain except for influences on moisture (Modahl, 1979; Barber and Mahrt, 1980). It is clear, however, that the role of the larger scale wind field varies quite significantly between individual storm days, as is evident by comparing Marwitz (1972), Fankhauser (1976) and Browning and Foote (1976).

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