Internal Waves in the Upper Ocean During MILE

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ABSTRACT

We describe the spectral analysis of temperature and velocity measurements made in the northeast Pacific as part of the Mixed Layer Experiment (MILE) and attempt to relate the observed fluctuations to internal-wave models of the upper ocean. From the inertial frequency to 1 cph there is good agreement between these upper-ocean data and typical deep-ocean observations as described by the WKB-scaled Garrett–Munk model. The largest deviations from the Garrett–Munk model occur in the vertical-displacement field at high frequency, 1–5 cph, where there is a spectral peak or shoulder and high vertical coherence. These high-frequency features in vertical displacement are successfully modeled using a few standing modes and uncorrelated noise, though the velocity spectra are poorly modeled—probably because of contamination by mooring motion. There are significant temporal fluctuations of the high-frequency energy that are not correlated with the local winds but are perhaps associated with the advection of an eddy-like feature.

1. Introduction

"And miles to go before I sleep." Robert Frost

The fluctuations of the velocity and temperature fields were monitored by an upper-ocean mooring of vector-averaging current meters (VACMs) for 19 days in August–September 1977 in the northeast Pacific Ocean as part of the Mixed Layer Experiment (MILE). Although the main goal of the experiment was to observe upper-ocean mixing processes (Davis et al., 1981a,b), valuable data relevant to the upper-ocean internal-wave field were also obtained.

In the deep ocean the kinematic model of Garrett and Munk (1972, 1975, hereafter referred to as GM) has provided the framework for comparing all types of internal wave measurements. Others have refined, modified and tested the model (see reviews by Munk, 1981; Garrett and Munk, 1979; Gregg and Briscoe, 1979) with the conclusion that the observed spectral characteristics of deep-ocean internal waves are nearly, but not exactly, “universal” in time and space (Wunsch, 1976; Wunsch and Webb, 1979). Data from the upper ocean are less extensive. From an examination of the limited amount of upper-ocean data, Roth et al. (1981) suggested that the upper-ocean internal-wave field can be described as additional energy superimposed on the base state of the deep ocean, such as that given by the GM model. The deviation of the upper ocean from the GM model occurs consistently at high frequency (Pinkel, 1975; Käse and Siedler, 1980) where there is a spectral peak or “shoulder” and high vertical coherence. This suggests that the number of vertical modes needed to describe the upper-ocean wave field decreases as frequency increases, a feature absent in the GM model.

In this paper we analyze four vertical-displacement and horizontal-velocity series constructed from the VACM data. When displacement is inferred by using a local mean temperature gradient calculated from CTD profiles, the variability of the gradient is carefully considered. Average frequency spectra and vertical coherences are calculated and compared with the WKB-scaled GM model. Although there are some differences in spectral shape, level and cutoff between the GM model and the observations, the most consistent deviation occurs at high frequency in the 1–5 cph band.

To explain these deviations at high frequency, a simple two-mode-plus-noise model is fitted to the vertical displacement data. The best fit to the observed spectra and coherences indicates dominance of mode 1 with a noise/signal ratio of ~1. The statistics of the vertical displacement are used as they are the most reliable quantities that can be estimated from the data. The velocity measurements from VACMs on a surface mooring are contaminated and difficult to interpret, especially at high frequency (Halpem et al., 1981). The difficulty in using contaminated velocity data to check the consistency of the model is discussed.

An analysis of the temporal variability of the high-frequency displacement and velocity was performed. This variability bears no resemblance to the wind forcing. The only significant temporal variation oc-
curred over the first 7 days of the experiment and appears to be coincident with the advection of a mesoscale eddy-like feature into the area.

2. Data

The temperature and velocity data used in this study came primarily from a single vertical string of VACMs attached to a surface mooring known as M1 in the northeastern Pacific at 49°37'N, 145°6'W for 19 days (19 August–6 September, 1977). The sampling rate of the instruments was 32 per hour. A complete description of the general oceanic conditions at the mooring site as well as specific details of the M1 mooring are given by Davis et al. (1981a).

The mixed-layer region above 30 m is strongly driven by heat and momentum fluxes from the atmosphere. As this direct forcing complicates the dynamical picture, time series from four depths below the mixed layer—41, 70, 92 and 175 m—of vertical displacement and horizontal velocity are used in this paper to describe the upper-ocean internal-wave field.

To model the internal-wave field, the vertical distribution of the Väisälä frequency \( N \) is required. Estimates of \( N \) were calculated from a series of CTD casts made at irregular intervals during MILE near the M1 site (Hayes, personal communication).

a. Currents

At all four depths the VACM velocity sensors exhibited no mechanical problems. These records were used directly in the analysis.

b. Vertical displacement from temperature

At the 70, 92 and 175 m depths the temperature time series from VACMs were converted to vertical-displacement series using appropriate temporal averages of vertical temperature gradient from CTD lowerings. This procedure is described in detail in Section 3.

At 41 m a single vertical-displacement time series was constructed as follows. The temperature records from the VACMs located every 3 m between 32 and 50 m can be easily and accurately converted to isotherm-displacement series by linearly interpolating positions of standard isotherms. These series show considerable long-period variation as the water-mass environment changed over 19 days (Davis et al., 1981a), so that some kind of high-pass filtering is necessary to isolate the internal-wave signal. The isotherm-displacement time series were divided into 56 half-overlapping 16 h blocks, and the 16 h average depth of each block calculated. We then chose that sequence of displacement blocks whose 16 h average depths lay closest to 41 m in order to create a displacement series at a constant mean depth. The overlapping displacement blocks are no longer identical in the regions of overlap, though visual inspection shows that the disparity is negligible.

The velocity and deep temperature records were also broken into overlapping blocks in exactly the same way as just described for subsequent high-frequency spectral analysis.

c. Väisälä frequency

The Väisälä frequency was estimated at 5 m intervals from temperature and salinity gradients calculated by linear least-square fit over 10 m segments of each daily-averaged shipboard CTD profile. A grand average profile, \( N(z) \), was obtained by averaging all the daily profiles (Fig. 1). A value of \( N = 0 \) was chosen for the surface, and linear interpolation was used to fill in values to the first data at 7.5 m. Below 200 m, deep casts to 1500 m indicate that an exponential form of the \( N \) profile is reasonable, i.e.,

\[
N = (2 \text{ cph}) \times \exp[(210 + z)/1500],
\]

where \( z \) measures depth in meters.

In the seasonal thermocline between 30 and 50 m the averaging process in forming \( N \) smears any sharp vertical structure. This occurs because the individual CTD profiles were taken randomly with respect to the phase of the large (∼10 m) vertical displacement of the semidiurnal tide.

3. Analysis

In this section we describe the average spectra and vertical coherences estimated for the entire 19-day experiment of the vertical-displacement and horizontal-velocity fields. Study of the high vertical coherence

![Fig. 1. Profile of average Väisälä frequency.](image-url)
and spectral shoulder in the high-frequency band (1–5 cp) is pursued in more detail. These observations serve as a basis for the internal-wave modeling that follows in Section 4.

a. Autospectra of temperature and vertical displacement

We first examine the average autospectra of temperature and vertical displacement calculated for the entire record length.

A more detailed examination reveals significant fluctuations in time of the temperature statistics at high frequency. An index of this temporal variability is constructed to quantify these variations. This index fluctuates greatly but bears semblance to long-period variations of the mean-square temperature gradient, indicating that the fluctuations may largely be due to changes in the mean gradient rather than in internal-wave energy. Despite the large variability in the spectral level of temperature, we find that the spectral shape at high frequency is nearly uniform in time.

1) AVERAGE AUTOSPECTRA

The average temperature autospectra, at 70, 92 and 175 m were estimated for the entire series. A correction was applied to account for the high-frequency response of the VACM temperature sensor (Levine, 1981). The effect of this correction is to boost the high-frequency spectral levels (ω > 1 cp) and flatten slightly the shape of the spectral roll-off above 5 cp. These spectra were converted to vertical-displacement spectra by multiplying them by (dT/δz)−2 where dT/δz is the 19-day average temperature gradient at the appropriate depth. They are shown in Fig. 2. The spectrum of the directly inferred vertical displacement at 41 m is also shown.

These spectra have common basic features. Below f (0.0805 cp) the spectra drop sharply and then increase rapidly down to the lowest resolvable frequency. Above f the spectra decrease with an −ω−1.6 dependence to about 1 cp where the spectral slope begins to flatten, forming a “shoulder”. At 4 cp there is a sharp break followed by a rapid roll-off. Near the Nyquist frequency at 16 cp the leveling of the spectra is apparent, especially at 175 m, putatively due to instrument noise and the correction made for sensor response. There are large peaks at the semidiurnal tidal frequency and marginally significant ones at twice the semidiurnal frequency.

2) TEMPORAL VARIABILITY AT HIGH FREQUENCY

Since the upper-ocean background conditions changed markedly during the experiment, the stationarity of the estimated spectra deserves further scrutiny. Indeed, significant fluctuations in the vari-

Fig. 2. Vertical-displacement autospectra from 41, 70, 92 and 175 m for the entire 19-day period. The 41 m spectrum was calculated directly from isotherm displacement, and the spectra at 70, 92 and 175 m were converted from temperature spectra using average temperature gradients of 2.22 × 10−2, 1.31 × 10−2 and 8.22 × 10−3°C m−1, respectively. The WKB-scaled GM model curves are also plotted for r = 300 m cph and the local values of N. The dashed line on the 41 m spectrum shows the model using a non-local N of 3.2 cph. Confidence limits at the 95% significance level are given at the bottom.
of the spectrum $S_k(\omega)$, $k = 1, 2, \ldots, 56$, was calculated. The variability index $a_k$ is a factor defined such that the "adjusted spectra" $a_k S_k(\omega)$ have the same variance $F$ in the frequency band $\omega_1 - \omega_2$ at each time block $k$. The precise definition of $a_k$ is given by

$$\int_{\omega_1}^{\omega_2} \log(a_k S_k(\omega)) d\omega = F,$$

where the logarithm of the spectrum is used to emphasize the lower magnitude of $S_k$ at higher frequencies and $\omega_1$ and $\omega_2$ are 0.6 and 6 cph, respectively. The index $a_k$ is a measure of the temporal variability of the inverse of the high-frequency variance.

The time series of $a_k$ for the temperature spectra at 70, 92 and 175 m are plotted in Fig. 4 as open circles. For comparison, daily estimates of $(\partial T/\partial z)^{-2}$, as available from limited CTD data, are also plotted for each depth. The indices have been best aligned with the corresponding temperature gradient curve by selecting an appropriate value of $F$ at each depth.

![Fig. 3. High-frequency (2–5 cph) vertical-displacement variance as a function of time from 41 m (thin solid line), 70 m (thin dashed line), 92 m (bold dashed line), and 175 m (bold solid line). Each value is estimated by averaging two half-overlapping 16 h blocks resulting in a data point every 24 h. The square of the wind speed is also plotted.](image)

![Fig. 4. The spectral conversion factors $(\partial T/\partial z)^{-2}$, needed to convert a temperature spectrum to a vertical displacement spectrum, as estimated from daily-averaged CTD profiles (dashed line). The uncertainty of these values is shown by the short horizontal lines. The indices of temporal variability are plotted as circles on the same scale as the conversion factors and shifted for best agreement. At 41 m, only the relative variation of the variability index is important.](image)
level, specifically those at the beginning of the record, are due to real changes of the internal-wave energy. This is demonstrated by examining the vertical displacement series at 41 m which avoids the problem of choosing a mean gradient, as the displacement of an isotherm is estimated directly. The variability index at 41 m therefore represents actual fluctuations in the displacement variance (Fig. 4). There is less scatter than in the variability index for the temperature spectra and, except for the increase over the first 7 days, the index is virtually constant.

Despite the temporal variability of the spectral level, the high-frequency spectral shape preserves itself strikingly well. To demonstrate this, the time-average adjusted spectra are plotted in Fig. 5 with bands indicating the range within which 80% of the data points from the individual spectra lie. The adjusted spectra $\omega_S S_k$ use the variability index to remove temporal variations in the spectral level whether due to mean temperature gradient (at 70, 92, 175 m) or actual energy variations (at all depths). If the shapes of individual spectra were identical, then the adjusted spectra would lie exactly over each other. As each spectrum is considered to be a realization of a random process, one expects variation among spectral estimates, the magnitude of which is indicated by the 95% confidence limits of an individual spectrum (Fig. 5, bottom). If the shape of the spectrum does not vary significantly with time, one would expect the confidence limits of about 95%, or 53 out of 56, of the adjusted spectra to overlap the average spectrum. Fig. 6 indicates that number of adjusted spectra that fulfill this criterion at 92 m as a function of frequency; the histogram of the unadjusted spectra is also shown. The fact that the adjusted spectra nearly always fall within 95% confidence limits of the average can be taken as acceptance of the hypothesis of constant spectral shape. The test is also applied to the spectra at 41 m where the adjustment removes the variation in actual internal-wave energy. As expected, the unadjusted spectra at 41 m show significantly less variability than the unadjusted spectra at 92 m. Hence, the shape of the temperature spectra at high frequency is nearly constant in time, though the spectral level varies considerably—largely because of mean-gradient variations rather than fluctuations in the energy of the internal-wave field.

**Fig. 5.** High-frequency vertical displacement spectra. At each depth the spectral estimates as a function of time were multiplied by the appropriate variability index and then ensemble-averaged. The dashed lines represent limits of which 80% of the individual spectral estimates lie. Confidence limits at the 95% significance level for a spectrum estimated for a single time segment are given at the bottom.

**Fig. 6.** Histograms of the number of individual adjusted temperature spectra $\omega_S S_k$, unadjusted spectra $S_k$, that fall within the 95% confidence limits of the average spectrum out of 56 total (see Fig. 5) as a function of frequency: 92 m adjusted (dashed line), 41 m adjusted (solid line), 92 m unadjusted (circles), 41 m unadjusted (asterisks).
b. Autospectra of horizontal velocity

Rotary spectra of horizontal velocity at the four depths 41, 75, 92, 175 m, were estimated from the entire series (Fig. 7). The response of the VACM velocity sensor on a mooring with surface flotation is complicated and poorly understood. There are two coupled effects that must be considered in assessing the velocity data. First, there is the question of the ability of the VACM on a surface mooring to measure accurately the oscillating horizontal velocity of the water flowing past the instrument in the presence of vertical mooring excursions. Second, mooring motion prevents the VACM from measuring a true Eulerian velocity. Tests have shown that spectral levels measured by the Savonius rotor of a VACM are increased by a nearly constant factor at frequencies < 5 cph (Halpern et al., 1981) when compared with measurements made with the propeller sensor of a vector-measuring current meter (VMCM) (Weller and Davis, 1980) which is not thought to have this problem. Comparison of two VACMs at 79 m on different moorings during JASIN showed that the spectrum measured from a subsurface-flotation mooring was significantly quieter above 1 cph.

Even with these reservations some general comments about the velocity spectra can be made. There are peaks in the clockwise rotary spectrum at the inertial and semi-diurnal frequencies at the three deep instruments. The counterclockwise spectra are substantially lower than the clockwise spectra at low frequencies and nearly the same at high frequencies. For free internal waves the ratio of counterclockwise to clockwise energy is given by \((\omega - f)^2/\omega + f)^2\) (Fofonoff, 1969). The data are consistent with this theoretical prediction (Fig. 8), but the counterclockwise component is systematically higher than the theory predicts. This was also found in results from IWEX (Müller, 1978).

Between 1–5 cph the velocity spectral slope flattens and forms a "shoulder"; this is especially evident at 175 m. At the 175 m level there is a sharp break at 5 cph followed by a rapid roll-off. This feature is remarkably similar to the spectral shoulder in the displacement spectra. Above 7 cph the spectral slope begins to level. We attribute this effect to the surface-floated mooring and not entirely to the response of the VACM, as a VMCM at 94 m on this same mooring also leveled out at high frequency (Halpern et al., 1981, Fig. 6c), although at a lower spectral level.

The time evolution of the high-frequency velocity spectra was estimated in the same manner as in the displacement spectra. The velocity variances of the 1–5 cph band are shown in Fig. 9a; since the two velocity components are nearly identical only the eastward component is shown. Overall, the velocity variances do not fluctuate as much as the temperature variances. The 41 m velocity variation compares favorably with the 41 m vertical-displacement fluctuations (Fig. 9b), indicating that the decrease in spectral level that occurred over the first 7 days is due to a decrease of high-frequency internal-wave energy. This feature is perceptible at 175 m but absent at 70 and 92 m.

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Fig. 7. Rotary spectra of the horizontal velocity from 41, 70, 92 and 175 m for the entire 19-day period. The solid line is the counterclockwise component; the dashed line is the clockwise component. The WKB-scaled GM-model curves are also plotted for \(r = 300 \text{ m}^2 \text{ cph}\) and the local values of \(N\). The bold dashed line on the 41 m spectrum shows the model using a non-local \(N\) of 3.2 cph. Confidence limits at the 95% significance level are given at the bottom.
Comparison of the current in the 1–5 cph band and the winds shows no consistent correlation at most depths. For instance, the peaks at 70 and 92 m on 23 August are not observed at 41 m (Fig. 9). They may be artifacts of increased relative mooring motion at depth during the high winds of 22–24 August.

c. Vertical coherence of vertical displacement

Average vertical coherences were calculated between all pairs of the four vertical-displacement series over the entire experiment. An example of the coherences is shown in Fig. 10. Between the most closely spaced sensors the coherence is fairly level beyond 0.1 cph to about 5 cph, where there is a dramatic cutoff. For the largest separation between 41 and 175 m the coherence is generally not significant above 0.1 cph until \(\sim 0.8\) cph where a very significant peak begins. This peak is centered around 3 cph and falls off sharply above 5 cph. The corresponding phase spectrum is near zero for all pairs in the neighborhood of this high-frequency peak.

To study the time evolution of the high frequencies, coherences were estimated from the 56 blocks of 16 h length with 8 h overlap. Each displacement series of each block was multiplied by the appropriate index \(a_k\) as determined in Section 3a to account best for the variations in the mean temperature gradient. This adjustment by using the index, however, did not significantly change these calculated coherences. To reduce variance in the coherence estimates, ensemble-averaging of six adjacent blocks as well as bandwidth-averaging in frequency was performed. This provides a detailed look at the high-frequency coherence at nine time segments of 56 h length.

To examine the general features of the high-frequency peak, the coherence was further averaged over the frequency range 1.9–4.7 cph for each vertical separation (Fig. 11). Generally, the coherence decreases as a function of time. There are only marginally significant differences among the coherences, and they are not always systematically ordered with the smallest separation the most coherent. Occasionally, the closest pair (70–92 m) with a separation of only 22 m, is the least coherent of all.

\[d.\] Vertical coherence of velocity

Vertical coherences were calculated between all pairs of the eastward velocity series. Several average coherences for the entire experimental period are shown in Fig. 12.

Features common to all the coherences are the peaks at the semiidiurnal tide and the high coherence at very high frequencies from 4 to 16 cph. This high coherence at high frequency is difficult to reconcile with physical ideas of the vertical structure of water motions at such frequencies. Most likely this feature is an effect of the VACM response to the mooring motion. Because the effect of the response on coherence at high frequency is unknown, a detailed analysis of the temporal variation of the velocity coherence was not pursued.
4. Modeling

This section examines how well the observed spectra and coherences of displacement and velocity accord with the ideas based on the superposition of dynamically linear, stochastically independent internal waves (Fofonoff, 1969; Garrett and Munk, 1972, 1975). First, we apply the Garrett-Munk WKB scaling to our data set and find that in the frequency range 0.1–1.0 cph, the GM-model autospectra fit the observed spectra well for displacement. The fit is not so good for velocity, possibly because of instrumental effects and mooring motion. Vertical modes in this frequency range are oscillatory throughout most of the water column and the lowest modes are fairly insensitive to the details of the N profile in the upper ocean (Fig. 1). This appears to be the reason that WKB ideas yield sensible results at low frequencies even in a region of rapidly varying N. Coherences between moderately separated instruments are low for both displacement and velocity, indicating numerous modes.

Second, we find that at frequencies > 1.0 cph the spectral structure of displacement can be modeled with only two vertical internal-wave modes plus incoherent random noise. The wave functions associated with modes above 2 cph are confined to the upper ocean, their turning depths lying above 200 m (Fig. 1), and are more dependent on details of the N profile so that the WKB methods are not appropriate. However, the high-frequency velocity is inconsistent with the displacement, most likely indicating the tainting of VACM velocity records from a surface mooring at high frequency.

a. WKB model

Although the details of the upper-ocean stratification are very complex (Fig. 1), it is likely that ver-

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Fig. 9. (a) High-frequency (2–5 cph) eastward velocity variance as a function of time from 41 m (bold dashed line), 70 m (thin solid line), 92 m (thin dashed line) and 175 m (bold solid line). Each value is estimated from a 24 h long segment. The square of the wind speed is also plotted. (b) The high-frequency variances of vertical displacement (from Fig. 3) and eastward velocity at 41 m are replotted over each other for easy comparison.

Fig. 10. Vertical-displacement coherences between 70 and 92 m (dashed line) and between 41 and 175 m (solid line) for the entire 19-day period. The 95% significance level is indicated.
vertical modes in the mid-frequency range \((0.1 < \omega < 1.0 \text{ cph})\), which extend through the entire water depth, are little affected by this near-surface complexity and that WKB ideas can be applied to model these mode shapes. Hence we attempt a fit of a superposition of stochastically-independent linear WKB internal waves in the fashion of Garrett and Munk (1972). We use Desaubies's representation of the GM model (Desaubies, 1976) where the four parameters \(E\) (energy level), \(f\) (effective mode number), \(b\) (vertical depth scale of \(N\)) and \(N_0\) (Väisälä frequency scale) have been recast into two independent parameters:

\[
r = Eb^2N_0, \quad t = f/2N_0b.
\]

The \(r\) parameter and the vertical displacement variance \(\sigma^2\) are related by \(\sigma^2 \approx r/2N\), where \(N(z)\) is the local slowly-varying Väisälä frequency; the parameter \(t\) is related to the vertical bandwidth determining the vertical coherence scale. The moored spectra of vertical displacement and velocity are given by

\[
S_q(\omega) = \frac{2}{\pi} \frac{f}{N(z)} \frac{(\omega^2 - f^2)^{1/2}}{\omega^3},
\]

\[
S_s(\omega) = 4\pi r \frac{fN(z)}{\omega^4(\omega^2 - f^2)^{1/2}} (\omega + f)^2.
\]

The GM-model displacement spectra are superimposed on the observed 19-day averaged spectra (Fig. 2) by using local values of Väisälä frequency from Fig. 1, viz., \(N(41 \text{ m}) = 12.5 \text{ cph}, N(70 \text{ m}) = 3.2 \text{ cph}, N(92 \text{ m}) = 3.3 \text{ cph}, N(175 \text{ m}) = 2.6 \text{ cph}\), and a typical deep-ocean value for \(r\) of 300 \(\text{ m}^2 \text{ cph}\) (Desaubies, 1976). Recall that except for the 41 m spectrum, these are temperature spectra converted by dividing by an average value of \((dT/dz)^2\) at each depth. We have noted the uncertainty that this procedure introduces (Fig. 3). The depth-to-depth variations of the levels of the observed and model spectra agree reasonably well at low frequency, indicating the success of the WKB scaling. The observed levels agree well with the absolute model levels based on \(r = 300 \text{ m}^2 \text{ cph}\) and conform with the “universal” level of

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**Fig. 12.** Eastward velocity coherences between 70 and 92 m (dashed line) and between 41 and 175 m (solid line) for the entire 19-day period. The 95% significance level is indicated.

The spectral slope of the model \( \omega^{-2} \) agrees fairly well with observations in this mid-frequency range, though a flatter spectral slope of \( \omega^{-1.6} \) would be better. The low-frequency spectral cutoff is near the inertial frequency, but details are obscured by the large tidal signal and its harmonics. At high frequency, \( \omega > 1.0 \) cph, the observed spectra show a distinct shoulder which is absent in the model. In the three deep spectra, the observed high-frequency cutoff occurs approximately at the local \( N \), while the spectrum at \( 41 \) m cuts off at a much lower frequency than \( N(41 \) m) = 12.5 cph. This discrepancy may be due to the inability to determine a representative local mean value of \( N \) at 41 m because of large time and space variability. However, if one chooses the appropriate value of \( N \) by the cutoff—at about 3.2 cph—then the model spectral level is about a factor of 4 too high (Fig. 2). Hence, the level of the continuum scales with the local \( N \) of 12.5 cph but the upper cutoff does not. Note that choosing a different value of \( r \) will shift the model spectra at all depths by the same factor.

The observed and model velocity spectra are shown in Fig. 7. The observed spectral shapes are generally similar to those of the model. As with the vertical-displacement spectra, a flatter frequency dependence is indicated by the data. The low-frequency cutoff at the inertial frequency is clearly demonstrated at all depths in both the clockwise and counterclockwise components. Above 1 cph the observed spectra exhibit a shoulder which the model spectra do not. The upper cutoffs are somewhat obscured by the noise of the sensor, but again the cutoffs of the three deep spectra scale reasonably well with their local \( N \) values, which are all \( \sim 3 \) cph. At 41 m, as we found with displacement, the predicted cutoff is clearly too high. Choosing a lower, 3.2 cph cutoff will lower the model spectral level by a factor of 4, putting it below the observed spectrum (Fig. 7). However, as discussed previously, determining the absolute level of the velocity spectra is difficult because of the effects of mooring noise. For example, it is known from comparison with the VMC3 moored at 94 m that the 92 m VMC3 spectrum is too high by a factor of 2. Note that at 175 m the discrepancy between theory and data is largest, possibly indicating large mooring contamination.

b. High-frequency mode models

In this section we focus on the observed high frequency (1–5 cph) features, the spectral shoulder and high vertical coherence. Similar features have been observed in both the deep ocean (e.g., Briscoe, 1975; Desaubies, 1975; Cairns, 1975) and the upper ocean (e.g., Pinkel, 1975; Käse and Siedler, 1980). The WKB version of the Garrett-Munk model cannot explain these features. Desaubies (1973) reformulated the GM model without using the WKB approximation and found peaks in spectra and coherences at high frequency. Alternatively, high coherence can be explained by assuming fewer modes than the GM model—fewer modes results in less destructive interference and less vertical-wavenumber bandwidth.

In order to examine these data in a theoretical framework, the observed spectra and coherences at high frequency were fitted with superpositions of vertically standing linear internal-wave modes and vertically uncorrelated noise. This acts as a test of the self-consistency of the data and provides a means for comparison with theories and other results.

1) Vertical displacement

Internal vertical displacements of the form \( \xi = \psi(z) \exp[i(k \cdot x - \omega t)] \) are governed by the internal wave equation

\[
\frac{d^2\psi}{dz^2} + k^2 \left[ \frac{N(z)^2}{\omega^2} - \frac{f^2}{\omega^2} \right] \psi = 0,
\]

with boundary conditions

\[
\psi = 0 \quad \text{at} \quad z = 0, -H.
\]

This equation has a set of solutions \( \{ \psi_n(z), k_n \} \), for a given \( \omega \), normalized by \( \int N^2 \psi^2_n dz = 1 \), ordered according to increasing \( k_n \), with \( (n - 1) \) zero-crossings between \( z = 0 \) and \( -H \) (not counting end points). The displacement autospectrum at a depth \( z \) of a statistically independent superposition of \( M \) such modes at each frequency can be written

\[
S_\psi(\omega, z) = A(\omega)\tilde{S}_\psi(\omega, z) + \eta_\psi(\omega, z),
\]

where \( \tilde{S}_\psi(\omega, z) = \sum_{n=1}^{M} E_n(\omega)[\psi_n(\omega, z)]^2 \)

and \( A(\omega) \) is a frequency weighting function, \( E_n(\omega) \) is the proportion of variance at frequency \( \omega \) attributed to mode \( n \) [such that \( \sum_n E_n(\omega) = 1 \)], and \( \eta_\psi(\omega, z) \) is the spectrum of noise as a function of depth. By defining a noise/signal ratio

\[
R_\psi(\omega, z) = \eta_\psi(\omega, z)/[A(\omega)\tilde{S}_\psi(\omega, z)],
\]

the displacement spectrum may be written

\[
S_\psi(\omega, z) = A(\omega)[1 + R_\psi(\omega, z)]\tilde{S}_\psi(\omega, z).
\]

Coherence of displacement between depths \( z_1, z_2 \) may be written as

\[
\gamma(\omega, z_1, z_2) = \left| \sum_n E_n(\omega)\psi_n(\omega, z_1)\psi_n(\omega, z_2) \right| \\
\times \{1 + R_\psi(\omega, z_1)\tilde{S}_\psi(\omega, z_1)^{-1/2} \}
\times \{1 + R_\psi(\omega, z_2)\tilde{S}_\psi(\omega, z_2)^{-1/2} \}.
\]

Note that this expression is independent of \( A(\omega) \).
In each of seven frequency bands between 1.1 and 4.3 cph, for a particular choice of the \( E_n \), we fit model coherences to observed coherences among 41, 70, 92, 175 m in a least-square sense by adjusting \( R_z \) at the four depths. The \( R_z \) are allowed to assume only six discrete values: 0, 0.2, 0.45, 1.0, 2.2, 5.0, evenly spaced on a logarithmic scale. We then fit the autospectra in a least-square sense by adjusting \( A(\omega) \). The fit was performed independently for each of the nine time segments of 56 h duration. We considered models with (i) one mode, \( M = 1 \), so that \( E_1 = 1 \), and (ii) two modes, \( M = 2 \), with various settings of \( E_1 \), \( 0 < E_1 < 1 \) \( (E_2 = 1 - E_1) \).

The goodness of fit to the coherences was judged by the measure

\[
C = \frac{1}{\sigma_\gamma(\omega)} \left( \frac{1}{6} \sum_{i=1}^{4} \sum_{j=1}^{4} \left[ \gamma_{\text{obs}}(\omega, z_i, z_j) - \gamma_{\text{model}}(\omega, z_i, z_j) \right]^2 \right)^{1/2},
\]

which we shall term the "coherence error" where \( \gamma' \) is coherence transformed by the hyperbolic arc tangent, i.e.,

\[
\gamma' = \text{arctanh} \gamma,
\]

and \( \sigma_\gamma(\omega) \) is the standard error of \( \gamma' \). The distribution of \( \gamma' \) is approximately normally distributed (Koopmans, 1974) with \( \sigma_\gamma = \left[ 2(n - 1)^{-1/2} \right] \) where \( 2n \) is the equivalent degrees of freedom. Hence, this transformation gives more weight to the more significant higher coherences. A value of O(1) for this measure indicates that the observed coherence was fitted to within one standard deviation.

Similarly, goodness of fit to autospectra was judged by the "spectrum error"

\[
S = \frac{1}{L(\omega)} \left\{ \frac{1}{4} \sum_{i=1}^{4} \left[ \log S_{\text{obs}}(\omega, z_i) - \log S_{\text{model}}(\omega, z_i) \right]^2 \right\}^{1/2},
\]

where \( L(\omega) \) is the logarithm of the length of the 95% confidence band of \( S_{\text{obs}} \).

![Fig. 13. Vertical-displacement coherences between all possible pairs for time-segment 1 (solid line) and time-segment 9 (dashed line). The best-fit model for 1 mode and noise is shown for segment 1 (asterisks) and segment 9 (triangles).](image)
Some of the results have been summarized in Table 1 for the first, fifth and ninth segments; results for the other segments were similar. We first discuss the fits to the one-mode-plus-noise model. Coherence and spectrum fits are shown in Figs. 13 and 14 for the first and ninth segments.

The model coherences fit the data pretty well (Fig. 13). This is not too surprising as four values of the noise/signal ratio $R_f$ have been adjusted to fit only six coherences. However, it is pleasing that the values of $R_f$ are fairly smooth functions of frequency, usually jumping by no more than one discrete value between adjacent frequency bands. Also, $R_f$ values are generally smaller than 1, although they are larger for later segments. The best fit requires an increase in noise with time reflecting the fact that coherence, in any frequency band, decreased over the duration of the experiment.

The autospectra (Fig. 14) also fit quite well. The parameter $A(\omega)$ is the same for all depths; hence a spectral shoulder at 41 m and no shoulder at 175 m is a consequence of the mode shapes.

The same fitting procedure was repeated for a two-mode model with noise using different proportions of modes 1 and 2. For 80% mode 1 and 20% mode 2 ($E_1 = 0.8$, $E_2 = 0.2$), the coherence and spectrum errors were about the same as with 100% mode 1 at all frequencies and times (Table 1). However, the amount of noise needed to obtain this fit tended to be smaller, possibly indicating that some of the noise of the one-mode fit is really mode-2 energy. The noise levels in this best-fit, while lower overall than for 100% mode 1, also show an increase with time. The coherence and spectrum errors are fairly sensitive to the amount of mode 2; increasing the proportions of mode 2 beyond 20% substantially increases the errors (Table 1).

Phase differences (not shown) between displacements at different depths are invariably indistinguishable from zero, when the corresponding coherence is statistically significant. This confirms the predominance of mode 1 which is in phase through the entire depth. Mode 2 has a zero-crossing between 50 and 250 m, depending on frequency. Presence of small amounts of mode 2 degrades the coherence without altering the phase relationship dominated by mode 1.

Model fits in terms of four internal-wave modes ($M = 4$), without any noise, were also attempted. Although spectra could be fairly successfully modeled in this way, coherences could not. Coherence error was always much larger than for one- or two-mode fits with noise. Even so, the best four-mode fits again show a predominance of mode 1 (Table 1).

2) HORIZONTAL VELOCITY

The model spectrum of horizontal velocity, corresponding to the displacement spectrum for a horizontally isotropic field of internal waves with noise [Eqs. (3) and (4)] is given by

$$S_d(\omega, z) = A(\omega)\tilde{S}_d(\omega, z) + \eta_v(\omega, z),$$

where

$$\tilde{S}_d(\omega, z) = \frac{1}{2}(\omega^2 + f^2) \sum_{n=1}^{M} E_n(\omega)[\phi_n(\omega, z)]^2,$$

$$\phi_n = k_n^{-1}d\psi_n/dz.$$

Note that the velocity noise $\eta_v$ is not necessarily related to displacement noise $\eta_c$.

As expected, the levels of model spectra with no noise ($4\tilde{S}_d$) are well below observed velocity spectra.
However, shapes of observed spectra are fairly well modeled. Fig. 15 shows observed spectra for first and ninth segments with the model spectral level arbitrarily shifted for better agreement. Generally, with the exception of 175 m, the model and the data follow each other from 1-3 cph. Several of the specific features of the observed spectra are mimicked well by the model. For example, during time-segment 9 the spectral slope is much flatter in the 1-3 cph band than during segment 1. Also, there is a sharp break and flattening of spectral slope at 2 cph during time-segment 1 that does not appear at the other time. At 175 m a mode in the velocity wavefunction for mode 1 at 1.9 cph accounts for the deep depression at 1.8 cph followed by a peak at 3 cph. The data do not show this dramatic shape but look much like the model with the depression filled in. This may be due in part to the fact that the spectra are estimated over a finite frequency band while the model is evaluated at a precise frequency.

The amount of arbitrary shifting required to force the model spectra levels in the 1-3 cph band to agree with the observed spectra ranged from a factor of 1.1-5 at the three shallowest depths to a factor of 12-20 at 175 m. This discrepancy may be due to a combination of non-internal-wave velocity signal (velocity finestructure) and instrumental effects. Other studies (Müller et al., 1978; Briscoe, 1977) have found that velocity finestructure can represent a significant fraction of the total variance at these high frequencies. Some of this discrepancy is undoubtedly due to the poor performance of the VACM. As discussed, at 92 m the spectral levels measured by the VACM were too large by a factor of 2 at frequencies below 5 cph. This leaves a discrepancy of only about a factor of 2 or less at 92 m between the data and the model. At 41 and 70 m the amount of shifting is comparable to that at 92 m. At 175 m there would have to be much larger contamination to explain the discrepancy. There is evidence that the overestimation of velocity variance measured by a VACM below 50 m increases with depth when moored beneath a surface toroid (Halpern et al., 1981, Fig. 8), especially at high frequency. However, factors of 12-20 from a combination of finestructure and mooring effects are difficult to reconcile completely and remain an enigma. The fact that the observed spectral level at 175 m is comparable to the levels measured at the other depths (within a factor of 2) seems to argue against high levels of instrument noise or finestructure at 175 m.

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**Four modes no noise**

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coherence and spectrum error [Eqs. (7) and (8)] at the best fit solutions. For the two-mode model with noise, $R_t$ is the noise/signal ratio 1.0 and 2.2, respectively. In the four-mode model with no noise the modal composition of the best fit is given in percent.

### Two modes with noise

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### Four modes no noise

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Perhaps some aspect of the mode model is incorrect, although no simple modification of the model can account for the large discrepancy at 175 m. Before an increase in the complexity of the model is warranted, better velocity measurements are needed.

At frequencies > 3 cph the model always rolls off more sharply than the data at 41, 70 and 92 m. There are several explanations that may account for this behavior. One is that there is more velocity fine-structure at the high frequencies; the ratio of fine-structure to internal-wave signal has been found to increase with frequency (Müller et al., 1978). Another possibility is that the surface mooring itself generates significant noise at high frequencies. This effect was noted in Section 3. Based on results from JASIN, at 5 cph (10 cph) a subsurface-floated instrument’s spectral level was nearly a factor of 5 (20) below a spectrum measured from a surface-floated VACM (Halpern et al., 1981, Fig. 3b). If this result is used to correct the high-frequency data to remove the mooring-induced energy, the data roll-off much more sharply and generally agree better with the model (Fig. 15b). It must be stressed that this “correction” is based on one comparison between surface and subsurface moored VACMs from a different experiment.

Though this is not a rigorous procedure, it does give a crude estimate of the mooring effect.

A lack in thoroughly understanding the velocity measurements at high frequency makes modeling of the velocity coherences tenuous. Coherences estimated from the VACM instrument and toroid mooring system are an unknown combination of internal-wave signal, mooring motion and sensor response. A detailed check of the consistency of the high-frequency velocity field would require cleaner data.

3) **Mean Flow**

In these simple models we have neglected the effect of the mean flow in modifying the internal waves that propagate through it. The nonlinear interaction of the mean flow with internal waves increases as the mean velocity approaches the horizontal phase velocity of the wave. The phase velocity for the modes calculated from the $N$ profile decreases rapidly with frequency (Fig. 16). For high-frequency waves the large, low-frequency inertial and tidal flows appear as a quasi-steady “mean” flow. These observed low-frequency flows at MILE were 10–20 cm s$^{-1}$ with peaks as high as 30–40 cm s$^{-1}$. These velocities are high enough to
affect dramatically linear internal waves, especially at modes > 2 and frequencies > 2 cph. Therefore, at these high frequencies the assumption of linearity for modes higher than 2 may not be valid. Hence, it is consistent to restrict the analysis to a few modes.

Mean-flow interaction might actually be responsible for the location of the observed high-frequency cutoff of 4 cph at 41 m. As the local $N$ at 41 m is 12.5 cph, higher frequency waves are allowed by linear theory but are subject to increasing nonlinear interaction with increasing frequency. Through crit-

**5. Discussion and summary**

We described velocity and temperature measurements in the internal-wave frequency band at MILE and attempted to model them with linear internal-

**Fig. 16. Phase velocity of the internal waves as a function of frequency for the first 5 modes. Values were estimated by calculating numerically the dispersion curves using the average Väisälä profile.
wave theory. We first compared velocity and vertical-displacement spectra with the universal deep-ocean description provided by the Garrett-Munk model (Figs. 2 and 7). Specific similarities and differences include:

1) The slope of the observed spectra is less steep than in the GM model. In the vertical-displacement spectra a frequency dependence of $\omega^{-1.6}$ was found, flatter than the $\omega^{-2}$ of the GM model; this has also been observed in the deep ocean (Wunsch, 1972; Barbee et al., 1975; Briscoe, 1975; Willebrand et al., 1977; Levine and Irish, 1981).

2) The relative levels of vertical-displacement spectra agree well with the local $N$-scaling of the GM model at all depths; the scaling of the velocity spectra is more uncertain on account of mooring motion and unknown sensor response.

3) The observed spectral cutoff occurs near the local $N$ as in GM, except at 41 m, in both velocity and vertical displacement.

4) The absolute spectral level of GM, set by using $r = 300$ m\(^2\) cph, agrees well with typical deep-ocean values.

5) The ratio of counterclockwise to clockwise velocity spectra is consistent with linear internal-wave theory.

6) There is a spectral peak or shoulder at high frequency, 1–5 cph, in both velocity and vertical displacement, not described by the GM model. Also, there is high vertical coherence in vertical displacement at high frequency, not found in the GM description.

As there are large differences between the GM model and the observations at high frequency, a detailed analysis of the high-frequency band was performed. The energy in the high-frequency band was found to fluctuate in time. The most significant temporal variation of the spectrum occurred during the first 7 days (19–25 August) (Figs. 4 and 9). The decrease in spectral energy by a factor of 3 is apparently related to the advection of an eddy-like feature (Davis et al., 1981a,b) rather than to any direct atmospheric forcing. This conclusion is supported by data from mooring M2, 25 km north of M1. This eddy-like feature apparently did not extend to M2 and the high-frequency variance from M2 showed a less significant decrease during this same 7-day period (Davis, personal communication; cf. Munk, 1981, Fig. 9.27), though M1 and M2 were both subject to the same wind conditions. Evidence that the energy in the internal-wave field can indeed be modulated by the mesoscale flow has been found by Brown and Owens (1981).

There is no significant correlation of the high-frequency, internal-wave displacement variance with the wind. Käse and Clarke (1978) proposed that the existence of a spectral peak at high frequency is a result of a resonant response of the upper ocean to wind forcing at these frequencies. There is a gap in their theory involving the detailed balance of spectral energy needed to relate a high-frequency source to a steady-state spectral peak. Nevertheless, if wind-forcing is important, one might expect to see more energy at high frequency during the 22–24 August storm. Even at a mean depth of 41 m, just below the 30 m mixed layer, there is no statistically significant change in spectral level during the storm (Fig. 9). This fact argues against the suggestion by Käse and Clarke (1978).

In order to understand better the spectral shoulder and high vertical coherence at high frequency, we abandoned the GM model in favor of simple linear models using a few vertical modes plus noise. The GM model represents vertical structure by using the WKB approximation which seems inappropriate given the rapid variation of $N$ in the upper ocean and the shallowness of high-frequency turning depths. The actual vertical modes were obtained by solving the internal wave equation (1) with the average $N$-profile (Fig. 1). In the model of two modes plus noise a consistent representation of the vertical-displacement spectra and coherences was found by adjusting the noise level at each depth (Figs. 13, 14). This "noise" may represent either non-internal-wave induced temperature fluctuations, i.e., fine-structure, or waves from higher modes not included in the model. The fit was performed for seven frequency bands at each of nine time segments of 56 h length to allow for the non-stationarity of the data. The best overall fit is obtained for 80% mode 1 and 20% mode 2. The noise/signal ratio at this best-fit is a function of frequency, depth and time. The noise/signal ratio usually increases in time (to model the observed decrease in coherence) and varies from an average of ~0.45 to ~1 over the duration of the experiment (Table 1). This is consistent with the order of magnitude found in studies in the deep ocean where fine-structure is expected to be less; Müller et al. (1978) and Levine and Irish (1981) report ratios of fine-structure noise to internal-wave signal of ~0.2 in the high-frequency band.

Because of the uncertainties of sensor response and mooring motion the velocity data were of limited use in modeling. The model velocity spectra with no noise, using the parameters fit to the vertical-displacement information, follow the shape of the data reasonably well (Fig. 15). As expected, the model levels with no noise are lower than the data. The ratios of the observed-to-model-predicted velocity spectra are 4, 1.7 and 3, at 41, 70 and 92 m, respectively. At 175 m, however, the ratio is 12–20. The smaller ratios ($<4$) may be reasonable as observed levels of velocity spectra have often been found to be above those expected from internal waves alone. The over-response of the VACMs on surface moorings may help explain
the elevated levels. Indeed, the 92 m VACM when compared with a VMCM at 94 m gives spectral levels twice as high (Halpern et al., 1981, Fig. 6). During IWEX in the quieter deep ocean, the current fine-structure signal was found to be comparable to and even greater than the internal wave velocities at high frequency (Müller et al., 1978). The large factor at 175 m may be partly a result of mooring-motion contamination increasing with depth (Halpern et al., 1981), or an overly simplified model—more data would be needed to make a more definite statement. Measured velocity coherences diverge significantly from the model coherences. A model dominated by the first vertical mode predicts a 180° phase change across the zero-crossing at ~130 m, but this is not observed in the phase spectrum. We suspect that mooring contamination may be responsible for this observation.

In this model high coherence at the high frequencies results from choosing a small number of significant modes. Dillon and Caldwell (1980) also found during MILE low-mode-dominated internal waves at high frequency—mode 1 on 26 August and mode 2 on 29 August—using vertical velocities inferred from drop rates of their microstructure recorder. Other upper-ocean internal-wave studies by Pinkel (1975) from FLIP off California and Hawaii and Käse and Siedler (1980) from moorings deployed during GATE in the tropical Atlantic have also concluded that the high-coherence feature is due to a reduced number of significant modes at high frequency. There is also an indication from the deep ocean during IWEX (Müller et al., 1978) that the number of significant modes decreases as a function of frequency.

An alternative explanation for high vertical coherence accompanied by a spectral shoulder at high frequency is found by modifying the GM model to treat correctly the region around the turning depth where the WKB approximation is not valid (Desaubies, 1973, 1975). This occurs because the vertical dependence is described by standing modes that for a given frequency are locked in phase at a common turning depth. There is enhanced vertical coherence at high frequency because there are no zero-crossings of the modes between the depths, at least for the lowest modes. However, this explanation cannot apply to the observations at MILE. For example, between 41 and 175 m at 2.5 cph (Fig. 10), in the center of this high-coherence band, the free internal waves are confined between the turning depths 10 and 175 m, and there are always zero-crossings between 41 and 175 m for all modes higher than 1.

We speculate that the deviations from linear models at high frequency may be caused by nonlinear interaction with large-amplitude inertial- and tidal-frequency internal waves. The high-frequency waves travel at slow horizontal phase speeds that are comparable to the particle speeds of the energetic low-frequency waves (Fig. 16). It may be that no more than a few vertical modes can exist, since the slower higher linear modes can never develop because of critical-layer absorption. Such a nonlinear mechanism may also explain why the observed frequency cutoff at 41 m occurs more nearly at a "deep water" rather than the higher local value of N.

Acknowledgments. We thank NOAA Pacific Marine Environmental Laboratory, Woods Hole Oceanographic Institution and Nova University current-meter groups for preparing and mooring the VACMs. S. Hayes generously provided the CTD data. We also thank David Mandel for his assistance with the computation. We are pleased to acknowledge the Office of Naval Research Code 480 for supporting this research.

REFERENCES


