

**FAIR ALLOCATIONS AS POLICY HANDLE TO DISCOURAGE FREE RIDING IN REGIONAL FISHERIES MANAGEMENT**

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**ABSTRACT**

This paper presents the feasible proportional allocation rule to discourage free riding for a special class of free riding problems. Some theoretical and practical properties of the rule are discussed. Applications to the management of the Baltic Sea cod fishery and the Norwegian spring-spawning herring fishery are presented.

**Key Words:** *partition function form game, feasible allocation, free rider game, regional fishery management, proportional allocation rule.*

**INTRODUCTION**

Free riding can be seen as a prisoner's dilemma. Common resource management, as in the case of international fish stocks, may take this form. Consider for instance one of the main problems for an international fishery, the new member problem. The 1995 UN Fish Stocks Agreement (UNFSA) allows any nation to fish outside the Exclusive Economic Zone. Although the agreement mandates that a Regional Fishery Management Organization (RFMO) should manage such an international fish stock in a sustainable manner, distant water fishing nations may decide not to join a RFMO, but rather, to harvest in an individually optimal fashion (Bjørndal and Munro, 2003; Munro et al., 2004). This creates an incentive for all fishing nations not to join the RFMO and for incumbents to leave the RFMO which may lead to the breakdown of the fishery. (Observe the similarity to the tragedy of the commons (Hardin, 1968)).

This paper discusses the proportional allocation<sup>1</sup> rule to discourage free riding. Assuming that the players can freely merge or break apart and are farsighted<sup>2</sup>, we formulate a free rider problem as a game in partition function form (Thrall and Lucas, 1963). Using the principle of distributive fairness "equals should be treated equally, and unequals unequally, in proportion to relevant similarities and differences" (Moulin, 2003), we propose proportional allocation as a solution concept to achieve stable coalition structures. We also analyze the feasible set of coalitions, their values and how application of the proportional rule discourages free riding in the case of international fish resources.

Our approach is an extension of the work by Pham Do and Folmer (2006) and Kronbak and Lindroos (2005) in a search for fair solutions to discourage free riding

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<sup>1</sup>Proportional allocation is not a new idea but it "is deeply rooted in law and custom as a norm of distributed justice" (Young, 1994).

<sup>2</sup>That is, it is the final and not the immediate payoffs that matter to the coalitions (Chander, 2003).

in RFMOs. Pham Do and Folmer (2006) apply the Shapley value<sup>3</sup> for a special class of fishery games and Kronbak and Lindroos (2005) apply the satisfactory nucleolus. Although these sharing rules can be considered as "fair" for distributing the total positive gains from the grand coalition and stimulate the players to join the grand coalition, they are not sufficient to discourage free-riding, since for certain coalition structures free riding may result in a higher payoff (see Pham Do and Folmer (2006) and section 4 of this paper). Our approach is also related to Eyckmans and Finus (2004) and Pham Do et al (2006). However, Eyckmans and Finus (2004) use the concept of internal and external stability of d'Aspremont et al. (1983) to propose a sharing scheme for the distribution of the gains from cooperation for the particular class of games with only one non-trivial coalition, all other players are singletons. Pham Do et al. (2006) consider the population monotonic allocation schemes for a special class of fishery games with and without transferable technologies. This paper, on the other hand, focuses on a feasible solution concept for a class of free rider games.

The next section presents some preliminaries, particularly the basic concepts of games in partition function form and free rider games. Section 3 deals with with the notion of feasible allocation and its properties for free rider games. Section 4 presents two applications. Concluding remarks follow in the last section.

## PRELIMINARIES

Let  $N = \{1, 2, \dots, n\}$  be a finite set of players. Subsets of  $N$  are called *coalitions*. Let  $\mathcal{P}(N)$  be the set of all partitions<sup>4</sup> of  $N$ . For a partition  $\kappa \in \mathcal{P}(N)$  and  $i \in N$ , let  $S(i, \kappa)$  be the coalition  $S \in \kappa$  to which player  $i$  belongs. It will be convenient to economize on brackets and suppress the commas between elements of the same coalition. Thus, we will write, for example, 124 instead of  $\{1, 2, 4\}$

A partition of a subset  $S \subset N$  is denoted by  $\kappa_S$ ; a singleton coalition by  $\{i\}$ , the coalition structure consisting of all *singleton coalitions* by  $[N]$ ; the grand coalition by  $N$  and the coalition structure consisting of the grand coalition only by  $\{N\}$ . Finally, let  $|S|$  be the number of players in  $S$  and  $|\kappa|$  the number of coalitions in  $\kappa$ .

For  $i \in N$  and  $\kappa \in \mathcal{P}(N)$ , we define a coalition structure arranged by *affiliating*  $S(i, \kappa)$  to  $T (\neq S) \in \kappa$ , denoted by  $\kappa_{+i}(T)$ , as :

$$\kappa_{+i}(T) = \{(\kappa \setminus \{S, T\}) \cup (S \setminus \{i\}) \cup (T \cup \{i\})\}. \quad (1)$$

A coalition structure  $\kappa_{+i}(T)$  is simply a merge of player  $i$  into  $T$ . (Observe that the number of coalitions does not increase, i.e.  $|\kappa| \geq |\kappa_{+i}(T)|$ ).

A coalition structure  $\kappa_{-i}(S)$  arranged by *withdrawing*  $i$  from  $S$  is defined as follows:

$$\kappa_{-i}(S) = \{(\kappa \setminus S) \cup \{i\} \cup (S \setminus \{i\})\} \quad (2)$$

<sup>3</sup>The Shapley value in that paper refers to the modified Shapley value for the class of partition function form games (see Pham Do and Norde (2007) for details.).

<sup>4</sup>A *partition*  $\kappa$  of  $N$  is a set of pairwise disjoint nonempty coalitions,  $\kappa = \{S_1, \dots, S_m\}$ , such that their union is  $N$ .

A coalition structure  $\kappa_{-i}(S)$  is simply a split of player  $i$  from  $S$ . (Again, the number of coalitions does not decrease, i.e.  $|\kappa_{-i}(S)| \geq |\kappa|$ ).

For each  $i \in N$ , let  $\kappa^i$  denote a coalition structure where player  $i$  plays as a singleton, and  $\kappa^i(N)$  be the set  $\{\kappa \in \mathcal{P}(N) \mid \{i\} \in \kappa\}$ , i.e the set of all coalition structures where player  $i$  plays as a singleton. So,  $\kappa^i = \{\{i\}, \kappa_{N \setminus i}\} \in \kappa^i(N)$ .

**Example 1** Consider the coalition structure  $\kappa = \{123, 45\}$ . For  $i = 1, T = \{123\}$  and  $S = \{45\}$ , then  $\kappa_{+1}(S) = \{23, 145\}$ ,  $\kappa_{-1}(T) = \{1, 23, 45\}$ , and  $\kappa^1$  can be one of the following coalitions  $\{1, 23, 45\}$ ,  $\{1, 25, 34\}$ ,  $\{1, 24, 35\}$ ,  $\{1, 2345\}$ ,  $\{1, 2, 345\}$ ,  $\{1, 3, 245\}$ ,  $\{1, 4, 235\}$ ,  $\{1, 5, 234\}$ ,  $\{1, 23, 4, 5\}$ ,  $\{1, 2, 34, 5\}$ ,  $\{1, 24, 3, 5\}$ ,  $\{1, 25, 3, 4\}$ ,  $\{1, 35, 2, 4\}$ ,  $\{1, 2, 3, 45\}$ ,  $\{1, 2, 3, 4, 5\}$ .

A pair  $(S, \kappa)$  which consists of a coalition  $S$  and a partition  $\kappa$  of  $N$  to which  $S$  belongs is called an *embedded coalition*. Let  $E(N)$  denote the set of all embedded coalitions, i.e.  $E(N) = \{(S, \kappa) \in 2^N \times \mathcal{P}(N) \mid S \in \kappa\}$ .

A mapping  $w : E(N) \rightarrow R$  that assigns a real value  $w(S, \kappa)$  to each embedded coalition  $(S, \kappa)$  is called a *partition function*. The ordered pair  $(N, w)$  is called a *partition function form game*<sup>5</sup> (pffg).

The value  $w(S, \kappa)$  represents the payoff of coalition  $S$  given that coalition structure  $\kappa$  forms. For a given partition  $\kappa = \{S_1, S_2, \dots, S_m\}$  and partition function  $w$ , let  $\bar{w}(S_1, S_2, \dots, S_m)$  denote the  $m$ -vector  $(w(S_i, \kappa))_{i=1}^m$ . The set of partition function form games with player set  $N$  is denoted by  $\Gamma(N)$ . For convenience, we write  $w$  as a pffg, instead of  $(N, w)$ .

**Definition 1** Let  $w \in \Gamma(N)$ . We call player  $j \in N$  a *free rider* when it expects to benefit from a merger of the other players by staying outside the coalition. Formally,  $j$  is a free rider if for  $\kappa^j \neq [N]$ ,  $w(j, \kappa^j) > w(j, [N])$ .

**Example 2** Consider the partition function form game  $w$  defined by:

$\bar{w}(1, 2, 3) = (0, 0, 0)$ ,  $\bar{w}(12, 3) = (2, 0)$ ,  $\bar{w}(23, 1) = (3, 2)$ ,  $\bar{w}(13, 2) = (2, 1)$ ,  $\bar{w}(123) = 10$ . This game has two free riders: player 1 and player 2, since  $\min_{\kappa^i \in \kappa^i(N)} \{w(i, \kappa^i)\} = 0, \forall i \in N$ ,  $\max_{\kappa^1 \in \kappa^1(N)} \{w(1, \kappa^1)\} = 2$  and  $\max_{\kappa^2 \in \kappa^2(N)} \{w(2, \kappa^2)\} = 1$ , whereas  $\max_{\kappa^3 \in \kappa^3(N)} \{w(3, \kappa^3)\} = 0$ .

**Definition 2** Let  $w \in \Gamma(N)$ .  $w$  is called a *free rider game* if the two following conditions hold

- (i)  $w(N, \{N\}) \geq \sum_{S \in \kappa \in \Gamma(N)} w(S, \kappa)$ , and
- (ii)  $\forall i \in N, \forall S, T \in \kappa^i \setminus \{i\}, w(i, \kappa^i \setminus \{S, T\} \cup \{S \cup T\}) \geq w(i, \kappa^i)$ .

<sup>5</sup>For an application of partition function form games to fisheries see Pintassilgo (2003).

Condition (i) implies that the grand coalition is the most efficient coalition, while condition (ii) implies that player  $i$  expects to benefit from the merger of coalitions by not joining the merger. The set of free rider games is denoted by  $\mathcal{FRG}(\mathcal{N})$ .

Let  $w \in \mathcal{FRG}(\mathcal{N})$  and  $i \in N$ . We define the minimum and maximum payoffs for player  $i$  as follows:

$$\begin{aligned}\eta_i &= \min_{\kappa^i \in \kappa^i(N)} \{w(i, \kappa^i)\}, \\ \theta_i &= \max_{\kappa^i \in \kappa^i(N)} \{w(i, \kappa^i)\}.\end{aligned}\tag{3}$$

The value  $\eta_i$  is the payoff guaranteed to player  $i$  if it stays alone regardless of what the partition of  $N \setminus \{i\}$  does, whereas  $\theta_i$  is the maximum payoff that player  $i$  can expect when all others cooperate. The interval  $[\eta_i, \theta_i]$  is called a *feasible right* for each player  $i \in N$ . For any free rider  $i \in N$ ,  $\theta_i > \eta_i$ .

One can easily see that for every  $w \in \mathcal{FRG}(\mathcal{N})$ ,

$$\eta_i = w(i, [N]), \text{ and}\tag{4}$$

$$\theta_i = w(i, \kappa_{-i}(N)).\tag{5}$$

Note that (4) implies that the worst payoff is obtained when all players behave non-cooperatively (act as singletons) whereas (5) indicates an incentive for players to free ride since a free rider expects to get the highest payoff if it is the only outsider of the grand coalition.

**Definition 3** Let  $w \in \mathcal{FRG}(\mathcal{N})$ . A coalition  $S$  is called *stable* if no sub-coalition can improve its payoff by breaking up from the coalition, *ceteris paribus*, i.e.  $w(S, \kappa) \geq \sum_{S_b \in \kappa_S} w(S_b, \kappa_S \cup (\kappa \setminus S))$ .

The 1995 UN Agreement calls for cooperative management through RFMOs. This implies that the extension of a RFMOs and its stability are crucial features. This translates into the following necessary requirements for a RFMO (as a stable coalition in the coalition structure  $\kappa$  with  $|\kappa| < N$ ).

(C1) Feasibility:  $w(S, \kappa) \geq \sum_{i \in S} w(i, [N])$ ;

(C2) Potential stability:  $w(S, \kappa) \geq \sum_{i \in S} w(i, \kappa_{-i}(S))$ ;

(C3) Strong stability:  $S$  is potentially stable and

$$w(S, \kappa) = \max_{\kappa_{N \setminus S}} w(S, \kappa_{N \setminus S} \cup S)$$

The conditions (C1) and (C2) are necessary for forming a coalition, while (C3) implies the stability of coalition  $S$  under a coalition structure  $\kappa$ . Note that if (C2) holds for all coalitions, the coalition structure can be considered as a potentially stable coalition structure.

**Definition 4** A game  $w \in \mathcal{FRG}(\mathcal{N})$  is called *potentially stable* if there exists a coalition structure  $\kappa$  such that  $w(S, \kappa) \geq \sum_{i \in S} w(i, \kappa_{-i}(S))$  for every  $S \in \kappa$ .

A potentially stable game implies the existence of a potentially stable coalition structure in the sense that no player is interested in leaving its coalition to adopt free rider behavior. Moreover, if the grand coalition is stable, then no player is interested to leave it.

### FEASIBLE ALLOCATIONS

We now turn to the notion of feasible allocation to induce free riders to give up their behaviour. We make use of Myerson (1978) who points out that the basic requirement of a fair solution is that its allocation is feasible. Moreover, he observes that the construction of fair allocations (settlements) should be based on the expected payoffs in all feasible coalitions, particularly the grand coalition, taking into account threats. Below we focus on the construction of a feasible allocation. We shall pay attention to the question whether the coalition is profitable and how profit should be divided so as to induce the players to form a coalitions such that a free rider has an incentive to cooperate.

For every coalition  $S$ , a *reasonable allocation* (with respect to  $S$  in  $\kappa$ ) is defined as a vector  $x = (x_i)_{i \in S} \in R^{|S|}$  satisfying  $w(S, \kappa) \geq x(S) = \sum_{i \in S} x_i$  and  $x_i \geq w(i, [N])$  for every  $i \in S$ . Reasonable allocation implies that for every coalition, the sum of its allocation values (awards) does not exceed the worth of the coalition, whereas on the other hand the payoff of player  $i$  exceeds its payoff if the coalition structure consists of singletons only. The set of all reasonable payoffs for  $S$  in  $w$  is denoted by  $X(S, w)$ .

The *semi-stable set* of  $w$  is defined by

$$SemS(N, w) = \{x \in X(N, w) | \forall S \in \kappa, x(S) = w(S, \kappa)\}. \quad (6)$$

Semi-stability implies that all players can form a coalition structure in such a way that every player can find a coalition for itself that meets the demand of all members, exactly divides total payoff and that the payoff for each  $i \in S$  is individually rational. The semi-stable set exists for every free rider game  $w$ , as this coalition structure consists of all singletons satisfying all conditions of Definition 3. (Note that the semi-stable set differs from the imputation set known from the characteristic function (TU) game. An imputation set is the payoff vector for the grand coalition, whereas a semi-stable set assigns a vector to every possible coalition (structure), specifying individual payoffs to coalition members and outsiders).

A *weighted scheme of coalition  $S$*  is a collection of real numbers  $\lambda_S = (\lambda_{S,i})_{i \in S} \in R^{|S|}$  satisfying  $\sum_{i \in S} \lambda_{S,i} = 1$  and  $\lambda_{S,i} \in [0, 1]$ .

For example, an *upper weighted value* is defined as the collection of

$$\lambda_S = \left( \frac{\theta_i}{\sum_{j \in S} \theta_j} \right)_{i \in S}; \quad (7)$$

and a *lower weighted value* is the collection of

$$\lambda_S = \left( \frac{\eta_i}{\sum_{j \in S} \eta_j} \right)_{i \in S}, \quad (8)$$

where  $\theta_i = \max_{\kappa^i \in \kappa^i(N)} \{w(i, \kappa^i)\}$  and  $\eta_i = \min_{\kappa^i \in \kappa^i(N)} \{w(i, \kappa^i)\}$ .

A valuation<sup>6</sup> is a mapping  $\Psi$  which associates to each coalition structure  $\kappa \in P(N)$  a vector of individual payoffs in  $R^N$ .

A *weighted valuation* is a valuation  $\Psi$  such that

$$\Psi_i(S, w) = a_i + \lambda_{S,i} G(S, \kappa), \quad (9)$$

for every coalition  $S(i, \kappa)$ , where  $a_i \in [\eta_i, \theta_i]$ ,  $G(S, \kappa) = w(S, \kappa) - \sum_{i \in S} a_i$ , and  $\lambda_S = (\lambda_{S,i})_{i \in S}$  is a *weighted scheme* of  $S$ .

A weighted valuation gives an expected value to each player with respect to the distribution among the players of all the free rider values and the gain from cooperation. A weighted valuation is called *proportional valuation* if  $\lambda_S$  is chosen such that  $\lambda_{S,i} = \frac{\lambda_i}{\sum_{i \in S} \lambda_i}$ , where  $\lambda_i \in R^+$ .

Let  $\Psi : \mathcal{FRG}(\mathcal{N}) \rightarrow R^N$  be a valuation. The weighted valuation  $\Psi$

- (i) is *individually rational (IR)* if  $\Psi_i(w) \geq w(i, [N])$  for all  $i \in N$ .
- (ii) is *relatively efficient (RE)* if for  $w \in \Gamma(N)$

$$\sum_{i \in S \in \kappa} \Psi_i(S, w) = w(S, \kappa) \text{ for all } S \in \kappa.$$

(iii) satisfies *fair ranking (FR)* if for players  $i, j \in N$ ,  $\theta_i \geq \theta_j$ , for  $S(i, \kappa)$  and  $S(j, \kappa)$ , then  $\Psi_i(S, w) \geq \Psi_j(S, w)$ .

(iv) satisfies *claim right (CR)* if for player  $i \in N$ ,  $\min_{\kappa^i \in \kappa^i(N)} \{w(i, \kappa^i)\} = \max_{\kappa^i \in \kappa^i(N)} \{w(i, \kappa^i)\}$ , then  $\Psi_i(S, w) \geq \max_{\kappa^i \in \kappa^i(N)} w(i, \kappa^i)$ .

(v) is *relatively proportional (RP)* if for every player  $i \in N$  and  $S(\kappa, i)$ ,  $w(i, \kappa_{-i}(S)) = \lambda_i \sum_{j \in S} w(j, \kappa_{-j}(S))$ , then  $\Psi_i(S, w) = \lambda_i w(S, \kappa)$  where  $\lambda_i \in [0, 1]$  and  $\sum_{i \in S} \lambda_i = 1$ .

Below we shall pay attention to proportional valuation where  $\lambda_S$  is an upper weighted value of  $S$ . Note that this valuation splits the surplus (if  $G(S, \kappa) > 0$ ) or loss (if  $G(S, \kappa) < 0$ ) proportionally to what could be obtained by each player as an outsider.

**Proposition 1** *For every potentially stable game  $w \in \mathcal{FRG}(\mathcal{N})$ , there exists a proportional allocation that satisfies the five properties IR, RE, FR, CR and RP.*

<sup>6</sup>The notion "valuation" indicates that each player is able to evaluate directly the payoff it obtains in different coalition structures. Valuations thus emerge when the rule of division of the payoffs between coalition members is fixed (for further details, see Bloch, 2003).

**Proof.** For every  $S \in \kappa$ , define  $\Psi_i(S, w) = w(\{i\}, \kappa_{-i}(S)) + \lambda_i G(S, \kappa)$ , where  $G(S, \kappa) = w(S, \kappa) - \sum_{i \in S} w(\{i\}, \kappa_{-i}(S))$ ,  $\lambda_i = \frac{\theta_i}{\sum_{j \in S} \theta_j}$ , and  $\theta_i = \max_{\kappa^i \in \kappa^i(N)} w(i, \kappa^i)$ . Since  $G(S, \kappa) \geq 0$ , it follows that  $\Psi_i(S, w) \geq w(i, [N])$ ,  $\Psi(S, w) = \sum_{i \in S} \Psi_i(S, w) = w(S, \kappa)$ . Thus, if  $w(i, \kappa_{-i}(S)) \geq w(j, \kappa_{-j}(S))$  implies that  $\theta_i \geq \theta_j$  then  $\Psi_i(S, w) \geq \Psi_j(S, w)$ . Now let  $i \in N$  be a player such that  $\min_{\kappa^i \in \kappa^i(N)} \{w(i, \kappa^i)\} = \max_{\kappa^i \in \kappa^i(N)} \{w(i, \kappa^i)\}$ . Thus,  $\eta_i = \theta_i = w(i, \kappa^i) \leq w(i, \kappa_{-i}(S)) + \lambda_i G(S, \kappa) = \Psi_i(S, w)$ . ■

**Example 3** Consider the oligopoly game defined as  $\bar{w}(1, 2, 3) = (36, 16, 9)$ ;  $\bar{w}(12, 3) = (57.78, 18.78)$ ;  $\bar{w}(13, 2) = (49, 25)$ ;  $\bar{w}(23, 1) = (25, 49)$  and  $\bar{w}(123) = 90.25$ . In this game,  $\eta = (\eta_i)_{i=1,2,3} = (36, 16, 9)$  and  $\theta = (\theta_i)_{i=1,2,3} = (49, 25, 18)$ . The proportional rule with upper weighted value  $\lambda_N$  leads to  $\Psi(N, w) = (47.66, 24.32, 18.27)$ . The modified Shapley value<sup>7</sup> for this game would lead to  $Sh(w) = (46.70, 24.71, 18.83)$ .

This example shows that the modified Shapley values assigns more value to players 2 and 3 than to player 1, while the proportional valuation assigns more value to player 1 and less value to players 2 and 3.

We define an *adjustment of proportional allocation*  $APV(w)$  as

$$APV(w) = \{\Psi(S, w) | \forall S \in \kappa, \forall \kappa \in \mathcal{P}(N) \text{ and } w \in \Gamma(N)\}, \text{ where}$$

$$\Psi_i(S, w) = \begin{cases} w(i, \kappa_{-i}(S)) + \lambda_i G(S, \kappa), & \text{if } S \text{ is potentially stable} \\ w(i, [N]) + \lambda_i (w(S, \kappa) - \sum_{i \in S} w(i, [N])), & \text{otherwise} \end{cases}, \quad (10)$$

**Proposition 2** Let  $w \in \mathcal{FRG}(N)$ , then  $APV(w) \subset SemS(N, w)$ .

**Proof.** Let  $\lambda_S = (\lambda_i)_{i \in S}$  be a weighted scheme. Since  $w \in \mathcal{FRG}(N)$ , it follows that  $w(S, \kappa) \geq \sum_{i \in S} w(i, [N])$ . Therefore,

(i) if  $S$  is not potentially stable, then  $\Psi_i(S, w) = w(i, [N]) + \lambda_i (w(S, \kappa) - \sum_{i \in S} w(i, [N])) \geq w(i, [N])$

(ii) if  $S$  is potentially stable, then  $w(S, \kappa) \geq \sum_{i \in S} w(i, \kappa_{-i}(S))$  implies that  $G(S, \kappa) \geq 0 \Rightarrow \Psi_i(S, w) = w(i, \kappa_{-i}(S)) + \lambda_i G(S, \kappa) \geq w(i, \kappa_{-i}(S)) \geq w(\{i\}, [N])$ .

Since  $\Psi(S) = \sum_{i \in S} \Psi_i(S, w) = w(S, \kappa) \Rightarrow APV(w) \subset SemS(N, w)$ . ■

The propositions above lead to the following Theorem.

**Theorem 3** For every free rider game, there exists a feasible allocation that satisfies individual rationality, relative efficiency, fair ranking and claim right. Moreover, if this game is potentially stable then this allocation is relatively proportional.

<sup>7</sup>Recall that the modified Shapley value (Pham Do and Norde, 2007) is the Shapley value for the class of partition function form games. It is calculated as the average of the marginal contributions for each player in all coalition structures consisting of one non-trivial coalition and others as singletons.

**Remark 1** Let  $w \in \mathcal{FRG}(N)$ . The grand coalition is strong stable if  $w(N, \{N\}) = \max_{\kappa \in P(N)} \sum_{S \in \kappa} w(S, \kappa) \geq \sum_{i \in N} w(i, \kappa_{-i}(N))$ .

## APPLICATIONS

This section presents applications of the feasible allocation rule to the Baltic Sea cod fishery and the Norwegian spring-spawning herring fishery. The underlying bioeconomic models and calculations are adopted from Kronbak and Lindroos (2005) and Lindroos and Kaitala (2000).

### The Baltic Sea Cod fishery<sup>8</sup>

In the Baltic Sea cod fishery there are three participants: four "old" EU member states (Denmark, Finland, Germany and Sweden), four "new" EU member states (Estonia, Latvia, Lithuania, Poland) and the Russian Federation. The International Baltic Sea Fishery Commission (IBSFC)<sup>9</sup> manages the Baltic Sea cod fishery. The countries participating in the Baltic Sea cod fishery are represented in the IBSFC by their coalitions (1: old EU member states, 2: new EU member states, and 3: Russian Federation). The optimal strategy of each coalition is to maximize its net present value, given the behavior of the non-members.

There are five possible coalition structures:  $[N] = \{1, 2, 3\}$ ,  $\{N\} = \{123\}$ ,  $\{12, 3\}$ ,  $\{13, 2\}$ , and  $\{23, 1\}$ . Table 1 show the payoffs of the coalition structures (Kronbak and Lindroos, 2005).

Table 1. The possible benefits (Dkr (mil.)) from five coalition structures in the Baltic Sea cod fishery

Coalition	Net benefit	Free rider value
1	23069	-
2	16738	-
3	15608	-
12	42562	20276
13	41250	21094
23	33544	28456
123	74717	-

Source: Adjusted from Kronbak and Lindroos (2005)

From Table 1, the free rider game  $w$  is obtained as follows:

<sup>8</sup>The Baltic Sea fishery is not a high sea fishery and is not facing the problem of new members. Nevertheless, there could be a problem of free riding, a situation that we analyze below.

<sup>9</sup>The IBSFC was abolished in January 2007 when the EU member states withdrew from it in 2006.

$$\bar{w}(1, 2, 3) = (23069, 16738, 15608); \bar{w}(12, 3) = (42562, 20276);$$

$$\bar{w}(13, 2) = (41250, 21094); \bar{w}(1, 23) = (28456, 33544); \bar{w}(123) = 74717.$$

This game is potentially stable since  $w(S, \kappa) \geq \sum_{i \in S} w(i, \kappa_{-i}(S))$ , for all  $S$ , and all  $\kappa$ . Moreover,  $w(N, \{N\}) = \max_{\kappa \in \mathcal{P}(N)} \sum_{S \in \kappa} w(S, \kappa) = 74717 \geq \sum_{i \in N} w(i, \kappa_{-i}(N)) = 69826$  implies the stability of the grand coalition.

In Table 2 the outcome of the proportional allocation rule (Proposition 1) is presented. We also present the outcomes of the alternative sharing rules modified Shapley value (Pham Do and Norde, 2007) and satisfactory nucleolus<sup>10</sup> (Kronbak and Lindroos, 2005) for comparison.

Table 2. The feasible allocations in the Baltic Sea cod fishery (value in Dkr (mil.), percentages in brackets)

Player	Free rider	Modified Shapley value	Satisfactory nucleolus	Proportional allocation
1	28456 (40.8)	29962 (40.1)	30111 (40.3)	30451 (40.8)
2	21094 (30.2)	23013 (30.8)	22714 (30.4)	22571 (30.2)
3	20276 (29.0)	21743 (29.1)	21892 (29.3)	21694 (29.0)

From Table 2 it follows that the proportional allocation rule preserves each coalition's share under free riding behavior. Moreover, in absolute terms each coalition is better off than under free riding.

### The Norwegian Spring-spawning Herring fishery

In the Norwegian spring-spawning herring fishery the following nations participate: Norway, Iceland, The Russian Federation, Faeroe Islands and some members of the EU. The latter is a distant water fishing nation. Lindroos and Kaitala (2000) argue that on the basis of historical developments the following coalitions are involved in the fishery: coalition 1 (Norway and the Russian Federation), coalition 2 (Iceland and the Faeroe Islands) and coalition 3 (EU). Table 3 shows the values of possible coalition structures.

From Table 3 the following free rider game  $w$  is obtained:

$$\bar{w}(1, 2, 3) = (4878, 2313, 986); \bar{w}(12, 3) = (19562, 14534);$$

$$\bar{w}(13, 2) = (18141, 17544); \bar{w}(23, 1) = (17544, 18141); \bar{w}(123) = 44494.$$

<sup>10</sup>The satisfactory nucleolus is a modified imputation calculated in a similar fashion as the nucleolus (for details, see Kronbak and Lindroos, 2005).

Table 3. The possible benefits (Dkr (mil.)) for five coalition structures

Coalition	Net benefit	Free rider value
1	4878	-
2	2313	-
3	896	-
12	19562	14534
13	18141	17544
23	17544	18141
123	44494	-

Source: Lindroos and Kaitala (2000)

Observe that the grand coalition is not potentially stable, as  $w(N, \{N\}) = 44494 \leq \sum_{i \in N} w(i, \kappa_{-i}(N)) = 50219$ . However, it is efficient, as  $w(N, \{N\}) \geq \sum_{S \in \kappa} w(S, \kappa)$  for all  $\kappa \in \mathcal{P}(N)$ . Since it is not potentially stable, Lindroos and Kaitala (2000) conclude that a multilateral agreement is not feasible. However, since

- (1) the RFMO can freely accept new members and that members can break apart;
- (2) the players are farsighted and aware of the fact that free riding ultimately will lead to the worst case scenario of a break-down of the fishery;

it makes sense to consider efficient allocation. Particularly, Chander (2003) argues that there are two alternative coalition structures in the long run under the assumption that all players are farsighted: full cooperation and no-cooperation (other outcomes can be considered as intermediate outcomes that will ultimately lead to either full cooperation or no-cooperation). The ultimate outcome depends on how each player evaluates its share from the final surplus. We apply the proportional allocation rule to calculate feasible allocations. For comparison we also present the outcome for the modified Shapely value<sup>11</sup>. The results are presented in Table 4.

Table 4. Allocations in the Norwegian fishery (value in Dkr (mil.), percentages in brackets).

Player	Free rider	Modified Shapley value	Proportional allocation
1	18141 (36.1)	16030 (36.7)	16074(36.1)
2	17544 (35.0)	14816 (33.3)	15540 (35.0)
3	14534 (28.9)	13348 (30.0)	12880 (28.9)

We observe that the outcome for each coalition under each allocation rule is smaller than the outcome under free riding. Moreover, the modified Shapley value is relatively more beneficial for player 1 (36.7 % vs 36.1) whereas players 2 and 3 are relatively worse

<sup>11</sup>Since the satisfactory nucleolus (Kronbak and Lindroos (2005)) is applicable to a stable game only, we do not use it in this comparison.

off. Finally, proportional allocation is the only rule that preserves the proportional shares under free riding. Therefore, its outcome is most likely to be accepted.

### CONCLUDING REMARKS

The purpose of this paper is to analyze the properties of the proportional allocation rule as “fair” sharing rule for a special class of free rider games and shows how this rule can be applied to stimulate cooperation and to discourage free riding. We present five conditions that a reasonable and fair sharing rule should meet: individual rationality, relative efficiency, fair ranking, claim right and relative proportionality. We show that the proportional rule satisfies all five properties.

We also point out that alternative allocation rules, particularly the modified Shapley value and satisfactory nucleolus, do not satisfy the relative proportional characteristic. Furthermore, we compare the proportional allocations to other sharing devices, notably the modified Shapley value. Two applications to international fisheries are presented. We have shown that if all players are free to merge or break apart and are farsighted, then all players have an incentive to cooperate, since they are ultimately better off than in a non-cooperative outcome. In particular, the proportional rule is the only one that preserves the proportional shares under free riding. Therefore, it's most likely to be accepted in a real world policy context.

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