Anchor-last Deployment Procedure for Mooring

by
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ANCHOR-LAST DEPLOYMENT PROCEDURE
FOR MOORING

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ABSTRACT

The anchor-last mooring procedure is investigated in order to determine the transient forces in the mooring line and the velocities of the anchor. Transient forces were determined and the results showed that no severe snap loads occurred for the cases investigated. In addition, it was found that the vertical velocity of the anchor can be small as it approaches impact with the floor of the ocean.

Both extensible (nylon and dacron) and inextensible (steel wire rope) lines were investigated. Lumped mass numerical models were developed for both cases. For the extensible line case the equations of motion were determined for each mass from Newton's Second Law, and they were integrated using a second order predictor-corrector integration technique. Hamiltonian techniques were utilized to determine the equations of motion for the inextensible line. The predictions from the numerical models show the line tensions and positions as a function of time.
INTRODUCTION

Oceanographic research buoys moored in deep water are often deployed by the anchor-last deployment procedure. The general sequence of events in this procedure is: the buoy is deployed from the ship with the mooring line attached and distributed in some manner on the ship; while the buoy drifts away from the ship the line is paid out until only the anchor, which is secured to the lower end of the mooring line, remains on board; finally the anchor is cast overboard. At sometime during the descent of the anchor it is possible for large transient forces to be exerted on the line, which can be destructive to the line or to attached conductors and instruments. One purpose of this research is to examine the possibilities of such large forces for line scopes greater than one. The line scope is the unstressed length of the line divided by the water depth.

The Woods Hole Oceanographic Institution has conducted a series of field measurements of the launching transient in mooring lines where the anchor-last deployment procedure was utilized (1). However, the study was restricted to the consideration of moorings where the scope was less than 1.0. It was found that a fairly steady increase in line tension occurred after the anchor drop and during the free-fall stage. Then a pendulum action followed where the final line tension was equal to the submerged weight of the anchor. During the pendulum action, the line (nylon) was stretching until the anchor reached bottom. After the anchor reached bottom a reduction in line tension occurred.

A similar problem was investigated by Froidevaux and Scholten (4) with a numerical model which considered the line to be a discrete number of lumped masses connected by weightless line segments. However, when the elongation properties of the line were included, a prohibitive amount of computer time resulted and the
investigation only calculated the first and last few seconds of fall. A short system was considered and results were extrapolated to apply to the 6500 Oceanic Telescope. One scope which was greater than 1.0 was investigated but the distance from the buoy to the anchor at launch time was not determined, nor the influence of scope and line type on line tensions. The conclusion from the study was that there may be an overstress shortly after the drop and that a severe transient may occur when the anchor made impact with the ocean floor. Therefore, the anchor-first mooring procedure was recommended. During this investigation it was found that large transient forces shortly after anchor deployment can be an artifact of the numerical program.

The Electronics Division of General Dynamics has employed the anchor-last deployment procedure for several moorings of the forty-feet diameter oceanographic buoy for the Office of Naval Research. It has been noticed that some conductors in the upper portion of the line have experienced large stresses during the deployment of some moorings but not for others. Thus it was suspected that large transient forces occurred during the anchor deployment either concentrated at the buoy, or propagating up the line to the buoy. This investigation predicts that large snap loads should not occur for such moorings.

Goeller (5) investigated snap loads in steel cables but the study was restricted to the consideration of straight lines with a scope equal to 1.0 where the mooring was already in place and the upper end was excited with a sinusoidal motion.

A review of the current techniques for the dynamic analysis of mooring line systems was presented by Casarella and Parsons (3).

The first technique to be employed during this study was like the one used by Nath (7, 8) where the line is considered to be a continuum and the partial
differential equations of motion are solved by the method of characteristics. Later the analysis technique was changed to a lumped mass approach that was very similar to the analysis made by Wang (10) except that the equations developed were in a different coordinate reference frame and the integration technique was different.
NUMERICAL MODELS FOR EXTENSIBLE LINES

The numerical models presented here pertain specifically to the anchor-last mooring procedure. The basic equations and procedures for the method of characteristics solution were presented by Nath (7, 8) and they will not be reproduced here. However, certain modifications to the earlier program will be presented which shows the attempts to utilize the method of characteristics for the anchor-last procedure. The lumped mass model for the extensible mooring line will be presented in detail. It is similar to the approach taken by Froidevaux and Scholten (4). The lumped mass model for the inextensible case will not be presented in detail because it is available in the work by Rupe (9).

Method of Characteristics

The computer program presented in (7) was modified to accept the new boundary conditions of the anchor-drop problem. The first runs produced results which gave unrealistic mooring line shapes. Attempts to improve on the numerical procedure met with only limited success. The problem was caused by the extremely tight curvature of the line in the first few seconds of fall. The calculations of line positions did not correspond to the line velocities.

The solution technique was to solve for the normal velocity, \( V_N \), the tangential velocity \( V_T \) and the local angle, \( \theta \), made by the tangent to the line and the horizon.
For simplicity and as a first approximation the buoy attachment point was assumed to be fixed. At first it was assumed that the mooring line was composed of a series of straight lines between calculation points. Then the coordinates of points along the line could be calculated using:

\[ X_i = X_{i-1} + \Delta S \cos \theta_i \]
\[ Z_i = Z_{i-1} + \Delta S \sin \theta_i \]  

(1)

where \( \Delta S \) is the distance between calculation points. It is easy to visualize why this technique would give unreasonable results if the mooring line were to be given a high curvature or "kink", which is exactly what takes place during the early stages of anchor drop.

In an attempt to eliminate this problem the following more accurate approximation was used to calculate the mooring line position:

\[ \frac{dx_i}{ds} = \cos \theta_i \]
\[ \frac{dz_i}{ds} = \sin \theta_i \]  

(2)

Since the \( \theta_i \)'s are known at constant intervals along the line, then all that is required is to integrate the above equations along the line from the attachment point to the anchor. For example, the trapazoidal integration formula gives

\[ X_i = X_{i-1} + \frac{\Delta S}{2} (\cos \theta_i + \cos \theta_{i+1}) \]
\[ Z_i = Z_{i-1} + \frac{\Delta S}{2} (\sin \theta_i + \sin \theta_{i+1}) \]  

(3)
which is equivalent to assuming that the line shape is quadratic between calculation points. Both the above formula and Simpson's rule were used, however, the curvature of the line is so steep that even these higher order approximations did not eliminate the problem. Therefore, the decision was made to proceed with the lumped mass model of the following section.

**Lumped Mass Model**

A mooring line can be modeled as a group of discrete masses interconnected by springs as illustrated in Fig. 1. The various distributed forces which act on a mooring line are gravity, hydrodynamic viscous drag and acceleration, and the line tension, which is influenced by the internal, or structural, damping and the stress-strain relationship. In the lumped mass model these forces are discretized and it is assumed that they act at each mass. Figure 2 shows the various forces acting on a typical mass. The magnitude of the various forces were calculated in the following manner:

**Spring Force.** From closed solution studies of taut line mooring dynamics it has been found that the external hydrodynamic forces are considerably larger than the internal damping force. For this study in particular, where periodic oscillations are not a factor, the internal damping force was negligible. Therefore it was not included into the equations of motion of the masses.

Hooke's Law was taken in the general form

\[ \sigma_i = f(\varepsilon_i) \]  \hspace{1cm} (4)

calculating the strain, using the straight line distance between masses, then

\[ \varepsilon_i = \frac{\Delta l_i}{l_i} = \left[ \frac{(x_{i+1} - x_i)^2 + (Z_{i+1} - Z_i)^2}{l_i^2} \right]^{1/2} - \frac{l_i}{l_i} \]  \hspace{1cm} (5)
where $\lambda_i$ is the unstressed distance between masses and $x_i$ and $z_i$ are the coordinates of the mass $M_i$. This gives the spring force $SF_i$ as:

$$SF_i = \text{Area} \cdot f \left\{ \frac{(x_{i+1} - x_i)^2 + (z_{i+1} - z_i)^2}{\lambda_i^2} - \lambda_i \right\}.$$

The net spring force acting on the mass $M_i$ is given by

$$\begin{align*}
\left\{ \text{spring force} \right\}_{x \text{ direction}} &= SF_{x_i} = SF_i \cos(\theta_i) - SF_{i-1} \cos(\theta_{i-1}) \\
\left\{ \text{spring force} \right\}_{z \text{ direction}} &= SF_{z_i} = SF_i \sin(\theta_i) - SF_{i-1} \sin(\theta_{i-1})
\end{align*}$$

(7)

The Hooke’s Law relationship was assumed to be in the form

$$\sigma_i = C_1 (e_i)^2 C_2$$

where $C_1$ and $C_2$ are constants. The values used for the various lines are given in the appendix.

Hydrodynamic Drag Forces. Using the quadratic drag law, the tangential and normal drag forces on the mass $M_i$ are given by

$$\begin{align*}
\left\{ \text{Tangential Drag} \right\} &= DFT_i = \frac{CDT \cdot \rho \cdot AT}{2} (VT_i) |VT_i| \\
\left\{ \text{Normal Drag} \right\} &= DFN_i = \frac{CDN \cdot \rho \cdot AN}{2} (VN_i) |VN_i|
\end{align*}$$

(8)

where tangent and normal are referenced to the angle $\theta AV_i$ as illustrated in Figure 2, and
CDT = Tangential Drag Coefficient
CDN = Normal Drag Coefficient
VT\textsubscript{i} = Tangential Velocity of Mass M\textsubscript{i}
VN\textsubscript{i} = Normal Velocity of Mass M\textsubscript{i}
\rho = Fluid Density
AT = Tangential Area = \pi D\textsubscript{\textit{i}}
AN = Normal Area = D\textsubscript{\textit{i}}

Defining VCX and VCZ to be the local velocities due to currents and waves, then the tangential and normal velocities of mass M\textsubscript{i} are given by

\[ VT\textsubscript{i} = (\dot{x}\textsubscript{i} - VCX) \cos (\theta_{AV\textsubscript{i}}) + (\dot{z}\textsubscript{i} - VCZ) \sin (\theta_{AV\textsubscript{i}}) \]
\[ VN\textsubscript{i} = - (\dot{x}\textsubscript{i} - VCX) \sin (\theta_{AV\textsubscript{i}}) + (\dot{z}\textsubscript{i} - VCZ) \cos (\theta_{AV\textsubscript{i}}) \]  

Added Mass Forces. The added mass forces due to the acceleration of the line masses through the still fluid was considered in the usual way. That is, the excitation force accelerates the line and the fluid medium around the line. The pressure distribution on the line from the accelerating fluid is characterized by the displaced mass of fluid times a coefficient called the added mass coefficient and the product is called the added mass. The sum of the line mass and the added mass coefficient for the line in the direction normal to the line was taken equal to that for a smooth cylinder (0.5) and in the tangential direction it was assumed to be zero.

Governing Equations for the Line. Substituting the above forces into Newton's second law and including the hydrodynamic added mass effect with the actual line mass (the sum is the virtual mass)
\[ \sum F_{\text{tangential}} = (M_i + CIT \cdot \rho \cdot Vol_i) \cdot ACT_i \]

\[ \sum F_{\text{normal}} = (M_i + CIN \cdot \rho \cdot Vol_i) \cdot ACN_i \]

where

- \( CIT \) = Added Mass Coefficient Tangential
- \( CIN \) = Added Mass Coefficient Normal
- \( \rho \) = Fluid density
- \( Vol \) = Displaced Line Volume = \( \frac{\pi D^2}{4} l_i \)
- \( ACN \) = Normal Acceleration of the Mass \( M_i \)
- \( ACT \) = Tangential Acceleration of the Mass \( M_i \)

Then the accelerations are given by

\[ ACT_i = \frac{(SFX_i \cos (\theta AV_i)) + (SFZ_i - WT_i \sin (\theta AV_i)) + DFT_i}{(M_i + CIT \cdot \rho \cdot Vol_i)} \]

\[ ACN_i = \frac{- (SFX_i \sin (\theta AV_i)) + (SFZ_i - WT_i \cos (\theta AV_i)) + DFN_i}{(M_i + CIN \cdot \rho \cdot Vol_i)} \]

In the \( x, z \) coordinate system the accelerations are given by

\[ \ddot{x}_i = ACT_i \cos (\theta AV_i) - ACN_i \sin (\theta AV_i) \]

\[ \ddot{z}_i = ACT_i \sin (\theta AV_i) + ACN_i \cos (\theta AV_i) \]

**Governing Equations for the Anchor.** Figure 3 shows a free body diagram of the anchor from which the following governing equations can be obtained.
\[ \dot{x} = \frac{T \cos(\beta) - \frac{1}{2} CD \cdot A \cdot \rho \cdot \dot{x} \cdot V}{(MA + CI \cdot \rho \cdot Vol)} \]

\[ \dot{z} = \frac{T \sin(\beta) - W - \frac{1}{2} CD A' \rho \cdot \dot{z} \cdot V}{(MA + CI \cdot \rho \cdot Vol)} \]

where $CD = \text{the anchor drag coefficient}$

$T = \text{line tension at the anchor}$

$CI = \text{the anchor added mass coefficient, constant in all directions}$

$A = \text{Area} = \pi d^2/4$ .

For simplification, the water velocities from current and waves have been ignored. They can be easily added if necessary. The anchor was assumed to be a sphere, with a high drag coefficient to recognize the fact it is not actually a sphere.

Numerical Integration. Numerical integration of Eqs. (12) and (13) was accomplished with the following second order predictor-corrector set (6).

Velocity Predictor:

Let

\[ X_i(t) = f(x_i(t), z_i(t), \dot{x}_i(t), \dot{z}_i(t)) \]

and

\[ Z_i(t) = g(x_i(t), z_i(t), \dot{x}_i(t), \dot{z}_i(t)) \]

where the functions $f$ and $g$ are equations (12) above. The velocity predictors are then

\[ \dot{x}_i^p(t) = \dot{x}_i(t-\Delta t) + 2\Delta t \cdot \dot{x}_i(t) \]
and \[ z^P_1(t) = \dot{z}_1(t-\Delta t) + 2\Delta t \ddot{z}_1(t) \tag{15} \]

Displacement Predictor: Similarly the predictors for displacements are

\[
\begin{align*}
\dot{x}^P_1(t) &= x_1(t) + \frac{\Delta t}{2} (\dot{x}_1(t) + \dot{x}^P_1(t+\Delta t)) \\
\dot{z}^P_1(t) &= z_1(t) + \frac{\Delta t}{2} (\dot{z}_1(t) + \dot{z}^P_1(t+\Delta t))
\end{align*}
\tag{16}
\]

Velocity Corrector: The corrector equations are for velocities

\[
\begin{align*}
\dot{x}^C_1(t+\Delta t) &= f(x^P_1(t+\Delta t), z^P_1(t+\Delta t), \dot{x}^P_1(t+\Delta t), \dot{z}^P_1(t+\Delta t)) \\
\dot{z}^C_1(t+\Delta t) &= g(x^P_1(t+\Delta t), z^P_1(t+\Delta t), \dot{x}^P_1(t+\Delta t), \dot{z}^P_1(t+\Delta t)) \\
\end{align*}
\tag{17}
\]

\[
\begin{align*}
\dot{x}^C_1(t+\Delta t) &= \dot{x}_1(t) + \frac{\Delta t}{2} (\ddot{x}_1(t) + \dot{x}^P_1(t+\Delta t)) \\
\dot{z}^C_1(t+\Delta t) &= \dot{z}_1(t) + \frac{\Delta t}{2} (\ddot{z}_1(t) + \dot{z}^P_1(t+\Delta t))
\end{align*}
\tag{18}
\]

Displacement Corrector: Likewise, the corrector equations for displacements are

\[
\begin{align*}
\dot{x}^C_1(t+\Delta t) &= x_1(t) + \frac{\Delta t}{2} (\dot{x}_1(t) + \dot{x}^C_1(t+\Delta t)) \\
\dot{z}^C_1(t+\Delta t) &= z_1(t) + \frac{\Delta t}{2} (\dot{z}_1(t) + \dot{z}^C_1(t+\Delta t))
\end{align*}
\tag{19}
\]

The corrector values are then considered to be the actual position and velocity. The appropriate error at each step of the integration for each mass \( M \)
can be calculated (6) using the following formula

$$\epsilon(\dot{x}_i) = (\dot{x}^{c}_i(t) - \dot{x}^{p}_i(t))/S.$$  \hspace{1cm} (20)

$$\epsilon(\dot{z}_i) = (\dot{z}^{c}_i(t) - \dot{z}^{p}_i(t))/S.$$  

These errors were monitored at all time during the integration and when errors exceeded a specified limit, the time step was cut in half and the integration continued; conversely, if they fell below some minimum limit the time step was doubled.

From the form of equations (14) through (19) it is evident that both position and velocity must be known for the time \((t - \Delta t)\) and \((t)\). However, when starting a solution or just after the time step has been changed the position and velocity will only be known for the last time step, and therefore some other equations must be used to generate this required data. In this particular case the Runge-Kutta third order single step integration formulas were used. Runge-Kutta: Equations (14) can be integrated to give

$$\dot{x}_i(t+\Delta t) = \dot{x}_i(t) + (k_1 + 4k_2 + k_3)/6$$  \hspace{1cm} (21)

$$\dot{z}_i(t+\Delta t) = \dot{z}_i(t) + (k_1 + 4k_2 + k_3)/6$$

and

$$x_i(t+\Delta t) = x_i(t) + (n_1 + 4n_2 + n_3)/6$$
\[ z_i(t + \Delta t) = z_i(t) + \frac{m_1 + 4m_2 + m_3}{6} \]  \hspace{1cm} (22)

where

\[ \xi_1 = \Delta t f(x_i(t), z_i(t), \dot{x}_i(t), \dot{z}_i(t)) \]

\[ \xi_2 = \Delta t f(x_i(t) + n_1/2, z_i(t) + m_1/2, \dot{x}_i(t) + \xi_1/2, \dot{z}_i(t) + \dot{k}_1/2) \]

\[ \xi_3 = \Delta t f(x_i(t) - n_2 + 2n_2, z_i(t) - m_1 + 2m_2, \dot{x}_i(t) - \xi_1 + 2\xi_2, \dot{z}_i(t) - k_1 + 2k_2) \]  \hspace{1cm} (23)

\[ \dot{z}_i(t) - k_1 + 2k_2 \]

\[ k_1 = \Delta t g(x_i(t), z_i(t), \dot{x}_i(t), \dot{z}_i(t)) \]

\[ k_2 = \Delta t g(x_i(t) + n_1/2, z_i(t) + m_1/2, \dot{x}_i(t) + \xi_1/2, \dot{z}_i(t) + \dot{k}_1/2) \]

\[ k_3 = \Delta t g(x_i(t) - n_2 + 2n_2, z_i(t) - m_1 + 2m_2, \dot{x}_i(t) - \xi_1 + 2\xi_2, \dot{z}_i(t) - k_1 + 2k_2) \]  \hspace{1cm} (24)

\[ n_1 = \Delta t \dot{x}_i(t) \]

\[ n_2 = \Delta t (\dot{x}_i(t) + 1/2 \xi_1) \]
Using the predictor-corrector set and the single step Runge-Kutta equation for the first time step the governing equation for the lumped mass mooring line and the anchor can be numerically integrated. In this routine, the error can be automatically controlled by increasing or decreasing the time step $\Delta t$ through Eqs. (20). The general outline of the calculation scheme is illustrated in Fig. 4.

It should be noted that higher ordered predictor-corrector schemes exist which increase the precision of the integration. However, the accuracy of the technique used may be well within the accuracy of the entire computations. Until data from physical models becomes available to prove otherwise it is felt that this technique gives adequate results.
A LUMPED MASS MODEL FOR INEXTENSIBLE LINES

A tentative model for the inextensible case has been developed jointly with the Oregon State University Sea Grant Program (9). Hamiltonian techniques were utilized to develop the equations of motion. The explicit detailing of the numerical procedures will not be presented here as for the inextensible line case because this has been accomplished in Reference (9).

The model assumed the line to be perfectly inextensible. Thus the motion of each mass is more directly influenced by the motion of each other mass than for the extensible line case. A listing of the program is presented in the Appendix B.
Prior to the production runs several preliminary tests were made. The first of these showed large dynamic loading in the segment of the line near the anchor at about 20 seconds after the anchor was released from the "standard catenary configuration" of Fig. 5a. This result was reported in the 1972 report to ONR together with results where the anchor was released from the "goose neck" catenary configuration of Fig. 5b. The second configuration showed much lower dynamic loading. Since that time the segment lengths were made considerably smaller and the control on the time step was improved by using the integration method described in this report. It is now evident that the high dynamic loading reported earlier was an artifact of the first lumped mass numerical program. The results presented in the following section show only a small amount of transient loading during the early stages of anchor deployment.

After this experience, each test case to be run was divided into several time segments and the computer results were examined at the end of each time segment. In this manner, links in a critical area along the line could be subdivided while links in areas where the line tensions were more constant could be consolidated. After these adjustments, the computer continued the solution from that time until the end of the next time segment where the output would again be examined and adjustments made in the link lengths. In this way, accurate results could be obtained without using an excessive number of links and computer time could be conserved.

Nylon Line

Figures 6 through 13 present the results for the 2.5" diameter nylon mooring line for both the 15° catenary and the 60° catenary initial condition. Figures
12 and 13 show results for the "goose neck" initial position. Figure 6 shows the line shape for various times during the drop for the 15° catenary. Figure 7 shows the tension contours for the 15° catenary as a function of position and time. The most notable feature of these results is that the line has essentially constant tension during most of the drop, and there is no snap load during the fall or after the anchor touches bottom. Figures 8 and 9 present the same plots for the 60° catenary. Although the line tension is not constant the tensions vary slowly with time and position due to the high hydrodynamic resistance forces acting on the line.

Figure 10 compares the tension for both catenaries at the anchor link for the first 140 seconds of drop. Figure 11 compares the vertical anchor velocities during the drop. It is interesting to note that the terminal velocity for the 12,000 lb. anchor in free fall (unrestrained by a mooring line and where drag acts only on the anchor) is about 50 ft/sec. When the anchor is attached to a mooring line, the velocity approaches the terminal free fall velocity during the first few seconds of fall and then reduces considerably due to the drag on the mooring line. Figure 11 also shows the horizontal anchor velocities. The maximum tension during the drop was approximately equal to the anchor weight.

In general, the long run times for these solutions (see Appendix E) were a result of the high curvature in the line near the anchor. When the anchor is released it drops straight down which causes a "kink" in the line just behind the anchor. Early during the drop the line motion tends to be mostly tangential especially near the anchor, which requires that the line on the buoy side of the "kink" must move around the "kink" at relatively large velocities. For the lumped mass model, this means that a lump on the buoy side of the "kink" is
accelerated toward the "kink" and obtains a relatively large horizontal velocity. As this lump moves around the "kink" its velocity must very rapidly change from nearly horizontal to vertical. Because of this large acceleration the time step is drastically reduced as a mass moves around the "kink".

The problem is compounded if the links are relatively large. As a large heavy link is accelerated toward the "kink" it obtains a considerable amount of momentum and it has a tendency to over-shoot the "kink" which causes jumps in the line tension. In one particular case the horizontal anchor velocity changed directions as a relatively large line mass overshot the "kink". This result is shown in Figure 11 where the horizontal anchor velocity becomes positive for a short time during the early portion of the drop.

Comparison of the output for the "standard catenary configuration" and the "goose neck catenary configuration" of Figure 5 showed about the same results provided sufficiently small link lengths were used. Figures 12 and 13 show the results for the "goose neck" catenary during the first 70 seconds of the drop. The comparison of anchor link tensions for the "goose neck catenary" and the "standard catenary" shown in Figure 13 indicates very similar results except that at any given time the "standard catenary configuration" has about 1000 lb. more tension. Since the "standard catenary configuration" seemed to be a more realistic starting configuration, and because the results are very similar, no further runs were made using the "goose neck" catenary.

Dacron Line

Figure 14 through 19 present the results for a 1.0 "diameter dacron mooring line for both the 15° catenary and 60° catenary initial conditions. Figure 14 shows the line shape for various times during the drop for the 15° catenary.
Figure 15 shows the tension contours for the 15° catenary as a function of position along the line and time. The computer runs for this case were terminated after 140 seconds of drop because the results in general were not greatly different from the 2.5" diameter nylon line presented in Figs. 6 and 7. There was no snap and the tensions seemed to be fairly constant and approximately equal to the anchor weight. Figures 16 and 17 present the same plots for the 60° catenary initial condition. For this case the computer runs were terminated after 100 seconds of drop because of the general similarity to the results obtained for the nylon line.

Figure 18 compares the tension in the anchor link for both catenary initial conditions during the first 140 seconds of drop. Figure 19 compares the anchor velocities during the drop and here again these results are very similar to the results for the nylon line.

**Steel Wire Rope**

At the time of this writing a computer program to model inextensible steel or chain mooring lines has just been completed. A few preliminary runs have been made for the anchor drop problem. The results of one of these runs is presented in Figs. 20 and 21. Figure 20 shows the line shape for various times during the drop for the 15° catenary initial configuration of a fictitious equivalent 2.5" diameter steel cable with a density of 350 lb/ft³. This diameter was selected initially to compare with the 2.5" diameter nylon line results. Subsequent runs will use more realistic diameters.

Although the program was only run for about the first 85 seconds of fall, the results are considerably different from the previous results as would be
expected due to the large line weight and the inextensibility conditions. The results show that the line falls almost as fast as the anchor, which is expected. Figure 21 shows the tension contours in the line and the results are markedly different with the maximum tensions occurring at the attachment point. Although these results are preliminary, in that the link lengths should be reduced and a more realistic case should be run, they do indicate general tendencies. The drop was run for only 85 seconds due a limited computer budget.
CONCLUSIONS AND DISCUSSION

1. The lumped mass model together with an appropriate integration scheme such as the predictor-corrector method is a useful tool for the dynamic analysis of extensible mooring lines. It is easily adapted to almost any line configuration and can readily handle non-linearities.

2. The results obtained for the anchor drop problem show no dynamic line snap for the configurations that were considered. It appears that the high drag of the long mooring line quickly dissipates the kinetic energy of the anchor, which slows the anchor to a velocity well below its free fall terminal velocity of 50 ft/sec.

3. The elastic properties of the line do not seem to play a great role in the line motion during anchor drop. The nylon and the dacron line showed similar position time histories.

4. Mooring line diameter did not seem to play an important role in the mooring line motion. Both the 2.5" line and the 1.0" line showed about the same position time histories.

5. For an anchor drop from the 15° catenary as modelled here, the line tension is essentially constant with respect to line position.

6. For an initial condition of a 60° catenary the tension increases more slowly than for the 15° catenary.

7. At no time, for any run, was the line tension greater than the anchor weight for the extensible mooring lines.

8. Time-velocity histories of the anchor show that the anchor settles onto the bottom at a vertical velocity of about 5 ft/sec for the 2.5" nylon rope.
9. Preliminary runs made for an inextensible steel line show a much different behavior. At least for the limited runs made, no snaps loads were observed. Further runs must be made to verify this result.

10. It is postulated that the high loads during buoy mooring implantment that have been experienced at sea for scopes greater than one were not caused by shock, or snap, loads in the line. Possibly, during the time of anchor fall, the line was relatively taut and the dynamic loads from waves acting on the buoy created the high tension forces close to the buoy.
REFERENCES


FIGURES
Figure 1. The Lumped Mass Model
\[ eAV_i = \frac{(6i-1 + 0. )}{2} \]

Figure 2. Free Body Diagram of a Typical Mass

\[ \Theta AV_i = \frac{(\Theta_{i-1} + \Theta_i)}{2} \]

Figure 3. Free Body Diagram of the Anchor
Figure 4. Computation Flow Chart
Figure 5a. Standard Catenary Configuration

Figure 5b. Goose Neck Catenary Configuration
Figure 6. Nylon Line 2.5" Dia (NVR05)

Figure 7. Tension Contours ~ KIP
15° Catenary Nylon Line 2.5" Dia (NVR05)
Figure 8. 60° Catenary Nylon Line 2.5" Dia. (NVR05)

Figure 9. Tension contours^KIP
60° Catenary Nylon Line 2.5" Dia (NVR05)
Figure 10. Anchor Link Tension for Nylon Line

Figure 11. Components of Anchor Velocity
Nylon Line 2.5" Dia. (NVR05)
Figure 12. 60° "Goose Neck" Catenary, Nylon Line, 2.5" Dia. (NVR06)

Figure 13. Anchor Link Tension for Nylon Lines Starting from 60° Standard Catenary and 60° "Goose Neck Catenary".
Figure 14. 15° Cat Dacron Line (VR08)
1.0" Dia.

Figure 15. Tension Contours ~ KIP
15° Catenary Dacron Line 1.0" Dia. (NVR08)
Figure 16. 60° Catenary Dacron Line 1.0" Dia (NVR08)

Figure 17. Tension Contours ~ KIP
60° Catenary Dacron Line 1.0" Dia. (NVR08)
Figure 18. Anchor Link Tension for Dacron Line

Figure 19. Vertical Component of Anchor Velocity
Dacron Line 1.0" Dia. (NVR08)
Figure 20. 15° Catenary Steel Line 2.5" Dia. (NVR07)

Figure 21. Tension Contours KIP 129 KIP at t = 0
15° Catenary Steel Line 2.5" Dia. (NVR07)
Appendix A

Listing for Extensible Line
PROGRAM LUMP
COMMON/TIME/DELTAT,TIME,TIMEMAX
COMMON/DTPT/PRINTIM,PRINTIV
COMMON/PNCH/PNCHCOD,READCOD
COMMON/FLAG/IFLAG,N2,JFLAG
IFLAG=0
JFLAG=0
CALL INPUT
TIME=J.
PRINTIM=PRINTIV-.JUL01
CALL CABLEIC
CALL OUTPUT
CALL LENGTH
100 IF (TIME.GE.TIMEMAX) GO TO 200
CALL END1
CALL NUMINTE
TIME=TIME+DELTAT
IF (TIME.GE.PRINTIM)10,20
10 CALL OUTPUT
PRINTIM=PRINTIM+PRINTIV
20 CONTINUE
GO TO 100
200 CONTINUE
IF (PNCHCOD.EQ.1) CALL PUNCHER
CALL LENGTH
STOP
END
SUBROUTINE INPUT
COMMON/CONST/PI,GC
COMMON/ACCEL/S(101),AREA,0,WATROEN,DENLINE,T(101),0
COMMON/STRAS/SOCN,SEXPT
COMMON/TIME/DELTAT,TIME,TIMEMAX
COMMON/FAIL/FAILS
COMMON/TMPT/PRINTIV,PRINTIV
COMMON/INPT/DP,NMASS,N1,STOTAL,SFINAL
COMMON/ERRORS/ERRMAX,ERRMIN
COMMON/PNC/PNCDD,PNCDD,READCD
COMMON/SANGLE/THET
COMMON/FLAG/IFLAG,M2,JFLAG
PRINT 111
111 FORMAT(1H1,58X,32HDISCRETE MASS MOORING LINE MODEL)
READ 102,STOTAL,DENLINE,DP,NMASS
102 FORMAT(3F16.0,E15.3,I10)
READ 103, DP, WATROEN
READ 107, FAILS
107 FORMAT(F10.0)
103 FORMAT(2F16.0)
READ 401, THET
READ 32, SOCN, SEXP
32 FORMAT(E15.3,F16.0)
READ 103, TIMEMAX, DELTAT
READ 103, ERRMAX, ERRMIN
READ 401, PRINTIV
READ 401, PNCDD
READ 401, READCD
401 FORMAT(F10.0)
PI=3.1415927
GC=32.17
AREA=PI/4.**(D/12.)**2
D=D/12.
PRINT 104,STOTAL,DENLINE,DP,GC,NMASS
104 FORMAT(1//14HLINE LENGTH =F6.1,4H FT./)
115HOLELINE DENSITY = F7.3,10H LB./FT**3/
216HOLELINE DIAMETER = F5.3,4H FT. /
319HOLELINE JUMPING CONSTANT = F10.3,13H L3 SEC/FT**2/
4 16HDL. OF MASSES = I3
N1=NM1ASS+1
N2=N1+1
PRINT 105,OP,WATROEN,TIMEMAX,DELTAT
PRINT 402,ERRMAX,ERMIN
PRINT 405,THET
405 FORMAT(24H Catenary Angle At Surface = F5.2,5H Deg.)
THET=THET*PI/180.
402 FORMAT(24H Velocity Error Bounds,F8.4,3H To,F8.4)
115 FORMAT( 14H WATER DEPTH = F9.2,3H FT /
11H DENSITY = F6.2,9H LB/FT-CU /
3 12H TIME STEP = F5.3,4H SEC)
PRINT 106,PRINTIV
106 FORMAT(13H PRINT EVERY F5.3,4H SEC)
PRINT 108,FAILS
108 FORMAT(24H Failure Load = F11.1,4H LBS)
PRINT 403,PUNCHC0D
415 FORMA1T(13H PUNCH CODE = F5.1,5X,56HPUNCH CODE = 1. PUNCHS ALL POS A
1ND VEL AT TIME = TIMEMAX )
PRINT +144, READCOD
414 FORMAT(13H READ CODE = F5.1,5X,57H READ CODE = 1. READS ALL POS A
1ND VEL FOR TIME = TIMEMAX )
PRINT 404,SCON,SEXF
414 FORMAT(13H STRESS-STRAIN RELATION STRESS = ,E10.3,9H*STRAIN
1**,F++.1)

C C
C
SEGMENT LENGTHS

SL=STOTAL/NMASS
S(I)=SL/2.
DO 20 I=2,NMASS
S(I)=SL
20 CONTINUE
S(NMASS+1)=SL/2.
RETURN
END
SUBROUTINE NUMINT
COMMON/TIME/DELTAT,TIME,TIMEMAX
COMMON/NUM/ XL (101), ZL (101), XD (101), ZD (101), XDO (101), ZDO (101), 1 XLN (101), ZLN (101), XDN (101), ZDN (101), ERRORX (101), ERRORZ (101)
COMMON/INPT/DP,NMASS,N1,STOTAL,SFINAL
COMMON/ACCEL/S (101), AREA, O, XTROEN, DENDLINE, T (101), D
COMMON/FAIL/FAILS
COMMON/ERRORS/ERRORMAX,ERRORMIN
COMMON/PRINT/PRINTIM,PRINTIV
COMMON/OLD/XOLD (101), ZOLD (101), XDOLOD (101), ZDOLOD (101), XDOOLD (101), ZDOOLD (101)
COMMON/FLAG/FLAGS, N2, JFLAG
DIMENSION XDON (101), ZDON (101)
DIMENSION AKX (101), AKZ (101), AKXZ (101), AKZ2 (101), AKX3 (101), AKZ3 (101)
X (101), QX1 (101), QZ1 (101), QX2 (101), QZ2 (101), QX3 (101), QZ3 (101)

PREDICT VELOCITY AND POSITION AT TIME T + DELTAT

1 CONTINUE
IF (JFLAG.EQ.0) GO TO 100
GO TO 101
IF (I.EQ.1) GO TO 73
CALL ACCEL (I, XL (I+1), XL (I), XL (I-1), ZL (I+1), ZL (I), ZL (I-1), XD (I+1), 1 XD (I), XD (I-1), ZD (I+1), ZD (I), ZD (I-1), XDO (I), ZDO (I))
GO TO 74
73 CALL END2 (XL (N2), XL (N1), ZL (N2), ZL (N1), XD (N2), ZD (N2), S (N1), AREA, 1 XD (N2), ZD (N2))
IF (I.EQ.N2 .AND. ZL (N2) .LT. DT) XDOLOD (I) = 0.
IF (I.EQ.N2 .AND. ZL (N2) .GT. -DT) ZDOLOD (I) = 0.
74 CONTINUE
XDON (I) = XDOLOD (I) + DELTAT * 2. * XD (I)
ZDON (I) = ZDOLOD (I) + DELTAT * 2. * ZD (I)
XLN (I) = XL (I) + DELTAT * (XD (I) + XDN (I)) / 2.
ZLN (I) = ZL (I) + DELTAT * (ZD (I) + ZDN (I)) / 2.
ERRORX(I) = -XDN(I)
ERRORZ(I) = -ZDN(I)

140 CONTINUE
CORRECTOR

X = XLN(I)  $ZM = ZLN(I)$ $SDM = XDN(I)$ $SZM = ZDN(I)$
DO 213 I = 2, N2
IF (I.EQ. N2) GO TO 75
CALL ACCEL(I, XLN(I+1), XLN(I), XM, ZLN(I+1), ZLN(I), ZM
1 XDN(I+1), XDN(I), XCM, ZON(I+1), ZON(I), ZDM, ODX, ODOZ)
GO TO 76
75 CALL END2(XLN(N2), XLN(N1), ZLN(N2), ZLN(N1), XDN(N2), ZDN(N2), S(N1),
1 AREA, ODX, ODOZ)
76 CONTINUE

X = XLN(I)  $ZM = ZLN(I)$ $SDM = XDN(I)$ $SZM = ZDN(I)$
XDN(I) = X(I) + DELTAT*(XDN(I) + ODX)/2.
ZDN(I) = ZDN(I) + DELTAT*(ZDN(I) + ODOZ)/2.

XLN(I) = XL(I) + DELTAT*(X(I) + XDN(I))/2.
ZLN(I) = ZLN(I) + DELTAT*(Z(I) + ZDN(I))/2.

ERRORX(I) = (ERRORX(I) + XDN(I))/5.*XON(I)
ERRORZ(I) = (ERRORZ(I) + ZDN(I))/5.*ZDN(I)
IF (I.EQ. N2 .AND. ZLN(N2) .LT. O) ERRORX(N2) = 0.
IF (I.EQ. N2 .AND. ZLN(N2) .LT. O) ERRORZ(N2) = 1.
ERFX = ERRMAX
ERFZ = ERRMAX
IF (ABS(XON(I)) .LT. 1. ) ERRX = 10.
IF (ABS(XON(I)) .LT. 1. ) ERRZ = 1.
IF (ABS(ERRORX(I)) .GT. ERRX .OR. ABS(ERRORZ(I)) .GT. ERRZ ) 140, 200
140 DELTAT = DELTAT/2.
IFLAG = 1
PRINT 903, DELTAT, TIME, I, ERRORX(I), ERRORZ(I)
918 FORMAT(19H0DELTAT REDUCED TO F7.4,3H AT,F8.3,4H SEC,3X,I5,2(E10.3,1DZ))
    IF(DELTAT.LT.0.001)155,156
155 PRINT 959
959 FORMAT(23H0 ERROR DELTAT LT .0001 )
    STOP
156 GO TO 1
203 CONTINUE
N COUNTER=0
DO 330 I=1,N2
XL OLD(I)=XL(I)
ZL OLD(I)=ZL(I)
XL OLD(I)=XD(I)
ZL OLD(I)=ZD(I)
XL(I)=X LN(I)
ZL(I)=Z LN(I)
XO(I)=XDN(I)
ZD(I)=ZDN(I)
IF(ABS(ERRORX(I)).LT.ERRMIN.AND.ABS(ERRFZ(I)).LT.ERRMIN)N COUNTER
    1 =COUNTER+1
333 CONTINUE
    IF(N COUNTER.EQ.N2)350,356
350 DELTAT=DELTAT*2.
    IFLAG=0
    DO 407 I=1,N2
XL OLD(I)=XLN(I)
ZL OLD(I)=ZLN(I)
XL OLD(I)=XDN(I)
407 ZL OLD(I)=ZDN(I)
PRINT 916,DELTAT,TIME
916 FORMAT(21H0DELTAT INCREASED TO F7.4,3H AT,F8.3,4H SEC)
910 CONTINUE
363 CONTINUE
GO TO 2010
1050 CONTINUE
RUNGE-KUTTA (THIRD ORDER)

DO 1100 I=2,N2
IF (I.EQ.N2) GO TO 1105
CALL ACCEL(I,XLOLD(I+1),XLOLD(I),XLCLUD(I-1),ZLOLD(I+1),ZLOLD(I),ZLOLD(I-1),XOOLD(I+1),XOOLD(I),XOOLD(I-1),ZOOLD(I+1),ZOOLD(I),ZOOLD(I-1),XOOLD(I),ZOOLD(I))
GO TO 1105

1105 CALL END2(XLOLD(N2),XLOLD(N1),ZLOLD(N2),ZLOLD(N1),XOOLD(N2),ZOOLD(N1),S(N1),AREA,XOOLD(N2),ZOOLD(N2))

1106 CONTINUE

AKX1(I)=DELTAT*XOOLD(I)
AKZ1(I)=DELTAT*ZOOLD(I)
QX1(I)=DELTAT*XOOLD(I)
QZ1(I)=DELTAT*ZOOLD(I)
XON(I)=XOOLD(I)+AKX1(I)/2.
ZON(I)=ZOOLD(I)+AKZ1(I)/2.
XLN(I)=XLOLD(I)+QX1(I)/2.
ZLN(I)=ZLOLD(I)+QZ1(I)/2.

1100 CONTINUE

DO 1200 I=2,N2
IF (I.EQ.N2) GO TO 1205
CALL ACCEL(I,XLN(I+1),XLN(I),XLN(I-1),ZLN(I+1),ZLN(I),ZLN(I-1),XON(I+1),XON(I),XON(I-1),ZON(I+1),ZON(I),ZON(I-1),XDON(I),XDON(I))
GO TO 1205

1205 CALL END2(XLN(N2),XLN(N1),ZLN(N2),ZLN(N1),XON(N2),ZON(N2),S(N1),AREA,XDON(N2),ZDON(N2))

1206 CONTINUE
AKX2(I) = DELTAT * XDON(I)
AKZ2(I) = DELTAT * ZDON(I)
QX2(I) = DELTAT * XON(I)
QZ2(I) = DELTAT * ZON(I)

1200 CONTINUE
DO 1300 I = 2, N2
XDON(I) = XDON(I-1) - AKX1(I) + 2. * AKX2(I)
ZDON(I) = ZDON(I-1) - AKZ1(I) + 2. * AKZ2(I)
XLN(I) = XLN(I-1) - QX1(I) + 2. * QX2(I)
ZLN(I) = ZLN(I-1) - QZ1(I) + 2. * QZ2(I)

1300 CONTINUE
DO 1400 I = 2, N2
IF (I .EQ. N2) GO TO 1305
CALL ACCEL(I, XLN(1+1), XLN(I), XLN(I-1), ZLN(I+1), ZLN(I), ZLN(I-1), XDON(I+1), XDON(I), ZDON(I+1), ZDON(I), ZDON(I-1), XDON(I), ZDON(I))
GO TO 1306
1305 CALL END2(XLN(N2), XLN(N1), ZLN(N2), ZLN(N1), XDON(N2), ZDON(N2), S(N1), 1 AREA, XDON(N2), ZDON(N2))
1306 CONTINUE
AKX3(I) = DELTAT * XDON(I)
AKZ3(I) = DELTAT * ZDON(I)
QX3(I) = DELTAT * XON(I)
QZ3(I) = DELTAT * ZON(I)

1400 CONTINUE
DO 1500 I = 2, N2
XJ(I) = XDON(I) + (AKX1(I) + 4. * AKX2(I) + AKX3(I)) / 6.0
ZJ(I) = ZDON(I) + (AKZ1(I) + 4. * AKZ2(I) + AKZ3(I)) / 6.0
XL(I) = XLN(I) + (QX1(I) + 4. * QX2(I) + QX3(I)) / 6.0
ZL(I) = ZLN(I) + (QZ1(I) + QZ2(I) + QZ3(I)) / 6.0
1500 CONTINUE
IFLAG = 1
2000 CONTINUE
DO 330 I = 2, N1
CALL TENSION(I, XL(I+1), XL(I), XL(I-1), ZL(I+1), ZL(I), ZL(I-1))
330 CONTINUE
DO 400 I = 1, N1
IF (T(I) .GT. FAILS) GO TO 410
400 CONTINUE
CALL OUTPUT
DUMMY = Rupt (1.)
410 CONTINUE
RETURN
END
SUBROUTINE PUNCHER
COMMON/TIM/T,DEL/TAT,TIM,TIMMAX
COMMON/ACCEL/S(101),AREA,D,KATROD,DELINE,T(101),D
COMMON/INIT/DP,NUMASS,MI,STOTAL,SPINAL
COMMON/NUMI/XL(101),ZL(101),XD(101),ZD(101),XOD(101),ZOD(101),
1 XLN(101),ZLN(101),XDN(101),ZDN(101),ERRORX(101),ERRORZ(101)
N2=M1+1
PUNCH 116,DEL/TAT,TIM
DO 23 I=1,N2
PUNCH 101,XL(I),ZL(I),XD(I),ZD(I)
23 CONTINUE
10 FORMAT(2F10.4)
111 FORMAT(2F10.5)
RETURN
END
SUBROUTINE ACCEL(I, XP, X, XM, ZP, Z, ZM, XDP, XD, XDM, ZDP, ZD, ZDM, XDD, ZDD)
COMM /ACCEL/S(I1), AREA, Q, WATROEN, DENVLINE, T(I1), D
COMM /INPT/DP, NMASS, N1, STOTAL, SFINAL
COMM /CONS/PI, GC
COMM /TIME/DDELAT, TIME, TIMEMAX
COMM /OUTPT/PRINTX, PRINTV
THETA=ATAN2((ZP-Z), (XP-X))  
THETAM=ATAN2((Z-ZM), (X-XM))

SPRING FORCE ON MASS I

EPM=SQRT((X-XM)**2+(Z-ZM)**2)/S(I-1)-1.
SFORCEX=AREA*(STRASS(EP)*COS(THETA)-STRASS(EPM)*COS(THETAM))
SFORCEZ=AREA*(STRASS(EP)*SIN(THETA)-STRASS(EPM)*SIN(THETAM))

WATER DAMPING FORCES ON MASS I
THAV=(THETA+THETAM)/2.

VX AND VZ ARE LOCAL VELOCITIES.
VX=0,
VZ=0,
VTANG = (X0-VX)*COS(THAV) + (Z0-VZ)*SIN(THAV)
VNORM = -(X0-VX)*SIN(THAV) + (Z0-VZ)*COS(THAV)
SL = (S(I) + S(I-1))/2.
IF (I.EQ.2) SL=S(I-1)+S(I)/2.
IF (I.EQ.1) SL=S(I-1)/2+S(I)
ARET=PI*O*SL
AREN=O*SL
CT=1.4*0.006
CN=1.4

DRAGT = -CT*WATROEN*ARET*VTANG*ABS(VTANG)/(2.*GC)
DRAGI = -CN*WATROEN*AREN*VNORM*ABS(VNORM)/(2.*GC)

VIRTUAL MASS CORRECTION

SMASS = SL*DENLINE*AREA/GC
BMASS = WATROEN*AREA*SL/GC
CTT = 1.5
CIN = 3
WT = (DENLINE-WATROEN)*SL*AREA
AUCTANG = ((SFORCEX ) )*COS(THAV) + (SFORCEZ ) -WT )*
1SIN(THAV)+DRAGT )/(SMASS+CIT*AMASS)
ACCNORM = (- (SFORCEX ) ) +SIN(THAV) + (SFORCEZ ) -WT )*
1COS(THAV)+DRAGN )/(SMASS+CIN*AMASS)
X00 = AUCTANG*COS(THAV) -ACCNORM*SIN(THAV)
Z00 = AUCTANG*SIN(THAV) +ACCNORM*COS(THAV)
RETURN
END
SUBROUTINE END2(X,XM,ZM,XD,ZD,S,AR,XDD,ZDO)
COMMON/FLAG/IFLAG,N2,JFLAG
COMMON/INPT/DP
REAL 1
IF(JFLAG.EQ.1) GO TO 95
BWEIGHT=12000.
VOL=24.5
CD=4
CI=1
AREA=12.6
WDEN=64.
RHO=2.
G=32.2
W=VOLUME-WDEN*VOL
PI=3.1415926
M=VOLUME/G+CI*VOL*RHO
A=CD*AREA*RHO/(Z.*M)
JFLAG=1
95 CONTINUE
IF(Z.LT.-DP) 50,56
50 XD=ZD=XDD=ZDO=0.
RETURN
T=AR*STRASS(EP)
THETA=ATAN2(Z-ZM,X-XM)+PI
FX=T*COS(THETA)
FZ=T*SIN(THETA)-W9
V=SQRT(XD**2+ZD**2)
XDD=FX/M-A*XD*V
ZDO=FZ/M-A*ZD*V
RETURN
END
FUNCTION STRASS(X)
COMM/STRAS/SCON,SEXP
C
GIVEN STRAIN X, FIND STRESS
IF(X.LT.0.)10,20
10 STRASS=0.
RETURN
20 CONTINUE
STRASS=SCON*X**SEXP
RETURN
ENTRY STRAIN
STRASS=(X/SCON)**(1./SEXP)
RETURN
ENTRY RUPT
PRINT 6
6 FORMAT(1X, '3RD LINE HAS BEEN RUPTURED')
STOP
END
SUBROUTINE EN1
COMMON/TIME/DELTAT,TIME,TIMEMAX
COMMON/NUMI/XL(101),ZL(101),XD(101),ZO(101),XDD(101),ZDO(101),
1 XLN(101),ZLN(101),XD(101),ZD(101),ERRORX(101),ERRORZ(101)
COMMON/LOD/XLOD(101),ZLOD(101),XDO(101),ZDO(101),XDDO(101),ZDDO(101),
1 ZDDO(101)
SET BOUNDARY CONDITIONS AT END 1

XLN(1)=., ZLN(1)=., XD(1)=., ZD(1)=.
XL(1)=XOLD(1), ZL(1)=ZLOD(1), XDO(1)=ZDO(1), ZDDO=O.
ERRORX(1)=O, ERRORZ(1)=0.
RETURN
END

SUBROUTINE LENGTH
COMMON/NUMI/XL(101),ZL(101),XD(101),ZO(101),XDD(101),ZDO(101),
1 XLN(1),ZLN(1),XD(111),ZD(111),ERRORX(111),ERRORZ(111)
COMMON/IND/DR,NMAD,NI,STOTAL,SFINAL

STRAINED LENGTH OF CABLE

SFINAL=1.
N2=N1+1
DO 110 I=2,N2
SFINAL=SFINAL+SQRT((XL(I)-XL(I-1))**2+(ZL(I)-ZL(I-1))**2)
110 CONTINUE
PRINT 101,SFINAL
101 FORMAT(/1H3,25X,12HLINE LENGTH =,F8.1)
RETURN
END
SUBROUTINE OUTPUT
COMMON/TIME/DELTAT,TIME,TIMEMAX
COMMON/NUMI/XL(101),ZL(101),XD(101),ZD(101),XDD(101),ZDD(101),
1 XLN(101),ZLN(101),XDN(101),ZDN(101),ERRORX(101),ERRORZ(101)
COMMON/ACCEL/S(101),AREA,Q,WATRDN,DENLINE,T(101)
COMMON/NINPT/OP,NMASS,N1,STOTAL,SPINAL
PRINT 100 ,TIME
100 FORMAT (1H1, //, 7HOTIME =F6.2)
PRINT 101
101 FORMAT (//,1H0,5X,6H LINK ,6H MASS ,6X,13H COORDINATES ,10X,7HTENS
1ION,10X,16VELOCITIES,14X,6HERRORS)
PRINT 112
112 FORMAT (1X,7X,3HNO.,3X,3HNO.,9X,1HX,9X,1HZ,29X,1HX,10X,1HZ,8X,
1 4HXDOT, 8X,4HZDOT)
PRINT 113,(I,XL(I),ZL(I),XD(I),ZD(I),ERRORX(I),ERRORZ(I),I,T(I),
1 I=1,N1)
113 FORMAT (13X,I3,2X,2(1X,F3.2),17X,2(1X,F10.4),2(2X,E10.3)/7X,I3,31X,
1 E11.4)
N2=NMASS+2
PRINT 113,N2,XL(N2),ZL(N2),XD(N2),ZD(N2),ERRORX(N2),ERRORZ(N2)
RETURN
END
SUBROUTINE CABLEIC
COMMON/ANGLE/THET
COMMON/OTPT/PRINTM,PRINTV
COMMON/PNH/PNHGUO,READGCD
COMMON/TIME/DELTAT,TIME,TIM MAX
COMMON/CONST/PI,GC
COMMON/ACCL/NS(101),AREA,Q,WATRED,DELNLE,T(101),D
COMMON/INPT/DF,NMASS,N1,STotal,SFINS
COMMON/NUMH/XL(101),XL(101),XD(101),ZD(101),XO(101),ZOD(101),
1 XLO(101),ZLNO(101),XDON(101),ZDON(101),ERRORX(101),ERRORZ(101)
COMMON/VLOD/XL0D(101),ZL0D(101),XDOLO(101),ZDOLO(101),XDOADOLO(101)
1,ZDOADOLO(101)
N2=NL+1
IF (READGCD.EQ.1) GO TO 566
XL=(DENLINE-WATRED)*AREA
SL=TOTAL
DO 212 L=1,3
TO=XL*SL / (2.*TANQ)
FL=TJ*2.*XL*SLH(TAQ)
SL=S
DO 233 J=1,N2
IF (J.EQ.1) SL=SL+S(J-1)
XL(J)=FL/2.+TO/2.*SLH(SL-WL/TO-Sinh(WL*FL/(2.*TO)))
ZL(J)=TO/2.*Cosh(WL/2.*XL(J)-FL/2.))
T2=TJ*Cosh(WL/2.*XL(J)-FL/2.)
EP2=STRAIN(T2/AREA)
IF (J.EQ.1) GO TO 36
T(J-1)=(T1+T2)/2.
DELTA=(EP1+EP2)*S(J-1)/2.
ANG=ATAN2(ZL(J)-ZL(J-1),XL(J)-XL(J-1))
XL(J)=XL(J)+DELTA*COS(ANG)
ZL(J)=ZL(J)+DELTA*SIN(ANG)
SL=SL+DELTA
CONTINUE
T1 = T2
EP1 = EP2
ZDOL(I) = 0
XDOL(I) = 0
ZD(I) = 0
XD(I) = 0
XLN(I) = XL(I) $ZLN(I) = ZL(I)
XLOL(I) = XL(I)
ZLOL(I) = ZL(I)
XON(I) = LN(I) $ZON(I) = ZN(I)
ERROR(X(I)) = 0, ERRORZ(I) = 0.
233 CONTINUE
202 CONTINUE
RETURN
500 CONTINUE
READ 300, DELTAT, TIME
PRINTH = TIME + PRINTIV
50 DO 630 I = 1, N2
READ 301, XL(I), ZL(I), XD(I), ZD(I)
T(I) = T.
XL(I) = XL(I) $ZLN(I) = ZL(I)
XD(I) = XD(I) $ZON(I) = ZD(I)
XLLO(I) = XL(I)
ZLLO(I) = ZL(I)
XZOL(I) = XD(I)
ZDOL(I) = ZD(I)
ERRORX(I) = 0, ERRORZ(I) = 0.
630 CONTINUE
READ 500, TIME, MAX
600 FORMAT(2F10.5)
601 FORMAT(4F10.5, 2E12.5)
RETURN
END
FUNCTION SINH1(P)
SINH1=ALOG(P+SQRT(P**2+1.))
RETURN
END

FUNCTION HYP(X)
ENTRY SINV
HYPS=5*(EXP(X)-EXP(-X))
RETURN
ENTRY COSV
HYPC=5*(EXP(X)+EXP(-X))
RETURN
END

SUBROUTINE TENSION(I,XP,X,ZP,Z,M)
COMMON/ACCEL/S(11),AREA,G,WTHRD,T(101),D
IF(I.NE.2)GO TO 10
EPM=SQRT((X-X)**2+(Z-Z)**2)/S(I-1).
T(I-1)=STRESS(EPM)*AREA
CONTINUE
T(I)=STRESS(EP)*AREA
RETURN
END
Appendix B

Listing for Inextensible Line
PROGRAM CABLE1
DIMENSION THETA(30), TD(30), THETA2(30), TD2(30), TDD(30), TDD2(30), TD1
C(30), THETA1(30)
DIMENSION TDK2(30)
DIMENSION VERROR(30), PERROR2(30)
COMMON/ ANCH/ MASS/ ANCH
COMMON/ PO8E/ X(30), Y(30)/ TEN/ TEN(30)/ VEL/ VX(30), VY(30)/ 0S/ X0, Y0, X0, Y0
CD, XDD, YDD, DENSITY, Y0
COMMON/ DRAG2/ DRAG(30)/ G/ GRAV(30)
COMMON/ TIME, DELTAT, DELTATN
X0=Y0=X0=Y0=3.
READ 5, TIMCUT, PRINT, CI, CN, CT, KORECT
FORMAT(F6.2, 1X, F5.2, 1X, F5.2, 1X, F4.2, 1X, F5.4, 1X, I2)
READ 6, DELTAT, EU, EL, LLOW
FORMAT(F6.4, 1X, F6.4, 1X, F11.9)
CALL INPUT(N, OIA, DENSITY, X0, Y0, TOTAL, ANCHOR)
CALL ADDMASS(N, OIA, CI)
CALL TN(CN, OIA, CN, CT, ANCHOR)
PRINT 10, OIA, DENSITY, TOTAL, ANCHOR, X:, Y:N
FORMAT(/, 11H LINE DIA= , F4.2, /, 14H LINE DENSITY= , F5.1, /, 14H LIN
CE LENGTH= , F4.3, /, 14H ANCHOR MASS= , F7.3, /, 7H SPAN= , F5.4, /, 14H WA
CTER DEPTH= , F5.2, /, 18H NO. OF SEGMENTS= , 12)
PRINT 10, TIMCUT, EU, LLOW
FORMAT(11H TIMCUT= , F6.2, /, 18H PRINT INCREMENT= , F5.2, /, 18H ADDED
COMPRESSED= , F4.2, /, 15H ORbeits COEF, /, 5X, ANNORMAL= , F4.2, /, 5X, 12HTAN
CINITIAL= , F6.4, /, 29H, NO. OF CORRECTIONS= , 12)
PRINT 10, EU, EL, LLOW
FORMAT(13H ERROR LIMITS, /, 5X, 7HUPPER= , F7.4, /, 5X, 7HLOWER= , F12.9, /
C/ //)
CALL INITIAL(N, X0, THETA1, TD1)
CALL ANGLES(N, THETA1)
CALL COORD(N, XQ, YQ, X0, Y0, TD1)
PRINT 20
20 FORMAT(32X,13HINITIAL CONDITIONS,///,10X,10HLINK MASS,8X,1HX,11X, 
CI=1Y,3X,7HTENSION,7X,5HTHETA,3X,9HTHETA DOT,///) 
A=1H.1,7,1.159 
DC 4! I=1,N 
THETA(I)=THETA1(I)*A 
TD2(I)=TD1(I)*A 
PRINT 33,I,TER(I),THETA2(I),TD2(I),I,X(I),Y(I) 
30 FORMAT(11X,I2,14X,5X,F7.2,3X,F5.2,///,18X,I2,5X,F7.2,5X,F7.2) 
40 CONTINUE 
PRINT 53,DELTA T 
50 FORMAT(///,12HDELTA T= ,FO.4,//////) 
TIME=TCHECK=2.0 
DELTA T=DELTA T 
JFLAG=0 
KFLAG=0 
NFLAG=0 
KCHECK=0 
60 IF(KFLAG.EQ.1) GO TO 62 
FAIL=1 
GO TO 135 
63 CALL BOTTOM(N,TD,TIME) 
KFLAG=1 
NFLAG=0 
GO TO 135 
62 IF(JFLAG.EQ.0) GO TO 70 
CALL CALL(N,THETA,TDO) 
CALL SOLVE(N,TDO,KORRECT) 
CALL PREDICT(N,DELTA T,TD1,THETA,TDO,TDO,THETA2,TD2) 
DC 61 I=1,N 
THETA1(I)=THETA(I)
TO1(I) = TC(I)
TO2(I) = TOO(I)

CONTINUE
CALL CALL(N, THETA2, TO2)
CALL SOLVE(N, TOO2, KORRECT)
CALL CORRECT(N, DELTAT, THETA1, TO1, TOO1, TOO2, THETA, TO)
DO 63 I = 1, N
VEFROR(I) = (TO2(I) - TD(I)) / (5.0 * TD(I))
PERFOR(I) = (THETA2(I) - THETA(I)) / (THETA(I) * 5.0)

65 CONTINUE
DO 112 I = 1, N
IF (ABS(VEFROR(I)) .LE. EUP) GO TO 112
IF (DELTAT .GE. 0.0001) GO TO 108
PRINT 105, DELTAT, TIME, I
105 FORMAT (17H DELTA T HELD TO , F9.7,10H AT TIME= , F11.7, 17H BECAUSE O
OF LINK , 12)
GO TO 112
108 DELTATN = DELTAT / 2.
TIME = TIME - DELTAT
TCHECK = TCHECK - DELTAT
DELTAT = DELTATN
PRINT 105, DELTAT, TIME
109 FORMAT (22H DELTA T DECREASED TO , F9.7,10H AT TIME= , F11.7)
KCHECK = KCHECK + 1
IF (KCHECK .GE. 5) CALL EXIT
IFLAG = 1
GO TO 60
112 CONTINUE
DO 113 I = 1, N
IF (ABS(VEFROR(I)) .GE. ELOW) GO TO 113
DELTATN = DELTAT + 2.
PRINT 90, DELTATN, TIME
80 FORMAT (22H DELTA T INCREASED TO , F9.7,10H AT TIME= , F11.7)
KCHECK = KCHECK + 1
IF (KCHECK GE 9) CALL .EXIT
JFLAG = 0
GO TO 115
110 CONTINUE
GO TO 115
70 CONTINUE
CALL RK(N, DELTAT, THETA1, TD1, T00, THETA, TD, KCORRECT)
JFLAG = 1

115 TIME = TIME + DELTAT
TCHK = TCHECK + DELTAT
DELTAT = DELTAT / N
KCHECK = 0
IF (TCHK GE TPRINT) GO TO 130
GO TO 60
130 TCHECK = TCHECK - TPRINT
135 PRINT 140, TIME
140 FORMAT (1H1, 7H TIME=, F11.7, //, 15X, 1H LINK MASS, 6X, 11H COORDINAT
CES, 11X, 1H VELLOCITY, 7X, 12H THETA, 7X, 7H TENSION, 13X, 5H ERROR, //, 2
6X, 14X, 5X, 1HY, 3X, 1HX, 7X, 1HY, 15X, 3HDOT, 23X, 5H THETA, 4X, 9H THETA DOT, /
G/)
CALL ANGLES(N, THETA)
CALL COORD(N, XC, YC, XD, YD, TD)
CALL TENSION(N, X00, Y00, THETA, TD, T00, DENSITY, XC, YC, YD)
A = 180. / 3.14159
DC 150 I = 1, N
THETA2(I) = THETA(I) * A
TD2(I) = TD(I) * A
PRINT 156, I, THETA2(I), TO2(I), TEN(I), PERROR(I), VERORR(I), I, X(I), Y(I), C), VX(I), VY(I)


160 CONTINUE
PRINT 156
155 FORMAT(/) GO TO 63
IF(NFLAG.EQ.1) GO TO 63
IF(TIME.GE.TIMECUT) GO TO 170
GO TO 60
170 CONTINUE
CALL EXIT
END
SUBROUTINE INPUT(N,DIA,DENSITY,X0,Y0,TOTAL,ANCHOR)
REAL L,M
COMMON/LENGTH/L(30)/MASS/M(30)/CURRENT/VC(30)/TOTMAS/TM(30)
READ 10, N,ANCHOR,DIA,DENSITY,TOTAL,X0,Y0
10 FORMAT(I2,1X,F7.3,1X,F4.2,2X,F5.1,1X,F5.0,1X,F5.0,1X,F5.0)
L=DENSITY*3.14159*(DIA/24.)**2
DO 30 I=1,N
READ 20, L(I),VC(I)
20 FORMAT(F5.1,1Y,F5.2)
CONTINUE
30 CONTINUE
M(I)=L(I)+.5*L(I+1)
N1=N-I
DO 35 I=1,N1
M(I)=.5*(L(I)+L(I+1))
35 CONTINUE
M(N)=L(N)+ANCHOR
TM(N)=M(N)
DO 40 I=2,N
K=N+1-I
TM(K)=TM(K+1)+Y(K)
40 CONTINUE
RETURN
END
SUBROUTINE ADDMASS(N, DIA, CI)
REAL L
COMMON/LENGTH/L(33)/ADMAS/AM(33)
A=CI*64.*3.14159*(DIA/24.)**2
DO 10 I=1,N
AM(I)=A*L(I)
10 CONTINUE
RETURN
END
SUBROUTINE TM(N, DIA, CN, CT, ANCHOR)
REAL L
COMMON /LENGTH/L(N), CCef/OT(N), ON(N)
COMMON /ACeff/CI
NEN=54,
B=OT*WEN*3.14159*DIA/12,
A=CN*WEN*DIA/12,
CC 1: I=1, N
ON(I)=A*L(I)
CT(I)=3*L(I)
CONTINUE
DENI=.286*12.***
VANCHOR=ANCHOR/DENI
RANCHOR=(3.*VANCHOR/(4.*3.14159265))**(1./3.)
AANCHOR=3.14159265*RANCHOR**2
DA=.5*WEN*AANCHOR/2.
RETURN
END
SUBROUTINE INITIAL(N, X, THETA, T)
DIMENSION THETA(30), T(30), TX(30), TY(30)
REAL L, M,
COMMON/LENGTH/L(30), MASS/M(30), TEN/TEN(30),
TMASS=6.0
N1=N-1
DO 10 I=1, N1
TMAS=TMAS+M(I)
10 CONTINUE
TO(1)=0.0
T(1)=1.0
TY(N)=0.5*TMAS
TX(N)=TY(N)
GAMA=1.2*TMAS
SAVE=1000000000.,
TXS=TX(N)
TYS=TY(N)
TY(1)=TMAS-TY(N)
TEN(1)=SQRT(TX(1)**2+TY(1)**2)
A=TY(1)/TX(1)
THETA(1)=ATAN(A)
EXO=L(1)*COS(THETA(1))
EYO=L(1)*SIN(THETA(I))
DO 20 I=2, N
TY(I)=TY(I-1)-A(I-1)
TX(I)=TX(I-1)
TEN(I)=SQRT(TX(I)**2+TY(I)**2)
A=TY(I)/TX(I)
THETA(I)=ATAN(A)
EXO=EXO+L(I)*COS(THETA(I))
EYO=EYO+L(I)*SIN(THETA(I))
20 CONTINUE
ERROR=SQRT((X0-EX) ** 2 + EY(1) ** 2)
IF(ERROR .LT. SAVE) GO TO 40
GAMMA = GAMMA / 2
GO TO 50
40 IF(ERROR .LE. C .10) GO TO 50  
EXS = EXI  
EY0 = EYO  
TXS = TX(N)  
TYSE = TY(N)  
SAVE = ERROR  
50 TY(N) = TXS + GAMMA * (X0 - EXS) / SAVE  

TY(N) = TYSE + GAMMA * EY0(S) / SAVE  
GO TO 20  
60 CONTINUE  
A = 3.14159265 * 2  
DC 71 I = 1, N  
IF (THETA(I) .GT. 3.14159265) THETA(I) = THETA(I) - A  
70 CONTINUE  
RETURN  
END
SUBROUTINE CALL(N,THETA,TD)
COMMON/X0,Y0,XDO,YDO,DENSITY,Y0
DIMENSION THETA(30),TD(31)
CALL ANGLES(N,THETA)
CALL COORD(N,XQ,YQ,XO,YO,TD)
CALL MASS(N,Y2)
CALL DRAG1(N)
CALL XYMASS1(N,XDO,YDO,YJ)
CALL GRAVITY(N,Y0,DENSITY)
CALL DRAGA(N,THETA)
CALL SETUP(N,TD)
RETURN
END
SUBROUTINE ANGLES(N,THETA)
DIMENSION THETA(30)
COMMON/ANG/S1(30),C1(30)/DIFFANG/S2(30,30),C2(30,30)
DO 1 I=1,N
   S1(I)=SIN(THETA(I))
   C1(I)=COS(THETA(I))
DO 1 J=1,N
   DIFF=THETA(I)-THETA(J)
   S2(I,J)=SIN(DIFF)
   C2(I,J)=COS(DIFF)
CONTINUE
RETURN
END
SUBROUTINE COORD(N,XO,YO,XD,YD,TD)
DIMENSION TO(3)
REAL L,
COMMON/LENGTH/L(3)/ANG/S1(3),C1(3)/FOS/X(3),Y(3)/VEL/VX(3),V
C(3)
X(1)=L(1)*C1(1)+XO
Y(1)=L(1)*S1(1)+YO
VX(1)=XO-L(1)*TD(1)*S1(1)
VY(1)=YD+L(1)*TD(1)*C1(1)
DO 10 I=2,N
X(I)=X(I-1)+L(I)*C1(I)
Y(I)=Y(I-1)+L(I)*S1(I)
VX(I)=VX(I-1)-L(I)*TD(I)*S1(I)
VY(I)=VY(I-1)+L(I)*TD(I)*C1(I)
10 CONTINUE
RETURN
END
Subroutine MASS(N,Y)
REAL W,L,MASS1,MASS2
COMMON/TCTMAS/TM(30)/ADMAS/AM(30)/DIFFANG/S2(30,30),C2(30,30)/LEN
CTH/L(30)/MASS/H(30)/NUMMAS/MASS1(30,30),MASS2(30,30)
DO 6J I=1,N
DO 6J J=1,N
K=1
SIGMA=0.
IF (I,J) GO TO 10
SIGMA=1.
K=J

10 CONTINUE
MASS1(I,J)=TM(K)*C2(I,J)-.5*SIGMA*AM(J)*C2(I,J)
MASS2(I,J)=-TM(K)*S2(I,J)
TOTALM1=0.
TOTALM2=0.
DC 2J L1=K,N
TOTALM1=TOTALM1+AM(L1)*C2(I,L1)*C2(J,L1)

20 CONTINUE
IF (J,J) GO TO 41
K1=J+1
IF (I,J) K1=I
DC 3J L1=K1,N
TOTALM2=TOTALM2+AM(L1)*C2(I,L1)*S2(J,L1)

30 CONTINUE
MASS1(I,J)=MASS1(I,J)+TOTALM1
MASS2(I,J)=MASS2(I,J)+TOTALM2
MASS2(I,J)=MASS2(I,J)*L(J)

40 CONTINUE
RETURN
END
SUBROUTINE DRAG1(N)
DIMENSION VN(30), VT(30)
COMMON/VEL/VX(30), VY(30)/ANG/S1(30), C1(30)/DIFFANG/S2(30,30), C2(30, 30)
COEFF/DT(30), ON(30)/CURRENT/VC(30)/DRAG2/DRAG(30)

VN(1) = .5*((VC(1) - VX(1)) * S1(1) + VY(1) * C1(1))
VT(1) = 0.0

DO 1 I = 2, N
  DO 10 J = I - 1
    VN(I) = .5*(VC(I) + VC(J) - VX(I) - VX(J)) * S1(I) + .5*(VY(I) + VY(J)) * C1(I)
    VT(I) = .5*(VC(I) + VC(J) - VX(I) - VX(J)) * C1(I) - .5*(VY(I) + VY(J)) * S1(I)
    10 CONTINUE

DO 20 J = 1, N
  DRAG(I) = 0.0
  DO 30 J = I, N
    DRAG(I) = DRAG(I) - ON(J) * VN(J) * ABS(VN(J)) * C2(I, J) - DT(J) * VT(J) * ABS(VT(J)) * C2(I, J)
    30 CONTINUE

20 CONTINUE
30 CONTINUE
RETURN
END
SUBROUTINE XYNASS1(N,XDD,YDD,YN)
REAL M,L
COMMON/CTCMAS/TM(30),ADMAS/A1(30),/LENGTH/L(30),/MASS/M(30),/XY/XYNAS
CS(30)/ANG/S1(30),C1(30)/DIFFANG/S2(30,30),C2(30,30)
DO 20 I=1,N
XYNASS(I)=0.0
DO 10 J=1,N
XYNASS(I)=XYNASS(I)+A1(J)*S1(J)*C2(I,J)*XDD-A1(J)*C1(J)*C2(I,J)*YN
10 CONTINUE
XYNASS(I)=XYNASS(I)+1.0*(XDD*S1(I)-YDD*C1(I))*TM(I)
20 CONTINUE
RETURN
END
SUBROUTINE GRAVITY(N,Y,J,JENITY)
REAL Y
COMMON/MASS/M(30)
COMMON/ANG/S1(30),C1(30)/TOTMAS/TM(30)/POS/X(30),Y(30),G/GRAV(30)
WDEN=64.
G=32.2*(DENSTY-WDEN)/DENSTY
DO 1 I=1,N
G*AV(I)=G*TM(I)*C1(I)
10 CONTINUE

IF(M(N).LT.1000000.) GO TO 60
DO 25 I=1,N
GRAV(I)=GRAV(I)-M(N)*C1(I)*G
25 CONTINUE
N1=N-1
DO 50 I=1,N1
IF(Y(I).LT.0.) GO TO 50
DO 40 J=1,N
GRAV(J)=GRAV(J)-M(I)*C1(J)*G
40 CONTINUE
50 CONTINUE
60 CONTINUE
RETURN
END
SUBROUTINE DRAGA(N, THETA)
DIMENSION THETA(32)
COMMON/ACOLT/J2/VEL/VX(30), VY(30)/ANC/ADrag(30)
IF(VX(N), NE.0.0) GC TO 20
PHI = 3.141592657/2.0
GC TO 30
20 H = VX(N)/ABS(VX(N))
PHI = ATAN(H)
IF(VX(N), LT.0.0) PHI = 3.14159265 + PHI
30 FORAG = DA*(VX(N)**2 + VY(N)**2)
DC 40 I = 1, N
ADrag(I) = FORAG*SIN(THETA(I) - PHI).
40 CONTINUE
RETURN
END
SUBROUTINE SETUP(N,TO)
REAL MASS1, MASS2
DIMENSION TO(30), TCSQ(30)
COMMON/G/GRAV(30)/NUMMAS/MASS1(30,30), MASS2(30,30)/XY/XYM(30)/D
CRAG2/DRAG(30)/EQUAL/EQUALS(30)
COMMON/ANG/A(30)
DC 1) I=1, N
TCSQ(I)=TO(I)**2
10 CONTINUE
DC 30 I=1, N
EQUALS(I)=0.0
DC 20 J=1, N
EQUALS(I)=EQUALS(I)+MASS2(I,J)*TOSQ(J)
20 CONTINUE
EQUALS(I)=EQUALS(I)+A(30)
EQUALS(I)=EQUALS(I)+GRAV(I)+XYMAS(I)+DRAG(I)
30 CONTINUE
RETURN
END
SUBROUTINE SOLVEX(N, T0D, KORRECT)
DIMENSION DELTA(30), T0D(30)
REAL M1SS1, M1SS2, L
COMMON/NUMMAS/M1SS1(30,30), M1SS2(30,30)/EQUAL/EQUALS(30)/RESID/R(30)/WORK(30,30)/LENGTH/L(30)
DO 11 I=1,N
WORK(I, N+1) = EQUALS(I)
DO 11 J=1,N
WORK(I, J) = M1SS1(I, J)
11 CONTINUE
CALL GAUSS(N, T0D)
IF (KORRECT .EQ. 0) GO TO 50
DO 20 K=1,KORRECT
DO 20 I=1,N
R(I) = -EQUALS(I)
20 CONTINUE
DO 23 J=1,N
R(I) = R(I) + M1SS1(I, J) * T0D(J)
23 CONTINUE
DO 30 I=1,N
WORK(I, N+1) = R(I)
DO 30 J=1,N
WORK(I, J) = M1SS1(I, J)
30 CONTINUE
CALL GAUSS(N, DELTA)
DO 40 I=1,N
T0D(I) = T0D(I) - DELTA(I)
40 CONTINUE
CONTINUE
50 CONTINUE
DO 60 I=1,N
60 T0D(I) = T0D(I) / L(I)
RETURN
END
SUBROUTINE GAUSS(N,X)
DIMENSION X(30)
COMMON /W/WORK(30,31)
N1=N+1
DO 21 I=1,N
WORKII=WORK(I,I)
IF(WORKII.EQ.0.00) GO TO 8
GO TO 5
8 PRINT 9
9 FORMAT(53H ABNORMAL HALT CAUSED BY DIVIDING BY ZERO IN GAUSS ROUTINE)
CALL EXIT
DO 11 J=1,N1
WORK(I,J)=WORK(I,J)/WORKII
11 CONTINUE
DO 30 J=1,N
WORKIJ=WORK(J,I)
DO 21 K=1,N1
IF(J.EQ.I) GO TO 30
WORK(J,K)=WORK(J,K)-WORK(I,K)*WORKIJ
20 CONTINUE
30 CONTINUE
DO 40 I=1,N
X(I)=WORK(I,N+1)
40 CONTINUE
RETURN
END
SUBROUTINE RK(N,DELTAT,THETA1,T01,TDD,THETA,TC,KORRECT)
DIMENSION THETA1(30),T01(30),TDDO(30),TDD2(30),THETA(30),T0(30),TDD(30),THETA2(30),T02(30),TDD2(30),AK1(30),AK2(30),AK3(30),O1(30),O2(30),G3(30)
CALL CALL(N,THETA1,T01)
CALL SOLVE(N,TDD1,KORRECT)
DO 1 I=1,N
AK1(I)=DELTAT*TDD1(I)
O1(I)=DELTAT*TDD1(I)
T02(I)=T01(I)+AK1(I)/2.
THETA2(I)=THETA1(I)+O1(I)/2.
CONTINUE
CALL CALL(N,THETA2,T02)
CALL SOLVE(N,TDD2,KORRECT)
DO 2 I=1,N
AK2(I)=DELTAT*TDD2(I)
O2(I)=DELTAT*TDD2(I)
T02(I)=T01(I)-AK1(I)+2.*AK2(I)
THETA2(I)=THETA1(I)-O1(I)+2.*O2(I)
CONTINUE
CALL CALL(N,THETA2,T02)
CALL SOLVE(N,TDD2,KORRECT)
DO 3 I=1,N
AK3(I)=DELTAT*TDD2(I)
O3(I)=DELTAT*TDD2(I)
T0(I)=T01(I)+(AK1(I)+4.*AK2(I)+AK3(I))/6.
THETA(I)=THETA1(I)+(O1(I)+4.*O2(I)+O3(I))/6.
CONTINUE
CALL CALL(N,THETA,T0)
CALL SOLVE(N,TDD,KORRECT)
RETURN
END
SUBROUTINE PREDICT(N, DELTAT, T01, THETA, TD, T00, THETA2, TD2)
DIMENSION T01(30), THETA(30), TD(30), T00(30), THETA2(30), TD2(30)
DO 10 I = 1, N
    T02(I) = T01(I) + 2. * DELTAT * T00(I)
    THETA2(I) = THETA(I) + DELTAT * (TD(I) + TD2(I)) / 2.
10    CONTINUE
    RETURN
    END
SUBROUTINE TENSION(N,X00,Y00,THETA,T0,T0D,DENSITY,X0,Y0,Y0)
REAL L,M,
DIMENSION THETA(30),T0(30),T0D(30),AT(30),VN(30),VT(30)
COMMON/LENGTH/L(30),/MASS/M(30),/COEFT/DT(30),/ON(30),/VEL/VT(30),/VY(30)
COMMON/ANG/S1(30),C1(30),/DIFFANG/S2(30,30),C2(30,30)/TEN/T(30)
COMMON/A,COEFD,A
AT(I)=-X00*C1(1)-Y00*S1(I)+L(I)*T0(I)**2
DO 1 I=2,N
II=I-1
AT(I)=-X00*C1(I)-Y00*S1(I)+L(I)*T0(I)**2
DO 1 J=1,II
AT(I)=AT(I)+L(J)*(T0D(J)*S2(J,I)+C2(J,I)*T0(J)**2)
1 CONTINUE
VN(I)=.5*L(I)*T0(I)-X0*S1(I)+Y0*C1(I)+VT(I)*S1(I)
VN(1)=.5*L(I)*T0(I)-X0*S1(I)+Y0*C1(I)+VT(I)*S1(I)
VT(I)=-X0*C1(I)-Y0*S1(I)+VT(I)*C1(I)
DO 2 I=2,N
II=I-1
VN(I)=.5*L(I)*T0(I)-VX(I)*S1(I)+VY(I)*C1(I)+VT(I)*S1(I)
VN(I)=.5*L(I)*T0(I)-VX(I)*S1(I)+VY(I)*C1(I)+VT(I)*S1(I)
VT(I)=-VX(I)*C1(I)-VY(I)*S1(I)+VT(I)*C1(I)
2 CONTINUE
DENS=D4.*T
G=32.2*(DENSITY-WDEN)/DENSITY
IF(VX(N) .LT. 0.00) GO TO 3
PHI=3.14159265/2.
GO TO 2
3 A=VY(N)/ABS(VX(N))
IF(VX(N) .LT. 0.00) GO TO 1
PHI=ATAN(A)
GO TO 2
1 A=ABS(A)
PHI=3.14159265-ATAN(A)
2 CONTINUE
T(N)=M(N)*AT(N)+M(N)*G*S1(N)+.5*DT(N)*VT(N)*ABS(VT(N))
+DA*(VX(N)**2)


\[ \text{IF } T(N) > 0 \text{ THEN } T(N) = T(N) - M(N) \times G \times S_1(N) - N(N) \times A_T(N) \]

\[ K = K + 1 \]

\[ K_1 = K + 1 \]

\[ T(K) = Y(K) \times A_T(K) + T(K_1) \times C_2(K, x_1) + M(K) \times S_1(K) + N(0) \times V_T(K) \times A_B \times V_T(K_1) + N(1) \times S_2(K, K_1) \times V_N(K_1) \times A_B \]

\[ \text{IF } (Y(K) > 0) \text{ THEN } N(K) = T(K) - M(K) \times G \times S_1(K) \]

\[ \text{CONTINUE} \]

\[ \text{IF } I = 1 \]

\[ T(I) = T(I) / 32.2 \]

\[ \text{CONTINUE} \]

\[ \text{RETURN} \]

\[ \text{END} \]
SUBROUTINE BOTTOM(T,TD,TIME)
REAL L,M
DIMENSION TD(30)
COMMON/LENGTH/L(30)/ANG/S1(30),S2(30)/DIFFANG/S2(30,30),S2(30,30)/
COMASS/M(30)/TOTMAS/M(30)
N1=N1+1
N2=N2+2
L=L+1,00
DO 10 I=1,N2
C=L(I)*TD(I)
A=A+C*S2(I,N)
B=B+C*S2(I,N1)
10 CONTINUE
TD(N1)=A/(L(N1)*S2(N,N1))
TD(N2)=B/(L(N2)*S2(N1,N))
PRINT 20,TIME
20 FORMAT(///,22H STOP CALLED AT TIME= ,F11.7)
M(N)=M(N)+1000000000,
DO 30 I=1,N
TM(I)=TM(I)+1000000000.
30 CONTINUE
RETURN
END
Appendix C
Program Sequence


NVRO2 - The first modification of NVRO1 to allow the boundary conditions for the anchor-drop problem.

NVRO3 - An improvement of NVRO2 which used equations (3) of the text for calculating line coordinates.

NVRO4 - An improved version of NVRO2 using a Simpson's integration formula to integrate equations (2) of the text.

NVRO5 - The first lumped mass program using a self starting predictor-corrector for the "Standard catenary configuration" of a Fig. G.

NVRO6 - The same program as NVRO5 except the initial condition is the "Goose neck configuration" of Figure E.

NVRO7 - The inextensible line lumped mass model of reference [9].

NVRO8 - The final version of the lumped mass model using the second order predictor-corrector of Section II-3.0.
Appendix D
Tabulation of Constants

Moorine Line

Nylon - 2.5" Dia.

Density = 71 lb./ft$^3$

\[ C_N = 1.4 \]

\[ C_T = (0.008)(1.4) \]

\[ C_I = 0. \]

\[ C_I = 0.5 \]

Length = 5660 ft. unstressed

Stress = $C_1$(strain)$^C_2$

\[ C_1 = 1.67 \times 10^8 \text{ PSF} \]

\[ C_2 = 4.37 \]

Dacron - 1.0" Dia.

Density - 1.0" Dia.

\[ C_N = 1.4 \]

\[ C_T = (0.008)(1.4) \]

\[ C_I = 0. \]

\[ C_I = 0.5 \]

Stress = $C_1$(strain)$^C_2$

\[ C_1 = 1.2 \times 10^9 \]

\[ C_2 = 3.7 \]

Length 5660 ft. unstressed
Steel - 2.5" Dia.

Density = 350 lb/ft$^3$

$C_N = 1.3$

$C_T = (.008)(1.3)$

$C_{I_t} = 0.$

$C_{I_{N}} = .5$

Inextensible

Anchor

Weight = 12000 lb.

Vol. = 24.5 ft$^3$

Frontal Area = 12.6 ft$^2$

$C_D = .4$

$C_{I} = 1.0$

Length = 5660 ft.

Water

Density = 64 lb/ft$^3$

Depth = 5000 ft.
Appendix E

Computer run times for computations made on the Bonneville Power Administration CDC-6400.

Runs made for nylon lines

1) 15° catenary
   a) Early portion of drop \( t = \frac{.14 \text{ computer sec}}{\text{Real Sec - Mass}} \)
   b) Late portion of drop \( t = \frac{.1 \text{ Computer Sec}}{\text{Real Sec - Mass}} \)

2) 60° catenary
   a) Early portion of drop \( t = \frac{.28 \text{ Computer Sec}}{\text{Real Sec - Mass}} \)
   b) Late portion of drop \( t = \frac{.21 \text{ Computer Sec}}{\text{Real Sec - Mass}} \)

Runs made for dacron lines

1) 15° catenary
   Early portion of drop \( t = \frac{.84 \text{ Computer Sec}}{\text{Real Sec - Mass}} \)

2) 60° Catenary
   Early portion of drop \( t = \frac{1.4 \text{ Computer Sec}}{\text{Real Sec - Mass}} \)
### Appendix F

#### List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Frontal Area of Anchor</td>
</tr>
<tr>
<td>ACN</td>
<td>Normal Acceleration</td>
</tr>
<tr>
<td>ACT</td>
<td>Tangential Acceleration</td>
</tr>
<tr>
<td>C_n</td>
<td>Constants</td>
</tr>
<tr>
<td>CD</td>
<td>Drag Coefficients</td>
</tr>
<tr>
<td>CDN</td>
<td>Normal Drag Coefficient</td>
</tr>
<tr>
<td>CDT</td>
<td>Tangential Drag Coefficient of the Line</td>
</tr>
<tr>
<td>CI</td>
<td>Added Mass Coefficient for Anchor</td>
</tr>
<tr>
<td>CIN</td>
<td>Added Mass Coefficient Normal for Line</td>
</tr>
<tr>
<td>CIT</td>
<td>Added Mass Coefficient Tangential for Line</td>
</tr>
<tr>
<td>D</td>
<td>Line Diameter</td>
</tr>
<tr>
<td>Da</td>
<td>Anchor Diameter</td>
</tr>
<tr>
<td>DFN</td>
<td>Drag Force in the Normal Direction</td>
</tr>
<tr>
<td>DFT</td>
<td>Drag Force in the Tangential Direction</td>
</tr>
<tr>
<td>M_i</td>
<td>The i\textsuperscript{th} mass representing the Line</td>
</tr>
<tr>
<td>MA</td>
<td>Anchor Mass</td>
</tr>
<tr>
<td>SF</td>
<td>Spring Force</td>
</tr>
<tr>
<td>SFX</td>
<td>Spring Force in the X direction</td>
</tr>
<tr>
<td>SFZ</td>
<td>Spring Force in the Z direction</td>
</tr>
<tr>
<td>T</td>
<td>Line Tension at Anchor</td>
</tr>
<tr>
<td>VCX</td>
<td>Local Water Velocity in X Direction</td>
</tr>
<tr>
<td>VCY</td>
<td>Local Water Velocity in Y Direction</td>
</tr>
</tbody>
</table>
VOL  Anchor Displaced Volume
VCN  Line Velocity Normal
VCT  Line Velocity Tangential
W    Anchor Weight
WT₀  Weight of the i\textsuperscript{th} Mass

\lambda_{i}  Line Segment Length

k₁ to k₄ Runge Kutta Parameters
\lambda₁ to \lambda₃ Runge Kutta Parameters
m₁ to m₄ Runge Kutta Parameters
n₁ to n₄ Runge Kutta Parameters

x    Horizontal Coordinate
z    Vertical Coordinate
S    Arc Distance Along the Line
ΔS   Equal Distance Taken between Calculation Point
Δt   Time Increment

β    Angle from the Horizontal to the Anchor Line Link
\epsilon_{i} Line Strain at the i\textsuperscript{th} link
θ    Angle Between the Horizontal and the Line Tangent
θ_{AV} The Average Between Two Adjacent Links
ρ    Mass Density of the Line
ρ₀   Mass Density of the Anchor
σ    Stress in the Line
ε    Error in Velocity Computation
The anchor-last mooring procedure is investigated in order to determine the transient forces in the mooring line and the velocities of the anchor. Transient forces were determined and the results showed that no severe snap loads occurred for the cases investigated. In addition, it was found that the vertical velocity of the anchor can be small as it approaches impact with the floor of the ocean.

Both extensible (nylon and dacron) and inextensible (steel wire rope) lines were investigated. Lumped mass numerical models were developed for both cases. For the extensible line case the equations of motion were determined for each mass from Newton's Second Law, and they were integrated using a second order predictor-corrector integration technique. Hamiltonian techniques were utilized to determine the equations of motion for the inextensible line. The predictions from the numerical models show the line tensions and positions as a function of time.
### Key Words

<table>
<thead>
<tr>
<th>LINK A</th>
<th>LINK B</th>
<th>LINK C</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROLE</td>
<td>WT</td>
<td>ROLE</td>
</tr>
</tbody>
</table>

- Numerical Models for Lines
- Anchor-Last Mooring Procedure
- Anchoring
- Lumped Mass Models