AN ABSTRACT OF THE THESIS OF

Luis Serrano Cid for the degree of Doctor of Philosophy in Statistics presented on June 10, 1985.

Title: Estimation of Feedback Parameters on a Closed-Loop System: A Geophysical Problem.

Abstract approved: Redacted for Privacy

Fred L. Ramsey

The presence of feedback in an input-output system associated with an oceanographic phenomenon is determined. A testing procedure to determine the presence of feedback is given. The ordinary transfer function model is determined not to be appropriate in modeling the system. The two and three stage least squares methods are used to estimate the parameters of a system of equations associated with the phenomenon. Computer simulation is used to verify the adequacy of the model. It is shown that without the inclusion of feedback parameters, the model fails to represent the physical nature of events. Several other methods of analysis are reviewed.

ESTIMATION OF THE FEEDBACK PARAMETERS IN A CLOSED-LOOP SYSTEM: A GEOPHYSICAL PROBLEM

by

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A THESIS

submitted to

Oregon State University

in partial fulfillment of
the requirements for the
degree of

Doctor of Philosophy

Completed June 10, 1985
Commencement June 1986

Approved :

Redacted for Privacy

Professor of Statistics in charge of major

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Chairman of the Department of Statistics

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Dean of Graduate School

Date thesis is presented <u>June 10, 1985</u>

AKNOWLEDGEMENTS

I would like to express my deepest gratitude to all the people who, in one way or another, help me to obtain this degree.

To my major professor Dr. Fred Ramsey for believing in me and for giving me the necessary guidance and support to achieve my goal.

To Dr. David Enfield for introducing me into the fascinating world of the ocean and for his patience with my ignorance.

To Dr. William Quinn for allowing me to use his invaluable files and for many hours of enlightening conversation.

To Ron Stillinger for having the courage of debugging my countless computer problems.

To my friends and comrades in the adventure, Mohamed Limam, Abderrazak Garoui and Mostafa Aminzadeh, for their companionship during endless weekends of hardwork and irreplaceable discussions.

To my mother and brother for supporting me from the distance and giving me all the encouragement so frequently needed.

To my daughters Maria A., Claudia and Marcela, for having the patience of tolerating me and for being my joy and inspiration during all this years.

To my loving wife Maruja for her companionship, tolerance, care and love, only through her help this thesis became possible. I dedicate this thesis to her.

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I. INTRODUCTION

Every few years, a phenomenon of unusual intensity affects the west coast of the Americas. Although originally defined as a sudden warming of the surface waters along the coast of South America, lately it has been described on a much larger scale, being blamed for provoking changes in weather manifested as intensified rains, stormier winters, stronger than usual wave activity, etc. Its best known characteristic, the anomalous increase of the sea surface temperature, alters the environment of many fisheries along the coast of South and North America with great damage to the economies of the countries involved.

The first symptom of the events is an anomalous increase of the sea surface temperature along the coast of Peru, where the phenomenon is called El Niño (The Child), due to the fact that it usually starts around Christmas time. But, it starts to develop several thousand miles away in the central equatorial Pacific Ocean as a relaxation of the easterly trade winds associated with fluctuations of the atmospheric pressure. The relationships between the atmosphere and the temperature of the surface of the ocean can apparently be classified as the cause-effect type and there are many oceanographers and atmospheric scientists involved in the study of them. We do not intend to analyze the phenomenon from a physical perspective. Rather we intend to examine its stochastic nature.

The cause-effect nature of the phenomenon requires the identification of the causal processes and their distinction from those associated with the effect. In what follows, we will refer to these as the input and output processes respectively. The search for the most adequate input series leads us to the study of the Southern Oscillation (SO), which is a long term atmospheric fluctuation associated with the atmospheric pressure over the South Pacific and with the pattern of winds along the central equatorial Pacific. A measure of these fluctuations is obtained by calculating the difference in the sea level atmospheric pressure between points located at the geographic extremes of the atmospheric "see-saw", that is, one near the center of the South Pacific anticyclone and the other within the Indonesian equatorial low pressure region. The close relationships between the El Niño and the SO have given the name El Niño-Southern Oscillation (ENSO) to the entire phenomenon (Quinn, 1984).

Although there is a general consensus on the nature of the interaction, i.e., that changes in the atmosphere lead changes in the ocean, there also exists evidence that ocean is able to respond and itself induce subsequent atmospheric changes. For example, warmer waters will produce an increase in the temperature of the overlaying atmosphere which is in turn associated with a further decrease in atmospheric pressure. These changes willbe more significant to the extent that the sea surface temperature anomaly persists. Similarly, the in water temperature along the equatorial band will persist insofar as the weather anomalies in the zone are maintained. We can thus conclude that there exists a potential for self-perpetuation of atmospheric and oceanic anomalies due to the occurrence of positive feedback air-sea interaction processes.

Data records from selected weather stations have been made available to us for use in modeling the type of interactions described. They include sea level atmospheric pressure, sea surface temperature, rainfall, and sea level height. These time series are comprised of monthly mean values. The station locations are shown in Figure 2.

Most of the current and past studies in this field have involved frequency domain analysis based on sample spectra and Fourier transformed time series (e.g., Enfield, 1980). Recently, however, Chu and Katz (1984), have approached the problem by using time domain analysis which is instead based on sample autocorrelations. We believe that the time domain analysis is a valid approach to the problem, without discarding the Fourier methods which can also be used. We intend, however, to make use of the Box and Jenkins (1970) ARMA models to describe the basic processes involved.

The development of the model was motivated by the desire for a description of the physical connections using time domain analysis as a way to obtain results that are easier to interpret and have at least the same validity as those that one can obtain using frequency domain analysis. Accordingly, the goals of this thesis are:

(1) To describe the time domain methodology for modeling input-output systems.

(2) To find a method for estimating the feedback parameters of a closed-loop system of equations.

(3) To apply the estimation procedures toward obtaining a model that represents the relationships between the El Niño events and the Southern Oscillation. (ENSO)

(4) By using computer simulations, to compare the actual input and output series with those generated from the estimated parameters.

The second chapter will be dedicated to the analysis of the transfer function models and their possible applicability to the case of closed-loop systems. A second method of estimation will then be considered, namely the two-stage least squares procedure. It will be compared to its extension, the three-stage least squares method as a simultaneous approach of solving the system of equations . It will be shown that in presence of feedback the use of regular transfer function models is not recommended.

In the third chapter we will apply the methods studied in Chapter two to the input and output series found to best represent the nature of the phenomenon. In doing so we present a modified version of the box-and whiskers plot (Tukey 1977), which we consider is more appropriate to display data that follows a yearly pattern. The transfer function model will be compared with the one obtained from the two-stage least squares method. Finally, a simulation program will be used to generate series based on the best model and they will be compared with the original series to determine the validity of their estimates. Several methods for the detection of feedback and the estimation of parameters will be reviewed in the last chapter. They will be shown to be of little use in the present context. A final section considers the general form of a prediction model based on the proposed closed-loop model. The necessary steps to obtain the predicted values are outlined, and a procedure to estimate the variance of the predictors is discussed.

II. REVIEW OF THE METHODOLOGY.

1. DEFINITION OF BASIC CONCEPTS.

A sequence of random variables E_t, E_{t-1}, E_{t-2}, ..., which is assumed to be normally distributed with mean zero and variance σ^2, is called a white noise sequence and it can be transformed to a process $\{Y_t\}$ by the linear filter operation

$$Y_t = u + E_t + p_1E_{t-1} + p_2E_{t-2} + \dots$$
$$Y_t = u + P(B)E_t \qquad (2.1.1)$$

where B is the backshift operator such that $y_tB^k = y_{t-k}$, with $k = 0, 1, 2, \dots$

$$P(B) = 1 + p_1B + p_2B^2 + \dots$$

is called the transfer function of the filter and u is ordinarily taken to be zero.

An autoregressive process of order p, is defined as a finite linear aggregate of previous values of the process, plus a white noise E_t, i.e.

$$Y_t = \phi_1Y_{t-1} + \phi_2Y_{t-2} + \dots + \phi_pY_{t-p} + E_t$$
$$Y_t = \phi(B)Y_t + E_t \qquad (2.1.2)$$

with transfer function $\phi(B)$:

$$\phi(B) = [1 - \phi_1B - \phi_2B^2 - \dots - \phi_pB^p]^{-1}$$

this process is denoted by AR(p).

A moving average process, defines Y_t as depending on a finite number of previous E's, thus

$$Y_t = E_t - \theta_1 E_{t-1} - \theta_2 E_{t-2} - \dots - \theta_q E_{t-q}$$

$$Y_t = \theta(B) E_t \tag{2.1.3}$$

where the moving average operator is defined by

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q$$

This is called a moving average process of order q and it is denoted by MA(q), q = 0, 1, 2, ...

A more flexible model is often necessary to adequately fit a process under study. This is achieved by including both autoregressive and moving average terms in the model obtaining:

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p}$$

$$+ E_t - \theta_1 E_{t-1} - \theta_2 E_{t-2} - \dots - \theta_q E_{t-q} , \tag{2.1.4}$$

or equivalently:

$$Y_t - \phi_1 Y_{t-1} - \dots - \phi Y_{t-p} = E_t - \theta_1 - \dots - \theta_q E_{t-q}$$

$$\phi(B) Y_t = \theta(B) E_t \tag{2.1.5}$$

where $\phi(B)$ and $\theta(B)$ are called the autoregressive and moving average operators respectively. The process (2.4) is called ARMA(p,q) process.

Consider now a second process $\{X_t\}$ that is related to the process $\{Y_t\}$ by the expression:

$$Y_t = V_0X_t + V_1X_{t-1} + V_2X_{t-2} + \dots$$

$$= (V_0 + V_1B + V_2B^2 + \dots)X_t$$

$$= V(B)X_t \qquad (2.1.6)$$

Here V(B) is a polynomial on B. This expression is also a linear filter and V(B) is the corresponding transfer function. The model is known as Transfer Function Model and the weights V_0, V_1, V_2, ... are called Impulse Response Function.

Henceforth, $\{Y_t\}$ will be called the output and $\{X_t\}$ the input of the Transfer Function Model.

Equation (2.1.5) is not very satisfactory because it contains an infinite number of unknown parameters V_0, V_1, ... To avoid this difficulty, a convenient way to parameterize the model is to express the system as a difference equation

$$(1 - d_1B - d_2B^2 - \dots - d_rB^r)Y_t =$$

$$(w_0 - w_1B - w_2B^2 - \dots - w_sB^s)X_t \qquad (2.1.7)$$

If we now specify that $d_r(B) = 1 - d_1B - d_2B^2 - \dots - d_rB^r$ and $w_s(B) = w_0 - w_1B - w_2B^2 - \dots - w_sB^s$ we can write (2.1.7) as:

$$d_r(B)Y_t = w_s(B)X_t$$

$$\text{for } s = 0, 1, 2, \dots ; r = 0, 1, \dots \qquad (2.1.8)$$

So we can express the infinite order polynomial V(B) of equation (2.1.6) as the ratio of two finite order polynomials

$$Y_t = \frac{(w_0 - w_1B - w_2B^2 - \ldots - w_sB^s)}{(1- d_1B - d_2B^2 - \ldots - d_rB^r)} X_t \quad \text{and}$$

$$Y_t = d_r^{-1}(B)\ w_s(B)X_t \tag{2.1.9}$$

Now comparing equation (2.1.6) with (2.1.9) we have that

$$V(B) = d_r^{-1}(B)\ w_s(B) \tag{2.1.10}$$

The output process $\{Y_t\}$ will never exactly follow the pattern dictated by the model (2.1.6), as in general there will be other influences affecting the system. These disturbances, or noise, could be caused by other variables not specifically included in the model. If we denote this noise term by N_t the model (2.1.6) may be written as

$$Y_t = V(B)X_t + N_t \tag{2.1.11}$$

But N_t itself can be represented by

$$N_t = E_t + a_1E_{t-1} + a_2E_{t-2} + \ldots$$
$$N_t = A(B)E_t \tag{2.1.12}$$

where $\{E_t\}$ is a white noise sequence and

$A(B) = (1 + a_1B + a_2B^2 + \ldots)$.

Now we can write (2.1.11) as

$$Y_t = V(B)X_t + A(B)E_t \tag{2.1.13}$$

Again we see that A(B) is a sequence of infinite terms that can be reduced to a ratio of two finite polynomials by using the same procedure as for equation (2.1.6), that is if N_t can be modeled by an ARMA(p,q) process, then

$$\phi(B)N_t = \Theta_q(B)E_t$$

$$N_t = \phi_p{}^{-1}(B)\Theta_q(B)E_t \tag{2.1.14}$$

that when related to (2.1.12) gives

$$A(B) = \phi^{-1}(B)\Theta_q(B). \tag{2.1.15}$$

Replacing (2.1.10) and (2.1.15) in (2.1.13, we then obtain:

$$Y_t = d_r{}^{-1}(B)w_s(B)X_t + \phi_p{}^{-1}(B)\Theta_q E_t \tag{2.1.16}$$

Systems of the type defined by (2.1.13) are called open-loop systems (Figure 1). The procedures for analyzing data that can be adequately fit by these models are in common use (Box and Jenkins, 1970, Montgomery and Weatherby, 1980, Tiao and Box, 1981). Frequently we suspect the presence of feedback which connects the output with the input. These are called closed-loop systems (Figure 2). They are represented by adding a second equation to (2.1.13)

$$X_t = g_0Y_t + g_1Y_{t-1} + g_2Y_{t-2} + \dots + M_t$$

$$X_t = G(B)Y_t + M_t \tag{2.1.17}$$

where M_t is an unobserved disturbance that can also be represented as

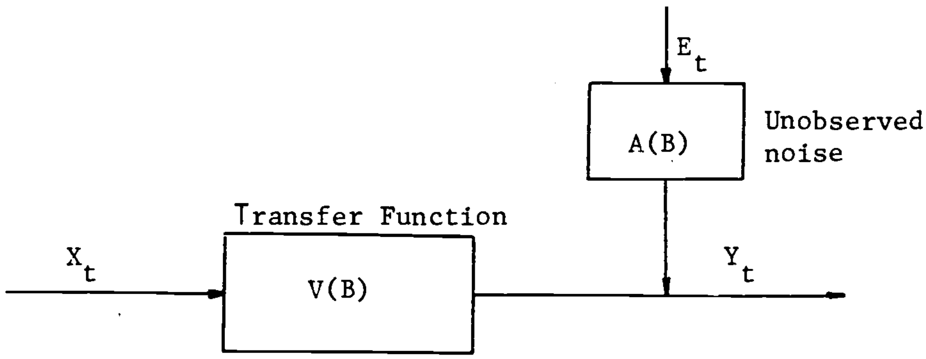


Figure 1a. Open-Loop System

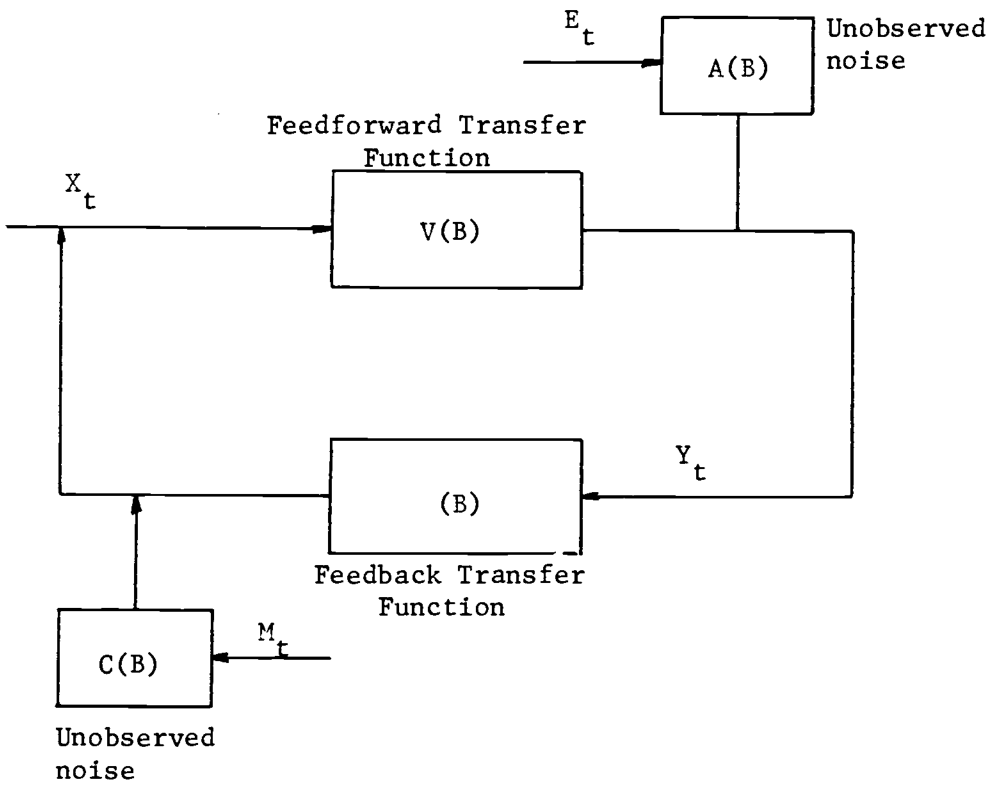


Figure 1b. Closed-Loop System

$$M_t = m_t + c_1 m_{t-1} + c_2 m_{t-2} + \dots$$

$$M_t = C(B) m_t$$

$\{m_t\}$ being a white noise sequence and $C(B) = B^o + c_1 B + c_2 B^2 + \dots$ So the closed-loop equations can now be represented as

$$Y_t = V(B)X_t + A(B)E_t \tag{2.1.18}$$

$$X_t = J(B)Y_t + C(B)M_t \tag{2.1.19}$$

Equation (2.1.19) is known as the Feedback Equation (Caines and Chan, 1975).

An alternative representation of the equations (2.1.18) and (2.1.19) is

$$Y_t = K(B)Y_t + L(B)X_t + E_t \tag{2.1.20}$$

$$X_t = P(B)X_t + R(B)Y_t + M_t. \tag{2.1.21}$$

These are equivalent to

$$Y_t = \sum_{i=1}^{k} \alpha_i Y_{t-i} + \sum_{i=1}^{p} \gamma_i X_{t-i} + E_t$$

$$X_t = \sum_{i=1}^{q} \beta_i X_{t-i} + \sum_{i=1}^{r} \theta_i Y_{t-i} + M_t$$

which in the absence of feedback become

$$Y_t = \sum_{i=1}^{k} \alpha_i Y_{t-i} + \sum_{i=1}^{p} \gamma_i X_{t-i} + E_t$$

$$X_t = \sum_{i=1}^{q} \beta_i X_{t-i} + M_t$$

The alternative representation displayed by the system of equations (2.1.20-21), represents a parametrization that includes both input and output parameters in both equations. This characteristic is particularly useful because it allows the identification of the feedback parameters θ_i, $i = 1, 2, \ldots, r$. This is useful when comparing systems with and without the inclusion of the feedback parameters.

2. IDENTIFICATION PROCEDURES

In this section we intend to describe briefly the procedures generally used to identify the type of processes involved and the type of model that, as a first approximation may be applied. The identification procedures to be used are of two types: One is a univariate approach, which is intended to determine the type of ARMA process that best fits the data is associated with a univariate time series. The second approach is bivariate and is used to determine the type of model that best fits an input-output type of relationship between two time series. For the univariate case, the identification of an autorregressive model is described and for the bivariate case, the identification of a transfer function model is discussed and associated with the identification of a closed-loop model.

2.1 Estimation of the Order of an Autoregression.

The main tools used in the process of identification of an ARMA model are the autocorrelation and partial autocorrelation functions, denoted ACF and PACF respectively and defined by the sequences $\{\rho_{xx}(k)\}$ and $\{\phi_t,\ t \in Z^+\}$, with Z being the set of all integers and Z^+ the set of all positive integers.

$$\rho_{xx}(k) = \frac{\gamma_{xx}(k)}{\gamma_{xx}(0)} \qquad (2.2.1)$$

where $\gamma_{xx}(k) = E(X_t\ X_{t+k})$, $\gamma_{xx}(0) = E(X_t{}^2)$

and $\phi_t = \phi_{s,s+t}$; $s \in Z$ and $t \in Z^+$

$$\phi_1 = \alpha_1{}^{(1)} = \rho_{xx}(1)$$

$$\sigma_1{}^2 = 1 - \phi_1{}^2$$

$$\phi_{k+1} = \alpha_{k+1}{}^{(k+1)} = \{\rho_{k+1} - \sum_{j=1}^{k} \alpha_j{}^{(k)}\ \rho_{k+1-j}\} \ /\ \sigma_k{}^2$$

$$\alpha_j{}^{(k+1)} = \alpha_j{}^{(k)} - \phi_{k+1}\ \alpha^{(k)}{}_{k+1-j}\ ; \quad j = 1, 2, \ldots, k$$

$$\sigma_{k+1}{}^2 = \sigma_k{}^2(1 - \phi^2{}_{k+1}) \qquad (2.2.2)$$

This method of obtaining the PACF is known as Durbin's procedure (see, e.g. Ramsey, 1973).

A preliminary step toward the determination of the order of an autoregressive model is the study of the structure of the PACF. The results can be studied using the method proposed by Hannan and Quinn (1979), who assign to the process the order k, with k being such that it minimizes the expression

$$\phi(k) = \ln \sigma^2{}_k + N^{-1}\ln(2k)C\ln(\ln N)\ ; \ C \geqslant 1 \qquad (2.2.3)$$

where $\sigma_k{}^2$ is the estimate obtained in (2.2.2) above.

2.2 Identification of the Transfer Function Model.

Recall that the identification of univariate AR or MA models requires the use of the ACF and PACF. Similarly, the identification of transfer function models requires the use of the crosscorrelation function

$$\rho_{xy}(k) = \frac{\gamma_{xy}(k)}{[\gamma_{xx}(0)\ \gamma_{yy}(0)]^{1/2}}$$

$$k = 0, 1, 2, \ldots \qquad (2.2.4)$$

where $\gamma_{xy}(k) = E(X_t Y_{t+k})$ is the crosscovariance between X_t and Y_t at lag k, and $\gamma_{xx}(0)$ and $\gamma_{yy}(0)$ are the variances of the input and output respectively.

The two processes, input and output, could have both correlative structure within the processes as well as between the processes. The within-process structure will frequently mask the interrelationships between input and output and will tend also to inflate the variances and covariances of the crosscorrelation function estimates. These problems are simplified to a great extent if the structure of the input is converted to that of a white noise process. This is achieved by prewhitening the input and transforming the output series. This procedure consists of replacing the input series by its residuals after an adequate ARMA model has been obtained for it. The transformation of the output uses the same ARMA model parameters used for the prewhitening of the input. Note that, in general, this will not produce a prewhitening of the output series.

If we assume that $\{X_t\}$ is a stationary process, that is, for all s, t > 0 the processes $\{X_t\}$ and $\{X_{t+s}\}$ have the same probability structure. In other words, if choosing any fixed point as the prigin, the ensuing process have the same probability law. (If the process is not stationary, a stationary derivative can often be obtained by a suitable differencing) and we also assume that the input can be modeled as an ARMA process, say

$$\phi_x(B)X_t = \theta_x(B)\alpha_t$$

where α_t is white noise. Then the prewhitened input is

$$\alpha_t = \theta_x^{-1}(B)\phi_x(B)X_t \tag{2.2.5}$$

If the same transformation is applied to the output series Y_t we obtain:

$$\beta_t = \theta_x^{-1}(B)\phi_x(B)Y_t \tag{2.2.6}$$

then the transfer function model (2.1.11) can be written as

$$\beta_t = V(B)\alpha_t + E_t \tag{2.2.7}$$

where $E_t = \theta_x^{-1}(B)\phi_x(B)N_t$ is the transformed noise process. If both sides of (2.2.7) are multiplied by α_{t-k} and we take expectations, we obtain

$$V_k = \frac{\gamma_{\alpha\beta}(k)}{\gamma_{\alpha\alpha}{}^2} = \rho_{\alpha\beta}(k)\left[\frac{\gamma_{\beta\beta}(0)}{G_{\alpha\alpha}(0)}\right]^{1/2} ; \quad k = 0,\ 1,\ 2,\ \ldots \tag{2.2.8}$$

where $\gamma_{\alpha\beta}(k)$ and $\rho_{\alpha\beta}(k)$ are the crosscovariance and cross-correlation of the transformed input and output, respectively.

This procedure will produce estimates, adequate enough to allow the specification of appropriate values of r and s for the transfer function model (2.1.9), which then leads to estimates of the noise N_t. The sample ACF and PACF will then be used to produce an univariate ARMA model for the noise term N_t. As a general rule rather simple ARMA models, usually of first or second order, are adequate (Montgomery and Weatherby 1980).

2.3 Testing for the presence of feedback.

Under usual circumstances the presence of feedback is not suspected unless one of the two following conditions (or both) are present: 1) the nature of the phenomenon indicates the possibility of feedback and 2) the crosscorrelation function between input and output exhibits significant values of crosscorrelation for negative values of k. In either, case a test statistic must be obtained to verify the presence of feedback.

Let $\{X_t\}$ and $\{Y_t\}$ be the input and output processes respectively and $\rho_{xy}(k)$ their crosscorrelation at lag k, where X leads Y by k time periods. This crosscorrelation can be estimated by

$$\hat{\rho}_{xy}(k) = r_{xy}(k) = \frac{c_{xy}(k)}{(c_{xx}(0)c_{yy}(0))^{1/2}} \quad ; \qquad k = 0, 1, 2, \ldots \qquad (2.2.9)$$

where $c_{xy}(k)$ is the sample crosscovariance at lag k.

Box and Jenkins (1970) derive the following expression for the covariance between two sample crosscorrelations $r_{xy}(k)$ and $r_{xy}(k+h)$ under the assumption of normality as

$$\mathrm{cov}\{r_{xy}(k), r_{xy}(k+h)\} =$$

$$(n-k)^{-1} \sum_{v=-\infty}^{+\infty} [\rho_{xx}(v)\ \rho_{yy}(v+h) + \rho_{xy}(-v)\rho_{xy}(v+2k+h)$$

$$+ \rho_{xy}(k)\rho_{xy}(k+h)\{\rho_{xy}{}^2(v) + 1/2\ \rho_{xx}{}^2(v) + 1/2\ \rho_{yy}{}^2(v)\}$$

$$- \rho_{xy}(k)\{\rho_{xx}(v)\rho_{xy}(v+k+h) + \rho_{xy}(-v)\rho_{yy}(v+k+h)\}$$

$$- \rho_{xy}(k+h)\{\rho_{xx}(v)\rho_{xy}(v+k) + \rho_{xy}(-v)\rho_{yy}(v+k)\}]$$

$$\text{for } k = 0, 1, 2, \ldots ;\ h = 0, 1, 2, \ldots \qquad (2.2.10)$$

Under the further assumption that $\rho_{xy}(j) = 0$ for $j < 0$,

$$(n-k)\mathrm{cov}\{r_{xy}(k), r_{xy}(k+h)\} =$$

$$\rho_{yy}(h) + \sum_{v=0}^{2k+h} \rho_{xy}(v)\rho_{xy}(2k+h-v)$$

$$+ \rho_{xy}(k)\rho_{xy}(k+h)\{\sum_{v=0}^{+\infty} \rho_{xx}{}^2(2) - 3\}$$

while by considering $k<0$ and $k+h < 0$,

$$(n-k)\mathrm{cov}\{r_{xy}(k), r_{xy}(k+h)\} = \rho_{yy}(h)\ . \qquad (2.2.11)$$

If we further assume that 1) $\rho_{xy}(k) = 0$ for $j < 0$, 2) $\{X_t\}$ is white noise, and 3) $Y_t = \sum_{h=0}^{+\infty} \alpha_h X_{t-h} + E_t$, with $\{X_t\}$

independent of $\{E_t\}$and $\{E_t\}$ being a ARMA process (assumption of no feedback(), we can conclude that:

$$R = \begin{bmatrix} r_{xy}(-1) \\ r_{xy}(-2) \\ \cdot \\ \cdot \\ \cdot \\ r_{xy}(-m) \end{bmatrix} \sim MVN_m(\underset{\sim}{0}, \underset{\sim}{\Sigma}) \qquad (2.2.12)$$

where the elements of the covariance matrix Σ are defined by (2.2.11).

Now, from (2.2.11) we see that:

$$N \sum_{j=1}^{m} r_{xy}^{2}(-j) \sim \chi^2 \; (m) \qquad (2.2.13)$$

This expression can be used as a test statistics to determine the presence of feedback.

An alternative method is suggested by Box and MacGregor (1974). They propose detecting the presence of feedback by testing for the presence of a significant crosscorrelation between the prewhitened input and the transformed output at lag k = 0.

3. ESTIMATION OF THE PARAMETERS

Once the form of the models has been tentatively identified, is necessary to estimate the parameters for both univariate and bivariate cases. The estimation of the parameters for the univariate ARMA models is required previous to the estimation of the parameters for the transfer function and closed-loop systems since they will be used later in the process of prewhitening of the original series.

3.1 Autoregressive Model.

Consider again the autoregressive model:

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \dots + \phi_p X_{t-p} + E_t$$

as defined in (2.1.2). After the tentative order p of the model has been identified, we can obtain estimates for the parameters ϕ_k, k = 1, 2, ..., p. The most popular way of estimating these parameters is by using the Yule - Walker equations, which provide approximations to the least squares and maximum likelihood estimates.

The Yule - Walker estimates are:

$$\hat{\underset{\sim}{\phi}} = \underset{\sim}{R}^{-1} \underset{\sim}{r}$$

where

$$\underset{\sim}{R} = \begin{bmatrix} 1 & r_{xx}(1) & \dots & r_{xx}(p-1) \\ r_{xx}(1) & 1 & & r_{xx}(p-2) \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ r_{xx}(p-1) & r_{xx}(p-2) & \dots & 1 \end{bmatrix} \; ; \quad \underset{\sim}{r} = \begin{bmatrix} r_{xx}(1) \\ r_{xx}(2) \\ \cdot \\ \cdot \\ \cdot \\ r_{xx}(p) \end{bmatrix} \tag{2.3.1}$$

with for , $k = 1, 2, \dots, p$,

$$r_{xx}(k) = \frac{c_{xx}(k)}{c_{xx}(0)}$$

and

$$c_{xx}(k) = \frac{1}{n} \sum_{t=1}^{n-p} (X_t - \bar{X})(X_{t+k} - \bar{X})$$

In particular for

$$\text{AR (1)} : \hat{\phi}_1 = r_{xx}(1)$$

$$\text{AR (2)} : \hat{\phi}_1 = \frac{r_{xx}(1)\ (1-r_{xx}(2))}{1 - r_{xx}(1)^2}$$

$$\hat{\phi}_2 = \frac{r_{xx}(2) - r_{xx}(1)^2}{1 - r_{xx}(1)^2}$$

Estimates of the variances and covariances of $\hat{\phi}_k$ can be obtained from

$$\text{Var}\ (\hat{\underset{\sim}{\phi}}) = n^{-1}\ (\underset{\sim}{1} - \underset{\sim}{r}_{xx}{}'\hat{\underset{\sim}{\phi}})\ \underset{\sim}{R}^{-1} \tag{2.3.2}$$

3.2 Impulse Response Function.

As mentioned before, the primary tools for estimation of the impulse response function are the crosscorrelations between input and output. By replacing the estimates (2.2.9) for the prewhitened input α_k and transformed output β_k into equation (2.2.8), the following preliminary estimates of the impulse response function can be obtained

$$\hat{V}_k = r_{\alpha\beta}(k)\ (c_{\beta\beta}/c_{\alpha\alpha})^{1/2} \quad ; \quad k = 0,\ 1,\ 2,\ \ldots \qquad (2.3.3)$$

where $c_{\alpha\alpha}$ and $c_{\beta\beta}$ are estimate of $\gamma_{\alpha\alpha}$ and $\gamma_{\beta\beta}$ respectively.

Equation (2.3.2) yields rough estimates of the V_k's, But from the pattern of these estimated values, one can guess the appropriate values of r and s for the transfer function model (2.1.9). Box and Jenkins (1970) present a table that examine the models for all combination of r, s, $<$ 2 for a given value of b, along with the typical impulse response function for each model.

After the form of the transfer function has been tentatively determined, the noise model must be considered. A simple estimate of the noise series will be obtained from

$$\hat{N}_t = y_t - \hat{V}(B)x_t$$

Similarly we can use equation (2.3.3) to estimate d(B)and w(B) from equation (2.1.10) so that the estimate of the noise series becomes

$$\hat{N}_t = y_t - \hat{d}^{-1}(B)\ \hat{w}(B)\ X_t$$

After $\hat{N}_t$ has been obtained, its sample autocorrelation can be analyzed to produce an univariate ARMA model of the residuals series. The preliminary model is now:

$$Y_t = \hat{d}^{-1}(B)\ \hat{w}(B)\ X_t - \hat{N}_t \qquad (2.3.4)$$

with

$$\hat{N}_t = \hat{\phi}^{-1}(B)\ \hat{\theta}(B)\ E_t$$

More efficient estimators can be obtained for the parameters d, w, ϕ and θ by minimizing the conditional sum of squares:

$$S_o\ (d,\ w,\ \phi,\ \theta) = \sum_{t=1}^{m} E_t^{\,2}(d,\ w,\ \phi,\ \theta/x_o,\ y_o,\ E_o) \qquad (2.3.5)$$

Under the assumption of the normality of the E's and for given initial values x_o, y_o and E_o, this method will provide a useful approximation to the maximum likelihood estimators of the parameters.

The minimization procedure was carried out using computer packages such as SAS (1982) which make use of the Marquardt non-linear estimation algorithm.

3.3 Feedback Parameters.

Once the presence of feedback has been determined, a method of estimation of the parameters of the two equations in the system (2.3.6) must be determined.

Consider then the closed-loop system

$$\begin{aligned} Y_t &= A_1(B)Y_t + C_1(B)X_t + E_{1t} \\ X_t &= A_2(B)Y_t + C_2(B)X_t + E_{2t} \end{aligned} \qquad (2.3.6)$$

The estimation of the parameters for simultaneous systems of equations can be either single-equation methods based on serial estimation of the parameters of one equation at the time, or complete-system methods that consider the system as a whole. The two-stage least squares (2SLS) correspond to the first type and the three-stage least squares method (3SLS) to the second. We will present briefly the foundations of each one:

3.3.1 Two Stage-Least Squares.

Let Z_t , X_t and Y_t, be matrices such that

$$Z_t = (Z_t^{(1)}, Z_t^{(2)})$$

$$X_t = (X_t^{(1)}, X_t^{(2)})$$

$$Y_t = (Y_t^{(1)}, Y_t^{(2)})$$

Define also the matrices A(B), C, and E_t as

$$A(B) = (A_1(B), A_2(B))$$

$$C(B) = (C_1(B), C_2(B))$$

$$E_t = (E_t^{(1)}, E_t^{(2)})$$

So that the feedback model can be written as

$$Z_t = A(B)Y_t + C(B)X_t + E_t \tag{2.3.7}$$

Each one of the equations in the system can be written now in the form

$$Z_t^{(i)} = A_i(B)Y_t^{(i)} + C_i(B)X_t^{(i)} + E_t^{(i)} \quad ; \quad i = 1,\ 2 \qquad (2.3.8)$$

To simplify notation, from now on we will denote Z_i for $Z_t^{(i)}$, X_i for $X_t^{(i)}$, Y_i for $Y_t^{(i)}$, E_i for $E_t^{(i)}$, A_i for $A_i(B)$, and C_i for $C_i(B)$, for $i = 1,\ 2$.

For our purposes, Z_i is the n x 1 vector of observed outputs (dependent) variables, Y_i a $n \times g_i$ matrix of observations on the rest of the lagged dependent variables, X_i a $n \times k_i$ matrix of observations of the input variables, including a column of ones if an intercept is desired, A_i a $g_i \times 1$ vector, corresponding to the coefficients of Y_i, C_i a $k_i \times 1$ vector of the coefficients of X_i, and E_i the vector of disturbances.

The essence of the 2SLS is the replacement of Y_i by its estimated $\widehat{Y}_i$, and then performing ordinary least squares of Z_i on $\widehat{Y}_i$ and X_i. The matrix $\widehat{Y}_i$ is computed in the first stage by regressing each $Y_{i,j}$, $j = 1,\ 2,\ \ldots,\ g_i$ on all X_i , $i = 1,\ 2$, that is,

$$\widehat{Y}_i = X_t(X_t'X_t)^{-1}X_t'Y_i \qquad (2.3.9)$$

In the second stage, Z_i is regressed on $\widehat{Y}_i$ and X_i, which yields the estimated equations

$$\begin{bmatrix} \widehat{Y}_i'\widehat{Y}_i & \widehat{Y}_i'X_i \\ X_i'\widehat{Y}_i & X_i'X_i \end{bmatrix} \begin{bmatrix} a_i \\ c_i \end{bmatrix} = \begin{bmatrix} \widehat{Y}_i'Z_i \\ X_i'Z_i \end{bmatrix} \qquad (2.3.10)$$

were a_i and c_i are the estimates of A_i and C_i respectively.

An expression involving only the matrices of actual observations is

$$Y_i = \hat{Y}_i - V_i \tag{2.3.11}$$

where V_i is the matrix of the residuals from the least squares regressions of Y_i on X_t. With this, equation (2.3.6) can now be written as

$$Z_i = \hat{Y}_i A_i + X_i C_i + E_i + V_i A_i$$

or

$$Z_i = WD_i + (E_i + V_i A_i) \tag{2.3.12}$$

where $W = (\hat{Y}_i \ X_i)$ and $D_i = \begin{bmatrix} A_i \\ C_i \end{bmatrix}$

Applying least squares to (2.3.10) gives the 2SLS estimator of the form

$$d_i = \begin{bmatrix} a_i \\ c_i \end{bmatrix} = (W_i'W_i)^{-1}W_i'Z_i$$

$$d_i - D_i = (W_i'W_i)^{-1}Z_i'E_i \tag{2.3.13}$$

and so the asymptotic variance-covariance matrix is:

$$s^2(W_i'W_i)^{-1} =$$

$$s^2 \begin{bmatrix} Y_i'X(X'X)^{-1}X'Y_i & Y_i'X_i \\ X_i'Y_i & X_i'X_i \end{bmatrix}^{-1} \tag{2.3.14}$$

where

$$s^2 = (Z_i - Y_i a_i - X_i c_i)' \ (Z_i - Y_i a_i - X_i c_i)/(n-g_i-k_i)$$

3.3.2 Three-Stage Least Squares.

This is an estimation method proposed by Zellner and Theil (1962), which takes account of all equations in the model and which under certain circumstances could have greater efficiency than the 2SLS. Consider again the system of equations defined in the previous section and in particular lets us consider again the i^{th} equation (2.3.8)

$$Z_i = Y_iA_i + X_iC_i + E_i \ ; \quad i = 1,\ 2 \quad ,$$

which can be now rewritten, as for the 2SLS case,

$$Z_i = W_iD_i + E_i \ ; \quad i = 1,\ 2 \qquad (2.3.15)$$

If we now premultiply (2.3.13) by X', where $X' = (X_1',\ X_2')$, we obtain

$$X'Z_i = X'W_iD_i + X'E_i \ ; \ i = 1,\ 2 \qquad (2.3.16)$$

Note that in this equation, the error term $X'E_i$ has variance-covariance matrix $\sigma_{ii}^2X'X$ which implies that the vector D_i should be estimated by generalized least squares

$$\hat{D}_i = d_i = (W_i'X(X'X)^{-1}X'W_i)^{-1}X'Z_i \ , \qquad (2.3.17)$$

which is just another way of writing the 2SLS estimator of (2.3.12).

Let's now write the complete set of equations

$$\begin{bmatrix} X'Z_1 \\ \\ X'Z_2 \end{bmatrix} = \begin{bmatrix} X'W_1 & 0 \\ \\ 0 & X'W_2 \end{bmatrix} \begin{bmatrix} D_1 & & X'E_1 \\ & + & \\ D_2 & & X'E_2 \end{bmatrix} \qquad (2.3.18)$$

Then the variance-covariance matrix of the error vector is

$$V = \begin{bmatrix} \sigma_{11}X'X & \sigma_{12}X'X \\ \\ \sigma_{21}X'X & \sigma_{22}X'X \end{bmatrix} ,$$

and if we make $\sum = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$

and define the tensor product $\otimes$ of two matrices A and B, where the elements of the matrix A are of the form a_{ij}, $i = 1, 2, \ldots p$ and $j = 1, 2, \ldots q$ by

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \ldots & a_{1p}B \\ \\ a_{21}B & a_{22}B & \ldots & a_{2p}B \\ \vdots & & \vdots & \vdots \\ a_{q1}B & a_{q2}B & \ldots & a_{pq}B \end{bmatrix}$$

then $\quad V = \sum \otimes (X'X)$

so that

$$V^{-1} = \sum^{-1} \otimes (X'X)^{-1}$$

Generalized least squares can be applied to (2.3.17) Zellner and Theit suggest that σ_{ij} should be estimated from disturbances calculated using (2.3.17) for each equation and then replacing the estimated d's into equation (2.3.15) to yield a calculated vector E_i from which estimates s_{ij} are calculated

The 3SLS estimator d is then

$$\hat{D} = d = \left(\begin{bmatrix} W_1'X & 0 \\ 0 & W_2' \end{bmatrix} \begin{bmatrix} s_{11}(X'X)^{-1} & s_{12}(X'X)^{-1} \\ s_{21}(X'X)^{-1} & s_{22}(X'X)^{-1} \end{bmatrix} \begin{bmatrix} X'W_1 & 0 \\ 0 & X'W_2 \end{bmatrix} \right)^{-1} \left(\begin{bmatrix} W_1'X & 0 \\ 0 & W_2'X \end{bmatrix} \begin{bmatrix} s_{11}(X'X)^{-1} & s_{12}(X'X)^{-1} \\ s_{21}(X'X)^{-1} & s_{22}(X'X)^{-1} \end{bmatrix} \begin{bmatrix} X'Z_1 \\ X'Z_2 \end{bmatrix} \right)$$

$$= \{W'XV^{-1}X'W\}^{-1}X'WV^{-1}X'Z \, , \qquad (2.3.19)$$

with $W'X = \begin{bmatrix} W_1'X & 0 \\ 0 & W_2'X \end{bmatrix}$

The variance-covariance for the estimates is then:

$$\{W'XV^{-1}X'W\}^{-1} \qquad (2.3.20)$$

In comparing the two methods, there is only a gain in asymptotic efficiency over the 2SLS method if the matrix is not diagonal, that is if the error terms of both equations are not independent.

4. MODEL DIAGNOSTIC

Following the parameter estimation, the adequacy of the models should be examined. The verification can be done by using one or several goodness of fit tests. In the case of the system under study, we focus on the residuals obtained from the ARMA model for the univariate series and from the transfer function model for the input-output process.

The tests for the adequacy of models of the type

$$\hat{\phi}(B)\ y_t = \hat{\theta}(B)\ E_t$$

that has been fit according to the procedures already mentioned, will consist basically in testing for white noise of the estimated residuals

$$\hat{E}_t = e_t = \hat{\theta}(B)^{-1}\ \hat{\phi}(B)\ y_t$$

In this respect, the sample autocorrelations of the residuals, $r_k(e)$, k=1, 2, ..., K, can provide useful information concerning the lack of fit.

Box and Jenkins (1970), showed that

$$Q = N \sum_{k=1}^{K} r_k^2(e_t) \sim \chi^2_{(k-p-q)} \qquad (2.4.1)$$

where p and q are the orders of the autoregressive and moving average parts of an ARMA model respectively. This test allows us to consider a number K of autocorrelations all together rather than one by one in testing for the adequacy of the model. A modification of this test , proposed by Ljung and Box (1978) was used for testing for goodness of fit. The modified expression is

$$Q = N(N+2) \sum_{k=1}^{K} r_k^2/(N-k) \sim \chi^2_{(k-p-q)} \qquad (2.4.2)$$

$$r_k = \sum_{k=1}^{n-k} E_t\ E_{t-k} \Big/ \sum_{k=1}^{N} E_t^2$$

This formula has the advantage of yielding a better approximation of the asymptotic Chi-square distribution.

The goodness of fit test for the transfer function model, is based on similar test statistics but now incorporates cross-correlations involving model residuals. To clarify this point, suppose that a transfer function model has been fitted so that it can be written as

$$y_t = \delta^{-1}(B)\ W(B)X_t + \phi^{-1}(B)\ \theta(B)\ E_t$$

$$y_t = \upsilon(B)\ X_t + \Psi(B)\ E_t$$

At this point, there are two possibilities of incorrect fitting that need to be revised: One is the transfer function itself, represented here by the impulse response function $\upsilon(B)$, and the other is the noise model $\Psi(B)$. The goodness of fit test for the noise model will involve then the autocorrelation function of the residuals e_t. The crosscorrelation function between the residuals and the input series will be used in testing the impulse response function.

The test for autocorrelation of the residuals is the same that was described before in equation (2.5.2). The rejection rule will be: If the auto-correlation function of the residuals shows marked correlation patterns, there is evidence that the noise model is inadequate. A similar test, but based on the cross-correlation function could be obtained to test for the adequacy of the impulse response function. If the transfer function is incorrect, and whether or not the noise model was correct, a crosscorrelation analysis could indicate the type of modifications needed in the transfer function. The test statistic to be used in this case is

$$S = m \sum_{k=0}^{K} r_{ee}^2(k) \sim \chi^2_{(K+1 - (r+s+1))} \qquad (2.4.3)$$

with $r+s+1$ being the number of parameters fitted in the transfer function model and $m = n - u - p$, with $u = \max\{r,s\}$, p, r, s defined as in (2.1.16).

In general, the criteria to apply will be that, if the auto-correlation of the residuals exhibits structure, and the cross-correlation does not, then the noise model is incorrect and if both exhibit structure, the the transfer function model is incorrect. The type of structure in the cross and autocorrelation will indicate the kind of modifications needed in one or both.

III. MODELING OF THE ENSO PHENOMENON.

1. DESCRIPTION OF THE PROBLEM.

The relationships between the atmospheric pressure over the South Pacific and the climatic changes in certain parts of the globe have been the subject of numerous studies, especially during recent years (see Wyrtki, 1975, Chen, 1982, Rasmusson and Carpenter, 1982, Quinn, 1984, Esbensen, 1984). The focus of most of these investigations is the Southern Oscillation (SO) and its associated short term climatic variations.

The SO is an irregular interannual and global-scale atmospheric fluctuation reflecting a shift of atmospheric mass between the Indonesian equatorial low pressure region and the South Pacific subtropical anticyclone. The most interesting consequence of the SO is the El Niño phenomenon, an occasional and rapid warming of the sea-surface temperature (SST) off the west coast of South America accompanied by alterations of several other atmospheric and oceanic variables such as rainfall, air temperature, sea level and wind along the tropical Pacific belt.

The El Niño phenomenon has its earliest manifestations along the west coast of South America, producing catastrophic effects on the local fisheries and climate with great damage to the economies of the countries involved, and although it originates at a distance of several thousand kilometers to the west, the physical connections during ENSO seem to be fairly clear.

Changes in the SO are associated with a relaxation of the easterly trade winds in the central and western equatorial Pacific. This breaks the balance of forces between westward wind stress and the eastward pressure gradient associated with the surface topography of the equatorial ocean, which slopes downward from west to east. This imbalance produces an oceanic internal wave that travels eastward along the equator to split into two parts upon arrival at the American west coast. One part travels northward along the west coast of North America and the other travels southward along the coast of Ecuador, Peru and northern Chile (Enfield 1980, 1981a). Along the west coast of South America, the depression of the isotherms associated with the Kelvin wave implies that unusually warm water is upwelled into the surface layers nearshore. The warmer water is generally impoverished in the nutrients needed for biological activity, so that the local fish stocks are forced to migrate in some cases and suffer high mortality rates in others. Countries whose economies depend to a great extent on these resources will suffer considerable hardship. Almost as great as the fishery impact are the setbacks imposed on agricultural activity and the interruption of vital services, highways, etc. that accompany the abnormal rainfall patterns resulting from El Niño. The ability to predict the occurrence of ENSO events could help the South American governments to implement policies that lessen the impacts of such catastrophes.

Although the El Niño phenomenon is typically described mainly in terms of an anomalous rise in ocean surface temperature, it is in fact rather complex and has a strong effect on rainfall, sea level, surface atmospheric pressure and wave activity among others.

To obtain accurate information about the fluctuations of the SO, it is necessary to consider both the wide area that it affects and the numerous variables involved in the process. In recent years, many indices have been obtained from different meteorological variables from the tropical and subtropical Pacific. Sea surface temperature, precipitation, sea level atmospheric pressure and atmospheric thickness among others, have been identified as useful in representing the SO (Chen, 1982 and Quinn, 1984). Among these, the most frequently used is the sea level atmospheric pressure (SLP).

Chen (1982) examined the ability of various SLP indices to adequately represent the SO by analyzing the series obtained from stations located at Easter Island (27^{o}S, 109^{o}W), Rapa (28^{o}, 144^{o}W), Tahiti (17.5^{o}S, 150^{o}W) and Darwin (12^{o}S, 131^{o}E). These stations are strategically located close to the centers of Indonesian equatorial low pressure region (Darwin) and to the South Pacific anticyclone (all others). The indices used are calculated, in most of the cases, as the difference in atmospheric pressure between stations located at each of these two geographic extremes of the tropical Pacific (Rasmusson and Carpenter, 1982). Quinn, (1979), provides an interesting comparison of several indices obtained from some of the stations already mentioned. Trenberth, (1984) discuss the utility of using a simple index based on the combination of sea level pressure data from two stations (Tahiti and Darwin)

An analysis shows that when the value of the SO index is high, we can expect the equatorial low to be deep and the subtropical high to be strong. This condition is associated with strong southeast trades and equatorial easterlies, which enhances coastal and equatorial upwelling, causing isotherms to shoal and lowering the sea surface temperature (SST). Along with this there is a characteristic rain pattern for the region. Altogether, these characterize the so-called anti-El Niño condition.

When the index falls from the anti-El Niño peaks toward the ENSO low values, the equatorial low pressure region fills and migrates eastward and the subtropical high weakens, the winds relax and an El Niño type of situation sets in, the equatorial upwelling decreases and the SST's rise, rainfall increases in the central equatorial Pacific and northern South American coast, while drought conditions set in along the central Andean highlands and northeastern Brasil, and large anomalous rises occur in the coastal ocean temperatures from ecuador to northern Chile.

Rasmusson and Carpenter (1982), Quinn (1979) and Chen (1982) have shown that changes in the phase of the Southern Oscillation Index (SOI) statistically lead changes in SST along the Peru coast by several months. This added to the physical connection already described suggests that the SOI may be very useful in predicting the variation of the SST if an adequate statistical model is found. This is consistent with often-cited arguments that the atmosphere usually leads (forces) the ocean (e.g., Davis 1976 and 1977).

The selection of the most adequate SOI to be used in the context of this analysis was made based on Rasmusson and Carpenter (1982) . They found that changes of the atmospheric pressure at Tahiti lead changes at Darwin by about one month. The lag between Rapa and Easter Island was also approximately one month. Both Rapa and Easter were found to lead Tahiti by several months. In general this means that changes in the surface atmospheric pressure near the center of the South Pacific high (Rapa and Easter Island) lead those at lower latitudes in the central Pacific (Tahiti) and also lead changes of the opposite sign in the vicinity of the Australian-Indonesian low (Darwin).

Since Tahiti leads Darwin only by one month, it seems to be more convenient to use SOI based on these two series to reduce the time lag between the two regions. The index to be used then is Tahiti-Darwin, which is also the index proposed by Trenberth (1984) as the one that best represents the characteristics of the southern oscilation. It must be understood that even when other similar indices such as Rapa-Darwin may explain certain portions of the variability somewhat more effectively, the proposed Tahiti-Darwin is the one that best serves the purpose of this study.

2.DESCRIPTION OF THE DATA.

The following time series were made available to us by Drs. David Enfield and William Quinn of the College of Oceanography at Oregon State University. They are based on monthly mean observations and, except for those of sea level pressure (SLP), they all correspond to meteorological stations located along the west coast of South America , mainly the coast of Peru and Ecuador (Figure 2).

Sea Surface Temperature (SST):	Puerto Chicama	(1925-1983)
	Talara	(1956-1983)
	San Juan	(1958-1983)
	Chimbote	(1956-1983)
	Don Martin	(1952-1983)
	La Punta	(1950-1983)
Sea Level Pressure (SLP):	Tahiti	(1935-1983)
	Rapa	(1951-1983)
	Easter	(1935-1983)
	Darwin	(1882-1983)

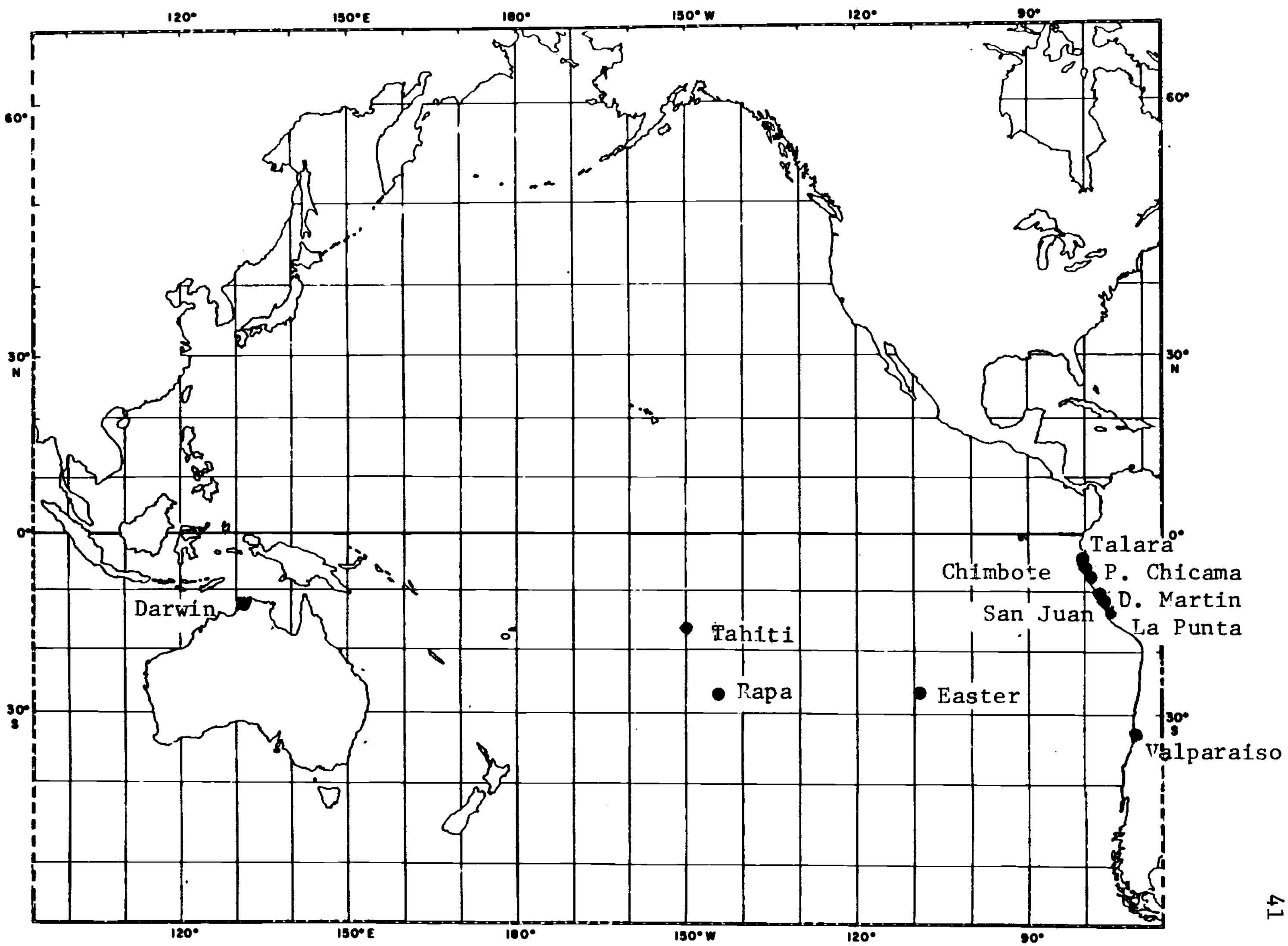


Figure 2. Location of Atmospheric Stations.

3. EXPLORATORY ANALYSIS.

The first stage of the analysis consists of a graphic study of the time series available as an attempt to determine patterns that can be used to characterize the occurrence of the ENSO events in each of the series.

Since the SST series are of different lengths, and with the purpose of using the maximum information available, the least squares method was used to extend the series to the same length starting from 1951. The longer series were used as predictors to estimate the values of the shorter ones, e.g.: Puerto Chicama and La Punta were used to estimate the year 1951 for the Don Martin series.

Another complication that needs to be solved is the fact that this type of data present in general a strong seasonality which can significantly affect further analysis. To avoid any problem, the seasonal factor was removed from all series by substracting the corresponding climatic mean for each month (i.e., each January minus the 24-year climatic mean for January, etc.). For the rest of the study, all series used will be deseasoned series.

The next step is to identify a single series that can represent the characteristics of the ENSO events. The idea is that if we use the SOI as the leading factor, it is necessary to have a response variable that can, in some way, summarize what is most characteristic of the ENSO event in the ocean. A natural choice for this so called response variable is the SST since it is the oceanic variable most clearly related to El Niño episodes and the one that we have more information about. Since 6 records of SST were made available to us, and because their correlations are high, a principal components analysis was used to summarize the information.

The first principal component explaines 88.92% of the variability. With loadings approximately equal for all stations, the first principal component represents the overall average SST of the region (Table 3.1).

Table 3.1. Loadings for the First Principal component of the SST series.

variable	station	loading
SST1	Puerto Chicama	.952
SST2	Talara	.928
SST3	San Juan	.904
SST4	Chimbote	.941
SST5	Don Martin	.975
SST6	La Punta	.956

This allows us to define the El Niño series (EN) as follows:

$$EN = .952SST1 + .928SST2 + .904SST3 + .941SST4 + .975SST5 + .956SST6$$

Note that since the loadings for the first principal component are proximately the same, the mean value of the SST surface temperature over the six stations should also yield an adequate representation for the El Niño series.

The graphical representation of the series was done by using a modified version of the box-and-whiskers plots (Tukey, 1977) which are frequently used to determine the presence of outliers. The modification of the standard box-and-whiskers plots is introduced here as a method for better visualization of monthly data by using a clock-like disposition of the months, with the month of January on top of the array and continuing clockwise with February, March, etc. Because of this disposition, the plot is will be called "clox-plot".

The box-and-whiskers plots display several values that have particular importance in detecting "far out" values (Figure 3).

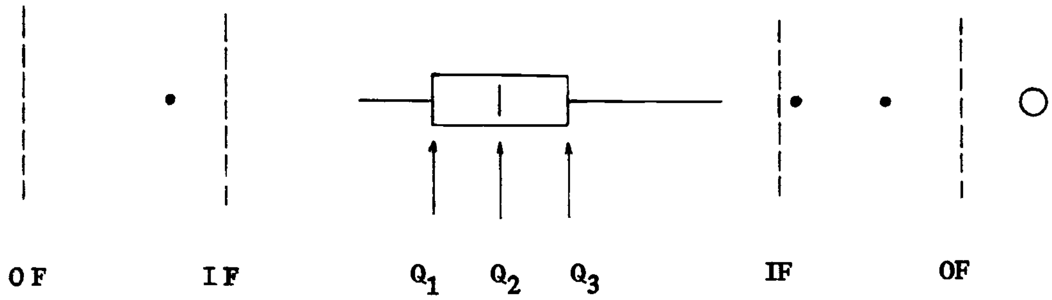


Figure 3. Box-and-whiskers plot.
OF = outer fence, IF = inner fence, Q_1 = first quartile, Q_2 = median, Q_3 = third quartile.

A useful algorithm for constructing the plots is:

- H = Difference between third and first quartile.

- Step = 1.5 times H.

- Inner fences = One step outside the quartiles.
- Outer fences = Two steps outside the quartiles (and thus one step outside inner fences)
- The whiskers extend up to the last point within the inner fences.
- The values between the inner fence and neighboring outer fence are said to be outside and are usually represented by a dot.
- The values beyond the outer fences are said to be far out and are represented by a circle.

As mentioned before, these plots are particularly useful in detecting outliers and can be used in the same manner to identify individual ENSO events (see Figures 4 and 5). For the purpose of this study we will replace the usual dots and circles by symbols that allow the identification of the outliers as corresponding to certain years (which coincide with the ENSO years). These are:

o for 1982-83

• for 1971-72

△ for other events

In Figures 4 and 5, the inner circle represents the minimum value of the series and the dashed circles represent the series means, and the special symbols correspond to the points laying beyond the inner or outer fences respectively. As expected, all symbols correspond to some of the well known ENSO events.

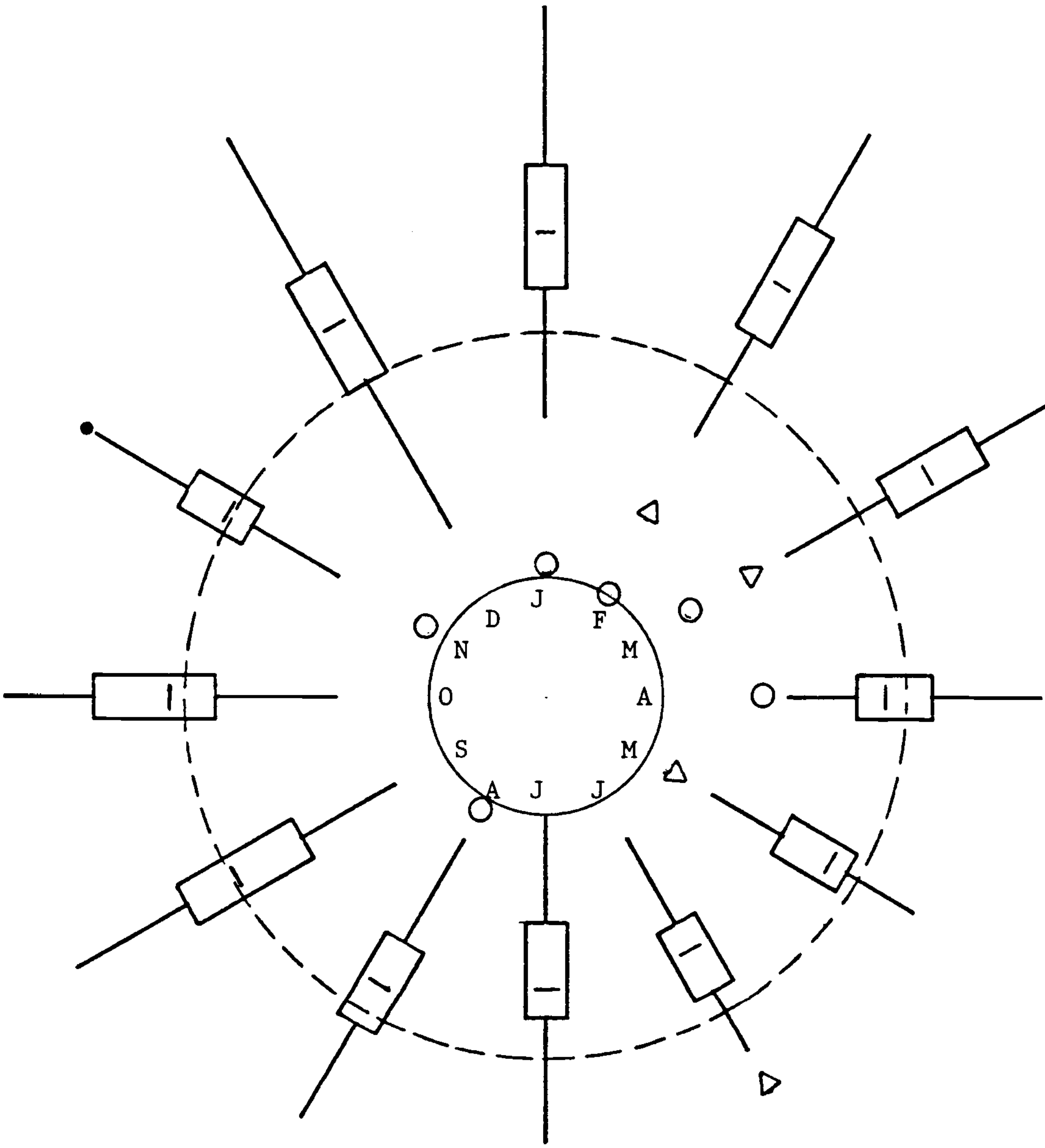


Figure 4. Clox-Plot of the TD series.
Distances from the inner circle (minimum value of the series) are proportional to the mean corrected values of the Tahiti-Darwin SOI.

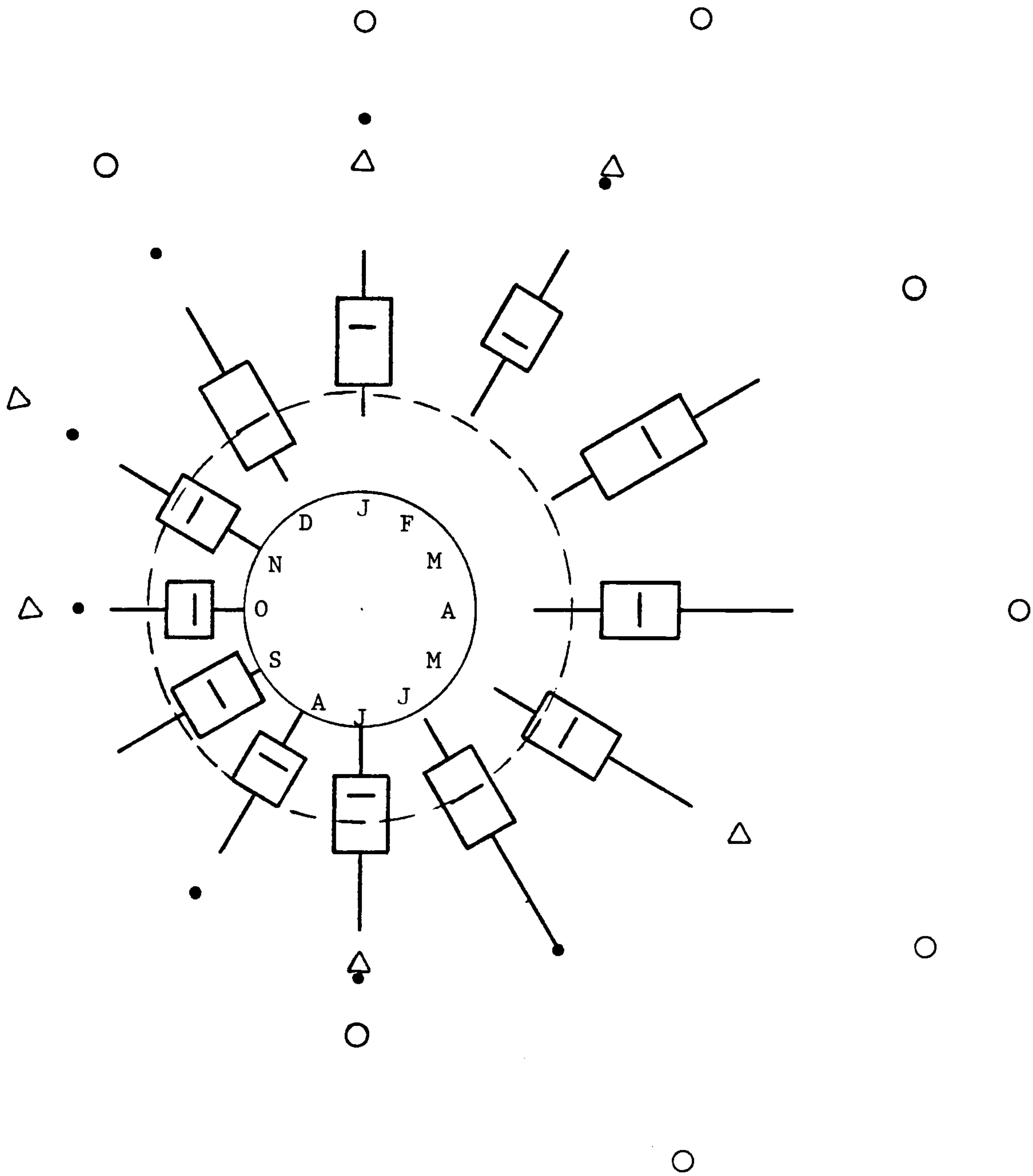


Figure 5. Clox-plot of the EN series.
Distances from the inner circle (minimum value of the series) are proportional to the mean corrected values of the first principal component of the six SST series.

Using such plots we expect to obtain a preliminary estimation of the magnitude of ENSO episodes as well as an approximate date of onset and termination. By comparing series of distinctive nature (e.g., SST vs. SOI) we also intend to determine approximate lags for the onset phases that can be used later as a basis for a predictive model (Figures 6 and 7).

The original data corresponding to the ENSO events was compared with the newly defined EN series (based on the first principal component) using the clox-plot. The same basic pattern was found for all cases studied. This supports the representativeness of the first principal component as the El Niño series.

Note, from Figures 4 and 5, that most of the "far out" points correspond to the 1982-83 ENSO, which was an event of unusual intensity (Quinn, 1984). Both the EN and SOI values for this event have been plotted in Figure 6. Most of the other events show a similar pattern (e.g. Figure 7 which displays the EN and SOI values for the ENSO of 1971 - 1972).

The fact that the original series exhibit a strong seasonality implies autocorrelation at non-zero lags with repeating maxima at the annual harmonics (12, 24, ...). After for both the EN and SOI series (Figure 8 and 9). Although after removing the corresponding long term mean for each calendar month, the strong seasonal pattern shown in Figure 8 and 9 practically disappeared (Figures 10 and 11), But since the SO is considered an inter-annual fluctuation (Esbensen, 1984 and Quinn

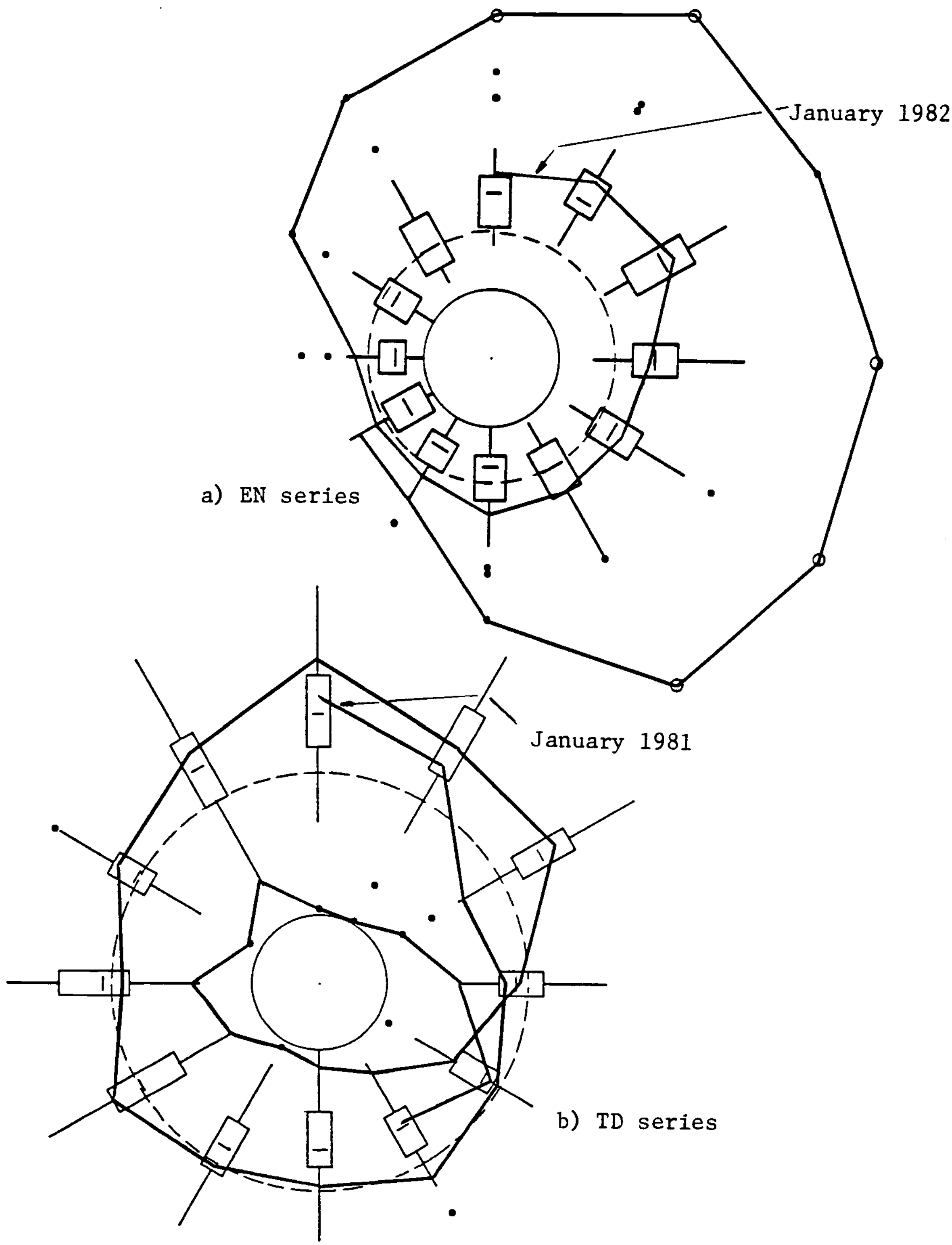


Figure 6. The 1982-1983 ENSO event.
a) Clox-plot displaying SST anomalies represented by the EN series, starting January 1982.
b) Clox-plot displaying the SOI anomalies, represented by the TD series, starting January 1981.

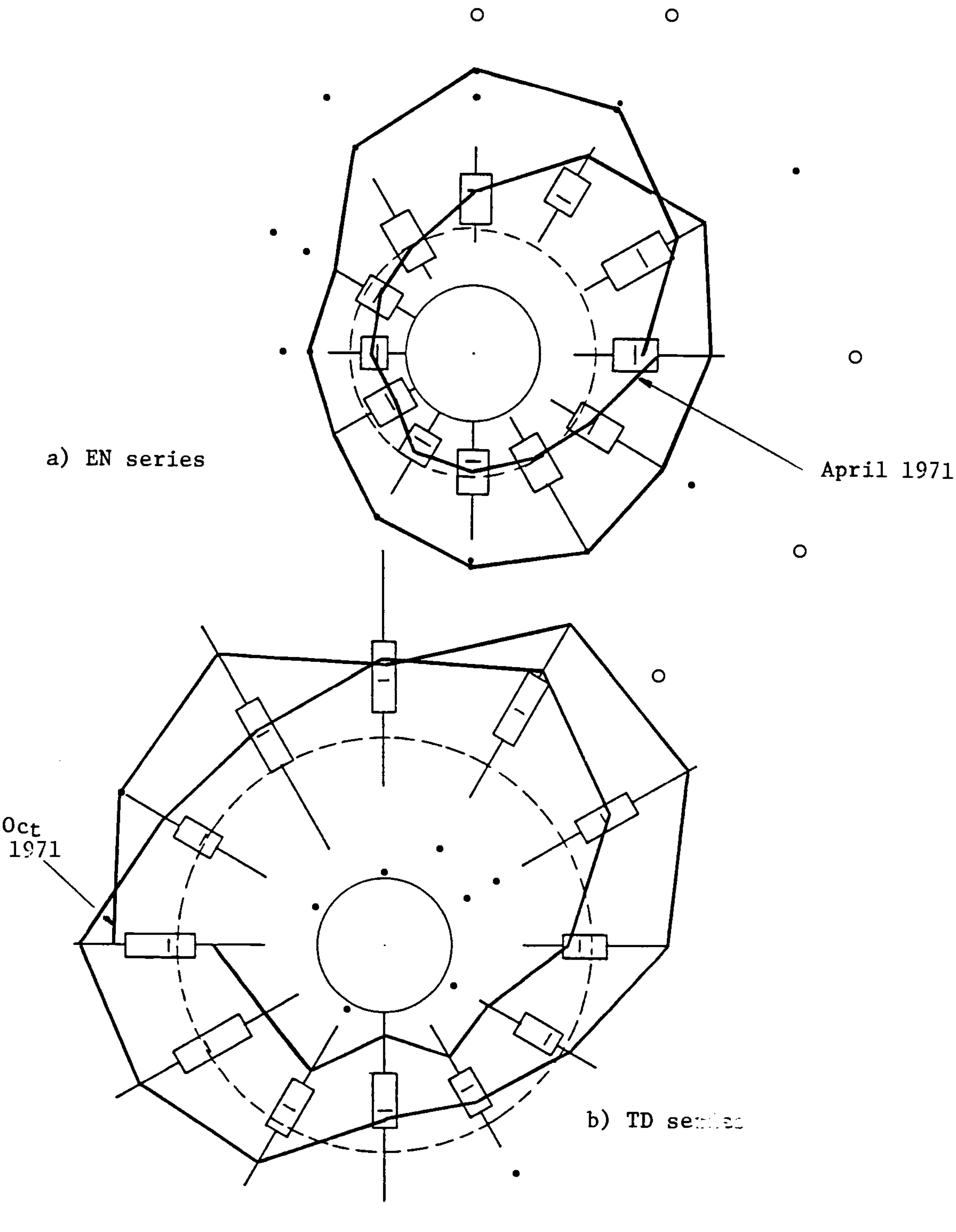


Figure 7. The 1971-1972 ENSO event.
a) Clox-plot displaying the EN series starting April 1971
b) Clox-plot displaying the TD series starting October 1970

and Neal, 1983), we expect to observe some traces of this non-annual peridocities in studying the by-month interannual auto-correlations (i.e., deseasoned). None of them however, appeared to be significant (Figure 12). In fact the autocorrelations for lags of more that one year were also revised but none of them was significant. A plausible explanation is that the occurrences of the El Niño events affect the patterns of the fluctuation so strongly that the evidence of their presence can not be assessed.

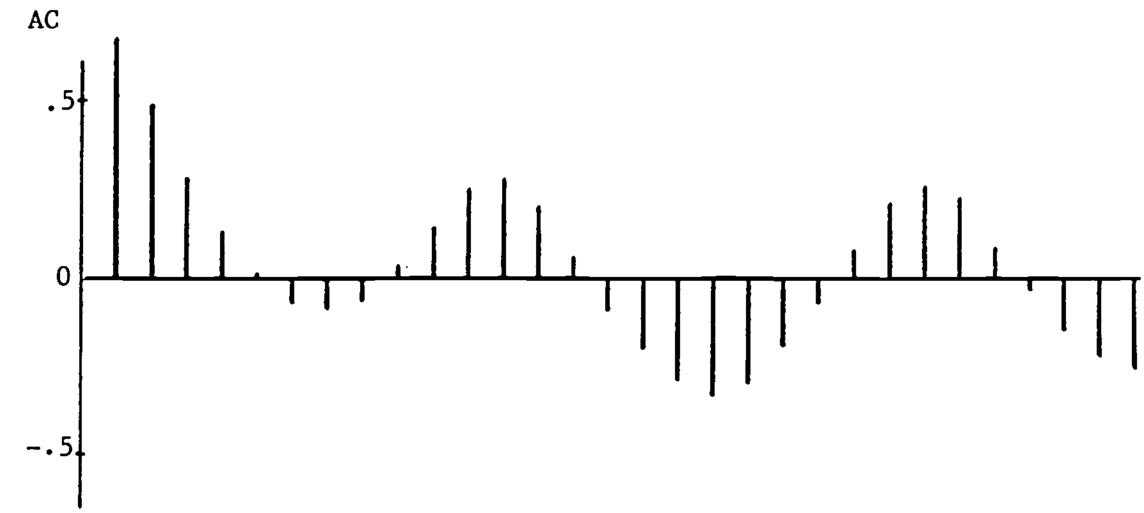


Figure 8a. Autocorrelation function for the TD series, showing a strong recurrence of positive correlation at lag intervals of 12 months

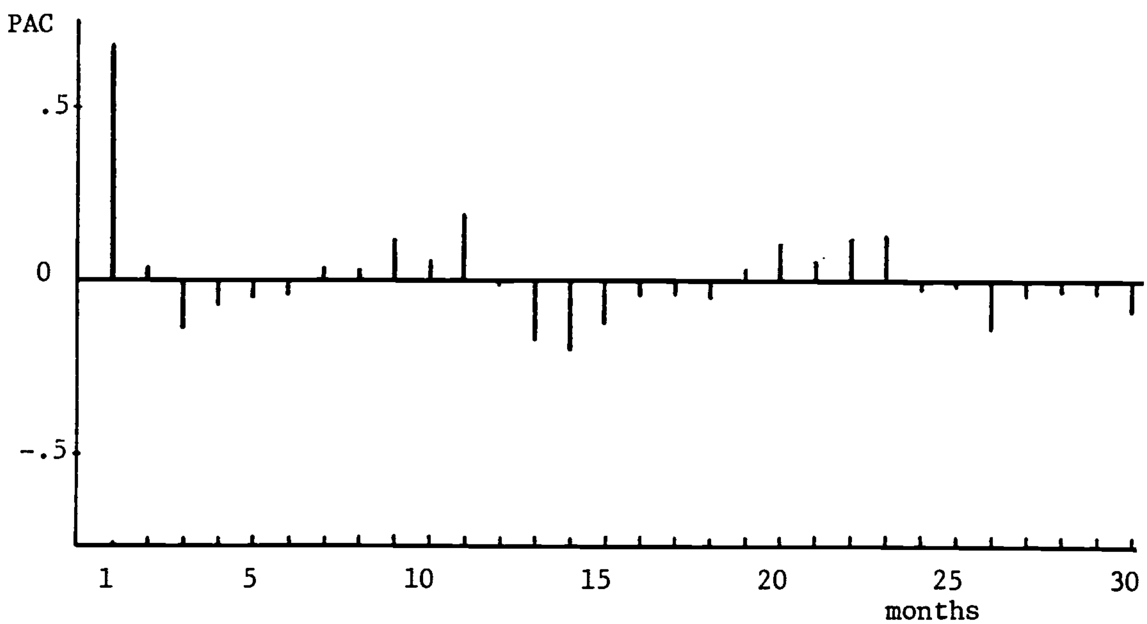


Figure 8b. Partial autocorrelation function for TD series showing some recurrence of positive correlation at intervals of 12 months.

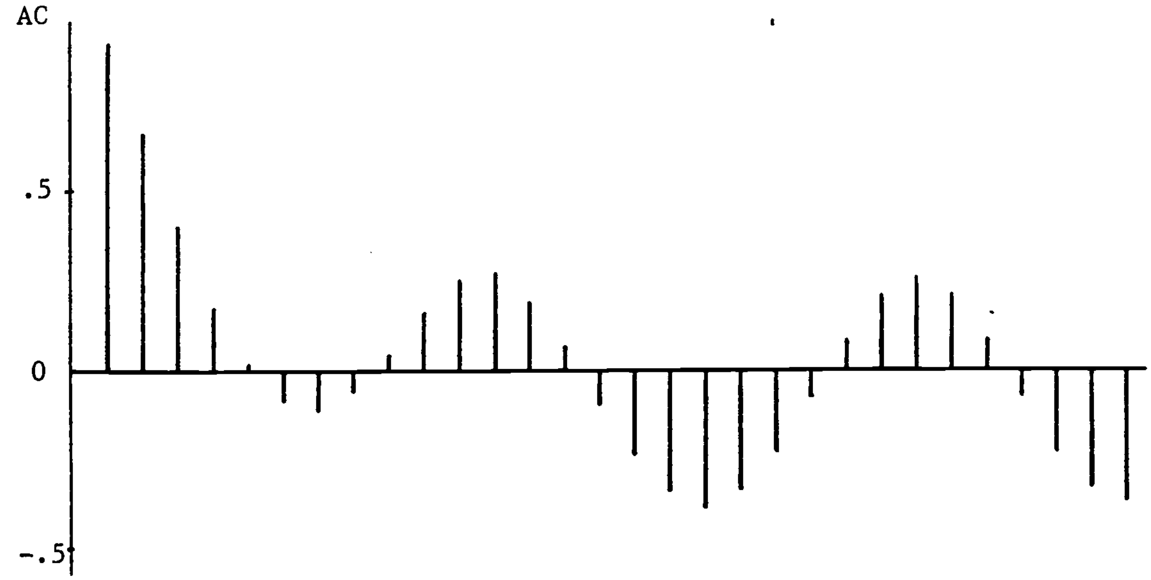


Figure 9a. Autocorrelation function for the EN series, showing a strong recurrence of positive correlation at lag inteervals of 12 months.

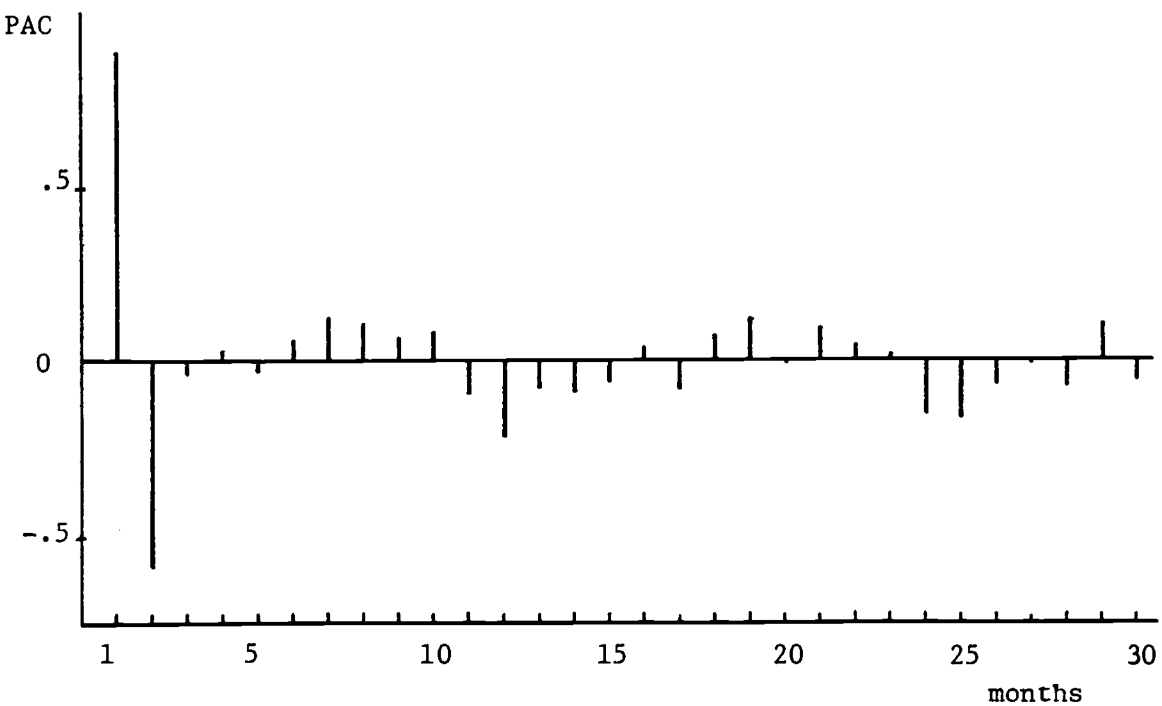


Figure 9b. Partial autocorrelation function for the EN showing series, showing some recurrence of positive correlation at intervals of 12 months.

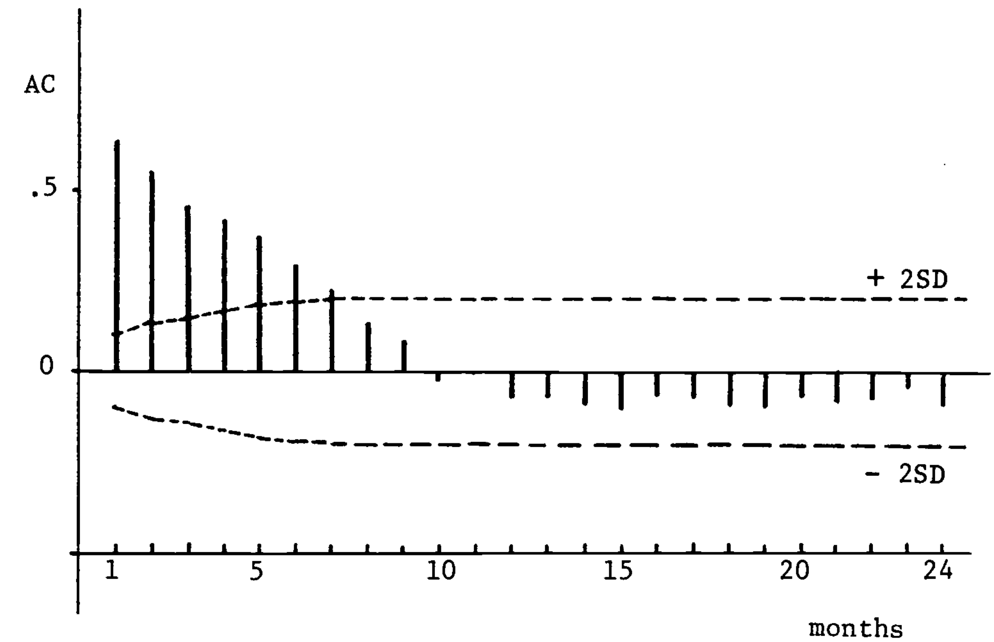


Figure 10a. Autocorrelation function for the TD series, after removing the monthly means, showing significant positive values only up to lag k = 7.

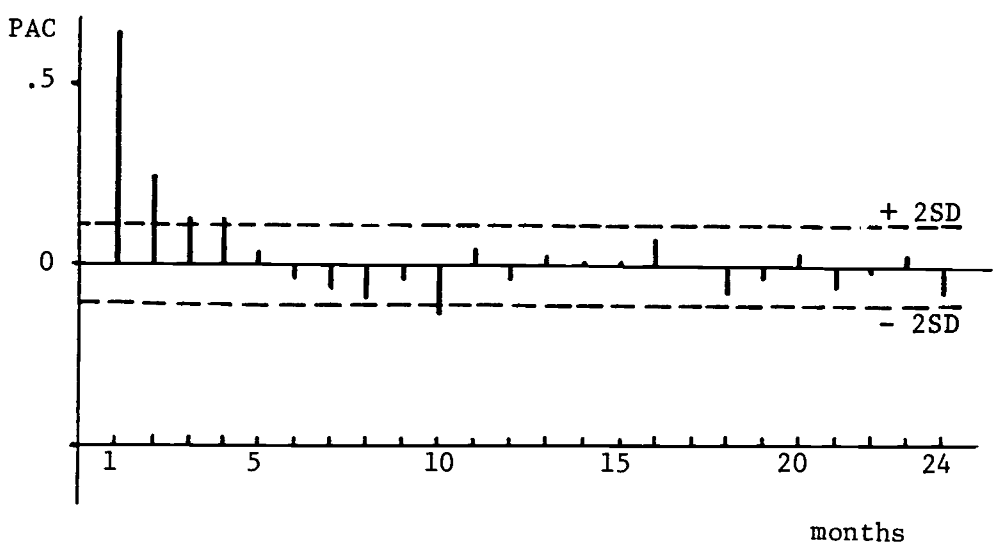


Figure 10b. Partial autocorrelation function for the TD after removing the monthly means, showing a strong indication of a possible AR(2) model for the series.

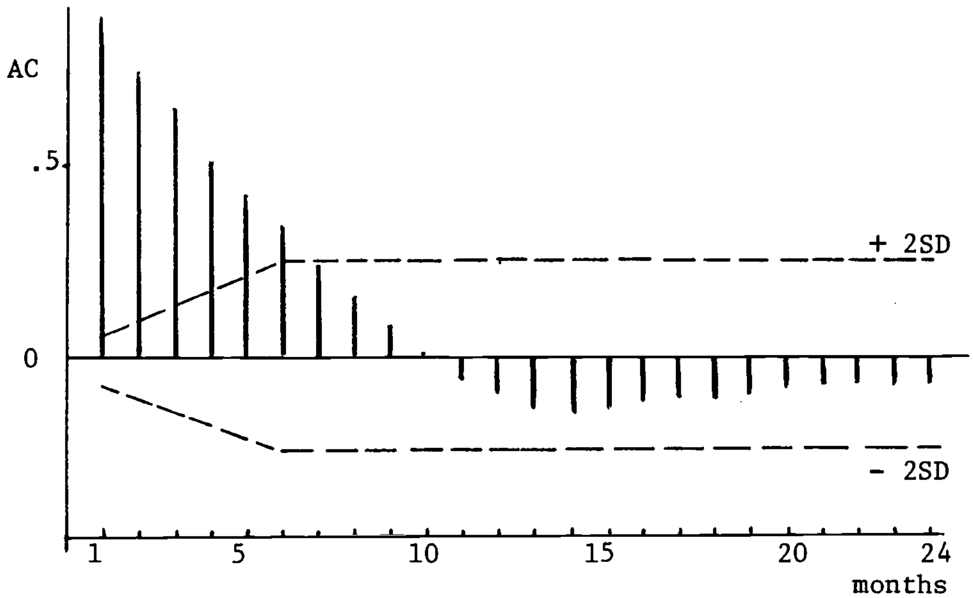


Figure 11a. Autocorrelation function for the EN series, after removing the monthly means, showing significantpositive values only up to lag k=6.

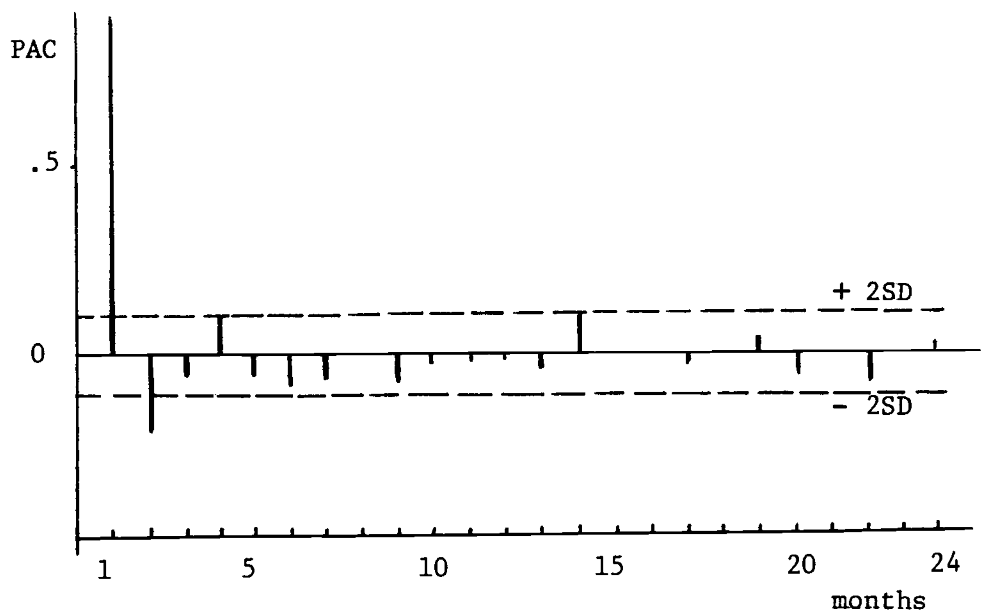


Figure 11b. Partial autocorrelation function for the EN after removing the monthly means showing series.

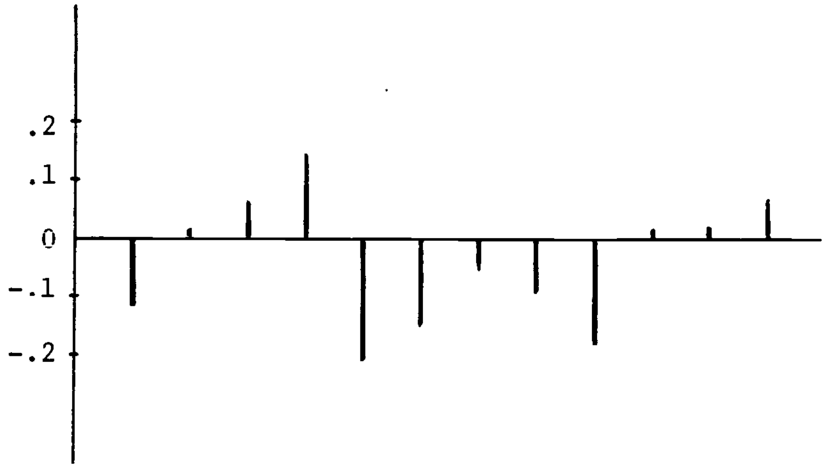


Figure 12a. Autocorrelations at one-year lag for the TD series, by month of the year, with the climatic annual cicle removed

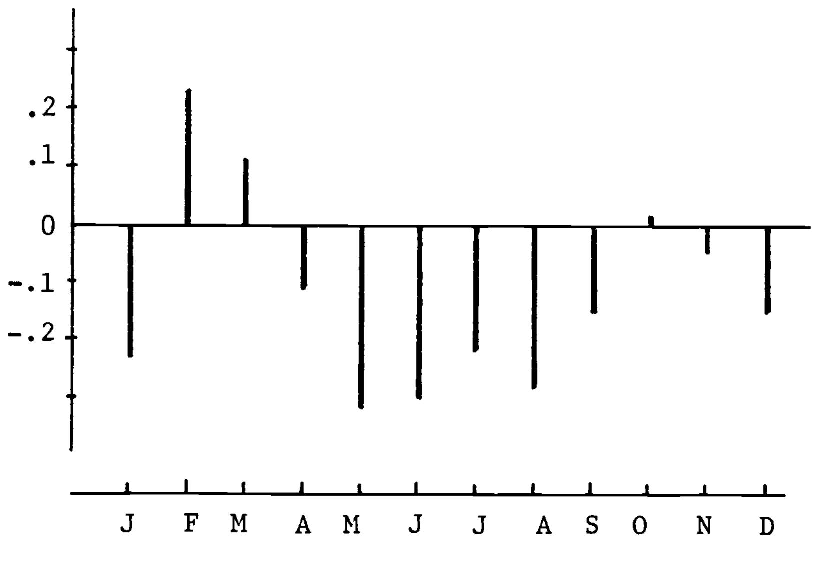


Figure 12b. Autocorrelations at one-year lag for the EN series, by month of the year, with the climatic annual cicle removed

4. INPUT MODELING

The objective is now to find an ARMA model to adequately fit the input series. Chu and Katz (1984), working with normalized monthly records of sea level pressure from 1935 until 1983, proposed an ARMA(1,1) model and fitted the monthly SOI based on Tahiti-Darwin. Using Hannan and Quinn's method to determine the order of an autoregression, we fit an autoregressive model of order 2 (AR(2))to the data. This model was compared with the corresponding ARMA(1,1) proposed by Chu and Katz using the same data. The first 25 lags of the residual autocorrelations were used in both cases to test for the goodness of fit. The test used here was the Ljung and Box statistics defined in Section 4 of Chapter II, where

For AR(2): Q = 27.86 (21 df), p-value = .144, and for

ARMA(1,1): Q = 26.66 (21 df), p-value = .182

Both p-values are high enough to conclude that the residuals from both models are only white noise which indicates an adequate fitting. The decision to choose one of them was based mainly on the simplicity of the parameter estimation and interpretation. We decided to use the AR(2) model to describe the input series. The estimated model is:

$$X_t = .07464 + .48464X_{t-1} + .24486X_{t-2} + E_t \qquad (3.4.1)$$

where E_t is white noise.

5. IDENTIFICATION AND ESTIMATION OF A TRANSFER FUNCTION MODEL.

After the model for the input series (TD) had been conveniently identified and its parameters estimated, the series was prewhitened using model (3.4.1). The output series (EN) was transformed in the same manner. This allowed us to obtain an estimate of the crosscorrelation function (Figure 13). The Ljung and Box test for white noise applied to the crosscorrelation of the transformed series was highly significant, with a p-value of less than .0001 for 24 degrees of freedom. It should be noted that the values at negative lags are also significant. This fact, as mentioned by Chatfield (1975) could be a clear indication of the presence of feedback.

Since the presence of feedback is suspected, the standard techniques of identification of the transfer function model may not be adequately used for use here because they may yield unrealistic results (Box and MacGregor, 1976). The first approach in estimating these parameters will be the use of the ordinary transfer function model estimation as defined before in equations (2.3.4) and (2.3.5).

Consider the initial estimates of the impulse response function obtained from:

$$V_k = r_{\alpha\beta}(k)\ S_\beta / S\alpha \ ; \qquad k = \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$$

Note that we have included negative values of k. We expect that

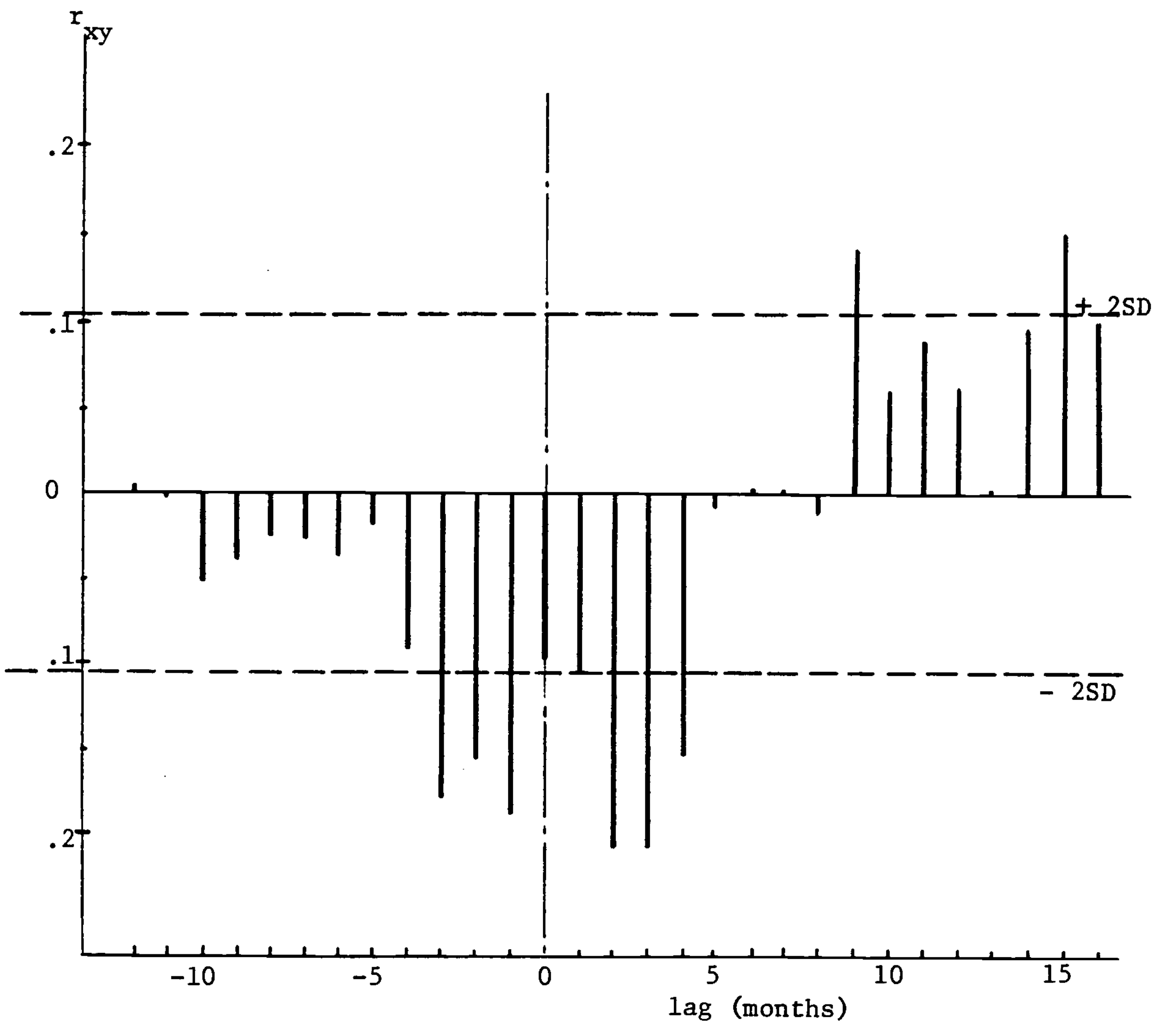


Figure 13. Crosscorrelation function for TD - EN, showing significative values of the crosscorrelation for negative lags and the recurrence of positive correlation at lag 12 months.

this procedure will give us a rough approximation of the order of the model. The estimates obtained are shown in Table 3.2.

Table 3.2 Estimates of the Impulse Response Function.

k	v_k
-4	-.1067
-3	-.2237
-2	-.1941
-1	-.2321
0	-.1218
1	-.1390
2	-.2871
3	-.2776
4	-.1907
5	-.0462
6	.0402
7	.0164
8	-.0526
9	.1797

This suggests a transfer function of the form (See Box and Jenkins, 1970)

$$Y_t = d^{-1}(B)\ w(B)X_t + N_t$$

with $d(B) = 1 - dB^{12}$

and $$w(B) = \sum_{j=-4}^{4} w_j\ B^{j+12}$$

Preliminary estimates of the parameters for this model were obtained by minimizing expression (2.3.5) using procedures from the Statistical Analysis System (SAS) computer package.

After this preliminary phase of the identification was completed, equation (2.3.4) was used to obtain an expression for the noise model

$$N_t = \hat{Y}_t - Y$$

with $\hat{Y}_t$ being determined by the expression

$$\hat{Y}_t = Y_t - \sum_{j=-3}^{4} w_j B^j / d_{12} B^{12} X_t$$

when d_j and w_{12} are the preliminary estimates of the parameters of the transfer function model already defined.

The study of the autocorrelation of the estimated residuals suggested once again the use of an AR(2) model for the noise series. The corresponding transfer function model is

$$Y_t = \frac{\sum_{k=-3}^{4} w_k B^k}{1 - d_{12} B^{12}} X_t + N_t$$

with $N_t = \alpha_1 N_{t-1} + \alpha_2 N_{t-2} + a_t$, where a_t is white noise.
Once again using SAS procedures, the following final estimates of these parameters were obtained (Table 3.3)

Table 3.3 Estimated Parameters for the Transfer Function Model.

Parameter	Estimate	Std Error	Estim/S.E.
α_1	.99538	.05226	19.05
α_2	-.20828	.05260	-3.96
w_{-3}	.04825	.01260	3.83
w_{-2}	.03954	.01408	2.81
w_{-1}	.05994	.01431	4.19
w_o	-.04122	.01374	-3.00
w_1	.03223	.01376	2.34
w_2	.07063	.01386	5.10
w_3	.06998	.01380	5.07
w_4	.04452	.01222	3.64
d_{12}	-.04116	.09348	-4.40

As indicated in Section 4 of Chapter II, after the estimates of the parameters have been obtained, we have to test for the adequacy of the model.

The goodness of fit test used here was the Ljung and Box white noise test for the autocorrelations of the residuals. The test showed that the residual series present no evidence of autocorrelation. The test for significance of the crosscorrelation between the prewhitened input and the transformed residuals gave a value of Q = 19.04 which compared with the corresponding χ^2 value with 15 degrees of freedom from the χ^2 table, yields a p-value $>$ .212. Hence, we conclude that both the transfer function and the noise model were adequate.

It must be noted that the negative lag parameters included in the model described in Table 3.3, although significant, do not configure the type transfer function model defined by Box and

Jenkins (1970). The fact that future values of the input function are needed to describe the present output constitutes an unrealistic aspect of the model and implies that the model cannot not be used for prediction purposes. Hence it is necessary to look for a model that accounts for the significance of the negative lag crosscorrelations and constitutes a valid representation.

6. ESTIMATION OF THE PARAMETERS FOR THE CLOSED-LOOP SYSTEM.

For practical purposes, the transfer function model whose parameters were estimated in the previous section, although they appearing to be technically correct, cannot be used either for description or for predictive purposes. The fact that it includes significant parameters for negative lags, implies that for representing the present or making any kind of prediction we must know in advance at least three future values of the input series. These facts lead us to look for a representation based only on past values of the series. The equations that accomplish this basic requirement are (2.1.20) and (2.1.21), i.e.

$$Y_t = \sum_{i=1}^{k} \alpha_i y_{t-i} + \sum_{i=1}^{p} \gamma_i x_{t-i} + E_t \qquad (3.6.1)$$

$$X_t = \sum_{i=1}^{q} \beta_i x_{t-i} + \sum_{i=1}^{r} \theta_i y_{t-i} + M_t \qquad (3.6.2)$$

As it will be shown later, the identification of the model to be fitted cannot be based on the structure of the cross-correlation function (CCF) between the prewhitened input and the transformed output. Doing so could lead us to select the wrong model to be fitted. Instead, we will use as a basís the model already fitted, using the usual techniques for estimating the parameters of transfer function models. The rationale is that if the parameters corresponding to negative lags are found to be significant for that model, we can find significant feedback parameters at least up to lag 1, with possibilities of being of even higher order. The rest of the parameters of the feedback equation are presumably of order 2 since the input series was already shown to be adequately modeled as an autoregression of order (AR(2)). The order of the parameters for equation (3.6.1) will be assumed to be of similar order. The general idea is to initially over-fit the model in such a way that we can obtain a vague idea about the order of the rest of the parameters. The CCF was considered in determining the number of the parameters for the first equation. The following first tentative model was fitted

$$y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \gamma_3 x_{t-3} + \gamma_4 x_{t-4} + \gamma_5 x_{t-5} \qquad (3.6.3)$$

$$x_t = \beta_1 x_{t-1} + \beta_2 x_{t-2} + \theta_o y_t + \theta y_{t-1} + \theta y_{t-2} \qquad (3.6.4)$$

Since we assume that in a closed-loop system there is no instantaneous response, we expect the parameter θ_o to be equal to zero.

Next, the two-stage (2SLS) and three-stage least squares (3SLS) methods were used to estimate the parameters of the system of equations. The two and three-stage least squares estimates for the parameters were obtained using SAS procedures (see SAS 1982). The matrix setup used in both 2SLS and 3SLS was as follows

$$Z_t = (Y_t, X_t)$$

$$Y_t^{(1)}(B) = Y_t^{(2)}(B) = (Y_{t-1}, Y_{t-2})$$

$$X_t^{(1)}(B) = (X_{t-3}, X_{t-4}, X_{t-5})$$

$$X_t^{(2)}(B) = (X_{t-1}, X_{t-2})$$

$$A_1 = (\alpha_1, \alpha_2) \ , \quad A_2 = (\beta_1, \beta_2)$$

$$C_1 = (\gamma_3, \gamma_4, \gamma_5) \ , \quad C_2 = (\theta_1, \theta_2)$$

The preliminary results show that testimates of the parameters α_2, α_3, γ_4, θ_2 were not significant, but the analysis of the residuals indicates the necessity of including additional terms in the models. A fourth term was then added, namely X_{t-6}, and found to be significant. The estimation was performed simultaneously by the 2SLS and 3SLS methods, and the results are shown in Table 3.4.

Table 3.4: Estimated Parameters from 2SLS and 3SLS for the Close-Loop system

	2SLS		3SLS	
Param	Estim.	Std. Err.	Estim.	Std. Err.
α_{11}	.928559	.024540	.928124	.024543
γ_{15}	.022990	.013678	.022524	.013692
γ_{16}	.030254	.014214	.030184	.042127
β_{11}	.404812	.049871	.408820	.049419
β_{12}	.175524	.049734	.182156	.049779
θ_{11}	-.481644	.093272	-.47007	.093279

The variance-covariance matrices, correlation and mean square error for the residuals for each of the methods are

2SLS:

$$\begin{array}{cc} & \begin{array}{cc} E_{1t}^{(1)} & E_{1t}^{(2)} \end{array} \\ \begin{array}{c} E_{1t}^{(1)} \\ E_{1t}^{(2)} \end{array} & \begin{bmatrix} 1.47342 & -.019073 \\ -.019073 & .127048 \end{bmatrix} \end{array}$$

Correlation $(E_{1t}^{(1)}, E_{1t}^{(2)}) = -.04448$

MSE for the System .078448.

3SLS:

$$\begin{array}{cc} & \begin{array}{cc} E_{1t}^{(1)} & E_{1t}^{(2)} \end{array} \\ \begin{array}{c} E_{1t}^{(1)} \\ E_{1t}^{(2)} \end{array} & \begin{bmatrix} 1.47360 & -.020327 \\ -.020327 & .12705 \end{bmatrix} \end{array}$$

Correlation $(E_{1t}^{(1)}, E_{1t}^{(2)}) = -.046979$

MSE for the system = .0788365

As can be seen, both methods behave similarly, with the 2SLS producing slightly better estimators, with smaller correlations between the residuals for both equations. Since the 2SLS also involves less calculations, it is advisable to adopt it for use.

To run comparative simulation studies, the feedback term was removed from the second equation and the parameters were estimated again using similar procedures. The results are shown in Table 3.5

Table 3.5: Parameter Estimators using 2SLS and 3SLS for Open-Loop system.

	2SLS		3SLS	
Param.	Estim.	Std. Err.	Estim.	.Std. Err.
α_{21}	.928124	.024542	.927450	.024543
γ_{25}	.022543	.013692	.022540	.013692
γ_{26}	.030184	.014227	.030127	.049297
β_{21}	.481562	.049279	.481201	.049278
β_{22}	.252110	.049298	.251315	.049297

The variance-covariance matrices for the residuals were

2SLS:

$$\begin{array}{cc} & \begin{array}{cc} E_{2t}^{(1)} & E_{2t}^{(2)} \end{array} \\ \begin{array}{c} E_{2t}^{(1)} \\ E_{2t}^{(2)} \end{array} & \begin{bmatrix} 1.57107 & -.003094 \\ -.003094 & .127048 \end{bmatrix} \end{array}$$

3SLS:

$$\begin{array}{cc} & \begin{array}{cc} E_{2}^{(1)} & E_{2t}^{(2)} \end{array} \\ \begin{array}{c} E_{2t}^{(1)} \\ E_{2t}^{(2)} \end{array} & \begin{bmatrix} 1.57108 & -.003361 \\ -.003361 & .127048 \end{bmatrix} \end{array}$$

Since both methods have provided the same values for the estimated parameters, the following system of equations is considered to be the most adequate

$$Y_t = \alpha_1 Y_{t-1} + \gamma_5 X_{t-5} + \gamma_6 X_{t-6} + E_t^{(1)} \qquad (3.6.5)$$

$$X_t = \beta_1 X_{t-1} + \beta_2 X_{t-2} + \theta_1 Y_{t-1} + E_t^{(2)} \qquad (3.6.6)$$

7. PREDICTION

For a transfer function model of the type described by equation (2.1.16) the predicted values are easily calculated by direct use of the difference equation

$$Y_{t+h} = d_r^{-1}(B) w_s X_{t+h} + \phi^{-1}(B)\theta_q E_{t+h}$$

assuming that the input series X_t can be represented by an ARMA model and assuming also that the noise component E_t is statistically independent of the input X_t. Expressions for the variance of the predictions are described in detail in Box and Jenkins (1976). This prediction process is also called "forecast using leading indicators" with the leading factor being the input series, which in our case corresponds to TD.

Since the regular transfer function was shown to be inadequate in present context, the prediction equations are based on the closed-loop equations

$$Y_t = \alpha_1 Y_{t-1} + \gamma_5 X_{t-5} + \gamma_6 X_{t-6} + E_t^{(1)}$$
$$X_t = \beta_1 X_{t-1} + \beta_2 X_{t-2} + \theta_1 Y_{t-1} + E_t^{(2)}$$

As an example let us consider first the predictive equations for h=1

$$Y_{t+1} = \alpha_1 Y_t + \gamma_5 X_{t-4} + \gamma_5 X_{t-5} + E_{t+1}^{(1)}$$
$$X_{t+1} = \beta_1 X_t + \beta_2 X_{t-1} + \theta_1 Y_t + E_{t+1}^{(2)}$$

These predicted values for Y_{t+1} can be used to obtain the predicted value Y_{t+2}. The same procedure can be repeated several times to obtain further predictions.

Recall at this point that one of the main features of the closed-loop systems is that the noise terms are not independent of the input variables (Akaike 1968). This problem constitutes a serious difficulty for the calculation of an estimate of the variance of the prediction. Further analysis of the nature of the dependency needs to be conducted in order to obtain an adequate estimate of the variance that allows us to determine a confidence region for the predicted values.

8. SIMULATION

As an improvement on the Box and Muller method, for generating normally distributed random numbers, random observations were generated from each of the two equations using the random number generator described by Law and Kelton (1982), known as the polar method. The algorithm allows for the generation of a pair of two independent and identically distributed (iid) Normal (0,1) random variables:

Step 1: Generate U_1 and U_2 as iid Uniform (0,1) and define

$$V_1 = 2U_1 - 1 \text{ and } V_2 = 2U_2 - 1 \text{ and let } W = V_1^2 + V_2^2$$

Step 2: If $W > 1$, go back to step 1. Otherwise, let

$$Y = \{(-2\ln(W))/W\}^{1/2},\ X_1 = V_1Y \text{ and } X_2 = V_2Y.$$

Then, X_1 and X_2 are iid N(0,1) random variables.

For each simulation, a set of N pairs of independent normal (0,1) random variables were generated. Each variable must be multiplied by the standard error of the estimates obtained for each of the equations, thus providing the appropriate sequence of random errors to be fed into each equation of the system.

The simulation procedures used the two systems of equations estimated in the previous section. The model to be studied will use the estimates of the parameters obtained from the 2SLS method which, as mentioned before, gives almost the same results as the 3SLS approach, the model for the closed-loop system is

$$Y_t = \alpha_{11}Y_{t-1} + \gamma_{15}X_{t-5} + \gamma_{16}X_{t-6} + E_{1t}^{(1)}$$
$$X_t = \beta_{11}X_{t-1} + \beta_{12}X_{t-2} + \theta_{11}Y_{t-1} + E_{1t}^{(2)}$$

where $E_{1t}^{(i)}$, $(i = 1, 2)$, are two independent white noise series, distributed N(0, 0.127048) and N(0, 1.4736), respectively, which are generated according to the procedures described in previous chapters. With the parameter values given in Table 3.4, for the model for open-loop system we have:

$$Y_t = \alpha_{21}Y_{t-1} + \gamma_{25}X_{t-5} + {}_{26}X_{t-6} + E_{2t}^{(1)}$$
$$X_t = \beta_{21}X_{t-1} + \beta_{22}X_{t-2} + E_{2t}^{(2)}$$

where $E_{2t}^{(i)}$, $i = 1, 2$ are two independent white noise series generated for the closed-loop system. But this time their variances have been set to .12705 and 1.57108, respectively, with parameters given in Table 3.5.

There are two main steps to be considered during the simulation of the processes. One is the analysis of the generated input and output series which should be studied to determine whether or not the model, with only the input of the generated noise series already described plus a few initial values obtained from the first six observations of the original series, is able to reproduce the behavior of the system under study to an acceptable degree. Figure 15 and Figure 16 show a typical result of the simulated series with and without the inclusion of the feedback term into the system respectively. Figure 17 and

Figure 18 show the crosscorrelation functions obtained from the simulated series, with and without the inclusion of the feedback, term respectively.

IV. CONCLUSIONS AND DISCUSSION.

1. TESTING FOR THE PRESENCE OF FEEDBACK

The focus of this study was the use of non-traditional closed-loop system modeling procedures to estimate the relationships between two time series classified as input and output respectively. Although only two approaches were mentioned in the search for the most adequate model, several other methodologies were tried and some of them are discussed now.

When the presence of feedback is suspected, the first step-- prior to the identification of the model-- must be to statistically determine whether or not feedback is present in the system. At this stage several procedures were considered to test for the presence of feedback. Some of them were shown not to be adequate given the nature of the data.

Box and MacGregor (1974) suggest that if there is some doubt as to the the type of data one has, a test for the presence of feedback can be obtained by testing for the presence of a significant crosscorrelation between the prewhitened input and the transformed output at lag $k = 0$. Although this is valid procedure to use (in the sense that if the test is significant we can conclude that we are dealing with a closed-loop system) it is also true that the test is not conclusive in determining that the system is feedback-free.

Chatfield (1975) extends the Box and MacGregor procedure to testing for significant values of crosscorrelation at negative lags. In fact, nearly significant crosscorrelation at lag $k = 0$, accompanied by highly significant values for $k < 0$, will be a strong indication of the presence of feedback in the system.

Considerable attention was given also to Caines and Chan (1975). They analyze the feedback system:

$$y = Kx + Lv$$

$$x = My + Nw$$

were K and M are the feedforward and the feedback transfer functions of the system, and v and w are unobserved noise sources. They use the maximum likelihood method to estimate the parameters of the impulse response function corresponding to K and M.

By expressing the previous model as in equations (2.1.20) and (2.1.21), that is:

$$y_t = K(B)y_t + P(B)x_t + e_{ct}^{(1)} \qquad (4.1.1)$$

$$x_t = Q(B)x_t + R(B)y_t + e_{ct}^{(2)} \qquad (4.1.2)$$

we can define the matrix:

$$N(\phi_c) = \begin{bmatrix} \{e_{ct}^{(1)}\}^2 & e_{ct}^{(1)}e_{ct}^{(2)} \\ e_{ct}^{(1)}e_{ct}^{(2)} & \{e_{ct}^{(2)}\}^2 \end{bmatrix}$$

We can now obtain an expression for the determinant:

$$V(\emptyset_c) = \det\{\xi^N(\emptyset_c)\}$$

$$= \{e_{ct}^{(1)}\}^2\{e_{ct}^{(2)}\}^2 - \{e_{ct}^{(1)}e_{ct}^{(2)}\}^2 \qquad (4.1.3)$$

Following a similar procedure for the open-loop model:

$$y_t = K(B)y_t + P(B)x_t + e_{ot}^{(1)}$$
$$x_t = Q(B)x_t + \qquad e_{ot}^{(2)}$$

we can obtain the following expression for the determinant $V(\emptyset)$:

$$V(\emptyset_o) = \det\{\xi^N(\emptyset_o)\}$$

$$= \{e_{ot}^{(1)}\}^2\{e_{ot}^{(2)}\}^2 - \{e_{ot}^{(1)}e_{ot}^{(2)}\}^2 \qquad (4.1.4)$$

The next step is to obtain estimates of the parameter $\emptyset$ that minimize the expression $V(\emptyset)$. These estimates can be obtained by using a IMSL minimization subroutine, provided that initial values of the parameters were obtained by ordinary least squares estimation applied to equations (4.1.1) and (4.1.2).

If $\emptyset_c$ and $\emptyset_o$ are the parameter vectors having dimension n_c and n_o respectively, then a likelihood ratio test can be used in testing the hypothesis $H_o : \emptyset = \emptyset_o$ versus $H_c : \emptyset = \emptyset_c$. The likelihood ratio test is:

$$\lambda = \left[\frac{V(\emptyset_c)}{V(\emptyset_o)}\right]^{N/2}$$

This expression is equivalent to

$$\lambda = \left[1 + \frac{n_c - n_o}{N - n_c} t \right]^{N/2}$$

$$\text{With } t = \frac{N - n_c}{n_c - n_o} \quad \frac{V(\emptyset_o) - V(\emptyset_c)}{V(\emptyset_c)} \sim F(n_c - n_o, N - n_c)$$

The statistic λ provides the following decision rule for testing the hypothesis of no feedback: If N is large enough to justify some asymptotic assumptions over the previous test, we can use the fact that $-2\log\lambda$ is asymptotically χ^2 distributed with $(n_c - n_o)$ degrees of freedom. That is, we would accept the hypothesis of feedback-free model if $-2\log\lambda < \gamma_\alpha$ with γ_α being the 100α % point of the χ^2 distribution with $n_c - n_o$ degrees of freedom.

Although the proposed method appeared to be very interesting, it failed to provide reasonable statistics to test for the presence of feedback, since the minimization of the determinant $V(\emptyset)$ systematically produced only a trivial solution for the parameters being considered. Further studies of this method are needed to obtain a reliable solution for the estimation problem.

2. ESTIMATION OF THE PARAMETERS

Hannan (1970) proposed estimating the parameters of the system using a matrix approach that requires the inversion of large matrices. Although the estimates obtained through his methods appeared to be very close to the ones obtained by the 2SLS and 3SLs methods, the sizes of the matrices to be inverted are so large that they induce a large cumulative roundoff error. As a direct consequence of this, the standard errors for the estimates are so big that the method fails to produce significant estimates for any of the parameters.

Another approach used initially to determine the appropriate model, was the fitting of a regular transfer function model and the estimation of the corresponding impulse response function. Although many of the estimated parameters were found to be significant and the residual analysis showed no evidence of lack of fit, there were some considerations that lead us to conclude that the transfer function model is not a valid approach in the present context.

In fact, let us assume that an appropriate identification procedure has been utilized and that the parameters of the impulse response function have been estimated. The inclusion of both positive and negative lags implies that to use the model for prediction purposes will require the knowledge of future values of the input to predict the actual output, which is evidently entirely inappropriate (Akaike,1968). The fact that the parameters corresponding to negative lags have been found significant is nothing but a confirmation of the closed-loop structure of the system.

3. SIMULATION

The simulation of the model for the closed-loop system was compared with a similar model generated without feedback parameters. Two aspects of this comparison are to be discussed. One is how well the models are able to reproduce the sample relationships between the SOI and the SST. As we expected, using the closed-loop system model (Figure 15), the generated EN appears to be responding to fluctuations of the simulated SOI approximately in the same way as the observed series does (Figure 14). When the index declines, it is inmediately followed by a rise of the SST. The magnitude of the anomalies are similar to those exhibited by the original series (Figure 14) and the elapsed times between the simulated events are similar to those observed for the real events.

The no-feedback simulation (Figure 16), showed that the generated EL Niño series presents the same patterns as for the feedback model with the only difference that this time there was no apparent connection between the input series (SOI) and the generated response (EN). We conclude that the inclusion of the feedback parameter is crucial in reproducing the true relationship between input and output.

A second and also important aspect to be considered in the analysis is the pattern of crosscorrelations for the simulated series. Recall from Figure 13 that a main feature of the CCF was

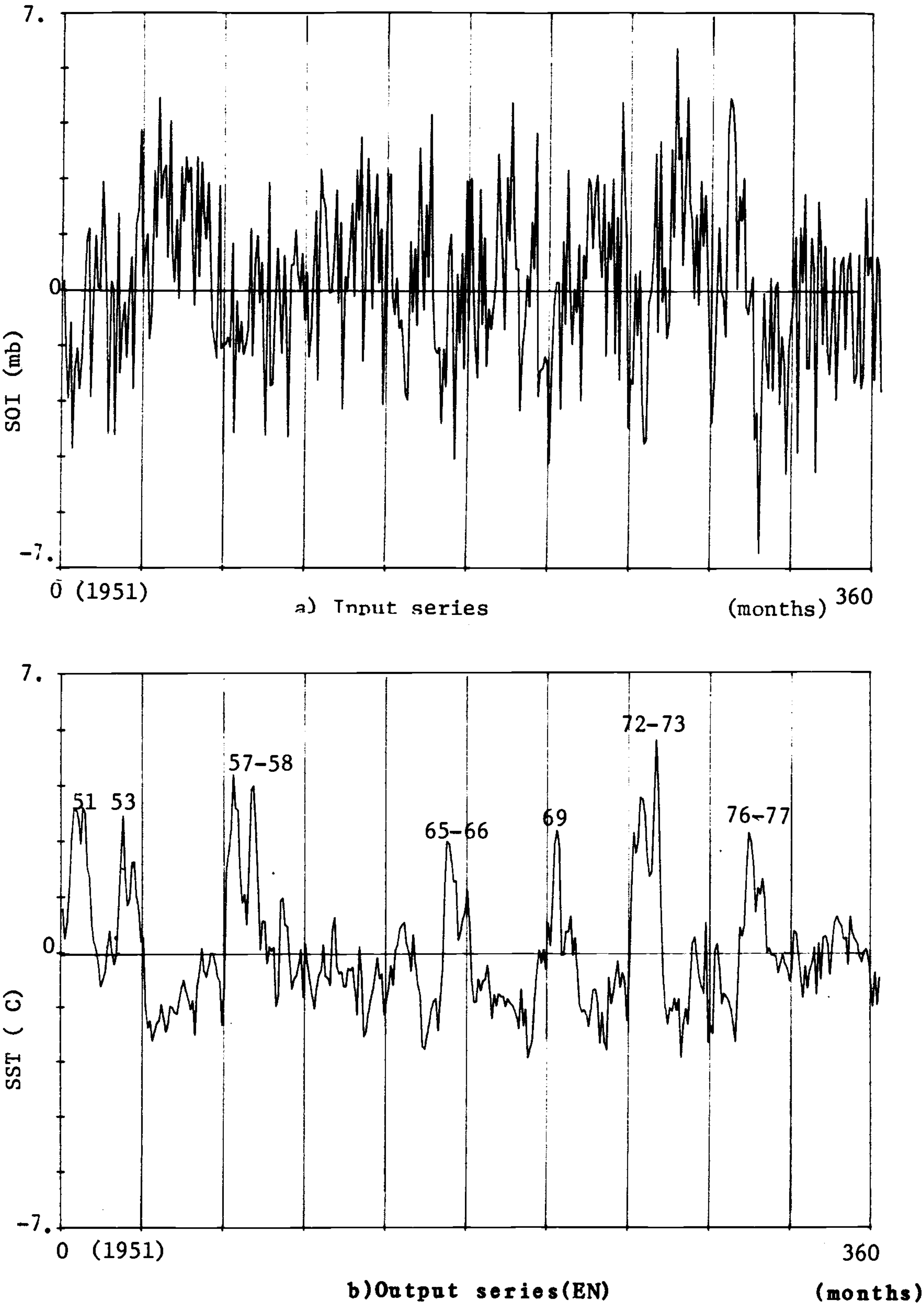
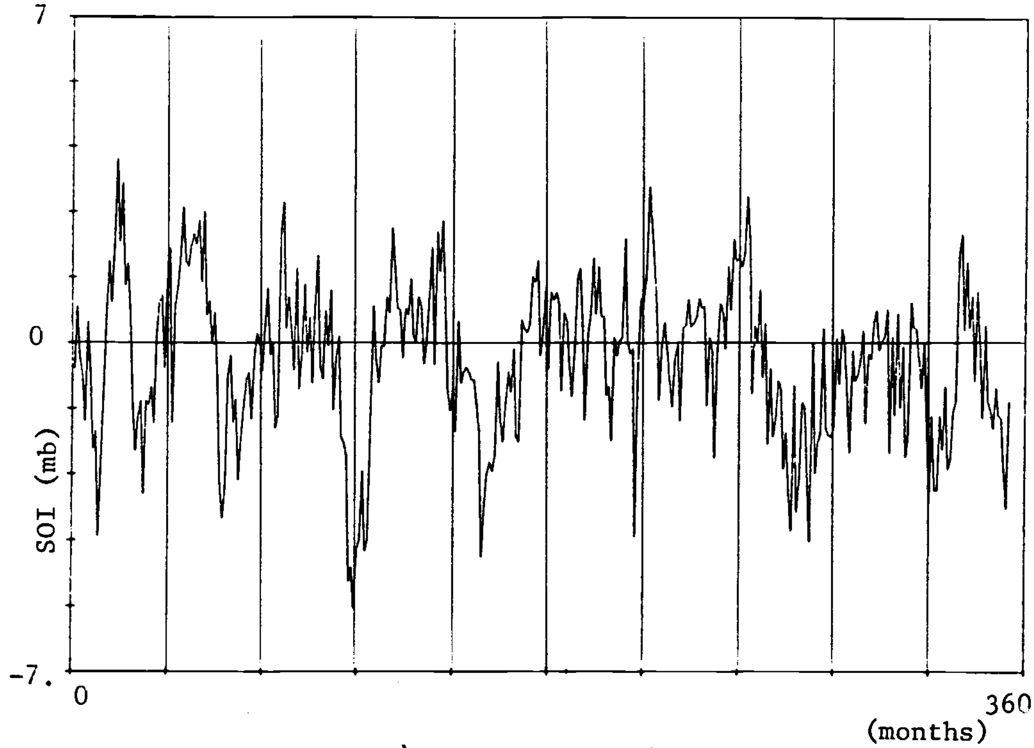
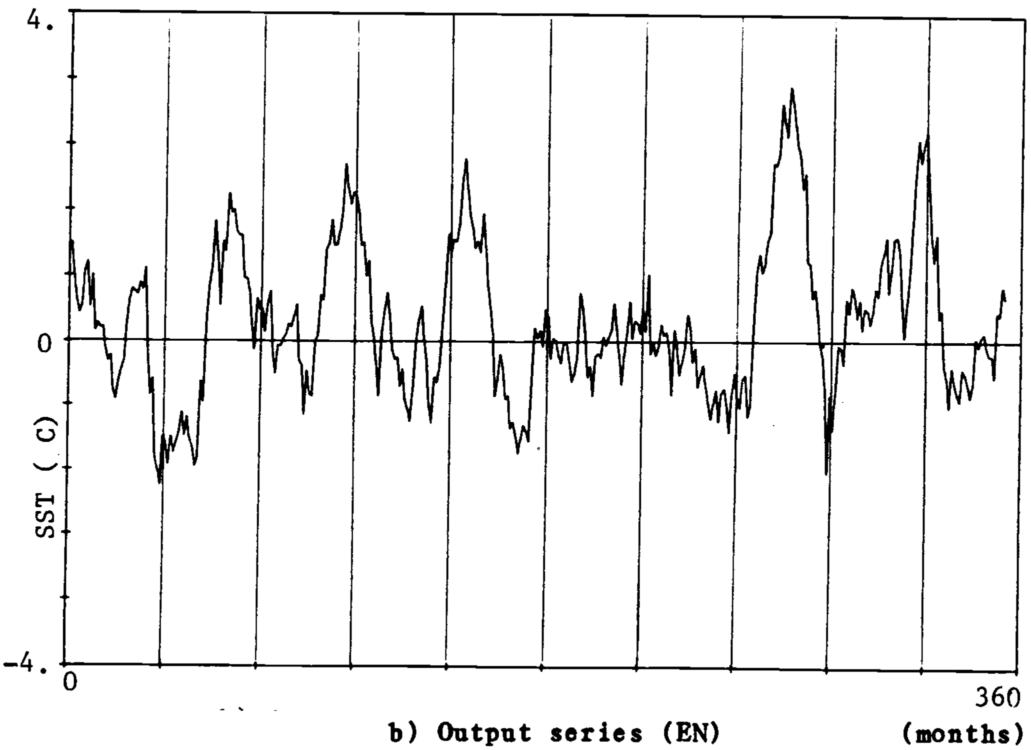


Figure 14. Original input and output series (TD and EN) after removing the annual monthly mean. Shows the first 360 months, starting January 1951.



a) Input series (TD)



b) Output series (EN) (months)

Figure 15. Typical example of input and output series generated by simulation using the closed-loop model (with feedback).

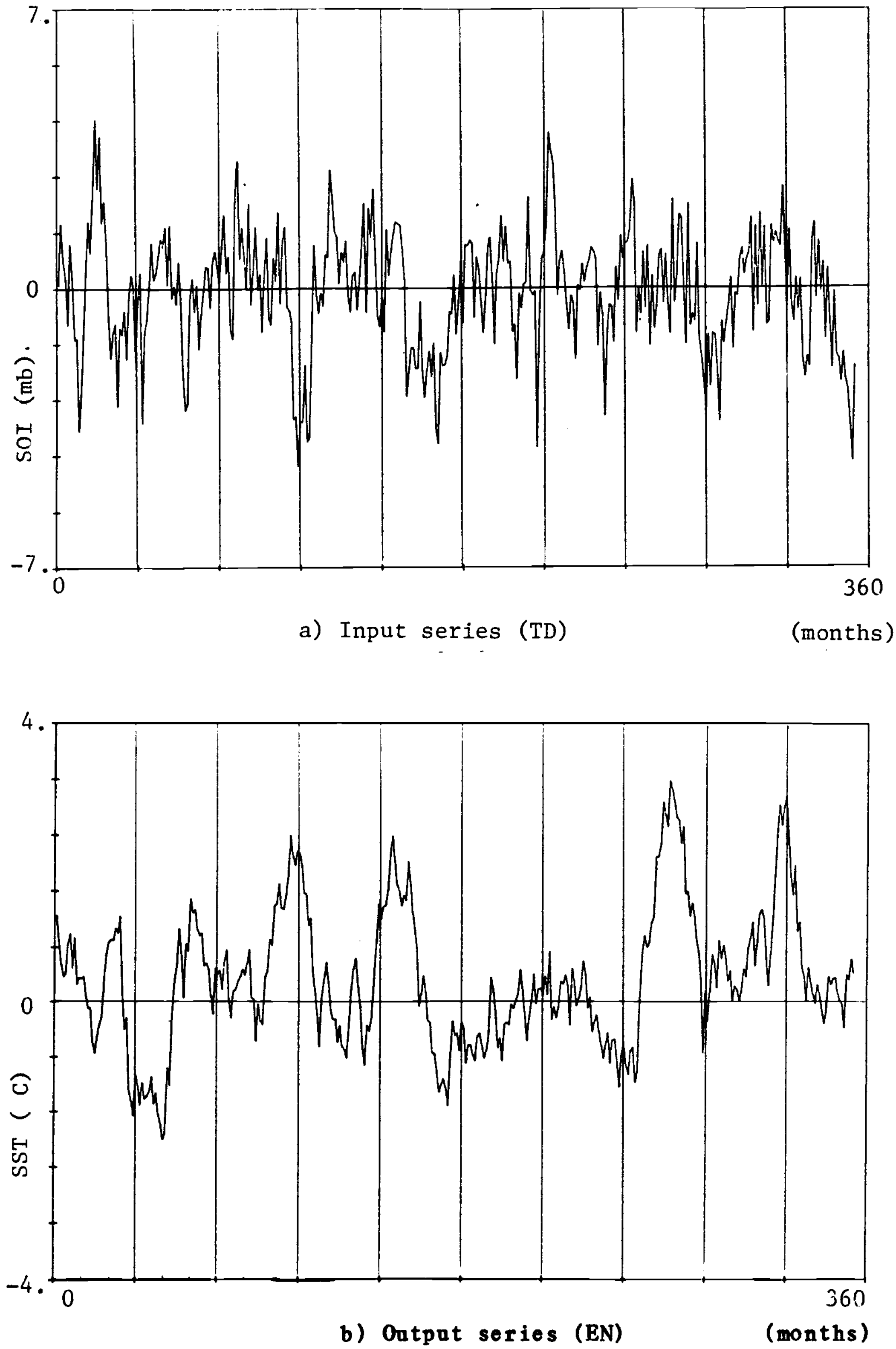


Figure 16. Typical example of input and output series generated by simulation using the open-loop model (no feedback).

the presence of significant crosscorrelation at negative lags. In this respect, when the feedback parameter is present in the system of equations, the closed-loop model is able to reproduce almost exactly the CCF patterns observed for the original series with only minor variations on the center of the peak of cross-correlation near lag zero. A measure of the crosscorrelation function between simulated input and output series is presented in Figure 17. It shows the mean values of the croscorrelations for each lag. The average was obtained from a sample of 10 independent simulations. Again the similarities between this and the original CCF (Figure 13) is evident. Although there are some differences between the CCF of the original series and the mean values, these discrepancies can be explained at least partially by the fact that the same original AR(2) model was used for prewhitening the input and output series prior to the calculation of the CCF. The open-loop model on the other hand, presents no pattern with significant values of cross-correlation in the neighborhood of $k = 0$ as shown in Figure 18, which includes also mean values of crosscorrelation between the simulated input and output, based on a sample of 10 simulations. All cases were highly consistent and crosscorrelation values were very similar. These facts confirm the conclusions obtained from the analysis of the simulated series, i.e. that the inclusion of feedback is necessary to adequately reproduce the relationship between the input and output.

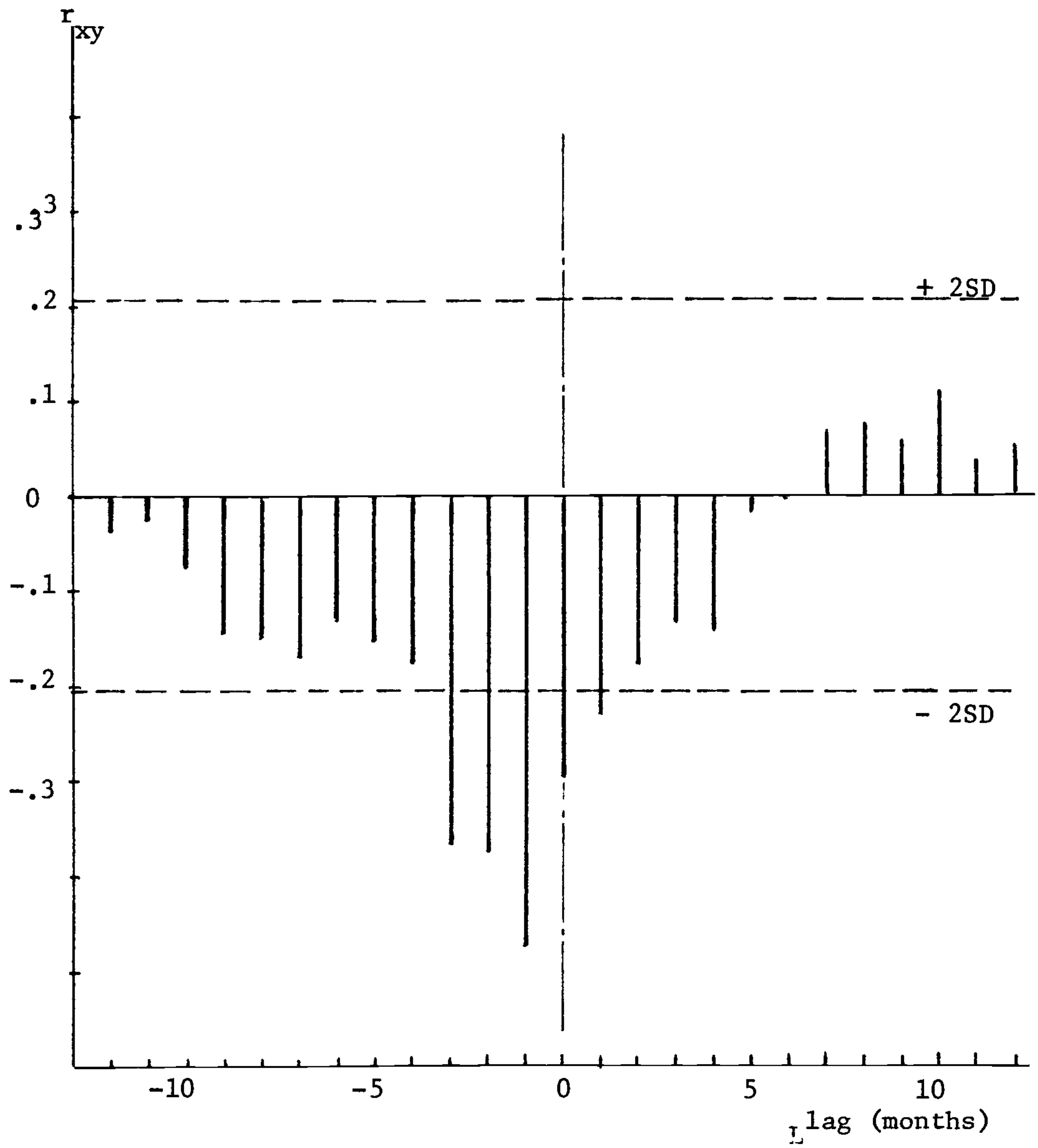


Figure 17. Crosscorrelation function of simulated input and output, using the closed-loop model (with feedback), Showing strong similitude with CCF of original series (Figure 13). Each bar represent the average crosscorrelation of 10 simulations.

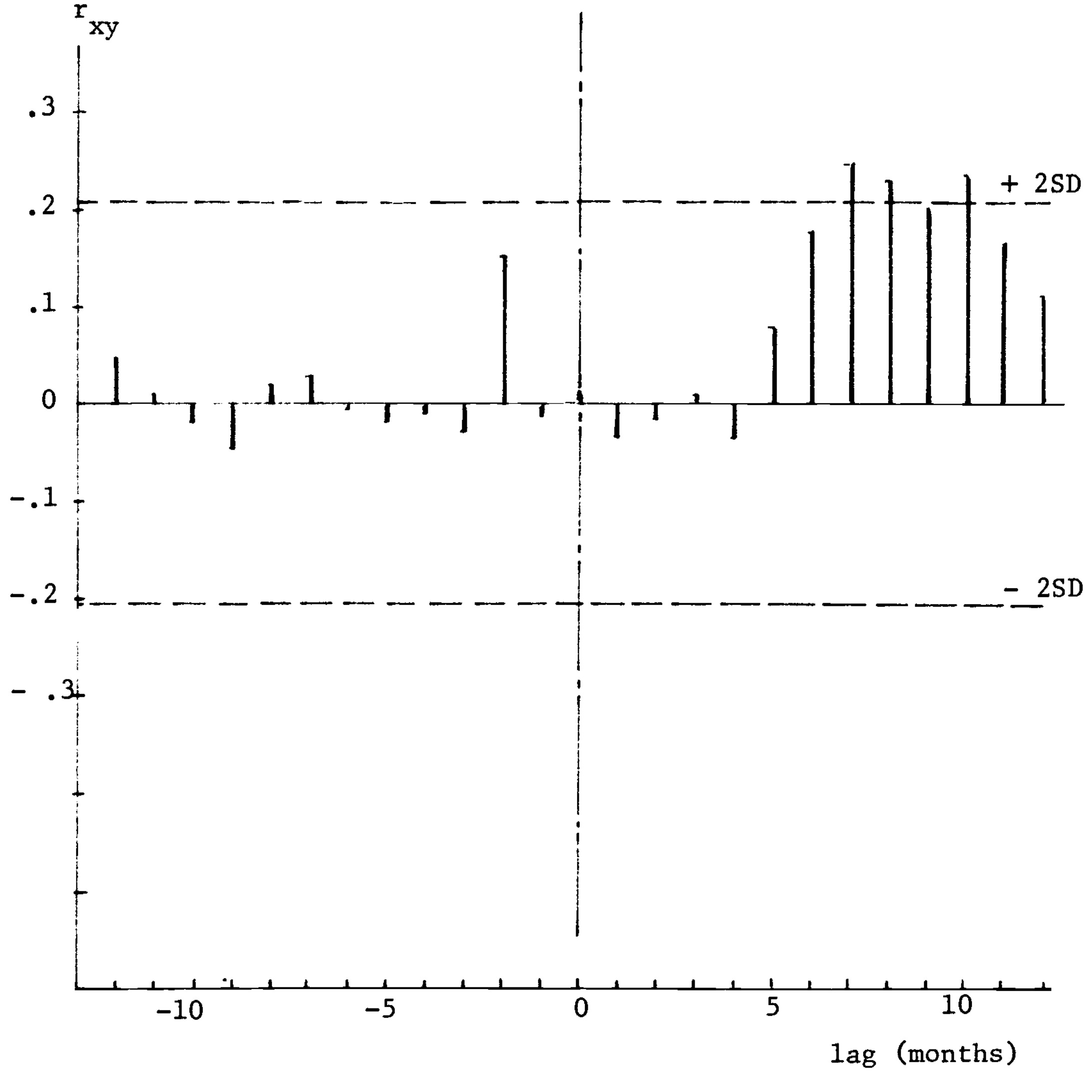


Figure 18. Crosscorrelation function of simulated input and output using the open-loop model (no feedback) showing no similitude with CCF of original series (Figure 13). Each bar represents the average crosscorrelation of 10 simulations .

The crosscorrelation function obtained from the simulated input and output provides the basis for an interesting point of discussion. Why is the proposed model able to reproduce the CCF patterns observed for the original series? And why is it that the same results are not obtained from the non-feedback model where the generated output appears to be almost exactly the same for both models? There are several possible explanations, among them the fact that the output series generated using the open-loop (non-feedback) model does not exhibit the physical connection that one could expect for the type of atmospheric phenomenon being studied. That is, we expect that a sudden decay of the the index will induce a rapid increase of the SST. This relationship shows up strongly and consistently for all simulated El Nino events, every time the closed-loop model is used in the simulation.

4. GENERAL COMMENTS

The use of the closed-loop system model for prediction purposes faces the problem of finding adequate estimates for the variances of the predicted values, to be used in computing confidence bands. Further studies need to be done to solve the problem of obtaining estimates of the mean square error, given that the noise term is not independent of the input variable of equation (3.6.5). If such estimates are obtained, some of the longer existing series (e.g. SST at Puerto Chicama) can be used to test the efficiency of the model by considering observations of the pre-1951 ENSO events and verifying how close the predicted values are to the actual ones.

The inclusion of the feedback parameter in the model appears to contradict certain studies that argue that the atmosphere leads the ocean, and that the reaction of the atmosphere to the ocean is negligible, (Davis 1976, 1977). In fact, we cannot expect to use only the South American SST as a prediction tool to forecast the future behavior of the atmosphere and in particular of the SOI. It is also very unlikely that a narrow portion of coastal waters located along the west coast of South America will influence the atmosphere all over the Pacific ocean to the degree of producing the Southern Oscillation fluctuations.

In the other hand, recent studies (Wright 1984,1985, McCreary 1983) have shown some evidence that the physical connection between ocean and atmosphere involves the presence of feedback which appears to be particularly strong in the equatorial Pacific zone. During the months of July to December, the proposed models include additional variables such as wind between the eastern Pacific and the Australasia region, as well as cloudiness and SST in the Indonesian region.

The problem of finding an adequate representation for the ENSO events is far from being solved. The addition of new information, in the form of time series of precipitation, sea level, winds along the equatorial zone, atmospheric thickness (which is a measurement of how the ocean feeds back into the atmosphere) need to be included in a more complex model. This might be accompanied by defining a new and more sophisticated El Niño series plus the addition of a variety of input series, under the form of several Southern Oscillation indices, coming from stations spread out over the South Pacific. After finding a adequate model, to obtain a reasonable interpretation, we must consider the El Niño series as representing a larger scale phenomenon with significant influence over the atmosphere.

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