AN ABSTRACT OF THE THESIS OF

TORBJORN SPURKLAND for the MASTER OF SCIENCE
(Name) (Degree)

in CIVIL ENGINEERING presented on May 9, 1968
(Major) (Date)

Title: THE EFFECT OF BOUNDARY GEOMETRY ON INTERNAL DENSITY CURRENTS IN A DENSITY STRATIFIED RESERVOIR

Abstract approved: [redacted]
Larry S. Slotta

The effects of boundary geometry on internal density currents in a density stratified reservoir were investigated in a laboratory model. When water was simultaneously withdrawn and admitted to the reservoir, a complex current regime was established. The general flow pattern is described, and the influence of physical variables on the current regime is demonstrated. No analytical treatment is attempted due to the complexity of the problem.

The Effect of Boundary Geometry on Internal Density Currents in a Density Stratified Reservoir

by

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A THESIS

submitted to

Oregon State University

in partial fulfillment of
the requirements for the
degree of

Master of Science

June 1968

APPROVED:

[redacted]

Associate Professor of Civil Engineering

in charge of major

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Date thesis is presented May 9, 1968

Typed by Clover Redfern for Torbjorn Spurkland

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INTRODUCTION

The world is facing a shortage of potable water. To amend the problem, selective withdrawal could be employed. Through selective withdrawal the quality of the water discharged from a reservoir can be controlled, enabling optimal usage of the released water. To develop means by which selective withdrawal can be obtained, additional fundamental knowledge about the current regime in a density stratified reservoir is required. This thesis is an experimental effort to investigate internal density current patterns in stratified reservoirs.

The Water Shortage

It is estimated that by 1980 the needs in the United States for fresh water will almost double the 1965 daily requirement of 300 billion gallons. Domestic usage will, in 1980, consume 120 billion gallons per day, while industry will need 380 billion gallons, most of which will be returned to the rivers and lakes in more or less polluted form (32). With an average daily runoff of 1160 billion gallons and a rapid deterioration of existing water supplies, the United States is faced with an increasing shortage of potable water. Rivers that previously provided drinking water to whole communities are today little more than open sewage lines; the ground water supplies are almost depleted in parts of the nation; and the surface reservoirs are being filled with sediments carried by the rivers discharging into them. It is estimated that 2×10^9 tons of sediments accumulated in the Hoover Dam on the Colorado River, during the first 14 years of operation, reducing the maximum depth of the reservoir from 565 feet to 460 feet. The storage capacity was reduced five percent (9). To rectify this situation, the need for increased knowledge in the field of water resources and reservoir operation is vital. With extended studies, new tools will be developed which can secure a more economic usage of the present and future water supplies.

Density Stratification

The problems of density stratification and density currents on reservoir operation have recently been subject to engineers' and scientists' studies. It has been observed that when a reservoir is stratified, that is, when a density gradient exists in the vertical within the reservoir, the flow regime and current patterns are greatly altered from the regime existing when the reservoir is homogeneous. A study made by the T. V. A. showed that polluted river water entering a T. V. A. reservoir would be propagated through the system as a distinct entity and not mix with the impounded water. The effect was that minimal selfpurification of the polluted water took place. The polluted water moved as a current with observed velocities from 0.15 to 0.35 fps. through the series of reservoirs. A two year detention time for the system has been estimated. However, during the time of the year when the reservoirs are stratified, water will flow through the system in less than six months (8).

The stratification of a reservoir is due to the cyclical heating and cooling of the impounded water. The density, or rather the specific gravity of water, is a function of the water temperature. When a temperature gradient exists in the reservoir, a density gradient will also exist. Figures 1.1 and 1.2 show plots of temperature distributions and the corresponding density profiles for the Brownlee Reservoir in Western Washington at four different dates in 1963. Most reservoirs which are located in a region with distinct seasons have similar characteristics as shown on these figures. In the winter the reservoir is homogeneous, with the coming of spring and summer the surface water becomes increasingly warmer. Through convection and absorbtion, heat energy is transferred down into the reservoir heating the water to successively greater depths. This can be classified as a stabilizing process. The specific gravity of water decreases with temperature and therefore the strata of lighter water will be above the heavier water. In the early fall, the reservoir will have obtained its maximum stratification. At this time of the year, the back radiation from the reservoir surface becomes

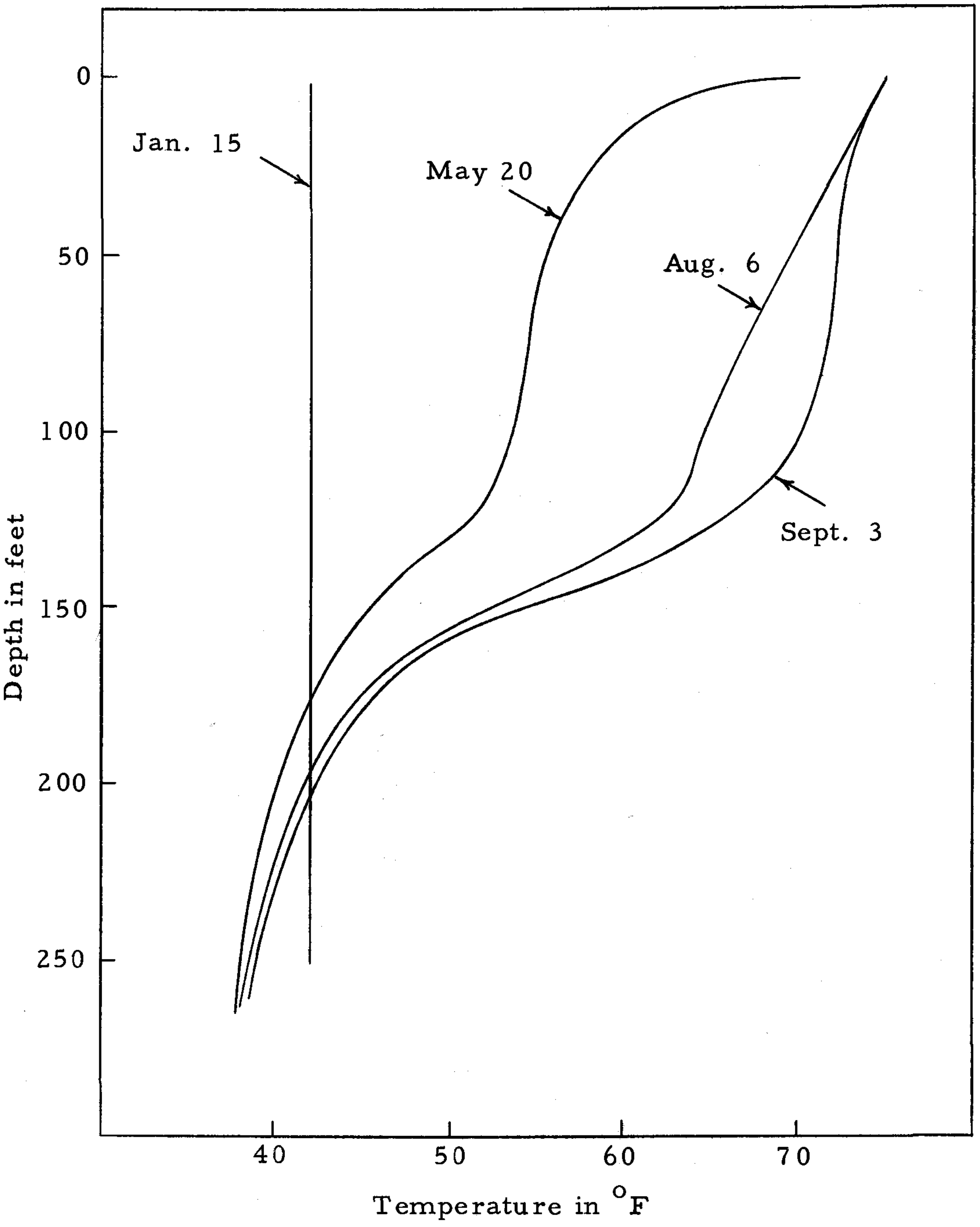


Figure 1. 1. Temperature profiles of Brownlee Reservoir at four different dates in 1963. Courtesy of FWPCA (Calloway).

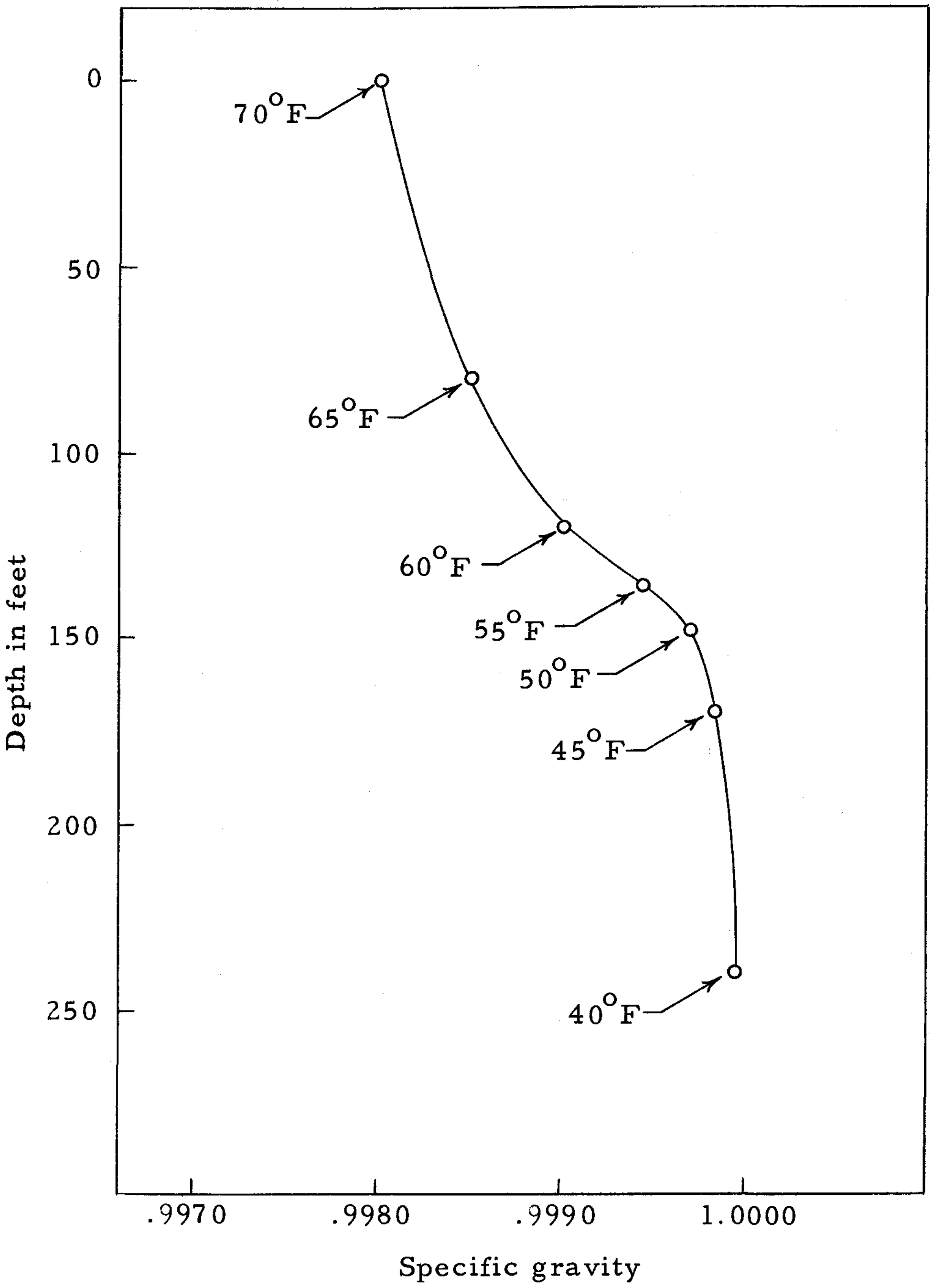


Figure 1.2. Thermal stratification of Brownlee Reservoir, plotted from a mean temperature profile for the period June-September 1963.

increasingly greater than the incoming radiation, resulting in a net heat loss. The surface water will become cooler and heavier than the water immediately below, so that an interchange or mixing of the top layers takes place. As time progresses, this mixing occurs at increasing depths until the whole reservoir is mixed and quasi-isothermal again.

Density Currents

The National Bureau of Standards defines the term "density current" as follows:

> A density current is the movement, without the loss of identity by turbulent mixing at the boundary surfaces, of a stream of fluid under, through, or over a body of fluid, with which it is miscible and the density of which varies from that of the current. The density difference being a function of the difference in temperature, salt content, and/or silt content of the two fluids (14).

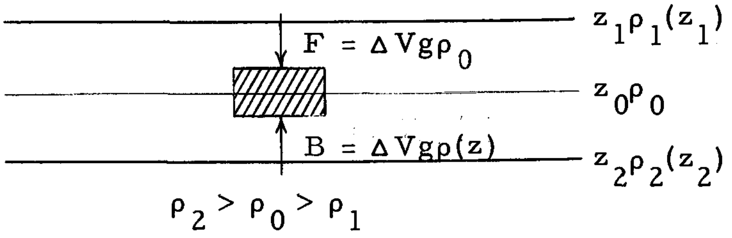
By this definition density currents are not restricted to water and water reservoirs only. The atmosphere is often stratified and the movement of a cold or a warm front is a density current phenomenon. The behavior of air pollutants may also be successfully analyzed as an analog to density currents in a reservoir. Density currents also exist in the ocean. Some scientists speculate that these ocean density currents are one of the main contributors to the shaping of the ocean floor. A generally recognized explanation for the rough sub-surface topography has not yet been given.

Density currents often create a problem in estuaries. During rising tides, a salt water wedge forces its way upstream under the outflowing river water. If the slope of the river is small, the wedge can advance far up-river. This salt water wedge, or under-current, may do damage to the surrounding land if the water is used for irrigation or if the brackish water seeps into the ground.

Selective Withdrawal

Most dams and reservoirs built today are designed to serve multiple purposes. Water quality must be controlled for domestic use, for recreation, for the preservation of fish and wildlife, for irrigation, and for industrial processes. The use of water for one of these purposes is often detrimental to the other users. A nuclear power plant, for example, needs cold water to cool the reactors. This water may be too cold for irrigation and for the recreational activities to take place in the area. Since water quality varies with depth, a reservoir may have water of the required qualities to satisfy all of the different users. A tool to predetermine and to control the quality of the released water is however necessary. As will be shown below, knowledge about stratified fluid flow may prove to be this tool.

Consider a stratified reservoir with some arbitrary, but positive, density gradient in the vertical. A fluid particle at elevation Z_0 with volume ΔV and density ρ_0 will be acted upon by two vertical forces; the bouyancy force and the force of gravity.



If this fluid particle were to change position in the vertical, the two forces would do an amount of work

$$F = \int_{z_0}^{z_1} \Delta V g \rho(z) dz - \Delta V g \rho_0 (z_0 - z_1) < 0$$

for a displacement in the upward direction, and

$$F = \Delta V g \rho_0 (z_2 - z_0) - \int_{z_0}^{z_2} \Delta V g \rho(z) dz < 0$$

for a downward displacement. In both cases, energy must be added to the particle if it is to move vertically. This is the reason for the "piston effect" in stratified fluid flow.

Compare the above argument to a fluid particle changing position in a homogeneous reservoir. The fluid particle can occupy any position or move to any elevation without energy being consumed or released. If a fluid is released horizontally into a stratified reservoir, the advancing fluid will act as a piston and move the fluid in front of it in a horizontal direction, unless energy in some form is transferred to the reservoir fluid to overcome the work required to make it move vertically.

The same phenomenon exists when water is released from a stratified reservoir. Instead of withdrawing from the total cross-section, the discharging fluid is taken from a narrow layer at the elevation of the outlet. If the variation of water quality with depth is known, one should theoretically be able to withdraw water of limited desired quality, if outlets are placed at different depths in the dam.

STRATIFIED FLUID FLOW

The densiometric Froude number has been found to be the most important concept in stratified fluid flow (10). The critical value of the densiometric Froude number has been analytically predicted for several geometrically simple cases. This study investigates the effect of boundary geometry on the flow field of a density stratified reservoir. Except for a dimensional analysis, no analytical treatment is undertaken.

The Densiometric Froude Number

Bell (2) was among the first to envision the benefits selective withdrawal could offer. As a result of flow phenomena observed in Lake Mead Reservoir, Bell conducted an experimental investigation and concluded that selective withdrawal was possible if the flow rate was low. Later Yih (34) derived the limiting conditions analytically. By assuming a quasi potential flowfield, Yih found that withdrawal would be selective if the densiometric Froude number was less than a critical value, depending on the geometry of the outlet. The densiometric Froude number is defined by Yih as:

$$F = \frac{V}{\sqrt{\frac{\rho_{max}-\rho_{min}}{\rho_{max}} gd}}$$

where

V = velocity at orifice

d = depth of orifice

g = gravitational constant

ρ_{max} and ρ_{min} are the maximum and minimum densities in the flowfield.

For a circular orifice, Yih found a critical Froude number, F_c, of $\frac{1}{\pi}$ or 0.318. When F was greater than F_c no selective withdrawal was possible. Later the same critical Froude number was determined experimentally by Debler (5).

After Yih's breakthrough, the critical Froude number was found for several types of outlet conditions. These results are summarized by Daubert (4).

Velocity profiles of density currents created by withdrawal from a stratified reservoir has recently been determined by Koh (22). Koh used very low withdrawal rates, on the order of 0.1 cm./sec. Considering the scaling effect, this was a realistic flowrate. Koh developed a Gaussian velocity profile for the internal current. His results also showed that the thickness of the withdrawal current was a function of the distance from the outlet.

Present Study

In this study, the effect of an obstruction on the internal density currents was investigated. The obstruction consisted of a "mountain" located in the flowfield. The mountain may represent a cofferdam or a mountain ridge on a land massif. In some reservoirs, like the Rapel in Chile, the cofferdam is left as a permanent structure to intercept turbidity currents that otherwise would have carried sediments to the main dam.

Figure 2.1 shows the mountain and the resulting density currents created by discharging fluid into the reservoir at the sloping end, and simultaneously withdrawing water through a circular outlet below the elevation of the mountain. The heavy lines show the deformation of the flow disturbed dye traces used to visualize the current pattern. Inflow and outflow were maintained equal, thus steady state conditions existed in the model reservoir.

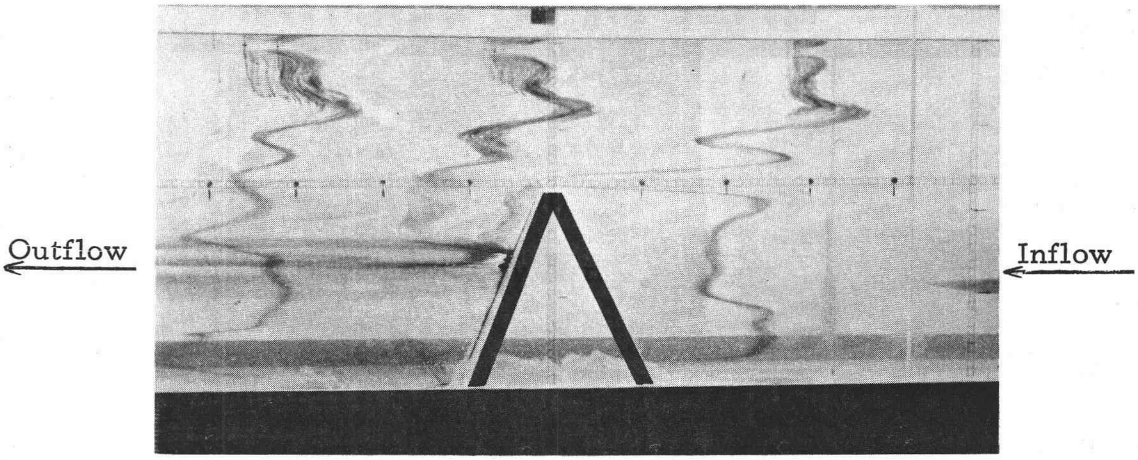


Figure 2.1. Current pattern at mountain.

In this investigation, the density profile, the density of the inflowing fluid, and the elevation of the inflow nozzle were the variables.

The data collected have been plotted as dimensionless quantities to seek correlations between the different physical variables.

Dimensional Analysis

No attempts were made to analyze the flow phenomena analytically, as this would be very complex. As a problem involving velocity-, pressure-, and density fields, these variables interact and a single factor or part of the flowfield cannot be isolated and analyzed without due regard to the rest of the system. A dimensional analysis follows, in an attempt to find the general form of correlations between the physical variables involved in this problem.

The complexity of the problem was realized early in the preliminary stages of the investigation. All physical variables shown on Figures 2. 2 and 2. 3 may be interdependent. To facilitate analysis, variables shown on Figure 2. 2 were kept constant; q_{in} and q_{out} were measured and balanced with rotameters to obtain steady state conditions.

Figure 2. 3 shows the physical parameters that were varied during the investigation. They are:

D = total depth in reservoir

d = depth of inlet nozzle

h_1 = depth over mountain

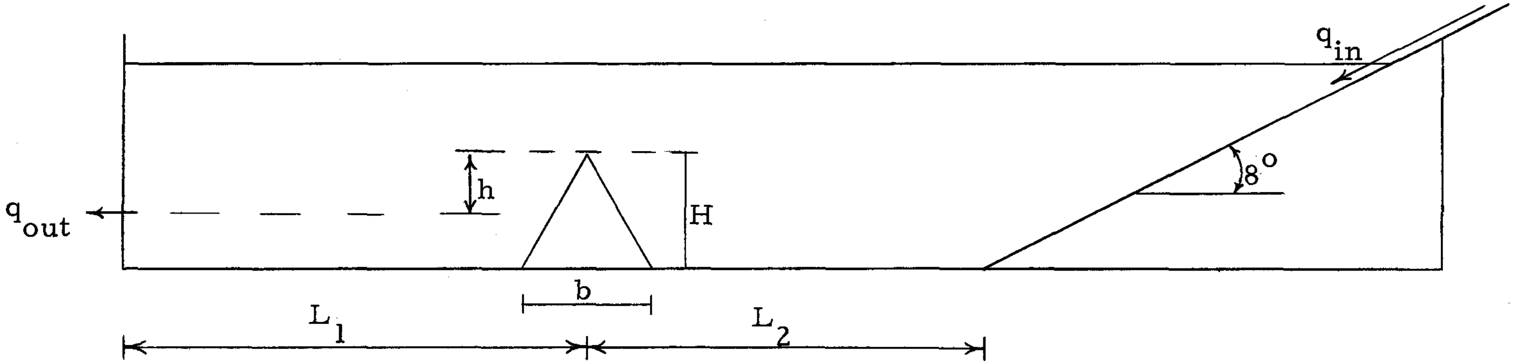


Figure 2. 2. Physical variables kept constant during the investigation.

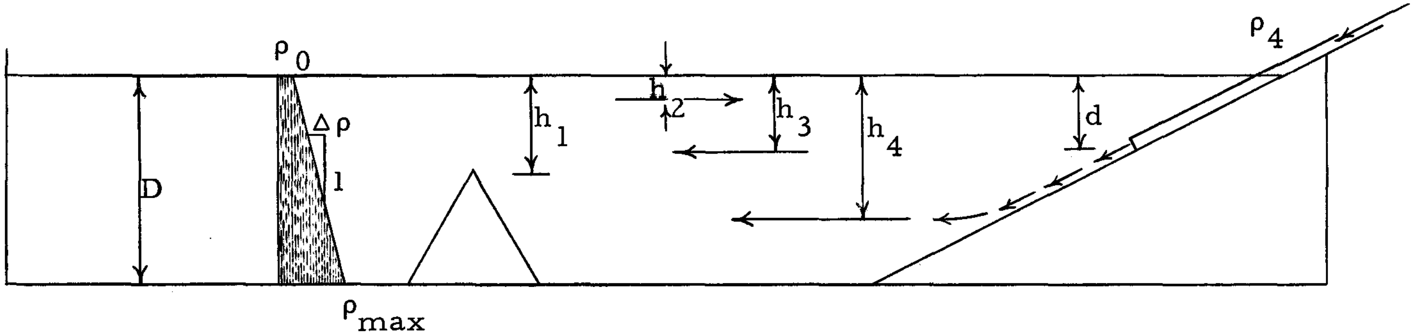


Figure 2. 3. Physical parameters that varied during the investigation.

h_2, h_3, and h_4 depth of the different currents

ρ_0 = density of top layer of water

ρ_{max} = density of bottom layer of water

ρ_4 = density of inflowing fluid

$\Delta\rho$ = density gradient in reservoir.

Ten variables are considered. Using the Buckingham Pi theorem, these variables can be combined into seven dimensionless parameters. This number can be further reduced by excluding D and ρ_{max} and consider h_2, h_3, and h_4 as dependent variables. Then for h_4:

$$h_4 = f(d, h_1, \rho_0, \rho_4, \Delta\rho)$$

By combining as follows, h_4 will be a function of three variables.

$$h_4 = f'((h_1 - d)^{x_1} \cdot (\rho_4 - \rho_0)^{x_2} \cdot \Delta\rho^{x_3})$$

The unknown powers are found by observing that this combination of parameters must have the dimension of length. By the usual procedure the following formula results:

$$h_4 = f'[(\frac{\rho_4 - \rho_0}{\Delta\rho(h_1 - d)})^{x_1} \cdot (h_1 - d)]$$

or

$$\frac{h_4}{h_1-d} = F((\frac{\rho_4-\rho_0}{\Delta\rho(h_1-d)})^{x_1})$$

$$h_3 = g(\rho_0, \rho(d), \rho_4, \Delta\rho, h_1, d)$$

where

$\rho(d)$ = density in reservoir at inflow nozzle,

and the other variables are as previously defined.

The number of variables can again be reduced by dropping $\rho(d)$ and by combining ρ_4 and ρ_0 and h_1 and d.

$$h_3 = g'((\rho_4-\rho_0)^{y_1} \cdot \Delta\rho^{y_2} \cdot (h_1-d)^{y_3})$$

Again this grouping of parameters must have the dimension of length, and the following formula will result:

$$\frac{h_3}{h_1-d} = G[(\frac{\rho_4-\rho_0}{\Delta\rho(h_1-d)})^{x}]$$

One of the goals of this investigation is to determine if h_3 and h_4 follow the functional relationship developed. This can be established by plotting values of $\frac{h_4}{h_1-d}$ and $\frac{h_3}{h_1-d}$ versus the corresponding values of $\frac{\rho_4 - \rho_0}{\Delta\rho(h_1 - d)}$. If one, or both, of the two plots defines a simple curve, the correlations between h_3, h_4, and the physical variables are as developed. However, if a family of curves results, then the correlations are more complex, and will include some variables that have been excluded in the analysis above.

EXPERIMENTAL INVESTIGATION

To investigate the effect of boundary geometry on internal density currents in a density stratified reservoir, a model, simulating a reservoir, was used. The density gradient was obtained by adding salt to the reservoir water. The current patterns were created by simultaneously injecting and discharging fluid from the reservoir. A steady state condition existed in the model during the experiment. Part of the time dependent data were collected photographically on film.

Model Design

The experiments were carried out in a clear walled, rectangular, open channel flume; 25 feet long, 18 inches wide, and 22 inches deep. The discharge was through a circular opening in the end plate. The end plate could be mounted in several positions, enabling one to vary the position of the outlet relative to the water surface. A "mountain," consisting of two plexiglas plates, hinged at one edge to form a ridge with adjustable height and base width, was located inside the channel. The mountain fitted snugly to the sides of the channel so that all flow took place over its top. The upstream end of the channel was sloped so that the depth varied from zero to full depth. The inlet slope-angle was eight degrees for these experiments, but could be varied. A storage tank was located at the upper end of the flume. During the experiments, water from this tank was released on the slope, simultaneously as water was discharged at the other end of the channel. The rate of the flows was balanced by rotameters equipped with needle valves. A schematic model is shown in Figure 3. 1.

Filling Procedure

To obtain the desired stratification, the channel was filled to a desired depth with distinct layers of water; each layer containing a pre-determined amount of salt in suspension. The desired density profile was achieved by mixing measured amounts of a stock salt-brine solution and a fixed amount of water in a mixing tank. When the salt-brine and the water were completely mixed, the tank was drained into the channel. The fluid was introduced into the channel by three standpipes placed on the channel floor. For each layer, progressively more salt brine was added so that the density of the fluid being introduced was higher than the fluid already in the channel. It was observed that if the channel was set on a very mild slope, less than one percent, the fluid would flow by gravity along the bottom of the channel under the other layers. Little mixing took place on the lower interfaces.

When the mixing tank had been drained, water and salt-brine for the next layer were mixed and released into the channel. Nine layers

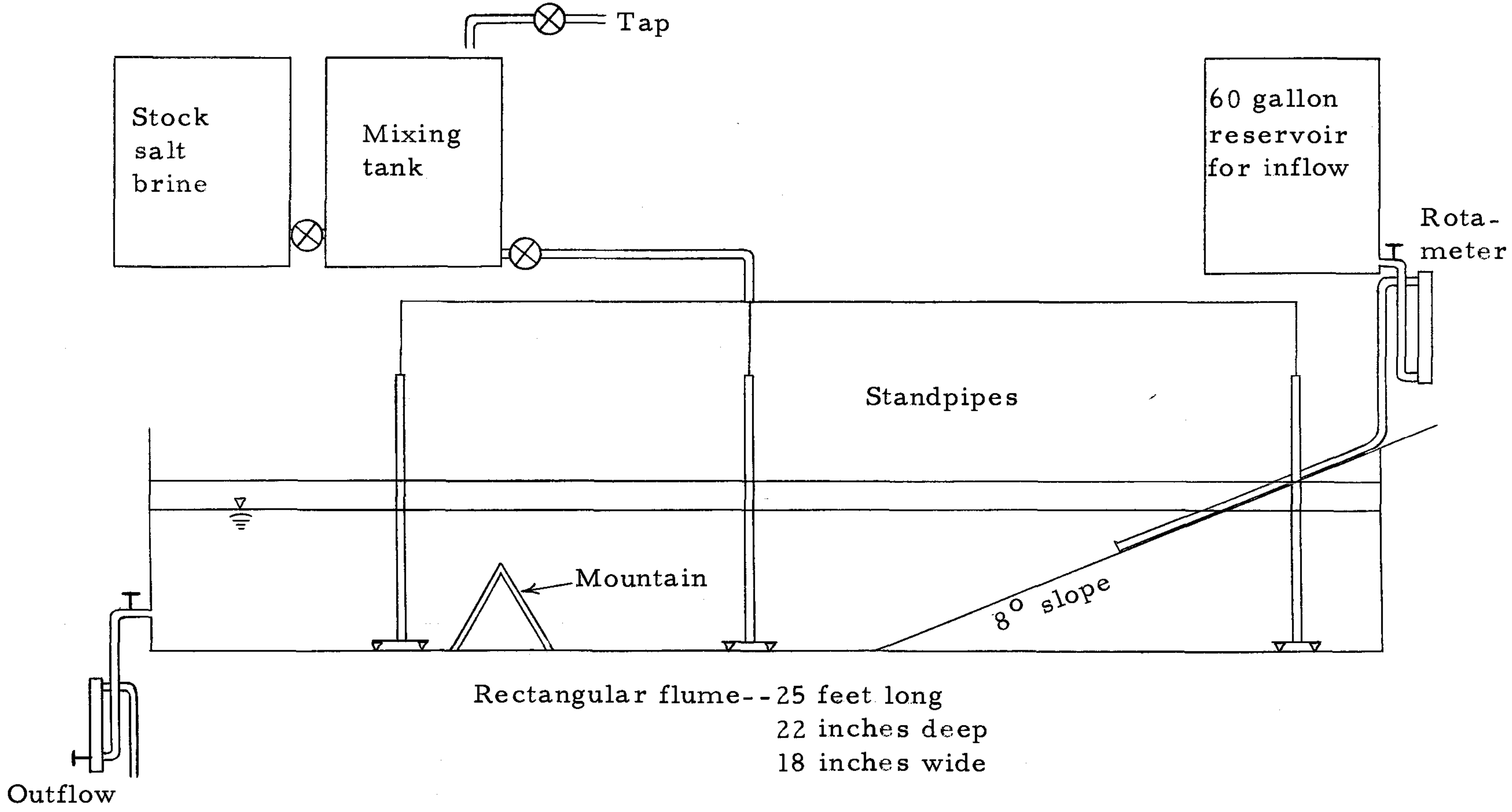


Figure 3. 1. Schematic plan of model.

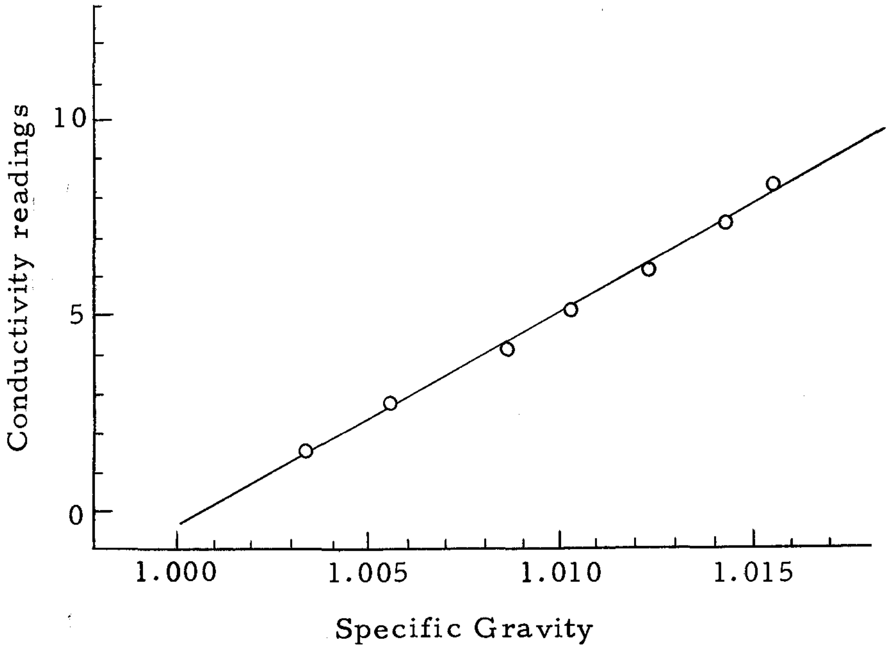
were introduced this way, resulting in a stairway type density profile. Complete filling time was six hours.

The standpipes were carefully removed and the mountain placed in position. Then the tank containing a dyed fluid to be discharged on the slope was filled. No salt was added to this water. The experiment was performed the following day. In the lapse between filling and experiment, the stairway density profile smoothed out and attained the form shown on Figure 3. 2 as determined by conductivity measurements. Both the stratified reservoir and the fluid in the storage tank were then in thermal equilibrium with the environment.

The filling process was fully automated. Solenoid valves, attached to the different tanks, were connected to banks of timers which would open and close the valves at set time intervals. The amount of water in the mixing tank was controlled by a micro switch that would shut off the water from the tap when the water surface reached a certain level. A block diagram of the automatic filling sequence is shown on Figure 3. 3.

Test Procedure

The density profile was determined before every run. This was done with a Serfass Conductivity Bridge and probes made specifically for this purpose. The probes were made of two 1 cm.^{2} platinum plates, spaced one cm. apart. The leads connecting the probes to the



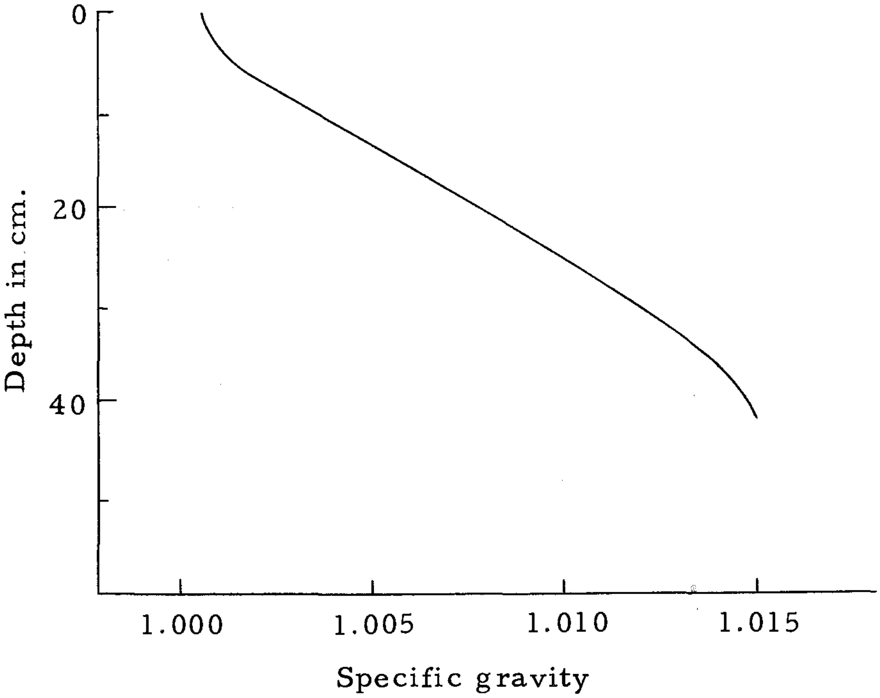


Figure 3. 2. Calibration curve and density profile for run 20.

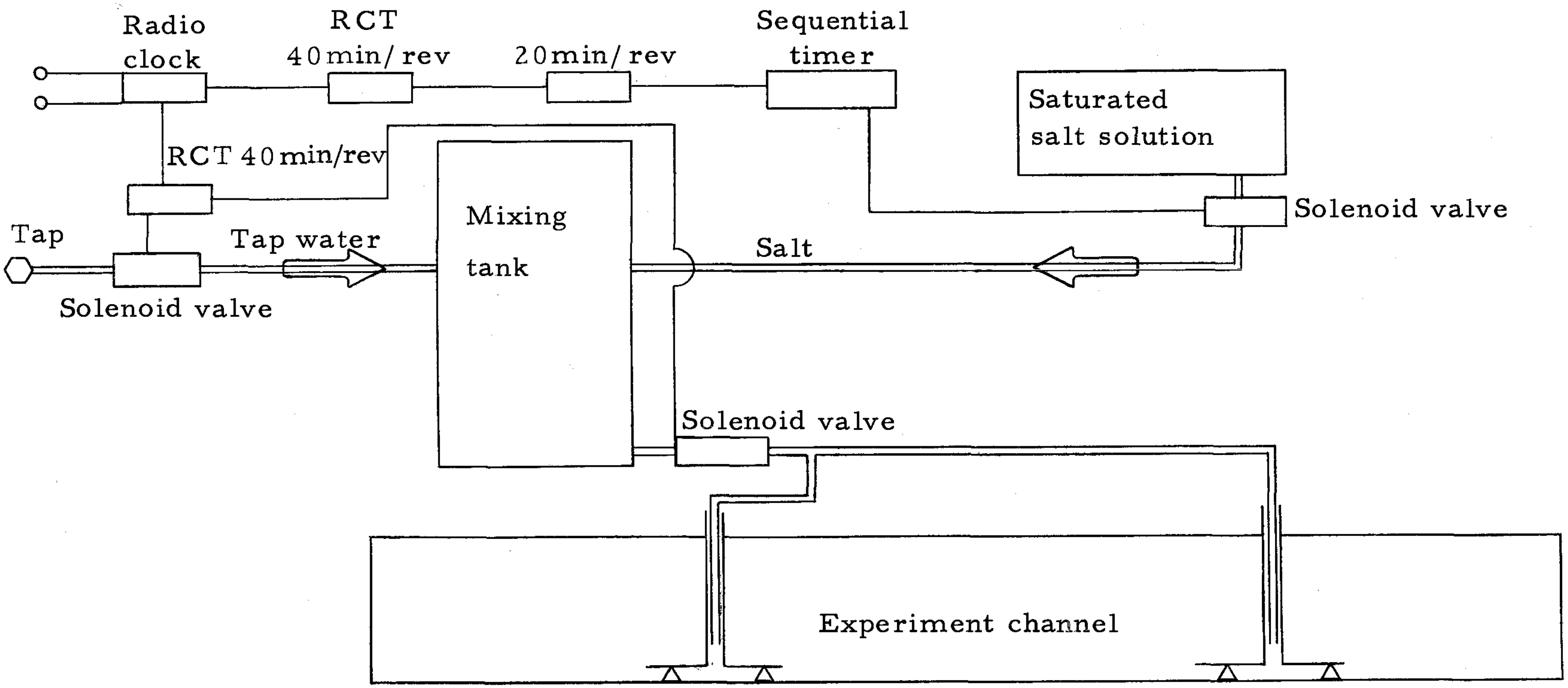


Figure 3.3. Automatic filling.

conductivity bridge ran inside a glass tube which was potted at each end to prevent water from coming in contact with anything but the platinum plates. Figure 3.4 shows a sketch of the probes.

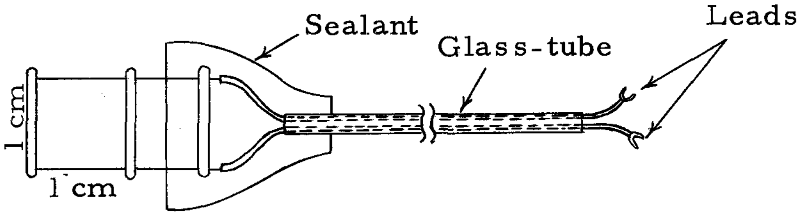


Figure 3.4. Conductivity probes.

The conductivity was measured at one cm. intervals in the vertical. To obtain the density corresponding with the conductivity readings, samples were taken after each run, the conductivity of the samples recorded and the density found with a Christian Becker density balance. In this way, the conductivity meter and the probes were calibrated for each run. Due to variations in temperature, the conductivity changed from run to run, and a standard calibration curve could not be used. A typical calibration curve and the corresponding density profile are shown on Figure 3.2.

Salt brine was mixed with the water in the storage tank until the conductivity of the fluid corresponded with the reading at the point where it was released. Erioglaucine A Supba dye was added to the fluid in order to trace its movements through the reservoir. To observe the current patterns created by the discharge and inflow of water, potassium permanganate particles were dropped on the water surface of the channel. The particles settled slowly in the water and left a bright red trace as they fell. Neither the dye nor the potassium permanganate had any noticeable effect on the conductivity of the fluid.

The valves on the rotameters were opened and the deformation of the dyelines recorded. A 16 mm camera with a 10 mm lens was used to record the progress of the dye traces in the current patterns of the reservoir. The camera (Bell and Howell) was triggered every second by a Samenco Movie Control. This controller activates a solenoid that in turn activates a plunger that hits the shutter release. The equipment was not quite accurate; however, a clock mounted against the channel was included in the picture, enabling time-dependent measurement to be made from the movies.

When the experiments were not recorded on film, selective data were obtained during the run with a point gage.

To obtain data from the films a grid was made and fixed to the wall. Sequences of the film were projected frame for frame on this grid and the pertinent measurements taken and recorded. The scale of the projected picture was found by measuring the picture distance between two fixed points in the flume. Measurements were taken mostly in the center of the picture to avoid parallax.

EXPERIMENTAL RESULTS

The current regime created in the reservoir by the simultaneous withdrawal and inflow of water is next described. Correlations between current parameters and physical variables are also established.

The general current regime existing in the density stratified reservoir during the test run is shown on Figure 4. 1. The behavior of each of the currents will now be briefly discussed.

The Inflow Current

The inflow current, designated q_4, flows down the slope until it reaches a reservoir depth having equivalent density. It then leaves the slope and flows horizontally into the reservoir. When h_4 is less than h_1, the depth over the mountain, the current flows through the reservoir uneffected by the presence of the mountain. When h_4 is larger than h_1, several things occur: after leaving the slope, the current will flow towards the mountain, its speed of advancement slowing down as it approaches the obstruction. The mountain blocks the further advancement of the current and the pool behind the mountain will fill up with the incoming fluid.

The current will flow into the reservoir in a two-dimensional manner before its progress is stopped by the mountain. When it is

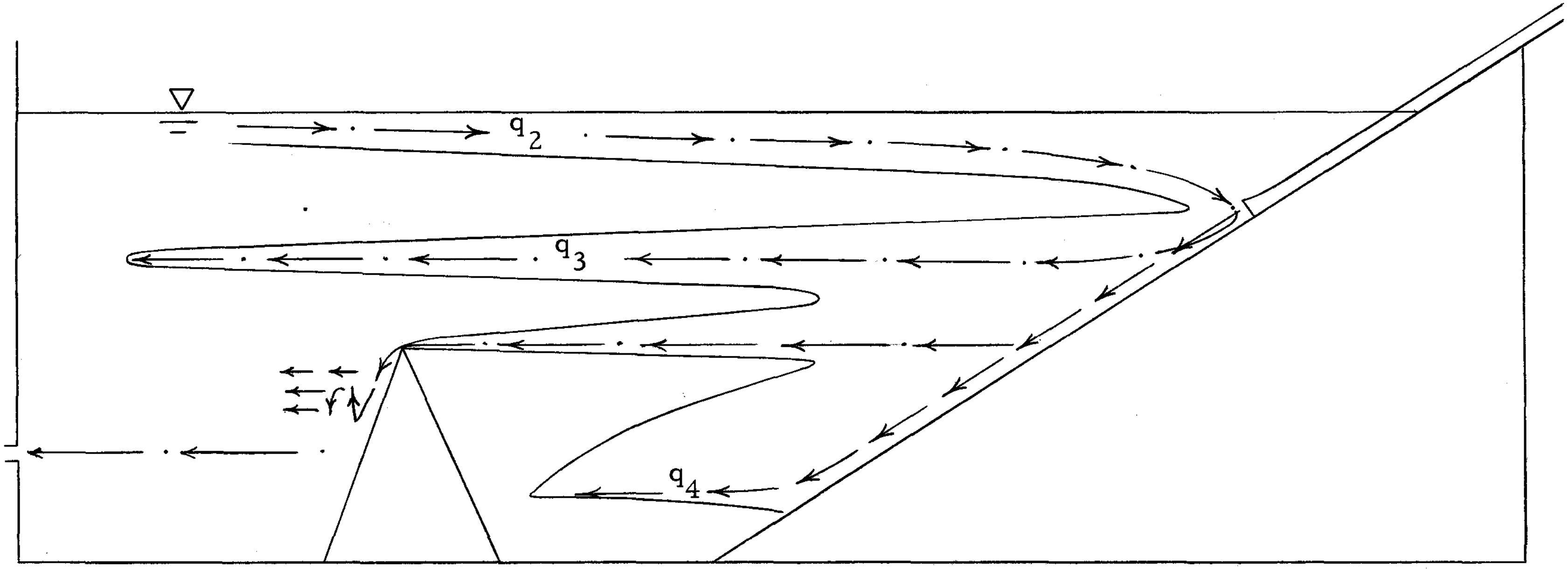


Figure 4. 1. General current pattern. Note: sketch not to scale.

blocked, the current is observed, in plan view, to meander from side to side in the reservoir and large eddies will form. Figure 4. 2 shows the flow patterns in the two cases.

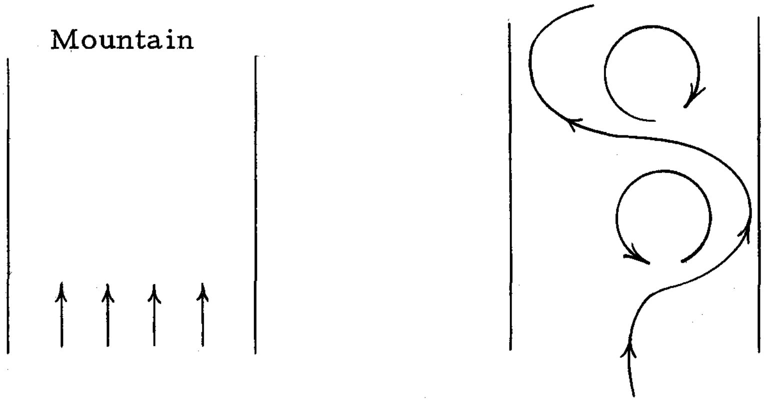


Figure 4. 2. Flow of inflow current initially before and after blocking (plan view shown).

During some of the experiments, horizontal oscillations were observed upstream of the mountain before blocking occurred. These oscillations were very slow and could not be detected during the actual test. However, when the films of the tests were run at 24 times the actual speed, the horizontal movement of the dye-traces became evident.

Figure 4. 3 is a dimensionless plot of the depth of the inflow current versus density parameters. The plot shows that the inflow will seek its own density level in the reservoir regardless of the location of the discharge point. This indicates that very little, if any, turbulent mixing takes place on the slope; mixing may however take place

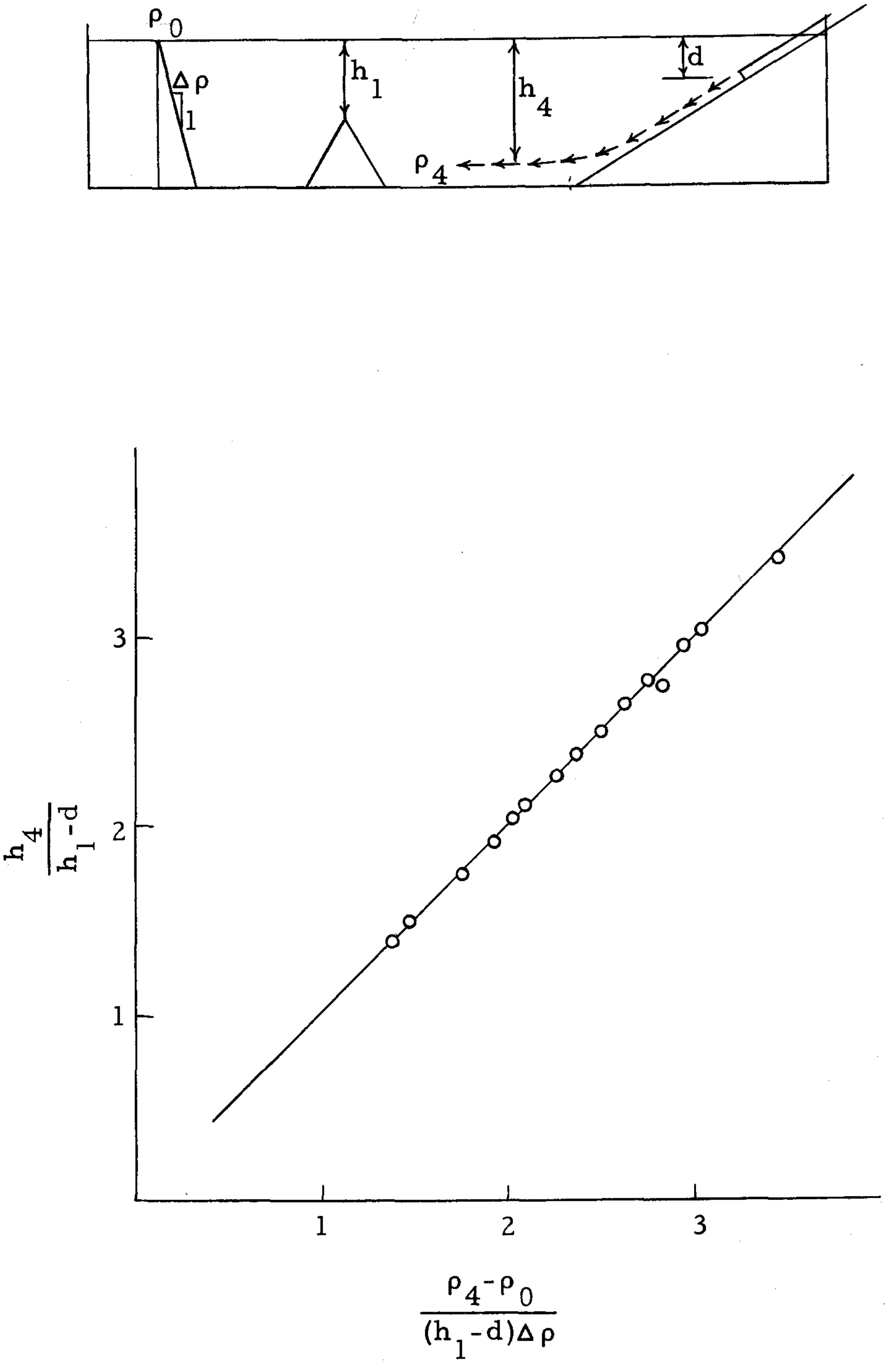


Figure 4. 3. Dimensionless plot of depth of inflow current versus density parameters.

at a higher discharge velocity or on a steeper slope.

The Mountain Current

As soon as withdrawal starts, a current is formed at the elevation of the mountain. This current is of the same magnitude as the withdrawal current. This was made clear when some tests were run without inflow. When water was withdrawn from the reservoir without simultaneous inflow, the mountain current was the only current that was formed. In tests with inflow and without withdrawal the mountain current did not form at all.

As long as the density distributions were equal on both sides of the mountain, the mountain current continued on the downstream side of the mountain as shown on Figure 4. 4.

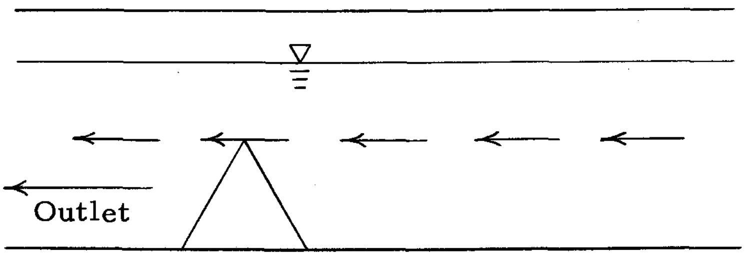


Figure 4. 4. Current over mountain with equal density distribution on both sides of mountain.

As the withdrawal continued, the layer at the elevation of the outlet was depleted and the higher laying layers moved down, causing a shift in the density profile. The void created by the withdrawal was filled by the mountain current, resulting in a wide band of water with uniform density below the mountain top. Figure 4. 5 shows the flow field in this situation.

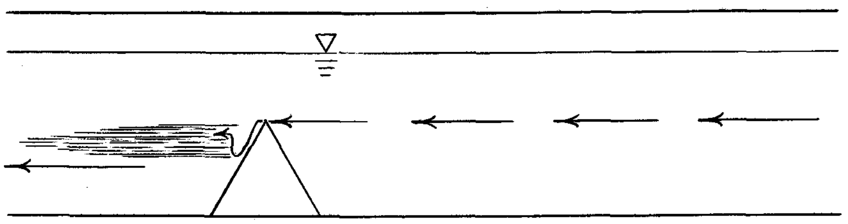


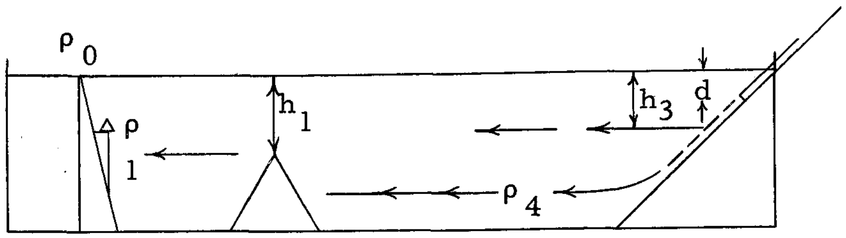
Figure 4. 5. Flow over mountain after extended period of withdrawal.

The Surface Current

With two currents flowing downstream, a return flow must be formed to satisfy continuity conditions. This return flow, q_2 on Figure 4. 1, took place in the upper layers of the reservoir. At the upper end of the reservoir an eddy was formed. This eddy caused the pathlines of q_2 to curve. Measurements made during several tests showed that the depth of the point of maximum velocity varied in the longitudinal direction of the reservoir. The extent of this current, i. e., its depth and thickness, was also found to be a function of the depth of the inflow nozzle. Bell mentions the existence of this surface current in Lake Mead. On some occasions this current will carry logs and debris to the upper end of the lake and completely block the entrance to the Colorado River (2).

The eddy formed at the upper end of the reservoir would shear on the incoming fluid and create a third current moving downstream. The depth of this current, designated h_3, is plotted in dimensionless form on Figure 4. 6. The data shown on this figure were collected from five sets of tests. In four of these sets, the parameter $\frac{d}{h_1-d}$, the depth of the nozzle in dimensionless form, was kept constant for the set, but was varied for each set. The density parameter $\frac{\rho_4-\rho_0}{(h_1-d)\Delta\rho}$ was different for each run.

For each set of tests, a linear relationship existed between the density parameter and the depth parameter $\frac{h_3}{h_1-d}$, which was the depth of the current in dimensionless form. This linear relationship appeared to be a function of the parameter $\frac{d}{h_1-d}$. The data from a fifth set of tests, with $\frac{d}{h_1-d}$ as a variable, confirmed this relationship. In four tests the parameter $\frac{d}{h_1-d}$ was varied while the density parameter $\frac{\rho_4-\rho_0}{(h_1-d)\Delta\rho}$ was kept constant. The data from these four tests plotted in accordance with the results from the other test runs.



Definition sketch $d^1 = \frac{d}{h_1 - d}$

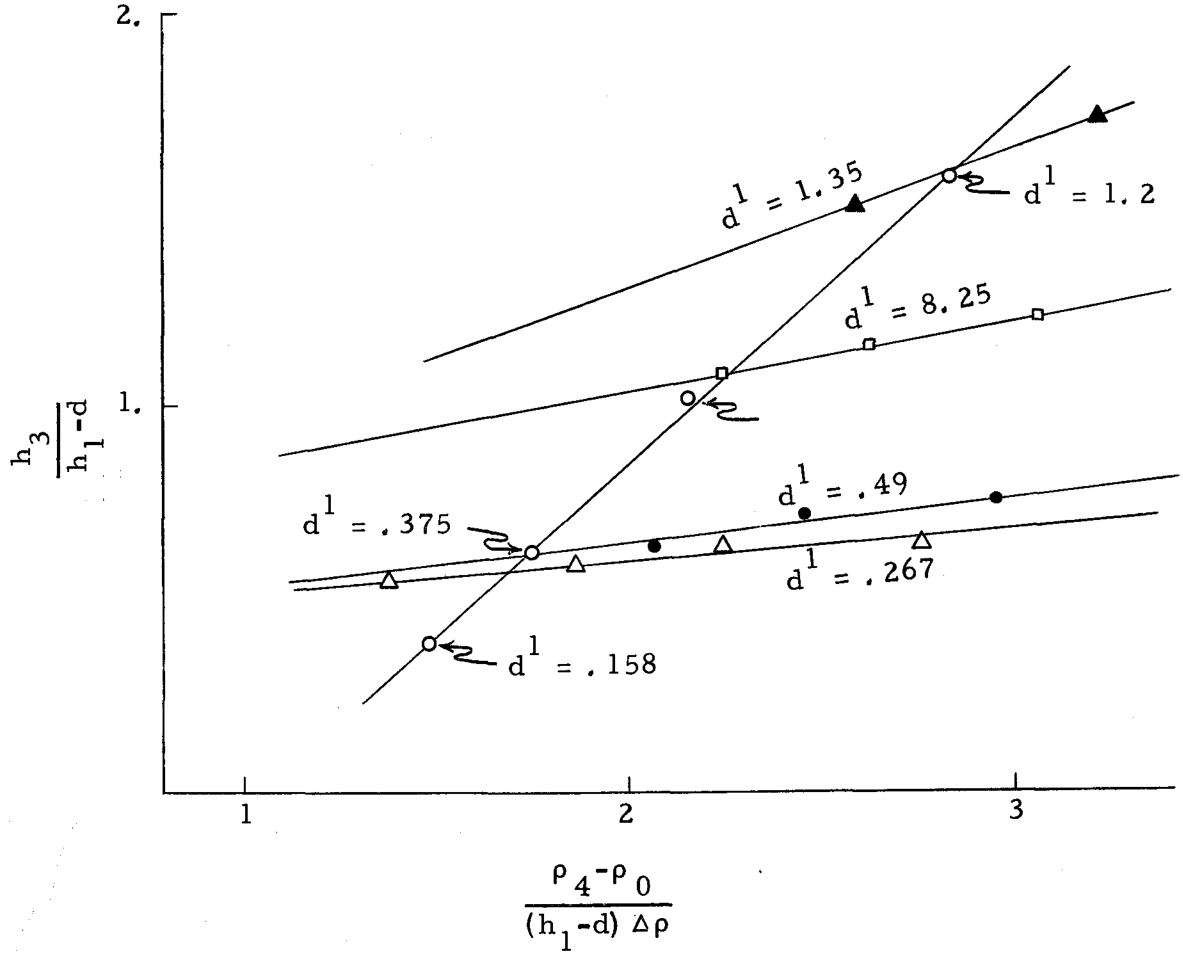


Figure 4.6. Depth of q_3 plotted versus density parameters.

DISCUSSION

The results of this investigation are here summarized. The validity of extrapolating results obtained in a model to prototype is also discussed.

Experimental Results

The effects of boundary geometry on internal density currents were demonstrated in this investigation. Other investigator's (4, 11, 13, 17) model studies on selective withdrawal from a stratified reservoir, failed to include consideration of geometrical irregularities at fluid boundaries. All the same, the results of these investigations have been extrapolated for prototype design, even though obstructions to the flow abounded in the proposed reservoir.

This investigation shows that the withdrawal layer will be at the elevation of the outlet as long as there is no obstruction to the flow. If there is an obstruction, in the form of a cofferdam or a submerged weir, the discharge on the upstream side will take place at the elevation of the top of the obstruction. Discharge will continue at the outlet level on the downstream side, but as water is withdrawn, it will be replaced by water drawn from the upstream side. The "upstream" water may differ in quality from the water initially discharged and when finally reaching the outlet, it may not meet the quality standards specified by the water user.

This investigation also shows that a submerged orifice, discharging fluid into the reservoir, had a considerable effect on the current regime. Even with a relatively high discharge velocity, little mixing took place between the reservoir fluid and the fluid being discharged. The discharging fluid behaved as an entity and progressed as a current through the model. A submerged dam would intercept this current, but another current would be created that would carry part of the discharging fluid past the obstruction.

As a measurement of turbulence and turbulent mixing in stratified fluid flow, the Richardson criteria is used. The Richardson number is defined as

$$R = \frac{-\frac{g}{\rho}\frac{\partial \rho}{\partial z}}{\left(\frac{du}{dz}\right)^2}$$

$\frac{\partial \rho}{\partial z}$ and $\frac{du}{dz}$ are the density and the velocity gradients, g is the gravitational constant, and ρ is the density of the fluid. For stability, i. e. no turbulence, Schlichting (30) gives a critical Richardson number, R_c, of 0. 0417. With a Richardson number greater than 0. 0417, the flow is laminar and little mixing will accordingly take place. In this investigation, velocity measurements were not made. To get an order of the magnitude of the Richardson number, velocity measurements were obtained from time lapse movies for one test. For the reservoir currents, a Richardson number of nearly ten was computed. For the flow down the slope, the Richardson number was about unity. Since no mixing was observed in the model, it must be concluded that the Richardson number always was greater than the critical value.

Field studies have shown that turbulent mixing always takes place when a river discharges into a reservoir. The fact that the model flow was stable was therefore surprising. In a few preliminary tests, the discharge nozzle was located above the water surface. In these tests, extensive mixing took place at the reservoir surface similar to that expected in the field. Even with densities far in excess of the maximum model reservoir density, a submerged current would not form when simulated stream discharge entered on the inlet slope above the reservoir surfaces.

Application to Prototype

If the data from a model study are to be extrapolated to prototype, similarity laws require the model and the prototype to be similar, geometrically, kinematically, and dynamically. Geometrical similarity exists if all significant geometrical parameters, in dimensionless form, are the same for the model and prototype. For dynamic similarity the Froude number, which is the ratio between the gravity force and the inertia force, and the Reynolds number, the ratio of inertia force to friction force, must be identical for the model and for the prototype. It is however impossible to simultaneously maintain both the Froude and the Reynolds criteria. As has been demonstrated by Yih (34), the Froude number, with the density gradient incorporated, is by far the most important of the two, as far as selective withdrawal from a stratified reservoir is concerned. Kinematic similarity exists when the streamline patterns in the model and in the prototype are the same.

The present study established that the geometry of the reservoir greatly effects the internal density current regime. These flow interactions can be modeled successfully when the boundary geometry is scaled to the prototype according to the Froude criteria. However, care must be exercised when model results are to be extrapolated to the prototype. Surface disturbances, caused by wind or wave action, are present in the prototype. These disturbances affect the current regime in the same way as bottom irregularities do. A natural extension of this experimental work would be to consider surface disturbances on the reservoir model.

As more factors are included in the modeling scheme, the combined effects on the flow may be more complex than expected from simple superposition of the separate effects.

Mathematical models are presently being generated and analyzed by digital and analog computers. These approaches may provide realistic results to serve as analytic descriptions of flow patterns.

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APPENDIX

Table 1. Value of physical constants.

Constant	Dimension	Value
L_1	feet	7
L_2	feet	5
H	inches	9.5
h	inches	4.5
b	inches	7.0
α	percent	8
q_{out}	cm^3/sec	45
q_{in}	cm^3/sec	45

Table 2. Experimental data.

Test	20a	20b	20c	21a	21b	21c	22a	22b
d(cm)	8.5	8.5	8.5	6.0	6.0	6.0	4.0	4.0
h_1(cm)	18.8	18.8	18.8	18.3	18.3	18.3	19.0	19.0
h_3(cm)	11.0	11.8	12.5	7.8	8.8	9.2	7.60	8.75
h_4(cm)	23.0	27.0	31.5	26.0	30.6	37.5	20.0	27.8
ρ_0	1000.6	0.6	0.6	0.3	0.3	0.3	0.4	0.4
ρ_4	1009.4	11.75	13.55	11.5	13.65	15.3	5.5	8.2
$\Delta\rho(cm^{-1})$	.38	.38	.38	.43	.43	.43	.24	.24
$\frac{h_4}{h_1-d}$	2.24	2.62	3.06	2.06	2.53	2.98	1.38	1.86
$\frac{\rho_4-\rho_0}{(h_1-d)\Delta\rho}$	2.24	2.62	3.06	2.07	2.44	2.95	1.38	1.86
$\frac{h_3}{h_1-d}$	1.07	1.145	1.21	.626	.715	.750	.506	.584
$\frac{d}{h_1-d}$	0.825	.825	.825	.49	.49	.49	.267	.267

Table 2. Continued

Test	22c	22d	23a	23b	24a	24b	24c	24d
d(cm)	4.0	4.0	10.7	10.7	3	6	9	12
h_1(cm)	19.0	19.0	18.6	18.6	22.0	22.0	22.0	22.0
h_3(cm)	9.05	9.05	11.95	13.9	7.48	9.63	13.15	15.8
h_4(cm)	35.4	41.2	17.2	23.7	27.7	27.7	27.7	27.7
ρ_0	0.4	0.4	0.6	0.6	0.6	0.6	0.6	0.6
ρ_4	10.0	11.3	6.1	7.4	6.8	6.8	6.8	6.8
$\Delta\rho$(cm^{-1})	.24	.24	0.27	0.27	0.22	0.22	0.22	0.22
$\frac{h_4}{h_1-d}$	2.38	2.76	2.59	3.20	1.45	1.73	2.13	2.77
$\frac{\rho_4-\rho_0}{(h_1-d)\Delta\rho}$	2.37	2.76	2.59	3.22	1.48	1.76	2.16	2.82
$\frac{h_3}{h_1-d}$	.604	.604	1.5	1.74	.393	.600	1.01	1.58
$\frac{d}{h_1-d}$	.267	.267	1.35	1.35	.158	.375	.69	1.2