AN ABSTRACT OF THE THESIS OF

Ellen Irene Burnes for the degree of Doctor of Philosophy in Agricultural and Resource Economics presented on June 7, 2001. Title: Three Studies on the Role of Uncertainty in the Valuation and Management of Renewable Natural Resources.

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William G. Boggess

These three papers address uncertainty in the management and valuation of renewable natural resources. The first paper develops a model to integrate both stock and price uncertainty to obtain optimal resource levels. The model is an extension of Reed (1974) and shows by proof and by example that when prices can fall below costs, considering stochastic prices is an important element of resource management.

The second paper considers the valuation of a United States Forest Service timber harvest contract. The tools and methods of mathematical finance are applied to a timber contract auction to obtain an arbitrage free minimum bid. These methods provide managers with an objective measure to integrate the effects of interest rates, costs, price volatility and contract duration into the contract bid.

The third paper fine tunes the exploration of arbitrage-free valuation of natural resource contracts. The paper shows that traditional market-based instruments are not sufficient to hedge contracts on undeveloped resources, such as standing trees in the case of the Forest Service. Rather, a market-based valuation proxy for the undeveloped resource must be obtained. The contribution of this approach is that it shows the consistency of financial theory for use with non-market assets. Also, the results provide a framework within which convenience yield measures for holding the developed, or marketable asset, may be determined. This is important because failure to account for convenience yields results in

arbitrage opportunities leaving the contract writer exposed to risk from price fluctuations.

These papers offer resource managers the rationale and tools for integrating price uncertainty into management plans. They suggest that under uncertainty, harvest levels should be more conservative when prices can fall below harvest costs, and that contract values should be adjusted to accommodate the writer's exposure to lost revenue.

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Three Studies on the Role of Uncertainty in the
Valuation and Management of Renewable Natural Resources

by

Ellen Irene Burnes

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I understand that my thesis will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my thesis to any reader upon request.

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Ellen Irene Burnes, Author

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CONTRIBUTION OF AUTHORS

I acknowledge the contributions of Enrique Thomann, Associate Professor, Department of Mathematics, Oregon State University and of Edward Waymire, Professor, Department of Mathematics, Oregon State University as co-authors on Chapters Two and Three.

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DEDICATION

Jaya Guru Devi

THREE STUDIES ON THE ROLE OF UNCERTAINTY IN THE VALUATION AND MANAGEMENT OF RENEWABLE NATURAL RESOURCES

CHAPTER ONE
INTRODUCTION

Managers of natural resources face difficult decisions in the valuation and management of natural resources due to the stochastic elements affecting both the resource and the markets that they serve. This dissertation deals with the optimal management of a natural resource given both price and stock uncertainty, the valuation of natural resource contracts, and the use of developed versus undeveloped assets as hedging mechanisms for natural resource contracts.

The dissertation is organized as a suite of three papers. A common theme of the papers is how price fluctuation relative to costs impacts harvesting decisions. The first paper utilizes dynamic optimization to determine optimal harvesting policies. The point is made that if prices are uncertain, and if prices can fall below costs, a more conservative stock management program must be engaged relative to that made under the assumption of deterministic prices. The second and third papers use applications of option pricing models to evaluate resource investments under arbitrage-free considerations.

In the first paper, *Optimal economic management of a renewable natural resource under price and stock uncertainty,* the question of the optimal economic management of a renewable natural resource is addressed as an extension of Reed (1974), *A Stochastic Model for the Economic Management of a Renewable Animal Resource*, and of Clark 1971, *Economically Optimal Policies for the Utilization of Biologically Renewable Resources*. The objective of the paper is to extend Reed's work and to show by proof and by example that stochastic prices matter in the economically efficient management of renewable resources. Proposition one says that under the assumptions of stochastic prices, the optimal harvest and stock management regimes derived by Clark and by Reed are not optimal. Proposition two states that the harvest rule made under the assumption of deterministic prices is too high if prices are stochastic in the sense that there is a positive probability that in any period, price lies below unit cost. By proof, it is shown that optimal harvest levels under the assumption of fixed prices are inaccurate when prices are stochastic. By example, an optimal harvesting rule is developed under both deterministic and stochastic prices.

The second paper, *Arbitrage-free valuation of a federal timber lease* applies the methods of option valuation from the finance literature to a US Forest Service timber lease to obtain an arbitrage-free bid value. The objective of the paper is to describe a quantitative framework for offering timber harvest bids on federal lands which includes special considerations of the volatility of timber indices and the harvesting costs involved in harvesting timber. The advantage of such an approach is that it provides a precise framework in which various underlying considerations, such as volatility and cost, may be systematically defined, measured and evaluated. The valuations derived in the paper provide a market standard against which additional value that encompasses social or environmental welfare may be evaluated. A specific USDA Forest Service timber sale is discussed as a case study to illustrate the approach. Sensitivity analyses are performed to show how price volatility, harvesting costs, and interest rates affect the model.

The third paper, *A comparison of hedging futures contracts with developed versus undeveloped assets for arbitrage-free resource harvest contract valuati*on is an extension of the second paper. In the second paper, it is assumed that timber (an undeveloped resource) is available for creating a hedge portfolio. Typically, developed assets such as stocks, or futures contracts are used in the hedge portfolio. The third paper quantifies the impact of using a developed versus undeveloped asset for valuing a futures contract for a natural resource. In this paper a real options approach is taken to natural resource contract valuation. A comparison is made between hedging with futures on developed (harvested) versus undeveloped assets. The risk-free Q martingale probability measure is obtained for the

developed asset, and modified to a cost-adjusted Q martingale in the undeveloped case. The convenience yield is examined and explicitly determined for a given price and cost structure.

CHAPTER TWO
THE OPTIMAL ECONOMIC MANAGEMENT OF A RENEWABLE NATURAL RESOURCE UNDER PRICE AND STOCK UNCERTAINTY

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2.1 Introduction

The economic management of renewable natural resources is challenging due to both environmental and market factors. Resource productivity fluctuates seasonally due to environmental variability such as weather, pestulance, and food supply. Prices that harvesters face fluctuate due to changes in demand caused by both macroeconomic and microeconomic factors. The prices of substitute goods, competition from imports, consumer incomes and preferences influence the prices that harvesters obtain. Research has made progress toward integrating the effects of price or stock uncertainty into management criteria. This paper contributes to the literature in that it provides a resource management plan that integrates price and stock uncertainty for renewable natural resource management. The importance of considering both price and stock uncertainty is made by proposition, mathematical proof, and example.

In his 1974 paper, *A Stochastic Model for the Economic Management of a Renewable Animal Resource*, Reed expands on Clark (1971), *Economically Optimal Policies for the Utilization of Biologically Renewable Resources* to include stochastic elements of renewable resource inventories in the optimal economic management of a renewable resource. However, both Clark and Reed assume a fixed unit price. The seminal paper by Arrow and Fisher (1974) as well as other work (e.g. Pindyck, 1991) since that time suggests that when prices are stochastic, it is critical that this uncertainty be included in the manager's decision framework when irreversible decisions are being made. If managers fail to account for stochastic prices, under-valuation, and over-exploitation occur. Arrow and Fisher performed their analysis on a non-renewable resource subject to irreversible development or exploitation. A consistent outcome is depicted in this paper using a renewable resource. Using Reed's paper as a starting point, it can be shown that by including stochastic prices in the same optimization problems, a more conservative harvest schedule is obtained. That is, failure to include uncertain prices in management decisions leads to an optimistic management plan for a renewable resource.

The objective of this paper is to extend Reed's work and to show by proof and by example that stochastic prices matter in the economically efficient management of renewable resources. A two-period model is developed for this extension. Proposition one says that under the assumptions of stochastic prices, the optimal harvest and stock management regimes derived under the assumption of deterministic prices are not optimal. Proposition two states that the harvest rule made under the assumption of deterministic prices is too high if prices are stochastic. By proof, it is shown that optimal harvest levels under the assumption of fixed prices are inaccurate when prices are stochastic. By example, an optimal harvesting rule is developed under both deterministic and stochastic prices.

The paper is organized as follows: section two provides background and an introduction to general models used in the paper; section three presents the work of Clark and Reed, and develops the model for the stochastic price extension. Section four presents the example of how stochastic prices affect harvest decisions by explicitly considering harvest levels under deterministic and stochastic prices. Also, sensitivity analyses are performed to show the impact of volatile prices on the harvest level. Section five presents the policy implications and conclusions of this work.

2.2 Background

In this section, the set up for the models used in this paper is developed. Harvest and stock evolution timing assumptions are presented; evaluation of the maximum stock size under no harvest, a maximum sustainable resource and the concept of an optimal stock size under harvesting regimes are developed.

Figure 2.1 provides an illustration of the assumptions regarding harvest and stock evolution timing following Reed's model.

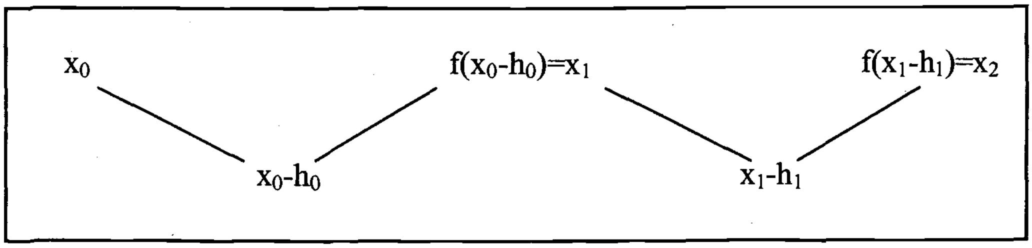

Figure 2.1: Harvest and Stock Evolution Timing

Figure 2.1 shows that the beginning level of the stock is x_0. The resource level is reduced by a harvest amount h_0. Then stock evolution, $f(x_0-h_0)$ occurs and the stock level for the next period, x_1, is obtained. The point is that first harvest occurs, and then, second, the stock grows.

2.2.1 Modeling the Stock

Before the stock's responses to harvest are considered, the evolution of the unperturbed stock is evaluated. The stock evolution model used in this paper is characterized by first order difference equation:

$$x_{t+1} = f(x_t) = \beta_{t+1} * x_t(m - x_t) \qquad \textbf{(1)}$$

where m is the environmental carrying capacity, x is the stock size, t+1 is a date and β is a growth parameter, such that $\frac{1}{m} < \beta < \frac{4}{m}$. The maximum growth occurs at $\frac{m}{2}$, and zero population growth occurs at $x_{zpg} = m - \frac{1}{\beta}$. That is, x_{zpg} is the population size where the 45° line crosses f(x) (May, 1976). Figure 2.2[1] describes the stock evolution f(x) relative to the 45° line. The maximum growth attainable, where $\beta = \frac{4}{m}$ is depicted by the heavy line, while the minimum growth,

---

[1] As an example, the environmental carrying capacity, m, is set m=5000 units

where $\beta = \frac{1}{m}$, is shown by the light-weight line. For growth at or below $\frac{1}{m}$, the stock will surely fall to zero, extinction.

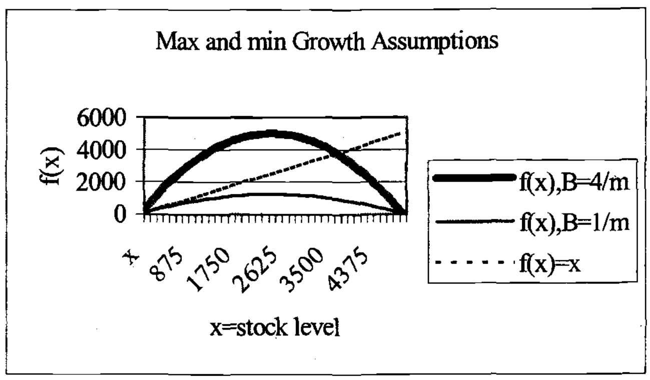

Figure 2.2: Evaluating the Maximum and Minimum Stock Growth Parameter; B

Let p be a measure of the unperturbed stock for which the growth is maximized. Its value is considered under a) deterministic stock evolution (a la Clark), b) stochastic stock evolution, with deterministic prices (via Reed) and c) stochastic stock evolution, with stochastic prices. Given Equation (1), it is the stock level, $x = \frac{m}{2} = p$ where f(x) attains its maximum on [0,m] Figure 2.3 depicts the function f(x) and the stock level p.

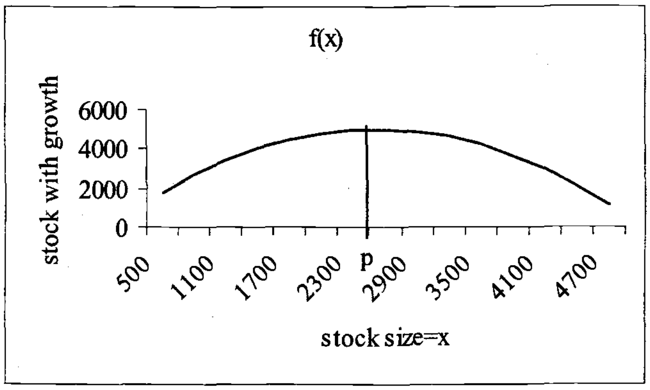

Figure 2.3: The Logistic Stock Evolution Model and the Stock Size p, that Maximizes Stock Growth

Since p is not a function of harvest, it will not be affected by prices and will remain unchanged under the model extensions considered in this paper.

2.2.2 Maximum Sure Sustainable Population Level

In Reed's extension, the stock grows stochastically via:

$x_{t+1} = f(x_t)(1+\varepsilon_{t+1}) = \gamma_{t+1} f(x_t)$, where $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_T$ is sequence of iid random variables with zero mean. Reed introduces a measure, r, which is a maximum sure sustainable population level. This is the maximum population size at which the population will, with probability one, not decrease. The measure r is evaluated under the deterministic stock evolution assumptions under harvest regimes in Clark, as well as with respect to Reed's stochastic stock evolution assumptions. r forms a basis for establishing regions where the deterministically- and stochastically-modeled-stock management plans coincide. Upper and lower bounds on stock size are considered via the parameters κ and λ.

The distribution of the stock size in each period is Φ, with expected value 1, and is supported on $[\kappa, \lambda]$; $0 \le \kappa \le \lambda$. To evaluate r, first consider the following definition of Φ.

$$P(X_{t+1} \le \tau \mid X_t = x) = P(\gamma_t * f(x) \le \tau)$$
$$= P\left(\gamma_t \le \frac{\tau}{f(x)}\right) \quad \textbf{(2)}$$
$$= \Phi\left(\frac{\tau}{f(x)}\right)$$

where $\Phi(y) = P(\gamma_t \le y)$, Φ is supported on $[\kappa, \lambda]$, $\int_{\kappa}^{\lambda} y d\Phi(y) = 1$, and $P(\gamma_t \ge \kappa) = 1$.

In words, we are looking for the stock level, such that a perturbation of magnitude γ, leaves the resource at least at the same size in the next period. Given the support on Φ, the smallest increment that the stock can fall in the next period is κ. Therefore, if $\gamma \ge \kappa$ with probability one, the stock will not fall.

The maximum sure sustainable level, r, is the solution in [0,m] to $\kappa f(r) = r$, which may exist if $\kappa > \frac{1}{f'(0)}$, otherwise r=0. It should be noted that 0 is a possible solution for r. r is subject to $0 = P\left(X_{t+1} < r | X_t = r) = \Phi\left(\frac{r}{f(r)}\right)\right)$; r= $\begin{cases} \kappa f(r) \\ 0, otherwise \end{cases}$. That is, we are looking for the r, that with probability zero will not yield a smaller stock size in the next time periods.

Under deterministic stock evolution assumptions, and under specific cases of stochastic stock development, the optimal stock level, x_0, is less than or equal to the level r, as long as r>0.[2] As long as $x_0 \le r$, the stock can *at least* be maintained at a level x_0. This result is a seminal insight from Reed, and he shows that under special considerations applied in this paper, the maximum sure sustainable resource level equals the optimal resource size (q), in the presence of harvesting. Further, for

[2] The optimal stock level is the one where g(x)=A, or the marginal cost of harvesting at x_0 exactly equals A, the price.

the specific region where $r \geq q$, optimal solutions in the deterministic and stochastic stock cases coincide.

2.2.3 The Optimal Resource Size

This optimal stock size is called q. q = x-h* (h* is the harvest level that maximizes economic value of the resource) is the resource level such that $f'(q) = \frac{1}{\alpha}$ (Clark, 1971; Reed, 1974). This is the familiar optimization result that the growth rate equals the discount factor, where $\alpha = \frac{1}{1+i}$, and i = the interest rate. When determining optimal harvest and stock levels, it is important to note that the resource is possessed by a single owner, or managed by a single management authority. That is, rent seeking and profit dissipation found in common property and open access resources are not occurring with this resource. Therefore, optimization occurs with profit maximization as the owner's objective. Further, the harvester is assumed to be operating in a perfectly competitive market, taking prices as exogenous. A contribution that this paper makes is to show that q_R*, the optimal resource level under deterministic prices is less than q_S*, the optimal resource level under stochastic prices.

2.3 The Models

In this section the Clark (1971), Reed (1974) and extension models are introduced. These models represent bioeconomic considerations of natural resource management. Clark considers a deterministic resource evolution model, and deterministic prices. Reed expands on Clark's model to consider the optimal economic use of a renewable natural resource under stochastic evolution assumptions, but maintains deterministic price assumptions. The extension of these models considers efficient stock management given both stochastic resource evolution and stochastic prices.

2.3.1 The Clark Model

Clark considers the economically efficient management of a renewable resource. As long as the growth rate of the resource is greater than the interest rate, and the resource growth function is concave, there exists an "economically optimal *equilibrium* utilization policy" (emphasis Clark). The resource evolves according to a concave, continuously differentiable stock evolution function, f(x) (Figure 2.3). Clark explores other stock evolution possibilities, but this discussion is limited to the concave case. The resource evolves according to:

$x_{t+1}=f(x_t)$

f(x) increases from 0=f(0) to f(p) for $0 \le x \le p$ and decreases for $p \le x \le m$. p represents the stock level that provides the greatest growth each year, ie where $x = \frac{m}{2}$ (e.g. see Figure 2.3).

Clark develops a two period policy in a discrete format. No price consideration is given, and stock evolution is deterministic. $C(\cdot)$ is the profit equation, where prices equal 1, and costs equal 0. The optimization becomes:

$$\max_h C_2(h) = \max_{0\le h\le x}\left[h_0 + \alpha f(x_0 - h_0)\right]. \quad \textbf{(3)}$$

Since this is a two period optimization, and costs are not a factor, it is optimal to harvest in the second (last) period (s), the intire stock that exists at the beginning of that period.[3] Recall that $x_1=f(x_0-h_0)$. Given (3), differentiate with respect to h to obtain the optimal management strategy for the first (t-1) period to obtain:

$f'(q) = \frac{1}{\alpha}$ where $q=x-h_C^*$, and q is the optimal stock level to maintain.

If q exists, the optimal two period, (h_0^*, h_1^*) policy (via Clark) becomes:

for

$$x_1 \ge q, h^* = \{x_0 - q, f(q)\} \quad \textbf{(4)}$$
$$x_1 < q, h^* = \{0, f(x_0)\}$$

If a q that solves $f'(q) = \frac{1}{\alpha}$ does not exist, then h*={x_0,0). That is, for stock levels greater than or equal to q, harvest down to q in the first period (x_0-q), and f(q) in the second period. If the stock level today is less than the optimal level, harvest nothing today, and f(x_0) in the second period. If a solution to $f'(q) = \frac{1}{\alpha}$ does not exist, this implies that the stock can not grow at the rate of interest. In this case it is optimal to harvest today, and to put the investment elsewhere.

The optimization model using equation (1) under Clark is:

$$\begin{aligned} &\max_{h \le x} C_2(h) = h + \alpha f(x-h) \\ &= h + \alpha\beta(x-h) * (m - x + h) \end{aligned} \qquad \textbf{(5)}$$

Now solve for the optimal first period harvest and subsequent resource to leave for the next period. Set $\frac{\partial C_2}{\partial h}\ \frac{\partial C}{\partial h} = 0$, and obtain $h^* = x + \frac{1}{2\alpha\beta} - \frac{m}{2}$. Further the optimal resource to leave is x-h*=q_C*= $\frac{m}{2} - \frac{1}{2\alpha\beta}$, where q_C* is the optimal resource level under deterministic stock evolution.

Comparing p with q it can be seen that $\frac{m}{2} > \frac{m}{2} - \frac{1}{2\alpha\beta}$ and that the model is consistent with the stipulation that the optimal stock level under a harvesting regime is lower than the optimal stock size under no perturbation. That is, the stock size for which growth is maximized. A manager seeks to maintain stock levels to the left of p to assure positive stock growth, and to maximize the present value of revenues, and to maximize the present value of revenues.

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[3] When costs are a function of stock size, costs can become prohibitively high (ie unit costs greater than price), as stock levels fall

Now consider r, the maximum sustainable resource level. In the deterministic stock evolution case, there is a binary distribution of stock outcomes.

That is $$\Phi\left(\frac{t}{f(x)}\right)=\begin{cases}1, if, \dfrac{t}{f(x)}\geq 1\\ 0, if, \dfrac{t}{f(x)}<1\end{cases}. \quad \textbf{(6)}$$

In this case, $\kappa=\lambda=1$. So, r=κ *f(r)=f(r). Now we can derive the r, such that $\frac{r}{f(r)}\leq 1$. Using the stock evolution equation, we get: $\beta * r(m-r)\geq r$, which gives as solution: $0<r\leq m-\frac{1}{\beta}$. As long as the scaling factor, β, satisfies $\beta>\frac{1}{m}$, this equation is satisfied.

2.3.2 The Reed Model

In addition to expanding on the Clark model to incorporate stochastic stock evolution, Reed also explicitly considers the role of prices, marginal costs, and fixed costs in his determination of the optimal harvesting strategy.

Reed assumes that the stock evolves according to:

$$x_{t+1}=\gamma_{t+1} * f(x_t)=f(x_t)+\varepsilon_{t+1} * f(x_t), \quad \textbf{(7)}$$

where $\varepsilon_1, \varepsilon_2, ..., \varepsilon_T$ is a sequence of iid random variables with zero mean, and the expected value of γ equals one. The price of the resource equals A, a constant price per unit. g(x) is the unit cost of harvesting the resource when the population level is x. K is a fixed set-up cost payable each time a harvest is undertaken. The profits earned in one period when harvest (h) is greater than zero are:

$$C=Ah-\int_{x-h}^{x} g(v)dv-K. \quad \textbf{(8)}$$

Reed assumes that the harvester operates as a profit maximizer and takes prices as exogenous. Under this set up, it is possible to obtain an (S,s) policy which describes when it is optimal to fish (x=s), and to what stock level one should fish (x=S). The (S,s) policy says that if a stock is at least of size s, harvest down to S.

For example, today, we can harvest x_0-S = h_0. Tomorrow, the stock will grow to f(x_0-h_0). Then we can harvest x_1-S = h_1. Then we are left with x_1-h_1 =S and so forth. Figure 2.4 depicts this relationship.

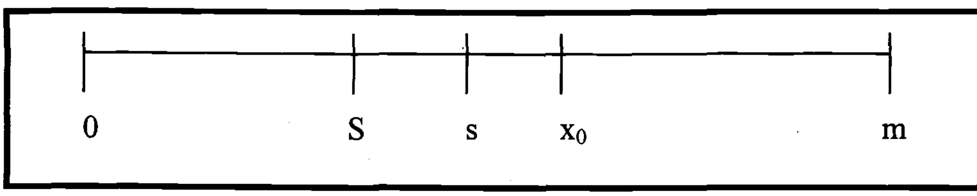

Figure 2.4: The Relationship Between S,s and the Stock Size x

In the example depicted in figure 2.4, there will be a harvest because x_0 is greater than or equal to s. The harvest will be of size x_0-S. m is the maximum environmental carrying capacity.

Simplifying the analysis, Reed develops his theorem 3(A):

"If f is strictly concave on [0,m] and g(x) is constant and there is no set-up cost (K=0) then [...] there exists an optimal pure Markovian policy determined by a sequence $\{S_T\}_{T=1,2,...}$"...where the harvest rule is: when there are T periods remaining, harvest down to S_T if and only if the population exceeds S_T.

That is, if g(x) = g for all stock levels, and there is no set-up cost K, we obtain the single S policy described above. Now the two period maximization problem may be developed:

$$\begin{aligned}\pi(a,x,h) &= \max_{h_0,h_1}\{\pi_0(a,x_0,h_0)+\alpha\pi_1(a,x_1,h_1\} \\ &= \max_{h_0,h_1}\{(a-g)h_0+\alpha(a-g)h_1\} \qquad \textbf{(9)} \\ &= \{(a-g)h_0^*+\alpha(a-g)h_1^*\}\end{aligned}$$

where a equals the price. Solve iteratively, beginning with the second period.

$$\max_{h \le x} \alpha\pi(a,x,h) = \alpha\pi(a,x,h_1^*(x))$$
$$\alpha\pi(a,x,h) = \alpha(a-g)h_1$$
$$h_1^*(a,x) = \begin{cases} x, if, a \ge g \\ 0, if, a < g \end{cases} \qquad \textbf{(10)}$$

That is, as long as a, prices, are greater than marginal costs, harvest the entire remaining stock in the second period (as in Clark).

The Bellman equation becomes:

$$C_2(x,h) = (A-g)^+ * h_0 + \alpha(A-g)^+ * f(x-h_0) \qquad \textbf{(11)}$$

Differentiating with respect to h_0 we get the Clark result that h^*_R is the harvest that gives the remaining stock (q) to solve:

$$\frac{1}{\alpha} = f'(x - h_R^0)$$

The implication is that under the limiting conditions of K=0, and g(x) =g, a constant, the stochastic stock case reduces to the same solution as the deterministic stock evolution scenario presented in Clark. Theorem 4 (Reed), states the sufficient condition that the stochastic stock solution collapses to the deterministic solution as long as $r \ge q$. However, under the class of models considered in this paper, where stock evolution follows a logistic function, the two solutions coincide for all values of r and q (e.g. q_R=q_C for all x) and while r>q is a sufficient condition in Reed's paper, it is not always necessary.

2.3.3 The Maximum Sustainable Population Level

According to Reed's stock model, $X_1 = f(x_0)*(1+\varepsilon)$. The stock disturbance, $(1+\varepsilon)$ can be at most λ, and at least κ. For the maximum sustainable population condition to be satisfied, $X_1 = f(x_0)*(1+\varepsilon) \ge f(x_0)*\kappa \ge X_0$ must hold, and, as before, $\kappa * f(r) = r$. For the stock evolution model in this paper, $f(q) = \beta q*(m-q)$, and

$$q = \frac{m}{2} - \frac{1}{2\alpha\beta} = x - h.$$

Computing r for the model, $f(q) = \beta q * (m - q)$, we obtain:

$$\kappa * f(r) = r$$
$$\Leftrightarrow r = \frac{\kappa\beta m - 1}{\kappa\beta} = m - \frac{1}{\kappa\beta} \quad \textbf{(12)}$$

Now we need to check that $r - q_R^* \geq 0$.

$$r - q = m - \frac{1}{\kappa\beta} - \frac{m}{2} + \frac{1}{2\alpha\beta} \geq 0$$
$$\Leftrightarrow \alpha\beta\kappa m + \kappa - 2\alpha \geq 0 \quad \textbf{(13)}$$

Since $r - q \geq 0$, it is possible to show that there is a region of parameter values where the stochastic (Reed) and deterministic (Clark) solutions for the optimal stock level q overlap. First, solve for α as a function of κ, β, m. Note that $0 \leq \kappa \leq 1$. This is because $E(1 + \varepsilon) = 1$, and κ represents the lower bound of ε. Obtaining the equation, $\alpha = \frac{\kappa}{2 - \beta\kappa m}$, and graphing it we see the exponential curve depicted in Figure 2.5, which asymptotically approaches the upper bounds of κ. The lower bound of α on the y axis is determined by setting $i = 1$ (recall that $\alpha = \frac{1}{1+i}$), to obtain $\alpha_{\min} = \frac{1}{2}$. The upper bound of α occurs when $\kappa = 1$, and $\alpha_{\max} = \frac{1}{2 - \beta m}$. These values are also graphed on figure 2.5, where $\alpha_{\min} = A\min$ is represented by a thick line at $\alpha = \frac{1}{2}$, and $\alpha_{\max} A\max$ is represented by the thin line.

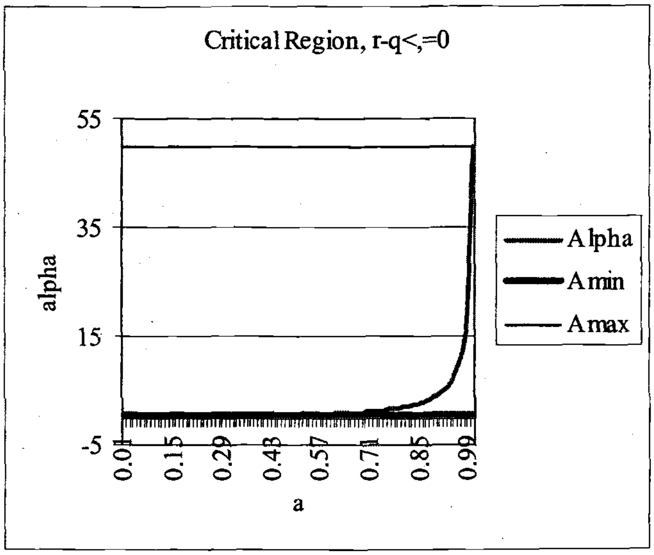

Figure 2.5 The Coincidence of the Deterministic and Stochastic Stock Management Solutions

Then, the region depicted by $\int_{\frac{2}{2+\beta m}}^{1} \frac{\kappa}{2-\beta\kappa m} d\kappa$ is the region where the parameter values in Reed's model that give q_R* coincide with the optimal stock level q_C* under the deterministic stock evolution case.

2.3.4 Extension: Stochastic Prices

Now consider the model when prices are stochastic. This is an important extension because if one looks at the price processes of marketable commodities such as fish, lumber, or cotton, it can be easily noted that prices are not deterministic, but rather stochastic. In this section, it is shown that incorporating price uncertainty into management decisions leads to more conservative management practices. In this extension, prices are no longer known with certainty, but rather, their inclusion in the model is based on expected values from an underlying exogenous distribution. It is shown through proof by contradiction that the optimal harvest level, h_R*, that satisfies $\frac{1}{\alpha} = f'(q)$, derived under the assumptions in section III.2, is no longer efficient under stochastic prices.

Under stochastic prices, the Bellman equation becomes:

$$C(a,x,h) = (a-g)^{+} * h + \alpha\gamma f(x-h)\{E_a A_1 \mid [A_1 \geq g] - gP(A_1 \geq g)\}$$

$$\Leftrightarrow \tag{14}$$

$$C(a,x,h) = (a-g)^{+} * h + \alpha f(x-h)\left\{ a\int_{\frac{g}{a}}^{\infty} nf_n(n)dn - g * P\left(N \geq \frac{g}{a}\right)\right\}$$

That is, the optimal value of harvesting the resource is the profits available today plus the discounted value (α) of the stock size next period given that the expected price (E_a is the expected value of A under an exogenous price distribution), A_1, is greater than or equal to the marginal cost, g. The pricing model in this paper is as follows:

$$\frac{A_{t+1} - A_t}{A_t} = \tau_{t+1};$$

$$A_{t+1} = (1+\tau)A_t$$

$$= N_{t+1} * A_t$$

The evolution of the price ratio $\frac{A_{t+1}}{A_t} = 1 + \tau$, is described by the density function $f_n(n)$. The density is used to illustrate the calculations. However, it is not necessary; the general model described in Equation 14 accommodates binomial tree models as well.

Notice that this model accommodates the stochastic inventory, but due to the nature of γ (that is, $E(\gamma) = 1$), we obtain $f(x_0\text{-}h)$ as the expected value of the inventory next period.

PROPOSITION ONE:

Unless price is greater than cost with probability one, the management policy which considers deterministic prices is never optimal if prices are stochastic.

It is shown via proof by contradiction, that h* obtained under Clark, and Reed's limiting cases is not optimal. Consider:

(i) f(x-h)=f(q)|h* ; note that $f'(q) = \frac{1}{\alpha}$

(ii) $f_n(n)$ is the density function which describes the distribution of price ratio

(iii) $E(N) = \int_0^\infty n f_n(n)dn = \mu$; let $\mu = 1$

Now substitute condition (i) into the Bellman equation to obtain:

$$C(a,x,h) = (a-g)^+ h + \alpha f(q) a \int_{\frac{g}{a}}^{\infty} n f_n(n)dn - \alpha f(q) g * P\left(N \geq \frac{g}{a}\right) \quad \textbf{(15)}$$

To prove by contradiction, maximize with respect to h and set the derivative equal to zero. We will show that the $\frac{\partial C}{\partial h}$, using the optimal h_R* solution under stochastic prices, where the $P(A_1 < g) > 0$, is strictly less than zero.

Proof

$$\frac{\partial C}{\partial h} = (a-g) - \alpha f'(q) a \int_{\frac{g}{a}}^{\infty} n f_n(n)dn + \alpha f'(q) g * P(N \geq \frac{g}{a}) \underset{set}{=} 0$$

Note: $\alpha f'(q) = \alpha \frac{1}{\alpha} = 1$

$$\frac{\partial C}{\partial h} = a\left(1 - \int_{\frac{g}{a}}^{\infty} n f_n(n)dn\right) + g\left(P(N \geq \frac{g}{a}) - 1\right)$$

$$= a\left(1 - \int_{\frac{g}{a}}^{\infty} n f_n(n)dn\right) - gP\left(N \leq \frac{g}{a}\right)$$

where

$$g\left(P(N \geq \frac{g}{a}) - 1\right) = -g\left(1 - P\left(N \geq \frac{g}{a}\right)\right) = -g * P\left(N \leq \frac{g}{a}\right)$$

$$\frac{\partial C}{\partial h} = a\left(1-\int_{\frac{g}{a}}^{\infty} nf_n(n)dn\right) - g\int_{0}^{\frac{g}{a}} f_n(n)dn$$

$$= a\left\{1-\left(\int_{0}^{\infty} nf_n(n)dn - \int_{0}^{\frac{g}{a}} nf_n(n)dn\right)\right\} - g\int_{0}^{\frac{g}{a}} f_n(n)dn$$

$$= a(1-\mu) + a\left(\int_{0}^{\frac{g}{a}} nf_n(n)dn\right) - g\int_{0}^{\frac{g}{a}} f_n(n)dn$$

$$= a(1-\mu) + \int_{0}^{\frac{g}{a}} (an-g)f_n(n)dn$$

$$= a(1-\mu) + a\int_{0}^{\frac{g}{a}} \left(n-\frac{g}{a}\right)f_n(n)dn << 0$$

Note: $(1-\mu) \le 0$ and $\int_{0}^{\frac{g}{a}} (n-\frac{g}{a})f_n(n)dn < 0$. Therefore, h_R* can not optimally solve the Equation 14 and is not optimal under stochastic prices.

PROPOSITION TWO:

If prices are modeled as stochastic, rather than deterministic as in Reed, Reed's optimal stock solution q_R* is too small and a more conservative harvesting policy must be invoked.

PROOF:

Consider again the Bellman equation that incorporates stochastic prices:

$$\max_{h \le x} C_2(a,x,h) = (a-g)^+ h + \alpha E(\gamma * f(x-h)) * \left[E_a A \,\|\, [A \ge g] - gP(A \ge g)\right]$$

$$= (a-g)^+ h + \alpha\beta(x-h)(m-x+h)\left[a\int_{\frac{g}{a}}^{\infty} nf_n(n)dn - g\int_{\frac{g}{a}}^{\infty} f_n(n)dn\right]$$

Differentiating with respect to h we get:

$$\frac{\partial C}{\partial h} = (a-g) + \alpha\beta(2x - m - 2h)[\varsigma - \vartheta] \overset{set}{=} 0$$

where

$$\varsigma = a \int_{\frac{g}{a}}^{\infty} n f_n(n) dn$$

$$and, \vartheta = g \int_{\frac{g}{a}}^{\infty} f_n(n) dn$$

Solving for h_S*, the optimal harvest under stochastic prices, we obtain:

$$h_S^* = x - \frac{m}{2} + \left(\frac{a-g}{\varsigma - \vartheta}\right)\frac{1}{2\alpha\beta}$$

which differs from the Reed solution, h_R*, by $\frac{a-g}{\varsigma-\vartheta}$. This means that if $(a-g) > (\varsigma - \vartheta)$, then h_S*>h_R*. Conversely, if $(a-g) < (\varsigma - \vartheta)$, then h_S*<h_R*.

Given (a-g), the value of prices and marginal costs today, the E(A-g), the value of prices and marginal costs in the next period, can be written as:

$$0 < a - g = E[(A-g) \mid [A \geq g]] + E[(A-g) \mid [A < g]]$$

$$E[(A-g) \mid [A < g]] < 0$$

therefore,

$$E[(A-g) \mid [A \geq g]] > a - g$$

also,

$$E[(A-g) \mid [A \geq g]] = \varsigma - \vartheta$$

Then, $\frac{a-g}{\varsigma-\vartheta} \leq 1$, with equality iff $A \geq g$, with probability 1.

Thus, h_R*>h_S*. □

2.4 Example

Having shown in the general case that stochastic prices require an alternative management regime, now it will be shown by specific example using the model developed in section III. Prices are typically modeled as following a lognormal distribution. The problem with this distribution is that a closed form solution is not possible. Instead, an exponential distribution is used since a closed form solution is possible. Using this distribution, it is possible to obtain an h^*_S, the optimal harvest size under stochastic prices, that can be compared with h^*_R, the optimal harvest under deterministic prices.

The Bellman equation for a two period profit maximization from the harvest of a renewable natural resource under stochastic prices and inventory is:

$$\max_{h\leq x} C_2(a,x,h) = (a-g)^+ * h + \alpha E(\gamma * f(x-h))[E_a(A_1 \mid A \geq g) - g * P(A \geq g)]$$

$$= (a-g)^+ * h + \alpha\beta(x-h)*(m-x+h)\left[a\int_{\frac{g}{a}}^{\infty} n f_n(n)dn - g\int_{\frac{g}{a}}^{\infty} f_n(n)dn\right] \tag{16}$$

as shown on page 19.

Using an exponential distribution for prices so that the final solution is tractable, $f_n(n) = \frac{1}{\mu}e^{-\frac{n}{\mu}}$, and $P(N \geq x) = e^{-\frac{x}{\mu}}$. For simplicity, let $\mu=1$. Now equation 16 becomes:

$$\max_{h\leq x} C_2(a,x,h) = (a-g)^+ h + \alpha\beta(x-h)*(m-x+h)\left[a\int_{\frac{g}{a}}^{\infty} n e^{-n} dn - g\int_{\frac{g}{a}}^{\infty} e^{-n} dn\right] \tag{16a}$$

Now set $\frac{\partial C}{\partial h} = 0$, and solve for h_S*. h_S* is explicitly derived in 2.1 to obtain:

$$h_S^* = x - \frac{m}{2} + \frac{1}{2\alpha\beta} * \left[\frac{(a-g)}{ae^{-\frac{g}{a}}}\right] = x - \frac{m}{2} + \frac{1}{2\alpha\beta} * \left[\frac{1-\frac{g}{a}}{e^{-\frac{g}{a}}}\right] \tag{17}$$

Recall that $h_R^* = x - \frac{m}{2} + \frac{1}{2\alpha\beta}$. Since $1 - \frac{g}{a} < e^{-\frac{g}{a}}$, for g>0, $h_S^* < h_R^*$, and the harvest level under stochastic prices is lower than the harvest level under deterministic prices.

Another interesting question is how do prices affect the harvesting decision, that is, what is $\frac{\partial h}{\partial a}$? When prices are modeled as deterministic, there is no relationship between prices and the harvest level and $\frac{\partial h_R^*}{\partial a} = 0$. However, when prices are modeled as stochastic, the relationship between prices and harvest becomes:

$$\frac{\partial h}{\partial a} = \frac{1}{2\alpha\beta}\left(ge^{\frac{g}{a}}\left(1 - \frac{g}{a}\right) + e^{\frac{g}{a}} g\left(1 - \frac{g}{a^2}\right)\right) > 0 \tag{18}$$

This means that as prices increase, so will harvests. This makes sense if one considers that the optimal resource size is $f'(q) = \frac{1}{\alpha}$. Higher harvests under higher prices result because as prices become higher, there is a higher opportunity cost to leaving a resource, so more is harvested, until the productive capacity of the resource ($f'(q)$) matches the opportunity cost of money $\left(\frac{1}{1+\alpha}\right)$.

2.5 Conclusions

We have presented the model of Clark (1971) which developed a bio-econonmic management plan for a renewable natural resource under the assumptions of stochastic stock evolution, and prices. Second, the Reed (1974) model and results were presented as extensions of the Clark paper. Reed extended Clark's model under the case of stochastic stock evolution and deterministic prices. Under Reed's theorem 3 and 4, it could be shown that for certain parameters, the management plan for a natural resource modeled under stochastic evolution assumptions coincided with that of a resource modeled under deterministic evolution assumptions. Third, from this set of results, stochastic prices were brought into the model to show that management decisions made under stochastic prices, even under limiting conditions, are more conservative than those made under the assumption of deterministic prices. A significant outcome of modeling prices as stochastic is that it is possible to obtain Reed's results as a special case. That is, when $A \geq g$ with probability one, the model collapses to the Reed outcome. Modelling the prices as stochastic provides the manager with a tool to address a greater range of stock management conditions. For example, the manager can evaluate how changes in stock evolution, and price change impact harvest levels. Under the framework presented here, the role of costs becomes a measurable parameter.

Finally, an example was provided to demonstrate the usefulness of these results, and to show the outcome of sensitivity analyses on the harvest level to price changes. The form of the models developed in this paper is useful to managers who wish to better understand how harvest and stock levels are dependent on prices. As one would expect, there is a positive relationship between prices and harvest levels. The model can be expanded to accommodate additional boundary conditions such that stock level not fall below a q_{min}.

In addition to the extensions of the Reed (1974) and Clark (1971) models, the paper provides additional insight into Arrow and Fisher's (1974) model. First, the model considers a renewable resource. Second, it provides a method to integrate the condition and evolution of the resource into the model. Third, it expands on the result that if benefits (prices) are uncertain, managers should proceed with more conservative development (harvest) plans. The results in this paper suggest that, when prices can fall below costs, incorporating price uncertainty into management decisions is important. Further research which develops appropriate price models for renewable resources is necessary, and a possible extension of this work (e.g. Plantinga and Provencher, 2001). Such research could generate further understanding of the relationship of harvest timing and price.

2.6 Literature Cited

Arrow, Kenneth and Anthony Fisher. 1974. *Environmental Preservation, Uncertainty and Irreversibility*. Quarterly Journal of Economics. vol. 88, no. 2. pp. 312-319.

Clark, Colin C. 1971. *Economically Optimal Policies for the Utilization of Biologically Renewable Resources*. Mathematical Biosciences. vol. 12. pp. 245-260.

May, Robert M. 1976. *Simple Mathematical Models with Very Complicated Dynamics*. Nature. vol. 261. Pp. 459-467.

Pindyck, Robert S. 1991. *Irreversibility, Uncertainty and Investment*. The Journal of Economic Literature. vol. 29. pp. 1110-1148.

Plantinga, Andrew J. and William Provencher. 2001. *Internal Consistency in Models of Optimal Resource Use Under Uncertainty*. Oregon State University, unpublished paper.

Reed, William. 1974. *A Stochastic Model for the Economic Management of a Renewable Animal Resource*. Mathematical Biosciences. vol. 22. pp. 313-337.

2.7 Appendix

Deriving h_s*

h_s* is derived as follows:

let $\varsigma = \int_{\frac{g}{a}}^{\infty} ne^{-n} dn$, and let $\vartheta = g\int_{\frac{g}{a}}^{\infty} e^{-n} dn$

The Bellman equation is:

$$\max_{h \le x} C_2(a,x,h) = (a-g)^{+} h + \alpha\beta(x-h) * (m-x+h)[\varsigma - \vartheta]$$

$$\frac{\partial C}{\partial h} = a - g + \alpha\beta(2x - 2h - m)[\varsigma - \vartheta] \overset{set}{=} 0$$

$$h_S * = x - \frac{m}{2} + \frac{a-g}{2\alpha\beta(\varsigma - \vartheta)}$$

$$\varsigma = a\int_{\frac{g}{a}}^{\infty} ne^{-n} dn = ae^{-\frac{g}{a}}\left(\frac{g}{a}+1\right)$$

$$\vartheta = g\int_{\frac{g}{a}}^{\infty} e^{-n} dn = ge^{-\frac{g}{a}}$$

$$\varsigma - \vartheta = e^{-\frac{g}{a}}(a+g) - ge^{-\frac{g}{a}} = ae^{-\frac{g}{a}}$$

$$h_S^{*} = x - \frac{m}{2} + \frac{1}{2\alpha\beta}\left[\frac{1-\frac{g}{a}}{e^{-\frac{g}{a}}}\right]$$

CHAPTER THREE

ARBITRAGE FREE VALUATION OF A FEDERAL TIMBER LEASE

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3.1 Introduction

The principle of no-arbitrage provides that two instruments with the same payoff will have the same price. That is, if selling trees or bonds tomorrow will yield the same profit then buying the trees or bonds today should cost the same amount. A timber lease provides the right to harvest the wood subject to a prescribed payment rule, as well as other parameters e.g. duration, base rates, and harvesting costs. In particular this payment rule depends on the type of contract, e.g. escalated or non-escalated, and a minimum bid rate, A. This agreed upon rule exposes both the lease writer and holder to a risk of revenue loss or gain depending upon the underlying price of wood at harvest. As the lease writer, the government measures this exposure relative to an underlying weighted average of timber prices, I, referred to as a timber index. The basic idea which will be pursued in this paper is as follows. The government receives a certain amount g(A) for the contract from the lease holder. To offset revenue losses, the government may invest an amount p(A) in a portfolio that matches the exposed revenue loss. Equating g(A) with p(A) leads to an arbitrage free minimum bid rate A appropriate to the terms of the contract and the amount received for the contract. That is the two assets, the timber contract and the off-setting portfolio, yield the same return. The no-arbitrage value for the cutting price A in terms of the contract parameters thus holds these values equal.

To address the issue of obtaining a no-arbitrage value for a federal forest lease we develop a theoretical framework in which to systematically determine a minimal bid rate A at which timber prices should be advertised, given the conditions currently prescribed by Federal contracts.

Of particular interest to us are: (i) The effect of fixed investment costs, and (ii) The problem of incorporating the volatility of timber indices into the determination of A when the risk assumed by the writer is fully diversifiable. The costs of harvesting standing timber are investment costs which, unlike the transaction costs which occur in financial securities markets, are one time costs that the government returns to the lease holder. We shall see that the incorporation of investment costs leads to reasonably simple modifications of the standard no-arbitrage pricing by a Black-Scholes-Merton equation. This is in significant contrast to the difficulties which arise in mathematical pricing theory in the presence of transaction costs; e.g. see Karatzas (pp.117-133,1997).

There are a number of approaches available to incorporate volatility. For example, one may attempt to stochastically model the underlying indices and compute expected present values. However when risk is fully diversifiable such an approach is well-known to lead to prices susceptible to arbitrage in securities markets and, as we shall show in this paper, in timber leases as well. We demonstrate that the no-arbitrage theory of mathematical finance provides a fairness standard (i.e. arbitrage free) from the point of view of the writers of the types of timber lease contracts found in practice.

While we will attempt to make this paper accessible to non-experts in the modern theory and methods of finance, a number of new textbooks have recently appeared which provide more detailed treatments of the essential aspects of the theory required here; e.g. see Dixit and Pindyck(1994), Hull (1993), and Baxter and Rennie (1997). In particular there are two rather deep results from mathematical finance theory which will play a primary role. One result is that the average rate of return of timber prices is transformed to the risk free interest rate in the final no-arbitrage pricing formula. In particular the numerical value of the average return plays no role beyond formulation of the model, where it is subject to the natural efficient market constraint that the average return be larger than the risk free interest rate; e.g. see Hull (p. 221, 1993). The second standard result is that the no-arbitrage price is a discounted expected value based on a mathematical expectation that involves a special risk-neutral weighting; e.g. see Baxter and Rennie (p. 120,1997). This new probability weighting, referred to as a martingale change of measure, is explicitly furnished by the theory for the type of timber index model being considered here; namely the geometric Brownian motion model, e.g. see Dixit and Pyndick (p.71,1994), Hull (p.198, 1993) or Baxter and Rennie (p.51, 1997). We will explain the essential ideas for these results as they arise in our treatment of timber lease pricing.

In the papers by Morck, Schwartz and Strangeland (1989), Reed (1993), Reed and Haight (1996) for example, the authors seek to compute optimal harvest schedules using geometric Brownian motion models to describe the stochastic evolution of such quantities as prices and timber inventories. While this problem is quite different from that of the present paper, the solutions are obtained under similar hypotheses on the evolution of underlying parameters. It will be of interest to determine time scales on which the statistics of timber indices are captured by this model, however its main justification is its simplicity.

Our interest is in a theoretical determination of the advertised price A based on no arbitrage principles. The fee collected may be viewed as an advertised price plus a premium determined by the auction value. The premium is an independent consideration which is determined by the sealed bid methods. In this regard we call attention to the body of literature available from auction theory for the setting of minimum bids; McAfee and McMillan (1987), Paarsch (1997). In particular the authors are grateful to an anonymous referee for pointing out the paper by Paarsch (1997) in which it is (empirically) shown how to calculate the minimum acceptable bid for timber sales in order to obtain the desired selling price. This paper complements the auction theory by furnishing a no-arbitrage value of the desired lease selling price.

The overall structure of this paper is as follows: In Section 3.2 we describe the elements of two particular types of Federal timber leases, escalated and non-escalated. Here standard contract terms are recast in mathematical terms. Since it is not obvious how one may view a timber lease as derivative contract, in Section 3.3 we provide a brief overview of some basic general principles of mathematical finance which will facilitate the identification of timber leases as certain types of options made in Section 3.4. Then in Section 3.4 we introduce a stochastic model for the timber indices and show precisely how the timber leases translate into option contracts. We then calculate the no-arbitrage values of escalated and non-escalated contracts, respectively. From the point of view of standard finance theory the incorporation of costs is a delicate matter and may lead to an incomplete market, see Baxter and Rennie (p.196-200, 1997). For the purpose of hedging the exposed risk, our calculation assumes that the writer has available the same investment opportunities as the bidder. That is, both have access to standing timber. We will see that this leads to a determination of the no-arbitrage value of escalated and non-escalated timber leases. A case study will be provided in Section 3.5 to numerically illustrate the theory with parameter values used in current practice. Section 3.6 is a summary of the paper offering discussion and concluding remarks.

3.2 Elements of a Federal Timber Lease

One of the last steps in timber sale planning is the appraisal which provides the agency's estimate of the value of the timber over the term of the lease. It is the appraised value which is the minimum harvest price (e.g. $\$/m^3$) for which the timber is ultimately advertised to the public for bid. There are two main ways in which the minimum bid is determined in practice, one is by transactions evidence (TE) and the other is by residual value (RV). The TE method begins by estimation of a fair market value (FMV) of stumpage to be sold and then applies a competition premium.

By contrast, RV does not seek a FMV but is the difference of the expected selling value of the end product and an industry average harvesting costs. While the TE method involves sampling error in the average it is not explicitly incorporated into the pricing. On the other hand, in Section 4 we will see that approaches based on expected selling prices, such as RV, are subject to arbitrage regardless of how well the model fits the data; see Baxter and Rennie (p.9,1997).

Under the National Forest Management Act, bids on timber leases offered by the Forest Service may be either sealed or oral. However the sample contracts we have analyzed were awarded in competitive sealed bid auctions.

The Forest Service sets an advertised price A which serves as the minimum allowable bid. The maximum bid M is awarded the contract and the awardee is required to post 20% M. This amount (without interest) will be applied to the payment for wood once 25% of the contracted timber amount is harvested. Thus the interest earned $(e^{-r\tau}-1).2M$, where r is the risk free interest rate and τ is the schedule time by which 25% harvesting occurs, may be viewed as the lease price paid by the awardee for the contract to cut during some specified period of time [0,T] covered by the lease agreement after adjustment to present value, i.e. $e^{-r\tau}(e^{-r\tau}-1).2M=(1-e^{-r\tau}).2M$.

The lease contract provides an implied obligation to harvest at a price K which is specified net of cost according to two different formulae depending on whether the contract is escalated or not. In an escalated contract the price K depends on the movement of an underlying base price index I. While this index typically lags the actual price of wood (Random Lengths,1997), it is this index which, after adjustment for cost, provides the government's benchmark price on which decisions are made. In particular, the escalated value of K is prescribed by

$$K=\begin{cases}\max\{B+c,I\}-c=\max\{B,I-c\} & \text{if } I\le M+c\\ \dfrac{I-c+M}{2} & \text{if } I\ge M+c\end{cases} \tag{1}$$

where B denotes a minimum acceptable price that the contract will accept for harvesting, referred to as the base price, c is a cost adjustment, and I is the index value at payment. The base price B and the bid M are both quoted net of cost, whereas the index I is not net of cost. There are two ways in which this can be reconciled in the formulae. Namely, one can either return the costs to the net values or one can make the index I net of cost. As it turns out the former adjustment is mathematically more convenient in the model formulation . The primary reason is that this keeps the index values positive, whereas net of cost indices may take negative values. Moreover, parameters such as the volatility should not depend on the particular operating costs, but rather be determined by the index price evolution. There are a number of different types of cost adjustments which may occur in a specific contract, e.g. average harvesting costs, base rate adjustments, buyer obligated add-ons etc. Note that the specific nature of these costs will be immaterial to our pricing method so long as we are consistent both mathematically and with the contracts we analyze with these adjustments. The point is that while the way in which these costs are distributed will obviously affect the final price, consistency is the only issue for the approach followed here. Finally, by our considerations of auction premiums described above we are interested in the valuation with $M = A$ throughout this paper.

Similar considerations apply to the non-escalated contract where the harvest price, K is prescribed in the lease and is fixed throughout the term of the lease. As we will see in the next section, this is a particularly interesting contract in the way that no-arbitrage considerations give a price which is independent of the stochastic evolution followed by the timber indices. Such a phenomenon is well-known in the pricing of forward contracts; e.g. see Hull (p.52,1993), or Baxter and Rennie (p.16, 1997).

3.3 Some Principles of Mathematical Finance

The principles of arbitrage theory have their origins in the valuation of financial instruments such as options, forwards, and other derivative contracts. The adaptation of this theory to the valuation of timber leases such as described in the previous section is based on the observation that it is the payoff function to the holder of a contract which essentially defines the "option". Equivalently, the payoff to the holder is an exposure to lost revenue for the writer. For example, a European call option is a contract which gives the holder the "option" to buy a security at some specified strike price K on some date T. Denoting today's value of the security by S_0, the value at a future time t is a stochastic quantity S_t whose evolution is often modeled by a geometric Brownian motion parameterized by an average return μ and a volatility σ; see Dixit and Pyndick (p.71,1994), Hull (pp.196-200,1993), Baxter and Rennie(p.51,1997).

Arbitrage pricing theory assumes rational decision making to the extent that the call option will be exercised by the holder if and only if the value S_T of the security at maturity T is above the strike price K. Thus the exposure to the writer is a function of the behavior of the underlying asset prices alone. In the present case of a call option it is $S_T - K$ if $S_T > K$, or 0 if $S_T \leq K$, which is typically expressed more briefly as $(S_T - K)^+$. Mathematically this translates into the working principle that it is the formula for the payoff function in terms of the underlying asset which determines whether or not an "option" is exercised or not. In this way the contract is no more an option than is a forward contract where one is " obligated" to buy a security at some specified strike price K at an agreed upon date T, i.e. a forward is defined by some other payoff function of the underlying prescribed by the rules of the contract, namely $(S_T - K)$ regardless of whether S_T is above or below K. A negative payoff is a loss to the holder.

In view of the above discussion, an option contract is defined by a prescribed payoff function to the holder or an equivalent exposure to lost revenue of the writer. Another example which will be of interest to us here is that of a put contract. The European put gives the holder the option to sell the underlying security at a specified strike price K and maturity T. Contracts which permit exercise of an option at any time prior to the maturity date T are referred to as American, rather than European. As one might expect, there is a put-call parity relation between the payoffs in that holding a call and writing a put is equivalent to a forward. Examples of more " exotic" contracts may be found in Hull (pp. 414-431,1993), Karatzas (pp.17-24,1997), for example. Particular exotics which will be observed include barrier options, see Hull (p.418,1993). Here they arise naturally in the context of escalated and non-escalated contracts. Also Asian put and call options, see Hull (p. 138. 1993), arise naturally in the context of escalated timber leases. Barrier options refer to contracts that are declared void if the underlying price crosses a specified value. On the other hand, Asian options refer to contracts in which the underlying asset or strike price is typically replaced by an average value, e.g. S_T or K replaced by $\int_0^T S_\tau d\tau$. Such designs discourage artificial manipulation of option values by massive short term speculations on the underlying asset, i.e. hostile takeovers.

As discussed in the introduction, the Principle of No -Arbitrage is applied to the valuation of financial derivatives via the construction of an off-setting hedging portfolio having the same payoff function as that of the derivative contract. For example, the no-arbitrage price of the European call option defined above is computed as the amount which the writer could invest at time 0 in a portfolio of risk free assets (e.g. US Treasury bonds) plus shares of the underlying asset at its present price S_0 in such a way that suitable management of the portfolio will yield a portfolio value at maturity T which exactly matches the payoff to the holder regardless of the future behavior of the underlying asset prices S_t. Thus one determines the present value of the option to be that amount required to initiate the portfolio investment. This method works to obtain the present price of the option for the two basic reasons that: (i.) There is a market for managing the portfolio of underlying asset and risk free bonds to create a hedge and (ii.) Mathematical methods are available to calculate the initial amount required to create the portfolio used to hedge the writer's exposure. For detailed illustrations for a wide variety of payoff functions see Hull (1993), Baxter and Rennie (1997), Karatzas (1997), for example.

3.4 Federal Timber Leases as Options: Mathematical Representation

In evaluating the government pricing policy it is assumed that the index I is the standard against which gains and losses are measured by the federal forest management agencies. The arbitrage theory applied here is formulated relative to this index. In particular, the lease is valued from the perspective of the writer, or government. From the lease holder's perspective, while we compute a no-arbitrage price relative to I, arbitrage opportunities may be available in the real timber market since I is not the actual market value for wood.

To formulate a pricing model we must identify the tradable asset and the time horizon of the contract. With regard to the time horizon, we simplify the payment schedule for the contract as follows:

CONDITION E: *Payment for the wood harvested is settled at the expiry T of the lease.*

As a result of this payment schedule the 20% is returned at time T rather than according to some other payment schedule which involves harvesting 25% of the prescribed volume.

We make a final assumption with regard to the development of the off-setting portfolio. Namely we assume that lumber prices track timber values and that standing timber can be used to hedge the risk in the timber contracts. There is currently a futures market for lumber made available by the Chicago Mercantile Exchange.

In order to obtain the government's assessment of a fair price relative to the underlying price index we require a model for the evolution of the underlying price index. For simplicity we assume that the base price index I evolves in time according to a standard stochastic model of the form

$$dI = \mu I dt + \sigma I dW(t) \qquad \textbf{(2)}$$

where μ is a mean yield rate parameter in the sense that $EI(t) = e^{\mu t} I_0$. The parameter σ defines the volatility in the base price index, and $\{W(t): t \geq 0\}$ is the standard Wiener's Brownian motion process governing random fluctuations. In addition, while the parameter values $I_0 > c$, T, σ found in our case study make it a rare event, if I happens to fall below c, the contract is void.

Given these assumptions, non-escalated and escalated timber leases may be formulated as option contracts as follows:

3.4.1 Non-escalated Contract

In a non-escalated contract the writer's exposure to lost revenue in terms of the price index I net of cost is simply $V = I - c - A$ for $I>c$ since the contracted price A will be paid regardless of the indices. That is, if $0<I-c<A$ then the writer has the positive payoff A+c-I, but if $I-c>A$, the writer is exposed to a revenue loss in the negative amount I-c-A. This is mathematically equivalent to a forward contract with strike price $\tilde{A}=A+c$ and knock-out barrier at c. The knock-out occurs if I falls below c.

3.4.2 Escalated Contract

In an escalated contract the writer's exposure to lost revenue in terms of the cost adjusted price index $I - c>0$ and the advertised price A is obtained from (1) as

$$V = \begin{cases} I - B - C & c < I \leq B + c \\ 0 \quad \text{if} & B + c \leq I \leq A + c \\ \dfrac{I - c - A}{2} & I \geq A \end{cases} \tag{3}$$

In particular, this is mathematically equivalent to the exposure to lost revenue in holding a put with strike price $\tilde{B} = B+c$ and one half that of writing a call with strike price $\tilde{A} = A+c$, with a knock-out barrier at c. We obtain this revenue loss by observing that: (1) Anytime the value of timber falls below B+c, the government will at minimum get B, resulting in a positive payoff, and (2) If the index rises to $I > A+c$, then the lease stipulates that the government receives one-half of the increase from the harvester. That is, the government is only exposed to one-half the lost revenue in a price increase. In particular in the case $\tilde{B} = 0$ this lost revenue V is, up to a factor of one-half, mathematically equivalent to writer's exposure in a European call option.

First let us consider the case of non-escalated contracts. In order to simplify the exposition we will first consider the case of zero costs. In view of the form of the writer's exposure in this case, the contract price $p^{(0)}(A)$ is given by the discounted expected payoff function under the risk-free martingale measure Q and is therefore calculated as

$$p^{(0)}(A) = e^{-rT} E_Q[I - A] = e^{-rT}(e^{rT} I_0 - A) \qquad \textbf{(4)}$$

The superscript 0 refers to the special case c = 0. In particular note that the price (4) is completely independent of the average yield μ and volatility parameter σ of the underlying model (3). As noted in the introduction, this is a departure from what one would get by discounting the expected value of wood in (3) to its present value $e^{-rT} e^{\mu T} I_0$. A financial argument which anticipates why no-arbitrage dictates this solution can be made as follows. Suppose that the writer receives an amount ν for the contract which is larger than the amount $p^{(0)}(A) = e^{-rT}(e^{rT} I_0 - A) = I_0 - e^{-rT} A$ for the contract. We may assume ν is smaller than I_0 for the argument without loss of generality. Then the writer will borrow $I_0 - \nu$ at the interest rate r and buy the timber to be harvested for the present value I_0. This portfolio will evolve to an amount $I_T - e^{rT}(I_0 - \nu)$. Now, at expiry he will effectively receive an amount A and give up an intrinsic timber value I_T, leaving a positive wealth $A - e^{rT} I_0 + e^{rT} \nu > 0$. The inequality is strict, meaning that the amount ν is sufficient to repay the loan, cover the exposure and pocket a positive remainder without risk. Thus, a price ν above $p^{(0)}(A)$ presents an opportunity for risk free profit when there are no costs. To see that less than this amount is also subject to arbitrage one takes the position of a buyer of such contracts in a similar way.

As discussed at the outset, the amount received by the government for the contract is the interest on the 20% contracted price described in Section 2, so that the no-arbitrage specification of A is the unique positive solution to:

$$(1 - e^{-rT}).2A = p^{(0)}(A) \qquad \textbf{(5)}$$

The left hand side of (5) is the interest $(e^{rT}-1).2A$ earned on the 20%A deposit discounted by e^{-rT} to its present value. Solving (5) using (4) one obtains the non-escalated advertised price as

$$A=\frac{I_0}{.2+.8e^{-rT}} \qquad \textbf{(6)}$$

As is typical for forward contracts, the value of A does not depend on the behavior of the underlying index I(t) beyond its initial value $I(0) = I_0$ at the start of the lease.

Now let us turn to the case of escalated contracts. The basic principles are the same but the writer's exposure depends on the underlying indices in a more complicated manner via (2). First consider the simplest case in which B = c = 0. Then the payoff function is equivalent to one-half a call. Thus in this case the no-arbitrage value of the contract in terms of the present timber index I_0 when A is the harvest price is given as a function of A by

$$p^{(0)}(A)=\frac{I_0}{2}\Phi\left(\frac{\left(\log\left(\frac{I_0}{A}\right)+\left(r+\frac{1}{2}\sigma^2\right)T\right)}{\sigma\sqrt{T}}\right)-\frac{A}{2}e^{-rT}\Phi\left(\frac{\left(\log\left(\frac{I_0}{A}\right)+\left(r-\frac{1}{2}\sigma^2\right)T\right)}{\sigma\sqrt{T}}\right) \qquad \textbf{(7)}$$

where $\Phi(z)=\frac{1}{\sqrt{2\Pi}}\int_{-\infty}^{z}e^{-\frac{1}{2}x^2}dx$ is the standard normal probability distribution function; see Hull(p.224, 1993) or Baxter and Rennie (p.91,1996) for the Black-Scholes-Merton formula for the price of a European call option. Here we use the superscript in $p^{(0)}(A)$ to denote that this is the price in the special case that B = c = 0. This is the amount needed to initiate a portfolio of standing timber and risk free bonds which will exactly match the government's exposure when there are no fixed harvesting costs.

The case in which the cost c is positive is a little more delicate. No-arbitrage dictates that the writer has at his disposal the same opportunity to purchase standing timber at I_0 - c as do the perspective holders. In particular, when there are harvesting costs, if the writer's hedge is limited to standing timber at the price I_0 then the cost allowance given to the purchaser of the lease will be insufficient to construct the hedge. This is a departure from standard formulae of finance theory and we must compute the investment required to construct this hedge.

Allowing $c > 0$, and thus $\tilde{B} = B + c \neq 0$, both the escalated and the non-escalated prices may be computed as a solution of the following cost-modified Black-Scholes-Merton equation:

$$\frac{\partial v}{\partial t} + rI\frac{\partial v}{\partial I} + \frac{1}{2}\sigma^2 I^2 \frac{\partial^2 v}{\partial I^2} - rv = cr\frac{\partial v}{\partial I} \quad \textbf{(8)}$$

with boundary condition

$$v(c) = 0 \quad \textbf{(9)}$$

with final values at time T given by (3). However the difference between this and the usual Black-Scholes-Merton equation is in the right hand side of (8); cf equation (10.20) in Hull (p.220,1993) or Baxter and Rennie (p.95,1996). A detailed mathematical derivation of (8,9) will be given in a separate paper. To qualitatively understand the appearance of this new term on the right hand side of (8) one must keep in mind that we assume the same opportunities are available to the writer as to the holder when making the hedge. So, while the underlying timber index values I evolve according to the usual model (3), the hedge is based on an off-setting portfolio of standing timber at prices I-c and bonds at the risk free interest rate r.

The cost-modified partial differential equation (8) may be solved as a discounted expected value using the Feynman-Kac formula and standard relationships between stochastic differential equations and parabolic partial differential equations; details are given in Appendix A. Specifically, the solution of (8) with final values at time T given by (3) may be expressed as a discounted expected value of the form:

$$v(I,t) = e^{-r(T-t)}E(V(X_{T-t})) \quad \textbf{(10)}$$

where X_t, $t \geq 0$, is the diffusion process defined by the stochastic differential equation

$$dX_t = r(X_t - c)dt + \sigma X_t dW_t, X_0 = I \quad \textbf{(11)}$$

and an absorbing boundary at c.

In the case of a non-escalated contract the price $p^{(c)}(A)$ is obtained from (10) by taking V=I-A-c and t=0. As discussed in the introduction, the advertised price A is obtained by solving

$$(1 - e^{-rT}).2A = p^{(c)}(A) \quad \textbf{(12)}$$

Let us now utilize the specific form of the lease payoff (3) together with some more standard finance theory to express the lease price (10) explicitly in terms of the underlying parameters for an escalated contract. As noted at the outset, the payoff (3) is equivalent to the writer holding a put option with strike price $\tilde{B} = B + c$ and one-half a call with strike price $\tilde{A} = A + c$. Recall that the call and put payoffs are, respectively, given by $V_{call}(I) = (I - \tilde{A})^+$, and $V_{put}(I) = (\tilde{B} - I)^+$.

Putting these results together we arrive at the general formula for the price of the escalated lease as

$$p^{(\tilde{B})}(A) = v(I_0,0) = e^{-rT}EV(X_T) = \frac{1}{2}e^{-rT}EV_{call}(X_T) + e^{-rT}EV_{put}(X_T)$$

$$p^{(\tilde{B})}(A) = v(I_0,0) = e^{-rT}EV(X_T) = \frac{1}{2}e^{-rT}EV_{call}(X_T) + e^{-rT}EV_{put}(X_T) \quad \textbf{(13)}$$

In particular we obtain the general pricing formula as that of one-half an Asian type call option plus an Asian type put option on an underlying distributed as an arithmetic average of a geometric Brownian motion; see Appendix B for details. While closed form analytic expressions for such prices are quite complicated, certain mathematical tricks are sometimes used to obtain simple approximations which may be implemented on a programmable calculator; e.g. see Hull (p.422,1993). More comprehensive Monte-Carlo simulation techniques may be found in Gruber (1997) and references therein.

With these results we are now equipped to determine the corresponding advertised price A according to the objective stated at the outset of this paper. As above, the fact that the price received for the contract is the interest on the 20% contracted price described in section two yields under Condition E that the fair specification of A is $A = \tilde{A} - c$, where $\tilde{A}$ is the unique positive solution to:

$$(1-e^{-rT}).2A = p^{(\tilde{B})}(\tilde{A}) \quad \textbf{(14)}$$

Numerical illustrations of solutions to equation (14) are provided in the next section. However one last calculation needs to be described before we can proceed to the case studies. With the exception of volatility, the parameter values required to determine A may be readily obtained from published resources. To determine the volatility from a time series $I_1, I_2, \ldots, I_{N+1}$ of N+1 sample values of the base price index evolving according to (2) one may take advantage of the form of the solution in the form

$$I(t) = I_0 \exp\left\{\sigma W(t) + \mu t - \frac{1}{2}\sigma^2 t\right\}. \quad \textbf{(15)}$$

In particular the sequence $L_j \log\left(\frac{I_{(j+1)\delta t}}{I_{j\delta t}}\right), j = 1,2,\ldots, N,$ comprise stationary independent values from a Normal distribution with mean $\left(\mu - \frac{\sigma^2}{2}\right)\delta t$ and variance $\sigma^2\delta t$. Thus, for the model (2) the minimum variance unbiased estimator of volatility based on this sample of size N is

$$\hat{\sigma}\sqrt{(\delta t)} = \sqrt{\frac{1}{N-1}\sum_{j=1}^{N} (L_j - \bar{L})^2} \quad \textbf{(16)}$$

where $\bar{L} = \frac{1}{N}\sum_{j=1}^{N} L_j$; e.g. see Hull (pp.214-215,1993).

The holder of the timber lease may harvest at any time τ prior to expiry T. This aspect does not effect the result under Condition E and $\tilde{B} = B + c = 0$ since it is well-known for the payoff function of a (non-dividend paying) American call option that there is no advantage to early exercise i.e. the valuation coincides with that of the European call option; see Hull (pp. 158-160,1993), Karatzas (pp. 28-29,1997).

3.5 Case Study

For a case study we will consider two sample leases furnished to us by the USDA Siuslaw National Forest Service in Philomath, Oregon; one escalated and one non-escalated. These examples furnish a range of realistic parameter values under which we shall calculate escalated and non-escalated prices in this section. As mentioned in the introduction, it is not our intention to reproduce the bid values finally advertised by the forest service, but simply to calculate the corresponding no-arbitrage values based on the information available and under various interest rate, costs and volatility scenarios for comparison. The numerical values are computed with MATLAB and the MATLAB Financial Toolkit.

For our purposes the essential ingredients are first the time series of timber indices available. The monthly Pacific Douglas Fir indices appearing in Table R0800 of the U.S. Forest Service Automated Timber Sales Accounting System, Region 06 Pacific Northwest, are reported in Table 3.1 for a period covering March,1996 through March, 1997.

Mar 96	Apr 96	May 96	Jun 96	Jul 96	Aug 96	
51.80	54.36	56.67	56.91	60.74	64.32	
Sep 96	Oct 96	Nov 96	Dec 96	Jan 97	Feb 97	Mar 97
63.42	59.92	61.74	60.66	60.55	62.20	61.76

Table 3.1: Timber Index Values ($\$/m^3$)

A plot of this time series is given in Figure 3.1.

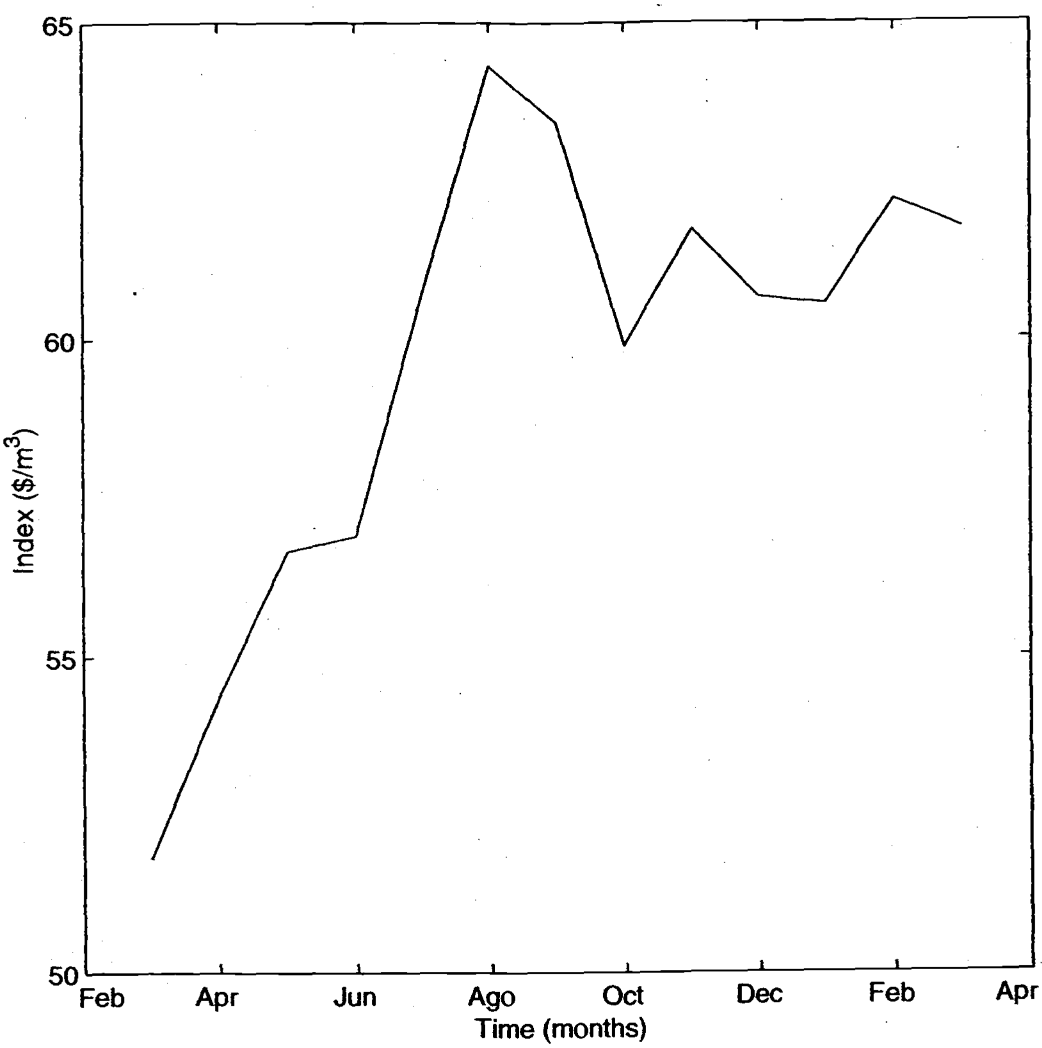

Figure 3.1: Time Series of Monthly Index Values

Based on this sample one obtains using (16) an estimate $\hat{\sigma} = .0363\sqrt{12} = .13$, or a volatility of 13 %.

The two leases which we consider have appraisal dates of July 9, 1997 and August 18, 1997. The July contract (USDA Forest Service R6-FS-2400-17, Sale Name EXAMPLE) was escalated and the August contract (USDA Forest Service R6-FS-2400-17, Sale Name ID THIN (#95234) was non-escalated. Both contracts are on Pacific Douglas Fir and prices are quoted in \$/mbf and are converted to metric units (\$/ m^3) for this paper. The July contract was not sold and was assigned the name EXAMPLE by the Forest Service. The initial values are I_0 = \$62 and I_0 = \$60, respectively. The complete set of parameter values which we require are summarized in Table 3.2 (rounded to nearest dollar).

	July Lease/ EXAMPLE	August Lease/ ID THIN
I_0	62/ m^3	60/ m^3
σ	13%	13%
C	29	13
B	27	20
T	5 yrs	5 yrs
A	24	20
$\tilde{A}$	53	32
$\tilde{B}$	33	15

Table 3.2: Lease Parameters (costs and values given in \$/ m^3)

The Forest Service priced EXAMPLE as an escalated contract and ID THIN as a non-escalated contract. The cost c refers to stump to truck, haul, road construction and other developments. Also we were not furnished the length of time intended for the lease EXAMPLE, so we assumed it to be T = 5 years for the sake of calculations.

In order to compute the advertised price we use a binomial tree to approximate the value of the contract with the cost correction as introduced in (8,9). For a reference on the use of binomial trees to evaluate options, see Hull pp. 335-362, 1993). As in Hull, we choose as binomial tree parameters $u = e^{\sigma\sqrt{\Delta T}}, d = \frac{1}{u}, p = \frac{\left(e^{r\Delta T} - d\right)}{(u-d)}$, and $\Delta T = \frac{1}{52}$. The choice of ΔT corresponds to taking price corrections every week for the duration of the contract. This choice is based on numerical considerations of discretizing a continuous time model.

3.5.1 Non-escalated Contract Prices

The non-escalated value computed from (12) and corresponding to the parameter values of ID THIN, c =13, I_0 = \$60, T = 5 with a sample interest rate r = 5% is, as an illustration, A = \$58 (as compared to the Forest Service price of \$20).

Additional solutions to (12) are depicted as points of intersection of the curves in Figure 3.2 for various cost adjustments c = \$15, 25, 35, 45 and contract length T = 5yrs. The interest rate is r = 5%.

Note that it is only for the large value of c=\$45that the relation between the advertised price A and the sale price ceases to be linear. In particular for cost values ranging from c=15 to 35 the advertised price changes in proportion to incremental changes in cost. This relationship does not hold for larger values of c. In fact, in the case of non-escalated sales, the price of the sale can be well approximated by the value of a lease that does not take into account the absorbing boundary condition (9). Moreover, in this case it is possible to solve the differential equation (8) with the final value ν(T,I)=(I-A-c) and thus obtain an approximation to the advertised price as

$$A \approx \frac{I_0 - e^{-rT} c}{.2 + .8e^{-rT}} \qquad \textbf{(17)}$$

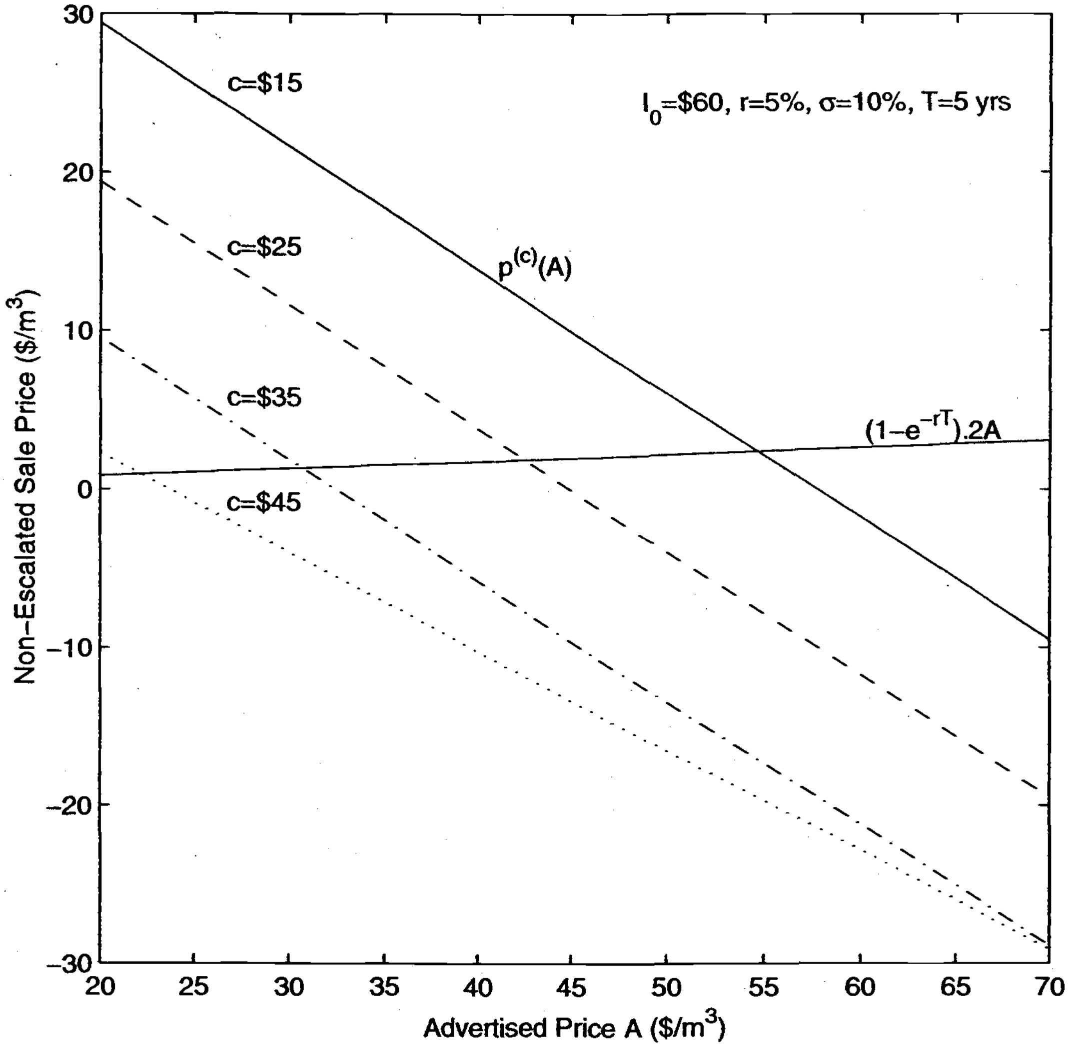

Figure 3.2 Non-escalated Sale: Effect of Cost Adjustment

When this approximation is valid, the relation between A and c is linear.

In Figures 3.3 and 3.4 we have graphically computed some non-escalated advertised prices A under a variety of different interest rates $r = 1\%$ to $r = 10\ \%$, and volatilities σ=1% to σ=20%, respectively.

Once again we note the almost linear relationship between the sale price and the advertised price. Moreover, Figure 3.4 shows that non-escalated contracts are almost insensitive to changes in the volatility. This observation agrees with the fact that, as noted above, the value of the non-escalated contracts is well approximated by the value given in (17).

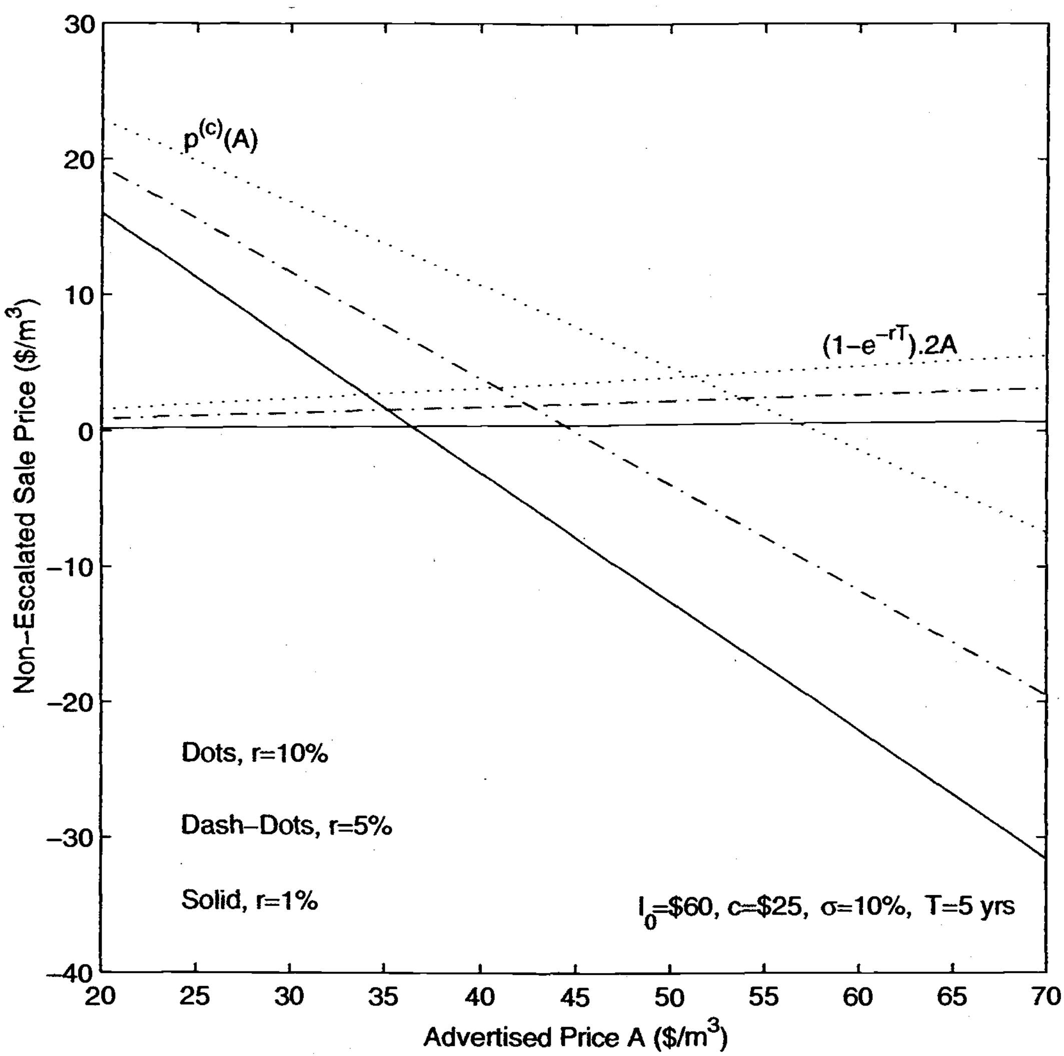

Figure 3.3: Non-escalated Sale, Effect of Interest Rates

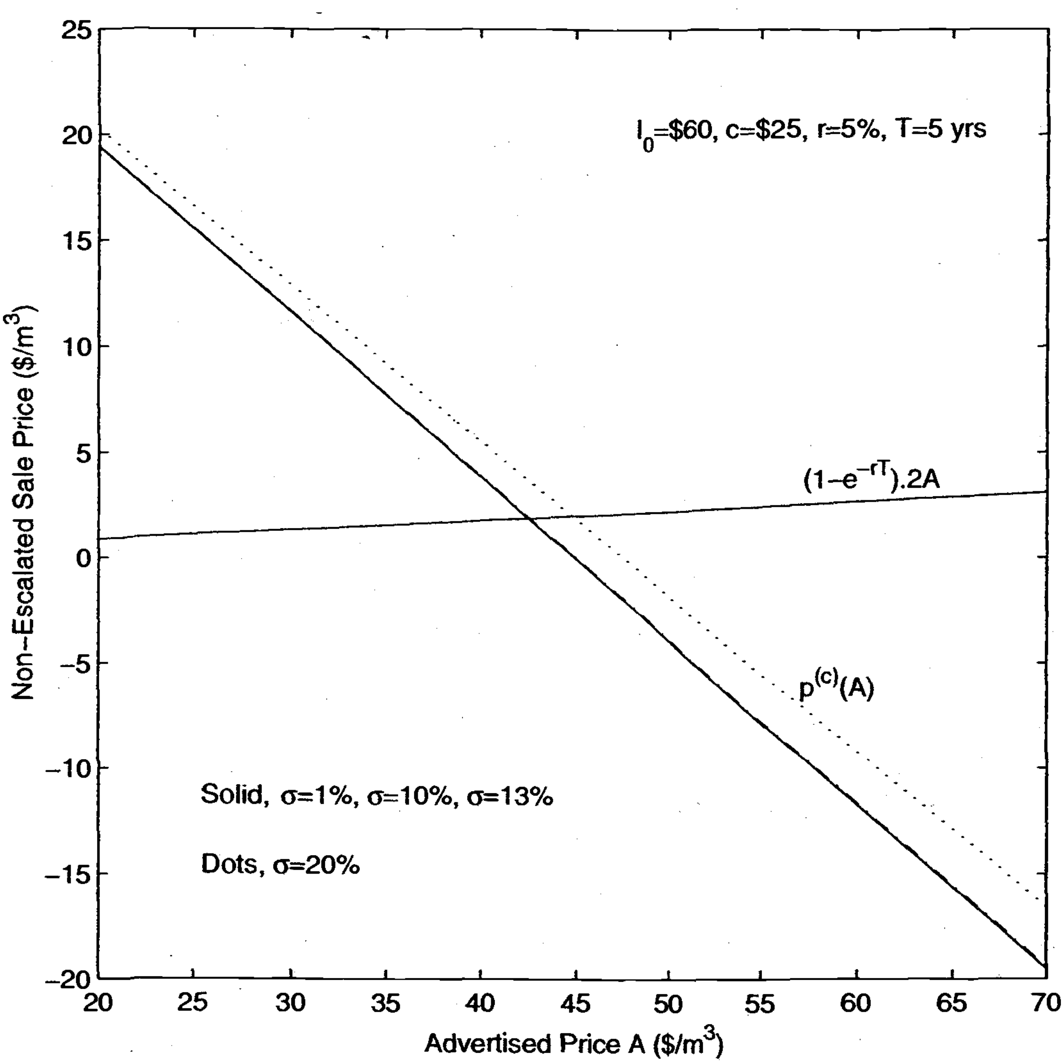

Figure 3.4 Non-escalated Sale, Effect of Volatility

In Figure 3.5 we illustrate the effect of the contract duration on the non-escalated sale. Notice that as the duration of the contract increases so does the value of the contract. This greater value captures the increases in the timber index over time.

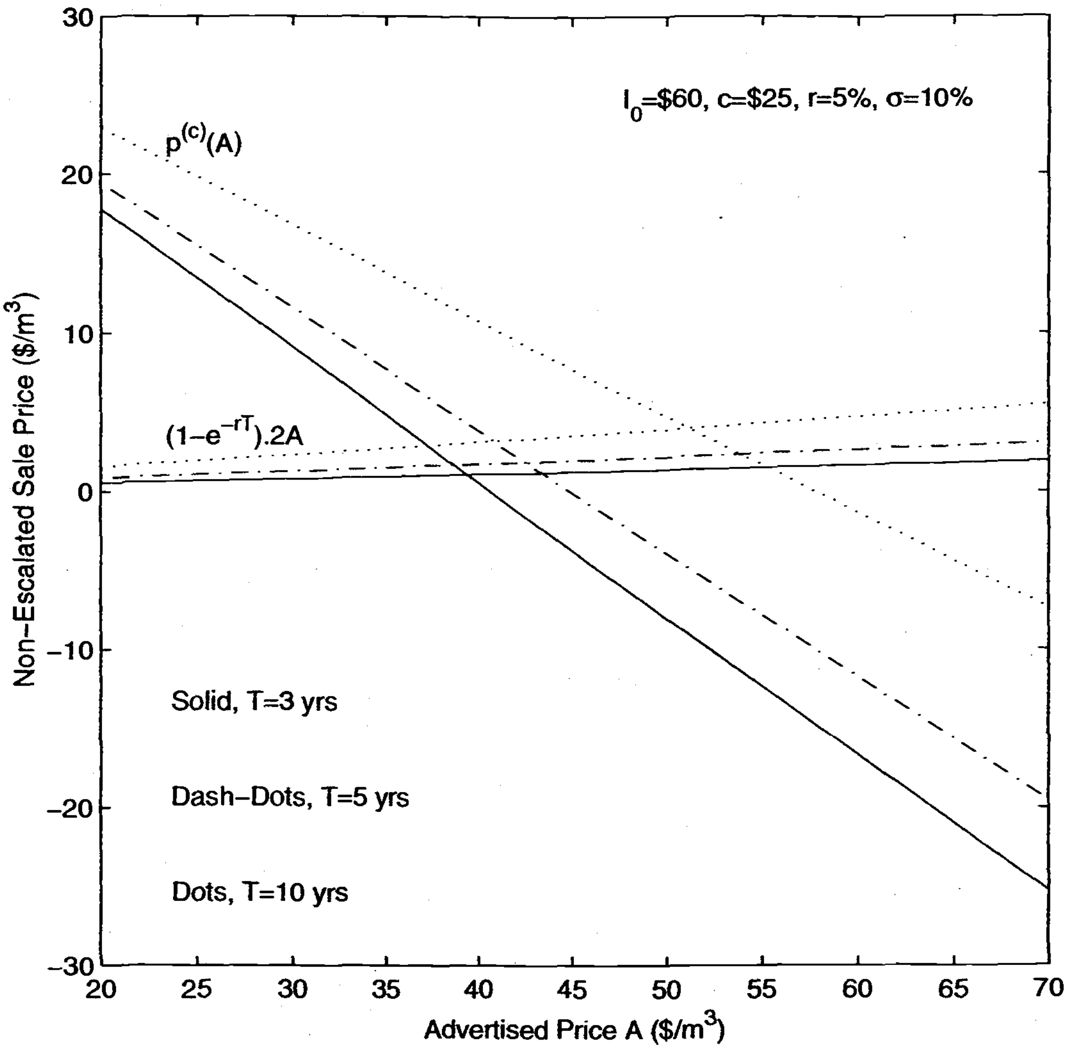

Figure 3.5 Non-escalated Sale, Effect of Contract Duration

3.5.2 Escalated Contract Prices

The escalated value numerically computed from (14) using the parameter values of EXAMPLE, $c = 29$, $I_0 = 60$, $T = 5$, $\sigma = 13$ % and the rate $r = 5\%$ is A = \$51 (as compared to the Forest Service price of \$24).

Let us remark that one may check for the range of parameter values considered here, that the contribution to the lease value from the "put" term is negligible. To validate this, Figure 3.6 shows the value of a put price when the base price B, varies from \$0 to \$50 and $I_0 = 60$, r= 5%, c= \$25 and $\sigma = 10\%$.

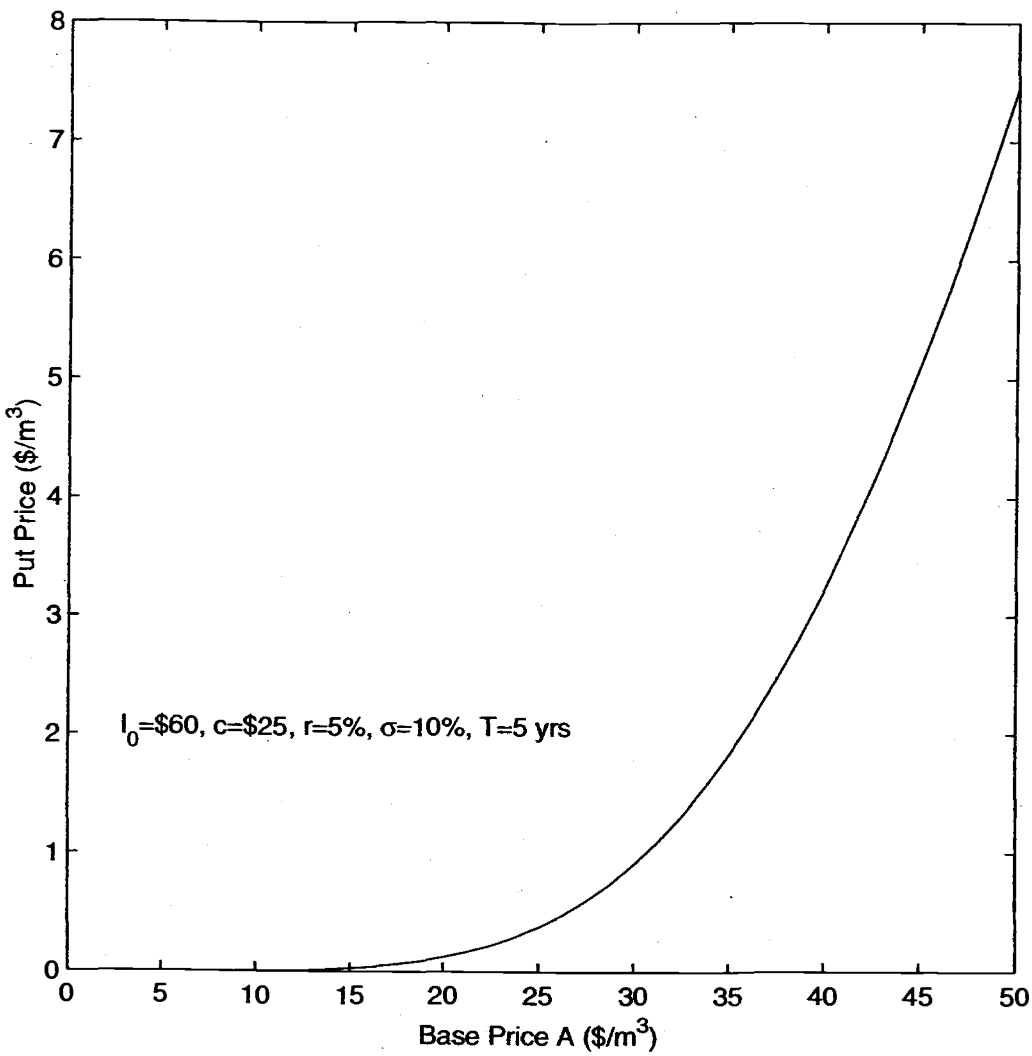

Figure 3.6: Escalated Sale, Effect of Base Price

As we can see from this graph it is only for B > \$ 30 that the effect that the "put" option has on the right hand side of (14) is more than \$1. We therefore ignore this term in the calculations that are presented below.

Additional solutions to (14) are depicted as points of intersection of the curves in Figure 3.7 for the interest rates, r= 1%, 5% , 10%. For these illustrations, the following values were used, the initial index I_0= \$ 60, the cost c = \$25, the contract length T = 5 yrs and the volatility $\sigma = 10\%$.

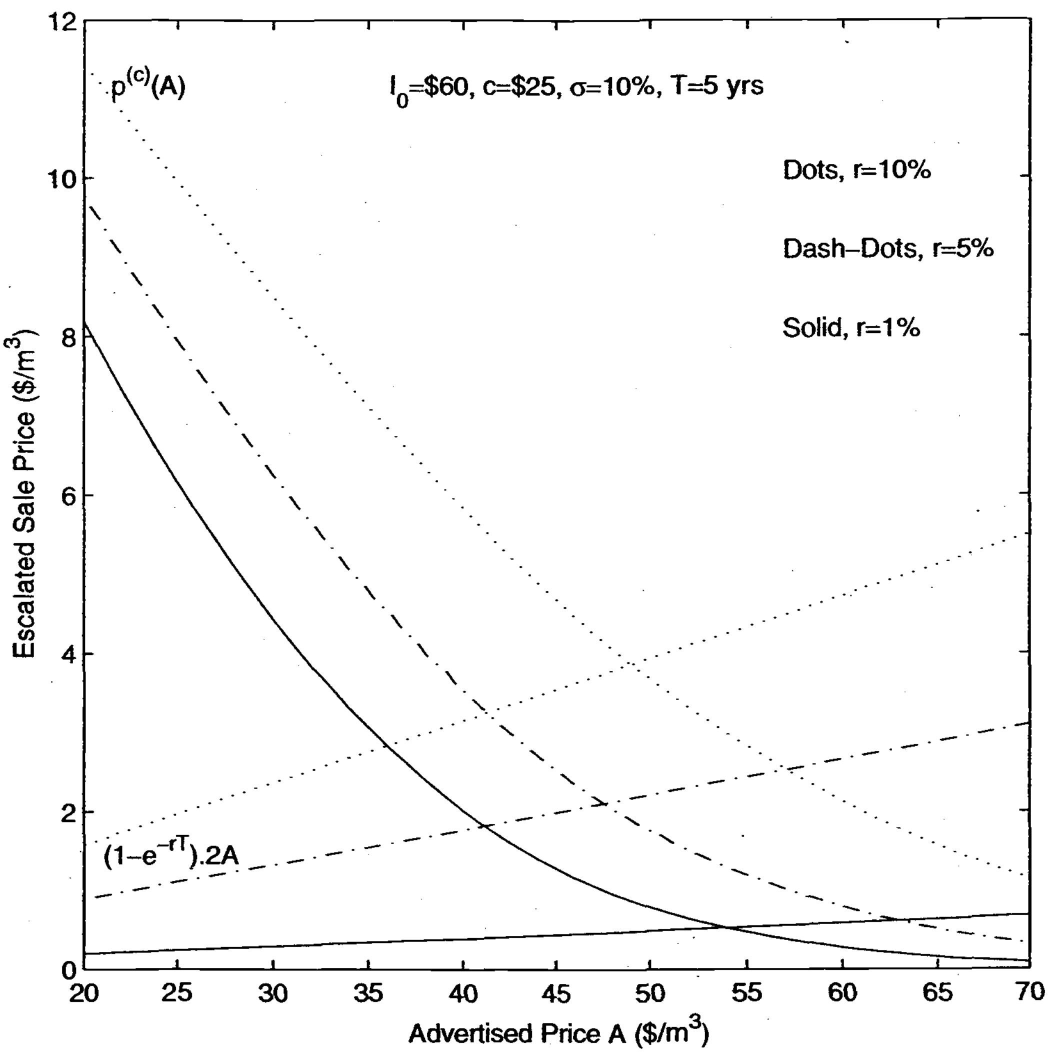

Figure 3.7: Escalated Sale, Effect of Interest Rates

We remark that changes in the interest rate have a nonlinear effect on the advertised prices. For example, increasing the interest rate from 5% to 10% only increases the price from \$48 to \$49. On the other hand, increasing the interest rate from 1% to 5%, causes a drop in price of approximately \$6 per mbf from \$54 to \$48.

We have graphically computed some escalated advertised prices A under a variety of different cost adjustments in Figure 3.8. The escalated advertised price under each adjustment is the point of intersection of each set of curves. We kept $I_0 = 60$, $r=5\%$, $T = 5$, and $\sigma = 10\%$.

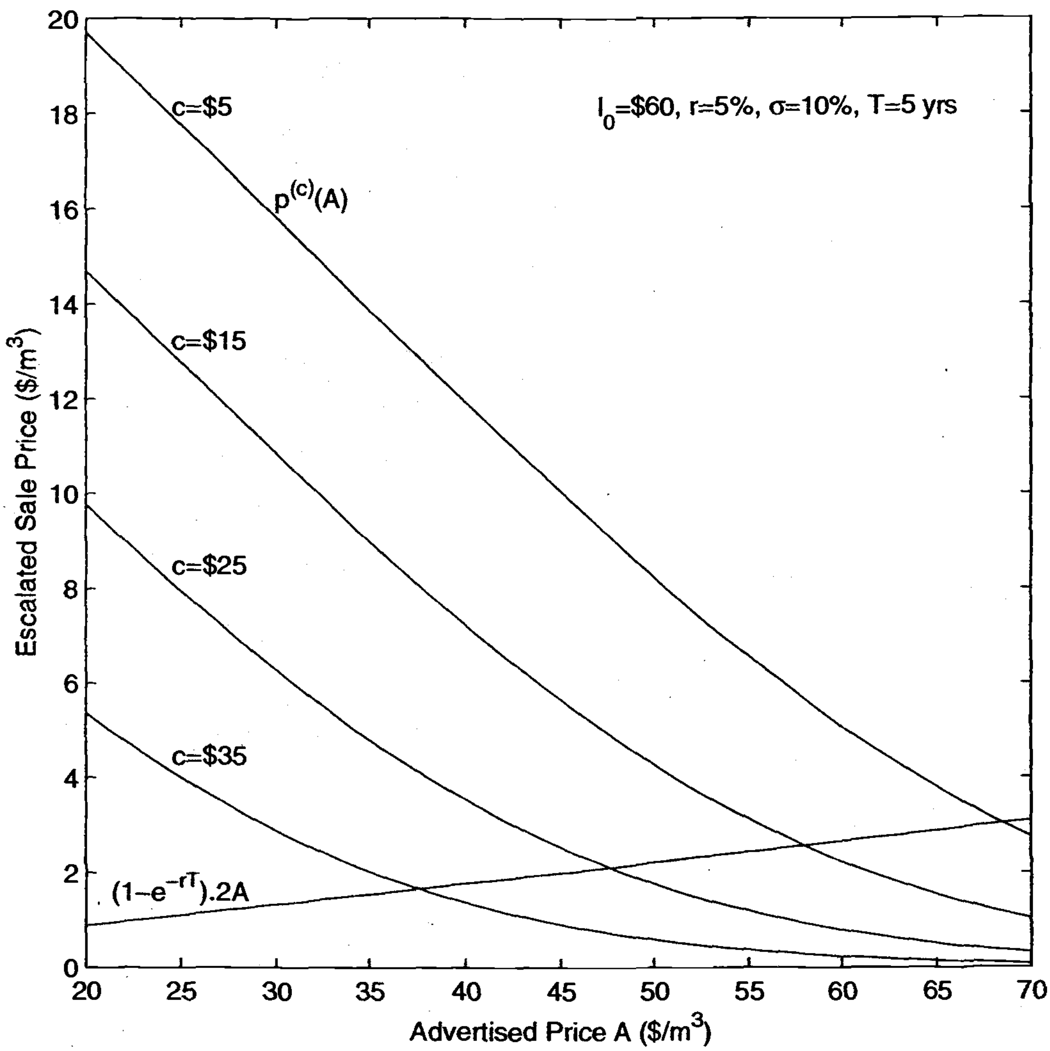

Figure 3.8: Escalated Sale, Effect of Cost Adjustment

The figures show that to a good approximation the advertised prices vary linearly with the cost adjustment c, namely an increase in the cost adjustment of \$10 corresponds to a decrease in the advertised price of the same amount. We remark however that this is only an approximation since it is also clear from the figure that the curve corresponding to c=100 is not simply the translation of the curve corresponding to c=15.

Finally, we also consider escalated pricing as a function of index volatility. Some escalated advertised prices as a function of volatility $\sigma = 1\%, 10\%, 13\%, 20\%$ for $r = 5\%$, $c = 25$, $I_0 = 60$, $T = 5$ are graphically depicted in Figure 3.9.

The nonlinear dependence of the advertised price on the volatility is apparent from Figure 3.9. For example, doubling the volatility from 10% to 20% changes the advertised price by more than \$15. On the other hand, a ten fold increase from 1% to 10% increases the advertised price by approximately \$6.

Comparing the Figures 3.7, 3.8 and 3.9, one may conclude that the advertised price is most greatly affected by changes in volatility.

Figure 3.10 illustrates the effect of the duration on the advertised price. As in the non-escalated case, we see that as the duration of the contract increases, the price does as well. This higher value captures the changes in the index over time.

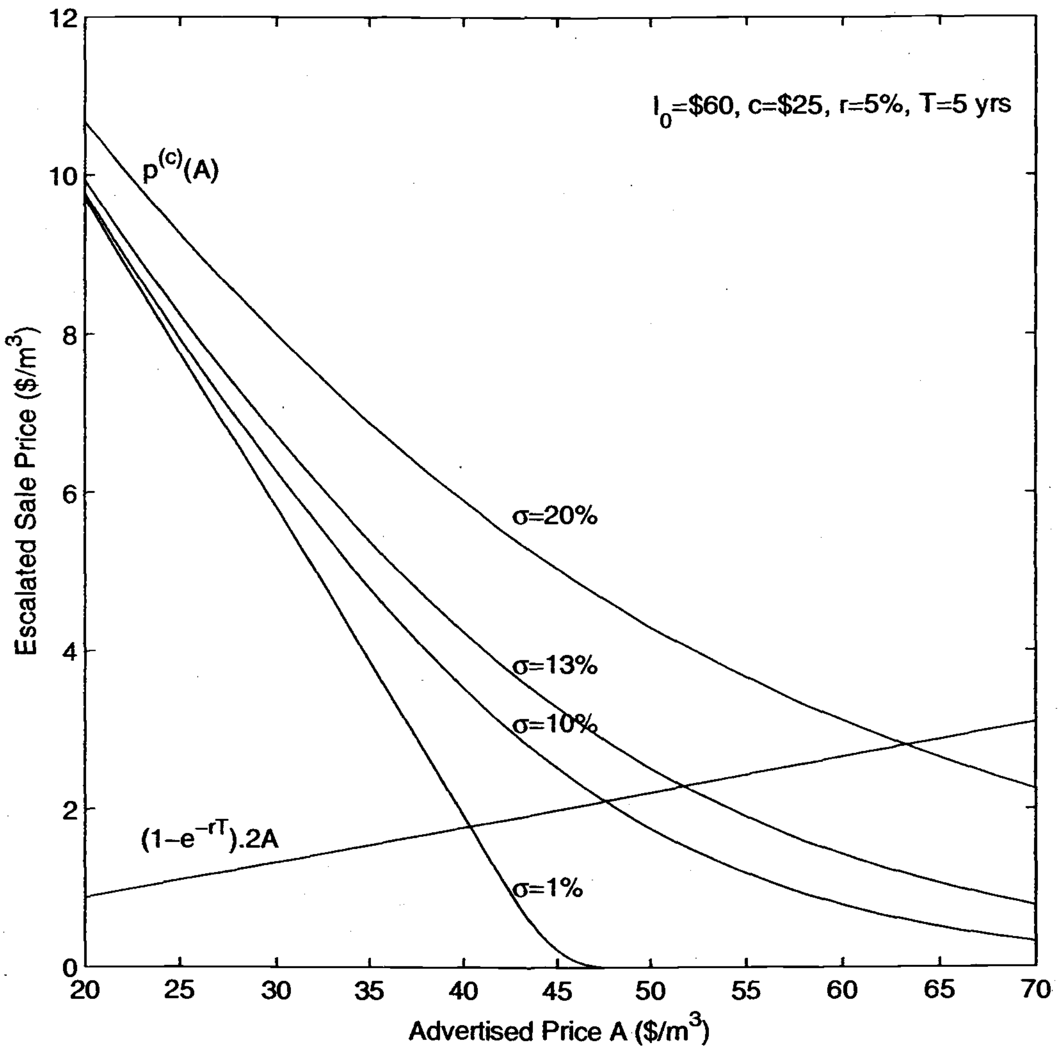

Figure 3.9: Escalated Sale, Effect of Volatility

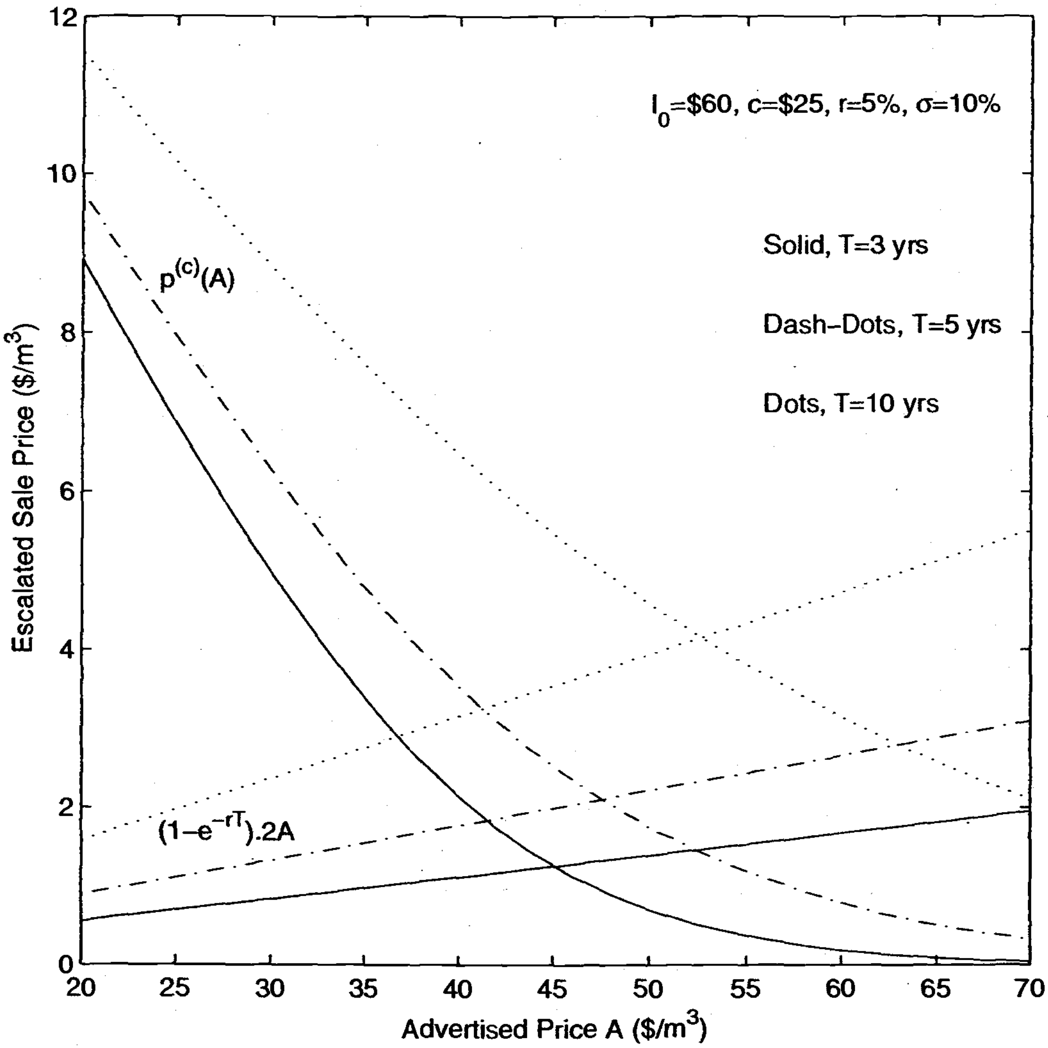

Figure 3.10: Escalated Sale: Effect of Duration of Contract

3.6 Conclusions and Discussion

In this paper we have provided a forest lease valuation tool that incorporates both price volatility and harvesting costs to determine arbitrage-free minimum bid. One strength of our method is that price changes are incorporated directly into the minimum bid such that the lease writer can cover itself against losses due to price fluctuations. An additional contribution of the paper is that we have developed a method for including harvesting costs in the Black Scholes-Merton option pricing theory.

The case study and corresponding graphs provide illustrations of the effect that interest rates, cost and volatility have on the advertised price. These examples clarify one of the main points of the paper: Volatility can critically affect the lease price and must be systematically included in the pricing structure.

The pricing method of this paper provides a market standard for the value of a timber lease. That is, we may have an objective benchmark against which we may consider valuations that include non-market values such as social and environmental welfare. We have chosen to use arbitrage free methods because they are based on formal mathematical solutions, and sound financial theory. Therefore, we treat the government as a participant in a competitive market in which the necessary trading exists for creating a hedge portfolio.

The reader may notice that our values differ from those that the Forest Service obtained. One reason for this is that in this iteration, we have not included other forest service objectives, such as community payments, or other environmental responsibility. Including these objectives would complicate our presentation of the arbitrage-free method that we seek to introduce with this paper. This observation suggests an interesting extension of this paper. That is, in future work, non-market values such as social and environmental welfare could be modeled within the stochastic, dynamic framework that we present. Thus, our intention here is not to show whether the Forest Service has over or under-valued timber leases, but rather to introduce a valuation procedure based on robust financial methods.

Weaknesses of our approach include the Condition E that the settlement occurs at the end of the contract. An additional caveat is the assumption that we have made regarding the stochastic model for the evolution of indices. Since we have based our model on the index, and not on the actual market, price discrepancy may arise between the writer and the holder (lessee).

Despite these shortcomings this approach introduces a quantitative framework in which policy makers and managers may simply and systematically examine the effects of some key parameter values, in particular interest rates, costs, and volatility, in appraisals of bid prices.

3.7 Literature Cited

Baxter, M. and A. Rennie. 1997. Financial calculus: An introduction to derivative pricing. Cambridge University Press, Cambridge, United Kingdom. 317 p.

Bhattacharya, R. and E. Waymire .1990. Stochastic processes with applications. Wiley,New York, USA.

Dixit, A. K. and R. S. Pindyck. 1994. Investment under uncertainty. Princeton University Press, New Jersey, USA.

MATLAB. 1995. Financial Toolbox, The Math Works Inc., Natick, Mass.

Gruber, Urs. 1997. Pricing barrier and Asian options with an emphasis on Monte-Carlo methods. M.Sc. paper, Oregon State University, Corvallis, Oregon, USA.

Hull, J. 1993. Options, futures and other derivative securities, Prentice-Hall, 2nd edition.

Karatzas, I. 1997. Lectures on the mathematics of finance. American Mathematical Society, Providence,Rhode Island, USA.

McAfee, R.P. and J. McMillan. 1987. Auctions and bidding, Journal of Economic Lit. XXV, 699-738.

Morck, R., E. Schwartz, D. Stangeland. 1989. The valuation of forestry resources under stochastic prices and inventories. Journal of Financial and Quantitative Analysis 473-487.

Paarsch, H.J. 1997. Deriving an estimate of the optimal reserve price: An application to British Columbian timber sales, Journal of Econometrics (78), 333-357.

Random Lengths. 1997. The Weekly Report on North American Forest Products Markets 53 (44).

Reed, W. 1993. The decision to conserve or harvest old growth forest, Ecological Economics (8), 45-69.

Reed, W. and R.G. Haight. 1996. Predicting the present value distribution of a forest plantation investment. Forest Science (42), 378-388.

3.8 Appendices

Appendix A

Application of the Cameron-Martin-Girsanov Theorem

(i) An application of the Cameron-Martin-Girsanov theorem, e.g. see Bhattacharya and Waymire (pp. 616-618, 1990), or Baxter and Rennie (1997), yields a path-dependent representation of the stochastic solution to (11) as for $t<T_c$ as

$$X_t = Ie^{\left\{\sigma W_t + \left(r - \frac{\sigma^2}{t}\right)t\right\}} - rc\int_0^t e^{\left\{\sigma(W_t - W_\tau) + \left(r - \frac{\sigma^2}{2}\right)(t-\tau)\right\}} d\tau$$

$$= IL_0^{(t)} - rc\int_0^t L_\tau^t d\tau$$

(A 3.1)

where for each

$$0 \le \tau \le t$$

the random factors $L_\tau^{(t)}$ are the lognormally distributed variables defined by

$$L_\tau^{(t)} = \exp\left\{\sigma(W_t - W_\tau) + (r - \frac{\sigma}{2})(t-\tau)\right\}$$

and T_c is the absorption time for X_t. In particular it is important to observe that the

$L_\tau^{(t)}$

and

$\int_0^t L_\tau^{(t)} d\tau$

two factors,

are statistically independent.

Appendix B

Application of Feynman-Kac Formula

(ii) It now follows from (A1) using the Feynman-Kac formula, e.g. see Bhattacharya and Waymire (pp. 606-607), or Baxter and Rennie (1997), that the general formula for the price of the escalated lease is

$$p^{(\widetilde{B})}(A) = e^{-rT}\left[\frac{1}{2}E\left\{\left(I_0 L_0^{(T)} - rc\int_0^t L_\tau^{(t)} d\tau - \widetilde{A}\right)^+ 1(\tau_c > T)\right\}\right]$$

$$+e^{-rT}\left[E\left\{\left(\widetilde{B} - I_0 L_0^{(T)} + rc\int_0^{(T)} L_\tau^{(T)} d\tau\right)^+ 1(\tau_c > T)\right\} + \left(\frac{A}{2} + B\right)P(\tau_c \le T)\right]$$

:

$$p^{(\widetilde{B})}(A) = e^{-rT}\left[\frac{1}{2}E\left\{\left(I_0 L_0(T) - rc\int_0^T L_\tau^{(t)} d\tau - \widetilde{A}\right) + 1(\tau_c > T\right\}\right] +$$

$$e^{-rT}\left[E\left\{\left(\widetilde{B} - I_0 L_0^T + rc\int_0^T L_\tau^T d\tau\right) + 1(\tau_c > T\right\} + \left(\frac{A}{2} + B\right)P(\tau_c \le T)\right]$$

(A 3.2)

CHAPTER FOUR
A COMPARISON OF HEDGING FUTURES CONTRACTS WITH DEVELOPED VERSUS UNDEVELOPED ASSETS FOR ARBITRAGE-FREE RESOURCE HARVEST CONTRACT VALUATION

4.1 Introduction

Tradable permits, leases, and contracts are methods of allocating the access to and use of valuable natural resources. The valuation issue of natural resources may be as broad as resource services, or as specific as the market value of the harvested commodity. An example of such a contract is a United States Federal timber lease. These leases allocate a pre-determined plot and quantity of timber to a harvester via auction. The forest service must determine an appropriate minimum bid for the stand.

From the perspective of market value, there are two points to address in valuing resources: price volatility and the appropriate discount rate. Traditionally, users have utilized net present value (NPV) methods of valuation to determine the investment value of resource harvest. NPV methods are insufficient for at least three reasons: First, they project a deterministic price path. This assumption over-values the project and results in harvesting too much, too soon (Arrow and Fisher, 1974). Second, by over-valuing the investment, they allow for arbitrage opportunities. That is, risk-free profits are possible. Third, the discount rate is subjective. Value discrepancies are often accounted for by subjectively altering the discount rate to account for price volatility. High discount rates place a low value on the future, and encourage early harvest. Conversely, low discount rates promote slower, more conservative use.

Methods which account for risk and uncertainty overcome the valuation problems associated with NPV (e.g. Arrow and Fisher, 1974; Dixit and Pindyck, 1994). By applying modern financial portfolio theory to the resource investment, stochastic parameters may be integrated into the valuation problem. Also, an objective discount rate may be obtained by establishing a self-financing, risk-mitigating portfolio of risk-free bonds and another asset. This "other asset" is typically the underlying asset (in the case of financial stocks). However, futures may also be used as the hedging mechanism (Brennan and Schwartz 1985; Morck, Schwartz and Stangeland, 1989). Most recently, Burnes, Thomann and Waymire (1999) have shown that it is possible to construct a hedging portfolio that uses the undeveloped or standing asset (i.e. trees) as the hedging mechanism. This is an important choice because if one hedges a contract for an undeveloped asset by holding the developed asset, the portfolio is not self-replicating and the contract writer will not have enough money to settle the contract at the time of expiry. The choices between not hedging, hedging with developed assets or hedging with an undeveloped asset impacts the contract value.

A convenience yield associated with holding an asset also impacts the contract value. Convenience yield is a value that accrues to the holder of the asset, but not to the buyer of the contract. For example, in the case of stocks, a dividend would accrue to the person who holds the stock, not to the person who holds the right to purchase it in the next period. Since this extra benefit accrues, the cost of the contract must be adjusted by the appropriate amount, or the payoff to the asset holder will be too high and arbitrage will be possible. The question is, how much is the convenience yield that accrues to the holder of an asset. Traditionally, a "fudge factor" is chosen to play the part of a convenience yield. We calculate the value, if any, which accrues to the holder of an asset for settling futures contracts. The comparison of the value of unique hedging mechanisms uncovers a way to measure the convenience yield directly subject to a given starting price and cost structure.

The objective of this project is to evaluate the use of undeveloped assets (i.e. standing trees) versus developed assets (lumber) as hedging mechanisms for uncertain resource harvest investment under no-arbitrage. This is important, because as it will be shown, the developed resource is not an appropriate hedging mechanism when the contract is for the undeveloped asset. The comparison will be accomplished by first considering the value of a timber lease under stochastic prices using developed assets as the hedging mechanism. Second, the value of the same timber lease is determined using the undeveloped resource as the hedging mechanism.

Section two provides a literature review of the motivating papers for this study. Section three develops the methods used to form the analysis and results. Specifically, arbitrage is discussed, the rationale for exploring various hedging instruments is introduced, hedging portfolios are modeled, and numerical examples of means and variances under three different expectation measures are provided. Section four provides the results of the portfolio types through an example based on U.S, Forest Service timber lease auctions, and section five offers conclusions and policy implications of this work.

4.2 Literature Review

Two pieces of literature form the basis for the exposition of this paper. Extending this literature motivates the models presented in this paper. Morck, Schwartz and Stangeland (1989) develop a method of using futures as the portfolio element in the hedge portfolio for natural resource investment. This is because there is often not a market in the underlying resource/asset, while active futures markets exist. Second, Burnes, Thomann and Waymire (1999) apply option methods to a natural resource harvest contract (a timber contract) given stochastic prices. Using option valuation methods, they develop a hedge portfolio based on an undeveloped asset.

Morck, Schwartz and Stangeland (1989) discuss the case of natural resource investment under price uncertainty using futures to replicate cash flows in the hedge-portfolio. Their motivation, however, is to determine an optimal harvesting policy given stochastic prices that are hedged using the methods of no arbitrage. They develop a continuous model to determine an optimal operating policy given stochastic output prices. Another contribution of their paper is to explore the notion of "convenience yield" with respect to the contract writer. Morck et. al. validate the use of futures to replicate cash flows, however they do not determine an explicit valuation solution for the natural resource contract. Nor do they explicitly develop an objective convenience yield measure. Rather, they rely on the "fudge factor" method.

Burnes, Thomann and Waymire (1999) model an arbitrage free value of a timber lease given a known inventory but stochastic prices. The authors also use a contingent claims approach. However, the hedging portfolio is comprised of a risk-free asset and the undeveloped asset. The authors explicitly value the harvest contract and perform sensitivity analyses with respect to harvesting costs, discount rates, volatility and contract duration. The authors work in a one-dimensional continuous case.

The impact of the alternative portfolio choices on the contract value in the Morck et. al. and Burnes et. al. papers is a key issue to be addressed in this paper. By examining how using developed assets versus undeveloped assets as hedging mechanisms affect the value of a renewable resource contract, one is led to a determination of convenience yield. The key parameter then becomes the conversion cost.

4.3 Methods

The net present value approach is to look at the probabilities of various outcomes and then to formulate an expectation of the price at a future time period. Once this expectation is formed, one must choose the appropriate discount rate with which to bring this expected value into current terms. However, simply calculating an expected value using the price process over-values the investment and leads to arbitrage opportunities. Also, choosing a discount rate is not straightforward. Doing so leaves one open to discrepancy regarding the appropriate rate for a given risk, and also may provide an opportunity for arbitrage. Finally, in the case where the valuation decision includes non-market entities, such as trees, one needs to substitute an appropriate market-valued, proxy-asset based expectation measure and an appropriate discount rate.

4.3.1 Portfolios

Option pricing methods based on the Black Scholes option pricing formula rely on a self-financing portfolio whose payoff exactly matches that of the investment at each time step (Black and Scholes, 1973). The combination of the sel-financing (hedging) portfolio and investment contract constitute the contract writer's total portfolio in the traditional financial sense. These methods are applicable not only to options, the right to buy or sell an asset, but to futures contracts, the obligation to buy or sell an asset, as well.

The value of these methods is that they address the points of interest rates, expected values, and the notion of a hedging portfolio which relies on an underlying asset and a risk-free bond. These tools are necessary to address the existence and implication of arbitrage. Product owners and sellers will not want to undervalue their products relative to the market. If they did, people could buy the products and return them to another vendor to risklessly profit from the transaction (i.e. arbitrage).

The motivation for using the underlying asset *and* bonds to develop pricing expectations is that we use all of the tools of the market to abdicate the existence of arbitrage from this investment opportunity. However, it is shown that the choice of hedging instruments is critical in order to develop a self-replicating portfolio strategy. This means that the money and positions held in the portfolio exactly matches the writer's obligations at the time of contract expiry. Another benefit of using the arbitrage-free approach is that it objectively determines the risk-free interest rate as the appropriate discount rate. A further contribution of the approach developed in this paper is that the framework incorporates harvesting costs into the portfolio and expectation measure.

With the hedge portfolios constructed, we calculate a risk-neutral probability measure, called Q, under which we can describe future price expectations. Q represents the evolution of the discounted value of the underlying asset as a martingale, which is a change of measure from the expectations under the actual price process called P.

4.3.2 Real Assets

The timber example relies on "real assets" rather than derivatives or stock values as a portfolio component. This is typical of the real options approach which uses the rigor of financial options theory to evaluate project values that have uncertain outcomes (Amram and Kulatilaka, 1999). In real options, we require a "twin security" that has a payoff equal to the asset in the portfolio (Trigeorgis, 1995). In the case of timber valuation, the hedge "portfolio" is comprised of trees, an undeveloped asset, and risk-free bonds. The twin security is lumber, the value of which we can track in the market. In order to match the value of lumber and trees a cost conversion factor is needed. Therefore, the measure Q is further modified to Q(c) to accommodate the undeveloped resource case. Q(c) represents a cost adjusted risk neutral martingale measure. In this latter case, one obtains a martingale for a period of time defined by the settlement conditions of the contract (Thomann and Waymire, 2000).

4.3.3 The Hedge Portfolios

The hedging mechanisms are derived to provide a basis for obtaining risk-neutral, arbitrage free expectations based on the market tools of assets, futures, and bonds. Using these instruments, the contract writer determines the value of the contract, and then uses that money to buy a designated amount of stocks and bonds to hedge exposures to price movements. The allocation of the portfolio between stocks and bonds may be traded continuously as asset prices change. This is the self-financing part of the portfolio: the portfolio holder is always able to cover changes in the contract payoff due to price changes. The portfolio is constructed such that when the contract is exercised, the writer has enough to buy the asset, and pay off any borrowings (bonds), and the payoff to the writer is exactly zero (Baxter and Rennie, 1996).

Three hedging cases will be considered. It will be shown that in the case of hedging a contract for an undeveloped resource, hedging with developed assets does not create a risk-free position. Case one is the traditional Black-Scholes binomial option pricing formula. In this case, the hedge portfolio is comprised of the underlying financial stock and risk-free bonds. Case two utilizes futures to formulate the hedge portfolio. This method is useful for resources, such as timber, oil, and agricultural products for which a "stock" market does not exist. We will show that when trading futures to obtain a risk-neutral portfolio, we still end up with the traditional Black-Scholes formula. However, these mechanisms are not sufficient when an undeveloped asset is used in the hedging portfolio. Finally, in case three, the hedge portfolio using the undeveloped resource is derived.

4.3.4 Case One: Hedging with the Underlying Stock

A variety of derivatives take their value based on an underlying asset such as a stock, or a futures contract. A futures contract stipulates a date and price at which the stock must be bought. Then the pertinent question is, how much should I agree today to pay for a stock in the future. To answer this question, I must be able to anticipate the value of my payoff, i.e. the difference between the market value of the stock and what I agree to pay. We denote this payoff as S_0-k -- where S_0 is today's stock or asset price, and k is the future agreed on price-- which is then equated with the hedge portfolio component, $W_{0s} = \varphi S_0 + \Psi B_0$. W_{0S} is the value of the portfolio of market instruments, φ is the amount of the portfolio held in the asset and Ψ is the amount of the portfolio held in bonds. The portfolio holder then has a risk-neutral position in that the holder's payoff is indifferent to increases and decreases in the asset's value as these movements affect the contract payoff. No arbitrage considerations give a price which is independent of the stochastic evolution of prices (Burnes et al, 2000)

In the next time period, the value of the stock can either go up by magnitude u (u>1) or down by d (d<1). In either case, the risk free bond moves with certainty by an increment r, where r equals one plus the risk free discount rate, R. The relationship u>r>d is assumed throughout. This is because prices of risky assets must increase at a greater rate than riskless assets, or people would only hold riskless assets. If the downside of holding risky assets offered a higher payoff than the riskless asset (i.e. if d>r) people would hold only risky assets.

This method of valuation is applicable to a non-escalated U.S. Forest Service timber contract. This means that the Forest Service will receive the agreed upon price for the timber at the time of harvest regardless of the current market price. This is like a futures contract.

Using the Black-Scholes formula, we can construct the appropriate risk-mitigating portfolio. Given the evolution of asset prices and bond values, the portfolio W, evolves according to:

$$W_1 = \begin{cases} \varphi S_0 u + \Psi B_0 r = V_1(u) \\ \varphi S_0 d + \Psi B_0 r = V_1(d) \end{cases} \quad \textbf{(1)}$$

Now subtract $V_1(d)$ from $V_1(u)$ and solve algebraically for φ and Ψ.

$$\varphi = \frac{V_1(u) - V_1(d)}{S_0(u-d)} \quad \textbf{(2)}$$

$$\text{and } \Psi = \frac{1}{rB_0}\left[\frac{V_1(d)u - V_1(u)d}{u-d}\right]. \quad \textbf{(3)}$$

Simultaneously solving the outcomes under up and down price movements for φ and Ψ results in the investor's indifference between increases and decreases in movements in the value of the underlying asset. That is, the portfolio allocations φ and Ψ determined by equating the outcomes in the next period, establish that the payoff of the hedging portfolio matches the payoff of the investment whether prices rise or fall.

Now substitute the values from (2) and (3) into W_{0s} to obtain the Black-Scholes formula.

$$W_{0s} = \left[V_1(u)\left(\frac{r-d}{u-d}\right) + V_1(d)\left(\frac{u-r}{u-d}\right)\right]\frac{1}{r} \text{ (after canceling).} \quad \textbf{(4)}$$

That is, the value of W_{0s} is a function of the value of the portfolio in the price rise case ($V_1(u)$), times the risk-neutral probability of the up case, plus the value of the portfolio in the price fall case ($V_1(d)$), times the risk-neutral probability of the down case. These probability weightings are called risk-neutral martingale probability measures. Notice that they are only functions of magnitude of the changes in value and of the risk-free interest rate. Given the Markov processes underlying the initial discrete models, the magnitude of value changes are constant over time.

Now consider the price of the futures contract, k. Substituting the payoffs for $V_1(u)$ and $V_1(d)$, we obtain:

$$\varphi = \frac{(S_0 u - k) - (S_0 d - k)}{S_0(u-d)} = \frac{S_0(u-d)}{S_0(u-d)} = 1, \quad \textbf{(5)}$$

$$\Psi = \frac{1}{rB_0}\left[\frac{(S_0 du - ku) - (S_0 ud - kd)}{u-d}\right] = -\frac{k}{rB_0} \quad \textbf{(6)}$$

Replace φ and Ψ in W_{0S} to obtain:

$$W_0 = S_0 - \frac{k}{r} = 0;$$
$$k = S_0 r$$

We can equate W_0 to zero, since the contract is a futures contract, no money is exchanged today. The result is the traditional Hotelling (1937) outcome, that prices (rents) rise at the rate of discount. We should be willing to pay k, in the future (tomorrow), considering prices today, S_0 and the risk-free discount rate. (For continuous timesteps this becomes the familiar e^{rt})[4].

4.3.5 Case Two: Hedging With Futures

In the case of futures, we are trading the obligation to buy a specified quantity at a specified price. Unlike most other derivatives: call, puts, etc., no money is traded at time zero. Rather, the asset must be purchased at the pre-determined future time, for a price k. Given the k determined in 4.4.1, we can determine the price of a derivative contract hedged by futures.

At time (t) =0, the value of the futures portfolio is:

$W_{f0} = \varphi Futures + \Psi Bonds = \varphi F_0 + \Psi B_0$.

Again, the bonds are risk-free government bonds. Since we are agreeing on a price k to pay in the future *regardless* of future asset prices, the payoff after one time step is:

$$W_{f1} = \begin{cases} \varphi F_0 u + \Psi B_1 = V_{f1}(u) = uS_0 - k \\ \varphi F_0 d + \Psi B_1 = V_{f1}(d) = dS_0 - k \end{cases} \qquad \textbf{(7)}$$

We will now show that this same outcome may be obtained using futures as the hedging instrument as using the asset itself (see also Brennan and Schwartz, 1985). Recall that $F_t = S_t - k$. Given the solution that $k^* = rS_0$, substitute into the W_{f1} outcomes to get:

$$W_{f1} = \begin{cases} \varphi F_1(u) + rB\Psi = \varphi(uS_0 - rS_0) + rB\Psi = V_1(u) \\ \varphi F_1(d) + rB\Psi = \varphi(dS_0 - rS_0) + rB\Psi = V_1(d) \end{cases} \qquad \textbf{(8)}$$

where F is the payoff of a futures contract, S_t-k*. Solving for φ we obtain:

$$\varphi = \frac{V_1(u) - V_1(d)}{(u-d)S_0} \qquad \textbf{(9)}$$

and solving for Ψ results in:

$$\Psi = \frac{1}{rB_0}\left[V(u)\left(\frac{r-d}{u-d}\right) + V(d)\left(\frac{u-r}{u-d}\right)\right]. \qquad \textbf{(10)}$$

Consider again the portfolio at time zero: $W_0 = \varphi F_0 + \Psi B_0$. As mentioned, in the case of futures, no asset is purchased at time zero, so F_0=0 and we are left

[4] The Hotelling result only applies to the case of the non-escalated contract. In the case of escalated contracts, the Black-Scholes model results in a unique value of k. See Appendix at the end of this chapter for details.

with

$$W_0 = \psi 0 + \frac{1}{rB_0}\left[V(u)\left(\frac{r-d}{u-d}\right) + V(d)\left(\frac{u-r}{u-d}\right)\right]B_0 = \left[V(u)\left(\frac{r-d}{u-d}\right) + V(d)\left(\frac{u-r}{u-d}\right)\right]\frac{1}{r}$$

(11)

which is the same Black-Scholes value for a derivative with intrinsic value W.

4.3.6 Assymetry Between Hedging Undeveloped Assets with Developed Assets vs Undeveloped Assets

One might expect that the hedging tools of a developed and undeveloped asset would be sufficient. However, it can be shown that by hedging a contract to deliver undeveloped assets with developed assets, the contract writer is left exposed, while hedging a contract to deliver developed assets with the undeveloped asset leaves one's exposure to price risk covered. As constructed in section 4.3.4, the value of the undeveloped asset may be depicted as S_0-c. That is, it is a cost-adjusted value of the price of the developed asset, where the cost is the harvesting cost. The payoff of writing a futures contract on the *un*developed asset is $V_0 = S_0 - c - \frac{k}{r}$. If this contract is hedged with the developed asset, the writer will hold one unit of the asset valued at S_0.

The portfolio today W_0, is the same as in case one. Again, $\varphi = 1$. However, now $\Psi = -\left[\frac{c+k}{rB_0}\right]$. The portfolio is $W_0 = S_0 - \left[\frac{c+k}{r}\right] = 0$. This implies that the writer will borrow S_0 for an asset that at expiry will be worth rS_0-c. The writer will receive rS_0-c, and will owe rS_0. Therefore, hedging with the developed asset is not appropriate. The implication of this outcome is the key to why one must be able to hedge with the underlying asset. The Black-Scholes valuation principle says that one can obtain an arbitrage-free price for a contract on a risky asset by constructing a portfolio of the asset, and a risk-free asset. This principle is violated when one assumes that the *form* of the asset is substitutable with respect to developed and undeveloped assets.

4.3.7 Case 3: Hedging with the Undeveloped Asset

Hedging with the undeveloped asset overcomes the exposure problem pointed out in section 4.4.3. The same process is utilized to construct the hedge portfolio using the undeveloped asset. However, the portfolio is different, namely $W_0(c) = \varphi(S_0 - c) + \Psi B_0$ (c= the conversion cost). In this case we use current prices of timber minus harvesting costs as the value of the undeveloped asset. That is current timber price minus harvesting cost equals the value of the standing resource. The proportion of φ and Ψ in this case is:

$$\varphi = \frac{V_1(u) - V_1(d)}{S_0(u-d)} \tag{12}$$

and $$\Psi = \frac{1}{rB_0}\left[V_1(u) - \frac{V_1(u) - V_1(d)}{u-d}\left(u - \frac{c}{S_0}\right)\right] \tag{13}$$

And the value of the portfolio ($W_0(c)$) is :

$$W_{0R} = \left[V_1(u)\left(\frac{r-d}{u-d} + \frac{c}{S_0}\left(\frac{1-r}{u-d}\right)\right) + V_1(d)\left(\frac{u-r}{u-d} + \frac{c}{S_0}\left(\frac{r-1}{u-d}\right)\right)\right]\frac{1}{r} \tag{14}$$

Equation (14) provides the adjusted contract value under the cost adjusted martingale measure, Q(c).

4.3.8 Expectations Under Uncertainty

This section develops the rationale for developing an arbitrage free contract price. The case of pricing futures, using developed and undeveloped assets is explored. Futures prices are considered under the probability measures P, Q, and Q(c). The measure P is taken as given. That is, it is possible to observe the actual probabilities with which asset values increase and decrease. The measure Q is based on the relationship between u, r and d which form the probabilities of up and down movements of asset prices when hedging is performed with the asset or with futures. When the standing resource is used as the hedging mechanism, the measure Q(c) relies on u, r, d, c and S_0, where c enters as a conversion factor that links the standing resource to the market.

Under each distribution, P, Q, and Q(c), prices are lognormally distributed. Consider an asset with value S_0, which at each time step under the measure P moves up with probability p and magnitude u and down with probability 1-p and magnitude d (see Figure 4.1).

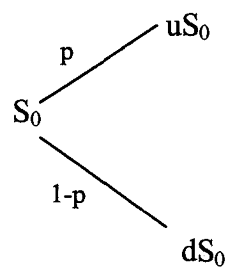

Figure 4.1: A binomial tree depicting asset price evolution after one time period

The following values are used to demonstrate the results of determining the value next period:

S_0	=	$50/unit
u	=	1.4
d	=	.8
p	=	.6
r	=	1.06
c	=	$10/unit

S_0 equals the value of the asset today, u is the magnitude of the upward movement, d is the magnitude of the downward movement, p is the probability of the upward movement ((1-p) is the probability of the downward movement), r is the risk-free interest rate and c is the marginal cost of harvesting one unit of the resource.

$$E_P\left(\frac{S_1}{S_0}\right)=\frac{puS_0}{S_0}+\frac{(1-p)dS_0}{S_0}=\frac{.6*(1.4*50)}{50}+\frac{.4*(.8*50)}{50}=1.16 \qquad \textbf{(15)}$$

That is the expected return under P of $(S_1/S_0) = 1.16$. 1.16 implies a 16% rate of return, and that one should be willing to pay $58 next period to obtain this asset. However, as asserted at the beginning of the paper, this price is too high and arbitrage is possible. The agreement is that the buyer will pay the seller $58 in the next period. This outcome is equatable to net present value methods. The probabilities used to build expectations may be accurate, but failure to fully utilize the market results in arbitrage and project overvaluation. Arbitrage is possible because the seller may borrow $50 today at the risk free interest rate of 6% to buy the asset, and in the next period pay back 1.06*50, or $53 and receive $58 for a risk-free $5 profit.

Consider instead the expectations under Q and Q(c), which fully utilize the market instruments of assets *and* risk-free bonds. When these measures are applied to the expectation of (S_1/S_0) the following is obtained:

$$E_Q\left(\frac{S_1}{S_0}\right)=\left(\frac{r-d}{u-d}\right)*\frac{uS_0}{S_0}+\left(\frac{u-r}{u-d}\right)*\frac{dS_0}{S_0}=$$
$$\left(\frac{1.06-.8}{1.4-.8}\right)*\frac{(1.4*50)}{50}+\left(\frac{1.4-1.06}{1.4-.8}\right)*\frac{(.8*50)}{50}=1.06 \qquad \textbf{(16)}$$

That is, the expected value under the measure Q of S_1/S_0 is 1.06. This means that the rate of return is 6%, exactly the risk free interest rate and that the appropriate price to contract for the asset in the next period is $53. Equation 16 represents a numerical illustration of the outcomes that one obtains when one applies the Black-Scholes model to obtain the risk-neutral Q martingale probability weightings to payoff outcomes.

We obtain Q(c) in the case where the "asset" is the undeveloped asset. Again, to obtain an arbitrage free valuation the probability measure must change due to the cost conversion factor. In this case, the expected value of S_1/S_0 moves up with probability $\frac{r-d}{u-d}-\frac{c}{S_0}\left(\frac{r-1}{u-d}\right)$ and down with probability $\frac{u-r}{u-d}+\frac{c}{S_0}\left(\frac{r-1}{u-d}\right)$.

EQ(c)(S_1/S_0)=1.048 -- that is a 4.8% rate of return implying that \$52.40 - c is an appropriate amount to contract for purchasing tomorrow. The relationship between the contract value with the futures hedge and portfolio hedged with the undeveloped asset is the basis for determining the convenience yield measure. This measure holds under the price and cost assumptions in the current case. The convenience yield outcome is further explored in the results section.

To reiterate, under the measure P, prices are lognormally distributed. Prices are exogenous to the contract writer. If the writer prices the contract based on the distribution P, the contract is over-valued and arbitrage is possible. The writer constructs the distribution Q, under which prices are also lognormally distributed in order to match the contract outcome with the return available by investing in the risk-free bonds. Under this setting, arbitrage is not possible. Finally, the case in which there is not a market for the contracted asset is considered. Again, a new distribution Q(c) is required to maintain the no arbitrage condition.

4.4 Results

In this section, the value of each contract is derived, and an example is presented to clarify how the hedge portfolio which maintains a risk neutral position for the contract writer is determined. A risk neutral position means that the writer is indifferent to upward and downward price movements.

As a motivation for this section, see chapter three, consider specifically , section 3.2: elements of a federal timber lease; section 3.3: some principles of mathematical finance; and section 3.4: federal timber leases as options: mathematical representation.

Particularly, consider the non-escalated contract, where the bid price is the bid price paid at lease expiry. This is like a futures contract: a future price is agreed upon regardless of the actual price at the time of expiry. The forest service is the contract writer. The valuation and hedge portfolio is constructed from the perspective of the writer.

Consider the same parameter values as in the previous section.

S_0 = \$50/mbf (thousand board feet)

u = 1.4

d = .8

p = .6

r = 1.06

c = \$10/mbf

As before, in the case of expectations under P, when no hedging occurs, the price that the writer agrees to for the asset in the next period is \$58. As long as the risk-free interest rate, r is less than the $E_P(S_1/S_0)$, there is no value to forming a contract to buy the resource in the future. This is because the value of the contract today is $W_0 = \frac{1}{1.06}[(70*.6)+(40*.4)] = \frac{58}{1.06} = \$54.72$. The writer would have to borrow \$54.72 today to hedge the \$58 expected return in one period. That is, the writer is spending \$54.72 today on an asset that is worth \$50.

Under the measure Q, the writer contracts to receive \$53 (see equation 16) for the asset in the next period. She borrows \$50 in bonds today to buy the asset. Tomorrow, if the asset price goes up to \$70, she still receives \$53 and pays off the bond debt which equals \$50*1.06=\$53. If the asset price goes down to \$40 she receives \$53 and again pays off the bond debt.

Now consider the portfolio under Q(c). Here we need to consider two cases. In case one, the contract is to deliver trees in the next period. That is, we are hedging the promise to deliver trees, with trees. In case two, the contract is to deliver harvested trees, but the contract is hedged with standing trees. An example of case one is the United States Forest Service writing a harvest contract. A harvester agrees today what to pay for the trees at the time of harvest. The forest service holds the trees until that time. An example of case two is the person (company) buying the contract to harvest the trees. They may have a futures contract to deliver *harvested trees* in the future to a private party. They have a contract with the Forest Service to harvest trees (which they deliver to the private party). If we apply the measure Q(c) derived in equation (14), the value of the contract from the perspective of the Forest Service is

$W_0 = \frac{1}{1.06}\left[(70 * .41\bar{3}) + (40 * .587)\right] = \$42.40$. This is the appropriate contract price when the deliverable is standing trees. However, from the perspective of the harvester who must deliver lumber the correct price is \$42.40 + \$10 (*the conversion cost*)=\$52.40.

Table 4.1 depicts the hedging relationships occurring in the above cases.

Hedging Method	*Forward Contract (the deliverable)*	*Contract Value*
Lumber	lumber	\$53
trees	trees	\$42.40
trees	lumber	\$52.40

Table 4.1: Contract Values under Various Hedging Methods

When we hedge undeveloped assets with undeveloped assets, the value of the contract today is $V_0=\varphi(S_0-c) + \Psi B_0$. We know from equation (6) that $\varphi=1$. We also know that $V_0=0$ since no money is exchanged in period 0 in the case of futures. We have also already derived Ψ in equation (7) as $-k/rB_0$. For simplicity, let $B_0=1$.

Therefore we have, $0=S_0-c-k/r$. That is $40(1.06)=k=\$42.40$, the amount the writer (the Forest Service) should receive in the next period. The writer borrows $40 at the risk-free rate today, and in the next period turns over the trees to harvest, receives $42.40 and pays back the bond loan. Notice that the traded asset and the hedging asset are *both* undeveloped assets.

When the undeveloped asset is used to hedge a contract to deliver the developed asset, the bidder agrees to buy the government contract for standing trees at $42.40 in the next period. This party must harvest these trees next period at a cost of $10. Therefore, the total costs which accrue to the party are $52.40. However, the party may also contract with "another" to buy harvested trees. We have already shown that the appropriate amount to charge for harvested trees hedged with harvested trees is $53. Next period the bidder earns $.60 or the interest earned on the $10 between today and the next time period ($10*.06=$.60), since the harvest occurs in the next time period.

The difference between these hedging methods suggest a convenience yield and that the $53 for a contract to deliver a developed resource is actually too high. $53 assumes that there is no convenience yield associated with holding the harvested asset (Brennan and Schwartz, 1985, discuss the implication of a convenience yield in the valuation of non-renewable assets under price uncertainty). However, there is a convenience yield since the holder of the asset may capitalize on shortages and higher than expected prices. The risk free rate r, may be adjusted by a value Λ to account for this possibility.

Based on the value of the contracts hedged with the undeveloped versus developed assets, we can derive the convenience yield associated with holding the developed asset. The binomial tree which describes the payoff is depicted in Figure 4.2.

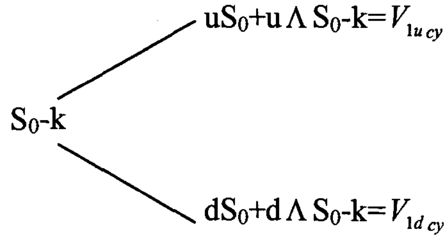

Figure 4.2: Contract Payoffs Under a Convenience Yield

The portfolio representation of this process is as follows:

$$W_0 - S_0(1-\Lambda) + \varphi S_0 \Lambda + \Psi r B_0 \tag{16}$$

Combining the payoffs of the binomial tree above with the portfolio's first time-step results we get:

$$W_{1cy} = \begin{cases} \varphi(uS_0 - u\Lambda S_0) + \varphi u \Lambda S_0 + \Psi r B_0 = V_{1ucy} \\ \varphi(dS_0 - d\Lambda S_0) + \varphi d \Lambda S_0 + \Psi r B_0 = V_{1dcy} \end{cases} \tag{17}$$

Solving for Ψ we get $1-\Lambda$, rather than 1. This implies that less must be borrowed. $\Psi = -\frac{W_0(c)}{r}$, where $W_0(c)$ is the value obtained for the contract under the cost converted Q martingale, Q(c). Inserting these values into the portfolio equation today, we obtain $(1-\Lambda)S_0 - W_0(c)/r = 0$ (the portfolio still equals 0 today since no money is exchanged). Now we can solve for Λ by equating $W_0(c) = (1-\Lambda)S_0 r$ as the amount that must be paid tomorrow given the convenience yield associated with holding the unharvested resource. Since $W_0(c)$, S_0 and r are given, it is possible to derive Λ. Namely, $\Lambda = 1 - \frac{1}{r}\frac{W_0(c)}{S_0}$. Given the specifications in this example, $\Lambda = .0113$ (Note that Λ is S_0 dependent, as Morck, Schwartz and Stangeland suggest).

We have shown that a harvesting contract hedged with futures on a harvested resource will be overvalued if a convenience yield is not considered. By hedging with the unharvested resource, we are able to replicate the payoff of the developed resource and determine a convenience yield measure. By incorporating this measure into the contract valuation an arbitrage-free valuation of the harvesting contract is obtained.

4.5 Conclusions and Policy Implications

This paper has shown that it may be necessary to change the measure on which we make price expectations in order to develop a risk-neutral, arbitrage free contract price. This is possible by creating an underlying hedge portfolio comprised of the asset and a risk-free asset. In cases where the underlying asset is not traded in the market, it has been shown that we can develop yet another measure to accommodate the necessary conversion costs in "bringing the asset to market". That is, we include the harvest costs of timber when the marketed asset is harvested trees but the contract writer is holding timber (standing trees). Finally, the relationship between the hedging portfolios of developed and undeveloped assets is used to derive the existence and measure of a convenience yield.

These results have policy implications for resource managers interested in setting contract rates for natural resources such as trees, fish and water. First, in the cost adjusted case, we can directly see the impact that costs have on the contract price. In instances where harvester costs are uncertain, managers can better model the impact that cost variations across harvesters may have on contract values. Second, using the risk-neutral case allows managers to use an objective interest rate measure (the risk free interest rate) in discounting contract values over time. Third, these results provide a market-based foundation for integrating non-market resources into a valuation system. These results are strictly based on observable market characteristics of prices and hedging opportunities. If one wishes to value social desires and resource benefits derived from different policy scenarios, these values suggest a starting value in the absence of public or environmental welfare. This framework provides a method for setting a reliable and justifiable benchmark for the value of undeveloped natural resources.

4.6 Literature Cited

Amram, Martha and Nalin Kulatilaka. 1999. *Real Options: Managing Strategic Investment in an Uncertain World.* Harvard Business School Press. pp. 245.

Arrow, Kenneth and Anthony Fisher. 1974. *Environmental Preservation, Uncertainty and Irreversibility.* Quarterly Journal of Economics. vol. 88, no. 2. pp. 312-319.

Baxter, Martin and Andrew Rennie. 1996. *Financial Calculus.* Cambridge University Press. pp. 233.

Black, Fischer and Myron Scholes. 1973. *The Pricing of Options and Corporate Liabilities.* Journal of Political Economy. vol. 81, no.3. pp. 637-654.

Brennan, Michael and Eduardo Schwartz.1985. *Evaluating Natural Resource Investments.* Journal of Business. vol 58, no. 2. pp. 135-57.

Burnes, Ellen, Enrique Thomann and Edward C. Waymire. 1999. *Arbitrage Free Valuation of a Federal Timber Lease.* Forest Science. vol. 45, no. 4.

Dixit, Avinash and Robert Pindyck. 1994. Investment Under Uncertainty. Princeton University Press. pp. 468.

Morck, Randall, Eduardo Schwartz and David Stangeland. 1989. *The Valuation of Forestry Resources under Stochastic Prices and Inventories.* Journal of Financial and Quantitative Analysis. vol 24, no. 4. Pp. 473-487.

Thomann, Enrique and Edward C. Waymire. 2000. *Contingent Claims On Assets With Conversion Costs.* under review.

Trigeorgis, Lenos. 1995. *Real Options in Capital Investment: Models, Strategies and Applications.* Praeger Publishers. pp. 361.

4.7 Appendix

Escalated contracts imply that if the price increases, the government receives $\frac{1}{2}(S_t - k)$, where S_t is the market price at the time of harvest, and k is the agreed upon "strike price". The payoff of this arrangement is like one-half of a call option. If the price falls, the government receives k.

We can use the Black-Scholes model (Equation 4) to value this contract as well. However, V_t has a new expression. When we price a call, we only exercise if S_0>k. Therefore, we are interested in the payoff $V_t = \frac{1}{2}(S_t - k)^+$ The interestting case is where, after one time step,

$$S_0 u - k > 0 \therefore V_1(u) = \frac{1}{2}(S_0 u - k)$$
$$S_0 d - k < 0 \therefore V_1(d) = 0$$

Solving for φ, as before, we obtain:

$$\varphi = \frac{V_1(u)}{S_0(u-d)}, \text{ and}$$

$$\Psi = -\frac{V_1(u)d}{rB_0(u-d)}.$$

The value of the portfolio is:

$$W_0 = \frac{V_1(u)}{u-d} - \frac{V_1(u)d}{r(u-d)} = \frac{1}{r}\left[V_1(u)\left(\frac{r-d}{u-d}\right)\right] \qquad \textbf{(A 4.1)}$$

That is, it is the Black-Scholes equation when $V_1(d)=0$.

Now we must determine the appropriate k, or price for the timber at time t. That is, we need to obtain the k, that will give the appropriate amount of money to build the contract as specified by equation 4.1. The Forest Service receives 20% of the value of the contract at the time that the contract is written, and returns this money to the harvester, minus interest, at the time of harvest (per certain timing considerations). Therefore, the money available to the Forest Service is .2kr-.2k.

Discounted, this is $.2k\left(1-\frac{1}{r}\right)$. Now equate this to W(t) to obatin the correct value of k. This is because the value of the portfolio must be equal to the money available to construct the portfolio. That is:

$$\frac{1}{r}\left[\frac{1}{2}(S_0u-k)\left(\frac{r-d}{u-d}\right)\right]=.2k\left(1-\frac{1}{r}\right) \qquad \textbf{(A 4.2)}$$

$$k=\frac{S_0u\left(\frac{r-d}{u-d}\right)}{.4(r-1)+\frac{r-d}{u-d}} \qquad \textbf{(A 4.3)}$$

CHAPTER FIVE
CONCLUSION

These papers have developed tools, applications, and frameworks for evaluating natural resource investment and use under uncertainty. The primary benefit of these approaches is that they use systematic, rather than ad hoc, methods of incorporating price risk into contract valuation. As a result resource use and investment may be better matched to market parameters, and managed for economic efficiency given the biologic characteristics of the resource.

In Chapter 2, the impact of price uncertainty on renewable resource management is considered. Previous work suggested that under certain assumptions coincidental management plans existed regarding deterministic and stochastic stock evolution cases. This paper is an important link between stock and price due to the role of costs in determining feasible solutions, partcularly when price can fall below costs with a positive probability. By showing that under price uncertainty, stock management must be more conservative, the groundwork has been laid to deepen the work on the role of price uncertainty in resource management. For example, recent work by Plantinga (2001) suggests that the assumption of an exogenous price distribution may, in fact, not yield efficient results. The model in chapter 2 could be extended to consider alternative price expectations.

Chapter 3 provides an example of how option valuation methods traditionally applied in finance may be applied to the valuation of renewable natural resource contracts. These methods provide an objective way to assess the role of uncertainty in contract valuation since they are based on mathematical formulations that incorporate market parameters, including the risk-free interest rate, directly into the valuation calculations. Price changes are incorporated directly into the minimum bid such that the lease writer can cover itself against losses due to price fluctuations. The sensitivity analyses presented depict the magnitude of impact that volatility, interest rates, harvest costs and contract timing have on valuation. An additional contribution of the paper is it develops a method for including harvesting costs in the Black Scholes-Merton option pricing theory.

In Chapter 4 it was shown that resource contract valuations based on option pricing theory are sensitive the to assets used to hedge the contract payoffs. In fact, if one hedges a contract to deliver the undeveloped asset by holding the developed asset, one is left exposed, despite the appearance of an arbitrage free result. This is due to the existence and timing of conversion costs to bring the undeveloped resource into a developed state. Due to irreversibility, one can not make the transition from the developed to undeveloped state. The value of the asset in the undeveloped asset is convertible to that of a developed asset through harvesting, but the value of a developed asset is not convertible to that of an undeveloped asset. The role of conversion costs and convenience yields is important in obtaining an arbitrage-free valuation. Failure to account for convenience yields leads to a value that may be arbitraged.

In summary, these papers have expanded on the tools and insights available for the valuation and management of renewable natural resources for economic benefit. The papers provide deeper insights into the roles of interest rates, price uncertainty and costs. They also provide an explicit framework for developing risk-management strategies to mitigate both price and stock uncertainty. Finally, by explicitly linking the biologic and economic criteria that affect stock management decisions they provide a framework for exploring the relationship between environmental and market factors.

BIBLIOGRAPHY

Amram, Martha and Nalin Kulatilaka. 1999. *Real Options: Managing Strategic Investment in an Uncertain World.* Harvard Business School Press. pp. 245.

Arrow, Kenneth and Anthony Fisher. 1974. *Environmental Preservation, Uncertainty and Irreversibility.* Quarterly Journal of Economics. vol. 88, no. 2. pp. 312-319.

Baxter, Martin and Andrew Rennie. 1996. *Financial Calculus.* Cambridge University Press. pp. 233.

Bhattacharya, R. and E. Waymire .1990. Stochastic processes with applications. Wiley,New York, USA.

Black, Fischer and Myron Scholes. 1973. *The Pricing of Options and Corporate Liabilities.* Journal of Political Economy. vol. 81, no.3. pp. 637-654.

Brennan, Michael and Eduardo Schwartz.1985. *Evaluating Natural Resource Investments.* Journal of Business. vol 58, no. 2. pp. 135-57.

Burnes, Ellen, Enrique Thomann and Edward C. Waymire. 1999. *Arbitrage Free Valuation of a Federal Timber Lease.* Forest Science. vol. 45, no. 4.

Clark, Colin C. 1971. *Economically Optimal Policies for the Utilization of Biologically Renewable Resources.* Mathematical Biosciences. vol. 12. pp. 245-260.

Dixit, Avinash and Robert Pindyck. 1994. *Investment Under Uncertainty.* Princeton University Press. pp. 468.

Gruber, Urs. 1997. Pricing barrier and Asian options with an emphasis on Monte-Carlo methods. M.Sc. paper, Oregon State University, Corvallis, Oregon, USA.

Hull, J. 1993. Options, futures and other derivative securities, Prentice-Hall, 2nd edition.

Karatzas, I. 1997. Lectures on the mathematics of finance. American Mathematical Society, Providence,Rhode Island, USA.

MATLAB. 1995. Financial Toolbox, The Math Works Inc., Natick, Mass.

May, Robert M. 1976. *Simple Mathematical Models with Very Complicated Dynamics.* Nature. vol. 261. pp. 459-467.

McAfee, R.P. and J. McMillan. 1987. Auctions and bidding, Journal of Economic Lit. XXV, 699-738.

Morck, Randall, Eduardo Schwartz and David Stangeland. 1989. *The Valuation of Forestry Resources under Stochastic Prices and Inventories.* Journal of Financial and Quantitative Analysis. vol 24, no. 4. Pp. 473-487.

Paarsch, H.J. 1997. Deriving an estimate of the optimal reserve price: An application to British Columbian timber sales, Journal of Econometrics (78), 333-357.

Random Lengths. 1997. The Weekly Report on North American Forest Products Markets 53 (44).

Reed, William. 1974. *A Stochastic Model for the Economic Management of a Renewable Animal Resource.* Mathematical Biosciences. vol. 22. pp. 313-337.

Reed, W. 1993. The decision to conserve or harvest old growth forest, Ecological Economics (8), 45-69.

Reed, W. and R.G. Haight. 1996. Predicting the present value distribution of a forest plantation investment. Forest Science (42), 378-388.

Thomann, Enrique and Edward C. Waymire. 2000. *A Black-Scholes-Merton Equation with Conversion Costs.* to appear in Decisions in Economics and Finance: A journal of Applied Mathematics.

Trigeorgis, Lenos. 1994. *Real Options in Capital Investment: Models, Strategies and Applications.* Praeger Publishers. pp. 361.