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The Very High Temperature Reactor (VHTR) design was one of six designs chosen for development under the Generation IV initiative. Thermal mixing of the coolant at the exit of the reactor has been a common concern for VHTR reactor designs. Thermal mixing processes are commonly investigated through the use of scaled experimental models. Accurate scaling methods are essential for designing experimental facilities that accurately reproduce the thermal mixing processes of the VHTR design under investigation. A scaling method that is based upon a two-equation turbulence closure model is developed in this paper. Numerical studies performed verify that this scaling method correctly identifies the similarity parameters of the k-ε turbulence closure model and preserves the turbulent processes within the flow.
Scaling of the Thermal Mixing Processes within the Lower Plenums of VHTR Designs

by

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Scaling of the Thermal Mixing Processes within the Lower Plenums of VHTR Designs

1 Introduction

1.1 Background

Today, nuclear power reactors are helping to meet energy demands around the world. Approximately 17% of the world’s electricity is supplied by 431 nuclear power reactors operating in 31 different countries. There are another 32 nuclear plants under construction outside of the United States [32]. Figure 1.1 shown below outlines the evolution of nuclear power generation technology.

![The Evolution of Nuclear Power](image)

Figure 1.1 Evolution of Nuclear Power. Courtesy of U.S. DOE [32].

The Pressurized Water Reactors (PWRs) and Boiling Water Reactors (BWRs) currently operating in the U.S. are Generation II designs. These designs have shown themselves to be economically competitive and reliable sources of electricity [32].
However, energy security and environmental concerns have prompted consideration of the future of current energy sources. Fuel availability, climate changes, and air quality concerns all suggest that nuclear power may have an important role in meeting future energy demands. The advanced designs, which have been designated as Generation IV, are intended to broaden the opportunities for use of nuclear energy.

In 1999, The U.S Department of Energy (DOE) established the International Nuclear Energy Research Initiative (I-NERI) to support bilateral and multilateral research with other nations.[33] The DOE’s Office of Nuclear Energy, Science and Technology initiated the international effort to examine the future of nuclear energy by beginning a wide ranging discussion between the world-wide research community, industry, and governments. The discussion focused on future advances in nuclear energy system designs. These designs represent the next generation of nuclear energy systems, and have been brought together under the “Generation IV” umbrella.

1.1.1 Generation IV

The U.S. DOE came up with several objectives for Generation IV nuclear energy system designs. Generation IV nuclear energy systems are required to be sustainable, economical, safe and reliable, and proliferation resistant. The Generation IV International Forum (GIF) was chartered in 2002 in order to develop nuclear energy system designs that would satisfy these Generation IV objectives. GIF consists of ten countries (Argentina, Brazil, Canada, France, Japan, the Republic of Korea, the Republic of South Africa, Switzerland, the United Kingdom, and the United States)
Republic of South Africa, Switzerland, the United Kingdom, and the United States) and Euratom. The joint efforts made by GIF are focused on developing future-generation nuclear energy systems that can be licensed, constructed, and operated in a manner that will provide competitively priced, environmentally responsible, and reliable energy products.

In December, 2002, GIF along with the U.S. DOE’s Nuclear Energy Research Advisory Committee issued *A Technology Roadmap for Generation IV Nuclear Energy Systems*, which provides a comprehensive evaluation of potential Generation IV nuclear energy concepts. With the input of more than 100 experts from numerous countries and international organizations, the Roadmap was completed over a two year period.

The culmination of the Generation IV roadmap process was the selection of six of the most promising designs. These six Generation IV systems are the focus of future collaborative research and development by GIF countries. The goal for the Generation IV program is for the Generation IV designs to be deployable by 2030 when many of the currently operating nuclear power plants will be at the end of their operating licenses [34].

The six Generation IV systems chosen by GIF are:

- Gas-Cooled Fast Reactor System GFR
- Lead-Cooled Fast Reactor System LFR
- Molten Salt Reactor System MSR
- Sodium-Cooled Fast Reactor System SFR
These Generation IV nuclear energy system designs were selected because of their promise in several areas. Each design has demonstrated the potential to meet the next-generation nuclear energy system goals. These designs address missions such as electrical generation, actinide management, hydrogen production, and process heat production.

1.1.2 VHTR – Very-High-Temperature Reactor System

The VHTR uses a thermal neutron spectrum and operates with a once-through uranium fuel cycle. The proposed plant design is rated at 600 MWt and will use helium as its coolant. The desired reactor outlet temperature is above 1000°C. The reactor core will be a prismatic core such as Japan’s operating High Temperature Engineering Test Reactor (HTTR), or a pebble-bed core such as China’s High Temperature Gas-cooled reactor-test Module (HTR-10) which is also operating.

The primary missions of the VHTR are hydrogen production and process heat for various other applications. The high outlet temperature satisfies the efficiency requirements for the thermo-chemical iodine-sulfur hydrogen production process. Of the Generation IV nuclear system designs that can potentially be used for hydrogen production, the VHTR system is the nearest-term deployable. It is postulated that the VHTR could be deployed as soon as 2020 [34].
Operational gas cooled research reactors now exist in China and Japan, and a VHTR design (Next Generation Nuclear Plant) is currently under development in the U.S.

1.1.3 NGNP- Next Generation Nuclear Plant

Idaho National Laboratory (INL) is currently working on preliminary designs for a Next Generation Nuclear Plant (NGNP) which is intended to meet the design goals of the VHTR. One of the NGNP designs under investigation is a 600 MWt prismatic fuel type helium gas cooled reactor. The prismatic fuel NGNP design draws from a broad set of data and operational experience. It is based directly on the General Atomics design submitted to the Generation IV Roadmap, which, in turn, was based on a modification of the General Atomics Gas Turbine-Modular Helium Reactor (GT-MHR) design [19].

The NGNP uses much of the core physics, thermal hydraulic, and fuel performance design analyses that were performed by General Atomics while they were designing the GT-MHR. The development of the NGNP reactor design has consisted mostly of modifying the GT-MHR design to raise the reactor outlet gas temperature from 850 °C to 1000 °C.

The GT-MHR core consists of an annular-fueled region surrounded by graphite reflectors. The fueled region consists of graphite blocks, which contain compacts that are impregnated with TRISO coated fuel particles, and the reflector is made up of non-fueled graphite blocks. This is shown below in figure 1.2, where the
dark area is the fueled region of the core and the white blocks are the non-fueled graphite reflectors.

![Diagram of GT-MHR annular reactor core](image)

Figure 1.2 GT-MHR annular reactor core [19].

The GT-MHR core is cooled by helium gas, which enters and exits the reactor via the hot gas duct. The cold gas is heated as it flows downward through the annular reactor core. When the hot coolant exits the reactor core it mixes in the lower plenum and exits the reactor pressure vessel via the hot gas duct. The system schematic shown in figure 1.3, illustrates the internals of the GT-MHR and the flow path of the coolant as it enters and exits the reactor.
1.1.4 Experimental VHTR Facilities

China's Institute of Nuclear Energy Technology at Tsinghua University is home to the 10 MWt High Temperature Gas-cooled Reactor-Test Module (HTR-10). The HTR-10 uses spherical fuel elements, graphite core structural material, and helium coolant. Like other high temperature gas cooled reactor designs, the coolant flows between the reactor vessel and the steam generator via a coaxial hot gas duct.

In Japan, the Japan Atomic Energy Research Institute (JAERI) has been working to advance high temperature gas-cooled reactor technology since the early
1980's. In order to assist the development of gas cooled reactor technology, JAERI has constructed the high temperature engineering test reactor (HTTR). The HTTR is a 30 MWt seven region, prismatic fuel type, helium-cooled reactor.

1.2 Objective

The desired coolant temperatures at the outlet of the VHTR designs including the NGNP are near 1000°C. These extremely high temperatures restrict the types of materials that could be used within VHTRs and their downstream components. In order to choose suitable materials, the operating environment of the component in question must be known. If excessive radial temperature differences exist within the coolant exiting the reactor, downstream components may be subjected to excessive thermal stress. This could lead to a greatly reduced operational lifetime or even catastrophic failure during operation [19].

One of the ongoing design issues for the NGNP is the examination of the mixing of the high temperature jets that exhaust into the lower plenum from the reactor core. There is concern that hot streaks in the lower plenum can move through the lower plenum and hot gas duct and still be present at the hot gas duct outlet. The performance of the components downstream of the hot gas duct outlet will be affected by excessively large temperature differences caused by coolant hot streaks. Figure 1.4 shown below provides an illustration of hot streaks within a flow.
The likely methods for the investigation of the coolant mixing phenomena in the lower plenum of the NGNP reactor design will be properly scaled physical models and CFD simulations. Scaled physical models can be used to benchmark CFD simulations which in turn can then model lower plenum mixing at full scale. Therefore it is vital that an appropriate scaling methodology be developed for the investigation of the turbulent mixing of gas jets in the lower plenum of the NGNP.

The purpose of this paper is to develop a scaling method that will preserve the turbulent mixing processes within the lower plenum of the NGNP. Turbulent flows of interest in engineering applications cannot be resolved exactly using existing analytical methods. Instead, the effects of turbulence within flows are determined numerically or experimentally.

Numerical solution techniques use methods that approximate the turbulent parameters in order to determine the effects of turbulence on the flow. The complexity of these approximations varies between the different turbulence closure models. The governing equations for turbulence models can be used as a foundation for developing scaling methods that will maintain turbulent processes. Zero equation and k-epsilon turbulence closure models will be examined in the development of a
scaling methodology that will maintain the thermal mixing processes within turbulent flows.

1.3 Assumptions

The following assumptions have been made in the following study:

1. The pressure drop across the region of interest is negligible.
2. The flow is at steady state.
3. The boundary of the domain is considered to be adiabatic.
4. The viscous effects on the transport of turbulent properties are assumed to be negligible.

This scaling study is intended to identify the factors affecting the mixing that takes place within the lower plenum of the NGNP. This study does not make any attempt to quantify the mixing that takes place. Quantitative mixing studies can instead be carried out on properly scaled experimental facilities or through the use of benchmarked CFD codes.

1.4 Outline

This first chapter has introduced the Generation IV initiative and some of the reactor designs that have been designated as Generation IV designs. The background and design of the NGNP has been presented as well as the objectives and rationale for the current scaling study.
Chapter 2 provides a basic overview of the study of turbulence and introduces Reynolds decomposition, which is widely used in engineering applications.

Chapter 3 introduces turbulence closure models. Zero, one, and two equation models are introduced in order to provide background information on turbulence modeling.

Chapter 4 introduces several experiments that have been used to investigate the mixing of flow in various reactor designs. These experiments used scaling methods that were based upon zero equation turbulence models and engineering judgment.

Chapter 5 contains the development of methods to scale the thermal mixing processes within fluid flow based upon zero and two equation turbulence closure models.

Chapter 6 provides a numerical study of the developed scaling method by presenting predictions for the temperature distribution within the flow in domains that have been scaled based upon the method developed within this paper.

Finally, chapter 7 summarizes the findings of this work. It points to the conclusions that can be obtained and the future work that can be undertaken.
2 Introduction to Turbulence

The tortuous, disordered fluid flow that we characterize as turbulence has been known for centuries [18]. Before mathematical descriptions of fluid flow were available, visual and descriptive models were used to describe turbulent flow. Leonardo da Vinci (1452-1519) was the first to attempt to describe or model turbulent flow (circa 1495 A.D.) [6]. His sketch pads contain many realistic drawings of turbulent flows. These sketches illustrate the qualitative understanding of fluid flow possessed by da Vinci. A compilation of these sketches can be found in Richter (1970) [26].

In the early nineteenth century, French mathematician L. M. H. Navier (1758-1836) and English mathematician Sir G. G. Stokes (1819-1903) developed the governing equations for incompressible Newtonian fluids [22]. The Navier-Stokes equations, as these equations are typically referred to, are usually made up of the continuity equation, the momentum equations with Newton’s viscosity law, and an energy equation that makes use of Fourier’s conduction law [24]. As developed, these equations provided a description of laminar flow. A mathematical model to describe turbulent flow had not yet been developed [6].

Nearly four hundred years after Leonardo da Vinci’s qualitative investigation of turbulent flows, Osborne Reynolds (1883) became the first to perform a quantitative investigation of turbulent flow. Reynolds made use of dye injected into water flowing through a glass tube. He observed that if the velocity of the mean flow was low
enough, the injected dye remained laminar. At higher velocities Reynolds observed that "... at some point in the tube, always at a considerable distance from the trumpet of intake, the colour band would all at once mix with the surrounding water." [23] The dimensionless parameter that he used to describe the flow in his experiments has forever been linked with his name.

Most of the research done on turbulent flows over the last century has been based on the concept of time averaging. In 1895, O. Reynolds purposed that since turbulent flow is in a randomly unsteady state, mean equations of motion should be used to describe the flow. Since the Navier-Stokes equations also apply to turbulent flow, the pressure, velocity, and temperature of the Navier-Stokes equations should be decomposed into average and fluctuating components. This decomposition gives rise to turbulent or "Reynolds" stresses.

2.1 Reynolds Decomposition

Today, engineers use the same approach as Reynolds used when he decomposed instantaneous properties within turbulent flow into a mean part and a fluctuation. For incompressible turbulent flow with constant transport properties the variables of concern are velocity, pressure, and temperature. When decomposed, the instantaneous velocity, pressure and temperature become:

\[
\begin{align*}
    u_i &= \bar{u}_i + u'_i \\
    p &= \bar{p} + p' \\
    T &= \bar{T} + T'
\end{align*}
\]
The variables with the overbar denote the time averaged components, while the turbulent fluctuations are denoted by primed variables. The time-averaged or mean component for all variables is defined in the following manner:

$$\bar{a}_i = \frac{1}{t} \int_{t_0}^{t_0+t} a_i dt$$  \hspace{1cm} (2.2)

where $a$ is the variable of interest and $t$ is sufficiently large such that the averaged component becomes independent of time. The mean value $\bar{a}_i$ may itself vary with time. In such a case, the flow is referred to as unsteady turbulent flow [37].

The above integral relation can verify the following averaging rules, which apply to any two fluctuating turbulent quantities:

$$f' = 0 \qquad \bar{f} = \tilde{f} \qquad \bar{fg} = \tilde{fg} + \tilde{f}'g' \qquad \bar{f} + g = \tilde{f} + \tilde{g}$$  \hspace{1cm} (2.3)

If the variation of the mean quantities in an unsteady turbulent flow is slow compared to the frequency of the fluctuations, then the following derivative and integral formulas are valid for both the space and time dimensions.

$$\frac{\partial \bar{f}}{\partial s} = \frac{\partial \tilde{f}}{\partial s} \qquad \int \bar{f} ds = \int \tilde{f} ds$$  \hspace{1cm} (2.4)

It is important to note in the averaging rules above, that the mean of the product of two instantaneous values is not simply the product of their means. This
point is essential to the statistical approach of studying turbulence. If this were not true, the fluctuating quantities would be lost in the averaging process and there would be no way to relate the turbulent fluctuations to the mean flow.

Reynolds Decomposition can be applied to the instantaneous steady-state continuity, momentum, and energy equations shown below.

Continuity:

\[
\frac{\partial u_i}{\partial x_i} = 0
\]  

Momentum:

\[
u \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p^*}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_i \partial x_j}
\]  

In equation 6.2, the body force term due to gravity is combined with the pressure term to form a new variable \( p^* \).

Energy:

\[
u \frac{\partial T}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \alpha \frac{\partial T}{\partial x_i} \right) + S
\]  

where S in the energy equation is a source term.

Substituting the decomposed velocities into the continuity equation, and taking the time average of the equation yields.

\[
\frac{\partial \overline{u_i}}{\partial x_i} + \frac{\partial u_i'}{\partial x_i} = 0
\]
The time average of the fluctuating velocity is zero and the average of the average velocity is found to satisfy:

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0$$  \hspace{1cm} 2.9

Next, by subtracting equation 2.9 from equation 2.8 we find that the fluctuations also satisfy the continuity equation.

$$\frac{\partial \bar{u}_i'}{\partial x_i} = 0$$  \hspace{1cm} 2.10

Now we turn to the momentum and energy equations. After the decomposed variables are substituted into the momentum equation, and the time average is taken, we obtain the mean momentum equation shown below in summation notation:

$$\bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{p}^*}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \nu \frac{\partial \bar{u}_i}{\partial x_j} - (\bar{u}_i' u_j') \right]$$  \hspace{1cm} 2.11

where $\rho$ is the fluid density, and $\nu$ is the kinematic viscosity of the fluid. This mean momentum equation, often called the Reynolds-averaged Navier-Stokes (RANS) equation, forms the basis for the analysis of turbulent flows, and is the governing equation for time averaged turbulent flow.
The energy equation must be considered if the mixing of various temperature streams is of concern. Like the momentum equations, a Reynolds decomposition of the velocity and temperature variables can be performed. After performing the decomposition and time averaging, the thermal energy transport equation will have the following form:

\[
\bar{u}_i \frac{\partial \bar{T}}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \alpha \frac{\partial \bar{T}}{\partial x_i} - \bar{u}'_i \bar{T}' \right) + S
\]

where \( \alpha \) is the thermal diffusivity and \( S \) is a source term.

Much of the information of the structure of turbulent flow is lost by time averaging the Navier-Stokes equations. The turbulent details of the flow are now represented by new terms in the mean momentum and mean energy equations 2.11 and 2.12 respectively.

2.2 Turbulence closure problem

The mean fluctuation terms that appear in the time averaged momentum equation, \((u'_i u'_j)\) are known as Reynolds stresses, and can be represented by the Reynolds stress tensor:

\[
\bar{u}_i' u'_j = \tau_{ij}
\]

Although this new term is referred to as the Reynolds stresses, it is important to note that it actually comes from the convective transport of fluctuating momentum.
The effect of turbulent mechanisms on the mean flow takes place via the Reynolds stresses.

Time averaging the Navier-Stokes equations provides a way to relate turbulent mechanisms within a flow to the mean flow itself through the Reynolds stresses. The Reynolds stresses are not known, however. This becomes a new source of analytic difficulty, as six new unknowns, which are contained in the Reynolds stresses, have been generated. The number of equations describing the flow is insufficient to solve for all of the unknowns that now exist. This issue has been dubbed the turbulence closure problem [23].

Resolving this problem remains an ongoing task. It requires special assumptions that must be based upon a fundamental understanding of turbulent mechanisms. All of the analytical techniques based upon Reynolds decomposition of flow variables require assumptions somewhere within their development. The assumptions used to provide closure cover a wide range of complexity. Some are very simple while others are so complex that they are impractical from an engineering standpoint. The ideal closure model would be relatively simple, easy to implement, and robust, while also being accurate and general enough to be applied to any type of turbulent flow.

Solutions to the mean equations of motion require that the Reynolds stresses be related to mean flow quantities. This is where much of the effort in turbulence modeling has been focused [37]. Reynolds decomposition provides information on the net effect of fluctuations on the mean flow. This provides only a partial
description of turbulent flow since only the effect of the fluctuating quantities is determined, and not their actual values. Nevertheless, Reynolds decomposition has proven itself as an effective method of describing the effects of turbulent flow in engineering applications where the actual values of the fluctuating quantities are not as important as their net effect.

The Navier-Stokes equations fully represent both laminar and turbulent flow. In fact, an emerging field of study known as Direct Numerical Simulation (DNS) of turbulence makes use of the increased computational power of modern computers to directly solve the Navier-Stokes equations. However, the flow scales involved limit the application of these techniques to problems with very low Reynolds numbers only.

In DNS numerical solutions, the mesh spacing and time steps must be sufficiently small such that appreciable changes in velocity do not occur over them. If they are too large, the evolution of turbulent flow will not be correctly predicted. The scales required by higher Reynolds number flows of practical engineering problems cannot be economically resolved [3]. In fact, Emmons (1970) illustrates this by considering flow in a pipe at a $Re_D=10^7$ [10]. According to his calculations, predicting the fine details of such a flow would require approximately $10^{22}$ operations. Using a computer capable of one operation each nanosecond to solve such a problem would take about 320,000 years.

Since solving the Navier-Stokes equations directly for typical flows in engineering problems is not practical or achievable, other methods must be attempted. Most engineering problems involving turbulent flows can be solved using statistical
calculation methods. In statistical methods, the turbulence is averaged out of the flow by use of the idea proposed by Reynolds in 1895. Typically, engineers are not concerned with the details of turbulent fluctuations, so statistical methods provide a prudent technique to solving engineering problems [3].
3 Turbulence Modeling

3.1 Turbulence Closure Models

Any investigation of turbulent flow requires that certain physical attributes of the flow be characterized. The physical foundation for any turbulence closure model is the ability to capture and represent the characteristic scales associated with the flow. More than 60 years ago, Kolmogorov suggested that turbulent flow could be characterized if development of representative turbulent velocity and length scales within a flow were adequately described. Based upon this idea, turbulence closure models that use a turbulent eddy viscosity to couple the RANS equations have been developed. The turbulent eddy viscosity of these models is a representation of the product of the velocity and length scales within the flow [3].

\[ \nu_t \propto u_e l_t \]

Over the years, a significant number of both linear and nonlinear eddy viscosity models have been proposed. Linear models relate the Reynolds stresses or second-moments to the mean strain rate by making use of an isotropic eddy viscosity. Nonlinear models use polynomial tensors to represent the second moments or Reynolds stresses. The turbulent models used in this paper are linear eddy viscosity models, so the following discussion will focus only on development of linear eddy viscosity models.
3.2 Linear Eddy Viscosity Closure Models

Linear eddy viscosity closure models range in complexity from the zero equation models up through two equation models. The zero equation models use simple algebraic expressions to specify the turbulent velocity and length scales. At the complex end of the linear eddy viscosity model spectrum, the two equation models use differential transport equations to specify both the turbulent velocity and turbulent length scales.

Linear eddy viscosity models use a Boussinesq-type assumption to relate the turbulent Reynolds stresses and the mean strain rate. These models assume that the turbulent transport of momentum and energy can be related to the gradient of the transported property. For incompressible flow, the Reynolds stress tensor:

\[
\overline{u_i u_j} = \tau_{ij}
\]

takes on the following form:

\[
\tau_{ij} = -\nu_t \left( \frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i} \right) + \frac{2}{3} k \delta_{ij}
\]

where \( \delta_{ij} \) is the Kronecker delta, \( \nu_t \) is the turbulent eddy viscosity, and \( k \) is the turbulent kinetic energy given by

\[
k = \frac{\tau_{ij}}{2} = \frac{1}{2} \left( \overline{u_i'^2} + \overline{u_2'^2} + \overline{u_3'^2} \right)
\]
The right hand term in equation 3.3 is included in the Reynolds stress tensor so that the sum of the normal stresses is 2k. Thus equation 3.3 is also applicable to normal turbulent stresses as well. Unlike molecular viscosity, the turbulent viscosity is not a fluid property. The turbulent viscosity is a property of the flow, and may be different at different points within the flow.

Modeling of the turbulent thermal transport term appearing in the Reynolds averaged energy equation 2.12 is performed in a similar manner. The turbulent thermal transport is approximated in the same manner as the Reynolds stresses, which describe the turbulent transport of momentum. The turbulent thermal transport is related to the gradient of the transported quantity, temperature, by:

$$-u'_iT' = \alpha_i \frac{\bar{T}}{\bar{x}_i}$$  \hspace{1cm} 3.5

Eddy viscosity models assume that turbulent flow can be characterized by a single time and a single length scale. The core of each turbulence closure model is the method by which the time and length scales are specified. A brief description of various linear eddy viscosity models will now be presented.

3.3 Zero Equation Models

Zero equation models make use of empirical experimental data and past experience to algebraically specify the eddy viscosity. Two of the most widely used zero equation models are the constant eddy viscosity model and the mixing length model.
Constant eddy viscosity models use experimental data or trial and error to specify a constant eddy viscosity, $\nu_t$. This highly idealized assumption leads to grossly erroneous velocity profile predictions, especially near boundaries. In fact, constant eddy viscosity models only yield satisfactory results in situations when the turbulent terms in the momentum equation are negligible compared to the convective and pressure gradient terms [3]. The development of more accurate closure methods has rendered constant eddy viscosity models virtually obsolete.

Prandtl (1925) used an analogy with the kinetic theory of gases to develop his mixing length model. He was the first person to propose a turbulence model based on the length and velocity scales of turbulence. From the kinetic theory of gases, Prandtl postulated that the eddy viscosity was proportional to the average velocity and mean free path of the turbulence. The turbulent length scale was taken to be equal to the mixing length, $l_m$, and the velocity scale was taken to be equal to the mean velocity gradient times the mixing length. In the mixing length model for plane shear flow, the eddy viscosity is given by:

$$\nu_t = l_m^2 \left| \frac{d\bar{u}}{dy} \right|$$  \hspace{1cm} (3.6)

For three-dimensional flows, the mean velocity derivative is replaced by the r.m.s strain rate of the mean velocity field.

$$\nu_t = l_m^2 \left( 2 \bar{S}_{xy} \bar{S}_{xy} \right)^{1/2}$$  \hspace{1cm} (3.7)

where

$$\bar{S}_{xy} = \frac{1}{2} \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)$$  \hspace{1cm} (3.8)
The turbulent diffusivity of heat is represented by $\alpha_t$, and like eddy viscosity, it is not a property of the fluid but is instead a property of the flow. Turbulent thermal diffusivity is generally described by the following relation: [30]

$$\alpha_t = \frac{\nu_t}{\sigma_t} \quad 3.9$$

where $\sigma_t$ is referred to as the turbulent Schmidt number or turbulent Prandtl number. Experimental data indicates that $\sigma_t$ varies little across a flow, and that variations from flow to flow are also small. So it is reasonable to define $\sigma_t$ as an empirically derived constant, which is often done [3].

The only thing left to complete the model is to relate the mixing length, $l_m$, to the flow. The mixing length is generally defined in three different regions. The sublayer, overlap layer, and outer layer all have their own very simple empirical equations to estimate $l_m$.

The governing equations for a mixing length turbulence model are given by:

Continuity:

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \quad 3.10$$

Momentum:

$$\bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p^*}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ (\nu + \nu_t) \frac{\partial \bar{u}_i}{\partial x_j} \right] \quad 3.11$$
Energy:

\[
\bar{u}_i \frac{\partial \bar{T}}{\partial x_i} = \frac{\partial}{\partial x_i} \left( (\alpha + \alpha_i) \frac{\partial \bar{T}}{\partial x_i} \right) + S
\]

where \( u_i \) and \( \alpha_i \) are specified by equations 3.6 and 3.9 respectively.

Mixing length models have been successfully used to predict a wide variety of simple flows. These models are very effective and will give an excellent prediction of the velocity profile of a turbulent flow [37]. The “law of the wall” is an example, and is often used near the boundaries in more complex closure models.

One of the problems with mixing length models is that it requires ad hoc determinations of \( l_m \). In complex flows, the actual mixing length may not be adequately specified by the simple algebraic mixing length model for every location within the flow.

3.4 One Equation Models

One-equation models make use of the turbulent kinetic energy to determine the eddy viscosity. The turbulent kinetic energy transport equation can be obtained through the Reynolds stress transport equation. The Reynolds stress transport equation is obtained by subtracting the Reynolds averaged momentum equation 2.11 from the instantaneous momentum equation 2.6. Next, the \( i \)th component of the resulting equation should be multiplied by the \( u'j \) velocity fluctuation and \( j \)th component should be multiplied by the \( u'i \) velocity fluctuation. This gives the Reynolds stress transport equation. To get the kinetic energy transport equation,
simply let \( i=j \) in the Reynolds stress transport equation and sum over all three directions.

The turbulent kinetic energy is determined by solving the following differential transport equation:

\[
\bar{u}_j \frac{\partial k}{\partial x_j} = -\tau_{ij} \frac{\partial u_i}{\partial x_j} - \frac{\partial}{\partial x_j} \left( \frac{1}{2} u'_i u'_j + \frac{1}{\rho} p'u'_j \right) + \nu \nabla^2 k \tag{3.13}
\]

where \( \varepsilon \) is the viscous dissipation term given by:

\[
\varepsilon = \frac{1}{2} \varepsilon'' = \frac{1}{2} 2\nu \left( \frac{\partial u'_j \partial u'_i}{\partial x_j \partial x_i} \right) \tag{3.14}
\]

Solving equation 3.13 is not an easy task however, and requires modeling of the turbulent transport of pressure gradient work and turbulent velocity fluctuation. These are modeled by a gradient term

\[
\left[ \frac{1}{2} u'_i u'_j + \frac{1}{\rho} p'u'_j \right] = -\nu \frac{\partial k}{\sigma_k \partial x_j} \tag{3.15}
\]

where \( \sigma_k \) is an empirical dimensionless constant and

\[
\tau_{ij} = -\nu \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \tag{3.16}
\]

The turbulent kinetic energy, \( k \), is a direct representation of the intensity of the turbulence in a flow. Since the turbulent kinetic energy is mostly contained in large-
scale fluctuations, the velocity scale for one-equation models is taken to be $k^{1/2}$. With this, the eddy viscosity can be written as:

$$

v_t = \sqrt{k l_0}

$$

3.17

The kinetic energy dissipation rate is also governed by large-scale turbulent motion, so it to can be characterized by $k$ and $l_0$. Through a dimensional analysis using $k$ and $l_0$, the kinetic energy dissipation rate can be written as:

$$

\varepsilon = C_D \frac{k^{3/2}}{l_0}

$$

3.18

where $C_D$ is an empirical constant.

The turbulent thermal diffusivity $\alpha_t$ required for the energy equation is defined in the same manner as it was for the zero equation model, equation 3.9. Once again the determination of the turbulent thermal diffusivity relies upon the specification of the eddy viscosity. The set governing equations for the one equation turbulence model is given by the time averaged continuity and energy equations, equations 3.10 and 3.12 respectively, and a new form of the momentum equation involving $k$ along with the kinetic energy transport equation, both shown below.

Momentum:

$$

\bar{u}_i \frac{\partial \bar{u}_i}{\partial x_i} = -\frac{\partial}{\partial x_i} \left( \frac{1}{\rho} \bar{p} + 2k/3 \right) + \frac{\partial}{\partial x_i} \left[ (\nu + \nu_t) \left( \frac{\partial \bar{u}_j}{\partial x_i} + \frac{\partial \bar{u}_i}{\partial x_j} \right) \right]

$$

3.19
Kinetic Energy:

\[
\frac{u_j}{\partial x_j} = \nu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_i}{\partial x_j} \right) - \frac{\nu}{\sigma_k \partial x_j} \left( \frac{\nu}{\partial x_j} \frac{\partial k}{\partial x_j} \right) - C_D \frac{k^{3/2}}{\ell_0} + \nu \frac{\partial^2 k}{\partial x_j \partial x_i} \quad 3.20
\]

where \(\nu_t\) is specified by equation 3.17. Closure for one-equation models is obtained by determining the turbulent length scale, \(\ell_o\). Empirical formulas to determine \(\ell_o\) have been developed, but the results obtained with these one-equation models are no better than the best zero-equation models [37]. The fact that zero-equation models can predict flows with comparable accuracy while requiring much less computational effort has not helped the popularity of one-equation models.

3.5 Two Equation Models

In two-equation models the turbulent length and velocity scales of the eddy viscosity are calculated using two separate transport equations. Many two-equation models have been proposed. The most common method of determining the turbulent velocity scale in two-equation models is by solving the same turbulent kinetic energy equation that was used in the one-equation models 3.20. The length scale is determined through a transport equation for a variable that is made up of any combination of \(k\) and \(\ell_o\). Some of the proposed variables are frequency \((k^{1/2}/\ell_o)\) [16], vorticity \((k/\ell_o^2)\) [27], and turbulent dissipation \((\varepsilon)\) [14].
3.6 The k-ε Model

Of the two-equation models, the k-ε is the most popular. Past experience also indicates that the k-ε model works better than other two-equation models of the same family [3]. The k-ε turbulence model is based upon modeled transport equations for turbulent kinetic energy $k$, and turbulent dissipation $\varepsilon$. The turbulent length and time scales are related to $k$ and $\varepsilon$ through the following dimensional arguments:

\[
I_o \propto \frac{k\sqrt{k}}{\varepsilon} \quad 3.21 \\
\tau_o \propto \frac{k}{\varepsilon} \quad 3.22
\]

Borrowing from the kinetic theory of gases, as Prandtl did while developing his mixing length model, the eddy viscosity can be represented by:

\[
\nu_t = C_\mu \frac{k^2}{\varepsilon} \quad 3.23
\]

where $C_\mu$ is an empirical constant. This turbulent kinetic energy equation is the same equation as the kinetic energy transport equation used in the one-equation model, equation 3.20.

The definition for the turbulent dissipation appears in derivation of the turbulent kinetic energy transport equation. No modeling or approximations are required to define $\varepsilon$. The transport equation for $\varepsilon$, however, relies heavily on empirical
data. If $P_{\varepsilon}$, $D_{\varepsilon}$, and $\phi_{\varepsilon}$ represent the production of dissipation, the turbulent diffusion of dissipation, and the turbulent destruction of dissipation respectively, the exact equation for $\varepsilon$ can be written as:

$$\bar{u}_i \frac{\partial \varepsilon}{\partial x_i} = \nu \nabla^2 \varepsilon + P_{\varepsilon} + D_{\varepsilon} - \phi_{\varepsilon} \quad 3.24$$

The turbulent diffusion term is modeled by a gradient approximation similar to the one used in the $k$ equation.

$$D_{\varepsilon} = \frac{\partial}{\partial x_j} \left( \frac{\nu_t \frac{\partial \varepsilon}{\partial x_j}}{\sigma_{\varepsilon}} \right) \quad 3.25$$

The destruction of dissipation term is modeled through the same approach used to develop an equation for $\varepsilon$ in the one-equation models. The destruction of dissipation is assumed to be a function of the length and time scales of the turbulence. This allows the dissipation to be related to $k$ and $\varepsilon$.

$$\phi_{\varepsilon} = C_{\varepsilon^2} \frac{\varepsilon^2}{k} \quad 3.26$$

The production of turbulent dissipation is modeled by assuming that the production of dissipation is proportional to the production of turbulent kinetic energy, and the length and time scales of the turbulence. A dimensional analysis based upon these assumptions leads to:

$$P_{\varepsilon} = C_{\varepsilon^1} \frac{\varepsilon}{k} \nu \frac{\partial \bar{u}_i}{\partial x_j} \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \quad 3.27$$
The modeled equation for dissipation of turbulent kinetic energy can then be written by making the appropriate substitutions,

\[
\bar{u}_i \frac{\partial \varepsilon}{\partial x_i} = \nu \nabla^2 \varepsilon + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial \varepsilon}{\partial x_j} \right) + C_{s1} \frac{\varepsilon}{k} \nu \frac{\partial \bar{u}_i}{\partial x_j} \left( \frac{\partial \bar{u}_j}{\partial x_i} + \frac{\partial \bar{u}_i}{\partial x_j} \right) - C_{s2} \frac{\varepsilon^2}{k}
\]

3.28

The closed set of equations that make up the k-\(\varepsilon\) turbulence model include the continuity equation 3.10, the momentum equation 3.19, turbulent kinetic energy transport 3.20, the turbulent dissipation rate transport equation 3.28, and the energy equations 3.12.
4 Scaling Studies in Gas Cooled Reactors

The degree of thermal mixing of the coolant prior to exiting the reactor was a concern in both the HTR-10 and HTTR reactor designs and still under investigation for the developing NGNP design. Historical efforts for investigating this concern are discussed in the following section.

In the past, the thermal mixing phenomena in gas cooled reactor designs have been investigated by performing experiments on scaled models. A large amount of useful data can be obtained from these types of experiments.

4.1 HTR-10

The HTR-10 is a 10 MWt gas cooled research located at China’s Institute of Nuclear Energy Technology at Tsinghua University. The HTR-10 is a pebble bed reactor that uses helium as its primary coolant. The average coolant temperature rise across the HTR-10’s core is about 450 °C. Within the core, large radial temperature deviations occur during normal operation. The large radial temperature differences, which can reach 210 °C, are due to the unevenness of the neutron flux and gas flow rates within the core. In order to prevent damage to downstream components, the coolant must be mixed prior to reaching the reactor outlet so that the temperature differences within the coolant are less than 15 °C. [38]

The analysis of the coolant mixing phenomenon in the core bottom of the HTR was performed through the use of scaled models. In Germany, Damm and Wehrlein
(1992) were able to use air in a 1:2.9 scaled Plexiglas model to simulate the mixing phenomena within the HTR. Plexiglas was chosen in order to allow for flow visualization. The Plexiglas model was placed in a 2 MW air loop at Interatom’s Aerodynamic Test Facility. The maximum flow rate of the air loop, 20 kg/sec and operating pressure, 1.1 bar were used to determine the model scale of 1:2.9. This scale allowed the Reynolds number of the flow in the model to be the same as that for actual facility. Geometric scaling and maintaining equal Reynolds numbers in the flow of the model and prototype were the basis for the scaling of this experiment. Figure 4.1 shows a simplified sketch of the 1:2.9 scaled experiment used by Damm and Wehrlein to study the mixing in the lower plenum of the HTR-10.

Figure 4.1. Simplified sketch of scaled model of HTR-10 low plenum.
The results of Damm and Wehrlein's experiment showed that a very high temperature mixing rate occurred in the hot gas header and hot gas duct which was not anticipated. The temperature mixing of the main flow in the hot gas header and hot gas duct were quantified using the temperature mixing coefficient, $\phi$ defined as:

$$\phi = \left(1 - \frac{\Delta T_{outlet}}{\Delta T_{inlet}}\right) \times 100$$  \hspace{1cm} 4.1$$

where $\Delta T_{outlet}$ is the maximum air temperature difference at the outlet of the hot gas duct and $\Delta T_{inlet}$ is the difference in temperatures of the air streams. The temperature mixing coefficients encountered in this experiment were between 96.5 and 97.5 for a $Re$ range of 600,000 to 1,800,000. [8]

Further investigation of the coolant mixing within the HTR hot gas chamber was performed by Yao et al [38]. Air was used in a 1:1.5 scaled model. This experiment investigated the effect of various hot gas chamber structural configurations on the overall coolant mixing within the hot gas chamber. The configurations studied were an empty chamber, a chamber with a baffle mixer, a chamber with a scaled HTR mixer, and a chamber with more complex mixer.

In this experiment the degree of mixing was quantified using the Temperature Mixing Degree defined as:

$$TMD = 1 - \frac{\Delta T_{max}}{\Delta T_{in}}$$  \hspace{1cm} 4.2$$
The range of Reynolds numbers investigated were 75,000 to 400,000. The HTR scaled mixer and the complex mixer resulted in the same degree of coolant mixing. The TMD for both these configurations was between 93% and 96% for the range of Reynolds numbers investigated. Increasing the complexity of the mixer did not lead to an increased degree of coolant mixing [38].

The scaling approach used here was based upon a direct assault of the governing equations for fluid flow and thermal transport. Scaling criteria was formulated based upon the dimensionless form of these equations. Simplifications were made to these criteria based upon ad hoc assumptions and order of magnitude analysis of the dimensionless numbers that appear in the dimensionless governing equations.

Following this analysis, it was determined that the dimensionless temperature difference $T^*$ within the flow is a function of the Reynolds number and Prandtl number of the flow.

$$T^* = f(Re, Pr)$$

This provided the scaling criteria for the experiment. The Reynolds numbers, based upon the hot gas duct dimensions, were maintained equal between the model and the actual HTR-10 reactor. The Prandtl numbers were approximately the same, with the HTR-10 helium coolant having a Prandtl number of 0.70, and the air of the scaled model having a Prandtl number of 0.67.
4.2 HTTR

The HTTR is a 30 MWt gas cooled research reactor run by Japan’s Atomic Energy Research Institute (JAERI). The average temperature rise across the HTTR’s core is about 550 °C, with an average reactor exit temperature of 950 °C. It was estimated from the core design of the HTTR that the temperature difference between the central and exterior regions of the core would be 60 °C. Exiting temperature differences of 20 °C were estimated among exterior core regions. The hot plenum of the HTTR contains a disk type mixing promoter to assist the mixing of the coolant [11].

4.2.1 Scaled Experiments on HTTR CBS

Numerous experiments to clarify the thermal mixing in the core bottom structure (CBS) of the HTTR have been performed at JAERI. Inagaki et al (1990) performed an experiment using water in a one seventh scale model of the CBS. Water was chosen for the coolant because it was convenient for flow visualization.

Conformal geometry was used between the model and the HTTR core bottom structure. Therefore the geometric dimensions of the model were obtained by reducing the dimensions of the HTTR core bottom structure by a factor of one seventh. Since water, instead of helium, was used as the test fluid, special consideration was made to the differences in the mixing characteristics of the two fluids. The scaling considerations for thermal mixing in this experiment were based
upon the Reynolds averaged momentum and energy equations 2.10 and 2.11, respectively. Closure for these equations was provided through the use of a zero equation linear eddy viscosity model. The eddy viscosity for this model was specified by the following empirical correlation for fully developed flow within a pipe given by Y. Katto [15].

\[
\frac{\nu_t}{\nu} = 0.0069 \text{Re}^{0.5} - 1
\]  

4.5

This experiment showed that there was adequate coolant mixing in the CBS when the hottest coolant flowed from the central core region, but mixing was inadequate if the hottest coolant flowed from an exterior region. The dimensionless temperature \( \theta \), is shown at various locations within the hot gas duct, where \( L \) is the axial position and \( D_0 \) is the diameter. \( \theta \) is defined by:

\[
\theta = \frac{(T_{\text{Hot}} - T_{\text{Cold}})}{\Delta T}
\]  

4.6

At \( L/D_0 = 2 \) position in the outlet duct the temperature of the mixed water was found to be uniform when measured radially across the outlet duct. The value of \( \theta \) was found to be \( \theta = 0.143 \). This result was consistent for all Reynolds numbers investigated. The range of Reynolds numbers used in this test ranged from \( \text{Re} = 44,000 \) to \( \text{Re} = 83,000 \). All subsequent \( L/D_0 \) positions in the outlet duct showed similar results [12]. Figure 4.2 below shows a simple diagram of the one-seventh scale experimental apparatus used by Inagaki et al.
The tests run on this experimental facility showed that the thermal mixing characteristics have little dependence on the temperature differences of the coolant in the hot and cold regions or changes in Reynolds number of the flow over the range investigated. The mixing was found to occur within the plenum itself, prior to entering the outlet duct. The experimental data also provided a benchmark for a three-dimensional time dependent flow and heat transfer computer code. The code used a $k-\varepsilon$ turbulence model and agreed with experimental data. [12]
4.2.2 Full Scale Test of HTTR CBS

A full-scale model of the HTTR CBS called the T₂ test section was also used to confirm that the coolant would be adequately mixed. Normal operating conditions of the HTTR could be duplicated by placing the T₂ test section in the helium engineering demonstration loop (HENDEL). Figure 4.3 shows a simplified top and crosscut view of the T₂ test section.

![Diagram of T₂ test section](image)

**Figure 4.3 Simplified top view and cross sectional view of the T₂ test section.**

The results of this experiment also showed there would be adequate mixing in the CBS if the hottest coolant flowed from the central region, and that a hot streak would be present along the wall of the outlet gas duct if the highest temperature coolant flowed from one of the circumferencial regions. This had been predicted by
Y. Inagaki et al using the one-seventh scaled experimental model with water as the test fluid. Good agreement between the thermal mixing data of the scaled model and the actual core bottom structure, the T2 test section, can be used help to support the scaling methodology used by Y. Inagaki et al in the development of their one-seventh scale model.

The locations at which the temperature was measured in the T2 test section and the one-seventh scaled model are not exactly the same. However, temperature measurements were taken at $L/D_o$ values of 5.0 and 5.5 within the outlet gas duct in the experimental model and T2 test section, respectively. The thermal mixing data from similar tests in which the hot fluid entered through the central region was in agreement. Both sets of data make use of the same definition for measurements of the degree of thermal mixing $\theta$ given by equation 4.7. In the full scale experiment performed in the T2 test section $\theta$, measured at $L/D_o = 5.5$ was $\theta = 0.143$, which was the same value measured at $L/D_o = 5.0$ in the one-seventh scaled model. This supports the scaling method used to design the one-seventh scaled experiment.

Another important finding of this experiment was that the mixing characteristics within the CBS have no strong dependency on Reynolds number or the temperature difference between hot and cold regions [11]. The range of Reynolds numbers used in this test ranged from $Re = 1.8 \times 10^5$ to $Re = 4.7 \times 10^5$. This also supports the findings of the tests performed in the one-seventh scaled model, which showed similar behavior for thermal mixing processes.
The data obtained in this experiment was compared to the predicted results obtained from a three-dimensional, time-dependent flow and heat transfer computer code. The code used a k-\( \varepsilon \) turbulence model and was found to be in good agreement with the experimental data [11]. This supports the idea that the prominent turbulent processes within the flow are captured by the k-\( \varepsilon \) turbulence model.
5 Scaling of Turbulent Processes

Scaling laws for any phenomenon can be based upon a dimensional analysis of the governing equations of that phenomenon. The idea behind dimensional analysis is simple: physical laws do not depend upon arbitrarily chosen basic units of measurement. Dimensional analysis makes use of this idea through the formation of dimensionless groups. Dimensionless groups allow for a reduction in the number of arguments in the governing equations, making it easier to obtain solutions. They also allow the governing equations to be written in a form that does not depend on any units of measurement. Dimensional analysis provides a basis for the derivation of scaling laws that maintain the phenomenon’s self-similarity, or ability to reproduce itself in space and time [2].

The development of scaling laws for turbulence must make use of experimental data [2]. Experimental data presents itself in the governing equations of the turbulence closure models. So when setting out to derive scaling laws for a turbulent process, it is important to start with a turbulent closure model that can accurately represent the type of turbulent phenomenon that is trying to be maintained.

5.1 Nondimensional Forms of the Equations of Motion

The dimensionless form of the continuity, momentum, and energy equations will provide the starting point for this analysis. The continuity, momentum, and energy equations are given by equation 3.10, 3.11, and 3.12, respectively. The dimensionless forms of these governing equations are derived through the use of characteristic
parameters. These characteristic parameters are determined through an analysis of the initial conditions, boundary conditions, and the equations that govern the flow and are summarized in Table 5.1.

Table 5.1: Characteristic parameters appearing in the eddy viscosity model equations for continuity, momentum and energy.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Characteristic for</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_0$</td>
<td>Velocity ($u_i$)</td>
</tr>
<tr>
<td>$L$</td>
<td>Length ($x_i$)</td>
</tr>
<tr>
<td>$T_0$</td>
<td>Temperature ($T$)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Viscosity ($\nu$)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Diffusivity ($\alpha$)</td>
</tr>
</tbody>
</table>

These characteristic parameters are used to form a set of dimensionless variables that will be used during this scaling analysis. The set of dimensionless variables is defined below in Table 5.2.
Table 5.2. Dimensionless variables appearing in the governing equation for a linear eddy viscosity model of turbulent flow.

<table>
<thead>
<tr>
<th>Dimensionless Parameter</th>
<th>Dimensionless Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity ($u_i^*$)</td>
<td>$\bar{u}_i^* = \frac{\bar{u}_i}{U_o}$</td>
</tr>
<tr>
<td>Length ($x_i^*$)</td>
<td>$x_i^* = \frac{x_i}{L}$</td>
</tr>
<tr>
<td>Temperature ($\theta$)</td>
<td>$\theta^* = \frac{T}{T_o}$</td>
</tr>
<tr>
<td>Pressure ($p^*$)</td>
<td>$p^* = \frac{p - p_o}{\rho U_o^2}$</td>
</tr>
<tr>
<td>Eddy Viscosity ($\nu_t^*$)</td>
<td>$\nu_t^* = \frac{\nu_t}{\nu}$</td>
</tr>
<tr>
<td>Eddy diffusivity ($\alpha_t^*$)</td>
<td>$\alpha_t^* = \frac{\alpha_t}{\alpha}$</td>
</tr>
</tbody>
</table>

The following dimensionless form of the governing equations can be obtained by substituting the dimensionless variables listed in table 5.2 into equations 3.10, 3.19, and 3.12.

Continuity:

$$\frac{\partial \bar{u}_i^*}{\partial x_i^*} = 0$$  \hspace{1cm} 5.1
Momentum:

\[ \bar{u}_i \frac{\partial \bar{u}_i}{\partial x_i} = -\frac{\partial (\bar{p} + \frac{2k}{3})}{\partial x_i} + \frac{1}{Re} \frac{\partial}{\partial x_i} \left[ (1 + \bar{\nu}_i) \left( \frac{\partial \bar{u}_i}{\partial x_i} + \frac{\partial \bar{u}_i}{\partial x_j} \right) \right] \]

Energy:

\[ \bar{u}_i \frac{\partial \bar{\theta}}{\partial x_i} = \frac{1}{Re \cdot Pr} \frac{\partial}{\partial x_i} \left[ (1 + \bar{\alpha}_i) \frac{\partial \bar{\theta}}{\partial x_i} \right] \]

The initial conditions considered in this analysis are set based upon the steady state values of the flow. The boundaries of the domain are considered to be adiabatic, and a no slip condition is assumed on all surfaces.

5.2 Justification for Simplified Form of Governing Equations

The dimensionless forms of the governing equations given above rely upon several simplifications. These simplifications include incompressible flow, negligible pressure drop across the region, and negligible buoyancy effects within the flow. These simplifications were made based upon the normal operating conditions in the Core Bottom Structure (CBS) of the HTTR.

Incompressible flows can be characterized by flows which have relatively low Mach numbers (Ma). The Mach number is defined as:

\[ Ma = \frac{V}{a} \]
where \( V \) is the velocity of the flow and \( a \) is the speed of sound within the fluid. It is generally accepted that flows with Mach numbers under approximately 0.3 can be treated as incompressible flows [22].

The incompressible assumption is justified based upon the normal operating conditions within the lower plenum, using the hot gas duct as the reference measurement location. Tests were run on the full scale model of the HTTR CBS in order to simulate normal operating conditions within the HTTR CBS [11]. These tests used a Reynolds number based on the flow and dimensions of the hot gas duct of \( \text{Re} = 2.7 \times 10^5 \). The temperatures of the flow streams within this test were 900 °C and 1000 °C, and the test pressure was 4.0 MPa. The fluid properties were therefore determined at 950 °C and 4.0 MPa. The Mach number calculated in the hot gas duct at normal operating conditions is 0.013. Therefore the flow can be treated as incompressible.

The buoyancy effects within a flow can be evaluated through the use of an effective Froude number. The effective Froude number, which is the ratio of gravity forces to inertial forces within the flow, is given by:

\[
Fr_{\text{eff}} = \frac{Gr}{Re^2} = \frac{\beta \Delta T g L}{V_0^2} = \frac{\beta \Delta T}{Fr}
\]

Where \( Gr \) is the Grashof number, \( Re \) is the Reynolds number, and \( \beta \) is the coefficient of thermal expansion, which is defined as:

\[
\beta = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_p
\]
With this definition, the effective Froude number takes on the form of the inverse of the usual definition for the Froude number, which is the ratio of inertial to buoyancy forces. The buoyancy effects within the flow will be negligible if the effective Froude number of the flow is small. This can be the result of small temperature differences across the streams within the flow, large flow velocities, or negligible differences in fluid density when evaluated over the range of temperatures within the flow. For normal operating conditions, the $F_{r_{eff}}$ was calculated to be on the order of $F_{r_{eff}} = 0.006$. Based upon this the buoyancy effects were neglected.

5.3 Similarity Relations

5.3.1 Geometric similarity:

Geometric similarity requires that the dimensionless position for corresponding points within the model and prototype be equal. This can be achieved by choosing a conformal geometric relationship between the model and prototype. By using a model with dimensions that are a direct reduction of the dimensions of the prototype, the dimensionless length ($x_i^*$) criteria will be satisfied at every corresponding location within the model and prototype, and geometric similarity will be obtained.

5.3.2 Kinematic similarity:

As previously mentioned, kinematic similarity requires that the dimensionless velocity profile given as a function of dimensionless position, $\tilde{u}_i^* = f(x_i^*)$, be the same for the model and prototype. We shall seek to satisfy this requirement through
analysis of the dimensionless form of the momentum equation 5.2. After only a quick

glance at this equation it becomes evident that the only way kinematic similarity

between the model and prototype can be achieved is if the forces acting on the fluid

are the same at corresponding points within both flows. Therefore, geometric and
dynamic similarity is required to ensure that kinematic similarity exists.

5.3.3 Dynamic similarity:

Dynamic similarity between the model and prototype requires that the relative

forces acting upon their flows are equal at corresponding points. Based upon this

requirement, several dimensionless parameters that are required for dynamic similarity
can be identified within the dimensionless form of the momentum equation 5.2. These
dimensionless parameters include, dimensionless length $x_i^*$, dimensionless pressure
$p^*$, dimensionless turbulent kinetic energy $k^*$, dimensionless eddy viscosity $\nu^*_t$, and
Reynolds number $Re$.

5.3.4 Thermal similarity:

Since we are concerned with the turbulent thermal mixing within the flow,

thermal similarity between the model and prototype must also be required. Thermal

similarity requires the dimensionless temperature as a function of dimensionless
position $\bar{\theta} = f(x_i^*)$ to be the same for both the model and prototype. This requirement
can be investigated through an analysis of the dimensionless energy equation 5.3.

From the dimensionless form of the energy equation, it can be seen that thermal
similarity requires the dimensionless length $x_i^*$, dimensionless thermal diffusivity $\alpha_i^*$, Reynolds number $Re$, and Prandtl number $Pr$ be equal at corresponding points within the model and prototype.

5.3.5 Summary of Similarity Requirements

A summary of the similarity requirements that have been identified through the dimensionless forms of the continuity, momentum and energy equations is listed below in Table 5.3.

Table 5.3 Similarity requirements from dimensionless governing equations.

<table>
<thead>
<tr>
<th>Similarity Type</th>
<th>Continuity</th>
<th>Momentum</th>
<th>Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric</td>
<td>$x_i^*$</td>
<td>$x_i^*$</td>
<td>$x_i^*$</td>
</tr>
<tr>
<td>Kinematic</td>
<td>$\bar{u}_i^*$</td>
<td>$\bar{u}_i^*$</td>
<td>$\bar{u}_i^*$</td>
</tr>
<tr>
<td>Dynamic</td>
<td>—</td>
<td>$p^<em>, k^</em>, \nu_i^*, Re$</td>
<td>—</td>
</tr>
<tr>
<td>Thermal</td>
<td>—</td>
<td>—</td>
<td>$\alpha_i^<em>, \bar{u}_i^</em>, Re, Pr$</td>
</tr>
<tr>
<td>Total</td>
<td>$x_i^<em>, \bar{u}_i^</em>, p^<em>, k^</em>, \nu_i^<em>, \alpha_i^</em>, Re, Pr$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.4 Similarity Requirements

5.4.1 Length

A conformal geometric relationship will be chosen for the model and prototype. This choice of geometric scaling fixes the characteristic length of the
model as some fraction of the characteristic length of the prototype, and ensures geometric similarity. This fraction will be referred to as the geometric ratio \( \Lambda \), where

\[
\Lambda = \frac{L_M}{L_p}
\]  

5.4.2 Pressure

For regions over which the pressure losses and pressure differences due to changes in elevation are negligible, pressure variations within the flow will be caused by changes in the fluid velocity. This allows us to describe the pressure at any point within the flow in terms of the time-averaged velocity at that point.

\[
p - p_o = \frac{1}{2} \rho (\bar{u}_i^2 - \bar{u}_{ref}^2)
\]  

Where \( \bar{u}_{ref} \) is the \( i^{th} \) component of the mean velocity averaged over the inlet or outlet of the region of interest. Since the pressure losses across the region have been assumed to be negligible, the inlet and exit static pressures are considered to be equal. So the \( i^{th} \) component of the mean inlet or exit velocity simply provides a reference velocity for the region, which can be used to calculate the pressure changes from point to point within the flow.

A new form for the dimensionless pressure can now be developed through the use of equation 5.8. The dimensionless pressure now becomes:

\[
p^* = \left( \frac{p - p_o}{\rho U_o^2} \right) = \left( \frac{1}{2} \frac{\rho (\bar{u}_i^2 - \bar{u}_{ref}^2)}{\rho U_o^2} \right)
\]  

5.9
Further simplification can be made if the characteristic velocity of the region is chosen in an intelligent manner. If the characteristic velocity is chosen to be the mean inlet or exit velocity of the region, the dimensionless pressure can be written in the following form:

$$p^* = \frac{1}{2} \left( \frac{\mu_i}{\mu} \right)^2 - 1$$  \hspace{1cm} 5.10

Writing the dimensionless pressure in this form implies that the dimensionless pressure can actually be shown to be equal between corresponding point in the model and prototype through obtaining kinematic similarity. Therefore, equality of the dimensionless pressure is no longer a dynamic similarity concern. Like the other kinematic similarity requirement, the dimensionless pressure equality will be satisfied if dynamic similarity is achieved.

5.4.3 Dimensionless Groups:

Two dimensionless numbers appear in the dimensionless form of the momentum and energy equations, the Reynolds number and the Prandtl number. Since the Reynolds and Prandtl numbers must be equal for the model and the prototype, the following conditions must be met:

$$\left( \frac{U_o L}{\nu} \right)_M = \left( \frac{U_o L}{\nu} \right)_P$$  \hspace{1cm} 5.11

$$\left( \frac{\nu}{\alpha} \right)_M = \left( \frac{\nu}{\alpha} \right)_P$$  \hspace{1cm} 5.12

Given that the ratio of the characteristic lengths can be expressed in terms of the geometric ratio \( \Lambda \), the conditions for Reynolds number equality can be expressed as:
5.5 Turbulent Kinetic Energy Similarity Requirements

The parameters that the turbulent kinetic energy depends upon can be seen in the definition of turbulent kinetic energy given by equation 3.4. By this definition, the turbulent kinetic energy is defined to be a function of the fluctuating velocity components. However, since there is no way of directly solving for the fluctuating velocity component, this is as far as we can go with only the use of exact mathematical definitions. Instead, experimental data, engineering judgment and the dimensionless transport of turbulent kinetic energy will be used to investigate the functional dependence of $k^*$.

5.5.1 Turbulent velocity scale in jet flow

The turbulent thermal mixing of the coolant streams within the lower plenum of VHTRs can be approximated as a series of axisymmetric jets. Due to the relative simplicity of axisymmetric turbulent jets, reasonably well-developed analytical models and experimental data exist. The actual thermal mixing processes within the lower plenum are more complicated than a series of axisymmetric jets. However, insight into the mixing processes can still be gained through use of this simplification. Figure 5.1 illustrates the physical dimensions of an axisymmetric turbulent jet.
One of the simplest ways to understand turbulent kinetic energy is to assume that turbulent thermal mixing is dominated by certain turbulent eddies within the flow. The largest eddies will be the most effective at transferring momentum and energy across the flow stream, so they can be thought of as the most important eddies in the thermal mixing process. These large eddies also contain most of the turbulent kinetic energy within the flow. The length and velocity scales of the large eddies can therefore be used to characterize the turbulent mixing processes within the flow. The velocity and length scales of the largest eddies are on the order of the local mean velocity and the width of the flow, respectively. [31]. With this simplification, the turbulent kinetic energy of the flow is given by:

\[ k = f(U_s) \]  

5.14
where $U$ is the local mean velocity. The dimensionless form of this expression for $k$ is given by:

$$k^* = f(\bar{u}^*)$$

This simple model suggests that kinematic similarity between the model and prototype will result in similarity in turbulent kinetic energy.

Experimental data can be examined to investigate the accuracy of this description of the turbulent kinetic energy. Panchapakesan and Lumley [33] performed a series of experiments in order to investigate the density fluctuations within axisymmetric turbulent jets discharging into ambient air. Their experiment consisted of two parts. The first part investigated a turbulent air jet, while the second part used helium as the test fluid. The momentum flux at the nozzle exit was made the same for both parts in order to facilitate a detailed comparison. Interference and composite probes were used to measure the intensity of the turbulent velocity fluctuations within the jet flows. See Panchapakesan and Lumley [25] for full details of their experimental setup.

Note that this experimental study is for fully developed self-similar turbulent jet. As a rule of thumb turbulent jets can be considered to be fully developed self-similar jets for $x/d > 40$. This is not the case in the HTTR CBS where $x/d < 1.2$.

In this study turbulent intensity in all three directions are measured experimentally. Turbulent intensity is defined by the following equation:
where \( U_x \) is the centerline velocity at axial position \( x \) within the jet.

This study suggests that for the fully developed self-similar helium jet, the turbulent intensity is a function of the local mean velocity and position. It is important to note that this study measures the turbulent intensity which takes into consideration turbulent eddies of all sizes in the flow. The turbulent eddies with large velocities are given the same weight as eddies with smaller velocities. This does not give proper importance to the eddies that are most effective at transporting energy. However, the suggestion that local mean velocity and position affect the turbulent intensity is important. This again suggests that if the same fluid is used, kinematic similarity between the model and prototype will lead to similarity in turbulent kinetic energy within their flows.

5.5.2 Modeled transport equations

The transport equations for turbulent kinetic energy and turbulent dissipation, given by equations 3.20 and 3.28, respectively, can be used to develop similarity criteria. The dimensionless forms of these governing equations are derived through the use of characteristic parameters. These characteristic parameters are summarized in Table 5.4.
Table 5.4: Characteristic parameters appearing in the transport equations for turbulent kinetic energy and turbulent dissipation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Characteristic for</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_0$</td>
<td>Velocity ($u_i$)</td>
</tr>
<tr>
<td>$L$</td>
<td>Length ($x_i$)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Viscosity ($\nu$)</td>
</tr>
</tbody>
</table>

Once again, these characteristic parameters are used to form a set of dimensionless variables that will be used during this scaling analysis. The set of dimensionless variables appearing in the turbulent kinetic energy and dissipation rate transport equations is defined below in Table 5.5.

Table 5.5. Dimensionless variables appearing in the governing equation for a linear eddy viscosity model of turbulent flow.

<table>
<thead>
<tr>
<th>Dimensionless Parameter</th>
<th>Dimensionless Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity ($u_i^*$)</td>
<td>$\vec{u}_i^* = \frac{\vec{u}_i}{U_0}$</td>
</tr>
<tr>
<td>Length ($x_i^*$)</td>
<td>$x_i^* = \frac{x_i}{L}$</td>
</tr>
<tr>
<td>Turbulent Kinetic Energy ($k^*$)</td>
<td>$k^* = \frac{k}{U^2}$</td>
</tr>
<tr>
<td>Turbulent Dissipation Rate ($\varepsilon^*$)</td>
<td>$\varepsilon^* = \frac{\varepsilon}{U^3/L}$</td>
</tr>
<tr>
<td>Eddy Viscosity ($\nu_t^*$)</td>
<td>$\nu_t^* = \frac{\nu_t}{\nu}$</td>
</tr>
</tbody>
</table>
Substituting the dimensionless variables of table 5.5 into equations 3.20 and 3.28 leads to the dimensionless turbulent kinetic energy and dissipation rate transport equations. The viscous term of the kinetic energy transport equation 3.20 has been assumed to be negligible and will be omitted from the dimensionless form of the turbulent kinetic energy transport equation. The functional dependency of $k^*$ can also be investigated through an analysis of the modeled $k^*$ and $\varepsilon^*$ transport equations shown below.

**Turbulent Kinetic Energy:**

$$
\overline{u_i^*} \frac{\partial k^*}{\partial x_i^*} = \frac{\nu_{t^*}}{Re} \frac{\partial \overline{u_j^*}}{\partial x_j^*} \left[ \frac{\partial \overline{u_i^*}}{\partial x_i^*} + \frac{\partial \overline{u_j^*}}{\partial x_j^*} \right] + \frac{1}{Re} \frac{\partial}{\partial x_j^*} \left( \frac{\nu_{t^*}}{\sigma_k} \frac{\partial k^*}{\partial x_j^*} \right) - \varepsilon^*
$$

5.17

**Turbulent Dissipation:**

$$
\overline{u_i^*} \frac{\partial \varepsilon^*}{\partial x_i^*} = \frac{1}{Re} \frac{\partial}{\partial x_j^*} \left( \frac{\nu_{t^*}}{\sigma_\varepsilon} \frac{\partial \varepsilon^*}{\partial x_j^*} \right) + C_{\varepsilon 1} \frac{\varepsilon^*}{k^*} \frac{\nu_{t^*}}{Re} \frac{\partial \overline{u_i^*}}{\partial x_j^*} \frac{\partial \overline{u_j^*}}{\partial x_i^*} - C_{\varepsilon 2} \frac{\varepsilon^2}{k^*}
$$

5.18

where $\nu_{t^*}$ is defined by the $k$-$\varepsilon$ turbulence model as:

$$
\nu_{t^*} = C_\mu \frac{k^{1/2}}{\varepsilon^*}
$$

5.19

Through analysis of the dimensionless transport equations shown above, a list of parameters upon which the turbulent kinetic energy depends can be compiled. This list is provided below in table 5.6 The requirements for $\varepsilon^*$ derived from the dimensionless turbulent dissipation transport equation are substituted into the list of parameters from the turbulent kinetic energy equation to obtain a complete list of parameter that $k^*$ depends upon.
It should also be noted that in each term of the transport equations 5.17 and 5.18 that the dimensionless eddy viscosity appears is divided by Re. Therefore the solutions for the dimensionless transport equations will not depend upon Re. Additionally, the modeling constants used in equations 5.17 and 5.18 are taken to be the same for the model and prototype. This is mentioned because it may potentially be a source of error in this scaling method.

Table 5.6 Similarity requirements for $k^*$ from turbulent kinetic energy and turbulent dissipation transport equations and experimental data.

<table>
<thead>
<tr>
<th>Source of Requirements</th>
<th>Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbulent kinetic energy transport equation**</td>
<td>$x_i^<em>, \bar{u}_i^</em>, \text{and } \varepsilon^*$</td>
</tr>
<tr>
<td>Turbulent dissipation transport equation**</td>
<td>$x_i^<em>, \bar{u}_i^</em>, \text{and } k^*$</td>
</tr>
<tr>
<td>Complete list for $k^*$ similarity from transport equations**</td>
<td>$x_i^<em>, \bar{u}_i^</em>$</td>
</tr>
<tr>
<td>Simple model</td>
<td>$\bar{u}_i^*$</td>
</tr>
<tr>
<td>Experimental Data</td>
<td>$\bar{u}_i^<em>, x_i^</em>$</td>
</tr>
</tbody>
</table>

** Assumes that model constants from $k$-$\varepsilon$ turbulence model are equal in both model and prototype.

The similarity requirements of the turbulent kinetic energy and turbulent dissipation rate transport equations are similar to the requirements that were determined from the dimensionless continuity, momentum and energy equations 5.1, 5.2 and 5.3,
respectively. For similarity of the dimensionless turbulent kinetic energy and turbulent dissipation rate, geometric and kinematic similarity are required. Additionally, similarity of the model constants used to develop the transport equations is also required.

5.6 Eddy Viscosity Similarity Requirements

The dimensionless eddy viscosity is given by:

\[ \nu_t' = \frac{\nu_t}{\nu} \]

The assumption that the turbulent characteristics of a flow can be described through the length and velocity scales of the flow is the basis for eddy viscosity turbulence models. Recall that an analogy with the kinetic theory of gases that was used to develop the following description of the turbulent eddy viscosity:

\[ \nu_t \propto u_t l_t \]

Application of this idea to the development of a scaling methodology to maintain the thermal mixing phenomena of a flow required that a method for describing the turbulent length and velocity scales be defined. This is the same issue that arose within all of the linear eddy viscosity turbulent closure models that were previously discussed.
5.6.1 Turbulent velocity scale

The simplest way to investigate the turbulent velocity scale is through an assumption that is similar to the one used in the analysis of turbulent kinetic energy. Here we assume that most of the momentum and energy transfer across a turbulent flow is due to large eddies. Since the eddy viscosity represents a measurement of the momentum transfer within the flow, it will depend on the physical characteristics of the large scale eddies. The large scale eddies have velocities that are of the same order of magnitude as the mean flow.

An earlier discussion showed that the turbulent kinetic energy of the flow can be used as a representation of the turbulent velocity scale within the flow. This idea is used in second order turbulence closure models such as the $k-\varepsilon$ turbulence model. In the $k-\varepsilon$ turbulence model, the turbulent velocity scale is defined by:

$$u_t = \sqrt{k} \quad 5.22$$

where $u_t$ is the turbulent velocity scale. Therefore the turbulent velocity scale will depend upon the same parameters as the turbulent kinetic energy and no new scaling criteria are obtained.

5.6.2 Turbulent length scale:

Visualization of turbulent flow, which was one of the earliest methods used to study turbulence, shows that a spectrum of eddy sizes exist within turbulent flow. Based upon the work of Kolmogorov, it can be determined that the largest eddies within the flow are where the production of turbulent kinetic energy occurs. These
largest eddies, which can be thought of as the energy containing eddies, are primarily responsible for the bulk of the turbulent mass and momentum transfer. Therefore, the size of these largest eddies will be used to represent the turbulent length scale of the flow. The largest eddy size within the flow will be of the same order as the domain of the flow. Therefore, the turbulent length scale can be represented by:

\[ l_t = x_i \quad 5.23 \]

This idea can be combined with the earlier description for the turbulent velocity scale to form scaling criteria that is based upon analogy with the kinetic theory of gas. With this analogy, given by equation 5.18, scaling of the eddy viscosity requires similarity of the product of local mean velocity and the size of the domain at that location. This simple representation of the turbulent length and velocity scales of the flow yields the following functional dependence for the eddy viscosity.

\[ \nu_t = f(\bar{u}_i x_i) \quad 5.24 \]

Using this definition, the dimensionless eddy viscosity can be written as:

\[ \nu_t^* = f(\bar{u}_i^* x_i^*) \quad 5.25 \]

The eddy viscosity can also be investigated through the definition used in the \( k-\varepsilon \) turbulence model. The \( k-\varepsilon \) turbulence model uses the following definition for the eddy viscosity:

\[ \nu_t = C_{\mu} \frac{k^2}{\varepsilon} \quad 5.26 \]
From this definition it becomes apparent that the eddy viscosity will be expressed with the same parameters as the $k$ and $\varepsilon$. The nondimensionalized form of the eddy viscosity will depend upon the parameters derived from the dimensionless turbulent kinetic energy and dissipation rate transport equations shown in table 5.6.

5.6.3 Comparison to empirical models:

Alpinieri's correlation for the eddy viscosity of a jet flow discharging into an external stream gives a similar functional dependence for $\nu_t$. Alpinieri's correlation is given by:

$$ (\rho \nu_t)_c = 0.025 \, b \, U_j \, \rho_e \left[ \frac{U_c}{U_j} + \left( \frac{U_e}{U_j} \right)^2 \right] $$

where $U_c$ is the centerline velocity, $U_j$ is the jet velocity, $U_e$ is the velocity of the external medium, $\rho$ is the density of the jet fluid, and $\rho_e$ is the density of the external medium [1]. The parameter $b$ is the half-radius of the mixing region, which is defined as the distance from the centerline to the point where the local velocity is one-half of the centerline value. Assuming that the densities of the fluid streams are similar allows Alpinieri's correlation to be written as:

$$ \nu_t = f(\bar{u}_j, x_i) $$

If the turbulent eddy viscosity given by 5.28 is nondimensionalized using the dimensionless variables of table 5.2, the dimensionless eddy viscosity is given by:

$$ \nu'_t = f(x'_i, \bar{u}'_j) $$
Therefore, the kinetic theory of gas is shown to be consistent with known correlations for the turbulent eddy viscosity for jet flow. The expression for $v_t^*$ obtained from the kinetic theory of gas requires more information about the jet to be specified, namely the jet temperature half width constant.

Table 5.7 lists the parameters that the turbulent eddy viscosity $v_t^*$ depends upon according to how the derivation of this dependence was performed.

Table 5.7 Similarity requirements for $v_t^*$ from kinetic theory of gas and empirical correlation.

<table>
<thead>
<tr>
<th>Source of Requirements</th>
<th>Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kinetic theory of Gas</td>
<td>$x_i^* \bar{u}_i^*$</td>
</tr>
<tr>
<td>Empirical correlation</td>
<td>$x_i^* \bar{u}_i^*$</td>
</tr>
<tr>
<td>Simple model</td>
<td>$x_i^* \bar{u}_i^*$</td>
</tr>
<tr>
<td>k-ε model**</td>
<td>$x_i^* \bar{u}_i^*$</td>
</tr>
</tbody>
</table>

** Assumes that model constants from k-ε turbulence model are equal in both model and prototype.

### 5.7 Turbulent thermal diffusivity

It is generally accepted that the turbulent thermal diffusivity $\alpha_t$ can be written as a function of the turbulent eddy viscosity and turbulent Prandtl number $\sigma_t$. [40]
Recall the definition for turbulent thermal diffusivity that was used for all of the linear eddy viscosity turbulence models that were previously discussed:

$$\alpha_t = \frac{\nu_t}{\sigma_t}$$  \hspace{1cm} 5.30

where the turbulent Prandtl number $\sigma_t$ is treated as a constant. This allows the functional dependence of $\alpha_t$ to be expressed in terms of the same parameters that $\nu_t$ depends upon with the addition of the turbulent Prandtl number. The dimensionless form of the turbulent thermal diffusivity, as defined by the $k$-$\varepsilon$ turbulence model, is shown below.

$$\alpha_t^* = \frac{\nu_t^*}{\sigma_t^*} = C_\mu \frac{\text{RePr} \ k^{\prime2}}{\sigma_t \ \varepsilon^*}$$  \hspace{1cm} 5.31

It should also be pointed out here that the term of the dimensionless energy equation in which the dimensionless turbulent thermal diffusivity appears is divided by RePr. So the turbulent transport of thermal energy is not expected to rely heavily on the product RePr.

5.8 Summary of requirements

After a review of experimental data, empirical correlations, and mathematical descriptions a set of parameters that the turbulent properties within a flow depend upon has been compiled. Table 5.8 summarizes the requirement for similarity between the model and prototype.
Table 5.8 Summary of similarity requirement derived through the use of experimental data, empirical correlations and mathematical models.

<table>
<thead>
<tr>
<th>Similarity</th>
<th>Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric</td>
<td>$x_i^*$</td>
</tr>
<tr>
<td>Kinematic</td>
<td>$x_i^<em>, u_i^</em>$</td>
</tr>
<tr>
<td>Dynamic Models:</td>
<td>$x_i^<em>, u_i^</em>$</td>
</tr>
<tr>
<td>k-ε Equations**:</td>
<td>$x_i^<em>, u_i^</em>, \text{Re}$</td>
</tr>
<tr>
<td>Thermal**</td>
<td>$x_i^<em>, u_i^</em>, \text{Re}, \Pr$</td>
</tr>
</tbody>
</table>

** Assumes that model constants from $k$-$\varepsilon$ turbulence model are equal in both model and prototype.

5.9 Scaling recommendations:

Through analysis of the dimensionless governing equations, a list of parameters of interest in the scaling of turbulent thermal mixing phenomena within turbulent jet flows was compiled. This list suggests that similarity between the model and prototype is driven by geometric and kinematic similarity.

5.9.1 Domain:

The domain of the prototype should be scaled using conformal geometry. This satisfies the dimensionless length equality required for geometric, kinematic, and
dynamic similarity. The models length dimensions will then be related to the dimensions of the prototype through a geometric scaling ratio \( \Lambda \).

5.9.2 Fluid:

The information about fluid properties in the dimensionless set of governing equations for the \( k-e \) turbulence model is contained in the Re and Pr. These dimensionless groups appear in only one term of the momentum and energy equations. The term that they appear in represents the molecular viscosity and molecular diffusivity terms of the momentum and energy equations, respectively. Based upon this, the Re and Pr are not expected to have significant influence upon the thermal mixing processes within the flow. Therefore it is not required to exactly match Re or the product RePr in the model and prototype.

5.9.3 Flow

The last parameter that must be set to ensure similarity between the model and prototype is the flow velocity. Since thermal mixing is mainly driven by turbulent processes, it is important that the turbulent processes are preserved when the flow is scaled. The turbulent processes within the flow were found to strongly depend upon the flow's velocity profile. Because of this, the velocity profile was preserved, while the mass flow rate through the facility was not. Preserving the velocity profile within the flow can be performed by using conformal geometry and specifying the same
dimensionless velocity boundary conditions for the model and prototype. Re should be used to verify that the scaled flow is still turbulent.

One additional parameter of interest was identified through simple descriptions for the turbulent length and velocity scales. This parameter is the product of the characteristic length and velocity, UL. UL is contained in the Reynolds number, but its equality is not required to achieve Reynolds number equality. This is mentioned only to draw attention to the fact that consideration should also be given to the product UL when scaling mixing phenomena.
6 Results

6.1 Purpose

Numerical simulations were performed in order to verify the accuracy of the scaling method developed in this paper. The k-ε turbulence closure model provided the foundation for this scaling method. If the governing equations were nondimensionalized correctly, then the same solution should be generated by the k-ε model in domains scaled by the method developed in this paper. Since thermal mixing is the primary turbulent process of interest, the solutions are compared through analysis of the generated temperature distribution.

The solutions were generated using the commercial computational fluid dynamics (CFD) code FLUENT. This code provides the ability to simulate, visualize or analyze fluid flows. It includes numerous models for analyzing turbulent flows including the k-ε model. The modeling constants of the k-ε model can be specified in the problem setup. For the analysis below, the default settings were used for each run.

The domain that I chose for this analysis was the core bottom structure of the HTTR. The dimensions and normal operating conditions of the flow in this structure were readily available. The size of the mesh spacing was scaled by the same factor as the geometric dimensions, \( \Lambda \). This choice of mesh spacing allowed the number of nodes in all models to be approximately the same, giving the same resolution for each solution. The default settings for convergence criteria were used for all models. Convergence occurred in less than 150 iterations for all models.
6.2 Numerical Simulations

Table 6.1 summarizes the test conditions for each of the numerical simulations that were performed.

Table 6.1. Test conditions for numerical simulations.

<table>
<thead>
<tr>
<th>Test</th>
<th>(\Lambda)</th>
<th>Pressure [MPa]</th>
<th>Fluid</th>
<th>(T_{\text{Hot}}) [°C]</th>
<th>(T_{\text{Cold}}) [°C]</th>
<th>(\Delta T) [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.0</td>
<td>4.0</td>
<td>He</td>
<td>1000</td>
<td>900</td>
<td>100</td>
</tr>
<tr>
<td>B</td>
<td>1.0</td>
<td>0.101</td>
<td>Ar</td>
<td>127</td>
<td>27</td>
<td>100</td>
</tr>
<tr>
<td>C</td>
<td>0.2</td>
<td>0.101</td>
<td>(N_2)</td>
<td>127</td>
<td>27</td>
<td>100</td>
</tr>
<tr>
<td>D</td>
<td>0.2</td>
<td>0.5</td>
<td>(N_2)</td>
<td>127</td>
<td>27</td>
<td>100</td>
</tr>
<tr>
<td>E</td>
<td>0.2</td>
<td>0.101</td>
<td>(N_2)</td>
<td>127</td>
<td>27</td>
<td>100</td>
</tr>
<tr>
<td>F</td>
<td>0.2</td>
<td>0.101</td>
<td>(N_2)</td>
<td>127</td>
<td>27</td>
<td>100</td>
</tr>
</tbody>
</table>

The hot gas duct was chosen as the reference location for calculating the flow conditions. Reynolds number equality was obtained by varying the average velocity of the flow and the system pressure. Table 6.2 shows the hot gas duct values for the parameters required for similarity for each of the four numerical simulations.

Table 6.2. Hot gas duct parameters.

<table>
<thead>
<tr>
<th>Test</th>
<th>(\text{Re}_0)</th>
<th>Prandtl Number</th>
<th>(L_0) [m]</th>
<th>(U_0) [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(3.6 \times 10^5)</td>
<td>0.66</td>
<td>0.35</td>
<td>32.61</td>
</tr>
<tr>
<td>B</td>
<td>(3.6 \times 10^5)</td>
<td>0.67</td>
<td>0.35</td>
<td>16.54</td>
</tr>
<tr>
<td>C</td>
<td>(3.6 \times 10^5)</td>
<td>0.72</td>
<td>0.07</td>
<td>105.46</td>
</tr>
<tr>
<td>D</td>
<td>(3.6 \times 10^5)</td>
<td>0.72</td>
<td>0.07</td>
<td>21.37</td>
</tr>
<tr>
<td>E</td>
<td>(1.0 \times 10^5)</td>
<td>0.72</td>
<td>0.07</td>
<td>20.8</td>
</tr>
<tr>
<td>F</td>
<td>(5.0 \times 10^5)</td>
<td>0.72</td>
<td>0.07</td>
<td>104*</td>
</tr>
</tbody>
</table>

Maximum \(U_0\) while still maintaining \(\text{Ma}<0.3\)
6.2.1 Test A

Test A was run to verify that the flow conditions within the HTTR CBS could be reproduced with the numerical model used in this analysis. The conditions of test A matched those used by Inagaki et al. in their experiment. The results of this test compared favorably with the results obtained by Inagaki et al. The same regions in which significant mixing occurred were identified. The results of test A are shown below in figure 6.2.

Figure 6.2. Test A. Numerical simulation of flow full scale HTTR CBS.
6.2.2 Test B

Test B was also performed at full scale. However, because it is desirable to be able to investigate thermal mixing processes at conditions closer to ambient conditions, test B was carried out at atmospheric pressure and much lower temperature. The results from test B are shown in figure 6.3.

Figure 6.3. Test B. Numerical simulation of flow in full scale HTTR CBS at reduced temperature and pressure.
6.2.3 Test C

Test C was conducted at one-fifth scale. The temperature and pressure were also reduced. Atmospheric pressure was chosen for this test. This choice of pressure required a high velocity in order to get Reynolds number equality. The results from test C are shown below in figure 6.4.

Figure 6.4. Test C. Numerical simulation of flow in one-fifth scale HTTR CBS. Velocity in hot gas duct was high.
6.2.4 Test D

Test D was run with the same configuration as test C. However, pressure of test D was increased slightly in an effort to reduce the velocity in the hot gas duct required for Reynolds number equality. The results of test D are shown below in figure 6.5.

Figure 6.5 Test D. Numerical simulation of flow in one-fifth scale HTTR CBS. Flow velocity in hot gas duct is significantly lower that test C due increasing test pressure.
6.2.5 Test E

Test E was run to investigate the effects of changing the Reynolds number of the flow on the thermal mixing. The Re of the flow was set at a value lower than that of the reference HTTR CBS normal operating Re. The results of test E are shown below in figure 6.6.

Figure 6.6 Test E. Numerical simulation of flow in one-fifth scale HTTR CBS. Reynolds number of the flow is lower than the Reynolds number of previous runs.
6.2.6 Test F

Test F was run to investigate the effects of changing the Reynolds number of the flow on the thermal mixing. The Re of the flow was set at a higher value than that of the reference HTTR CBS normal operating Re. The Re of this run was increased until it was limited by maintaining a Ma < 0.3 for the flow. The results of test F are shown below in figure 6.7.

Figure 6.7 Test F. Numerical simulation of flow in one-fifth scale HTTR CBS. Reynolds number of the flow is higher than the Reynolds number of previous runs.
6.3 Conclusion

The results obtained in these test show good agreement with each other. This supports the conclusion that the scaling method developed in this paper captures the similarity requirements of the k-ε turbulence closure model.
7 Conclusions

7.1 Summary

This paper introduced proposed VHTR designs. The presence of hot streaks within the coolant is a concern common to all of the designs that were introduced. Operational VHTR designs, the HTR-10 and HTTR, made use of scaled experiments to investigate the thermal mixing phenomena within their respective lower plenum. The NGNP being developed by INL will also most likely make use of scaled models to investigate the presence of hot streaks within the coolant flow. With this in mind, a scaling method that focuses on maintaining the thermal mixing processes within a flow was developed.

The $k-\varepsilon$ turbulence closure model provided the basis for the scaling method developed in this paper. The $k-\varepsilon$ turbulence closure model was chosen because it is one of the most popular and widely used two-equation linear eddy viscosity models available in commercial CFD codes, and it has shown good success in predicting the effects of turbulence [3].

The scaling method developed in this paper was intended to capture the parameters required for similarity of the $k-\varepsilon$ turbulence model equations. The scaling method was verified by using a CFD code with the $k-\varepsilon$ turbulence model to generate temperature profiles for several steady-state flows. The domains and flow parameters were determined through use of the scaling method developed in this paper, and the temperature distributions generated by the CFD code showed good agreement between all tests.
7.2 Conclusion

The scaling criteria developed in this paper were developed through analysis of the governing equations for fluid motion, the \( k-\varepsilon \) turbulence model equations and simple physical arguments. Prior scaling methods used in VHTR designs make use of empirical correlations and engineering judgment to capture turbulent processes within the coolant flow. In this analysis, the scaling of thermal mixing phenomena was found to depend primarily upon geometric and kinematic similarity. This result assumed that modeling constants appearing in the \( k-\varepsilon \) turbulence model equations be equal in the model and prototype.

The turbulent processes which are responsible for the thermal mixing within turbulent flow are preserved in the scaling method developed within this paper. Numerical simulations of flows in scaled models show good agreement with the reference flow, and support this conclusion.

7.3 Future work

Simple arguments used to investigate the turbulent length and velocity scales suggest that the product \( UL \) might also be an important parameter to maintain when scaling turbulent mixing processes. This parameter was not investigated within this paper. Future work on the scaling of thermal mixing processes should include investigation into the need to maintain equality of \( UL \).

Future work should also include experiments that can provide enough useful data to serve as benchmarks for CFD codes. These experiments must resolve flow
parameters in enough locations such that meaningful comparisons can be made with
the CDF code predictions.

Effort should also be made to measure the local turbulent velocity and length
scales within the flow. These measurements should then be used to further investigate
the effects of local domain size and local mean flow velocity on the turbulent velocity
and length scales.

Finally, an investigation should be made into the potential for using scaled
numerical models to reduce the computational time required for CDF codes to obtain
solutions for turbulent flows. It may be possible to scale the mesh size of the model
by a different scale than that used to scale the domain. This would result in a
reduction of the number of mesh volumes and potentially allow the solution to be
generated more rapidly.
Works Cited


