

General algorithm to optimize the diffraction efficiency of a phase-type spatial light modulator

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We present a general approach for optimizing the diffraction efficiency of a phase-type spatial light modulator (SLM). While the SLM displays a one-dimensional phase grating, the phase shift of one pixel in the grating is varied and the first-order diffraction efficiency is measured. This is repeated pixel-by-pixel to find the optimum phase encoding for the device that maximizes the diffraction efficiency. This method compensates for nonlinearity of the modulator phase response and is especially useful for optimizing modulators with less than 2π phase shift. © 2013 Optical Society of America

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Spatial light modulators (SLMs) are commonly used to implement diffractive optical elements (DOEs) for a variety of applications. In particular, optical tweezer trapping has greatly benefited from the advent of phase-only and phase-liquid crystal SLMs to write holograms that produce multiple traps that can be reconfigured at video frame rates [1–4]. The utility of SLMs depends on their diffraction efficiency, which is impacted by effects such as pixel size, quantization, amplitude modulation, polarization, nonlinear phase modulation, and phase modulation less than 2π . We address the last two of these effects with a general method for optimizing the diffraction efficiency of a SLM on a pixel-by-pixel basis. A different method was recently demonstrated that addressed the limited phase depth of SLMs, but it did not address the nonlinearity and it used a global rather than a pixel-level adjustment [5]. Our method is limited to the phase aspect, in contrast to others that consider phase and amplitude modulation [6].

The simplest DOE is a one-dimensional (1D) blazed phase grating, with a transmission (or reflection) amplitude $t(x) = e^{ikx}$ over each unit cell of the periodic DOE or hologram. For an ideal SLM, the phase shift is linear and covers 0 – 2π in each unit cell, yielding a diffraction efficiency of unity. For a realistic SLM, a nonlinear or reduced phase shift reduces the diffraction efficiency. The general SLM response is $t(x) = e^{ikx+i\Delta\phi(x)}$, where $\Delta\phi(x)$ is the phase difference or error from the ideal case. The amplitude of the first-order diffracted beam from the device is given by the Fourier coefficient:

$$a_1 = \frac{k}{2\pi} \int_0^{2\pi/k} e^{ikx+i\Delta\phi(x)} e^{-ikx} dx = \frac{k}{2\pi} \int_0^{2\pi/k} e^{i\Delta\phi(x)} dx. \quad (1)$$

In the small phase error limit, the diffraction amplitude becomes (to second-order in $\Delta\phi$)

$$\begin{aligned} a_1 &\cong \frac{k}{2\pi} \int_0^{2\pi/k} \left[1 + i\Delta\phi(x) - \frac{1}{2}\Delta\phi^2(x) \right] dx \\ &\cong 1 + i \frac{k}{2\pi} \int_0^{2\pi/k} \Delta\phi(x) dx - \frac{1}{2} \frac{k}{2\pi} \int_0^{2\pi/k} \Delta\phi^2(x) dx \\ &\cong 1 + i\langle\Delta\phi(x)\rangle - \frac{1}{2}\langle\Delta\phi^2(x)\rangle, \end{aligned} \quad (2)$$

where the averages $\langle \rangle$ are over one unit cell of the periodic hologram. The diffraction efficiency into the first-order beam is

$$\eta_1 = |a_1|^2 \cong 1 + \langle\Delta\phi(x)\rangle^2 - \langle\Delta\phi^2(x)\rangle \cong 1 - \sigma_{\Delta\phi}^2. \quad (3)$$

Hence, the diffraction efficiency is reduced by the variance of the phase error. This simple expression agrees with more detailed calculations in a variety of cases. For example, if the phase is quantized into Z equally spaced levels over the range 0 – 2π , as shown in Fig. 1, then the first-order diffraction efficiency is given by [7]:

$$\eta_1 = \left[\frac{\sin(\pi/Z)}{\pi/Z} \right]^2 \cong 1 - \frac{\pi^2}{3Z^2}. \quad (4)$$

The variance of the phase error of the quantized levels can be calculated analytically and is $\sigma_{\Delta\phi}^2 = \pi^2/3Z^2$, which, using Eq. (3), yields the same approximate efficiency as Eq. (4).

Adding an overall constant phase to the SLM response does not alter the diffraction efficiency, so the average phase error may be taken to be zero and the variance of the phase error is simply the mean square phase error. For a device with N pixels or rows of pixels per unit cell, the efficiency is thus

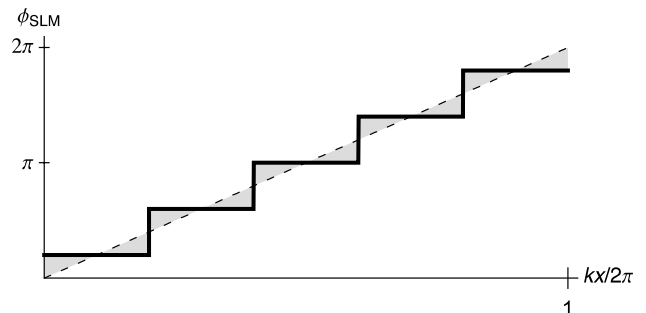


Fig. 1. One unit cell of the quantized phase of a SLM, showing the ideal linear phase (dashed) and the phase error (shaded).

$$\eta_1 \cong 1 - \sum_{i=1}^N \Delta\phi_i^2 / N \quad (5)$$

to which each pixel or row i contributes independently. This suggests an algorithm to maximize the efficiency by adjusting the phase encoding of the SLM on a pixel-by-pixel basis to minimize the phase error variance. This algorithm is depicted schematically in Fig. 2 through a numerical simulation. For the purpose of the simulation, we model the nonlinear, reduced phase shift of the SLM with a sigmoid function as shown in Fig. 2(a), where g is the gray value of the eight-bit video signal input to the SLM. Some SLM manufacturers provide a gray-scale look-up table (LUT) that compensates for the phase nonlinearity so that the displayed DOE is linear. Our proposed algorithm generates such a table, so we assume that a simple identity LUT $g_{\text{out}} = g_{\text{in}}$, shown in Fig. 2(b), is used initially.

With the identity LUT in place, a 1D blazed phase grating is input to the SLM, utilizing the full gray-scale (0–255) over each unit cell of the periodic hologram. In the ideal case, this would produce 100% diffraction efficiency in the first-order, but for our chosen parameters it produces about 83% efficiency, due to the phase error shown in the inset of Fig. 3. We then alter the phase shift of a single pixel in the unit cell of the grating by choosing one particular input gray level of the LUT and varying the output gray level (just for that single input value) over the full 8 bit range. We measure the first-order diffraction efficiency of the SLM as the output gray level is varied. The resultant efficiency curves are shown in Fig. 2(c) for three particular choices of the input gray value (60, 128, 225). For these single pixel changes, the efficiency varies by about 1.5% over the range of the gray-scale changes, with the peak or optimal value shifted in general from the chosen input value. For example, for the three input values chosen in Fig. 2, the optimal output values are 84, 128, and 255. In the vicinity of the optimal gray level, the efficiency exhibits a quadratic dependence, as expected from Eqs. (3) and (5).

This measurement is repeated for each of the possible input gray values and the optimal output gray values are collected to create the optimized LUT shown in Fig. 2(d). This LUT exhibits the nonlinearity required to produce a linear phase grating and the saturated phase encoding that is known to maximize the efficiency of phase limited SLMs [5,8]. One unit cell of the resultant, optimized phase shift is shown in Fig. 3. Once the optimized LUT for all gray values is implemented, the diffraction efficiency increases to 95%, which is consistent with the phase error variance of 0.05 shown in Fig. 3, and is in agreement with the efficiency for phase saturated encoding $\eta_1 = [\phi_{\text{max}}/2\pi + \sin(\phi_{\text{max}}/2)/\pi]^2$ [8].

We have applied this optimization method to two different SLMs using different laser wavelengths, a Holoeye LC-R 2500 at 532 and 1064 nm and a Cambridge Correlators SDE 1024 at 658 nm. The experimental configuration is shown in Fig. 4. With the identity LUT in place, a linear blazed grating is displayed on the SLM using a LabVIEW interface [9]. The diffracted orders are focused in the Fourier plane and then reimaged in the detector plane, where an aperture isolates the first-order beam

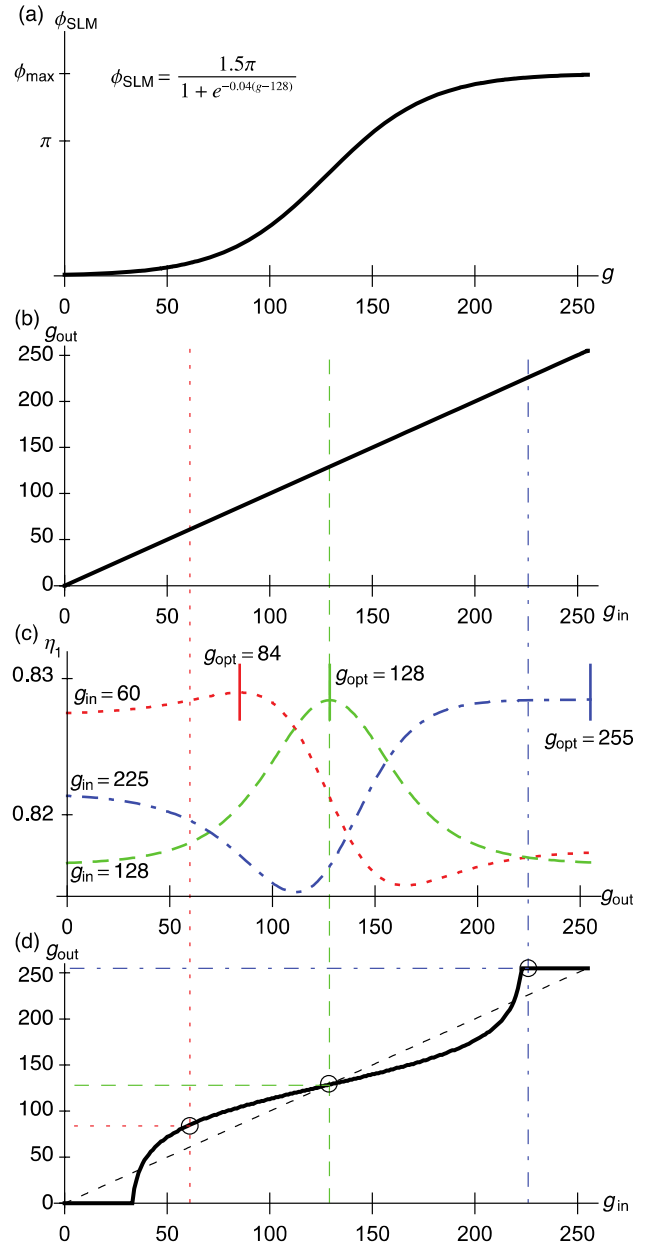


Fig. 2. Schematic of SLM optimization algorithm. (a) Simulated nonlinear SLM response with reduced phase shift. (b) Initial identity LUT showing three example gray values that are varied (red dotted; green dashed; and blue dotted-dashed). (c) Resultant diffraction efficiency curves and (d) optimized LUT.

that is detected with a silicon photodiode. To improve the signal-to-noise ratio of the measured diffracted power, we employ two strategies. To increase the signal, we group several adjacent input gray levels together and simultaneously vary their common output gray level. This approach also solves the problem that some input gray levels are not used in a hologram if the unit cell does not contain 256 pixels. We typically use a hologram with 10–20 unit cells, corresponding to 50–100 pixel rows per unit cell, which provides sufficient separation of diffraction orders and minimal phase quantization error. The major source of noise is the temporal phase fluctuation caused by the pulse modulation used to address the

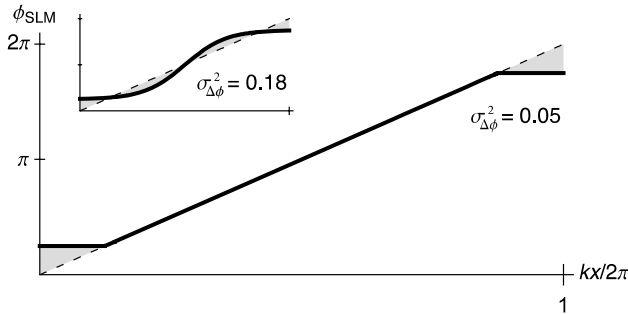


Fig. 3. One unit cell of the linearized, phase saturated output of the SLM after employing the optimized LUT. The inset shows the phase obtained with the identity LUT. In both cases, the phase error is shown as shaded.

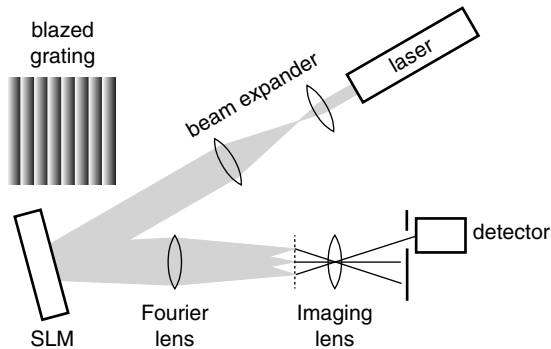


Fig. 4. Experiment to measure first-order diffracted power.

SLM [10]. To reduce this noise, we direct a separate laser beam at one isolated corner region of the SLM upon which we write a constant phase. The temporal phase flicker in this extra beam is detected and used to synchronize our diffraction measurements.

Some representative measurements of the diffracted power P_1 are shown in Fig. 5, in analogy with Fig. 2(c) of the simulation. These measurements were taken with the Holoeye LC-R 2500 SLM at 1064 nm, which is outside of the wavelength range where 2π phase modulation is possible. The powers shown in Fig. 5 have been normalized to the common maximum measured diffracted power. The power for each input gray level is fit to a parabola in a limited region near the maximum to find the optimal output gray level. The collection of optimal output gray levels comprises the LUT shown in Fig. 6. The full measurement of the LUT takes about 30 minutes, limited by the hologram frame rate.

As expected, the optimized LUT exhibits nonlinearity and saturated phase encoding because the SLM does not produce 2π phase shift at 1064 nm. The inset in Fig. 6 shows the optimized LUT derived from the measurements at 532 nm, showing that this method also adapts to the case where the SLM produces more than 2π phase shift. In practice, we smooth the LUT further before implementation. Using the optimized LUT for 1064 nm, the relative diffraction efficiency, defined as $P_1/(P_1 + P_0 + P_{-1})$, increased from 42% to 47%. The overall efficiency is reduced by other effects such as the fill factor, absorption, and surface reflection. At 532 nm the relative efficiency increased from 57% to 98%. We obtained similar results for the SDE 1024

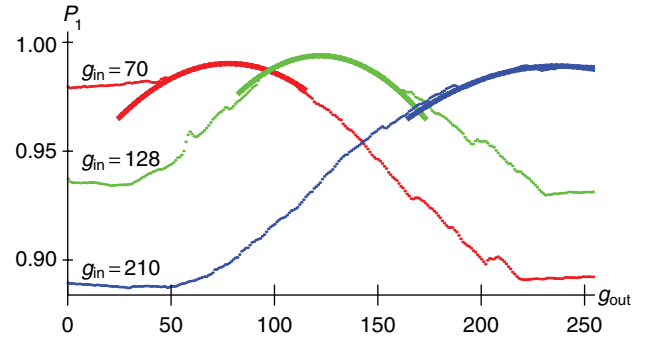


Fig. 5. Measured diffracted power (normalized) as a function of output gray level for three choices of input gray level. The parabolic fit for each data set is also shown.

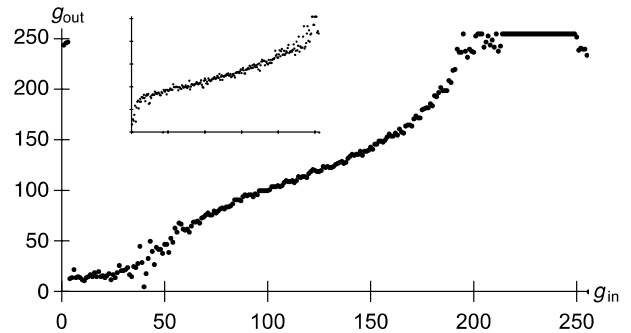


Fig. 6. Optimized SLM gray level LUT for 1064 nm showing nonlinearity and saturated phase encoding. The inset shows the optimized LUT for 532 nm where the phase shift is greater than 2π .

SLM, with the relative efficiency increasing from 17% to 39%. In all three cases, the results were equal to or better than the method of [5], which is to be expected because our method has more parameters (all pixels) available for adjustment.

In summary, we have presented a generally applicable method to maximize the diffraction efficiency of a SLM by reducing the phase error of a blazed grating on a pixel-by-pixel basis.

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