AN ABSTRACT OF THE THESIS OF

Yoko Iizuka for the degree of Master of Science in Agricultural and Resource Economics presented on December 8, 1995.

Title: Supply of Milk Components on U.S. Dairy Farms: A Cost Function Analysis

Redacted for Privacy

Abstract aproved:

Steven T. Buccola

Recently, a new pricing system for dairy products has been introduced in many Federal Milk Marketing Orders in the United States. This new pricing system, commonly called multiple component pricing, places a dollar value on protein, fat, and other solid components in the milk, while the old pricing system priced only two components: skim and butterfat. The multiple component pricing system is designed to give dairy farmers an incentive to produce more of the kinds of milk components demanded by consumers.

In this thesis, the structure of technology in milk production, and dairy farmers' short-run production response of milk and milk components to multiple component pricing, are investigated.

To achieve these objectives, a translog hedonic feed cost function and feed concentrate share equation are formulated and jointly estimated. Next, hypothesis tests are conducted in conjunction with the estimated hedonic cost function. By duality theory, testing the cost function is equivalent to testing the underlying technology.

Finally, marginal costs, supply functions, and supply elasticities are estimated for milk, milk protein, and milk fat. From the supply elasticities, dairy farmers' likely response to multiple component pricing are examined.

© Copyright by Yoko Iizuka
December 8, 1995

All Rights Reserved

Supply of Milk Components on U.S. Dairy Farms: A Cost Function Analysis

by

Yoko Iizuka

A THESIS

submitted to

Oregon State University

in partial fulfillment of
the requirements for the
degree of

Master of Science

Completed December 8, 1995

Commencement June 1996

Master of Science thesis of Yoko Iizuka presented on December 8, 1995

APPROVED:

Redacted for Privacy

Major Professor, representing Agricultural and Resource Economics

Redacted for Privacy

Chair of Department of Agricultural and Resource Economics

Redacted for Privacy

Dean of Graduate School

I understand that my thesis will become part of the permanent collection of Oregon State University libraries. My signiture below authorizes release of my thesis to any reader upon request.

Redacted for Privacy

Yoko Iizuka, Author

ACKNOWLEDGMENT

I would like to thank Dr. Steven T. Buccola, my major professor. He gave me the interesting thesis topic, a lot of ideas and advice, and also a financial support which allowed me to continue my study. Without him, I could not have presented even one sentence of this thesis.

I thank my committee members and a graduate representative, Dr. Patricia Lindsey, Dr. Richard Johnston, and Dr. Russell Karow, for spending time to review my thesis and giving me valuable comments.

I am grateful to Dr. Joe Kerkvliet, who gave me precious advice especially when conducting econometric estimation.

I would like to express my gratitude to Mr. John Clay, extension specialist for program development at North Carolina Dairy Record Processing Center for providing me useful data.

I deeply appreciate all my friends for their useful advice for my thesis.

Finally, I want to thank my parents for their encouragement throughout my study.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

SUPPLY OF MILK COMPONENTS ON U.S. DAIRY FARMS: A COST FUNCTION ANALYSIS

1. INTRODUCTION

Multiple Component Pricing

Recently, the pricing system for dairy products has been changed in many federal marketing orders.[1] Under the old pricing system, milk[2] is priced based on two components: skim and butterfat. In contrast, the new system prices milk on two or more of the solid components, such as butterfat and solids-not-fat (SNF). This new system is commonly called multiple component pricing (MCP). In essence, MCP is designed to price the skim components instead of skim. MCP is considered to give better economic incentives for farmers to produce the kinds of milk components that consumers demand. For example, it would help the industry respond to the nationwide increase in demand for the nonfat portion of the milk product, especially protein. Under the old pricing system, farmers received the same price per unit of skim no matter how much protein or other SNF the skim contained. In contrast, the new pricing system values each major category

[1] Federal marketing orders are designed to provide stable milk market by requiring manufacturers to pay at no less than specified prices and also by assuring dairy farmers to receive the price at no less than the blended classified value. Today, 40 federal milk marketing orders covers most of the population in the US (McKinley, 1993).

[2] Milk consists of water, fat, and solids-not-fat where solids-not-fat includes mostly protein, lactose, and minerals. Skim milk is the whole milk less butterfat.

of nonfat components, and is expected to encourage farmers to respond better to market signals (McKinley,1993).

How handlers and manufacturers would pay under this new system is complicated and is therefore worthwhile discussing. Different federal orders use different MCP systems. Here, the Northwest Federal Marketing Order is used as an example.

The price fluid handlers and manufacturers are obliged to pay is based on classified pricing.[3] The price fluid handlers pay for Class I milk is the sum of the following (Equity Newsletter, 1993):

1. The butterfat price per pound times total pounds of butterfat.
2. Class I differential price[4] per hundredweight times total milk purchased.
3. Price of Class III skim milk per hundredweight times total hundredweights of skim purchased.

The price manufacturers pay for Class II, III, or III-A milk is the sum of the following factors:

1. The butterfat price per pound times total pounds of butterfat.
2. Price differential (Class II or Class III-A differential according to their end use) times the quantity of milk purchased.

[3] Milk is classified and priced according to its end use. Class I price is the price used for milk used in fluid purpose. Class II price is the price for soft products such as yogurt and ice cream. Class III price is also called the Minnesota-Wisconsin manufactured milk price (M-W price) and is the same all over the US. Northwest Federal Order also sets Class III-A price which is the price for milk powder. Class I price, Class II price, and Class III-A price will shift according to any price change in Minnesota-Wisconsin price (McKinley, 1993).

[4] Class differential means each class price less Class III price. Class I differential therefore is (Class I price - class III price).

3. Price of Class III solids-not-fat per hundredweight times the total quantity of solids-not-fat.

Because different prices apply to milk depending on its final use, a method of pooling, or distributing the total class value of milk among producers, is used in conjunction with the classified pricing. A marketwide pooling procedure is used in most of the orders and it assures milk producers receive a uniform price for the milk they sell. In the Northwest Order, producers are paid based on three factors:

1. The butterfat price per pound produced times the total pounds of butterfat produced.
2. Total pounds of solids-not-fat produced times the producer solids-not-fat price[5] per pound.
3. Total volume of milk production times the weighted average differential price[6] per hundredweight. The weighted average differential represents each producer's share of the Class I, II, and III-A fluid differentials.

---

[5] $A = \frac{(B \times C) + (D \times E)}{F}$

where A = producer nonfat milk solids price,
B = hundredweights of skim milk in Class I,
C = skim milk price,
D = nonfat milk solids in Classes II, III, and III-A,
E = nonfat milk solids price, and
F = total number of nonfat milk solids pounds in producer milk for the market for the market.

[6] $a = \frac{(b \times c) + (d \times e) + (f \times g)}{h}$

where a = weighted average differential price,
b = pounds of whole milk in Class I,
c = (Class I price) - (Class III price),
d = pounds of whole milk in Class II,
e = (Class II price) - (Class III price),
f = pounds of whole milk in Class III-A,
g = (Class III-A price) - (Class III price), and

Objective

The general objective of this thesis is to investigate the technology of milk production and to see how the dairy farmer's production of whole milk, milk protein, and milk fat responds to the multiple component pricing system in the short run.

The specific objectives are as follows:

1. Using a cost function approach, to provide empirical estimates of the structure of milk production technology. These tests will include (ii) a test of homotheticity in technology; (ii) a test of whether the technology has constant returns to scale; and (iii) a test of whether all outputs can be aggregated as a single index.
2. Again using a cost function, to estimate supply functions for milk, milk protein, and milk fat.
3. To estimate supply elasticities, and from these to investigate whether the producer responds to MCP significantly when feed ration but not dairy breed or sire is allowed to change.

To achieve these objectives, general theories of multiple-product technology and cost will first be discussed. Next, the dairy science literature on factors affecting the production of each milk component will be reviewed. Third, the multiple-product hedonic translog cost function will be constructed from 1993 cross-sectional data from the North Carolina Dairy Record Processing Center. The cost function relates total feed

---

$h =$ quantity of producer hundredweight whole milk in the market.

cost to concentrate and forage price; to whole milk, milk protein, and milk fat quantities; and to a variable reflecting the milk production capacity of the cow. Parameters are estimated together with a share equation using maximum likelihood. Next, I test selected hypotheses with the cost function. By duality theory, hypothesis tests on the cost function are equivalent to hypothesis tests on technology. Finally, I estimate supply functions and supply elasticities for each output and show how the multiple component pricing system likely will affect US milk production in the short run.

2. ECONOMIC THEORY

A Multiple-Product Cost Function

A multiple-product technology is commonly represented as an implicit function of inputs and outputs. This function is called a transformation function and represented as

$$(1) \qquad T(\mathbf{x}, \mathbf{y}) = 0,$$

where $\mathbf{x}$ and $\mathbf{y}$ are vectors of inputs and outputs respectively. Equality (1) holds if and only if the technology is efficient. Just as a production function shows the maximum output as a function of inputs, the transformation function shows the most efficient transformation of input vector $\mathbf{x}$ to an output vector $\mathbf{y}$ (Brown et al., 1979; Denny and Pinto, 1980).

Input requirement set $V(\mathbf{y})$ is the set of all input combinations that is capable of producing at least $\mathbf{y}$. The border of $V(\mathbf{y})$ is the isoquant, which represents the most efficient combination of input bundles. Formally, an input requirement set can be expressed as

$$(2) \qquad V(\mathbf{y}) = \{\mathbf{x}\colon T(\mathbf{x}, \mathbf{y}) \leq 0\}$$

(Chambers, 1991). Now, suppose transformation function (1) has a regular, convex input requirement set; that is, $V(\mathbf{y})$ is nonempty, convex, and closed. The nonemptiness of $V(\mathbf{y})$ simply means it is always possible to produce a positive amount of output. The assumption of closedness states that $V(\mathbf{y})$ includes its boundary and that there are no "holes" in the boundary. A hole is the region in $V(\mathbf{y})$ where it is not possible for the technology to produce output bundles **y**. The principle of duality states that if the input requirement set satisfies the above restrictions, there exists a unique multiple-product cost function $C(\mathbf{w}, \mathbf{y})$, which has a one-to-one relationship to the transformation function.

The cost function measures the minimum cost of producing **y** and can be expressed as

$$(3) \qquad C(\mathbf{w}, \mathbf{y}) = \min_{\mathbf{x}} \{\mathbf{w} \cdot \mathbf{x} \colon \mathbf{x} \in V(\mathbf{y})\},$$

where **w** is the vector of input prices. In order that the cost function behave well, restrictions called regularity restrictions must hold. First, it is assumed that **w** is exogenous and strictly positive. Second, $C(\mathbf{w}, \mathbf{y}) > 0$ since $\mathbf{w} > 0$ and $\mathbf{y} > 0$ (nonnegativity). The cost function satisfies further properties. These are positive linear homogeneity in **w**, nondecreasingness in **w**, and concavity in **w**. A cost function is said to be linearly homogeneous in **w** if $C(t\mathbf{w}, \mathbf{y}) = tC(\mathbf{w}, \mathbf{y})$ for all $t > 0$. Nondecreasingness in **w** means increasing any input prices will not decrease cost. Concavity in the factor prices implies that the chord between any two points on the cost function, when graphed over input prices, is no higher than the function itself. Finally, the cost function obeys Shephard's lemma,

$$(4) \qquad \frac{d\,C(\mathbf{w}, \mathbf{y})}{d\,\mathbf{w}} = \mathbf{x}(\mathbf{w}, \mathbf{y});$$

that is, the vector of derivatives of the cost function with respect to factor prices is equal to the vector of cost-minimizing conditional factor demands. Factor demands are conditional in the sense that they depend on the level of **y** produced. Conditional factor demand is always positive, given that the cost function is nondecreasing in **w** (Chambers, 1991).

Duality

It is straightforward to derive a unique, well-behaved cost function from a well-behaved production function; that is, we solve for the cost minimization problem (3). Shephard (1953) proved that the reverse is also true. Given a cost function, it is possible to derive a unique production function from which the cost function can be generated. This implication tells us that the technology holds all the information necessary to construct the cost function and vice versa. The specification of a well-behaved cost function is equivalent to specifying a well-behaved technology. This principle is known as the "duality" theorem.

Duality of the cost function and the technology can be demonstrated by showing that the cost function can be used to reconstruct the input requirement set from which it was generated. It is easy to show how duality works by using the notion of a half-space (Chambers, 1991). A half-space $H(l, m)$ can be defined as

(5) $$H(l, m) = \{x: l \cdot x \leq m\},$$

where $l \in R^n$, $l \neq 0$, and $m \in R$. Given this definition, the cost function can define a half-space; that is

(6) $$C(\mathbf{w}, \mathbf{y}) = \{\mathbf{x}: \mathbf{w}^t \cdot \mathbf{x}^t \leq \mathbf{w}^t \cdot \mathbf{x} \text{ for all } t \text{ such that } \mathbf{y}^t \leq \mathbf{y}\}.$$

The cost of the observed choice of inputs is no greater than the cost of any other level of inputs that would produce at least as much output (Varian, 1992). Any point at the boundary of (6), $\mathbf{w}^t \cdot \mathbf{x}^t = C(\mathbf{w}, \mathbf{y})$, is tangent to a hyperplane and represents the cost-minimizing input bundles to produce a given level of $\mathbf{y}$. The feasible bundle cannot lie below the hyperplane on which $\mathbf{x}^t$ is situated, since if it does, it means $\mathbf{x}^t$ does not represent the cost-minimizing bundle. Since, by definition, input requirement set $V(\mathbf{y})$ is the combination of input bundles that produce at least $\mathbf{y}$, a half-space above the hyperplane should contain $V(\mathbf{y})$ and at least one point of $V(\mathbf{y})$ should share a point in common with the hyperplane. If we examine the infinite number of alternative price vectors $\mathbf{w}^t$, that is, as the number of sets of $\mathbf{w}$ gets very large, we can obtain at least the convexified form of the input requirement set $V(\mathbf{y})$. This implicit input requirement set, $V^*(\mathbf{y})$, is defined as

$$(7) \qquad V^*(\mathbf{y}) = \{\mathbf{x}\colon \mathbf{w} \cdot \mathbf{x} \geq \mathbf{w} \cdot \mathbf{x}\,(\mathbf{w}, \mathbf{y}) = C(\mathbf{w}, \mathbf{y}) \text{ for all } \mathbf{w} \geq 0\}.$$

$V^*(\mathbf{y})$ always strictly contains input requirement set $V(\mathbf{y})$. Indeed, if the original technology is convex and monotonic, $V^*(\mathbf{y}) = V(\mathbf{y})$. This is because each point on the boundary of $V(\mathbf{y})$ represents a cost-minimizing factor demand. This proposition shows it is possible to reconstruct the original technology directly from a cost function if the original technology is convex and monotonic. When the technology does not satisfy convexity and monotonicity, the technology derived from the implicit input requirement set (7) does not yield information about the nonconvex or nonmonotonic region. However, these regions are never utilized by cost-minimizing firms and hence they are negligible for economic analysis. Ignoring them, we can construct $V^*(\mathbf{y})$, which is a convexified and monotonized version of the original technology.

Hence, whether $V(\mathbf{y})$ is well-behaved or not, we can construct implicit input requirement set $V^*(\mathbf{y})$ which is monotonic and convex. It does not reflect any nonconvex and nonmonotonic region of $V(\mathbf{y})$ and therefore the cost function used to derive $V^*(\mathbf{y})$ is always equal to that of $V(\mathbf{y})$ (Varian, 1992).

Separability

On many occasions, a multiple-product technology has too many inputs and outputs to be handled with limited data sets. Hence, economists frequently group several inputs or outputs and specify the set as a single index, if possible (Chambers, 1991).

(a) *Separability in Outputs*

A technology is separable in outputs if one can specify the multiple-output as a single function of the outputs. That is, if the technology is separable in outputs, we can aggregate the outputs $\mathbf{y}$ into a single variable $g(\mathbf{y})$. Hence, if $(\mathbf{x}, \mathbf{y}) \in T$ implies there exists a technology $\hat{T}$ such that $[\mathbf{x}, g(\mathbf{y})] \in \hat{T}$, the technology is said to be separable.[7] If this is the case, one can specify the input requirement set as $V(\mathbf{y})$, a generalization of the single-output case (Chambers, 1991).[8]

(b) *Separability in Inputs*

Just as separability in outputs can aggregate multiple-outputs into a single-output index, we can characterize the multiple-inputs as a single-input if separability in inputs holds. That is, separability in inputs holds if it is possible to aggregate inputs $\mathbf{x}$ into a

[7] Production possibilities set T is the set of firm's all feasible combinations of $(\mathbf{x}, \mathbf{y})$ (Chambers, 1991).

[8] If the multiple-product technology is separable, we can define input requirement set $V(\mathbf{y})$ as $V(\mathbf{y}) = \{\mathbf{x}: (\mathbf{x}, \mathbf{y}) \in T\} = \{\mathbf{x}: [\mathbf{x}, g(\mathbf{y})] \in \hat{T}\} = \{\mathbf{x}: (\mathbf{x}, g) \in \hat{T}\} = V(g)$ (Chambers, 1991)

single variable $p(\mathbf{x})$. Therefore, a technology is separable in inputs if there exists $\widetilde{T}$ such that $[p(\mathbf{x}), \mathbf{y}] \in \widetilde{T}$ whenever $(\mathbf{x}, \mathbf{y}) \in T$. If this is the case, we can specify a producible-output set $Y(\mathbf{x})$ as a generalization of the single-input case.[9]

If we write the output separability in the form of transformation function (1), the implicit function theorem[10] implies we can explicitly solve for $g(\mathbf{y})$ as a function of $p(\mathbf{x})$; that is

$$(8) \qquad T[\mathbf{x}, g(\mathbf{y})] = 0 \qquad \text{implies} \qquad g(\mathbf{y}) = p(\mathbf{x}).$$

We can also apply a similar argument to input separability. Assuming the implicit function theorem holds, we can write separability in inputs as

$$(9) \qquad T[p(\mathbf{x}), \mathbf{y}] = 0 \qquad \text{implies} \qquad g(\mathbf{y}) = p(\mathbf{x}).$$

Hence, from (8) and (9), the notions of output separability and input separability are equivalent. Separability implies that the transformation function can be written in the familiar form where output vectors are on the left-hand-side and input vectors are on the right.

Further, the cost function when separability holds is generally expressed as a single-output function

[9] Producible-output set $Y(\mathbf{x})$ is the set of all output bundles that can be produced at a fixed input bundle. Mathematically, $Y(\mathbf{x}) = \{\mathbf{y}: (\mathbf{x}, \mathbf{y}) \in T\}$. When T is input-separable, the producible output set $Y(\mathbf{x})$ becomes $Y(\mathbf{x}) = \{\mathbf{y}: (\mathbf{x}, \mathbf{y}) \in T\} = \{\mathbf{y}: [p(\mathbf{x}), \mathbf{y}] \in \widetilde{T}\} = \{\mathbf{y}: (p, \mathbf{y}) \in \widetilde{T}\}$ (Chambers, 1991).

[10] The implicit function theorem holds iff (1) $F(\mathbf{x}, \mathbf{y}) = 0$ has continuous partial derivatives F_y and F_x, and (2) at a point $(\mathbf{x_0}, \mathbf{y_0})$ satisfying $F(\mathbf{x}, \mathbf{y}) = 0$, F_y is nonzero. If (1) and (2) hold, then $F(\mathbf{x}, \mathbf{y}) = 0$ is called an implicit function and can be explicitly solved for $h(\mathbf{y}) = f(\mathbf{x})$ (Chiang, 1984).

(10) $C(\mathbf{w}, \mathbf{y}) = C[\mathbf{w}, g(\mathbf{y})]$.

This equality holds since $V(\mathbf{y})$ in equation (3) can be replaced with $V(\mathrm{g})$ (see footnote 8) (Chambers, 1991).

Constant Returns to Scale

The concept of returns to scale allows us to analyze the nature of the firm in the long run. The measure of increased outputs associated with increases in all inputs by the same proportion is called returns to scale.

For a single-output case, this notion is quite intuitive. Returns to scale are said to be increasing, constant, and decreasing when doubling all inputs more than doubles, doubles, and less than doubles the outputs, respectively. Mathematically, constant returns to scale can be written as

(11) $f(mk, ml) = m \cdot f(k, l)$ where $m \geq 0$.

When the technology exhibits constant returns to scale, all isoquants are evenly spaced and parallel (Nicholson, 1992).

In the multiple-output case, if the transformation function $T(\mathbf{x}, \mathbf{y}) = 0$ implies that $T(m\mathbf{x}, m\mathbf{y}) = 0$ for all $m \geq 0$, the technology is said to exhibit constant returns to scale. As in the single-output case, constant returns to scale in the multiple-output technology still implies evenly spaced and parallel isoquants. That is, the input requirement sets are

(12) $V(t\mathbf{y}) = tV(\mathbf{y}).$[11]

Thus, from equation (12), the input requirement set $V(t\mathbf{y})$ is just t times the input requirement set $V(\mathbf{y})$. It follows that the boundaries of these two sets are parallel and the distance between these two boudaries in input space is just t.

Constant returns to scale implies that the producible sets are also parallel. That is

(13) $Y(t\mathbf{x}) = tY(\mathbf{x}),$[12]

so that the producible output set associated with $t\mathbf{x}$ is just t times the producible output set $Y(\mathbf{x})$ (Chambers, 1991).

[11] $V(t\mathbf{y}) = \left[\mathbf{x}: (\mathbf{x}, t\mathbf{y}) \in T\right] = \left[\mathbf{x}: \left(\frac{\mathbf{x}}{t}, \mathbf{y}\right) \in T\right] = t\left[\frac{\mathbf{x}}{t}: \left(\frac{\mathbf{x}}{t}, \mathbf{y}\right) \in T\right] = tV(\mathbf{y})$ where the second equality is the consequence of constant returns to scale (Chambers, 1991).

[12] $Y(t\mathbf{x}) = \left[\mathbf{y}: (t\mathbf{x}, \mathbf{y}) \in T\right] = \left[\mathbf{y}: \left(\mathbf{x}, \frac{\mathbf{y}}{t}\right) \in T\right] = t\left[\frac{\mathbf{y}}{t}: \left(\mathbf{x}, \frac{\mathbf{y}}{t}\right) \in T\right] = t\left[\mathbf{z}: (\mathbf{x}, \mathbf{z}) \in T\right] = tY(\mathbf{x})$, where $\frac{\mathbf{y}}{t} = \mathbf{z}$ (Chambers, 1991).

3. FACTORS WHICH INFLUENCE PRODUCTION OF MILK COMPONENTS

In recent years, consumers have demanded relatively less milk fat and more milk protein, reflecting their increasing awareness of nutrition. This tendency will likely continue. In response to this shift in consumer demand, many researchers and producers have sought to change the composition of milk products toward lower fat and higher protein percentages. There are two ways to alter milk components namely, by changes in feed rations and by genetic manipulation. Altering feed rations is a faster but more short-run way of changing milk yield and composition, while changes through genetic selection are slower but more permanent (Sutton, 1989).

Feed Ration

One can alter protein and fat percentages in the milk by changing the ratio of forages to concentrates in the cow's diet. It is possible to alter milk fat concentration by about 3% through nutritional means. Alteration of protein percentage is limited to a much smaller range, roughly to a little more than one-fifth of this, or 0.6% (Bachman, 1992; Sutton, 1988). Many dietary factors can affect milk protein and milk fat percentages. However, for simplicity, I confine our attention to the effects of changing the forage-concentrate ratio.

(a) *Milk Fat*

Although reducing the ratio of forage to concentrate in a cow's diet usually reduces the milk fat percentages, the pattern of response varies widely. Recent reviews of the literature (Thomas et al., 1984) have shown that fat concentration is fairly stable until the proportion of forage in the ration decreases to approximately 50%. However, if one decreases the forage proportion further, a decrease in milk fat percentage occurs. These results tend to vary depending upon the contents of the forages and concentrates (Sutton, 1989).

(b) *Milk Protein*

Changes in milk protein percentage occasioned by dietary manipulation are smaller and less well understood than are manipulations of the fat component.

There is widespread agreement that a negative relationship exists between the percentage of forage in the diet and the percentage of protein in the milk, and a positive relationship between the percent of concentrate in the diet and the percent of protein in the milk. Hence, increased feeding of concentrates relative to forages increases the protein concentration. However, this result is not conclusive. In some studies, decreasing the forage-concentrate ratio from 40:60 to 10:90 increased protein percentage by 0.4% (Sutton, 1988). In a different study (Flatt et al.,1969), no response was found in milk protein concentration when the feed ration was altered in similar fashion.

Genetic Manipulation

Milk yield and composition can be altered by selection of sire (Gibson, 1989). Daughters of a bull whose half-sisters produce milk with high protein and/or fat percentage also tend to produce milk with similar traits. This heritability of fat and protein percentages is very high, twice as high as that of milk yield. Therefore, it is easier to manipulate protein and fat percentages than to manipulate milk yield through sire selection. Researchers have shown that protein percentage and fat percentage are highly and positively correlated genetically. Also, while the yield of milk and of each of the two components are all highly and positively correlated with one another, genetic correlations between fat percent and milk yield, and between protein percent and milk yield, are negative. Hence, a simultaneous improvement in milk yield, and composition percentages is difficult. If, for example, one chooses to increase milk yield while ignoring the milk components, the result will be an increase in milk yield with a decrease in fat and protein percentages. Although protein and fat concentration are highly correlated, Kennedy found that it would be possible to reduce fat percentage while maintaining the protein concentration through genetic manipulation (Kennedy, 1982; Gibson, 1989).

4. MODEL SPECIFICATION AND MARGINAL COSTS

Hedonic Translog Cost Function and Share Equation

The choice of which flexible functional form to use, when analyzing the cost structure, depends on which hypotheses one wants to maintain and test.

In my analysis, in order to investigate the structure of milk production technology, several tests are to be conducted on cost function. Hence, I chose a functional form which allows me to perform a statistical test for separability and constant returns to scale. One such cost function is the transcendental logarithmic (or translog) cost function, in which these restrictions must be maintained as side conditions. The translog cost function can be envisaged as a quadratic approximation in natural logarithms to an arbitrary multiple-product cost function around a point of expansion (Brown et al., 1979).

As an extension of this translog cost function, Spady and Friedlaender (1978) have introduced the notion of a quality-separable hedonic cost function, combining the hedonic method with the translog. They argued that if the firm varies the qualities of its product, it is important to take these differences into account when estimating the cost function. Spady and Friedlaender specified output as a hedonic function of the output's attributes and characteristics. They concluded that failure to take into account the quality differences may result in serious specification error (Spady and Friedlaender, 1978).

In my model, it is possible to treat the amount of fat and protein in the milk as the measure of quality of the milk. This is because how much fat or protein the milk contains determines much of the milk's quality. The quality-separable hedonic cost function in my model can be constructed as

$$(14) \quad C = C\left\{H\left[M,\left(\frac{P}{M}\right),\left(\frac{F}{M}\right)\right],\mathbf{w}\right\},$$

where M is output of milk in pounds, P is quantity of protein in the milk in pounds, F is quantity of fat in the milk in pounds, P/M and F/M indicate the percentages of protein and fat in the milk, **w** is the vector of input prices, and $H[\cdot]$ represents the function which measures effective output. [13] Here, it is assumed that doubling the physical output M at given protein and fat percentages doubles the effective measure of output $H[\cdot]$. Thus, $H[\cdot]$ is linearly homogeneous in M; that is

$$(15) \quad H\left[M,\left(\frac{P}{M}\right),\left(\frac{F}{M}\right)\right] = M \cdot N\left[\left(\frac{P}{M}\right),\left(\frac{F}{M}\right)\right].$$

No *a priori* restrictions have to be placed on equation (15) (Spady and Friedlaender, 1978).

One can regard equation (14) as an output-separable cost function if F/M and P/M, as well as M, are thought of as physical outputs, since these three outputs can be aggregated to a single index, $H[\cdot]$.

[13] The reason for dividing and fat pounds protein pounds by milk pounds is because multicollinearity is severe among fat, protein, and milk output levels. By dividing fat and protein by milk quantity, this problem is eliminated.

In my model, I estimate a cost function for total feed cost per cow, using the quality-separable hedonic translog model (14). Dummy variables for cow breed, and for the state in which the farm is located, were used to take account of breed and regional differences. A variable for sire was also included in the model to investigate how much the genetic factor affects total feed cost. Imposing symmetry restrictions on the cross-product terms by Young's theorem, the model becomes

$$\begin{aligned}
(16)\quad \ln TFC = {} & \phi_0 + \phi_h \ln H + \phi_c \ln W_c + \phi_r \ln W_{fr} + \phi_s \ln S + \delta_b DB \\
& + \delta_v DV + \delta_p DP + \delta_i DI + \delta_s DS \\
& + (1/2)\,[\gamma_{hh} (\ln H)^2 + \gamma_{cc} (\ln W_c)^2 + \gamma_{rr} (\ln W_{fr})^2 + \gamma_{ss} (\ln S)^2] \\
& + \zeta_{hc} \ln H \cdot \ln W_c + \zeta_{hr} \ln H \cdot \ln W_{fr} + \zeta_{sh} \ln S \cdot \ln H \\
& + \zeta_{cr} \ln W_c \cdot \ln W_{fr} + \zeta_{cs} \ln W_c \cdot \ln S + \zeta_{rs} \ln W_{fr} \cdot \ln S,
\end{aligned}$$

where $\ln H$ is the translog approximation of equation (15); that is

$$\begin{aligned}
(17)\quad \ln H = {} & \ln M + \beta_p \ln\left(\frac{P}{M}\right) + \beta_f \ln\left(\frac{F}{M}\right) + \frac{1}{2}\left\{\beta_{pp}\left[\ln\left(\frac{P}{M}\right)\right]^2 + \beta_{ff}\left[\ln\left(\frac{F}{M}\right)\right]^2\right\} \\
& + \beta_{pf} \ln\left(\frac{P}{M}\right) \cdot \ln\left(\frac{F}{M}\right).
\end{aligned}$$

Variables in (16) and (17) are

TFC = total feed cost;

W_c = price of concentrate;

W_{fr} = price of forage;

S = sire quality score; [14]

M = quantity of whole milk in pounds;

P/M = protein pounds/ milk pounds;

F/M = fat pounds/ milk pounds;

DB = 1 if the breed is Holstein and 0 otherwise;

DV = 1 if the state is Vermont and 0 otherwise;

DP = 1 if the state is Pennsylvania and 0 otherwise;

DI = 1 if the state is Indiana and 0 otherwise; and

DS = 1 if the state is South Carolina, Georgia, Florida, Alabama, Mississippi, Louisiana, or Texas and 0 otherwise.

The base region consists of the Border states; that is, Virginia, North Carolina, Kentucky, Missouri, and Tennessee.

In equation (16), it is assumed that the sire variable is fixed in the short run. That is, rather than assume that all inputs adjust instantly to the equilibrium level, we treat the

[14] Predicted Transmitting Ability (PTA) is used to indicate the genetic superiority or inferiority which the bull transmits to its offspring. From PTA, economic indexes called PTA dollars are computed. Sire quality score in my model is the average PTA dollars for the sire of the cows presently in the herd (Aitchison, 1989).

sire variable as a quasi-fixed input. Equation (16) is a variable cost function in the sense that at least one input is fixed (Berndt, 1991). Buildings, equipment, and labor are ignored, and it is assumed that these costs have no effect on total feed cost.

For a cost function to be well-behaved, it must, among other things, be homogeneous of degree one in input prices. This implies the following restrictions on equation (16):

(18) $\phi_c + \phi_r = 1$; $\gamma_{cc} + \zeta_{cr} = 0$; $\gamma_{rr} + \zeta_{cr} = 0$; $\zeta_{rs} + \zeta_{cs} = 0$; and $\zeta_{hc} + \zeta_{hr} = 0$.

Homogeneity restriction (18) implies that equation (16) can be written as[15]

(19)
$$\ln\left(\frac{TFC}{W_{fr}}\right) = \alpha_0 + \alpha_h \ln H + \alpha_w \ln\left(\frac{W_c}{W_{fr}}\right) + \alpha_s \ln S + \varphi_b DB + \varphi_v DV + \varphi_p DP$$
$$+\varphi_i DI + \varphi_s DS + \frac{1}{2}\left\{\psi_{hh}(\ln H)^2 + \psi_{ww}\left[\ln\left(\frac{W_c}{W_{fr}}\right)\right]^2 + \psi_{ss}(\ln S)^2\right\}$$
$$+\rho_{hw} \ln H \cdot \ln\left(\frac{W_c}{W_{fr}}\right) + \rho_{sh} \ln S \cdot \ln H + \rho_{ws} \ln\left(\frac{W_c}{W_{fr}}\right) \cdot \ln S.$$

[15] This is because when the cost function is linearly homogeneous in input prices, $\lambda C = C(\lambda w_1, \lambda w_2, \mathbf{y})$. Suppose $\lambda = \frac{1}{w_2}$. Then, $\frac{C}{w_2} = C\left(\frac{w_1}{w_2}, \mathbf{y}\right)$. Taking the natural log on both sides and using the translog form on the right-hand-side will yield equation (19) (Sil, 1991).

One can impose additional restrictions on (19) corresponding to further restrictions on the underlying technology. If the technology is homothetic, it is necessary to impose the restriction $\rho_{hw} = 0$ on (19). The technology is homogeneous if, in addition to the homotheticity restriction, one can impose $\psi_{hh} = 0$. The degree of homogeneity in that case is $1/\ \alpha_h$. Finally, if the technology is constant returns to scale, one must impose the further restriction $\alpha_h = 1$ on (19) (Berndt, 1991).

One can estimate translog cost function (19) directly. However, gains in efficiency can be realized by estimating it along with the cost-minimizing input demand equations, transformed into input cost share equations.

If one logarithmically differentiates the cost function with respect to input price and then employs Shephard's lemma, one obtains a cost share equation of the form

$$(20) \qquad S_i = \frac{\partial \ln C}{\partial \ln W_i} = \frac{W_i}{C} \cdot \frac{\partial C}{\partial W_i} = \frac{W_i}{C} \cdot X_i^*,$$

where X_i^* is the cost-minimizing level of X_i, W_i is the ith input price, and where $\Sigma W_i X_i^* = C$ and $\Sigma S_i = 1$ (Berndt, 1991).

For (19), the cost share equation with respect to feed concentrate price S_{con} is

$$(21) \qquad S_{con} = \frac{\partial \ln TFC}{\partial \ln W_c} = \alpha_w + \psi_{ww} \ln\left(\frac{W_c}{W_{fr}}\right) + \rho_{hw} \ln H + \rho_{ws} \ln S.$$

The share equation for forage price must be dropped to avoid perfect linear dependence (see chapter 5).

I take the sample mean of each variable as the point of approximation in equations (19) and (21). This is done by dividing each variable by its sample mean before taking logs. The logarithm of each transformed variable will then be zero when evaluated at the sample means; and at the mean, cost share equation (21) will simply be the linear input price coefficient α_w. Such procedure simplifies the interpretation of my results in terms of the "average" dairy farm (Cowing and Holtmann, 1983).

Marginal Costs

Marginal costs of whole milk, protein, and fat can be derived by estimating cost elasticities of each of these three outputs, then multiplying the elasticities by $T\hat{F}C / Y_i$, where $T\hat{F}C$ is the predicted total feed cost and Y_i is the ith output. The cost elasticities themselves are obtained by differentiating the predicted values of the translog cost function (19) with respect to each separate output. This process is easier if one uses the chain rule when differentiating the natural log of $T\hat{F}C$ with respect to each output. For equation (19), assuming the quantity of whole milk (M) is the sum of protein quantity (P), fat quantity (F), and other components (L) (*i.e.* $M = F + P + L$), the cost elasticity of milk is

$$(22)\quad \frac{\partial \ln TFC}{\partial \ln M}=\frac{\partial \ln TFC}{\partial \ln H}\cdot\frac{\partial \ln H}{\partial \ln M}$$

$$=\alpha_h+\psi_{hh}\ln H+\rho_{hw}\ln\left(\frac{W_c}{W_{fr}}\right)+\rho_{sh}\ln S.$$

The cost elasticity of protein is

$$(23)\quad \frac{\partial \ln TFC}{\partial \ln P}=\frac{\partial \ln TFC}{\partial \ln H}\cdot\frac{\partial \ln H}{\partial \ln P}$$

$$=\left[\alpha_h+\psi_{hh}\ln H+\rho_{hw}\ln\left(\frac{W_c}{W_{fr}}\right)+\rho_{sh}\ln S\right]\cdot\left\{\beta_p+\left(1-\beta_f-\beta_p\right)\cdot\left(\frac{P}{M}\right)\right.$$

$$+\beta_{pp}\ln P+\beta_{fp}\ln F-\left(\beta_{pp}+\beta_{fp}\right)\cdot\left[\ln P\cdot\left(\frac{P}{M}\right)+\ln M\right]$$

$$\left.+\left(\beta_{ff}+\beta_{pp}+2\beta_{fp}\right)\cdot\left(\frac{P}{M}\right)\cdot\ln M-\left(\beta_{ff}+\beta_{fp}\right)\cdot\left(\frac{P}{M}\right)\cdot\ln F\right\}.$$

The cost elasticity of fat is

$$(24)\quad \frac{\partial \ln TFC}{\partial \ln F}=\frac{\partial \ln TFC}{\partial \ln H}\cdot\frac{\partial \ln H}{\partial \ln F}$$

$$=\left[\alpha_h+\psi_{hh}\ln H+\rho_{hw}\ln\left(\frac{W_c}{W_{fr}}\right)+\rho_{sh}\ln S\right]\cdot\left\{\beta_f+\left(1-\beta_f-\beta_p\right)\cdot\left(\frac{F}{M}\right)\right.$$

$$+\beta_{ff}\ln F+\beta_{fp}\ln P-\left(\beta_{ff}+\beta_{fp}\right)\cdot\left[\ln F\cdot\left(\frac{F}{M}\right)+\ln M\right]$$

$$+\left(\beta_{ff}+\beta_{pp}+2\beta_{fp}\right)\cdot\left(\frac{F}{M}\right)\cdot\ln M-\left(\beta_{pp}+\beta_{fp}\right)\cdot\left(\frac{F}{M}\right)\cdot\ln P\biggr\}.$$

Cost elasticities (22), (23), and (24) can be transformed into marginal cost equations by multiplying each by ($\hat{TFC} / Y_i$), where Y_i is M, P, and F respectively and $\hat{TFC}$ is the exponentiation of the predicted value of translog cost function (19) (Brown et al., 1979).

Short-Run Supply Functions and Supply Elasticities

A short-run supply function for a firm shows us how much output it will produce at various output prices, given the fixity of certain inputs. A perfectly competitive firm always produces at the point where price equals marginal cost, as long as the marginal cost is upward sloping and above average variable cost (AVC). Thus, supply curve S can be defined as

$$(25)\qquad \begin{aligned} S&=S(p) &&\text{for } P\geq \min \text{AVC, and}\\ S&=0 &&\text{for } P<\min \text{AVC.}\end{aligned}$$

The reason for the second line of equation (25) is that the firm will shut down if the price is below average variable cost. The marginal cost curve cuts the average cost curve at the average cost curve's minimum point. Therefore, at positive output, the firm's supply curve is equal to the portion of the marginal cost curve that lies above the minimum point of the average cost curve. For this reason, any factor that shifts the firm's short-run marginal cost curve will also shift the short-run supply curve (Pindyck and Rubinfeld, 1992; Henderson and Quandt, 1980).

A supply elasticity measures the responsiveness or sensitivity in quantity supplied of a commodity to a change in its price. Mathematically, the supply elasticity is

$$(26) \qquad \varepsilon^{s}_{P_i} = \frac{\text{percentage change in quantity supplied}}{\text{percentage change in price}} = \frac{\partial Q}{\partial P_i} \cdot \frac{P_i}{Q}.$$

where subscript i refers to the ith output. When the supply curve is upward sloping, supply elasticity (26) is always positive since quantity and price move in the same direction (i.e. $\partial Q / \partial P$>0). If this is the case, supply is said to be elastic when $\varepsilon > 1$, unitary when $\varepsilon = 1$, and inelastic when $\varepsilon < 1$ (Salvatore, 1991).

In my model, estimates of supply functions for each output will be calculated from equations (22), (23), and (24). A very simple model of supply functions for milk, protein, and fat can be obtained by nonstatistically regressing ln M, ln P, and ln F against ln MC_i by ordinary least squares, where and MC_i is the marginal cost of milk, protein, and fat, respectively. First, I obtain marginal costs for arbitrary levels of milk, milk fat, and milk protein. Then I equate marginal costs to the corresponding output prices, and express the quantities as functions of the output prices. The resulting functions are supply functions. They are

$$(27) \quad \ln M = e_m + e_{mm}\ln MC_m + e_{mp}\ln MC_p + e_{mf}\ln MC_f\,;$$

$$\ln P = e_p + e_{pm}\ln MC_m + e_{pp}\ln MC_p + e_{pf}\ln MC_f\,; \text{ and}$$

$$\ln F = e_f + e_{fm}\ln MC_m + e_{fp}\ln MC_p + e_{ff}\ln MC_f\,.$$ [16]

Both the dependent and independent variables take the log form to make the calculation of supply elasticity simpler.

Supply elasticities are then derived by simply differentiating ln Y_i with respect to each output price.

[16] Because the random error term on the translog cost function was removed prior to deriving marginal cost functions (22) - (24), and because functions (27) represent the inverse of (22) - (24), the latter regressions are, in Diewert's phrase,"nonstatistical." As a consequence, *t*-tests and R^2 in (27) would be inappropriate (Diewert, W. E., 1981).

5. ECONOMETRIC ESTIMATION

Data

The 1993 data used to estimate the feed cost function in equation (19) were obtained from the North Carolina Dairy Record Processing Center. Data included annual feed cost, quantity of whole milk produced, quantity of fat in the milk, quantity of protein in the milk, expenditures on and quantities of feed concentrates and forages, number of cows, breed of herd, location of farm, and average sire score. Output variables are measured in pounds per cow per year, input prices in dollars, and sire score in PTA dollars (see footnote 14 about PTA dollars). The data were collected from the Northeastern states (Vermont and Pennsylvania), North Central Region (Indiana), the Border states region (Missouri, Kentucky, Virginia, Tennessee, and North Carolina), and Southern states (South Carolina, Georgia, Alabama, Mississippi, Louisiana, Florida, and Texas). These areas cover approximately half the area of the United States, so that forage type and quality differ widely in the data. Such feed quality differences affect total feed cost. In hedonic translog cost function (19), three state-level dummy variables were included, namely Vermont (*DV*), Pennsylvania (*DP*), and Indiana (*DI*). Each of these states contains more than 200 observations. I combined all Southern states into one variable (*DS*) and combined the Border states together as the base dummy.

Dummy variables for breed were also constructed to investigate whether cow breed affects total feed cost. In the data set which I use in this study, 87.1% of the herds are Holstein, 6.23% are Jersey, and 5.85% are Guernsey. Jersey and Guernsey cows are combined together as the base dummy. When the dummy variable equals unity, its effect is to shift the intercept by the amount of the parameter estimate (Kennedy, 1992).

The data set contains 1924 observations after deleting observations which lack information on any one of the cost, quantity, or price variables.

Maximum Likelihood

When random variable **x** has probability density function $f(\theta \mid \mathbf{x})$ characterized by parameter vector θ, the maximum likelihood method chooses the particular value of unknown parameter θ that gives the greatest probability of randomly drawing the sample that was actually obtained. Mathematically,

$$(28) \quad \underset{\theta}{\mathrm{Max}} \prod f(\theta \mid \mathbf{x});$$

where $\prod f(\theta \mid \mathbf{x})$ equals the likelihood function $L(\theta \mid \mathbf{x})$. However, one usually maximizes the natural log of the likelihood function instead of maximizing likelihood function (28) itself. The reason is that maximizing the log-likelihood function is the same as maximizing the likelihood function itself and the former task is easier.

A maximum likelihood estimator is, under fairly general conditions, asymptotically efficient, consistent, asymptotically unbiased, and distributed asymptotically normally (Kennedy, 1992).

I use the maximum likelihood method to jointly estimate hedonic translog cost function (19) and share equation system (21). Joint estimation of these equations increases the degrees of freedom without additional parameter estimates and, hence, increases efficiency. However, when conducting the estimation, one must drop 1 share equation from the model. The reason is that at each observation, the dependent variables in all share equations taken together sum to one. It follows that if there are n factor share equations, only n - 1 of them are linearly independent. This result implies that, for each observation, the sum of the disturbances across equations must always equal zero. That is, let the vector of the observed share equation be denoted $\hat{\mathbf{S}}$ and the vector of the true share equation be $\mathbf{S}$. One can write the relationship between $\hat{\mathbf{S}}$ and $\mathbf{S}$ as $\mathbf{S} = \hat{\mathbf{S}} + \mathbf{e}$, where $\mathbf{e}$ is the vector of errors. Since observed shares always sum to unity, as do the true shares, then $\sum \mathbf{e} = 0$. This implies that the disturbance covariance and residual cross-product matrices will both be singular and hence the maximum likelihood parameter estimates are indeterminate (Chambers, 1991).

To avoid this singularity of the disturbance covariance, I arbitrarily drop one equation from the cost share equation system and solve for this equation after estimation. In my model, I drop the cost share equation for forages, and estimate the coefficients of the cost share equation for concentrates (21) along with hedonic translog cost share function (19).

A problem may arise about which equation to delete. If the parameter estimates vary with the choice of share equation dropped, the researcher may report only those estimates appealing to her beliefs or judgments. Fortunately, all parameter estimates, log-likelihood values, and estimated standard errors are invariant to the choice of which n - 1 equations are directly estimated (Berndt, 1991).

Starting Values

The full information maximum likelihood (FIML) procedure in TSP, which obtains maximum likelihood estimates for the system of equations, is used to estimate hedonic cost function (19) and share equation (21). The FIML procedure can be used with linear as well as with nonlinear models. Equations (19) and (21) are each nonlinear since one has to substitute ln$H$ of equation (17) into each and jointly estimate all coefficients in (17), (19), and (21). A common problem when estimating highly nonlinear models is that unless one has good starting values, parameter estimation becomes very difficult. Providing good starting values is important because poor starting values not only increases computational time but may even prevent convergence to the least-square estimates. Finding good starting values is often difficult in itself. I first estimated equation (19) with ordinary least squares, then used the OLS estimates as starting values in the maximum likelihood (Hall, 1994).

Testing for Separability

In the next chapter, I conduct hypothesis tests on equation (19) to investigate the structure of the underlying dairy feeding technology. The hypotheses to be tested are for: (1) homotheticity; (2) constant returns to scale; and (3) separability. Methods for conducting tests (1) and (2) were discussed in chapter 4. Constant returns to scale are possible if and only if homotheticity restriction (1) holds in the hedonic cost function. Below, I discuss the separability test in the context of the translog cost function.

(a) *Hypothesis Test for Separability*

The separability test is to determine whether outputs can be aggregated into a single output measure $H[\cdot]$. This test can be conducted by specifying: (i) the alternative general (non-separable) model; and (ii) general model (i) with separability restrictions imposed. A likelihood ratio test is then performed comparing separable cost function (i) and (ii). General nonseparable cost function (i) can be constructed as

$$(29) \quad \ln\left(\frac{TFC}{W_{frg}}\right) = B_0 + B_m \ln M + B_p \ln\left(\frac{P}{M}\right) + B_f \ln\left(\frac{F}{M}\right) + B_w \ln\left(\frac{W_c}{W_{fr}}\right) + B_s \ln S$$

$$+ D_b DB + D_v DV + D_p DP + D_i DI + D_s DS$$

$$+ \frac{1}{2}\left\{ C_{mm}(\ln M)^2 + C_{pp}\left[\ln\left(\frac{P}{M}\right)\right]^2 \right.$$

$$\left. + C_{ff}\left[\ln\left(\frac{F}{M}\right)\right]^2 + C_{ww}\left[\ln\left(\frac{W_c}{W_{fr}}\right)\right]^2 + C_{ss}(\ln S)^2 \right\} + G_{pm}\ln\left(\frac{P}{M}\right)\cdot \ln M$$

$$+G_{fm}\ln\left(\frac{F}{M}\right)\cdot\ln M+G_{wm}\ln\left(\frac{W_c}{W_{fr}}\right)\cdot\ln M+G_{sm}\ln S\cdot\ln M$$

$$G_{pf}\ln\left(\frac{P}{M}\right)\cdot\ln\left(\frac{F}{M}\right)+G_{pw}\ln\left(\frac{P}{M}\right)\cdot\ln\left(\frac{W_c}{W_{fr}}\right)+G_{ps}\ln\left(\frac{P}{M}\right)\cdot\ln S$$

$$G_{fw}\ln\left(\frac{F}{M}\right)\cdot\ln\left(\frac{W_c}{W_{fr}}\right)+G_{fs}\ln\left(\frac{F}{M}\right)\cdot\ln S+G_{ws}\ln\left(\frac{W_c}{W_{fr}}\right)\cdot\ln S.$$

Following Denny and Pinto, certain restrictions must be imposed in order that equation (29) be separable. These are

(30) $G_{wm}\cdot B_f=G_{fw}\cdot B_m$;

$G_{fw}\cdot B_p=G_{pw}\cdot B_f$; and

$G_{wm}\cdot B_p=G_{pw}\cdot B_m$.

One can use the likelihood ratio test to compare the maximum values of the likelihood (or log-likelihood) functions of equations (29) and (29) with restrictions (30) imposed. Separability restriction (30) is correct if the two likelihood values are not significantly different.

Hedonic separable cost function (19) is one form of separable cost function. If the separability restrictions (30) on general cost function (29) are true, it follows that the hedonic separable cost function (19) is a reasonable specification for dairy feed cost function.

(b) *Likelihood Ratio Test*

The hypotheses of homotheticity, constant returns to scale, and separability can also be tested with the likelihood ratio method. To implement the likelihood ratio test, first estimate the maximized value of the likelihood function of the restricted and unrestricted model. The likelihood ratio test is then computed as

$$(31) \quad \mathrm{LR} = 2[L(H_1) - L(H_0)],$$

where $L(H_0)$ and $L(H_1)$ are the maximum values of the (log-)likelihood functions with and without restrictions, respectively. LR is always positive since the likelihood value of the unconstrained model is always higher than the likelihood value of the constrained model. If the restrictions are true, $L(H_1) - L(H_0)$ should not be significantly different from zero. LR is asymptotically distributed as chi-square, with degrees of freedom equal to the number of constraints. We reject the H_0 when LR is greater than the critical value (Griffith et al., 1992).[17]

[17] Alternatives to the likelihood ratio test are the Wald Test and Lagrange multiplier test. The Wald test examines whether the unrestricted estimates violate the restriction by a significant amount, and the Lagrange multiplier test examines whether the slope of the log-likelihood function, when evaluated at the restricted coefficient, significantly differs from zero. Some tests are easier to compute in certain circumstances than others are. Although in limited samples, these test results differ, they are asymptotically equivalent (Kennedy, 1992).

6. ECONOMETRIC RESULTS AND DISCUSSION

Estimation Results

Hedonic translog cost function (19) and feed concentrate share equation (21) were estimated jointly. This required estimating parameters α_h and βs, ψ_{hh} and βs, ρ_{hw} and βs, and ρ_{sh} and βs jointly in nonlinear fashion. The maximum likelihood method, specifically the FIML procedure of TSP, was used. Results of estimating equations (19) and (21) jointly are listed in Table 1.

In Table 1, one can observe very low t-values for all the terms involving sire quality variables. A test for whether the sire quality terms are jointly significant is therefore necessary. This test should be

(32) H_0: $\alpha_s = \psi_{ss} = \rho_{sh} = \rho_{ws} = 0$; against

H_1: At least one of the sire coefficients is nonzero.

I used the likelihood ratio method to test the significance of the sire terms. To perform this test, the alternative model, which excludes all sire terms, needs to be estimated. Then, the maximum value of the (log-)likelihood function for the restricted model (without sire terms) was compared to that of the unrestricted model (with sire terms). If the value for the restricted model is significantly smaller than that for the unrestricted model, the H_0 is rejected and, therefore, sire terms are not jointly significant.

Table 1. Joint Estimates of Hedonic Cost Function (19) and Share Equation (21).

variable	Parameter	Estimates	*t*-value
intercept	α_0	10.5837	562.7
$\ln H$	α_h	0.4317	10.7274
$\ln\left(\frac{W_c}{W_{fr}}\right)$	α_w	0.533917	113.78
$\ln S$	α_s	0.017069	1.018
$\ln\left(\frac{P}{M}\right)$	β_p	0.224655	0.682932
$\ln\left(\frac{F}{M}\right)$	β_f	0.458599	2.98697
$\frac{1}{2}\left[\ln\left(\frac{P}{M}\right)\right]^2$	β_{pp}	0.653116	0.063378
$\frac{1}{2}\left[\ln\left(\frac{F}{M}\right)\right]^2$	β_{ff}	- 0.822731	- 0.474894
$\ln\left(\frac{P}{M}\right)\cdot\ln\left(\frac{F}{M}\right)$	β_{pf}	- 1.59198	- 0.43109
DB	φ_b	0.104229	5.71557
DV	φ_v	- 0.049346	- 3.52541
DP	φ_P	- 0.095301	- 7.66224
DI	φ_i	- 0.071495	- 4.42946
DS	φ_s	- 0.035804	- 2.3402
$\frac{1}{2}(\ln H)^2$	ψ_{hh}	- 0.088251	- 0.341235
$\frac{1}{2}\left[\ln\left(\frac{W_c}{W_{fr}}\right)\right]^2$	ψ_{ww}	0.187502	31.267
$\frac{1}{2}(\ln S)^2$	ψ_{ss}	0.012563	1.02089
$\ln H\cdot\ln\left(\frac{W_c}{W_{fr}}\right)$	ρ_{hw}	- 0.06824	- 3.40687
$\ln \mathrm{S}\cdot\ln H$	ρ_{sh}	- 0.052781	- 0.951763
$\ln\left(\frac{W_c}{W_{fr}}\right)\cdot\ln S$	ρ_{ws}	- 0.007366	- 1.40895

maximum value of log likelihood function = 1315.7
R^2: translog hedonic function : 0.640048; share equation: 0.29133

The coefficients of the restricted model are listed on Table 2. The result of the hypothesis test is listed in Table 3. Using formula (31) and calculating the log-likelihood ratio from Tables 1 and 2, I get LR = 5.28. If the H_0 is true, (31) is distributed as Chi-square with 4 degrees of freedom. At the 5 % level of significance, the H_0 cannot be rejected and hence I conclude that the sire terms jointly have no effect on the cost function. Therefore, I drop the sire terms from my model and, in the following discussion, concentrate on the reduced model with sire terms eliminated. The reduced hedonic cost function is

$$(33) \quad \ln\left(\frac{TFC}{W_{fr}}\right) = \alpha_0 + \alpha_h \ln H + \alpha_w \ln\left(\frac{W_c}{W_{fr}}\right) + \varphi_b DB + \varphi_v DV + \varphi_p DP + \varphi_i DI + \varphi_s DS$$
$$+ \frac{1}{2}\left\{\psi_{hh}(\ln H)^2 + \psi_{ww}\left[\ln\left(\frac{W_c}{W_{fr}}\right)\right]^2\right\} + \rho_{hw} \ln H \cdot \ln\left(\frac{W_c}{W_{fr}}\right),$$

and the associated share equation for feed concentrates, S_c, is

$$(34) \quad S_c = \frac{\partial \ln TFC}{\partial \ln W_c} = \alpha_w + \psi_{ww} \ln\left(\frac{W_c}{W_{fr}}\right) + \rho_{hw} \ln H.$$

Table 2. Joint Estimates of Reduced Hedonic Cost Function (33) and Share Equation (34).

variable	Parameter	Estimates	t-value
intercept	α_0	10.5843	559.71
$\ln H$	α_h	0.434377	10.9042
$\ln\left(\frac{W_c}{W_{fr}}\right)$	α_w	0.534648	115.354
$\ln\left(\frac{P}{M}\right)$	β_p	0.211603	0.65278
$\ln\left(\frac{F}{M}\right)$	β_f	0.467523	3.04545
$\frac{1}{2}\left[\ln\left(\frac{P}{M}\right)\right]^2$	β_{pp}	0.656814	0.060458
$\frac{1}{2}\left[\ln\left(\frac{F}{M}\right)\right]^2$	β_{ff}	- 0.699491	- 0.417712
$\ln\left(\frac{P}{M}\right)\cdot\ln\left(\frac{F}{M}\right)$	β_{pf}	- 1.91955	- 0.508481
DB	φ_b	0.103592	5.64386
DV	φ_v	- 0.046922	- 3.35748
DP	φ_P	- 0.092709	- 7.58416
DI	φ_i	- 0.072772	- 4.54065
DS	φ_s	- 0.035579	- 2.33106
$\frac{1}{2}(\ln H)^2$	ψ_{hh}	- 0.103615	- 0.404996
$\frac{1}{2}\left[\ln\left(\frac{W_c}{W_{fr}}\right)\right]^2$	ψ_{ww}	0.187758	31.7464
$\ln H\cdot\ln\left(\frac{W_c}{W_{fr}}\right)$	ρ_{hw}	- 0.069305	- 3.54657

maximum value of log likelihood function = 1313.06
R^2: translog hedonic function : 0.63891; share equation: 0.290415

Table 3. Hypothesis Test of Sire Quality Terms

Null Hypothesis: Sire Terms are Jointly Zero	Log Likelihood Ratio	Critical Chi-Square (At 5 %)	Degrees of Freedom
	5.28	9.488	4

At 10 % level of significance, critical chi-square is 7.78. So, the null hypotheses still cannot be rejected at this level.

Table 4. Hypothesis Test of Homotheticity Using Reduced Model (33).

Null Hypothesis: Homotheticity holds	Log Likelihood Ratio	Critical Chi-Square (At 5 %)	Degrees of Freedom
	9.6	3.84146	1

Due to rejection of homotheticity, the constant returns to scale restriction is not tested.

Coefficient estimates of reduced model (33) and share equation (34) are listed in Table 2. The R^2 of the reduced hedonic translog cost function (33) is approximately 0.64. Although the R^2 for the concentrate share equation (34) is low (0.29), this is not unusual in translog cost studies (Spady and Friedlander, 1978).

In Table 2, the dummy variable for breed (*DB*) is positive and strongly significant, indicating feed costs for a Holstein are higher than for other breeds. According to the dairy science literature (Webster, 1993), a Holstein is heavier than other dairy cows, has higher maintenance energy requirements, and hence lower feed efficiency. My results are consistent with the literature. All the dummy variables for geographic regions have negative and strongly significant signs, indicating that the Border states have the highest total feed costs of all regions in the model.

The fact that all variables are divided by sample means before logs are taken in (33) and (34) facilitates the interpretation of results in terms of the average farm. That is, at the mean, all the logarithmic terms go to zero and the predicted value of the dependent variable simply equals the intercept. Hence, at the mean, the intercept of (33), α_0, equals the predicted value of ln (TFC/ W_{fr}), and α_w is the estimated cost share for feed concentrates on the average farm. Estimated total feed cost at the mean is the exponentiated value of α_0 times W_{fr} evaluated at the mean. Using the results from Table 2 and descriptive statistics from Table 5, this value equals \$1010.73 per cow-year, which is close to the actual mean of total feed cost in Table 5 (\$976.86).

Table 5. Descriptive Statistics of Variables in Hedonic Cost Functions (19) and (33).

Variable	Unit	Sample Mean	Standard Deviation	Minimum	Maximum
Total Feed Cost (*TFC*)	($/cow/year)	976.86	182.15	304	1955
Milk Quantity (*M*)	(lbs./cow/year)	18067.02	2799.83	7144	26208
Protein Quantity (*P*)	(lbs./cow/year)	584.58	81.01	141	827
Fat Quantity (*F*)	(lbs./cow/year)	665.69	96.16	275	996
Price of Concentrate (W_c)	($/lbs.)	0.089	0.025	0.0134	0.47
Price of Forage (W_{fr})	($/lbs.)	0.026	0.009	0.0035	0.11
Sire Score (*S*)	(index)	145.95	48.18	1	296
Breed Dummy Holstein (*DB*)		0.871	0.335	0	1
State Dummy Vermont (*DV*)		0.115	0.319	0	1
State Dummy Pennsylvania (*DP*)		0.247	0.431	0	1
State Dummy Indiana (*DI*)		0.156	0.363	0	1
State Dummy Southern States (*DS*)		0.148	0.458	0	1

The insignificant t-statistics for β_p in Tables 1 and 2 may reflect the fact that the current pricing system under the Federal Marketing Orders values only the amount of fat percentage in the milk, not the protein percentage.

Results of Hypothesis Testing

To investigate the structure of technology, I first test whether the technology is homothetic, and if so, whether it has constant returns to scale. As noted in chapter 4, homotheticity holds if $\rho_{hw} = 0$ in equation (33). Constant returns to scale holds if, in addition to that restriction, $\psi_{hh} = 0$ and $\alpha_h = 1$. The null and alternative hypotheses for homotheticity are

(35) H_0: $\rho_{hw} = 0$; against

H_1: $\rho_{hw} \neq 0$.

Results of the likelihood ratio test for homotheticity are reported in Table 4. The maximum value of the log likelihood function in the homothetic model is 1308.26. The result of the likelihood ratio test in Table 4 shows that at the 5% level of significance and 1 degree of freedom, the null hypothesis is rejected and, therefore, the technology appears to be nonhomothetic. Because the homotheticity restriction is rejected, constant returns to scale cannot hold either.

Next, a test is conducted of whether the technology is separable. One can do so by comparing the likelihood value of the estimate of generally nonseparable model (29), dropping sire terms, with the likelihood value of generally nonseparable model (29) in which separability restrictions (30) are imposed. Again, these two models are estimated together with the share equation for feed concentrates, and the maximum likelihood method is employed. The estimates of generally nonseparable model (29) are listed on Table 6, and the estimates of (29) with separability restrictions (30) are listed on Table 7. Generally, if the maximized likelihood value of the separable model in Table 7 is close to that of generally nonseparable model in Table 6, it can be concluded that the separable specification is correct.

The test is conducted and the result reported in Table 8. The null hypothesis is that the separability specification is correct. At the 5 % level of significance and 3 degrees of freedom, the null hypothesis is not rejected. Hence, I conclude that separability restrictions (30) are correct.

Checking Regularity Conditions

In order that the translog hedonic cost function (19) and (33) behave well, the regularity conditions I discussed in chapter 2 must be satisfied.

The first condition, that w be exogenous and strictly positive, is satisfied since w does not depend on the value of other variables in the cost function and since prices of forages and feed concentrates in the data set are all positive as shown in Table 5.

Second, the nonnegativity condition holds since all inputs and outputs are positive. The third condition, linear homogeneity in input prices, is imposed on hedonic translog model (19) and (33). Next, concavity in input prices requires that the Hessian matrix be negative semidefinite. Following Young et al., the input price Hessian matrix of (19) and (33) is

$$(36) \qquad |H| = \begin{vmatrix} \dfrac{T\hat{F}C\left(\psi_{ww} + S_c^2 - S_c\right)}{W_c^2} & \dfrac{T\hat{F}C\left(-\psi_{ww} + S_c \cdot S_{fr}\right)}{W_c \cdot W_{fr}} \\ \dfrac{T\hat{F}C\left(-\psi_{ww} + S_c \cdot S_{fr}\right)}{W_c \cdot W_{fr}} & \dfrac{T\hat{F}C\left(\psi_{ww} + S_{fr}^2 - S_{fr}\right)}{W_{fr}^2} \end{vmatrix}.$$

(Young et al.) Negative semidefiniteness requires that $|H_1| \leq 0$ and $|H_2| \geq 0$. I will calculate whether this condition holds at the mean since it is impossible to determine whether a translog cost function satisfies the conditions globally. At the mean, $T\hat{F}C$ = $exp\,(\alpha_0) \cdot \overline{W}_{fr}$ where $\overline{W}_{fr}$ = mean price of forage. $S_c = \alpha_w$, and $S_{fr} = 1 - S_c$. Substituting these values into (36), I get $|H_1| = -7835.39$ and $|H_2| = 0$. Thus, concavity in input prices is satisfied.

Estimation of Marginal Costs

Marginal costs of milk, protein, and fat in hedonic translog cost function (33) and share equation (34) were estimated by multiplying cost elasticities (22), (23), and (24) by ($\hat{TFC} / Y_i$), where $\hat{TFC}$ is the predicted value of total feed cost. These marginal costs are estimated holding other outputs and inputs constant at their mean values. Although the estimates of a translog function tend to be precise at the mean, the further the estimates are away from the mean, the more imprecise they become. Thus, marginal cost estimates far from those of the average farm tend to be imprecise. Hence, I calculated marginal costs in the neighborhood of mean variable values, namely within 2 standard deviations above and below each mean output. Marginal costs of milk and fat were estimated at the mean output, one standard deviation above and below the mean output, and two standard deviations above and below the mean output. Marginal costs of protein were calculated for a slightly narrower range (1.84 standard deviation above and below the mean) since at two standard deviations below the mean protein output, marginal cost is negative. Results are listed in Table 9 and graphed in figures 1, 2, and 3.

Table 6. Estimates of General Non-Hedonic Cost Function (29).

variable	Parameter	Estimates	t-value
intercept	B_0	10.5778	538.036
ln M	B_m	0.4534957	11.2387
$\ln\left(\frac{P}{M}\right)$	B_p	0.256025	1.37386
$\ln\left(\frac{F}{M}\right)$	B_f	0.196817	2.21643
$\ln\left(\frac{W_c}{W_{fr}}\right)$	B_w	0.535089	115.356
DB	D_b	0.110764	5.85876
DV	D_v	- 0.048778	- 3.47715
DP	D_p	- 0.093322	- 7.5645
DI	D_i	- 0.073229	- 4.56915
DS	D_s	- 0.035575	- 2.3254
$\frac{1}{2}(\ln M)^2$	C_{mm}	- 0.134701	- 0.489799
$\frac{1}{2}\left[\ln\left(\frac{P}{M}\right)\right]^2$	C_{pp}	0.365459	0.057853
$\frac{1}{2}\left[\ln\left(\frac{F}{M}\right)\right]^2$	C_{ff}	- 0.167057	- 0.191007
$\frac{1}{2}\left[\ln\left(\frac{W_c}{W_{fr}}\right)\right]^2$	C_{ww}	0.188229	31.7177
$\ln\left(\frac{P}{M}\right)\cdot\ln M$	G_{pm}	- 0.013018	- 0.013847
$\ln\left(\frac{F}{M}\right)\cdot\ln M$	G_{fm}	- 0.09382	- 0.19895
$\ln\left(\frac{W_c}{W_{fr}}\right)\cdot\ln M$	G_{wm}	- 0.044069	- 2.01425
$\ln\left(\frac{P}{M}\right)\cdot\ln\left(\frac{F}{M}\right)$	G_{pf}	- 0.928728	- 0.425318
$\ln\left(\frac{P}{M}\right)\cdot\ln\left(\frac{W_c}{W_{fr}}\right)$	G_{pw}	0.184885	2.01777
$\ln\left(\frac{F}{M}\right)\cdot\ln\left(\frac{W_c}{W_{fr}}\right)$	G_{fw}	- 0.04226	- 0.931994

maximum value of log likelihood function = 1315.78
R^2: translog cost function = 0.640881; share equation = 0.291224

Table 7. Estimates of General Non-Hedonic Cost Function (29) with Separability Restrictions (30) Imposed.

variable	Parameter	Estimates	t-value
intercept	B_0	10.5802	540.502
ln M	B_m	0.453482	11.0169
$\ln\left(\frac{P}{M}\right)$	B_p	0.018176	0.125947
$\ln\left(\frac{F}{M}\right)$	B_f	0.267384	3.27083
$\ln\left(\frac{W_c}{W_{fr}}\right)$	B_w	0.535246	115.658
DB	D_b	0.107575	5.7231
DV	D_v	- 0.048392	- 3.45213
DP	D_p	- 0.093403	- 7.6024
DI	D_i	- 0.073211	- 4.57374
DS	D_s	- 0.033869	- 2.21719
$\frac{1}{2}(\ln M)^2$	C_{mm}	- 0.14596	- 0.532497
$\frac{1}{2}\left[\ln\left(\frac{P}{M}\right)\right]^2$	C_{pp}	0.118377	0.018897
$\frac{1}{2}\left[\ln\left(\frac{F}{M}\right)\right]^2$	C_{ff}	- 0.10363	- 0.119987
$\frac{1}{2}\left[\ln\left(\frac{W_c}{W_{fr}}\right)\right]^2$	C_{ww}	0.187977	31.6757
$\ln\left(\frac{P}{M}\right)\cdot\ln M$	G_{pm}	- 0.103763	- 0.111727
$\ln\left(\frac{F}{M}\right)\cdot\ln M$	G_{fm}	- 0.079063	- 0.168505
$\ln\left(\frac{W_c}{W_{fr}}\right)\cdot\ln M$	G_{wm}	- 0.057817	- 2.83375
$\ln\left(\frac{P}{M}\right)\cdot\ln\left(\frac{F}{M}\right)$	G_{pf}	- 0.804342	- 0.37046
$\ln\left(\frac{P}{M}\right)\cdot\ln\left(\frac{W_c}{W_{fr}}\right)$	G_{pw}	- 0.002377	- 0.125799
$\ln\left(\frac{F}{M}\right)\cdot\ln\left(\frac{W_c}{W_{fr}}\right)$	G_{fw}	- 0.022233	- 0.477449

maximum value of log likelihood function = 1312.97
R^2: translog cost function = 0.640127; share equation = 0.290404

Table 8. Hypothesis Testing of Separability by Comparing General Nonseparable Model (29) and Model (29) with Separability Restrictions (30) imposed.

Null Hypotheses: Separability Holds	Log Likelihood Ratio	Critical Chi-Square (At 5 %)	Degrees of Freedom
	5.72	7.81473	3

Table 9. Estimates of Marginal Costs

Marginal Cost of Milk

Mean - 2 SD	Mean - 1 SD	Mean	Mean + 1 SD	Mean + 2 SD
0.033618	0.028408	0.02525	0.023238	0.02191

Marginal Costs for Protein

Mean - 1.84 SD	Mean - 0.92 SD	Mean	Mean + 0.92 SD	Mean + 1.84 SD
0.016322	0.11193	0.17318	0.21435	0.24298

Marginal Costs for Fat

Mean - 2 SD	Mean - 1 SD	Mean	Mean + 1 SD	Mean + 2 SD
0.62762	0.45176	0.32848	0.24314	0.1824

SD = standard deviation

Figure 1. Marginal Cost of Milk

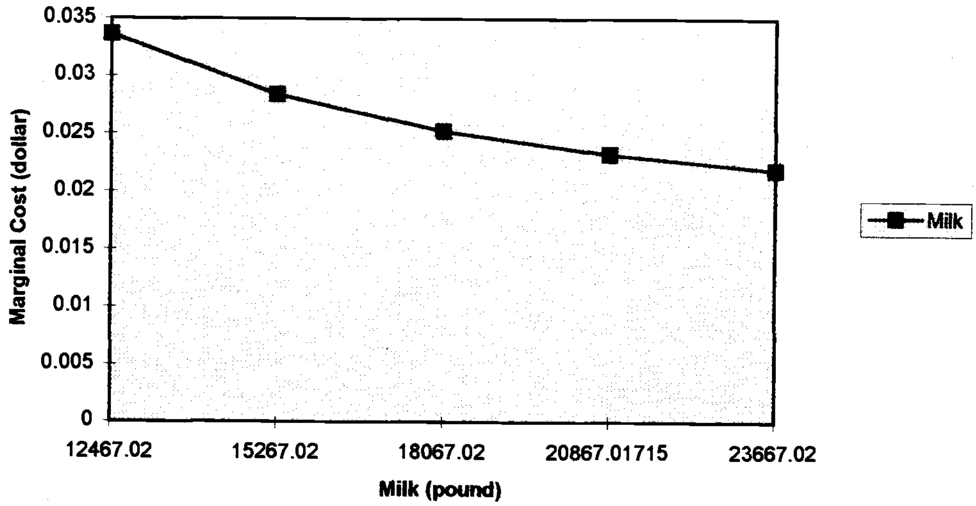


Figure 2. Marginal Cost of Protein

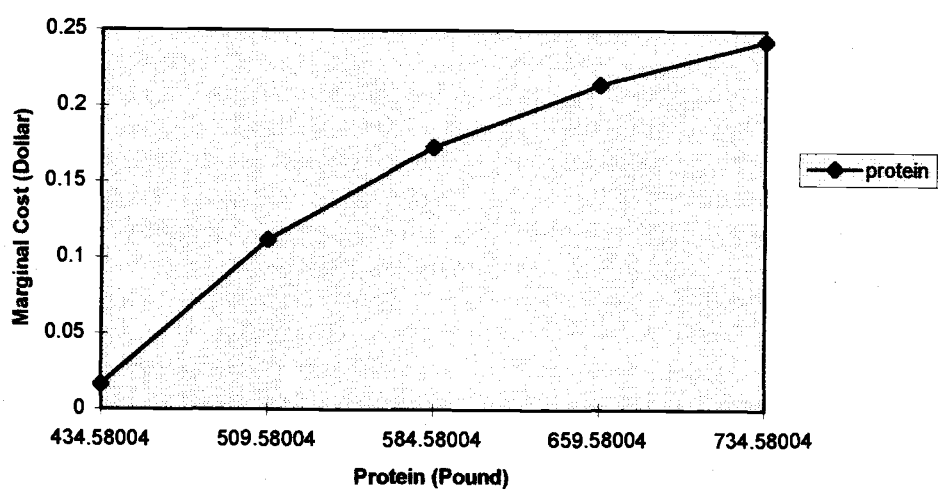
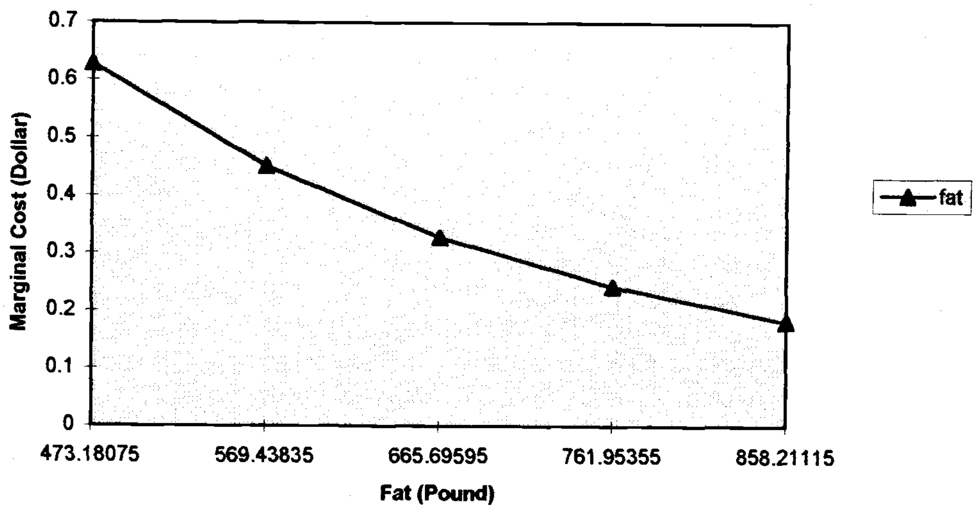


Figure 3. Marginal Cost of Fat



Implication of Downward Sloping Marginal Costs

One can see from Table 9 and Figures 1 and 3 that the marginal cost function of milk and fat are downward sloping. This result suggests that the dairy farm is producing in stage I of the production function.

Wilson showed that beef cattle in feedlots operate in stage I of their production function also. He stated that cattle-feeding production relationships are constrained by the animal's appetite to stage I. That is, as feed intake rises, "there comes a point while in stage I of production, where rumen capacity of the cattle ceases to be the limiting factor but chemostatic and thermostatic factors take over to inhibit more feed intake." Although the exact causal mechanism is not fully understood, this phenomenon is widely accepted in the literature. Hence, the animal's appetite limitation precludes movement further down the average feed-cost function (Wilson, 1976). My result suggests that the Wilson's result applies for dairy cows as well as beef cattle. That is, the appetite constraint prohibits dairy cows from producing milk and milk fat beyond stage I of their production functions.

Estimation of Inverse Marginal Cost Functions

According to conventional microeconomic theory, a firm does not wish to produce in stage I of production. If it is constrained to operate in stage I, it will produce at maximum volume as long as profit is positive. Thus, instead of the term "supply functions" for equation (27), I will use the term "inverse marginal cost functions."

Kirkland and Mittelhammer (1986) investigated supply response of fat and solids-not-fat (SNF) in experiments with 100 hypothetical Holsteins, using nonlinear programming methods. They concluded that the own-price supply elasticity of fat was 0.07, and the supply elasticity of SNF with respect to fat price was 0.014. Results of estimating equation (27) are listed on Table 10. The own-price marginal cost elasticity of fat is $d \ln F / d \ln MC_f = e_{ff} = - 0.04$, which is negative and fairly inelastic. The marginal cost elasticity of protein with respect to fat price is $e_{pf} = - 0.124$, close to the elasticity of SNF with respect to butterfat price in Kirkland and Mittelhammer. The cross-price elasticity of fat with respect to protein $e_{fp} = - 0.0576$. The own-price marginal cost elasticities of milk and protein in Table 10 are $e_{mm} = - 0.938$ and $e_{pp} = 0.03$, respectively. These are inelastic marginal cost responses, consistent with the elasticities which Kirkland and Mittelhammer computed.

Hence, production response of milk components to varying component prices is small. It is nearly impossible for farmers to respond significantly through feeding management to output price changes, at least in the short run. Additional research to test the robustness of these results would be useful.

Table 10. Inverse Marginal Cost Functions for Milk, Protein, and Fat.

dependent variable: ln Q_m

Variable	parameter	estimates
intercept	e_m	6.4002
ln P_m	e_{mm}	- 0.937885
ln P_p	e_{mp}	- 0.00037655
ln P_f	e_{mf}	0.026747

dependent variable: ln Q_p

Variable	parameter	estimates
intercept	e_p	4.66386
ln P_m	e_{pm}	- 0.44916
ln P_p	e_{pp}	0.029662
ln P_f	e_{pf}	- 0.124361

dependent variable: ln Q_f

Variable	parameter	estimates
intercept	e_f	3.14993
ln P_m	e_{fm}	- 0.871171
ln P_p	e_{fp}	- 0.057593
ln P_f	e_{ff}	- 0.039998

7. CONCLUSIONS

In this thesis, I have investigated how dairy farmers' incentives to produce milk, milk protein, and milk fat likely would change under multiple component pricing. To achieve this objective, I first estimated a hedonic translog cost function and share equation using maximum likelihood. The cost function satisfied regularity conditions, implying that dairy farmers are cost-minimizers.

Low significance of sire quality terms suggested that sire quality has no influence on a cow's feeding efficiency, or therefore on feed cost at given milk, milk protein, and milk fat levels.

To investigate the structure of the milk production technology, test of homotheticity and separability were conducted. Homotheticity in the feeding technology was rejected, implying also that the technology does not exhibit constant returns to scale. The test of whether the feeding technology is separable was conducted by comparing the maximum log-likelihood values of generally nonseparable model (29) in which general model (29) in which separability restrictions (30) imposed. The null hypothesis of separabilty was not rejected. This result implies that hedonic specifications (19) and (33), which impose separability on the cost function, are a reasonable way to characterize dairy feeding costs.

Using the estimated hedonic cost function, marginal costs of milk, milk protein, and milk fat were derived at various output levels. Results showed downward-sloping marginal cost curves for milk and milk fat, and an upward sloping marginal cost curve for milk protein. Although protein production is at stage II of its production function, a downward-sloping marginal cost curve for milk implies that dairy farms operate in stage I of their production function. The reason is straightforward. Feed must first be used to satisfy a cow's maintenance requirements before any residual feed is available for milk production. Further, the cow's appetite is satisfied before its marginal product can begin to fall and thus marginal cost begin to rise.

Inverse marginal cost functions for milk, milk protein, and milk fat were then estimated from the marginal costs of each output. From these inverse marginal cost functions, marginal cost elasticities for milk, milk protein, and milk fat were derived. Marginal costs of all three outputs are inelastic, consistent with Kirkland and Mittelhammer's results based on experimental data with Holstein herds.

My results suggest that milk component production response to varying component price changes will be small if feed rations alone are permitted to change. Thus, the multiple component pricing plans currently proposed in many federal marketing orders, which will increase protein prices substantially, will not raise protein concentrations much unless breeding programs succeed in developing cows that produce higher-protein milk.

BIBLIOGRAPHY

Aitchison, Thomas E. *The USDA Animal Model Genetic Evaluations.* Iowa State University Extension Bulletin. DyS-3091, April 1989.

American Agricultural Economics Association Policy Task Force on Dairy Marketing Orders. *Federal Milk Marketing Orders: A Review of Research on Their Economic Consequences.* Occasional Paper no.5, 1986.

Bachman, K. C. "Managing Milk Composition" in H. H. Van Horn, and C. J. Wilcox ed., *Large Dairy Herd Management.* Illinois: Management Services, American Dairy Science Association (1992): 336-346.

Berndt, Ernst R. *The Practice of Econometrics: Classic and Contemporary.* Massachusetts: Addison-Wesley Publishing Company, 1991.

Berndt, Ernst R., and Mohammed S. Khaled. "Parametric Productivity Measurement and Choice Among Flexible Functional Forms." *Journal of Political Economy* 87(1979): 1220-1245.

Binger, Brian R., and Elizabeth Hoffman. *Microeconomics with Calculus.* Illinois: Scott, Foresman and Company, 1988.

Brown, Mike. *Comparison of Six Different Multiple Component Pricing Plans: Impacts on Milk Prices and Manufacturing Margins.* Presented in Northeast Federal Order MCP Meeting Massachusetts, 1994.

Brown, Randall S., Douglas W. Caves, and Lautris R. Christensen. "Modelling the Structure of Cost and Production for Multiproduct Firms." *Southern Economic Journal* 46(1979): 256-270.

Chambers, Robert G. *Applied Production Analysis: A Dual Approach.* Cambridge: Cambridge University Press, 1991.

Chiang, Alpha C. *Fundamental Methods of Mathematical Economics.* third edition New York: McGraw-Hill Book Company, 1984.

Christensen, Laurits R., and William H. Green. "Economies of Scale in U.S. Electric Power Generation." *Journal of Political Economy* 84(1976): 655-676.

Cowing, Thomas G., and Alphonse G. Holtmann. "Multiproduct Short-Run Hospital Cost Functions: Empirical Evidence and Policy Implications from Cross-Section Data." *Southern Economic Journal* 49(1983): 637-653.

Deller, Steven C., David L. Chicoine, and Norman Walzer. "Economies of Size and Scope in Rural Low-Volume Roads." *The Review of Economics and Statistics* 70(1988): 459-465.

Denny, Michael, and Cheryl Pinto. "An aggregate Model with Multi-Product Technologies." in M. Fuss, and D. McFadden, eds., *Production Economics: A Dual Approach to Theory and Applications*, Amsterdam: North Holland Publishing Company, 1980 vol. 2: 249-267.

Diewert, W. E. "The Theory of Total Factor Productivity Measurement in Regulated Industries." in T. G. Cowing and R. E. Stevenson eds., *Productivity Measurement in Regulated Industries.* New York: Academic Press (1981): 17-44.

Dobson, S. H. "Grasslands and Forage in the Agricultural Economy of the Middle States Region." in Howard B. Sprague, ed., *Grasslands of the United States: Their Economic and Ecologic Importance*. Iowa: The Iowa State University Press (1974): 113-120.

Equity Newsletter. *Multiple Component Pricing (MCP) Recommended for Two More Federal Milk Orders.* (vol. XVIII no. 6 December 1993).

Gibson, J. P. "Altering Milk Composition Through Genetic Selection." *Journal of Dairy Science* 72(1989): 2815-2825.

Green, William H. *Econometric Analysis.* New York: Macmillan Publishing Company, a Division of Mcmillan, 1990.

Griffiths, William E., R. Carter Hill, and George G. Judge. *Learning and Practicing Econometrics*. New York: John Wiley & Sons, 1993.

Hall, Bronwyn H. *Times Series Processor Version 4.2.: Reference Manual*. California: TSP International, 1993.

Hall, Bronwyn H. *Time Series Processor Version 4.2.: User's Guide*. California: TSP International, 1994.

Hall, Robert E. "The Specification of Technology with Several Kinds of Outputs." *Journal of Political Economy* 81(1973): 878-892.

Heath, Maurice E. "Significance of Forages to the Economy of the North Central Region." in Howard B. Sprague, ed., *Grasslands of the United States: Their Economic and Ecologic Importance*. Iowa: The Iowa State University Press (1974): 140-150.

Henderson, James M. and Richard E. Quandt. *Microeconomic Theory: A Mathematical Approach.* third edition New York: McGraw-Hill Publishing Company, 1980.

Judge, George G., R. Carter Hill, William E. Griffiths, Helmut Lütkepohl, and Tsoung-Chao Lee. *Introduction to the Theory and Practice of Econometrics.* second edition New York: John Wiley & Sons, 1988.

Kennedy, B. W. "Reducing Fat in Milk and Dairy Products by Breeding." Journal of Dairy Science 65(1982): 443-449.

Kennedy, Peter. *A Guide to Econometrics.* third edition Massachusetts: The Mit Press, 1992.

Kirkland, Jack J., and Ron C. Mittelhammer. "A Nonlinear Programming Analysis of Production Response to Multiple Component Milk Pricing." *American Journal of Agricultural Economics* 68(1986): 44-54.

Mays, David A. "Contribution of Forages to Agricultural Income in the Humid South." in Howard B. Sprague, ed., *Grasslands of the United States: Their Economic and Ecologic Importance.* Iowa: The Iowa State University Press (1974): 121-139.

McKinley, Myron. *Multiple Component Pricing.* Presented in International Livestock Congress Texas, 1993.

Nicholson, Walter. *Microeconomic Theory: Basic Principles and Extensions.* fifth edition Fort Worth: The Dryden Press, 1992.

Pindyck, Robert S., and Daniel L. Rubinfeld. *Micro Economics.* second edition New York: Macmillan Publishing Company, 1992.

Salvatore, D. *Microeconomics.* New York: Harper Collins Publishers, 1991.

Shumway, C. Richard. "Supply, Demand, and Technology in a Multiproduct Industry: Texas Field Crops." *American Journal of Agricultural Economics* 64(1983): 748-760.

Sil, Jayashree. "Cost Function Analysis of Fruit and Vegetable Processing in an Oregon Cooperative." M.S. Thesis, Oregon State University, 1991.

Silberberg, Eugene. *The Structure of Economics: A Mathematical Analysis.* second edition New York: McGraw-Hill Publishing Company, 1990.

Spady, Richard H., and Ann F. Friedlaender. "Hedonic Cost Functions for the Regulated Trucking Industry." *The Bell Journal of Economics* 9(1978): 159-179.

Sutton, J. D. "*Altering Milk Composition by Feeding.*" Journal of Dairy Science 72(1989): 2801-2814.

Thomas, P. C., and P. A. Martin. "The Influence of Nutrient Balance on Milk Yield and Composition" in P. C. Garnsworthy, ed., *Nutrition and Lactation in the Dairy Cow.* London: Butterworths (1988): 97.

Varian, Hal R. *Microeconomic Analysis.* third edition New York: W. W. Norton & Company, 1992.

Washko, John B. "Forages and Grasslands in the Northeast." in Howard B. Sprague, ed., *Grasslands of the United States: Their Economic and Ecologic Importance.* Iowa: The Iowa State University Press (1974): 98-112.

Webster, John. *Understanding the Dairy Cow.* second edition Oxford: Blackwell Scientific Publications, 1993.

Wilson, Ewen M. "Implications of Short-Run Rising Average Productivity in Cattle Feeding." *American Journal of Agricultural Economics* 58(1976): 305-310.

Young, Douglas L., Ron C. Mittelhammer, Ahmad Rostamizadeh, and David W. Holland. *Duality Theory and Applied Production Economics Research: A Pedagogical Treatise.* Pallman: Washington University Cooperative Extension. EB 1473, September 1987.