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A variational integral method in the Fourier transform domain is used to calculate the characteristic normal mode parameters of a coupled pair of nonsymmetrical microstrip transmission lines in an inhomogenous dielectric medium under quasi-TEM conditions. These together with the theory of such structures are used to show that ideal directional couplers with impedance transforming capabilities can be physically realized by utilizing coupled nonsymmetrical microstrips deposited on composite substrates or a single substrate with a dielectric overlay. The numerical results obtained agree quite well with known results for special cases which have been analyzed using other methods. Numerical results for coupler parameters are presented for some typical structures. The techniques and results presented in this thesis should be quite helpful in the design of ideal couplers and other microwave circuit elements in the form of planar structures that are amenable to integrated circuit design.

NONSYMMETRICAL COUPLED MICROSTRIP LINES FOR APPLICATIONS AS IDEAL DIRECTIONAL COUPLERS

by

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I. INTRODUCTION

Transmission lines have been an important topic of investigation for many years. The types of lines that are currently of greatest interest result from speed and packaging advances in digital computers and an increase in frequency capabilities and sophistication in microwave systems. The propagation of high speed digital signals makes similar demands on the propagating medium. Impedance levels must be constant to minimize reflections, coupling to neighboring transmission lines should be controllable and losses must usually be low. Microstrip can fulfill these requirements and is suitable for high density digital systems as well as microwave applications. The planar geometry of microstrip is rugged, easily fabricated, and particularly appropriate for use with integrated circuits.

The work to be presented here deals with the coupling properties of two coupled microstrip transmission lines. The properties of nonsymmetrical coupled microstrip lines on composite substrates and with dielectric overlay for applications as ideal directional couplers are presented in this thesis.

Designing with microstrips obviously requires knowledge of its' propagation characteristics and circuit properties. Evaluation of the propagation characteristics of a specific class of structures is the basis for this thesis. The propagation parameters result from solution of the Maxwell equations.

The technique used to solve the field problem depends on the boundary conditions involved. The Maxwell equations can be solved in closed form only for simple structures and charge distributions. As structures become more complicated, by use of multiple dielectrics and nonuniform conducting surfaces, it soon becomes very difficult to solve the Maxwell equations directly and techniques must be developed to obtain the required information efficiently. In general one uses the simplest method that yields the required system parameters. For the class of problems considered here, nonsymmetrical coupled lines in an inhomogeneous dielectric medium, the boundary conditions make the application of most techniques impractical, i.e., conformal mapping, finite difference equation, potential function expansion.

A spectral domain approach has been shown in the past [6, 7, 8] to be an easily implemented efficient way to get accurate results for quasi-TEM transmission line parameters for single microstrip and coupled symmetrical microstrip structures in simple dielectrics. The technique has been applied to the case of nonsymmetrical coupled microstrip lines in composite and overlay dielectric structures.

Until recently the study of transmission line directional couplers has been confined to symmetrical and nonsymmetrical coupled lines in a homogeneous medium [5] and symmetrical coupled lines in an inhomogeneous medium [3,4]. Tripathi [9] has analyzed the general nonsymmetrical inhomogeneous case, found the general four port parameters and applied them to show that various circuit elements can be realized. From the general four port scattering matrix Tripathi [10] has shown that ideal directional couplers are realizable if the line parameters are interrelated in a specified manner.

In this thesis Yamashita's variational method [6,7] is used to find the normal mode parameters and show that the conditions required by Tripathi can be met and an ideal directional coupler using nonsymmetrical coupled lines in an inhomogenous dielectric is physically realizable. This work, by applying the general results, makes possible a new option in the design of directional couplers that was not previously available.

II. DIRECTIONAL COUPLERS

Background

Uniformly coupled transmission line systems can be classified into four groups according to the conductor cross section and the dielectric medium variations. These are coupled symmetrical and nonsymmetrical lines in a homogenous medium, coupled symmetrical lines in an inhomogeneous medium and coupled nonsymmetrical lines in an inhomogeneous medium. Symmetric homogeneous couplers have been investigated by Oliver [1], for the case of wire pairs, and Jones and Bolljahn [2] for the case of striplines. Their results show that for a homogeneous dielectric medium an ideal directional coupler is possible and is impedance matched at all frequencies. This type of coupler has equal port impedances and typically has a half power bandwidth of three to one for small coupling [2].

Coupled symmetrical lines in an inhomogeneous dielectric medium, such as microstriplines, have been investigated [13] and results show that the even and odd normal mode velocities must be made equal to obtain an ideal coupler. Dalley's work with non-ideal couplers utilizing broadside coupled lines indicate that directivities greater than 20db can be achieved over a relatively narrow bandwidth, approximately 20%, when care is taken in choosing the port impedances.

Nonsymmetrical lines in a homogeneous medium have been studied by Cristal [5]. Ideal matched couplers can be designed and have the same expressions for coupling and bandwidth as the symmetric case. The advantage here over the symmetrical case is that the two lines can have different impedances, allowing the coupler to also perform as a transformer. The planar structures discussed in this thesis are amenable to integrated circuit or hybrid design with their associated advantages. Sun [12] has derived the general port voltages for a pair of nonsymmetrical coupled inhomogeneous lines terminated in impedances satisfying Cristal's homogeneous case conditions for an ideal coupler.

The non ideal coupler is not matched at all frequencies. However, for symmetric coupled microstrip lines the mode velocities can be matched by using a dielectric overlay [14]. This was shown to result in an ideal coupler. Bandwidths of the order predicted by Oliver were measured for 6db and 10db couplers using two cascaded quarter wave sections. Examination of the expression for the eigenvalues of the [L] [C] or [Z] [Y] matrix characterizing the coupled distributed system reveals that for an inhomogeneous nonsymmetrical coupled line case the eigenvalues cannot be made degenerate, i.e., the mode velocities cannot be synchronized. For a proof of this refer to Appendix C.

The general properties of coupled nonsymmetrical inhomogeneous lines has been investigated by Tripathi [9], as mentioned previously.

The conditions required to produce an ideal coupler of this type have been found by examining the four port scattering matrix [10]. Synchronization of the mode velocities satisfies the scattering matrix but can be shown to be physically impossible for an inhomogenous dielectric medium, (Appendix C). Relations between the normal mode impedances (Appendix A) were also found to lead to an ideal coupler. Two distinct cases can be identified: 1. A co-directional coupler and 2. Contra-directional coupler. Figure 1 is a schematic representation of two coupled transmission lines terminated in impedances Z_1 and Z_2. A co-directional coupler will transfer power incident at port 1 to ports 3 and 4 with no power out of port 2 while a contra-directional coupler will transfer power to ports 2 and 4 with no power out of port 3. The co-directional condition between the normal mode impedances is:

$$Z_1 = Z_{c1} = Z_{\pi 1} \text{ and } Z_2 = Z_{c2} = Z_{\pi 2} \qquad (1)$$

It is easily shown that for the system being considered in this thesis, a planar inhomogeneous system, this condition cannot be satisfied. To realize such a co-directional coupler requires the addition of capacitive loading to make the capacitive coupling constant equal in magnitude but of opposite sign to the inductive coupling constant [1, 16]. This requires a non planar four line system as shown by Speciale [16]. The relation between the normal mode impedances for the contra-directional case is:

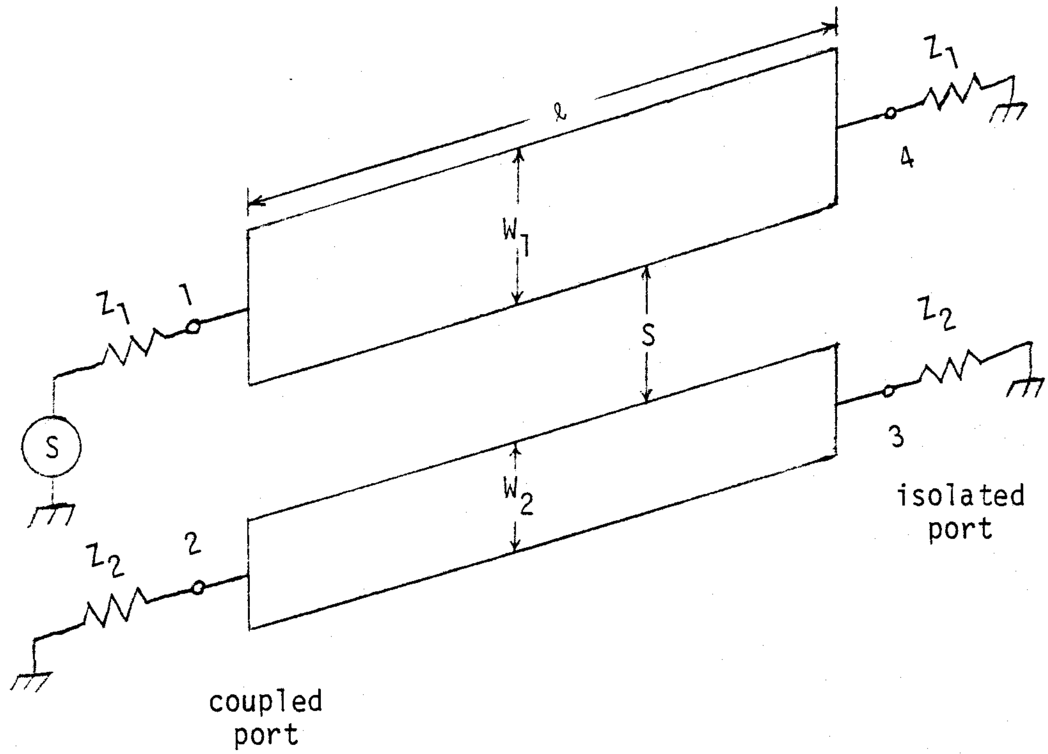

Figure 1. Directional coupler terminated in characteristic impedances Z_1,Z_2 and driven at port 1.

$$Z_1 = Z_{c1} = -Z_{\pi 1} \text{ and } Z_2 = -Z_{c2} = Z_{\pi 2} \tag{2}$$

$$\text{or } Z_1 = -Z_{c1} = Z_{\pi 1} \text{ and } Z_2 = Z_{c2} = -Z_{\pi 2} \tag{3}$$

Utilizing the expressions for normal mode impedances in terms of the per unit length line constants from Appendix A, equations (A13) - (A16) and (A21) - (A24), and using equations (A6), (A8), (A19), (A20) results in the following simple conditions for the lossless quasi-TEM case:

$$\frac{z_m}{z_1 z_2} = \frac{y_m}{y_1 y_2} \tag{4}$$

$$\text{or} \quad \frac{L_m}{L_1 L_2} = \frac{C_m}{C_1 C_2} \tag{5}$$

Where $z_{1,2}$, $y_{1,2}$, $L_{1,2}$, $C_{1,2}$ are the self impedance, admittance, inductance and capacitance per unit length of lines 1 and 2, while z_m, y_m, L_m, C_m are the mutual impedance, admittance, inductance, and capacitance per unit length of lines 1 and 2. This condition merely states that the inductive and capacitive coupling constants must be equal to realize an ideal directional coupler. An examination of the coefficient of coupling for nonsymmetrical microstrip lines reveals that the inductive coefficient of coupling is always larger than the capacitive coefficient of coupling. The latter can, however, be increased by utilizing the composite substrate or a dielectric overlay and can be made equal to or larger than the inductive coefficient of coupling.

Use of Quasi-TEM Approximation to Express Line Parameters in Terms of Capacitances

To predict the performance of a coupled line system it is necessary to know the four port parameters. For the case of coupled nonsymmetrical lines in an inhomogeneous medium Tripathi [9] has found the general four port impedance and admittance matrices and the scattering parameters [10] for the ideal directional coupler in terms of the normal mode parameters. The normal mode parameters are given in terms of the line impedances and admittances per unit length [Z] and [Y]. Therefore to be able to analyze specific coupled line configurations these matrices must be known.

If the Maxwell equations could be solved for the geometries considered, microstrip lines on composite substrates or with a dielectric overlay, then the line impedance and admittance matrices could be found. Solving the Maxwell equations for the structures of interest here is very difficult due to the boundaries involved. It has been shown that to match boundary conditions the field solution must be a hybrid [21] that can be expressed in terms of a TEM mode and a series of TE and TM modes. At low frequencies if losses are small and transverse dimensions are much less than the wavelength the waves propagating on the system may be approximated by a TEM mode. The system is then called quasi-TEM. Using this approximation the three dimensional problem is reduced to a two dimensional static problem and the impedance and admittance matrices are simplified. Since the object here is to show that the requirements for an ideal directional coupler can be met using certain specific geometries the frequency can be chosen small enough so that non TEM effects, e.g., dispersion, may be ignored. An analysis that accounts for dispersive effects and can be used to design such couplers at higher frequencies would be worthwhile after it is established that ideal couplers are realizable. For a TEM wave the impedance and admittance matrices can be written simply in terms of the line inductance and capacitance:

$$[Z] = j\omega[L] = j\omega\begin{bmatrix} L_1 & L_m \\ L_m & L_2 \end{bmatrix} \tag{6}$$

$$[Y] = j\omega[C] = j\omega\begin{bmatrix} C_1 & -C_m \\ -C_m & C_2 \end{bmatrix} \tag{7}$$

In a vacuum the phase velocity is equal to the velocity of light, c, so [L] can be expressed in terms of [C] by use of:

$$[L]\,[C] = \frac{1}{c^2} \tag{8}$$

$$\begin{bmatrix} L_1 & L_m \\ L_m & L_2 \end{bmatrix} = \frac{1}{c^2(C_{1a}C_{2a} - C_{ma}^2)} \begin{bmatrix} C_{2a} & C_{ma} \\ C_{ma} & C_{1a} \end{bmatrix} \tag{9}$$

The capacitances per unit length when the dielectric is removed are C_{1a}, C_{2a}, C_{ma}. The problem is now reduced to finding the capacitance per unit length for the given cross sectional geometry with and without the dielectrics.

Computational Method to Find Capacitance

Many methods have been devised for solving this type of field problem. The relative usefulness of some methods that might be considered will be discussed. Some that have been useful in solving strip line problems are conformal mapping, Green's function, finite difference and a variational integral method.

Conformal mapping can be used to find exact solutions for field problems when there is a high degree of symmetry, e.g., symmetric coupled lines in a homogeneous medium. Since the system here has a high degree of asymmetry this method is not feasible.

The Green's function integral equation approach attempts to directly find the charge distribution on the conductors. Then, knowing the strip potentials the capacitances can be calculated.

This method was considered since it was used successfully for the problem of asymmetric microstrip lines on a simple dielectric [11]. However, finding the Green's function would be difficult for the case of composite or overlay dielectric and the computing time to get the charge distribution would likely be relatively large.

The finite difference method is an attractive straight-forward approach used to find potentials at a large number of grid points. This is done by expanding Poisson's equation as a difference equation and iterating to find the potentials at the grid points. This method is simple and has been shown to give useful results to microstrip problems [22] but it was felt that the added complexity of composite substrates and dielectric overlays would dictate a very large number of grid points for accurate calculations which would make the computer time excessive.

The variational integral method of Yamashita [6] overcomes most of the difficulties listed above while still being relatively straight-forward so it was implemented to find the capacitances. This method uses the variational expression:

$$\frac{1}{C} = \frac{1}{Q^2} \int_S \phi(x,y)\ \rho(x,y)\ ds \tag{10}$$

where ϕ is the strip potential, Q is the total charge on the strip, $\rho(x,y)$ is the charge function, S is the x-y plane, and the direction

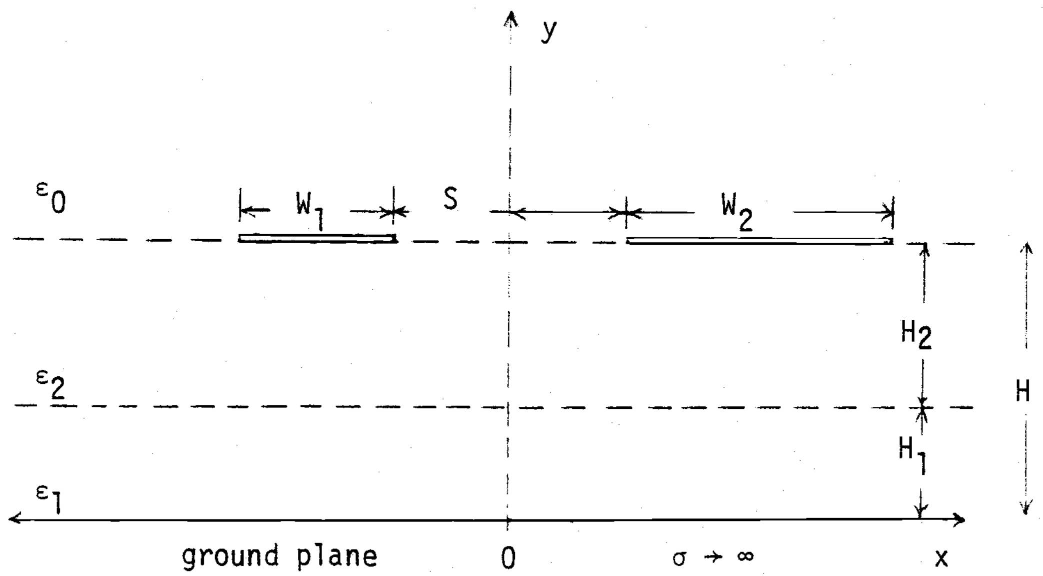

Figure 2a. Cross section of composite substrate structure

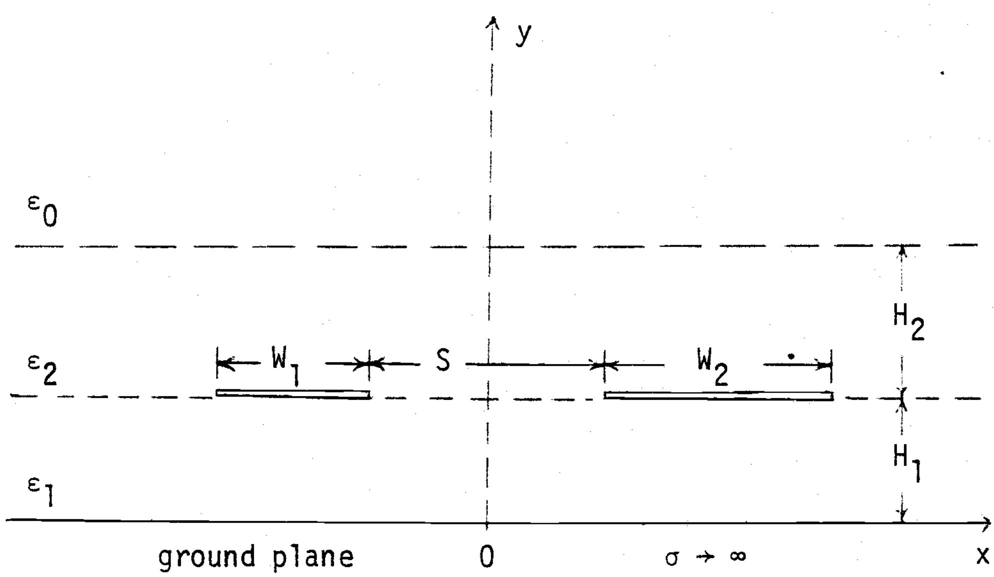

Figure 2b. Cross section of overlay dielectric structure.

of propagation is z. Refer to figure 2. The charge distribution $\rho(x,y)$ is the variational parameter and is chosen to maximize capacitance, that is, to minimize energy stored in the electric field. This results in a lower bound estimate for capacitance. For the calculation of capacitance, however, the integral is rewritten, using Parseval's theorem, in the Fourier transform domain to make the derivation of the potential function much easier. The particular functions used in the integral depend on geometry and are described in the next chapter.

III. EVALUATION OF NORMAL MODE PARAMETERS

Introduction

Coupled microstrip transmission lines with compound dielectrics (Figure 2) can be characterized in terms of the normal mode phase velocities and partial mode impedances. For the quasi-TEM, low frequency case, these parameters can be expressed in terms of the self and mutual capacitances of the coupled lines calculated with and without the dielectrics. In this chapter Fourier transforms of the charge and potential functions will be found and it will be shown that the capacitances per unit length of the coupled system can be calculated by utilizing the variation integral. These capacitances are then used to evaluate the characteristic normal mode parameters of the coupled system.

Potential Function

The potential function is found by using Poisson's equation and the specific boundary conditions (from Figure 2).

$$\nabla^2 \phi(x,y) = -\frac{1}{\varepsilon} \rho(x,y) \tag{11}$$

If the conducting strips are assumed to be infinitesimally thin then the charge can be written:

$$\rho(x,y) = f(x)\,\delta(y-H) \qquad (12)$$

where $\delta(y-H)$ is the Dirac delta function.

Define the one dimensional Fourier transform of $f(x)$ to be:

$$F(\beta) = \int_{-\infty}^{\infty} f(x)\, e^{j\beta x}\, dx \qquad (13)$$

Then the transform of the potential $\phi(x,y)$ may similarly be written

$$\Phi(\beta,y) = \int_{-\infty}^{\infty} \phi(x,y)\, e^{j\beta x}\, dx \qquad (14)$$

and Poisson's equation may be transformed to be:

$$\left(-\beta^2 + \frac{\partial^2}{\partial y^2}\right) \Phi(\beta,y) = 0 \quad , \; y \neq H \qquad (15)$$

This may now be solved separately for the composite and the overlay cases. The boundary conditions can be deduced from Figure 2.

Case 1: Composite Substrate

$$\Phi_1(\beta,0) = 0 \qquad (16)$$

$$\Phi_1(\beta,H_1) = \Phi_2(\beta,H_1) \qquad (17)$$

$$\varepsilon_1 \left[\frac{d}{dy}\,\Phi_1(\beta,y)\right|_{y=H_1} = \varepsilon_2 \left[\frac{d}{dy}\,\Phi_2(\beta,y\right|_{y=H_2} \tag{18}$$

$$\Phi_2(\beta,H) = \Phi_0(\beta,H) \tag{19}$$

$$\varepsilon_2 \left[\frac{d}{dy}\,\Phi_2(\beta,y)\right|_{y=H} = \varepsilon_0 \left[\frac{d}{dy}\,\Phi_0(\beta,y)\right|_{y=H} - F(\beta) \tag{20}$$

$$\Phi_0(\beta,y\rightarrow\infty) = 0 \tag{21}$$

Φ_0, Φ_1, Φ_2 are the transformed potentials in regions 0, 1, and 2 respectively. In the regions $0 \le y \le H_1$ and $H_1 \le y \le H$ the solutions are linear combinations of $e^{-\beta y}$ and $e^{\beta y}$. In the region $y \ge H$ the solution is of the form $e^{-|\beta|y}$. Writing these solutions and applying the above boundary conditions results in a set of simultaneous equations which may be solved for the coefficients in the three regions. Using these coefficients $\Phi(\beta,H)$ may be written:

$$\Phi(\beta,H) = \frac{F(\beta)}{\varepsilon_0}\left[\frac{\varepsilon_1 \coth(\beta H_1) + \varepsilon_2 \coth(\beta H_2)}{\coth(\beta H_2)\left[\varepsilon_2|\beta| + \varepsilon_1\varepsilon_2\beta \coth(\beta H_1)\right] + \varepsilon_1|\beta| \coth(\beta H_1) + \beta\varepsilon_2^{\,2}}\right] \tag{22}$$

For later simplification define $\Phi(\beta,H) = \frac{1}{\varepsilon_0} F(\beta)\, G(\beta)$ (23)

Case 2: Dielectric Overlay

The boundary conditions are:

$$\Phi_1(\beta,0) = 0 \tag{24}$$

$$\Phi_1(\beta,H_1) = \Phi_2(\beta,H_1) \tag{25}$$

$$\varepsilon_1 \left[\frac{d}{dy}\Phi_1(\beta,y)\right|_{y=H_1} = \varepsilon_2 \left[\frac{d}{dy}\Phi_2(\beta,y)\right|_{y=H_1} - F(\beta) \tag{26}$$

$$\Phi_2(\beta,H) = \Phi_0(\beta,H) \tag{27}$$

$$\varepsilon_2 \left[\frac{d}{dy}\Phi_2(\beta,y)\right|_{y=H} = \varepsilon_0 \left[\frac{d}{dy}\Phi_0(\beta,y)\right|_{y=H} \tag{28}$$

$$\Phi_0(\beta,y\to\infty) = 0 \tag{29}$$

The solutions to the overlay geometry are similar to those of the composite geometry and the coefficients are obtained in the same manner. The result is:

$$\Phi(\beta,H) = \frac{F(\beta)}{\varepsilon_0} \frac{\dfrac{|\beta|}{\varepsilon_2\beta} + \coth(\beta H_2)}{\varepsilon_1\beta \coth(\beta H_1)\left[\coth(\beta H_2) + \dfrac{|\beta|}{\varepsilon_2\beta}\right] + |\beta| \coth(\beta H_2) + \beta\varepsilon_2} \tag{30}$$

As before define $\Phi(\beta,H) = \dfrac{1}{\varepsilon_0} F(\beta)\, G(\beta)$ (31)

Both solutions of the transformed Poisson equation were compared with Yamashita's result after taking the limit as $H \to 0$ and $\varepsilon_2 \to 1$ and were the same.

The Variational Integral in the Transform Domain

To avoid the necessity of finding the inverse transforms of $\Phi(\beta,H)$ for the two cases the variational integral, equation (10), can be transformed into the Fourier transform domain by use of Parseval's theorem. The resulting integral is:

$$\frac{1}{C} = \frac{1}{2\pi Q^2} \int_{-\infty}^{\infty} |F(\beta)\ \Phi(\beta,H)|\ d\beta \qquad (32)$$

$$\text{or} \quad \frac{1}{C} = \frac{1}{\pi \varepsilon_0 Q^2} \int_{0}^{\infty} |F(\beta)|^2\ G(\beta)\ d\beta \qquad (33)$$

Equation (33) follows from the definition of $G(\beta)$ and the fact that $|F(\beta)|^2$ and $G(\beta)$ must be even functions.

The Charge Function, $F(\beta)$

The transform of the charge function is the variable used to maximize capacitance so the function $f(x)$ that gives the largest value is the best. Yamashita [6] had good results using polynomial functions that increased steeply at the strip edges so for this work a function was chosen that could increase rapidly near the edges.

$$f(x) = \begin{cases} A_1 \cosh \; K_1 \left[\left(x - \dfrac{S+W_1D_1}{2}\right)\right] , & - \; W_1 + \dfrac{S}{2} \leq x \leq - \dfrac{S}{2} \\ A_2 \cosh \; K_2 \left[\left(x - \dfrac{S+W_2D_2}{2}\right)\right] , & \dfrac{S}{2} \leq x \leq \dfrac{S}{2} + W_2 \end{cases} \tag{34}$$

A_1 and A_2 are amplitudes, $K_{1,2}$ are constants and from Figure 2; S is the spacing between strips, $W_{1,2}$ are strip widths and $D_{1,2}$ determine the position of minimum charge on the strips. In addition, A_2 is chosen so that both strips have the same total charge by solving the following equation.

$$Q = \int_{-W_1+\frac{S}{2}}^{-\frac{S}{2}} f_1(x)\, dx = \int_{\frac{S}{2}}^{W_2+\frac{S}{2}} f_2(x)\, dx \tag{35}$$

The charge function Fourier transforms are: (36)

$$F_1(\beta) = A_1 e^{-j\beta\frac{S}{2}} \left[\frac{e^{\frac{-K_1W_1}{2}}}{K_1-j\beta} \left(e^{W_1(K_1-j\beta)} - 1\right) - \frac{e^{\frac{K_1W_1}{2}}}{K_1+j\beta} \left(e^{-W_1(K_1+j\beta)} - 1\right)\right]$$

$$F_2(\beta) = A_2 e^{j\beta\frac{S}{2}} \left[\frac{e^{\frac{-K_2W_2}{2}}}{K_2+j\beta} \left(e^{W_2(K_2-j\beta)} - 1\right) - \frac{e^{\frac{K_2W_2}{2}}}{K_2-j\beta} \left(e^{-W_2(K_2-j\beta)} - 1\right)\right]$$

(37)

Transmission Line Capacitances in Terms of Calculated Capacitances

The capacitances calculated from the variational integral can be related to the line capacitances through the definition of the capacitance matrix and electrostatic energy.

$$\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} C_1 & -C_m \\ -C_m & C_2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{38}$$

$$W = [Q]\,[V] \tag{39}$$

where $Q_{1,2}$ and $V_{1,2}$ are the charge and voltage on strips 1 and 2, and W is the system electrostatic energy.

The calculated capacitances, C_i, can now be written in terms of W.

$$\frac{1}{C_i} = \frac{W}{Q^2} \tag{40}$$

Using equations (38) and (39) to substitute for W in equation 40 gives the desired result.

$$\frac{1}{C_i} = \frac{1}{Q^2}\,[Q]\,[C]^{-1}\,[Q] \tag{41}$$

$$\frac{1}{C_i} = \frac{Q_1^2\, C_2 + 2Q_1Q_2C_m + Q_2^2\, C_1}{Q^2(C_1C_2 - C_m^2)} \tag{42}$$

Since the lines are in general not symmetric, $C_1 \neq C_2$ and three independent calculations are required to be able to find C_1, C_2, and C_m.

Case a: $Q_1 = 0,\ Q_2 = 0;\ F(\beta) = F_1(\beta)$ (43)

$$\frac{1}{C_a} = \frac{C_2}{C_1C_2 - C_m^2} \tag{44}$$

Case b: $Q_1 = 0,\ Q_2 = Q;\ F(\beta) = F_2(\beta)$ (45)

$$\frac{1}{C_b} = \frac{C_1}{C_1C_2 - C_m^2} \tag{46}$$

Case c: $Q_1 = Q_2 = Q;\ F(\beta) = F_1(\beta) + F_2(\beta)$ (47)

$$\frac{1}{C_c} = \frac{C_1 + C_2 + 2C_m}{C_1C_2 - C_m^2} \tag{48}$$

These equations for C_a, C_b, C_c in terms of C_1, C_2, C_m may be solved to find the line capacitances C_1, C_2, C_m in terms of C_a, C_b, C_c.

$$C_1 = \frac{C_b}{\frac{C_b}{C_a} - \frac{1}{4}\left(\frac{C_b}{C_c} - \frac{C_b}{C_a} - 1\right)} \tag{49}$$

$$C_2 = \frac{C_b}{C_a} C_1 \tag{50}$$

$$C_m = \frac{C_1}{2}\left[1 + C_b\left(\frac{1}{C_a} - \frac{1}{C_c}\right)\right] \tag{51}$$

When the line capacitances are known for the system with and without dielectric present, i.e., the inductance and capacitance matrices are known, then the normal mode parameters can be found from equations (A19) - (A24) in Appendix A.

IV. NUMERICAL RESULTS

Introduction

A Fortran program was written to solve numerically the variational integral to find the line capacitances and calculate the normal mode parameters. Usually five to twenty different geometries were analyzed for each run of the program so that a range of parameters was available to find conditions where inductive and capacitive couplings were equal. Successive runs were used to narrow down the range of couplings calculated until k_L and k_C were very close. For the overlay case H_2 was varied and for the composite case both H_1 and H_2 were varied so that total height, H, stayed constant.

Optimization of the Charge Function, F(β)

The criteria for choosing the best charge function is that the calculated capcitances be maximized. There are four variables in F(β) that are used to maximize the C_i: $K_{1,2}$ and $D_{1,2}$. The variables $K_{1,2}$ multiply the argument of the hyperbolic cosine function controlling the steepness of f(x) near the strip edges. The terms $D_{1,2}$ are used to shift the charge minimum from the strip center. These are always set to one for the calculation of C_a and C_b since only one strip has charge and therefore there is no reason to have an asymmetric charge distribution. This was verified by the computer by making the distribution asymmetric and noticing that C_a, C_b had decreased. To calculate capacitance when both strips have charge, C_c, $D_{1,2}$ can be chosen to be less than one to maximize C_c. This effect was also verified by the computer. Even though the function $F(\beta)$ could have been easily optimized for this last case the optimum distribution for the first two calculations, $C_{a,b}$, was not easily found. The matrix capacitances depend strongly on C_c and less strongly on $C_{a,b}$ so if C_c was optimized by choosing appropriate $D_{1,2}$ and $C_{a,b}$ were not quite as good estimates of the real values then the resulting capacitances C_1, C_2, C_m could become meaningless. The best compromise was to choose $D_1 = D_2 = 1$ for all cases. This had the effect of maximizing C_1, C_2, C_m which were compared, for some test cases for which results were available, to results found independently using a different method [11]. The best values of K_1 and K_2 were found to depend on the strip width as $K = 4/W$. Tables 1 and 2 contain comparisons of mode parameters and capacitances calculated using this and a Green's function method for the case of a single substrate material and no overlay. The intermediate capacitances, $C_{a,b}$, for the cases when only one strip has charge were found to agree closely with previously published results for single lines. Capacitances calculated using the variational method described in this paper, setting $\varepsilon_2 = \varepsilon_0$, and the corresponding coupling constants are presented in Table 1. Capacitances for the same structure calculated using a different method by Chang and Tripathi [11] are included for comparison. There are relatively large differences between corresponding values of capacitance, the largest being 18% for the C_{ma} results. This results in errors in coupling constant, k, of the order of 10%. Also it may be noticed that all the values calculated here are lower than their counterpart. This is reasonable since the variational integral used here results in a lower bound estimate [6]. However, the ratios of coupling constant with $\varepsilon = \varepsilon_0$ to that when $\varepsilon = 10$ are very close, only 2.5% difference. Since the condition of interest can be expressed as $k_L/k_C = 1$ the trial function $F(\beta)$ may be considered to be near optimum for the purposes of showing that the ideal coupler condition can be realized.

Normal mode parameters calculated from the capacitances in Table 1 are presented in Table 2. The percent differences are also shown and are a maximum of 10%.

A variety of cases of ideal directional couplers have been found for both overlay and composite dielectrics. The curves in Figures 3 and 4 can be used to choose H_2 or ε_2 given H_1, ε_1 and ε_2 or H_2.

Dielectric Overlay Ideal Coupler Results

Data for the case of overlay dielectric, Figure 3, has some interesting and useful aspects in addition to describing ideal coupler conditions. For each curve the impedances, $Z_1 = -Z_{c1} = Z_{\pi 1}$ and $Z_2 = Z_{c2} = -Z_{\pi 2}$, are constant over the range. This is a potentially

Table 1: Comparison of calculated capacitances for a single dielectric using the variational integral of this thesis and a Green's function integral [11] for: $W_1 = 2$, $W_2 = 1$, $S = 0.2$, $H = H_1 = 1$, $\varepsilon = \varepsilon_1 = 10\ \varepsilon_0$, and $H_2 = 0$. Capacitances are in pf/cm.

	C_1	C_m	C_2	K_c	C_{1a}	C_{ma}	C_{2a}	K_L
Variational Integral	202	-58	297	0.235	33	-13.5	44.4	0.355
Green's Function	183	-49	276	0.218	29	-11	41	0.321

$\varepsilon = \varepsilon_0$

Table 2: Comparison of normal mode parameters calculated by the variational integral of this thesis and by a Green's function integral for: $W_1 = 1$, $W_2 = 2$, $S = 0.2$, $H = H_1 = 1$, $\varepsilon = \varepsilon_1 = 10$, $H_2 = 0$. Impedances Z_{c1}, $Z_{\pi 1,2}$ are in ohms and c is the free space velocity of light.

	V_c	V_π	R_c	R_π	Z_{c1}	Z_{c2}	$Z_{\pi 1}$	$Z_{\pi 2}$
Green's Function	0.36c	0.41c	1.06	-0.55	65.5	38.1	34.4	20
Variational Integral	0.37c	0.42c	1.05	-0.55	68	39.2	38	22
Δ%	2.7	2.4	1	0	3.7	2.8	9.5	9.1

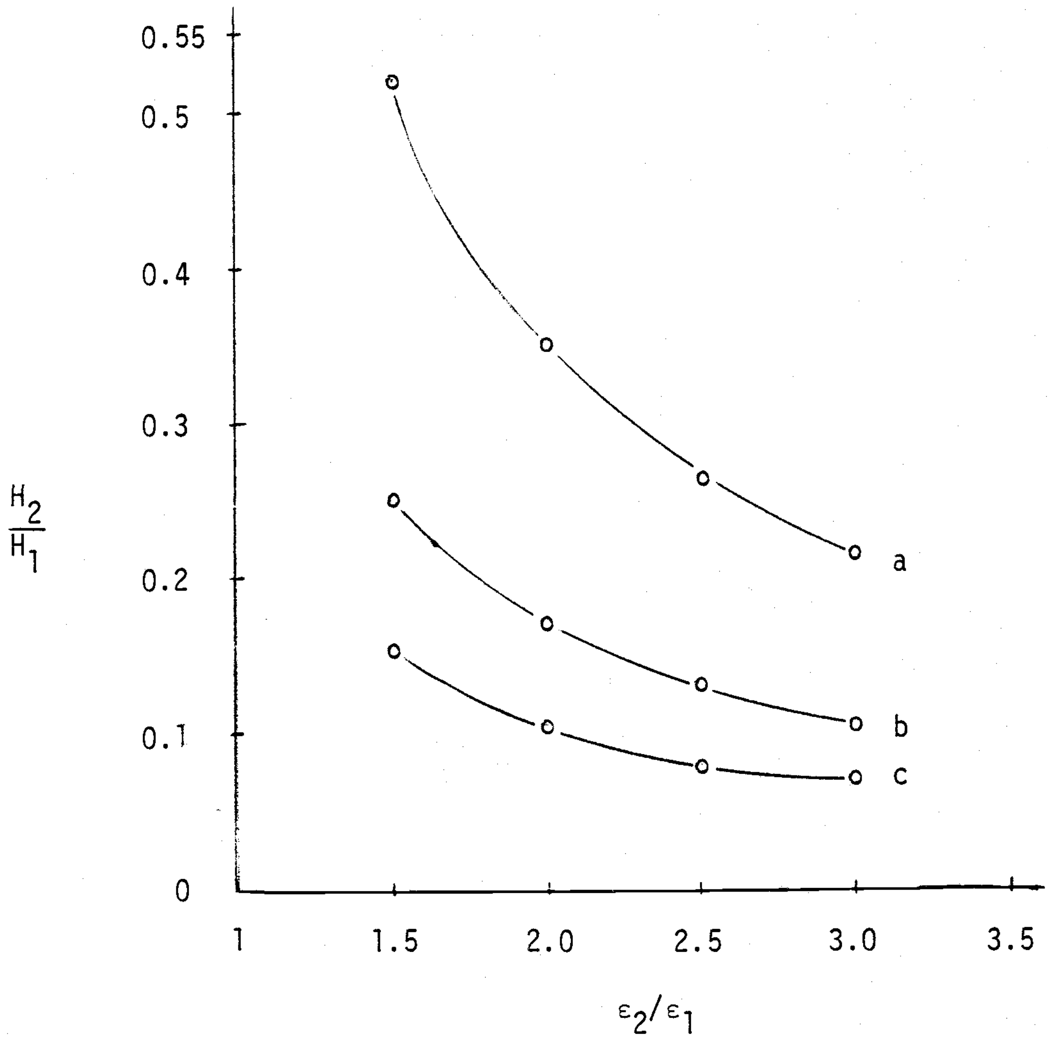

Figure 3. Some solutions for H_2 and ε_2 required to satisfy ideal nonsymmetrical directional coupler conditions using a dielectric overlay.

(a) $Z_1 = 19\Omega$, $Z_2 = 29\Omega$, $k = 0.207$

$W_1/H_1 = 4$, $W_2/H_1 = 2$, $S/H_1 = 0.4$, $H_1 = 0.5$

(b) $Z_1 = 29\Omega$, $Z_2 = 43\Omega$, $k = 0.321$

$W_1/H_1 = 2$, $W_2/H_1 = 1$, $S/H_1 = 0.2$, $H_1 = 1.0$

(c) $Z_1 = 30\Omega$, $Z_2 = 55\Omega$, $k = 0.381$

$W_1/H_1 = 2$, $W_2/H_1 = 0.5$, $S/H_1 = 0.1$, $H_1 = 2.0$

very useful property that would allow a designer to choose the impedance ratio and either Z_1 or Z_2, find the appropriate curve and then choose H_2 and ε_2 freely without needing to consider changing impedances. Also it is obvious that the coupling constant, k, does not vary along each curve. From S_{12} the maximum coupling can be found to depend on the ratio R_π/R_c (Appendix B). In all cases the coupling calculated this way was very close to k. This result means that the nonsymmetrical inhomogeneous coupler has approximately the same maximum coupling as the symmetrical homogeneous type.

Composite Substrate Ideal Coupler Results

The composite substrate case, Figure 4, also has some very interesting properties. The most striking feature of these curves is that for a given $\varepsilon_1, \varepsilon_2$ with $\varepsilon_2 > \varepsilon_1$ there is more than one H_1/H_2 that satisfies the ideal coupler condition. This may be easily understood by noting that for each curve $H = H_1 + H_2$ is a constant. If $H_1 > H_2$ then H_2 can be adjusted to set $k_L = k_C$ since intuitively adding a thin layer of high permitivity material will increase coupling faster than it will increase capacitance to the ground plane. If $H_2 > H_1$ then we have just the opposite situation and H_1 is used to reduce the self capacitance while having relatively little affect on coupling. Since the effective dielectric constant is being altered when H_1/H_2 is adjusted the impedances are expected to change and indeed they do.

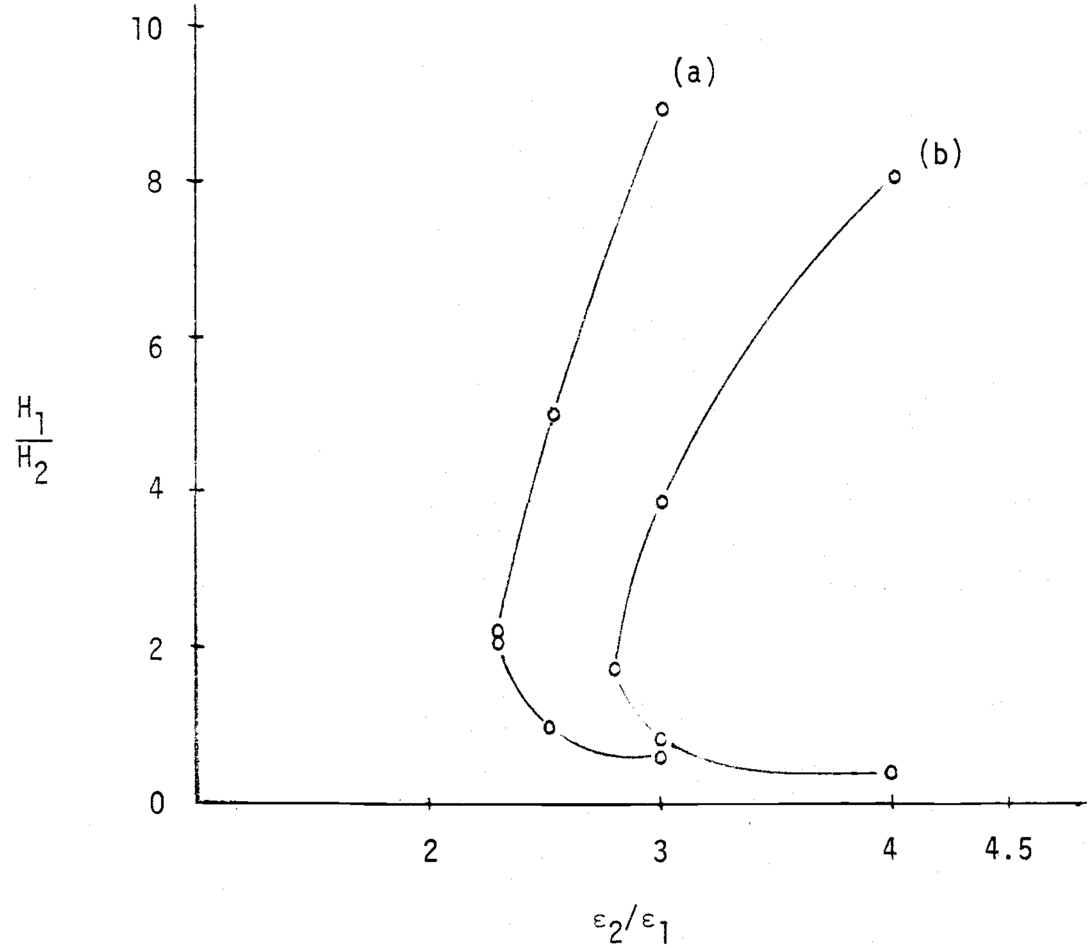

Figure 4. Some solutions for H_2 and ε_2 required to satisfy ideal nonsymmetrical directional coupler conditions using a composite substrate. ($\varepsilon_1 = 10$)

(a) $W_1/H = 2$, $W_2/H = 0.5$, $S/H = 0.05$, $H = 2$
$k = 0.406$ coupling $\cong$ 8db

(b) $W_1/H = 2$, $W_2/H = 1$, $S/H = 0.2$, $H = 1$
$k = 0.321$ coupling $\cong$ 10db

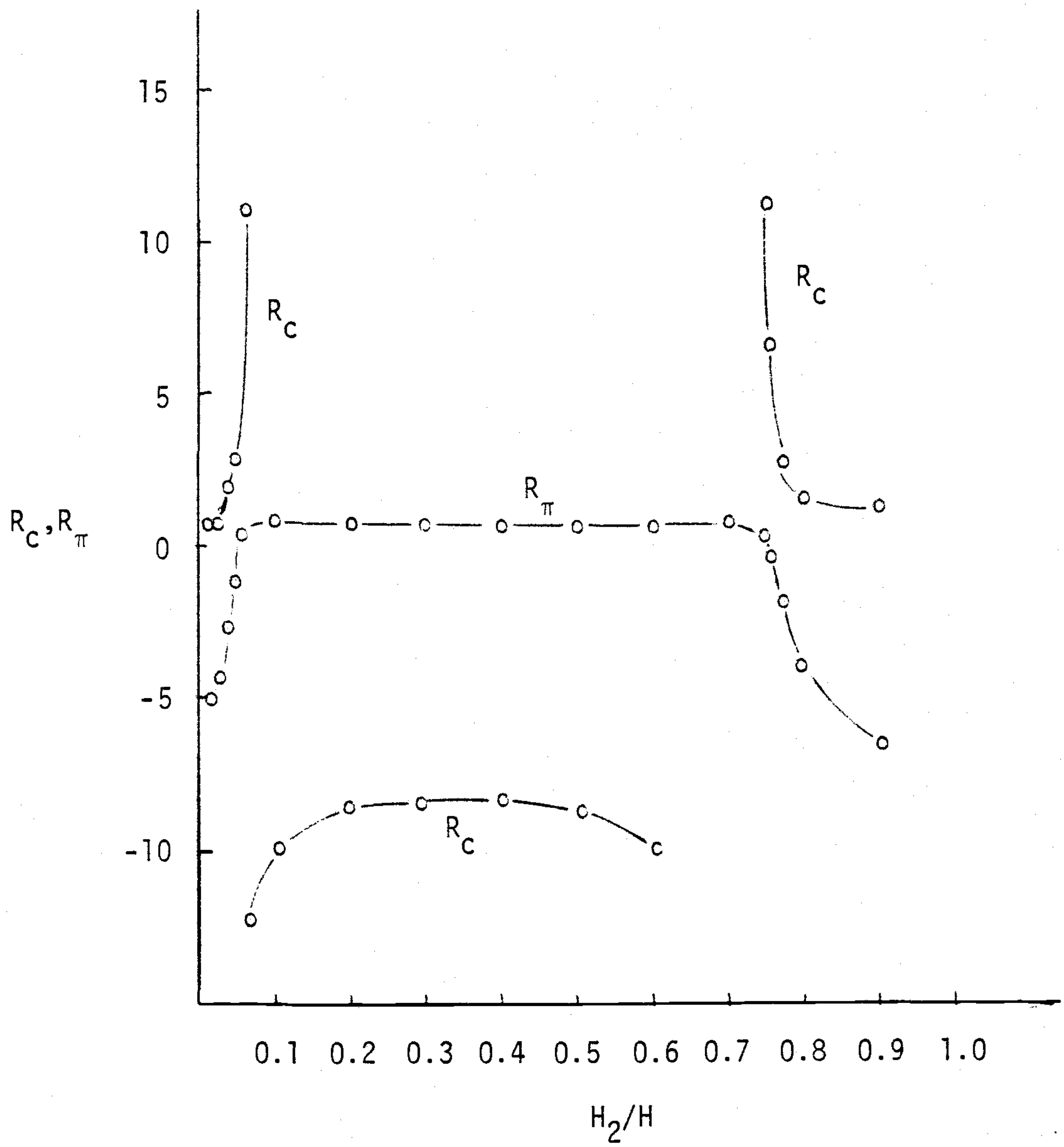

Figure 5. Mode voltage ratios R_c, R_π for a composite substrate versus normalized dielectric height H_2/H.

$H = 2,\ \varepsilon_1 = 2.5,\ \varepsilon_2 = 10,\ W_2 = 8,\ W_1 = 1$

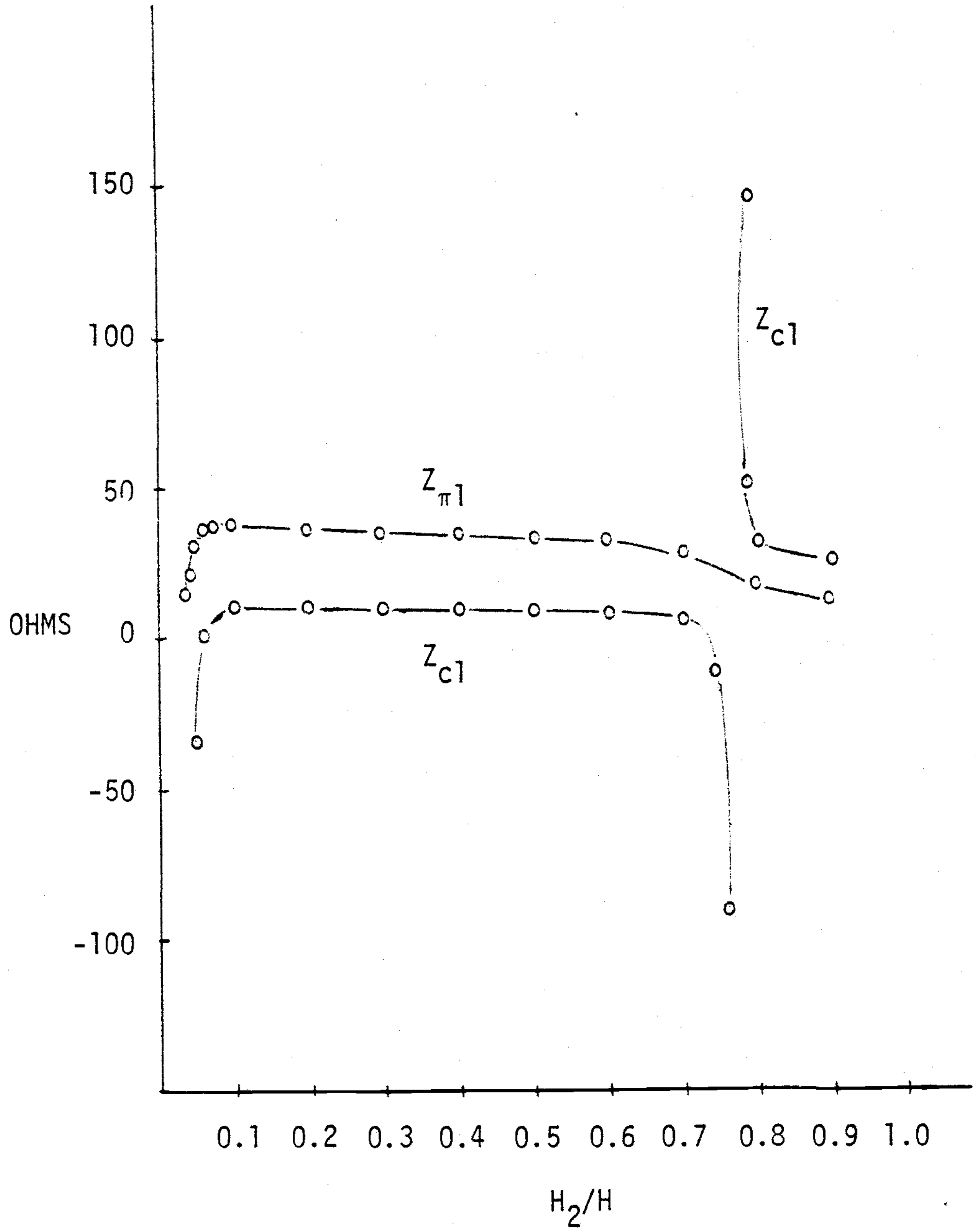

Figure 6. Normal mode impedances, Z_{c1}, $Z_{\pi 1}$, versus normalized H_2 for a composite substrate.

$W_1/W_2 = 8$, $W_2 = 1$, $H = 2$, $\varepsilon_1 = 2.5$, $\varepsilon_2 = 10$

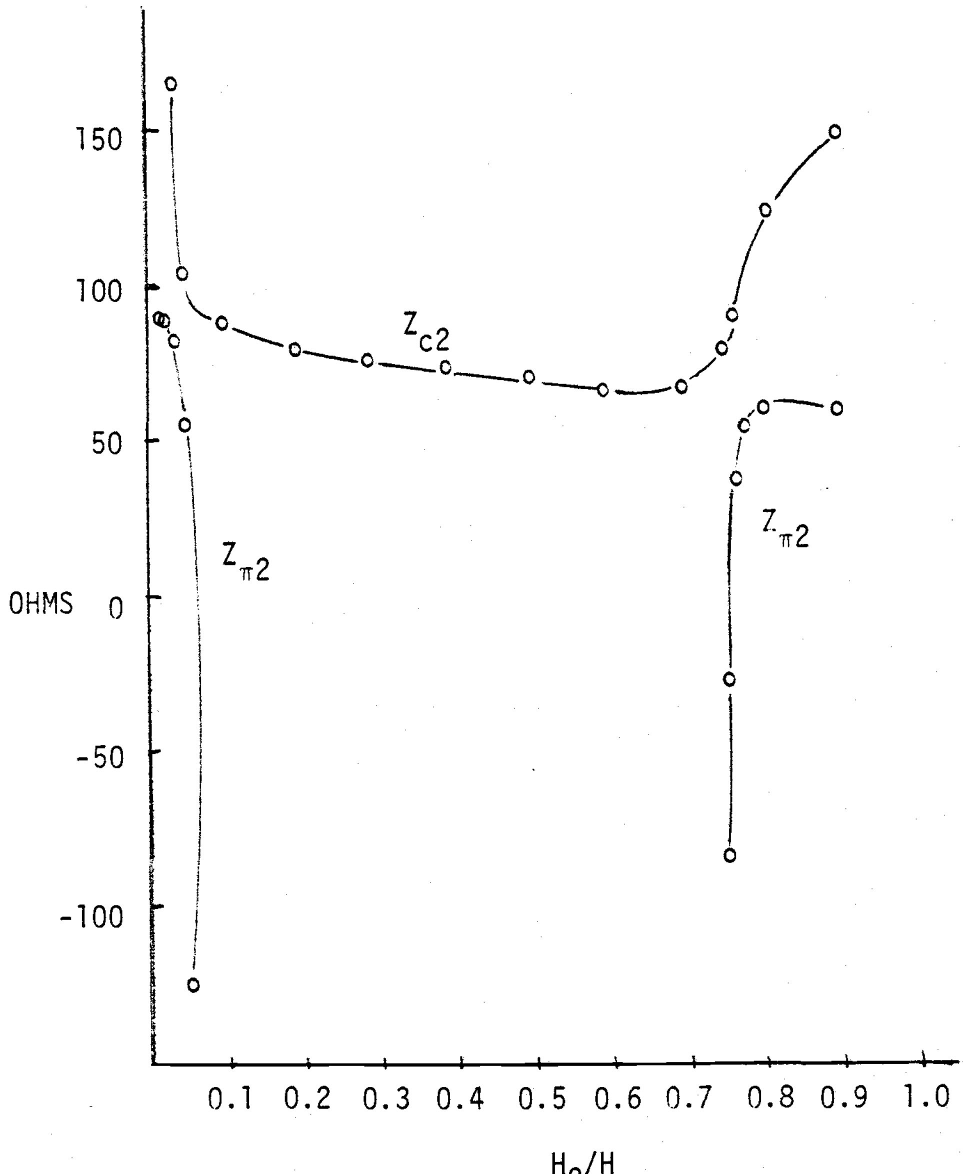

Figure 7. Normal mode impedances, Z_{c2}, $Z_{\pi2}$, versus normalized H_2 for a composite substrate.

$W_1/W_2 = 8$, $W_2 = 1$, $H = 2$, $\varepsilon_1 = 2.5$, $\varepsilon_2 = 10$

It is interesting, and not surprising, that there is a minimum $\varepsilon_2/\varepsilon_1$ to allow $k_L = k_C$ that depends on total height, H, and strip separation S. As height decreases the minimum $\varepsilon_2/\varepsilon_1$ will increase and as separation decreases the minimum $\varepsilon_2/\varepsilon_1$ should decrease also. There is not enough information in Figure 4 to evaluate each affect independently but curve 4a has a smaller minimum $\varepsilon_2/\varepsilon_1$ than curve 4b and also has larger dielectric height and smaller strip spacing.

Although the magnitudes change, the impedance ratios along the curves are nearly constant, within the resolution of the calculated parameters.

The parameters R_c, R_π, $Z_{c,\pi 1}$, $Z_{c,\pi 2}$ as a function of H_2/H are plotted in Figures 5, 6, 7 for a composite type geometry. From the expression for the mode voltage ratios (Appendix A) it is seen that R_π will be zero when either $C_{ma}/C_{1a} = C_m/C_1$ or $C_{ma}/C_{2a} = C_m/C_2$ and R_c is singular when $C_{ma}/C_{2a} = C_m/C_2$. So the zero crossing of R_π is not required to be coincident with the singularity of R_c. These voltage ratios are important design parameters since coupling depends on R_π/R_c and the impedance ratios depend on $R_c R_\pi$. Bandwidth can be found from S_{12} and is found to depend on the ratio R_π/R_c also. The coupled power is plotted as a function of line length for some of the cases found here in Figure 8. The resulting bandwidth is found to be close to that for a homogeneous coupler, approximately 3:1.

Normal mode parameters corresponding to the equal coupling cases investigated are tabulated in Table 3. The impedances are seen to vary widely depending on strip width, dielectric height, permitivity and strip spacing. Coupling only varies from 8db to 14db but this could easily be increased by increasing the strip separation.

From a practical standpoint the large variations in impedance near equal coupling points requires that the dielectric thickness by closely controlled.

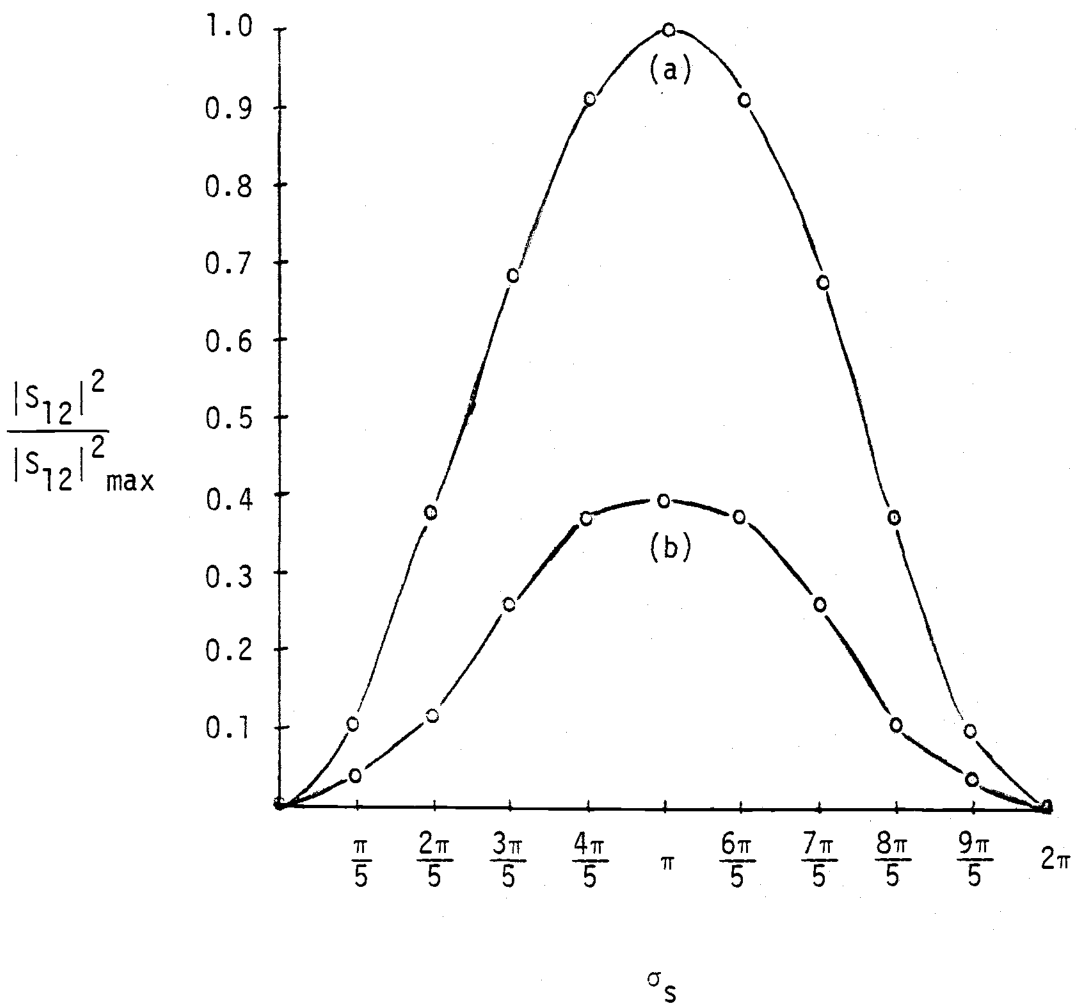

Figure 8. Normalized coupled power magnitude.

$|S_{12}|^2/|S_{12}|^2_{max}$ versus line electrical length, σ_s.

(a) 10db coupler, $|S_{12}|^2_{max} = 0.1$, $R_c = 7.5$, $R_\pi = 0.2$

(b) 14db coupler normalized to 10db coupler

$|S_{12}|^2_{max} = 0.39$, $R_c = 12.8$, $R_\pi = 0.13$

Table 3: Normal mode parameters for the composite coupler cases in Figure 4.

Figure	H_1/H_2	$\varepsilon_2/\varepsilon_1$	Z_1	Z_2	R_π	R_c	V_c/C	V_π/C
4a	9	3	28	53	0.23	5.33	0.304	0.309
	5	2.5	27	50	—	—	0.310	0.316
	2.4	2.3	26	49	0.28	6.5	0.283	0.286
	2.1	2.3	26	49	0.28	6.5	0.283	0.286
	1	2.5	23	45	0.09	3.7	0.268	0.271
	0.58	3	22	41	0.32	7.1	0.235	0.238
4b	8.1	4	28	40	0.25	8	0.329	0.331
	3.9	3	27	39	0.20	7.5	0.320	0.321
	1.9	2.7	25	36	0.13	5.4	0.303	0.304
	0.8	3	24	33	0.19	7.1	0.274	0.275
	0.4	4	20	28	0.20	7.5	0.233	0.234

V. CONCLUSION AND SUGGESTIONS FOR FURTHER WORK

Conclusion

It has been shown that in a lossless, quasi-TEM system, planar ideal directional couplers utilizing nonsymmetrical coupled lines in an inhomogeneous dielectric medium are possible by using either a dielectric overlay or a composite substrate. A variety of specific cases were presented that indicate that coupling, impedances, and impedance transformation ratios can be easily controlled by strip widths and spacings and dielectric heights and permitivity. With the technique shown in this thesis it will be possible to generate sets of design curves like Figures 3 and 4 which will, for example, allow a particular design to be a matter of choosing the desired coupling and impedances and then finding on the curve the required dielectric thicknesses and permitivities.

The variational integral method in the Fourier transform domain was shown to be particularly efficient and easy to use for this type of planar multiple dielectric structures. For example, finding the potential function in the transform domain is much easier than finding the Green's function directly for these problems considering the multitude of boundary conditions to satisfy.

Suggestions for Further Work

The numerical calculations done are sufficient to verify the physical behavior of these coupled line systems but for actual design work the accuracy should be improved. This can be done by optimizing the trial function to agree with known capacitance values for some special cases. It should be possible to get very good agreement with published results. To give more weight to such calculations it is also possible to calculate an upper bound on capacitance using a similar variational technique as shown by Araki and Naito [20].

Perhaps the most important step at this point is to verify experimentally these analytical results. The design could begin with more accurate calculations or could be guided by the data presented here if some empirical fine tuning is allowed. Since this theory is based on lossless quasi-TEM propagation the frequency should be chosen low enough to minimize high frequency effects.

Since the intent of this thesis was to use the variational integral approach to show that overlay and composite substrate structures can be made to produce ideal couplers no effort was made to go beyond the limitations of a quasi-TEM system. Now that the feasibility has been demonstrated it would be very useful from a practical standpoint to know how these devices act at higher frequencies, where the quasi-TEM approximation is not valid. In particular dispersion effects are not known and could have a significant effect on bandwidth and directivity.

With two coupled lines it is difficult to achieve large values of coupling so it has been necessary to use three or more lines for 3db couplers [18, 19]. The use of a composite substrate or dielectric overlay, as illustrated here for two lines, should also improve the performance of these couplers. The variational integral technique could easily handle the added complexity. For the interdigitated couplers referenced, alternate lines are connected together with jumper wires so it would only be necessary to split the charge function into two pieces before taking the Fourier transform. The rest of the procedure would remain the same.

As it stands now a coupler design would result from using graphs to first narrow the range of dielectric parameters and cross sectional geometries and then use the analysis technique shown here to get precise numerical values to build a prototype. A much more convenient scheme is to have a synthesis procedure which could accept certain requirements, e.g., coupling and impedance transformation, and arrive at several possible geometrical configurations from which a convenient one may be chosen. Conceivably, such a procedure could be used for more general microstrip applications since the variational technique is quite general. The obvious obstacle is the requirement for many computer iterations to optimize a particular design. Even with a relatively efficient way to calculate the mode parameters a large amount of computer time might still be required.

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APPENDICES

APPENDIX A: COUPLED LINE THEORY

A brief outline of coupled transmission line theory and some of the important results [9] is presented here.

Begin with the transmission line equations:

$$\frac{d}{dx}[V] = -[Z]\,[I] \tag{A1}$$

$$\frac{d}{dx}[I] = -[Y]\,[V] \tag{A2}$$

[V] and [I] are the two dimensional voltage and current vectors and [Z], [Y] are 2x2 impedance and admittance matrices.

$$[Z] = \begin{bmatrix} z_1 & z_m \\ z_m & z_2 \end{bmatrix} \qquad [Y] = \begin{bmatrix} y_1 & y_m \\ y_m & y_2 \end{bmatrix}$$

The voltages and currents, when the lines are uniformly coupled, are the solutions of:

$$\frac{d^2}{dx^2}[V] - [Z]\,[Y]\,[V] = 0 \tag{A3}$$

$$\frac{d^2}{dx^2}[I] - [Y]\,[Z]\,[I] = 0 \tag{A4}$$

The characteristic equation

$$\gamma^2 - [Z]\,[Y] = 0 \tag{A5}$$

can be solved to get the normal mode propagation constants:

$$\gamma_{c,\pi} = \frac{\omega}{\sqrt{2}} \left[(Z_1 Y_1 + Z_2 Y_2 - 2 Z_m Y_m) \pm \sqrt{(Z_1 Y_1 - Z_2 Y_2)^2 + 4(Z_m Y_1 - Z_2 Y_m)\,(Z_m Y_2 - Z_1 Y_m)} \right]^{1/2} \tag{A6}$$

From the voltage eigenvectors are found the mode voltage ratios,

$$R_{c,\pi} \triangleq \left(\frac{V_2}{V_1} \right)_{c,\pi} . \tag{A7}$$

$$R_{c,\pi} = \frac{(Z_2 Y_2 - Z_1 Y_1) \pm \sqrt{(Z_2 Y_2 - Z_1 Y_1)^2 + 4(Z_m Y_m - Z_2 Y_m)\,(Z_m Y_2 - Z_1 Y_m)}}{2(Z_m Y_2 - Z_1 Y_m)} \tag{A8}$$

The general solution for the voltages on the two lines is then

$$v_1 = A_1\, e^{-\gamma_c x} + A_2\, e^{\gamma_c x} + A_3\, e^{-\gamma_\pi x} + A_4\, e^{\gamma_\pi x} \tag{A9}$$

$$v_2 = A_1 R_c\, e^{-\gamma_c x} + A_2 R_c\, e^{\gamma_c x} + A_3 R_\pi\, e^{-\gamma_\pi x} + A_4 R_\pi\, e^{\gamma_\pi x} \tag{A10}$$

Where A_1, A_2, A_3, A_4 are amplitude coefficients. The solutions for currents are found by using equations (A9), (A10) in equation (A1). They are

$$i_1 = A_1 Y_{c1}\, e^{-\gamma_c x} - A_2 Y_{c1}\, e^{\gamma_c x} + A_3 Y_{\pi 1}\, e^{-\gamma_\pi x} - A_4 Y_{\pi 1}\, e^{\gamma_\pi x} \tag{A11}$$

$$i_2 = A_1 R_c Y_{c2}\, e^{-\gamma_c x} - A_2 R_c Y_{c2}\, e^{\gamma_c x} + A_3 Y_{\pi 2} R_\pi\, e^{-\gamma_\pi x} - A_4 R_\pi Y_{\pi 2}\, e^{\gamma_\pi x} \tag{A12}$$

Partial mode admittances Y_{c1}, Y_{c2}, $Y_{\pi 1}$, $Y_{\pi 2}$ are the ratios of current to voltage on a given line when the corresponding mode is excited. They should not be confused with uncoupled normal mode wave admittances which are always positive real for lossless lines.

Partial mode admittances and impedances are given below.

$$Y_{c1} = \frac{1}{Z_{c1}} = \gamma_c \frac{z_2 - R_c z_m}{z_1 z_2 - z_m^2} \tag{A13}$$

$$Y_{c2} = \frac{1}{Z_{c2}} = \frac{\gamma_c}{R_c} \frac{z_1 R_c - z_m}{z_1 z_2 - z_m^2} \tag{A14}$$

$$Y_{\pi 1} = \frac{1}{Z_{\pi 1}} = \gamma_\pi \frac{z_2 - z_m R_\pi}{z_1 z_2 - z_m^2} \tag{A15}$$

$$Y_{\pi 2} = \frac{1}{Z_{\pi 2}} = \frac{\gamma_\pi}{R_\pi} \frac{z_1 R_\pi - z_m}{z_1 z_2 - z_m^2} \tag{A16}$$

Mode Parameters for the Lossless Quasi-TEM Case

Equations (A6), (A8), (A13) - (A16) can be used to express the normal mode parameters in terms of self and mutual capacitances with and without dielectric present by substituting for z_1, z_2, z_m and y_1, y_2, y_m.

$$[Z] = j\omega[L] = j\omega[C_a]^{-1}\frac{1}{c^2} = \frac{j^{\omega}}{c^2\Delta a}\begin{bmatrix} C_{2a} & C_{ma} \\ C_{ma} & C_{1a} \end{bmatrix} \tag{A17}$$

$[C_a]$ is the matrix of self and mutual capacitance per unit length of the two lines with the dielectric removed and c is the velocity of light, and Δa is the determinant of the capacitance matrix.

$$[Y] = j\omega[C] = j\omega\begin{bmatrix} C_1 & -C_m \\ -C_m & C_2 \end{bmatrix} \tag{A18}$$

The normal mode parameters can now be written:

Mode propagation velocities

$$V_{c,\pi} = \frac{\omega}{\beta_{c,\pi}} = \sqrt{2\Delta a}\; c \left[C_1C_{2a} + C_2C_{1a} - 2C_mC_{ma} \pm \sqrt{(C_{1a}C_2 - C_{2a}C_1)^2 + 4(C_{ma}C_1 - C_{1a}C_m)\,(C_{ma}C_2 - C_{2a}C_m)}\right]^{-1/2} \tag{A19}$$

Mode voltage ratios

$$R_{c,\pi} = (-1)\,\frac{(C_{1a}C_2 - C_{2a}C_1) \pm \sqrt{(C_{1a}C_2 - C_{2a}C_1)^2 + 4(C_{ma}C_1 - C_{1a}C_m)\,(C_{ma}C_2 - C_{2a}C_m)}}{2(C_{ma}C_2 - C_{2a}C_m)} \tag{A20}$$

Mode impedances

$$Z_{c1} = \frac{1}{v_c}\left(\frac{1}{C_1 + R_c C_m}\right) \tag{A21}$$

$$Z_{\pi 1} = \frac{1}{v_\pi}\left(\frac{1}{C_1 + R_\pi C_m}\right) \tag{A22}$$

$$Z_{c2} = -R_c R_\pi Z_{c1} \tag{A23}$$

$$Z_{\pi 2} = R_c R_\pi Z_{\pi 1} \tag{A24}$$

Four Port Parameters

The four port impedance or admittance parameters can be found from the expressions for voltage and current on the two lines. Equations (A9) - (A12) can be used to find port voltages and currents by replacing the variable "x" with either zero or the line length ℓ.

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ R_c & R_c & R_\pi & R_\pi \\ R_c\, e^{-\gamma_c \ell} & R_c\, e^{\gamma_\pi \ell} & R_\pi\, e^{-\gamma_\pi \ell} & R_\pi\, e^{\gamma_\pi \ell} \\ e^{-\gamma_c \ell} & e^{\gamma_c \ell} & e^{-\gamma_\pi \ell} & e^{\gamma_\pi \ell} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} \tag{A25}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{c1} & -Y_{c1} & Y_{\pi 1} & -Y_{\pi 1} \\ R_c Y_{c2} & -R_c Y_{c2} & R_\pi Y_{\pi 2} & -R_\pi Y_{\pi 2} \\ R_c Y_{c2}\, e^{-\gamma_c \ell} & -R_c Y_{c2}\, e^{\gamma_c \ell} & R_\pi Y_{\pi 2}\, e^{-\gamma_\pi \ell} & -R_\pi Y_{\pi 2}\, e^{\gamma_\pi \ell} \\ Y_{c2}\, e^{-\gamma_c \ell} & -Y_{c2}\, e^{\gamma_c \ell} & Y_{\pi 2}\, e^{-\gamma_\pi \ell} & -Y_{\pi 2}\, e^{\gamma_\pi \ell} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} \tag{A26}$$

Now the amplitude coefficients may be eliminated to end up with an equation relating V_1, V_2, V_3, V_4 to I_1, I_2, I_3, I_4.

$$[V] = [Z]\,[I] \tag{A27}$$

Where now [Z] is a 4x4 matrix relating port parameters.

APPENDIX B: DIRECTIONAL COUPLER THEORY

The condition for ideal directional couplers can be written simply in terms of the four port scattering parameters expressing infinite directivity and impedance matching. Referring to Figure 1

$$S_{13} = S_{24} = S_{31} = S_{42} = 0 \qquad \text{(B1)}$$

$$\text{and } S_{11} = S_{22} = S_{33} = S_{44} = 0 \qquad \text{(B2)}$$

From the general four port scattering parameters two conditions that satisfy the above requirements can be found. They are the equalization of mode velocities and relating the mode impedances in the following way:

$$Z_1 = Z_{c1} = -Z_{\pi 1} \text{ and } Z_2 = -Z_{c2} = Z_{\pi 2} \qquad \text{(B3a)}$$

$$\text{or } Z_1 = -Z_{c1} = Z_{\pi 1} \text{ and } Z_2 = Z_{c2} - -Z_{\pi 2} \qquad \text{(B3b)}$$

Refer to Appendix A for the definition of mode impedances. For the non symmetrical case of interest here it can easily be shown that mode velocities cannot be made equal (Appendix C). It has been shown in this thesis that the second condition can be met. If the mode impedances are equal and have the same sign corresponding to $k_c = -k_L$, then a co-directional coupler results. This condition cannot be satisfied with microstrip-like transmission lines that share a ground plane.

Substituting in (B3) from equations (A13) - (A16) and using the expressions for $\gamma_{c,\pi}$ and $R_{c,\pi}$ in terms of the line constants leads to a simple form:

$$\frac{L_m}{\sqrt{L_1 L_2}} = \frac{C_m}{\sqrt{C_1 C_2}} \tag{B4a}$$

or

$$\frac{C_{ma}}{\sqrt{C_{1a} C_{2a}}} = \frac{C_m}{\sqrt{C_1 C_2}} \quad . \tag{B4b}$$

This is the criteria used to find the ideal coupler cases.

When ports 2, 3, and 4 are terminated as in Figure B1 and power applied to port 1 then $V_2 = -Z_2 I_2$, $V_3 = -Z_3 I_3$, and $V_4 = -Z_4 I_4$. Using this along with equations (A25) and (A26) and condition (B3) the amplitude coefficients turn out to be:

$$A_2 = A_3 = 0 \tag{B5}$$

$$A_4 = -A_1 \, (R_c/R_\pi) \, e^{-j(\theta_c + \theta_\pi)}, \qquad \theta_{c,\pi} = \gamma_{c,\pi} \ell \tag{B6}$$

Again using (A25) and (A26) with these results shows that there are no reflections at the ports (all ports are matched) and the port voltages are:

$$\frac{V_2}{V_1} = \frac{R_c\left(1 - e^{-j(\theta_c + \theta_\pi)}\right)}{1 - \frac{R_c}{R_\pi} e^{-j(\theta_c + \theta_\pi)}} \tag{B7}$$

$$\frac{V_3}{V_1} = 0 \tag{B8}$$

$$\frac{V_4}{V_1} = \frac{(1 - R_c/R_\pi)\, e^{-j\theta_c}}{1 - R_c/R_\pi\, e^{-j(\theta_c + \theta_\pi)}} \tag{B9}$$

The scattering parameters are:

$$S_{11} = S_{22} = S_{33} = S_{44} = S_{13} = S_{31} = S_{24} = S_{42} = 0 \tag{B10}$$

$$S_{12} = S_{21} = S_{34} = S_{43} = -\frac{\sqrt{R_\pi/R_c}\left[1 - e^{j(\theta_c + \theta_\pi)}\right]}{1 - R_\pi/R_c\, e^{j(\theta_c + \theta_\pi)}} \tag{B11}$$

$$S_{23} = S_{32} = \frac{(1 - R_\pi/R_c)\, e^{j\theta_c}}{1 - (R_\pi/R_c)\, e^{j(\theta_c + \theta_\pi)}} \tag{B12}$$

$$S_{14} = S_{41} = \frac{(1 - R_\pi/R_c)\, e^{j\theta_\pi}}{1 - (R_\pi/R_c)\, e^{j(\theta_c + \theta_\pi)}} \tag{B13}$$

The coupling magnitude is

$$|S_{12}|^2 = \frac{2\ R_\pi/R_C\ (1 - \cos\theta_s)}{1 + (R_\pi/R_C)^2 - 2\ R_\pi/R_C\ \cos\theta_s},\quad \theta_s = \theta_C + \theta_\pi \tag{B14}$$

$$|S_{12}|^2_{max} = \frac{4\ R_\pi/R_C}{1 + R_\pi/R_C\ (2 + R_\pi/R_C)} \tag{B15}$$

Coupling can be defined as $C = 10\ \log_{10}\ (|S_{12}|^{-2})$ (B16)

Coupling bandwidth can be defined to be at the half power points where:

$$\frac{|S_{12}|^2}{|S_{12}|^2_{max}} = \frac{1}{2} \tag{B17}$$

that is

$$\theta_{SBW} = \cos^{-1}\left[\frac{2\ R_\pi/R_C}{1 + (R_\pi/R_C)^2}\right] \tag{B18}$$

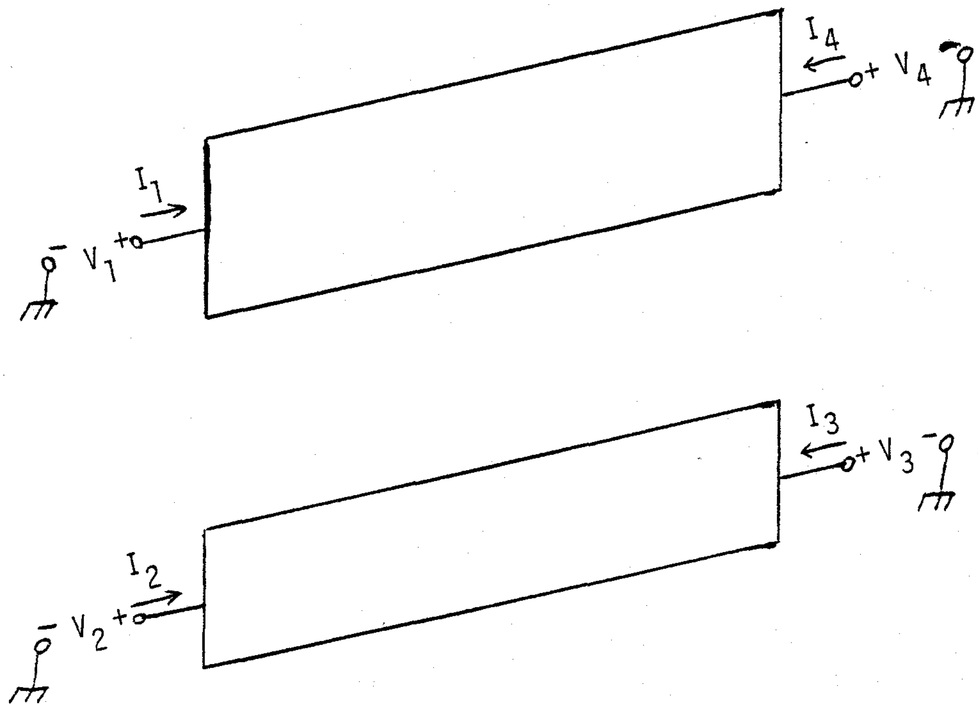

Figure B1. General coupled line four port schematic.

APPENDIX C: SHOW THAT MODE VELOCITIES CANNOT BE MADE EQUAL IN AN INHOMOGENEOUS MEDIUM

To show this first rewrite equation (A19) for the mode velocities in terms of capacitance.

$$v_{c,\pi} = \sqrt{2\Delta a}\, c \left[C_1 C_{1a} + C_2 C_{1a} - 2C_m C_{ma} \pm \sqrt{(C_{1a}C_2 - C_{2a}C_1)^2 + 4\,(C_{ma}C_1 - C_{1a}C_m)\,(C_{ma}C_2 - C_{2a}C_m)} \right]^{-1/2} \tag{A19}$$

If the mode velocities v_c, v_π are to be equal then the square root term must be zero. It is easy to see that this is not the case if the two lines are not in a homogeneous medium. To make this clear find the condition for the velocities to be equal.

$$(C_{1a}C_2 - C_{2a}C_1)^2 + 4\,(C_{ma}C_1 - C_{1a}C_m)\,(C_{ma}C_2 - C_{2a}C_m) = 0 \tag{C1}$$

The first term is obviously zero if the lines are symmetric or the medium is homogeneous since in both cases $C_2/C_{2a} = C_1/C_{1a}$. The second term is more subtle. Rearranging and rewriting simplifies it:

$$4\, C_{ma}C_m \left[\left(\frac{C_1}{C_m} - \frac{C_{1a}}{C_{ma}} \right) \left(\frac{C_2}{C_m} - \frac{C_{2a}}{C_{ma}} \right) \right]$$

Clearly this term will be zero only when the dielectric is homogeneous.