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for Capital Expenditures
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In this study, three main issues are discussed: possible bias arising when inflation effects are neglected in engineering economy studies, capital projects authorization forms which are not designed to include inflation effects in the calculations, and the introduction of a new graphical technique for capital expenditure sensitivity analysis.

Approaches are described which include inflation in economic studies by the use of current and constant dollars. The fartitioning of cost into individual components is proposed in order to include differential frice trends. Numerical examples are presented. A current dollar analysis using partitioned factors is recommended.

Capital authorization forms were obtalned from industries in order to analyze the characteristics of in:ormation actually used in these orgarizations. By extracting the more relevant elements from them and adding improvements, a new set of forms is developed. The set incluced worksheets for the calculation of the cash flow in current and constant dollars that are alalyzed to determine the present worth, rate of return, and pay:u... feriod. Space for a
graphical sensitivity analysis display is also provided.
A graphical technique, the trilinear chart, is adapted to represent the different elements in economic studies. Its characteristics are discussed with numerical examples which focus on capital projects. A sequence of charts is constructed to analyze the sensitivity of investment proposals with respect to possible deviations from expected values. SENSOR, a computer program, is developed to assist in the plotting of the charts on a flatbed plotter or in a time sharing terminal.

It is concluded that the cost partitioning gives a more realistic measurement of the inflation effects for the evaluating of investment proposals. The forms presented can be implemented in most industries with minor changes and will be helpful in minimizing calculation errors. Trilinear charts have considerable potential for use in customized sensitivity analysis in standard situations and for presentation of recommendations.

Improved Justifications and Sensitivity Analysis for Capital Expenditures
by
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TABLE OF CONTENTS
Chapter Page
I INTRODUCTION ..... 1
II INFLATION EFFECTS AND ENGINEERING ECONOMY ..... 4
Example Problem A ..... 5
Example Problem B ..... 8
Basic Concepts of Engineering Economy ..... 9
Depreciation ..... 13
Inflation in Engineering Economic Analysis ..... 22
Including Inflation in Capital Expendi- ture Analysis ..... 26
III ECONOMIC METHODS AND DIFFERENTIAL PRICE TRENDS ..... 35
Cash Operating Profit After Tax as a Percentage of Gross Investment Capital ..... 35
Including Inflation in the Analysis ..... 40
IV INVESTMENT ANALYSIS FORMS AND SENSITIVITY ANALYSIS TECHNIQUES ..... 47
Investment Analysis Forms ..... 47
Calculating Rate of Return and Payback Period Graphically ..... 52
Sensitivity Analysis Techniques ..... 54
Trilinear Charts ..... 61
$V$ CAPITAL EXPENDITURE ANALYSIS FORMATS ..... 67
VI EXTENSIONS ..... 79
Cost Reduction Proposals ..... 79
SENSOR (Computer Program) ..... 85
VII CONCLUSIONS AND RECOMMENDATIONS ..... 102
Conclusions ..... 102
Recommendations ..... 105
REFERENCES ..... 107
APPENDIX A ..... 109
APPENDIX B ..... 118

## LIST OF FIGURES

Figure Page1 Comparison of Book Values for DifferentDepreciation Methods17
Comparison of Annual Depreciation Chargesfor Different Methods17
Consumer Price Index for the Last 38 Years ..... 25 ..... 34COPAT/GIC for Example Problem A UsingDifferent Depreciation Methods39
5
Graphical Interpolation to Obtain the Rateof Return53
Graphical Calculation of the Payback Period ..... 54
Net Present Worth for Different Annual Savings ..... 56
Sensitivity Graph on the Effects of Net Present Worth When Factors Deviate From Original Esti- mates ..... 57
Isoquant for Annual Savings and Years of Useful Life that Make the Present Worth Equal Zero ..... 58
Graphical Annual Savings for a Given Investment ..... 59
Acceptable Investment Diagram ..... 61
Trilinear Chart Theorem ..... 62
Analysis of Food Values ..... 63
Trilinear Chart for a Linear Relationship ..... 64
Trilinear Chart for a Nonlinear Relationship ..... 65
Graphical Representation of the Growth Rate ..... 66
Capital Project Authorization Form Number CPA-1 ..... 68
Economic Analysis Worksheet Form Number CPA-2 ..... 69
Cash Flow Worksheet Form Number CPA-3 ..... 70
Trail 1 ..... 73
Example for Completing the Form Set ..... 76-78
Graphical Relationships in the Equation Sales - Cost of Sales = Earnings ..... 89
Graphical Relationships in the EquationWorking Capital + Capital Investment = TotalInves tments90
Graphical Relationships in the Equation
$\frac{\text { Sales }}{\text { Total Investment }} \times \frac{\text { Earnings }}{\text { Sales }}=\frac{\text { Earnings }}{\text { Total Investment }}$91

## LIST OF FIGURES (CONT.)

Figure Page
25 Graphical Relationships in the Ratio
$\frac{\text { Sales }}{\text { Total Investment }}$ ..... 92
26
Graphical Relationship in the Equation Future Worth Factor $=(1+i)^{n}$ ..... 93
27 Future Worth Factor Chart with Modified Scales ..... 94
28 Graphical Relationship of the Present Worth Calculation ..... 97
29 Graphical Relationship of the Present Worth Deviations ..... 98
30 How to Run SENSOR With a Logorithmic Scale From a CRT ..... 100
31 How to Run SENSOR With a Nonlogorithmic Scale From a CRT ..... 101

## LIST OF TABLES

Table ..... Page
I Before Tax Cash Flow Example Problem A ..... 7
II Cash Flow for Example Problem A Using Straight Line Depreciation ..... 20
III Cash Flow for Example Problem A Using Sum- of-the-Years Digits Depreciation ..... 21
IV Cash Flow for Example Problem A Using Double- Declining Balance Depreciation ..... 22
V Factors for Converting Quantities in Constant Dollars to Quantities in Current Dollars ..... 30
VI After Tax Cash Flow for Example Problem A Using the Double-Declining Balance Depreci- ation Method ..... 32
VII
Modified After Tax Cash Flow for Example Prob- lem A Using the Double-Declining Balance Depreciation Method ..... 43
VII Modified After Tax Cash Flow for Example Prob- lem A Using the Double-Declining Balance Depreciation Method and Including Inflation ..... 44
IX Comparisons of Cash Flows When Inflation is Included Individually by Cost Components ..... 46
$X$ Project's Letter of Justification ..... 49
XI Worksheet Summary ..... 50
XII Physical Characteristics Summary ..... 51
XIII Optimistic-Pessimistic Approach ..... 55
XIV Cost Saving in Current Dollars Assuming Different Situations ..... 84

# IMPROVED JUSTIFICATIONS AND SENSITIVITY ANALYSIS FOR CAPITAL EXPENDITURES 

## I. INTRODUCTION

During the last decade, economies throughout the world have suffered from unexpected surges in inflation and declines in productivity. The industrial engineering profession has responded by devoting greater attention to productivity improvement. Conventional ways to increase productivity include increasing production capacity, making processes more efficient, and reducing production costs. All require capital.

Inflation and other uncertainties tend to obscure whether an investment is recovered with an adequate return for the investors. Engineering Economics is concerned with the practical applications of economic principles in the evaluation of technically feasible productivity improvements, so as to select the optimal alternative.

The traditional engineering economic methods are incapable of distinguishing between returns representing increases in intrinsic value and returns reflecting inflation. In industrial practice, capital investment decisions are among the most critical decisions that a manager must make. Typically, these decisions involve vast amounts of funds committed for relatively long periods.

Managers have to deal with lack of information, uncertainty, and calculation errors while the decision is analyzed. To cope with this situation, the characteristics of the traditional analysis methods
are discussed in this study and the possible consequences of ignoring the effects of inflation in the analysis are described with numerical examples.

Some evidence exists that investors include an element in their return requirements to protect themselves against anticipated inflation. The suggestion of some writers as to how to include inflation in the calculations are presented.

Uncertainty arises when future cash flows are estimated. During periods of inflation, the amount of goods that can be bought for a particular amount of money decreases as the time of purchase occurs further in the future. However, the creation of indexes of specific costs is suggested in this study in order to enable the analyst to consider the tendencies of prices. It is recognized that the creation of such indexes requires an additional amount of calculation and their accuracy is questionable; nevertheless, it is at least a feasible way to trace price changes.

Economic analyses of proposals are often standardized by a set of forms. A new format is proposed to include inflation in the calculations by means of simple tableaus. Characteristics of the forms were obtained from actual capital expenditure authorization forms used in several industries. A visual aid is included in the new version to analyze the sensitivity of a proposal.

Sensitivity analysis is useful when an economic study is to give consideration to possible future price changes. Thereby, the risks involved in accepting a given investment proposal can be
identified. The analysis of the sensitivity can become discouraging because of the proliferation of numbers. However, graphic displays assist decision making by presenting the information in a compact form.

A trilinear chart is a graphical device where the relationships among three variables can be analyzed. This kind of chart is not new, but their adaptation for use in capital expenditure analysis is original. In this study, characteristics of the charts are presented. The possible use of the trilinear charts in economic analyses is to portray graphically the sensitivity of a proposal to unexpected changes in the conditions, providing the decision maker with maximum information and understanding of the analysis.

## il. inflation effects and engineering economy

Engineering Economics proposes quantitative methods to measure the attractiveness of capital investments. Among the proposed techniques, the payback period, rate of return, and present worth are the most commonly used in practice. Each one of these methods represents a different analysis criterion. For instance, the payback period is a rough measure of the speed with which invested funds are returned to the firm. The rate of return measures the break even point of the cost of money in a particular project. The present worth criterion provides today's value of the project's estimated cash flow.

Even though these measurements are consistent under assumed conditions, some problems arise when inflation is taken into consideration. There is some evidence that inflation and the cost of capital and, therefore, the discounting rate at which the present worth is calculated, are positively related.

In this chapter some characteristics of the engineering economy techniques and the suggestions by some writers as to how to incorporate inflation effects in these techniques will be discussed.

For illustrative purposes, two example problems are used throughout the study to allow convenient comparisons of the concepts presented.

## Example Problem A

A well known company is considering the prospect of buying a new machine in order to increase production output. They estimate that there is a very good market for these products because in the western part of the country, no one is producing them yet.

The price of the machine is $\$ 450,000$, payable within 12 months (a $\$ 150,000$ down payment and two semestral payments of $\$ 150,000$ each). Installation of the equipment takes 12 months. Start up costs during the first six months of operations are $\$ 70,000$. Working capital requirements are $\$ 60,000$ and are needed in the first year of operations. Periodical maintenance is required every two years, with a current cost of $\$ 2,000$.

After making a study of the process and the market, the company estimates that the annual operating costs are going to be $\$ 250,000$ :

| Labor | $\$ 70,000$ |
| :--- | ---: |
| Energy | 120,000 |
| Raw Material | 60,000 |
|  | $\$ 250,000$ |

The revenues from sales will be $\$ 500,000$ per year. The life of the project is estimated to be six years. The tax rate for the company is $50 \%$. The salvage value of the machine is $\$ 50,000$.

Summarizing with an arrow diagram:
Salvage Value ..... 50
Working Capital Recovery ..... 60
RevenuesStart Up CostMaintenance CostOperating Cost


The before tax cash flow for each year of the useful life of the project is shown in Table I. Note that all of the expenses and revenues are assumed to be at the end of the year. The useful life of the machine is estimated at six years but because of the installation period of 12 months, the project life is seven years.

Amounts (\$x1000)

| End of Year | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Receipts: |  |  |  |  |  |  |  |  |
| Revenues | - | - | 500 | 500 | 500 | 500 | 500 | 500 |
| Working Capital Recovery |  |  |  |  |  |  |  | 60 |
| Salvage Value |  |  |  |  |  |  |  | 50 |
| Total Receipts | - | - | 500 | 500 | 500 | 500 | 500 | 610 |
| Disbursements: |  |  |  |  |  |  |  |  |
| Investment | 150 | 300 |  |  |  |  |  |  |
| Working Capital |  | 60 |  |  |  |  |  |  |
| Start Up Cost |  | 70 |  |  |  |  |  |  |
| Maintenance |  |  |  | 2 |  | 2 |  |  |
| Operating Costs: |  |  |  |  |  |  |  |  |
| Labor |  |  | 70 | 70 | 70 | 70 | 70 | 70 |
| Energy |  |  | 120 | 120 | 120 | 120 | 120 | 120 |
| Material |  |  | 60 | 60 | 60 | 60 | 60 | 60 |
| Total Disbursements | 150 | 430 | 250 | 252 | 250 | 252 | 250 | 250 |
| Before Tax Cash Flows | -150 | -430 | 250 | 248 | 250 | 248 | 250 | 360 |

Table I. Before Tax Cash Flow Example Problem A.

## Example Problem B

A company is considering the purchase of an accessory for a machine in the production line. The accessory's price is $\$ 7,500$. The accessory is expected to save operating costs but not increase productivity. At the present time, the operating cost is $\$ 15,000$ and the estimated operating cost with the accessory would be $\$ 10,600$. The expected life of this equipment is three years. There is no salvage value on this equipment once it is removed. The tax rate for this company is $50 \%$.

Summarizing in an arrow diagram,

Annual Savings
Years
Investment


## Basic Concepts of Engineering Economy

Traditionally, the most common decision criteria in Engineering Economy are the payback period, the internal rate of return, and the net present worth. There have been many criticisms of these methods. I will just present the basic fundamentals of these criteria and some of their characteristics.

Payback Period Criterion. This criterion is widely used in the United States and throughout the world [23] because it is relatively easy to calculate and because a manager, intuitively, would like to recover capital as quickly as possible. The payback period (sometimes called the payout period) is the number of years required for the cost (savings) and receipts from an investment to equal the amount of the investment, disregarding depreciation and interest changes. It is a rough measure of the speed with which invested funds are returned to the firm. The payback period is calculated by dividing the required investment by the net annual revenues (i.e., annual receipts minus annual disbursements); then,

$$
\text { Payback Period }=\frac{\text { required investment }}{\text { annual receipts -annual disbursements }}
$$

or,

$$
\text { Payback Period }=\frac{\text { first cost }}{\text { net annual savings }}
$$

In this method, the data is the actual estimations, not discounted, and salvage values are not included. This criterion will
recommend any project having a payback period of less than a specified, low number of years. Where several alternatives are available, it would recommend choosing the one with the shortest payback period.

However, when the payback period criterion is used as the sole criterion for evaluating projects, an "uneconomical" alternative may be chosen over a more "economical" one. For example, if an alternative has high cash flows in its early years and nothing more thereafter, it will have a relatively small payback period. On the other hand, if another investment promises to return a smaller cash flow but over a longer period of years, it will be more profitable than the first alternative, although it will have a longer payback period.

The payback period criterion is used as a supplementary criterion when funds are very limited and where a fast recovery allows capital to be reallocated again.

Rate of Return Criterion. The rate of return criterion (sometimes called the internal rate of return or the return on investment) is the criterion that provides a percentage figure which represents the critial cost of money at which a project breaks even. That is, the interest rate at which the present value of receipts equals the present value of the disbursements. There is no single formula to solve it, therefore, the only procedure is one of cumbersome trial and error.

When the selection criterion is based on the rate of return, and one has to choose among several alternatives, an incremental rate of return is required. That it, each alternative is compared against
the last acceptable alternative on an incremental basis. The incremental rate of return can be defined as the interest rate which causes the present value of the incremental cash flow between two projects to be zero (i.e., the break even cost of money for incremental investment).

Some problems may arise from the use of the rate of return technique. One critical assumption is that all cash flows can be reinvested at that calculated rate of return. That is, this method assumes that, for the period between the base point and the time when the funds are spent or collected, the funds are or could be invested at the rate of return being calculated for the proposal.

This reinvestment assumption has caused many criticisms of the method and some modifications have been proposed. R.H. Baldwin ([1], pp. 185-187, and [20], p. 212) has proposed the Baldwin Rate of Return Method in order to calculate a more realistic rate. This method includes an interest rate that the organization earns, on the average, on its assets.

Another problem may arise when the cash flow of an investment proposal follows a certain pattern (i.e., the cash flow sign changes two or more times) provoking multiple rates of return. In order to eliminate this possibility, another variation of the rate of return has been derived and is called the modified rate of return ([2], pp. 317-321).

Some authors have related both problems to the same cause. This is: the multiple rates of return are the result of the assumed
reinvested rate of return ([23], pp. 271-274). The author proposes the application of an explicit reinvestment rate of return to a limited portion of the cash flow in order to eliminate one of the sign reversals, affecting as little as possible the cash flow pattern. The explicit reinvestment rate may be the minimum attractive rate of return employed by the organization or a rate suggested by the present worth profile.

Present Worth Criterion. The present worth criterion is the one that provides the today's value of the estimated cas'n flow for a proposal. It can be used for comparing expenditures, annual cost savings, and income producing outlays.

This kind of analysis requires that assets' economic lives, or increments thereof, be equal. It also requires an interest rate at which the estimated cash flow will be discounted. (This interest rate is called the discount rate.)

The present worth is defined as the value, today, of a future payment or stream of payments discounted at the appropriate discount rate.

Some experts consider the present worth criterion theoretically superior to the rate of return criterion (described in the preceding section) when the discount rate to be used in the present worth method reflects the cost of money; but there is no agreement as to how to define the discount rate.

Discount Rate. As mentioned before, there is no agreement among the experts on how the discount rate is to be defined. The discount
rate should reflect the cost of money, but there have been many approaches suggested. These approaches are not going to be examined in detail in this research because considerable controversy exists and the author feels there is no conclusive answer. Nevertheless, for purposes of information, the following references are recommended: [1], [6], and [30].

## Depreciation

Tangible fixed assets are subject to depreciation. Other assets and intangibles with limited periods of usefulness (such as trademarks, patents, goodwill, etc.) are subject to amoritization. The best definition for depreciation accounting is the one used by accountants. According to the American Institute of Certified Public Accountants, "Depreciation accounting is a system of accounting which aims to distribute cost or other basic values of tangible capital assets, less salvage value (if any), over the estimated useful life of the unit in a systematic and rational manner. It is a process of allocation not of valuation."

In other words, instead of charging the price of a tangible fixed asset as an expense in that period's income, it is spread over the life of the asset in the accounting records as an expense of operations. There are two allocation processes. The most commonly used is book-entry depreciation, the accumulated depreciation account appears in the record as "other assets". This is when the depreciation accrual stays within the company as a source of funds because they are reinvested. The funds can be invested anywhere within the
company. Usually, it is not possible to know exactly where they are.
The other kind of allocation process, seldom used in practice, is the sinking fund. Here, a separate fund is posted to accumulate the depreciation funds and invest them outside the company at an interest rate. This fund is dedicated to the replacement of the asset in study.

The rate of depreciation, regardless of the process of allocation used, depends on the life assigned to a particular asset and the estimated salvage value at the end of its useful life. Unfortunately, it is related to the original price of the asset, not to the replacement price; and because money depreciates every day, by the time the asset should be replaced (end of its useful life), the replacement cost becomes more expensive than the accumulated depreciation due to high inflation rates.

The regulatory body (the Internal Revenue Service in the United States) allows different kinds of depreciation methods. As long as they are consistently applied, the organization can choose any one, depending on its convenience. 1 The most commonly used are the straight line, the sum-of-the-years digits, and the declining balance methods.

Before describing the depreciation methods, it is necessary to

[^0]define some terms and notations that will be used:
Depreciable Base (DB): Purchasing price minus salvage value.
Purchasing Price (PP): The actual amount paid for the asset, not including installation costs.

Salvage Value(SV): The net amount that can be realized when the asset is disposed of.

Useful Life (iv): The time elapsed from the date on which the asset is first placed in use until its final disposition. (Note that it refers to the usefulness to the firm and not to the total life of the asset.) The Internal Revenue Service has set some guidelines to determine the service life of an asset (see Publication 534 (revised October 1974), Department of the Treasury, Internal Revenue Service).

Annual Depreciation Charge (ADC): The amount to be charged to the period's income as an operating cost. Used specifically for tax purposes.

Book Value (BV): For assets, refers to the net valuation account. That is, the purchasing price--cumulative depreciation.

Straight Line Depreciation Method. In this method, the annual depreciation charge (ADC) is constant. It is calculated by dividing the depreciable base (DB) by the useful life (N).

$$
A D C=\frac{P P-S V}{N}
$$

Sum-of-the-Years Digits Method. This method is one of the accelerated methods allowed. It provides a larger annual depreciation charge (ADC) during the early years of the asset's life than in the
later years. The annual depreciation charge varies each year. It is calculated by the ratio of the remaining years of the asset's life (i.e., $N-n+1$ ) to the sum of the digits corresponding to the number of years of the estimated useful life (i.e., $1+2+3 \ldots$...N, times the depreciable base (DB).

$$
A D C=\frac{(N-n+1)}{(1+2+3 \ldots+N)}(D B) \quad \text { for year } n .
$$

Declining Balance Depreciation Method. Another accelerated depreciation method allowed by the Internal Revenue Service is the declining balance method. The only restriction is that the salvage value has to be greater than zero. Under this method, a fixed percentage rate is applied each year to the book value of the asset at the beginning of the year. In the final year, sufficient depreciation is taken to reduce the book value to, but not below, salvage value. The annual depreciation charge ( $A D C$ ) is calculated by multiplying the book value (BV) of the asset by the depreciation rate (\%). The fixed depreciation rate can arbitrarily be chosen or calculated by the formula ([18], p. 225)

$$
\text { Depreciation Rate }(\%)=\left(1-\sqrt[N]{\frac{\text { Net Salvage Value }}{\text { Purchasing Price }}}\right) \times 100
$$

As mentioned earlier, the depreciation rate can be chosen arbitrarily, not taking into consideration the salvage value. One of the many declining balance methods is the double-declining balance. It is merely a declining balance method using, as a depreciation rate, $200 \%$ of the straight line rate.

The book values of the asset in Example Problein $A$, for each of
the years of its useful life, are graphed in Figure 1. It compares the methods described in the preceding paragraphs. The annual depreciation charges are plotted in Figure 2.


Figure 1. Comparison of Book Values for Different Depreciation Methods.


Figure 2. Comparison of Annual Depreciation Charges for Different Methods.

Depreciation and Taxes. For engineering economic analysis, depreciation is important because depreciation charges are considered to be an operating expense and are therefore tax deductible; taxes directly alter cash flows (in after tax analysis). However, the effect in the after tax cash flow depends on the schedule for depreciation. Also it is necessary to note that in the case where the actual salvage value is lower or higher than the book value, for tax purposes it is considered as a loss or a gain, respectively. In general, if the asset has been held less than six months, it is a short term loss or gain and is taxed at the regular rate. If it is held more than six months, it is a long term loss or gain and the maximum taxation rate is 30\%.

There is a new tax depreciation procedure in the United States. It is known as the Asset Depreciation Range System. It has a special advantage, under certain circumstances, to firms which use group depreciation accounting. Under the Asset Depreciation Range System, the taxpayers are allowed to elect a fixed tax life, which is different from the guidelines life, for a group of assets. ${ }^{2}$

The tax law is very complex. For simplicity purposes, it is going to be assumed that a corporate income tax of $50 \%$ is applicable. The other aspect of the tax law that is going to be relevant for this study is the Investment Tax Credit.

The Investment Tax Credit is a direct subsidy given by the
${ }^{2}$ For more details on the Asset Depreciation Range System, see [2], pp.187-194.

Federal Government in the form of a tax deduction. This incentive allows the deduction of a certain percentage amount of dollars on a new investment as a credit against income taxes. The Investment Tax Credit allowances depend on the useful life of the asset as follows:

| Asset With a <br> Useful Life of | Tax Credit Allowed <br> Up To |
| :--- | :---: |
| 8 years or more | $10 \%$ |
| 6 to 8 years | $2 / 3$ of $10 \%$ |
| 4 to 6 years | $1 / 3$ of $10 \%$ |
| Less than 4 years | $0 \%$ |

The Investment Tax Credit also has the characteristic that it can be carried forward or backward for a limited number of years to apply against tax liabilities in those years.

Another feature of the tax regulations is that the interest paid for funds borrowed are tax deductible. This has an important effect on the source of funds for a project because some other sources are not tax deductible.

After Tax Economic Analyses. Sometimes before tax studies give appropriate solutions for the project under analysis, but when tax considerations are introduced, their attractiveness may differ greatly.

The degree of effect will vary based on the estimation of the amount and the timing of some particular elements of the future cash flow. These particular elements are depreciation schedule, tax incentives (when applicable), and interest payment deductions. To illustrate this effect, the present worth, the payback period, and
the rate of return for the investment period in Example Problem $A$ is going to be calculated.

Recalling the before tax cash flow from Table I (page 7) and calculating the after tax cash flow, using the straight line depreciation method, the cash flow would be as follows:
$(\$ \times 1000)$

| End of Year | Before Tax Cash Flow | Depreciation Charges | $\begin{gathered} \text { Taxable } \\ \text { Income } \\ {[(2)-(3)]} \end{gathered}$ | $\left[\begin{array}{c} \text { Taxes } \\ {[(4) \times \text { Tax Rate }]} \end{array}\right.$ | After Tax Cash Flow $[(2)-(5)]$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| (1) | (2) | (3) | (4) | (5) | (6) |
| 0 | -150 |  |  |  | -150 |
| 1 | -430 |  | - 70 | - 35 | -395 |
| 2 | 250 | 67 | 183 | 91.5 | 158.5 |
| 3 | 248 | 67 | 181 | 90.5 | 157.5 |
| 4 | 250 | 67 | 183 | 91.5 | 158.5 |
| 5 | 248 | 67 | 181 | 90.5 | 157.5 |
| 6 | 250 | 67 | 183 | 91.5 | 158.5 |
| 7 | 250 | 67 | 183 | 91.5 | 268.5 |

Table II. Cash Flow for Example Problem A Using Straight Line Depreciation

Calculating the after tax present worth, assuming $i=15 \%$ : $P W_{A T}=\$ 68.30$.
Calculating the after tax rate of return: $\operatorname{RoR}_{A T}=19.36 \%$.

If, instead, the after tax cash flow is calculated using the sum-of-the-years digits depreciation method, the cash flow would be as follows:
(See Table III on following page)


Table III. Cash Flow for Example Problem A Using Sum-of-the-Years Digits Depreciation

Calculating the after tax present worth, assuming $i=15 \%: \quad \mathrm{PW}_{A T}=\$ 80.50$ Calculating the after tax rate of return: RoR $_{A T}=19.3 \%$

If the double-declining balance depreciation method is used, then the after tax cash flow would be:
(See Table IV on following page)

| (\$ $\$ 1000$ ) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| End of Year | Before Tax Cash Flow | Depreciation Charges | $\left\lvert\, \begin{gathered} \text { Taxable } \\ \text { Income } \\ {[(2)-(3)]} \end{gathered}\right.$ | $\left[\begin{array}{c} \text { Taxes } \\ {[(4) \times \text { Tax Rate }]} \end{array}\right.$ | After Tax Cash Flow $[(2)-(5)]$ |
| (1) | (2) | (3) | (4) | (5) | (6) |
| 0 | -150 |  |  |  | -150 |
| 1 | -430 |  |  | - 35 | -395 |
| 2 | 250 | 150 | 100 | 50 | 200 |
| 3 | 248 | 100 | 148 | 74 | 174 |
| 4 | 250 | 67 | 183 | 91.5 | 158.5 |
| 5 | 247 | 4.4 | 203 | 102 | 146 |
| 6 | 250 | 30 | 220 | 110 | 140 |
| 7 | 250 | 9 | 241 | 120.5 | 239.5 |

Table IV. Cash Flow for Example Problem A Using Double-Declining Balance Depreciation

Colculating the after tax present worth, assuming $\mathrm{i}=15 \%$ : $\mathrm{PW}_{\mathrm{AT}}=\$ 85.93$.
Calculating the after tax rate of return: RoR $_{A T}=21.4 \%$.
In summary, it is advantageous to depreciate the asset using an accelerated method. In some circumstances, switching from one method to another is more profitable, especially when the group depreciation is being used ([2], pp. 189).

## Inflation in Engineering Economic Analysis

First, some engineering economic techniques for before tax investment proposal studies were discussed. One assumption was that all estimated cash flows remain constant throughout the project's life. If it were assumed that all the cash flows are inflated at the same rate, the selection of alternatives would be the same.

In the preceding section, tax considerations were introduced and it was illustrated that they influence the attractiveness of proposals. Unfortunately, there is another factor that can affect the attractiveness of the projects; it is called inflation.

Inflation is a rising general level of prices. It has been of great concern during the last half of this decade because the rate has been much higher than ever before.

It is necessary to distinguish between two types of price changes: inflation and differential price changes. Inflation price change is really a change in the purchasing power of the monetary instrument, whereas differential price change refers to the difference between the price trends of the goods or services and the general price trend. That is, during inflation some prices decrease while others remain fairly constant or exceed the general trend.

Two causes of inflation have been identified ([13], p. 36). One, the so called "demand-pull inflation", is caused by the demand for goods and services increasing much more than the available supplies. Most of the time, the government is blamed for this kind of inflation because it often occurs when large government expenditures are undertaken. However, individuals and businesses also contribute to this inflation by demanding more than the supply available. The other kind of inflation is caused because wages increase more than the labor productivity; it is called "cost-push inflation". There is some controversy about the relative effects of cost-push and demand-pull causing inflation, but inflation would have been less severe during
the last years if wages had not increased more rapidly than the productivity of labor.

Indexes. Indexes are developed to measure the historical rate of price changes in portions of the economy. Inflation is usually described in terms of an annual percentage that represents the rate at which the current year's prices have increased over the previous year's prices. It has a compound effect that can be quite severe when applied over an extended period of time.

Historical rates of price changes in different portions of the economy are measured by government organizations such as the Department of Labor, Bureau of Labor Statistics, in the United States; the Banco de Mexico, in Mexico; and some private organizations. An index is developed by sampling that segment of the economy that the index is designed to measure. The sample is called "market basket". The index is the ratio of the cost of the goods in one year divided by the cost of the same goods in some base year.

Some representative indexes of price changes in the United States are the Consumer Price Index, the Wholesale Price Index, and the Implicit Price Index for the Gross National Product. These indexes, and some others, are compiled by the Department of Commerce, Bureau of Economic Analysis; and the Department of Labor, Bureau of Labor Statistics.
(See Figure 3 on following page)


Figure 3. Consumer Price Index for the Last 38 Years.
[Source: Economic Report of the President, 1979.]

It is important to be aware that these indexes are a reasonable measure for their purpose; but goods included in the index, and the weight assigned to each, may not accurately reflect the buying habits of a business enterprise. The Consumer Price Index is the most commonly used measure of prices in the United States. It is designed to show the effect of retail price changes on a selected fixed standard of living. The Wholesale Price Index measures the impact of inflation at the wholesale level for both consumer goods and industrial products. The Implicit Price Index for the Gross National Product is a measure of the effect of general price level changes on the Gross National Product. The Gross National Product is the total market value of final goods and services produced in the economy during a certain period of time.

## Including Inflation in Capital Expenditure Decisions

In evaluating capital expenditure decisions, an analyst must consider not only the possible effects of inflation but also the effects of long run trends on the relative prices of his products on his important categories of expenditures. Perhaps the best way to illustrate the inflation component is by means of an example.

Consider the investor as a consumer who is willing to postpone present consumption in order to gain additional consumer goods in the future. This gain in goods is his/her real return or profit. Ignoring any income tax, for simplicity, if the prices of the goods are increasing because of inflation, the investor must earn a rate of return equal to the rate of inflation just to stay even, so he/she will be able to buy the same amount of goods in the future. To illustrate this, assume the investor has $\$ 10,000$ which could be used, now, to buy 100 units of goods at $\$ 100$ each. However, this investor may be willing to invest the $\$ 10,000$ and forego buying these goods for five years. If prices increase at a rate of, say five percent, then, this investor will need $\$ 12,763$ to buy the same 100 units in five years. (That is, now the price is $\$ 100$; five years hence the price would be $\$ 100(1+.05)^{5}=\$ 127.63$; then, $\$ 127.63 \mathrm{x}$ $100=\$ 12,763$.

There has been no real gain if the investor gets only $\$ 12,763$. However, the investor expects a reward for foregoing consumption. Assume that this investor wants to acquire a ten percent gain in goods each year, then the investor will need to receive $\$ 16,100$ if
the price of goods were not increasing. That is, now he/she can buy 100; five years from now, he/she would like to buy $100(1+.1)^{5}=161$, at $\$ 100$ each then, $161 \times 100=\$ 16,100$.

This is just a ten percent rate of return. To achieve the same result when the rate of inflation is seven percent, the investor must have $\$ 20,548(161 \times \$ 127.63=\$ 20,548)$. The apparent rate of return is,

$$
\begin{aligned}
10,000(F / P, i, 5) & =20,548 \\
(F / P, i, 5) & =20,548 / 10,000=2.0548
\end{aligned}
$$

With the help of an interest table or a pocket calculator, it is found that the rate of return equals $15.5 \%$.

The reason is that a portion of the $\$ 20,548$ is needed to compensate for the decrease in the purchasing power of those future dollars. Furthermore, this is the return the investors will demand to assure the additional compensation needed for protection against inflation. This return ( $15.5 \%$ in the example) is the cost of money and will be called the nominal rate of return (i), whereas the inflation-free rate, this means the effects of inflation are removed (ten percent in the example) will be called the real rate of return $(r)$.

Several authors have built this relationship to add to the original work of Irving Fisher. He demonstrated, by means of numerical examples, that the real rate of return from an asset fixed in monetary terms would fall short of the real rate of return from a comparable asset fixed in real terms by an amount equal to the inflation rate. His illustrations were made with the assumptions of
unanticipated inflation, certainty, and perfect market adjustments. For a continuous compounded rate of inflation $k$, market interest $i$, and real interest $r$ :

$$
\begin{aligned}
& \mathbf{r}=\mathbf{i}-k \quad o r \\
& \mathbf{i}=r+k .
\end{aligned}
$$

In [3] the author discusses the work done by other authors, who have verified Fisher's equation, such as Ralph Jones. Also, it discusses the case of discrete inflation. Some writers, such as Reisman and Rao, have developed the relationship between purchasing power, $P(T)$, and a discrete rate of inflation concluding that:

$$
P_{T+1}=P_{0}(1+k)^{T}
$$

where $T$ = number of periods.
Other authors ([4],[5]) have expressed Fisher's equation for the discrete form as:

$$
\mathbf{i}=r+I+r I
$$

where $I=$ inflation rate, $r=$ real rate, and $i=$ nominal rate, suggesting that the nominal rate of return (i) should contain two components: one related to inflation (I) increases and the other to intrinsic values increases.

Most of the authors mentioned have based their studies considering two types of monetary units, dollars in this case. The money that is exchanged for goods and services at the actual time of purchase is called Current Dollars, and the money that would be exchanged for the same goods and services in some reference year is called Constant Dollars; hereafter, they will be denoted CR\$ for

Current Dollars and CN\$ for Constant Dollars.
The expected cash flow of a project is also affected by inflation in several ways. Future cash flows simply are estimated on the basis of existing prices. However, due to a reduction in purchasing power, the consumer is unable to buy the same amounts of goods as before. The cash inflows generally arising from the sale of products are affected by expected future prices. In cash outflows, inflation affects both expected future wages and material costs. In some cases, cash inflows through price increases will rise faster than cash outflows while in other cases, the opposite will hold. Note that inflation does not affect depreciation charges on existing assets or income taxes.

The problem here is how to relate one kind of dollars to the other kind of dollars. An author ([4], p.84) has developed a very useful table (see Table V) that is used when the information available is the nominal rate of return or the real rate of return, and the inflation rate. He based his calculations for this table upon the relationship that if he chooses the reference year for constant dollars as the year zero in the rate of return calculations, then the present worth in current dollars is the same as the present worth in constant dollars. The table first converts a cash flow in constant dollars to the corresponding present worth and then converts this present worth to the desired quantity in current dollars.
(See Table $V$ on following page)


Table V. Factors for Converting Quantities in Constant Dollars to Quantities in Current Dollars.

It is important to note that this table has limitations because it is assumed that the inflation rate is constant, and the author concludes that in the case where the inflation rate changes are arbitrary, as often happens in real life, then the results in the table involving a uniform flow or a gradient would be invalid because it should be given a unique rate of return for the corresponding dollar flow in a given year.

To illustrate the effects of inflation in after tax analysis,
consider Example Problem A (page 5), again. Using the before tax cash flow from Table I and constructing the table for after cash calculations using the double-declining balance depreciation method, assuming no inflation, the present worth was calculated as $\$ 85,930,000$. The internal rate of return is equal to $21.4 \%$, considering a discount rate of $15 \%$.

Suppose that the general price level is increasing at $11 \%$ per year and that this price change is reflected in receipts as well as in disbursements. Then, the before tax cash flow will be modified as follows:

| Years | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total Receipts |  |  | 616.0 | 683.8 | 759.0 | 842.5 | 935.2 | 1202.0 |
| Disbursements | 150 | 430 |  | 2.7 |  | 3.4 |  |  |
| Operational Costs |  |  |  |  |  |  |  |  |
| Labor |  |  | 86.2 | 95.7 | 106.3 | 118.0 | 130.9 | 145.3 |
| Energy |  |  | 147.8 | 164.1 | 182.2 | 202.2 | 224.4 | 249.1 |
| Material |  |  | 73.9 | 82.1 | 91.1 | 101.1 | 112.2 | 124.6 |
| Total Disbursements |  |  | 307.9 | 344.7 | 379.6 | 424.7 | 467.5 | 519.0 |
| Before Tax |  |  |  |  |  |  |  |  |
| Cash Flow | -150 | -430 | 308.1 | 339.2 | 379.4 | 417.8 | 467.7 | 683.0 |

The after tax cash flow will be calculated as follows (Note that the depreciation charges are the same as using the double-declining balance method; also, the tax rate is the same):
(See Table VI on following page)


Table VI. After Tax Cash Flow for Example Problem A Using the Double-Declining Balance Depreciation Method.

If only the values in current dollars are compared, it can be concluded that inflation is beneficial to the project because the after tax cash flows have increased. But with $11 \%$ per year inflation, the current dollars cash flow does not have the equivalent purchasing power referred to in a base year. Therefore, it is necessary to compare the same kind of dollars.

Many authors have proposed different ways to make economic analysis under these circumstances. They have based their discussions on two basic techniques. The first technique, which is the simplest to understand, is to explicitly define future cost that reflects relevant cost trend. Then the cash flows in current dollars can be converted to present worth by using time value factors based
on the firm's cost of money rate; this is the nominal rate. Earlier, it was pointed out that the cost of money already includes a component to protect investors against inflation. Then in this case, both the cash flow and the discount rate will have included inflation, otherwise it would exist as a bias in the calculations of the present worth [11].

The other basic technique is to remove inflation from the cash flows (i.e., convert the cash flow to constant dollars with reference to a base year), then the real cost of money must be used in studies. However, the real cost of money cannot be determined exactly. For the last two decades, financial analysts, statisticians, mathematicians, and economists have analyzed the relationship between inflation in the economy and the performance of the stock and bonds markets with no absolute conclusions drawn.

Among the authors that discuss these techniques are [2], [4], [5], [9], [11], [12], and others. Some of the writers have discussed a special case where the base year refers to the constant dollars in year zero. Therefore, the present worth in current dollars is the same as the present worth in constant dollars ([4], [5], [7]).

A discussion suggests the possibility of two rates of return in a mixed mode computation. Because all estimates in the cash flow have both current and constant dollars components it is possible to choose between current present worth or constant present worth.

In [2] a mathematical short cut for the first basic technique has been discussed. It derives a "convenient rate" to apply to the
cash flow that will be inflated by the rate I and discounted to the present worth by the rate $i$, then the convenient rate is:

$$
\text { Convenient rate }=\frac{i+I}{1+I}
$$

A word of caution to avoid the possible misuse of this rate points out that it is used to find the present worth of an inflating series. It is not a discount rate. It assumes a geometrical cost increase.
III. ECONOMIC METHODS AND DIFFERENTIAL PRICE TRENDS

In the preceding chapter, troubling aspects of monetary depreciation and possible bias arising from ignored effects of inflation were discussed. Two types of price changes were defined: inflation and differential price change.

To be discussed in this chapter are the problems encountered in a technique called COPAT and the advantages of using a customized price index to reflect inflation in the economic analysis.

For these considerations, the following assumptions are made: -The cost of money is determined by one of various methods, and is $15 \%$ after taxes.
-No technological advances are expected to occur during the useful life of the project.
-Investment is internally financed.
-Perfect capital market.
-Revenues, variable costs, and salvage value are fixed in real terms and grow with inflation.
-Depreciation charges are fixed in constant terms and do not grow with inflation
-Tax rate is $50 \%$.

Cash Operating Profit After Tax as a Percentage of Gross Investment Capital

A measurement called "cash operating profit after tax as a percentage of gross investment capital" (COPAT/GIC) is used by some
companies. It is based on an internal publication of the Chase Manhattan Corporation, "Analitical Methods in Financial Planning" written by Joel Stern ${ }^{3}$.

The calculation is a percentage of cash operating income after taxes divided by the average investment capital during the period, then

$$
\text { COPAT/GIC }=100 \times \frac{\text { COPAT for the period }}{\text { Average GIC. }}
$$

Where Average GIC $=\frac{\text { Beginning of year GIC }+ \text { End of year GIC }}{2}$
COPAT is an after tax cash flow excluding interest; tax depreciation is used rather than book depreciation. GIC is defined as investment in fixed assets at cost plus total working capital.

The purpose of this measure is to emphasize the impact of the cash operating profit after taxes during specific periods, as opposed to the rate of return and present worth techniques which consider the timing of the cash flow over the life of the project.

To illustrate the use of COPAT/GIC, consider Example Problem A (page 5). Recalling from Table IV, the after tax cash flow using double-declining balance depreciation is:

Year
0
1
2
3
4
5
6
7

After Tax Cash Flow
$-150.0$
-395.0
200.0
174.0
158.5
146.0
140.0
239.5
${ }^{3}$ The author could not obtain the original publication
and these are considered to be the COPAT for each year. Then, the average GIC is calculated as follows:

GROSS INVESTMENT CAPITAL


However, if the straight line depreciation method is used, instead, the COPAT would be (from Table II):

|  | Project Years |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| COPAT/GIC (\%) | - | - | 158.5 | 157.5 | 158.5 | 157.5 | 158.5 | 268.5 |

and calculation of COPAT/GIC, using the same average GIC as previously
computed is:

| Project Years |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|  | COPAT/GIC (\%) | - | - | 27.1 | 27.1 | 27.2 | 27.0 | 27.1 | 445.9

Recalling from Table III, the after tax cash flow using the sum-of-the-years digits depreciation method is:

Project Years

which leads to a COPAT/GIC of:

|  | Project Years |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| COPAT/GIC (\%) | - | - | 31.3 | 29.5 | 28.0 | 26.1 | 24.7 | 41.9 |

Figure Four displays the values of the COPAT/GIC for each of the depreciation methods. Note that the double-declining balance method makes the COPAT/GIC higher at the beginning of the project's life than the other two methods. As a consequence, the straight line depreciation method has a higher COPAT/GIC than the other two methods in the final years. The results are logical because the doubledeclining balance method deducts the biggest part of the investment in the early years; therefore, the cash flow, after tax, is higher. Also, it is important to note that because COPAT/GIC does not take into consideration the time value of money, the average of COPAT/GIC for each depreciation method is the same, $25.9 \%$.


Figure 4. COPAT/GIC for Example Problem A Using Different Depreciation Methods.

Some deficiencies are apparent in this method. The COPAT/GIC assumes that the same amount of GIC will be reinvested each year of the project's life. Therefore, the method does not consider that the money has a cost (Cost of Capital or Opportunity Cost).

If a modification of the method is proposed to reflect the cost of money in the GIC, as well as in the COPAT, the problem becomes a regular rate of return exercise in which a different interest rate can be assessed each year. However, it would be a time consuming operation.

It is concluded that modifying the COPAT/GIC does not justify the extra work reflecting the cost of money and causes the method to lose its simplicity. The COPAT/GIC is a rough measure of the rate of return in a given period and can be useful if the decision maker is aware of the significant difference that exists between the real return and the return shown by the unmodified COPAT/GIC. The real rate of return is necessarily lower because the cost of
money has been considered.

## Including Inflation in the Analysis

In the preceding chapter, possible ways to include inflation in economic analyses were discussed. An illustration was presented, assuming an inflation rate of ten percent per year, which identically affected the cash flows.

However, the effect of general price level changes will vary for each type of cash flow. That is, cash inflows are affected by expected future prices whereas cash outflows are affected primarily be expected future wages, material costs, and energy costs.

At present, there is no reliable technique to predict the inflation rate. Although, as mentioned earlier, some indexes have been developed by sampling the segments of the economy that the index is designed to measure. But these indexes may not accurately reflect the buying patterns of an organization. For instance, geographic location can make a big different (i.e., highly developed metropolitan areas are more affected by inflation than rural areas). Also, the cash flow components for a particular organization may not be the same as those considered in the market basket used to develop the index; thus, it is suggested here that each organization should build its own price indexes.

The trend that a particular price has followed for the last ten years can be traced using the accounting records of an organization. Five main components should be distinguished: capital, labor expenses, material costs, energy expenses, and product selling
price.
Capital will probably follow the general inflation rate. Labor wages can be considered to have a steady growth. Material cost trends will be unique for each organization. Energy expenses have recently increased at a rate higher than the inflation rate and depends on the proportion and type of this component used within the organization. The product selling prices could be highly sensitive to inflation but are sometimes restriced by regulations. In the case where a component does not represent a significant portion of the total cost, there is no need to construct its index, a general inflation rate is sufficient because the impact is small.

An index is calculated by dividing the price or cost of the goods in one year by the price or cost of the same goods in some base year. For example, if the base year is 1970 and the price of the product in the base year was $\$ 50.00$, last year the price was $\$ 85.00$, and this year the price is $\$ 90.00$ then, last year's index is 170 (i.e., $85 / 50 \times 100=170$ ) and this year's index is 180.

The cost trend change implied by an index is the rate of change in that index. For instance, in the example above, the cost trend rate is:

$$
\frac{180-170}{170}=\frac{10}{170}=.059 \text { or } 5.9 \%
$$

A one-year cost trend rate is not very useful for measuring price inflation. The average cost trend rate for a period of time is better. For instance, if the same index as above was 116 seven
years ago, the cost trend (CT) rate for the last seven years has been:

$$
\begin{aligned}
116(1+C T)^{7} & =180 \\
(1+C T)^{7} & =1.5512 \\
C T & =\sqrt[7]{1.5512}-1 \\
C T & =.065 \text { or } 6.5 \%
\end{aligned}
$$

If the index was 133 four years ago, the cost trend rate has been:

$$
\begin{aligned}
133(1+C T)^{4} & =180 \\
C T & =7.86 \%
\end{aligned}
$$

One of the two cost trends can be used to represent the historical trend in the index.

To illustrate the importance of these considerations, assume that in Example Problem A (page 5), even with no change in general price level--no inflation--the labor expenses will increase nine percent yearly. This modifies the before tax cash flow as follows:

| (\$x1000) Year | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total Receipts |  |  | 500 | 500 | 500 | 500 | 500 | 610 |
| Disbursements | 150 | 430 |  | 2 |  | 2 |  |  |
| Operational Costs: |  |  |  |  |  |  |  |  |
| Labor |  |  | 83.2 | 90.6 | 98.8 | 197.7 | 117.4 | 128 |
| Energy |  |  | 120 | 120 | 120 | 120 | 120 | 120 |
| Material |  |  | 60 | 60 | 60 | 60 | 60 | 60 |
| Total Disbursements | 150 | 430 | 263.2 | 270.6 | 278.8 | 287.7 | 297.4 | 308 |
| Before Tax Cash Flow | -150 | -430 | 236.8 | 229.4 | 221.2 | 212.3 | 202.6 | 302.0 |

Then, the after tax analysis table using double-declining balance depreciation would be as follows:


Table VII. Modified After Tax Cash Flow for Example Problem A Using the Double-Declining Balance Depreciation Method.

Note that before tax cash flow has decreased, taxes have decreased, and after tax cash flow was reduced.

Now suppose that labor cost will increase at nine percent, the price level is increasing at eleven percent per year, and this change is reflected in proportional changes in the product's price and the material cost. Then, modifying the before tax cash flow and calculating the after tax cash flow using double-declining balance depreciation, the following figures are:
(See Table VIII on following page)


Table VIII. Modified After Tax Cash Flow for Example Problem A Using the Double-Declining Balance Depreciation Method and Including Inflation.

Now suppose that inflation is still eleven percent, labor expenses increase at nine percent, but material costs actually are increasing at fourteen percent, energy costs are rising at twenty percent, and selling price increases at the inflation rate. Following the same procedure as before, the after tax cash flow is:

## End of Year

0
1
2
3
4
5
6
7

## After Tax Cash Flow

-150.0
-395.0
216.0
197.1
188.5
180.6
178.8
397.9

Summarizing in Table VII, the five cash flows are compared in current dollars. This means that the cash flow is the actual amount that is expected to be paid or received in those years.

The total cash flow is discounted at a zero percent rate for comparison purposes. Note that when inflation is not considered, the total cash flow is higher than the situation where labor is assumed to be inflating at a nine percent rate. However, when the selling price is considered to increase at the inflation rate, labor and material are increasing, too, at different rates--the price is higher. The more realistic situation is presented in the fourth column where the cost components are assumed to inflate at different rates. Note that when inflation is considered to affect all costs equally, a considerable deviation arises.

## Assumptions

| Selling Price | No Change | No Change | $11 \%$ Increase | $11 \%$ Increase | $11 \%$ Increase |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Labor Cost | No Change | $9 \%$ Increase | $9 \%$ Increase | $9 \%$ | Increase | $11 \%$ Increase |
| Energy Cost | No change | No Change | No Change | $20 \%$ Increase | $11 \%$ Increase |  |
| Material Cost | No Change | No Change | $11 \%$ Increase | $14 \%$ | Increase | $11 \%$ Increase |
| Inflation | No Change | No Change | $11 \%$ Increase | $11 \%$ | Increase | $11 \%$ Increase |

After Tax Cash Flow (\$x1000)

Year

0
1
2
3
4
5
6
7

| -150.0 | -150.0 | -150.0 |
| ---: | ---: | ---: |
| -395.0 | -395.0 | -395.0 |
| 200.0 | 195.2 | 244.4 |
| 174.0 | 164.7 | 244.5 |
| 158.5 | 144.1 | 258.0 |
| 146.0 | 128.1 | 277.8 |
| 140.0 | 116.3 | 307.7 |
| $\underline{239.5}$ | $\underline{210.5}$ | $\underline{451.5}$ |
| 513.0 | 413.9 | $1,238.9$ |


| -150.0 | -150.0 |
| ---: | ---: |
| -395.0 | -395.0 |
| 216.0 | 229.0 |
| 197.1 | 219.0 |
| 188.5 | 223.2 |
| 180.6 | 230.9 |
| 178.8 | 248.9 |
| $\underline{397.9}$ | $\underline{460.2}$ |
| 813.9 | $1,066.2$ |

Table IX. Comparisons of Cash Flows When Inflation is Included Individually by Cost Components

## IV. INVESTMENT ANALYSIS FORMS AND SENSITIVITY ANALYSIS TECHNIQUES

Mostmanagers consider the calculation of economic justifications as a drain on their time. When engineering judgment has produced a technically sound proposal, there is reluctance to expend additional effort preparing back up data for economic analysis. Engineers and managers need quick estimating techniques which will easily detect an economically unsound proposal.

Discussed in the first section of this chapter are investment analysis forms provided by several companies for this study. They are not going to be criticized as to whether they are good or bad. Each organization has its own systems and their forms may be suitable for their objectives. The forms are to be analyzed as to the kind of information they employ and how they are utilized.

In the second section, sensitivity analysis techniques are discussed. These techniques are not unique to this study. The graphic techniques have not changed for many years, although some have been adapted for other purposes. In this study, they will be applied to capital investment expenditure decision problems.

## Investment Analysis Forms

Possible mistakes in a report are calculation errors while making the report and estimation errors in forecasting future cash flows. The former are caused mainly by eye fatigue ([16], p. 174), and can be minimized•if the investment analysis form is well designed and easy to process. For the latter, there is no single solution
because many uncontrollable factors affect future values; therefore, it is advisable to make sensitivity analysis in order to answer "what if" type questions. Some graphic sensitivity analysis techniques are discussed in the next section.

The 13 forms available for this study are not necessarily a representative sample. Some organizations did not send their forms for this study because they either did not want to participate or they did not have one. However, some conclusions can be drawn from the sample.

Decision makers like to have a short form to analyze, concentrating relevant information in one or two pages, with supporting information attached. The summary page(s) will be called "letters of justification" and the support information will be called "worksheets". In a few cases, an additional questionnaire is required, especially when analyzing expenditures exceeding $\$ 25,000$.

Table Ten summarizes the information included in the forms. Most forms do not include information about space availability (facilities considerations) for the new equipment, nor if it is necessary to have additional personnel to operate the equipment (personnel considerations), or if there is any other alternative to the problem. (See Table Ten on following page)


Table X. Project's Letter of Justification General key for All Tables

$$
\begin{aligned}
& \text { - included within the form li.E. tot specitied } \\
& \text { x Bot included within :ne form } \quad \text { I.A. Vot avallable }
\end{aligned}
$$

Key for Table $x$ - Letter of justification
(a) Es Essay Eype

F! Filt-in (inc!uding nonquantitative option)

Table XI summarizes the worksheets. It can be noticed that the rate of return technique is the most widely used, and the payback period is preferred over the present worth technique. Wo other techniques or indexes are used besides the ones mentioned above.

Only two companies use graphical devices for calculations and none of them make an explicit sensitivity analysis. Three companies out of thirteen include inflation factors in the analysis and after tax analysis is used more than before tax analysis. The tax rate is
$48 \%$ or $50 \%$. The rest of the summary is self explanatory.


Table XI. Worksheet Summary

Key for Table XI - Horksheet
(a) AT After tax analysis BT Before tax analysis
Q1 Qualitative analysis (questioner)
(b) In years
(c) The discount rate has been specified
(d) i Thousands
$s$ Amounts in dollars, cents omitted
0 It is optional
(e) HW Handwritten

TW Typewritten
EN Either way (hand or type written)

Table XII is a summary of the physical characteristics of the forms. Most of them are printed on letter size, $8-1 / 2 \times 11$ inch, bond paper or carbonless copy paper (when extra copies are needed). Four of these forms are in Appendix A.


Table XII. Physical Characteristics Summary

Key for Table XII - Physical Characteristics
(a) p Printed NC

Ph Photocopies
C Carbon paper required to reproduce
HC
(b) LT Letさer size, 8-1/2 $\times 11$

LG Legal size, $8-1 / 2 \times 14$
(c) C Colored paper

4 Heavy paper
W White paper
(a) HW Handwritten
iw Typewritten
EN Either way (hand or type written)

## Calculating Rate of Return and Payback Period Graphically

The two graphical aids for calculating the rate of return and the payback period are the profitability index method and the payback plot. The former is based on a method developed in 1954 [22]. It is an accurate and dependable guide which is capable of straight forward calculations of rate of return for any type of investment. The latter is a simple, two-dimensional graph in which is plotted the ratio of the cumulative after tax cash flow divided by the initial investment. The cash flow is not discounted ([60], p. 404].

To illustrate the use of these graphical aids, consider Example Problem A (page 5). Recalling from Table IV, the after tax cash flow using double-declining balance depreciation is:

| $\frac{\text { Year }}{0}$ | After Tax Cash Flow |
| :---: | :---: |
| 1 | -150.0 |
| 2 | -395.0 |
| 3 | 200.0 |
| 4 | 174.0 |
| 5 | 158.5 |
| 6 | 146.0 |
| 7 | 140.0 |
|  | 184.5 |

and the initial investment is $\$ 450,000.00$. Then, different values for the ratio,

$$
\begin{gathered}
\frac{\text { Initial Investment (II) }}{\text { Discounted Present Worth at } \mathbf{i}(\mathrm{DPW})} \\
\text { where } \mathbf{i}=\text { discount rate }
\end{gathered}
$$

are calculated and plotted in the interpolation chart, Figure 5. At the point where the line connecting the II/DPW ratios crosses the
vertical line at $I I / D P W=1.0$, the value of $\mathbf{i}$ that makes the initial investment equal to the present worth is identified as the rate of return.


Figure 5. Graphical Interpolation to Obtain the Rate of Return.

Note that, in this case, the entire investment outlay is assumed to take place at time zero; when the investment cash outflows are over a period of years, the discounted present worth of the investment is used.

The payback period plot is similarly calculated, but here the ratio used is:

Cumulative After Tax Cash Flow
Initial Investment
After the ratios are calculated, they are plotted on a chart, Figure 6. At the point where the line connecting the plotted ratios crosses the horizontal line, the payback period is found on the horizontal axis.


Figure 6. Graphical Calculation of the Payback Period.

Note that the cash flow is not discounted in this example. However, a discounted cash flow payback period is easily computed when a value for $i$ can be decided upon in order to discount the cumulative cash flow in the numerator.

## Sensitivity Analysis Techniques

As mentioned earlier, one possible error in making economic analyses is to make an incorrect forecast of future cash flow.

There are many techniques used to forecast. Some of these techniques are based on subjective predictions, others on historical records, and others on underlying causes (i.e., models that include all relevant causal relationships) ${ }^{4}$. But because some factors cannot always be anticipated, like inflation, there is always a possibility that the data will be misleading or inaccurate.

In such circumstances, it is convenient to measure the relative $\overline{4}$ For more detail, see [23], p. 304-321
change in the decision parameters while varying one or more critical factors in the model. This is called "sensitivity analysis". It is possible to create a proliferation of numbers which will cause confusion, rather than clarification, for the decision maker. Therefore, it is advisable to create visual aids to assist the interpretation of the sensitized data.

One sensitivity analysis technique is the optimistic-pessimistic approach. Here, one or more variables are changed in a favorable outcome direction (optimistic) and in an unfavorable outcome direction (pessimistic). The effects of these changes on the decision are investigated.

For instance, recalling from Example Problem B (page 8), the most likely estimates and calculating the present worth on a before tax basis, for simplicity, a present worth of $\$ 491$ results. If the optimistic viewpoint is assumed, the present worth is $\$ 4,691$; with the pessimistic viewpoint, the present worth is $\$ \mathbf{- 2 , 9 5 0}$. Table XIII summarizes these results.

|  | Optimistic |  | Most Likely |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Pessimistic |  |  |  |
| Investment | $\$ 7,500$ |  | $\$ 7,500$ |  |
| Salvage Value | 50 |  | 0 |  |
| Useful Life | 5 years |  | 3 years | 1 year |
| M.A.R.R. | $30 \%$ | $30 \%$ | $30 \%$ |  |
| Annual Savings | $\$ 5,000$ | $\$ 4,400$ | $\$ 3,500$ |  |
| Present Worth | $\$ 4,691$ | $\$ 491$ | $\$-2,950$ |  |

However, these are narrow outcomes because the optimistic
present worth assumes that all variables turn out to be favorable and the pessimistic present worth assumes all variables turn out to be unfavorable. There are ways to make combinations of the pessimistic and optimistic estimates in tabular form, but if there are many variables to be analyzed, the matrix becomes discouragingly complex.

Managers may be interested in exploring the sensitivity of the decision parameters over a wide range of estimates for one or more variables, not only the extreme outcomes. An effective way of displaying and examining sensitivity is to graph it. Figure 7 shows, in a two-dimensional graph, the present worth for each of the annual savings estimates in the optimistic-pessimistic approach for a range of possible useful life.


Figure 7. Net Present Worth for Different Arınual Savings.

A graph like Figure 7 has to be constructed for each variable that warrants analysis. A more effective procedure, when a single proposal is to be analyzed, is to construct a dimensionless graph. Then, the abscissa is the percentage deviation from original value, generated by substituting various values for one factor in the present worth formula, while holding constant the values of all other factors. The sensitivity graph in Figure 8 reveals that deviations up to $50 \%$ in annual savings can give a present worth of $\$ 3,600$, instead of $\$ 500$.


Figure 8. Sensitivity Graph on the Effects of Net Present Worth When Factors Deviate From Original Estimates

The life of the asset and the size of annual savings are the most sensitive factors in the evaluation of the proposal and also are
essentially uncontrollable. If it is desired to explore possible combinations of these factors' variations that will make the present worth of the proposal equal to zero, an isoquant should be constructed. The isoquant in Figure 9 forms the indifference line that shows the possible combinations of the project's life and annual receipts at which the present worth is zero (indifference condition). A project should be acceptable if the actual combination falls above the indifference line, when other factors are maintained constant ([23], pp. 321-326).


Figure 9. Isoc̣uant for Annual Savings and Years of Useful Life That Make the Present Worth Equal Zero.

In the case where more than one alternative exists, a sensitivity analysis for each one has to be performed, changing the same parameters in order to examine the sensitivity of each alternative
to a specific factor.
Other types of charts to assist the decision maker have been discussed in ([34], p.671). A chart is developed to evaluate initial investment amounts and annual savings. Figure 10 shows a similar chart assuming the cost of capital to be $12 \%$. For instance, if the Example Problem B (page 8) is to be analyzed using this chart, it reveals that for an investment of $\$ 7,500$, if the estimated useful life is three years, the minimum annual savings required is $\$ 3,100$.


Figure 10. Graphical Annual Savings for a Given Investment

Where several alternatives exist, a chart similar to Figure 10 can be used.

In [24], a graphical display of useful concepts in evaluating alternatives is discussed. It is called the Acceptable Investment Diagram (AID). Figure 11 shows an example.

The Acceptable Investment Diagram is based on the assumption that attitude toward risk can be defined roughly by two points. The loss coefficient is the desired probability that the rate of return will exceed a specified minimum percentage. The other point, the payoff coefficient, is the desired probability that the rate of return of an investment will exceed a preselected percentage.

In Figure 11, the loss coefficient is shown as $95 \%$ probability that the investment will not lose more than $5 \%$; while the payoff coefficient is $30 \%$ probability that the investment rate of return will be greater than $15 \%$. The straight line connecting the two coefficeints is called the aspiration level. This is a linear combination between the given limits.

The other entries on the chart are curves representing alternative investment. To illustrate the usefulness of the Acceptable Investment Diagram two curves are assumed, each represents two theoretical investments. Project A should be accepted and Project B rejected because it does not meet the requirements.


Figure 11. Acceptable Investment Diagram.

## Trilinear Charts

As mentioned earlier, graphic techniques have not changed much but they have been adapted to different purposes. The so called "trilinear chart" or "100\% triangle" is a good example.

This triangle is based upon the same trigonometric principles as are nomographs. The theorem in the trilinear chart is that in an equilateral triangle, the sum of the three perpendiculars extended from a point within the triangle to the side of the triangle is constant and always equal to the altitude of the triangle. Since this rule is limited to equilateral triangles, these types of charts are always made in equilaterals ${ }^{5}$ ([14], p.588).

[^1]

Figure 12. Trilinear Chart Theorem

The scale used for the axes of the chart may have any range, but the customary scale is an arithmetic one, calibrated in percentages and ranging from zero to one hundred percent. Data to be charted upon it must first be turned into precentages of the total of the three elements charted. It is an additive chart, the three elements are combined by addition to form a total.

The classical use of the chart is when interest is not in the whole but in the proportions of the three elements. For instance, the analysis of food values in terms of calories of fats, proteins, and carbohydrates.


Figure 13. Analysis of Food Values

As far as the author knows, the only attempt to use this trilinear chart in capital investment decision problems is discussed in [27], p. 268-275. Here, the procedure to construct a single triangle in a linear relationship is developed. Figure 14 shows a trilinear chart that analyzes the relationship among return on investment, selling price, and raw material price. They are based on a factory the produces coconut oil at a capacity rate of 7,500 tons per year.


Figure 14. Trilinear Chart for a Linear Relationsinip

In Figure 15 a chart with a nonlinear relation case if presented. The variations of the unitary profit are analyzed as a function of selling prices and raw material price. An illustrative example is given. When the oil selling price is $\$$ pesos 6,040 per ton (1), and the paste price is $\$$ pesos 360 per ton (2), from the base scale a unitary loss of $\$ 140$ per ton of raw material processed is obtained (3). When raw material price is $\$$ pesos 1,975 per ton (4) a unitary profit increment of $\$ 90$ is obtained (5); therefore, the loss of $\$ 50$ is expected (i.e., $90-140=-50$ ).


Figure 15. Trilinear Chart for a Nonlinear Relationship.

The other application found for these charts is in an advertising brochure of a Mexican bank (Multibanco Comermex s.a.). In this case, the chart is used to analyze the growth rate in a company. Figure 16 shows the chart with an example.
(See Figure 16 on following page)

rate of growth (\%)

Figure 16. Graphical Representation of
the Growth Rate.

1. Profit/Sales $=10 \%$ Sales/Assets $=100 \%$ Return on Investment $=10 \%$
2. Debt Equity $=1$ Potential Growth $=20 \%$
3. Dividends/Profits $=50 \%$ Growth Rate $=10 \%$

## V. CAPITAL EXPENDITURE ANALYSIS FORMATS

Reports are the most expensive class of paperwork within an organization. Therefore, it is convenient for time and cost purposes to standardize the report formats. If the format is simple and easy to complete, the procedure is likely to be efficient and economical. On the other hand, a format that is difficult to process or poorly planned will lead to higher procedural cost and a higher probability of mistakes.

In this chapter, a form is presented for capital expenditure authorization analysis that includes possible effects of inflation and a graphical sensitivity analysis.

A capital expenditure analysis format is designed to facilitate the decision by providing the decision maker with relevant and accurate information. The characteristics of the forms designed in this study are described below in the sequence that they should be completed.

For identification purposes, a form title and a form number are given;

Capital Project Authorization, Form Number CPA-1 (Figure 17)
Economic Analysis Worksheet, Form Number CPA-2 (Figure 18)
Cash Flow Worksheet, Form Number CPA-3 (Figure 19)

The physical appearance of the forms is important because the workers' attitude may vary depending on the design and the process used to reproduce the forms. It is suggested to print the form set


Figure 17. Capital Project Authorization Form Number CPA-1


on letter size ( $8-1 / 2 \times 11$ inch $)$ bond paper, but for space reasons, they have been reduced in this study.

Sufficient writing space is allowed for typewriting or handwriting. It is advisable to type Form Number CPA-1, because it is the summary used for authorization. The worksheets are designed for calculations and data tabulations.

Project identification is common information shared by all the forms. The project name is given by the originator in a short, descriptive sentence. The name and extension number of the person who prepares the forms is given in case additional information is needed.

The completion of the forms is an interactive process. First, the worksheets (Figures 18 and 19) are completed by hand and then the Capital Project Authorization Request form (Figure 17) can be typed summarizing information generated in the worksheets.

The first step is to define the assumptions on the lefthand side of the Economic Analysis worksheet (Figure 18). All items are self-explanatory, but care should be exercised with discount rates and with inflation factors for each of the cost components (labor, material, and energy). Generally speaking, the nominal rate is the one that will be known in most cases; therefore, it is necessary to calculate the real rate using the formula

$$
r=\frac{N-I}{1+I}
$$

where $r=$ real rate, $N=$ nominal rate, $I=$ inflation.
The next step is to complete the Cash Flow Worksheet (Figure 19).

Constant dollars (CN\$) are dollars that do not include inflation. That is, these amounts will be at today's prices; therefore, the most likely to be known. It is enlightening to work in both kinds of dollars; constant (CN\$) and current (CR\$) dollars.

To convert constant dollars into current dollars, it is necessary to multiply the amount in current dollars times one plus the factor that applies to that kind of prices to the nth power. That is, Constant Dollars $\times(1+\text { Factor })^{n}=$ Current Dollars where $\mathrm{n}=$ number of periods after year zero, Factor $=$ inflation rate that applies.

For instance, to convert CN\$ 100.0 in labor costs into CR\$, five years hence, assuming a nine percent per year increase in labor wages:

$$
C N \$ 100 \times(1+.09)^{5}=C R \$ 153.86
$$

Also, CR\$ can be converted into $C N \$$ by dividing the amount in CR\$ by one plus the factor to the nth power. That is,

$$
\frac{\text { Current Dollars }}{(1+\text { Factor })^{n}}=\text { Constant Dollars }
$$

or
Current Dollars $\times 1 /(1+\text { Factor })^{n}$
where $n=$ number of periods after year zero, Factor $=$ inflation rate the applies.

The present worth for the cash flows is calculated in the last six rows within the form. It is important to distinguish between the real rate and nominal rate. The real rate is the constant dollars' discount rate, whereas the nominal rate is the current dollars' discount rate. In the last column of the Cumulative Discounted Cash

Flow, rows 20 and $T$, are the present worths for current dollars and constant dollars, respectively. These values are entered in the summary table (Form Number CPA-2, lines 1 and 2).

The third step is to calcualte the rate of return on the Economic Analysis Worksheet. From the Cash Flow Worksheet (Form Number CPA-3) the current dollars' and constant dollars' after tax cash flows (rows 17 and Q) are entered in the "Trail 7 "columns (see Figure 20). Because the discount rate is assumed to be zero, a direct summation for each column is made and the profitability ratio (Total Investment/ Summation of Cash Flow) is calculated. Two additional present worths are calculated, discounting the cash flow at $10 \%$ and $25 \%$.

Then, the profitability ratios are plotted on the interpolation chart and connected with straight lines. At the point where the line connecting the plotted ratios crosses the horizontal 1.0 line, the rate of return is obtained by reading across from that intersection to the discount rate axis. This value is entered in the summary table (Form Number CPA-2, lines 3 and 4).

| Yrair | $\begin{aligned} & \text { TRIAL } 1.70 \% \\ & \text { CRI } 1 . \\ & \hline \end{aligned}$ |  |
| :---: | :---: | :---: |
| -1 |  |  |
| 0 |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 9 |  |  |
| 9 |  |  |
| 10 |  |  |
| total | (Y). |  |
| Ti/Y |  |  |

Figure 20. Trail 1

The payback period is calculated in the fifth step. The cumulative amount is the summation of the constant dollars column in Trial 1; when the ratio, Cumulative Constant Dollars/TI, is equal to or greater than $100 \%$, the year in which this takes place is entered in the summary table (Form Number CPA-2, line 5). That is, the payout will occur during this period.

Finally, the senstivitiy of the project is analyzed in Step Six. The sensitivity analysis is very important whenever uncertainty exists. In most of the capital expenditure problems, the expected values assumed have some degree of uncertainty, especially when inflation is included.

The procedure is simple but cumbersome. The final purpose of the analysis is to provide the decision maker with a range of possible outcomes in order to facilitate the decision.

A table can be used to calculate manually the sensitivity of the project. Since there are several ways to perform the calculations, no form is included in the Capital Project Authorization set. A computer program could be developed to calculate the sensitivity for factors, and from the printout, the graph could be drawn.

The purpose of the sensitivity analysis is to identify the possible changes in the net present worth when the value of one factor is varied while the values of the other factors are held constant. The most prominent factors that can affect the net present worth are receipts, disbursements, salvage value, useful life, and discount rate. However, the analyst can determine which factors are the most
sensitive and analyze just these factors.
The last step is to complete the Request Form (Form Number CPA-1). Most of the information has been calculated in the worksheets. This is the form which will be submitted for authorization consideration. The worksheets should also be sent as supporting documents.

Prior to final authorization, a staff review and approval is necessary. Only in the cases where additional personnel or layout rearrangements are needed will approval from the responsible department be needed.

The level of final authorization may vary from one organization to another. In general, the following levels are used:

Project Total
Up to $\$ 10,000$
from $\$ 10,000$ to $\$ 25,000$
from $\$ 25,000$ to $\$ 50,000$
from $\$ 50,000$ and up

Level of Authorization Chief, Requesting Unit Division Manager
Chief Executive Officer Board of Directors

To illustrate the use of the set of forms, Example Problem $A$ (page 5) will be used. The data used to complete the forms are hypothetical.


Figure 21. Example for Completing the Form Set





## VI. EXTENSIONS

To be discussed in this chapter are extensions of analysis methods for investments that reduce total production cost, particularly when the effects of inflation are incorporated. Also, the characteristics of trilinear charts and their uses are presented. A computer program is developed to graph the charts.

## Cost Reduction Proposals

The traditional manner to analyze cost reduction proposals, using the present worth criterion, is to discount to the present the net yearly savings, at the nominal rate, and subtract from them the initial investment. That is, with a uniform series of savings;

Present Worth $=-$ Investment + Net Yearly Savings ( $P / A, i, n$ ) where $\mathbf{i}=$ discount rate, $n=$ periods

However, during inflation periods, a bias may arise because the net savings does not reflect the inflation. Even if the savings were considered to be in constant dollars and the real rate of return was used as the discount rate, considerable deviations would exist. This is especially true due to changes in the weights of various components in the inflation index provoked by the nature of the inflation function.

In Chapter II it was proposed that production costs be classified into labor cost, material cost, and energy cost. It was suggested that they should have their own index because each one will have
different price change rates.
Cost reduction proposals are no exception; savings should be increased at the inflation rate of the specific cost component affected by the investment. That is, an investment will be more attractive if the savings are produced in a cost component with high inflation index, than in a component with a lower inflation rate. To illustrate these concepts, consider the following example.

A new investment of $\$ 50,000$ will lead to a reduction in labor cost of $\$ 30,000$ per year and an increase of $\$ 5,000$ per year in energy costs for five years. The components of total production cost are:

| Labor cost | $\$ 500,000$ |
| :--- | ---: | ---: |
| Material cost | 300,000 |
| Energy cost | 200,000 |
| Total cost | $\$ 1,000,000$ |

Assuming the inflation rates to be constant throughout the project's life and the indexes are known to be:

General Inflations 11\%
Labor Inflation 9\%
Material Inflation 14\%
Energy Inflation 20\%
the weighted average inflation rate for the production cost for the next year will be:

| Cost Component | Weight | Inflation | W X I |
| :--- | :---: | :---: | :---: |
| Labor Cost | $50 \%$ | $9 \%$ | $4.5 \%$ |
| Material Cost | $30 \%$ | $14 \%$ | $4.2 \%$ |
| Energy Cost | $\frac{20 \%}{100 \%}$ | $20 \%$ | $\frac{4.0 \%}{12.7 \%}$ |

Assuming a nominal rate of $23 \%$ and a rounded inflation rate of $13 \%$, the real rate must be $10 \%$.

Then, if in the preceding example, the present worth is calculated assuming the discount rate to be the nominal rate, we have:

Net Present Worth $=-50,000+25,000(P / A, 23 \%, 5)$
Net Present Worth $=-50,000+25,000(2.8035)$
Net Present Worth $=\$ 20,087.00$

Now, if the discount is assumed to be the real rate of ten percent, the present worth would be,

Net Present Worth $=-50,000+25,000(P / A, 10 \%, 5)$
Net Present Worth $=\$ 44,769.00$

However, the attractiveness of the proposal may vary depending upon which cost component is affected. For instance, in the example above, the investment will reduce labor cost by $6 \%$ and increase energy cost by $2.5 \%$. The attractiveness of the proposal should be different if the investment would, instead, reduce energy cost by $15 \%$ and increase material cost by $1.7 \%$, even though in both situations, the overall saving will be $\$ 25,000$.

The savings will effectively increase when inflation increases. Actually, the savings and the cost will be increasing at different inflation rates. Consider the present situation in the example where the total cost is increasing for the next five years according to the given inflation rate for each cost factor.

|  | Present | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 500 | 545 | 594 | 648 | 706 | 769 |
| Labor | 300 | 342 | 390 | 445 | 507 | 578 |
| Material | 200 | -240 | -288 | $\underline{346}$ | $\frac{415}{}$ | $\frac{498}{1,42}$ |
| Energy | 1,000 | 1,127 | 1,272 | 1,439 | 1,628 | 1,845 |

Assuming the investment is realized, then the labor cost is reduced by $\$ 30,000$ or $6 \%$ and energy cost is increased by $\$ 5,000$ or $2.5 \%$. Then the total cost will be increasing as follows:

|  | Present with <br> Investment | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 400 | 512 | 558 | 609 | 663 | 723 |
| Labor | 300 | 342 | 390 | 445 | 507 | 578 |
| Material | $\underline{205}$ | $\underline{246}$ | -295 | -354 | -425 | -510 |
| Energy | 975 | 1,100 | 1,243 | 1,408 | 1,595 | 1,811 |

If the savings are calculated year by year, the following results are obtained:

|  | 1 | 2 | 3 | 4 | 5 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Total Cost (Present) | 1,127 | 1,272 | 1,439 | 1,628 | 1,845 |
| Total Cost (Investment) | $\underline{1,100}$ | $\frac{1,242}{}$ | $\underline{1,408}$ | $\underline{1,595}$ | $\frac{1,811}{31}$ |
| Savings (Current Dollars) | 27 | 29 | 31 | 33 | 34 |

Now assume the investment will, instead, reduce energy cost by $\$ 30,000$ or $15 \%$ and increase material cost by $\$ 5,000$ or $1.7 \%$. Then, the total cost and annual savings would be:

|  | Present with Investment | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Labor | 500 | 545 | 594 | 648 | 706 | 769 |
| Material | 305 | 348 | 396 | 452 | 515 | 587 |
| Energy | 170 | 204 | 245 | 294 | 353 | 423 |
| Total | 975 | 1,097 | 1,234 | 1,394 | 1,574 | ,779 |

Savings in
$\begin{array}{llllll}\text { Current Dollars } & 30 & 37 & 45 & 54 & 66\end{array}$

Now, consider the situation in which the investment will, instead, reduce energy cost by $\$ 20,000$ or $10 \%$, reduce labor cost by $\$ 10,000$ or $2.0 \%$, and increase material cost by $\$ 5,000$ or $1.7 \%$. Then, the total cost and annual savings would be:

|  | Present with <br> Investment | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 490 | 534 | 582 | 635 | 692 | 754 |
| Labor | 305 | 348 | 396 | 452 | 515 | 587 |
| Material | $\underline{180}$ | $\underline{216}$ | $\underline{259}$ | $\underline{311}$ | $\underline{373}$ | $\frac{448}{1,098}$ |
| Energy | 975 | 1,237 | 1,398 | 1,580 | 1,789 |  |
| Total |  | 29 | 35 | 41 | 48 | 56 |

Summarizing in Table XIV, and calculating the net present worth (the $23 \%$ nominal rate of return is used because the amounts are in current dollars), major deviation in the present worths of the basic data become apparent.

| Labor | $6 \%$ decrease | No change | $2 \%$ decrease |
| :--- | :--- | :--- | :--- |
| Material | No change | $1.7 \%$ increase | $1.7 \%$ increase |
| Energy | $5 \%$ increase | $15 \%$ decrease | $10 \%$ decrease |


| Year | Savings |  |  |
| :--- | ---: | ---: | ---: |
| 1 | $\$ 27,000$ | $\$ 30,000$ | $\$ 29,000$ |
| 2 | 29,000 | 37,000 | 35,000 |
| 3 | 31,000 | 45,000 | 41,000 |
| 4 | 33,000 | 54,000 | 48,000 |
| 5 | 34,000 | 66,000 | 56,000 |
| Net Present Worth |  |  |  |
| at 23\% | $\$ 34,270$ | $\$ 70,060$ | $\$ 59,600$ |

Table XIV. Cost Saving in Current Dollars
Assuming Different Situations.

Table XIV shows that the investement that will reduce energy expenses by $15 \%$, while increasing material cost by $1.7 \%$, would be much more profitable than the situation where labor cost decreases by $6 \%$ and energy costs increase by $5 \%$. Note that in both cases, the net saving is still $\$ 25,000$ (year zero constant dollars). A more realistic situation is where all three cost components are affected by the investment at different levels; nevertheless, the attractiveness of the proposal still differs from the $\$ 44,679$ present worth obtained with the traditional procedure.

It is concluded that an investment that reduces the cost of a component with a high inflation rate is more attractive than investments that affect components with lower inflation rates.

Unfortunately, to recognize this effect in the analysis, the present worth has to be calculated using current dollars and nominal rates; the calculations are cumbersome. When the analysis is done in constant dollars and real rates are used, a bias exists because the weighted inflation rate changes each year; therefore, a constant discount rate is inappropriate.

## SENSOR (Computer Program)

Trilinear charts have many uses for presentation and education purposes. Basic concepts of this chart will be presented with some illustrations.

The original version of the chart was an additive device in which three elements are combined to form a single component; the equation is $X+Y+Z=100 \%$. Figure 13 in Chapter IV (page 60), shows the chart used to represent the nutrients--fats, carbohydrates, and proteins--in a component. In the example, milk carbohydrates are $30 \%$, fats are $52 \%$, and proteins are $18 \%$. The chart can also be used for subtraction. The equation would then be $1=X+Y-Z$ or $1=X-Y-Z$. The graphic solution relies on another geometric theorem. The algebraic sum of the perpendiculars from the side of an equilateral triangle (or extensions of the sides) to any point in the slant of the triangle, is constant and equal to the altitude of the triangle ([14], p.595). The method involves the use of the area outside the triangle.


Using the scales outside the chart, the equation can be transformed to $Y=X+Z$ or $X+Y=Z$. The procedure to construct a chart with external scales is simple.

Assume a chart is to represent the equation:

$$
\begin{equation*}
\text { Sales }(Y) \text { - Cost of Sales }(X)=\text { Earnings } \tag{Z}
\end{equation*}
$$

First, the range of variation for one of the independent variable is selected. For instance, if the cost of the sales variable varies from $\$ 2,000$ to $\$ 7,000$, then the $X$ Axis can be set as follows:


Then, the minimum (or maximum) value of the other independent variable is assigned to one extreme point on the $Y$ Axis. Consider the maximum sales to be $\$ 25,000$. Then,


Now, using the existing relationship between the variables, in this case a subtraction, the middle point and one extreme point of the third variable are calculated. In the example, $\$ 25,00-\$ 7,000=$
$\$ 18,000$ and $\$ 25,000-\$ 2,000=\$ 23,000$.


Using the middle point value of $Z$ and the maximum value of $X$, the maximum value of $Y$ can be obtained as follows:

$$
\begin{aligned}
Y-2,000 & =18,000 \\
Y & =18,000+2,000 \\
Y & =\$ 20,000
\end{aligned}
$$

then,


Finally, using the maximum value of $Y$ and the minimum value of $X$, the maximum value of $Z$ can be obtained. The scales are divided for reading convenience (see Figure 22 ).


Figure 22. Graphical Relationships in the Equation Sales - Cost of Sales = Earnings.

In the addition case, the procedure would be similar. As an example, a chart is constructed to represent the following equation: Working Capital $(Y)+$ Capital Investment $(X)=$ Total Investment The completed chart is shown in Figure 23.

T. INVEST

Figure 23. Graphical Relationships in the Equation Working Capital + Capital Investment Equals Total Investments.

Furthermore, the $X$ axis can be on either side of the chart.
However, the values on one axis have to be in the opposite direction.
That is,
W.CAPITAL


By using logarithmic scales, the chart can be made factorial instead of additive; then, the three elements are combined by multiplication to form the product. The equations would be $C=\log X+$ $\log Y+\log Z$, or in the outside scales chart $\log Z=\log X+\log Y$ (see Figure 24).

The chart can also be used for division by the use of reciprocals. The equation then is $\log Z=\log X-\log Y$, or $Z=X / Y$ (see Figure 25).


Figure 24. Graphical Relationships in the Equation

$$
\frac{\text { Sales }}{\text { Total Investment }} \times \frac{\text { Earnings }}{\text { Sales }}=\frac{\text { Earnings }}{\text { Total Investment }}
$$



Figure 25. Graphical Relationships in the Ratio $\frac{\text { Sales }}{\text { Total Investment }}$

The chart can also be used for an equation such as the future worth factor: $(1+i)^{n}$. However, the value obtained is not the factor but its logarithm. That is, if $E=(1+i)$ and $X=n$, then the formula is;

$$
F=E^{X}
$$

Its logarithm is;

$$
\log F=X \log E
$$

which is in the form of a multiplicative expression. In this form, the chart converts it into logarithms again as;

$$
\log \log F=\log X+\log \log E
$$

Figure 26 is a chart that represents the relation of interest and years for the future worth factor.


Figure 26. Graphical Relationship in the Equation Future Worth Factor $=(1+i)^{n}$

Note that as long as the scales are correctly plotted, the labels can be changed to any desired value. For instance, in Figure 27, the labels have been changed for the interest values and for the actual factors; then, the following chart is obtained;


Figure 27. Future Worth Factor Chart with Modified Scales.

Note that the scales remain the same, only the values at the axes change. The axes values can also be converted to percentage deviations, based on the most likely estimates. For instance, consider the following values for the total investment axis,


Assuming the expected value of the investment to be $\$ 44,000$, the percentage deviations can be calculated as follows:

$$
\text { Deviation }(\%)=\left(X / X_{0}-1\right) 100
$$

where $X=$ value of the deviation, $X_{0}=$ value of the most likely estimate.

Then, the axis' values are


It is important to mention that these charts are not adequate for precise calculations. They are useful in conducting a rough sensitivity within a system. For example, in an economic analysis, a sequence of charts can be constructed to represent the sensitivity of the present worth of a proposal as follows.

Assume the present worth of an investment is desired to be analyzed and the expected outcomes are:

| Investment | $\$ 90,000$ |
| :--- | :--- |
| Receipts | $\$ 59,000$ |
| Expenses | $\$ 39,000$ |
| Salvage Value | $\$ 10,000$ |
| Useful Life | 7 years |
| Discount Rate | $15 \%$ |

The net present worth is calculated by;
Present Worth $=-90,000+10,000(p / f, 15 \%, 7)+(59,000-39,000)(P / A, 15 \%, 7)$
Present Worth $=\$ 3,027$
A sequence of charts can be built, where the sensitivity of the project can be analyzed. Figure 28 is one possible arrangement that can be constructed.

It is not a simple procedure because the ranges vary independently from one type of chart to the other. Therefore, separate charts
must be built for each situation, in a manner that the ranges are similar.

When a logarithmic chart is related with a nonlogarithmic chart, some accuracy is lost because the scales are not necessarily the same. However, the use of the chart is to analyze how much effect a change in one of the variables has on the net present worth. It is not a precise calculation device.

In the chart, some speculation can be considered, such as, what happens when the useful life decreases by one year and the salvage value increases by $\$ 5,000$ ? With the chart, the question can be answered with some accuracy: Present worth is read as $\$-8,000$, compared with the calculated net present worth of $\$-7,827.05$ (see Figure 28).

If the investment is to be analyzed based on deviation from the most likely estimates of the variable involved; the analysis chart can be changed. Figure 29 represents the same analysis but the percentage values are deviations from the expected values of the variables involved. For instance, if the salvage value increases by $50 \%$ of its expected value and the useful life decreases by $14 \%$, then, from Figure 29, it can be determined that the present worth will decrease by $133 \%$ of its most likely outcome.

Preprinted forms can be used for the nonlogarithmic charts. For multiplication, division, and exponentiation cases, the charts' scales will be different for each situation. A computarized plotting device is useful in this case.


Figure 28. Graphical Relationship of the Present Worth Calculation


A computer program was designed to plot the charts. It is given in Appendix B. The name of the routine is SENSOR. It can draw, with the use of a Gerber flatbed plotter or a cathodict rays terminal (CRT) as output devices, five kinds of charts. Figures 22, $23,24,25$, and 26 are examples of the charts drawn by SENSOR.

SENSOR is generalized format. It will plot the chart but it will not calculate the values. That is, the inputs required in the program are the actual number that will appear on the chart. Figure 30 is the sequence followed when running the SENSOR from a CRT to obtain the chart within the figure.

Note that the chart has to be calculated prior to the use of the SENSOR because the input requires the number of values that will be plotted in the chart as well as their values. This has the advantage of determining exactly which values are to be included. Nevertheless, the procedure is long.

For the nonlogarithmic relations, the input is simpler.
Figure 31 is an example of how the data is entered when a subtraction is made. Note that the separation of labels can be determined in the input data.

The program can be modified to have the computer calculate the values. It can also change the labels for the user's convenience.

GIUE NUMBER OF UALUES TO BE IN-BASE,SIDE,TOP. (IN ORDER)
? 19, 10, 10
UALUES FOR BASE UARIABLE ( FROM LEFT TO RIGHT....)
? $100,90,80,70,60,50,40,30,20,10,9,8,7,6,5,4,3, \dot{2}, i^{-}$
UALUES FOR SIDE UARIABLE (FROM BOTTOM TO TOP......)
? $1,2,3,4,5,6,7,8,9,10$
UALUES FOR TOP UARIABLE (FROM RIGHT TO LEFT......)
? $1,2,3,4,5,6,7,8,9,10$
GIÚ UARIABLE NAMES. 1. BASE UARIABLE.
2. SIDE UARIABLE.
3. TOP UARIABLE.

EACH.. NAMES BETUEEN QUOTTATION MARKS.
? "WEIGHT", IMPORTANCE", "RATE"
IF TEKTERMINAL. RESET PAGE AND DEFAULT GERBER ONLY......ICODE $=3$, MODEL $=4010$, IRATE $=2400$

MAXIMUM 10 LETTERS

AND GERBER. $\ldots$ ICODE $=4$, MODEL $=4010$, IRATE $=2400$ $?$

30. How to Run SENSOR With a Logorithmic Scale From a CRT.

GIUE NUMBER OF DIUISIONS ON bASE-AXE (INTEGER) 45 give number of diuisions side axes. (Integer) ? 15
GIUE. 1.BOTTOM-LEFT UALUE (BASE).
2. THE INCREMENT FOR THE BASE. (INTEGER)
3. THE SEPARATION BETUEEN LABELS. (INTEGER)

ORDER........
? 0,10,5
GIUE. 1.SIDE-BOTTOM UALUE (SIDE).
2. THE INCREMENT FOR THE SIDE. (INTEGER)
3. THE SEPARATION BETUEEN LABELS. (INTEGER)

P 150,10,2
GIUE. 1.TOP-RIGHT UALUE (TOP).
2. THE INCREMENT FOR THE TOP. (INTEGER)
3. THE SEPARATION bETUEEN LABELS. (integer) ........in

ORDER.......
? 300,10, 5
give variable names. 1. base variable.
2. SIDE UARIABLE.
3. TOP UARIABLE.
maximum 10 Letters
EACH.. NAMES BETUEEN QUOTTATION MARKS.
? "TOTAL", 'SUMMATION", "UEIGHT"
IF TEKTERMINAL. RESET PAGE AND DEFAULT IF GERBER ONLY. ICODE $=3$, MODEL $=4014$, IRATE $=2400$ GERBER. ICODE $=4$, MODEL 4014 , IRATE $=2400$ ICODE, MODEL, IRATE
?

31. How to Run SENSOR With a Nonlogorithmic Scale From a CRT.

## VII. CONCLUSIONS AND RECOMMENDATIONS

## Conclusions

The objective of this thesis was to analyze the effects caused by inflation on capital expenditure decisions. Possible ways to cope with uncertainty were considered. Characteristics of several engineering economy techniques were discussed. First, the theoretical concepts for before tax analyses, and then the importance of the depreciation charges and income taxes were discussed.

Depreciation charges are allowed, by law, to be deducted from operating profits. Therefore, they are tax deductible. However, the bases to determine the charges are historical costs and the useful life is set by the regulatory body guidelines. Among the depreciation methods allowed by the Internal Revenue Service are the straight line, sum-of-the-years digits, and declining balance. The last two methods are preferred because they recover the capital faster.

The existing relationships between inflation and after tax analyses were discussed. On the basis that inflation is reflected in neither depreciation charges nor income tax rates, an after tax analysis is preferred to before tax comparisons.

Because investors protect themselves against anticipated inflation by increasing their return requirements, a similar and consistent anticipation of trends in the economic studies must be included.

It is possible to do studies either by explicitly calculating future expenditures and receipts or by combining this calculation with discounting procedures. The latter approach is more difficult to
apply because the forecasting of real rates of return is uncertain. The former is the simplest technique to apply and to understand. For either type of analysis, a firm must somehow estimate the effects of inflation of future costs in order to make reasonable decisions between alternatives.

The analysis of cost trends may be an important help in making the best possible estimate of future costs. Past cost trends are measured by the use of price indexes. Several government indexes are available. However, the tendency of a nationwide index, like the Consumer Price Index, does not necessarily reflect the buying patterns of a specific business.

Cost partitioning is proposed to create individual price indexes by dividing the cost into four components: capital, labor, material, and energy. Since each one will likely be affected by different rates of inflation, the individualized indexes will reflect more accurate inflation effects within the organization. It was deomonstrated that the present worth calculated with factored rates deviates from the present worth computed with a general inflation rate applied evenly to all costs.

Preprinted forms for capital expenditure analysis were analyzed in order to determine the most common industrial practices. It was concluded that managers like to have as much information as possible concentrated in one or two summaries. No attempt to include inflation in engineering economic methods was found. None of the forms had any kind of sensitivity analysis.

A new set of forms was designed in order to include inflation in the present worth criterion as well as in the rate of return calculation. Within the forms, a cash flow worksheet is included. It can be worked out with current dollars as well as with constant dollars. These forms are considered to be easily implemented in almost any kind of organization where capital investment decisions are made.

Since the forecast of inflationary trends are quite uncertain, at best, it is necessary to experiment with ranges of value to ascertain potential impacts. A wide variety of creative and conventional ways to display quantitative estimates and to conduct sensitivity analyses are presented. Among them, a graphical device to test the sensitivity of the proposal is incorporated into the designed set of forms; it consists of a two-dimensional chart in which percentage deviations from the most likely estimates are plotted against the effect caused by these changes in the net present worth, while the rest of the elements are held constant. The resulting curves provide the decision maker with information about which elements are more sensitive to changes.

Possible approaches to include inflation in cost reduction proposals were analyzed. It was suggested that the inflation effects be included individually be cost components. That is, when an investment leads to a reduction in cost, the cost component affected is more relevant than the net saving produced. As mentioned earlier, each cost component will be inflating at a different rate. Therefore,
when savings are produced in a low inflation rate component, the attractiveness of the proposal will be less than when the investment reduces the cost of a component with a higher inflation rate. Some numerical examples were presented. It was concluded that cost partitioning gives a more realistic measurement of the net present worth of a proposal.

However, it is recognized that the computational burden increases considerably; the only way to accurately include these effects in the analysis was by use of current dollars and a nominal interest rate approach.

Finally, a graphical technique was adapted to represent capital expenditure proposals. This technique was originally developed to analyze the relationship of three elements that form a $100 \%$ component. Some characteristics of the chart were discussed with numerical examples. A sequence of charts were constructed to analyze capital expenditure proposals. It is mentioned that these charts are somewhat limited for these types of analysis because of the difficulty in relating logarithmic scales to nonlogarithmic scales. However, a chart can be constructed for a standardized range of proposals in order to analyze, roughly, the sensitivity of small investments.

## Recommendations

Extensions. Each of the three primary developments in this thesis deserve additional investigation.

A short cut method for calculating the effects of inflation should be explored. Considerable effort was made to develop a
procedure that utilized a single, weighted inflation factor in a onestep computation for calculating the present worth. The difficulty with this approach is that the weighted inflation factor varies as a function of the predominant cost factor in the problem. That is, over a period of time, the weighted factor tends to approach the value of the single inflation factor that represents the primary cost category in the cost reduction proposal. If a single, weighted percentage could be applied in a constant dollar analysis, much of the computational burden of the current dollar approach would be alleviated.

The improved format for preprinted economic evaluation forms should be sent to industrial organizations to obtain their opinions about its worth. If they have suggestions, their consensus modifications should be included in a revised version.

SENSOR. The SENSOR, a computer program developed to assist in the implementations of the trilinear charts in capital expenditure analysis, can be modified in order to generate, by itself, more information. A good advance would be to have the program calculate all the values for specific ranges in a given sequence of charts. Also, it would be convenient for it to change the values of the axes when the information required is different than the actual scale value. Further analysis to implement this chart in practical applications in order to improve the utility of the information available to decision makers would be of great value.

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APPENDIX A


## PRESENT CONDITION:

PROJECT DETAILS:

PROJECT JUSTIFICATION:

| PROJECT APPROVAL |  |  |  |
| :---: | :---: | :---: | :---: |
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FINANCIAL DATA AND ASSUMPTIONS
FOR
QUANTITATIVE PROJECTS


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C. Operational Impact - Profit/Loss

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2. Sales Revenue analysis (Specify)


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|  |  | Present Proposed Change |  | Present Proposed Change] |  |
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    3
    I=H+2
    J0}=(\frac{2}{30
    Y(N)=Y(N-1)+YINC
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        JO tE N=I,J
        X(N)=x(N-1)-(XINC/2)
    40 Y(N)=Y(N-1)-YING
    x<0;gg= colululate the axesz cooroinates.
    XKO:Eg=COS(RAO)
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        003
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        TOCux=xAXE (K)-J:5
        IOCGY=YAXE(K)+2+YKINK
        J=K+1
        YAXE(J)=TOCOX-KKIMK
        K=j+M
        J=j+
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$I K A T E 2400$
$I C O C E=1$
$M C D E L=424$
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    IF{I.LT.J)GO TO IO
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    I=I+1
    LA(I)=M+1-MVEK
    SB(I)=M+MVER+3
    20
        IA(I)=IJ(I- I)+1
        IFII.EGOJ;GOTO 29
    I=I:I
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        J=2*M+
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        IB(I)=M+1
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    YHAE{I)=Y(1)=KKINK
    K=4\geq1
    OC j0 N=2,K
    AMXE(N)= AAXE(N-1)+XINC
    E YAXE(N)= YAXE(N-I)
    KLUKX=XAXE(K)+1
    XACYY=YAXE(<)
    ```


```

    L=K+2
    K=x+1+1,\ER
    00 6.N N=.KK
    AnxE(N)=人maE(%-1)-(XINC/2)
    S YL\lambdaE(N)=YANE (N-E) +YINC
    TFUO-A=人\dot{AE}(K)-XK=\iJK
    İCO-Y=Y\dot{A}\tilde{C}(KJ+YKINN
    ג\dot{A}AE{K+1}=1nculsג-0.う
    YAXE (K+1)=TFCG冒多
    L二K+E
    K=K+1+(M-MVEN)
    OO 7! \J=6.
    XAKE(N)=人AXE (H-1)-4工流
    7\ YAAE (N)=YANE (N-1)
    EETUEH
    ENO
    ```
```

    SUYROUTINE LINTLE (A,Y,XAXE,YAXE,M,MVER,RCORX,RCORY,XKIAK)
    ```

```

    PRIHT, & GZVE NUMJÉfi UF DIUISIONS UN BASE-AAE (INTEGER)z
    PEAO**M
    ```

```

c

* THisj sevtiun Galculates the triang.eys ccoroinates.*
3ASE=10%3.1415926530/180.

```

```

    SFM60=SEM(KADIG
    XINC=3ASE/M
    YINC=ALT U/M
    y(1)=0.
    Y(1)=0.
    J=4+1
    x (H) 2S N = = % % 
    X(H)=X(N-1) +XINC
    Y(N)=r(N-1)
    I=J+1
    J=S+MVER
    OC 35 N={,J
    x(N)=x(id-1)-(xINCC/2)
    Y(N)=Y(N-1)+YINL
    NPTS=G*M+10
    I=J+1
    J=J+(M-4VEF)
    U0 4S N=`!J
    M(N)=x{N=1}-xINC
    i=j+1
    J=J +MVEK
    X(N)=x(N-i)-(AINC/2)
    41 Y(H)=r (iN-1)-rINC
    THIS SECTION CALCULATES THE AXESA COOROINATES.*
    AKOS60=COS (R4O)
    XK&NK=0.5*XKOS60
    YKINK=0.5*SENGO
    Y\DeltaXE{1}=A{11)+XKINKK
    Y\DeltaA=11
    00 5u N=2,K
    ```

```

    5J YAXE (H)=YAAE (N-1)
    <CORA=A4x E(<)*0.5
    \しURY=YAXE(K)
    XAXE(K+1)={隹X-XKINK
    \MXE(K+1)={&UKX=XKINK
    !=K+2
    K=K+1+MVEत
    00 50 N=L,K
    \lambdaAXE(N)=\lambdaAAE(N-1)-(XINC/2)
    60 YムגE(N)=YAXE(N-1)+YINC
    XAXE(K+1)=A4XE(K)-XKINK
    YAXE(K+1)=\AXEE(K) +YKINK
    l=KCZ
    K=K+1+(M-HvER)
    00 70 H=L:K
    XAAE(N)=AAXE(N-1)-XINC
    70 YA人E(N)= YA人三隹-1)
TLGJ-A =^4\lambdaE(K)-0.5
TLGOGY=YAXE(K)
XAXE (K+1)=ILGO\&A-(AINL/2)
YAAE (K+1)=T-CDKY Y-YKI:NK
L=K\2
K=K+1+11VER
OO 30 N=L\&K
\lambdaA^E (N)=\lambdaAXE(i,-1)-(KINC/?)
80 YAXE (N)=YAA ミ(G-: )-YINL
EETUFHI
ENO

```
```

        SUsmuUTINE LIRIV
        GMMLN/PLT,IJUUE,WEJTH:HEIGHT,MUUEL,TQATE
        , (10), (AN(1000),I3(1000)
    C**
    KENUGINFGFIAATGON ANJ CSEATE THE COJROINATES FUR THE SEIHEOF
    LiNFUKHITTIUN is= NUM3E& UF OIVISICNS ON 3\muSE-AXE (M).
        NU:\ERR OF OLVISI (NS ON SIDE-AXE (MVEK).
    CA_L LINTRI(X,Y,XAKE,YAXE,M,MVER,KCORX,RCORY,THCUKY,TKCUEX)
    FIHC THE FEA. HIENITUM aNE MANIMUM.
        X.HIN=XAXE(1)-0.5
        AMAA=FCON
        YMMAX=TrGCJP苗
        YMINEQ&ORY
    ```


```

    * PEINT *, LGIVE. 1.JIUF-3OTTQM VALUEG (SICE). THE &NCREMENTFUR THESIOE. (INTEGER)
    ```


```

    * PFINT *, &GIVE. 1.TOF-RIGHT JALUE (TOP). THE INCFEMENT FGR THE TOP. (INTEGER)
    ```

```

    EEAD*,ULGMAA* ENKTH;HOT
    *PKANT **GHJE MAIABLE NAMES. 1. JASE VAFIABLE.
    ```

```

    FEAD VIUCHVNEWVAROIOLUV
    PFINT &,AIF IEKTENMINALOG GESET OAGE ANO DEFAULT
    ```

```

L*** MAKEFHYSILAL SET-UP FUR PLOTTING.*
Cm_L SETUP(XMIN.MMAX, YMIN, YMAX)
C** CMEL SETUP(XMIN.MMA, YMIN, YMAX)
(ALL SIOPES(1,MVEH,1A,I3)
L**** PLUT POSiTIVE SLOPES.*
J=4+1
I=0
j=k19 K=1,\mp@code{1,2}
CALL PEOT(AAAC(IAG(I)),YAXE(IA(I)),0,0)

```

```

    0-2mpime
    6***
23
M=J*1
I=K 1 <=LOJ,2
ELO-F!OT(A(FA(i)),Y(IN(Z)

```

```

    l&-FPLOE(AMAE(rA(I)
    ```

```

    1: CUNTINUE
    C**
21 L=J+1
J=2*4+3
OO 12 K=ニ.J.2
=K
*M(I)),Y(FA(I)), J,0)

```

```

        i = i +1
        CHL FLUP(AAXE(IA(:I),YAXE(IN(Z)),0,0)
        CALGFILUT(x(IU(L)),Y(İ(I)I,1,0)
        JECTIONTTU LABEL PHE SENSO:R. *
    ```
    DC 50 iN=1, J, NLE
    YAB \(\mathrm{Y}=\mathrm{XAA} \mathrm{E}(1)=0.35\)
    \(Y_{6} A S=Y A \mathcal{E}(i d)=j 061\)
    50 GALL NUITBER(XLA3, YLAJ,60., 0. i, 7, JALUE)
    \(\forall A_{L} U E=V A_{L} U E+(I N K \pm \# N J B)\)
    \(k=j+1\)
    \(J=M+M \vee E \cdot R+2\)
    JALU \(=1\) M
    00 七0 \(N=K\), J, iNus
    人 \(649=\wedge \Delta \times\) ( H )
    YLAB=YAXE (H)
    CALL NUMGER(ALAJ,Y-AJ,0.,J.1,7,VALUE)
    0 VACUE=VALUE-íAAKSANUSi
    \(k=j: 1\)
\(j=2\)

    \(\therefore \operatorname{LA} 3=x \mu x \in(i j)\)
    YLAB=rax
    79 VAL NUH BERIALAS, PLAD,90.,0.1,7,VALUEI
    VAEUE =VALUK-TENKT FOTO
    IVAF=IULUV


    VAF =NEHVAK
    MLA \(=A A X E(M+M \forall E R+1)+1) \cdot 35\)

    CALLSYM3GL(ALA3,YLA子,-60.,0.2,10,IVARI
    IVAm=IGALV
    ALAB=人4x
    KA \(=Y\) Y

    CAETURN
    RETO
ENO
```

    Sugfoutine lilev
    COMMUN/PLT/IGOUE, NLJTH,HEIGHT,MOJEL,IFATE
    ```



```

* INFOFMmTLON is= NUMBER OF CIVISIONS ON 3ASE-AXE (M).
LAL NU:YER OF DIVISJCNS ON SIDE-AXE (MVEK).
GGNL THELLEAA,Y,AAXE, YAXE,Y,YNEKR,RCOPX,REOFY,XKIMKI
XNEN=X\&AE\Z*M+4+i\VERI-גKINK
AM\mux=FíCJEx
YM\&A=Y程 (1+1+3+MVER)
YMIN=E.CORY
PRINT ,\&GZVE. 1.BOTTOM-LEFT VLLUE. (3ASEI

```

```

    * REAO*:NALMAXOINKÖNNO
    ```


```

    &EAU, WHAA,IVKS.NOSN UKUER.......*
    ```

```

    REAO*,OLOALN,&NKT:NUT
    *FLINT , FGIVE VARIAGLENNAMES. 2. BASE VARIABLE.
    *
    * TEAO MAKINUM 1O LETTEZS EACH. NANES BETWEENBLEGOTTATION MARKS.*
    MEAO PATCGALV,NEWVNARSSOLDCH.. NAMES
    *     * MAKE PHYSIUAL SET-UP FUN, PLOTTING
*     * PLOT THE UATA INXTHE GIVEN SEGUENCE.*
* CALLI PEFSLUYM,MVER,IA,E3I
FLUT PGSITIVE SLUPES.*
J=4+1
I=0
OOKK 1CK=1,J,2
EAK fal fugryxane
CALL FLUT(XMAEIIA(I), YAXE(IA(I):,0,0)
CALG F\&OT(AIFj(I)\,Y(IB(I)|,1,0)
IFII.Eu.J\GU IO 20
I=I+1

```

```

    10 CONfITuE
    ```

じ
\(L=j+\)
\(j=j+1\) NEGATIVE دLOPES．
JO，MーMVER＋2
＝K 11 ィンレ・Jって
CAK
EALL FLOT（A（IA（I））Y Y（IA（I））O，O）

IFIE．EL．JIGO TO 21
\(I=I+1\)
CALL F－OT（AAXE（IA（i）），VAXE（IA（I）），J，0）

CONTIMUS
21 PLOT NOOLOPE LINES．＊
\(21 L=j+1\)
J0 12
\(\sum_{i=k} 12 \kappa=\llcorner, J, 2\)

IF
\(I=\bar{j}+1\)

12 CCITINUE TUG LASE．THE SENSOR．

CO 100 it \(=1, J, N U S\)
ALAG＝入AAE（N）
YiAÓO \(=\) YXE（H）－0． 1
GALL NUMSEK（ALAG，YLA3，－0U．0．0．1．7．VALUE）
VALUE＝VALUE－（IIKE＊NOHi
IVAF＝ICAL

CALL SYMUULIXーAX，YLA3，J．，U．2，1U，IVARI
\(K=1+M \vee E R+3\)
\(J=2+4+3\)
VALUと＝OLJHIN
\(00119 \quad N=K, J\) ，HOT
XL4 \(4=X A X E(N)\)
YLAJ＝YAXE（N）－0．05



\(\hat{Y}\) ㄴ \(A=X A X E(2 * M+3)+3 . L\)
\(Y\) LAE \(A K X E(Z+M+3)+0.9\)
EALL SYMUUL（XLAG，YLAJ，J．，0．2，10，IVAR）
\(K=J+1\)
MVE？
VALUE＝WY立A

\(X L \operatorname{Hin}=X A X E(N)\)
\(Y L\) M \(3=Y A E(N)\)
LiLL IUUMBEF（A，AB，YLA3，J．，0．1．7．VALUEI
120
VALUE＝VALUE（ I AKS－INUSJ
IVAR＝TEWHAK
\(X L\) M \(J=X A X E(2+H+4)-1.6\)
\(Y L A E=Y A X E(2+H+-1-1.5\)
CALL SY 1 SUL（ALASOYLAB，60．，0．2．10．IVAR）
CALL SY 1 PULIALAO，YLAS，60．．．0．2．10．IVARI
CALY PLOTENO
ENO

SUGROUTINE LOGRIVIXG，Y \(3, X J, Y E, X T, Y T, x A A E G, Y A X E S, X A X E S, Y A X E S, X A X E T, ~\)
－YLXET，VALOKJ，VALORS，VALURT，M，MVEQ，MOPOI

－XES（1），YAAES（1），XAXETII），YAXET（1），VALORG（1），VALORSIII，VALORT（I）

FEAS M，MVER，MTOP
REALNT：M，MVER，VAGUES FQR \(3 A S E\) VARIABLE I FROM LEFT TO RIGHT．．．．IA
PEMD ，（VALURH（N），NEL，M）
PKINT ，\(\ddagger\) VALUES FOR SIUE VAFIABLE IFROM BOTTOM TC TOP．．．．1F
KEAO ，（VALJRSIN），N二 1 \＆MVER）
PRINT＊ \(\mathcal{I}\) VALUES FJR FOP VARIAJLE IFROM RIGHT TO LEFT．．．It
KEMO ，（VALOKI（N），N＝1，HTOP）
KAD \(=60: 3.14154265361130\) ．
SENGO＝SIN\｛RAU）
XKOS60＝COS（RAO）
גKINK \(=0.5+x K 0 S 60\)
XKINK \(=0.5 * \times K O S 60\)
\(Y K \sum N K=0.54 \sum E N 60\)
BASE＝10．
SIUEX＝BASE／2＊xKOS60
SIUEY＝3ASE／2＊KENGO
XPUINT \(=0\).
\(Y P O I N T=0\).
Xg（1）\(=x P O I N T+d A S E\)
\(X G(1)=X P O I N T\)
\(Y B(1)=Y P O I N T\)
```

    C****
    callulate the covroinates of the gase-axe ano the tcp trian.*
        AAXEO(1)=APOINT-XKINK
        YAAEO(I)=YPOINT-YKINK
        xT(1)=xPOINT +SIUEX
        CALMAX = VALORGI(1)
        DMEX=ALDGIU(CALMAX)
        CALMIN=VALORB(M)
        DMIN=ALOLIO(GALMIN)
        OOIV=OMAX-JMITN
        <<D=0
        N=1
    N=N+1
    VALUE=VALURG(N)
    OVAL=ALOG1U(VALUE)
    AUCUM=XINC+AXD
        XINC=(OUVAL-OMIN)/ODIV)*QASE
        XUIST= BASE-XINC
        XAXEG(N)=AAXEB(1)+XOIST
        YAAEB(N) = YAXEB(1)
        IF(XOLST:EU: 5:0) GO TO 30
        XT(N)=XT(1)+AEIST
        VIN)=YT(1
        GO TO 10
        \lambdaIN)=x/(1)+BASE/2
        YT(N)=rT(1)
        AR=AT(N)
        N=N+1
        VALUE = VALORO (N)
        OVALLALOLINI VALUEI
        XOOUM=XINE+AXD
        XINC=((OVAL-UMIN)/DOIV)*3ASE
        AOIST=BASE-XINC
        XmXEd(N)=xaxE3(1)+XOIST
    ```

```

        XAO OXDOUM-AINC
        XT(N)=XTR + (XXD* \KOSj0)
        YT(N)=rTR-(xAD*SEN60)
        IF(N.EGOH)60 TO 4T
        G0 T0 3i
        35 XTP=XT(1)+EASE/C
        rTr=r\(1)
        XOOUH= ЗASE/?
        AKC=XOOUM-AINC
        GO TO 3G
    C****
C***
4) xaxes(1)=xB(1)+0.5
YAAES(1)= YG(1)
XS{1)=XPOINT
YS(1)=rpuINT
WMAX=VALOKS(MVER)
OMAX=ALOG1O (WMAA)
WMI N=V\&\URS (1)
OMIN=ALJGIO(WHIN)
OMIN=ALMASDGHIN
K=MVEF
OO 50 N=2,K
VALUE=VALOKS (N)
OVAL=ALOG10 (VALUE)
YINC=((OVA=-UMIN)/OUIV)*SIJEY
XLNC=YING/TAN(RAO)
\triangleO:SI= 人INC
YO\perpST= Y:NC
XAXES(N)=AAXES(1)-XOIST
YANES(N)=YARESI
50 YS(N)=\S(1)+XOIST
C****
calculate the couroinates of the top-axe ano gase-thian.*
XAXET(1)=XTR-XKINK
YGAET(I)=YTN+YKINNK
OLOMAA=VÁLORT(MT SP)
OMAX=ALUG10(OL[MAX)
OLOHIN=VALURTIII
UMIN=ALUGIG (OLDMINI
HOI V=OMAX-0MIN
K=4 TOP
DO 60 N=?,K
VALUE=VABORTT (NI
VVLUE=ALOLONT(NINUE)
GVAL=ALOG1O(VALUE)
TOP = SASE/2
XINC=(IOVAL-JMINNI/DUIV)*TOP
XOLST=XINC
xAXET(N) = AAXET(1)-XDIST
YAXET(N)= YAXET(1)
XB(N)=XJ(1)-XDIST
60 YB(N)=r9(1)
RETURN
ENO

```

\section*{SUbFUUTINE NLIRIV}

CCMMUN／PLTHEUEE，WIUTH，HELGHT，MOJEL，LRATE


FEAD INFURMATIUN ANO GKEATE THE COOROL，IATES FOK SENSUR



C＊＊＊VLOKY，VALURد，VA CORT，M，HVEK，Mf OFI
Ç．＊yake the rean minimuy aho maximuy＊
TRCORY＝YAAES（MVEF）＋ 7.25
KCOOr＝YA，E \(3(1)\)
FCOFP \(x=\lambda \lambda E j(y)+1\)
LCOPA＝スA入E日（1）－0．5
XMIN＝LCO
MMA \(=\kappa\) CORA
YMAX
YMAX＝TEDUAY
YMIN＝OCJTr
＊MiNT＊tGive vaciab－e names．\(\frac{1}{2}\) ：jase variable．

＊KEAD MAXIMUH L
KEAGT：PCALYPENVARGOLJV RESET PAGE AND OEFAULT

Co＊＊＊＊MAKE PHYSIGAL SET－UP FOR PLOTING．＊
GALL \(\operatorname{setuF}(i m i n, x M A x, y m I N, Y m A X)\)
Co＊＊＊Plut pusitive siopes．．．．．．．．．．．．．
\(J=4\)
00
\(\sum_{i=k}^{0} 10 \quad k=1, v, 2\)
CALL PLOTG（AAAES（I）PAKE3（i），J，GI
CALL POT（AT（I），YTII）， 1,01
ifir．EG．JIGO TO 20
cíl
CALL FLOT（xT（I），YT（I），íJ）
10 continue

20
\(j=m T u P\)
\(\pm=K 11 K=1, J, 2\)
I＝k
CALL PLOTA子（i），Ya（I），子a）

\(\stackrel{+}{\mathrm{F}}=\mathrm{i}+1\)
CALL FLUT（AMAET，I），PAAET（I），O，O）

11 GONTINUE

3］J＝mver
20 \(12 k=:, J, 2\)
I＝K


+1 － 1 （

12
\(40 \mathrm{~K}=\)
\(0050 \mathrm{~N}=1\) K

rLAEEYMXE3（N）－C．61
50

\(\therefore \forall A F=I C A-V\)
XLA \(=\) A\＆AC3（1）＋4
YLAE \(\mathrm{YAXEB}(1)-4.90\)

K＝MVE．
\(0060 \quad \mathrm{H}=1, \mathrm{~K}\)

YLAGEYAXES（N）
60 CALL MUALERSN）
60 CALL inUHJEP（ALAB，YLAJ．J．，0．1，7，VALUEI
```

            IVAF=NEWVAI
            ALAG=xAXES({VEN)+1.73
            YLA =Y&XES (4\cupEK)-1.05
            KALLUSTMJOL(xLAS,Y_A3,-60.,0.2,1目,IVAR)
            00 7ij N=12K
            KLAB=x&AET(N)
            YLA J=Y&XET(N)
    7 0
            VALUE=VA ONT
        CALL NUMJC(NA
            IVAR NUMBEK(XLAB,YLAB,90.,3.1,7,VALUE)
            IVAR=ICLOY
            XLAG=XAXET(1)-3.5
            YLAE=YAXET(1)-3.5
            CALLL SYISUL(ANGMB,YLAJ,J.,0.2,10,IVARI
            CALL PLOTEND
            END
            SuBgOUTINE NLILE:
            COMMON/PLT/ICOUE,WIOTH,HEIGHT,MOUEL,IRATE
    ```


```

    *ODi.VALORS(300), Jalovt(300)
    C**
*VALOKEOGRIVIXZ,YG,XALORS,VALJKT,MS,MYGYTGAAXEB,YAXEB,XAXES,YAXES,XAAET, YAXET,
C**
MAKE THE REAL MINI:TU:I ANC MAXIMUM*
TFCOFY=YAXES(MVER) +0.25
RCOPY= YAXEB(M)
HORX=XAXEO (M) +1
LCORX=AAXES(1)-0.5
XMIN=LCURX
XMAX=\&CORX
YMAX=INCORY
Y:AIN=FCORY
PRIHT *, \&GIVE VAFIABLE NAMES. I. UASE VARIABLE.
* MAXIMUM IO S. SIDE VARIABLE.
REAOM, ICALV,NEWVTTEKS EACH.. NA,HES GETHEEN GUOTTATION MARKS.A
KEAO*, ICALV,NEWVAK,IOLDV

```

```

C:****MAKE PHYSICAL SET-UP FUR PLOTING.*
CALL SETUP(AMIN,XMAX, YMIN,YMAXI
C**
C**** PLOT POSITIVE ふLOPES............
J=M
\=K}10\quadK=1,J,
= = K
XAAEB(I)=AMAX-XAXE3(I)
XT(I)=\lambdaMAA-XT (I)
CALL FLOT(XAXES(I),YAXEB(I),0,0)
CALL PLOT(AT(I),YTi(I),1,O)
IFII.EQ.JIGO TO 20
I=I+1
M, =XMAX-XAXEG(II
XT(II=XMAX-XT(i)
CALL PLOf (XIG (I),YT(I!,O,O)
O CONTINUE AAAES(I), YAXEB (I),1,0)
C**
C**
29
J=MTOP
00 11 K=1,J,2
I=K
XA(I)=XMAX-XG(I)
XAXETII!=AMAX-AAXETIII
GALL PLOT(AG(I),YB(I),J,OI
CALL FLuT(AAAETIII,YAXEf(I),1,0)
IF(I.EQ.JI vO TO 30
I= I+1
XAXET(I) = МMAA-XAXET(I)
xG(I)=xMAX-x}(i)
CALL FLUT(AAXET(I),YAAET (I),G,C)
CALGFLOT(AZ(I),YB(I),1,0)
11 CONTINUE
C=*
C**** flUT NU-jLUPE LINES...................
30 J=MVER
00 12 K=1,J,2
\&=K
xS(I)=人4ム^-xS{ま゙

```
```

    \(X \dot{A} X E S(I)=X M A A-X A X E S(I)\)
    CALL PLOT(XSTA), YS(I), 0, OI
    CALL PEOTXXAXES IST,YAXES(I), 1, 6)
    \(I F T\)
    $I=T$
$I=T+1$
$X S(I)=X M A X-X S(I)$
XAAES (I) $\equiv \therefore$ AAX-XAXES (I)
CALL PLOT (AAXES(I), YSXES(I), 0,0)
CALLTPLOT(ASII), YS(I),1,0)
*
C* + +
C
40
$\begin{aligned} & K=4 \\ & 00 \\ & K 0\end{aligned} \quad N=1 \cdot K$
XLA3=XAXEG(N)
YLAB=YAXEB(N)-0. 1
value=valuk ${ }^{2}$ (N)
gal mumijeri(xLab, ylag,-60..0.1,7,VALUE)
$V A R=I C A L V$
ALAg=AAXEB(1)-6
rLAS=YAXEB(1)-0.90
CALL SYM JUL (XLAB, YLAB, J., 0.2, 10, LVAR)
K=MyEF
$0050 \quad N=1, K$
XLA $3=x \Delta X E S(N)-0.8$
YLA $3=Y A X E S(N)$
Válue valuis (N)
50 WALL NUMJEK(ALA3, YLA3, 3.,0.1,7,VALUE)
IVAR =NENVAR
$X L A 3=X A X E S(M V E R)=3.1$
$Y L A X=Y A X E S(M V E R)=3.1$
CALL SYMJOL (ALAB, YLAJ, $\mathrm{O} O ., 0.2,10$, IVAR)
$K=M$ TOP
$0070 \mathrm{~N}=1 \mathrm{~K}$
XLATB=XAXETIN
$X L A Y=X A X E T(N)$
$Y L A S=Y A X E T(N)$
$Y L A B=Y A X E T(N)$
$V A L U E=V A L U R T(N)$
70
CALL NUMUEK (XLAB, YLAH, 30., 0.1,7, VALUE)
IVA $H=$ IOLUV
XLAB=xAXET(1)+1.5
CALL SYMBOL IXLAD. YLA3,0.,0.2,10.IVARI
CALL SYMBOLIX
CAL PLOTEND
EETUKN
ENO
ENO

```
```


[^0]:    ${ }^{1}$ Consistency requires that any one of these methods can be used but in infrequent cases, the generally accepted accounting principle allows switching from one method to another. It has to be noted in the auditor's opinion and justified in the financial statement's footnotes.

[^1]:    ${ }^{5}$ Any other triangular form can be used, but the intersection of the three sets of coordinates becomes less sharp and well-defined. See [14], Chapter 43.

[^2]:    * Assumptions relating to all of the casb flow components on reverse side must be bighlighted

