AN ABSTRACT OF THE THESIS OF


Abstract approved: ____________________________

Margaret L. Niess

This study described how and why a Korean elementary teacher taught mathematics as he did. Specifically, the study sought to describe his beliefs about the teaching and learning of mathematics and relate them to patterns of classroom interaction and norms.

An ethnographic inquiry guided the study of one third grade 10-year veteran teacher over three months in Korea. Through participant observation, the researcher observed Teacher Lee’s teaching paying special attention to the mathematics lessons for one class of 45 students daily Monday through Saturday. Formal and informal interviews were used to collect data on the teacher and 17 of his students as well as other teachers (such as four teachers in the third grade), two principals, two mothers, and three beginning teachers. In addition to participant observation and interviews, a variety of documents were also collected, including newspapers, articles from journals, test items used in the teacher’s classroom, daily worksheets, curriculum guide book, mathematics
textbook, the school's newspaper. All videotapes and audiotapes were transcribed for inductive analysis.

The analysis generated six major themes of the teacher's beliefs about the teaching and learning of mathematics and how those beliefs impacted the interactions and norms: (a) behave orderly, think freely; (b) teaching mathematics with understanding; (c) manipulative activities and games; (d) discourse-oriented teaching practices; (e) mathematical tasks; and (f) professional development. The teacher's beliefs about the teaching and learning of mathematics were closely related to the interaction patterns and classroom norms. This close relationship implies that identifying interaction patterns and classroom norms may shed light on understanding teachers' beliefs and teaching practices. The teacher's study group activity was a major professional development factor in promoting the consistent relationship.

Implications and recommendations included (a) the need for more study of classroom norms and interactions as practical knowledge of teaching mathematics, (b) the need for investigating the effect of a study group to support teacher change, (c) the importance of the relationship between pedagogical content knowledge and teachers' beliefs, and (d) the need for more study of classroom management for teaching mathematics using understanding and discourse as an instructional strategy.
A KOREAN ELEMENTARY TEACHER'S BELIEFS ABOUT TEACHING AND LEARNING AND ITS IMPACT ON INTERACTIONS AND NORMS IN MATHEMATICS CLASSROOM

by

Cheong-soo Cho

A THESIS

submitted to

Oregon State University

in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

Completed May 3, 2000
Commencement June 2000
Doctor of Philosophy thesis of Cheong-soo Cho presented on May 3, 2000

APPROVED:

Redacted for privacy

Major Professor, representing Mathematics Education

Redacted for privacy

Chair of Department of Science and Mathematics Education

Redacted for privacy

Dean of Graduate School

I understand that my thesis will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my thesis to any reader upon request.

Redacted for privacy

Cheong-soo Cho, Author
ACKNOWLEDGEMENT

I would like to thank my friend, the teacher in this study, Teacher Lee, for his time, energy, and interest to participate in this study. I am thankful to his third grade students who let a stranger sit beside them and watch.

I would like to thank my major professor Dr. Maggie Niess for her guidance, helpful comments, providing financial support for allowing me to concentrate more on my studies. I would like to express my appreciation to the members of my committee, Dr. Norm Lederman, Dr. Dianne Erickson, and Dr. Thomas Dick for their support over five years. In addition, my deepest appreciation for their caring goes to the faculty, staff, and fellow graduate students in the Department of Science and Mathematics Education.

I would like to express gratitude to Marnie and Clell Conrad, whose unconditional love, support, and encouragement were invaluable over a period of five years as I completed my study in the U. S. Especially, I would like to thank Clell Conrad for his generous love and care, who passed away June 22, 1999 while I was conducting this study in Korea.

I would like to express my appreciation to Joan Knapp, who edited this bulky dissertation and voluntarily helped broaden my knowledge of English.

Finally, I wish to thank and dedicate this dissertation to my wife and our parents whose loving support and encouragement have always been without limits.
# Table of Contents

## Chapter I: The Problem

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Statement of the Problem</td>
<td>2</td>
</tr>
<tr>
<td>Significance of the Study</td>
<td>9</td>
</tr>
</tbody>
</table>

## Chapter II: Review of the Literature

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>12</td>
</tr>
<tr>
<td>Definitions and Descriptions of Beliefs</td>
<td>13</td>
</tr>
<tr>
<td>Distinctions between Beliefs and Knowledge</td>
<td>18</td>
</tr>
<tr>
<td>Teachers' Beliefs about Mathematics</td>
<td>25</td>
</tr>
<tr>
<td>Social and Cultural Perspective of Mathematics and the Teaching and Learning of Mathematics</td>
<td>33</td>
</tr>
<tr>
<td>Teachers' Beliefs about the Teaching and Learning of Mathematics</td>
<td>47</td>
</tr>
<tr>
<td>The Relationships between Teachers' Beliefs and Teaching Practices</td>
<td>58</td>
</tr>
<tr>
<td>Ethnographic Research Tradition</td>
<td>76</td>
</tr>
<tr>
<td>Conclusions and Recommendations</td>
<td>87</td>
</tr>
</tbody>
</table>

## Chapter III: Methodology

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>93</td>
</tr>
<tr>
<td>Design of the Study</td>
<td>94</td>
</tr>
<tr>
<td>Locating a Teacher</td>
<td>95</td>
</tr>
<tr>
<td>Teacher Lee and His Students</td>
<td>101</td>
</tr>
</tbody>
</table>
# TABLE OF CONTENTS (Continued)

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Description of the Community and the School</td>
<td>105</td>
</tr>
<tr>
<td>The Community</td>
<td>105</td>
</tr>
<tr>
<td>The School, Student Body, and Staff</td>
<td>106</td>
</tr>
<tr>
<td>Curriculum Organization of the School</td>
<td>110</td>
</tr>
<tr>
<td>Gaining Entry: The First Day of Fieldwork</td>
<td>112</td>
</tr>
<tr>
<td>Data Collection</td>
<td>117</td>
</tr>
<tr>
<td>Participant Observation</td>
<td>117</td>
</tr>
<tr>
<td>Ethnographic Interviews</td>
<td>125</td>
</tr>
<tr>
<td>Documents</td>
<td>130</td>
</tr>
<tr>
<td>Data Analysis</td>
<td>131</td>
</tr>
<tr>
<td>The Researcher Role</td>
<td>138</td>
</tr>
<tr>
<td><strong>CHAPTER IV: RESULTS</strong></td>
<td>144</td>
</tr>
<tr>
<td>Introduction</td>
<td>144</td>
</tr>
<tr>
<td>Current Educational Reforms in Korea</td>
<td>145</td>
</tr>
<tr>
<td>A Day of Teacher Lee’s Classroom</td>
<td>149</td>
</tr>
<tr>
<td>“Behave Orderly, Think Freely”: Regulations of Teacher Lee’s Classroom</td>
<td>157</td>
</tr>
<tr>
<td>Teaching Mathematics with Understanding</td>
<td>170</td>
</tr>
<tr>
<td>Students’ Own Ways of Understanding</td>
<td>173</td>
</tr>
<tr>
<td>Development of Understanding of Concepts and Procedures</td>
<td>180</td>
</tr>
<tr>
<td>Objectives Stated by Students</td>
<td>186</td>
</tr>
<tr>
<td>Using Students’ Everyday Experiences</td>
<td>190</td>
</tr>
<tr>
<td>Process-Oriented Practice</td>
<td>195</td>
</tr>
<tr>
<td>Summary</td>
<td>203</td>
</tr>
<tr>
<td>Manipulative Activities and Games for Conceptual Understanding</td>
<td>205</td>
</tr>
<tr>
<td>Using Manipulative Activities</td>
<td>207</td>
</tr>
</tbody>
</table>
TABLE OF CONTENTS (Continued)

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Using Games for Fun and Interest</td>
<td>222</td>
</tr>
<tr>
<td>Summary</td>
<td>234</td>
</tr>
<tr>
<td>Discourse-Oriented Mathematics Teaching</td>
<td>235</td>
</tr>
<tr>
<td>Interaction Patterns and Classroom Norms for Discourse</td>
<td>239</td>
</tr>
<tr>
<td>On-Board Presentation</td>
<td>258</td>
</tr>
<tr>
<td>Standing-Up Presentation</td>
<td>269</td>
</tr>
<tr>
<td>Students’ Responses to Discourse</td>
<td>275</td>
</tr>
<tr>
<td>Group Learning</td>
<td>277</td>
</tr>
<tr>
<td>Summary</td>
<td>285</td>
</tr>
<tr>
<td>Mathematical Tasks for Understanding and Discourse</td>
<td>288</td>
</tr>
<tr>
<td>Open-Ended Tasks</td>
<td>289</td>
</tr>
<tr>
<td>Using Students’ Mistakes and Ideas for Tasks</td>
<td>301</td>
</tr>
<tr>
<td>Summary</td>
<td>307</td>
</tr>
<tr>
<td>Teacher Lee’s Professional Development</td>
<td>308</td>
</tr>
<tr>
<td>The Study Group of Elementary School Mathematics (SGESM)</td>
<td>312</td>
</tr>
<tr>
<td>The Study Group of Mathematics Teaching (SGMT)</td>
<td>318</td>
</tr>
<tr>
<td>Other Resources for Professional Development</td>
<td>322</td>
</tr>
<tr>
<td>Summary</td>
<td>325</td>
</tr>
<tr>
<td>CHAPTER V: DISCUSSION AND CONCLUSIONS</td>
<td>327</td>
</tr>
<tr>
<td>Introduction</td>
<td>327</td>
</tr>
<tr>
<td>Discussion of the Main Findings</td>
<td>328</td>
</tr>
<tr>
<td>Teacher Lee’s Instructional Sequence of a Lesson</td>
<td>328</td>
</tr>
<tr>
<td>The Change in His Pedagogical Beliefs and Teaching Practices</td>
<td>334</td>
</tr>
<tr>
<td>The Relationship Between His Pedagogical Beliefs and Interaction Patterns and Norms</td>
<td>339</td>
</tr>
<tr>
<td>Facilitators of the Relationship Between Beliefs and Interaction Patterns and Norms</td>
<td>345</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
</tr>
<tr>
<td>--------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>Limitations of the Study</td>
<td>353</td>
</tr>
<tr>
<td>Implications and Recommendations for Future Research</td>
<td>356</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>364</td>
</tr>
<tr>
<td>APPENDIX: Informed Consent Forms</td>
<td>374</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Schematic Diagram of <em>Mi-dong</em> Elementary School</td>
<td>107</td>
</tr>
<tr>
<td>2. Teacher Lee’s Classroom</td>
<td>150</td>
</tr>
<tr>
<td>3. The Sequence of On-Board Presentation</td>
<td>260</td>
</tr>
<tr>
<td>4. Teacher Lee’s Instructional Sequence of a Lesson</td>
<td>329</td>
</tr>
<tr>
<td>5. Teacher Lee’s Belief System of the Teaching and Learning of Mathematics</td>
<td>340</td>
</tr>
</tbody>
</table>
A Korean Elementary Teacher’s Beliefs about Teaching and Learning and Its Impact on Interactions and Norms in Mathematics Classroom

CHAPTER I
THE PROBLEM

Introduction

Once the list of students’ solutions was up on the board, they were open for discussion and revision. Students often began by explaining why they gave the answer that they did. If they wanted to disagree with an answer that was up on the board, the language that I have taught them to use is, “I want to question so-and-so hypothesis.” (Until the group arrived at a mutually agreed-upon proof that one or more of the answers must be correct, all answers were considered to be hypotheses.) (Lampert, 1990, p. 40)

Her explanations were brief and aimed at demonstrating the procedures that the students were to use in working out the day’s assignment. The bulk of the remaining class time was given to independent seatwork during which the students practiced the procedures taught... She regarded mathematical understanding as tantamount to one’s ability to follow and verbalize a specified procedures to obtain the correct answer or solution to a given task. (Thompson, 1984, pp. 117-118)

These excerpts clearly illustrate how different the two mathematics classrooms are. In one classroom, there is a lot of talk between teacher and students or among students. It might appear a noisy classroom. In the other classroom, little talk can be heard. It might appear a silent and well disciplined classroom. Looking at these classrooms closely, a more interesting difference between them is visible. One teacher considers doing mathematics as conjecture and refutation of mathematics ideas and knowing mathematics means participating in argumentation as a member of a class. On
the other hand, the other considers that doing mathematics means following the rules laid down by the teacher; knowing mathematics means remembering and applying the correct rule when the teacher asks a question; and mathematical truth is determined when the answer is ratified by the teacher. Two questions are raised at this point: “Why are these mathematics classrooms so different?” and “What created these differences?” Many scholars (Ernest, 1989a, 1989b; Hersh, 1997, 1998; Lerman, 1983; Polya, 1981; Steiner, 1987; Thompson, 1984, 1992) argue that these differences originate from classroom teachers’ beliefs about mathematics and the teaching and learning of mathematics. Ernest (1989a) points out that the teacher’s view of the nature of mathematics provides a basis for his or her mental models of the teaching and learning of mathematics. He further claims that views of the nature of mathematics are likely to correspond to views of its teaching and learning in classrooms. Thompson (1992) also notes: “Although the complexity of the relationship between conceptions and practice defies the simplicity of cause and effect, much of the contrast in the teachers’ instructional emphases may be explained by differences in their prevailing views of mathematics” (p. 119).

Statement of the Problem

Current reform in mathematics education has included discussion of and inquiry into the nature of mathematics, mathematics learning, and mathematics teaching (Simon, 1994). Simon points out that reform efforts have been shaped by a number of influences including: humanistic and quasi-empiricist view of the nature of mathematics, constructivist views of mathematics teaching and learning, and research into the culture
of the mathematics classroom. A partial consensus has developed with regard to the nature of mathematics, mathematics learning, and mathematics teaching. This consensus is represented by the National Council of Teachers of Mathematics standards documents (National Council of Teachers of Mathematics [NCTM], 1989, 1991).

The standards documents promote a vision of classroom mathematics in which students engage in exploration of mathematical situations, oral and written communication of ideas, and modification and validation of these ideas. Thus students actively participate as mathematicians, creating mathematics, evaluating mathematics that has been created by members of the mathematical community, and negotiating shared approaches to and standards for these activities. This vision represents a radical departure from traditional mathematics classes, where the teacher and textbook serve as the source of mathematics and the evaluation of mathematical validity (Simon, 1994).

Moreover, the Professional Standards for Teaching Mathematics (NCTM, 1991) specifies five “major shifts” that must take place in mathematics classroom for this new vision to become a reality:

(a) toward classrooms as mathematical communities-away from classrooms as simply collections of individuals;
(b) toward logic and mathematical evidence as verification-away from the teacher as the sole authority for right answers;
(c) toward mathematical reasoning-away from merely memorizing procedures;
(d) toward conjecturing, inventing, and problem solving-away from an emphasis on mechanistic answer finding;
(e) toward connecting mathematics, its ideas, and its applications-away from treating mathematics as a body of isolated concepts and procedures. (p. 3)
According to these reform documents, teachers’ beliefs about mathematics and the teaching and learning of mathematics are viewed as important agents of change in the reform effort currently underway in mathematics education and thus are expected to play a key role in changing schools and classrooms. Paradoxically, however, Prawat (1992a) points out that teachers’ beliefs are also viewed as major obstacles to change because their adherence to outmoded forms of instruction that emphasize factual and procedural knowledge at the expense of deeper levels of understanding. Thus, understanding teachers’ beliefs about mathematics and the teaching and learning of mathematics is a starting point to improve the quality of instructional practices in mathematics classrooms.

Such importance of teachers’ beliefs has already been predicted even 20 years ago by Fenstermacher. Fenstermacher (1978) indicated that teachers’ beliefs are the single most important construct in educational research. Kagan (1992) indicates that teachers’ beliefs “lie at the heart of teaching” (p. 85). As these scholars have suggested, several studies, but unfortunately not many in mathematics education, have shown that teachers’ beliefs about mathematics and the teaching and learning of mathematics significantly affect the form and type of instruction they deliver (Cooney, 1985; Cooney, Shealy, & Arvold, 1998; Prawat, 1992b; Raymond, 1997; Stein, Baxter, & Leinhardt, 1988; Thompson, 1984). Most of the studies have focused on consistencies between teachers’ beliefs and their teaching practices. Some research (Peterson, Fennema, Carpenter, & Loef, 1989; Stein, Baxter, & Leinhardt, 1988; Thompson, 1984) has described consistencies, whereas others have identified inconsistencies (Cooney, 1985; Raymond, 1997). In spite of consistency or inconsistency, teachers’ beliefs about mathematics and the teaching and learning of mathematics will provide insight into understanding how and
why teachers teach mathematics in a certain way. Thus, the importance of teachers’ beliefs and the inconclusive findings suggest that more studies need to investigate this topic in mathematics education.

On the other hand, some studies (Bush, Lamb, & Alsina, 1990; Ernest, 1989a, 1989b; Mura, 1993, 1995) have identified the categories of teachers’ beliefs about the nature of mathematics. Perry’s (1970) scheme of intellectual and ethical development (i.e., dualism, multiplism, relativism, and commitment in relativism) and Ernest’s (1989b) model have been used as conceptual frameworks for data analysis. For example, Ernest (1989b) categorized teachers’ conceptions of the nature of mathematics: (a) the dynamic, problem-driven view of mathematics as a continually expanding field of human inquiry; (b) the view of mathematics as a static but unified body of knowledge; and (c) the view of mathematics as a useful but unrelated collection of facts, rules, and skills. However, these studies did not attempt to associate the nature of mathematics with teaching practices. Although these studies could contribute to understanding the categories of beliefs about the nature of mathematics, categorizing teachers’ beliefs does not appear an appropriate way to study classroom teachers’ beliefs because teachers’ professed beliefs might not be quasi-logical or psychologically central (Green, 1971). In essence, teachers rarely are able to express their beliefs through questionnaires (Mura, 1993, 1995). Most studies on teachers’ beliefs (e.g., Cooney, 1985; Prawat, 1992b; Raymond, 1997; Thompson, 1984) used both interviews and classroom observations. Thompson’s (1992) review of research on teachers’ beliefs about mathematics also indicates that research should more closely examine links between conceptions of mathematics and instructional practice. According to Rokeach (1968), beliefs, which may
be conscious or unconscious, may be inferred from what a person does or says. Thus, classroom teachers’ beliefs about mathematics or the teaching and learning of mathematics can be investigated through classroom observations of instruction and interviews.

One limitation of the current studies on teachers’ beliefs in mathematics education is oversight of the social and cultural aspect of teaching and learning. Bauersfeld (1992) argues that learning is a process of an adaptation to a culture through active participation in social and cultural activity, and, through engaging in constant activities with students, the teacher establishes and maintains a classroom culture. On the contrary, teaching and learning in mathematics classrooms is considered as a process of negotiating mathematical meanings and establishing sociomathematical norms (Cobb, Yackel, & Wood, 1995; Lampert, 1990; Lo, Wheatley, & Smith, 1994; Lo & Wheatley, 1994; Yackel & Cobb, 1996; Yackel, Cobb, & Wood, 1991). Vogit (1995) argues that the terms, mathematical norms or sociomathematical norms are used to describe a criteria of values with regard to mathematical activities. Sociomathematical norms are not obligations that students have to fulfill; these norms facilitate the students’ attempts to direct their activities. For example, the study conducted by Yackel and Cobb (1996) described classroom norms for individual students in small-group activities: (a) Students should figure out solutions that are meaningful to them; (b) students should explain their solution methods to their partner; and (c) students should reach consensus as they work on the activities. A question is raised here: “Where are the norms from?” Norms are always connected to beliefs and values in a society, according to cultural anthropology (Scupin, 1995).
In mathematics classrooms, norms facilitating social interaction implicitly or explicitly are connected to teachers’ beliefs about mathematics and the teaching and learning of mathematics. Lampert’s (1990) study is a guiding example of this line of investigation because the classroom norms of her fifth-grade mathematics classroom came from her conceptions of the philosophy of mathematics based on Lakatos and Polya. According to Lakatos (1995, 1998) and Polya (1954, 1981), doing and knowing mathematics involve a process of proofs and refutations following a path from induction to generalization through bold conjectures. Despite theoretical traditions, these scholars describe the importance of social interaction between teacher and students and view a mathematics classroom as a microcultural community.

The teacher is a major agent in participating in social interaction and negotiating mathematical meaning in this community. At this point, the participation and interaction patterns and levels in mathematics classrooms may be greatly different, depending on the teachers’ beliefs. Nickson (1992) argues that the level of interaction within the mathematics classroom has been found to be constrained by teachers’ beliefs about mathematics. The teachers in two studies aforementioned (e.g., Lampert, 1990; Thompson, 1984) demonstrate the constraints of classroom interaction between the teacher and students due to the teachers’ beliefs about mathematics. Thus, the studies on the social and cultural aspect of the teaching and learning of mathematics indicates that teachers’ beliefs are reflected in engaging social interactions in the classroom. They suggest that patterns and levels of social interaction between teachers and their students should be focused on studying the teachers’ beliefs. Since teachers usually are not able to express their beliefs (Thompson, 1992), describing and identifying the patterns and
norms will bring their unconscious beliefs to the conscious level and help them reflect on their actions in the teaching and learning of mathematics.

Another important incentive to drive this research is that, although Korean students have done well in international mathematics tests (e.g., the Third International Mathematics and Science Study [TIMSS]), no studies exist on how Korean teachers teach mathematics and what their views of mathematics and the teaching and learning of mathematics are. The result of TIMSS conducted between 1994 and 1995 revealed that Singapore and Korea were the top-performing countries at both the third and fourth grades (Mullis et al., 1997). Japan and Hong Kong also performed well at both grades. The seventh and eighth grade, Korea and Japan performed similarly to each other and better than all of the other participating countries except Singapore (Beaton et al., 1996). Several researchers have investigated the mathematics education of these Asian countries, specifically Japan, Hong Kong, Taiwan and China (e.g., Robitaille & Garden, 1989; Rohlen & LeTendre, 1996; Stevensen, Chen, & Lee, 1993; Stevenson & Stigler, 1992). However, mathematics education has not been studied in Korea.

Therefore, the purpose of this investigation, in general, was to describe how and why a Korean elementary teacher teaches mathematics in an everyday classroom as he does. Specifically, the study sought to describe the following questions:

1. What patterns of classroom interaction and social-mathematical norms exist in an elementary Korean mathematics classroom?

2. What beliefs about the teaching and learning of mathematics does the elementary teacher hold?
3. Are the teacher’s beliefs related to the patterns of classroom interaction and social-mathematical norms? What facilitates or constrains the relationship?

For this study, the definition of beliefs includes conceptions, views, perspectives, implicit or personal theories in a broad sense because they share the same meaning of a person’s unique way of looking at, interpreting, and understanding the world. More specifically, the focus of this study was to describe a teacher’s beliefs about the teaching and learning of mathematics through classroom norms and interaction patterns rather than beliefs about mathematics or the nature of mathematics. The teacher in this study expressed difficulty articulating beliefs about mathematics or the nature of mathematics, although literatures maintained that teachers’ beliefs about mathematics or views of the nature of mathematics provide the models of the teaching and learning of mathematics (Ernest, 1989a, 1989b; Hersh, 1997; Nickson, 1992; Thompson, 1984, 1992). In addition, it was learned in the fieldwork that the teachers’ beliefs about the teaching and learning of mathematics were much richer than beliefs about mathematics or the nature of mathematics.

Significance of the Study

This study focused on how the way in which mathematics teachers teach mathematics is influenced by their beliefs about the teaching and learning of mathematics. The study of beliefs is critical to education precisely because, as Kagan (1992) concluded, “the more one reads studies of teacher belief, the more strongly one
suspects that this piebald of personal knowledge lies at the very heart of teaching” (p. 85). Further, Pajares (1992) put the importance of studying teachers’ beliefs in this way:

When they [teachers’ beliefs] are clearly conceptualized, when their key assumptions are examined, when precise meanings are consistently understood and adhered to, and when specific belief constructs are properly assessed and investigated, beliefs can be as Fenstermacher (1978) predicted, the single most important construct in educational research. (p. 329)

Raymond (1997) also pointed out that there are still debates on consistency and inconsistency between teachers’ beliefs and their instructional practices in the mathematics classroom. Thus, this study will add empirical evidence to “clean up the messy construct” (Pajares, 1992).

One distinctive feature of this study is to combine two different research traditions in mathematics education: that is, teachers’ beliefs of the teaching and learning of mathematics with (1) classroom interactions and (2) norms in the mathematics classroom. None of the aforementioned studies has associated them. It seems reasonable that the teacher and students bring their beliefs into the classroom and then, based on these beliefs, social-mathematical norms in the mathematical classroom are mutually established between the teacher and students through their social interaction. Thus, the combination of two traditions could provide a new insight to understanding the teaching and learning of mathematics.

This study will provide the international community of mathematics education with useful information about teaching and learning mathematics in Korea. Much effort to improve educational quality and students’ achievement in mathematics has been made by comparing and contrasting mathematics instructions of different countries, including
Japan, the United States, and Germany. The TIMSS' video and ethnographic study is a good example. This study on a Korean elementary teacher's beliefs will provide an additional and valuable resource in order for mathematics educators to begin to understand the teaching and learning of elementary mathematics in Korea. This study will describe an elementary mathematics classroom in Korea to aid the mathematical community in better interpreting the TIMSS' results.
CHAPTER II

REVIEW OF THE LITERATURE

Introduction

The purpose of this study was to describe how and why a Korean elementary teacher teaches mathematics in an everyday classroom and why he teaches mathematics in a certain way. Specifically, the study sought to describe:

1. What patterns of classroom interaction and social-mathematical norms exist in an elementary Korean mathematics classroom?

2. What beliefs about the teaching and learning of mathematics does the elementary teacher hold?

3. Are the teacher's beliefs related to the patterns of classroom interaction and social-mathematical norms? What facilitates or constrains the relationship?

To describe teachers' beliefs, it was necessary to note the definitions and descriptions of beliefs, in general. In addition to general definitions of beliefs, teachers' beliefs about mathematics were discussed. Since the study aimed to describe teachers' beliefs, distinctions between beliefs and knowledge need review. With teachers' beliefs about mathematics, teachers tended to have beliefs that mathematics should be taught and learned in a certain way. To understand teachers' teaching practices it was necessary to review teachers' beliefs about the teaching and learning of mathematics. And the current view of philosophy of mathematics emphasized social, cultural, and human aspects of the epistemology of mathematical knowledge. This review examined the view of
mathematics and the teaching and learning of mathematics. To understand and describe teachers’ beliefs about their instructional practices, the researcher needed a methodological perspective. This study took the perspective of ethnographic research tradition because in this perspective, a classroom where the teaching and learning of mathematics take place acts primarily as an agent of the culture, transmitting a complex set of values, beliefs, and norms enabling students to participate.

Definitions and Descriptions of Beliefs

Within Green’s (1971) and Fenstermacher’s (1978) philosophical conceptualization of teaching and teacher education, beliefs play a central role. According to them, beliefs are an individual’s understandings of the world and the way it works or should work; these beliefs may be consciously or unconsciously held but they do guide one’s actions. Anthropologists and social psychologists have also contributed to the understanding of beliefs. Rokeach (1968) defines beliefs from the viewpoint of the observer. According to him, beliefs, which may be conscious or unconscious, may be inferred from what a person does or says. Beliefs, he states, are propositions that may begin with the phrase: “I believe that…” (p. ix). Scupin (1995) defined beliefs in the term of cultural anthropology. According to him, beliefs are cultural conventions that concern true or false assumptions, specific descriptions of the nature of the universe and humanity’s place in it. Values are generalized notions of what is good and bad; beliefs are more specific and, in form at least, have more content.
Green (1971) provided the following conceptualization of how beliefs are structured and identified three dimensions of belief systems, not having to do with the content of the beliefs themselves, but with the way in which they are related to one another within the system.

We may, therefore, identify three dimensions of beliefs systems. First there is the quasi-logical relation between beliefs. They are primary or derivative. Secondary, there are relations between beliefs having to do with their spatial order or their psychological strength. They are central or peripheral. But there is a third dimension. Beliefs are held in clusters, as it were, more or less in isolation from other clusters and protected from any relationship with other sets of beliefs. Each of these characteristics of belief systems has to do not with the content of our beliefs, but with the way we hold them. (pp. 47-48)

The first dimension of beliefs systems suggests observation that a belief is never held in total independence of all other beliefs, and that some beliefs are related to others in the way that reasons are related to conclusions. Thus, belief systems have a quasi-logical structure, with some primary beliefs and some derivative beliefs. As an illustration, consider a teacher who believes it is important to present mathematics “clearly” to the students, a primary belief. To this end, the teacher believes it is important (a) to prepare lessons thoroughly, to ensure a clear, sequential presentation, and (b) to be prepared to answer readily any question posed by the students. These are both derivative beliefs.

Green’s (1971) second dimension is related to the degree of conviction with which beliefs are held or to their psychological strength. According to Green, the beliefs in the system can be viewed as either central or peripheral—the central ones being the most strongly held beliefs, and the peripheral ones those most susceptible to change or
examination. He noted that logical primacy and psychological centrality are orthogonal dimensions, arguing that they are two different features or properties of a belief. In the example given earlier, the derivative belief in the importance of being prepared to answer student questions may be more important or psychologically central to the teacher for reasons of maintaining authority and credibility than for clarifying the subject to the students.

The third of Green’s (1971) dimensions has to do with any relationship with other sets of beliefs. People hold beliefs in clusters, and each cluster within a belief system may be protected from other clusters; there is little cross-fertilization among them. As long as the incompatible beliefs are never set side by side and examined for inconsistency, the incompatibility may remain. Such incompatibility may be seen in the example of a teacher who believes that when students read out loud accurately, they are not necessarily comprehending the passage; on the other hand, reading silently contributes more to comprehension than reading out loud (Richardson, 1994). At the same time, the teacher may see a need to ask students to read out loud because the teacher believes that reading out loud is the only way to ensure that the students are actually reading. These two beliefs may be held within different clusters—one related to learning to read, and one related to classroom management. Since they are held in different clusters, the teacher may not have considered these two beliefs together nor confronted the contradictions.

Green (1971) distinguished quasi-logical from psychological strength. This distinction can be described in the following way (Cooney & Shealy, 1997). A teacher may hold that the use of alternate assessment items is an important way to assess learning, yet rarely use such items because of the difficulty in creating them or concern
over the amount of time it takes to grade them. His or her belief in the use of such items may be primary and, consequently, yield derivative beliefs about the importance of students demonstrating their ability to reason and communicate. But this belief may not be psychologically central; that is, its intended actions dissipate in the face of other demands that are valued more highly (e.g., coverage of content). Beliefs that are psychologically central in one context may not be psychologically central in another context—as in the case when clusters of beliefs are isolated. For example, Jones (1991, cited in Cooney & Shealy, 1997) found that Darla, a middle school teacher, believed strongly in different perspectives about teaching mathematics, yet this psychologically central belief was peripheral with respect to her limited beliefs about mathematics. Indeed, teachers’ beliefs about mathematics may be held more strongly than their beliefs about the teaching of mathematics.

Rokeach (1960) talked about beliefs being psychologically central and about the notion of primary beliefs. He wrote:

The concept ‘primitive belief’ is meant to be roughly analogous to the primitive terms of an axiomatic system in mathematics or science. Every person may be assumed to have formed early in life some set of beliefs about the world he lives in, the validity of which he does not question and, in the ordinary course of events, is not prepared to question. Such beliefs are unstated but basic. (p. 40)

According to Rokeach, these primitive beliefs interfere with acceptance of other, more peripheral, beliefs. Like Green, Rokeach discussed the notion of beliefs held in isolation from each other. Thus, according to both Green and Rokeach, it is possible for a teacher to hold simultaneously that problem solving is the essence of mathematics and that students learn mathematics best by taking copious notes and mastering every detail or by
having the teacher explain each step. Isolation occurs when contradictory beliefs are not explicitly compared, perhaps reflecting the existence of beliefs held from a nonevidential perspective, a perspective immune from rational criticism. Nonevidentially held beliefs encourage one to see the world in terms of polarities (Cooney & Shealy, 1997).

Green (1971) distinguished beliefs nonevidentially held from those evidentially held in the following way:

When beliefs are held without regard to evidence, or contrary to evidence, or apart from good reasons or the canons for testing reasons and evidence, then I shall say they are held nonevidentially. It follows immediately that beliefs held nonevidentially cannot be modified by introducing evidence or reasons. They cannot be changed by rational criticism. The point is embodied in a familiar attitude: “Don’t bother me with facts; I have made up my mind.” When beliefs, however, are held on the basis of evidence or reasons, they can be rationally criticized and therefore can be modified in the light of further evidence or better reasons. I shall say that beliefs held in that way are held evidentially. (p. 48)

As Green pointed out, beliefs held in a quasi-logical relationship suggest that a person could indicate that he believes A, giving as his reason belief B, which implies belief A. If, however, a person encounters a demonstration that belief B is not a good reason for belief A and consequently fails to question belief A, then belief A is held nonevidentially (Cooney, Shealy, & Arvold, 1998).

Sigel (1985) proposed the following definition of beliefs:

Beliefs are knowledge in the sense that the individual knows that what he (or she) espouses is true or probably true, and evidence may or may not be deemed necessary; or if evidence is used, it forms a basis for the belief but is not the belief itself ... In sum, beliefs are construction of reality. They may incorporate knowledge of what and knowledge of how, but do not necessitate evidential propositions. Beliefs are considered as truth statements even though evidence for their veridicality may or may not exist. (pp. 348-349)
Peterman (1993) suggests “beliefs are defined as an individual’s mental construction of experience-often condensed and integrated into schemata or concepts that are held to be true and may guide personal action” (p. 229). He further points out that the assumptions about schemata and concepts may apply, if beliefs are mental representations integrated into schemata and concepts. His first assumption is that beliefs may be held as semantic networks similar to concepts and schemata. The second one is that contradictory beliefs may exist within different knowledge domains. The last assumption is that certain beliefs may be core beliefs, and, like core schemata, these core beliefs may be difficult to change. Despite their differences, most definitions of beliefs and belief systems, aforementioned, contain referents to concepts or other linguistic representations, to truth, and in some instances, to action.

Distinctions between Beliefs and Knowledge

In the traditional philosophical literature, knowledge depends on a “truth condition” that is outside the individual with the particular thought (Green, 1971; Leher, 1990). Knowledge is not, then, viewed by philosophers as a psychological concept. A proposition is knowledge if there is rigorous evidence for the premise, and the procedures for developing the argument as well as the conclusions are agreed on by a community of scholars, scientists, or other professions. By contrast, when a proposition is held psychologically by an individual and drives his or her actions, it is a belief. Beliefs do not require a truth condition. If the belief is derived from knowledge, it is an evidential belief.
Percesepe (1991) describes philosophically that beliefs are a habit of mind, a disposition to respond in a particular way to the demands of the world. The duration of a belief may be short or long. In contrast to beliefs, knowledge is a special kind of believing. The class of things that people believe is always larger than the class of things that people know. He further argues that knowing takes more effort and requires a stronger disposition. One can believe without knowing. Believing something does not count as knowing unless what is believed is in fact certain. Thus, knowing requires certainty.

Such a differentiation between knowledge and beliefs is not evident, however, in much of the research on teaching and learning literature (Fenstermacher, 1994). For example, Alexander, Shallert, and Hare (1991) described 26 terms that are used in the literature on literacy to denote different types of knowledge. These include procedural knowledge, content knowledge, and syntactic knowledge. They equate knowledge with belief: “Knowledge encompasses all that a person knows or believes to be true, whether or not it is verified as true in some sort of objective or external way” (p. 317).

Kagan (1992) also made the decision to use the terms beliefs and knowledge interchangeably in the analysis of methodological issues inherent in studying teachers’ beliefs and knowledge: “I do so in light of mounting evidence that much of what a teacher knows of his or her craft appears to be defined in highly subjective terms” (p. 421). The following terms are also often used interchangeably: beliefs, attitudes, world views, perceptions, ideologies, theories, and values.

Defining beliefs is at best a game of a researcher’s choice. Pajares (1992) mentioned that beliefs are used in the literature with the same meaning of attitudes, values, judgements, opinions, ideology, perceptions, conceptions, dispositions, implicit
theories, personal theories, internal mental processes, perspectives, etc. In addition, Pajares based on his review suggested that most of the constructs were simply different words meaning the same thing.

Distinguishing knowledge from belief is hard because it is difficult to pinpoint where knowledge ends and beliefs begin. Nespor (1987) identified four feature characteristics of beliefs that distinguish them from knowledge – a) existential presumption, b) alternativity, c) affective and evaluative loading, and d) episodic structure. Existential presumptions are the incontrovertible, personal truths everyone holds. Rokeach (1968) suggested that they are the taken-for-granted beliefs about physical and social reality and self and that to question them is to question personal sanity. As such, they are deeply personal, rather than universal, and unaffected by persuasion. They can be formed by chance, an intense experience, or a succession of events, and they include beliefs about what oneself and others are like. For example, a teacher may believe that students who fail are lazy; another teacher may believe that learning math is a function of drilling. Existential presumptions are perceived as immutable entities that exist beyond individual control or knowledge (Nespor, 1987). People believe them because they are beliefs that are taken for granted.

Sometimes individuals, for varying reasons, attempt to create an ideal, or alternative, situation that may differ from reality. Nespor (1987) explained how Ms. Skylark, due to traumatic experiences as a student, attempted to create the ideal teaching environment she had fantasized about as a child. Because her fantasies were carried out with teaching practices inconsistent with effective classroom procedures, they resulted in unfinished lessons and frequent interruptions.
Nespor (1987) suggested that beliefs have stronger affective and evaluative components than knowledge and that affect typically operates independently of the cognition associated with knowledge. Teachers often teach the content of a course according to the values held of the content itself. Other theorists have suggested the evaluative nature of beliefs. Nisbett and Ross (cited in Pajares, 1992) conceptualized generic knowledge as a structure composed of a cognitive component, and a belief component, possessing elements of evaluation and judgement. For example, a teacher’s knowledge of what typically happens in a school or his understanding of the faculty handbook are instances of cognitive knowledge. But knowing that Jim is a troublemaker or that boys are better at mathematics than girls are examples of beliefs because they are affective and evaluative. All human perception is influenced by this generic knowledge structure-schemata, constructs, information, beliefs-but the structure itself is an unreliable guide to the nature of reality because beliefs influence how individuals characterize phenomena, make sense of the world, and estimate covariation. They influence even cognitive knowledge. Ernest (1989a) suggested that knowledge is the cognitive outcome of thought and belief is that of the affective outcome, but he acknowledged that beliefs also possess a slender but significant cognitive component.

Nespor (1987) further contended that knowledge system is semantically stored, whereas beliefs is episodic memory with material drawn from experience or cultural sources of knowledge transmission. Nespor argued that beliefs drew their power from previous episodes or events that colored the comprehension of subsequent events. Such episodes played key roles in the practices-Ms. Skylark’s efforts to create a friendly classroom environment were rooted in her vivid childhood memories, and Mr. Ralson
based his math methods on memories of the teaching techniques that his high school math teacher used. Calderhead and Robson (1991) reported that preservice teachers held vivid images of teaching from their experiences as students, images that influenced interpretations of particular courses and classroom practices and played a powerful role in determining how they translated and utilized the knowledge they possessed and how they determined the practices they would later undertake as teachers. The importance of critical episodes and images helps explain how teachers develop their educational belief structure. Nespor (1987) found it likely that “crucial experience or some particularly influential teacher produces a richly detailed episodic memory which later serves the student as an inspiration and a template for his or her own teaching practices” (p. 320).

Nespor (1987) also contended that belief systems, unlike knowledge systems, do not require general or group consensus regarding the validity and appropriateness of the beliefs. Individual beliefs do not even require internal consistency within the belief system. This nonconsensuality implies that belief systems are disputable, more inflexible, and less dynamic than knowledge systems. Beliefs are basically unchanging, and when they change, it is not argument or reason that alters them but rather a “conversion or gestalt shift” (p. 321). Knowledge systems are open to evaluation and critical examination, but beliefs are not. Nespor (1987) describes the characteristic of beliefs, nonconsensuality:

Belief systems often include affective feelings and evaluations, vivid memories of personal experiences, and assumptions about the existence of entities and alternative worlds, all of which are simply not open to outside evaluation or critical examination in the same sense that the components of knowledge system are. (p. 321)
Thompson (1992) also pointed out the characteristics of nonconsensuality of beliefs. She argued that from a traditional epistemological perspective, a characteristics of knowledge is general agreement about procedures for evaluating and judging its validity. That is, knowledge must meet criteria involving canons of evidence. Beliefs, on the other hand, are often held or justified for reasons that do not meet those criteria, and thus, are characterized by a lack of agreement over how they are to be evaluated or judged.

Nespor (1987) added that belief systems are also unbounded in that their relevance to reality defies logic, whereas knowledge systems are better defined and receptive to reason. He concluded that beliefs are far more influential than knowledge in determining how individuals organize and define tasks and problems and are stronger predictors of behavior.

Lewis (cited in Pajares, 1992) argued that the origin of all knowledge is rooted in belief, that ways of knowing are basically ways of choosing values. Even when learning is due to personal discovery or insight, for example, individuals begin by believing their own senses, their intuition, the laws of nature, logic. Lewis insisted that the two constructs are synonymous, that the most simple, empirical, and observable thing one knows will, on reflection, reveal itself as an evaluative judgement, a belief. But acquiring knowledge and choosing, developing, and maintaining beliefs may not involve the same cognitive processes, at least not in the same ways.

In sum, the following lists can be considered not as a compendium of categorical truths but as fundamental assumptions that may reasonably be made when initiating a study of teachers’ educational beliefs (Pajares, 1992, pp. 324-326). The first assumption is about formation and change of individuals’ beliefs. Individuals develop a belief
system early through the process of cultural transmission. This belief system is persevered even against contradictions caused by reason, time, schooling, or experience. In the same way individuals tend to hold onto beliefs based on incorrect or incomplete knowledge, even after scientifically correct explanations are presented to them. The earlier a belief is incorporated into the belief structure, the more difficult it is to alter. That is, newly acquired beliefs are most vulnerable to change. However, belief change during adulthood is a relatively rare phenomenon. If it happens, the change is a gestalt shift.

The second assumption is about functions of a belief system. The belief system helps individuals define and understand the world and themselves. But since beliefs strongly influence perception, they can be an unreliable guide to the nature of reality. The potent affective, evaluative, and episodic nature of beliefs control thought processes (e.g., defining, planning, interpreting, making decisions). Hence, beliefs play a critical role in understanding behavior and the organization of knowledge and information.

The third assumption is about relationships among beliefs. Beliefs are prioritized according to their connections or relationships to other beliefs. Apparent inconsistencies may be explained by exploring the functional connections and centrality of the beliefs. The centrality of beliefs explains why some beliefs are more incontrovertible than others.

The last assumption is about inference of beliefs. Beliefs must be inferred, and this inference must take into account the congruence among individuals' belief statements, the intentionality to behave in a predisposed manner, and the behavior related to the belief.
Teachers’ Beliefs about Mathematics

Philosophical positions and epistemological theories related to mathematics, such as logicism, formalism, constructivism, empiricism, have always had a significant influence on the guiding ideas and leading principles in mathematics education (Steiner, 1987). The positions and theories not only hold for curriculum development and teaching methodology but also for theoretical work and empirical research related to the mathematical learning process. René Thom emphasizes the relationship between teachers’ philosophy of mathematics and the teaching and learning of mathematics: “In fact, whether one wishes it or not, all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics” (Steiner, 1987, p. 7). Arguing that it is possible for two teachers to have very similar knowledge, but while one teaches mathematics with a problem-solving orientation, the other has a more didactic approach, Ernest (1989b) emphasizes the importance of teachers’ beliefs about mathematics.

Teachers’ beliefs of the nature of mathematics form the basis of the philosophy of mathematics, although some of the views likely to be held by teachers may not have been elaborated into fully articulated philosophies. Teachers’ conceptions of the nature of mathematics by no means have to be consciously held views; rather they may be implicitly held philosophies. According to Thompson (1992), these implicit and personal philosophies can be viewed as that teacher’s conscious or subconscious beliefs, concepts, meanings, rules, mental images, and preferences concerning the discipline of mathematics.

Ernest (1989a) distinguished three conceptions of mathematics:
First of all, there is a dynamic, problem-driven view of mathematics as a continually expanding field of human creation and invention. Thus mathematics is a process of inquiry and coming to know, adding to the sum of knowledge. Mathematics is not a finished product, for its results remain open to revision (the problem-solving view). Secondly, there is the view of mathematics as a static but unified body of knowledge, a crystalline realm of interconnecting structures and truths, bound together by filaments of logic and meaning. Thus mathematics is a monolith, a static immutable product, which is discovered, not created (the Platonist view). Thirdly, there is the view that mathematics, like a bag of tools, is made up of an accumulation of facts, rules and skills to be used by the trained artisan skillfully in the pursuance of some external end. Thus mathematics is a set of unrelated facts, rules and skills of but utilitarian rules and facts (the instrumentalist view). (p. 21)

These three philosophies of mathematics, as psychological systems of belief, can be conjectured to form a hierarchy (Ernest, 1989b). Instrumentalism is at the lowest level, involving knowledge of mathematical facts, rules and methods as separate entities. At the next level is the Platonist view of mathematics, involving a global understanding of mathematics as a consistent, connected and objective structure. At the highest level the problem-solving view sees mathematics as a dynamically organized structure located in a social and cultural context.

A number of studies (Bush, Lamb, & Alsina, 1990; Copes, cited in Thompson, 1992; Neyland, 1995a) have used Perry’s (1970) scheme of intellectual and ethical development or adaptations of it, as a framework for analyzing and characterizing teachers’ beliefs and conceptions of mathematics (Thompson, 1992). Perry describes a series of stages for the intellectual and ethical development of college students from the viewpoint of their conceptions of knowledge. Bush et al. (1990) describe case studies of three teachers (two elementary teachers and one art teacher, all female) enrolled in a program to gain certification to teach secondary mathematics. They classified the
teachers' beliefs about mathematics according to Perry’s developmental framework, which involves four major states: dualism—any proposition or act must be right or wrong; multiplicity—a plurality of viewpoints exist, but no internal structure of external relationships exist; relativism—a plurality of viewpoints exist, context is very important; and commitment—one personally commits to a mode of action and belief. The conceptions of mathematics displayed by the three teachers observed by them ranged from dualistic to relativistic and proved to be stable.

A similar study using an adaptation of Perry’s scheme for the study of conceptions of mathematical knowledge was done by Copes (cited in Thompson, 1992), who proposed four types of conceptions: absolutism, multiplism, relativism, and dynamism. Copes described each type as corresponding to a conception of mathematical knowledge prevailing at different periods of its historical development. For example, absolutism prevailed from the time of the Egyptians and Babylonians until the middle of the nineteenth century. From an absolutist perspective, mathematics was viewed as a collection of facts whose truth is verifiable in the physical world. Multiplism emerged with the advent of non-Euclidean geometries. Mathematical facts no longer needed to be verified by observable physical phenomena. Multiplism was characterized by the coexistence of different mathematical systems that might contradict each other. Relativism was marked by the abandonment of efforts to prove the logical consistency of the different systems and the concomitant acceptance of their coexistence as equally valid systems. Dynamism was characterized by a commitment to a particular system or approach within the context of relativism. Copes discussed applications of his framework to the teaching of mathematics, and suggested ways in which different teaching styles can
communicate different conceptions. For example, a teaching style that emphasizes the
transmission of mathematical facts, right versus wrong answers and procedures, and
single approaches to the solutions of problems may communicate an absolutist or dualist
view of mathematics. Thompsom (1992) raised a question based on the studies using
Perry's scheme: "Whether teachers' mathematical beliefs can be predicted by their level
of intellectual development" (p. 133).

Neyland (1995a) discussed beliefs and values of mathematics based on Ernest's
and Perry's model. He distinguished personal and public philosophy of mathematics. The
personal beliefs (e.g., "There is only one way to teach maths. It is the way I was taught. I
cannot conceive of any other approach") are not the same as the public philosophies of
mathematics, which are explicitly stated and subject to critical analysis (p. 140). Personal
beliefs about mathematics are more private and some are tacit; and they influence how
teachers teach mathematics. For example, if a teacher believes mathematics to be a highly
organized body of knowledge, out there in a special mathematical world of abstract
symbols and concepts, the teacher will probably teach mathematics in a different way
from the person who believes that mathematics is what mathematicians do, and that
mathematical knowledge is just the socially agreed upon product of this activity.

Based on Perry's (1970) model, unlike Copes' (cited in Thompson, 1992) study
that applied Perry's scheme to the historical development of mathematical knowledge,
Ernest described three personal beliefs of mathematics: dualistic, multiplistic, and
relativistic views of mathematics (Neyland, 1995a, p. 142). Dualistic views of
mathematics regard it as concerned with facts, rules, correct procedures and simple truths
determined by absolute authority. Mathematics is viewed as fixed and exact; it has a
unique structure. Doing mathematics is following the rules. In multiplistic views of mathematics, multiple answers and multiple routes to an answer are acknowledged, but regarded as equally valid, or a matter of personal preference. Not all mathematical truths, the paths to them or their applications are known, so it is possible to be creative in mathematics and its applications. However, criteria for choosing from this multiplicity are lacking. Relativistic views of mathematics acknowledge multiple answers and approaches to mathematical problems, and that their evaluation depends on the mathematical system, or its overall context. Likewise mathematical knowledge is understood to depend on the system or frame adopted, and especially on the inner logic of mathematics, which provide principles and criteria for evaluation.

In addition to the personal beliefs, there are publicly debated philosophies of mathematics: absolutism and fallibilism (Neyland, 1995a). Lerman (1983) argued that the absolutist and fallibilism views correspond to two competing schools of thought in the philosophy of mathematics: Euclidean and Quasi-empirical. Absolutism viewpoint holds that mathematics contains certain, universal, absolute, and unchallengeable truths; that it is a body of certain, absolute, value-free, and abstract knowledge, with its connection to the real world perhaps of a platonic nature. Axioms, definitions, and rules of inference are used to provide a precise description of the development of, and justification for, mathematical truth. Fallibilism viewpoint sees mathematical truth as fallible. Mathematics develops through conjectures, proofs, and refutations, and uncertainty is accepted as inherent in the discipline. Mathematical concepts and proofs can never be regarded as beyond revision and correction; they may require renegotiation as standards of rigor change or new meanings emerge. Mathematics is what mathematicians do with
all the imperfections inherent in any human activity or creation. Mathematics is a dialogue between people exploring mathematical problems, and it must be viewed in its historical and social context.

Nimier (cited in Mura, 1993) analyzed the role that mathematics plays in each individuals' life and personal history from a psychoanalytic perspective. His research, based on 1100 questionnaires and 30 interviews, had produced a wealth of detailed descriptions of the representations of mathematics held by secondary teachers in France. He identified four “axes” or “modalities” that form the framework of these representations. The first modality concerned beauty and harmony. Mathematics appears as an idealized object, it is a source of wonder, it affords refuge from the disappointments of life. The second modality was about laws and rules. These statements may signify prohibitions as well as permissions; they organize thought. Mathematics is something serious, coherent, unifying; it favors correct reasoning. The third modality opposed the view of mathematics as an internal or an external object. Mathematics is either an invention, a game of the mind with no reference to reality, or, on the contrary, it is discovered through reality and applied back to it. The last modality contrasted the representation of mathematics as a given, as a truth to be unveiled, with its representation as a construction in progress.

Two studies conducted by Mura (1993, 1995) investigated the views of mathematics held by university teachers of mathematics (mathematicians) and mathematics educators. The major question of the questionnaires was an open-ended one that asked the definition of mathematics. Analysis of data produced a list of 14 themes. Among them the two images of mathematics as a formal abstract system ruled by logic
and as a model of the real world are both quite widespread. Mathematics is also considered to be both an art and a science, both a language (a form) and a set of specific contents. Methodologically, Mura commented that defining mathematics and expressing one's beliefs about mathematics are extremely hard to be investigated by questionnaires:

By writing a few words about mathematics in response to a questionnaire, one cannot display the richness of one's vision of this subject, as one might in the course of a personal interview. Moreover, and perhaps more seriously, mental images are often diffuse, incoherent and partly unconscious, hence difficult to articulate. (p. 396)

Although Ernest's (1989a) and Perry's (1970) model discussed earlier would be useful to categorize teachers' beliefs about mathematics, the models do not seem to explain successfully why some beliefs about mathematics can be changed and others cannot. Along with Green's (1971) conceptualization of beliefs, the conceptual change model by Posner, Strike, Hewson, and Gertzog (1982) might provide a useful perspective of understanding teachers' beliefs change. Posner et al. have suggested that four conditions must be fulfilled in order for accommodation or conceptual change of students to occur. Although these conditions have received considerable attention, especially in science education, this model can be applied to explain why some teachers' beliefs are likely to be changed and others are not.

1. The student must become dissatisfied with his or her existing conceptions. Individuals are unlikely to make major changes in the way they conceptualize or think about something unless they believe that their prior conceptions are no longer functional and that less radical changes will not work.
2. The new conception must be intelligible. The student must acquire a minimal initial understanding of the new conceptual structure in order to explore the possibilities that exist within it.

3. The new conception must appear initially plausible. Any new conceptual system must appear capable of solving the problems generated by its predecessor for an individual to consider it as having sufficient plausibility to warrant an attempt to establish its validity.

4. The student must see the new concept as a fruitful or useful one for purposes of understanding a variety of situations. New possibilities for understanding and explaining things must be apparent to the student.

In sum, mathematics certainly means many things to teachers (Orton, 1994): an organized body of knowledge, an abstract system of ideas, a useful tool, a key to understanding the world, a way of thinking, a deductive system, an intellectual challenge, a language, the purest logic possible, an aesthetic experience, and a creation of the human mind being some of the many possible elements of a definition. For Davis and Hersh (1983) the definition of mathematics changes over time, for each thoughtful mathematician within a generation formulates a definition according to his lights. However, as a classroom teacher it is critical for teachers to have their own beliefs about the nature of mathematics because the beliefs greatly influence their instructional practices. In a theoretical discussion of the relationship between philosophy of mathematics and teaching mathematics, Tymoczko (1998) argued that the quasi-empirical view of mathematics—what Lerman called the fallibilist view—is the only one appropriate for teachers.
Social and Cultural Perspective of Mathematics and the Teaching and Learning of Mathematics

Historically, mathematics has long been viewed as the paradigm of infallibly secure knowledge (Ernest, 1998). Euclid and his colleagues first constructed a magnificent logical structure about 2,300 years ago in the Elements, which at least until the end of the nineteenth century was taken as the paradigm for establishing incorrigible truth. Descartes, Newton, Spinoza, Whitehead and Russell modeled their work on the method and style of Euclid. Thus mathematics has long been taken as the source of the most infallible knowledge known to human kind, and much of this infallibility of mathematical knowledge is due to the logical structure of its presentation of justification.

Currently, philosophy of mathematics emphasizes social, cultural, and human aspect of epistemology of mathematical knowledge. References to the social aspects of mathematics can be found in the work of some mathematicians and philosophers, who offer a challenge to the traditional conceptions of mathematics. Lakatos (1995) regards mathematics knowledge as speculative and fallible, and mathematics as a growing science like natural science through conjectures, criticism, counterexamples, refinement of theories, refutation, and further refinement. He does not reject formal mathematics but rejects the idea that mathematics grows in the deductive pattern of formalization. He criticizes the deductivist style:

Deductivist style hides the struggle, hides the adventure. The whole story vanishes, the successive tentative formulations of the theorem in the course of the proof-procedure are doomed to oblivion while the end result is exalted into sacred infallibility. (p. 142)
Lakatos (1998) distinguishes between two kinds of theories, Euclidean theories and quasi-empirical theories. The basic statements of a Euclidean theory are its axioms; its rules of inference are precisely determined. Truth (or acceptability for formalists) is injected into the system at the axioms and “flows downward” to their deductive consequences. An image of Euclidean theories is that they begin by stating the essential nature of their subjects and go on to describe its detailed variations. Knowledge, as given by proof, is infallible. The student of mathematics is obliged to the Euclidean ritual, to attend the conjecturing act without asking questions either about the background or about how this sleight-of-hand is performed. If the student by chance discovers that some of the unseemly definitions are proof-generated, if he simply wonders how these definitions, lemmas and the theorem can possibly precede the proof, the conjuror will not accept him as a member of mathematical community for this display of mathematical immaturity (Lakatos, 1995).

The image of quasi-empirical theories, on the other hand, is that they begin while their subjects are still indeterminate (Lakatos, 1998). They can describe and manipulate many variations and their goal is to get to the underlying principles. Knowledge is fallible. The basic statements of a quasi-empirical theory are a special set of theorems, traditionally, observation sentences or experimental outcomes, and its rules of inference might be less precisely formulated. Truth and falsity are injected into the basic statements but logically, in quasi-empirical theories, it is not truth that flows downward but falsity that flows upward. Thus, the axioms or basic principles of quasi-empirical theories are usually the results of bold speculation that have survived the test of severe criticism. Lakatos’ underlying argument is that mathematical theories, like those of science, are
quasi-empirical. The development of a quasi-empirical theory starts with problems followed by daring solutions, then by severe tests, refutations. The vehicle of progress is bold speculations, criticism, controversy between rival theories, problem shifts. The slogans are growth and permanent revolution, not foundations and accumulations of eternal truths.

While Lakatos made no specific references to the role of social processes, Putnam (1998) and Tymoczko (1986) put their arguments on the social aspect of construction of mathematical knowledge. Putnam argues, like Lakatos, mathematical knowledge is not a priori, absolute and certain, rather it is quasi-empirical, fallible and probable, much like natural science. He notes that the emphasis on quasi-empirical methods leads us to rely on social processes for establishing knowledge in addition to rigorous proofs. Quasi-empirical methods, he adds, are analogous to the methods of the physical sciences except that the singular statements which are ‘generalized by induction,’ used to test ‘theories,’ etc., are the product of proof or calculation rather than being ‘observation reports’ in the usual sense. Since proof has the great advantage of not increasing the risk of contradiction, Putnam argues that proof will continue to be the primary method of mathematical verification.

Tymoczko (1986) states that the prime concern of the philosophy of mathematics should be to argue for the role of the community of mathematicians instead of focusing on one individual, isolated mathematician. He argues that mathematics is a human activity, and he urges for recognition that mathematics is public knowledge and that it is communication among mathematicians in their community that grounds mathematical knowledge.
Ernest (1992, 1998) calls “social constructivism” as a philosophy of mathematics, and Hersh (1997) speaks of the “humanist” view of the nature of mathematics. The social constructivist view of mathematics, according to Ernest, is both conventionalist and empiricist, in that human language, agreement and experience play a role in establishing its truths. Central to this view is the fact that over the course of time, mathematical knowledge changes, just as knowledge in the empirical sciences evolves. At any time, mathematics is an intersubjective agreement, rather than an objective body of knowledge. This view of mathematics emphasizes human endeavor at making mathematics. Ernest (1992) puts it in this way:

The start point for any social constructivist account of mathematics is the assumption that the concepts, structures, methods, results and rules that make up mathematics are the invention of humankind. (p. 93)

According to this view, the concepts of mathematics are derived by abstractions from direct experience of the physical world, from the generalization and abstraction of previously constructed concepts (Lakatos, 1995; Polya, 1954, 1981), by negotiating meanings with others during discourse (Bauersfeld, 1992, 1995; Cobb & Bauersfeld, 1995; Vogit, 1994, 1995), or by some combination of these means. Mathematicians form a community with a mathematical culture, that is a more or less shared set of concepts and methods, a set of values and rules, within the contexts of social institutions and power relations. If this view is accepted as a new epistemology of mathematics, mathematics could then be re-perceived as humans, responsive, negotiable and creative (Burton, 1995).
Hersh (1997) argues that philosophy of mathematics and teaching of mathematics influence each other. The teaching of mathematics should affect the philosophy of mathematics, in the sense that philosophy of mathematics must be compatible with the fact that mathematics can be taught. Hersh stresses the “teachability of mathematics” as a philosophy (p. 237). He argues that Platonists and formalists ignore this question. If mathematical objects were an other-worldly, nonhuman reality (Platonism), or symbols and formulas whose meaning is irrelevant (formalism), it would be a mystery how to teach it or learn it. Its teachability is the heart of the humanist conception of mathematics. In other words, adoption by teachers of a humanist philosophy of mathematics could benefit mathematics education. Hersh’s point of view of philosophy of mathematics is that mathematics must be understood as a human activity, a social phenomenon, part of human culture, historically evolved, and intelligible only in a social context.

Then, what is mathematics in terms of social constructivism and humanist view of mathematics? According to Hersh (1998), mathematics deals with ideas, not pencil marks or chalk marks, not physical triangles or physical sets, but ideas (which may be represented or suggested by physical objects). Based on sociocultural and historical aspect of mathematics, Hersh (1997, 1998) suggests the following as the main properties of mathematical activity or mathematical knowledge:

(a) Mathematics is human. It’s part of and fits into human culture.

(b) Mathematical knowledge is not infallible. Like science, mathematics can advance by making mistakes, correcting and recorrecting them.

(c) Mathematical objects are invented or created by humans.
(d) Mathematical objects are created, not arbitrarily, but arise from activity with already existing mathematical objects, and from the needs of science and daily life.

(e) Once created, mathematical objects have properties that are well-determined, which we may have great difficulty in discovering, but which are possessed independently of our knowledge of them.

(f) Mathematical objects are a distinct variety of social-historic objects. They are a special part of culture.

(g) There are different versions of proof or rigor, depending on time, place, and other things. The use of computers in proofs is a nontraditional rigor. Empirical evidence, numerical experimentation, probabilistic proof all help us decide what to believe in mathematics.

In order to implement the social constructivism (Ernest, 1998) and humanist mathematics (Hersh, 1997, 1998) to the mathematics classroom, teacher and students should take dramatically different roles than traditional ones. In *Proofs and Refutations*, Lakatos’ (1995) argument is that mathematics develops as a process of “conscious guessing” about relationships among quantities and shapes, with proof following a “zig-zag” path starting from conjectures and moving to the examination of premises through the use of counterexamples or “refutations.” Doing mathematics through conscious guessing (or conjectures) is taking a risk (Lampert, 1990); it requires the admission that one’s assumptions are open to revision, that one’s insights may have been limited, that one’s conclusions may have been inappropriate. What do teacher and students need to participate in such mathematical activity? In other words, what classroom norms do they
need? The teacher in Lakatos’ book said, “I respect conscious guessing, because it comes from the best human qualities: courage and modesty” (p. 30).

Polya (1954) also emphasizes the similar classroom norms in more detail for the inductive attitude when a mathematics classroom is considered as a micro-mathematical community. Polya admires human beings’ experience. He describes induction as the procedure of extracting the most correct beliefs from a given experience. For a mathematics classroom the inductive attitude is a social norm for participating in a game of conscious guessing and zig-zag passing. Polya asserts the inductive attitude in this way:

> In our personal life we often cling to illusions. That is, we do not dare to examine certain beliefs which could be easily contradicted by experience, because we are afraid of upsetting our emotional balance. There may be circumstances in which it is not unwise to cling to illusions, but in science [or participating mathematics discussion in a classroom] we need a very different attitude, the inductive attitude. This attitude aims at adapting our beliefs to our experience as efficiently as possible. It requires a certain preference for what is matter of fact. It requires a ready ascent from observations to generalizations, and a ready descent from the highest generalizations to the most concrete observations. (p. 7)

This inductive attitude requires the following three attitudes among many other things (Polya, 1954, p. 8): intellectual courage – a readiness to revise any one of our beliefs; intellectual honesty – ability to change a belief when there is a compelling reason to change it; and wise restraint – a resistance to change a belief wantonly, without some good reason. They are so-called “moral qualities” of doing mathematics and participating in mathematical activities. According to Lakatos and Polya, it is likely to assert that these norms necessarily do mathematics in classrooms come from teachers’ beliefs about mathematics. These norms drawn from teachers’ beliefs about the nature of mathematics
are transmitted to students who are participating jointly in classrooms or small-group discussion and discourse with the teacher or peers (e.g., Lampert, 1990). For example, Lampert described whether and how it might be possible to bring the practice of knowing and doing mathematics in school closer to what it means to know mathematics with the discipline by deliberately altering the roles and responsibilities of teacher and students in classroom discourse. She developed and implemented new forms of teacher-student interaction as a teacher of fifth grade. In this interaction, the words knowing, revising, thinking, explaining, problem, and answer took on new meanings in the classroom context. To accomplish this change, she took three classroom norms from Lakatos’ and Polya’s philosophy of mathematics and then taught the norms to her students through classroom discourse. Her first norm was that students’ knowing and doing mathematics is a process of conjecture and proof. According to Lampert, to maintain this kind of mathematics classroom, selecting a problem would be a very important and initial step because a traditional mathematics problem could not retain classroom discourse that she had envisioned. The problem should be problematic to students. The “problematization” is well expressed by Hiebert et al. (1996). Hiebert et al. argued that students should be allowed to make the subject problematic. Allowing the subject to be problematic means allowing students to wonder why things are, to inquire, to search for solutions, and to resolve incongruities.

The second norm was mathematical argument as mathematical knowledge. In this classroom until the group arrived at a mutually agreed-upon proof that one or more of the answers must be correct, all answers were considered to be hypotheses. She always asked students to give reasons for why they questioned the hypothesis, so that their challenge
took the form of a logical refutation rather than a judgement. The person who gave the answer was free to respond or not with a revision. This routine was a way of modeling talk about thinking. It also made thinking into a public and collaborative activity, wherein students would rehearse the sort of intellectual courage, intellectual honesty, and wise restraint that Polya considered essential to doing mathematics.

The third one was about the teacher’s role in this classroom. The teacher was a representation of what it means to know mathematics. Given her goal of teaching students a new way of knowing mathematics, she demonstrated what it would look like for someone more expert than they themselves to know mathematics in the way she wanted them to know it. The role she took in classroom discourse, therefore, was to follow and engage in mathematical arguments with students. She needed to know more than the answer or the rule for how to find it, and to do something other than explain to them why the rules worked. She needed to know how to prove it to them, in the mathematical sense, and she needed to be able to evaluate their proofs of their own mathematical assertions. Lampert emphasized the importance of establishing classroom norms to teaching new forms of doing and knowing mathematics. She expressed it in this way:

Changing students’ ideas about what it means to know and do mathematics was in part a matter of creating a social situation that worked according to rules different from those that ordinarily pertain in classrooms, and in part respectfully challenging their assumptions about what knowing mathematics entails. (p. 58)

Social and cultural aspects of teaching and learning mathematics is usually credited to Vygotsky’s (1978) sociocultural theory. According to Vygotsky, cognition is a
profoundly social phenomenon. Social experience shapes the ways of thinking and interpreting the world available to individuals. And language plays a crucial role in a socially formed mind because it is a primary tool of communication and mental contact with others, serves as the major means by which social experience is represented psychologically, and is an indispensable tool for thought (Berk & Winsler, 1995). The important role of language is also well summarized by von Glasersfeld (1990) as follows:

Knowledge is the result of an individual subject’s constructive activity ... language is not a means of transporting conceptual structures from teacher to student, but rather a means of interacting that allows the teacher here and there to constrain and thus to guide the cognitive construction of the student. (p. 37)

Through collective dialogues with more knowledgeable members (i.e., teacher in a classroom) of their society during challenging tasks, students learn to think and behave in ways that reflect their community’s culture (i.e., classroom norms). Vygotsky (1978) believed that as more mature partners—both teacher and peers—offer guidance to students mastering culturally meaningful activities in the zone of proximal development, the communication with these partners becomes part of students’ thinking. Once students internalize the essential features of these dialogues, they can use the strategies embedded in them to guide their own actions and accomplish skills on their own (Berk & Winsler, 1995). Students learn classroom norms as an appropriate way of thinking and behaving in a micro-mathematical community through social interaction with the teacher and peers. Then these norms guide and facilitate students’ mathematical activities. However, mathematics classroom norms should not be taught as a set of rules that students should follow when they engage in mathematical activities. Teaching classroom norms requires
the teacher to be sensitive in providing careful scaffolding and attaining intersubjectivity.
The teacher’s role in this process is more likely what Lampert (1990) calls a “dance instructor,” who requires some telling, some showing, and some doing mathematics with students along with the mathematics classroom norms.

Bauersfeld (1992), based on Vygotsky’s sociocultural theory, speaks of the culture of a mathematics classroom (p. 22). According to him (pp. 20-21), learning is a process of personal life forming, a process of an interactive adaptation to a culture through active participation, rather than a transmission of norms, knowledge and objectified items. Mathematics is a practice based on social conventions rather than the application of an universally applicable set of eternal truths. And teaching is the attempt to organize an interactive and reflexive process, with the teacher engaging in a constantly continuing and mutual differentiating and actualizing of activities with the students, and thus the establishment and maintenance of a classroom “culture,” rather than the transmission, introduction, or even re-discovery of pre-given and objectively codified knowledge.

Several scholars from the interactionist perspective (e.g., symbolic interactionism or ethnomethodology) have investigated the process of teaching and learning from the social and cultural conceptions of the nature of mathematics. The most frequently referred to terms in their studies are “social and mathematical norms,” “negotiation of mathematical meanings,” “language and communication,” “classroom discussion,” “classroom culture,” or “social interaction,” and so on (Cobb & Bauersfeld, 1995; Lo & Wheatley, 1994; Lo, Wheatley, & Smith, 1994; Yackel & Cobb, 1996; Yackel, Cobb, & Wood, 1991). For instance, Yackel et al. focused on the construction of classroom norms
for cooperation that arose in the course of interactions when children work in small groups to complete mathematical activities. The teacher initiated and guided the mutual construction of a variety of social norms for social cooperation: (a) that students should cooperate to solve problems; (b) that meaningful activity is valued over correct answer; (c) that persistence on a personally challenging problem is more important than completing a large number of activities; and (d) that partners should reach consensus as they work on the activities. In addition, there were classroom norms for individual activity: (a) that children should figure out solutions that are meaningful to them; (b) that they should explain their solution methods to their partner; and (c) that they should try to make sense of their partner’s problem-solving attempts. In this classroom these norms were continually negotiated in concrete situations and did not exist apart from the interactions. It was the classroom teacher who guided and directed the construction of the norms. The norms came into being through the expectations that the teacher and children had for each other and the largely implicit obligations that they had for themselves in specific situations.

Unlike the establishment of classroom norms as mutual negotiation between teacher and students through social interaction in concrete situations (Yackel et al., 1991), Lo et al. (1994) described classroom norms in terms of a student’s participation in class discussion and beliefs about mathematics. The features of this classroom were that students’ interpretations of a mathematical task are the focus of the activity, that student-to-student interaction is encouraged, and that the teacher’s primary role is to facilitate student-to-student communication rather than to explain or evaluate. A norm for presentation was that students should explain different strategies and methods. Just
rephrasing other’s strategies would not be considered as different ones. A norm for small
groups was that students should make sense of another student’s explanation and assist
other students with the activity of sense making. Sharing their solutions in small groups
was another norm in this classroom. A distinctive norm in this study is that students
should explain different strategies and methods for solving problems. It is different from
other norms because it requires reflections of both one’s own and other’s methods to
meet the obligations of the norm.

Yackel and Cobb (1996) argued that the sociomathematical norms are distinct
from general classroom social norms in that they are specific to the mathematical aspects
of students’ activity. More specifically, the understanding that students are expected to
explain their solutions and their ways of thinking is a social norm and the understanding
of what counts as an acceptable mathematical explanation is a sociomathematical norm.
Three sociomathematical norms were identified in this classroom. First, students should
explain their methods in different ways (mathematical difference). This position means
that just little more restatements of previously given solutions were not regarded as
mathematical differences. Second, students should provide a more sophisticated or more
efficient solution (mathematical sophistication). The third norm was about what counts as
an acceptable mathematical explanation and justification. In this classroom, to be
considered as an acceptable mathematical explanation and justification, the explanation
and justification should be (a) a mathematical basis for explanations, (b) explanations as
descriptions of actions on experientially real mathematical objects, and (c) explanations
as objects of reflection. Children in the classroom had to mutually figure out a way to
fulfill their general obligations within the context of their ongoing interactions.
According to these studies, individual students are seen as actively contributing to the development of both classroom mathematical practices and the classroom norms, and both enable and constrain their individual mathematical activities (Cobb & Bauersfeld, 1995). The notion of reflexivity of ethnomethodology is useful to explain the process of the establishment of classroom norms. It implies that neither an individual student’s mathematical activity nor the classroom norms can be adequately accounted for without considering the other. From a sociological perspective, Vogit (1994) argues mathematical meaning is taken as a product of social processes, in particular as a product of social interactions. From this point of view, mathematical meanings are primarily studied as emerging between individuals, not as constructed inside or as existing independently of individuals. Vogit argues that philosophers like Lakatos or Wittgenstein emphasize the argumentation processes and “language games” among persons when they explain the development of mathematical meanings. According to him, these philosophical works support the sociological assumption that mathematical meaning can be studied as emerging in social relationships among individual subjects.

In sum, the shift from the paradigm of absolutist to the paradigm of sociocultural and historical nature of mathematics is significantly influencing the teaching and learning mathematics. The emphasis on humanistic nature of mathematical knowledge provides all students with a path to access this knowledge in that they can see mathematics in their everyday life rather than existing somewhere the students never perceive what it is. One of NCTM’s (1989) goals, “mathematics for all,” can be accomplished only when teachers change their beliefs to this new paradigm of the nature of mathematics.
The analysis of social-mathematical norms indicated that the teacher plays a critical role in providing quality of classroom interactions and in establishing classroom norms that regulate teacher's and students' roles for participation of mathematical activities. A limitation of these studies, however, was that they did not examine the relationships between classroom norms and teachers' beliefs about mathematics and the teaching and learning of mathematics. Classroom norms are formed through social interactions between teacher and students when they are participating in mathematical activities. The quality of social interactions depends on teachers' beliefs about the nature of mathematics (Nickson, 1992). Nickson describes how teachers' differing perspectives of mathematics (e.g., absolutist view or problem solving view) makes classroom activity difference. Thus, if we understand classroom norms that regulate teacher and students interactions then teachers' beliefs about mathematics should be considered.

Teachers' Beliefs about the Teaching and Learning of Mathematics

Several scholars (Brownell, 1935; Ernest, 1989a; Neyland, 1995b; Kuhs & Ball, 1986) suggested different models or theories of the teaching and learning of mathematics. Selection of the models is likely to be closely related to and influenced by the teacher's beliefs about the nature of mathematics (Ernest, 1989a). More than a half century ago, Brownell distinguished two theories: drill theory and incidental learning theory. According to the drill theory, arithmetic consists of a vast host of unrelated facts and relatively independent skills. The students acquire the facts by repeating them over and over again until he is able to recall them immediately and correctly. The students develop
the skills by going through the processes in question until they can perform the required operations automatically and accurately. The teacher needs to give little time to instructing the students in the meaning of what they are learning. On the contrary to the drill theory, the incidental learning theory argues that children will learn as much arithmetic as they need, and will learn it better, if they are not systematically taught arithmetic. The assumption is that children will themselves, through natural behavior in situations which are only in part arithmetical, develop adequate number concepts, achieve respectable skill in the fundamental operations, discover vital uses for the arithmetic they learn, and attain real proficiency in adjusting to quantitative situations. The learning is through incidental experience.

Neyland (1995b) suggests that there seems to exist eight different approaches to the teaching and learning of mathematics: new math, behaviorist, structuralist, formative, integrated-environmentalist, problem solving, cultural, and social constructivist. Neyland categorizes these approaches based on not only the nature of mathematics but educational psychology. A distinctive feature of the categorization is to include cultural and social aspects of teaching mathematics that are stressed by a new philosophy of mathematics, that is, social constructivism.

The new math approach to teaching is an attempt to radically improve mathematical attainment. It was thought that giving the subject a foundational, conceptual, unity would be a major step towards achieving this goal. Just prior to this time mathematicians had been exploring the way set theory and logic could be used to give mathematics a unifying structure. Accordingly sets, relations, axioms and logic were chosen to build a framework for school mathematics. This method was an important new
idea in mathematics education: to present mathematics as a coherent, logically organized and consistent body of knowledge (Neyland, 1995b).

The behaviorist approach to teaching is to carefully organize school mathematics into a precise sequence of small steps in such a way that the learning path will be optimal. This approach rejected the idea that some people are born to study mathematics and others are not. It was believed that a student can learn almost anything, given enough time and the proper prerequisite learning. If the instructional tasks could be arranged into their ‘proper’ learning sequence, almost all students would eventually be able to accomplish each objective on the chain. Assessment would focus on the mastery of objectives. This approach can be seen as anti-mathematical if one sees mathematics as rule challenging, or rule transcending, rather than rule learning. Or if one wishes to present mathematics as a science-like endeavor involving experimentation with mathematical ideas, forming conjectures and attempting to prove or refute them (Neyland, 1995b).

The structuralist approach to teaching has origins in both mathematics and psychology. The idea is that if teachers can introduce their students to the essential, underlying, structures and processes within mathematics, these structures and processes can be used as a framework to design around which mathematical understanding can be developed. Learning in this way will be optimal. The learners explore and discover these structures via a series of embodiments and through a spiral program that revisits these key mathematical structures in a cycle. This approach can be criticized for sometimes using contrived and even confusing embodiments in the attempt to help students discover
the predetermined structures. It can also be criticized for not putting enough emphasis on students forming their own structures (Neyland, 1995b).

The formative approach has a basis in developmental psychology and focuses on the natural process of personal development. It is entirely learner-centered and aims to match learning opportunities in mathematics with the learner’s natural cognitive abilities. This approach is built on the work of Piaget who emphasized that learners actively construct their own knowledge rather than receive it through their senses ready made. The teacher aims to help learners develop mathematical concepts in tune with their development in thinking. Because this approach is based on the learner’s idiosyncratic structuring of knowledge and not on the revelation of the essential structures of mathematics, it requires the teacher to take an active role in helping each student make sensible connections linking their ideas (Neyland, 1995b).

The integrated-environmentalist approach is based on the view that mathematical knowledge cannot be separated from the contexts from which it is extracted and from which it achieves its meaning. Mathematics is seen as being integrated with other areas of knowledge, and knowledge is thought of as an integrated web. The learner’s environment is used as a source of inspiration and meaning, and as the basis for the abstraction process. Teachers using this approach commonly rely on mathematical modeling, statistics, thematic units, and project work, in an attempt to develop mathematical ideas in context (Neyland, 1995b).

The problem-solving approach places an emphasis on mathematical processes as well as the mathematical content. Problem solving presents mathematics in context and provides a reason for doing mathematics - to solve problems. Problem solving
emphasizes strategies rather than rules. It allows a range of solution methods to problems, and so students learn that mathematics is not just the use of fixed and predetermined rules. Problem solving was seen to be similar to what practicing mathematicians actually do. The focus on using problems in context enables students to ground their mathematics in something meaningful to them. Encouraging students to find their own solution strategies is empowering; they learn that mathematics is something they can explore using methods of their own choosing. The opportunity it provides for students to investigate new ideas gives them a taste for making knowledge rather than just receiving it. It should be noted that many teachers treat problem solving as a stand-alone unit alongside other content units (Neyland, 1995b).

The cultural approach is based on the belief that all cultural groups engage in activities that exhibit mathematical elements. This approach views mathematics as a social and cultural product based on certain activities. It is primarily focused on linking mathematics with the lives and culture of the learners concerned. It is not essential that one particular set of activities be universal for all cultures. What is important is that the activities reflect mathematical thinking, and be harmonious with the cultural context concerned. The emphasis on mathematics as a cultural product is empowering, reduces the mystery many associate with mathematics, and gives traditional school mathematics a useful base upon which to build. The teachers need to be confident and secure with their own mathematical knowledge in order to transform it into a new structure, and they need to be familiar with the cultural practices of the group concerned (Neyland, 1995b). This approach to the teaching and learning of mathematics is similar to Brownell’s (1935)
incidental theory because it puts emphasis on the relationship between the student's everyday experience and mathematics learning.

The social constructivist approach is the belief that mathematical knowledge is socially constructed and validated, and that classroom teaching should reflect this understanding. Mathematics is seen as a part of human culture; it is a social, cultural and historical entity. Mathematics develops as a result of a range of human activities and the discourse these activities generate. To be engaged in mathematical activity is to participate in the culture of mathematizing. Mathematical knowledge that is accepted as making sense in mathematics is the knowledge that can be justified in relation to empirical evidence, the knowledge that seem to work in practice, the knowledge that result from inductive reasoning and that is resistant to falsification, the knowledge that has a high probability of being true, and the knowledge that can be formally proven. Learning is seen not just as individuals forming mathematical concepts, but as the learners becoming involved in a community discourse. The students are seen as a fledgling community who are becoming enculturated or socialized by the teacher into the mathematizing culture. Teaching focuses on classroom discourse in the context of relevant mathematical investigations, problems and tasks, with the teacher, the agent of enculturation, playing a key role. Students are encouraged to form new understandings of mathematics using their interpretations of the existing ones as part of their reference frame. In a similar way, mathematics is presented as a problem solving tool that can contribute to the solution of more general problems posed by students (Neyland, 1995b). It should be noted that teachers teach mathematics using either a single model or a
combined model. The selection of a teaching model would be largely dependent upon the teacher’s beliefs about mathematics and the teaching and learning of mathematics.

Based on a review of the literature in mathematics education, teacher education, the philosophy of mathematics, the philosophy of education, and research on teaching and learning, Kuhs and Ball (1986) identified at least four dominant and distinctive views of how mathematics should be taught: a) learner-focused, b) content-focused with an emphasis on conceptual understanding, c) content-focused with an emphasis on performance, and d) classroom-focused. A constructivist view of mathematics learning typically underlies the learner-focused view of mathematics teaching. Because this view centers around the students’ active involvement in doing mathematics—in exploring and formalizing ideas—it is the instructional model most likely to be referred by those who have a problem-solving view of mathematics (Ernest, 1989b). The teacher is viewed as facilitator and stimulator of student learning, posing interesting questions and situations for investigation, challenging students to think. Students are viewed as ultimately responsible for judging the adequacy of their own ideas.

The content-focused view with emphasis on understanding is similar to Ernest’s (1989b) Platonist. From this view instruction makes mathematical content the focus of classroom activity while emphasizing students’ understanding of ideas and processes. This view emphasizes students’ understanding of the logical relations among various mathematical ideas and the concepts and logic underlying mathematical procedures. Unlike the learner-focused model, in which students’ ideas and interests are primary considerations, content is organized in the content-focused model according to the
structure of mathematics, following some notion of scope and sequence the teacher may have.

In the content-focused view with emphasis on performance, the view of teaching would follow naturally from the instrumentalist view of the nature of mathematics (Ernest, 1989b). Some of the central premises of this view are: (a) that rules are the basic building blocks of all mathematical knowledge and all mathematical behavior is rule-governed; (b) that knowledge of mathematics is being able to get answers and do problems using the rules that have been learned; (c) computational procedures should be automatized; (d) it is not necessary to understand the source or reason for student errors—further instruction on the correct way to do things will result in appropriate learning; (e) that in school, knowing mathematics means being able to demonstrate mastery of the skills described by instructional objectives. In the instrumentalist view of teaching, the content is organized according to a hierarchy of skills and concepts; it is presented sequentially to the whole class, to small groups, or to an individual, following a pre-assessment of students’ mastery of prerequisite skills. This view is also similar to Brownell’s (1935) drill theory that emphasizes the mastery of a large amount of unrelated facts and skills and automatization.

The classroom-focused view of teaching is the notion that classroom activity must be well-structured and efficiently organized according to effective teacher behaviors identified in process-product studies of teaching effectiveness. Unlike other models of mathematics teaching, this model does not address questions about the content of instruction. Rather, it assumes that content is established by the school curriculum. In addition, this model is not necessarily grounded on any particular theory of learning. The
assumption is that students learn best when classroom lessons are clearly structured and follow principles of effective instruction (e.g., maintaining high expectations, insuring a task-focused environment). The teacher is viewed as playing an active role directing all classroom activities, clearly presenting the material of the lesson to the whole class or to subgroups, and providing opportunities for students to practice individually. From this perspective, effective teachers are those who skillfully explain, assign tasks, monitor student work, provide feedback to students, and manage the classroom environment, preventing, or eliminating, disruptions that might interfere with the flow of planned activity. Accordingly, the student’s role is to listen attentively to the teacher and cooperate by following directions, answering questions, and completing the tasks assigned by the teacher.

Polya (1981) once mentioned his firm conviction: “Teaching to think” as a primary goal of teaching mathematics (p. 100). Based on this belief he argues that mathematical thinking is not purely “formal” but rather “informal”; it is not concerned only with axioms, definitions, and strict proofs, but many other things belong to it. It includes generalizations from observed cases, inductive arguments, arguments from analogy, recognizing a mathematical concept in, or extracting it from, a concrete situation. He describes three principles of teaching and learning mathematics: the principle of active learning, the principle of best motivation, and the principle of consecutive phases. In the consecutive phases he speaks of his view of learning: “Learning begins with action and perception, proceeds from thence to words and concepts, and should end in desirable mental habits” (p. 103). Action and perception, Polya explains, should suggest manipulating and seeing concrete things such as pebbles,
or apples, or Cuisenaire rods, or ruler and compasses, or instruments in a laboratory, and so on.

Polya (1981) proposes three phases in the consecutive phases for teaching and learning mathematics: exploration, formalization, and assimilation. A first exploratory phase is closer to action and perception and progresses in a more intuitive, more heuristic level. A second formalizing phase ascends to a more conceptual level, introducing terminology, definitions, proofs. The phase of assimilation comes last: there should be an attempt to perceive the “inner ground” of things, the material learned should be mentally digested, absorbed into the system of knowledge, into the whole mental outlook of the learner; this phase paves the way to applications on one hand, to higher generalizations on the other. He mentions the importance of the teachers’ philosophy:

These principles proceed from a certain general outlook, from a certain philosophy, and you may have a different philosophy. Now, in teaching ... it does not matter much what your philosophy is or is not. It matters more whether you have a philosophy or not. And it matters very much whether you try to live up to your philosophy or not. The only principles of teaching which I thoroughly dislike are those to which people pay only lip service. (p. 106)

Hersh (1998) also emphasizes the importance of teachers’ beliefs about the nature of mathematics: “The issue, then, is not, What is the best way to teach? but, What is mathematics really all about?” (p. 13). Hersh calls the teacher who does not challenge formalism but advocates a compromise in quality “a sort of pedagogic opportunist, who wants to offer the student less than the real thing” (p. 13). Ernest (1989b) emphasizes the teacher’s level of consciousness of his or her own philosophy and the extent of the teacher’s reflections on practice of teaching mathematics. Prawat (1992a) points out that
a new form of belief (e.g., constructivist teaching) places greater demands on teachers and students. As Cohen (cited in Prawat, 1992a) points out: “Teachers who take this path must work harder, concentrate more, and embrace larger pedagogical responsibilities than if they only assigned text chapters and seatwork” (p. 357). Prawat argues that teachers are unlikely to complicate their lives in this way without undergoing a significant change in their thinking.

Ernest (1989a) describes the model of teaching mathematics as the teacher’s conception of the type and range of teaching actions and classroom activities contributing to the teacher’s personal approaches to the teaching of mathematics. It includes mental imagery of prototypical classroom teaching and learning activities, as well as the principles underlying teaching orientations. This model of teaching mathematics includes the following views:

(a) A narrow, instrumental and basic skills view of mathematics teaching;
(b) A broader, creative and exploratory view of mathematics teaching;
(c) A meaning, understanding, and unified body of knowledge view of mathematics teaching;
(d) A facts and skills mastery view of mathematics teaching, that focuses on performance and correctness of response;
(e) An approach in which mathematics is based on strictly following a text or scheme;
(f) An approach in which the teacher supplements or enriches the textbook with additional problems and activities;
(g) An approach in which the teacher or school constructs virtually all of the mathematics curriculum materials.
Ernest (1989a) also describes a model of learning mathematics. This model consists of the teacher's view of the process of learning mathematics, what behaviors and mental activities are involved on the part of the learner, and what constitutes appropriate and prototypical learning activities. Thus the model is made up of aims, expectations, conceptions and images of learning activities and of the process of learning mathematics in general. This model involves the following views:

(a) A view of learning as the active construction of knowledge as a meaningful connected whole;

(b) A view of learning mathematics as the passive reception of knowledge;

(c) A view of the development of autonomy and the child's own interests in mathematics;

(d) A view of the learner as submissive and compliant;

Thompson (1992) suggested that these models of mathematics teaching and learning are useful in describing major differences among current views of mathematics teaching and learning. A given teacher's conception of mathematics teaching and learning is more likely to include various aspects of several models than it is to fit perfectly into the description of a single model. And instructional practices of mathematics teachers should be understood and interpreted based on their beliefs of the nature of mathematics.

The Relationships between Teachers' Beliefs and Teaching Practices

The differing views held by teachers in relation to the nature of mathematics are an important component in the culture of a mathematics classroom, since they are linked
with the way mathematics is taught and received (Nickson, 1992). Nonetheless, teachers’ beliefs about mathematics itself, pedagogy, and students’ learning are not always reflected in instructions in classrooms. Ernest (1989a) argues that the different beliefs of mathematics have practical classroom outcomes. And he further points out that, although two mathematics teachers would have the same content knowledge of mathematics, they would teach very differently. For example, an active, problem-solving view of mathematical knowledge can lead to the acceptance of children’s methods and approaches to tasks. In contrast, a static Platonist or instrumentalist view of mathematics can lead to the teacher’s insistence on the existence of a single ‘correct’ method for solving each problem.

Thompson’s (1984) study demonstrated the clear relationship between teachers’ beliefs about mathematics and their instructional practices. As Ernest (1989a) points out that views of the nature of mathematics are likely to correspond to views of its teaching and learning, Thompson found consistency in the views of mathematics expressed by three junior high mathematics teachers she studied. Lynn, Jeanne, and Kay fit nicely into Ernest’s (1989a) three categories of beliefs of the nature of mathematics (i.e., the problem-solving view, the Platonist view, and the instrumentalist view).

Jeanne had been teaching junior high school mathematics for 10 consecutive years and was the mathematics coordinator for the middle school. Kay had taught for five years and was in charge of the mathematics component of a program for gifted students. Lynn had been teaching junior high mathematics for three and one-half years and was also mathematics coordinator for her middle school.
Lynn saw mathematics as a static collection of facts to be transferred verbally to the students. Mathematics was characterized by certainty, predictability, absoluteness, and freedom from emotional content. Lynn believed that her students learned primarily by watching the teacher’s demonstrations attentively and then practicing the presented procedures. She sought to produce students who could perform the mathematical tasks specified in the curriculum, using standard procedures or methods. In her instruction, Lynn was concerned with managerial aspects of teaching and allowed little interaction. She also had low expectations for her students, blaming difficulties in learning on the students’ dispositions and backgrounds.

Jeanne (Thompson, 1984) was closer to Ernest’s (1989a) Platonist. She emphasized mathematics as a logical system with concepts coherently related. She, like Lynn, saw mathematics as fixed and predetermined, but emphasized concepts and structure rather than facts and procedures. For Jeanne, instruction meant providing a logical, coherent presentation emphasizing justification and reasoning, relating new concepts to previous ones. Jeanne talked about the importance of students participating during class, but student participation generally involved responding to her questions.

In contrast, Kay demonstrated some aspects of Ernest’s (1989a) problem-solving approach to mathematics. She tended to see mathematics as a mental exercise and involved students in problem-solving sessions, encouraging them to guess, conjecture, and reason on their own. She emphasized the creation of an open and informal classroom atmosphere and the importance of being receptive to suggestions, ideas, and intuition.

Thompson concluded that teachers’ beliefs and views about mathematics and their instructional decisions and behaviors, regardless of whether they are consciously or
unconsciously held, play a significant, albeit subtle, role in shaping the teachers’
characteristic patterns of instructional behavior. In particular, the observed consistency
between the teachers’ professed conceptions of mathematics and the manner in which
they typically presented the content strongly suggests that the teachers’ views, beliefs,
and preferences about mathematics do influence their instructional practice. This
conclusion is an evidential restatement of René Thom that all mathematical pedagogy,
whether or not coherent, is influenced by a philosophy of mathematics (Steiner, 1987).

However, teaching mathematics in classroom based on teacher’s beliefs about
mathematics and the teaching and learning of mathematics is not a simple matter. For
instance, a beginning teacher of Cooney’s (1985) study showed that his beliefs about
problem solving could not be translated into teaching practices because he lacked the
means to accommodate a greater array of students and content. This study revealed
conflicts between his idealism and the reality of classroom practice, as his students were
not always receptive to his problem-solving teaching strategy. Cooney conducted a
study of a beginning mathematics teacher’s view of problem solving and how that view
affected, and was affected by, his first three months of teaching. It was concerned with
the beliefs of a young mathematics teacher, as he progressed through his preservice
master’s degree program and his initial year of teaching. The informant, a 25-year-old,
was interviewed seven times. Hypothetical situations, called episodes, were used during
the interviews to stimulate in-depth discussions about mathematics and the teaching of
mathematics. Two of the 19 episodes used with the informant were the following: (a)
Describe a particular anecdote during your student teaching that held special meaning
for you; (b) If you could be another person (or famous person) when teaching, whom
would you pick? Why?

Each interview lasted approximately 45 minutes. The first two dealt entirely with
preselected episodes of various types and intent. The third and fourth focused on
elaboration of discussions from the first two interviews and on episodes suggested by
these discussions. After reviewing transcriptions of the first four interviews, the
informant was asked in the fifth interview to identify those of his statements that capture
what he felt were important aspects of his beliefs about mathematics and the teaching of
mathematics. The author wrote each of the selected statements on a card to use in a
sixth, “clustering” interview. The informant was requested to group the cards into
categories of his choosing. The 28 statements he had identified, combined with the titles
and descriptors, played a major role in the analysis of what he believed about
mathematics and the teaching of mathematics. There were nine consecutive classroom
observations. Six stimulated recall interviews were conducted with him, each based on
the preceding classroom observations. Several students from his classes were
interviewed individually or in groups of two or three.

During preservice training he proclaimed that what he would most like students
to learn was “solving problems, which is the essence of mathematics” (p. 328). He made
frequent reference to the enjoyment he derived from working recreational problems and
to their potential for motivating students. To him, a teacher’s chief responsibility is to
motivate students, which he felt could best be done by using recreational problems. That
math is fun was enough justification for him to study math and teach it.
When teaching mathematics in the classroom, he gave the distinct impression that he enjoyed mathematics and its teaching and that he expected students should enjoy its study as well. He expressed frustration over the extensive time demands a problem-solving orientation required of him. He complained about having little time to consider genuine problems that would in some ways excite students and get them involved. He confessed that it was much easier to teach by the book, and left heuristics out completely. He characterized textbook word problems as nothing more than disguised routine exercises, yet the demands of teaching impeded his ability to create real problems. His uses of problems as interest creators were consistent with his previously espoused beliefs about the use of recreational problems. But somewhat inconsistent with his view that problem solving is the essence of mathematics in light of the fact that his lessons were clearly textbook oriented and were handled in a rather cookbook fashion. He placed little emphasis on teaching problem-solving heuristics, which he had earlier stated was the central point of teaching problem solving. However, the general mathematics students characterized his efforts as making a joke out of mathematics and complained that their numerous failing grades occurred because they never got down to serious work. The more advanced students appreciated his puzzles and historical commentaries. They saw him as someone who really loved mathematics and kept the class interesting. He realized that the means by which he had decided to teach mathematics conflicted with the expectations of many students, particularly the less able ones, about what constituted mathematics and how it should be taught. Although he enjoyed success with the advanced class, that success was overshadowed in his mind by his perceived failure to motivate those students who held the view that mathematical
games or puzzles have no place in the teaching of mathematics. He described that those students who failed to appreciate his approach lacked internal motivation.

Cooney (1985) concluded that this beginning teacher had somewhat dualistic notions about teaching. At this early stage of his professional development the teacher seemed to envision only two teaching styles: a highly authoritarian approach, as exemplified by his cooperating teacher during student teaching, and a problem-solving approach. The notion that a teacher could be a strong and forceful classroom leader and yet use a problem-solving approach did not seem to fit this teacher’s conception of teaching mathematics. According to Ernest (1989b) a great disparity between this teacher’s espoused and enacted models of teaching and learning mathematics would be caused by two factors: the influence of the social context (e.g., the expectations of students, parents, fellow teachers, and superiors) and the teacher’s level of consciousness of his or her beliefs. This beginning teacher’s mismatch between his beliefs and teaching practices seems to be mostly affected by the students’ expectations.

The problem-solving view held by the teacher in Cooney’s (1985) study appears more pedagogical beliefs rather than beliefs about the nature of mathematics because his psychologically central beliefs would be that teachers’ chief responsibility is to motivate students using recreational problems. Ernest (1989a) and Nickson (1992) argue that beliefs about the nature of mathematics (that is, the subject itself) influence the way the teachers teach mathematics, not pedagogical beliefs. This is what Hersh (1989) calls “a sort of pedagogic opportunist.” As aforementioned, he emphasizes the importance of teachers’ beliefs about the nature of mathematics: “The issue, then, is not, What is the best way to teach? but, What is mathematics really all about?” (p. 13).
Brown and Borko (1992) explain the inconsistency of the teacher of Cooney's (1985) study in a different perspective. According to them, this inconsistency reflects the struggle of the beginning teacher's "socialization within the teaching profession." As Brown and Borko explain, "teacher socialization is seen as involving a constant interplay between choice and constraint, between individual and institutional factors" (p. 221).

Like the beginning teacher in Cooney's (1985) study, the first- and second-year teachers in Raymond's (1997) study also showed the struggle of the beginning teacher. Raymond investigated relationships between a beginning elementary school teacher's beliefs and mathematics teaching practice. Data were gathered over 10 months through audio-taped interviews, classroom observations, document analysis, and a beliefs survey. For beliefs about mathematics, the fourth-grade teacher described mathematicians constantly working with numbers and equations. She viewed mathematics as predictable, certain, absolute, and fixed and as having no aesthetic value. She believed that mathematics is mostly facts and procedures that needed to be memorized and that mathematics was not a creative endeavor.

For beliefs about learning mathematics, she insisted that learning mathematics was equally the responsibility of the student and the teacher. Students should discover mathematics on their own without it being shown to them. They learn mathematics better when they work together on mathematics problems. They should be able to figure out for themselves whether or not an answer is mathematically reasonable. She believed that all students are motivated and can learn mathematics using manipulatives and that students should frequently engage in problem solving.
For beliefs about teaching mathematics, she believed teachers did not have to follow textbooks closely to be effective. Rather, teachers should provide more activities from a variety of sources. She was a strong advocate of hands-on learning with manipulatives and believed that good mathematics teachers demonstrated a variety of ways to look at the same question.

For factors influencing her beliefs, she named her experienced as a student in school as the primary influence on her beliefs about the nature of mathematics and her own teaching experience as the main influence on her beliefs about teaching and learning mathematics. She confessed that she disliked mathematics. However, despite her negative view of mathematics, she wanted to afford her students every opportunity to like mathematics and to experience it in ways she never had. Raymond speculated that her desire to make things different for her students may account for her nontraditional view of mathematics pedagogy even though her personal beliefs about the nature of mathematics were so traditional.

For her mathematics teaching practice the teacher was authoritative. She believed that learning would take place only if students were quiet, remained in their seats, and paid attention to her at all times. The students in her classroom routinely took out their paper and pencils and opened their books to the page number written on the board, indicating that they had learned the expected daily classroom pattern and their role in it. Most of the mathematics class period was designated as time for individuals to work quietly on problems from the textbook. What factors could explain such inconsistencies between her beliefs and teaching practice? She described factors, such as constraints, scarcity of resources, concerns over standardized testing, and students’ behavior as
potential causes of inconsistency. She explained that a traditional approach to teaching mathematics is generally more efficient and requires fewer resources. She mentioned that time constraints and classroom management were the greatest sources of inconsistency.

Raymond (1997) concluded that social teaching norms (e.g., school philosophy, administrators, standardized tests, curriculum, textbook, other teachers, resources) and immediate classroom situation (e.g., students abilities, attitudes, and behavior, time constraints, the mathematics topic at hand) play a key role in influencing mathematics teaching practice. Thus, they were likely to play a role in creating inconsistencies between beliefs and practice. Gregg (1995) showed how a beginning high school mathematics teacher is acculturated into the school mathematics tradition (i.e., the beliefs and practices that characterized the traditional approach to school mathematics). For example, the other teachers suggested this teacher tell students that geometry would help them out in real life and that geometry was helping them to learn to reason.

In addition, the results of this study suggested that deeply held, traditional beliefs about the nature of mathematics have the potential to perpetuate mathematics teaching that is more traditional, even when teachers hold nontraditional beliefs about mathematics pedagogy. The teacher in this study simultaneously holds two isolated beliefs. One is that mathematics is predictable, certain, absolute, and fixed. The other is that students should discover mathematics on their own without being shown and they learn mathematics better when they work together on mathematical problems. Such isolation occurs when contradictory beliefs are not explicitly compared (Cooney & Shealy, 1997).

Is it possible that such deeply held traditional beliefs about the nature of mathematics can be changed? Sigel (1985) suggests that certain beliefs may be core
beliefs and these core beliefs may be difficult to change. Pajares (1992) also assumes that belief change is a relatively rare phenomenon and that individuals tend to hold on to beliefs based on incorrect or incomplete knowledge, even after scientifically correct explanations are presented to them. A fifth-grade teacher in Prawat's (1992b) study confirmed the difficulties of beliefs change. With the subject of the case study, Karen, Prawat skillfully described that Karen's views about mathematics teaching had undergone change as a result of using the new, more conceptually-oriented math curriculum adopted by the district. The major theme found by Prawat was that, although there was important change in the teacher's views about mathematics teaching over the course of the year, this change did not appear to be reflected in her classroom practice.

Karen had traditional views of mathematics: Math learning proceeds hierarchically, with children first needing to master the “basics,” certain math facts and procedures learned in rote fashion, before getting into problem solving and other application sorts of activities. She further described rules and procedures as powerful tools not in isolation. They must be learned in context: “I use context all the time.” In her classroom it was important for students to have reasons for learning rules and procedures: “Most of my lessons, I start out with a statement of why we’re doing, what we’re doing” (p. 199).

Karen's hierarchical view of mathematics appeared to be related to her views about how to teach mathematics. She was a strong advocate of the “demonstrate-test-apply” model of instruction. Karen summarized her approach in this way:

I think the teacher should present them [i.e., rules and procedures] in total as to what it is you are doing. This is the way you do this, and you show them. You show them a million times and among those times, you get
Karen expressed some reservations about the use of manipulatives. The use of manipulatives was not at the heart of the program; rather, “it is the icing on the cake” (p. 201).

Karen’s approach to teaching could be categorized as traditional and teacher directed. In her classroom, there is no doubt about who is in charge. Although there was a good deal of verbal give-and-take between the teacher and students, she was firmly in control at all times. However, her views had changed. The notion that students can know or understand mathematics in different ways was a novel one for Karen. It was not evident at all in her comments previously. Karen was asked if she frequently called on students to work through problems on the board. “I only do it when it’s review” (p. 205). When presenting something new, she indicated that she prefers to demonstrate how to do it herself.

I don’t like to embarrass them. It’s focusing an awful lot on the negative of this child’s inaccurate answer, so he’s embarrassed... So then you get kind of this negative thing, and after a while they don’t want to raise their hands because they’re afraid they’re going to say something wrong. (p. 205)

Another problem with having students discuss their solution strategies was that it frequently misleads other students: “If somebody’s giving a wrong answer or they’re leading somebody through the problem wrong... all you’ve done is confuse everybody” (p. 205).
Karen came to recognize the legitimacy of student discourse during mathematics. Prawat assumed that the teacher’s growing appreciation for the role of student discourse during mathematics did not appear to be inspired by arguments. Rather, according to Prawat the impetus for change was to lie in her daily interactions with students as, together, they worked their way through the new mathematics curriculum. The teacher made a significant shift in her thinking.

The idea is—and I agree with that—that there are many different ways to find answers to things or to look at things. I mean, maybe I would come up with the same answer—but there are different ways to look at it. But, I mean, when you just overwhelm kids and give them too many all at once, they don’t get it and are just confused. (p. 207)

Karen more fully accepted the notion that students can, and should, be encouraged to develop their own solutions to problems. She would say, “This is fantastic, and why don’t you stand up and tell the class how you did this—how you came to this conclusion?” (p. 207). Karen’s views about mathematics and the teaching of mathematics appeared to have change in significant ways. There was some evidence that these changes were beginning to affect her teaching. Prawat points out, however, that Karen’s new beliefs have only been translated into a new teaching behaviors. Significant change in Karen’s teaching may require far more sweeping change in her beliefs about the nature of mathematics and the learning process than what has occurred in her classroom. Prawat concludes:

If teachers are to alter their teaching of mathematics, they may need to reexamine a whole network of beliefs extending far beyond their views about the craft of teaching, narrowly defined; they may need to change their views about the nature of knowledge and how one acquires that knowledge. (p. 210)
Cooney and Shealy (1997) argue that contradictory beliefs should be explicitly compared unless isolation occurs, perhaps reflecting the existence of beliefs held from a nonevidential perspective, a perspective immune from rational criticism. The teacher’s beliefs in this study might not be psychological central beliefs but peripheral (Green, 1971). As the conceptual change model of Posner et al. (1982), her existing beliefs about mathematics may not provide her much dissatisfaction. Or her experience with the new form of curriculum may not be enough intelligible, plausible, and fruitful to change her beliefs.

According to Clarke’s (1997) case study, consistency between teachers’ beliefs and their teaching practices depends largely on their tendency and opportunity to reflect on their actions, in the company of supportive colleagues. Clarke conducted a case-study research to investigate changing teacher roles associated with two teachers’ use of innovative mathematics materials at Grade 6. The two teachers had taught for about 20 years, mostly at the Grade 6 level. During the study, they participated in a professional development program with 30 other teachers, part of which involved teaching a 6-week unit of nonroutine problems. Data were gathered using daily participant observation and regular interviews with the teachers over a period of seven months. The author found seven themes of teachers’ role in these classrooms and related these themes to beliefs about the teaching and learning mathematics. Themes and beliefs reflect the relationship between what teachers do in mathematics classrooms and what they believe. The first theme was the use of nonroutine problems as the starting point and focus of instruction, without the provision of procedures for their solution. The related beliefs about the
teaching and learning of mathematics was that students can solve nonroutine problems without first being taught a procedure. The second theme was the adaptation of materials and instruction according to local contexts and the teacher’s knowledge of students’ interests and needs. The related beliefs about the teaching and learning of mathematics was that mathematics needs to be studied in living contexts that are meaningful and relevant to students, including their languages, cultures, and everyday lives. The third theme was the use of a variety of classroom organizational styles (individual, small-group, whole-class). The related beliefs about the teaching and learning mathematics was that differences in mathematical tasks and preferred learning styles of individuals demand variety in classroom organization. The fourth was the development of a “mathematical discourse community,” with the teacher as “follow player” who values and builds on students solutions and methods. The related beliefs about teaching and learning mathematics was that an atmosphere of conjecture and justification of mathematical ideas enhances learning. Teachers should be open about their own struggles with mathematical problems. Students’ solutions and methods provide the basis for discussion of problems. The fifth theme was the identification and focus on the big ideas of mathematics. The related beliefs about the teaching and learning mathematics was that important mathematical ideas are not confined to specific procedures in isolated content areas; rather, mathematics is seen as an integrated whole, in which the processes of problem solving, reasoning, and communication are central. The sixth theme was the use of informal assessment methods to inform instructional decisions. The related beliefs about the teaching and learning mathematics was that observing and listening to students provides a “window” into their thinking that can be used to plan further instruction. The
last theme was the facilitation of students’ reflection on activity and learning. The related beliefs about the teaching and learning mathematics was that reflection provides the opportunity to revisit recently encountered concepts and procedures and to identify connections both within subject content and between content and experience.

Like Clarke’s conclusion, Thompson (1992) also points out the importance of teachers’ reflection on their teaching practices as an agent that makes teachers’ beliefs change. The extent to which teachers’ beliefs about mathematics and the teaching and learning of mathematics is consistent with their classroom practice depends on their tendency to reflect on their actions. She further claimed:

This is not to suggest, however, that, upon reflection, all tensions and conflicts between beliefs and practices will be resolved. However, it is by reflecting on their views that teachers gain an awareness of their tacit assumptions, beliefs, and views, and how these related to their practice… [They] develop coherent rationales for their views, assumptions, and actions, and become aware of viable alternatives. (p. 139)

On the contrary to examine consistencies and inconsistencies between inservice teachers’ beliefs and their instructional practices, the study on preservice teachers’ beliefs change by Cooney, Shealy, and Arvold (1998) suggested that beliefs change is more likely to occur to the open-minded person than the closed-minded person. Their analysis was based on Rokeach’s (1960) notion of an open-mind to closed-mind continuum and Green’s (1971) notion of centrally held beliefs. Rokeach’s notion explicitly addresses context as a lessening factor in holding beliefs. The more open-minded person attends to context. In contrast, a closed-minded person sees no shades of gray because the world is seen from a perspective in which context is considered largely irrelevant. A by-product of this perspective is that an open-minded person is more likely to see the world as a
friendly place whereas a closed-minded person is more likely to see the world as unfriendly. Greg, a preservice teacher participating in this study, held tightly to the notion that, for him, teaching was a matter of “preparing people for life.” Although he initially opposed the use of technology in the teaching of mathematics, he came to see that technology, in fact, could be a vehicle for preparing people for life. In contrast, Henry, another preservice teacher, initially opposed the use of technology and maintained that view throughout the preservice program. It might appear that both Greg and Henry held dualistic views about the teaching of mathematics. Although their surface statements were quite similar, the structures in which those “surface” beliefs were embedded varied dramatically. Greg was open to and valued others’ voices. He enjoyed the fact that different opinions were expressed in class, an element of at least multiplism if not relativism. From this foundation, assimilation and adaptation are possible - accounting for the fact that he later saw technology as an integral part of preparing students for life. His peripheral belief about technology was incorporated into his more centrally held belief primarily because his beliefs were held evidentially.

Henry, on the other hand, described mathematics only as the accumulation of definitions and rules, the truths that were passed on by teachers and textbooks. The notion of “authority or truth” was integral to his thinking (p. 322). He equated knowing mathematics with “remembering what the teachers told you” and did not believe that it was necessary to investigate novel situations (p. 322). He believed that if he remembered the right answers, then all would go well. His thinking fit a dualistic model. He tended to either accept or reject opinions – holding tightly to his own. Henry grew frustrated when he perceived that his beliefs were being challenged. He viewed this challenge with alarm.
All these elements indicate an authoritarian view, one in which assimilation is possible but accommodation is unlikely. Based on these elements he could not incorporate his peripheral belief about technology into his more centrally held belief (authority or truth) primarily because his beliefs were held nonevidentially.

In sum, a few studies reviewed sought to document the relations between teachers' beliefs and teaching practices. Schmidt and Buchman (1983) found consistency between elementary school teachers' beliefs about school subjects and the amount of instructional time allocated to them. Thompson (1984) found a subtle, but somewhat consistent, relationship between teachers' classroom behavior and their beliefs of mathematics and mathematics instruction. And Cooney (1985) reported inconsistencies between what teachers' beliefs about mathematics, teaching and learning and how they behave in classrooms. Prawat's (1992b) confirmed inconsistencies and the difficulties of beliefs change. Raymond (1997) also showed inconsistencies through the struggle of a beginning teacher. Thus, in this small set of studies, some consistency and inconsistency can be observed.

What are plausible explanations for these findings? First, beliefs may represent what teachers want but cannot achieve because they do not possess the requisite knowledge of skills for operationalization (Underhill, 1988). The lack of appropriate knowledge to implement beliefs is the case of the preservice teacher in Cooney's (1985) study. Although his beliefs about mathematics centered on problem solving, he would not be able to teach mathematics based on his beliefs because he did not have many capabilities to deal with an array of students' abilities and content of mathematics. Second, like Green's (1971) central and peripheral beliefs, Underhill (1988) argues that
teachers' beliefs may be hierarchically arranged so that when beliefs conflict or when they require considerable work and effort to operationalize, teachers go with the ones that are the best compromise between importance in their personal belief hierarchies and ease of operationalizing. Third, Clarke (1997), Ernest (1989b), and Thompson (1984) suggest that consistency and inconsistency between teachers' beliefs and instruction depend largely on their tendency to reflect on their actions. In addition, Clarke notes the importance of supportive colleagues in school. Fourth, teacher socialization is also seen as a factor that causes inconsistencies (Brown & Borko, 1992; Gregg, 1995). Last, similar to teacher socialization, social context (e.g., school philosophy, administrators, standardized tests, curriculum, textbook, the expectations of students, parents, fellow teachers) and immediate classroom situations (e.g., students' abilities, attitudes, and behaviors, time constraints, the mathematics topic at hand) play a key role in influencing inconsistencies between teachers' beliefs and practices (Cooney, 1985; Ernest, 1989b; Raymond, 1997).

Ethnographic Research Tradition

Ethnographic research tradition is a major framework for this study that investigates teachers' beliefs and their teaching behaviors. This tradition serves two roles: the researcher's view of mathematics education and a research tool. In this study the basic assumption is that teachers' beliefs are transmitted to students through social interaction in participating in mathematical activities in mathematics classroom. Wilcox (1982) provides this point of view of education. Ethnographers have most frequently
framed their view of schools around the concept of cultural transmission. In this view, the school acts primarily as an agent of the culture, transmitting a complex set of beliefs, attitudes, values, norms, behavior, and expectations that will enable a new generation to maintain the culture as an ongoing phenomenon. This view of education is also found among several scholars’ work. Vygotsky (1978) argues that children learn cultural knowledge (e.g., thinking) from more advanced members of a society through participating in culturally meaningful activities. Rogoff (1990) used the term “guided participation.” By this phrase, she means active involvement by children in culturally structured activities with the guidance, support, and challenge of companions who transmit a diverse array of knowledge and skills. Through participation in the activities children internalize signs, symbolic systems including mathematical knowledge that are so-called the “tools of mind.”

This conceptualization of cultural transmission is also advocated by mathematics researchers (Bishop, 1988; Connoers, 1990). Bishop has creatively combined mathematics and anthropology to make an important point: that anthropology is a useful tool for understanding the transmission of mathematical knowledge in today’s culture. Connoers suggested that Bishop’s notions of teaching as enculturation, of adopting a cultural approach to mathematics curricula, and of teachers as people responsible for the enculturation process are innovative and have far reaching implications for educational reform.

Methodologically, many of the tenets of ethnography derive from a philosophical position sometimes referred to as interpretivism that is quite different from the logical positivism underlying traditional educational research. Central to interpretivism is the
idea that all human activity is fundamentally a social and meaning-making experience, that significant research about human life is an attempt to reconstruct that experience (Eisenhart, 1988). Eisenhart added that the purpose of doing interpretivist research is to provide information that will allow the investigator to “make sense” of the world from the perspective of participants (p. 103); that is, the researcher must learn how to behave appropriately in that world and how to make that world understandable to outsiders. Thus, the researcher must be involved in the activity as an insider and able to reflect upon it as an outsider. Conducting research within this tradition is an act of interpretation on two levels: The experiences of participants must be explicated and interpreted in terms of the rules of their culture and social relations, and the experiences of the researcher must be explicated and interpreted in terms of the same kind of rules in the intellectual community in which he or she works (Bredo & Feinberg, 1982).

The research questions posed by interpretivists are intended to get at the intersubjective meanings of participants’ and researchers’ worlds. The questions ask first, “What is going on here?” and second, “What intersubjective meanings underlie these ‘going on’ and render them reasonable?” (Eisenhart, 1988, p. 104). The important point is that such meanings are invisible using empiricist approaches to social research. In the usual questionnaire study, for instance, one asks individuals about their beliefs, attitudes, or value and then finds out about collective facts by aggregating individual response. In this case social consensus is represented by the extent to which individuals share the same beliefs, attitudes, or values. However, measures of “consensus” of this type do not get at common meanings (Bredo & Feinberg, 1982).
The most important point about intersubjective meanings is that they are a type of meaning that falls out of the grid of the usual empiricist approaches to social research. Intersubjective meanings cannot be measured by aggregating data on individual beliefs or attitudes, or by standardized recording of individual behavior, just as the grammar of a language cannot be mapped by averaging individual usages. (Bredo & Feinberg, 1982, p. 124)

Ethnography is the work of describing a culture (Spradley, 1979). The essential core of this activity, according to Spradley, to understand another way of life from the native point of view. Rather than studying people, ethnography means learning from people. The essential core of ethnography is the concern with the meaning of actions and events to the people the researcher seeks to understand. Some of these meanings are directly expressed in language; many are taken for granted and communicated only indirectly through word and action. But, in every society, people make constant use of these complex meaning systems to organize their behavior, to understand themselves and others, and to make sense out of the world in which they live. These systems of meaning constitute their culture: ethnography always implies a theory of culture.

Ethnographies are analytic descriptions or reconstructions of intact cultural scenes and groups (LeCompte, Preissle, & Tesch, 1993). LeCompte et al. describes that ethnographies re-create for the reader the shared beliefs, practices, artifacts, folk knowledge, and behaviors of some group of people. An ethnographic product is evaluated by the extent to which it recapitulates the cultural scene studied so that readers envision the same scene that was witnessed by the researcher (LeCompte, Preissle, & Tesch, 1993). In addition to being a product, ethnography is also a process, a way of studying human life. Ethnographic design mandates investigative strategies conducive to cultural reconstruction.
Wolcott (1975) defined ethnography as the science of cultural description. Ethnography is first and foremost a descriptive endeavor in which the researcher attempts to accurately describe and interpret the nature of social discourse among a group of people. Geertz (1973) discusses the task of description with great sensitivity. He suggests that the ethnographer is aiming at "thick description," in which a wink can be distinguished from a twitch, or parody of a wink from a wink itself, and contrasts thick description with thin description, in which a wink may be described as the rapid contraction of an eyelid. The thinner the description, the more it is stripped of multilayered social meaning. The practice of ethnography enables one to discover the cultural knowledge possessed by people as natives (members of groups or communities) as well as the ways in which this cultural knowledge is used in social interaction (Wilcox, 1982).

According to the definitions of ethnography, describing culture is an essential activity. Cultural theorist Raymond Williams writes that culture is one of the most difficult words to define (Chiseri-strater & Sunstein, 1997). Spradley (1979) defined culture as acquired "knowledge that people use to interpret experience and generate social behavior" (p. 5). In this scheme, culture embraces what people do, what people know, and things that people make and use. To describe culture from this perspective a researcher might think about events in the following way: "At its best, an ethnography should account for the behavior of people by describing what it is that they know that enables them to behave appropriately given the dictates of common sense in their community" (McDermott, cited in Bogdan & Biklen, 1982, p. 35). Chiseri-strater and Sunstein (1997) defined culture as "an invisible web of behaviors, patterns, rules, and
rituals of a group of people who have contact with one another and share common languages” (p. 3). In other words, definitions of culture can be both metaphorical ("webs" and "lenses") and structural (patterns of belief and behavior as well as untidy deviations from those patterns). Scupin (1995) provides a general definition of culture from cultural anthropology: “Culture is a shared way of life that includes values, beliefs, and norms transmitted within a particular society from generation to generation” (p. 33).

While ethnography has traditionally been thought of as the description of the culture of a whole community, it has been and is equally applicable to the description of social discourse among any group of people among whom social relations are regulated by custom. Erickson (1973) claimed that classrooms and schools are both well-suited to ethnographic inquiry, although the difference in scope and setting requires certain adaptations. LeCompte et al. (1993) mentioned that educational ethnographers examine the processes of teaching and learning, the intended and unintended consequences of observed interaction patterns, and the relationships among such educational actors as parents, teachers, and learners and the sociocultural contexts within which nurturing, teaching, and learning occur. They examine patterns of language use, interpersonal interactions, transactions, relationships, and participation through which cultural processes are expressed and created in educational settings. According to them, the purpose of educational ethnography is to describe educational settings and contexts, to generate theory, and to evaluate educational programs. It has provided rich, descriptive data about the contexts, activities, and beliefs of participants in educational settings. Nickson’s (1992) argument provides that ethnography is a useful tool for studying a culture in mathematics classroom. Claiming that no two mathematics classrooms are
exactly alike, she notes that the quality of the mathematics teaching and learning depend on unseen beliefs and values in a culture of classroom.

As a methodology, Wilcox (1982) notes, ethnography involves far more than a set of easily described and readily adopted data-gathering techniques. Its conceptual underpinnings and mode of use reflect heavily the characteristics of the discipline within which it was conceived and developed. According to Wilcox, ethnography is not synonymous with participant observation, fieldwork, or qualitative research. A thorough understanding of ethnography requires an understanding of the discipline of anthropology as well. Spindler (1982b), the father of educational anthropology, also makes the same argument and warns the danger of merely borrowing ethnographic techniques instead of accepting ethnographic world view. He suggested the following ethnographic world view. First, the ethnographic world view emphasizes the importance of the native, the actor in the scene, as informant-literally as instructor. Too often social scientists have assumed a superior stance in relation to their "subject." In ethnography, people are not subjects; they are experts on what the ethnographer wants to find out about and accordingly are treated with great respect and always in good faith. A trust relationship must be built between researcher and informant that cannot be violated by insensitivity or misuse of information. Agar (1996) points out this relationship between ethnographer and informants. The ethnographer is part of an asymmetrical relationship, especially at the beginning of work. The ethnographer is in the "one-down" position (p. 119). This initial one-down position is reflected in two of the metaphors ethnographers sometimes used to explain themselves-child and student. Agar describes that both child and student are learning roles; they are roles whose occupants will make mistakes, which is perfectly
acceptable as long as they do not continue to make the same ones. They can be expected to ask a lot of questions. They need to be taught; both will look to established membership of a group for instruction, guidance, and evaluation of their performance.

Second, the ethnographic world view emphasizes that natives do not, in fact cannot, realize the full implications of their own cultural knowledge and social behavior (Spindler, 1982b). Their knowledge is necessarily partial and often ambiguous. Explicit, verbalizable cultural knowledge is never complete, even when the lapses of any one informant are corrected or filled in by the knowledge of other informants. There are always tacit, nonverbalized understandings or unforeseen consequences of behavior. Ethnographers should realize that the native, the informant, the actor, is the ultimate source of all data upon which any and all inferences must be based. The untranslated knowledge and behavior of the native constitute the only surely valid data to which the ethnographer has access.

Third, the ethnographic world view holds that all behaviors occur in contexts, and that not only do contexts continuously change but people change with contexts (Spindler, 1982b). So we do not assume that a child in school is behaviorally the same child at home or on the street. Ethnographers are committed to the study of behavior in contexts and of cultural knowledge held by natives as relevant to contexts. Ethnographers chase contexts as far as time and resources permit, but acknowledge the importance of the single context as an object of intensive study. This view is very similar to the symbolic interactionist research tradition initially promulgated by Herbert Blumer (1969). Symbolic interactionists assume that individuals create meaning for their experiences in the course of social interaction. In this view, people’s actions are not caused by objects, other
people, or inner forces, but by the socially derived meanings for objects and actions that people construct to make sense out of a given situation.

Fourth, the ethnographic world view assumes that any classroom, any school, any group or community is a variant adaptation with a regional, national, and world-wide variation in culture and social organization (Spindler, 1982b). The culture ethnographic study at any time is one of many. This assumption calls for a perspective that makes the familiar strange, that exoticizes behavior and meanings that are all too familiar because they are once his own. The ethnographic world view places high value on being “thrice born” (Turner, cited in Spindler, 1982b). One is born into one’s own culture and enculturated within it. As an anthropologist is studying outside one’s own culture, one is born again—one must learn a new culture and learn to think like a native. Making the strange familiar, or familiarizing the exotic, is done both in the field and in one’s interpretation. Upon returning to his own native land, the ethnographer is born for the third time as he refamiliarizes himself with what has now become exotic but was once familiar. This process, so violent at times that it, like the experience of a foreign culture, is termed “culture shock,” gives him a perspective that can never be laid aside. He sees his own culture with new eyes.

Lastly, the ethnographic world view assumes that the function of schooling is to transmit the culture(s) (Spindler, 1982b). The school transmits what is, not what should be. Even when innovations are attempted and educators are dedicated to change, if these innovations and intentions stray too far from the center of the culture and social system they will be defused and mitigated in the process of transmission and cultural acquisition.
Based on Spindler’s (1982b) ethnographic world view, the goal of doing ethnography is to focus on a setting and to discover what is going on there (Wilcox, 1982). In doing so, first, the researcher should attempt to set aside one’s own preconceptions or stereotypes about what is going on and to explore the setting as it is viewed and constructed by its participants.

Second, the researcher should attempt to make the familiar strange, to notice that which is taken for granted either by the researcher or by the participants, to assume that that which seems commonplace is nonetheless extraordinary and to question why it exists or takes place as it does, or why something else does not (Erickson, 1973). Erickson calls this process making it strange; it allows even investigators who study familiar scenes to discern the detail and the generality necessary for credible description. Spradley (1980) also pointed out making the familiar unfamiliar. According to him, the more the researcher knows about a situation as an ordinary participant, the more difficult it is to study it as an researcher. In other words, the less familiar the researcher is with a social situation, the more he is able to see the tacit cultural rules at work. An excellent example of such defamilization is found in Miner’s (1956) “Body Ritual among the Nacirema.” Miner described everyday bathroom objects and grooming practices that seemed like something people never before engaged in. He described the medicine chest as a “shrine” that holds magic potions and “charms.” The toothbrush is “a small bundle of hog hairs” for the application of “magical powers.” Miner focused on describing American’s cultural attitudes about bathing and cleansing habits, American belief in dentists, doctors, and reliance on hospitals and diets. Miner defamiliarized everyday behaviors to see us as
outsiders might describe us: a highly ritualized people who believe in magical customs and potions.

Third, the researcher should assume that, to understand why events happen as they do, the researcher must look at the relationship between the setting and its context-for example, between the classroom and the school as a whole, the community, the teachers, the economy, and so on (Spindler, 1982a; Wilcox, 1982). Spindler notes that observations are contextualized. A judgement of relevant context must always be made, and the character of this context must be explored to the extent that resources allow. Fourth, the researcher should utilize one’s knowledge of existing social theory to guide and inform one’s observations (Wilcox, 1982).

According to Spindler (1982a) a major task of the ethnographic research is to understand what sociocultural knowledge (e.g., beliefs, values, expectations) participants bring to and generate in the social setting being studied. Sociocultural knowledge held by social participants makes social behavior and communication sensible to oneself and to others. Another significant task of the ethnographic research is to make explicit what is implicit and tacit to informants and participants in the social settings being studied.

The success of a good ethnographic research greatly depends on an intimate, long-term acquaintance (Agar, 1997; Spreadley, 1979, 1980; Wolcott, 1995). How long is long enough? How intimate is intimate enough to do ethnographic research? The ideal of 12 months-minimum now having become maximum, as it often does-remains well established (Wolcott, 1995). However, according to Wolcott, that does not mean a 12-month minimum is always observed, but fieldworkers whose stay must be brief usually go to some length to explain their circumstances. He recommends that one way to do
short-term fieldwork is to pay close attention to identifying and observing through whatever constitutes a "cycle" of activity, and to recognize how short recurring cycles may be nested in larger ones. He raises a question: "Does time alone guarantee breadth, depth, or accuracy of one's information?" (p. 78). He surely said that it is not true. Mere presence guarantees little, and most assuredly there have been fieldworkers and research problems that requir less time to get the job done. Then, how do ethnographers do good ethnographic research in a short period of time? According to Wolcott, an intimate relationship with the informants is a key concept.

If it is accepted that teaching and learning mathematics is participating in a social and cultural activity in a classroom as an abstract, small community, that teaching and learning mathematics is negotiating mathematical meanings through social interactions among members in a classroom, and that teaching and learning mathematics is a process of enculturation or of transmitting a culture of a society from generation to generation, then the perspective of ethnography provides a new insight to understanding the process of teaching and learning mathematics in school.

Conclusions and Recommendations

The purpose of this study was to describe how a Korean elementary teacher taught mathematics in everyday classroom and why he taught mathematics as he did in terms of his beliefs about the teaching and learning of mathematics as well as classroom interactions and norms. In order to gain information about what research has been done in the area, what theoretical frameworks help understand the topic, and what research design
was appropriate for the research purpose, the review of literature traversed the definitions and descriptions of beliefs, distinctions between beliefs and knowledge, teachers’ beliefs about mathematics, social and cultural perspective of mathematics and the teaching and learning of mathematics, teachers’ beliefs about the teaching and learning of mathematics, the relationships between teachers’ beliefs and the teaching and learning of mathematics, and ethnographic research tradition.

Beliefs are an individual’s mental construction or understandings of the world through experience and the way it works or should work, may be consciously or unconsciously held, and guide one’s actions. In cultural anthropology, beliefs are cultural conventions that deal with true or false assumptions. Beliefs are taken for granted about physical and social reality and self. Beliefs are immutable entities that exist beyond individual control or knowledge. Beliefs appear to be connected to each other in a network system in which some beliefs are psychologically central whereas others are peripheral or derivative. Beliefs are affective and evaluative while knowledge is cognitive. A knowledge system is semantically stored, whereas beliefs are episodic memory with material drawn from experience or cultural sources of knowledge transmission. While knowledge requires objectivity and consensuality, beliefs do not require group consensus regarding the validity and appropriateness. Since beliefs strongly influence perception, they can be unreliable guides to the nature of reality. Beliefs control thought processes and thus, play a critical role in understanding behavior and the organization of knowledge and information.

The differing beliefs held by teachers in relation to the nature of mathematics are an important component in the culture of a mathematics classroom, since they are linked
with the way mathematics is taught and received (Nickson, 1992). Ernest (1989a) distinguished three conceptions of mathematics (i.e., the problem-solving view, the Platonic view, and the instrumentalist view). Lerman (1983) and Neyland (1995) proposed absolutism and fallibilism (i.e., quasi-empirical view of mathematics) and Tymoczko (1986) argued that the quasi-empirical view of mathematics is the only one appropriate for teachers. It is likely that when mathematics instruction is taken beyond establishing facts and practicing skills to an approach using more openness, investigation, problem-solving, and critical discussion, there will be more social interactions, more negotiation, and more emphasis upon shared interpretation and evaluation of what goes on in the mathematics classroom (Nickson, 1992). From the research on beliefs about mathematics, Perry’s (1970) scheme of intellectual and ethical development was used for analysis. With Green’s (1971) conceptualization of beliefs, the conceptual change model by Posner et al. (1982) might provide a useful perspective of understanding teachers’ beliefs change. According to Mura’s (1993, 1995) study even mathematicians had difficulty in defining what mathematics is. He points out the limitations of the use of questionnaires in studying beliefs about mathematics. He recommends that studying beliefs about mathematics held by elementary teachers would be much more difficult in using questionnaires because they might not be able to articulate their thoughts without prompts. Thus, it is apparent that using interviews is a more appropriate way to investigate teachers’ beliefs. Since beliefs are a personal perspective of understanding the world, the interview should be like friendly conversations, keeping away any judgment of whether the teachers’ beliefs are correct or incorrect.
The current view of philosophy of mathematics emphasizes social, cultural, and human aspects of mathematical knowledge. Mathematics is invented or created by humans, mathematical knowledge is not infallible, and mathematics is an intersubjective agreement, rather than an objective body of knowledge. Doing and knowing mathematics is a process of conscious guessing, with proof following a zig-zag path starting from conjectures and moving to the examination of premises through the use of counterexamples or refutations. Like Lakatos, Polya describes that doing mathematics is following from induction to generalization with an inductive attitude (i.e., intellectual courage, intellectual honesty, and wise restraint) as a social norm for participating in a game of conjectures and refutations. In this perspective the mathematics classroom is a micro-mathematical community and teacher and students are members of the community who are participating in mathematical activities through intersubjective agreement on axioms, definitions, and theorems. From the research on social-mathematical norms it is apparent that the teacher plays a critical role in establishing quality of classroom interactions and classroom norms that regulate teacher’s and students’ roles for participation of mathematical activities. The patterns of classroom interactions between teacher and students and classroom norms are part of manifestations of the beliefs held by teacher and students. Thus, examining teachers’ beliefs should focus on the patterns and norms that exist in each mathematics classroom in different forms through classroom observations. In addition to observing the patterns and classroom norms, the models or theories of the teaching and learning of mathematics teachers select for their lesson would be a valuable resource to identify teachers’ beliefs because selecting the models or
theories is likely to be closely related to and influenced by the teacher’s beliefs about the nature of mathematics (Ernest, 1989a).

Although it is argued that teachers’ beliefs about mathematics and the teaching and learning of mathematics seem to be closely related to each other, teachers’ beliefs about mathematics itself, pedagogy, and students’ learning are not always reflected in instruction in classrooms. From the research on the relationships between teachers’ beliefs and their teaching practices, no conclusive findings are reported. The findings can be observed consistency, some consistency, and inconsistency. The first plausible explanation is that teachers do not possess the requisite pedagogical knowledge. Second, teachers might have core beliefs but they are not psychologically central. Thus, teachers tend to have isolated clusters of beliefs that make commitment impossible. Third, consistency and inconsistency between teachers’ beliefs and instruction depend largely on their tendency to reflect on their actions and supportive colleagues in school as well. Fourth, teacher socialization is also seen a factor that causes inconsistencies. Last, similar to teacher socialization, social context (e.g., school philosophy, administrators, standardized tests, curriculum, textbook, the expectations of students, parents, fellow teachers) and immediate classroom situations (e.g., students’ abilities, attitudes, and behaviors, time constraints, the mathematics topic at hand) play a key role in influencing the creation inconsistencies between teachers’ beliefs and practices. These plausible explanations suggest foci of classroom observations and interviews. Most studies on teachers’ beliefs have focused on individual teachers and their data sources are mostly dependent upon teacher’s self-report. Thus, they have not considered social context and classroom situations. It suggests that studying teachers’ beliefs should include interviews
with principal, fellow teachers students, and parents and analysis of documents (e.g., textbook, curriculum, tests). And observation of students’ attitudes, and behaviors, time constraints, the mathematics topic would be good sources to identify and describe teachers’ beliefs.

Ethnographic research tradition is particularly appropriate for this study. The basic assumptions of the study are that the mathematics classroom is a micro-cultural community, that mathematical knowledge is transmitted from the teacher who is a more knowledgeable member of a community, to students while participating in culturally meaningful activities, and that teaching and learning mathematics is the process of negotiating and renegotiating intersubjective meanings, definitions, assumptions of mathematical objects within the member of the discourse community. Ethnographic world view encompasses these assumptions for this study. Ethnographies are analytic descriptions or reconstructions of the culture, shared beliefs, practices, behaviors, artifacts of some group of people. Ethnographic study provides people with a way of seeing the world, understanding another reality. Since some beliefs are taken for granted, the process of “making them strange” for ethnographic research deals with this issue. Moreover, since beliefs are personal construction of the world and a unique way of understanding the reality, there do not exist any standardized questionnaires and interview questions. The researcher should follow the path the informants lead to understand their reality. Beliefs cannot be understood from outsiders because it is not easy for even the informants themselves to articulate their beliefs. Ethnographic research fits into describing teachers’ beliefs through friendly conversation with rapport building efforts.
CHAPTER III

METHODOLOGY

Introduction

The purpose of this study was to describe how a Korean elementary teacher teaches mathematics in his everyday classroom and why he teaches mathematics as he does. The assumption of this study was that teachers' beliefs and teaching practices of mathematics would be better understood through studying the patterns of classroom interaction and social-mathematical norms in mathematics classroom. Thus, the guiding questions were:

1. What patterns of classroom interaction and social-mathematical norms exist in an elementary Korean mathematics classroom?
2. What beliefs about teaching and learning of mathematics does the elementary teacher hold?
3. Are the teacher's beliefs related to the patterns of classroom interaction and social-mathematical norms? What facilitates or constrains the relationship?

This chapter presents the methodology used to conduct this three month study in Korea. This study used an ethnographic inquiry with a single informant. A description of the community and the school provides general background of the educational settings. The description includes the community, the school, student body and staff, curriculum organization of the school, and the informant and his students. The process of locating the
informant describes the difficulties the researcher faced in conducting this study in Korea, where teachers are unwilling to share their knowledge with an educational researcher. The section about gaining entry to the school describes the first day of fieldwork. In the data collection section, three strategies (participant observation, ethnographic interviews, and documents) are explicated.

Inductive analysis was conducted to identify themes of the teacher's beliefs and classroom practices. Describing the researcher's role provides his educational background and how he took different roles (teacher, supervisor, assistant teacher, and protector) in the field. Pseudonyms were used for all persons involved in this study in order to provide anonymity. The italic words were used for actual Korean language followed by literal meanings in English.

Design of the Study

The rationale for pursuing the ethnographic approach was that almost no attention has been given in the literature on how elementary teachers in Korea teach mathematics on a day-by-day basis. Another reason for this ethnographic account was to provide a description of a culture of a school and classroom by analyzing the teacher and students' behavior in their classroom. This ethnographic inquiry was focused on describing and analyzing beliefs about the teaching and learning of mathematics and classroom norms from an ethnographic perspective during a particular period of time.

The essential core of this activity was to understand another way of life from the
teacher's point of view (Spradley, 1979). A culture of a school and classroom was defined as acquired knowledge that teacher and students use to interpret experience and generate classroom behaviors. Keeping with this definition of a culture, the researcher focused on the identification of patterns of classroom interaction, social- mathematical norms in the classroom. To conduct this study, fieldwork approach was used to selectively record certain aspects of teacher and student activities in order to construct explanations of their behavior in cultural terms.

The selection of a single informant was based on Wolcott’s (1992) recommendation that studies of multiple informants reduce the total attention that can be given to any one of them, weakening rather than to strengthening the study. He expressed a strong preference for studying just one informant in depth, especially when the researcher is not experienced in this ethnographic type of research. Moreover, it would be a critical factor to establish a solid and dependable relationship with the teacher and students in the classroom in order to be accepted as a member of the classroom. Since establishing such relationships requires considerable time and effort, a single informant was selected.

Locating a Teacher

Locating an elementary teacher willing to participate in this study was clearly a crucial step in its accomplishment. Although the attempt was made as systematically as possible to select a representative classroom teacher, Teacher Lee, the informant of this
study, was not the first choice. It is not appropriate to address a teacher by his or her last name such as Mr. Lee in Korean educational settings. Formal vocabulary is used to speak to the teacher, who is called sun-sang-nim (teacher) rather than by name. “Teacher” is not the name of a job but should demand respect from others. This respect is expressed by “-nim,” which is attached to sun-sang.

Although almost any teacher would be an informant, not everyone makes a good informant. Before beginning this search, criteria for selecting a teacher were identified. A good informant is one who has had years of formal and informal experience with the social situations that the researcher is studying (Spradley, 1979). The teacher should know his school and classroom culture and no longer consciously think about it. One way to estimate how thoroughly the teacher has learned a cultural scene is to determine the length of time they have been in that scene. Berliner (1987) suggests that after teaching for five years, a teacher may be designated as experienced and it may be assumed that these teachers are consistent in their instructional strategies. Thus, at least five years of teaching experience as an elementary teacher was a criterion.

A series of ethnographic interviews were conducted to gather data for the study. In fact, seven one-and-half hour interviews with Teacher Lee were conducted during August. So, it was important to estimate whether a potential informant had adequate time to participate in the interviews. The willingness exhibited by a potential informant does not always give a good clue to whether that person has adequate time. Thus, the teacher who is willing and has adequate time to participate was considered as a potential informant.
The focus of this study was to describe teacher's beliefs and norms and interaction patterns in classroom. In order to investigate them, a teacher who uses classroom discourse and small group activities more frequently was considered as an appropriate informant because such classroom settings would provide more chances for identifying and describing the beliefs and norms of the classroom. In addition, the teacher who places more emphasis on mathematics as his professional teaching subject in elementary level would be considered as a good candidate. In fact, Teacher Lee and his classroom settings sufficiently satisfied these criteria to be studied. He was the only teacher in the school, who had constantly used small group settings with a group of five or six. And since his professional subject is mathematics, he was in charge of training teachers of the school to improve mathematical pedagogy and parents helping teachers in mathematics lessons.

Before this dissertation proposal, the researcher made several contacts by phone with colleagues who are elementary teachers or university professors in Korea in order to obtain a purposeful sample satisfying the criteria mentioned earlier. The network used included two elementary teachers, two professors at a university of education, and one principal of an elementary school. Since this study was not intended to be carried out for six months or a whole year of ethnographic study, a search was attempted to find an elementary teacher, who was willing to participate in the study and to allow the researcher to observe teaching practices in his or her classroom. By doing this, more emphasis could be on guiding research questions instead of spending painstaking time to establish rapport with an unfamiliar teacher.
Initial contacts, however, were not successful. After listening to the plan of the study, those contacted refused to participate in or recommend another teacher for the research. They indicated it is the customary behavior of Korean people that they are reluctant to show themselves to the public. A female fourth-grade teacher in a telephone conversation pointed out that she would be really uncomfortable being observed in her classroom. She was persuaded to consider the research, indicating that the intention was not to evaluate her teaching but to understand her teaching practice. Then she revealed the reason, saying “I have the least confidence in teaching mathematics. It is not only in my case but other teachers, too. They would not readily participate in such research for whole three months.” It was an indication of the difficulty in getting a teacher for the study.

There was no apparent hope to locate a teacher even in Korea. The whole first week of June was devoted to visiting several professors at the university of education to ask for teachers who would be able to participate in the study. At last, four elementary teachers names and telephone numbers were given. They were acquainted with the researcher because they were fellow students at the master program of mathematics education for two years. In the course of friendly conversations, participating in the research was discussed. Fortunately, they were willing to discuss the possibility. Three of them had at least seven-year teaching experiences at the elementary level and were enrolled in the doctoral program of mathematics education while they were classroom teachers at elementary schools. This high qualification created an unexpected problem. They were good enough as informants with a deep understanding of mathematics and
pedagogical content knowledge in mathematics. However, they used the same educational terms as the researcher knew, such as constructivism, qualitative research, norms, conceptual understanding, and so on. They could not articulate their thoughts without using these terms. It was of importance to know how they viewed their teaching practices and themselves as a classroom teacher. The theoretical terms they used tended to block their thoughts creating a disadvantage for the researcher to understand and interpret their culture in a school in terms of their everyday language. This phenomenon is what Spradley (1979) warns, "the beginning ethnographer will do well to locate informants who do not analyze their own culture from an outsider’s perspective" (p. 54).

Teacher Lee was the last hope at that point after all attempts came to failure. In fact, Teacher Lee and the researcher were in the elementary teacher education program of mathematics for four years. After graduating from the program, he became an elementary teacher and the researcher went to a graduate program of mathematics education at a different university. This relationship was enough for him to accept the request to participate. Another reason he readily accepted the request was that he wanted to learn how to conduct educational research because he was planning to do his action research in the next year. Doing action research is part of the requirement of Korean teachers, who want to be promoted.

In a conversation with him one night in the first week of June, there was an opportunity to explain the research: classroom observation on everyday basis, interviews with students, parents, teachers in the school as well as him, and videotaping lessons. The researcher was in no position to insist that Teacher Lee participate. He showed some
interest. There was hesitation to say whether he would be willing to participate in the study because it was already established that Korean teachers would not take part in such time demanding research. It was a delightful moment when Teacher Lee said, "That is not going to be any problem. Come to my classroom. I don't care about being observed by you." At this moment the anxiety suddenly faded away, but the excitement had to be kept until the researcher asked a few more questions about his classroom to check if Teacher Lee would meet the selection criteria mentioned earlier. He was a third-grade teacher and there were 45 students in his classroom. The arrangement of students' desks were groups of five or six. The situation seemed perfect except that the school in which he was working was a little below average in terms of achievement. "The school does not have good facilities, either. I wonder if this circumstance would be helpful for your study," said Teacher Lee, showing his concern about becoming a representative teacher of public schools in Korea.

Student's achievement in the school was not a factor for the study, once the school was a public school and it turned out later there was no means of comparing students' achievement among elementary schools because the Ministry of Education decided to prevent elementary school students from taking any standardized tests. It was also observed in Teacher Lee's school that the first standardized test was administered at the end of June during the first semester of the year but this test was not reported neither to students nor their parents. The test scores were only used by teachers for the purpose of assigning students to classrooms in the next school year.
Teacher Lee and His Students

Teacher Lee is a 33-year-old male, born in the southwestern region of Korea so he has a somewhat distinctive dialectal accent that students seem to understand. He is about 170 cm (5 feet 8 inches) tall and is thin. A description of him might be a “smoking man,” based on his students. He usually smokes about a pack of cigarettes a day. He did not drink alcohol which is exceptional in Korea.

Teacher Lee has 10 years of teaching experience and he is his third year at Midoong elementary school. This year is his second as a third grade classroom teacher. He will be appointed at a different school next year and did not know where he would be. He is now the head teacher of five third grade teachers and is in charge of curriculum planning and implementing, as well as evaluation at the grade level. He is acknowledged as a competent teacher by the principals. According to the assistant principal, he earned the highest score in a 60-hour science in-service training program. Perfect scores do not often happen. One beginning teacher stated that Teacher Lee is considered an able teacher in the school and does his job well at all times. Specializing in mathematics, Teacher Lee is the vice president of the “Study Group of Mathematics Teaching” (SGMT) supported by the school district and an active, organizing member of the “Study Group of Elementary School Mathematics” (SGESM) as well, which has about a 10 year history. Elementary teachers voluntarily join these two study groups to improve their pedagogic content knowledge of mathematics.

Born in a small farming village, during his elementary years, Teacher Lee did not think of being a teacher but was influenced by his cousin and uncle, who are elementary
teachers. He gradually realized that he needed to study engineering to make money.

However, he failed an entrance examination at the university and had to study another year for this exam. Many Korean high school graduates wait for another chance to enter a university where they want to study. During those years he had never considered being an elementary school teacher, to earn a smaller amount of salary compared with other jobs after four years college education. His parents urged him to go to a university of education because they could not afford his college education except in a university of education whose tuition is one-third of other universities. In addition, he was the eldest son of his family and had to consider his two sisters and one brother. Another reason he chose to become an elementary teacher was that he would be exempted from military service requiring all men in Korea to serve in the army for three years. Due to lack of male teachers in elementary schools, the Korean government offered to exclude those who applied as an elementary teacher at that time.

He received a bachelor degree of elementary education, specializing in mathematics, in 1989. By regulation, every first-year student in a teacher education program must select one area of specialization based on the subject matters relevant to elementary school. Because of specialization in mathematics, he had to study six pure mathematics courses, including set theory, linear algebra, real analysis for two semesters, geometry, topology, probability and statistics and pedagogic content knowledge and teaching materials of mathematics as well. However, many times he felt that taking these courses had nothing to do with his pedagogy in teaching elementary school mathematics. He joined the mountain climbing club for four years and experienced the importance of
the leader's responsibility for club members' activities. Such sense of responsibility of a leader was sufficiently reflected by his actions as a classroom teacher.

Like most Korean university students, he had done much tutoring of mathematics from fifth grade through high school students. From the time of his senior year of the teacher education program until his assignment to a classroom teacher by the school district, he had taught fifth and sixth graders in Sok-sem-hak-won, a learning center. This teaching experience was quite unique to him because most prospective teachers do not have that experience in teaching mathematics in front of about 25 students, although they usually have some experience of tutoring with one or two students. Such experience influenced his beliefs about "emphasizing conceptual understanding" and provided critical knowledge to decide what additional topics his students attending a learning center needed in his mathematics class.

His wife is also an elementary teacher in the city, they met each other at the same school. They were classroom teachers in the same grade and married when he was 31 and his wife 28. They have a two-year-old girl and will have a new baby in January next year. He lives in a small apartment in the city and is gaining stable financial status. He is satisfied with his profession as an elementary teacher.

In his third grade classroom, 45 students (24 boys and 21 girls) come from predominantly working class families. Ten of the parents have university education including one two-year college graduate. Thirty-five have a high school education and one has middle school education. For occupations 36 parents are either factory workers or company employees. Two of the students are living with their mothers, which is unusual in Korea. One girl in this classroom indicated that she had to pick up her five-year-old
brother at a kindergarten because her mother is a saleswoman employed by an insurance company and usually gets home about six in the evening.

About 15 students in Teacher Lee’s class attend a variety of Sok-sem-hak-won, literally meaning, a place of learning speed arithmetic. They learn mathematics here in advance of the teaching schedule of their school, including science and writing. The focus of teaching at this place is to memorize and practice facts, rules, and algorithms to solve problems in mathematics. This type of learning technique is called “machinelike solving problems” mentioned by the teachers. The only valuable activity here is to solve as many problems as possible correctly. Some schools located in the good communities have more than 50% students, who attend learning centers. Attending this place brings difficulties in teaching mathematics in the classroom because they already know what the teacher is trying to teach. A major problem these students bring to the classroom is a lack of motivation. Besides motivation, these students only valued is finding correct answer quickly. Teacher Lee dealt with this problem skillfully, but one beginning teacher was overwhelmed by these students.

I cannot motivate them. They are simply not interested in what they are doing. They know it already... They don’t care about activities. They want to know the results. (Conversation, 7/1)

Another problem regarding parents is that Teacher Lee is not able to consult often with parents about their child’s behavior or academic problems. He is only able to communicate with parents through a “letter to parents.” He does not attempt to call or visit them. Moreover, visiting parents is prohibited by the Ministry of Education due to possibly receiving a little token of parents’ gratitude. This prohibition might be
understood as an effort to maintain impartiality. Such a situation makes it difficult for Teacher Lee not only to understand his students’ family backgrounds, but to get parents’ support for more appropriate treatment for his students.

A Description of the Community and the School

The Community

The city, Inchon, is the third largest city in South Korea. Approximately 2 million people live here. It is a port city adjoining Seoul, the capital city of Korea which has a population of 46 million people. The city is divided by four administrative districts (eastern, western, southern, and northern) and the community is in the western district. Many factories (e.g., mostly automobile part and furniture factories) are located in this district due to the proximity to Seoul. There is an export industrial complex in this community, and thus these factories supply many of the jobs in the area. According to the teachers, this district is the poorest of the city and the residents are predominantly working class. A city highway to Seoul runs nearby the community and the streets are one-way on each side in front of the school. Many shops and business markets surround the community resulting in roads with busy traffic. The teachers are concerned about possible car accidents. This environment requires that the school must take strong disciplinary actions to protect the students. During an interview with the principal, a phone call from a parent reported that some students of the school had been seen entering an adults’ bar nearby the school to play electronic game machines. Students of Teacher
Lee also confirmed that this problem is a concern of classroom teachers: “Teacher Lee used to get harsh and if he knew the student who went to the game room, he ordered him or her to run five rounds of the playground,” said one of the students in an interview.

The School, Student Body, and Staff

This *Mi-dong* elementary school was built in 1988 with 18 classrooms. In February of 1999 the seventh graduation was held. As with other schools in Korea, this school’s motto indicated it is “a school helping children cherish a big future dream and to enjoy their school life.”

Figure 1 provides a schematic diagram of *Mi-dong* elementary school. There is a street on the left side of the wall of the school. A cement road that is 10 meters (33 feet) long and 5 meters wide leads to the main entrance of the school. The school is surrounded by a brick wall, so the main entrance is the only way to enter the school. The gate is a heavy, metal bar, making it unwelcome and impenetrable. The school is four stories, concrete and darkish gray. The site for the building is 13,612 square meters (148,230 square feet), 7,778 square meters (84,700 square feet) for the playground, and 1,576 square meters (17,160 square feet) for the building. The school building houses first- through sixth-grade students with a total of 1148. There are 29 classrooms in the building, including two classrooms for children with special needs and other rooms are used as a science lab, a computer lab, educational material rooms, a library, a nursing room, and so on. Average class size is 42.5. The size of this school is relatively small, compared to other elementary schools where each grade has approximately 8 or 10 classrooms.
**Figure 1.** Schematic diagram of *Mi-dong* elementary school
Each classroom window faces the direction of the playground, which is bare. On the windows Teacher Lee’s classroom is marked with white-color paint that says “3-3,” literally means “Room third of the third grade.” On the left and right side are two stands in front of the building. In the center of the steps there is a podium, which is usually used for a “school assembly” at 8:40 every Monday morning. The purpose of this assembly is mostly to make announcements by the principal and to award a certificate of merit to great readers, or the best essay writers for the month. There are three front doors, in the center, the left, and the right of the building. Students are allowed to use any doors near their classrooms.

Because the Ministry of Education discourages the use of standardized tests in elementary school, no means exist to compare the school’s achievement with other elementary schools, though teachers and principals acknowledged that students’ achievement of the school is somewhat below average. According to the teacher’s “Curriculum Guide Book” published by the school, the average test scores administered by the school in the 1998 school year was 68.85 out of 100 in math and 73.14 in science. The analysis of the mathematics test revealed a lack of conceptual understanding in geometry, measurement, relation, and word problems. To improve students’ achievement in these areas, this report suggests the need of process-oriented pedagogy and using a variety of concrete manipulatives in mathematics.

According to a survey completed by the school in the spring of 1999, the majority of the parents of the school are in their 30’s and dependent on the school program for their children’s education. Since more than 50% of them are double-income families,
parents are not actively involved in their children’s education. For the school newspaper, one sixth-grade girl wrote a letter to her mom about how much she wanted her mom to stay at home:

... Mom, please don’t go to work. On a rainy day, I used to ask my friends to share umbrellas because nobody could bring it to me. Otherwise I was in the rain crying all the way home. I had to open the door with my key and said “I’m home!” but there was no response... (July 16, 1999 in the school newspaper)

With many double-income families, the school has a lunch program even for the first- and second-grade students. First- and second-graders do not usually participate in the program in the other elementary schools. About 72% of the parents are high school graduates and more than 80% are blue-collar workers. More than 71% of the parents have fewer than two children and tend to be protective of their children.

Two principals oversee the school: Kyo-jang (the head principal) in charge of all decision-making for administrative concerns and Kyo-kam (the assistant principal) who assists him. The head principal has 32 years of educational background and is in his first year at this school. The assistant principal has 33 years of educational background and is in his second year at this school. Each teacher, including principals, is rotated to a different school every four years. The principals have no teaching load. They are in charge of teacher evaluation. In the end of the school year each teacher is evaluated by the principals and the evaluation is recorded in the assessment of each teacher’s performance. Curriculum coordinator, Kyo-moo, is the most important person in school in terms of school management. The curriculum coordinator, usually a male teacher, is in charge of academic affairs: design and management of curriculum, curriculum
evaluation, principals’ advisory role. The curriculum coordinator of the school has 21 years of educational background and is in his third year at this school.

In the school there are 34 teachers including one nurse teacher (14 male and 20 female) and eight general staff members (e.g., four technicians, one dietician, one licensed cook, one assistant for science lab). Of the 25 (6 male and 19 female) classroom teachers, seven teachers, including two first year teachers, have less than three years of teaching experience. The 25 classroom teachers are divided into six divisions of academic duties, each led by a division head. Teacher Lee is in charge of the “research” division, dealing with teacher training for new pedagogical content knowledge, in-school supervision, development of test items, and research on teaching.

The climate of the teachers is mostly relaxed and friendly. According to observations, the teachers have a volleyball game every Wednesday afternoon. The assistant principal said this kind of sport activity with the teachers helps him break bureaucratic relationship with teachers and form a cooperative spirit among the teachers. One teacher, who has 30 years of teaching experience and is seriously considering taking early retirement, said that playing volleyball is a boost of solidarity for the teachers. He also mentioned that the teachers are cooperative. Trying to reduce authority by school administrators seemed to result in a friendly environment.

Curriculum Organization of the School

The number of school days in a Korean elementary school is 212 per year (118 the first semester and 94 the second semester). The first semester (22 weeks) runs from
March through August except for a 30-day summer vacation between July and August. The second semester (18 weeks) runs from September through February of the next year except for a 60-day winter vacation between December and the next February.

Third through six graders must study 10 subjects: moral education, Korean language, mathematics, social studies, science, physical education, music, arts, practical home economics course, and English. The most school hours are assigned to learning the Korean language, 238 hours. Mathematics is next with 136 hours, and with 102 hours for social science and science.

Each grade level of this school has its own project, which the teachers wish to accomplish every year. For the third grade, the project has been decided by five teachers in the beginning of school year, including one teacher (not a classroom teacher) who takes full charge of English. The project for third grade of this year is "establishing solid classroom discussion through small group learning."

The goal of curriculum management in the third grade, especially in mathematics, is "to improve students' mathematical thinking ability and foster students' disposition toward mathematics." The focus of teaching mathematics to achieve these goals are: process-oriented teaching and learning, promoting mathematical reasoning, becoming autonomous learners with creative thinking, and providing individualization. The project, goal, and focus of teaching mathematics were stated by Teacher Lee as the head teacher. Interview and observation data suggest that these positions reflect Teacher Lee's beliefs about the teaching and learning of mathematics.
Gaining Entry: The First Day of Fieldwork

The day before the research was supposed to begin, the researcher made a call to Teacher Lee to ask if there was anything to be prepared before going to his school. The researcher was concerned about how the principal of the school would respond to this research. Although Teacher Lee had already accepted the job as an informant and an insider to guide the fieldwork, the principal and the assistant principal were important persons to approve access to the school. They were gate keepers of the study. “Don’t worry about conducting your research in the school. I already told the principal and he knows about it. He said there would be no problem in doing educational research unless you are not a political offender,” said Teacher Lee on the phone.

A written summary of the research was prepared to inform the staff of the presence of the researcher at the school so that everybody accepted him as a member of the school and a researcher. Teacher Lee said that there is a “faculty meeting” at four o’clock as usual. The meeting was held in a conference room every Monday at four o’clock and all teachers of the school attended this meeting. Teacher Lee recommended that the researcher explain the study and get to know them. The researcher needed to bring a pair of slippers to wear inside the school building but was not necessary to bring lunch due to the availability of a lunch program. All students and teachers of the school joined the lunch program for about $20 a month.

After 40 minutes on a subway and more then 20 minutes on a public bus, the researcher arrived at the school. Crossing the playground, the anxiety level rose as if entering into a different world. At the entrance hall a woman working in the
The administrative office saw the researcher through a window and asked his business. She was asked where Teacher Lee’s classroom is and she looked at a sheet on her desk and said bluntly, “Room three of the third grade on the second floor.”

It was a sunny day of 27 degrees Celsius (80 degrees Fahrenheit) with some breeze. The researcher did not wear a suit but a well-ironed and short-sleeved shirt with dark blue lines on white and pants with sharp creases. Black shoes were shining enough to reflect the sunlight. It was 10:30 and the researcher was waiting in the hallway near the back door until the class was completed. It was a math class and seven children were at the board working on a division problem. Teacher Lee was at the left side of them watching how they solved the problem on the board. This process continued about 10 more minutes. Teacher Lee came out of his classroom, saw the researcher and took him to a place on the same floor just 10 meters (33 feet) away from his classroom along the hallway. The sign said, “language lab.” The room was for an English class.

The lab served as a teacher’s lounge for five third-grade teachers. At the front left corner of the lab, three female and one male teachers enjoyed conversation, standing around a table and sipping coffee when Teacher Lee and the researcher approached them. “I’d like you to meet my friend as I told you last week,” said Teacher Lee. The researcher exchanged nods with them in ordinary Korean manner. A male teacher about 40 years old asked, “Where are you studying?” At that moment the researcher was not sure how much information Teacher Lee had told these teachers about the researcher. The researcher restrained himself from showing them that he is a researcher from university. He did not want to appear as a selfish educational researcher, who was doing research for the sake of their professional career or dissertation, not caring about teachers and students in the
study. He felt they would not talk and behave naturally unless they accepted the researcher as a member of their culture. These four teachers were the most important persons in the school concerning this study. Like Teacher Lee, they generously welcomed the researcher, patiently and thoughtfully responded to questions, and gracefully tolerated his intrusions.

Teacher Lee discussed work they had to complete this day or this week. After five minutes of discussion and friendly conversation about their weekend, the male teacher asked the researcher again, “You aren’t going to use this kind of conversation as your data source, are you?” The researcher took this opportunity to explain what the research was about and what he was planning. They were told that the researcher would listen to all their conversations and observe them and that a tape-recorder would also be used all the time. The researcher did not ask them to sign the Informed Consent Form since he could not decide at this point if these teachers might be informants for the study. But in fact they all were excellent informants and supporters through this study. At the end of fieldwork, all the teachers, including the principals, signed the forms.

When the school bell chimed the teachers left for their classrooms. It was 11:00 a.m., the time for the Korean language class of Teacher Lee’s classroom. Teacher Lee stopped by his classroom and told the students to read the textbook for about five minutes until he returned. Teacher Lee and the researcher walked down the hallway on the same floor to meet the assistant principal and the principal of the school. The sign of the room read “the teachers’ conference room” and appeared twice as large as a regular classroom. Teachers’ desks of the same grade level were placed together as a group. Two columns of desks were arranged at the left and right side of the room and the arrangement extended
to the end of the room. The assistant principal’s desk was at the front and center of the room. Teachers came in this room on Monday afternoon for the “faculty meeting” or for using one of the two computer printers and a copying machine. Since teachers usually stayed in their classrooms all day long, the big room looked like the assistant principal’s office.

Leaning his upper body and head forward about 15 degrees and putting his arms by his sides, Teacher Lee made a bow to the assistant principal, as a way of showing respect to him. The researcher had difficulties with this way of greeting, perhaps because of a five-year exposure to American culture. “I’d like to introduce my friend to you,” said Teacher Lee. Standing up, the assistant principle said, “I heard about you from him. I wish your research goes well in our school.” He put out his hand and shook hands with the researcher.

Teacher Lee and the researcher entered the principal’s office through another door. He was reading a newspaper and was sitting at a black-colored sofa. He recognized the researcher, “Where are you studying? I couldn’t remember, although Teacher Lee told me about you.” He already knew why the researcher was there. It was a good beginning. Teacher Lee and the researcher were invited to sit on the sofa. The principal’s oral permission was the only necessary action for conducting educational research in Korea. It was not necessary to get permission from the school district. “I wish that this research becomes a learning process for both you and the teachers of our school through solid cooperation. Good luck for your study.” Once out of the office, the researcher gave a sigh of relief, thinking that his accomplishments had gone through two stages, the third-
grade teachers and the principals. Although Teacher Lee arranged many things for him, the researcher still felt anxious.

When Teacher Lee and the researcher were returning to his classroom, there was noise coming from the classroom. The front and back doors of the classroom had been left open. Two or three students were chasing each other, a few students were reading the Korean language textbook as Teacher Lee had instructed them to do. Most students were enjoying chatting with each other and laughing. When they saw Teacher Lee and the researcher coming into the classroom, all sounds were stopped immediately. All 45 pairs of small eyes looked at a strange man they had never seen before in this school.

“As I told you last week this teacher is an old friend of mine. We studied together in college and he is now studying in the United States. Look at my lips. I will tell his name without sound. Guess it,” said Teacher Lee. Teacher Lee tried to make distinct lip movements for the researcher’s name. But it was not as successful as he hoped. “This is Teacher Cho, Cheong-Soo. He will visit our classroom for his research, so he will stay here until summer vacation. He will sit over there in the back of this room and observe what we are doing. Can you say ‘Hello’ to him as loud as possible so you can welcome him to this classroom?” said Teacher Lee to the students.

“Hello!” the students exclaimed at the top of their voices. Teacher Lee stepped aside from his lectern so that the researcher could say greetings. It was the first time that he had stood in front of such small children since his student teaching in 1988 and 1989. The researcher was embarrassed when he realized that he could not speak before them. Since the previous Thursday, all he had thought about were the teachers, principals and
gaining entry for the study. However, children were more important once the researcher was in the classroom. They truly became his companions throughout the research.

"Hello, everybody. I hope we get along with each other. I am really happy to meet you all," the researcher said, hearing his heartbeat. He maneuvered through children's desks and sat down at the desk, which Teacher Lee put at the back of the classroom. Although the desk was for little bit taller children in this classroom, it was small but not uncomfortable. Thereafter, Teacher Lee continued his Korean language lesson. The researcher felt his body was going to collapse with freedom from great tension.

Data Collection

Three main data sources were used for the study. The first data source was from participant-observation, the second from formal and informal interviews, and the third from a variety of documents. Each data source is separately described. The data for the study were collected over a three-month period from June through August 1999.

Participant Observation

The researcher adopted the role that Gold (cited in Wolcott, 1973, p.7) describes as the "participant-as-observer," a role in which the observer is known to all staff in the school and is allowed to observe rather than perform as teachers perform in their school and classrooms. Most of the time it was possible to be an observer, and during most of the time in the school it was possible to write notes. The main role that the researcher took was to observe Teacher Lee, the students, and the other four teachers of the third
grade. In the field, the researcher was an observer shadowing Teacher Lee in different situations and places either in school or outside school.

The researcher visited Teacher Lee’s classroom everyday except Saturday and Sunday. But if Teacher Lee had mathematics class on Saturday, the researcher visited the school. Such Saturday classes happened three times during the study. The researcher arrived at the school around 9:30 in the morning and stayed until five o’clock. The first class assigned to moral education began at 9:10 a.m. and was not observed. But during the last two weeks, the researcher visited the school at 8:00 in the morning to observe what Teacher Lee and the students were doing from their arrival until the first class.

During the first class, the researcher stayed on the stands in front of the school building, a place covered by wisteria providing cool shade. Otherwise, he stayed in the “teacher’s lounge” on the third floor of the school. In both places, he wrote thoughts, working hypotheses, or something on the fieldnotes he did not remember from the previous day.

The researcher did not bring a tape-recorder until the second week of fieldwork. It was felt that Teacher Lee and the four teachers of the third grade would not behave naturally if they knew the tape-recorder was with him. Without the tape-recorder the researcher had to totally depend on his memory. He listened carefully to what people said and watched how they interacted, and then whenever possible noted key words on his palm-sized notebook. He consistently carried both a yellow pad for fieldnotes and a palm-sized notebook for jottings and memos. Jotting down key words were usually done after conversations. As fieldwork continued, it was recognized that many words written in the fieldnotes were not exact words that the teachers used in their conversations. It was
realized that this process was somewhat of a disadvantage when studying teacher’s beliefs and needing to reflect on their words; so two weeks later, the teachers including Teacher Lee were informed that they would be audio-taped. In fact, audio-taping did not affect their behavior and conversation because they were already accustomed to being observed by the researcher.

In this school teachers have the “morning meeting” everyday for 20 minutes from 10:40 to 11:00 between the second and the third class. In the morning meeting, Teacher Lee would bring a teachers’ daily information sheet containing events and instructions for the work of the day. Teacher Lee delivered them to the teachers of the third grade. Besides, teachers talked freely with each other about curriculum management, their family, difficulties of dealing with some work, their students’ misbehaviors, teaching techniques, and so on. There is also a “same-grade teachers’ meeting,” (a meeting for all teachers’ of the same grade level) held on Thursday afternoon from 4:00 to 5:00. Contrary to the morning meeting, this meeting seems to be more official. Four teachers of the third grade came to Teacher Lee’s classroom and discussed curriculum implementation and modification for the next week, evaluation planning, or checked the progress of their work such as preparing report cards. These two types of teachers’ meetings were observed for two reasons: first, to understand the events of the third grade level, and second, and more importantly, to observe Teacher Lee in a variety of social situations.

The researcher also joined the “faculty meetings,” held on Monday afternoon from 4:00 to 5:00. Observations for these meetings were to identify school events. Here the leader teachers of six academic affair divisions proposed their plans and types of
work and sometimes asked for cooperation of other teachers. One important role of this meeting was that teachers in the “lesson research” division held short workshops for in-service training. The content dealt with teaching techniques or pedagogical content knowledge of subject matters. On one Monday the leader of that division, who had 20 years of teaching experience specializing in music, held a 40-minute workshop about Korean traditional music and differences between Korean music and Western music.

Teacher Lee indicated that he held his workshop in May about using mathematical games for elementary students. In this workshop, he demonstrated addition of integers and comparing magnitude of fractions using mathematical games. He wrote in this paper that:

A variety of mathematical games in mathematics classroom should be used more frequently so that students have interest in learning mathematics. They bring students’ active involvement in the process of teaching and learning to the classroom... Through mathematical games teachers will be able to get away from the old-fashioned pedagogy of mathematics, transmission of mathematical knowledge, and students will become autonomous learners of mathematics.

In fact, data collected revealed that this statement describes Teacher Lee’s beliefs about the teaching and learning of mathematics. The researcher joined all these meetings during the study to understand other situations in which Teacher Lee was involved.

The researcher remained in Teacher Lee’s classroom all day from the second class through five o’clock, when Teacher Lee left the school. He observed all 10 subjects taught by Teacher Lee, except English. The first class was assigned to moral education or Korean language. No mathematics lessons were missed during six-week field work. Guided by research questions, however, classroom observations were intensively focused on mathematics lessons. There were four mathematics classes a week. Of 22 mathematics
lessons observed, 12 lessons were videotaped and four lessons were audio-taped, which were taught in the first week of fieldwork. The other six lessons involved individual seatwork where the students used worksheets, workbook, or a mathematics textbook; these lessons were observed and fieldnotes were prepared. Videotaping was done after the first week because Teacher Lee and his students needed to be familiar with the equipment before actual taping. Even such cautious effort did not prevent Teacher Lee and his students from being uncomfortable. It took three or four video tapings before they were not distracted.

During classroom observations, the researcher remained as unobtrusive as possible in the back of the classroom. The video camera was placed beside him. The researcher operated the camera to pick up Teacher Lee’s writings on the board, students’ presentations on the board, or for close ups of the teacher-student interactions as they engaged in conversation. Fieldnotes were written immediately after a class during the 10 minute recess time. But it was not always possible. During recess time, several students came to the researcher and talked, distracting him from writing fieldnotes. And sometimes Teacher Lee and the researcher had a conversation in the hallway about his teaching.

Over the period of the study, the researcher had opportunities to observe the mathematics teaching of other elementary teachers. Teacher Lee suggested that the researcher needed to observe other teachers’ methods of teaching mathematics. In order to avoid a distorted description of teaching practices in Korean elementary schools, the researcher wanted to record many different teaching styles. In fact, these unique opportunities provided the researcher with a broader perspective from which to
understand and interpret Teacher Lee's teaching practices in his classroom. One observation made in the fourth grade classroom was about fractions (e.g., 2/2, 3/3, 4/4...). This teacher was a beginning teacher, who had just graduated from a university of education specializing in elementary mathematics education.

Another observation was made in the first grade mathematics class about the concept of 10. This female teacher, with eight years of teaching experience, was one of the three teachers who remained in the final stage of the "research lesson contest"¹ in the school district. Teacher Lee substituted for one of the judges in the judging committee consisting of about 10 elementary principals and master teachers. The members of the judging committee were appointed by the school district. Later it was learned that this teacher had won the first place in this contest. There was an opportunity to meet this teacher and Teacher Lee in August. Her specialization was not mathematics, but the Korean language. In conversation with her, she stated that preparing for the contest surely promoted her teaching and understanding of mathematics. She had to study how children learned number concepts and the base 10 system.

The third classroom observation of teachers other than Teacher Lee was made in August during summer vacation. This class demonstrated mathematics teaching by the

¹ The "research lesson contest" is held in the school district. Teachers who want to participate in this contest first submit their lesson plans to the school district. The school district screens the lesson plans and selects about seven to eight teachers. Then the selected teachers submit the videotapes of their actual teaching based on the lesson plans. The school district screens the videotapes and selects three or four for the final stage. The teachers at this stage teach the lesson again in their classroom on a specific day in front of a judging committee. The committee submits their evaluation and comments on the teacher's teaching. The school district grades and awards the teachers. The grades of this contest give advantages for teachers' promotion. Teacher Lee earned the second place in the school district contest. Another contest for teachers' profession development is called the "action research contest." This contest is held each year on three levels in Korea. On the first level, teachers submit their six-month or one-year action research to the school district. On the second level, the school district selects one best research and submits it to the Board of Education of the province. Finally, the Board of Education of each province submits one best research to the nation wide contest. Earning the first place in the nation wide contest is
“study group of elementary school mathematics,” led by professors of a university of education in the city. Teacher Lee was a member of this study group. This mathematics class was taught by a female teacher and the content was on finding a variety of polygons with different areas using a geoboard. These study groups are described in detail in the section on Teacher Lee’s professional development.

Along with these observations, the researcher participated in every in-service training program regarding mathematics provided by the school district accompanying Teacher Lee. The school district required that at least one teacher from each school should attend the programs. Teacher Lee was always sent as a representative of his school in mathematics. One of the programs was about the new curriculum of mathematics, expected to begin implementation in the year of 2000 for the first and second grade students.

The researcher had experience in teaching mathematics twice during the study. Teacher Lee requested that the researcher teach his students for a whole day. He claimed that the researcher would not fully understand Teacher Lee as a classroom teacher unless he had some teaching experience in the classroom. The researcher taught Korean language, social studies, two-hour arts class, but there was no mathematics class on the day. Though the researcher had teaching experience of mathematics and computer science at university level and the elementary teacher certificate, he did not have actual teaching experience in the elementary classroom.

The researcher had another experience in teaching mathematics during summer vacation. Teacher Lee’s study group of mathematics teaching held a four-day summer most honorable and advantageous to a teacher’s promotion. Teacher Lee is trying to do his action research on the effect of mathematical games in fostering students’ conceptual understanding of mathematics.
school during summer vacation. The main theme of the summer school, "Exploring Mathematics," was to teach mathematics differently from textbook problems. The researcher taught a two-hour mathematics lesson to four classes. This event is described in detail in the section of Teacher Lee’s professional development.

All these observations, including classroom observations and participation, were recorded in the fieldnotes. The researcher wrote as many ‘observer comments’ (O.C.) and working hypotheses as possible in order to later understand and interpret the occurrences in reviewing the data. Although the hypotheses turned out later to be either true or sometimes totally wrong, they served as a guide in the inductive process of the study. The researcher wrote his perceptions, interpretations, feelings, mistakes, and frustrations in the palm-sized notebook while commuting daily by subway. The fieldnotes and notebook were word-processed in a chronological order in the evening of every day.

For videotapes of mathematics lessons, all teacher-student interactions were not fully transcribed due to the limited time. Rather these transcriptions were more focused on a narrative description of the each activity to identify general patterns of teaching practices and classroom interaction, and social-mathematical norms. For instance, there were different presentation norms between mathematics and other subject matters. For mathematics lesson, students presented their solutions on the board in front of the class and explained how they solved the problems, followed by question-and-answers. More emphasis was placed on the students’ explanation of the process of obtaining an answer in this mathematics classroom. While reviewing the videotaped lessons, the researcher made observer comments regarding interaction patterns and norms. Observer comments
and working hypotheses guided the next observation and encouraged a topic of conversation of the next day with Teacher Lee either at lunch time or after school.

**Ethnographic Interviews**

Two types of interviews, formal and informal, were used in the study to collect multiple data sources in order to get Teacher Lee's knowledge, thoughts and interpretations. Informal interviews were more like friendly conversations regarding almost everything about Teacher Lee, such as his family, the school, the teachers in the school, his past experience with parents and principals, his mathematics class, his professional development, his involvement of the study groups, his roles as a classroom teacher and in the school, his beliefs about the teaching and learning of mathematics, his students and their parents, current educational reforms, teachers' hardship under IMF (International Monetary Foundation) since 1997, curriculum management, and so on. The researcher did not bring any specific questions to ask, but rather the topics covered in the informal interviews were developed from observer comments and working hypotheses of the previous observations and interpretations. The flow of the conversation was controlled by Teacher Lee and the researcher's role was to listen after suggesting a topic. However, the researcher had a clear direction of the conversations (Spradley, 1979). These conversations occurred whenever and wherever possible. They occurred in the hallway during the 10-minute recess time, during lunch time in the classroom, in the teachers' lounge, or after school.

The researcher also had several conversations with other teachers in the teachers' lounge. The topics covered their views about current educational reform in Korea, the
school, a variety of educational problems, parents, and so on. These conversations provided the researcher with different perspectives on the same phenomena of education. All the informal interviews were written in fieldnotes whenever possible when the researcher could find a time and a place. Fieldnotes was typically written in Teacher Lee’s classroom, the teachers’ lounge, or on a subway, and word-processed in the evening.

All other interviews were done in a more formal manner. Several domains, but not specific questions, were identified and prepared for the interview in advance and a tape recorder was used with the tapes transcribed later. The persons participating in the formal interviews were the principal and the assistant principal, two students’ mothers, and 17 students of Teacher Lee’s classroom. The researcher conducted interviews with both the assistant principal and principal to understand their roles, school management principles, and educational philosophy. This information was useful to grasp Teacher Lee’s action in the school. For example, Teacher Lee mentioned that many aspects of teaching practices in a classroom depended upon the two principals’ intentions. Buying educational materials and books must be obtained by their approval. If they decline, there was no way of using the materials in classroom unless the teacher bought them with their own money. For example, there was about $1500 funded to each school by the school district for purchasing educational materials for creativity in the early month of this school year. The assistant principal asked Teacher Lee what he wanted to buy and he suggested geoboards and tangrams for creative activity in mathematics. Now one geoboard and a set of tangrams were placed on the chalkboard in every classroom in the school. "It was fortunate enough to purchase these materials. It is not possible if they [the
principals] say no," said Teacher Lee. This approval reflected the assistant principal’s beliefs about mathematics. In the interview with him, he stated that elementary mathematics should be taught by hands-on activity using concrete materials. He added that students, especially in elementary school, should perceive mathematics as fun activities. To support this activity he claimed each school should furnish a “mathematics laboratory” like a science lab.

Fortunately, the researcher had an opportunity to talk with three beginning teachers in the school. It was near the end of June, the busiest time for teachers to prepare students’ report cards. According to Teacher Lee, making students’ report cards is a big challenge for beginning teachers because so many items must be carefully evaluated for each student’s academic improvement as well as physical checkup (e.g., the number of cavities, heights, weights, and the like). This two-hour conversation occurred in Teacher Lee’s classroom after the students were released. The researcher asked some questions developed by previous observations of Teacher Lee and conversations with him including several other teachers. Because mathematics was their specialized subject matter, most of questions were related to mathematics teaching and learning. For example, the sample questions that the researcher asked were, “Do you use a small group setting for mathematics teaching?” “What way do you think that mathematics should be taught in elementary school?” “What are some your concerns when you teach mathematics?” “How do you evaluate students’ achievement?” The beginning teachers expressed their beliefs, views, and struggles. Their stories were good prompts for Teacher Lee to reflect on and share his experience as a beginning teacher. He provided the three beginning teachers with many concrete situations and advice, providing valuable information for the
researcher to understand Teacher Lee’s beliefs, thoughts, and interpretations. A tape recorder was used with their permission.

The researcher had an interview with two mothers, who are housewives, during the first week of July. Teacher Lee wrote a letter to three parents about this research and asked for their participation. One mother could not make the interview. The purpose of the interview with them together was to listen to their concerns about education, students’ daily life, their expectations of teaching and learning in school, and their views of Teacher Lee and his classroom. This interview with mothers was conducted in Teacher Lee’s classroom after school and lasted for two hours. Teacher Lee joined the interview for about 30 minutes and then left so that the mothers could talk candidly. A tape recorder was used with their permission.

Seventeen students of Teacher Lee’s classroom were interviewed over two weeks in July. Through classroom observations and daily conversations with students, the researcher identified and selected eight student informants. They were selected based on their ability to articulate their thoughts. Their level of achievement was not considered as a factor for this selection because some students were not able to speak their thoughts in detail, though they are in a high-ability group. Student interviews were conducted four times, each with a group of four or five students. The composition of interview groups was made by students’ social groups. There existed a complicated social network among the third graders in Teacher Lee’s classroom and the network affected their learning and school life. The researcher capitalized on this network in order to make them more comfortable to talk freely and honestly. The researcher asked the student to choose students whom they wanted to share their thoughts. Interviews with students were
conducted in the teacher’s lounge during either music or art class, not mathematics, with permission of both Teacher Lee and students’ parents. Students were allowed to talk whenever they wanted to talk and a tape recorder was used. The purpose of the interviews with students was to understand students’ perspectives, perceptions, and interpretations about classroom events in order to identify interaction patterns and social-mathematical norms. The topics asked in the interviews were about presentation, learning mathematics, group learning, tests, Teacher Lee as a classroom teacher, the researcher, punishment, and the like. The students provided fresh interpretations about classroom events that helped the researcher triangulate the data Teacher Lee offered. Each interview lasted for 40 minutes.

Formal interviews of Teacher Lee were conducted after the fieldwork. Seven interviews lasting an hour and a half or two hours were conducted during the five-week summer vacation at Teacher Lee’s home or a coffee shop. The purpose of the interviews was to find Teacher Lee’s beliefs about the teaching and learning of mathematics and how he related his beliefs to teaching practices and classroom events. This interview using ethnographic interview techniques (e.g., descriptive, structural, and contrast questions) (Spradley, 1979) was the final stage of data collection for the study. Before the interviews, most of the data collected in the school were transcribed and initial analysis was finished. The domains and questions asked were based on this analysis and when new domains emerged from the interviews, more in-depth interviews were made. Examples of emerging domains were “early years of school experience,” “conceptual understanding,” “the aims of math textbook,” “perceptions of students,” and so on. In addition, the researcher identified his interpretations of classroom events, reflecting
Teacher Lee’s beliefs about teaching and learning and requested clarification as to whether the researcher’s interpretations were correct. For example, the researcher asked a question about his interpretation: “You had used mathematical games several times. Is using the games for improving student’s conceptual understanding?” Another example was: “Asking your students to solve mathematics problems on the chalkboard seems to be a way of checking students’ understanding. Did I get it right?” The interviews were tape-recorded and each tape was listened to several times to identify domains and meanings prior to the next interview. The tapes were not fully transcribed at this time, but were transcribed later prior to formal analysis.

The researcher wrote a summary, a narrative description, hunches, observer comments, methodological problems, feelings, and memos just after both formal and informal interviews. For informal interviews with no recording equipment, it was critical to record conversation on fieldnotes as soon as possible. More attention was paid on the site to write exact words or conversations before forgetting. For formal interviews writing fieldnotes was more focused on hunches, observer comments, and feelings.

Documents

In addition to observations and interviews, a variety of documents were also collected to provide multiple data sources. The potential usefulness of different documents was determined by the research questions. The documents included: newspapers, articles from journals, test items used in Teacher Lee’s classroom, daily worksheets, curriculum guide books of the school, brochures of the school, materials for in-service training programs, mathematics textbooks, workbooks, correspondences
between Teacher Lee or the school and parents, the school’s newspaper, and the like.

Newspapers, journal articles, and materials for in-service training programs were helpful in understanding current educational reforms in Korea. Test items and daily worksheets were beneficial for investigating how Teacher Lee practiced his beliefs about the teaching and learning of mathematics. Curriculum guide books and brochures of the school provided useful information about descriptions of curriculum management, the school, staff, and the community. One or two photos of each day were also taken to describe physical settings of the school, Teacher Lee’s classroom, the students’ various activities in mathematics lessons, and the students. The photos were helpful to remember events and activities on a daily basis and were useful for describing physical settings in detail.

Data Analysis

The analysis proceeded in two phases: analysis in the field and analysis after data collection (Bogdan & Biklen, 1992). Data analysis was done by an on-going process that helped the researcher move back and forth between thinking about the existing data and generating strategies for collecting new, or better data (Miles & Huberman, 1994). Since all of the data collected in the field were to eventually become larger, the researcher created a provisional starting list of categories prior to fieldwork. The starting list of categories was developed by focusing on Teacher Lee as a classroom teacher from broad perspective to narrow perspective. Using this method, the focus of the researcher was moving back and forth from a broader context to a smaller, concrete context. In doing so, the start list of 12 categories was developed as follows:
This provisional list of categories provided a means of resolving the problem of data overload and making the data collection more manageable.

Once the fieldnotes from observations, interviews, or documents were word-processed, two copies of the fieldnotes were printed out: one for coding, the other for data retrieval. The fieldnotes were written in a chronological order based on school hour with date, time, the number of fieldwork, and page numbers. Then descriptive codes were assigned in the left-hand margin and reflective remarks were usually put in the right-hand margin. The reflective remarks were short sentences for new hypotheses, questions, doubts, or the researcher's interpretations. At this stage of analysis, descriptive codes were mostly used instead of inferential codes because the researcher did not want to draw premature inferences. The fieldnotes were coded by sentence or paragraph instead of a line-by-line analysis. This approach to coding was especially useful when the researcher had several categories already defined and wanted to code around them (Strauss & Corbin, 1990). Inferential codes were used in the analysis after data collection.
For descriptive codes the researcher underlined key terms in the text and restated key phrases. For example, the following excerpt demonstrated how descriptive coding was done.

How can the way of teaching mathematics be changed? For example, the way of teaching division has not been changed for a long period of time. Pedagogy of mathematics is unchangeable, but the teachers should develop a new way of pedagogy to make students understand better mathematics. That’s the reason I often use mathematical games in my classroom. (Informal Interview with Teacher Lee, 6/17, p.5)

These underlined terms were restated, such as “unchangeable mathematics pedagogy,” “developing a new way of pedagogy for better understanding,” and “using mathematical games.” The date and page numbers of the fieldnotes were also recorded in the left-hand margin with the descriptive codes, in order to provide evidence of the data. The data and page numbers served as an index when the researcher had to read the original fieldnotes to understand the context. When descriptive codes were finished, the code list of the day was made. All codes of the day in the fieldnotes were listed on the date. This code list was particularly useful later to remember what had happened and what was said by people without looking at the original fieldnotes. After the code list of the day was made, the descriptive codes were categorized according to the provisional category list. Then each descriptive code was recorded with its date and page number under each provisional category folder. For example, the codes, “unchangeable mathematics pedagogy,” “developing a new way of pedagogy for better understanding,” and “using mathematical games,” were recorded under the folder of “Teacher Lee’s teaching and learning” with 6/17, p. 5. This coding and recording process was recurrent.
After being in the field five or six times, usually on the weekend, visual memos
(Hubbard & Power, 1993) or diagrams (Strauss & Corbin, 1990) of the week were drawn
with a couple of paragraphs as an initial product of analysis in order to make any possible
relationships among codes. Glaser defines a memo as “the theorizing write-up about
codes and their relationships as they strike the analyst while coding… it can be a
sentence, a paragraph, or a few pages” (cited in Miles & Huberman, 1994, p. 72). Writing
a memo was a way to connect the bits and pieces of seemingly unrelated data into a
whole. The researcher used diagrams as memos because a graphic display was better at
making connections among codes than words. The diagrams were developed on the basis
of the observer comments on the fieldnotes. The following is an example of a paragraph
in a visual memo based on an observer comment.

Teacher Lee often used “fun” in his mathematics classroom. This word,
“fun,” is always associated with a certain manipulative activity. At this
point, using manipulative materials is more for “fun,” rather than for
conceptual understanding of mathematics as he insist. (Fieldnotes, 6/22)

With this memo the word “fun” was connected to “manipulative activity” with a solid
line, and to “conceptual understand” with a broken line. As the research continued, the
visual memos were more developed and complicated. This initial analysis method (daily
and weekly analysis) continued through the end of the study.

During the fieldwork, the videotapes of mathematics lessons were not transcribed
in a word-by-word manner. Instead, they were coded by topics, reflecting interaction
patterns and norms. Before reviewing the tapes, the researcher designed an analysis sheet,
in which digital counters, topic, participants, conversation, and action were the elements
of this analysis. Topics in this study included:
• Questioning-answering
• Teacher explanation
• Students presentation
• Call on
• Feedback
• Evaluation
• Using materials and examples
• Motivation
• Lesson introduction, and lesson closure

Teacher Lee and the student names involved in the topic were recorded in the participants column. In the conversation column, the important content of the conversations was summarized, and in the action, brief descriptions about Teacher Lee and his students’ actions or movements were written (e.g., laugh, pause, point to, eye contact). In addition, observer comments and visual memos were written, which frequently became conversation or discussion subjects of the next day. Since there were no post-instructional interviews about his lesson, the conversation accounted for Teacher Lee’s point of view of a certain aspect of his lesson.

After all data were collected, formal analysis was undertaken. A duplicate of the original copy of field notes with codes was made for a master copy. The data, about 2000 pages, were arranged in chronological order and each day’s data had its own page numbers. Then, the researcher read over the data twice along with the 12 provisional categories. In doing so, these general categories were broken into more subcategories. The category, for example, “Teacher Lee’s teaching and learning,” included:

• Introduction
• Reading lesson objectives
• Understanding concepts and algorithms
• Learning and learners
• Open-ended approach
• Using daily experience of students
• Student presentation
• Teacher Lee’s explanation
• Seatwork
• Showing various ways of doing mathematics
• Group Learning
• Constraints of teaching math

All fieldnotes were sorted by cutting and sorting them in the folder for each category. The researcher read each folder again and if necessary, more subcategories were developed. Then an attempt was made to see any patterns and themes in the data. This process helped understand the contents of the folder better and write a summary about the category. The researcher continued to read the folders of the data to develop a holistic picture, writing summaries and drawing diagrams. In doing so, themes of Teacher Lee’s beliefs about mathematics teaching and learning emerged from the data. Once the data were coded, the researcher pursued confirming and disconfirming evidence from the multiple data sources (e.g., fieldnotes, interview transcripts, documents) (Erickson, 1986). At this point of the data analysis, videotapes of classroom practices were fully transcribed and analyzed again for confirming and disconfirming the themes. A way of confirming and disconfirming used in the study was triangulation.

For triangulation a matrix of findings was made by data sources and methods. Data sources used for triangulation included: different persons (e.g., Teacher Lee, other teachers, principals, parents, students), different places (e.g., classroom, in-service training programs and study groups, teachers’ meetings, social meetings after school), and different times (e.g., early stage of fieldwork, middle stage of fieldwork, final stage of fieldwork, after fieldwork). Methods used for triangulation included: classroom observations, observations made outside classroom, formal and informal interviews, and
documents. The important findings that represented Teacher Lee’s beliefs and teaching practices were listed by these sources and methods. Contrasts and comparisons were made in order to indicate how well supported they were and to note any inconsistencies and contradictions. When any inconsistencies and contradictions happened, data weighting was conducted (Miles & Huberman, 1994). The findings by observations of classroom and teaching practices were given more weight than those by interviews. The findings at the later stage of observations were given more weight than those at the early stage of observations. This decision was made especially by conversations with students. Students were asked whether they had noticed any changes in Teacher Lee’s behaviors after participating in this study. The following transcripts indicated that Teacher Lee’s classroom practices had been changed at the early stage of observations and returned to normal at the later stage of observations:

Sung-don: Since you [the researcher] have been here, our sun-sang-nim did not give us as much punishment as usual. (Fieldnotes, 6/17)

Ji-hyun: But now he is getting stricter regardless of your presence.
Jung-ha: Because you did not say anything about what our sun-saeng-nim does, he seems to do the same way he did before. (Interview, 7/9)

Tae-min: I said our sun-saeng-nim did not spank us with his rod since you came, but lately he behaves as usual whether you are in classroom or not. (Interview, 7/13)

The final analysis generated six major themes of Teacher Lee’s beliefs about mathematics teaching and learning and classroom practices:

1. Behave orderly and think freely
2. Teaching mathematics with understanding
3. Manipulative activities and games
4. Discourse-oriented teaching practice
5. Mathematical tasks
6. Professional development

The description of Teacher Lee’s beliefs about mathematics teaching and learning and classroom practices was developed based on these themes to answer the research questions. A variety of direct quotations and narratives were to illustrate the findings of the study.

The Researcher Role

The researcher’s primary role was as a participant observer participating in as many school events (e.g., classroom observations, teachers’ meetings, in-service training programs) as possible. In the fieldwork, observation had two functions: making sense and interpretation. Whatever the researcher in the field saw and heard, he tried to interpret and make sense within his conceptual and theoretical framework. In addition, observation as well as data analysis in qualitative research was purely selective. This selectivity was constantly influenced by the researcher’s institutionalized background. Thus, this background knowledge drove the focus of collecting data in the field of the study.

This study of the relationship between teachers’ beliefs and interaction patterns and norms has been generated from Vygotskian psychology (Moll, 1990; Rogoff, 1990; Wertsch, 1985), which put an emphasis on teacher’s sensitive guidance for students’ learning as a member of a society. In particular, two concepts, scaffolding and
intersubjectivity, stand for teacher's role in classroom learning. Such importance of the teacher's role that provides gradual transfer of responsibility of learning cognitive strategies to students and shares understanding with them are not readily demonstrated without the teacher's beliefs about teaching and learning. Since the realization of the important role of teachers' beliefs, teacher's beliefs have provided the researcher with powerful constructs to understand classroom practices and to improve the quality of teaching.

Another driving force of this study was the recognition of the fact that teaching practice of mathematics should pay more attention to negotiating mathematical meanings with students, not transmitting meanings to them which are already formed by teacher (Cobb & Bauersfeld, 1995; Cobb, Yackel, & Wood, 1995; Yackel & Cobb, 1996; Yackel, Cobb, & Wood, 1991). The researcher believes that social-mathematical norms are central constructs for negotiating mathematical meanings. It is believed that establishing such norms improves students autonomous learning in mathematics classroom.

The researcher envisions that the mathematics classroom is a community with its own culture, and in which teachers teach cultural knowledge to students. In this community, teacher and students should mutually construct social-mathematical norms, guiding their behaviors for interaction and doing mathematics. The researcher claims that mathematics is not perceived as an immutable body of truths, but it is a body of knowledge through tentative agreement among community members at a certain time. In this community, mathematics should be viewed as an empirical science, so each teacher should demonstrate mathematics as human activity. Mathematical communication among
members in a classroom should be a major way of doing mathematics, so that students are able to learn "morality" of doing mathematics in a classroom (e.g., accepting and acknowledging others' assertions, willingness to revise their own assertions, and keeping and defending their own assertions). Each student in a mathematics classroom should be perceived as an individual meaning-maker with his/her own world view, while actively participating in a mathematics activity. Thus, teaching and learning mathematics is a process of mutual construction of mathematical meanings through teacher-student interaction. This perspective of mathematics and mathematics teaching and learning framed and influenced the researcher's preconceptions throughout the study.

While having an elementary teacher certificate, the researcher had no experience in teaching elementary grades. He earned a Bachelor's degree in elementary education specializing in elementary mathematics education and Master's degree in elementary mathematics education in Korea, where he taught mathematics and computer science to preservice teachers for several years. At the time of this study he was a doctoral student in mathematics education at a northwestern university in USA.

No experience of teaching elementary grades and five years detachment from Korean society while staying in USA to pursue his doctoral degree, provided a unique researcher role. First of all, Teacher Lee and other teachers in the school considered the researcher as a novice teacher, who needed guidance from them in order to behave in an appropriate manner to be accepted as a member of this culture. They did not avoid obvious questions to them that the researcher asked. For example, one afternoon Teacher Lee and the researcher had to make a 30-minute trip to the mathematics education department of a university of education to pick up educational materials. He went to the
assistant principle’s office and asked for his permission to leave school early. The researcher asked, “Do you have to do that all the time when you leave school for a while?” Teacher Lee answered that teachers need permission if they leave the school before 5:00 p.m.

The researcher was gradually accepted as a member of the school as the study continued. The principals treated the researcher as a teacher; the researcher was asked to participate in all events related to the school such as in-service training programs, which were held inside and outside the school, social meetings after the school, and mathematics study group activities. One afternoon Teacher Lee asked the researcher to go a social meeting with teachers and said, “You should go with me everywhere if you want to study me.” He pointed out exactly the aims of this study.

Other teachers considered the researcher as a supervisor, who was watching and evaluating Teacher Lee’s teaching practices. They seemed to be surprised at what Teacher Lee was doing. It was often heard that they would not let even a colleague watch their teaching in the classroom for about two months. A female beginning teacher said to Teacher Lee, “You are being supervised every day, aren’t you?” And one teacher in the third grade, who had 20 years experience of teaching, said, “Don’t you [Teacher Lee] know how much your teaching techniques are getting improved by his [the researcher] supervision? That’s what supervision is all about.” The term “being supervised” was an exact expression of what all teachers in Korea think about educational researchers when they participate in research, especially in qualitative research, which remains an unusual type of research methodology in mathematics education of Korea.
Another role Teacher Lee and other teachers assumed was that the researcher was an assistant teacher; the researcher was not concerned with this assumption. Teacher Lee requested the researcher check students’ solution of mathematics problems from the textbook and workbook all the time. The researcher graded students’ test papers and helped students clean up the classroom after school. Teachers often said, “Teacher Lee, you should give half of your allowance to him,” “You have your assistant teacher, haven’t you?,” or “You earned the bachelor degree and he, your assistant teacher, has a PhD. That’s funny.” All these phrases suggested how the researcher was considered by the other teachers in the school.

Students, however, saw the researcher dramatically different from teachers. They tended to consider him as a “protector” from Teacher Lee’s spanking with a small wooden rod (about 1 cm diameter and 30 cm length). Teacher Lee used this rod only for disciplinary purposes. He struck the student palms twice, if they did not bring their textbooks or homework notes. He used the rod when students broke classroom rules such as fighting each other or disrupting lessons. “Spare the rod, spoil the child” has been Korea’s school credo for centuries. It is a cultural practice of schooling and the rod is referred to as the “rod of teacher’s love.” According to the students’ daily conversations and formal interviews, Teacher Lee had not used the rod as many times as he did before this study started. But they mentioned in the middle of the study that Teacher Lee was again using the rod as usual. Teacher Lee once said in the hallway during recess time, “They [the students] are recognizing that I am not going to use the rod when you [the researcher] are in the classroom. They know it. That’s why they sometimes misbehaved a lot.” However, the students complained when the researcher could not come to the
classroom in the first class or sometimes on Saturday. "Why didn't you come little bit earlier in the morning, so we can avoid the rod," said a student one morning as the researcher went into the classroom. Certainly there was some change in Teacher Lee's classroom practices when he participated in this study, but it was related to disciplinary actions, not to teaching practices, which was the focus of the study. On the other hand, the students fully accepted the researcher as a classroom teacher, not as a researcher.

In sum, in this fieldwork, the researcher took four roles at the same time: teacher, supervisor, assistant teacher, and protector. The role of supervisor and protector was not expected by the researcher before the study. Thus, the researcher was aware of the two roles because they were apt to influence Teacher Lee and students' behaviors in classroom practices. He attempted to avoid making any suggestions for effective teaching of mathematics. At the same time, he exerted himself to remain detached from the students, but not to the extent of damaging rapport with students. Teacher Lee needed to be in charge of classroom; they were his "children." The roles that the researcher took in the fieldwork changed occasionally, but eventually provided different perspectives to reflect upon the same events that had taken place in the classroom and the school.
CHAPTER IV
RESULTS

Introduction

The purpose of this study was to describe how and why a Korean elementary teacher teaches mathematics as he does. Specially, the study sought to describe Teacher Lee’s beliefs about the teaching and learning of mathematics and relate them to patterns of classroom interaction and social-mathematical norms. The guiding research questions were:

1. What patterns of classroom interaction and social-mathematical norms exist in an elementary Korean mathematics classroom?

2. What beliefs about teaching and learning of mathematics does the elementary teacher hold?

3. Are the teacher’s beliefs related to the patterns of classroom interaction and social-mathematical norms? What facilitates or constrains the relationship?

This chapter presents the findings of an ethnographic study of how Teacher Lee’s beliefs about the teaching and learning of mathematics were implemented into his classroom practices. In order to answer these research questions, this chapter consists of two descriptive sections, and six major themes identified through data analysis.

The first section depicts the current educational reforms in Korea in order to provide a broad perspective to understand the teaching practices. The second section describes Teacher Lee’s classroom, his activities for a day, and curriculum organization.
The descriptions of six major themes are to provide evidence to answer the research questions. These themes are as follows: a) behave orderly, think freely: regulations of Teacher Lee’s classroom; b) teaching mathematics with understanding; c) manipulative activities and games for the development of conceptual understanding; d) discourse-oriented teaching practices; e) mathematical tasks for teaching practices; and f) Teacher Lee’s professional development. The account of each theme begins with Teacher Lee’s beliefs about the teaching and learning of mathematics followed by actual classroom episodes. Each theme is supported by a variety of data sources such as classroom episodes, conversations, formal interviews, and documents.

Current Educational Reforms in Korea

Three types of educational reforms were undertaken. The one was economical restructuring in all areas of the country since IMF (International Monetary Fund). A newspaper expressed IMF in this way:

The financial crisis we faced in 1997 was perhaps our biggest challenge since the Korean War. The whole economy was on the verge of collapse and then was dragged into recession. (“Responsibility For,” 1999)

In order to overcome this financial crisis, the government and companies have reorganized their structures. As a consequence of reorganization, many workers were requested to apply for earlier retirement. The government cut the civil servant’s allowance up to 125% in a year, including public school teachers.
Such discouragement not only came from the issue of money but from the government. In the middle of reorganization, the Ministry of Education announced that the retirement age of public school teachers from elementary through high school would be shortened from 65 to 62. The purpose of adopting the early retirement system was to accelerate educational reforms for which the older teachers were blamed as obstacles of the reform movements. Many teachers decided to retire early because the government offered additional and higher pension. Another driving force of early retirement was from parents. Most parents had complained about the lack of absorbing new knowledge by the older teachers due to the adoption of new technology and English education. Parents were becoming distrustful of these teachers.

The enforcement of the early retirement plan, in August 1999, was an unprecedented replacement of principals in Korean educational history. Of approximately 8000 principals of elementary through high school in the nation, about 3800 (48%) were replaced (Kang, 1999). While classroom teachers expected that this replacement would bring a radical reform in education, there were some concerns about the difficulties of school administration. Indicating that there would be no educational reform unless teachers were changed, Teacher Lee anticipated the results of the reforms.

However, the rush of the early retirement resulted in a limited number of teachers in the nation. In the case of Teacher Lee’s school, the average class size had increased to 45 from 40. Simultaneously, since the teachers’ work load had increased, each had to spend more time doing classroom chores instead of preparing teaching lessons.

The other type of educational reform was to renovate the traditional goal and practice of education in Korea. It was acknowledged by Koreans that the essential
purpose of school education was for getting into a prestigious university. The traditional teaching practice used the strict teacher-oriented teaching with incessant memorization and practice. The teaching practice reform initiated and supported by the Ministry of Education since 1995 was referred to as “Yel-lin-koy-yuk,” which literally means open education in a Korean style. The incentive of this reform was to break with and shift from the traditional goal and teaching practice to cultivate the education of the whole person and the respect for human life and dignity. The new teaching practice was to meet and maintain individualized learning, an autonomous learner, and a creative thinker (Ham, 1999). This reform discouraged paper-and-pencil tests and corporal punishment in the elementary school level. In addition, while the reform released elementary students from oppressive exams, it caused a decline of students’ performance. Teacher Lee mentioned that students tended not to exert themselves to learn since the reform was put into practice in schools. On the other hand, Teacher Lee indicated that an active interaction between teacher and students had increased in the course of a lesson since the reform.

The third educational reform was that the Ministry of Education adopted and enforced “performance-based assessment” in all grade levels in order to improve the quality of teaching and learning. The school where Teacher Lee worked followed the regulation by designing a schedule and a rubric for this assessment. According to the schedule and rubric, the third grade mathematics was divided into five areas:

- Number (Unit 1. Four-digit whole number- the third week of March)
- Operations (Unit 2. Addition and Subtraction, Unit 3. Multiplication, Unit 4. Division- the first and third week of April, the second week of June)
- Geometry (Unit 4. Plane geometry- the third week of May)
• Measurement (Unit 7. Length and Time- the fourth week of June),
• Relationship (Unit 9. Various types of math problems- the fourth week of May)

From some unknown reason or by mistake, there was no schedule and rubric for Unit 8, Fraction. In addition, the assessment objectives and perspectives of each area were described. For example, the assessment objective of Unit 7, Length and Time was written: “Students will be able to measure the lengths of a variety of things around them (e.g., one side of the length of an eraser, the length of an pencil, the length of a span) by getting accustomed to using a ruler.” The assessment document described the assessment procedures and recommended the use of different assessment techniques (e.g., observation, paper-and-pencil test, performance). Three levels of the scoring scale were used, high, middle, and low level.

Despite the effort of the reform by the government, teachers expressed their frustration and difficulties to implement this new type of assessment. The following exchange occurred during a teachers’ morning meeting.

Song: Yesterday I participated in the in-service training program for implementing performance-based assessment. All teachers complained about the difficulties of that assessment. Professors and experts from the Ministry of Education, however, did not provide useful information but they said “do it this way, do it that way.” All teachers were really angry due to these people’s disregard of the frustration of classroom teachers.

Lee: Classroom observation of students’ performance is one technique recommended for performance assessment. However, that’s impossible. The school district would ask for the evidence of students’ evaluation when the supervisor comes to school. How do teachers provide such evidence they observed during classes? (Fieldnotes, 6/29)
The teachers needed sufficient information to understand and fulfill the performance assessment successfully in their classrooms. In addition to the information, ample administrative supports also needed to be provided.

The recent situations of education in Korea, especially at the elementary school level, apparently seemed to be in confusion with the lack of the number of teachers, the students' performance decline, the new reforms, and the discordance between educational policy makers and classroom teachers.

### A Day of Teacher Lee’s Classroom

On the wall next to the wooden front door, there was a framed picture of two students folding colored papers. Just over the picture was a title, “a class enjoying paper folding,” written in blue. This title was the motto of Teacher Lee’s classroom. The school district recommended that classroom teachers select a topic as a classroom project that they wanted to accomplish this school year. The reason Teacher Lee chose the motto was that the paper folding activity would provide students with fun and manipulative skills. Figure 2 illustrates the schematic diagram of Teacher Lee’s classroom.

Above the chalkboard, an amplifier and a national flag in a golden frame hung on the white colored wall. On the left side of the chalkboard, “a class enjoying paper folding” was written in big letters and below it the benefits of paper folding: (a) enhancing creativity; (b) improving cooperation; (c) enhancing dimensional ability; and (d) improving manipulative ability. Five or six students’ desks were put together as a group. Four groups were in the front row and another four groups in the back row. On
Figure 2. Teacher Lee’s classroom
every Monday morning each group had to move to the right-hand side group place in the
same row. Each group had their own name such as "White mind," "Little five," "Winter,"
"Snowman," and "Einstein." The composition of a group was heterogeneous. Each group
had a leader, who was one of the high-ability students in the class and appointed by
Teacher Lee.

There was a computer table in front of the chalkboard along with the teacher's
desk. The table was covered by a thick glass under which a monitor was placed with the
screen facing up. The table had three compartments. The VCR and main computer were
in one compartment. A small projector like an overhead projector with a video camera,
was in another. This equipment was connected to a 32 inch television. Teacher Lee often
used the Powerpoint for science, social science, or art class but did not use for math class
because his overhead slides were just copies of the math textbook. After group discussion
and when students were asked to explain their groups' findings in the science or social
science class, Teacher Lee used the projector. Students put their notebooks on the
projector to display the image on the TV screen.

Two electric fans clung on both left and right side of the wall. This classroom
furnished a variety of books including encyclopedias, children novels, biographies of
great men and women, workbooks, videotapes for math, science, and English. There was
an organ for music class but Teacher Lee never used it, because he was not confident in
playing it. There are no music teachers in public elementary schools in Korea so every
classroom teacher teaches his own music class. This requirement was somewhat of a
burden to Teacher Lee.
Beside the windows toward the playground, several plants such as a kidney bean, an onion, were in plastic pots displayed for observation of their germination and growth for science class. There was a small aquarium, in which several small tadpoles were reared for science class. On the backboard, students’ self-portraits, works of paper folding, three-dimensional works made with cardboard were displayed.

When Teacher Lee arrived at his classroom by 8:20 a.m., both front and back doors were already open and about 20 students were in the classroom. Two students were appointed as Dangbun, with duties to come to school before many students and the teacher to open classroom doors and clean the classroom. Teacher Lee sent about 10 students to pick up litter on the playground where his classroom was responsible for cleaning. Students continued coming to classroom and greeted him. Prior to the first class started, there was a “morning self-learning period,” where students completed pre-assigned task. Students could chose one of tasks such as reading a book, folding a paper, or constructing a tangram. The tangram construction would be checked by Teacher Lee. He put copies of tangram sheets containing about 20 varieties of shapes in silhouette on his desk such as a boat, a cat, a running person. Students picked them up and had to construct the shape using pieces of tangram. If students could construct the shapes, they drew lines on the shapes to identify how tangram pieces should be put together. Once a week Teacher Lee collected and checked them. Another important activity Teacher Lee had to do everyday was to check students’ diaries. Everyday two groups would put their diaries on the teacher’s desk in the morning and Teacher Lee checked them. In this manner, each student’s diary was checked once a week. Since the diaries needed to be back before school hours, he only checked whether the students missed writing in their
diaries. The purpose of checking diaries everyday was to “form the habit of writing a
diary.” He did not expect educational benefit from writing a diary such as writing skills.
He stated that a goal of elementary education should be to form a habit of studying, not
transmitting knowledge.

To obtain an overall picture of activities in Teacher Lee’s classroom, refer to the
table on page 154, Teacher Lee’s weekly timetable. He taught 27 periods of 29 academic
periods Monday through Saturday. Two periods for English were taught by an English
teacher. Seven periods were assigned to Korean language every week, four periods for
math, and three periods for science and social science. Each period was 40 minutes with a
10-minute recess after each period. There were lunch periods everyday except Saturday
and cleanup times everyday, including Saturday, in which all the cleaning tasks were
carried out by the students.

After the second period, there was a “teachers’ morning meeting of the same
grade” for 20 minutes. During this meeting, students had a 10-minute recess and did
activities (e.g., folding papers, constructing tangrams, reading books) for another 10
minutes under a student monitor. The five teachers of third grade gathered in the
language lab because this grade did not have a teacher’s lounge. Teacher Lee brought a
teachers’ daily information sheet that he picked up at the teachers’ conference room when
he arrived at the school in the morning. Picking up the information sheet was his first
routine as a duty of a head teacher of his grade. This sheet informed directions, activities,
in-service training from principals and six leaders of academic divisions. Teacher Lee’s
duty as a leader of his grade was to deliver this message to the teachers of the same
grade. In this meeting, the teachers’ conversation usually began with the message or they
<table>
<thead>
<tr>
<th>Period</th>
<th>Time</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8:00-8:40</td>
<td><strong>Dangbun</strong></td>
<td><strong>Dangbun</strong></td>
<td><strong>Dangbun</strong></td>
<td><strong>Dangbun</strong></td>
<td><strong>Dangbun</strong></td>
<td><strong>Dangbun</strong></td>
</tr>
<tr>
<td>1</td>
<td>8:40-9:05</td>
<td>School assembly</td>
<td>Morning self-learning period</td>
<td>Morning self-learning period</td>
<td>Morning self-learning period</td>
<td>Morning self-learning period</td>
<td>Morning self-learning period</td>
</tr>
<tr>
<td>1</td>
<td>9:50-10:00</td>
<td>Recess</td>
<td>Recess</td>
<td>Recess</td>
<td>Recess</td>
<td>Recess</td>
<td>Recess</td>
</tr>
<tr>
<td>2</td>
<td>10:00-10:40</td>
<td>Math</td>
<td>Science</td>
<td>Physical education</td>
<td>Korean language</td>
<td>Social science</td>
<td>English</td>
</tr>
<tr>
<td>2</td>
<td>10:40-11:00</td>
<td>Teachers' morning meeting</td>
<td>Teachers' morning meeting</td>
<td>Teachers' morning meeting</td>
<td>Teachers' morning meeting</td>
<td>Teachers' morning meeting</td>
<td>Teachers' morning meeting</td>
</tr>
<tr>
<td>3</td>
<td>11:00-11:40</td>
<td>Korean language</td>
<td>Math</td>
<td>Arts</td>
<td>Music</td>
<td>Home economics</td>
<td>Math</td>
</tr>
<tr>
<td>3</td>
<td>11:40-11:50</td>
<td>Recess</td>
<td>Recess</td>
<td>Recess</td>
<td>Recess</td>
<td>Recess</td>
<td>Recess</td>
</tr>
<tr>
<td>4</td>
<td>11:50-12:30</td>
<td>Physical education</td>
<td>English</td>
<td>Arts</td>
<td>Science</td>
<td>Science</td>
<td>Music</td>
</tr>
<tr>
<td>4</td>
<td>12:30-13:30</td>
<td>Lunch</td>
<td>Lunch</td>
<td>Lunch</td>
<td>Lunch</td>
<td>Lunch</td>
<td>Cleaning</td>
</tr>
<tr>
<td>5</td>
<td>13:30-14:10</td>
<td>Social science</td>
<td>Club activity</td>
<td>Social science</td>
<td>Math</td>
<td>Physical education</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>14:10-14:40</td>
<td>Cleaning</td>
<td>Cleaning</td>
<td>Cleaning</td>
<td>Cleaning</td>
<td>Cleaning</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Faculty meeting</td>
<td>Faculty volleyball game</td>
<td>The same-grade teachers' meeting</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Weekly timetable for Teacher Lee’s third-grade class at Mi-dong elementary school
talked about school problems, parents, or concerns about education. The followings are some examples of the conversations.

Kim: Is there anything important today?
Lee: [Look at the teachers’ daily information sheet] Not quite. There is a demonstrative lesson to the public at Song-won kindergarten starting at three o’clock. So it is ok for first, second, and third grade teachers to go there if they want...
Kang: Kindergarten lessons are always good because the teacher uses a lot of teaching materials and prepares really well.
Lee: I hope I can observe as many classes by other teachers as I can.
Song: What time does it begin?
Lee: Three o’clock. (Fieldnotes, 6/9)

Lee: The first period was so hard. Sweat dripped on my back, standing outside this hot summer morning for 20 minutes. [He complained about Monday morning’s school assembly.]
Kang: I had a terrible morning... One parent came to my classroom and got angry and shouted harsh words at me because her son lost his backpack on Saturday. She suggested it was my responsibility as a classroom teacher...
Song: What on earth do the parents think of teachers?
Lee: Not many parents show respect to teachers... That’s not unusual in recent years.
Kang: How dare did she do that... [Her face was still ablaze with anger.]
Lee: Never try to keep students in the classroom to teach something after school. The principal said some parents would call his office and scold, saying, “Why don’t you allow students come home as soon as possible after school?” The reason they gave was that their children should go to the learning center.
Jeon: [Sarcastically] Going to the learning center is more important than studying in the school?
Kang: Is there anything important we should be aware of?
Lee: Not really. Here is a good one. Teachers in our grade do not need to take in-service training programs for science in this summer.
Teachers: Whoa.... That’s good, so during summer vacation we can enjoy it without being worried about the training programs. (Fieldnotes, 6/22).
Lee: Teachers, we have to announce projects to students for the summer vacation within this week. Please don’t tell them what
and how much they have to do. Just let them choose for themselves and have them report possible ones. One of the students in my class turned in his possible projects that included visiting the cultural assets of this city and writing a description of his impressions. What we have to do is to record the titles of their projects for each student and grade their achievements based on their plans and results between before and after summer vacation.

Kim: Do we have to record all 45 students' projects?
Lee: Yes, as my understanding Kyo-moo (curriculum coordinator) said he has to report to the school district about summer vacation projects of our school.
Kim: This should be done in this school, but what has the school district to do with students' projects? Every educational activity should be controlled by the school district although they allow a school itself to do that autonomously...
(Fieldnotes, 6/29)

When it was time for lunch, two groups were appointed for the lunch-duty roster: one group for fetching the food and trays and another group for serving the food. The food was set out on five students' desks (extra for lunch time) at the back of the room. The serving roster of five or six students quickly put on yellow aprons and caps and stood next to the food they were going to serve. Each group called by Teacher Lee (he always called a group name by a leader's name such as "Tae-min's group") lined up for their servings, carrying them on a tray to their own places and started eating their meal in the classroom. The lunch time was a convivial occasion, with students conversing. Teacher Lee joined them and chatted away with several students as he ate. After the students were finished eating, they returned trays and leftovers to the desks and cleaned their own desks.

At the end of the fifth period (at 2:10 p.m.), Teacher Lee wrote homework and items to bring for the next day's lessons on the chalkboard said, "Take out Al-lim-jang
and write this.” (Al-lim-jang is a small notebook that students write homework and things to bring for next day’s lessons so that parents would help them.) On June 29, he wrote:

2. Bring a jump rope. (Fieldnotes, 6/29)

After the fifth period, all students were released except a group in charge of cleaning up the classroom. Each group was assigned to clean up the classroom after school hours with this assignment rotated to each week. Everyday several students had to participate in cleaning up even though their groups were not on cleaning duty that day. The cleaning-up students were released at least before 3:00 p.m. After that Teacher Lee stayed in his classroom to prepare tomorrow’s lessons or participate in meetings. There are two official meetings that teachers needed to attend: the teachers’ meeting of the school on Monday from 4:00 to 5:00 p.m. and the teachers’ meeting of the same grade from 4:00 to 5:00 p.m. The official time that teachers left school at 5:00 p.m.

“Behave Orderly, Think Freely”: Regulations of Teacher Lee’s Classroom

An instant catching phrase to express Teacher Lee’s classroom is “behave orderly, think freely.” Keeping 45 students in order is a formidable and inevitable job as a classroom teacher. Teacher Lee had to have his 45 students “behave orderly” in order to deliver his teaching effectively otherwise his classroom would be a most bustling place with these vigorous third graders spoiled by young Korean parents, who are too indulgent
with their children. Teacher Lee’s priority during March when the school year started was establishing this norm.

It is very important as a classroom teacher to make students get familiar and follow smoothly my expectations and rules. They have to forget everything they had learned before. As long as they are in my classroom they should follow my rules. Whether I will have a good year or horrible year with these kids totally depends on what I did in the first day, the first week, and the first month. (Interview, 8/1)

One instance clearly represented how Teacher Lee and other teachers developed classroom rules. The students loved having physical education class (there were no gyms in public elementary school in Korea.) in which they can play a soccer game. The following incident demonstrates how Teacher Lee utilized this fact to form a regulation.

It was on Friday and teachers just finished their morning meeting. Teachers looked at the students of Teacher Kim through windows in the language lab. Teacher Kim’s students gathered in front of the podium located before the school building, but they were not ready for the class. They chattered, shoved, or chased each other. Teacher Kim stuck his head out of the window and shouted at the students, “Make a straight line!” Teacher Lee said to Teacher Kim, “Do not let them go to the physical education class if they didn’t follow your rule like what I did since March. I would not have a physical education class if they did not make a straight line.” (Fieldnotes, 6/11)

In his classroom, most norms for classroom management were regulations established by the teacher, who can change them (Much & Shweder, cited in Cobb, Wood, Yackel, & McNeal, 1992). For example, when students needed materials to do activities in math class or experiments in science class, Teacher Lee always said, “Group leaders, come to the front and fetch the materials to give your group members.” Only group leaders were allowed to fetch the materials.
Most regulations in his classroom, however, were for the purpose of discipline. The consequence of breaking the regulations was typically a punishment of either “go to the front or back of the classroom and kneel down raising both hands above your ears,” “being hit on palms twice by a rod,” or “cleaning-up the classroom.” Teacher Lee sometimes used cleaning-up as a punishment. When students misbehaved or disrupted their lesson or did not do their homework or did not bring textbooks for the day’s lessons, Teacher Lee put them on the cleaning duty. Teacher Lee’s intent for this action was to provide an opportunity to repent for what had been done. One girl talked about this punishment in the following way:

Teacher Lee orders us to clean up the classroom as a punishment to students, who did not do their homework. He is different from other teachers, who do not make such students to clean up and just overlook their faults. Cleaning-up punishment gives opportunity to reflect on our wrongdoing. That’s why I like him. (Interview, 7/2)

Unlike this girl, most students did not like the cleaning-up duty simply because they could not go home earlier. In this case, he was a strict disciplinarian. Such disciplinary actions caused students to be frightened of him. The students expressed their experiences during March.

Sung-don: Our sun-sang-nim was the most horrifying teacher in the school. (Conversation, 7/2)

Tae-min: I got really scared when I came in his classroom. We all had punishments.

Ji-hyun: When we misbehaved there was a punishment at all time like raising hands or kneeling down in front of the classroom. (Interview, 7/9)

Han-jin: I was scared when our class had to assume pushup positions on the playground as a group punishment because we did not
make a straight line although the bell already rung.
(Interview, 7/13)

Students identified which regulations were punished: when they did not do or bring homework, when they did not bring educational materials for a class (e.g., jumping rope for a gym class), when textbooks were not brought by students, when fighting each other, when they did not pay attention and whispered during a lesson. Through class observations these examples were identified and the following incidents described how Teacher Lee established the regulations for classroom management.

Ji-hyun and U-min in the same group fought each other before lunch time just after the fourth period. Ji-hyun gave him a pinch and U-min cried. Teacher Lee made them to come to the front of classroom and to kneel down with raising their hands. This punishment went on for about 10 minutes. (Fieldnotes, 6/14)

Just as the bell sounded, Teacher Lee sat upright and was to be ready to begin his lesson in the computer table. Hoy-jang (student monitor) got up from his chair and commanded to the class, "Attention. Rest. Attention. Bow." Students bowed to Teacher Lee and he also bowed to them. Then Teacher Lee stood up and said, "Each group is supposed to make a newspaper about environment protection today. If the members of a group are going around and doing nothing, all group members will get a punishment of cleaning-up classroom." (Fieldnotes, 6/17)

It was a science lesson. Teacher Lee was ready to begin his lesson, but his students weren't. He needed students' attention. So he suddenly commanded, "Attention. Rest. Attention. Don't move your body." Students quickly followed his command. "Sang-jong, you don't pay attention and continue to move your body. Go to the back and kneel down." (Fieldnotes, 7/3)

It was an art lesson. Yesterday Teacher Lee asked students to bring materials for today. Some students had drawing pens and white sheets of paper to draw, others had empty cardboard boxes to build models. But some did not have anything on their desks. "Stand up and come to the front those who didn't have any materials for this class," Teacher Lee said. Six students lined up in front of the chalkboard. Teacher Lee asked them
why they didn’t bring materials and had them kneel down. (Fieldnotes, 7/7)

In contrast to these regulations for classroom management, the students in Teacher Lee’s classroom demonstrated their orderliness. Each class began with bowing between Teacher Lee and the students. Teacher Lee explained:

I think I have had my student bow since the first year of teaching. Bowing has three purposes in my class. First, it means the class is going to be underway officially. Second, it means the students should show their respect to the teacher. And lastly, it means the students make a promise to themselves for studying hard. (Interview, 8/7)

The bow was commanded by a student monitor. The command was “Attention. Rest. Attention. Bow.” The students followed this command orderly and Teacher Lee sat upright to receive bow. The students and Teacher Lee exchanged bows simultaneously. The students said in unison, “We shall study hard,” while bowing to Teacher Lee. During this ritual, the students were not supposed to move their bodies or talk. It seemed to be a solemn moment. At the end of a class, the students followed the same ritual, but saying a different word, “Thank you for teaching.” But this ritual of the end of the class rarely happened.

The bell just sounded and Teacher Lee was sitting upright on the computer desk. He called on the student monitor to begin his lesson. Eun-ho, the student monitor, rose from his seat and commanded to the class, “Attention. Rest. Attention. Bow!” with looking around the students to check whether they were ready for the lesson. The students followed his command quietly and orderly, saying together “We shall study hard!” Teacher Lee bowed to the students as a response. This ritual was performed very solemnly. (Lesson transcript, 6/14)

Eun-ho rosed from his seat to command the class, but the students were not quite ready for the lesson. Although the bell already sounded, some of
the students still chatted. Eun-ho called on one of the students, “Tae-su!” and Teacher Lee said, “Tae-su, I think you should sit upright and stop chatting when your name is called by the student monitor.” Now, everybody was ready to follow Eun-ho’s command. Eun-ho proceeded, “Attention. Rest. Attention. Bowl!” All the students bowed to Teacher Lee, saying together “We shall study hard!” (Fieldnotes, 7/2)

Another way of showing orderliness of this classroom was clapping. The students clapped in two different ways. One was used for praise and the other was used for a transitional signal of the instructional activity. In particular, clapping for the transitional signal showed the students' orderliness. Teacher Lee had the students clap when he needed to get attention of the whole class or when a presenter on the board explained his or her method of solution to the class. In both cases, Teacher Lee said, “Clap once.” When the students heard this word, they quickly stop what they were doing and clapped. They rhythmically clapped five times that combined two and three clapping like “clap-clap, clap-clap-clap.” After this clapping, the students were supposed to sit upright and get ready to listen to a presenter or Teacher Lee. Teacher Lee explained this activity:

I had used a bell to get the whole class’ attention when I have to explain or instruct something important. But I decided not to use it because it reminded me of a drooling dog. Thus, I think using a bell was not appropriate for my students’ emotion. So, now I have the students clap and it seemed to work well so far. (Conversation, 7/8)

Several teachers of the fifth and sixth grades in this school still used a bell for a signal of getting students’ attention. Most teachers of the lower graders had the children sing a song accompanying body movements when they needed the children’s attention.

Parents’ responses to Teacher Lee’s disciplinary actions were supportive. They appreciated his classroom discipline that gave their spoiled children a hard time. Two
mothers in an interview mentioned that corporal punishment for the education of their child was fine. Some children also told how their parents respected Teacher Lee’s disciplines when they reported their hard time to parents.

Mother 1: I don’t think corporal punishment is a bad thing in school. Using it for my child education would be fine to me if the teacher uses it properly like spanking the palms or hips.

Mother 2: I often have visited classrooms last year and been surprised at watching some students’ extreme misbehaviors. So, I realized why teachers need punishment in a classroom… I would not reject the using of corporal punishment of Teacher Lee unless he uses excessive spanking. (Interview, 7/7)

In-ah: My mother was pleased when I said I got a really disciplined teacher this year. She said she wants Teacher Lee to correct my bad habits.

Ji-min: I told my mom I am scared of my teacher. But she said she was happy because this year I had a male and strict teacher. She thought female teachers of the first and second grade spoiled me too much. (Interview, 7/6)

In all these senses, Teacher Lee was a strict disciplinarian. For Teacher Lee, all these regulations had educational purposes. His major concern was how he has his students learn and follow certain classroom regulations to form their habits of behavior as a member of the classroom. For him, establishing classroom regulations is a means of nurturing self-controlled students who have responsibility for their own behaviors.

Writing a diary everyday is one of the regulations or habits I had tried to teach them. I check each student’s diary once a week. In order for them to write a diary for themselves I should continuously check it. I hope they will appreciate it when writing a diary later. (Interview, 8/1)

I know making the students who broke the regulations to cleaning-up as a punishment would not look good. However, I hope they can have a chance to reflect about what they have done while doing classroom cleaning-up. (Fieldnotes, 7/6)
If Teacher Lee’s students were scared because of the obedience of the regulations, they would not actively participate in learning and there would be no fun and interesting things in their school life. This result was not Teacher Lee’s intent of keeping classroom regulations. He thought of learning as a “fun activity.” Thus, he needed some activity so that the students did not seriously perceive him as a disciplinarian which might make the classroom a safe and comfortable place to them. That is, he wanted his students to behave orderly, but to think freely in the classroom. The phrase, “behave orderly,” is not enough for accomplishing his beliefs about teaching and learning. While “behave orderly” is a regulation for classroom management, “think freely” is one for teaching and learning activity. According to classroom observations, Teacher Lee’s lessons were not disrupted by students’ misbehaviors and he scarcely had to say words to require students’ attention. Such students’ orderliness gave Teacher Lee ample opportunities to focus more on his lesson. Teacher Lee was well aware of what he was supposed to do for these students to “think freely” while “behaving orderly.” In order to have students “think freely,” Teacher Lee needed to build a safe classroom environment for them. To do so, he slowly loosened the strict regulations as soon as he decided students showed evidence of the extent of their adherence.

I don’t keep strict regulations during a whole semester. After March when students somewhat form the regulations, I usually loosen them. I don’t want to have my students follow the regulations all the time. They are just small kids and like running around classroom or playground. (Interview, 8/1)

As soon as he had students perceive him as an impartial teacher, who gives punishment to those who break the regulations regardless of their academic performance
or their positions in the class, he relaxed his discipline. In his classroom, there were one
student monitor, two assistant monitors, and eight group leaders.

The bell sounded. The last period of today’s school hour was over. Teacher Lee was about to announce homework and materials they have to bring tomorrow. “Put your hands on your laps,” He said. It is one of his commands when he needs students’ attention. Students were supposed to be quiet and follow it. The student monitor was laughing with one student in his group. Teacher Lee called on him in front and reprimanded him by hitting his palms twice with a rod. “You are a student monitor. You should follow this rule better than others.” (Fieldnotes, 6/16)

Partiality is one action a classroom teacher should avoid demonstrating to students in classroom. Perhaps it would be true that a classroom teacher has favorite students. Teacher Lee’s conviction, however, was: “Don’t show your partiality openly, but put it in your heart.” While this conviction seemed impossible, he managed it well. As a consequence, students seemed to recognize that Teacher Lee was fair to everybody in his classroom.

In-ah: Our sun-sang-nim is very fair to everybody. If you break a rule, you will get a punishment. There is no exception. That’s why I like him. (Interview, 7/6)

Sung-don: All of us have had certain punishments. (Interview, 7/2)

Another strategy he used to establish a “think freely” environment was to share his experiences with his students. He shared his experience with his students by telling stories about his childhood or everyday experiences, using games with singing children’s songs for lessons, participating in a soccer game with his students in a physical education class, accepting students’ suggestions, or expressing his confusion or frustration when he
did not figure out students’ presentations. Many elementary teachers used singing children’s song as a technique of gaining the students’ attention during lessons.

Teacher Lee’s class had been working the division unit over last two weeks. Teacher Lee started today’s lesson with a division game by asking a question, “How many students are in our classroom?” “Forty five,” students responded loudly in unison. “Okay, all of you get up and stand next to your desk.” Students grinned and made a low noise, expecting some fun. “Sun-sang-nim, are we going to play a game?” “Yes, we are. Now, look at my fingers and make a group with the number of friends corresponding to them,” Teacher Lee told the class. “What song did we learn in the last music class?” “Spaceship!” “Okay, let’s sing the song.” Students sang the song cheerfully by looking at Teacher Lee’s raised hand in air. In the middle of the song, he suddenly spread six fingers. All of a sudden, students got boisterous and quickly moved with a rush to get in a group of six. (Fieldnotes, 6/8)

The third graders really liked singing children’s songs that they learned in the music class. Teacher Lee had the students sing the songs when they were bored, likely to lose their interest in lessons, or playing games.

Teacher Lee frequently expressed his personal feelings to his students. In this episode, he candidly admitted his confusion and his mistake in order to share his feeling with the class. This action appeared to break the authoritative image of a classroom teacher and to indicate that making mistakes and being confused were a natural part of doing mathematics.

Hanjin was explaining his findings of patterns in Pascal’s triangular number problem. Today’s math lesson was about finding patterns. Teacher Lee and his students were listening to his explanation. Hanjin’s explanation of how he figured that out was not complete and did not make sense. Watching his explanation where he stood next to the board, Teacher Lee grinned with confusion and said to the class, “His explanation is so hard and I cannot follow him. Hmmm…” (Fieldnotes, 7/9)
The next episode also illustrates how Teacher Lee reduced his strict discipline to make the students feel comfortable so that they were able to “think freely.” The students were cutting colored paper ribbons for learning fractions. The ribbons were red, yellow, blue, pink, green, and purple. Teacher Lee circulated and helped them to cut them in appropriate length.

Lee: Are your ribbons ready?
Students: Yes! [In unison]
Lee: What is this? [Holding a strip of the colored ribbon]
Students: A colored ribbon! [In unison]
Lee: Right, what does it remind you of?
Students: [Called out] Me, Me, Me!

The students presented voluntarily their experiences related to the colored ribbons. They said that the ribbons reminded them of earthworms, headbands, measuring tapes, and the like.

Lee: Now, please put your hands down. All of you have some memories about these ribbons. For me, it reminds me of my wedding day.
Students: [Called out with laugh, giggle, and boo] Whoa…
Lee: So many colored ribbons tied around my body. It was wonderful. You have seen it in a wedding hall, haven’t you?
Students: [Cheerfully] Yeah!
Lee: Now, we are going to study fractions with these colorful ribbons. (Lesson transcript, 6/29)

In this episode, Teacher Lee invited his students’ experiences with colored ribbons. Talking about such experiences provided his mathematics lesson with a comfortable atmosphere. In addition, the friendly comment about his wedding day was enough to ameliorate his strict image.

The strategies he used to have his students “think freely” were integrated with “behave orderly” and provided the students with a safe classroom, in which students
actively participated in the teaching and learning activity. Students gradually adapted Teacher Lee’s regulations and realized his sense of responsibility as a classroom teacher.

Yun-ha: Whatever he does, I like our sun-sang-nim. I am not scared by him because he gets strict only because he is trying to teach us well.
Jun-ho: He plays a soccer game with us and tells stories too.
Min-jung: I still feel little bit frightened but he is really funny and tells exciting stories. (Interview, 7/2)

In-ah: I think getting a punishment is an expected result because we misbehaved... Students in other teachers’ classroom get so spoiled because the teachers are too nice. They are boisterous. However, our sun-sang-nim is so clear about good and bad. He surely corrects spoiled students. That’s why I like him.
Ji-min: I like him because he teaches us well and smiles all the time. (Interview, 7/6)

Students: No, we are not really scared of him now. (Interview, 7/9)
Ji-hyun: I like him and our classroom because he tells interesting stories a lot. (Interview, 7/13)

Students in Teacher Lee’s classroom may still be frightened of being punished, but such feelings did not seem to interfere with their learning processes. Rather, obeying the regulations that Teacher Lee had established helped them concentrate more on their work and made Teacher Lee’s teaching efficient.

It was quite apparent that the regulations for classroom management described here were directly related to Teacher Lee’s experience. The phrase, “behave orderly,” can be drawn from his experience of leadership and a sense of responsibility where he was a leader in a mountain climbing club in his college years. Teacher Lee and three beginning teachers had a conversation in his classroom. Through this conversation, Teacher Lee expressed leadership and a sense of responsibility.
I was a leader of a mountain climbing club and had responsibility of the members' safety. Sometimes we had climbed cliffs by hanging ourselves on a rope. I had two experiences I narrowly missed death because of one member's mistake. My minor mistake could cause others' death. Once you were on a cliff there were always life and death situations. So, my responsibility as a leader was extremely important for others' safety and the members had to obey my orders even though my judgement might be wrong... I think this can also be applied to a classroom teacher. Students can complain about our rules, but we should have them stick to the rules. (Fieldnotes, 6/22)

Teacher Lee's experience reflected on his disciplinary actions in classroom. He was a captain of his classroom and in charge of his students' academic progress. He wanted them to accomplish individual goals. Students needed to follow his regulations in order to work as a team member. Teacher Lee once indicated his responsibility, "My students' performance absolutely depend on me."

The phrase, "think freely," did not seem to be related to his beliefs about teaching and learning. Rather, it was a genuine approach to solve a practical problem in his classroom that he wanted his students to "think freely" while keeping them in order. The strategies he used were not theoretical, but they were practical knowledge. In the words of Feiman-Nemser and Floden (1986), teachers' "beliefs, insights, and habits that are derived from their experiences and that enable to them to do their work in schools" (p. 512). Teacher Lee's strategies mostly came from his 10-years of experience teaching.

"Behave orderly, think freely" appeared to be a paradoxical phrase that one action cannot go along with another. Classroom observations, however, verified the paradox would work well. Teacher Lee's teaching practices in a mathematics classroom well demonstrated how he resolved this paradox and implemented his beliefs about mathematics teaching with understanding.
Teaching Mathematics with Understanding

"Teaching mathematics with understanding" was the core of Teacher Lee’s beliefs about the teaching and learning of mathematics. Once he achieved the regulations of classroom management, he greatly emphasized students’ understanding of concepts, algorithms, procedures as well as rules. The development of understanding concepts and procedures was his major goal of mathematics teaching.

Teacher Lee stated that mathematics is an interconnected and unified discipline that is structured hierarchically. When asked to further elaborate the relationship between this conception of mathematics and his teaching practice, he said, “In fact, I don’t know what mathematics is. I have not thought of that seriously.” It was evident that his belief about teaching mathematics with understanding was not associated with his conception of mathematics. His beliefs were immensely influenced by his early school experience.

The following excerpt was recorded from the conversation with three beginning teachers in Teacher Lee’s classroom one afternoon. All of them specialized in elementary mathematics during their teacher education program. They exchanged their teaching ideas, frustrations, and difficulties with Teacher Lee. Teacher Lee told how desperately he tried to make sense of mathematics and how much it was important to “understand” mathematics.

Although I tried really hard to figure out math problems in high school years, I just couldn’t solve them. I didn’t understand the problems even referring to the answer sheet. I think at that time I tried to solve math problems by formulas and algorithms that I had memorized, instead of understanding them. For that reason, I would give up the problems that did not directly apply to the formulas and algorithms. (Conversation, 6/22)
It was during his college years that he finally realized the importance of understanding mathematics. Like many Korean university students who are tutoring for supplementing their educational expenses, Teacher Lee also tutored elementary through high school mathematics during his college years. About four years of tutoring experience provided a great time to reflect about his mathematics learning in high school. Through the tutoring experience, he finally realized why he did not previously understand mathematics.

I couldn’t make any sense of the math problems related to trigonometry and integral and differential calculus because I memorized the formulas. Without the formulas, I couldn’t solve them. Memorizing these formulas and applying them to the problems were useless. However, when I was tutoring math, my job was totally opposite. I don’t need to memorize the formulas, but I had to understand them first to teach. Once I grasped the concepts and formulas, everything was so clear and made sense to me. (Conversation, 6/22)

Teacher Lee believed that the mechanistic ability of solving problems by incessant practice was not an appropriate learning of mathematics. He stated that he was not concerned about the number of correct answers his students obtained. Instead, he emphasized the importance of understanding mathematics.

I agree that students need to memorize and get familiar with basic formulas, rules, algorithms, or procedures. But I do not emphasize them in my mathematics teaching. From my tutoring experience, I am convinced that the students were apt to not understand the problem itself even though they solved it. I believe, by my observations, that understanding is an essential factor for students’ mathematics learning. (Conversation, 7/26)
From these quotes, Teacher Lee’s belief about “teaching mathematics with understanding” were evidently related to his earlier experience as a mathematics learner and a tutor.

Another conversation with a first-grade female teacher (Teacher Park) also illustrated his belief of the importance of understanding mathematics. Her specialized area was Korean language but she was interested in mathematics learning.

Park:  I am interested in the relationship between mathematics learning and reading comprehension. I observed that the students who have a high ability of reading comprehension tended to solve math problems well.

Lee:  I think in the lower grades students will be capable of learning mathematics by memorization because most of the content was related to numbers. However, as the students went through higher grades, they have to have solid understanding about mathematical concepts in order to learn mathematics rather than simple computations. (Conversation, 8/17)

Although Teacher Lee put a great deal of emphasis on understanding mathematics, he stated the importance of individual student’s understanding instead of understanding from group learning. He stressed that students themselves should understand and be convinced about mathematics. It was essential to him that students strove to find the meanings of mathematics for themselves.

I think that after all, in learning mathematics, students have to understand for themselves. It does not mean I totally reject the group learning. In some situations I think group learning might be better. But although students learn something in a group, I think they still should make the effort to make sense of mathematics for themselves. Without such individual effort, their learning will be evaporated. (Interview, 8/17)
The belief about individual student's understanding was also formed from his early school experience. Since he planned to study engineering areas in college, he had to take additional higher mathematics courses in high school. There were many difficult and complex concepts in these courses. He did not quite understand them until he met a new mathematics teacher who explained clearly. He tried so hard to make himself grasp these concepts and eventually figured them out. "I got my own way of understanding," he said. For him, it was an unforgettable moment and he was delighted and proud of himself when he understood. This belief influenced his teaching practice so that he did not often make use of group learning or cooperative learning in his mathematics lesson except for hands-on activities or playing games. Another significance of this incidence was to impact his belief about perseverance of learning mathematics. He believed that grasping the concepts in mathematics required a great deal of effort.

Based on these beliefs, Teacher Lee's teaching practice of mathematics focused on the development of mathematical understanding. In the following sections, communication patterns and social-mathematical norms for developing understanding in mathematics are described in more detail with classroom episodes.

**Students' Own Ways of Understanding**

Teacher Lee emphasized that students should have their own a unique way of understanding of mathematics and even an idiosyncratic way of understanding would be valid in their mathematics classrooms. He constantly told his students that their own way of understanding would be valued.
I think our educational system has too much uniformity. For example, when students draw a house, the houses have almost the same shapes with the same types of windows and roofs. Our teachers have taught them what house should look like. We usually said, “This is what house should be.” Although students imagine countless types of houses in their minds, teachers tend to make such imagination uniform. I really try hard to break this uniformity and let my students think freely. In math class, whenever possible, I tell them there exist many different ways of understanding and solutions. I tell them that the most valuable thing in my class is their own ways of understanding and thinking. (Interview, 8/1)

Through classroom observations, Teacher Lee’s belief about “own way of understanding” was not just delivered to his students in the mathematics lesson. He expressed this belief all the time in almost every subject area that he taught. He said he did not like it when his students asked his permission to do something related to their academic learning.

I always tell them what they think is best for them and do not copy what others do. Nonetheless, they keep asking, “May I write this or that?” or “May I draw this or that?” I am displeased when they ask for these permissions. (Interview, 8/1)

Teacher Lee’s belief of “own way of understanding” was best demonstrated in mathematics teaching. He accepted each student’s own way of understanding as a valid justification and verification of their mathematical knowledge. As a consequence of this belief, he tried to have his students perceive many different ways to identify the correct answer. This teaching practice conflicted with his belief about mathematics that mathematics is a unified discipline with only one correct answer. This inconsistency seemed to happen because students’ “own way of understanding” was his more central belief of teaching mathematics.
To implement his belief about “own way of understanding” in his mathematics classroom, he emphasized individual student’s efforts for seeking understanding. The following episode illustrates Teacher Lee’s beliefs about vigorous effort for understanding. It was a mathematics class and he had the students solve the practice problems in the textbook and workbook. Just before letting the students begin, he gave this brief lecture.

Attack the problems until you clearly understand them. If you don’t, ask those who are next to you. Don’t just skip the problem because you don’t figure it out. If you can only solve one problem, that’s absolutely okay... You must make yourself understand. (Lesson transcript, 7/14)

Valuing each student’s own way of understanding, Teacher Lee did not require mathematical formality when his students presented their ideas, thoughts, or ways of thinking. When he was in charge of teaching high-achieving sixth graders in mathematics several years ago, the mathematics problems in a workbook were all related to nonroutine problems. He was not able to solve all of them, but sometimes the students did.

I was so surprised at how they solved it. So, I asked them to explain how they figured it out. What shocked me was they did not use any formulas or equations. What they did was trial and error. What I did was that I tried to solve the problem in the standardized form of mathematics. I witnessed many instances where some of the students solved math problems in their own way and understood without any formulas or equations. However, when I asked them to explain their solution methods, that really made sense to me. In that case, I would accept the explanation as a valid verification. (Conversation, 7/14)

In the following episode, Teacher Lee utilized a situation to inform his class of his belief, the importance of “own way of understanding,” when one student questioned him.
Teacher Lee and the students had discussed about how to calculate the difference between two lengths. The problem was:

\[ a) \ 4 \text{ cm} \ 6 \text{ mm} = 46 \text{ mm} \]
\[ b) \ 4 \text{ cm} \ 8 \text{ mm} = 48 \text{ mm} \]
The difference: \(48 \text{ mm} - 46 \text{ mm} = 2 \text{ mm}\)

Lee: Now, we are going to find out the total length of the two. [About 8 students raised hands to solve the problem on the board. He called on two of them.] U-jung and Ji-min, please come to the board to solve it. The rest of you solve it on your notebooks.

The two students began to solve the problem and Teacher Lee circulated and observed the students' works. Jun-ho asked him a question.

Jun-ho: Do I have to solve it in a column method?
Lee: Actually, it doesn't matter if it is either a column or row method. [Look around the class] I told you it is not a good thing to ask "Do I have to it in this way or in that way?" It is important for you to solve the problem in your own way of understanding.
(Lesson transcript, 6/22)

In the next episode, Sung-don solved a problem related to find elapsed time. He explained his method of solution on the board. The problem was:

\[ \begin{array}{c}
\text{3 hours 20 minutes} \\
- \text{50 minutes} \\
\hline \\
\text{2 hours 30 minutes}
\end{array} \]

Sung-don: 3 hours 20 minutes minus 50 minutes equals 2 hours 30 minutes.
Lee: Does anybody want to question him? [Min-jung shot her hand up and Teacher Lee called on her name.] Min-jung?
Min-jung: [Rising from her seat] How can you subtract 20 minutes of 3 hours 20 minutes from 50 minutes? You can't subtract it.
Sung-don: I borrowed 1 hour from 3 hours, so I added 60 minutes. Then I could subtract.

Eun-ho raised his hand, but he forgot what he was supposed to ask when Teacher Lee called on him. Min-jung raised her hand again and Teacher Lee called on her.
Min-jung: I think if you borrowed 1 hour, it would be better to write down 60 minutes over 20 minutes. So you would not make a mistake.

Teacher Lee summarized and highlighted Min-jung’s point. Then he asked the class.

Lee: What do you think about writing 60 minutes over here or not write it at all? Which one will be better?

Students: [Called out in disagreement] Writing it over there. Or not writing over there.

Lee: [Smiled to the class] If you think you can calculate it without writing 60 minutes over 20 minutes it would be good. Equally, if you think you need to write 60 minutes over here in order to avoid making a mistake, it also would be good. So, there is no rule of thumb. You should do it in your own way that you can make yourself understand. (Lesson transcript, 6/24)

Min-jung’s suggestion was a unique opportunity to compare the two methods. Teacher Lee could tell them which method was better based on his knowledge, but he refrained from doing that because he believed that each student had different ways of understanding and that they should make sense of the ideas in their own ways.

Based on his belief of valuing each student’s own way of understanding, Teacher Lee was trying to establish an additional norm: “presenting the same or similar response as the previous one was not valuable.” Since he wanted his students to think and present their ideas differently, he continually informed the students of this intention in the mathematics class. In his classroom, the students were well aware that Teacher Lee would not respect their ideas when they just restated the same or similar questions, responses, or justifications as other students did before. The following episodes illustrates how Teacher Lee attempted to establish this norm in his mathematics class. Teacher Lee and his students had just finished constructing the number lines on the board. Today’s
lesson was about writing fractions on the number line. He tried to connect the students' previous knowledge about the number line with writing fractions on the number line. He drew the following number lines with the start and end numbers and asked the students how many skip-counts there were. The number lines looked like:

![Number Lines](image)

Lee: What are differences you noticed on these number lines? [Several students shot their hands up.] Shin-yung?
Shin-yung: [Rising from his seat] The numbers between the start and the end are different.
Lee: [Repeated Shin-yung's response to the class] Ha-sun? [Called on Ha-sun among about eight students raising their hands]
Ha-sun: [Rising from her seat] The end numbers are different.
Lee: [Repeated Ha-sun's response to the class] Ji-eun? [Then called on Ji-eun among those who still raised their hands]
Ji-eun: The first one skip-counts by 5s, the second one by 20s, and the third one by 1s.
Lee: [Repeated Ji-eun’s response to the class.] U-min? [Called on U-min among those who raised their hands]
U-min: The numbers at the end of the lines are different.
Lee: [Repeated U-min’s response to the class] That’s the same idea as Ha-sun said. Other ideas? (Lesson transcript, 7/1)

Teacher Lee always called on five or six students each time when he asked a question because he was interested in the students’ different ideas in order to elicit their thinking. But when a student just restated another students’ idea like what U-min said, he did not
show much interest in the student response and quickly moved to call on another student.

In such case, his formal language was “It’s the same as…”

The following episode illustrates how his students recognized this norm to obtain permission to present their ideas. Teacher Lee and his students were playing a game to make a group related to division. He asked the class to make a group of two, six, and eight. After playing the game, he posed a question.

Lee: Our class has 45 students and here is the number 8. Let’s make a word problem using these two numbers. Make any word problems whatever you want. Ji-min?

Ji-min: [Rising from her seat] There were 45 balloons and 8 of them burst. How many are left now.

Lee: [Repeated Ji-min’s word problem] Ji-min made a subtraction problem. Who else?

Han-jin: [Interjected while raising his hand high] I have a different one.

Lee: Okay. Han-jin, what is your word problem?

Han-jin: [Rising from his seat] Young-su has 45 pencils in his pencil case. He wants to give these pencils to 8 students. How many pencils will each student have? (Lesson transcript, 6/8)

In this episode, Han-jin knew that he would not have a chance to present his idea unless it would be a different one. He recognized the norm that Teacher Lee wanted to build up in his mathematics class.

As mentioned earlier, Teacher Lee’s core belief about the teaching and learning of mathematics was to foster his students’ understanding. One way for him to meet this objectives was to value each student’s own way of understanding by struggling. This belief led him to help his students develop the understanding of concepts and procedures in mathematics.
Development of Understanding of Concepts and Procedures

As mentioned earlier, Teacher Lee could be referred to as a disciplinarian. His belief about a strict discipline seemed to come from a strong sense of responsibility as a classroom teacher. Moreover, many teachers including Teacher Lee used "a sense of mission" to represent their responsibility. He always said that his students' learning of mathematics totally depended on him, and that a teacher's role was to create a learning environment in which each student could do his or her best according to his or her ability.

"I think my role as a classroom teacher is to create a learning environment in which my students can achieve small things for themselves." (Conversation, 7/8)

"I am trying to do everything for my students. I think their academic performance as well as habits of mind really depends on me. They will follow the road I paved for them." (Conversation, 7/1)

With such a strong feeling of responsibility as a teacher, Teacher Lee orchestrated his mathematics lesson to develop the students' understanding of concepts and procedures. The development of understanding of concepts and procedures meant that he rarely told the students about concepts and procedures in mathematics. He deliberately led them to grasp the concepts and even simple procedures that most teachers just wrote down on the board or told the students the concepts and how to apply them. The following episode illustrates how Teacher Lee guided his students to understand comparing the magnitude of fractions. Using colored paper ribbons, Teacher Lee put the following strips on the board and wrote fractions next to them. He planned to compare the length of fractions. With this activity the students began to know what the fractions had to do with the lengths of ribbons.
Teacher Lee put the fractions in a row.

\[
\frac{4}{4} > \frac{3}{4} > \frac{2}{4} > \frac{1}{4}
\]

Lee: I wrote the fractions in order from the largest to the smallest. Let’s look at it carefully and tell us what patterns you can find. [Several students shot their hands up.] Okay, Han-jin?

Han-jin: [Rising from his seat] 4/4 eats 3/4 and 3/4 eats 2/4 and so on...

Lee: [Repeated Han-jin’s idea] Han-jin said the bigger fraction eats the smaller one. Who else? Ju-ri?

Ju-ri: [Rising from her seat] I think it’s getting smaller by 1.

Lee: What do you mean getting smaller by 1?

Ju-ri: The numerators are getting smaller by 1 like 4, 3, 2, 1.

Lee: [Repeated Ju-ri’s idea] I see. The numerators seem to get smaller by 1.

Teacher Lee invited more students to present their ideas. But their ideas failed to find how to compare the magnitude of the fractions. Thus, Teacher Lee helped them find the procedure by reducing the number of the fractions, refraining from telling the students what it was.

Lee: Well, I will erase these. [Now only two fractions were left.] Now, look at it carefully. We are going to compare these two.

\[
\frac{3}{4} > \frac{2}{4}
\]

Let’s look at the denominators. What about them?

Students: They are same. [Several students said.]
Lee: Why is 3/4 bigger than 2/4 when the denominators are same? [About 8 students raised their hands.] Let’s listen to Ha-sun’s thinking. Ha-sun?

Ha-sun: [Rising from her seat] Because the top is bigger.
Lee: What do you mean ‘the top’?
Ha-sun: The numerator...
Lee: Because the numerator 3 is bigger than 2. Who else? In-ah?
In-ah: [Rising from her seat] The 3/4 is bigger than 2/4 because the numerator 3 is bigger than 2.
Lee: [Repeated In-ah’s idea to the class] I see. Who else? Min-jung?
Min-jung: [Rising from her seat] If the denominators are the same and the numerators are different, the fraction of the bigger numerator is bigger.
Lee: [Repeated Min-jung’s idea to the class] Okay. Who else? Yung-hee?
Yung-hee: [Rising from her seat] I think 3/4 is bigger than 2/4.
Lee: Why do think that?
Yung-hee: I drew circles as a pizza and made pieces to represent 3/4 and 2/4 like we did today.
Lee: Good thinking. Yung-hee drew circles and found 3/4 is bigger. Okay, let’s summarize what we have done. When fractions have the same denominators, which numerator should be bigger if the fraction is bigger than another?
Students: Bigger! [In unison]
Lee: The fraction having the bigger numerator is bigger. (Lesson transcript, 6/29)

According to the episode above, Teacher Lee intentionally led the students to understand a generalization of comparing the magnitude of fractions. In doing so, he invited more than six students to present their ideas and accepted all as valid reasoning. Drawing was also accepted as a valid reason because he valued each student’s idea. He was trying to develop the students’ understanding of the procedure. Thus, although his students could not find the procedure, he did not give up the task but increased the possibility of success by making the task simpler and asking a question (i.e., “Let’s look at the denominators. What about them?”). He also made clear the students’ correct use of mathematical
language instead of everyday language (i.e., "What do you mean ‘the top’?"). This type of classroom discourse was typical in Teacher Lee’s classroom.

The following episode also illustrates how Teacher Lee guided the students to understand the procedure for the addition of fractions. As with the episode above, this activity utilized classroom discourse as a means of developing the students’ understanding. Teacher Lee began this lesson with the number line from which his students learned skip-counts in the first and second grades. So far in today’s lesson, the students learned to write the addition of fractions on the number line but they did not develop the generalization of the procedure. On the left-hand side of the board, there were number lines and the addition of fractions. The followings were the fraction facts.

\[
\frac{2}{5} + \frac{1}{5} = \frac{3}{5}, \quad \frac{1}{5} + \frac{3}{5} = \frac{4}{5}
\]

Lee: Let’s look at another one. I will draw one more number line.

\[
\begin{array}{c}
0 \\
-\hline
-\hline
-\hline
1
\end{array}
\]

Lee: [Asked to the class] Now, look at the number line. What fraction is here? [Pointing the first skip-count]

Students: One fourth! [In unison]

Lee: [Wrote 1/4 on the first skip-count] What about here?

Students: Two fourths! [In unison]

Lee: [Wrote 2/4 on the third skip-count] So, how can you write this number line?

Students: One fourth plus two fourths equals three fourths! [In unison]

Lee: [Wrote 1/4 + 2/4 = 3/4] Okay. Let’s try one more. [Drew the lines on the same number line] How can you write this number line?

Students: Two fourths plus two fourths equals four fourths! [In union]

Lee: [Write 2/4 + 2/4 = 4/4. The number line looked like this diagram]
Lee: Class, we are all through this, but I don’t still get how it worked. We have four addition facts of fractions. [Pointing the facts on the board] How did you get $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$? I seem to me it should be $\frac{3}{10}$.

Lee: I think $\frac{3}{10}$ might be reasonable, don’t you think?

Students: No! [In unison]

Lee: Why not $\frac{3}{10}$? Ji-hyun?

Ji-hyun: [Rising from her seat] You should add only the numerators.

Lee: Why should we add only the numerators? [Elicited Ji-hyun’s idea.]

Ji-hyun: Because the bottom things are same and the numerators are different, so you should add the top ones.

Lee: I still don’t get it. Why do we have to add the numerators while keeping the denominators? Han-jin? [Called on him among the students raising their hands]

Han-jin: [Rising from his seat] The denominators are same, so you have to add the numerators.

Lee: That’s the same as Ji-hyun said. Min-jung? [Called on her among the students raising their hands]

Min-jung: [Rising from her seat] Because one fifth is on the number line. There are 5 skip-counts. So, if you add the denominators the total number of the skip-counters should be 10. So, you should add only the numerators instead of adding the denominators.

Lee: That’s why we have to add only the numerators. But what if I write this fraction fact... [Wrote $\frac{1}{2} + \frac{2}{3} = ?$ and asked the class] What about this?

Students: Three fifths! [In unison]

Lee: [Wrote $\frac{3}{5}$ on the board] How did you get $\frac{3}{5}$? Yun-ha? [Called on her among about eight students who raised their hands]

Yun-ha: [Rising her seat] You can get the answer when you just add only the numerators and only the denominators.
Lee: Then just before some of you said we have to add only the numerators and now you said we have to add both numerators and denominators. I am getting confused. [The students still kept raising their hands up.] Eun-ho?

Eun-ho: [Rising from his seat] One half means 1 skip-counts of two and two thirds means 2 skip-counts of three. So, since the total skip-counts are five, you can add them.

Lee: Now, let’s clap once. [Said to the class to transit for explanation and the students quickly clapped and sat upright.]

Lee: You will learn how to add the fractions in the fourth grade. So, I am not going to explain that. But I will explain what we have learned about fractions. Let’s think about two fifths. How many 1/5 makes 2/5?

Students: Two! [In unison]
Lee: How many 1/5 makes 1/5?
Students: One! [In unison]
Lee: So, if we add 1/5 plus 2/5, how many 1/5 can we have?
Students: Three! [In unison]
Lee: There are three 1/5. [Wrote the following on the board]

\[
\begin{align*}
2/5 & \quad = \quad 2 \text{ of } 1/5 \\
+ & \quad 1/5 \\
\hline
2/5 + 1/5 & \quad = \quad 3 \text{ of } 1/5 \quad = \quad 3/5
\end{align*}
\]

Lee: So, what does it mean there are 3 of 1/5?
Students: Three fifths! [in unison]
Lee: [Wrote 3/5 on the board] (Lesson transcript, 7/3)

In the lengthy episode above, Teacher Lee did not provide the procedure of how to add two fractions. It would be much easier to tell them the procedure and have them apply and practice with similar problems. In fact, he knew that some of his students attended a learning center already and knew how to add fractions, but he knew that these students did not know the concepts of the procedure. The clarification by Ji-hyun, one of the students, showed that she already knew some of the procedures. Even for all these reasons, he tried to develop the students’ understanding of the procedure because he believed that understanding should be first developed before applying and practicing mechanistic procedures.
In this episode, some distinctive features could be identified. Teacher Lee always asked the students a probing question, “Why?” This question must be followed by the students’ ideas. It encouraged them to explain their reasoning to the class. In fact, providing their reasoning when the students were called was one of the norms in this mathematics classroom. Another feature was “clapping.” Clapping was used for getting the students’ attention when he intended to transit the flow of his lesson. Here he used it for his explanation after listening to students’ ideas and their reasoning. Lastly, sometimes he deliberately allowed the students to be puzzled and confused and to have them rethink what they had known. He used his knowledge about students’ misunderstanding that $2/5 + 1/5 = 3/10$. Many students in his class knew this result was not true, but they were not able to verify their reasoning. They knew the procedural knowledge, but they did not know the conceptual knowledge. With this starting place, Teacher Lee used the discourse to develop his students’ understanding of the concepts. Through classroom observations, it was evident that classroom discourse was a major means of delivering and explaining the content of mathematics as well as his expectations, and developing his students’ understanding of concepts and procedures in mathematics.

Objectives Stated by Students

One of distinctive features in Teacher Lee’s mathematics class was that Teacher Lee had the students read the objectives or a problem for the day together. Just after exchanging bows to begin his lesson, he had the students read the objectives for about three minutes and then asked what they were going to study that day. His intention was to
provide a broad picture of the day's lesson so that the students could readily understand
the concepts and procedures.

The school district recommended that teachers write objectives of the lesson on the board. "Date" and "Objectives of the Lesson" were permanently written with white paint on the board of Teacher Lee's classroom. But he never wrote the objectives here and explained:

The school district and principals urged teachers to write objectives before the class in the past. I don't think that helps my students' learning. And I don't like the idea because I know what I am going to teach. (Interview, 7/26)

As an alternative way, he had the students read the objectives from the textbook and present their own ideas about what the day's lesson would be. He had different reasons for this action:

I use this for several reasons, but I don't expect that many of my students understand objectives and problems by reading and presenting them to the class. However, at least they will have a broad and rough idea about what the lesson will be. It might be possible that some high-ability students quickly figure out what we are going to study. For some low-ability students, this might be a signal to begin the class. (Interview, 8/1)

In the mathematics textbook, one unit consists of small sections that cover about three or four pages. Each section starts with one objective. For example, in Unit 7, Length and Time, there are four small sections:

Let's measure length.
Let's write length accurately.
Let's add and subtract lengths.
Let's study time and hours.
Teacher Lee usually covered only one main concept, problem, or task during each lesson. The following episode illustrates how Teacher Lee had the students read the objective or problem for the day. It was the third lesson of Unit 7, Length and Time. Teacher Lee and the students exchanged bows each other. Teacher Lee just had the students read the textbook to know what they were going to study in this lesson.

Lee: Did you finish reading? Now, who can tell us what we are going to study? [About seven students raised their hands and called on In-ah.] In-ah?
In-ah: [Rising from her seat] I think it’s about kilometers.
Lee: [Repeated In-ah’s idea] Kilometers. Who else? Su-jong?
Su-jong: [Rising from his seat] We are going to study length.
Lee: [Repeated Su-jong’s idea] It’s about length. Who else? Sung-don?
Sung-don: [Rising from his seat] I guess it is to convert centimeters to kilometers… [Presented his idea hesitantly]
Lee: Who else? Min-jung?
Min-jung: [Rising from her seat] To convert meters to kilometers.
Lee: [Repeated Min-jung’s idea] To convert meters to kilometers. Now, you presented very good ideas and we want to study all of them. But we cannot study converting centimeters to kilometers in the third grade that Sung-don said because it is too hard now. Then, I noticed that there was a common word in these ideas. Do you know what it is?
Students: Kilometers! [In unison]
Lee: Right. The word, kilometers, was mentioned several times. So, today we are going to study kilometers which is one of the standard metric units for measuring length. Let’s talk a little bit about metric units you already knew. (Lesson transcript, 6/21)

In this episode, after having the students present their ideas about the day’s lesson, Teacher Lee asked them to find a common language that was a key term in the lesson. Sometimes the students presented advanced ideas that were not covered in the third grade like Sung-don’s idea. In such case, Teacher Lee usually provided a comment and reason that explained when the students were going to learn about it.
The following episode also illustrates how Teacher Lee tried to enhance the students’ understanding in the beginning of his lesson by having the students present their ideas about the lesson. The class was beginning Unit 9, Problem Solving, the last unit of the first semester of the third grade. Teacher Lee had the students read the day’s objectives written on the page and began his lesson by asking a question.

Lee: There is a word, ‘patterns.’ What does it mean? Han-jin?
Han-jin: [Rising from his seat] Something fixed in order, I think.
Lee: [Repeated Han-jin’s idea for the class] Something fixed in order... Ok. Who else? Ji-eun?
Ji-eun: [Not quite confidently] Like 1, 2, 3, 4... numbers in mathematics... Something that has sequence.
Lee: [Repeat the answer to the class] I see. Something has sequence like numbers. Who else? Ha-sun?

Teacher Lee continued asking the students the meaning of ‘patterns’ and four more students provided their definitions. He summarized and began his lesson.

Lee: Today we are going to learn patterns in sequence. Like what Jieun said, patterns is something displayed in a sequence. That’s a good definition... (Lesson transcript, 7/9)

In this episode, Teacher Lee continuously asked the students to present their ideas about the lesson dealing with patterns in number sequences. Having the students read and present their ideas about the lesson in the beginning was evidence of his endeavor to foster the students’ understanding in mathematics. When the objectives of the day contained a term such as “patterns” that needed to be explained to his third graders, he usually asked them to present their ideas in order to be cognizant of his students’ previous understanding related to the objectives.
Using Students’ Everyday Experiences

Along with believing mathematics as an interconnected and unified discipline, Teacher Lee seemed to believe that mathematics is a immutable body of knowledge. This belief supported his belief about mathematics pedagogy.

How can mathematics be changed? No, I don’t think so. For example, the principles of division were not changed at all from the past to now. Because mathematics principles are unchangeable, I am trying to use a variety of resources to foster my students’ understanding in mathematics. That’s why I use the students’ experiences, manipulative materials, games, and the like. (Conversation, 6/7)

Because of the belief about immutability of mathematics, Teacher Lee used his students’ everyday experience to help them understand the mathematics. For him, his students’ everyday experiences, manipulative materials, or games served as vehicles to enhance their mathematics understanding. He stated that the final destination of mathematics teaching was to reach the mathematics (e.g., concepts, procedures, algorithms, rules, and facts) suggested in the textbook and that as a classroom teacher, he could not change them. However, he added that because there were so many different paths to reach the textbook mathematics, he could change the path at any time and wanted to lead his students to any path as long as it would enhance the students’ understanding. In order to boost his students’ understanding, Teacher Lee consistently made use of the students’ everyday experiences. To bring their experiences into the mathematics classroom appeared to provoke their interests and help them connect mathematical languages and representations with their everyday experiences.
In addition to the purpose of improving his students’ mathematics understanding by using their everyday experiences, Teacher Lee believed that mathematics teaching should show the students that mathematics is related to their everyday lives.

I think that especially elementary mathematics should be related to our lives rather than strict and formalized mathematics. It is important to help the students realize that mathematics is a part of their everyday knowledge to live in our society. (Interview, 8/12)

By including the students’ everyday experiences, he wanted the students to perceive that mathematical knowledge is a part of their lives.

Teacher Lee stated that he was consciously trying to find everyday experiences that were related to the students’ mathematics learning so that he could provide the students with more meaningful contexts. In addition, he added that such experience would give rise to the students’ interest in learning mathematics.

For their interest I am trying to bring their everyday experiences into my mathematics classroom. For example, when teaching three-digit addition and subtraction, some students showed difficulty with regrouping, borrowing, or carrying. But they understood the concepts quickly and easily when I used various values of money. In addition, they had interest in the activities because they all knew about the money system. (Interview, 8/12)

When I introduce a new concept, I often ask about my students’ experiences and feelings. For instance, if the problem is 12 divided by 3, I draw 12 circles on the board to represent 12. But the circles do not have meanings at all to the students. So, I say, “Let’s assume these are watermelon,” then have them talk about their experiences related to watermelon. In doing so, I can keep their interest and I hope that the students have more meaningful contexts. (Interview, 8/23)

Through classroom observations, Teacher Lee consistently utilized the students’ everyday experiences to provide interest and to enhance their understanding of concepts.
The following episode illustrates how he related the students’ everyday experiences to the metric unit about kilometers. He and the class had just discussed how much more convenient the kilometer unit was to represent long distances and wrote 1000 m = 1 km on the board. He knew that the students had no idea how long 1 kilometer was.

Lee: How long is it if you run once around the playground?
Students: Two hundred meters! [In unison]
Lee: Right, it is two hundred meters. Then how many rounds do you have to run for one kilometer?
Students: Five rounds. [Several students called out and In-ah and Jung-hae spread their right hands to represent the five turns.]
Lee: Yes. If you run five rounds, the distance will be one kilometer. [The students showed a surprise, saying ‘Whoa!’] Did anybody run five rounds around the playground?
Eun-ho: [Called out with exaggeration] I ran nine rounds one day.
Lee: Really? You ran nine rounds. You must be tired, weren’t you?
Eun-ho: Not really...
Lee: Eun-ho said that he ran nine rounds around the playground. Then how long did he run if he ran nine rounds? (Lesson transcript, 6/21)

In this episode, Teacher Lee did not follow the sequence of the textbook. Instead, he introduced 1 kilometer with the distance of the playground with which the students were quite familiar. Moreover, he took Eun-ho’s experience to expand the students’ sense of 1 kilometer. It was noticed that the students showed interest whenever Teacher Lee asked questions related to their everyday experiences.

The next episode also illustrates how Teacher Lee consistently made use of the students’ everyday experiences in his mathematics teaching. The topic of the day was about how to add and subtract hours and minutes. The class had discussed what the day’s topic was about.
Lee: So, today we are going to study how to add and subtract hours and minutes. Now, does anybody know what time you got up this morning? [About 30 students raised their hands because it was an so easy question. Called on Yun-ha among them.]
Yun-ha: [Rising from her seat] At seven twenty.
Lee: What time did she get up?
Class: At seven twenty! [In unison]
Lee: Right, she said that she woke up at seven twenty. [Still the students raised their hands.] Please, put your hands down.
Yun-ha, what time did you leave for the school?
Yun-ha: Huh... at eight ten, I think.
Lee: [Wrote what Yun-ha said like this]

| Time to get up:          | 7 o'clock 20 minutes |
| Time to leave for school: | 8 o'clock 10 minutes |

Lee: Then what did she do between these times?
Students: [Called out] Eating breakfast. Washing her face. Brushing her teeth...
Lee: Now, as you said she did a lot of things before coming to school. How long did it take her to do these things? [The students quickly raised their hands.] Jung-hae?
Jung-hae: [Rising from his seat] Fifty minutes.
Lee: Why do you think it took fifty minutes? [Jung-hae’s explanation was inaudible.] Who else, U-jung? (Lesson transcript, 6/24)

Although it was the beginning of the lesson, Teacher Lee began with asking about Yun-ha’s everyday experience. Many students were eager to present their experiences. When she presented the time to get up and leave for the school, Teacher Lee changed the everyday experience to a mathematical situation. Before that, he invited the students to explain their experiences so that everybody was involved in the situation. Once the everyday experience was set up for mathematical investigation, Teacher Lee orchestrated classroom discourse to find the procedure of adding and subtracting times.
The last episode is the second lesson on fractions. The lesson was about writing fractions on number lines. Before introducing this topic, Teacher Lee wanted to review the concept of fractions by using the students' everyday experience.

Lee: Did everybody open the textbook? Do you like a pizza?
Students: Yes! [In unison]
Lee: Has everybody eaten pizza? [Everybody said yes except for one student.] Jin-yung, you have not eaten pizza? Well, I will buy it for you. [The students booed.] I don’t like a pizza, so I didn’t have many chances to eat it. But it was strange.
Students: [Showing interest] What’s so strange?
Lee: How does it look like?
Students: Like round! [In unison]
Lee: Right. What did you have to do to eat it?
Students: Cut it first! [In unison]
Lee: How many pieces are cut?
Students: [Called out in disagreement] Eight. Four.
Lee: I think it usually has eight pieces. [Drew a circle to represent a pizza] Here is a pizza. What does each piece look like?

Teacher Lee had the students present their experiences about a piece of pizza. They said that it looked like a triangle or that it had a round edge.

Lee: Well, who wants to come up to the board to cut this pizza into eight pieces? [About 15 students raised their hands.] Jung-hae, Su-jong, and Ju-ri, would you try to cut it? I am really wondering how the pizza was cut equally.

Jung-hae  Su-jong  Ju-ri

Lee: [About 2 minutes later] Did you watch how they cut the pizza?
Students: Yes! [In unison]

Teacher Lee and the class talked about how they cut it similarly and differently with laugh.

Lee: Now, when you eat the pizza you eat one piece each time. Let’s assume I ate one piece. [Shaded one piece of Su-jong’s pizza] Thinking about fractions that you learned during the
second grade, how can you write it in a fraction? [About 10 students quickly raised their hands, saying “Me!”] U-min is really excited about this pizza problem. Okay, U-min?

U-min: [Rising from his seat] One eighth.
Lee: Why do you think like that?
U-min: Because one piece was gone among eight pieces. (Lesson transcript, 6/29)

In this episode, Teacher Lee wanted to review and connect the students’ previous knowledge of the concept of fractions with the day’s topic. Instead of simply telling what fractions were about, he used the students’ everyday experience regarding pizzas. When he used a pizza for fractions, all students showed interest and expected what Teacher Lee was going to tell about it. From the students’ experience of a pizza, he gradually moved to a mathematical situation to connect their everyday experience with the fraction concept. In doing so, the students would recognize relevance between their experience and fractions so that they might have meaningful contexts that enhance their understanding of fractions.

Process-Oriented Practice

In addition to a great emphasis on teaching mathematics with understanding, Teacher Lee also stressed the importance of practicing and applying concepts and procedures. One distinctive feature in his mathematics classroom was that he stressed the students’ understanding in practice sections both at the end of each class and a unit. To make sure of his students’ understanding, he required them to explain their methods of solution.

According to him, understanding did not always ensure his students’ successful experience in mathematics. The following incidence demonstrated how Teacher Lee
perceived the importance of practice in mathematics learning. One female (Teacher Kang) of the third grade teachers described the difficulty of her students in learning division in the morning meeting.

Kang: Many students in my classroom have difficulty in studying division. Even some students in the high-ability group make simple mistakes.

Lee: Elementary students tend to forget easily what they have learned even in the last week if they do not practice it. They need continuous practice, especially like division. (Conversation, 6/10)

He indicated, however, that understanding concepts and procedures had priority over practicing them. The following conversation demonstrates this point. Teacher Park was the female first-grade teacher who earned the first place in the research lesson contest in the school district. Although Teacher Lee acknowledged the importance of memorizing some basic facts in mathematics such as the multiplication table, he indicated that students should understand basic concepts before memorization.

Park: Since my specialization is Korean language, I don’t know much about teaching mathematics. But I have always wondered whether I have to make my students memorize the multiplication table. I know memorizing the table does not improve the students’ understanding of multiplication. However, without memorizing it they cannot solve many problems regarding multiplication in a limited time like tests.

Lee: I think understanding concepts of multiplication should be advanced to memorizing the table. Once the students understand the concepts, memorization would not result in a serious problem. The student who memorized the table well might be good at solving problems now, but they would face with difficulty learning mathematics at later grades.

Park: I think every teacher knows that understanding should be first. But we don’t have enough time to teach mathematics based on every concept (Conversation, 8/17)
Teacher Lee was consistently saying that memorization in mathematics was important and was likely to result in a temporary effect of good grades, but eventually it would ruin mathematics learning of the students if understanding concepts and procedures was overlooked. In an interview, Teacher Lee again contrasted the importance of practice in mathematics with understanding, but understanding was emphasized over practicing.

Students should understand concepts first but understanding concepts does not guarantee fluent calculation. Once they understand concepts, they need to practice in order to apply them fluently and accurately when solving problems. (Interview, 8/7)

I don’t think that it would be good mathematics teaching to use manipulative activities all the time. I am sure that the activities would be beneficial for my students’ understanding. But they should be able to do basic calculations and solve problems using algorithms at the end of a unit. So, from my teaching experience, about 70% of my teaching activities are devoted to improvement of the students’ understanding with manipulative activities and games, and the rest 30% to practice. (Interview, 8/12)

Through classroom observations, he usually wrote the numbers of several problems in the mathematics textbook at the end of each lesson. Sometimes he gave worksheets to the students, too. Teacher Lee indicated that he provided worksheets two or three times during a unit. The purpose of using worksheets with the textbook problems was to check the students’ understanding and to redirect the rest of his teaching in a unit based on this ongoing assessment. At the end of a unit, the students should finish the problems both in the textbook and worksheets in order to make sure whether they understood the contents of the unit. When many students failed to comprehend the
content, Teacher Lee had these students stay in the classroom after school and taught
them again.

Most practice sections during each lesson were assigned for less than 10 minutes.
The students worked the problems and worksheets during the practice section, otherwise
they had to finish them during recess times. All students’ work was checked by Teacher
Lee before being released to go home. Some students who could not solve them during
school hours had to work on the problems after cleaning-up the classroom. Because there
was no homework in Teacher Lee’s mathematics class, he required the students to finish
the practice problems during school hours. The following episode illustrates how the
practice section proceeded in his mathematics classroom. The lesson was supposed to be
finished in about eight minutes.

Lee: Now, in the textbook page 97 and 98, and 94 through 96 in
the workbook. [Wrote the pages on the board] When you
finish them, your work should be checked by me as usual.

He circulated to each group and monitored the students’ work. He asked a
student how she obtained an answer and answered the students’ questions.
Tae-min asked a question of him.

Tae-min: Sun-sang-nim, what should I do with 0 millimeter?
Lee: [Looked at the class and asked] Do you think we need to
write 0 millimeter as an answer? What do the rest of you
think?
Students: No! Don’t need it. [Several students responded.]
Lee: Yes, I think so too.

Teacher Lee engaged in conversations with Tae-su, No-jae, and Ko-min.
Suddenly he went up to the front and instructed.

Lee: Let’s clap once. [The students quickly stopped their work and
clapped, sitting upright.] What is 1 centimeter in millimeters?
Students: 10 millimeters! [In unison]
Lee: Right, you all are doing well the problems related to this.
[Wrote 1 cm = 10 mm] But although we have already learned
this, some of you have difficulty dealing with it. What is 1 meter in centimeter?

Students: Hundred centimeters! [In unison]
Lee: Right. [Wrote $1 \text{ m} = 100 \text{ cm}$] What about 1 kilometer?
Students: Thousand meters! [In unison]
Lee: [Wrote $1 \text{ km} = 1000 \text{ m}$] When you solve the problems, you have to keep this in mind.

After one minutes later, the bell chimed.

Lee: Okay, the class is over and your work has to be checked by me before going home. Solve them during recess times or after lunch time. (Lesson transcript, 6/22)

In this episode, Teacher Lee had the students solve the problems related to the day’s lesson at the end of the class. When he wrote the problems with the page numbers, the students quickly began solving them. The students appeared to be accustomed to this kind of ritual. They all took for granted that they must solve them and be checked by Teacher Lee prior to being released. One noticeable activity was that Teacher Lee in this practice section did not tell how to solve the problems when he observed some students’ difficulty. Rather, he convened the class and only reminded them of the facts that were necessary for solving the problems.

In addition to the practice section of every mathematics class, Teacher Lee also assigned one or two lessons for practice at the end of a unit. In order to review concepts, procedures, algorithms, or rules, he provided the problems in the textbook, workbook, and several worksheets. During this practice section at the end of a unit, Teacher Lee checked the methods of solution for every student. The practice sections focused on how each student obtained the answer rather than the answer itself. Whenever the students finished the problems, they came up to Teacher Lee’s desk to explain their work. For
him, being able to explain the methods of solution meant that the students understood the concepts and procedures they employed to solve the problems.

They have to understand the procedures related to solving the problems. It is not important to me how many correct answers they obtained, but it is important to know how they solved them. When they come to me, I ask them the process of the methods of solution to make sure if they understand the problems or not. I require them to explain to me how they obtained the answers. If they could explain their solution methods, it meant that they understood the problems in their own way. (Interview, 8/7)

The following episode illustrates how much Teacher Lee emphasized the capability of explaining their methods of solution. He conveyed that the outcome of understanding was to be able to explain the student’s ways of thinking and the methods of solution. It was the third lesson on Unit 9, Problem Solving.

Lee:  We are going to solve the problems in the textbook, workbook, and worksheets. When you finish them, as you know, you have to be checked. I will ask you to explain how you solved them. If you are not able to explain, you have to solve them again. So, please make sure you understand them clearly so that you can explain the methods of solution to me. (Fieldnotes, 7/14)

Teacher Lee stated that emphasizing the process of solving mathematics problems came from his high school experience. This experience framed his role of a classroom teacher who guided the students to understand mathematics. He added that his major role of practice sections was to ask their explanations of the process. Asking explanations provided him with the starting place of scaffolding.

I believe that my role is very important for their mathematics learning. When I was in high school, my mathematics teacher always asked us where we could not understand the problems that we were working on. I
was working on the problems so hard, but sometimes I could not figure out how to solve them. Then he only explained the very place where I gave up and then had me try it again. I am doing the same way he did to me. I always ask my students the process of solving problems, and if they can not go further I just explain that point instead of showing them how to solve the problem. (Interview, 8/23)

After Teacher Lee pinpointed the place where the student did not understand, the student worked the problem again until he or she finally figured it out. Teacher Lee refrained from quickly jumping into the students’ learning process. Rather, he provided a gradual scaffolding to lead the students to understand.

When I tell the student an answer or the method of solution, I used to give a similar problem again in order to make sure that the student understand my explanation. If the student cannot solve it again, it means that he or she still does not understand the concepts and procedure of the problem. If you tell answers or solution methods, the students would simply accept your ways and would not try to understand the problem. They just want to get the correct answers. (Interview, 8/12)

One of the students verified that if they could not explain their methods of solution because they did not sufficiently understand the problems or copied the answers from other students, Teacher Lee usually gave them similar problems.

Ho-rae: Yes, I have copied the answer of In-ah or Jun-ho. Sometimes I can explain it to Sun-sang-nim, but most of time he gave me other problems.

Jun-ho: I never copied other’s answers because Sun-sang-nim would spot it when I explained it. (Interview, 7/2)

In interviews with the students, they, as third graders, expressed how difficult it was to explain their ways of thinking and the method of solution.
Ji-min: I am afraid that I will have to stay in the classroom to solve the problems again after the cleanup. (Interview, 7/6)

Ji-hyun: In today’s math class, I didn’t copy any other’s answers. I got correct answers but Sun-sang-nim demanded that I explain how I solved them. I was so afraid because I couldn’t explain it well.

Jung-ha: Because I am not good at mathematics, I am having difficulty explaining my method of solution. (Interview, 7/9)

Being asked to explain their methods of solution was a decidedly unpleasant experience to these students. Especially, some students who were not good at mathematics expressed their frustrations when they had to work on the problems again after school. Such frustrating experiences might force the students to attend a learning center. Although teachers including Teacher Lee disliked the mechanistic, drill and practice style of mathematics learning in a learning center, the students obtained their confidence.

Yun-ha: I like math most... Because I learned how to solve the problems in the textbook in advance in the learning center, I don’t worry about math class.

Min-jung: I feel sorry for those who have to work on the problems again after the cleanup. I think they will be smart and good at mathematics if they attend a learning center. (Interview, 7/13)

However, several students in a high-ability group in mathematics mentioned that they did not and would not attend a learning center because they wanted to understand mathematics for themselves. It appeared that most students who attended a learning center were in a middle-ability group. For them, attending a learning center was a place to gain their confidence in mathematics regardless of the drill and practice oriented teaching. Like Yun-ha who established confidence in mathematics, one mother said in an interview that her son also gained confidence in mathematics by attending a learning center.
center and mathematics became his favorite subject. Teacher Lee indicated that because about 15 students in his classroom were attending a learning center and already knew the answers and the methods of solution of the problems in the textbook without understanding, he avoided the same problems or tasks in the textbook in his mathematics class. His great emphasis on explaining the process of obtaining answers in the practice sections would prevent his students from perceiving that obtaining a correct answer was a only valuable thing in doing mathematics.

Summary

"Teaching mathematics with understanding" was the core of Teacher Lee's beliefs about the teaching and learning of mathematics. The development of understanding concepts and procedures was his major goal of mathematics teaching. While this belief came from his informal teaching experience such as tutoring, the belief about the importance of a learner's own understanding resulted from his early school experience and later was transformed to his belief as a teacher. He stressed that students themselves should understand and be convinced about mathematics. It was essential to him that students strove to find the meaning of mathematics for themselves. The theme of teaching mathematics with understanding was supported by the five major communication patterns and social-mathematical norms described in this section.

First, Teacher Lee emphasized that all students should have their own unique way of understanding of mathematics. He accepted each student's own way of understanding as a valid justification and verification of their mathematical knowledge. In addition, he emphasized student's vigorous effort for understanding. In order to implement his beliefs,
he established a mathematics classroom where a) it was okay for the students to solve only one problem if they clearly understood it, b) it was not necessarily mathematical formality when the students presented their ideas, thoughts, or ways of thinking, and c) different ways of understanding was valued, whereas presenting the same or similar response as the previous one was disregarded.

Second, Teacher Lee orchestrated his mathematics lesson to develop the students’ understanding of concepts and procedures. The development of conceptual and procedural understanding meant that he rarely told the students about concepts and procedures in mathematics, but led them to grasp these. He deliberately allowed them to be puzzled and confused. When the students were not likely to find or generate the concepts and procedures, he increased the possibility of success by making the task simpler with consistent use of why and how questions. In addition, he usually began his lesson by connecting the students’ previous knowledge with the day’s main concept or procedure.

Third, Teacher Lee had the students read the objectives from the textbook and present their own ideas about what the day’s lesson would be. His intention was to provide a broad context of the day’s lesson so that the students could readily understand the concepts and procedures. He usually covered only one main concept, problem, or tasks during each lesson.

Fourth, Teacher Lee consistently made use of the students’ everyday experiences to provide them with more meaningful contexts. Bringing their experiences into his mathematics classroom provoked their interests and helped them connect mathematical language and representations with the experiences. In addition, he believed that
mathematics teaching showed the students that mathematics is related to their everyday lives.

Lastly, in addition to a great emphasis on understanding, Teacher Lee also stressed the importance of practicing in mathematics. He indicated that understanding did not always ensure his students' successful experience in mathematics. Thus, he had the students solve mathematics problems both at the end of each class and a unit. In these practice sections, he required the students to explain their methods of solution to check their procedural understanding. He believed that understanding meant being able to explain how to do. His major role of practice sections was to ask the students' explanations of the process and to pinpoint the place where they did not understand in order to provide appropriate scaffolding for each student.

Manipulative Activities and Games for Conceptual Understanding

Teacher Lee viewed mathematics as a dull discipline, that had no fun and practical use for students' life. He mentioned that trigonometry, logarithm, and calculus are only for tests and have nothing to do with our lives. Moreover, he added that except for four numerical operations, people do not use mathematics in everyday life. It was apparent that his beliefs came from his early school experiences in a mathematics classroom where doing mathematics meant to memorize and practice many facts and rules to solve textbook problems. Tests for every mathematics class were followed by punishment for poor performance. Due to those experiences, learning mathematics for
him was only for getting a good grade in mathematics and there was no fun in a mathematics class.

In order for his students to avoid tedious and repulsive attitudes toward mathematics, Teacher Lee strongly believed that mathematics should be taught with fun and that mathematics teaching should be based on students' interest. In addition, he believed that fun and interest were primary vehicles to enhance students' understanding of mathematics.

I think many students feel bored and anxious about mathematics. It's not because the third grade's mathematics is quite complex for computing, but because teachers do not seem to be concerned about students' interest while teaching mathematics. I think teaching mathematics means to provoke their interest. I want to hear "Ah-ha!" when students finally understand something for themselves. (Interview, 7/26)

Teacher Lee's views about mathematics itself appeared not to be related to his beliefs about the teaching and learning of mathematics. Rather, he wanted to provide his students with a different perspective that he had had. The different perspective he was trying to nurture with the students was that doing mathematics is not just to solve problems by applying memorized algorithms, but is to have fun and interesting activities. The discrepancy between the beliefs about mathematics and the beliefs about the teaching and learning of mathematics contributed to his strong commitment as a classroom teacher. To implement the beliefs, he consistently used manipulative activities and games in his mathematics class.
Using Manipulative Activities

Teacher Lee strongly believed that the students should understand mathematical concepts first. Although he did not articulate a meaning of understanding, he viewed the teaching of concepts as the teaching of understanding.

From my experience, without understanding mathematical concepts, students failed to apply mathematical knowledge to solve problems. I think teaching concepts means to enhance understanding. For example, students should understand first why they need a division fact in a problem situation. After that they could learn the algorithm of division. (Interview, 8/27)

He noted that students were likely to fail to grasp basic concepts if manipulative activities were not used consistently. The conversation with a female teacher (Teacher Park) who earned the first place in the research lesson contest revealed Teacher Lee’s belief about manipulative activities for enhancing conceptual understanding of mathematics.

Park: I think students really have difficulty understanding concepts in geometry.
Lee: The major reason of the difficulty in geometry results from a lack of using manipulative activities.
Park: There are almost no manipulative activities...
Lee: Whenever I teach cube or rectangular prism of solid shapes I have the students make the figures for themselves. There are only a couple of examples of nets in the textbook. Students have really hard time to figure out which net can make a cube or prism. So, I instructed the students to cut any edges of the rectangular prism. Depending on which edge they cut first, there are many different ways to make solid shapes from nets. It was really interesting to watch. (Conversation, 8/17)
In this conversation, Teacher Lee claimed that the examples in the mathematics textbook were too limited to help the students' conceptual understanding of the relationship between solid shapes and their nets.

For Teacher Lee, manipulative activities had several different meanings in his mathematics teaching: (a) a means of bringing fun and interest to a mathematics lesson, (b) a means of devising algorithms and different methods of solution, (c) a means of fostering understanding of concepts and preventing misconceptions, (d) a means of enhancing transferability, and (e) a means of providing confounding and tedious experience in order to introduce procedures, algorithms, or rules.

As mentioned earlier, Teacher Lee stated that mathematics is a dull discipline without any fun. To help the students perceive that mathematics is filled with fun and is interesting, he mentioned that he had used manipulative activities. It appeared to him that being interested in and having fun with mathematics were inevitable for his understanding-oriented teaching of mathematics.

Whatever the reasons are, mathematics class should be fun. Manipulative activities can bring fun to the classroom. The third graders love making models, folding and cutting papers, or drawing lines. Even simple activities would provoke the students' interest in mathematics. (Interview, 8/12)

The following episode illustrates how Teacher Lee used manipulative activities in his mathematics teaching to bring fun and interest and foster understanding of a concept, "estimation." It was the first lesson on the Unit 7, Length and Time. The class had discussed estimating length in centimeters by using a ruler. Several students presented
their ideas to express the lengths using “approximately,” “less than,” “more than.”

Teacher Lee summarized their ideas.

Lee: So, today we have learned a new term, ‘approximately.’ Now, close your textbook and workbook because some students already wrote answers on them.

Teacher Lee gave a worksheet, a ruler, and a tape measure to each student. On the worksheet he provided several names of objects such as length of the width and height of the textbook, length of a span, length of a step, length of two arms. In addition, he provided blanks where the students could put any names of objects they wanted to measure. For example, Su-jong wanted to measure the length of his slippers, pencil, pencil case, leg, and height. There were two columns: estimated length and actual length.

Lee: When you measure the length of your step and two arms, please help each other. One more thing. Look at the tape measure. You have to read the numbers in black because the numbers represent inches.

Teacher Lee explained and demonstrated how to use the ruler and tape measure. Now, the students got busy measuring the objects on the worksheet. Some students were measuring the length of their desks. Others were measuring the size of their friends’ heads. The students helped each other to measure their heights and length of steps. The classroom suddenly turned out to be a boisterous place. Teacher Lee circulated the classroom and helped the students use the equipment correctly. At the end of the lesson he informed the students that they had to finish and turn in before going home. (Lesson transcript, 6/14)

Although there were several names of objects in the textbook, Teacher Lee did not use them because he knew that the students needed actual manipulative activities of measurement. Moreover, he wanted the students to actively engage in the process of his teaching. The list he made had several blank rows and allowed the students to put the names of objects they wanted to measure. In doing so, he provided the students with the opportunity of autonomous learning. This worksheet activity was useful to assess the students’ ability of measurement.
Second, Teacher Lee considered manipulative activities to be a means of devising algorithms and different methods of solution. He did not anticipate formal mathematical thinking of his third graders, but tried to encourage their own ways of methods and thinking.

Learning algorithms might improve the students' logical thinking in mathematics. I witnessed, however, elementary students usually had a very hard time solving mathematics problems logically and formally. I don't think that elementary students need strict logical thinking in mathematics. Rather, it is more important to me to foster their own ways of thinking and understanding. I am trying to help them think mathematics diversely. (Interview, 8/1)

Teacher Lee believed that different methods and algorithms exist in mathematics. Thus, he stated that understanding concepts made the students recognize this fact. In addition, accepting and valuing his students' different methods and understanding were one of his major roles as a discourse facilitator.

I think students should understand concepts first. Based on solid conceptual understanding, they can devise unique algorithms. I don't think that there is a single algorithm in mathematics. If they understand concepts, I am sure that they can solve problems using distinct algorithms that are not similar to the textbook ones. Thinking and solving problems in different ways is what I want to teach through my mathematics teaching. (Interview, 8/7)

Before introducing algorithms, I have the students engage in manipulative activities to explore and understand concepts. If they understand the concepts well, they can solve problems in different ways. By doing so, they would perceive that there are many different methods to get an answer in mathematics. (Interview, 8/12)

This view of manipulative activities was greatly contrasted with the view of teaching and learning algorithms. He stated that without establishing substantial understanding of
concepts, teaching and learning algorithms was to force the students to accept only one way of doing mathematics.

If you teach mathematics just based on algorithms, rules, and facts, the students learn only one way of doing mathematics, that is, following what is given by teachers. (Interview, 8/7)

Third, Teacher Lee believed that manipulative activities are a means of fostering understanding of concepts and preventing possible misconceptions. He indicated that mathematics embedded with manipulative activities would enhance the students' conceptual understanding.

I think my students need manipulative activities when learning concepts prior to algorithms. Manipulative activities would enhance their understanding of concepts and principles. For example, in the case of teaching $1/4 + 2/4$, if the students have an experience of cutting pieces of cake and putting them together, they can understand the concept of addition of fractions. They need to see the results of their actions. In addition, this kind of manipulative activities could prevent their misconceptions like $1/4 + 2/4 = 3/8$. That’s why I strongly believe that manipulative activities are necessary to teach mathematics, especially on an elementary level. (Interview, 8/17)

Teacher Lee consistently emphasized that teaching concepts should proceed before introducing rules or algorithms. As mentioned earlier, one main reason for using manipulative activities was to bring fun, interest, and active involvement of the students to his mathematics class. The following quotes illustrate Teacher Lee’s belief that gaining the students’ interest was a prerequisite of teaching concepts. In the first quote, he was providing advice about using manipulative activities while talking about the teaching of mathematics with three beginning teachers in the school.
I think it would be hard to explain a mathematical concept at the beginning of a certain unit by using formalized mathematics. Students readily get bored and lost their interest about what you are going to teach. That makes it harder for you. So, from my experience, I think using manipulative activities makes it much easier to teach mathematical concepts and students seem to make sense of them better. I don’t think we have to use manipulative activities for every mathematics class but at least we need them at the beginning of a unit. (Conversation, 6/22)

I use manipulative activities at least once or twice at the beginning of any unit. Once is enough for units like addition but geometry and problem-solving units need several times. So, it depends upon the unit. (Interview, 8/27)

The following episode illustrates how he used manipulative activities to enhance the students’ understanding the concept of fractions. It was the first lesson on Unit 8, Fractions. Teacher Lee began the lesson by having the students read the objective. Then he showed a red colored-paper ribbon about 12 inches long.

Lee: Now, I am going to make magic. [Suddenly the students’ eyes seemed to pop up and they got interested.] I am not sure this magic works well.
Tae-min: Don’t play a trick. [The students giggled.]
Lee: [Folded it in half] Look at this. The ribbon suddenly shrunk in half. [The students hooted.] Why? Isn’t it magic?
Students: You folded it in half. [In unison]
Lee: Well, how much is it shortened?
Students: Half! [In unison]
Lee: Okay, all of you said it is folded in half. How can you express it in a fraction? [More than 10 students quickly shot their hands up.] Ho-rae?
Ho-rae: [Rising from his seat] One half.
Lee: Do the rest of you think like that?
Students: Yeah... [In unison]

Teacher Lee demonstrated several times more and had the students present their ideas.

Lee: You had already learned these fractions in the second grade. Now, look at the page 104. Let’s read the problem.
Students: There are 8 colored papers. Su-jin wanted to have 1/4 of them. How many papers can she have. [In chorus]

Teacher Lee distributed eight colored papers (about 5 inches square) to each group.

Lee: How many papers can she have? Try to find it with your group. [The students worked for about 4 minutes.] Which group wants to present? Min-jung’s group?

Min-jung arranged the papers on the board.

Lee: Does anybody want to ask a question to Min-jung? Nobody? Then I will ask a question to Min-jung. Why did you make a group of two?

Min-jung: Because I have to make four groups.

Lee: Why do you think you need four groups?

Min-jung: Because the fraction is one fourth.

Lee: Does any group have a different arrangement or answer?

Teacher Lee continuously asked for a different arrangement or answer and provided more activities using these colored papers. (Lesson transcript, 6/29)

In this episode, instead of just a brief review on the board, Teacher Lee began his lesson with “magic” to get his students’ attention and make a comfortable classroom environment. He used a colored ribbon to review and connect the students’ previous knowledge about fractions with the day’s lesson. The manipulative activity with colored papers followed by on-board presentations and discourse appeared to bring fun and interest as well as fostering understanding of the concept of fractions. In addition, the students were trying to find different ways of solving the problem.
In the following episode, Teacher Lee showed that using a manipulative activity was a means of fostering understanding of a concept and preventing a misconception. He used colored paper ribbons to teach how to compare the magnitude of fractions. He distributed a colored paper ribbon and scissors to each student.

Lee: Now, please cut the ribbon in about 30 cm (12 inches). [He circulated to help the students cut it; about 3 minutes passed.] Let’s clap once.

Students: [Quickly stopped their work and clapped]

Lee: Cut your ribbon in four equal pieces this time. [Another 3 minutes passed.] Did you finish cutting your ribbon and have four pieces?

Students: Yes! [In unison]

Lee: Okay. Now, show me 1/4. [Most of the students quickly raised one piece of the four.] Tae-su, please rise from your seat, holding yours?

Tae-su: [Rising from his seat, holding four pieces]

Lee: How many pieces is he holding?

Students: Four! [In unison]

Lee: What is a fraction to represent them?

Students: Four over four! [About half students responded.]

Lee: Tae-su, what do you think about that? [Tae-su did not respond to this question and paused for a while.] How can you make one fourth?

Ha-sun: [Called out] One.

Lee: I asked to Tae-su. That means that he has to answer the question. [Tae-su raised one piece.] Okay. Show me 2/4. [He continued 3/4 and 4/4.] Now, show me 1/8.

Students: No, we can’t. Or we don’t have that many pieces. Or We have to cut more. [The students’ responses varied.]

Lee: [About 10 seconds passed and still nobody raise their hands. Another minute passed and several students raised hands.] Yung-jin, would you stand up and show yours?

Yung-jin: [Rising from her seat, holding a piece folded in half]

Lee: What did she do?

Students: Folded! [Murmured in unison as if it was not fair to fold it in half]

Lee: Now, everybody understands how she made 1/8?

Students: Yeah… [More students responded.]
Teacher Lee put each piece representing 1/4, 2/4, 3/4, and 4/4 on the board, wrote 4/4 > 3/4 > 2/4 > 1/4, and asked to the class to present any ideas about it. (Lesson transcript, 6/29)

In this episode, it was noteworthy when Tae-su made a mistake. Teacher Lee took the mistake as an opportunity to have the class think through the concept again. In addition, when Ha-sun called out, he reminded the class of the norm that the presenter had to figure out the answer for himself. Teacher Lee purposely asked them to show 1/8 to see whether the students could devise a unique method to represent it. Through manipulative activities in this lesson, he wanted the class to draw a conclusion of the rule for comparing fractions.

Fourth, Teacher Lee viewed understanding concepts through manipulative activities as a means of enhancing transferability of mathematical knowledge. His view about understanding concepts appeared to mean that these concepts furnished the basic tools of mathematical knowledge for the students to enable them to expand their understanding further. The following conversation with a female teacher delivered his view about transferability of conceptual understanding.

For example, if students are able to solve two-digit addition problems they should be able to solve three-digit addition problems too. But that doesn’t happen easily because they do not sufficiently understand the concepts of addition. If they understand the concepts, I think they should be able to solve not only three-digit addition but even five-digit. (Conversation, 8/17)

In an interview, he also mentioned that understanding concepts was a priority of his teaching. For teaching concepts, he indicated a variety of activities that were likely to engage the students in active learning.
I am trying to have the students explore and understand concepts. For example, when I teach division such as $36 \div 3$, I do not teach algorithms at the beginning. I have them solve it by drawings, counting rods, or marbles. I have them draw $36$ small circles and then group them by $3$s. They can see $12$ groups. They don’t need algorithms at this time. Further, they can solve $72 \div 3$ in the same way. In other words, if they understand the concepts of division by grouping, they can solve division problems beyond the third grade. (Interview, 8/7)

His view about high transferability of conceptual understanding was contrary to the view about low transferability of learning algorithms. For him, teaching algorithms was to show the students a single way of doing mathematics given by the teacher. And if the students followed that way, they could be successful but they would lose their own perspectives.

The students are usually good at calculations like $23$ times $4$ because of learning algorithms. But they don’t know why they have to use multiplication if the same problem is given by a story problem. I think that’s because they did not understand the concept of multiplication. If they understand the concept, the $23$ times $4$ can be solved by various ways like adding $23$ four times. (Interview, 7/26)

I think that recognizing why algorithms are needed in doing mathematics will make big difference for students’ learning and their attitudes about mathematics. From my experience, when I taught a unit based on an algorithm, the students could solve only the problems that could be directly answered by applying the algorithm. They could not solve application problems at all. (Interview, 8/7)

The following episode illustrates how Teacher Lee used a manipulative activity for enhancing transferability. He utilized three different representations to improve transferability of the procedural knowledge regarding the addition of two fractions. Using a manipulative activity is about adding and subtracting two fractions. On the day before this lesson, Teacher Lee informed the class that they had to bring a “Cho-ko Pie,” a small
and rounded chocolate cake with about 1.5 inches as a radius. The students also brought a small plastic knife to cut it. He started this lesson with the students' previous knowledge about a simple number line. He asked the class to draw a number line to represent $2 + 4$. In the middle of the lesson, the class discussed the reason for adding only numerators of $1/4 + 2/4 = 3/4$. Teacher Lee drew a number line as illustrated.

![Number Line Illustration]

Lee: Can anybody explain this? How did we get $2/4$? [Yun-ha quickly raised her hand.] Yun-ha?

Yun-ha: [Rising from her seat] $3/4$ means three $1/4$s and $1/4$ means one $1/4$. When subtracting, there are two $1/4$s. So, the answer is $2/4$.

Lee: Let's give her a hand. [The students clapped.] Now, let's experiment with this by using pieces of a cake. Cut your cake in four pieces. [The students got busy to cut the cake and about 2 minutes passed.] Show me the piece of $1/4$ and put it on the right side.

Students: [Raised and put one of four pieces]

Lee: Show me the piece of $2/4$ and put it on the right side.

Students: [Raised and put two of four pieces]

Lee: What does $2/4$ mean?

Students: Two pieces of $1/4$. [Several students responded.]

Lee: Now, how many pieces are on your right side?

Students: Three pieces! [In unison]

Lee: How can you represent it in a fraction fact?

Students: One fourth plus two fourths equals three fourths! [In chorus]

Lee: [Wrote this on the board, $1/4 + 2/4 = 3/4$]

Teacher Lee and the students did the same activity for $3/4 - 1/4 = 2/4$ and $1/4 + 1/4 = 2/4$.

Lee: Now, think about what we just have done. When you add two fractions, did you add denominators?

Students: No! [In unison]

Lee: Then, what do you have to do?
Teacher Lee made use of number lines to introduce the addition and subtraction of fractions. He wanted to make sure that the students understood fractions based on a unit fraction. Like Yun-ha’s explanation, the students had to know that 3/4 meant three pieces or parts of 1/4. To foster the students’ understanding of addition and subtraction of fractions, he decided to use a manipulative material, a “Cho-ko Pie.” This manipulative activity provided a different representation of operations of fractions from a number line. Moreover, until the lesson was completed, he did not mention how to add or subtract two fractions. Through different representational or manipulative activities, he had the students find the rule, “only add the numerators when the denominators are same.”

Lastly, Teacher Lee viewed manipulative activities as a means of providing confounding and tedious experience in order to introduce concepts, procedures, algorithms, or rules. He stated that he did not readily introduce the easy way of doing mathematics by stating concepts, rules, or algorithms. Rather, he wanted his students to feel inconvenience and tedium in doing mathematics through manipulative activities.

The students can solve 15 times 6 by adding 15 six times or 12 + 4 by drawings or counting rods. But when the numbers are getting bigger, they would recognize the inconvenience of such methods and the need of an algorithm. (Interview, 7/26)

They might solve 300 divided by 3 using drawings. In this case, the students usually complain, ‘Too many circles. How can I draw 300 circles?’ It is the time to introduce the algorithm of division. They need such inconvenient experiences of doing mathematics. Then if I introduce algorithms, they recognize the need of algorithms. They appreciate
convenience, easiness, and the effectiveness of using algorithms in mathematics. (Interview, 8/7)

Through manipulative activities, Teacher Lee intended to provide his students with confusing experiences in order for them to realize the necessity for concepts, rules, or algorithms and appreciate the effectiveness and convenience of them.

I know that involving manipulative activities might be complicated or confusing to them. Through these experiences, they could finally recognize why algorithms are convenient and why they need them. (Interview, 8/12)

In an interview, he demonstrated how he made use of a confounding and tedious experience of manipulative activity to introduce the concept of a circle.

I taught the concept of a circle last year for a demonstration lesson for parents. I have to do this kind of a lesson in September again. Introducing a circle seems to be very easy if the students use a compass and draw a circle. But I asked the students to go out and find two treasures I had hidden underneath the sand in the corner of the playground. I gave them strings and told them the treasures were hidden 2 meters apart at one point and 5 meters apart at another point. In marking points, they naturally said, ‘There are too many points here. How can we find them?’ Watching this activity, the parents and even the principals did not know what the students were going to do. In a reflection and discussion section after the class, they said that they were really confused at the beginning but later understood what the activity was about and how it was related to the concept of a circle. (Interview, 7/26)

Teacher Lee did not tell the students what a circle was. Instead, he let them explore the concept of a circle by a manipulative activity. After the activity, he might ask the students to reflect upon their actions and gradually introduce the concept of a circle.
In the following episode, Teacher Lee utilized tedious and inconvenient mathematical situations to introduce new concepts. The class was discussing how to add distances.

Lee: How far is it from the farm to the temple?
Students: [Looked at the picture in the textbook] 500 meters! [In unison]
Lee: How far is it from the temple to the turtle rock?
Students: 400 meters! [In unison]
Lee: [Wrote 400 m on the board] Then how far is it from the turtle rock to the top of the mountain?
Students: 100 meters! [In unison]
Lee: Now, we want to know how far it is from the farm to the top of the mountain. How can we know it?
Students: We have to add them together!
Lee: Right, we have to add them together. What is the distance if we add them together?
Students: 1000 meters! [In unison]
Lee: [Wrote 1000 m on the board] But don’t you think this number is too big?
Students: Yes!
Lee: So, I think we need a more simple number to represent this big number? Do you know what it is?
Students: 1 kilometer! [In unison] (Lesson transcript, 6/21)

In this episode, Teacher Lee did not just introduce the concept of 1 km = 1000 m. He showed the students in the tedious and inconvenient situation in order to introduce the new concept. Although the answer seemed to be straightforward, as usual, he asked a “how” question for the students to select an appropriate operation.

The next episode also illustrates how Teacher Lee used an inconvenient situation to teach the conversion of a metric unit. Jun-ho solved a problem regarding addition of two lengths (4 cm 6 mm + 4 cm 8 mm) and the answer was 8 cm 14 mm.

Lee: Now, let’s read the answer.
Students: 8 centimeters 14 millimeters! [In unison]
Lee: Although this answer isn’t wrong, it seems a little bit inconvenient. What is 1 km in meters?

Students: 1000 meters! [In unison]
Lee: Then, what is 1 km in millimeters?
Students: Thousand. Or ten thousand. Or one million centimeters. [Called out in disagreement]
Lee: [Wrote 1 km = 1,000 m = 100,000 cm = 1,000,000 mm]
Students: One million millimeters! [Several students called out with surprise.]
Lee: Right. Can you imagine how inconvenient it is to use one million millimeters instead of one kilometer?
Students: Awfully inconvenient. Or too difficult. [Called out]
Lee: It would be really inconvenient for both a reader and user. So, let’s get back to Jun-ho’s answer. It is not a wrong answer, but I think we can make it simpler. What is one centimeter in millimeters?
Students: 10 millimeters! [In unison]
Lee: Then, what is this 14 millimeters in centimeters and millimeters?
Students: 1 centimeter and 4 millimeters! [In unison]
Lee: Now, what are you going to do with this 1 centimeter?
Students: Move it up to the front. Or add 1 to 8. [Called out]
Lee: Do you want to move it up and add it to 8?
Students: Yes! [In unison]
Lee: So, what is Jun-ho’s answer now?
Students: 9 centimeters and 4 millimeters. [In unison] (Lesson transcript, 6/22)

In this episode, Teacher Lee wanted the students to understand the procedure of the addition of two lengths. When Jun-ho wrote 8 cm 14 mm as his answer, he accepted it as a reasonable answer. Then instead of instructing the class how to convert 14 mm to centimeters, he made it an inconvenient situation so that the students recognized the convenience of converting metric units.

Based on his strong belief about the importance of his students’ conceptual understanding, Teacher Lee criticized the current mathematics teaching practices.

Even though I have 10 years of teaching experience, I still have a hard time to help the students understand mathematical concepts and
procedures. This is my second year of teaching the third graders, but my teaching mathematics is not the same. I have to reflect upon the students’ misunderstandings and some problems of my teaching during last year. I have to prevent them this year. Teaching children mathematics is very difficult, and you should never assume that it would be easy. But some teachers said that teaching mathematics is easy. They just explain the concepts and procedures for about 20 minutes and have the students practice and solve problems. However, the students feel that mathematics is hard and even then they had serious anxiety toward mathematics. This means that their ways of teaching mathematics might have something wrong. (Conversation, 7/1)

There are the methods of solution and answers in the mathematics textbook. Every teacher has no problem to teach how to solve the problems. Giving a quiz or test frequently and scolding poor performers should improve the students’ test scores and performances. That’s what I did when I was a beginning teacher for several years. But what can these students learn from such a manner of teaching mathematics? (Interview, 7/26)

From these quotes, Teacher Lee surely expressed his belief about the enhancement of the students’ understanding of concepts. For him, simply verbalizing concepts and procedures was an easy way of teaching mathematics that would only have a temporary effect on the students’ learning, but eventually would hamper the students’ development of mathematical understanding.

**Using Games for Fun and Interest**

Teacher Lee stated that the students would be willing to actively participate in the teaching and learning of mathematics if there were fun activities in a mathematics class. In addition, he added that even some students who were not interested in mathematics were inclined to do their best. Accordingly, he wondered what made his mathematics teaching interesting and fun, and decided to utilize mathematics oriented games.
I am trying to implement as many games as possible in order to get the students' interest. Most of all, a mathematics class should be fun. I have observed a lot that most of the students displayed their interest and actively engaged in a lesson when I presented it in a game. That's the reason I am planning to do my action research on the effect of the use of games in a mathematics classroom. (Conversation, 6/7)

According to Teacher Lee, the term, "game," meant to facilitate an activity, to give rise to students' interests and fun, and to require competition.

I think a game means to provoke students' interests and to bring fun into the mathematics classroom. A game encourages students' participation... I use competition in playing a game because students really like it. I do not mean that solving many problems quickly is competition. But I encourage the students to compete as individuals or groups to cut five pieces of a colored paper quickly. (Interview, 8/7)

In addition, Teacher Lee indicated that teaching concepts would not be successful by playing games because he was not able to observe how the students solved problems and to correct when they made mistakes and misunderstood.

I think games should be used near the end of a unit. I don't think that playing games would enhance the students' understanding of concepts. For example, how do the students understand the concepts and algorithms of 234 times 34 while playing games. After understanding the concepts and algorithms, the students could play a game to foster their understanding by applying such knowledge to novel situations. (Interview, 7/26)

Consequently, Teacher Lee employed games near the end of a unit because he stated that playing games was solving problems by applying what the students learned in the unit. In addition, he added that playing a game required good understanding of concepts and algorithms.
Teacher Lee said that tomorrow they were going to play a division game. The students were delighted to hear this. He distributed two worksheets to each student, saying that they had to be good at solving division problems in order to play the game. During the whole class, the students solved problems on the worksheets and Teacher Lee checked each student’s work. (Fieldnotes, 6/11)

To me, playing games is a means of practicing because the students have to learn certain contents of mathematics to play the game. For example, in order to play a game regarding division the students should be good at division first. So, I have the students play a game when I am convinced that they are ready to play it. (Interview, 8/7)

The students in Teacher Lee’s class had to practice and master concepts and algorithms of a unit prior to playing a game. Without practicing and mastering concepts and algorithms, the students could not play a game with the rules that they were supposed to follow. Teacher Jeon, a female teacher of the third grade, explained in the morning meeting what happened in her mathematics class when the students played a division game without sufficient practice. In this division game, the marker moved forward corresponding to the remainder of a division problem when the students made a division with number cards.

Do you remember the board game for division on the workbook? I put in several hours to make number card sets for the game. But they played it with the rock-scissors-paper because they were not able to solve division problems. They just put away the stack of number cards. I really was frustrated to watch that. (Fieldnotes, 7/9)

Although Teacher Lee expected that the students would find concepts and algorithms while engaging in playing a game, he was well aware of how the students perceived playing a game. They did not think that playing a game was a part of learning mathematics.
My students often ask me whether they can play a game in mathematics class. They do not consider playing a game as solving mathematics problems. But, in fact, playing a game is similar to solving problems. Thus, it is very important to lead them to reflect upon what they learned from playing a game. (Interview, 8/12)

The observation data verified that he always encouraged the students to find hidden concepts and meaning of algorithms after a game. The following episode illustrates how a division game was used in Teacher Lee's mathematics classroom. With just two minutes left to begin the mathematics class, the third period, Teacher Lee sat and read a textbook and a lesson plan from the teacher's guide book on the lectern. He was summarizing the lesson. The students were jostling, joking, laughing, and running around the classroom. It was a boisterous place. It was the routine that the textbook should be opened on the very page of the day's lesson, but about half students did not follow the routine. Teacher Lee and his students had studied Unit 6, Division during the last two weeks and were almost at the end of this unit. The chimes sounded to begin this period. Teacher Lee and his students bowed to each other and the lesson was officially underway. He began his lesson by asking a question.

Lee: How many students are in our classroom?
Students: Forty five! [In unison]
Lee: Now, all get up and stand beside your desk.
Tae-min: [Interjected] Are we going to play a game?
Lee: Yes, we are. [The students looked at each other's face with grin and seemed to expect some fun.] Look at my fingers carefully and make a group of the numbers corresponding to my fingers. If you are successful in making a group, please sit down quickly on the floor, otherwise, sit down on your seats. Now, this is a practice. [Raised his right hand in air and suddenly spread two fingers]
The students made loud noises to make groups of two. Those who made the group rejoiced with laughs and talk.

Lee: Very good. Now, this time is a real one. [Spread six fingers in air]

The students quickly moved again to make a group of six. Those who made the group sat down on the floor and the three students who did not make it looked around to join any unsuccessful groups.

Lee: Please sit down in your seat those who did not make a group. Now, how many students did not make a group?

Students: Three! [In unison]

Lee: Right. Three people did not make a group. Then let’s make a division fact, using what we just have done. Eun-ho, can you tell me what it would be?

Eun-ho: Forty five minus three equals forty two.

Lee: Forty two. What does the forty two mean?

Students: Remaining people, remainders. [Called out]

Lee: Who else? In-ah? [Called on In-ah raised her hand]

In-ah: [Rising from her seat] If you divide forty five by six, then six (the number of one group members) times seven (the number of groups) is forty two and the remainder is three. So, forty two is those who remained.

Lee: [Asked the class] What is the forty two?

Students: Grouped people! [In unison]

Lee: Who else? U-min?

U-min: [Rising from his seat] Because six times seven equals forty two, three cannot make a group.

Lee: Now, three different expressions were suggested by Eun-ho, In-ah, and U-min. [Teacher Lee wrote the students’ suggestions on the board.] Let’s sing a song, ‘Spaceship.’

\[
\begin{align*}
45 - 3 &= 42 \\
45 \div 6 &= 7 \ldots 3 \\
6 \times 7 &= 42 \ldots 3
\end{align*}
\]

The students sang the song in loud voices and in the middle of the song, Teacher Lee raised both of his hands, spreading nine fingers. They quickly moved to make a group of nine with great noise.

Lee: How many were in the game?

Students: Forty two. [Several students responded.]

Lee: Forty two. How many should be in a group this time?

Students: Nine! [In unison]

Lee: Now, if you made a group of nine, how many dropped out?
Students: Three. Or four.
Lee: Who can make an expression? U-jung?
U-jung: [Rising from his seat] Forty two minus six.
Lee: Why do you think the expression would be forty two minus six?
U-jung: Because six people couldn’t make a group.
Lee: Why do you think six students were not able to make a group?
U-jung: There were not enough people...
Lee: Okay. Who else can make an expression with a division fact?
Yun-ha?
Yun-ha: [Rising from her seat] Forty two divided by nine.
Lee: What is the quotient? [Asked to the class]
Students: Four! [In unison]
Lee: Then, what is the remainder?
Students: Six. [About half students responded]
Lee: So, how many people made groups?
Students: Thirty six. [About half students responded]

Teacher Lee wrote the students’ ideas on the board and provided the students with two more grouping games using 8 and 9.

Lee: So far, we have played a grouping game. According to your presentations, there were three different expressions, that is, subtraction, division, and multiplication. When you make a group of a certain number of people, which expression can tell you the remainder?
Students: Division! [In unison]
Lee: Using division, we can easily find the number of groups and the number of people who did not make a group. Now, here are two numbers, 45 and 8 and let’s make any word problems you want. Jie-un?
Jie-un: [Rising from her seat] There were forty five balloons and five of them flew away. How many are left now?
Lee: Who else? Ji-min?
Ji-min: [Rising from her seat] There were forty five students in our classroom and five of them transferred to other schools. How many students are in our classroom?

Teacher Lee had several students make an expression for these problems.

Lee: Now, can anybody make different expressions with 45 and 8? [Several students call out, “Me!”] Han-jin?
Han-jin: [Rising from his seat] Su-jin had forty five pencils in her pencil box and gave the equal numbers to eight friends. How many pencils does she have now?
Lee: [Wrote Han-jin's problem on the board] What did this problem ask you?
Students: The number of pencils Su-jin has! [In unison]
Lee: Which expression do you need to solve this problem? Ji-su?
Ji-su: [Rising from her seat] Forty five divided by eight.
Lee: Do the rest of you agree with her?
Students: Yes! [In unison]
Lee: Okay, who wants to solve this problem? Yun-ha?

Yun-ha came up to the board and solved the problem.

\[
\begin{array}{r}
 \frac{5}{8} & 45 \\
 & 40 \\
 & 5
\end{array}
\]

Lee: Let's give her a hand. What does 5 mean here?
Students: Quotient! [In unison]
Lee: Right, it is quotient. But what is it in this problem?
Students: The pencils she gave. [Several students responded.]
Lee: What about this 5?
Students: The pencils she has. [Several students responded.]

Teacher Lee provided different numbers (e.g., 36 and 5) and the students made various expressions and solved them. The lesson continued in this way. (Lesson transcript, 6/8)

Teacher Lee began this lesson by asking the number of students in the classroom. It was obviously a familiar context for the students. The students showed great excitement while being involved in this division game. In the middle of or after the game, Teacher Lee asked questions about the relationship between divisor, quotient, and remainder. These questions required the students to reflect on their actions mathematically. He accepted all ideas (e.g., subtraction, division, and multiplication) as valid. Based on the students' ideas, at the end of the game he questioned the benefit of the division fact in this grouping game. It was noteworthy that he frequently asked several students to make problems and others to solve them. It was a kind of open-ended approach and he decided
which problem should be pursued in depth. In doing so, the students would perceive the problems as their own and make better sense of the contexts.

Another division game was observed four days later. After assigning two whole lessons for the practice of division problems with worksheets, Teacher Lee decided to have another division game for application. It was the last lesson for the division unit. This game was different from a previous one. Before the day of this lesson, Teacher Lee printed eight sets of 1 to 9 numbers with 48 font size from a computer printer, coated the numbers with a transparent plastic, and cut them separately. There were no number cards in the material room, so he had to make them for himself.

Lee: In the last two classes we practiced division and I think most of you can do division problems. So, today we are going to play a game using division. Let’s read the materials in the workbook.

The students read the materials in chorus. They needed two sets of number cards. One set was for making a dividend and the other for a divisor. There was a board game in the workbook and the students were going to use small objects for a marker. Teacher Lee demonstrated how to make a dividend and a divisor.

Lee: You have to choose two numbers from one of the sets. I will choose one. [Picked up one number card and showed it to the class] What is it?

Students: Six! [In unison]
Lee: [Put the six on the board held by a magnet] What about another?

Students: Three! [In unison]
Lee: [Put the three next to the six on the board held by a magnet] So, what number do I have now?

Students: Sixty three! [In unison]
Lee: Now, I will pick up another card from this stack. [Picked up a number] What is it?

Students: Nine! [In unison]
Lee: Let’s think about division. What is the quotient?

Students: Seven. [Less than half students responded.]
Lee: Then, what about a remainder?
Students: Zero. [Less than half students responded.]
Lee: So, how many steps can I move my marker?
Students: Seven steps. Or zero steps. Or you can’t move. [Called out in disagreement]
Lee: Your marker can only move the number of steps corresponding to the remainder. So, in this case, I cannot move my marker and will lose my turn.

Teacher Lee instructed that they had to solve division problems quickly after picking up the cards because this was a game. In addition, he recommended that they were going to play it with their group members and that some students in a group might not be able to solve the problems quickly and they needed to help each other. Each group leader came up to the front and fetched two stacks of the number cards. Each group put the stacks upside down and started playing the game. Teacher Lee circulated to help some students who had difficulty solving division problems.

In Min-jung’s group, Tae-su chose 99 and 2 and seemed to have hard time to figuring it out. Yung-jin and Min-jung were solving this problem using their notebooks. Then they told Tae-su that he could move his marker one step forward.

In Jun-ho’s group, Ho-rae made a dividend 72. When he was about to pick up another card, In-ah said “one, one.” It appeared that she already knew if Ho-rae picked up the card 1 he would lose his turn. But he got 5 and quickly copied the problem in his notebook and was trying to solve it. Other five members counted “one, two,…” until fifteen. They made their rule that each student in this group had to solve the problem within 15 seconds.
In Sung-don’s group, they made a different rule to play the game. One person made a division problem and gave it another person to solve. He picked up 57 and 4 and gave the division problem to Sun-kang. Sun-kang put the cards in front of him and looked at for about 5 seconds. He was solving it mentally. Sun-kang said ‘one’ and moved his marker one step forward.

In Eun-ho’s group, Yun-ji just solved the problem, 53 divided by 4 and moved her marker. Suddenly, the rest of members started arguing who was next.

Lee: [About 20 minutes later] Let’s clap once. [The students quickly stopped playing the game and clapped and sat upright.] I don’t want you just to play it, but to find some interesting rules. Your marker moved the number of steps corresponding to the remainder. What do you have to do if you want to get a bigger remainder so you can move your marker further?

Tae-min: [Interjected] Pick up the bigger divisor card.

Lee: Why does the divisor card have to be bigger? U-min, can you tell us why? [But he could not present his idea and several students raised their hands.] As Tae-min said, the divisor card should be bigger. But some students were happy when they picked up 8 and 9 for a dividend. It doesn’t matter what numbers you have for dividend. The remainder is dependent on the divisor. So, if your divisor is 1 and even though you got 89 for dividend, what is the remainder?

Students: Zero! [In unison]

Lee: Right. Now, let’s think about another thing. If I chose 9 for divisor, what numbers can be remainders?

Students: One. Or Three. Or Two. [Called out in disagreement]

Lee: What if I chose 9 for divisor and 9 or 18 for dividend? What is remainder in this case?

Students: Zero. [Several students responded]

Lee: [Wrote 0, 1, 2, 3 on the board and the chimes sounded to tell that this period was over.] If you have 9 for a divisor, the remainders should be 0, 1, 2, 3, 4, …, 8. There should be no 9 because 9 is divided by 9 again. Let’s finish here today.

(Lesson transcript, 6/12)
Teacher Lee informed the class that most of them were able to solve division problems. He began his lesson by having the students read the materials needed to play the game. Reading objectives, problems, or materials together in chorus was a distinctive feature of Teacher Lee’s mathematics class in order to keep the students on-task. He demonstrated how to play the game because he indicated that for even a simple activity or game he had to demonstrate it to the class in order to avoid confusion. He recommended cooperative work among the group members. Unlike his expectation, each group played the game a little bit differently. In addition, some students who were not good at mathematics were not able to actively participate in the game except for picking up cards and handing them to others to solve. As mentioned in interviews, he did not want the class just to play a game but to find important relationships in division. From this game, he tried to show them two relationships: (a) remainder is dependent on divisor, and (b) remainder is always less than divisor. But it appeared that the students could not sufficiently understand them because there was not enough time for Teacher Lee to guide the students’ understanding about the relationships. The following test item on the achievement test evidenced such lack of understanding of these relationships. Most of the third graders could not solve problem 14 in this test.

Problem 14: The following divisions require your mental calculation to find quotients and remainders.

<table>
<thead>
<tr>
<th>18 ÷ 6</th>
<th>19 ÷ 6</th>
<th>20 ÷ 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>21 ÷ 6</td>
<td>22 ÷ 6</td>
<td>23 ÷ 6</td>
</tr>
<tr>
<td>24 ÷ 6</td>
<td>25 ÷ 6</td>
<td>26 ÷ 6</td>
</tr>
</tbody>
</table>

Look at them carefully and write all of the remainders when a number is divided by 6. (Achievement Test on June 30, 1999)
In interviews with the students, the data verified that they really liked playing games in mathematics class.

Jun-ho: I like the division board game.
U-min: I don't like math class because it is hard to me to understand, but it is fun to play a game.
Sung-don: I like the grouping game. (Interview, 7/2)
Su-hee: Because it is okay to make mistakes when we are playing a game.
In-ah: I like playing a game because I can do mathematics in action. When playing the grouping game, I could understand division better. (Interview, 7/6)
Min-Jung: I like playing games, especially the division board game. I think some friends who were not good at division could do it better while playing the game.
Han-jin: I am not much interested in mathematics because it is easy. But when I am playing a game, I really like it. (Interview, 7/13)

According to these students, playing games provided fun and interest in a mistake free environment. In addition, as Teacher Lee expected, playing games would enhance the students’ understanding of concepts and algorithms. Throughout this study, he tried to integrate games into most of his mathematics teaching. However, there seemed to be a big obstacle in implementing his belief. He indicated that since there was a serious deficit of manipulative materials and guidebooks for mathematics games, he had to make the materials by himself and spent several hours making materials for the games. He mentioned that a lack of materials and guidebooks made teachers reject games or manipulative activities and stick to traditional ways of teaching mathematics.
Summary

Teacher Lee viewed mathematics as a dull discipline, which had no fun and practical use for students’ lives. It was apparent that the belief came from his early school experiences in a mathematics classroom where doing mathematics meant to memorize and practice many facts and rules to solve textbook problems. In order to prevent such mundane attitudes toward mathematics from his students, he believed that mathematics should be taught with fun and that mathematics teaching should be based on students’ interests. In addition, he believed that fun and interest were primary vehicles to enhance students’ understanding of mathematics.

Teacher Lee strongly believed that the students should understand mathematical concepts first. For conceptual understanding of his students, he consistently used manipulative activities. He stated that without establishing substantial understanding of concepts, teaching and learning algorithms was to limit the students to accept only one way of doing mathematics. For him, manipulative activities had several different meanings in his mathematics teaching: a) a means of bringing fun and interest to a mathematics lesson, b) a means of devising algorithms and different methods of solution, c) a means of fostering understanding of concepts and preventing misconceptions, d) a means of enhancing transferability, and e) a means of providing confounding and tedious experiences in order to introduce concepts, procedures, algorithms, or rules.

While manipulative activities were mainly used for the development of conceptual understanding, using mathematics embedded games were focused on bringing fun into the mathematics classroom and practicing concepts, procedures, algorithms, or rules. For Teacher Lee, the term, “a game,” meant to facilitate an activity, to give rise to
students' interest. Teacher Lee employed games near the end of a unit because playing games was solving problems by applying what the students learned in the unit. In addition, playing a game required good understanding of concepts and algorithms. He tried to integrate games into his mathematics teaching, but a deficit of manipulative materials and guidebooks for mathematics games were major obstacles.

Discourse-Oriented Mathematics Teaching

One of the themes that represents Teacher Lee's teaching practice of mathematics was "discourse-oriented." In his classroom, discourse was the central way to present, exchange, agree and disagree about mathematical ideas. Through discourse, he fostered the development of the students' understanding of mathematics and established classroom interactions and social-mathematical norms. These interactions and norms built up a learning environments for the teaching of mathematics with understanding. He consistently used discourse for reviewing and practicing, not only in a mathematics lesson, but other subjects. This teaching practice demonstrated his beliefs about the teaching and learning of mathematics.

Teacher Lee emphasized the importance and potential benefit of using discourse in teaching and learning mathematics. He believed that students should communicate their methods of solution and ways of thinking in order to share with the class.

Because we discuss and exchange our ideas and thoughts in a society, I think students should communicate and discuss their mathematical knowledge with each other. I like my students to exchange their ideas with each other. It would be good if they learned something from their
classmates through participating in communication in mathematics. In order for the students to communicate each other, they have to present their ideas and ways of thinking. It is important to show the rest of the class what he or she was thinking. That's the first step for classroom discourse. (Interview, 8/12).

Another reason Teacher Lee considered the discourse to be important in his mathematics classroom was that participating in discussion, responding to a question posed by him meant that the students were paying attention to a lesson and actively engaging in learning activities.

I usually don’t say “Be quiet” to the class, but I want them to follow my lesson. In other words, presenting ideas and answering questions mean that they are listening to my questions and are with me. That’s why I encourage them to communicate their ideas. (Interview, 8/1)

Thus, there was communication all the time in this classroom, but the conversation was orderly since the students knew when they should talk and when they should listen.

Even though Teacher Lee put a great deal of emphasis on his students’ participation and engagement in discourse, he indicated that many obstacles existed. Most of all, he mentioned two of them with which he was concerned: a) the deficit of home and society support, b) the deficit of the connection between the elementary and the higher grade levels.

Since family members are getting busy in our society, parents tend not to have enough time to sit down and talk with their children. Home is the first place that makes students’ dispositions of discourse possible in school. So, I think that education is not only the teachers’ duty in school but the home and our society should be a part of educating our children. It would be just an outcry in vain without the engagement of home and society… In order that teachers could focus more on classroom discourse, both students and parents should overcome the obsession of test grades or scores. Currently, the situation is getting better in elementary school
because of the prohibition of standard tests. More elementary teachers are inclined to use discussion and presentation in mathematics classes, but in middle and high school there is no exchange of ideas and the single purpose of mathematics teaching is to solve as many mathematics problems as students can. If students do not solve the problems quickly and accurately, the teachers of these grade levels would blame our mathematics teaching. (Interview, 8/12)

It was not unusual that he emphasized home involvement for school education. When he talked about two of the lowest-ability students in his class, his primary concern was the student’s home environment. He had not met the student’s parents and did not have enough information about them. Teacher Lee mentioned that parents’ support was one of the most important factors for students’ academic performance once the student had perseverance of learning.

Where did Teacher Lee get this discourse-oriented teaching practice? Was it from his early school experience, or from his four-year teacher education program, or from his 10-year teaching experience? The story went back to his middle school years. In the third year of middle school, he had a severe teacher, who brought a thick plastic ruler and hit the students’ palms whenever they misbehaved in his class. Nobody dared to make noise in his mathematics class. The teacher always asked the students to solve mathematics problems on the board and then explain the methods of solution to the class. It was the teacher’s routine and there was no exception. Especially, Teacher Lee’s experience of solving a mathematics problem on the board in the teacher’s research lesson influenced Teacher Lee’s way of teaching mathematics.

It was a day the teacher demonstrated his research lesson, so many people were there: a principal, an assistant principal, other math teachers, and even a superintendent in our classroom. I remember the lesson was about factoring polynomials. The teacher put a problem on the chalkboard and
called on a volunteer. Everybody was scared to solve the problem in front of so many unpleasant people. Nobody volunteered. I don’t know why I raised my hand. Anyway I walked to the front and solved it. And then the teacher asked the class if they have any questions to ask me. He always did that. His students should defend their solutions after solving problems. Unfortunately, there were a couple of hands raised reluctantly and pointed out my mistake. At the very moment, I realized I had made a mistake. I didn’t know that. I solved it again and got a correct answer. I was so sorry for my teacher because a research lesson should be perfect... The research lesson was very important for the teacher’s evaluation. I thought I had spoiled his lesson and I couldn’t look at his face... However, he surprised me by praising my mistake when he taught the next lesson. He said many students tended to make mistakes with that particular problem and my mistake made other students be aware of possible mistakes. Since that experience, I never felt afraid of making a mistake when I solved math problems on the chalkboard. (Interview, 7/26)

This teacher’s influence, along with other such experiences, shaped Teacher Lee’s beliefs about the discourse-oriented teaching practice of mathematics. It was obvious, however, that this experience did not provide him with sufficient knowledge to adhere to the discourse-oriented teaching. The experience surely changed his perspective of mathematics teaching but he needed professional knowledge to implement his beliefs. It was not possible until he joined the study groups where he learned the techniques of open-ended questioning in mathematics class. Study groups are discussed more completely in the section of professional development.

To implement the discourse-oriented teaching, Teacher Lee used two different methods of presentation in his mathematics class: One could be referred to as “On-Board Presentation,” the other “Standing-Up Presentation.” In On-Board Presentation, the students solved the problems on the board posed by Teacher Lee and presented their ideas and the methods of solution to the class. After the presentation, either Teacher Lee or the class asked questions of the presenter. On the other hand, in a Standing-Up
Presentation, the students presented their ideas and reasoning to the class after being called on by Teacher Lee. When Teacher Lee posed a question, the students who wanted to present and share their ideas and reasoning, raised their hands. When Teacher Lee called a student's name, the student had a permission to share the ideas and reasoning.

Interaction Patterns and Classroom Norms for Discourse

There were common patterns and norms that applied to both types of presentation. Once the regulations for classroom management, "behave orderly and think freely," were established, these patterns and norms created a learning environment that fostered the development of students’ understanding of mathematics.

Mistake free environment. Teacher Lee had to have the students feel comfortable for making mistakes so that they were willing to present and communicate their ideas, ways of understanding, justifications, and clarifications. Since discourse was Teacher Lee's essential method of teaching mathematics, building up a mistake free environment was really necessary for the students’ subsequent learning. As mentioned earlier, Teacher Lee valued each student’s own way of understanding. This belief meant that possible chances for making mistakes always existed. Thus, establishing a mistake free environment was equally important to valuing each student’s own way of understanding.

I always encourage every student in my class to participate in discourse. I am trying to work hard to convince them that I don’t mind whether their answers are correct or not and that making mistakes is absolutely okay. (Conversation, 6/8)
The following episode illustrates how Teacher Lee established this norm in his mathematics class. The lesson was about the concept of fractions. Teacher Lee had the students read a problem in the textbook. The problem was: "There are 8 colored papers. Su-jin wants to have 1/4 of them. How many colored papers can Su-jin have?" Min-jung just solved this problem on the board. Teacher Lee asked a couple of probing questions.

Lee: Does anyone have the same answer but a different way to explain it? Eun-ho? [Called on Eun-ho]
Eun-ho: [Seemed not to be confident] I think... my method might be wrong...
Lee: That's absolutely fine. I don't care whether you have a correct idea or not. If you have a wrong idea, I really want to listen to how you solved it. (Lesson transcript, 6/28)

In the episode, since the lesson was the first one on fractions, Teacher Lee used concrete materials. He invited any students' ideas and ways of understanding. Eun-ho hesitated to present his idea because he did not want to be embarrassed by his possible incorrect idea. Teacher Lee convinced him that he valued even a wrong idea.

The following episode also illustrates how Teacher Lee dealt with one students' incorrect idea to build up a mistake free environment. The problem was about finding elapsed time between 7:20 a.m. and 8:10 a.m. Jung-hae just finished explaining his idea.

Lee: Who else? U-jung?
U-jung: [Rising from his seat] One hour.
Lee: Why do you think the elapsed time is one hour?
U-jung: 20 minutes plus 60 minutes...
Lee: [Repeated U-jung's response] What would it be if you add 20 minutes and 60 minutes?
U-jung: 8:20 am...
Lee: Who else? Yung-jin?
Yung-jin: [Rising from her seat] I am not sure I got it right...
Lee: That's fine even though you have an incorrect idea. Simply tell us what you are thinking. That is important.
Yung-jin: I think it is 50 minutes.
Lee: Why do you think the elapsed time is 50 minutes?
Yung-jin: Because 8:10 minus 7:20 gives 50 minutes.
Lee: I see. Now, Jung-hae, U-jung, and Yung-jin, please come up to the board and solve the problem as you told the class. And the rest of you work in your notebook. (Lesson transcript, 6/24)

In this episode, Teacher Lee also encouraged Yung-jin to present her idea even though she thought that she might get it wrong. He said what was important was to present her own idea and understanding. He constantly asked probing questions to help the students elaborate and reflect their thinking and understanding. Whenever the students presented their ideas, methods of solution, justifications, and ways of understanding, how and why questions followed. In this episode, after listening to the students’ ideas, he had them solve the problem on the board so that he could use their methods of solution and justification for classroom discourse.

Through classroom observations, Teacher Lee never evaluated the students' ideas as correct or incorrect. Instead, he encouraged them to find an alternative way of understanding. Sometimes he had the class find the flows of the presenter's idea and help them think in a different perspective.

When my students have wrong ideas or concepts, I never say, “That's wrong. This is it.” When such cases happen, I encourage them to find a different way of understanding by saying, “I think your idea seem to have some problem. Would you think it over again?” Sometimes I have the rest of the class think about the possible flaws together and suggest some ideas so that the presenter can recognize the flaws. If I say, “Your idea might be wrong,” I can image how truly they get embarrassed. They never try to present and communicate their ideas in the class again. I don’t care whether their ideas or answers are correct or not. What I say is, “Good thinking. But I think if you think in this way, your idea would be better. How about that?” If I say so, they usually nod their heads. (Interview, 8/7)
The following episode illustrates how Teacher Lee dealt with incorrect reasoning by having the rest of the class help the presenter realize some misconceptions or misunderstanding. Teacher Lee put Pascal's Triangle on the board and had the class find as many patterns as possible. Ji-eun found one pattern and was explaining her reasoning. The illustration was her method to find the pattern.

Ji-eun: Here 1 plus 2 equals 3. Here 2 means twice, so 3 [Pointing another 3] comes twice.
Lee: [Repeated Ji-eun’s explanation] Okay...
Ji-eun: Here 1 plus 3 equals 4, 3 plus 3 equals 6, and 3 plus 1 equals 4.
Lee: Let’s give Ji-eun hands. [The students clapped.] I think there seems to be a flaw in Ji-eun’s explanation. Who can help Ji-eun figure it out? [Several students raised hands.] Eun-ho?
Eun-ho: 1 plus 2 equals 3 and the other 3 comes by adding 1 and 2.
Lee: Let’s give Eun-ho hands. Eun-ho was really a good listener.
(Lesson transcript, 7/9)

In this episode, Teacher Lee had the class give Ji-eun hands regardless of her answer. It was more valuable in his mathematics classroom for the students to present their ideas on the board. After applauding her effort to make her ideas public, Teacher Lee refrained from explaining her flaw in her reasoning. Rather, he assumed that if the students were listening carefully to her explanation, they could find it. In fact, Eun-ho obeyed the listener’s norm which was “listen carefully” when somebody presented on the board. By praising Eun-ho’s effort to adhere to the norm, Teacher Lee consciously delivered his
expectation that he respected a good listener. In his mathematics classroom, the students’ mistakes were always welcomed. By making the mistakes a public matter the class found the errors cooperatively, instead of treating them as a personal error. In this manner, Teacher Lee could reduce the students’ anxiety of making mistakes in presenting their ideas or ways of thinking.

Sometimes Teacher Lee engaged more actively in one-to-one discourse with a presenter when the student made a mistake. The following episode illustrates how Teacher Lee established a mistake free discourse environment by engaging in personal discourse to lead the presenter to understand. The lesson was near the closure stage. Teacher Lee had the students solve some practice problems from the textbook and now called on each student to present the answer. The problems were related to converting metric units.

Lee: What about 5410 meters? No-jae?
No-jae: [Rising from his seat] 5 kilometers 410 meters.
Lee: No-jae, would you speak loudly while you’re presenting your answer next time so everyone can understand? The next problem is 1 kilometer 60 meters. Who wants to do it? Yun-ji?
Yun-ji: [Rising from her seat] 160 meters.
Lee: [Repeated Yun-ji’s answer] 160 meters. [There were some murmuring among the students and several students raised their hands.] Please put down your hands. This is a problem where many students make mistakes. I think Yun-ji was not careful to look at the problem. Let’s look at why Yun-ji made such a mistake. [Wrote 1 km 60 m] Now, Yun-ji, how many meters are equal to 1 kilometer?
Yun-ji: 1000 meters.
Lee: Yes, 1 kilometer is equal to 1000 meters. And what meters do we have more?
Yun-ji: 60 meters.
Lee: So, how many meters equal to 1 kilometer 60 meters?
Yun-ji: 1060 meters.
Lee: Right. It should 1060 meters, not 160 meters. (Lesson transcript, 6/21)
In this episode, Teacher Lee called on Yun-ji, who was not good at mathematics because he thought the problem was not hard. When Yun-ji made a mistake of simple conversion, he commented on her mistake due to her inattentiveness in order to comfort her embarrassment. In addition, he had other students put down their hands and help Yun-ji understand the conversion. This teaching behavior also seemed to be associated with his belief that each student should make himself or herself understand mathematics.

The following episode illustrates a different approach to establishing a mistake free environment. This time, Teacher Lee utilized the students’ previous knowledge when one student presented incorrect reasoning. It was the third lesson on fractions. The students worked on how to write fractions on the number line. Teacher Lee and his students discussed how many skip-counts would be between the starting and ending point when the ending point was set such as 5, 10, or 100.

Lee: Let’s look at this number line. What is the starting point?
Students: Zero! [In unison]
Lee: What is the ending point?
Students: One! [In unison]
Lee: So, can we divide this number line as we just did?
Students: No. We can’t.
Lee: So, we cannot divide this number line but I really want to do it. Would you think it over again?
Eun-ho: [Interjected] Yes, we can!
Lee: [Showing interest] Really, Eun-ho? How can we do it?
Eun-ho: We can divide it by zero.
Lee: By zero… Eun-ho said we can divide it by zero. Okay, class. Let’s think about it in a little different way. When we was learning about division, was it possible to divide a number by zero?
Students: No, we cannot. [Several students responded.]
Lee: That’s right. We cannot divide a number by zero. Does anybody have an idea of how to divide this number line?
(Lesson transcript, 7/1)
In this episode, Teacher Lee did not provide any feedback regarding Eun-ho’s incorrect idea. Instead, he posted the idea to the class and encouraged them to think it together while relating to their previous about division. In doing so, the students might feel comfortable in making mistakes because Teacher Lee considered their mistakes as the matter of their classroom, not as a personal matter.

Making a mistake free environment was an essential piece that enabled Teacher Lee to accomplish the discourse-oriented teaching practice. He continually expressed the value of incorrect answers, had the class help the presenter understand his or her errors, engaged in one-to-one discourse with the presenter, and connected the students’ previous knowledge.

_Respect all students’ ideas._ Teacher Lee respected every student’s idea and way of thinking presented in his mathematics lesson. He conveyed his respect by probing questions for eliciting students’ thinking, by showing interest in understanding the students’ ideas and approaches, by mentioning presenters’ names and ways of thinking, and by providing equal opportunities for presentation. To do this, he asked “why” and “how” questions at any time when he decided to pursue in depth from among the ideas that his students brought up during discourse. In fact, he asked this type of probing questions of every student when he or she presented ideas and ways of understanding.

Teacher Lee always mentioned the presenter’s name and ways of thinking when he discussed, summarized and highlighted some important points on the presentation with the class. Mentioning presenters’ names and ways of thinking to the class was to show his students that he acknowledged the ownership and source of the knowledge. The
following episode illustrates how Teacher Lee demonstrated his respect by mentioning a presenter’s name and ways of thinking. The class worked at solving word problems.

Teacher Lee and the class discussed and practiced two word problems together. The class just finished the second one.

Lee: Today we learned about how to solve word problems. Was it hard?

Students: No! [In unison]

Lee: But, why do many of you solve the word problems incorrectly?

Students: [Laughing]

Lee: What reasons do you think? [Invited several students to talk about it]

In-ah: Because I did not check my solution when I finished it.

Ji-hyun: I think this time I can do it, but on the tests I didn’t pay much attention to read the problems carefully.

Min-jung: Because I want to solve them quickly, I made many mistakes.

Yun-ha: When I couldn’t solve them, I just skipped them.

Ji-min: There always wasn’t enough time on tests, so I hurried to finish them and my work was incomplete.

Lee: Okay. Please put down your hands. I have been thinking why many of you are not able to solve word problems well. If I give you enough time to think about the problems carefully like Ji-min said, I think all of you can solve them. So, I won’t give you many problems although I prepared many word problems for today’s practice. [Showed a copied work sheet containing about 20 problems on both side] Now, let’s solve only 4 problems on page 122 of the textbook. Remember what In-ah said. When you finish each problem, you should check the answer again. (Lesson transcript, 7/13)

Teacher Lee asked the students for the reasons that they made mistakes in solving word problems, instead of just telling that the students had to be cautious. Based on the students’ information, he changed his plan to give a small number of problems. In doing so, he mentioned Ji-min’s idea. In addition, the importance of checking the answer and
method was addressed by mentioning In-ah's idea. This teaching behavior, such as mentioning the students' names and ideas and changing his instructional plan based on the ideas, seemed to show the students that Teacher Lee cared for and respected their ideas.

Another pattern identified through classroom observations to show his respect of the students' ideas was to provide equal opportunities for presentation. He explained the reason of this communication pattern.

In my class, about 15 students always raise their hands. They have many opportunities to present their ideas, the methods of solution, and ways of thinking. So, to give equal opportunities of presentation to my students, I call the names of those who do not raise their hands often. In addition, if the questions are easy, many students want to present their ideas. In this case, I also call the names of those who do not have many opportunities to present. (Interview, 8/1)

Call-on could be a simple strategy that every teacher use in his or her classroom. But for Teacher Lee it was not just a simple approach to encourage the students to participate actively in discussion. It was a signal to express his respect and caring of the students' learning. The following episode illustrates how Teacher Lee arranged the opportunities of presentation for low-ability students. The class was working on finding the difference between two lengths. Teacher Lee put the following problem on the board which was a real story about the length of his pencil (a) and that of his brother (b).

(a) 5 cm 4 mm
(b) 3 cm 8 mm

Lee: Then how long is the difference between my pencil and my brother's?
Students: That's too easy. [About 15 students raised their hands.]
Teacher Lee called on the two students, Su-kyung and Jung-ha who were in the low-ability students in mathematics based on some students’ comment, “That’s too easy” and his knowledge of his students’ ability. Because of their low ability in mathematics, the two students had not have many opportunities to present their ideas on the board. Teacher Lee was interested in knowing their ways of thinking and understanding. By providing equal opportunities for presentation, he conveyed his respect to his students.

In the another episode, Teacher Lee called on one student in each group to draw 10-centimeter line segment on the board. He recognized that the student still had difficulty approximating the length of line segments. So, he decided to give one more practice on the board.

Teacher Lee intentionally called on one student from each group who was the low-ability students in mathematics. He made this decision based on the level of difficulty of the task and knowledge of the students. In doing so, he provided equal opportunities for presentation and showed his respect to the class.

Teacher Lee also had the class applaud the students who were the low-ability students in mathematics, when they presented their reasoning or understanding. In the following episode, three students solved a problem of finding elapsed time. One student,
You young-Jin, in the high-ability group in mathematics, solved the problem on the board.

Instead of having Young-Jin present her method of solution, Teacher Lee was asking a question to the class.

Lee: Let’s look at Young-Jin’s solution. She subtracted 7 hours 20 minutes from 8 hours 10 minutes. So she got 50 minutes. Now, I think we have to think carefully here. Is there any way to subtract 20 from 10?

Students: No! [The class responded in unison.]

Lee: If you say no, how did she subtract 20 minutes from 10 minutes? [About 10 students shot their hands up immediately.] Actually I want to ask Young-Jin but many of you seem to know how she did it. Then, Sang-sun, would you tell me how you think? [Teacher Lee called on Sang-sun’s name who were one of the low ability students in mathematics.]

Sang-sun: [Rising from his seat.] I think she borrowed 1 hour from 8 hours...

Lee: So, what happened when she borrowed 1 hour?

Sang-sun: 60 plus 10 is 70, so she can subtract 20. So, she has 50.

Lee: Good! Let’s give him big hands.

Students: [Applaud Sang-sun.] (Lesson transcript, 6/24)

In this episode, Teacher Lee deliberately called on Sang-sun, one of the low-ability students, to give him an opportunity to present his idea. Moreover, Teacher Lee had the class applaud his explanation of his idea when he provided his clarification of ways of understanding. It was unusual for him to give such quick feedback unless the presenter was in a low-ability group.

In order to foster the students’ active participation of classroom discourse, Teacher Lee showed his respect to and interest in the students’ ideas by mentioning the presenter’s name, by providing equal opportunities for presentations.
How and why questions. In Teacher Lee’s mathematics class, how and why questions were always heard at any time when the students presented their ideas. He never missed these kinds of questions for probing and eliciting his students’ thinking and understanding. He wanted to listen to the students’ ideas for redirecting his teaching and select the best appropriate tasks for his students. For him, how and why questions were a major method of gathering information about the students’ understanding. In addition, probing questions using how and why were another way of showing his respect and interest. He wanted his students to recognize that he really liked to listen to their ideas and ways of thinking and understanding. For the students, how and why questions provided considerable opportunities to reflect their thinking, elicit and elaborate on their ideas and understanding, and improve their mathematical communication skills by clarifying and defending their presented ideas.

The following episode illustrates the how and why questions he used in teaching mathematics. The class was working on a word problem: Min-ku and his brother picked tomatoes with their father. Min-ku picked up 147, his brother 94, and their father picked up tomatoes twice as much as his brother. How many tomatoes did their father pick up? Teacher Lee asked several questions about this problem so the class could identify key information.

Lee: How many conditions are there in this problem?
Students: Three! [In unison]
Lee: Is there the condition we can find the number of tomatoes the father picked up?
Students: Yes! [In unison]
Lee: What condition do we need?
Students: The number of tomatoes Min-ku’s brother picked up.
Lee: Why do we need this information?
Students: Because it’s twice… [In unison but discernable]
Lee: Now, let’s raise your hands to present your ideas. (Lesson, 7/13)

Teacher Lee continuously asked questions to develop the students’ understanding of the problem situations and information. Rather than explaining the word “twice” which means “multiply 2,” he asked a why question to know what the students knew about this word.

The next episode illustrates how he asked how and why questions to elicit and make sense of the students’ ideas. The class was trying to find elapsed time. Teacher Lee asked Yun-ha when she got up and left for school. Yun-ha said she got up at 7:20 in the morning and left for school at 8:10. From this information, Teacher Lee asked the class how long Yun-ha needed to get ready to come to school.

Students: [Responded loudly sitting on their chairs] 50 minutes, 80 minutes, 30 minutes...
Lee: [Call on a student among those who raised their hands up] Jung-hae, what did you get?
Jung-hae: [Rising from her seat] 50 minutes.
Lee: [Repeated the answer to the class] 50 minutes. Why do you think that? ... [Jung-hae could not verify his answer.] U-jung, can you tell your answer?
U-jung: [Rising from his seat] One hour.
Lee: Can you tell the class how you figured it out?
U-jung: Twenty minutes plus sixty minutes... (Lesson transcript, 6/24)

Jung-hae might think that she had a correct answer, but she was not able to clarify her understanding when Teacher Lee asked the why question. U-jung provided his reasoning that was obviously incorrect. Later in this class, Jung-hae and U-jung realized what they had misunderstood. What was important in this episode was that whatever answers and reasoning the students presented, Teacher Lee accepted them and then asked how and
why questions. In doing so, he reduced the students' anxiety of making a mistake and was able to help the students understand for themselves.

The next episode also illustrates how intensively Teacher Lee utilized how and why questions during classroom discourse. The class was working on finding patterns of sequences. Teacher Lee just posted two problems which his students solved easily. The students really liked to solve these kinds of problems. So, he decided to give a more challenge and impromptu problem.

```
1, 1, 2, 3, 5, 8, 13, __, __.
```

Lee: Let's think about this problem. It does look like a little bit odd, doesn't it? [About 10 students raised hands quickly.]

Students: Yes or no.

Lee: [Showed surprise] I thought this would be a very hard problem. Okay, who can try this? What kind of a pattern is here? Ji-hyun?

Ji-hyun: [Rising from her seat but not confidently] Multiply...

Lee: What did you multiply?

Ji-hyun: [Changed her mind] No, I think it might work to add...

Lee: How did you add and what did you add?

Ji-hyun: [Hesitated to say and gave up] I haven't quite figured it out.

Lee: Okay. [About 10 students raised hands again quickly.] U-jung?

U-jung: [Rising from his seat] 1 plus 1 equals 2, 1 plus 2 equals 3, 2 plus 3 equals 5, 3 plus 5 equals 8, 5 plus 8 equals 13. So, the first blank should be 8 plus 13. That's 21.

Lee: Do the rest of you understand what U-jung said? (Lesson transcript, 7/9)

Teacher Lee tried to elicit Ji-hyun's idea by asking how and why questions even though her idea was not correct. In doing so, he showed his respect and interest to Ji-hyun and the class. In addition, he wanted her to understand why the method of solution made sense. He indicated once that explaining their ideas and providing justification showed
their own way of understanding. Teacher Lee did not try to evaluate Ji-hyun’s ideas, rather what he did was to lead her to make sense by herself by asking probing questions.

The next episode illustrates how Teacher Lee used how and why questions in even a simple computation problem. It might be considered to be inefficient instruction but his intention was to make sense out of the students’ understanding and ways of thinking, to probe questions for developing understanding, and to assess their progress.

Teacher Lee was about to explain and summarize the method of solution on the board after three students solved a problem. It was like this.

\[
\begin{align*}
4 \text{ cm} & \quad 6 \text{ mm} \\
+ & \quad 4 \text{ cm} \quad 8 \text{ mm} \\
\underline{9 \text{ cm} \quad 4 \text{ mm}}
\end{align*}
\]

Lee: Now, I want to ask some questions. Jun-ho explained his method well. What is 6 millimeters plus 8 millimeters?
[ Circed 6, 8 ]

Students: 14 millimeters! [In unison]

Lee: Then, you said 14 millimeters but why he wrote only 4 here? Where is 10?

Students: Moved up.

Lee: Why did he move 10 up? [The students said some reasons in disagreement.] Let’s raise your hands to present your ideas.
[About 15 students raised hands.] Why was 10 moved up? U-min?

U-min: [Rising from his seat] 6 plus 8 millimeters went beyond 10 millimeters.

Lee: [Repeated U-min’s reasoning] Because 6 plus 8 millimeter went beyond 10 millimeters. Who else? Why was 10 moved up? Ha-sun?

Ha-sun: [Rising from her seat] Because 1 centimeter is 10 millimeters.

Lee: Because 10 millimeters is 1 centimeter. Who else? Ji-hyun?

Ji-hyun: [Rising from her seat] Because it would be inconvenient to read 14 millimeters.

Lee: Inconvenient to read 14 millimeters. Who else? Shin-yung?

Shin-yung: [Rising from his seat] Because 14 millimeters consists of 1 centimeter and 4 millimeters, so 1 centimeter can be added to…
Teacher Lee wanted the students to understand Jun-ho’s method of solution. Instead of asking clarifications to him, he decided this time to ask the simple but important concept, $1 \text{ cm} = 10 \text{ mm}$, to the students. In this episode, Teacher Lee asked a probing question using why and the students provided different ideas to explain their reasoning. It seemed that since the question was simple, there were no obvious benefits from it. But Teacher Lee always made the question a problematic situation so that the students might not take it for granted.

Using how and why questions was a major method of eliciting, encouraging, challenging, and assessing the students’ ideas and ways of thinking. These questioning could be heard in every situation from the beginning of Teacher Lee’s mathematics lesson to the end of the lesson.

**Norms for presenters and listeners.** “Loud enough and clearly” was a norm that was expected of presenters. Presenters either on the board or by standing-up at their seats had a responsibility and an obligation of talking loud enough so that the class clearly understood the presenters’ ideas, the methods of solutions, and ways of thinking. In addition, presenting loudly enough and clearly was important in this classroom because Teacher Lee and the class members had to make sense of what was presented and ask questions to help the presenters develop an understanding of the contents of the presentation. Moreover, because of his concern about fostering his students’ ability of mathematical communication, he attached importance to presentation. He said, “When
the students were able to present ideas clearly, it means that they understood something in their own ways.

The following two episodes illustrate how Teacher Lee established this norm. In the first episodes, Kyung-su was just finished solving a problem, 5 cm 4 mm – 3 cm 8 mm on the board. It was time to explain to the class his solution method.

Lee: Now, let’s listen to Kyung-su’s explanation.

Kyung-su: We cannot subtract 8 from 4, so I borrowed 1 from 5. It means 10 mm. So, it will be 14 mm… [However, Kyung-su’s voice was so low and he talked to the board, instead to the class.]

Lee: Kyung-su, you should present your method in a louder voice next time, facing the class, not the board. So the class could question you to help your understanding, if needed. (Lesson transcript, 6/22)

Lee: [Called on A-young, seated in the back row.] A-young, I can’t hear what you are saying. I don’t mind whether your answer is correct or not. Don’t worry about it. However, you should present your idea clearly so that we can understand if we needed to ask questions. (Lesson transcript, 6/24)

According to both episodes, Teacher Lee constantly reminded the students of the presenter’s norm. In addition, he commented that his concern was not to see whether the presenter had a correct answer or not but to understand the method of solution, justifications for their reasoning, or ways of thinking.

On the contrary to the presenters’ norm, “listen carefully and then ask questions” was a norm for listeners, including Teacher Lee. When presenters either on the board or by standing-up provided or explained their ideas and ways of thinking, both the rest of the class and Teacher Lee had a responsibility and an obligation of listening carefully in order to understand and ask questions to help the presenters elaborate their thinking and reasoning. Listening carefully was a starting place where discourse took place between
the class, the presenter, and Teacher Lee. He valued the importance of this norm in his mathematics class.

The following episode illustrates the process established by this norm. Han-jin was explaining how he figured out a word problem on the board. He appeared to be in a little bit of trouble while explaining his idea. And about five students raised their hands to get a chance to present their ideas.

Lee: Are you supposed to raise your hands when a student presents his idea on the board? What do you have to do when somebody presents something on the board?

Students: Listen carefully! [Responded in unison] (Lesson transcript, 6/28)

The students were not supposed to raise their hands when somebody made a mistake on the board. Instead, they had to listen carefully. In addition, Teacher Lee never cut the presentation underway even though it had flaws, errors, or incorrect logic because he wanted to listen to his students’ own way of understanding.

Sometimes Teacher Lee asked the rest of the class what the presenter said to ensure that all the students were paying attention to the presentation. In the first episode, Teacher Lee observed the two students’ inattentive behavior to the presentations. To make sure the listeners’ norm, he intentionally called on them. In the second episode, Tae-su was chatting with a student in his group while Su-hee responded Teacher Lee’s question. He used this as an opportunity to remind the class what the listener’s norm was.

Teacher Lee was interested in the students’ ideas about fractions before beginning his lesson about Unit 7, Fractions. Three students presented their ideas about fractions. Then he called on two students and asked what common things were in their ideas. Both students had not paid attention to
the presentations. Teacher Lee said, "Listening carefully is your important responsibility." (Lesson transcript, 7/2)

Lee: Tae-su, what did Su-hee say about this question? (Lesson transcript, 7/12)

Another important reason to listen carefully to the presenter’s idea, reasoning, or justification as listeners was that the students could provide detailed, better developed, and different ideas. As mentioned earlier, Teacher Lee believed that the students’ own understanding was the essential aspect of learning mathematics. Thus, he valued different ideas developed on the basis of previous ideas, instead of simply restating other’s ideas. This norm was related to the importance of their own ways of understanding because it was not possible without listening to other ideas carefully to present their own ideas. The following episode illustrates how Teacher Lee established this norm. The lesson was about the metric unit, kilometer. Teacher Lee asked the class the distance of running around the track on the playground. Students said it would be about 200 m. Eun-ho said he ran nine rounds of the track once. Using what Eun-ho said, Teacher Lee asked the class the total distance Eun-ho ran.

Ho-rae: [Rising from his seat] 1 kilometer and 800 meters.
Lee: [Repeated Ho-rae’s idea] 1 kilometer and 800 meters. Who else? Min-jung?
Min-jung: [Rising from her seat] 1800 meters.
Lee: [Repeated Min-jung’s idea] 1800 meter. Who else? Yung-jun?
Yung-jun: [Rising from his seat] 2 kilometers.
Lee: [Repeated Yung-jun’s idea] 2 kilometers. Who else? Shin-yung?
Shin-yung: [Rising from her seat] 1800 meters.
Lee: [Repeated Shin-yung’s idea] 1800 meters. That’s the same as previous one. (Lesson transcript, 6/21)
When Shin-yung presented her idea which was the same as Min-jung's, Teacher Lee repeated her idea as usual, but conveyed the norm that the same idea would not be valued in his class by saying "That's the same as ..."

Through classroom observations, four different types of communication patterns and classroom norms were identified: mistake free environment, respect of all students' ideas, how and why questions, and norms for presenters and listeners. These patterns and norms governed Teacher Lee's instructional behaviors and decisions as well as the students' learning.

**On-Board Presentation**

In every mathematics class, Teacher Lee called on students to have them solve mathematics problems on the board. Stating that getting an answer is not the end of solving mathematics problems, he always encouraged his students to explain and defend their methods of solution after solving any problems on the board. His purpose was to provide his students with more opportunity to reflect on their understanding and mistakes of the problems through the process of presentation.

I consistently have my students solve mathematics problems on the board because I want to know their methods of solution and ways of thinking. Getting an answer is not that important to me. Rather, my purpose is to provide the students with opportunity to explain and elaborate their understanding and thoughts. A number of third graders do not know how they solved the problems regardless of the correct answers they got because they memorized the procedures. I always ask for their verification and clarification by asking "Why does the method make sense to you?" In addition, I often have the rest of the class ask a question to the presenter on the board. (Interview, 8/7)
In addition, having students solve problems on the board was beneficial to him for checking students’ understanding and redirecting his teaching. It was an ongoing process of assessment.

I want to know the process of how the students understand and what they know from my teaching. Through the board presentation, I could identify where and why they have misconceptions and then, based on that information, I am able to correct them. Sometimes when there was a common misunderstanding, I share it with the students and teach it again. So, I will have as many students as possible solve mathematics problems on the board. (Conversation, 6/7)

When his students were at the board to solve mathematics problems, his role was to be a careful listener and to make sense of when and how to attach mathematical notation and language to the student’s ideas, what to pursue in depth from among the ideas that the students bring up during a discussion, when to provide information and to ask for clarification, when to lead, and when to let the students struggle with a difficulty.

For third graders, their mathematical language and grammar are not quite accurate. They could communicate their ideas and ways of thinking, but there are always errors of notations and equations. For instance, they say “2 plus 3 equals 5,” writing $2 + 3 = 5$. When they have to add 3 more, they write $2 + 3 = 5 + 3 = 8$. When such a case happens, I correct their notation and language errors. (Interview, 8/11)

The on-board presentation was a major method to implement Teacher Lee’s beliefs about discourse-oriented mathematics teaching. It was a unique opportunity for the students to present, defend, justify, and verify their ideas and methods of solution. For Teacher Lee, it was a process of assessing the students’ understanding.
The process of classroom discourse. The On-Board Presentation had a distinctive sequence. Figure 3 illustrates the sequence.

![Diagram](image)

**Figure 3.** The Sequence of On-Board Presentation

The selection of a task or problem depended on the situation. Generally, the main problem of the day in the textbook, the students' everyday experience, the students' comments or misunderstanding during discussion, or Teacher Lee's extemporaneous problem were used as the task or problem for discourse.

More than two students were voluntarily called upon to solve a task or problem on the board. When the students solved problems on the board and the rest of the class were ready to listen to them, the presenters began presenting their methods of solution. As mentioned earlier, the presenters needed to speak loudly enough and clearly so that everyone in the class could understand and if possible, ask some questions. Teacher Lee's expectation was that his students would help each other by asking questions. While the presenter explained his or her method of solution to the class, both Teacher Lee and the rest of the class listened to it carefully so that they could ask questions for justification and clarification. By asking questions for clarification or justification, the class could help the presenter realize what they did not understand clearly. After classroom discussion, Teacher Lee summarized and highlighted some major points of the presentation.
The following episode illustrates how the process of the classroom discourse proceeded. Teacher Lee’s class was in the third lesson on Unit 8, Fractions. The day’s lesson objective was to write a fraction on the number line. He and his students had just discussed why it was important to mark 0 and 1 as a starting and an ending point on the number line.

Lee: Now, we have about 10 minutes left. [Teacher Lee said, looking at the clock which clung on the left-side wall of the classroom. And he wrote the following fractions on the board which were not in the textbook.] Write these fractions, three over five, two over six, and seven over eight on the number lines. You can draw the number lines in your textbooks. Who wants to draw number lines with these fractions on the board?

$$\frac{3}{5}, \frac{2}{6}, \frac{7}{8}$$

Lee: Mi-ju, Ju-ri, U-jung. Please come to the board and mark them on the number line. [Teacher Lee called on three students’ names. The students, who were in the middle-ability group, came to the board.] Jung-ha, you solve three over five. Ju-ri, you solve two over six. And U-jung, you solve seven over eight.

The rest of the students worked on the problems individually at their seats. It was a norm when some students worked with problems on the board, the rest of the class would solve them individually. Teacher Lee circulated and observed the students’ attempts. He also asked questions about how they wrote. The three students appeared to have a hard time figuring out how to write the fractions on the number lines. They continued erasing and writing. About four minutes elapsed.

Lee: Now, please stop your work. Clap once. [Teacher Lee walked to the front of the classroom and called to order to have the class ready to listen to the presenters’ methods of solution.]
Students: [The students stopped their work quickly and clapped their hands and sat upright.]

Lee: Let’s listen to Mi-ju’s presentation. [Teacher Lee said to the class, but the students were not ready to listen to Mi-ju’s presentation. For a moment they still murmured.] What are you supposed to do when someone presents something? [Teacher Lee reminded the class of the norm of the board presentation.]

Students: Listen carefully! [In unison]

They were now ready to proceed with the presentation. Standing in front of her method of solution on the board, Mi-ju held the thin wooden rod as a pointer. The following was Mi-ju’s number line.

\[
\begin{array}{c}
0 \\
\hline \\
3/5 \\
\hline \\
5
\end{array}
\]

Mi-ju: [Explained, pointing the 3/5 with the rod] Since the fraction is three over five... the denominator is 5. It means that the end point of this line is 5. So, I divided this line into 5... and here is three over five.

She explained her ideas in a low voice. Teacher Lee used this opportunity to remind the class of the norm of presentation on the board. Mi-ju was supposed to explain her ideas in a loud enough and clear voice so that everyone in the class could hear.

Lee: I think Mi-ju has a little problem in presenting her idea.

Students: Her voice is so low.

Lee: Now, does anyone want to ask her a question? [Teacher Lee invited the students’ questions to the presenter. About five students shot their hands up in air.] Ji-eun?

Ji-eun: [Rising from her seat and pointing to 3/5 on the number line on the board with her finger, Ji-eun asked Mi-ju’s justification] Why did you write three fifths there?

Lee: Ji-eun asked you why you wrote three fifths here.
Standing next to Mi-ju and pointing to 3/5 on the number line on the board with his finger, Teacher Lee repeated Ji-eun’s question both to Mi-ju and the class.

Mi-ju: Among the total of 5 skip-counts there are three skip-counts... [She clarified her ideas.]

Lee: [Restated Mi-ju’s clarification] Because there are three skip-counts in a total of five skip-counts, she said she marked 3/5 here. Any other questions? In-ah? [Called In-ah’s name when she raised her hand.]

In-ah: [Rising from her seat after he called her name] I don’t understand why the ending point of the number line is 5.

Lee: [Repeated In-ah’s question] She asked why you put 5 at the ending point of the number line.

Mi-ju: The fraction is three over five, so if I divided it into five skip-counts the end point should be 5. [Mi-ju justified her method again.]

In-ah’s face appeared to be in confusion because she could not make sense of Mi-ju’s justification.

Lee: [Repeated Mi-ju’s justification] Well, since the fraction is three fifths, if you divide five skip-counts, the ending point of the number line becomes 5, she said. Any others questions? Let’s give them a hand. [Teacher Lee let the three students back to their seats.] Clap once.

Teacher Lee said, “Clap once” in order to make a transition from the phase of the board presentation to the phase of his explanation.

Students: [The students stopped their work quickly and clapped their hands and sat upright.]

Lee: [Pointing Mi-ju’s number line, Teacher Lee asked the class] Does anybody have a different number line from this? [No hands up.] Jun-ho? Is your number line same as this? Is it different, isn’t it?
Teacher Lee called Jun-ho, the most mathematically talented student in his class, based on his observation when he was circulating. Jun-ho nodded his head. Meanwhile, In-ah raised her hand up again. Teacher Lee called on her name.

In-ah: I found something different. I put 1 at the end of the number line, but Mi-ju said she put 5 there.
Lee: Now, what In-ah said was Mi-ju put 5 at the end of the number line but she put 1 there. Let’s think over their argument.

Teacher Lee took over this argument and provided his explanation. Before starting his explanation, he erased 3/5 that Mi-ju wrote on her number line and wrote 3, instead. The number line looked like this now.

![Number Line](image)

Lee: What’s the number at the end point? [Asked to the class]
Students: Five! [In union]
Lee: How many skip-counts are there?
Students: Five! [In unison]
Lee: [Pointing the first skip-count with the thin wooden rod, he asked the class] What is here?
Students: One! [In unison]
Lee: [Wrote 1 on the first skip-count] Then what is here where Jung-ha wrote 3/5? [Pointing the third skip-count, he continued asking]
Students: Three! [In unison]
Lee: This becomes 3, not three over five. [Wrote 3/5 on the third mark] It should be 3, instead three over five. Ok, then what should be here? [Erased 5 at the ending point]
Students: One! [In unison]
Lee: What should be here? [Asked the same question again to the class]
Student: One! [More students joined this time.]
Lee: Class, you have to remember that the ending point of the number line should be 1, neither 5 nor 6. We'll discuss it more later. Now, solve the problems on the page 112 of the textbook and on the page 109 of the workbook. [Wrote the page numbers on the board] Your work should be checked before going home. (Lesson transcript, 7/1)

This episode illustrated the process of classroom discourse. When the presenter finished solving a problem on the board, Teacher Lee had the class clap to get ready for listening to the presentation. If the class was not ready, he reminded them of the norm, “listen carefully.” Asking questions by the class was followed by the presenter’s explanation. Teacher Lee always invited the class to ask questions in order to engage the presenter in developing and eliciting mathematical ideas and understanding. The presenter clarified and defended her method of solution when the class member asked questions.

Teacher Lee’s role in the process of discussion between the presenter and the class was to repeat and make clear to them the questions and answers. In other words, he was a facilitator of classroom discourse. When the class did not recognize the fact that the ending point should be 1, Teacher Lee refrained from telling the rule. Instead, he invited those who might draw different number lines. It was important to him to have as many different methods of solution as possible so that the class could recognize the similarity and difference between them in order to find a concept, procedure, or fact. In this sense, he was an orchestrator of classroom discourse.

If the class did not ask any questions or missed important aspects of concepts, procedures, or facts which were essential to understanding the problem, Teacher Lee took over the role. In this case, his role changed from a facilitator of classroom discourse to active participant of the discourse. The following episode illustrates how he was involved
in a discourse as an active participant. It was the fifth lesson on Unit 7 Time and Length.

Teacher Lee had the class read the problem of the textbook in unison.

“Kang-su is traveling by train. It is 3:20 p.m. now. The train left 50 minutes before. What was the time this train left?”

Sung-don’s solution:

\[
\begin{array}{c}
3 \text{ hours} \ 20 \text{ minutes} \\
- \ 50 \text{ minutes} \\
\hline
2 \text{ hours} \ 30 \text{ minutes}
\end{array}
\]

Lee: Does anybody want to ask a question to him? Min-jung?
Min-jung: [Rising from her seat] How did you subtract 50 minutes from 20 minutes?
Lee: [Repeated the request of Min-jung’s verification] Min-jung asked how you subtracted 50 minutes from 20 minutes.
Sung-don: I borrowed 1 hour from 3 hours, so I added 60 minutes and did subtraction.
Lee: He borrowed 1 hour and then did subtraction. Who else? Nobody? Okay, then I am going to ask him a question? Sung-don, why did you use subtraction to solve this problem?
Sung-don: Because ‘before’ means subtraction and ‘after’ means addition.
Lee: Very good! Let’s give him a hand. [Sung-don grinned.] Sung-don said that even I didn’t know. “Before” means subtraction and “after” means addition. However, does that always work?
Students: Yes. [Several students responded.]
Lee: What do you think about it?
Students: Yes and No.
Lee: We will talk about it later. (Lesson transcript, 6/24)

In this episode, Teacher Lee repeated and made clear Min-jung’s question and Sung-don’s answer. Then when there were no more students to ask questions and he recognized that an important concept of the problem was not questioned, he participated in asking questions to elicit Sung-don’s mathematical thinking. When Sung-don provided a good reason, he was praised by having the class clap. However, Sung-don’s assertion was considered as a hypothesis in his classroom until more agreements, arguments, or
refutations by the class were presented. In this sense, Teacher Lee’s mathematics classroom could be considered to be a mathematical community where mathematical knowledge was developed by the members’ agreement.

Clapping. Teacher Lee used clapping in two different situations. As mentioned earlier, one was used to get the students’ attention for a transition. This type of clapping seemed to be very orderly, in that the students suddenly stopped their work, clapped five times rhythmically, and then sat upright. This kind of clapping was used for the transition between solving problems on the board and class discussion, or between class discussion and summary and highlight (see Figure 3). The other was used for praising the students’ presentation on the board. This type of clapping was similar to applauding. Teacher Lee indicated that regardless of correctness of the answer and methods of solution on the board, he had the class clap to praise the presenter. In this manner, he accepted all assertions on the board as valid in order to encourage many students to participate in presenting their ideas comfortably.

The following episode illustrates how Teacher Lee used clapping to get the students’ attention for instructional transition and praise a presenter as well. Ho-rae finished the problem and began explaining his method of solution.

\[
\begin{align*}
3 \text{ hours } 20 \text{ minutes} & \quad + \quad 40 \text{ minutes} \\
\hline
4 \text{ hours} &
\end{align*}
\]

Lee: Let’s clap once.
Students: [Stopped their work quickly and clapped]
Lee: Let’s listen to Ho-rae’s explanation.
Ho-rae: I added 40 minutes to 3 hours 20 minutes, so I got 4 hours.
Lee: Ho-rae, could you clarify a little bit more how you get 4 hours?

Ho-rae: [Repeated his previous explanation] 3 hours 20 minutes plus 40 minutes...

Lee: Are you saying you got 4 hours by simply adding 3 hours 20 minutes and 40 minutes?

Students: I think I can explain it. [Several students murmured.]

Lee: Okay, good job. Ho-rae. Let’s give him a hand. [The students clapped.] Now, let’s look at how Ho-rae solved it. (Lesson transcript, 6/24)

Teacher Lee had the students clap once to get their attention for Ho-rae’s explanation. Listening carefully was important for the presentation on the board. Teacher Lee asked Ho-rae’s clarification when he just repeated his explanation. However, Teacher Lee had the class clap for him to praise his effort.

The next episode also illustrates how Teacher Lee used clapping to get the students’ attention and praise the presenter. Jun-ho was explaining his method of solution of 4 cm 6 mm + 4 cm 8 mm = 9 cm 4 mm.

Lee: Let’s clap once.

Students: [Quickly stopped their work and clapped]

Lee: You should listen carefully in order to ask good questions.

Jun-ho: 6 millimeters plus 8 millimeters comes to 14 millimeters. Since 10 millimeters is 1 centimeter, I added this 1 centimeter to 8 centimeters. So I got 9 centimeters.

Lee: Is there anyone who wants to ask a question to Jun-ho? [No hands up.] Okay, let’s give him a hand. Good job. Now, let’s listen to Ji-min’s explanation of her method of solution. (Lesson transcript, 6/22)

In this episode, Teacher Lee used clapping for getting the students’ attention and praise as well as reminding them of the listener’s norm.

Through classroom observation, it was apparent that whenever a student solved problems, explained his or her method of solution, provided clarification and justification
on the board, Teacher Lee had the class clap for praise. As he indicated, he accepted
every single idea as valuable. In doing so, he encouraged the students to participate in
classroom discussion. In Teacher Lee’s classroom, clapping was a distinctive way of
praising, getting the students’ attention, and making instructional transition go smoothly.
In some way, it seemed very effective because there was no need to say “sit upright” or to
struggle to get the students’ attention.

Standing-Up Presentation

In addition to the presentations on the board, Teacher Lee consistently called on
his students to present their ideas at any time. Having the students present their ideas was
an essential piece consisting of his teaching practice. He stated that he wanted his
students to present their ideas clearly in front of people and to engage in discussions
confidently. It was important to him that the students stand up and speak their thoughts
regardless of correctness and that they participate in class discussion in any way.

Our Koreans are not good at presenting ideas, opinions or thoughts in
front of people although we do well in speaking when we are seated.
Don’t murmur when you are sitting. If you want to say something to the
class, present your ideas clearly by rising from your seat. (Lesson
transcript, 6/24)

He gave this short lecture in the middle of a lesson because he thought that
standing-up presentation in his mathematics class is a way of making students’
ideas public. The students’ ideas had to be formally recognized in order to be
discussed.
This “standing-up presentation” required careful work by Teacher Lee. In other words, he needed to ask each student or the class valuable questions that elicit, engage, and challenge each student’s thinking. He explained some difficulties and concerns about this matter:

I am always aware that I should ask more open-ended questions that elicit the students’ thinking, instead of “yes” or “no” questions. I am trying to form the habit of thinking mathematically by asking good questions. I still struggle to improve my questioning skills, but it’s hard to provide these kinds of questions that engage each student’s thinking in math class. (Interview, 8/27)

In Teacher Lee’s classroom, including the mathematics class, the students would stand up to present their ideas. To say something when they were sitting was not considered to be an appropriate manner. Teacher Lee stated that he was trying to provide his students with as many opportunities for presenting their ideas in the class as possible so that they were capable of discussing the ideas with others in the future. In fact, it was one of the norms of presentation to stand up. Teacher Lee repeated what a presenter said by careful listening. Repeating the presenter’s answers, ideas, and methods of solution was an official procedure of making their mathematical assertions public. The following episode illustrates how Teacher Lee reminded the students of the norm.

Lee: Now then, let’s look at the problems on the page 96. [All the students looked at the problems] Su-jong?
Su-jong: [Showed little bit being surprised to hear his name] Yeh?
Lee: I think Su-jong has studied hard recently. What about 2600 meters?
Su-jong: 2 kilometers and 600 meters. [Responded at his seat. In-ah sitting next to him tapped his shoulder to tell him that he should stand up to present his answer]
Lee: You should stand up to present your answer.
Su-jong: [Rising from his seat and answered again] 2 kilometers and 600 meters.
Lee: What do the rest of you think about Su-jong’s answer?
Students: It’s correct. [In unison]
Lee: I think so. (Lesson transcript, 6/21)

The students, like In-ah, knew the norm, that they had to stand up to present their ideas, when being called by Teacher Lee. He utilized this incidence to remind the class of the norm for stand-up presentation. Noticeably Teacher Lee usually let the class decide whether the presenter’s answer was reasonable or not. The class had to make sense of the presenter’s answer and reasoning. After that he provided his feedback to the presenter.

Another important norm was that when the students raised their hands to present ideas and were called on, they had to provide not only an answer but reasoning. To do this, he asked how and why questions so that the class could share ideas and understanding. The following episode illustrates how this norm was established in his mathematics class.

Lee: According to what you had learned in the second grade, I am going to divide this number line. [Drew and divided a number line] Now, this number line contains 0 as the starting point and 10 as the ending point. What is this point? [Pointed at the middle]
Students: Me! [Called out several students]
Lee: Jun-ho?
Jun-ho: [Rising from his seat] It is 5.
Lee: Why do you think it is 5?
Jun-ho: If we divide 10 into two, the middle should be 5.
Lee: Is that so? What does anyone else think? (Lesson transcript, 7/1)
Teacher Lee asked Jun-ho to clarify his reasoning about why the answer was 5. Teacher Lee felt the requirement that the presenter provide justification or clarification to the “how” and “why” question encouraged them to develop understanding.

The following episode illustrates two norms of the stand-up presentation. The first one was that the students should raise their hands to present their ideas or answers, and the second was that they had to provide their reasoning by Teacher Lee’s “how” and “why” question. The lesson was about estimating time to read children’s books. Teacher Lee wrote the following on the board.

<table>
<thead>
<tr>
<th>Pages</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 pages</td>
<td>1 minute and 20 seconds</td>
</tr>
<tr>
<td>10 pages</td>
<td>2 minutes and 40 seconds</td>
</tr>
</tbody>
</table>

Lee: If you read 10 pages each time, how many times do you have to read to finish this 125-page book?

Students: 12 times. Or 13 times. Or about 13 times. [Called out in disagreement]

Lee: Okay, let’s raise your hand to present your answer. [About 10 students raised their hands.] Ji-hyun?

Ji-hyun: [Rising from her seat] I think it’s 13 times.

Lee: Why do you think it’s 13 times? [She could not explain her reasoning] Han-jin?

Han-jin: It has to be 12 times. 12 multiply 10 equals 120 pages and the rest 5 pages cannot be counted because it’s not 10 pages. So, 12 times are needed to finish reading it.

Lee: Who else? [Several students still raised their hands.] In-ah?

In-ah: [Rising from her seat] We have to read 12 times and 5 pages more.

Lee: 12 times and 5 pages more. Why do you think that?

In-ha: Because 100 has 10 tens and 20 has 2 tens, that’s 12 tens. But we still did not read 5 pages, so I think we have to read 12 times and 5 pages more. (Lesson transcript, 7/15)

The students in Teacher Lee’s classroom liked presenting their ideas, but sometimes they did not follow discourse norms. In this episode, when the students called out, Teacher Lee reminded them of raising their hands to make their ideas public. When Ji-hyun
presented her answer, Teacher Lee asked for her reasoning instead of providing feedback for her answer. Like Han-jin, some students were aware that they had to present their answer as well as reasoning. Whenever his students provided their answers or ideas, Teacher Lee’s “how” and “why” questions were followed. Through the process of establishing the norms, the students could perceive that mathematical assertions had to be maintained by their reasoning, not by answers.

It happened that Teacher Lee demonstrated his authority to force the students to present their ideas. When Teacher Lee thought that a question or task he posed was easy and expected many students’ participation in discussion, but the students did not, or when the students were not paying attention to the lesson, he used his authority to present their ideas. In such case, he called students randomly.

The following episode illustrates how Teacher Lee called on nonvolunteers to present their ideas. Teacher Lee noticed that some of his students were not good at estimating length by using different metric units such as cm and mm and decided to provide more practice. In addition, because of the nearness of the summer vacation and hot weather, the students’ attention spans were getting short.

Lee: I think some of you still are not good at dealing with the estimation of length. What is this? [Holding the math workbook]

Students: Workbook. [In unison]

Lee: Some of you are confused with centimeters and millimeters. What would be the length of this workbook?

Students: 18 centimeters. 23 centimeters. 20 millimeters. [Students called out.]

Lee: Jie-eun? [Suddenly called her on] What do you think is the width of this workbook?

Jie-eun: [Rising from her seat] 29... [In low voice as if she got nervous.]

Lee: What’s the unit?
In this episode, the students who were good at estimating the length and thickness of the workbook using an appropriate metric unit were eager to present their ideas. On the contrary, the students who were not good at mathematics and whom Teacher Lee expected to raise their hands, did not raise their hands. Because the question was easy to answer for everyone in the class, he intentionally decided to randomly call on the students who were not willing to present ideas to provide opportunities to give their ideas in the class.

Despite all his effort to foster his students' mathematical understanding through discourse, he confessed to the difficulties of establishing discourse norms in the third grade.
I have emphasized how they must present and listen since the beginning of this school year. But it doesn’t look successful. The students know that they are supposed to present their ideas and listen to the presentation. Sometimes it works and sometimes it doesn’t. If my students stick to these rules by just my telling, they would not be third graders. That’s why they need education that forms their habits of mind for learning. (Conversation, 7/8)

Teaching mathematics through discourse was Teacher Lee’s goal that he attempted to achieve in his mathematics classroom with his students. According to him, it was not a simple task to establish a discourse-oriented mathematics classroom. It necessitated his consistent effort to form the norms for discourse.

Students’ Responses to Discourse

The students’ perceptions of solving and explaining their methods of solution, ideas, and ways of thinking were varied. The following responses through interviews showed their feelings.

In-ah: I had an experience and I got a wrong answer on the board. I felt my legs were trembling. It was awful.
Ji-min: I am anxious not to make a mistake on the board because such mistakes might be recorded. (Interview, 7/6)
Ju-hyun: I know I am not good at mathematics, so I am worried about getting a wrong answer when I am at the board.
Tae-min: I can hear my heart is beating when I solve a problem I don’t quite understand.
Jung-ha: I feel nervous when I solve hard problems but I like solving problems on the board that I know. (Interview, 7/9)
Han-jin: I really like solving math problems on the board. I can explain everything I know and the class will clap for me when I get a correct answer or explain my method of solution well.
Min-jung: When I get a correct answer, the class claps for me. I am really happy because I feel like I am a singer on the stage. (Interview, 7/13)

According to these students’ responses, most of them were concerned and anxious about getting an incorrect answer even though Teacher Lee tried to establish a mistake free discourse environment that making mistakes was absolutely fine in his mathematics class. Through classroom observations, however, Teacher Lee worked hard to reduce such anxiety by calling on volunteers and providing a task or problem on the board based on their academic abilities.

Two mothers indicated that their children’s abilities of presentation were improved since the third grade.

Mother 1: He said he really likes presentation. He didn’t raise his hands in the first and second grade because the teacher barely called on him. So, he couldn’t have much experience in presenting. But he said Teacher Lee calls on everybody and he is happy to present his ideas.

Mother 2: He is quite a shy boy and did not present his thinking in the past grades. But since the third grade, he had many experiences and now he said he doesn’t feel shy when he presents his idea. (Interview, 7/7)

Perhaps only two mother’s ideas might not be sufficient information to verify the extent of the students’ improvement of presentation and communication skills in mathematics classroom. But it could be verified that these two boys had many chances of presentation and became confident of presenting.
According to the interviews with some of the students, the students were well aware of the norms of discourse. The students easily identified the norms and feelings when they were called on.

Eun-ho: If there are not many hands up, sun-sang-nim usually calls on anybody in the class. (Interview, 7/2)

In-ah: I completely lost my words when sun-sang-nim called on me all of a sudden.

Su-jung: Presenting my idea is comfortable because sun-sang-nim doesn’t mind if we make mistakes. But recently he asked us to present in more detail.

In-ah: However, when sun-sang-nim says something is wrong in my presentation my face turns red. (Interview, 7/6)

Jung-ha: Say clearly… I can hear my heart pounding when he calls on our names randomly.

Tae-min: We have to speak loudly. (Interview, 7/9)

Yun-ha: Sun-sang-nim told us that we have to speak loud and clear and it is acceptable to make mistakes. (Interview, 7/13)

As the students’ responses showed, they clearly recognized discourse norms and Teacher Lee’s expectations. Although Teacher Lee worked hard to establish a mistake free environment for his discourse-oriented mathematics teaching and his students knew the norms and expectations, the students still were uncomfortable when they made mistakes in front of the class. They appeared to be more concerned about getting embarrassed by classmates, not by Teacher Lee.

Group Learning

Although Teacher Lee’s major method of teaching mathematics was discourse, the discourse meant an interaction between him and the whole class, not among the
students. As mentioned earlier, he acknowledged that the students needed to communicate with each other to learn mathematics. In addition, the students’ desks were arranged in group settings. However, what he meant was that the students’ interaction was in the whole class, not in groups. In this sense, his major pedagogy of mathematics could be referred to as “teacher guided.” This conception of group learning was influenced by his experience as a classroom teacher.

He indicated that for years, he tried several different methods to help his students’ mathematics learning. He observed each group in order to check the students’ understanding but it was enormous work during the limited class hour. Because he was only able to check the work of less than 10 students for one class, he had to abandon it. Next, he knew that the high-ability students understood the concepts and procedures of a lesson and solved problems quickly. He stated that the composition of the group in his classroom was heterogeneous in which one student from the high-ability group was a group leader, two or three students came from the middle-ability group, and another two or three students came from the lower-ability group. His purpose with this group arrangement was that he wanted the high-ability students to teach the low-ability students so that both of the students learned from each other. He checked the eight group leaders first and then had them teach their group members. He explained his intention about this:

The perspective of a learner is very different from that of a teacher. While a learner might understand the contents vaguely, the teacher, the high-ability student must understand them clearly. So, in order to provide teaching experience with the high-ability students, I had them teach the group members. In addition, I had them make problems for themselves and teach the group members. (Interview, 7/26)
The method of having the high-ability students teach their group members appeared to come from his tutoring experience. As mentioned earlier, he indicated that the tutoring experience made him realize the importance of understanding mathematics when acting as a teacher. He intended to let the high-ability students have the same experience he had. Moreover, he used the problem posing activity that the high-ability students made mathematical problems for themselves to teach the group members. The benefit of this activity was to provide the students with a deeper understanding of concepts, procedures, rules, or algorithms (Brown & Walter, 1990). Regardless of such benefit, he decided to abandon this mutual learning method.

I did not release the group unless the members had mastered the problems related to the days’ lesson because the students should at least know what they learned. It was good for each group leader to teach the members eagerly, but a serious problem was that the leaders treated the low-ability students badly. As a consequence, the low-ability students became passive and tended not to present their ideas. I didn’t anticipate this problem. (Conversation, 6/8)

I did a trial and error approach to resolve the problems related to group learning. I knew there were advantages and disadvantages. The most disadvantage was that the high-ability students often said to the low-ability students, “How come you don’t get such an easy thing?” or “Why is it so hard to understand this?” Although the low-ability students were performing better, that was not my intent for group learning. (Interview, 7/26)

In addition to the passivity of the low-ability students, there was another reason for not using group learning. Teacher Lee believed that the groups in mathematics classroom worked together only for sharing materials or for playing games, not for helping their mathematics learning. Obviously, the group settings, especially in mathematics class, were not associated with learning because the ideas to solve
In science, the students have to work together in a group in order to conduct experiments. Likewise, in social science, there are many topics that the students discuss together. I think the students' ability does not really matter in these subjects. But, except for games or manipulative activities in sharing materials, I don't think there are many topics in mathematics for the students to work together as a group. Especially, in math class, most answers come from the high-ability students and the low-ability students tend to simply copy the answers. Although I give a problem and have them work in a group, the low-ability students are usually passive. (Interview, 8/7)

Teacher Lee indicated that group discussion would be impossible in mathematics class because unlike other subjects such as science or social science, solving mathematics problems considerably depended upon both the students' ability and the level of problems.

In science or social science class, the students can work together in a group and draw conclusions based on their findings or observations. But in mathematics class, I think it would be very difficult... I think solving simple addition or multiplication problems by group discussion would not be useful because the students already know the answers. Similarly, many students already know the answers of the problems in the textbook. It is not necessary for group discussion to find the answers. So, the mathematics problems for encouraging group discussion should not be the same problems in the textbook nor simple problems. Rather, the problems could be difficult problems. However, if the problems are difficult, there is no group discussion because the low-ability students can not understand or solve them. They just accept the high-ability students' answers and ideas... From my experience, the high-ability students do not need group discussion because they already know and can solve most of the problems in the math textbook. On the other hand, the low-ability students never understand the problems anyway. I think group learning is usually good for the students who are in the middle-ability group. (Interview, 8/28)
According to this quote, Teacher Lee had difficulty resolving practical issues related to group learning in mathematics classes. The issues were mostly related to the dominance of the high-ability students, the passivity of the low-ability students, and the appropriateness of mathematical problems and tasks. As mentioned earlier, Teacher Lee believed that understanding mathematics was decidedly a personal matter and each student should construct and develop his or her own understanding. As a consequence, without resolving these issues, he adopted alternative ways of encouraging the students’ mathematical communication, that is, on-board and stand-up presentation guided by him.

It was obvious that the composition of groups had nothing to do with mathematics learning except for sharing materials for manipulative activities or playing games. Teacher Lee’s intention of using group settings in his classroom was clearly demonstrated in conversation with three female beginning teachers.

Teacher 1: The arrangement of my classroom used to be groups, but now I have them in pairs. My students’ sitting postures were becoming bad when I put them in groups. Also, they made too much noise and quarreled.

Teacher 2: [To Teacher Lee] Do you think group settings are beneficial for the students’ learning?

Lee: I have an important reason for doing this. From my teaching experience, the students are so selfish and seem not to consider others. I don’t like it. So, I want my students to help each other, consider others, and learn cooperative spirit through participating in a group. This is much more important than any other for the students’ education. Of course, they make much noise and sometimes have serious arguments, or fight with each other. However, despite these problems, I think they would learn something each other in a group.

Teacher 3: I told my students that they had to help each other in a group. But the high-ability students didn’t care about others. They just tried to finish their work. They were so selfish.

(Fieldnotess, 6/22)
The beginning teachers struggled with classroom management and the high-ability students. Because of the struggling, they indicated that they gave up group settings and changed to pairs. Teacher Lee’s priority was first social development, then learning. He was more concerned that his students became valuable persons for others in a society. His concern also was well demonstrated in his foreword for the students’ collection of essays that his sixth-grade students made for celebrating their graduation two years before.

Dear my six-grade students,
Thinking back over the last year with you all, I regret that I scolded, punished, and was angry with you. Please forgive me and I hope you do not take it personally. I want to tell you two things. First, love each other. You cannot live alone in the society or the world. You have to live together in our community. Please don’t be too selfish, but think of your friends and neighbors and love them. Second, do your best. Always do your best whatever your duties are and put great effort into improving your self. (Students’ collection of essays in December, 1997)

Through interviews with the students, the students indicated that they did not have group learning in a mathematics class except for playing games.

Jun-ho: Except for playing games, we do not study in a group in mathematics class.
Sung-don: Uh... I think barely but sometimes sun-sang-nim had the group leaders check the members’ work in mathematics class.
(Interview, 7/2)

As Teacher Lee mentioned about the passivity of the low-ability students and the dominance of the high-ability students, these students obviously did not prefer to work with the low-ability students in the mathematics classroom. The reason was that the low-ability students only tried to copy the answers from the high-ability students and were not being helpful members in group learning.
Jun-ho: I don’t like Jung-ha and Su-jong in my group. I won’t help them to learn how to solve math problems. But they keep asking me the answers.

Eun-ho: Most classmates don’t like to study mathematics with the students in the low-ability group.

Sung-don: All my friends in my group don’t want to work with Ji-su because she is the worst in math so she is always trying to copy our answers. (Interview, 7/2)

In-ah: I hate studying with the low-ability boys in my group because they always copy my answers. (Interview, 7/6)

Tae-min: I don’t like studying in a group because the members in my group are not good at mathematics so they are not helpful to me. I can solve most of the math problems for myself. (Interview, 7/9)

However, when a group member was helpful they liked working together in a group.

Han-jin: When I was solving a math problem on a worksheet, I was really confused. Ha-sun was trying to solve the same problem too. She asked me several questions and her questions gave me a hint, that was cool. So, the next time I helped her when she couldn’t solve math problems. I like solving math problems with her. She is helpful. (Interview, 7/13)

Han-jin was one of the high-ability boys in mathematics and Ha-sun in the middle-low ability group. Ha-sun was trying to solve the problem for herself unlike other low students who simply wanted to copy the answer. When she had difficulty, she questioned Han-jin and that helped him get a hint. He appreciated her help and perceived her as a helpful member in a group.

Unlike the students in the high-ability group mentioned above, most of the students participating in interviews liked working together in groups because they could help and better understand each other.
Sung-don: I like studying in a group because we can help each other, but there are too many noises, not paying attention to Sun-sang-nim.

Ho-rae: Too noisy when we study in a group. (Interview, 7/2)

Ji-min: I like studying in a group because we can get along with friends, talk to each other, and ask for their help when I don’t know.

Su-hee: I can ask for friends’ help in my group when I can’t solve math problems. (Interview, 7/6)

Su-jong: I can finish solving math problems quickly. That’s why I like studying in a group.

Jung-ha: When we have to discuss something in social science class, it is difficult to think of good ideas by myself. But if we work together as a group, we can think of many good ideas and conclusions. (Interview, 7/9)

Ki-su: I don’t like studying in a group when there are quarrels in my group. But most of time I like it because we can play and help each other.

Yun-ha: I think studying together in a group is better than studying alone because we can understand each other. (Interview, 7/13)

For them, group learning in mathematics class was only related to obtaining a correct answer quickly. When they talked about group learning, they did not often mention mathematics. It was apparent that the conflict and discord among group members greatly hampered the students’ group learning even though they liked to work together in groups. As a consequence, some students indicated that they wanted to study together with their close friends in a group.

Teacher Lee’s beliefs about group learning clearly demonstrated that his major way of classroom discourse was not group learning, but the whole class discussion. Although the reasons were the passivity of the low-ability students, the dominance of the high-ability students, and the difficulty of monitoring the students’ understanding in a group, perhaps his strong sense of responsibility as a classroom teacher was responsible
for the shift from the group learning to the whole class discussion. He believed that his responsibility was to develop the students' mathematical understanding of concepts and procedures. Thus, through group learning in mathematics class, he seemed to expect that the students would not develop solid mathematical understanding. Moreover, as mentioned earlier, he believed that students should develop their own individual understanding and ways of thinking. In other words, for him, learning mathematics was a purely personal matter. Accordingly, simply accepting others' understanding in a group would not be considered an appropriate way of doing mathematics in his classroom.

Summary

One of the themes that represents Teacher Lee's teaching practice of mathematics was "discourse-oriented." Through discourse, he fostered the development of the students' understanding of mathematics and established classroom interactions and social-mathematical norms. He believed that students should communicate their methods of solution and ways of thinking in order to share with the class. In addition, for him, participating in discussion, responding to a question posed by him meant that the students were paying attention to a lesson and actively engaged in learning activities. In order to implement discourse-oriented mathematics teaching, he emphasized the support of home and parents.

His belief about discourse-oriented mathematics teaching came from his middle school experience. This experience did not provide him with sufficient knowledge to implement his beliefs about discourse. It was not possible until he joined the study groups where he learned the techniques of open-ended questioning in mathematics. For this
teaching practice, he used two different methods of presentation: On-Board Presentation and Standing-Up Presentation.

There were common interaction patterns and classroom norms for the presentations. These patterns and norms created a learning environment that fostered the development of students’ understanding of mathematics. Teacher Lee had to have the students feel comfortable when they made mistakes so that they were willing to present and communicate their ideas, ways of understanding, justifications, and clarifications. He never evaluated the students’ ideas as correct or incorrect. Instead, he encouraged them to find an alternative way of understanding. Sometimes he had the class find the flaws of the presenter’s idea and helped them think in a different perspective. He engaged more actively in one-to-one discourse with a presenter when the student made a mistake. In addition, he utilized the students’ previous knowledge when a student presented incorrect reasoning.

Teacher Lee respected every student’s idea and way of thinking presented in his mathematics lesson. He conveyed his respect by using probing questions for eliciting students’ thinking, by showing his interest in understanding the students’ ideas and approaches, by mentioning presenters’ names and ways of thinking, and by providing equal opportunities for presentation. To do this, he asked how and why questions at any time when he decided to pursue in depth from among the ideas that his students brought up during discourse.

Teacher Lee asked “how” and “why” questions every moment in his mathematics lesson. He wanted to listen to the students’ ideas for redirecting his teaching and selected the best appropriate tasks for his students. For him, how and why questions were a major
method of gathering information about the students’ understanding. For the students, how and why questions provided considerable opportunities to reflect their thinking, elicit and elaborate their ideas and understanding, and improve their mathematical communication skills by clarifying and defending their presented ideas.

“Enough loudly and clearly” was a norm expected of presenters. Presenters had a responsibility and an obligation of speaking loudly enough so that Teacher Lee and the class clearly understood the presenters’ ideas, the methods of solutions, and ways of thinking. In addition, this norm was important in this classroom because he and the class could make sense of what was presented and ask questions to help the presenters develop understanding of contents of the presentation.

“Listen carefully and then ask questions” was a norm for listeners, including Teacher Lee. He and the class had a responsibility and an obligation of listening carefully in order to understand and ask questions to help the presenters elaborate their thinking and reasoning. Another important reason for listening carefully was to present ideas, reasoning, or justification as listeners so that the students could provide detailed, better developed and different ideas.

Teacher Lee called on students to have them solve mathematics problems on the board. He stated that his students would have one more opportunity to reflect upon their understanding and mistakes of the problems through the process of presentation. Having the students solve problems on the board was beneficial to him for checking their understanding and redirecting his teaching. This activity was an ongoing process of assessment.
Although Teacher Lee acknowledged the necessity and benefit of discourse, the discourse meant an interaction between him and the whole class, not among the students. He believed that the group in the mathematics classroom worked together only for sharing materials or for playing games, not for helping their mathematics learning. He indicated that group discussion would be impossible in mathematics class because solving mathematics problems, unlike social science and science, considerably depended upon both the students' ability and the level of problems. The issues related to group learning that he was concerned about were the dominance of the high-ability students, the passivity of the low-ability students, and the appropriateness of mathematical problems and tasks for group learning in mathematics. He emphasized social development over learning in group discussion. He was more concerned that his students became valuable persons for others in society. Through group learning in mathematics class, he might expect that the students would not develop solid mathematical understanding. Moreover, as mentioned earlier, he believed that each student should develop his or her own understanding and ways of thinking with a greater endeavor. In other words, for him, learning mathematics was purely a personal matter. Accordingly, simply accepting others' understanding in a group would not be considered an appropriate way of doing mathematics in his classroom.

Mathematical Tasks for Understanding and Discourse

Teacher Lee was responsible for the quality of the mathematics tasks in which his students engaged. He chose and developed the tasks that were likely to promote the
development of his students' understanding of concepts and procedures, to foster their abilities to solve problems, and to reason and communicate mathematically. Based on his beliefs about the development of mathematical understanding and discourse, he usually employed two different types of tasks: open-ended tasks and tasks from students' mistakes and comments during discourse. The first type of mathematical tasks were apparently based on his beliefs. The second type was identified by classroom observations as a pattern of his teaching practice of mathematics. However, Teacher Lee did not always followed the sequence of the mathematics textbook. He did follow the sequence, but frequently modified the problems and tasks in the textbook for understanding and discourse. In fact, although he viewed the mathematics in the textbook as the final destination of his teaching mathematics, he considered the mathematics laid out in the textbook as a guideline, not an authoritative rule.

My final goal of teaching mathematics is to teach the contents of the textbook because it shows the final conclusions. For example, in the case of division, the final goal is to teach the students to apply the division algorithm which is in the textbook by solving problems efficiently. However, I am trying to teach mathematics by different ways from the textbook. At least, I want my students to perceive there are different ways of doing mathematics. (Interview, 8/7)

In the following sections, two types of mathematical tasks are discussed in more detail with classroom episodes and Teacher Lee's beliefs.

Open-Ended Tasks

Teacher Lee strove to foster the ability of his students' mathematical thinking. For this purpose, he consistently had his students solve tangram problems in the morning self-
learning session before school started. Tangram activities were one way of forming his students' mental habits for mathematical thinking. On the other hand, in his teaching practice, he believed that his role was to show his students that many different ways of solving a mathematics problem existed. He emphasized the change of the students' perceptions about mathematics.

I think there are not only many different ways of methods of solution in mathematics, but many different ways of representing an answer. So, I don't think it is good teaching of mathematics to lead students to the teacher's own way. That's why students feel that mathematics is difficult and a nonsense subject in school... Most of the students tend to think that only one answer exists in mathematics and the answer should be correct. I think this tendency of mathematical practice is the fault of the teachers, who did not show them different ways of solutions and answers in mathematics. (Interview, 8/12)

Teacher Lee, in this quote, articulated that his role and responsibility was to show different methods of solution and different ways of representing an answer in his mathematics classroom. Because this belief was drawn from his primary belief, the students' own ways of understanding, it was considered to be a derivative belief (Green, 1971). To implement the belief about his role and responsibility, he utilized the open-ended tasks or approaches in his mathematics classroom.

Teacher Lee's belief about different ways of doing mathematics was demonstrated in his teaching and assessing mathematics. The following test items reflected his belief. Teacher Lee was in charge of making problems for this test. The purpose of this test was to gather information about mathematics understanding of the third graders and this information would be used for assigning them to different classrooms for the next year.
Problem 18-20: Read the following problem and answer the questions.

Ho-jun bought 10 stamps. How much did he have to pay for the stamps?

18. Do you think this problem can be solved? What is your reason? If it cannot be solved, what is your reason?

19-20. Make appropriate conditions so that you can solve this problem.

Condition:
Fact:
Answer:

(From Achievement Test, the first semester of 1999)

Less than half of the third graders were able to solve the problem 18-20. His intention was to assess the students' understanding of the problem. He stated that it was important for the students to make up mathematics problems for themselves and change conditions of the problems. In doing so, he made the problems open-ended so that the students were able to control and approach it in different ways rather than to follow the path that he identified.

Another test example of Teacher Lee's belief about different ways of doing mathematics was also found in the test items of performance assessment. In this problem, Teacher Lee made the problem open-ended by using a set of number cards.

By using a set of number cards as below, answer the problems.

0 1 2 3 4 5 6 7 8 9

1. Using the number cards, make a division problem of (two digits) ÷ (one digit) having zero quotient and solve it.

2. Using the number cards, make a division problem of (two digits) ÷ (one digit) having no zero quotient and solve it. (From Performance Assessment Test, the first semester of 1999)
Teacher Lee discussed these assessment items with the three beginning teachers. One of them, Teacher Song, claimed that many teachers were too dependent upon the problems and tasks in mathematics textbook to teach mathematics.

Song: I think that many teachers tend to force students to think mathematics conventionally. Teachers are introducing the problems and methods of solution from the textbook.

Lee: I agree with you so I do not give such problems to my students on the performance assessment. I just let them make and solve multiplication and division problems for themselves by using number cards. It is important for teachers to provide such problems and tasks which have several methods of solution or answers. I think these are open-ended problems and tasks. And also it is important to let them modify the conditions of the problems so that they can feel that the problems are theirs, not being posed by us... I really don’t like textbook style problems. For example, if a problem is about finding an area of a rectangle, the width and height are always provided. I think it would be a better way to let students decide the lengths. According to the lesson I observed, that was ranked at the first place in the research lesson contest, when the teacher asked the students to find the area of polygons, he didn’t give the length of each side. Rather, he asked them which side of length they had to know to find the area. He had the students write any numbers for the sides of length and solve them. (Conversation, 6/22)

In fact, Teacher Lee frequently had the students generate mathematical problems using number cards. In doing so, he wanted the students to understand that mathematics problems and answers were not given by the authoritative teacher and textbook. Rather, he wanted his students to take over the authority of doing mathematics. On the other hand, he observed a similar lesson with what he envisioned good mathematics teaching should be. This kind of experience consolidated his open-ended approach.
Teacher Lee consistently emphasized open-ended tasks and approaches in his mathematics teaching. It should be noted that these open-ended tasks and approaches were greatly associated with his teaching practices of discourse-oriented and conceptual development. The following episode was observed in his mathematics teaching of the second grade. A female classroom teacher was sick and missed that day. It was the fourth period when Teacher Lee’s students had an English lesson so he was available to teach this period. There is no substitute teacher system in Korea. If a teacher misses a day for some reason such as sickness or all day in-service training program, other teachers in the school are supposed to teach the class. He brought several stacks of number cards.

Lee: I teach the third graders but I am going to teach math at this period for you.

Teacher Lee had them make a group of four or five and nine groups were formed. He wanted to know these second graders’ knowledge of mathematics. So he wrote 1, 2, 3, 4, 5, 6, 7, 8, 9 on the board.

Lee: Now, I am going to ask a question. What is the sum if you add all of them?

Within less than 20 seconds, several students called out “45!” Teacher Lee asked a student to present his idea.

Student 1: I tried to make 10s first and then find the rest.
Lee: What about 56? [Wrote 56 on the board]
Student 1: It’s not 56. I made tens by adding two numbers and got four tens. There were 5 left. So I got 45.
Teacher Lee showed a little bit of surprise at these second graders. So he decided to put more challenging problems on the board. He wrote \(11 + 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19\) on the board.

Lee: What is the sum if we add all of them?

Of course, there were hands up again within 20 seconds. Teacher Lee called on the students who raised their hands.

Student 2: There are nine tens so it’s 90. Like what we did just before, the sum of 1 to 9 was 45. So I got 135.

Student 3: First I separated tens and 1s. So I got nine tens and then I added the rest, it is 45. So I got 135.

After the students’ presentation, Teacher Lee handed a set of number cards containing 1 to 9.

Lee: Now, each group should have 1 to 9. Using these cards, you are going to make the number I write on the board. If your group makes it, bring the cards to the front quickly. It’s a game. Okay, the first number is 21. (Lesson transcript, 6/22)

Soon five students from each group lined up in front of the classroom. The first group made 21 with 1, 2, 3, 7, 8 and Teacher Lee wrote the numbers. Another group made 21 with 1, 4, 7, 9. The next number he put on the board was 37. One group made it with 3, 4, 6, 7, 8, 9 and the students in this group said, “Yeah, we are first.” Another group made 37 with 1, 4, 5, 7, 8, 9 but the sum was 34. Teacher Lee put a check mark for any group that got an correct combination. The next number was 23. One group made it with 1, 3, 4, 6, 9 and another group with 1, 2, 3, 8, 9. Both groups made 23 with 5 cards but the third group
made it with 3 cards, 6, 8, 9. Teacher Lee commented that there were many different ways of making a combination for a number.

Teacher Lee had no experience with the first and second graders in his 10 years of teaching experience. Once he mentioned that he wanted to teach them because he had no knowledge about their learning. In this lesson, his beliefs about the teaching and learning of mathematics were demonstrated. First, he used a set of number cards to generate open-ended problems. Second, when the second graders presented their ideas, he asked for their reasoning by probing questions. Third, the problems were not in the textbook, but he modified them in order to motivate the students' interests by using a game. It confirmed that he consistently implemented his belief about open-ended tasks and approaches and his teaching practices were evidently stable across grades.

Teacher Lee's open-ended tasks and approaches did not always go well. He tried to avoid following the sequence of the textbook because about 15 students in his class attended the learning center and already knew the answers to the problems and the methods of solutions. To implement his belief about showing the existence of different ways of doing mathematics was sometimes an instructional disaster. The following episode illustrates his frustration with his open-ended approach. From his experience with the third graders last year, he knew that the textbook approach was not enough to teach the writing of fractions on the number line. At the end of the lesson, he asked the students to write $\frac{3}{7}$ on the number line. He explained:

Last year I gave the students various number lines with 5, 7, 9, and so on as the ending points. And like the textbook I divided the number of skip-counts corresponding to the ending numbers. For example, if the ending number is 7, the number line has 7 skip-counts. Then I asked the students to write the fractions like $\frac{3}{7}$, $\frac{2}{7}$, $\frac{5}{7}$ and they all got it correct. So I gave
them a number line without the starting and ending point nor skip-counts and then asked to write 3/8. Can you imagine what happened? Only one or two students barely got it correct. That was a horrible day. (Conversation, 7/1)

Thus, he decided to introduce the lesson this year with the students’ previous knowledge of the number line which they learned in the first and second grades. Teacher Lee began this lesson with drawing four number lines with 10, 100, 5, and 1 as the ending points. The class explored and discussed why the number lines had different skip-counts. Some students suggested that the skip-counts (e.g., 5s, 20s, 1s) were dependent on how many segments they divided the number lines. Then Teacher Lee called on two students to draw any number line they wanted. The class discussed these number lines too. Teacher Lee finally asked the class whether the last number line with 0 and 1 as the starting and ending points, respectively, could be divided. Teacher Lee called on another two students to divide the number line.

Lee: Let’s look at them. In how many pieces did Eun-ho divide between 0 and 1?
Class: Two! [In unison]
Lee: What is this? [Pointing the middle point]
Class: One! [In unison]
Lee: So what is it in fraction?
Class: One half! [In unison]
Lee: Let’s look at the number line of Tae-min. In how many pieces did he divide between 0 and 1?
Class: Ten! [In unison]
Lee: What is it in fraction? [Pointing the first skip-count]
Class: One tenth! [In unison]
Lee: What about here? [Pointing the fourth skip-count]
Class: Four tenths! [In unison]
Lee: What about here? [Pointing the seventh skip-count]
Class: Seven tenths! [In unison]
Teacher Lee and the class studied two more number lines that represented \( \frac{3}{4} \) and \( \frac{7}{7} \).

Then he wrote three fractions, \( \frac{3}{5} \), \( \frac{2}{6} \), \( \frac{7}{8} \), on the board and called on three students, Mi-ju, Ju-ri, and U-jung who were in the middle-ability group. These three students' number lines looked like below.

Teacher Lee asked them to present their ideas and class discussion followed. (Lesson transcript, 7/1)

Teacher Lee thought that the students were ready to construct the fractions, \( \frac{3}{5} \), \( \frac{2}{6} \), and \( \frac{7}{8} \) on number lines. For him, writing these fractions was considered to be open-ended tasks because there were no starting and ending points given. All similar problems in the textbook and workbook provided the number lines with 0 and 1 as a starting and ending point and the number of skip-counts corresponding exactly with a fraction. These problems only required the location of certain fractions on the number lines. The mathematics textbook provides some problems as follows.
Write the following fractions on the number line.
(1/4, 2/4, 3/4, 4/4)

Teacher Lee’s experience was that these problems did not contribute to the third graders’ grasp of why they needed rational numbers to express the segments between 0 and 1.

During the class discussion, only two students suggested that the ending points should be 1. After this class, Teacher Lee was very frustrated about this lesson.

Was there something wrong with my explanation? I taught this concept last year and followed the textbook. But at the end of the lesson when I asked the same questions as today’s they made exactly the same mistakes. Most of them put the denominator of a fraction as the ending point of the number line. So this time I thought I almost got them but it just slipped away again. These kids are really good at writing fractions if the number lines are given. But they cannot draw a number line to represent a fraction. What else can I try? What a day! (Conversation, 7/1)

Teacher Lee did have some good days with his open-ended tasks and approaches. When he decided to give these third graders Pascal’s Triangle, it appeared to be a difficult task for them. His decision was based on his ongoing assessment through discourse. It was the Unit 9, Problem Solving chapter. The lesson was to find patterns from sequences. He put the following sequences on the board and had the students present their ideas, reasoning, and justifications. Although there were several problems related to finding patterns in the textbook, he made them up extemporaneously.

1) 1, 3, 9, 27, ___, ___
2) 1, 4, 7, 10, 13, ___, ___
3) 1, 1, 2, 3, 5, 8, 13, ___, ___
4) 1, 2, 5, 10, 17, ___, ___
The students had no difficulty identifying the patterns of each sequence. In particular, they identified the pattern of the third one, a Fibonacci sequence that appeared difficult for them. The students enjoyed these problems. After these problems, most of the students understood what a pattern meant. Thus, Teacher Lee put up Pascal’s Triangle to encourage the students’ intellectual challenge.

![Pascal's Triangle]

Lee: Now, there are lots of patterns and find them. Write this figure in your notebook and find as many patterns as you can.

Yun-ha: Uhh... I did not learn this thing in the learning center.

Lee: I know. The teacher in a learning center did not teach this, did he?

Students: No. [Several students responded and became busy.]

Lee: I do not teach you the problems you learned in the learning center. I don’t deal with the problems in the textbook because you already know the answers. [After five minutes he convened the class.] Does anybody want to present the patterns you found? Ji-un?

Ji-un: [Rising from her seat] 1 plus 1 comes to 2, 1 plus 2 comes to 3, and 1 plus 3 comes 4. Because of 2, there are two threes. 3 plus 3 comes to 6.

Lee: Do the rest of you understand what Ji-un said?

Students: No... [Several students responded.]

Lee: Okay. Ji-un, can you come up to the board and explain your method again by pointing? [Ji-un explained her method again.] Let’s give her a hand. Does everybody agree with her? Eun-ho?

Eun-ho: [Rising from his seat] I think 1 plus 2 comes to 3 and 2 plus 1 comes to another 3, but Ji-un said there are two threes because of 2.

Lee: Eun-ho was listening carefully to Ji-un’s presentation. I think we found one pattern. [He explained the pattern Ji-eun
and Eun-ho found and drew the following figure.] Who else? Yun-ha?

Yun-ha: [Came up to the board] 1 plus 1 is 2, 2 plus 1 is 3, 3 plus 1 is 4, 4 plus 1 is 5, 2 plus 6 comes to 8. Here 1, 2, 3, 4, and the next should be 5.

Lee: Let’s look at the patterns Yun-ha found. [He drew the following figure.] Does everybody agree with Yun-ha? Han-jin?

Han-jin: 1 plus 2 is 3, 3 plus 3 is 6, and 4 plus 6 is 10, not 8. [Teacher Lee wrote what Han-jin said.]

Yung-jin: [Came up to the board] Here 1 plus 1 is 2, 1 plus 1 plus 1 is 3, 1 plus 1 plus 1 plus 1 is 4, and 1 plus 1 plus 1 plus 1 plus 1 plus 1 is 5. Adding 1, 2, 3 is 6.

Lee: Everybody understand what Yung-jin said? [He drew the following figure and explained Yung-jin’s method]
Teacher Lee had several more students present their methods of finding patterns. (Lesson transcript, 7/9)

The sequences Teacher Lee put on the board were not in the textbook. As he said in the episode, he knew that some of the students already had the answers. Thus, if he used the textbook problems the students would not be interested in this lesson. Using this open-ended problem, Teacher Lee had the students perceive that doing mathematics was not obtaining a correct answer. Rather, the students actively participated in doing mathematics by making, communicating, and verifying mathematical assertions that were remarkably different activities in the traditional mathematics classroom.

To implement his belief about open-ended tasks, Teacher Lee consistently modified a problem or task in the textbook. Because the problems in the textbook narrowly defined mathematical situations, he indicated that such problems would not engage the students in discourse. Teacher Lee refused to simply introduce the problem in the textbook. He was aware that it was not a meaningful activity for his students. Thus, he modified the problem as an open-ended task so that the task could bring the students' active engagement in classroom discourse. It was apparent that open-ended tasks that Teacher Lee used fostered the students' participation and enhanced their mathematical discourse.

Using Students' Mistakes and Ideas for Tasks

Besides open-ended tasks and approaches, Teacher Lee frequently used the students' mistakes and ideas during discourse. This pattern demonstrated how he dealt
with the students' mistakes and what he had to pursue in depth from among the ideas that the students brought up during a discussion.

Using the students' mistakes and comments for discussion provided the students with more a familiar context so that they could actively engage in a classroom discussion. The problems or tasks were produced from the students, not posed by the teacher. Another significant effect was that using the students' mistakes and comments conveyed Teacher Lee's interest in and care for their ideas and ways of thinking.

The following episode illustrates how Teacher Lee utilized a student's mistake for discussion. Teacher Lee's class worked on the issue of whether a number line between 0 and 1 could be divided by any number. Teacher Lee's intent was to have the students understand the need of fractions in representing parts between 0 and 1 on a number line. Now, in the middle of the lesson, he asked two students to try to divide the number line between 0 and 1. Tae-min was one of them. Jun-ho divided two parts but Tae-min did in a different way.

Lee: Let's listen to Tae-min's idea. Wow, Jun-ho divided it in two parts but Tae-min did in many parts.

Tae-min's method:

\[
\begin{array}{c}
0 \\
\hline
5 \text{ mm} \\
\hline
1
\end{array}
\]

Tae-min: [Grinned] On the ruler this 1 means 1 centimeter. 1 centimeter has 10 millimeters, so the middle is 5 millimeters.

Lee: Let's give him a hand. [The students clapped.] This is a very unique way I never thought of it. Okay, let's study Tae-min's idea. How many parts did Tae-min divide?

Students: Ten! [In unison]
Lee: Then what is the first skip?
Students: One millimeter! [In unison]
Lee: Now, since we are studying fractions let’s denote the skip-counts in the fraction. How many skips are there?
Students: Ten! [In unison]
Lee: Then what is a fraction for the first skip-count?
Students: One tenth! [In unison]
Lee: What about here? [Pointing the fourth skip-count]
Students: Four tenths! [In unison]
Lee: What about here? [Pointing the seventh skip-count]
Students: Seven tenths! [In unison]
Lee: So, do you think we can divide the number line between 0 and 1 in as many parts as we can?
Students: Yes! [In unison]

Teacher Lee and the class examined a couple of more number lines between 0 and 1 to make sure their conjecture. (Lesson transcript, 7/1)

In this episode, Tae-min thought about the number line between 0 and 1 as the length of 1 centimeter. Since the class had studied about measuring length prior to fractions, his idea appeared to be related to his previous knowledge. He was a good student in mathematics and told the researcher that he did not go to a learning center. Perhaps not being exposed to a learning center made him think in such a unique way. Teacher Lee accepted his idea as a valuable way of thinking and decided to pursue in depth his idea so that some students were able to connect the concepts between fractions and measuring units.

Teacher Lee made use of the students’ ideas during discussion time. Sometimes he probed the students’ ideas even though they went beyond what many of the students were trying to do at the point. He could have just mention that the idea was beyond the curriculum of the third grade. However, he never ignored the ideas that the students brought up during a discussion. Rather, he took the opportunity to stimulate discourse and to expand the students’ interests. The following episode illustrates how Teacher Lee made an idea beyond their ability turn into a useful discourse task. Teacher Lee and the
class had discussed whether a number line between 0 and 1 could be divided by using any numbers they knew.

Lee: So far, we divided number lines with 5, 8, 10, and so on as ending points. I wonder if this number line could be divided like what we did just before. Do you think it is possible?

Students: No! Or No way! [In unison]

Lee: We don’t… [Several students raised their hands to present their ideas.] Ho-rae? What do you think?

Ho-rae: [Rising from his seat] I think it is divided by 0.5.

Lee: You can divide it by 0.5? Would you come up to the board to show us what you mean?

Ho-rae drew a number line like below.

![Number line](image)

Lee: Now, let’s listen to Ho-rae’s explanation. You know you should listen carefully to it in order to ask a good question.

Ho-rae: Since the starting point is 0 and the ending point is 1, this middle point is 0.5.

Lee: Does anybody want to ask a question of him? Han-jin?

Han-jin: [Rising from his seat] How did you get the number 0.5?

Ho-rae: Because 0.5 means halfway between 0 and 1.

Lee: Who else? [No hands up.] Let’s give him a hand. Now, what is the name of the number 0.5? [The class murmured.] I will write similar numbers to 0.5. [He wrote 0.5, 0.3, 0.9.] Do you know what these are?

Min-jung: [Called out] Prime numbers.

Lee: Prime numbers. Where did you get that name? [Grinned to Min-jung and she did not respond] I am surprised that Ho-rae knows such a difficult concept in mathematics. These kind of numbers are called ‘decimals.’ [Wrote the term, decimals, on the board] Now, since we have not studied about decimals, let’s think about it with what we already knew. (Lesson transcript, 7/1)

In this episode, most of the students did not connect the concept of fractions and number lines between 0 and 1. They easily recognized fractions in parts of bars, circles, or
triangles but had difficulty imagining the fact that fractions are the numbers represented between 0 and 1. Since Ho-rae’s idea of decimal was interesting and such an idea might be a fruitful task to expand the students’ understanding of fractions, Teacher Lee decided to have him present his idea to the class. Han-jin’s request for clarification was interesting because he used the same kind of questions Teacher Lee asked when the students were at the board to solve mathematics problems. It was observed that some of Teacher Lee’s students asked this type of questions to the presenter for clarification, justification, or verification.

In addition, the following episode illustrates how Teacher Lee combined both the open-ended approach and the students’ comments for vitalizing their understanding of a concept and classroom discourse. Before beginning his lesson on addition of fractions using number lines, he drew the following number line to connect the lesson with the students’ previous knowledge.

![Number Line](image)

Lee: I taught one six-grade class during the second period when you were in the English class. I gave them some math problems for third-grade mathematics, but about half of them did not solve them quickly.

Students: [Giggling] What kinds of problems did you give them? [Several students asked.]

Lee: Now, I am going to ask you about first-grade mathematics. Did you all know about this number line?

Students: [With loud and cheerful voice in unison] Yes!

Lee: Who can tell us equations for this number line? [About 15 students quickly raised their hands.] Sung-don, can you tell us what an equation would be?
Sung-don: [Rising from his seat] Three plus two equals five.
Eun-ho: [Rising from his seat] Five minus three equals two.
Lee: [Repeated Eun-ho’s response and wrote $5 - 3 = 2$] Who else?
Yun-ha:
Lee: [Repeated Yun-ha’s response and wrote $5 - 2 = 3$]

Several students still kept their hands up and said, “I have another one!”

Lee: Okay, Jun-ho?
Jun-ho: [Rising from his seat] Two plus three equals five.
Lee: [Repeated Jun-ho’s response and wrote $2 + 3 = 5$]

Meanwhile, several students called out, “We already have that.” Teacher Lee invited more ideas but there were no more hands up. The following equations were on the board.

$$
\begin{align*}
3 + 2 &= 5 \\
5 - 3 &= 2 \\
5 - 2 &= 3 \\
2 + 3 &= 5
\end{align*}
$$

Lee: Now, these equations are correct if you think without this number line. But… look at this number line carefully. [Waited for 5 seconds] Do you draw the subtraction number line like this?

Students: No! [Several students responded]
Lee: Then, how do you draw the number line representing subtraction?

Students: Go to opposite direction. Or go down. [Called out in disagreement]
Lee: Let’s draw five minus three. [Drew a number line without arrows]

Students: Go the opposite direction! [About 10 students called out.]
Lee: What do you mean go the opposite direction?
Students: Go five skips and then back three skips.
Lee: [Drawing arrows] Go five skips and then back three skips. This number line is representing $5 - 3$. (Lesson transcript, 7/3)
In this episode, Teacher Lee caught the students' interest by telling how the sixth graders did third-grade mathematics problems. When he put this number line problem on the board, his students became interested because, as several students said in interviews, they liked to solve easy mathematics problems. Although the number line problem appeared to be a simple task, Teacher Lee's intention was to build up conceptual understanding of addition of fractions from the students' previous knowledge. As soon as he made the task an open-ended one, the class readily became engaged in discourse. Through discourse, he furthered the students' understanding about the relationship between the number line and algebraic equations. It could have been a boring review without enhancing the students' understanding and discourse unless he made the task open.

The tasks he chose, consequently, facilitated significant classroom discourse and the students' understanding because they required that the students reasoning about different strategies and outcomes, and weighed the pros and cons of alternatives.

Summary

Teacher Lee was responsible for the quality of the mathematics tasks in which his students engaged. He chose and developed the tasks that were likely to promote the development of his students' understanding of concepts and procedures, to foster their abilities to solve problems, and to reason and communicate mathematically. Based on his beliefs about the development of mathematical understanding and discourse, he usually selected tasks from two different sources: open-ended tasks and students' mistakes and comments in discourse.
Teacher Lee strove to foster the ability of his students’ mathematical thinking. He believed that his role was to show the students that many different ways of solving a problem existed to change their perceptions of mathematics. He made the tasks and problems open-ended so that the students were able to control and approach them in different ways rather than to follow the path that he laid out.

Besides open-ended tasks and approaches, Teacher Lee frequently used the students’ mistakes and ideas during the classroom discourse. These approaches provided the students with a more familiar context so that they might actively engage in a classroom discussion. Another significant effect was that using the students’ mistakes and comments would convey his interest in and caring for their ideas and ways of thinking. Teacher Lee made use of the students’ ideas during discussion time. Sometimes he probed the students’ ideas even though they went beyond what many of the students were trying to do. He never ignored the ideas that the students brought up during a discussion. Rather, he took it as an opportunity to stimulate discourse and to expand the students’ interests.

Teacher Lee’s Professional Development

Through this study, one question bothered the researcher: “How did Teacher Lee develop his own teaching practice over 10 years? And what made it possible?” It was amazing to watch how different teachers taught the same mathematics content in markedly different ways. The answer emerged from delving into his professional development from his beginning teaching until now. Although there were in-service
training programs both inside and outside of school, the major resources of his professional development were the two study groups: the Study Group of Elementary School Mathematics (SGESM) and the Study Group of Mathematics Teaching (SGMT).

He had about two years experience of teaching fifth and sixth grader’s mathematics in a learning center. Even this experience did not change his teaching style because the students attending at a learning center only wanted to obtain high test scores. Thus, doing mathematics at the learning center meant constant memorization and practice to get a correct answer quickly. On the other hand, a good benefit of this experience was that he knew the curriculum organization of these two grades and that he learned how to deal with elementary students.

Enhancing the students’ mathematical thinking was not an interest in the learning center. My job was to make them good test scorers. I don’t think that my pedagogical knowledge improved at all from teaching here. However, I was familiar with the mathematics topics and the students’ behaviors (Interview, 8/23)

Like other beginning teachers, Teacher Lee did not have a clear image of teaching mathematics even through the teaching experience in a learning center. He remembered his beginning years of teaching as a “busy and impatient” teacher.

I had no idea how I was going to teach mathematics. I thought I was teaching eagerly. My mind was too occupied with my own teaching. I did not think of my students’ feelings and differences of their learning styles. I was the only person to speak in the classroom for an entire hour... I was impetuous because I always felt that I did not have enough time to teach everything in the textbook. (Interview, 8/7)

Teacher Lee indicated that he did not know how to teach mathematics even though his specialized subject was mathematics. Since most of the course work during the four-year
program for teacher education focused on pure mathematics, he mentioned that there were barely opportunities to learn how to teach elementary mathematics and how elementary students learn mathematics. So, he resorted to the teaching methods that he had observed for 16 years in his school experience as a learner.

I really worked hard for the first two or three years to improve the students’ achievement of mathematics. I devoted myself to make my students solve as many problems as possible. I gave them several copies of problems everyday and did not allow them to go home until they got all correct answers. My teaching mathematics was only for incessant practice and memorization of rules and algorithms to be perfect. I didn’t care about manipulative activities and the students’ understanding of concepts. (Interview, 7/26)

As his knowledge of teaching mathematics grew in about three years, he realized that he should not have taught mathematics in the drill and practice style. He was not satisfied with the way he taught. This dissatisfaction perhaps occurred by gaining confidence for controlling students’ learning of mathematics. He was convinced that he was able to make the students’ test scores higher if he and his students worked hard the week before a test. Accordingly, he tried to teach mathematics in a different way but it was not successful because of his lack of pedagogical knowledge of mathematics. He mentioned his frustration because he was not improving his professional development.

Meanwhile, one of his one-year seniors, who was in the same program for teacher education, suggested that he join the SGESM (Study Group of Elementary School Mathematics). At that time, this study group had a three-year history. Several mathematics education professors of the university of education where Teacher Lee graduated organized this group to improve pedagogical content knowledge of mathematics.
I just wanted to learn something to improve my mathematics teaching. I thought that it would be good because professors joined the study group and discussed mathematics teaching with the teachers. We met once a month. (Interview, 8/12)

However, after the first several months, it was not easy to actively participate in the group. Since the slogan of the group was "to foster mathematical thinking and attitude," the first thing he had to learn was to write lesson plans based on this slogan.

It was so hard to write such lesson plans because I did not know even how to write lesson plans at that time. When I needed a lesson plan, I usually modified it from the teacher guidebooks. Once I wrote it, the professors suggested a lack of consistency with my lesson. After several years of such training, the study group finally published two books. One was for number and operations and the other for geometry. (Interview, 8/12)

In addition, the teachers videotaped their own teaching and discussed it with the professors and other teachers. He acknowledged that participating in this study group boosted his pedagogical content knowledge of mathematics.

I have learned a lot from this study group while discussing teaching ideas. If I had not joined it, I would teach mathematics in the same way I had taught before. I would not even know how to ask good questions and how to use manipulatives. (Interviews, 8/12)

In fact, Teacher Lee's belief about discourse-oriented teaching of mathematics was formed by the study group. The group members had taken seminars studying several books regarding the techniques for asking good open-ended questions in mathematics class. He indicated that the techniques of the books really impacted his teaching mathematics. Since then, he continued to ask open-ended questions in his mathematics class.
The Study Group of Elementary School Mathematics (SGESM)

This study group had about a 10-year history and the members were mostly elementary school teachers who voluntarily participated. About 200 elementary teachers were members at present. Some members were principals and superintendents who were interested in elementary mathematics education. No benefits were given to the teachers to participate in the study group because it was organized by professors, not by school districts. Thus, some members had difficulty participating in every meeting because their principals would not allow the teachers to leave the school before the official leaving hour of school which was 5:00 p.m.

Each city in the province had a branch and the members of the city gathered at a designated school after the school day once a month. Each professor joined a city’s monthly meeting and guided the discussion. Usually the meeting place was provided by a principal who was devoted to improving mathematics teaching. During summer vacation, all members participated in a one-day conference. Since there was a similar study group in Japan, every two years the professors and some members of the two countries visited each other and demonstrated mathematics teaching and shared teaching methods.

The researcher had a chance to participate in the annual conference of the year. An elementary school of the southern city in the province hosted this fifth conference. The title of this conference was “The Reform of Mathematics Teaching to Foster Mathematical Thinking and Attitudes.” The founder of this study group, who was an emeritus professor, stated that recent in-service training programs would not be successful because the intent and content were only delivered by speeches. In other
words, professors from universities delivered new theories and reforms, and then left the implementation of them the classroom teachers. He stated:

This means that classroom teachers should be better than professors. They have to not only know theories and reforms but apply them into their mathematics teaching. I believe that if you are a professor who knows theories and reforms, you should demonstrate them by teaching mathematics to elementary students in their classrooms. By doing that, teachers will be able to learn the new theories and the directions of current reforms. I don’t think that university professors teach elementary mathematics better than teachers, but at least we have to show them what we know about mathematics teaching. (Fieldnotes, 8/19)

According to Teacher Lee, this professor demonstrated his ideas of mathematics teaching in the elementary classroom several times.

Moreover, the founder stated that the most important part of the reform mathematics education was classroom teachers’ interest and passion to change their old-fashioned drill and practice styles of mathematics teaching. He added that the goal of the study group was “to nurture elementary students to be autonomous mathematical thinkers with their interest and curiosity of mathematics.” Thus, he indicated that the purpose of the demonstrative lessons in this conference was to show the teachers a lesson that guides students to think mathematically by accepting their perspectives and their own ways of understanding. In addition, he maintained that to accomplish these goals, teacher’s open-ended questioning techniques were considered the most important ingredient. These statements were similar with Teacher Lee’s beliefs about the teaching and learning of mathematics.

Another feature of the demonstration lesson of this study group was that the teachers who taught the demonstration lesson would not work with their own students on
the conference day. They had to teach the students of the hosted school. They met the
students just the day before the conference and explained what they were going to teach.
The intent of this method of demonstration was to prevent them from practicing the
lesson with the students in order to show a good lesson.

Usually two teachers of a city volunteered or were designated to do the
demonstration lessons. Once the teachers were selected, they worked with professors to
develop lesson plans for months. The topics of the lessons were selected on the basis of
the possibility of demonstrating open-ended approaches that involved different methods
of solutions. There were two demonstration lessons on that day. The objective of the fifth
grade mathematics lesson was “to cover triangles with pattern blocks.” The objective of
the sixth grade mathematics lesson was “to find patterns regarding polygons’ areas using
geoboards.” The researcher decided to observe the lesson with geoboards because some
of the activities of the new mathematics curriculum, to be implemented in the year 2000,
required the use of geoboards. The following episode illustrates how the demonstration
lesson proceeded. No recording equipment was used. Two sixth graders shared a
geoboard and were recommended to work together. The size of the geoboard was about 6
inches square and was green. The teacher was a female in her late 20s.

Teacher: Let’s make any polygons with the area of 1. I think many
polygons can have the area of 1. [Circulated the students’
desks]

Student: Is this polygon’s area 1? [ Asked to the teacher]

The teachers invited several students to present their polygons. Five different polygons
were presented on the projector which was connected a big TV screen.
Teacher: Does anybody find a different polygon with the area of 1? [Nobody raised hands.]
Okay, there were five different polygons which have the area of 1. Can anybody tell the common and different features of these polygons?

Student: All polygons have four boundary points.
Student: All polygons have no interior points.
Student: Two of them have four sides and three of them have three sides.
Teacher: What else? [Scanned the class] Now, let's find polygons with the area of 2.

The teacher let the students find as many different polygons as possible. She frequently encouraged the students to actively engage in the activity, saying, “Aha, this polygon has the area of 2. I didn’t know that,” or “I didn’t think about this one but it has an area of 2, too,” or “Yours is so unique.” She had the students present their ideas on the board and discussed each of the polygons with the area of 2 with the class.

Teacher: Let’s look at them carefully. Does anybody find a pattern?
Student: The boundary points. The interior points.
Teacher: Then, who can make a full sentence?
Student: The polygons with the area of 2 have the different number of points for the boundary and interior.
Student: Some polygons have 6 boundary points and no interior points, but others have 4 boundary points and 1 interior point.

The teacher had the students summarize what kinds of patterns they found in the case of the area of 1 and 2. These assertions were considered as hypotheses at this stage. The students were working on finding the polygons with the area of 3. About 5 minutes later, they presented their findings.
Teacher: How can we find the polygons with the area of 4? If you think about the patterns of the area of 1, 2, 3, I think you can find the polygons without using geoboard?

The teacher encouraged the students' thinking to move from concrete to abstract. One student presented his finding: The polygons with the area of 4 have to have 8 or 10 boundary points. Several students presented possible generalizations using a table or a tree diagram. The class discussed the validity of the generalizations. At the end of the lesson, summarizing all ideas presented by the students, the teacher closed her lesson.

Teacher: Today we have studied the relationship among the area, boundary points, and interior points of polygons. (Fieldnotes, 8/19)

This demonstration lesson lasted for 80 minutes, twice as long as a regular 40-minute lesson. This lesson illustrated several similar features of mathematics teaching that were identified in Teacher Lee's mathematics classroom. First, there was the extensive use of open-ended questions. Both teachers did not simply transfer their mathematical knowledge to the students. Rather, they asked questions to lead and guide the students to understand a key concept, fact, rule, or algorithm. The questions were open-ended (e.g., there were many different polygons with the area of 1, 2, or 3) so that every idea was welcomed. Second, the manipulative activity was a major vehicle for doing mathematics. In this lesson, the geoboard was a mediator between the teacher's knowledge and students' knowledge. It was impossible that the teacher could provide appropriate scaffolding without the geoboard in order to encourage the students to find important mathematical ideas. Third, finding patterns was a main mathematical activity. As Teacher Lee also asked his students to find the patterns of concepts, rules, or
algorithms, this lesson used inductive reasoning to find patterns of the relationship among area, boundary and interior points. Through an observation of this demonstrative lesson, it was apparent that Teacher Lee's teaching practices were substantially influenced by this study group. Once he changed his beliefs from the paper-and-pencil computation to discourse and manipulative-oriented teaching, participating in the study group nurtured his pedagogical knowledge of mathematics to maintain his beliefs.

After the demonstration lessons, each teacher and observers joined for an hour to listen to the teacher's reflection and discuss possible improvement and modification. This learning process was mutual between the presenter and observers. Two or three professors recounted their interpretations of the lesson through theoretical frameworks and classroom teachers provided useful classroom incidents. After the discussion and reflection section, the teachers reconvened again at the main conference room for a 30-minute workshop that demonstrated how various patterns blocks could be used in teaching mathematics.

During the conversations, the teachers indicated that a main attraction in the conference was to learn something new for their own professional development. Teacher Lee also indicated this beneficial aspect in an interview several days later.

One of the good things in the study group is that every aspect is related to practical knowledge of teaching mathematics. In particular, the demonstration lesson and the workshop are good for me. Attending the conference is a time for reflection about my teaching. I always learn different ways of teaching mathematics to be implemented into my classroom. Who can teach this kind of knowledge in school? (Interview, 7/26)
As Teacher Lee mentioned, many teachers stated that they were trying to implement what they learned here into their mathematics classrooms. One fourth-grade teacher explained:

Most of elementary teachers tend to think that mathematics is the easiest subject in elementary school curriculum. There is no problem for them to teach mathematics with their mathematical knowledge. However, teaching how to solve problems does not mean that the teacher is teaching mathematics. The mathematics classroom should enhance students’ mathematical thinking, attitude, and understanding of concepts and algorithms. This study group and the conference provide me useful knowledge to teach in that way. (Fieldnotes, 8/19)

This teacher shared the views of Teacher Lee’s mathematics teaching and the views also were reflected on the themes of the study group. Teacher Lee indicated that it was not important what mathematics the teacher taught in mathematics class but how the teacher encouraged students to think mathematically. By participating in the conference and having conversations with other teachers, it became evident that Teacher Lee’s beliefs and teaching practices were influenced by this group.

The Study Group of Mathematics Teaching (SGMT)

Along with the SGESM, Teacher Lee joined another study group, the Study Group of Mathematics Teaching (SGMT). Unlike SGESM, this study group was organized by the school district. Each teacher was required to participate in one study group for the development of the pedagogical content knowledge of a specific subject. About 40 teachers voluntarily took part in the group and Teacher Lee was an assistant president. The school district financially supported the activities of the group.

The members met at a designated school once a month and Teacher Lee said that the major purpose of this group was to share teaching ideas. Because any experts of
mathematics education were not involved in the group, some active members prepared and shared useful ideas for teaching mathematics with the members that were gathered from books or internet web sites. The researcher observed the monthly meeting of this group twice. This meeting was held at 4:00 p.m. and lasted for an hour. One afternoon at the end of June, the teachers joined a computer lab of an elementary school to learn several computer software programs related to mathematics learning (e.g., tick-tack-toe and computation enhancing games, geoboard activity games). They discussed and shared what mathematics areas the programs might be related to and how they might be used in mathematics classroom. Teacher Lee said that this study group sometimes invited professors of mathematics education so they could learn the current reform movements or the improvement of pedagogical knowledge of mathematics.

Another meeting at the beginning of July was held to discuss this year’s summer camp. This four-day summer camp was the biggest event of each year and represented the theme of the group, “Exploring Mathematics.” About 10 members who were actively involved in the group discussed possible instructors and topics for this year’s camp. Eight (three female and five male teachers including the researcher) of them volunteered to be instructors and suggested temporary topics. Within a week these instructors needed to submit their lesson plans but the lesson plans were not fully discussed among the members. It assumed that they respected each other’s lessons. However, all suggestions and comments were welcomed at any time. Teacher Lee suggested to the researcher that joining one of instructors would be a helpful experience to understand the management process and Teacher Lee’s role in this camp.
One week after summer vacation, 135 sixth graders from 65 elementary schools in the school district took part in the camp for this year. The number of participants was slightly increased compared to last year's 120. Since two students from each school were recommended by the principal, all the participants were considered to be high achievers in mathematics. Because of the school district’s support, the registration of this camp was free. About 33 students were assigned to one of four classes and the four instructors took charge of classroom teachers.

The lesson started at 9:30 a.m. and went until 12:30 p.m. and each lesson lasted for 80 minutes. Each classroom studied two topics a day. The topics were as follows: tangram activities, solving mathematical contest problems, calculator activities, making rectangles with pentominoes, fraction addition and subtraction using geoboards, Hanoi tower, problem posing activities to make problems which have many answers, measuring by using body parts, and making polyhedra with counting rods and finding Euler formula. Each year different topics were selected. While Teacher Lee taught the Hanoi tower problem, the researcher taught polyhedra and Euler formula. After the lessons of the day, the teachers shared their teaching experiences and difficulties, and discussed possible improvements for next year’s camp.

Teacher Lee and the teachers were asked what inspired their commitments for the camp and what possible professional developments they could have. Teacher Lee explained:

Without this kind of teaching experience of mathematics outside the textbook, my teaching would simply follow the sequence of the textbook. Learning new theories, observing other teachers’ methods, and discussing teaching ideas provided a good reference for my teaching. I think this kind
of experience would expand my knowledge of teaching mathematics.
(Fieldnotes, 7/19)

Here Teacher Lee again stated that participating in the study group would promote his pedagogical knowledge of mathematics.

The researcher observed Teacher Lee’s Hanoi tower lesson to find out whether Teacher Lee would teach mathematics in similar ways as in his third grade classroom. Moreover, the researcher wanted to know if his teaching practices and beliefs were demonstrated in mathematics classroom with the students from different schools. On the second day of the camp, with no teaching experience of elementary students, the researcher was overwhelmed with classroom management problems to keep them on task. Some students frequently disrupted the lesson and the researcher complained about the students’ misbehavior to Teacher Lee.

I think that happened because you don’t have disciplinary techniques to control them. Always several students tend to dominate and spoil the class, especially when the students came from different schools. So, before starting your lesson, you should show them your confidence and determination to have control over their behavior. (Fieldnotes, 7/20)

This remark revealed similarity with his teaching practices. As mentioned earlier, one theme of Teacher Lee’s mathematics classroom was to “behave orderly and think freely.” He obviously demonstrated this theme in the Hanoi tower lesson. The following short lecture was delivered in a strict manner just before beginning his lesson.

It’s really hot these days and you came here to learn something about mathematics. You can persevere in this kind of weather if you are keeping in mind that you are here to learn. I won’t be patient with those who disrupt the lesson or interfere with other’s learning. I don’t want to teach such kind of students. Some of you appeared not to understand what you
have to do in class. So, if you don’t like to study and want to play, please
go out of my classroom now. I don’t mind it. [Scanned the class and
nobody moved and made noise.] Okay. All of you promised to study hard
in this class, then let’s clap to keep up this spirit. [All students clapped
energetically.] You are representatives of your schools and have to be
proud of yourselves... I will give you a tip for mathematics learning. If
you intend to getting a correct answer by simply memorizing and
practicing algorithms, your performance in mathematics will not be
improved. Learning mathematics is not like get a correct answer quickly in
a quiz game. You have to make sense of the concepts, rules, facts, or
algorithms for yourself and think logically. In addition to being self-reliant, you need perseverance when learning mathematics. (Lesson
transcript, 7/21)

In this short lecture, Teacher Lee revealed his beliefs about the learning of mathematics
that were identified in his third grade classroom. He emphasized the well-disciplined
behavior in his classroom and required the students to behave orderly. Like his third
grade classroom, he also used clapping as a means of forming orderliness. He stated that
learning mathematics was a making-sense activity with perseverance. Through
observations of the four-day summer camp, his beliefs and teaching practices were
consistent with different grades and places.

Other Resources for Professional Development

Other than the study groups in which Teacher Lee was involved, there were
several different resources related to his professional development. The in-service
training programs mostly played this role. According to Teacher Lee, teachers could
participate in two types of in-service training programs. The type of programs that were
supported by the school district were free but mandatory. About one or two teachers from
each school attended the programs from 4:00 p.m. to 5:00 p.m. after school. The school
district suggested the subjects for the program and the school selected the teachers who
were specialized in that subject. Almost every week, one or two teachers would participate in such programs.

One afternoon during the first week in July, there was an in-service training program related to the new mathematics curriculum that was supposed to be implemented in the year 2000 for the first and second graders. Teacher Lee attended this program accompanied by the researcher and another teacher who was also specialized in mathematics in the school. About 100 teachers participated in this program and the instructor was the president of Teacher Lee’s study group (SGMT). The most distinctive feature of the new mathematics curriculum was the focus of “fostering mathematical power.” Each grade curriculum consisted of six curriculum standards: Numbers and Operations, Geometry, Measurement, Probability and Statistics, Algebra, and Patterns and Functions. To accomplish this, the mathematics curriculum of each grade was divided by 10 performance levels and each level had two sub-levels. Based on individual student’s achievement at the end of the school year, the student could proceed toward the next level or stay at the same level. However, neither Teacher Lee nor the other teacher from the school was not observed to report this training program to the teachers in the school. It was not clear how the intentions of the new mathematics curriculum were delivered to each classroom teacher.

An important in-service training program was to update the level of the teaching certificate. Teachers earned the second level teaching certificate when graduating from the four-year teacher education program. After five years teaching, the teachers were qualified to pursue the first level of the teaching certificate. Either during summer or winter vacation, the teachers had to attend the program held at a national university of
education in a city or province. There were 11 national universities of education in Korea where the faculties were in charge of providing the program. According to Teacher Lee, the teachers had to take tests for all subject areas in elementary school, given at the end of the program, and the test scores were substantially important for later evaluation and promotion.

However, these types of in-service training programs would not influence Teacher Lee's professional development because, as he stated, the contents and materials of them were out dated. Rather, he preferred to take some programs at his own expense. Last year a course of the “cultural and historical development of mathematics” was opened by a professor of mathematics education in a national university of education. It was supposed to be a 15-week course and a two-hour class was held twice per week. The registration fee was about $100. But Teacher Lee indicated that the course was canceled because of the shortage of enrollment.

The fee might be a little bit expensive, but I think this kind of course should be opened so that teachers learn a new perspective and knowledge of teaching mathematics. It would be worth paying $100. I was really disappointed at hearing about the cancellation. It seems that most teachers tend to conceive that learning mathematics is hard and not fun by reflecting their early school experiences of mathematics classes. Another reason for being canceled was that the principals did not allow teachers to take the course because they had to leave school before 5:00 p.m. I enrolled in the course but the assistant principal did not allow me to take the course even though it was for profession development of improving pedagogical knowledge of mathematics. (Interview, 8/7)

It was frequently heard that the principals were major obstacles for teachers' professional development. For the purpose of school management, they only allowed teachers to go to the in-service training programs officially supported by the school district. Such strict
regulations by school administrators frustrated Teacher Lee. Because of the circumstances, Teacher Lee tried to improve his professional development during either summer or winter vacation. Through telephone conversations after the data collection, he mentioned that he was learning how to use the internet, electronic mail, homepage, and the like at a private computer learning center during this winter vacation at his own expense.

**Summary**

Although there were in-service training programs both inside and outside school, the major resources of his professional development were the two study groups. His two-year teaching experience in a learning center was not advantageous for the development of his mathematics teaching. He was a "busy and impatient" teacher during his beginning years of teaching. Since most of his course work during his four-year program for teacher education focused on pure mathematics, he resorted to the teaching methods that he had observed for 16 years in his school experience as a learner. After three years of a teaching career, his beliefs gradually changed by his dissatisfaction of teaching mathematics. He tried to teach mathematics in a different way but it was not successful until he joined the study groups because of his lack of pedagogical knowledge of mathematics.

Teacher Lee's beliefs about the teaching and learning of mathematics and teaching practices were evidently influenced by participating in the study groups. When he changed his beliefs from the traditional memorization-and-practice computation to student, discourse, understanding-oriented teaching mathematics, the study groups provided sufficient and satisfying knowledge and successful experiences to adhere to his
changed beliefs. He was considered to be an open-minded teacher because he was eager
to learn new ways of teaching mathematics from colleagues and experts, and to
implement his new knowledge into his classroom. He consistently reflected his teaching
practices and made all possible effort to improve his professional development.
CHAPTER V
DISCUSSION AND CONCLUSIONS

Introduction

The goal of this study was to describe how a Korean elementary teacher teaches mathematics in an everyday classroom and why he teaches mathematics as he does. More specifically, this study described interaction patterns and classroom norms to understand the teacher’s beliefs about the teaching and learning of mathematics. In addition, this study sought to identify what facilitated or constrained the relationship between the teacher’s beliefs and his teaching practice. The teacher in this study taught mathematics by emphasizing conceptual understanding as well as procedural understanding. His major teaching method was discourse which reform documents advocate as an appropriate way of teaching mathematics. He utilized manipulative activities and mathematics embedded games to enhance his students’ mathematical understanding. Worthwhile mathematical tasks were indispensable ingredients to make it possible for him to teach mathematics with understanding through discourse. The teacher’s study group activity was identified as an essential source for providing him with sufficient knowledge for teaching mathematics. Based on the data, the first section presents a discussion of the main findings. The second section describes possible limitations of the study to help appropriate interpretations of the findings and conclusions. The third and last section suggests the implications for elementary mathematics education and provides
recommendations and directions for future research in the area regarding teachers' beliefs and teaching practices, including classroom norms and interactions.

Discussion of the Main Findings

Several issues emerged through this study regarding mathematics education and mathematics teacher education. These issues are discussed based on the main findings of the study and research questions, compared to previous research. The discussion follows (a) Teacher Lee's instructional sequence of a lesson, (b) the change in pedagogical beliefs and teaching practices, (c) the relationship between pedagogical beliefs and interaction patterns and norms, and (d) facilitators of the relationship between beliefs and interactions and classroom norms.

Teacher Lee's Instructional Sequence of a Lesson

Teacher Lee's mathematics lessons usually proceeded in a sequence suggested in Figure 4. The sequence presents Teacher Lee's instructional activities in a lesson. In addition, as the study described, each instructional activity is related to his beliefs about the teaching and learning of mathematics.

After Teacher Lee and the students exchanged bows, a lesson was officially underway. At the beginning of the lesson, three different types of activities were prevalent: (a) having the students read objectives and discussing the day's lesson. If the day's lesson was about mathematical concepts such as "patterns," or "time and hour,"
Begin the lesson by exchanging bows

Having students read objectives & discussing about the day’s lesson

Having students read the day’s problem or task

Reviewing students previous knowledge to connect to the day’s concept or procedure

Presenting the problem or task in the textbook for the day

Students working individually

Manipulative activities in the beginning of a unit

Mathematical games in the end of a unit

Discourse – On-Board Presentation
Standing-Up Presentation

Discussing, highlighting & summarizing solution methods and the major points of the students’ presentations

Providing another problem outside textbook

Students practice problems in the textbook or workbook

Check students’ methods of solution

Figure 4. Teacher Lee’s instructional sequence of a lesson
Teacher Lee had the students read the day's objectives for one minute. Then the students presented their understanding of what the lesson was and Teacher Lee had the students find the commonalities in their ideas. After the students presented their conjectures, Teacher Lee summarized the objectives;

(b) having the students read the day's problem or task. If the lesson was in the middle of a unit, meaning that key concepts were already learned, Teacher Lee had the students read the day's problem or task; and

(c) reviewing the students' previous knowledge to connect to the day's concept or procedure. When the concept or procedure was connected to the students' previous knowledge, Teacher Lee began his lesson with simple problems to develop a conceptual understanding. All three of these activities in the beginning of the lesson were proceeded with classroom discourse.

After about five minutes for the beginning instructional activities, Teacher Lee presented the problem or task in the textbook for the day's lesson. These activities were involved in this stage of instruction:

(a) having the students work individually. When the class discussed the problem or task, the students worked individually. Although they were allowed to work in a group or pairs, Teacher Lee rarely encouraged them to work together;

(b) using manipulative activities in the beginning of a unit. When key concepts of a unit were usually presented in the beginning of a unit, Teacher Lee utilized manipulative activities to help his students' conceptual understanding; and

(c) using mathematical games at the end of a unit. Mathematics games typically were used at the end of a unit for application of the concepts and procedures. Before the
games, Teacher Lee assigned one or two lessons to practice problems because the students needed to master the concepts and procedures in order to play the games.

When the students had completed the problem, manipulative activities, or mathematical games, the whole-class discourse followed. The students presented their methods of solution, verification, or justification of their answers, and the rest of the class asked questions or clarifications. Teacher Lee asked the presenters questions when the class missed important concepts or procedures that should be discussed or when the class did not ask questions at all.

After the students' presentations, Teacher Lee explained, highlighted, and summarized the major points of the presentations. At this stage, he guided the class to assure that everyone in the class understood the key concepts and procedures of the day's lesson. Teacher Lee provided the class with one or two more problems on the board that were closely related to the objectives of the day. These problems usually were not in the textbook, but he made them up, based on his observation of the students' presentations. Again, the students worked individually and classroom discourse followed.

At the end of the lesson, usually with five minutes left, Teacher Lee assigned several problems in the textbook or workbook for practice. He circulated among each group to answer questions while monitoring the students' understanding. When a common misunderstanding was observed, he reassembled the class to address it, to prevent the same types of errors in the future. Every student's work was checked during the lesson or during the day before the students were released to go home. Some students who did not understand the day's concept and procedure had to work after cleanup time. There were no homework problems in mathematics. However, more extensive practice
for solving problems was usually given at the end of a unit before playing mathematical games. Although one or two lessons were totally assigned for practice, the focus of the practice was not on obtaining correct answers. Rather, the students were required to explain their methods of solution. Teacher Lee gave more similar problems when the students were not able to explain sufficiently the concepts and procedures although they had obtained correct answers.

Teacher Lee's instruction consisted of considerably different sequences from the analyses of the recent TIMSS (the Third International Mathematics and Science Study) (Stigler & Hiebert, 1999). First, there were not many review sections in the beginning of a lesson. In Teacher Lee's mathematics classroom, reviewing meant to use the students' previous knowledge to connect that knowledge to the concept or procedure of the day's lesson. To do so, Teacher Lee wrote a simple problem the students had already learned in the first or second grade and gradually developed and connected it to the day's lesson. Second, Teacher Lee extensively utilized classroom discourse. The classroom discourse was observed everywhere, from the beginning, middle, and end of the lesson. It was not a fixed sequence, but an essential tool for communicating and teaching mathematical knowledge. Lastly, despite no mathematics homework problems, the students in this classroom had to show their understanding of the concept and procedure in the day's lesson before being released. Based on such evaluation of the students' understanding, Teacher Lee modified and redirected his next lesson.

Some sequence of Teacher Lee's instruction is shared with the findings of Stigler and Hiebert (1999). They stated their surprise in finding in the Japanese lesson that the teacher presented a problem to the students without first demonstrating how to solve the
problem, but U.S. teachers almost never followed this approach. Teacher Lee’s mathematics lesson had the same feature; he almost never demonstrated how to solve the problem and taught the concept or procedure of the day’s lesson after his students presented their ideas. Allowing his students to experience confusion and the search for meanings for themselves was one of Teacher Lee’s beliefs about student’s mathematics learning.

Lesson coherence was another important finding of Teacher Lee’s mathematics lessons. According to Stigler and Hiebert (1999), the lesson coherence is the connectedness or relatedness of the mathematics across the lesson. They reported:

U.S. lessons contained significantly more topics than did Japanese lessons, and significantly more switches from topic to topic than both German and Japanese lessons... Only the Japanese teachers routinely linked together the parts of a lesson. In fact, 96 percent of Japanese lessons contained explicit statements by the teacher connecting one part of the lesson with another, whereas only 40 percent of German and U.S. lessons contained such statements. (pp. 61-63)

It was not possible to provide statistical evidence, but as with the Japanese mathematics lesson, each of Teacher Lee’s mathematics lesson was significantly connected around one theme and related to the students’ previous knowledge. Such coherence of a mathematics lesson may be explained by textbook organization. With the strict limitation of the number of pages, mathematics textbooks in Korea are logically organized with only one or two concepts or procedures per lesson. Although Teacher Lee used open-ended problems, manipulative activities and games, his instruction was based on the concepts and procedures that were laid out in the textbook. Since he organized his mathematics
lessons around the key concepts and procedures, there was no shift to change topics in each lesson.

In conclusion, Teacher Lee’s instructional sequence of a lesson was coherently organized to enhance the students’ conceptual understanding in mathematics by using discourse. Although discourse was a major method of his mathematics teaching, group learning was rarely used in his teaching. His roles were to lead class discussions, ask questions about the solution methods presented, and point out important features of the students’ methods. Open-ended mathematical tasks by modifying textbook problems or from the students’ mistakes and ideas were utilized to teach mathematics with understanding and discourse. The focus of practicing problems was on gaining the capability of explaining the students’ own methods of solutions, instead of simply obtaining correct answers. In these aspects, Teacher Lee’s mathematics teaching was compatible with the current reform recommendations which place great emphasis on discourse and mathematical understanding (National Council of Teachers of Mathematics, 1989, 1991).

The Change in Teacher Lee’s Pedagogical Beliefs and Teaching Practices

Two major factors appeared to contribute to Teacher Lee’s change in beliefs about the teaching and learning of mathematics. One would be the gain of pedagogical content knowledge of mathematics, and the other would be reflection on mathematics teaching practices.

According to Teacher Lee, his mathematics teaching during the first three years was clearly associated with his beliefs about the teaching and learning of mathematics
and his teaching practices that he reproduced the same way he had been taught. His beliefs about the teaching and learning of mathematics were drawn from previous vivid episodes or events in his own experience as a student (Pajares, 1992). He viewed mathematics as a collection of facts and rules that were transferred verbally from the authoritative teacher to the students. He believed that his students learned primarily by observing his demonstrations attentively and then practicing the presented procedures. He sought to produce students who could perform the mathematical tasks specified in the textbook, using standard procedures or algorithms. He was concerned with managerial aspects of teaching and allowed little interaction in his mathematics classroom.

According to Artzt (1999) who described the development of changes in mathematics teaching, Teacher Lee’s teaching, during those years, was at the initial stage that could be characterized by traditional instruction. At this stage, he was driven by the belief that the students learned best by receiving clear information transmitted by a knowledgeable teacher.

Teacher Lee believed that mathematics must be taught to enhance students’ mathematics understanding. The terms, “understanding” and “conceptual understanding,” were ubiquitous in his mathematics teaching. The belief about teaching mathematics to enhance understanding was drawn from his informal teaching experience, such as his long-term tutoring experience. Many preservice teachers in Korea supplement their college expenses by tutoring, but it is not a requirement of a teacher education program. In tutoring, they teach elementary through high school mathematics and this teaching experience provides not only content knowledge of mathematics, but opportunities to reflect upon their own learning process. Six pure mathematics courses, including set
theory, linear algebra, real analysis for two semesters, geometry, topology, probability and statistics as a requirement for his specialization in elementary mathematics education, supported by the tutoring experience, provided Teacher Lee with solid content knowledge of mathematics beyond the elementary level and helped him provide mathematics problems outside of the textbook. He made up the problems extemporaneously in order to promote classroom discourse and pursue, in depth, the students' understanding. Through tutoring experiences, he encountered the shifting role from student to teacher for the first time. According to Teacher Lee, the tutor's role required him to understand mathematics in order to teach his students mathematics conceptually instead of through mere memorization. He said, "If you want to teach something, you have to understand it broadly and clearly. You cannot teach mathematics by parroting formulas, rules, and facts that you memorized."

However, his belief about teaching mathematics with understanding was not demonstrated during his beginning years of teaching. He had not been able to implement his beliefs because of a lack of pedagogical knowledge of mathematics and general pedagogical knowledge concerning classroom management. Teacher Lee had acquired some knowledge about classroom management through his teaching experience in the learning center before becoming a teacher. However, the knowledge was not sufficient to manage the 45 students in an actual classroom while following standardized curriculum sequence and progress. Regardless of the solid content knowledge of elementary mathematics, the lack of pedagogical knowledge of mathematics and classroom management skills were major obstacles preventing him from implementing his beliefs about teaching mathematics with understanding.
After three years of teaching, classroom management skills had grown and he was able to reflect on his mathematics teaching practices. These concerns were no longer the focus of his teaching of mathematics after the three years. Around this time, he participated in the study group (SGESM) where he developed and enhanced his pedagogical knowledge to teach mathematics for enhancing students' understanding. The increased general pedagogical knowledge through classroom experience and pedagogical content knowledge of mathematics through the study group seemed to help him to seriously rethink his teaching. Teacher Lee had become dissatisfied with his existing teaching practice of mathematics. He had realized that his teaching practice had no longer been functional for enhancing mathematical understanding (Posner et. al., 1982).

Through reflection, he might be embarrassed when he realized the way he had taught mathematics. This belief change could be explained by his strong sense of mission as a classroom teacher who considered that his students' knowledge and attitudes in mathematics solely depended on him, and who held tightly to the notion that teaching was a matter of preparing his students' for their lives (Cooney, Shealy, & Arvold, 1998). Since reflecting on his teaching practices, Teacher Lee’s beliefs about the teaching and learning of mathematics experienced a dramatic change. This change can be referred to as gestalt shift (Nespor, 1987; Pajares, 1992). According to Artzt (1999), Teacher Lee was at the subsequent stage of the development of mathematics teaching. This stage was characterized by instruction that was more focused on helping students build on what they understood and less focused on helping them simply acquire facts, rules, and algorithms.
Over subsequent years, he actively participated in the study group and acquired the pedagogical content knowledge that supported his beliefs about teaching mathematics with understanding. Teacher Lee’s current mathematics teaching was at the final stage of the development of mathematics teaching, according to Artzt (1990). This stage was characterized by instruction where he arranged activities that involved both the “hows” and “whys” of mathematical concepts and procedures. In this stage, he was motivated by the belief that, given appropriate settings, the students were capable of constructing deep and full mathematical understanding. In terms of Posner and the colleagues (1982), the changed beliefs and teaching practices provided him with a successful perspective in understanding a variety of situations. Teacher Lee indicated his satisfaction with his current teaching practice of mathematics: “I will continue teaching in this way for awhile until I realize the need of changing my mathematics teaching.”

As mentioned earlier, pedagogical content knowledge and reflection from the study group activity were major driving forces for Teacher Lee’s change in beliefs about the teaching and learning of mathematics. Acquiring general pedagogical knowledge and pedagogical content knowledge of mathematics assumed to provide Teacher Lee with opportunities for reflecting upon his teaching practices. Shulman (1987) defined pedagogical content knowledge as the distinctive bodies of knowledge of teaching.

Pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding... It represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction. Pedagogical content knowledge is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue. (p. 8)
As a knowledge-base of teachers, pedagogical content knowledge of mathematics is a way of representing and formulating mathematics that make it comprehensible to the students (Shulman, 1986). According to the change of Teacher Lee’s beliefs and teaching practices, pedagogical content knowledge also appeared to be an indispensable teachers’ knowledge and play an essential role for supporting and maintaining beliefs about the teaching and learning of mathematics. In addition, recapitulating research on pedagogical content knowledge and beliefs of experienced teachers, Borko and Putnam (1996) claimed that although experienced teachers had generally acquired a good deal of pedagogical content knowledge, their knowledge and beliefs often were not sufficient or appropriate for supporting teaching that emphasized students’ understanding.

Teacher Lee’s beliefs change in the teaching and learning of mathematics suggests that teachers would not implement the current reform recommendations into their mathematics classroom without sufficient and specific pedagogical content knowledge of mathematics, despite accepting the reform ideas. Moreover, to ensure the possibility of fulfilling their reform-minded beliefs into mathematics classroom, the beliefs should be consistent with their pedagogical content knowledge of mathematics.

The Relationship Between Teacher Lee’s Pedagogical Beliefs and Interaction Patterns and Norms

This section describes the interaction patterns and classroom norms in Teacher Lee’s mathematics classroom with his beliefs about the teaching and learning of mathematics. In addition, this section attempts to connect the relationship between them. In conclusion, Teacher Lee’s beliefs about the teaching and learning of mathematics were
closely associated with interaction patterns and classroom norms. Figure 5 illustrates his belief system of the teaching and learning of mathematics.

The first and essential part of his belief system was classroom management. The theme of his classroom management was “behave orderly, think freely.” Although acquiring the knowledge and skills for managing a classroom is an especially salient task for new teachers (Borko & Putnam, 1996), it was also important for Teacher Lee as an experienced teacher to implement his pedagogical beliefs into his classroom. He was confident in his skills of classroom management for orchestrating activities and keeping his students engaged. In the sense of classroom management, he was a strict disciplinarian who wanted the 45 students to be orderly. He kept his distance from the students to maintain discipline. On the other hand, for the students’ learning, he stated

**Figure 5.** Teacher Lee’s belief system of the teaching and learning of mathematics

that he usually reduced his strictness gradually after the first month of the school year and tried to become comfortable over his years of teaching. In order to do that, he maintained
impartiality so that his students felt they were treated fairly. In addition to maintaining impartiality, he shared his experiences and personal feelings with his students, admitted his mistakes, told stories, or used games to teach mathematics. He needed to form personal bonds with his students in order to motivate them to learn. Feiman-Nemser and Floden (1986) suggested that managing the tension would be a most difficult task for a beginning teacher, but it remains a central issue for experienced teachers as well. In the case of Teacher Lee, as an experienced classroom teacher, he skillfully resolved this tension. These two concerted roles of a classroom teacher made it possible for his students to behave in an orderly manner and to think freely, simultaneously.

Once the general beliefs and practices about classroom management were formed, Teacher Lee established particular social and mathematical norms for teaching mathematics with understanding. Many classroom norms identified in Teacher Lee’s mathematics classroom bore a resemblance to mathematical activity as characterized by several researchers (Cobb, Yackel, & Wood, 1995; Lo & Wheatley, 1994; Lo, Wheatley, & Smith, 1994; Yackel & Cobb, 1996; Yackel, Cobb, & Wood, 1991). On the other hand, some classroom norms and interaction patterns were not articulated in the previous studies. They could be referred to as the “norms and interaction patterns for teaching mathematics with understanding” because they were focused more on the development of mathematics understanding. As illustrated in Figure 5, Teacher Lee believed that mathematics should be taught to enhance mathematical understanding and that mathematical understanding should be first developed prior to applying and practicing procedures for automatization. The interaction patterns and classroom norms related to these beliefs included that:
(a) individual student's effort for seeking understanding was valued;  
(b) making sense of a single problem with persistence was more valuable than completing many problems (Yackel, Cobb, & Wood, 1991);  
(c) presenting the same or similar response as the previous one was not valuable;  
(d) presenting different ways of solution and student's own way of understanding was valuable (Lo, Wheatley, & Smith, 1994; Yackel & Cobb, 1996);  
(e) students need to be deliberately led to grasp concepts and procedures rather than being told;  
(f) teachers should give problems and not give an answer or explain procedures of getting a solution (Lampert, 1990; Yackel, Cobb, & Wood, 1991);  
(g) the students read the objectives of the day and discussed the day's lesson prior to beginning the lesson;  
(h) the students were allowed to be puzzled and confused;  
(i) the students' experiences with mathematical situations were consistently modified by the teacher to provide more meaningful context for them; and  
(j) the students needed to explain their methods of solution in the practice sections.

Another component of Teacher Lee's belief system was discourse. In Teacher Lee's mathematics classroom, including other subject areas, discourse was the central way to present, exchange, agree and disagree about mathematical ideas. Through discourse, he fostered the development of the students' understanding of mathematics and established interaction patterns and classroom norms. He believed that the students should communicate their methods of solution and ways of thinking to share with the class. The interaction patterns and classroom norms for discourse included that:
(a) each student's own way of understanding was valued (Yackel, Cobb, & Wood, 1991);
(b) the class had to find the flows of the presenter's idea or reasoning to help him or her think in different perspective;
(c) Teacher Lee utilized the students' previous knowledge when they presented incorrect reasoning;
(d) Teacher Lee asked "why" and "how" questions whenever he decided to pursue in depth from among the ideas that the students brought up during discourse;
(e) the presenters either on the board or by standing-up from their seats had a responsibility and an obligation of talking loud enough so that the class clearly understand the presenters' ideas, the methods of solution, and ways of thinking;
(f) both the rest of the class and Teacher Lee had a responsibility and an obligation of listening carefully to understand and ask questions to help the presenters elaborate their thinking and reasoning (Lo, Wheatley, & Smith, 1994);
(g) Teacher Lee always invited the class to ask questions in order to engage the presenter in developing and eliciting mathematical ideas and understanding;
(h) the presenter had to clarify and defend his or her methods of solution when the class member asked questions;
(i) Teacher Lee's role in the process of discourse between the presenter and the class was to repeat and make clear to them the questions and answers;
(j) clapping was used to get the students' attention for a transition and praise the presenter;
(k) Standing-up presentation in Teacher Lee's mathematics classroom was a way of making the students' ideas public. The students' ideas had to be formally recognized to be discussed; and

(l) the students who raised their hands to present their ideas had to provide not only answers but reasoning. Teacher Lee asked "why" and "how" questions so that the class could share the ideas and ways of thinking.

Although the other two components of Teacher Lee's belief system, manipulatives activities and games and mathematical tasks, were not related to the interaction patterns and classroom norms, they were essential ingredients for implementing his beliefs of teaching mathematics with understanding. Moreover, these teaching practices were closely related to his pedagogical beliefs. He strongly believed that the students should understand mathematical concepts first by using manipulative activities. And he also stated that mathematics should be taught with fun and interest so that students perceived mathematics as a discipline with fun and practicality. For Teacher Lee, a game meant to facilitate the students' engagement in mathematics activity, to give rise to their interests and fun, and to require competition. He employed games near the end of a unit because he stated that playing games was solving problems by applying what the students learned in the unit.

Posing worthwhile mathematical tasks was an important activity for Teacher Lee to implement his pedagogical beliefs and ensure the quality of mathematics activity. Mathematical tasks were central to the students' learning because "tasks convey messages about what mathematics is and what doing mathematics entails" (NCTM, 1991, p. 24). He chose and developed mathematical tasks to promote the development of his
students’ understanding of concepts and procedures and to help his students reason and communicate mathematically. Based on his beliefs about the development of mathematical understanding and discourse, he mostly employed two different types of tasks: open-ended tasks and tasks from the students’ mistakes and comments during discourse.

In conclusion, Teacher Lee played a central role in establishing the norms and interaction patterns for mathematical activity for students’ learning and in determining the quality of mathematical instruction. Teacher Lee’s beliefs about the teaching and learning of mathematics appeared to be closely related to the interaction patterns and classroom norms. This close relationship between his beliefs and classroom norms and interaction patterns implies that identifying interaction patterns and classroom norms may shed light on understanding teachers’ teaching practices and beliefs about the teaching and learning of mathematics. Identifying invisible classroom norms that make it possible for teachers and the students to act appropriately in concrete situations would give rise to observable interaction patterns related to teachers’ beliefs about the teaching and learning of mathematics.

Facilitators of the Relationship Between Beliefs and Interaction Patterns and Norms

This study described why Teacher Lee’s beliefs and teaching practices in mathematics have been changed and why his beliefs about the teaching and learning of mathematics were consistent with interaction patterns and classroom norms. Then, another question must be answered: What facilitated the change? Through this study, several factors were identified that made this change possible. Most of all, Teacher Lee’s
study group activity was a major factor in promoting the consistent relationship between his pedagogical beliefs and interaction patterns and classroom norms. The study group was a place to refresh teachers’ pedagogical content knowledge of mathematics teaching, share teachers’ own practical knowledge, and promote teachers’ reflection of their teaching practice through observations of demonstration lessons. Indeed, Teacher Lee’s knowledge about teaching mathematics can be called “craft knowledge” (Leinhardt, 1990), the knowledge that very skilled teachers have about their own teaching practice. The term “craft knowledge” has been used to refer specifically to the knowledge that teachers acquire within their own classroom practice, the knowledge that enables them to employ the strategies, tactics, and routines that they do (Calderhead, 1996). Although his teaching of mathematics with understanding was consistent with the current reform movements that require the development of students’ conceptual understanding and discourse, some of the interaction patterns and classroom norms that he wanted to establish for his students’ mathematics learning were personal insights, or habits from his teaching experiences (Feiman-Nemser & Floden, 1986) and were practical knowledge. For example, reading objectives by the students, explaining their methods of solution to Teacher Lee in practice sections, listening carefully and asking questions, or presenting their answers with reasoning were some of the interaction patterns and norms that were identified in Teacher Lee’s mathematics classroom. Carter (1990) used the term “practical rationality” to refer to this type of personal knowledge acquired from teaching-practical knowledge, differentiating from the “technical rationality” of teaching, the knowledge derived from research.
The study group activity provided Teacher Lee with practical knowledge. In fact, he disregarded technical, research-based knowledge. When the researcher and he had a chance to attend a conference in which most presenters were university professors and delivered research-based knowledge, his response was: "How can I put this stuff into practice in my mathematics classroom? It seems to me that they did not care about teachers who have 45 students running around the classroom. Instead, they do care about their research." Contrarily, participating in study groups provided him with opportunities to reflect upon and to share his practical knowledge with members as a supporting group. It appeared that they enjoyed and appreciated their craft knowledge instead of being intimidated by technical knowledge. Demonstration lessons were a major way of reflecting, sharing, and learning their practical knowledge. Watching and discussing the lessons provided opportunities to reflect upon and share their own knowledge of teaching mathematics. It was a place where teachers obtained help, support, and advice by revealing their own struggles and mistakes.

Another reason that the study group activity facilitated Teacher Lee's belief change may be the close relationship between members and university professors. Except for some teachers who transferred from other cities or provinces, most of members in Teacher Lee's study group spent four years in the same teacher education program with the professors. In addition, they all studied elementary mathematics education as their specialization area. Since they were familiar with each other, their conversation was candid in expressing their successes and failures while teaching mathematics. It appeared that university professors played a role of providing theoretical knowledge of teaching
mathematics and discussing the teachers' practices, whereas other teachers played supportive colleagues who had the same experience.

Lastly, the study group activity was a career-long process for professional development. Like Teacher Lee, most experienced teachers stayed more than five years with SGESM (the Study Group of Elementary School Mathematics). Although it was not uncertain how many teachers experienced a change of beliefs and teaching practices like Teacher Lee, the process of teacher change and development in the study group was gradual and took considerable time. Through long periods of involvement in the study group, the teachers gained sufficient pedagogical content knowledge of mathematics that could support their change of beliefs and practices. This result might indicate that teachers' change of beliefs in the teaching and learning of mathematics needs several years involvement for professional development.

Along with the study group, educational policy influenced the consistent relationship between Teacher Lee's pedagogical beliefs and interaction patterns and classroom norms. The prohibition of the use of standardized tests in elementary schools appeared to facilitate Teacher Lee's belief about teaching mathematics with understanding. Without standardized tests, there was no big concern about students' achievement. Students' performance was not an index to compare the quality of teachers' teaching, but only a matter of deciding the teachers' instructional pace. It was assumed that without the anxiety of students' test scores, mathematics teaching might move more toward the development of conceptual understanding and student-centered learning. Indeed, Teacher Lee utilized this policy to teach mathematics with understanding and discourse. However, other third-grade teachers' teaching mathematics did not seem to be
considerably changed from their traditional way of teaching. A possible explanation might be their lack of pedagogical content knowledge based on the current reform recommendations. Because their specialization was other than mathematics (e.g., music, Korean language), they might not have sufficient subject matter knowledge and support for changing their teaching practice of mathematics.

The last factor that facilitated the relationship between Teacher Lee’s pedagogical beliefs, interaction patterns and classroom norms is the perceptions of students and students’ successes in mathematics. About 15 students who led classroom discourse mostly had attended a learning center. Based on his teaching experience there, Teacher Lee was aware that these students learned algorithms, facts, concepts by memorization and practiced many mathematics problems in the textbook in advance of curriculum progress without sufficient understanding. This awareness made him avoid the same type of mathematics teaching in the learning center and move toward more conceptual understanding with discourse. Because he thought that the students would not be motivated and interested in his class if he followed the same sequence as the textbook laid out, he frequently used open-ended tasks. Thus, perceptions of his students sustained Teacher Lee to implement his beliefs about teaching mathematics with understanding.

Another factor that may have facilitated the relationship might be his students’ success in mathematics. Indeed, Teacher Lee taught mathematics by focusing on the development of conceptual understanding, but on the other hand, he was concerned with his students’ performance. For this reason, he placed great emphasis on practicing problems at the end of a unit. Instead of checking his students’ answers, however, by asking the process of methods of solution, his mathematics teaching seemed to be
considerably balanced between conceptual and procedural understanding. In doing so, he observed the students' successful performance in mathematics. This factor of students' success appeared to partly support Guskey's (1986) assumption that "significant changes in teachers' beliefs and attitudes are likely to take place only after changes in student learning outcomes are evidenced" (p. 7). The students' successful performance in Teacher Lee's mathematics classroom may influence on consolidating his beliefs about teaching mathematics for understanding. However, the students' performance was not a single factor that caused change in his beliefs about teaching mathematics.

In addition, it is worth discussing Teacher Lee's assumption of teaching mathematics with understanding. From the perspective of mathematical understanding as connections among conceptual and procedural knowledge, Nesher (1986) argued that conceptual knowledge should be thought of as the control structure for procedural, or algorithmic, knowledge. Because conceptual knowledge is, in essence, knowledge about the procedures, it can be developed only by reflecting, in part, on the procedures themselves. Nesher pointed out that there is little solid evidence for the belief that solid conceptual knowledge will produce correct procedures and, in at least some cases, procedural knowledge must form the basis for conceptual understanding. As described, Teacher Lee believed that solid conceptual understanding of mathematics played a critical role of creating different rules, algorithms, or methods of solution by his students. His approach to teach mathematics certainly proceeded from conceptual to procedural, and having the students solve practice problems to improve their conceptual understanding. Teacher Lee's mathematics classroom demonstrated that focusing on conceptual understanding by discourse, manipulative activities and games, and open-
ended tasks prior to teaching procedures, brought active student engagement in doing mathematics. After conceptual understanding, Teacher Lee’s students had to master procedural or algorithmic knowledge by solving many problems. Indeed, Teacher Lee’s students outperformed other classes in mathematics on the standardized test administered at the end of the first semester. It was not officially reported, but when the third-grade teachers discussed their students’ performance in the morning meeting, it was learned that Teacher Lee’s students performed about an average of seven points better than the other classes. Although Teacher Lee contended that his students’ solid conceptual understanding resulted in better grades, it was not evident if his conceptual-oriented teaching had made it happen. As Nesher argued, it was not clear that both conceptual and procedural understanding contributed to his students’ performance. In addition, Teacher Lee’s positive attitude toward mathematics certainly had a synergistic effect on his students’ success in mathematics.

Of course, Teacher Lee identified some constraints that threatened to implement his beliefs. Most of all, the principals’ support was a key element for him to maintain the consistent relationship between his pedagogical beliefs and interactions and norms. Because his pedagogical knowledge of mathematics was substantially drawn from the two study group activities and sometimes in-service training programs, he needed to leave the school earlier than 5:00 p.m. Even though the assistant principal acknowledged the value and importance of hands-on activities in mathematics and the atmosphere of the school was cooperative, his bureaucratic perspective did not allow Teacher Lee to leave the school. It assumed that bureaucratic administration of the school was valued over
teachers' professional development. Teacher Lee frequently expressed his frustrations about the principals' attitudes toward professional development.

Another constraint came from other teachers in the school. The lack of other teachers' solidarity might have been a permanent threat to the relationship between his pedagogical beliefs and classroom norms and interaction patterns. Like Lampert (1990) who deliberately altered the roles, responsibilities, and perceptions of teacher and students, Teacher Lee intended to change his students' perceptions about mathematics. Through classroom discourse and open-ended tasks, he wished that the students perceived mathematics as a discipline with many different methods of solution, a discipline requiring them to make assertions public, and a discipline that allowed mistakes as human intellectual activity. As Teacher Lee stated, however, altering students' perceptions about mathematics would not be possible with one individual teacher's effort. His great effort might change his students' perceptions, but when the students moved up to advanced grades and meet other teachers who have different beliefs about mathematics teaching and learning from Teacher Lee, their perceptions would likely return to traditional views of mathematics. In fact, it was frequently heard from many of the teachers in the school that they could not teach mathematics like Teacher Lee simply because of (a) the lack of pedagogical knowledge of mathematics, (b) the requirement of considerable time to prepare lessons, or (c) the seemingly inefficiency of teaching mathematics with understanding. Lampert (1990) also pointed out the importance of other teachers' solidarity.

There is convincing evidence that my students learned to do mathematics in a way that is congruent with disciplinary discourse. I do not claim that this result is entirely attributable to my teaching... There are other
teachers in the school where I work who also have tried to engage their students in mathematics discourse over the past 4 years. Until this year, there was a mathematics coordinator in the district who understood what mathematical discourse is about, and believed it to be an appropriate teaching method. (p. 58)

It is possible that Teacher Lee’s beliefs about the teaching and learning of mathematics will become more coherent with his teaching practice and classroom norms, if other teachers appreciated and supported Teacher Lee’s effort.

Limitations of the Study

The current study has limitations in conjunction with its design. In order to accomplish the general research goal of how one Korean elementary teacher teaches mathematics in his everyday classroom, the study has described mathematics activities in depth in one classroom with one teacher. Although this study might provide a broad picture about mathematics teaching and learning currently practiced in Korea through the teacher’s classroom activity, generalizability of the findings is not justified.

Most of all, although the school was a public elementary school where the national curriculum was administrated, it might not be a representative school. The economic levels and parents’ status in the community where a school belongs greatly influences not only school activity but also the teaching and learning activity in the classroom. Since the school where this study was conducted is in a working-class community, the description of school and classroom activity should be understood with regard to the context.
This study intended to show the mathematics teaching and learning activity through the teacher's perspective as well as others. As this study has described, the teacher did not have a Master's degree and his background experience was much different from the other teachers. For example, tutoring experience during several years and teaching experience in a learning center surely affected his teaching practices and formed his beliefs about the teaching and learning of mathematics. Thus, the teacher in this study might not be a representative teacher in public elementary school in Korea. Interpretations of the findings should consider this fact.

The findings from the study were limited by the data collection process. To be considered a good ethnographic study it needed sufficient participant observation in the field. Although one-full year or at least six months staying in the field is recommended, this study was conducted for three months and over three units in mathematics (e.g., measurement and time, fractions, problem-solving). This shortage of duration might attribute to some of the inconsistent findings. For example, the three units appeared to be appropriate mathematics topics to teach using manipulatives and discourse. The observations of different topics requiring mastery of heavy computational and basic skills, such as long digits of operations, might provide more complete information about mathematics activity in this classroom.

The students were valuable informants for the researcher to understand invisible norms and patterns of classroom activity. The students who were selected for interviews might not be representative for the classroom. Although the selection process utilized the students' social network in order to establish a comfortable and friendly environment, as the students stated in the issue of group learning, these students did not invite low-ability
students to the interviews. Because most of them were in either middle or high-ability group, the interpretations based on data from these students might not describe total aspect of the members of the classroom.

In any qualitative research, especially in an ethnographic study, the researcher was the major instrument of collecting data, interpreting findings, making conjectures and working hypotheses, and making final conclusions. The influence of the researcher in doing this research implies that almost all bias of the study was generated from the researcher’s perceptions. The researcher tried to learn how to behave appropriately in the school (Eisenhart, 1988) and tried to learn from the teachers (Spradley, 1979). The researcher took all possible precautions to maintain objectivity by keeping the outsider’s perspective and defamiliarizing familiar cultural events (Miner, 1956). Regardless of the researcher’s strict cautions, the findings and interpretations of this study might convey prejudice about values and norms in Korean educational settings.

Lastly, there were unresolved ethical issues in describing the collected data. Many unpleasant events and stories could not be described in this study although the data were significant to interpret and understand the culture of the school as well as Teacher Lee’s teaching practice. Because it is the researcher’s responsibility to protect personal information and keep strict confidentiality, some important data could not be revealed. Thus, the description of this study may not convey all aspects of the teaching activity in the school.
Implications and Recommendations for Future Research

Teacher Lee’s case of pedagogical beliefs and teaching practice of mathematics has particularly strong implications for elementary mathematics education, especially in light of current reform movements in mathematics education that has included discourse of and inquiry into the nature of mathematics, mathematics learning, and mathematics teaching (Simon, 1994). In addition, Teacher Lee’s case also provides recommendations for future research about teachers’ beliefs and teaching practices in mathematics. The following implications and recommendations for elementary mathematics education can be made from this study.

First, the norms identified in Teacher Lee’s mathematics classroom were more appropriate for teaching understanding, whereas previous research (Cobb, Yackel, & Wood, 1995; Lo & Wheatley, 1994; Lo, Whaeatley, & Smith, 1994; Yackel & Cobb, 1996; Yackel, Cobb, & Wood, 1991) described social and mathematical norms for inquiry, discourse, and the small group form of instruction. These studies investigated the process of establishing classroom norms through negotiation of meanings and expectations when teacher and students were engaging in mathematical activity. Contrarily, this study described classroom norms and interactions to identify teachers’ beliefs about the teaching and learning of mathematics. Consequently, these norms and the process of being established can be useful knowledge for both in-service and preservice teachers if they attempt to establish their classroom learning environments in reflecting the current reforms in mathematics education. For preservice teachers, these norms provide practical knowledge about how to create their own learning environments.
based on their beliefs. The description of the norms and the process of being established provide preservice teachers with various perspectives for their future classrooms. For in-service teachers, these norms provide them with opportunities to reflect upon their own teaching practices. An important factor to teachers’ belief changes is reflection upon their teaching (Clarke, 1997; Schon, 1983; Thompson, 1992). If these norms and interaction patterns with specific mathematics topics are illustrated in staff development, teachers may be more likely to reflect upon their teaching practices because the materials are more relevant to their real situations and more concrete, vivid episodes to provoke their interests. Therefore, more systematic analyses are needed about how such practical knowledge can influence the change of preservice and in-service teachers’ beliefs about the teaching and learning of mathematics.

Second, the significance of this study for mathematics education is that it describes practical knowledge of teaching mathematics with understanding and discourse. By reviewing research on teaching, Fenstermacher (1994) acknowledged the existence of “practical knowledge.” Leinhardt (1990) also described the significance of practical knowledge of teachers. According to Leinhardt, teachers possess a practical knowledge of their craft, sometimes called the wisdom of practice. She explained:

This craft knowledge encompasses the wealth of teaching information that very skilled practitioners have about their own practice. It includes deep, sensitive, location-specific knowledge of teaching, and it also includes fragmentary, superstitious, and often inaccurate opinions. (p. 18)

These scholars suggest the significance of practical knowledge of teaching and the possibility of delineating that knowledge. At the outset of this study, the focus was to describe what classroom norms existed in a mathematics classroom by using participant
observation in order to identify the possible relationship with the teacher’s beliefs about the teaching and learning of mathematics. This study was grounded in a high degree of regard for Teacher Lee. But the researcher was careful not to regard all that he said or did as worthy of acknowledgement as wisdom or knowledge (Fenstermacher, 1994).

Although his knowledge of teaching mathematics was unique, personal, and grounded on specific classroom situations, the norms and interactions identified in his classroom were compatible with the current reforms. Therefore, this study may suggest that more studies are needed to delineate the practical knowledge of “crafty teachers” (Grimmett & MacKinnon, 1992) who possess knowledge of teaching mathematics with understanding and discourse. In doing so, it may be possible to understand how teachers construct personal knowledge and what strategies they use to do so. Moreover, it may be possible to describe teachers’ tacit theories, beliefs, and values that guide their actions in classroom teaching. The corpus of such research on practical knowledge will provide valuable resource for teacher development. The significant implication of research on practical knowledge will be, as Fenstermacher (1994) stated, “not for researchers to know what teachers know but for teachers to know what they know” (p. 50). On the other hand, in this sense, ethnographic study may be a valuable tool to investigate teachers’ knowledge because the goal of this methodology is to attribute meanings teachers constructed in their classroom through instructional interactions with students to patterns and regularities that teachers otherwise take for granted in everyday classroom until the meanings are pointed out, highlighted, and given broader significance by associating them with other teachers’ experience, other classroom situations, and even literature (LeCompte & Schensul, 1999).
Third, this study suggests that a study group would be a new type of teacher development program. Whereas the "research lesson contest" and action research are the requirements of the teachers who want to have good teacher evaluation for promotion in Korea, participating in a study group is self-effort for improving their own teaching. A study group is not a place where teachers are taught or delivered new pedagogical content knowledge of mathematics which is compatible with the reform recommendations, but a place where teachers watch and participate in how to teach mathematics in a different way. In staff development and workshops which are broadly practiced for teacher development in the U.S., the teachers' role is being a learner who is supposed to learn and implement what university professors recommend. In addition, most programs appear to assume that teachers' belief changes can be accomplished over a couple of years or several months. In contrast to this type of teacher development, teachers in a study group are voluntary participants, based on their interest and willingness to refresh their knowledge of teaching mathematics. It is a long, gradual process of improvement of teaching. The study group provides the social context in which teachers' own level of consciousness about their beliefs, influences their disposition to realize change. It affords significant opportunities and references for teachers to reflect upon their own teaching practice by watching other teachers' teaching. Participating in study group may encourage teachers to be reflective practitioners (Schon, 1983). Another significance of study group for teachers' professional development is the possibility of reducing teachers' idiosyncratic experience which researchers have portrayed teachers' knowledge. In Feiman-Nemser and Floden's (1986) review of teacher knowledge, Lortie argued that teachers lack a technical culture, a set of commonly held, empirically derived
practice and principles of pedagogy. As a result, teachers must individually develop practice consistent with their personality and experience. Sarason, in this review, tied teacher isolation to account for the reason that teachers lack a shared body of practical knowledge. Because most teachers work apart from their colleagues, they have little opportunity to articulate and compare what they know and believe. Consequently, the study group may resolve teachers’ isolation and lack of shared practical knowledge by giving opportunity to reflect upon and compare their beliefs and teaching practices. Therefore, more studies are needed to account for the effect of a study group for teachers’ change.

Is it possible to implement such study groups into the U.S.? Stigler and Hiebert (1999) suggest from the analysis of TIMSS (the Third International Mathematics and Science Study) that “lesson study” in Japan can be an alternative way of improving teachers’ professional knowledge. Their significant finding is that the current reform effort should be not focused on teachers’ change, but on improvement of teachers’ teaching practice. Lesson study is for demonstrating the effort of improving teaching. In lesson study, groups of teachers meet regularly over long periods of time ranging from several months to a year to work on the design of a lesson, implementation, testing, and improvement. Stigler and Hiebert cautiously suggest the possibility of adopting the lesson study as a process for professional development of teachers. Likewise, the study group suggested in this study should be cautiously considered as an alternative way of improving teachers’ teaching because all educational systems are culturally value-laden.

Fourth, the findings of this ethnographic study might suggest that a change in teachers’ beliefs about the teaching and learning of mathematics will occur only when
they have sufficient pedagogical content knowledge of mathematics. In the case of Teacher Lee, although he had the pedagogical belief about teaching mathematics with understanding when he was a beginning teacher, he could not implement this belief into his mathematics classroom because of lack of pedagogical content knowledge corresponding to the belief. His belief about the teaching and learning of mathematics had been changed as his pedagogical content knowledge of mathematics grew. The change of Teacher Lee’s pedagogical beliefs seems to support the assumption made by Guskey (1986) who challenged the assumption that changes in practice follow changes in beliefs and instead suggested that beliefs depend on practice. Although Cobb, Wood, and Yackel (1990) argued Guskey’s linear causal relationship between pedagogical beliefs and teaching practice, gaining pedagogical knowledge through teaching experience and pedagogical content knowledge through the study group appeared to made it possible for Teacher Lee to change his pedagogical beliefs. Teacher Lee stated that demonstrating and teaching how to solve mathematics problems was an easy task, but teaching the students to understand mathematics was the hardest task of a classroom teacher. It implies that more emphasis on the relationship between pedagogical content knowledge of mathematics and changes in teachers’ beliefs about the teaching and learning of mathematics needs to be studied.

Lastly, it is learned from this study that classroom management plays a considerable role in promoting the successful teaching corresponding to beliefs about the teaching and learning of mathematics. Following the theme for classroom management in Teacher Lee’s classroom, “behave orderly, think freely,” his students’ orderliness helped his mathematics teaching to proceed smoothly. Although some researchers have
suggested that novice teachers need to become competent in the skills of classroom management before they can successfully turn their attention to other aspects of their teaching (Borko & Putnam, 1996), the importance of classroom management was also applicable to Teacher Lee who was an experienced teacher. He mentioned establishing management as a major goal in the first few weeks of the year. It was interesting, however, to observe how Teacher Lee’s strict discipline to control his classroom was compatible with his discourse-oriented teaching in mathematics. According to research on teacher control and student thought processes, Soar (cited in Fenstermacher, 1978) stated that close teacher control of student behavior did not necessarily interfere with complex cognitive or creative growth, contending that it was commonly supposed that in order to free students’ thought processes for complex cognitive tasks, teachers must also desert control of their behavior. Fenstermacher expressed the theme, “behave orderly, thinking freely,” for classroom management in a similar tone: “The body need not wander in order for the mind to wonder” (p. 162). More recently, the findings of management research agree about the conclusion that teachers who approach classroom management as a process of establishing and maintaining effective learning environments, tend to be more successful than teachers who place more emphasis on their roles as authority figures or disciplinarians (Good & Brophy, 1997). Good and Brophy suggested that classroom management should be designed to support teaching and learning to help students to gain the capacity for self-control. However, not much attention has been paid to what types of classroom management need to be used for discourse-oriented mathematics teaching. Does a well-disciplined learning environment hinder students’ involvement in discourse? Do teachers need to have a close relationship
with students in order to teach mathematics by using discourse? The current reform recommendations and research on classroom norms have not provided answers to these questions. Thus, more systematic research on the relationship needs to be done about the ways in which teachers organize and manage their teaching practice.
REFERENCES


APPENDIX
Appendix

Informed Consent Form
(Teacher Form)

Dear __________________;

My name is Cheong-Soo Cho, and I am a doctoral student in mathematics education at Oregon State University. I am writing to invite you to participate in a mathematics education doctoral research project. This project will focus on teachers’ beliefs about the teaching and learning of mathematics.

Participation will be from June, 1999 through August, 1999. If you volunteer to become involved in this study, you will be asked to participate in an interview each week with the researcher and also to allow the researcher to observe and videotape your mathematics lessons. It will also be necessary for the researcher to view some of your lesson plans and a variety of students’ work. You will be asked to recommend several students in your classroom and their parents to be interviewed regarding their views of the teaching and learning of mathematics.

All information gathered in this study will be held strictly confidential. The anonymity of your participation will be of utmost importance and no one except the researcher and major professor will view the classroom videotapes, these will be destroyed after the project is finalized. The data collected during interviews and classroom observations will be coded to protect yourself and students, pseudonyms will be used so that participants will not be identifiable in any publication of the results of the study. If at any time you feel the need to drop out of this research project, you will certainly have the freedom to do so.

Your participation in this project would be greatly appreciated. You will have access to all information gathered and may read transcripts at any time.

Questions about the research, personal rights, or research-related injuries should be directed to: Dr. Maggie Niess, (541) 737-1817.

Thank you for your time and participation in this research project.

Cheong-Soo Cho, (02) 635-6038

I agree to participate in this research project and understand the general intent of the study, the types of data to be collected, and the time commitments involved in the study.

____________________________  _______________________
Signature                  Date
Informed Consent Form
(Principal, Fellow Teacher Form)

Dear ___________________

My name is Cheong-Soo Cho, and I am a doctoral student in mathematics education at Oregon State University. I am writing to invite you to participate in a mathematics education doctoral research project. This project will focus on teachers’ beliefs about the teaching and learning of mathematics. In addition to the teacher’s beliefs, your views and concerns about the teaching and learning of mathematics in elementary school are also important factors to understand the teacher’s beliefs.

This project will be conducted from June, 1999 through August, 1999. If you volunteer to become involved in this study, you will be asked to participate in an one-hour interview with the researcher a couple of times. The interviews will be conducted at a convenient place for you.

All information gathered in this study will be held strictly confidential. The anonymity of your participation will be of utmost importance and no one except the researcher and major professor will access to the interview audiotapes and transcripts, these will be destroyed after the project is finalized. The data collected during interviews will be coded to protect yourself, pseudonyms will be used so that participants will not be identifiable in any publication of the results of the study. If at any time you feel the need to drop out of this research project, you will certainly have the freedom to do so.

Your participation in this project would be greatly appreciated. You will have access to all information gathered and may read transcripts at any time.

Questions about the research, personal rights, or research-related injuries should be directed to: Dr. Maggie Niess, (541) 737-1817.

Thank you for your time and participation in this research project.

Cheong-Soo Cho, (02) 635-6038

I agree to participate in this research project and understand the general intent of the study, the types of data to be collected, and the time commitments involved in the study.

____________________________  ____________________
Signature                      Date
Informed Consent Form  
(Parents Form)

To Parents:

Your child’s teacher at ___________________ Elementary School will soon be involved in a doctoral dissertation research project. This project is through Oregon State University where I am a doctoral student in Mathematics Education. The focus of my research will be the teacher’s beliefs about the teaching and learning of mathematics. The students in the classroom will not be involved in the study; the focus of this research is the teacher. However, your child might be selected to be interviewed regarding their views about mathematics and the learning of mathematics.

During the course of my investigation, I will be videotaping the classroom and the teacher’s interactions with students and audiotaping interviews with your child. It is possible that your child may be videotaped and audiotaped at sometime during this project. When the videotapes and audiotapes are transcribed, pseudonyms for all students, their teacher, their school, and their community will be used. Anonymity of all participants in this study will be preserved at all times. Once the videotapes and audiotapes have been analyzed, they will be destroyed; no one but my major professor and myself will have access to these tapes during their analyses.

If you have any questions or concerns regarding this research project, please contact me at (02) 635-6038 or feel free to contact my major professor, Dr. Maggie Niess, at (541) 737-1817

Please sign and date the form supplied below and return it to the teacher as soon as possible. I greatly appreciate your permission to allow me to videotape and audiotape students in the classroom.

Sincerely yours,

Cheong-Soo Cho

I agree to allow my child to be videotaped and audiotaped during the course of this research project and realize that all information will be kept confidential and pseudonyms will be used for all names.

Signed _________________________________________ (Parents)

Date ______________________

Student’s Name ____________________________________