AN ABSTRACT OF THE THESIS OF

Sergio Serrano Velazquez for the degree of Master of Science

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Title: RISK MANAGEMENT IN PARTICLEBOARD PRODUCTION SYSTEM DESIGN: A SIMULATION STUDY OF THE STOCHASTIC OPTIMIZATION PROBLEM

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Abstract approved:

Dr. Michael S. Inoue

Effects of stochastic variations of parameters in the planning and design of a particleboard production system are studied. The solution obtained from a linear deterministic optimization model is compared against both the solution derived from the traditional stochastic programming techniques and the distribution of optimal objective function values obtained from models whose parameters were Monte Carlo generated.

Resource Planning and Management System was used to produce a network representation of a particleboard operation that is planned in the Yucatan region of Mexico. The plant will use the cactus plants that naturally grow in the area and produce particleboards for housing construction. The stochastic elements are introduced in the process variability, risks associated with the quality and the quantity of supply of materials and labor, and the market demand where the

products must compete against imported particleboards. The network model included the risk elements as triangular distributions using the three parameters (L = minimum, M = most likely, U = maximum) similar to the beta distribution assumption commonly made in Program Evaluation and Review Technique.

Three major goals are pursued in this thesis: (1) practical contribution: to ascertain the viability of constructing and operating a particleboard production facility in Mexico; (2) theoretical contribution: to determine the effects of risk upon optimal solutions; and (3) industrial engineering contribution: to develop a practical approach to planning, scheduling and control of production systems under stochastic considerations. Four hypotheses were proposed for testing: (1) though only a few empirical applications of stochastic programming are now available, a practical industrial model can be constructed by modifying a linear programming model to incorporate the stochastic features; (2) Monte Carlo simulation provides more objective and meaningful data to management than the use of expected values in the linear programming techniques; (3) variations in objective function are proportional to the variations in the parameters; and (4) the problem of estimating parameters in the modeling phase, in terms of suitable sample functions, are nontrivial and practically insurmountable.

The first goal and the first two hypotheses were achieved through RPMS and Monte Carlo simulation by graphical representation of decision processes involved. The second and third goals and the third hypothesis were achieved by interpreting the results in such a way as to be useful and meaningful to management. Finally, by the use of management experience, machine tolerances and direct estimate methods, the fourth hypothesis was rejected.

Two major models were constructed and experimented by utilizing RPM1 (linear programming) and RPM2 (simulation) packages developed by Steve Shu-Kang Chou. The first model, containing 35 activity processes and 41 resource constraints, was used, first to validate selected activity levels observed from LP against historical records, and second to obtain the expected effects of risk in three phases of management consideration: (1) variations only in costs and prices: two stage programming simulation); (2) consideration of process variability: chance-constrained programming simulation; and (3) combination of the above two: stochastic programming simulation. The second model was used to prove and remedy the drop in production output and profit due to stochasticity. This was made possible by bounding the processes with the solutions depicted from the deterministic linear programming run. This model contained 35 activity processes and 71 resource constraints.

Following is a summary of the conclusions drawn from this study: (1) computer simulation facilities for stochastic programming are now available and can be used at a relatively low cost ($450.00 for this entire project); (2) the manner in which the models and techniques were utilized would constitute a viable tool for planning production systems; (3) no consideration of risk in production planning could result in underachievement of profit of, say, 28.28%, and of production yields of, say, 7.18% as in the case of stochastic programming simulation. However, the resulting payback resulted to be 7.83 years that, in comparison with the deterministic LP, is 2.21 years higher; and (4) the resulting drop in profit was found practically solved by increasing the resource availability by the same percentage of underachievement of profit.

# Risk Management in Particleboard Production System Design: A Simulation Study of the Stochastic Optimization Problem

by

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Professor of Industrial Engineering
in charge of major

Redacted for Privacy

Head of Department of Engineering

Redacted for Privacy

Dean of Graduate School

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Typed by Mary Jo Stratton for Sergio Serrano Velazquez

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# DEDICATION

To My Lovely Parents,

Jesus and Olga

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# RISK MANAGEMENT IN PARTICLEBOARD PRODUCTION SYSTEM DESIGN: A SIMULATION STUDY OF THE STOCHASTIC OPTIMIZATION PROBLEM

## I. INTRODUCTION

The increasing societal awareness of the scarcity of resources on our planet earth is starting to change the role of industrial engineers. Instead of being called upon to act as an efficiency expert, now an IE is apt to be called upon to solve the problems of converting wasted resources into productive materials that the society needs. One such example is the subject of this thesis.

Henequen cactus is a plant that grows widely in Mexico and for which very little use has been found in the past. The same country is experiencing a growing pain as it is moving from a developing nation into a mature industrial nation with the majority of its population becoming classified as the "middle class" citizens. The shift in industry from the labor intensive products to technology intensive outputs necessitates highly educated workers to move into new areas where their inventive minds are needed. The accompanying affluence is changing the living patterns of the entire population and making them more aware of the natural resources and environment. Construction of buildings is one of the foremost priorities that the nation is facing, and tons of particleboards are being imported yearly from Brazil and other neighboring nations for this purpose.

It is little wonder, therefore, that the National Council for Science and Technology of Mexico (CONACYT) has invited Mexican universities to participate in a feasibility analysis of starting up a cactus trunk particleboard industry in a region of Mexico.

The present thesis, therefore, is an outgrowth of one such study by Carrasco, Curiel and Serrano (1976), a bachelor of science thesis from the Anahuac University, entitled "Project of a Henequen Cactus Trunks Particleboard Industry." This report included results of both technical and economic studies and recommended that such an industry be started in the region of Yucatan where the raw materials are abundant, the region is suitable for comfortable living, and where such an industry will be appreciated.

The contribution of the present thesis, however, goes beyond the immediately useful data that the study has generated for CONACYT. The methodology used in the course of the study has proved to yield some unexpected results. The problem of trying to adapt deterministic results off our optimized linear programming solution, for example, was far worse than we had anticipated (Chapter V). The stochastic simulation using the Monte Carlo technique (Chapters IV and V), on the other hand, turned out to be far more reasonable both in cost and difficulty of experimentation. Other applications to stochastic programming problems confirmed the effectiveness of this new approach (Chapter IV).

In this chapter, we shall begin by identifying the original problem, discuss the proposed solution, and comment on the structure of this thesis.

## A. Problem Identification

According to Moslemi (1974, p. 1), the existence of the particleboard industry is nothing more than a few decades old, in fact in the U.S.A. initial unsuccessful efforts were developed at the beginning of this century to manufacture particleboard. Finally, the first industrial production of particleboard using synthetic resins is believed by Hunt (1962, F-42) to have occurred in 1941 in Bremen, Germany.

This industry came about as a result of the considerable interest displayed in western Europe in profitably manufacturing products from residues that are otherwise wasted. Wood wastes such as shavings, sawdust, trimmings, scraps, barks and logging waste, and other lignocellulosic residues have in the past been disposed of by the inefficient method of burning. This is becoming intolerable due to the pollution it generates. Awareness of environmental quality coupled with the need for intensified forest conservation efforts makes imperative the intelligent use of these residues.

More recently, similar problems arose from the disposal of these cactus trunks that are by-products of the production of fibers utilized for the manufacturing of twines and cords for jutes and wrapping packages.

According to Carrasco, Curiel and Serrano (1976), the yearly disposition in tons of cactus trunks is 715,881.46 scattered all over the Mexican territory. On the other hand, Mexico wants to eliminate the importation of Eucatex particleboards from Brazil by constructing a new facility that will generate profitable employment. Also it has to be pointed out, that the manufacture of particleboard offers many advantages not only for the variety of raw materials which can be used, but also in the product properties it can offer.

At this point, and despite the general acceptance of the so-called "particleboard," there still exists some confusion of the term and the precise definition of the material it describes. Akers (1966, p. 3; F.A.O., 1958) has defined particleboard as "a sheet of material manufactured from small pieces of wood or other ligno-cellulosic materials such as chips, flakes, splinters, strands, shives, etc., agglomerated by using an organic binder together with one or more of the following agents: heat, pressure, moisture a catalyst, etc." Similarly, Mitlin (1968, p. 121) defined, in context, much the same idea: "a homogeneous material composed of small units of timber or other fibrous material bonded together with synthetic resin and cured under pressure."

## B. Proposed Solution Approach and Thesis Structure

Once we have identified that a problem exists, we propose to build a Henequen cactus particleboard industry, not only because the wide variety of uses that the product to be produced offers as shown in Table 1-1, but also for creating new sources of work that would benefit the economic development of the desertic zones in Mexico, and to the "industrialization of renewable natural resources program" that the National Counsel of Science and Technology of Mexico is developing since 1976. The advantages of manufacturing particleboard not only lie in the variety of raw materials which can be used as raw material, but also in the method of manufacture and the product properties it can offer, for which we present in Chapter II the selection and description of the manufacturing process, as well as the model design.

Therefore, what we have in mind in this thesis is to analyze the particleboard production system, in such a way as to prove the feasibility and profitability of constructing this plant, by subjecting it to stochastic (risk) considerations to aid managers more realistically in their decision making tasks, than subjecting it under deterministically (certainty) assumptions.

Optimization techniques such as linear programming and stochastic programming are proposed to improve the allocation of

Table 1-1. Particleboard uses.

| | |
|---|---|
| Kitchen cabinets | Display fixtures |
| Bench tops | Plaques, toys and other novelty items |
| Boxes | |
| Institutional furniture | Various limited exterior uses |
| Floor underlayment | Billiard tables |
| Instrument and jewelry cases | Speaker enclosures |
| Trailer liners | Household furniture |
| Industrial jigwork | Radio, TV., and hi-fi cabinets |
| Signs | Doors |
| Table tops | Professional and institutional cabinets |
| Soffits | |
| Flooring | Pre-built houses |
| Industrial and farm uses | |
| Various case goods | |

of resources in the particleboard production system, the models are presented in mathematical expressions and in the form of RPM (Resource Planning and Management) networks in Chapters III and IV.

The RPM networks include linear relationships and random independent variations of the coefficients values. These assumptions were used to facilitate the construction and understanding of the models; therefore, they reflect an approximation of the reality.

The application and experimentation of such optimization techniques at the particleboard production plant by means of Monte Carlo simulation is presented in Chapter V. In that chapter, the computer results derived discussion and analysis is made.

Finally, Chapter VI presents the conclusions drawn from this thesis, in particular the advantages and disadvantages of the systems tools utilized, as well as the areas for future research.

In the Appendix, computer inputs and output of linear programming and stochastic programming simulation for the system analyzed are presented.

# II. PARTICLEBOARD PRODUCTION MODEL

## A. Manufacturing Process Selection and Description

There are presently two major processes for making particleboard: the flat-press process and the extrusion process. In the flat-press process, the board is pressed "flat-wise" while in the extrusion process, it is continuously extruded through a hot die. The flat-press boards account for over 95% of the total particleboard production. The flat-press process produces a large variety of boards thanks to the versatility of its engineering design and layout, and the large variety of particle shapes and sizes it can accept. On the other hand, the extrusion process normally utilizes hammer-milled particles, and its flexibility in plant design and layout is very limited.

The product manufactured by the plant to be analyzed is classified as hard boards with 75 $lb/ft^3$ density (Carrasco, Curiel and Serrano, 1976, pp. 23-24). Flat-press boards are made with density as low as 25 $lb/ft^3$ to as high as 90 $lb/ft^3$, while the bulk is currently being produced in a density ranging from 35 to 50 $lb/ft^3$.

The properties of particleboards depend on many factors. In strength, stiffness and dimensional stability, most flat-press boards are more uniform over the plane of the board when compared to extruded boards. However, the thickness is more stable in the extruded particleboard than in those made by the flat-press process. The plant layout in the flat-press process is less complex and requires a considerably smaller initial investment. However, due to the above-mentioned inherent deficiencies of this process, and also to the accelerated improvements in the flat-press process, the extrusion process is falling in popularity (Moslemi, 1974, pp. 3-4).

For the sake of the selection of the production equipment and process, the ones investigated for the Mexican plant were the "Baehre-Bison" (West Germany), the "Fahrni" (Belgium), the "Fratelli Pagnoni" (Italy), and the "Siempelkamp" (Austria) machineries. The one finally selected was the "Baehre-Bison" process which possessed three of the qualities suggested by Mitlin (1968, pp. 106-107):

1. The use of a special mat-laying process.
2. The large settling area of the chamber permits a very fine and uniform mat to be obtained. The above two features contribute to the production of higher quality boards with a fine, closed surface which can be coated or laminated without special treatment.
3. The raw materials are prepared in a single line continuous process. This is reflected in the lower investment and maintenance costs as compared to those of other plants which need

several preparation lines to give a comparable level of output for the equivalent quality boards.

The "Baehre-Bison" manufacture process falls into the category of flat-press process, which, in brief, consists of the following basic steps (Moslemi, 1974, pp. 2-3; Carrasco, Curiel and Serrano, 1976, pp. 37-45):

1. Reduction of the raw material to the desired size and shape. This is accomplished by hogging, grinding, hammermilling or flaking.
2. Drying the particles to a predetermined and uniform moisture content.
3. The separation of oversized and fine particles by screening or other means of particle segregation. Control is thus exercised over the size of particles going into the board structure. The fine particles are later deposited on the flat-press boards so that a smooth surface is generated. The coarse particles are redirected into a reduction system for further refinement.
4. Blending of calculated amounts of adhesive binders (mainly Urea-Formaldehyde) and other additives through a spraying process.
5. Forming the blended particles into a "mat" in the flat-press process. The procedure is controlled so that coarser particles are placed in the center layer of the board while finer particles

are deposited closer to the surfaces. This is called "graduating layering," and is necessary in order to produce homogeneous boards.

6. The hot press operation. The "mat" is solidified under controlled heat and pressure.
7. Post hot press operations. As the hot board emerges from the press, preparatory operations such as cooling, trimming and moisture equalization may be performed.
8. Sanding or planing. Such operations are needed to meet close thickness tolerances. The dust and shavings created during the process are either burned or returned to the production.
9. Further operations, Other activities are also undertaken in accordance with the customer requests. These may involve cutting to sizes, overlaying, routing or filling the surface.

The flow of this process is depicted in Figure 2-1.

## B. Data Collection

Some important concepts that must be understood and applied in order for the selected operations research project to succeed are stated by Riggs and Inoue (1975, p. 63):

> The identification of major resources and activities must preceed any model-design effort. Only resources that are scarce and therefore constitute a restriction on the solution space need to be considered. Similarly, we are interested only in activities that affect the scarcity

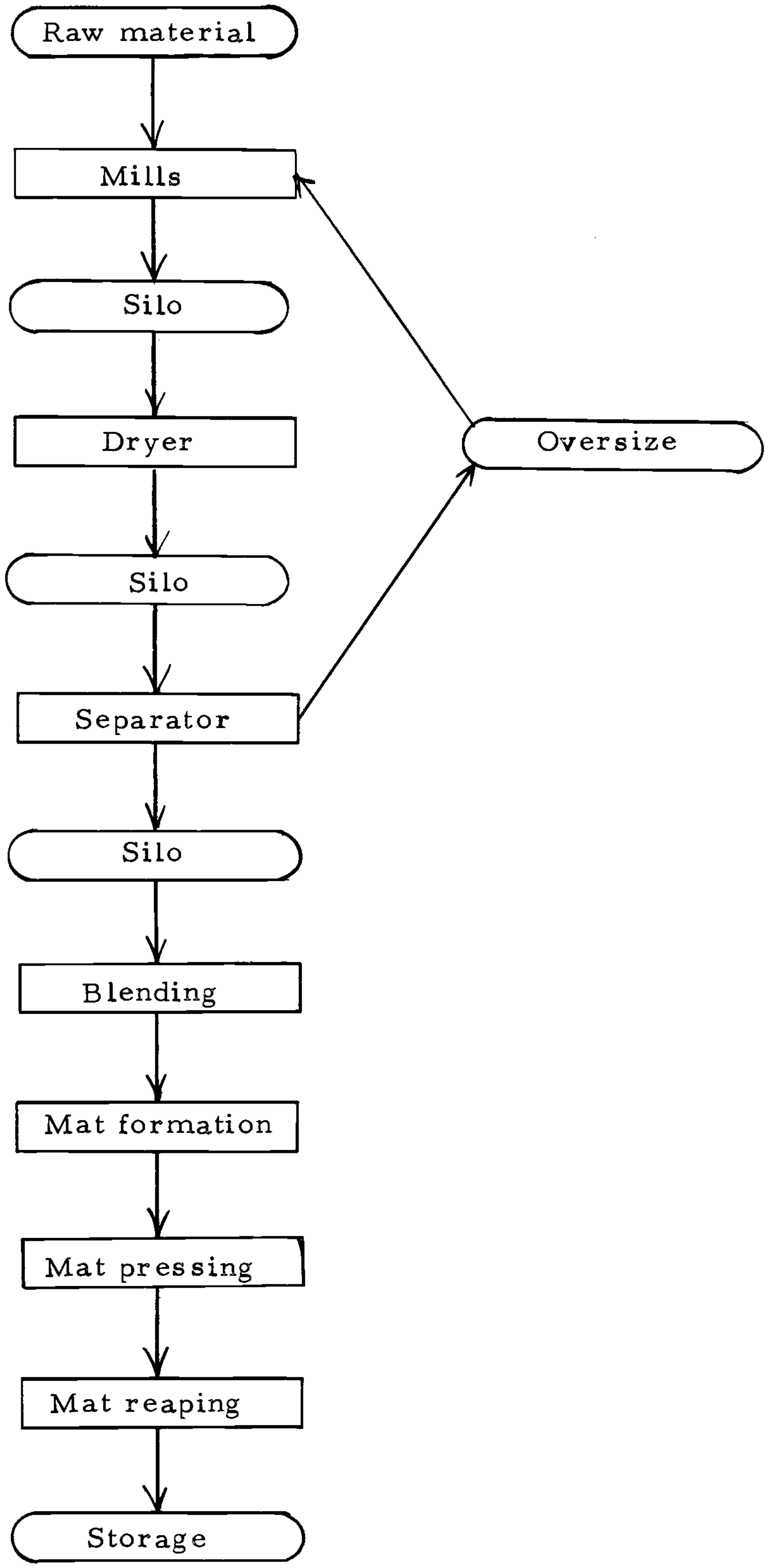



Figure 2-1. Flat-press particleboard flow process.

> of such resources. . . . Objective and pertinent data can be obtained through cooperative efforts of all concerned personnel, which means obtaining not only management authorization but also agreement with individuals who work directly with the system.

Therefore, what is appropriate to do now is to sort out the data and quantify them. We do this by analyzing the process flow diagram of the manufacturing system (Figure 2-2). Pertinent data are given by Carrasco, Curiel and Serrano (1976, p. 46) and their correspondence with "CONACYT" (National Counsel of Science and Technology of Mexico).

In summary, some of the more relevant information is given in Tables 2-1, 2-2, 2-3 and in Figure 2-2. The data were converted from the daily figures to yearly figures, in order to utilize the demand forecasts which were estimated on the yearly basis.

Table 2-1. Particleboard Forecasted Demands, Indirect Costs and Transfer Costs.

| Type of board (mm) | | 1975 Forecasted demand (tons/yr) | Indirect costs ($/ton) | Transfer costs ($/ton) |
|---|---|---|---|---|
| DODEM | 10 | 1120 | | |
| | 13 | 1162 | | 352.77 |
| | 16 | 2446 | | |
| | 19 | 1180 | | |
| | | | 749.63 | |
| EXDEM | 10 | 1286 | | |
| | 13 | 1738 | | 277.19 |
| | 16 | 3652 | | |
| | 19 | 1670 | | |

Source: Carrasco, Curiel and Serrano (1976).

Table 2-2. Data on the characteristics and price for the variety of boards sold in Mexico.

| A | B | C | $D=\frac{1000}{A}$ | E |
|---|---|---|---|---|
| Type of board (per mm) | Weight ($kg/m^3$) | Selling price per sheet ($) | Conversion factor (units/$m^3$) | Volume ($m^3$) |
| 10 | 850 | 33.43 | 100.00 | 0.03733 |
| 13 | 830 | 36.96 | 76.92 | 0.04853 |
| 16 | 830 | 41.91 | 62.50 | 0.05973 |
| 19 | 830 | 46.86 | 52.63 | 0.07093 |

| | $F=\frac{B}{D}$ | $G = C \times D$ | $H=\frac{G}{B\times 1000}$ | I |
|---|---|---|---|---|
| | Board weight (kg/sheet) | Selling price per $m^3$ ($/$m^3$) | Selling price per ton ($/ton) | Distributor selling price ($/ton) |
| 10 | 8.5 | 3343.00 | 3932.94 | 3362.66 |
| 13 | 10.79 | 2842.96 | 3425.25 | 2928.59 |
| 16 | 13.28 | 2619.39 | 3155.89 | 2698.29 |
| 19 | 15.77 | 2466.24 | 2971.37 | 2540.52 |

Source: Carrasco, Curiel and Serrano (1976).

Table 2-3. Blending components per type of boards (% per ton).

| Type of board (per mm) | Fine Fibers | Normal Fibers | Aglutinant | Emulsionated Paraffin | Crude Ammoniac | Pure Ammoniac |
|---|---|---|---|---|---|---|
| 10 | 0.2672 | 0.3837 | 0.1392 | 0.0074 | 0.0006 | 0.0019 |
| 13 | 0.2055 | 0.6797 | 0.1071 | 0.0057 | 0.0005 | 0.0015 |
| 16 | 0.1670 | 0.7398 | 0.0870 | 0.0046 | 0.0004 | 0.0012 |
| 19 | 0.1406 | 0.7809 | 0.0733 | 0.0039 | 0.0003 | 0.0010 |

Source: Carrasco, Curiel and Serrano (1976).

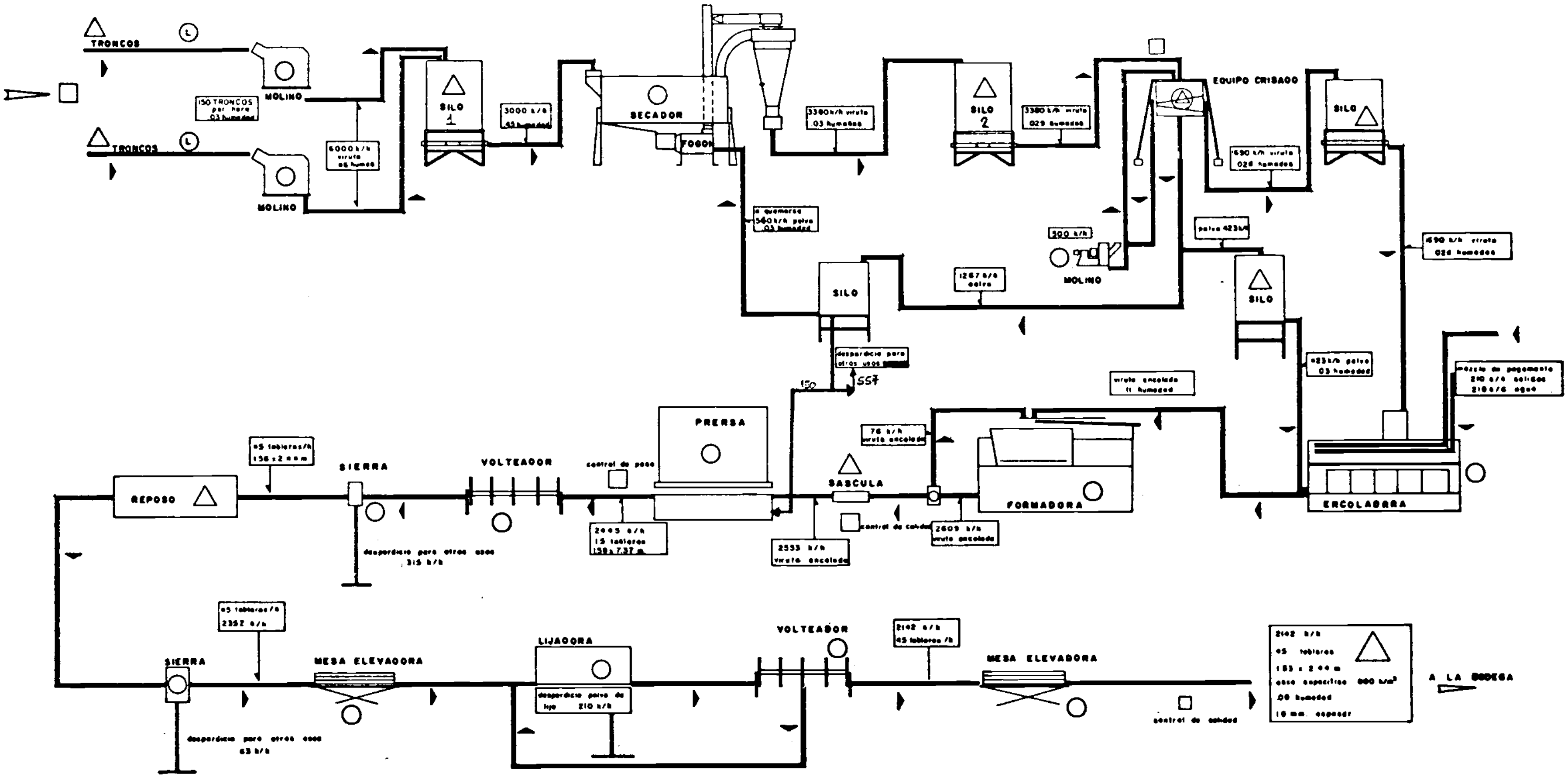



Figure 2-2. Manufacturing process flow diagram.

In the remaining portion of this chapter, we will propose and illustrate the use of RPM system to facilitate the particleboard production system analysis and design.

## C. Resource Planning and Management System (RPMS) Model Design

One of the major reasons for using a model in an OR/MS study is to clearly identify and communicate a problem.

> RPMS encourages more precise formulation of problems and provides a format for improved analysis and display of the solutions. It is visually apparent in the form of RPM networks, which are used both in describing and solving the problems (Riggs and Inoue, 1975, pp. ix-63).

The RPM networks are proposed as a tool in the particleboard production system analysis and design because of the following features:

1. RPM system is a general systems tool that can help decision makers deal with the complexity of today's organizations.
2. RPM networks encompass four types of process:
   a) Relational process by depicting any casual relationship among members of the system.
   b) Precedence process by depicting the chronological and technological relationships among elements of the system.
   c) Mathematical process, such as optimization, which has an objective for the system that can be portrayed mathematically.
   d) Stochastic process, such as simulation, which is used in experimentation.

3. Communication is achieved among management numbers and the plannification of the level of operations is facilitated by a better understanding of the limitations and costs implications of the production area.
4. Through RPM and linear programming techniques we may obtain optimal production plans, product mix (blending), scheduling, control, in-process inventory, opportunity costs, and making and buying decisions information.

Each major phase of the particleboard plant was separately analyzed to create cause and effect diagrams, and those were coupled to form an RPM network. The RPM network of the entire plant is shown in Figure 2-3. Notice that the numbers in the arrows correspond to the code number of the coefficients ($b_i$, $a_{ij}$, $c_j$) in the data Table 2-4. Each major phase of the RPM network will be explained in the next section for the better understanding of the system.

## D. Description of the Model

The RPM approach has been implemented over the particleboard manufacturing process. The model was built based on the information mentioned before. The information included data for the eight different products to be produced according to the following fields: raw material requirements (tons); raw material costs (\$/ton); labor requirements (hr/ton); labor costs (\$/hr); labor rates

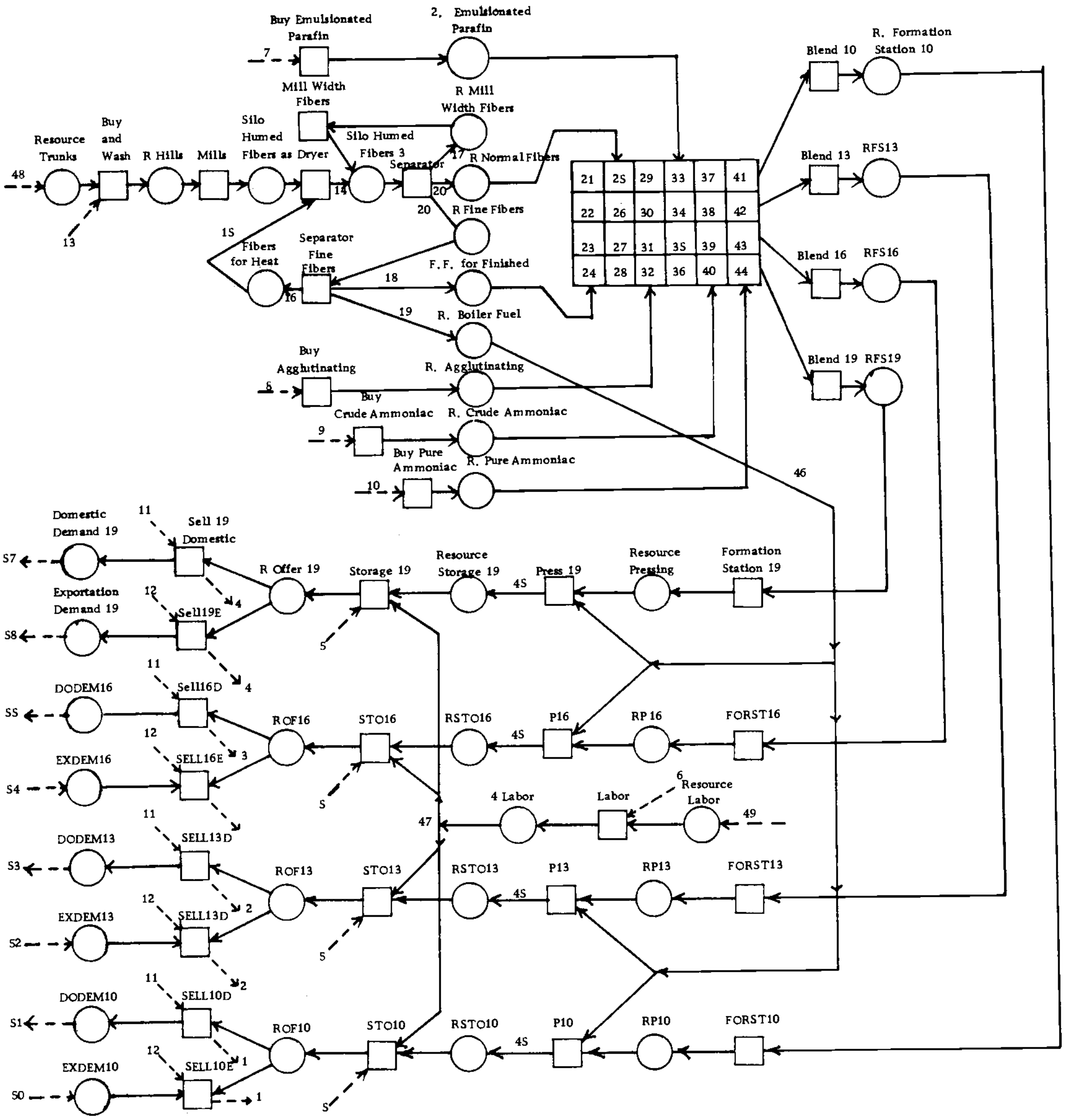



Figure 2-A. RPM Network of the particleboard production system.

Table 2-4. Most likely, lower limit and upper limit estimates of the coefficients ($b_i$, $a_{ij}$, $c_j$).

| | Code no. | Most likely (M) | Lower limit (L) | Upper limit (U) |
|---|---|---|---|---|
| $c_j$ | 1 | 3362.66 | 3327.70 | 3468.85 |
| | 2 | 2928.59 | 2897.76 | 3021.07 |
| | 3 | 2698.29 | 2669.88 | 2783.50 |
| | 4 | 2540.52 | 2513.78 | 2620.75 |
| | 5 | 749.63 | 749.63 | 811.55 |
| | 6 | +112.1941 | 112.1941 | 136.88 |
| | 7 | 8150.00 | 8150.00 | 9943.00 |
| | 8 | 6206.25 | 6206.25 | 7190.00 |
| | 9 | 2750.00 | 2750.00 | 3355.00 |
| | 10 | 8300.00 | 8300.00 | 10126.00 |
| | 11 | 277.19 | 277.19 | 299.72 |
| | 12 | 352.77 | 352.77 | 367.79 |
| | 13 | 75.00 | 70.00 | 90.00 |
| $a_{ij}$ | 14 | 0.5633 | 0.5351 | 0.5915 |
| | 15 | 0.09 | 0.081 | 0.099 |
| | 16 | 0.4538 | 0.4356 | 0.472 |
| | 17 | 0.1288 | 0.12236 | 0.13524 |
| | 18 | 0.2503 | 0.2403 | 0.2603 |
| | 19 | 0.2959 | 0.2841 | 0.3077 |
| | 20 | 0.4353 | 0.41354 | 0.45707 |
| | 21 | 0.2672 | 0.25384 | 0.28056 |
| | 22 | 0.2055 | 0.19728 | 0.21372 |
| | 23 | 0.167 | 0.16199 | 0.17201 |
| | 24 | 0.1406 | 0.13779 | 0.14341 |
| | 25 | 0.5837 | 0.55452 | 0.61289 |
| | 26 | 0.6797 | 0.65251 | 0.70689 |
| | 27 | 0.7398 | 0.71761 | 0.76199 |
| | 28 | 0.7809 | 0.76528 | 0.79652 |
| | 29 | 0.1392 | 0.13224 | 0.14616 |
| | 30 | 0.1071 | 0.10282 | 0.11138 |
| | 31 | 0.087 | 0.08439 | 0.8961 |
| | 32 | 0.0733 | 0.07183 | 0.07477 |
| | 33 | 0.0074 | 0.00703 | 0.00777 |
| | 34 | 0.0057 | 0.00547 | 0.00593 |
| | 35 | 0.0046 | 0.00446 | 0.00474 |

(Continued on next page)

Table 2-4. (Continued)

| | Code no. | Most likely (M) | Lower limit (L) | Upper limit (U) |
|---|---|---|---|---|
| | 36 | 0.0039 | 0.00382 | 0.00398 |
| | 37 | 0.0006 | 0.00057 | 0.00063 |
| | 38 | 0.0005 | 0.00048 | 0.00052 |
| | 39 | 0.0004 | 0.00039 | 0.00041 |
| | 40 | 0.0003 | 0.00029 | 0.00031 |
| | 41 | 0.0019 | 0.00181 | 0.00200 |
| $a_{ij}$ | 42 | 0.0015 | 0.00144 | 0.00156 |
| | 43 | 0.0012 | 0.00116 | 0.00124 |
| | 44 | 0.001 | 0.00096 | 0.00122 |
| | 45 | 0.8456 | 0.82023 | 0.87097 |
| | 46 | 0.0592 | 0.05802 | 0.06036 |
| | 47 | 2.1504 | 2.1504 | 2.4370 |
| | 48 | 41175.0 | 32940.0 | 43150.0 |
| | 49 | 48037.5 | 33092.5 | 48037.5 |
| | 50 | 1286.0 | 1137.0 | 1435.0 |
| | 51 | 1120.0 | 993.0 | 1247.0 |
| $b_i$ | 52 | 1738.0 | 1668.0 | 1808.0 |
| | 53 | 1162.0 | 1104.0 | 1220.0 |
| | 54 | 3652.0 | 3506.0 | 3798.0 |
| | 55 | 2446.0 | 2324.0 | 2568.0 |
| | 56 | 1670.0 | 1603.0 | 1737.0 |
| | 57 | 1180.0 | 1121.0 | 1239.0 |

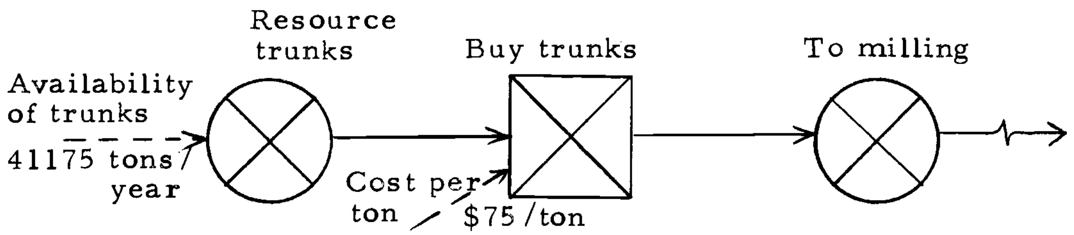
These estimates were gathered from three sources: CONACYT, machine specifications, and direct estimate methods. These sources will be further discussed in this thesis.

(hr/ton); direct costs ($/ton); indirect costs ($/ton); selling prices ($/ton); demands (tons/yr); processing operations (cause-and-effect relationships); and production yields.

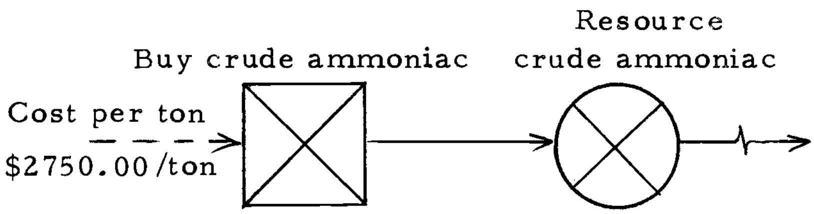
In order to understand the RPM description of the system, let us consider each segment of the network separately.

## Raw Material Requirements and Costs

The only major restriction of raw material constitutes the cactus trunks, that is 41,175 tons/yr due to the capacity of the mills (6000 kg/hr). On the other hand there is practically unlimited supply of emulsionated paraffin, agglutinating, crude ammoniac and pure ammoniac. The RPM segment of Figure 2-3 illustrates these purchasing operations of the model.




a. Limited raw material




b. Unlimited raw material

Figure 2-3. RPM segment illustration of raw materials requirements and costs.

Labor Requirements, Costs and Rates

The information about labor requirements, costs, and labor rate was provided on per unit bases. They are shown in the RPM segment of Figure 2-4.

Indirect Costs, Selling Prices and Demands

The indirect costs and selling prices were computed on a per unit basis ($/ton) and the demands per type of board in tons/yr. Figure 2-5 illustrates these concepts as represented by a segment of the RPM network.

Production Process

Information on the production process was provided and arranged into the percentage form. The illustration of the production process is provided in another RPM segment of the network (Figure 2-6).

## E. Model Verification

The model was evaluated in terms of its effectiveness as a documentation and communication tool; the model is simple, clear, understandable and representative of the problem as it appears to management.

The evaluation as an experimental tool will be discussed in Chapter V.

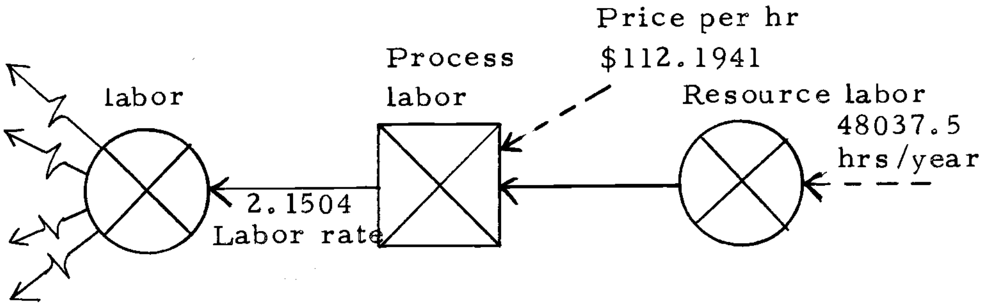



Figure 2-4. RPM segment showing labor requirements, costs and rates.

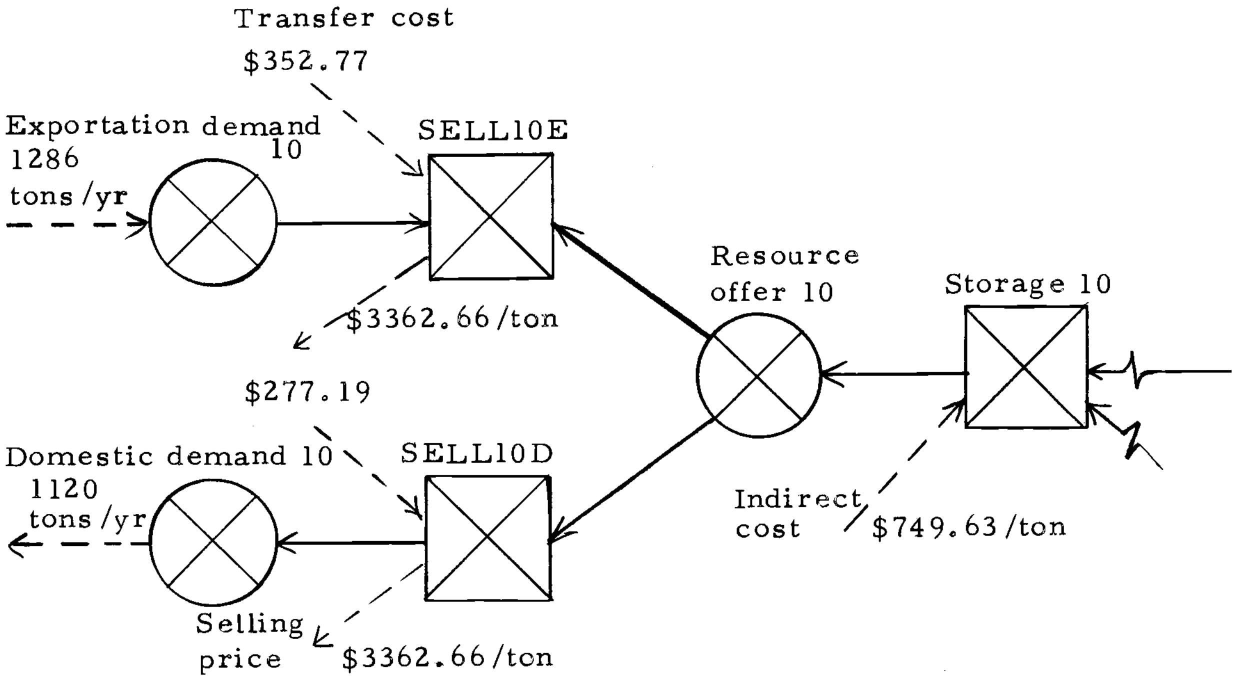



Figure 2-5. RPM segment illustrating indirect costs, selling prices and demands.

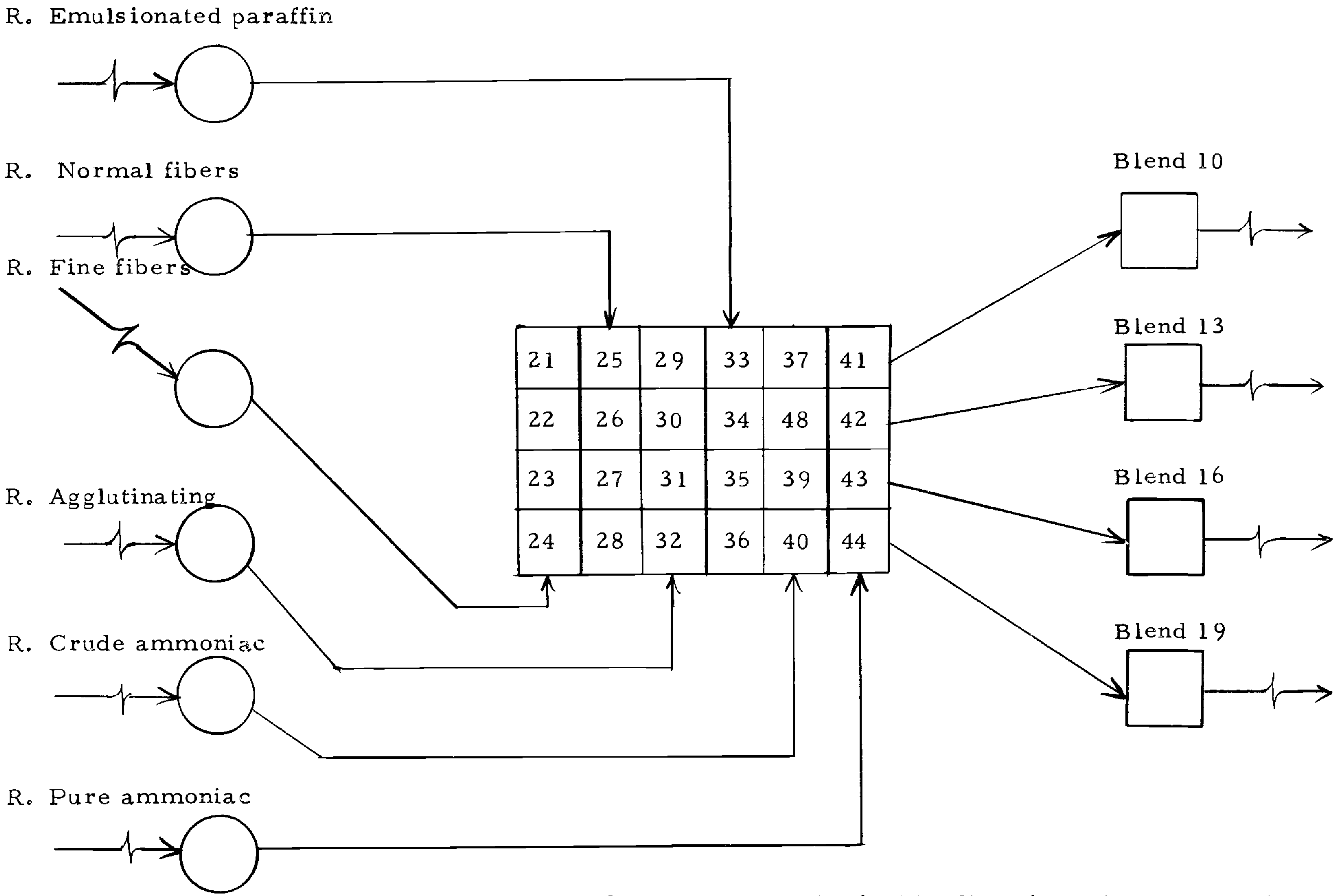



Figure 2-6. RPM segment illustration of production process in the blending phase (percentages).

# III. LINEAR OPTIMIZATION TECHNIQUES FOR IMPROVED PLANNING OF PRODUCTION

## A. The Need for Optimization Techniques

The production scheduling problem has been defined by Von Lanzenauer (1970, p. 104) as the question of knowing when and how much of what to produce in order to meet market requirements and to optimize some well-defined objective function.

As an organization grows in complexity and specialization, it becomes increasingly difficult to allocate resources in a manner that is most effective for the organization as a whole. The peace time use of operations research grew out of the necessity to utilize a scientific approach to optimization problems in industry (Hillier and Lieberman, 1967, p. 3).

Recognizing the advantage of using operations research techniques, the Inband Signaling Shop of Western Electric Company initiated the development of a computer-based mathematical model for production scheduling and control in January 1972 (May, 1974, p. 277). The continuing and steadily growing number of applications of operations research, especially linear programming, in a wide variety of industries such as petroleum, forestry, mining, manufacturing, etc. (Wagner, 1975, p. 53) amply demonstrates that it is practical and profitable to use mathematical models for management planning.

Before applying operations research techniques to the particleboard production problem, however, we must first examine two aspects of our study in relation to the applicability of such techniques. In this chapter, we shall first examine the type of information available for decision making, and then the type of operations research techniques available to assist the management in planning the production.

## B. Management Decision Theory

Hundreds of decisions daily go into scheduling jobs, hiring labor, ordering supplies, negotiating with subcontractors, planning production facilities, managing inventories, etc. in a big industry like the one to be analyzed. A human mind cannot possibly consider all alternatives and weigh the manifold complexities and interactions of all factors of production at once.

Industrial engineers, operation researchers, engineering economists, and management scientists are among those who develop tools to aid managers in their decision-making tasks. These tools, usually quantitative, fall into one of the following three categories depending upon the availability of information (Whitehouse and Wechsler, 1976, pp. 23-25); (1) decision under assumed certainty; (2) decision under risk; and (3) decision under uncertainty.

George A. W. Boehm wrote for the _Fortune_ magazine in April 1962 that:

> Some of the techniques are best suited to situations in which though all the factors are known or predictable, the complexity is so confusing that the human mind cannot arrive at a wholly rational decision. Other techniques cope with "risks"--chances that can be accurately measured or calculated, such as the probability that a given number of insurance holders will die within a year. Still others deal with "uncertainties"--chances that can be estimated only roughly at best, because, for example, they depend on future developments or the behavior of a competitor. All decision theory, however, has a common purpose: to show decision makers purer ways to attain goals.

To summarize the distinctions between certainty, uncertainties, and risk situations in management decision theory, we may quote from a wide variety of literature, such as Van Gigch (1974, p. 69), Easton (1973, p. 130), and Plane and Kochenberger (1972, p. 17).

## Decision under Assumed Certainty

A decision is made under certainty when the decision maker knows what the result will be for each course of action he might follow. The difficulty of decision making under certainty is that there are often so many courses of action that it may be impossible to consider each of them individually, determine the result for each course of action, and choose the best result for each of these courses of action. Linear programming is primarily an optimization tool related to decision making under certainty.

Decision under Risk

A decision is said to be under risk if for each course of action available to the decision maker there is a meaningful probability distribution over the outcomes; a decision maker in almost any organization is often faced with this kind of decision problem. Stochastic programming is related with decision making under risk.

Decision under Uncertainty

The third category of decision problems is decision problems under uncertainty. We say a decision problem is under uncertainty if there is no meaningful probability distribution over the outcomes that may occur for each course of action available to the decision maker. In this kind of decision problem, the decision maker simply has no idea what is likely to happen. He has no feeling, no judgement, no hunch about the relative likelihoods of the occurrence of the various outcomes that he might experience. Game theory models cope with uncertainty conditions.

Given the quality of the information collected for the particle-board production system, we shall deal only with decisions under certainty and risk.

## Optimization Techniques

Three broad categorizations are possible under the optimization techniques applicable to conditions of assumed certainty and risks: typically, those are called Linear Programming, Resource Planning and Management, and Stochastic Programming.

### Linear Programming

Linear programming is a mathematical means for providing the decision maker with a basis for resolving complex operational alternatives. It is applicable to a general category of optimization problems involving the interaction of many variables subject to certain constraints. These constraints usually arise because the activities under consideration compete for scarce resources. A basic supposition in linear programming is the existence of linearity. The objective is to optimize some linear effectiveness function subject to linear constraints. This may require minimization of time, of cost, or of distance, or it may require the maximization of profit depending upon the problem under consideration (Fabrycky, Ghare and Torgersen, 1972, p. 440).

The general mathematical formulation of the linear programming problem can be written as follows (Riggs and Inoue, 1975, p. 114):

$$\text{maximize} \quad z_x = \sum_{j=1}^{n} c_j x_j \qquad (3.1)$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \qquad 1 \leq i \leq m \qquad (3.2)$$

$$x_j \geq 0 \qquad 1 \leq j \leq n \qquad (3.3)$$

Notice that the non-negativity constraint (3.3) maintains that the primal variable, $x_j$, must be either positive or zero; there is no such restriction imposed upon the sign of parameters and constraints, $c_j$, $a_{ij}$, and $b_i$ .

The dual problem for the same linear programming problem (3.1) through (3.3), can be written as:

$$\text{minimize} \quad z_y = \sum_{i=1}^{m} b_i y_i \qquad (3.4)$$

subject to

$$\sum_{i=1}^{m} a_{ij} y_i \geq c \qquad 1 \leq j \leq n \qquad (3.5)$$

$$y_i \geq 0 \qquad 1 \leq i \leq m \qquad (3.6)$$

The use of RPM system methodology jointly with RPM1 linear programming software package and RPM2 stochastic simulation software package developed by Steve Shu-Kang Chou (1977) are proposed here to handle the mathematical optimization techniques in a more communicative and understandable way to analyze and propose a proper particleboard production planning scheduling and control.

Throughout this thesis we make use of the RPM networks assuming that the reader has a basic knowledge of such symbology. In this chapter we present a brief summary of some fundamental concepts and notations of the RPM representation. For a more detailed description of this methodology, the reader should refer to Riggs and Inoue (1975), Inoue and Eslick (1976) and to Engesser, Inoue and Mercer (1976).

RPM Approach to Linear Programming

Consider the conventional definition of a linear programming model (1), (2) and (3). The free parameters and constraints ($b_i$, $a_{ij}$, $c_j$) in an RPM representation are made non-negative by distinguishing the positive (+) components from the negative (-) components. The components value equals the absolute value of the parameter when the sign matches, otherwise, it is considered to be zero. Thus, equations (1) through (3) can be written as:

$$\text{maximize} \quad z_x = \sum_{j=1}^{n} c_j^+ x_j - \sum_{j=1}^{n} c_j^- x_j \tag{3.7}$$

subject to

$$\sum_{j=1}^{n} a_{ij}^- x_j + b_i^+ \geq \sum_{j=1}^{n} a_{ij}^+ x_j + b_i^- \tag{3.8}$$

where

$$b_i = b_i^+ - b_i^- \quad ; \quad b_i^+ \cdot b_i^- = 0 \tag{3.9}$$

$$a_{ij} = a_{ij}^+ - a_{ij}^- \quad ; \quad a_{ij}^+ \cdot a_{ij}^- = 0 \tag{3.10}$$

$$c_j = c_j^+ - c_j^- \quad ; \quad c_j^+ \cdot c_j^- = 0 \tag{3.11}$$

and all variables, parameters, and constraints are restricted to be non-negative values:

$$b_i^+,\ b_i^-,\ a_{ij}^+,\ a_{ij}^-,\ c_j^+,\ c_j^-,\ x_j \geq 0 \tag{3.12}$$

for $$1 \leq i \leq m \quad \text{and} \quad 1 \leq j \leq n \tag{3.13}$$

The dual for the same linear programming problem (1) through (3), can be expressed as:

$$\text{minimize} \quad z_y = \sum_{i=1}^{m} b_i^+ y_i - \sum_{i=1}^{m} b_i^- y_i \tag{3.14}$$

subject to

$$\sum_{i=1}^{m} a_{ij}^+ y_i + c_i^- \geq \sum_{i=1}^{m} a_{ij}^- u_i + c_i^+ \tag{3.15}$$

and $$y_i \geq 0 \tag{3.16}$$

Again the same ranges (3.13) and non-negative conditions (3.12) apply.

The basic linear RPM network is graphically portrayed by three node symbols and a network structure created with two types of arrows. Thus, the RPM symbology uses circles to represent resource nodes, squares to represent process nodes and triangles to represent the maximizing and minimizing nodes. And, the solid arrows that relate resource nodes to process nodes with the arrow-head indicating the direction of the inequality. Dotted arrows connect the activity and resource nodes with the terminal nodes as shown in Figure 3-1.

In the General Standard LP-RPM Network, Figure 3-2, the dotted arrows indicate the exogenous inputs into resource modes from

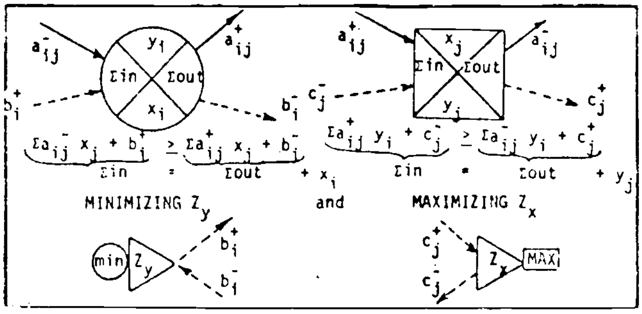



Figure 3-1. Symbology for RPM linear programming representation.

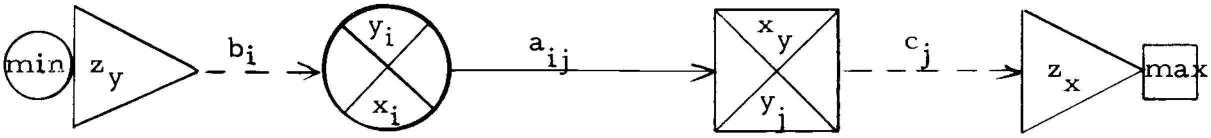



Figure 3-2. General standard LP-RPM network.

the resource objective function $z_y$, or the endogenous outputs from the process nodes to the process objective function $z_x$.

The unique ability of the RPM networks to show both primal and dual values, also provides a direct source of information so that the system can be translated to a LP format of standard LP packages without the need of writing equations (Riggs and Inoue, 1975).

Generally, after solving the linear programming model, the solutions are inserted back onto the network according to the notation of Figure 3-1, for checking feasibility and optimality.

What we have in mind, taking into account that all the coefficients ($b_i$, $a_{ij}$, $c_j$) are known exactly, is to run three kinds of deterministic models: first, deterministic run using as coefficients the most likely values (M). In general, the graphical portrayal of these cases is given in Figure 3-2. Second, deterministic run using as coefficients the estimated mean computed using pert technology. These coefficients estimate (TCE) mean is computed as:

$$TCE = \frac{U + 4M + L}{6} \qquad (3.17)$$

where:

U = Upper limit of the coefficient

M = Most likely value

L = Lower limit of the coefficient

To illustrate this case in general by means of RPM network representation, you should refer to Figure 3-3. And third, bounded deterministic run with the most likely values (M), using as upper limit, the resulting activity level values from the solution in case 1. We are going to run this (in order) to prove the drop in profit where simulating with the three estimators. In order to show a graphical display of this concept, Figure 3-4 illustrates a segment of Figure 2-2, applying the upper bounds over the process nodes. Notice that the only resource node with upper bound in Figure 3-4 is MILLS, because BAW is already bounded by the raw material availability restriction.

Thus for the RPM networks used in this approach include linear relationships and deterministic values. This basic supposition of linearity in the LP models has four major limitations: (a) the continuity limitation--implies that a variable may assume any non-negative fractional value between zero and positive infinity; (b) proportionality--means that all relations are linear when plotted in a graph; nonlinear relationships must be approximated by piecewise, linear relationships or a set of overlapping constraints and processes these relationships must all be deterministic; (c) additivity--implies that all flows into a resource mode must be homogeneous and indistinguishable; and (d) linearity of the objective function--assumes that the trade-offs are possible among processes as to their contribution to the objective function.

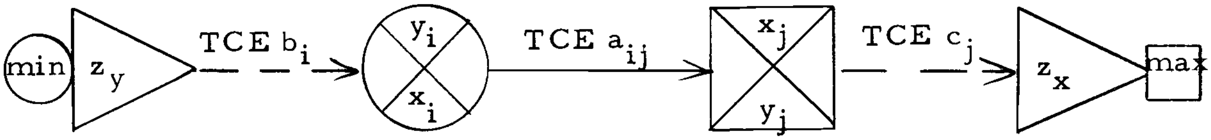



Figure 3-3. General RPM network for (TCE), the coefficients estimate mean.

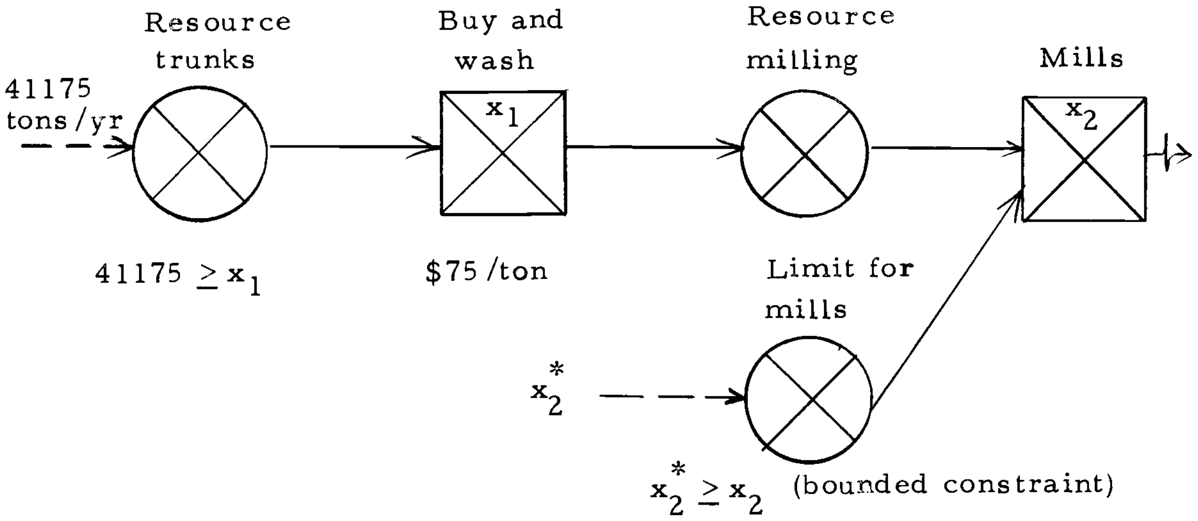



Figure 3-4. Segmented RPM showing the application of upper bounds over the processes nodes.

The assumption stated above facilitates the construction and understanding of the model, but also reflect crude approximations and may hide the effects of not including stochastic phenomena in the model.

## Stochastic Programming

The deterministic model used in linear programming may be an excellent tool to analyze in retrospect the decisions that we should have made, if all information is known exactly. In any management planning process, there is a large degree of stochasticity which renders any decision a risky proposition. Instead of a deterministic constant or coefficient to use in a linear programming model, we are insted given a range of values that such a parameter may assume. At best, what we have is a well-behaved statistical distribution. In most cases, we shall be so lucky as to have three estimates: an optimistic value, a most likely value, and a pessimistic estimate.

There are three major approaches to handle this problem of stochasticity. First, it is possible to break the problem in two stages and consider the decisions on outcomes based on expected values. Second, it is possible to incorporate a margin of safety within constraints to correspond with a given level of confidence. The latter model is called a chance-constrained programming model. And third, it is possible to break the problem into stages and consider the decisions as information becomes available on outcomes of our previous decisions. Such a model is commonly known as a multi-stage decision model. Finally, our purpose is to utilize Monte Carlo sampling techniques to simulate the risk conditions and the behavior of the optimal values of decisions under such varying conditions.

In the following chapters, these approaches will be examined one by one, together with computer models and their solutions. The comparisons provided valuable information on the relative precision, computer costs, and practicality of each approach, as well as to serve as a validation tool to reconfirm our results before recommending the implementation of our study results.

# IV. STOCHASTIC PROGRAMMING TECHNIQUES ENHANCEMENT FOR IMPROVING PRODUCTION PLANNING, SCHEDULING AND CONTROL

## A. Stochastic Programming Problem

A linear programming problem is said to be stochastic if one or more of the coefficients or constants in the objective function, the system of constraints, or resource availabilities is known only by its probability distribution. The available approaches to deal with this problem may be classified into three broad types: "two stage programming (TSP)," "chance constrained programming (CCP)" and "stochastic linear programming (SLP)" (Sengupta, Tintner and Millham, 1963). At this point, it is necessary to make clear that the terms "probabilistic programming" (Vajda, 1972; Sengupta and Fox, 1969) and "stochastic programming" (Wagner, 1975; Sengupta, Tintner and Millham, 1963) refer in essence to the same set of algorithms.

Following Tintner (1955), we distinguish two basic types of stochastic programming, the passive and the active approaches. The passive stochastic program arises when we follow the "wait and see" approach. More specifically, we wait for the observations of the random variables to occur and by utilizing these realized values in a suitable manner, we identify the proper probability distribution of the maximand (i.e., the maximum value of the objective function) and of the optimal decision. The second method of stochastic programming, called the active approach, defines a "here-and-now" attitude. The decision is made at once without waiting for the realizations of the random variations of the coefficients ($a_{ij}$, $b_i$ and $c_j$) (Sengupta, 1972, pp. 1-2).

In this study, we are going to deal primarily with LP, TSP, CCP and SLP approaches by means of Monte Carlo simulation in order to apply the "here-and-now" attitude for finding the proper production plan of particleboards depending upon management considerations of the random variations acting within the resources ($b_i$), the costs and selling proces ($c_j$) and the technological coefficients ($a_{ij}$) limitations.

## The Need for Stochastic Programming

The need for developing methods of probabilistic (or stochastic) programming in the context of linear programming models arose from at least three different sources: (1) the errors and deviations in parameters which sometimes can be associated with probability measures; (2) the presence of risk and uncertainty which sometimes allow a meaningful numerical representation of the utility function of a decision maker; and finally, (3) the requirement of developing optimal decision rules, which is essentially related to the theory of statistical decision functions (Sengupta and Fox, 1969, p. 197).

Inherent in the solution of many linear programming problems is the tacit assumption that the parameters involved are deterministic in nature. But, in many situations, production decisions must be made in the face of varying demand, the costs or profits to be expected are fluctuating, and the technological coefficients may be subjected to stochastic variation. George Dantzig (1962) seems to have been the first to note that current practice is to try to avoid the random character of the parameters by providing "plenty of fat" (to use his terminology) in the system, in the hope that this will provide enough "excess" capability to execute the program without failure. If demands, for example, can be shifted in time or the capabilities are well above requirements, this may be adequate. However, it is clear that if the problem is not tightly constrained, then decisions based on the model will very likely not be optimal (Evers, 1967, p. 680).

Two-stage Linear Programming (TSP) Model

Let us assume that in a deterministic version of the LP, the coefficients in the objective function have random variations, and that all the levels of the variables have to be determined prior to learning the actual values, $c_j$, of the random variables for costs and selling prices. Wagner (1975, p. 668) provides the following mathematical definition (eqs. 4.2 to 4.4) of the two-stage linear programming problem where the coefficients $a_{ij}$ and $b_i$ in the linear programming model are known exactly, and $c_j$ is a random variable independent of all activity levels $x_j$. If the levels of $x_j$, for $j = 1, \ldots n$, must be set prior to knowing the exact values of $c_j$, then a solution that

$$\text{maximize} \quad E\left[\sum_{j=1}^{n} c_j x_j\right] \tag{4.1}$$

is given by levels for $x_j$ that

$$\text{maximize} \quad \sum_{j=1}^{n} E[c_j]\, x_j \tag{4.2}$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \qquad 1 \leq i \leq m \tag{4.3}$$

$$x_j \geq 0 \qquad 1 \leq j \leq n \tag{4.4}$$

and where $E[\ ]$ indicates an expected value function.

Notice that the optimal solution can be found from an equivalent deterministic linear program, where the corresponding expected values are used in the objective function.

Chance-constrained Linear Programming (CCLP) Model

The problem of chance-constrained programming has been defined by Charnes and Cooper (1962) as follows: "Select certain random variables as functions of random variables with known distributions in such a manner as to maximize a functional of both classes of random variables subject to constraints on these variables which must be maintained at prescribed levels of probability."

We describe this model by writing down the objective function which is the same as for TSLP:

$$\text{maximize} \quad \sum_{j=1}^{k} E[c_j]\ x_j \tag{4.5}$$

We now proceed replacing the constraint (2) by the chance-constraints:

$$P\left[\sum_{j=1}^{m} a_{ij}\, x_j \leq b_i\right] \geq \beta_i \qquad 1 \leq i \leq m \tag{4.6}$$

$$\text{and} \qquad x_j \geq 0 \qquad 1 \leq j \leq n \tag{4.7}$$

We interpret (4.6) as constraining the unconditional probability to be no smaller than $\beta_i$, where $0 \leq \beta_i \leq 1$, meaning that the actual value of $b_i$ is at least as large as $\sum_{j=1}^{m} a_{ij}\, x_j$.

## B. Stochastic Linear Programming Model

Consider the LP problem (3.1) through (3.3), which we can write in matrix form as (Hillier and Lieberman, 1967, p. 531):

$$\text{maximize} \quad x_0 = \underline{c}\underline{x} \tag{4.8}$$

subject to

$$\underline{A}\underline{x} \leq \underline{b} \tag{4.9}$$

$$\underline{x} \geq \underline{0} \tag{4.10}$$

where:

$\underline{c}$ = row vector = $[c_1, c_2, \ldots, c_n]$

$\underline{x}$, $\underline{b}$, and $\underline{0}$ are the column vectors such that:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}, \quad 0 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

and $\underline{A}$ is the matrix,

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & & a_{mn} \end{bmatrix}$$

Now suppose that some or all of the parameters (the $c_j$, $a_{ij}$, and $b_i$) are random variables rather than constants. This necessitates a reformulation of the objective function. Since $x_0$ becomes a random variable if any of the $c_j$ are random variables, and it is meaningless to maximize a random variable, $x_0$ must be replaced by some deterministic function. There are many possible choices for this function, each of which may be very reasonable under certain circumstances. Perhaps the most natural choice, and the one assumed traditionally, is maximize the expected value of $x_0$, and we write this as:

$$\text{maximize} \quad E(x_{\underline{0}}) = E\,(\underline{cx}) \tag{4.11}$$

subject to

$$\underline{A}^{(k)}\,\underline{x} \leq \underline{b}^{(k)} \qquad (4.12)$$

$$\underline{x} \geq 0 \qquad (4.13)$$

where

$A^{(k)}$ = Matrix of $a_{ij}$ random elements

$b^{(k)}$ = Column vector of $b_i$ random elements

One interpretation is that a solution is considered feasible only if it satisfies all of the contraints for all possible combinations of the parameter values. No practical solution procedure has yet been developed for solving the general problem described above (Hillier and Lieberman, 1967, p. 532).

## General Problems in Stochastic Programming

Wagner (1975, p. 654) states that probabilistic models are inherently harder to use than deterministic versions. First, there are new conceptual difficulties, such as the interpretation of the probabilities themselves and the meaning of optimality. In other words, consider the impact on your immediate and future decisions choices if you cannot know for certain what will happen as a result of your actions. Second, there are new technical difficulties relating to the mathematics of optimization. To illustrate, even when the stochastic model is a straightforward generalization of the deterministic version, the computational burden increases, since you must consider each possible event instead of only a single estimate. And third, there are increased data requirements for the specification of the probability distributions. For example, a manager may see that the price of his competitor's product fluctuates, but he may find it difficult to state a meaningful probability distribution for this variation.

The stochastic models described in the previous sections assume that the decision maker can state probability distributions to describe the elements of risk in the model. But in reality, stating probability distributions is not an easy task. Wagner (1975, p. 662) as a pragmatic matter, suggests four approaches to obtain probability distributions:

1. Use introspection--use experience to bear in quantifying your judgment, and as the system operates for a while and data are accumulated, then apply numerical techniques of Bayesian analysis to update your probability assessments.
2. Employ historical data--to compute empirical distributions.
3. Find convenient approximations--calculating empirical mean and perhaps the variance, making whatever judgmental corrections. Then in a computerized stochastic control model, employ, as an approximation, a normal distribution having such mean and variance.

4. State descriptive axioms--for instance, the manager chooses a model describing the process by which demand is generated. Obviously, this method is more complicated than approach 3, but it can be effective when the resultant analysis provides an explicit form for the probability distribution of demand, such as a Poisson or a Binomial distribution. Then the historical data and judgmental corrections are used to obtain the few parameters needed to describe the derived probability law.

Due to the characteristics of the data available for this study, we are going to combine some of these approaches with machine specifications and tolerances, management experience and direct estimate methods in order to get three estimators (upper limit, most likely, and lower limit) to describe these random variations that were shown in Table 2-4.

## C. RPM Approach for Stochastic Programming

Because of the risk elements involved in this approach no single deterministic model could represent the situation adequately. Indeed, since an "approximate solution" of a stochastic linear programming problem is defined usually by replacing each random element by its expected value and then solving the resulting non-stochastic program (Sengupta, Tintuer and Millham, 1963, p. 143), in order to obtain a fair statistical evaluation of the complexity and risk situation, a stochastic software simulation package "RPM2" was built using PERT network technology and Monte Carlo simulation technique (Chou, 1977).

The network is then subjected to Monte Carlo simulation and is constrained to act randomly within the coefficients ($b_i$, $a_{ij}$ and $c_j$) limitations (U, M, and L).

Implications and Assumptions

The RPM2 software simulation package handles the risk factors by assuming that the random coefficients ($b_i$, $a_{ij}$ and $c_j$) are beta distributed.

The beta distribution was selected because it fulfills the following characteristics (Greer, 1970, p. 103): (a) the shape of the distribution is flexible. There is no reason to think that probabilities will always follow the same pattern. This means, we will want a distribution with parameters which allow for shifting the central tendency to produce changes in symmetry or skewness; (b) the distribution has a discrete range and is easy to use. This is necessary if management is to feel comfortable and confident that the results of the analysis are realistic.

Mathematically the beta distribution can be written as:

$$f(t) = k\,(t-a)^{\alpha}\,(b-t)^{\beta} \qquad (4.14)$$

where the parameters a and b define the end points of the distribution while the exponents alpha ( $\alpha$ ) and beta ($\beta$) determine the shape of the distribution, and k is merely a normalizing factor to let the total area under the curve add up to unity (100% cumulative probability).

As a matter of illustration of the flexibility and non-symmetry shape, we present in Figure 4-1 three beta distributions having the same set of ending points (U and L). Notice that while $M_1$ is skewed to the right and $M_2$ to the left, $M_2$ is symmetrical. Notice also that all three M's represent their respective modes (most likely values).

Applications to the beta distribution to problems dealing with risk is not new. This is the distribution that has been used with PERT (Project Evaluation and Review Technique).

Like PERT, RPM2 uses only three parameters, upper (U), most likely (M) and lower (L). With U, M, and L values, we have only three degrees of freedom and in order to determine the beta distribution, the four parameters (a, b, $\alpha$, and $\beta$ ) must be known, then the beta distribution cannot be uniquely defined. To eliminate the fourth degree of freedom from the distribution, RPM2 as well as the PERT developers made the assumption that the standard deviation of the distribution is 1/6 of the range between upper (U) and the lower (L). Since in unimodal distributions, the mean plus or minus three standard deviations ($\mu - 3\sigma$ to $\mu + 3\sigma$) do in fact cover 95.05% of the area according to the Camp-Meidel extension of the famous

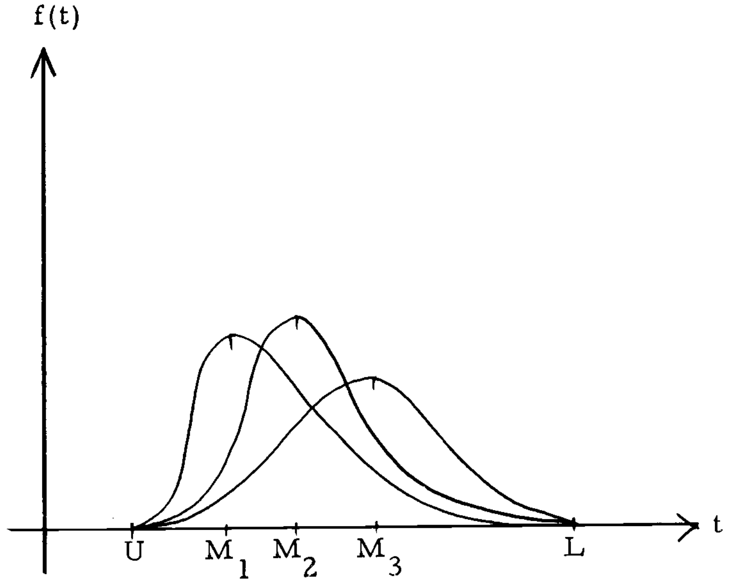



Figure 4-1. Examples of beta distributions.

Tchebycheff's theorem, the assumption appears somewhat justified (Riggs and Inoue, 1975).

In practice, there are several ways of obtaining the values of U (the best performance), M (the mode performance) and L (the worst performance) by means of the management experience, the machine specifications and by the direct estimate method.

## Direct Estimate Methods

Direct estimate methods that were used in obtaining the three parameters (U, M, L) were by means of confidence intervals and point estimates, using regression analysis, that is defined by Neter and Wassermann (1974, p. 21) as "A statistical tool which utilizes the relation between two or more quantitative variables so that one variable can be predicted from the other or others."

In fact, many examples of the use of regression analysis for prediction are found in business, such as estimating costs and forecasting sales (Neter and Wasserman, 1974, p. 30).

In general, and without going into detail, we are going to present the general linear multiple regression model, that is one of the most widely used of all statistical tools, and that uses the method of least squares. The general linear regression model is:

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_{p-1} x_{i,p-1} + \epsilon_i \qquad (4.15)$$

it can also be written as

$$y_i = \beta_0 + \sum_{k=1}^{p-1} \beta_k x_{ik} + \epsilon_i \qquad (4.16)$$

where

$y_i$ = the value of the independent (response) variable in the $i^{th}$ trial

$\beta_0$ , $\beta_k$ = parameters

$x_{ik}$ = known constants, dependent variable

$\epsilon_i$ = independent normally distributed with mean zero and variance ($\sigma^2$)

i = 1, 2, . . ., n

Assuming that $E(\epsilon_i) = 0$, the response function for the model (4.15)

$$E(y) = \beta_0 + \sum_{k=1}^{p-1} \beta_k x_{ik} \qquad (4.17)$$

The parameter $\beta_0$ is the y intercept when all $x_k$ are zero, and the parameter $\beta_k$ indicates the change in the mean response E(y) with a unit increase in the independent variable $x_k$, when all other independent variables $x_1$, $x_2$, etc. included in the model are held constant.

The predicted regression function is

$$y_k^+ = b_0 + \sum_{k=1}^{p-1} b_k x_{ik} = M \qquad (4.18)$$

where $y_k^+$ is the value most likely to occur, and $b_0$ and $b_k$ are unbiased estimators of $\beta_0$ and $\beta_k$, where $1 \leq k \leq (p-1)$.

The $1 - \alpha$ prediction interval for $y_k^+$ can be obtained by means of the t distribution, and is

$$y_k^+ - t_{(1-\alpha/2;\ n-p-1)} s(y_k^+) \leq y_k^+ \leq y_k^+ + t_{(1-\alpha/2;\ n-p-1)} s(y_k^+) \tag{4.19}$$

$$\text{lower limit} \leq \text{most likely} \leq \text{upper limit} \tag{4.20}$$

where $s(y_k^+)$ = sample standard deviation of $y_k^+$.

It is important to mention here that in many cases, the estimation of the parameters (L, M, U) can be found by other means as polynomial regression models, time series, etc. depending upon the behavior of the historical data used to predict or forecast the costs, future demands, etc.

These three parameters constitute the basic data for the RPM2 software package, in order to simulate the conditions of risk over the particleboard production system model.

## Simulation

In this section we will present the analysis of the particleboard model network via the RPM2 simulation techniques in light of the risk considerations.

As in RPM in LP, the use of Monte Carlo simulation to overcome many of the limitations of PERT has been proposed by many researchers (Inoue, 1977). The computer program that was used in this study employs this technique, under the assumption that the parameters (L, M, U) follow a triangular distribution. MacCrimon and Ryavec (1964) justified and advocated the use of triangular distribution as a less equivocal alternative to the uncertain beta distribution:

> . . . since there is no a priori justification for either function (beta or triangular) as an activity distribution, and since the actual standard deviations are unknown, the fact that the mean and standard deviation can be given exactly for a triangular distribution makes it an equally meaningful and more manageable distribution. It would be equally meaningful if its mean and standard deviation were used in a similar way to the approximate expressions used now, it would be more manageable if it was necessary to use the whole distribution, say in an analysis or a Monte Carlo study.

The effects other than normal distributions have led us to the logical conclusion of using Monte Carlo simulation assuming triangular distribution of the parameters, because of its simplicity and flexibility, as was stated before, and allowing us the opportunity of evaluating the effects of risk on this project, where the distribution of the coefficients is given.

Proposed Stopping Criteria for Simulating. In RPM2 (for simulating risk), that is an extension of RPM1 (for certainty), we obtain in general the following data results:

$n$ = Number of simulations made in a run. $n \geq 2$

$\overline{Z}^*_x$ = Average of the maximum objective function

$s(\overline{Z}^*_x)$ = Standard deviation of the maximum objective function

$U(\overline{Z}_x^*)$ = Maximum of the maximum objective function

$L(\overline{Z}_x^*)$ = Minimum of the maximum objective function

Since we have a random sample of n profit observations $Z_{x_1}, \ldots, Z_{x_n}$ from a normal population with mean $\mu$ and variance $^2$, the sample mean of profit $\overline{Z}_x^*$ and sample variance $s(\overline{Z}_x^*)$ are computed as follows (Neter and Wasserman, 1974, p. 9):

$$\overline{Z}_x^* = \frac{\sum_{i=1}^{n} Z_{x_i}^*}{n} \tag{4.21}$$

and

$$x(\overline{Z}_x^*) = \left[ \frac{\sum_{i=1}^{n} (Z_{x_i}^* - \overline{Z}_x^*)^2}{n - 1} \right]^{1/2} \tag{4.22}$$

The confidence interval for the profit expectation $\mu$ (population mean), with a confidence coefficient of $(1 - \alpha)$ is obtained by:

$$\overline{Z}_x^* - t_{(1-\alpha/2;\, n-1)}\, s(\overline{Z}_x^*) \leq \mu \leq \overline{Z}_x^* + t_{1-\alpha/2;\, n-1)} s(\overline{Z}_x^*) \tag{4.23}$$

because

$$\frac{\overline{Z}_x^* - \mu}{s(\overline{Z}_x^*)}$$

follows a t distribution with n-1 degrees of freedom because the random sample is from a normal population. Usually when simulating the question that continuously arises is, when shall we stop ?. In order to answer this question and due to the obtained results, we are going to make use of the chi-square statistical test of variances.

In general the question to be answered concerning a single variance (Dunn, 1967, p. 133) is Is $^2$less or equal to $^2_0$ (null hypothesis $H_0$) or is $^2$ greater than $^2_0$ ?

Mathematically this can be expressed as:

$$H_0 \quad ^2 \leq \, ^2_0$$

$$H_A \quad ^2 > \, ^2_0 \qquad (4.24)$$

If $^2 \leq \, ^2_0$ and the population is normally distributed, then $(n-1)\, s^2/\, ^2_0$ has a chi-square distribution with n-1 (number of simulations minus one) degrees of freedom; therefore, calculate:

$$P_c = \frac{(n-1)\, s^2}{^2_0} \qquad (4.25)$$

$$P_c^* = P_{(1 - \alpha,\ df)} \qquad (4.26)$$

where:

$s^2$ = Variance of the maximum objective function

$^2_0$ = Highest variance that management is willing to accept

$\alpha$ = Significance level (error type I)

df = Degrees of freedom = n-1

The decision rule is that if the value of $P_c$ is less than $P_c^*$, then accept the variance of the maximum objective is less than the one stated by management, otherwise reject the null hypothesis and simulate at least n + 1 times in the following run, and do hypothesis testing again.

Scope of the Suggested RPM Simulation

"In recent years the linear programming models have increasingly incorporated concepts of risk and uncertainty through the various approaches or probabilistic programming, e.g., chance-constrained programming, stochastic programming, etc." (Sengupta and Portillo-Campbell, 1976).

In this section we are going to show by means of generalized RPM networks, the different kinds of stochastic linear programming approaches based on the requested necessities by management.

Bounded Two-stage Program Simulation. This run consisted primarily in simulating the LP particleboard model, bounding the processes nodes with the upper limit equals to the activity levels resulting from the solution in the deterministic run with most likely values, subjecting the $c_j$ coefficients to random variations.

The objective of this two-stage bounded run was to prove the drop in profit, resulting from the idealistic deterministic run subjected under the stochastic two-stage simulation as we will show in the next chapter in more detail.

The graphical illustration of this run is presented over a segmented RPM in Figure 4-2.

Two-Stage Program Simulation. Here, we present the graphical portrayal of the two-stage programming by means of the general

RPM network representation discussed in Chapter III, section B. This general representation of the two-stage linear program by means of RPM2 nomenclature is shown in Figure 4-3.

Chance-constrained Program Simulation. Here, in the CCLP, we are going to simulate over the coefficients $c_j$ and $b_i$, because we consider only random variations in both and not in $a_{ij}$.

The general RPM representation of this case is portrayed in Figure 4-4.

Stochastic Program Simulation. In this case, we are going to take into account what the reality really is, we are going to assume random variations over the three coefficients $b_i$, $a_{ij}$, and $c_j$. In fact these random variations are over raw materials, availability of labor, demand forecasts, technological coefficients, costs, selling prices, etc.

The graphical representation of this case for RPM2 simulation in general is given in Figure 4-5.

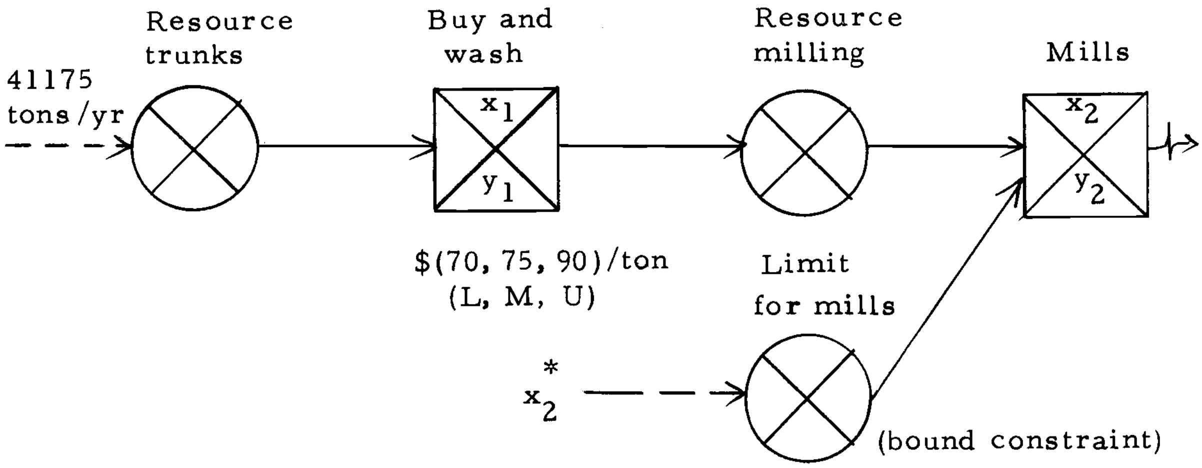



Figure 4-2. Segmented RPM showing two-stage bounded programming illustration.

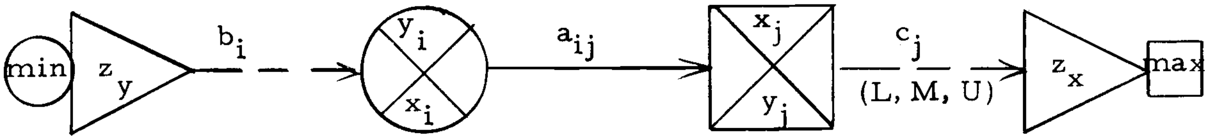



Figure 4-3. General RPM representation of the two-stage programming model with simulation notation.

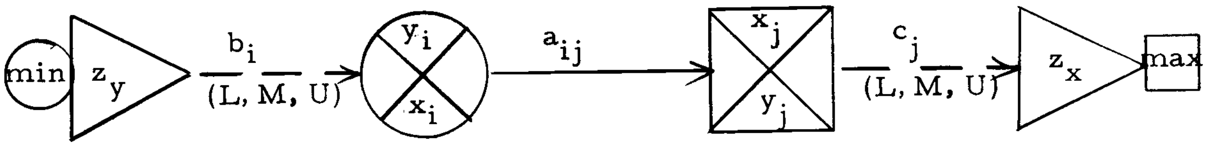



Figure 4-4. General RPM network for chance-constrained programming with simulation notation.

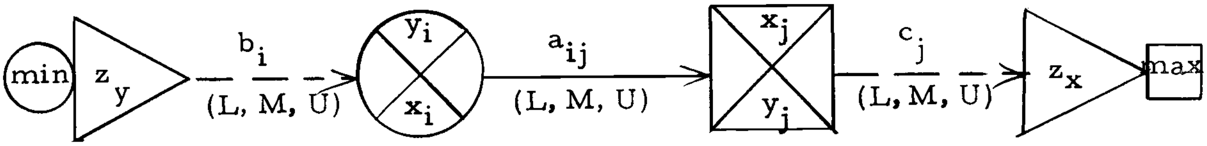



Figure 4-5. General RPM network for stochastic programming with simulation notation.

# V. EXPERIMENTATION

As important as the development of an accurate model is the use of that model to derive useful information for planning, scheduling and control of the system it portrays. The operations research management science study focuses attention on what to do, selecting basic variables out of all possible alternatives. The application of stochastic programming by means of stochastic simulation to the RPMS network model of the particleboard production had to provide just this type of information.

This chapter discusses this task in two phases: (1) deterministic (certainty) considerations; and (2) stochastic (randomly independent variation of the parameters) considerations as stipulated by management. The first phase validated the model, while the second was used to plan future management actions.

An attempt has been made to demonstrate advantages and limitations of both the traditional approach and the proposed Monte Carlo simulation procedure, and to defend the use of the latter technique for the purpose of planning the particleboard production system in Mexico.

## A. Deterministic Solutions

### Deterministic Run using Most Likely Values

In this case we consider that the raw materials, labor requirements, costs, selling prices and forecasted demands are fixed and known values. The fulfillment of the Mexican (domestic) demand and satisfying of the labor and raw materials restrictions must be complete and clearly portrayed. The segment of the RPM network model shown in Figure 5-1 gives a graphical portrait of the basic variables selected for production planning.

Validation of the Model. The model was validated by using the pattern of demands for the production period of 1975. The labor and raw material resources were set at the levels then available and the Mexican demands were to be met exactly. The resulting levels of activities obtained by this computer optimization were found to be consistent with the expectations by Carrasco, Curiel and Serrano at that time. The profit objective function value was found to be 14% lower than the one at that time, but this difference was judged by CONACYT managers to be reasonable. They are attributable to the approximations used in the earlier study; the numerical errors of parameters, and by the different levels of optimization.

Optimality Analysis. The linear programming solution in Figure 5-1 shows the optimal activity levels and optimal input expenditures needed to obtain such production outputs.

The optimization of the particleboard plant yielded ($7,318,723.33 profit per year, processing 41,175 tons of henequen trunks, 80.38 tons of emulsionated paraffin, 1,512.69 tons of

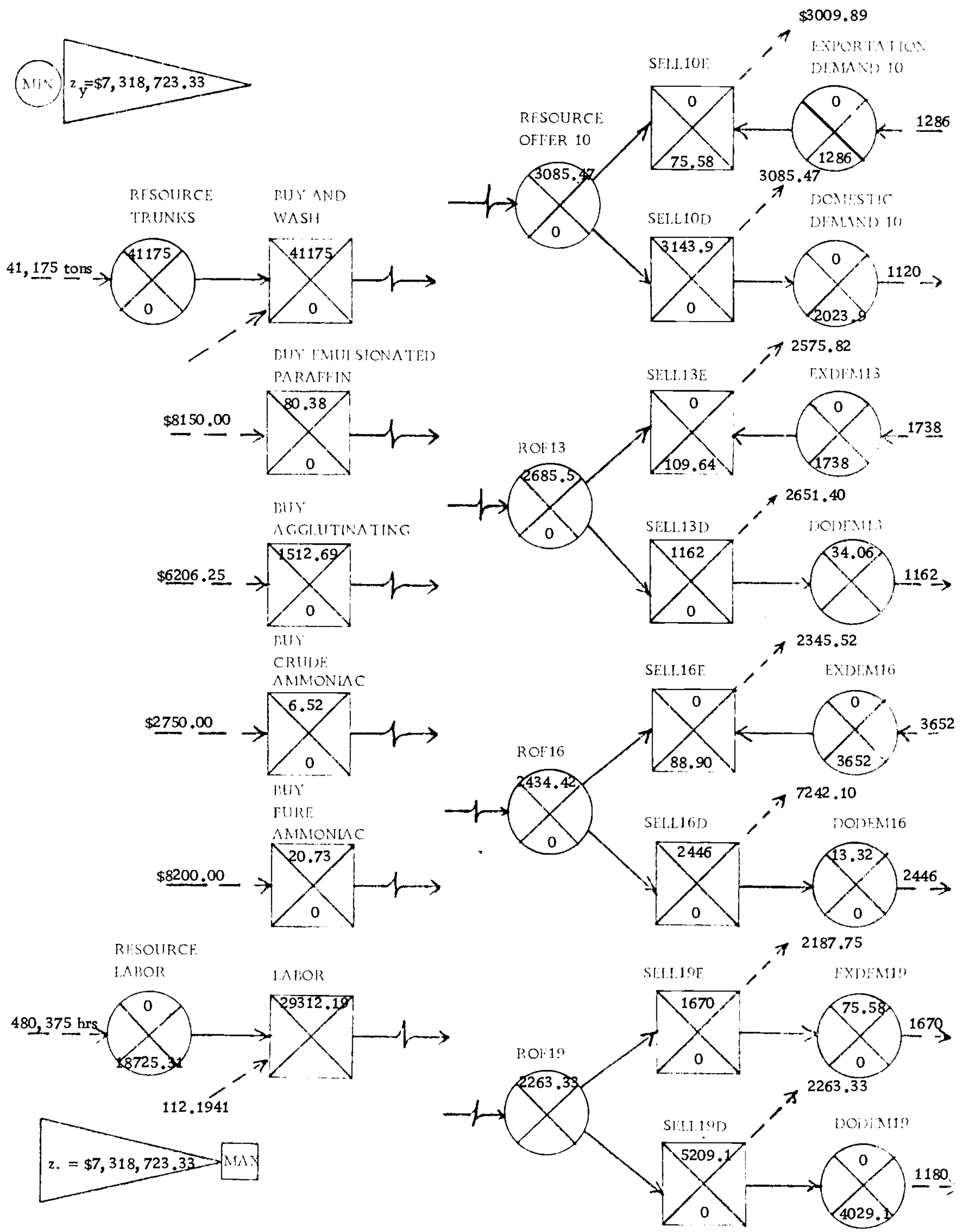



Figure 5-1. Segmented RPM network illustrating the deterministic run results over the selected basic variables for planning purposes.

agglutinating, 6.52 tons of crude ammoniac and 20.23 tons of pure ammoniac. It would have sold 3143.9 tons of 10 mm WPB (width of particleboard), 1162.00 tons of 13 mm WPB, 2446 tons of 16 mm WPB, and 5209.1 tons of 19 mm WPB, satisfying the Mexican requested demand. On the other hand, would have designated only 1670 tons of 19 mm WPB requested for exportation. Note that no 10 mm, 13 mm and 16 mm WPB were produced because the opportunity costs would have been very high ($75.58, $109.64 and $88.90 respectively). Note also that the Lagrange multiplier value for 13 mm WPB for exportation ($2685.56) exceeded the selling price ($2575.82) and any amount exported would have meant losing $109.64 per ton. The labor requirements for this production were 29312 hrs, equivalent to 32 workers in direct labor. The details of the composition of these layups can be found in Figure 5-1 and in the Appendix.

Post-optimality Analysis. By postoptimality analysis, we mean the study of the environmental effects upon the system. This is also called adaptivity analysis or sensitivity analysis by Riggs and Inoue (1975). In computer processing such a printout is known as a range report. This range gives the values within which a given coefficient can vary without affecting the basis of the optimal solution.

The range report provided for the model after running it in the computer is shown in Tables 5-1 and 5-2. In these tables, it can be observed, for instance, that the availability of trunks to be processed

Table 5-1. Ranges report for resources in the deterministic run.

| | CONSTANT | - RANGE - | |
|---|---|---|---|
| RT | 41175.0000 | 23490.8684 | 67478.6821 |
| RM | 0.0000 | -17684.1316 | 26303.6821 |
| S1HF45 | 0.0000 | -17684.1316 | 26303.6821 |
| FFH | 0.0000 | -1556.9408 | PINF |
| S2HF3 | 0.0000 | -6861.7929 | 14816.8641 |
| REP | 0.0000 | MINF | 80.3792 |
| RMWF | 0.0000 | -2987.3714 | 14816.8641 |
| RNF | 0.0000 | -2257.3598 | 2154.9620 |
| RFF | 0.0000 | -1550.1320 | 4128.4469 |
| FFF | 0.0000 | -387.9980 | 1033.3503 |
| BFR | 0.0000 | -2477.2323 | PINF |
| RAG | 0.0000 | MINF | 1512.6850 |
| RCA | 0.0000 | MINF | 6.5155 |
| RPA | 0.0000 | MINF | 20.7318 |
| RFS10 | 0.0000 | -2393.4894 | 10297.8071 |
| RFS13 | 0.0000 | -4667.8743 | 1374.1722 |
| RFS16 | 0.0000 | -6019.8268 | 2892.6206 |
| RFS19 | 0.0000 | -4764.7905 | 10297.8071 |
| ALABOR | 0.0000 | -18725.3083 | 29312.1917 |
| RST010 | 0.0000 | -2023.9346 | 8707.8257 |
| RST013 | 0.0000 | -3947.1545 | 1162.0000 |
| RST016 | 0.0000 | -5090.3656 | 2446.0000 |
| RST019 | 0.0000 | -4029.1069 | 8707.8257 |
| RP10 | 0.0000 | -2393.4894 | 10297.8071 |
| RP13 | 0.0000 | -4667.8743 | 1374.1722 |
| RP16 | 0.0000 | -6019.8268 | 2892.6206 |
| RP19 | 0.0000 | -4764.7905 | 10297.8071 |
| RLABOR | 48037.5000 | 29312.1917 | PINF |
| ROF10 | 0.0000 | -2023.9346 | 100000.0000 |
| EXDEM10 | 1286.0000 | 0.0000 | PINF |
| DODEM10 | -1120.0000 | -3143.9346 | PINF |
| ROF13 | 0.0000 | -3947.1545 | 1162.0000 |
| EXDEM13 | 1738.0000 | 0.0000 | PINF |
| DODEM13 | -1162.0000 | -5109.1545 | -.0000 |
| ROF16 | 0.0000 | -5090.3656 | 2446.0000 |
| EXDEM16 | 3652.0000 | 0.0000 | PINF |
| DODEM16 | -2446.0000 | -7536.3656 | -.0000 |
| ROF19 | 0.0000 | -4029.1069 | PINF |
| EXDEM19 | -1670.0000 | -5699.1069 | 0.0000 |
| DODEM19 | -1180.0000 | -5209.1069 | PINF |

Table 5-2. Ranges report for processes in the deterministic run.

| | CONSTANT | - RANGE - | |
|---|---|---|---|
| RAW | -75.0000 | -257.5650 | PINF |
| MILL | 0.0000 | -182.5650 | PINF |
| DRYER | 0.0000 | -182.5650 | PINF |
| MWF | 0.0000 | -398.3502 | PINF |
| SFEP | 0.0000 | -611.2811 | PINF |
| BEP | -8150.0000 | -96423.2071 | 0.0000 |
| SEPAR | 0.0000 | -282.3552 | PINF |
| BAG | -6206.2500 | -10896.3517 | 0.0000 |
| BCA | -2750.0000 | ***** ****** | 0.0000 |
| BPA | -8300.0000 | ***** ****** | 0.0000 |
| BLEND10 | 0.0000 | -54.0392 | 272.9886 |
| BLEND13 | 0.0000 | MINF | 28.8033 |
| BLEND16 | 0.0000 | MINF | 11.2675 |
| BLEND19 | 0.0000 | -14.2353 | 529.6438 |
| FORST10 | 0.0000 | -54.0392 | 272.9886 |
| FORST13 | 0.0000 | MINF | 28.8033 |
| FORST16 | 0.0000 | MINF | 11.2675 |
| FORST19 | 0.0000 | -14.2353 | 529.6438 |
| PRESS10 | 0.0000 | -54.0392 | 272.9886 |
| PRESS13 | 0.0000 | MINF | 28.8033 |
| PRESS16 | 0.0000 | MINF | 11.2675 |
| PRESS19 | 0.0000 | -14.2353 | 529.6438 |
| LABOR | -112.1941 | -278.9234 | -.0000 |
| ST010 | -749.6300 | -813.5364 | -426.7958 |
| ST013 | -749.6300 | MINF | -715.5675 |
| ST016 | -749.6300 | MINF | -736.3051 |
| ST019 | -749.6300 | -766.4646 | -123.2774 |
| SELL10E | 3009.8900 | MINF | 3085.4700 |
| SELL13E | 2575.8200 | MINF | 2685.4625 |
| SELL16E | 2345.5200 | MINF | 2434.4249 |
| SELL19E | 2187.7500 | MINF | 2263.3300 |
| SELL10D | 3085.4700 | 3021.5636 | 3408.3042 |
| SELL13D | 2651.4000 | MINF | 2685.4625 |
| SELL16D | 2421.1000 | MINF | 2434.4249 |
| SELL19D | 2263.3300 | 2246.4954 | 2889.6826 |

can vary from 23,470.87 tons to 67,478.68 tons, and that the availability of direct labor can vary from 29,312.19 hours to plus infinity, singly, without affecting the base of the solution. The variation is just one at a time, and if over the range specified, would not result in a different basis. Also, analyzing the dual values, it can be observed that we are able to pay up to $255.57 per ton of henequen trunks and up to $278.92 per hour of direct labor in order to put the production outputs in the market without losing money.

## Deterministic Run using PERT Estimated Mean Criteria

This is the case where, contrary to the last one considered, almost all the data affecting the particleboard production system are known only by their probability distributions. These probability distributions are described by means of the three estimates L, M and U, as shown in Table 2-4. This section deals with the fluctuations applying PERT formula for estimated means (3.17), and by utilizing these estimated means run a deterministic linear program.

The optimization of this case in which the coefficients were assumed having a PERT behavior, yielded $6,540,278.33 of profit per year. We would plan to produce 1670 tons of 19 mm WPB for exportation and 3036.25 tons of 10 mm WPB, 1162 tons of 13 mm WPB, 2446 tons of 16 mm WPB and 4971.4 tons of 19 mm WPB for the Mexican consumption, by processing 40,131.67 tons of henequen cactus with 78.34 tons of emulsionated paraffin, 1476.35 tons of agglutinating, 6.35 tons of crude ammoniac and 20.21 tons of pure ammoniac. For the above purposes, 29,228.42 of direct labor would be needed, that means to use 32 workers.

Note that we would not plan to produce boards of 10, 13 and 16 mm width because the opportunity costs would have been very high, e.g., \$74.32, \$108.52 and \$87.68 respectively. This decision would have been based upon the comparison of the Lagrange multiplier against the selling price, for instance, for the 16 mm WPB (exportation) the shadow price (\$2440.18) exceeded the selling price (\$2352.49) by \$87.69, which would have resulted in a loss for every ton thus sold.

For more details of the resultant layups, the reader should refer to Figure 5-2, where the selected basic variables for management production plans are presented, and to the Appendix, where the range reports and the complete figures are also presented.

Comparison of this case with the others to be analyzed will be done in Chapter VI.

## B. Stochastic Programming Solution Approaches

To incorporate the risk elements that are caused by the probabilistic nature of parameters and coefficients in our mathematical model, we must advance from the deterministic optimization model to

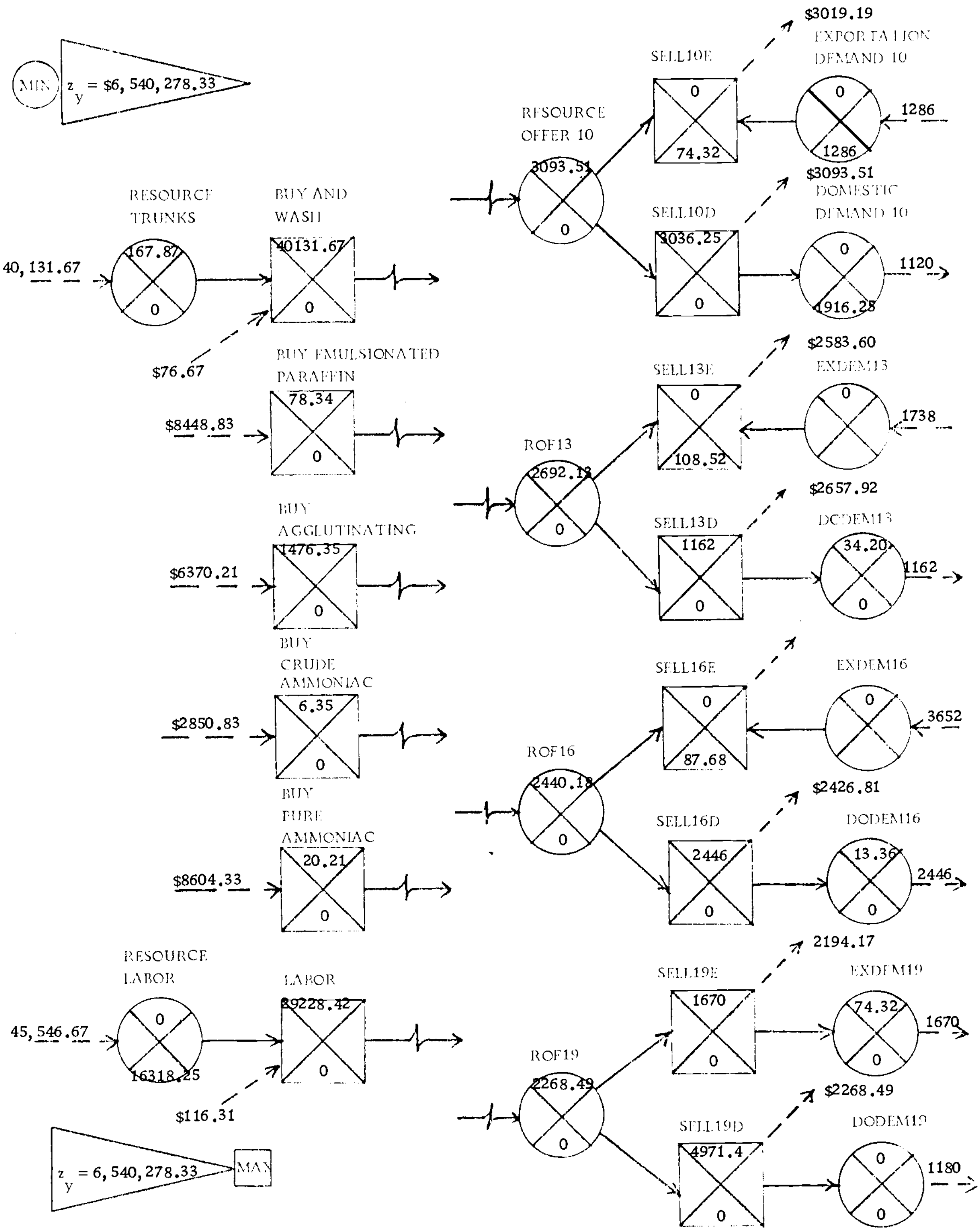



Figure 5-2. Deterministic run results using PERT estimated mean criteria.

a stochastic programming model. Traditional approaches available in the solution of such problems are two-stage programming, and chance-constrained programming. Our proposed approach is to utilize Monte Carlo sampling to generate parameters and repeatedly optimize the simulation model.

Three stochastic conditions are discerned and observed as to the appropriateness of their problems for solution by the above techniques.

## Known Resource Availability and Technological Coefficients, Unknown Costs and Prices

The condition where the costs and prices are the only probabilistic parameters represents the simplest case of stochastic programming problems. We assume that the management has *a priori* knowledge of the process yields and the amount of resources and demands that are prevalent at the time of model experimentation.

The traditional tools available in solving such problems are the chance-constrained programming and two-stage programming. Since the chance-constrained programming is applicable to more advanced cases of stochastic programs, we shall now discuss the two-stage programming approach.

*Two-stage Programming*. The traditional two-stage programming approach, as discussed in Chapter IV, would handle the optimization model by dividing the decision process in two or more stages. A set of decisions is made prior to having obtained all data concerning the parameters. Additional decisions are made after additional data are obtained.

Since the two-stage approach utilizes a decision-tree like development, the number of decision branches is directly proportional to the number of discrete events that are discerned. Thus, if a probability distribution for an unknown parameter has three possible outcomes, three decision branches are necessary in the optimization model. Most of the parameters in the particleboard production model are continuous, as shown in Figure 5-3, and would require a large number of discrete outcomes to describe the system adequately.

The results thus obtained would in any case be inferior to the optimal solutions generated from models where the cost and price parameters are allowed to vary according to the given probability distributions. Thus, the approach taken by this thesis is to utilize the Monte Carlo simulator to provide a series of optimal solutions and to consider those in lieu of the cruder two-stage computations. The data discussed below can, therefore, be considered the upper-bounds of the values that would have been obtained through the application of the more traditional two-stage programming approach.

Two-stage Programming Simulation. Let us, therefore, suppose that management is certain about all the values of the

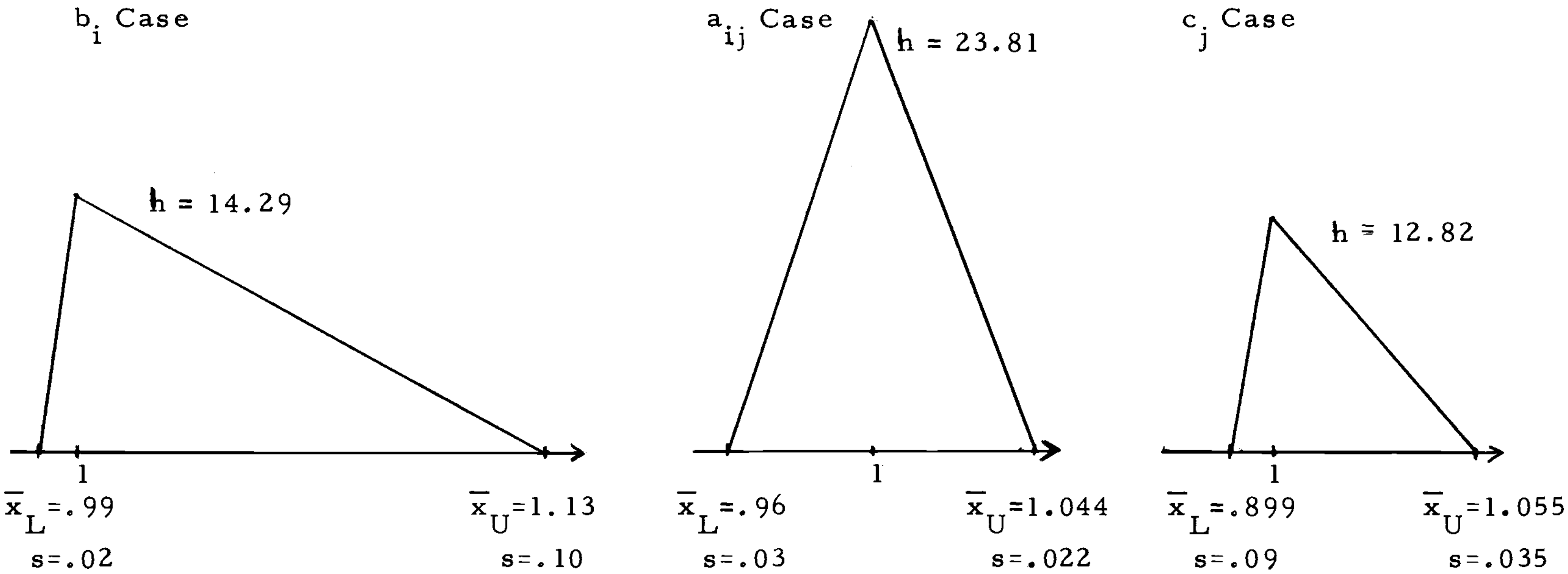



Figure 5-3. Illustration of the parameters beta distributed (on the average).

coefficients of the linear programming model except for the prices and cost parameters. Assume, however, that the management has some knowledge of the expected random variations and can assume them to be independent of one another. More specifically, management supposed that the material and processing costs, and selling prices are not known with certainty but are predictable with enough data to describe the probability distributions.

The two-stage linear programming simulation results of the runs are shown in Table 5-3 for which hypothesis testing of variances were done in order to select the appropriate run that meets management requirements.

For this case management required a level of significance ($\alpha$) of 5% and a variance ($\sigma_0^2$) as high as $(2.4 \times 10^5)^2$. This $\alpha$ was fixed at the value of 5%, critically considered according to Ingram (1974, p. 154).

Stopping Rules for Simulation. As a matter of illustration let us consider the hypothesis testing with four and five simulations.

First, let us set the hypothesis at the 95% level of confidence:

$H_0 : \sigma^2 \leq (2.4 \times 10^5)^2$ (n=4 simulations is sufficient)

$H_A : \sigma^2 > (2.4 \times 10^5)^2$ (n=4 simulations is not sufficient)

Second, we start the simulation and compute the statistic:

$$P_{calc} = \frac{(n-1)\ s^2}{\sigma_0^2} = \frac{(4-1)\ (351,972.09)^2}{(2.4 \times 10^5)^2} = 8.42$$

Table 5-3. Two-stage programming simulation and hypothesis testing results.

| Cost per run n sims ($) | Number of simulations (n) | Average max. objective ($) | Std. dev. (s) of the max. objective | $P_{calc} = \frac{(n-1)\, s^2}{\sigma_0^2}$ | Null hypothesis decision[a] |
|---|---|---|---|---|---|
| 0.983 | 2 | 5,776,450.59 | 535,536.27 | 4.98 | Reject |
| 1.306 | 3 | 5,597,838.33 | 488,985.24 | 8.30 | Reject |
| 1.517 | 4 | 5,621,545.16 | 402.060.23 | 8.42 | Reject |
| 1.790 | 5 | 5,644,545.28 | 351,972.09 | 8.60 | Accept |
| 2.048 | 6 | 5,791,542.78 | 478,285.56 | Stop simulating | Stop simulating |
| 2.364 | 7 | 5,885,533.13 | 502,464.13 | | |
| 2.560 | 8 | 5,997,760.04 | 533,338.02 | | |
| 2.843 | 9 | 5,927,717.05 | 520,991.33 | | |
| 3.110 | 10 | 5,954,833.83 | 498,524.16 | | |
| 4.433 | 15 | 5,991,992.07 | 460,694.68 | | |
| 5.782 | 20 | 5,984,026.71 | 550,464.21 | | |

[a]Significant at the $\alpha = .05$ level.

and in the chi-square distribution table get the value:

$$P^*_c = P_{1-\alpha), (n-1)} = P_{.95, 4} = 7.81$$

Since $P_{calc} > P^*_c$, there is sufficient evidence for rejecting the null hypothesis. Therefore, simulate (n+1) times in the next run of the model.

Again, we set the hypothesis at the 95% level of confidence, and start the simulation. Thus, compute the statistic:

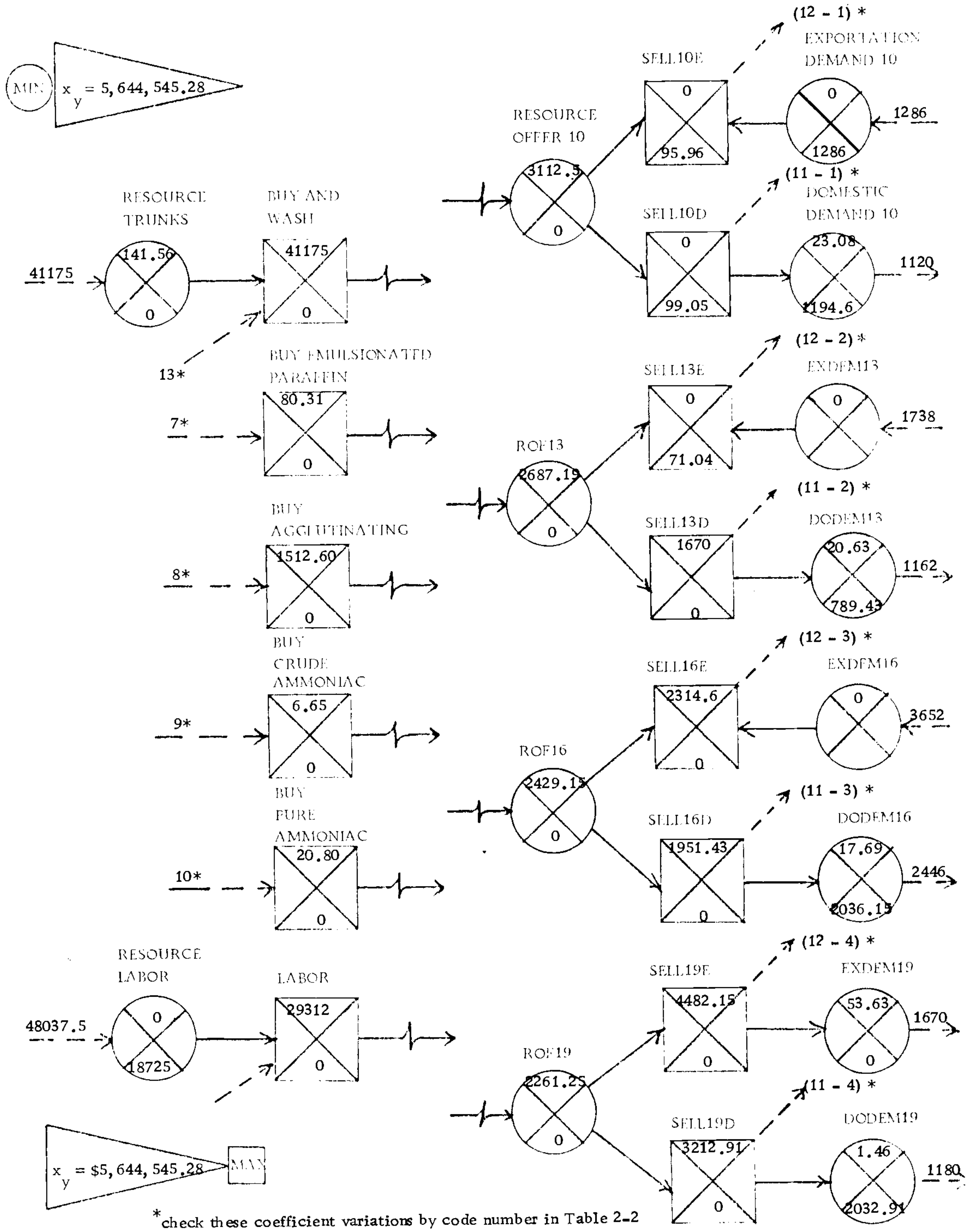
$$P_{calc} = \frac{(5-1)\ (351,972.09)^2}{(2.4 \times 10^5)^2} = 8.60$$

and get the $P^*_c$ from the chi-square distribution table:

$$P^*_c = P_{.95, 5} = 9.49$$

Since $P_{calc} < P^*_c$, there is no sufficient evidence for rejecting the null hypothesis. Thus, we could have concluded to stop simulating at five runs, and meet the management requirements for precision.

<u>Discussion of the TS Simulated Results</u>. The two-stage LP simulation computer output is shown in the Appendix. This solution is in terms of averages of the activity levels, shadow prices, primal residues and opportunity costs, all randomly selected by the computer (within the ranges given for the coefficients) when simulating the model. Therefore, due to these random variations, the basic variables selected as shown in Figure 5-4 indicate that on the average you should expect to use 41, 175 tons of trunks, 29, 312 hrs of




*check these coefficient variations by code number in Table 2-2

Figure 5-4. Segmented RPM network illustrating two-stage stochastic programming results over the selected variables.

direct labor equal to 32 workers, and that you would not expect to sell your products of 10, 13 and 16 mm WPB for exportation. On the other hand, you should expect to sell 1,670 tons of 19 mm WPB to exportation and 2314.60 tons of 10 mm WPB, 1951.43 tons of 13 mm WPB, 4482.15 tons of 16 mm WPB and 3212.91 tons of 19 mm WPB, for the complete satisfaction of the Mexican demand. The inprocess inventory at the end of the year is expected to be zero because we are allocating all of the production to either Mexico or exportation. You should also plan to buy on the average 80.31 tons of emulsionated paraffin, 1512.60 tons of agglutinating, 6.65 tons of crude ammoniac and 20.80 tons of pure ammoniac.

Since we had a random sample of five profit observations (simulations) $z_{x_1}, \ldots, z_{x_5}$ from a normal population, the confidence interval for the profit expectation $\mu$ (population mean) with $(1-\alpha)$ confidence coefficient by means of (4.23) was:

$$5,644,545.28 - t_{(.975;4)}(351,972.09) \leq \mu \leq 5,644,545,28 + t_{(.975;4)}(351,972.09)$$

$$\$4,667,470.76 \leq \mu \leq \$6,621,619.80$$

This means that 95% of the time the expected mean of the profit is going to fall under this interval.

A simple "payback" after income taxes was defined by Ireson and Grant (1964, p. 347) as the number of years required for net cash flow to equal zero without consideration of interest. In this TSPS case this payback in terms of expectations turned out to be 7.29 years, computed from the investment (\$41, 124, 389.25) over the expected profit (\$5, 644, 545.28). Now, suppose that under the conditions and suppositions of this case, in the reality we had that the observation of the profit for a certain year did not fall within this interval, then what is suggested in this case is to investigate if there exist other parameters with variations (for instance, variations in availability of resources and inprocess transformations), and check the ones that you already have considered. In other words, check which parameters in the model have stochastic characteristics and apply one of the methods suggested in this thesis that fulfills this condition for a more accurate production planning.

### Known Technological Coefficients, Uncertain Resource Availability, Demand, and Cost Parameters

A more realistic consideration of a stochastic management tool is the one which not only the cost and price coefficients, $c_j$, but also the resource demands and availabilities, $b_i$, are also subject to stochastic fluctuations.

The two-stage programming technique could be extended to cover such conditions, though this is not usually done. In any case, the curse of dimensionality would become an even more compounded problem, and the solution technique would not be applicable to a real-life problem.

The chance-constrained programming technique discussed in Chapter IV alleviates the objection of the dimensionality by merely tightening the constraints and objective function to allow for the probabilistic fluctuations. Unfortunately, the chance-constraint programming also has its shortcomings. For example, there is no provision for the stochastic variations to take place jointly, and the solution does not provide any measure of the distribution of solution variables. Again, the approach taken in this thesis is to utilize the Monte Carlo simulation as the vehicle to produce distributions of solutions that would give us the criterion needed to judge the limitation of chance-constraint programming.

<u>Chance-constrained Programming Simulation.</u> In this case management realized that some of the coefficients $b_i$ (resource availabilities) and $c_j$ (cost and selling prices) have random variations independent from one another, but that the process transformations (technological coefficients, $a_{ij}$) are known with certainty. The resulting runs with different simulations and the hypothesis testing of variance of the maximum objective function, are shown in Table 5-4. These results give the idea of what profit and production yields should be expected as well as when to stop simulating. Notice there that the

Table 5-4. Chance-constrained programming simulation and hypothesis testing results.

| Cost per run n sims ($) | Number of simulations (n) | Average max. objective ($) | Std. dev. (s) of the max. objective | $P_{calc} = \frac{(n-1)\,s^2}{\sigma_0^2}$ | Null hypothesis decision[a] |
|---|---|---|---|---|---|
| 0.726 | 3 | 5,328,135.01 | 857,248.65 | 8.96 | Reject |
| 0.857 | 4 | 5,470,832.80 | 755,888.53 | 10.45 | Reject |
| 1.001 | 5 | 5,547,097.72 | 676,466.77 | 11.16 | Reject |
| 1.165 | 6 | 5,652,108.54 | 657,456.81 | 13.18 | Reject |
| 1.295 | 7 | 5,699,914.68 | 613,356.24 | 13.76 | Reject |
| 1.453 | 8 | 5,682,461.14 | 569,999.29 | 13.87 | Accept |
| 1.593 | 9 | 5,834,796.96 | 702,241.13 | Stop simulating | Stop simulating |
| 1.750 | 10 | 5,851,659.32 | 664,223.14 | | |
| 2.511 | 15 | 5,842,140.85 | 550,741.67 | | |
| 3.254 | 20 | 5,872,280.72 | 540,676,54 | | |

[a]Significant at the $\alpha = .05$ level.

standard deviation (s) did not show a fixed pattern such as cyclic or linear trends.

For this case, the highest variance that management is willing to accept is $(4.05 \times 10^5)^2$, and again with $\alpha$ level of significance of 5%.

Discussion of the C.C. Stimulated Results. Figure 5-5 gives us a segmented RPM network of the model, illustrating the selected basic variables, useful for the decision maker to observe the effects of risk over the available resources, selling prices and costs of materials and labor.

It is necessary to remember at this point that the figures in the RPM cells are averages. For instance, any average primal value ($x_j$) is the result of the sum of the primal values resulted at each simulation divided by the number of simulations (n):

$$\bar{x}_j = \frac{\sum_i^n x_{j_n}}{n} \tag{5.1}$$

The risk effects in this simulation suggested 38,612.72 tons of trunks, 75.34 tons of emulsionated paraffin, 1418.50 tons of agglutimating, and so on. He should employ 30 workers on the average for direct labor with 27,488.16 hrs assigned to obtain the production outputs.

These production outputs for selling purposes proposed under these risk considerations to sell 2493.52 tons of 10 mm WPB,

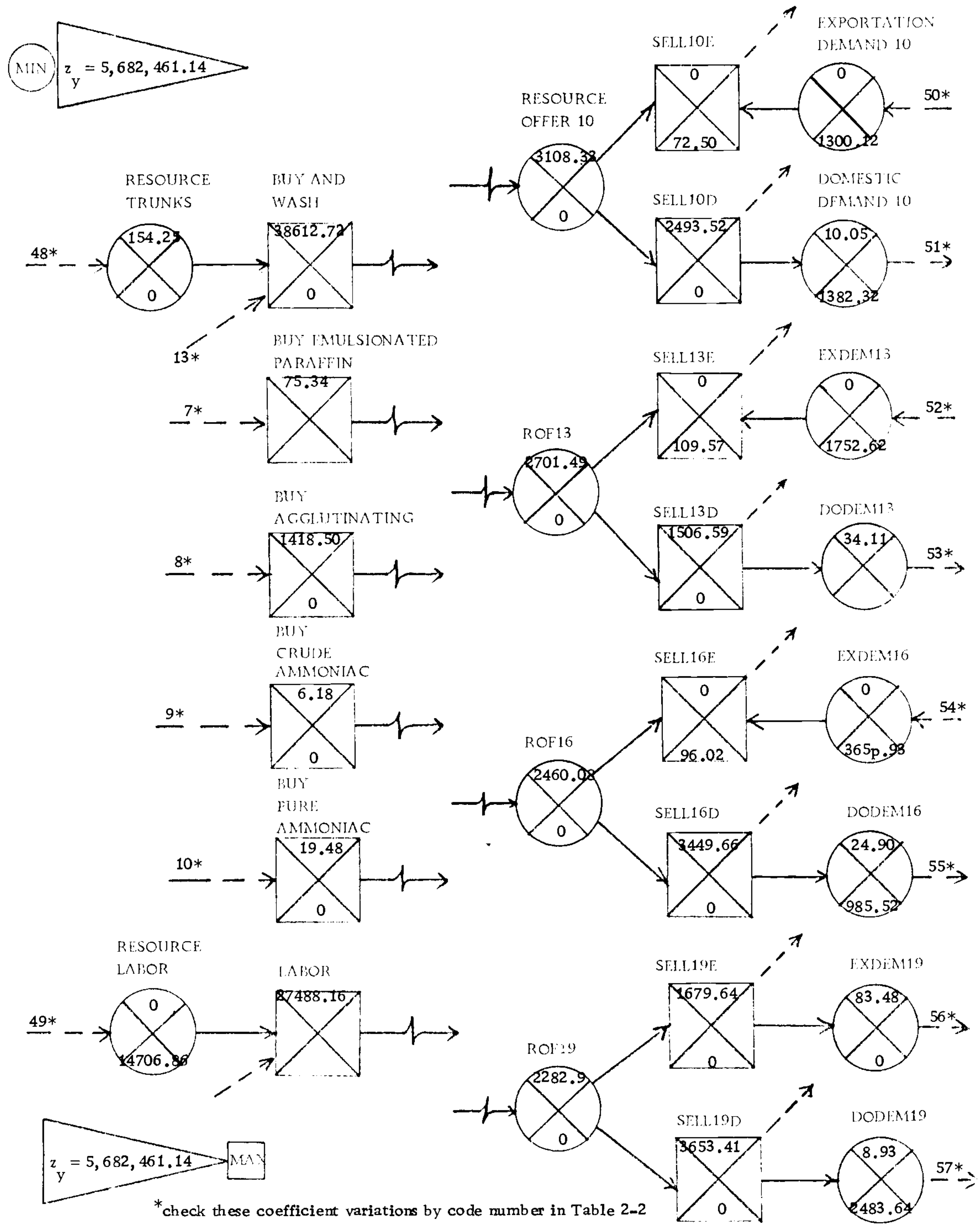



Figure 5-5. Chance-constrained programming simulation segmented RPM of the model illustrating the effects of risk over $b_i$ and $c_j$.

1506.59 tons of 13 mm WPB, 3449.66 tons of 16 mm WPB and 3653.41 of 19 mm WPB in Mexico and 1679.64 of 19 mm WPB for exportation. For more detailed figures in the whole model, the reader should refer to the Appendix, where he can find the computer outputs.

The $(1-\alpha)$ 100% confidence interval for the profit expectation $\mu$ (population mean), since we had a random sample of eight profit observations (simulations) from a normal population, was computed readily by means of (4.23):

$$5,682,461.14 - t_{(.975);7)} (569,999.29) \leq \mu \leq 5,682,461.14 + t_{(.975;\ 7)} (569,999.29)$$

$$4,334,412.82 \leq \mu \leq 7,030,509.46$$

This interval means that 95% of the time an observation of the profit is going to fall within these two values. The expected "payback" after income taxes in this CCPS case was found to be 7.24 years (\$41,124,389.25/\$5,682,461.14). Again, as in the last case, if under the supposition and considerations of risk in this case, in a certain year we fall outside these intervals, will be pertinent to make exhaustive analysis of the random variations considered and investigate if other random variations (inprocess transformations) should be considered to ascertain management to follow a given production plan.

## Uncertain Resource Availability, Technological Coefficients, and Costs and Prices

Finally, the most realistic consideration of a stochastic management model is the one where the different coefficients in the model are subjected to stochastic fluctuations. Due to the approach taken in this thesis, by means of Monte Carlo simulation we will have a provision for the stochastic variations to take place jointly; therefore, the solution will provide a measure of the distribution of such variables.

Stochastic Programming Simulation. Finally, as often is the case, management may realize that some or all the parameters $b_i$ (availability of resources), $a_{ij}$ (inprocess transformations) and $c_j$ (selling prices and costs associated with the production of the goods) have random variations. For the sake of our study, they were considered independent from each other. We simulate the effects of this variations obtaining the results shown in Table 5-5.

The testing of hypotheses was conducted to decide how many simulations are needed to achieve significant test results for getting management production plans for the year. In this table, observe that these standard deviations (s) did not appear to show any particular trend, but rather varied up and down randomly. We did not perform any further study of this tendency due to the limitations in computer budget. But it would be appropriate for analyzing the effects of

Table 5-5. Stochastic LP simulation and hypothesis test results.

| Cost per run n sims ($) | Number of simulations (n) | Average max. objective ($) | Std. dev. (s) of the max. objective | $P_{calc} = \frac{(n-1)\,s^2}{\sigma_0^2}$ | Null hypothesis decision[a] |
|---|---|---|---|---|---|
| 1.101 | 2 | 5,272,901.39 | 544,414.55 | 5.85 | Reject |
| 1.479 | 3 | 5,188,773.14 | 411,614.25 | 6.69 | Reject |
| 1,784 | 4 | 5,248,906.42 | 356,952.25 | 7.55 | Accept |
| 2.177 | 5 | 5,473,789.20 | 590,273.24 | Stop simulating | Stop simulating |
| 2.494 | 6 | 5,608,525.20 | 622,624.07 | | |
| 2.776 | 7 | 5,454,471.67 | 699,412.63 | | |
| 3.100 | 8 | 5,365,786.59 | 694,417.60 | | |
| 3.457 | 9 | 5,399,515.77 | 657,402.27 | | |
| 3.875 | 10 | 5,405,950.15 | 620,138.70 | | |
| 5.949 | 15 | 5,659,719.96 | 635,712.04 | | |
| 7,289 | 20 | 5,777,831.36 | 661,177.39 | | |

[a]Significant at the $\alpha$ = .05 level.

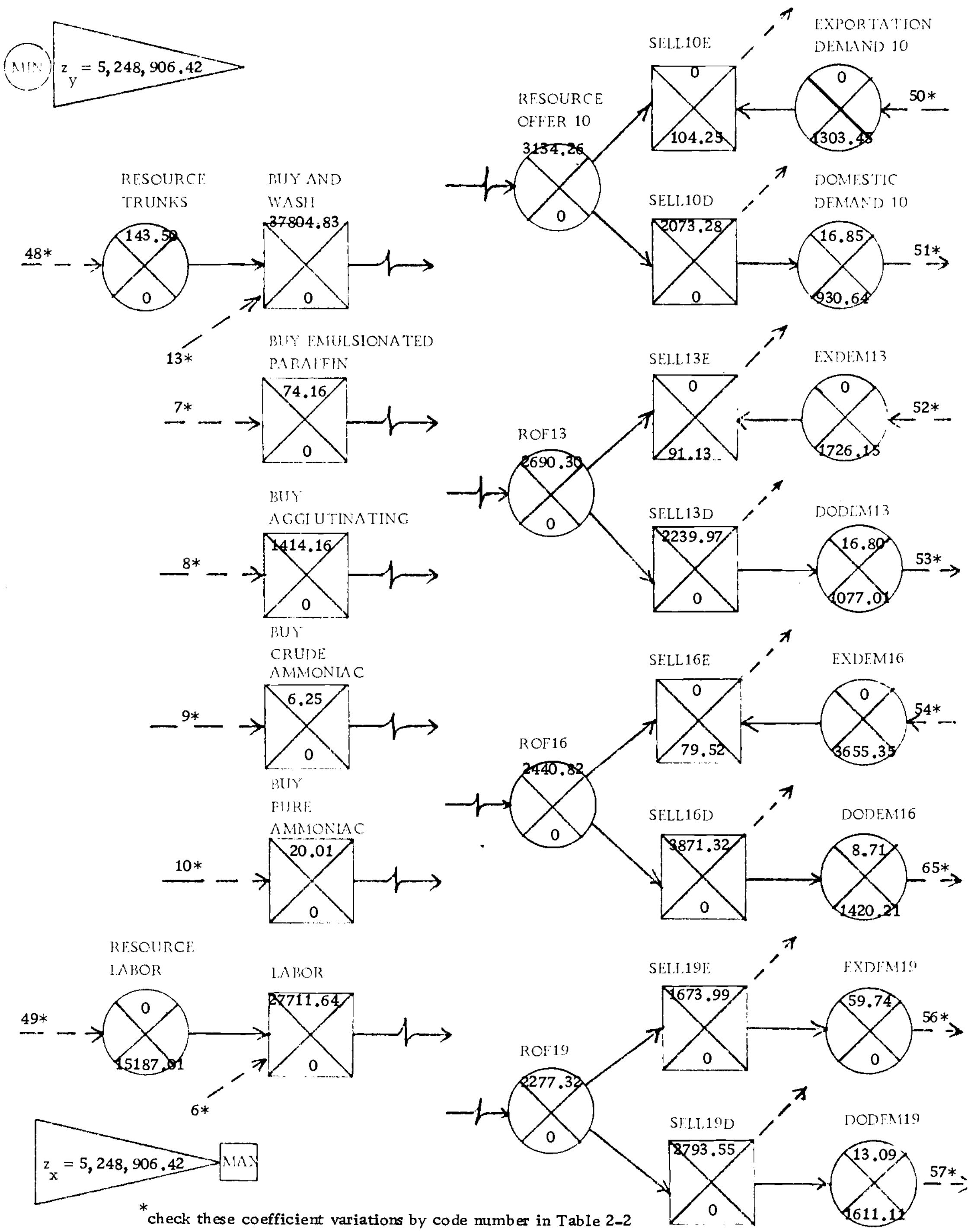
randomness in the standard deviations at a high number of simulations.

Discussion of the SP Simulated Results. The results can be interpreted using the segmented RPM simulated network containing the SLP results (Figure 5-6) after running the model for four simulations; 37,804.8 tons of trunks are going to be processed in the average, using 74.16 tons of emulsionated paraffin, as well as 1416.16 tons of agglutinating, etc. The manager will have to utilize on the average 31 workers for direct labor resulting in 27,711.64 hours of direct labor in the year, in order to obtain the following production outputs: 2073.28 tons of 19 mm WPB; 2239.97 tons of 13 mm WPB, 3871.32 tons of 16 mm WPB, and 2793.55 tons of 19 mm WPB for satisfying the Mexican demand; and 1673.99 tons of 19 mm WPB for exportation. For further information about these figures the reader is referred to the Appendix.

Management $(1-\alpha)$ 100% confidence interval for the profit expectation $\mu$ (population mean) due to the four simulations are random observations from a normal population, is computed readily from (4.23) as follows:

$$5,248,906.42 - t_{(.975,\,3)}(356,952.25) \leq \mu \leq 5,248,906.42 + t_{(.975,\,3)}(356,952.25)$$

$$4,113,084.36 \leq \mu \leq 6,484,728.48$$




*check these coefficient variations by code number in Table 2-2

Figure 5-6. Segmented RPM simulated network illustrating stochastic linear programming results with four simulations.

Where these interval means that 95% of the time an observation of the profit in a certain year is going to fall within these two limits. In other words, management should expect a profit within this figures with a .95 probability of assurance, for which the expected payback period would be \$41, 124, 389.25/\$5, 248, 906.42 or 7.83 years.

Again as in the other cases, suppose that at the end of the year management's profit falls outside these limits. What is suggested is review the model and the random variations, to see if we did not make any wrong assumptions, and make corrections, if any, in order to give reliable production plans to management.

## C. Effects of Stochasticity

### Drop in Profit

Let us suppose that management believes that all the coefficients ($b_i$, $a_{ij}$, and $c_j$) are known exactly as in our historical data case, and runs the LP model. It will yield \$7, 318, 723.33 of profit, and it will have the optimal activity levels and input expenditures in order to obtain the consequent optimal production outputs. But let us suppose that upon implementing this result in reality , they encounter random variations in costs and selling prices. In order to evaluate the effects of this risk fluctuation, considering that particleboard management is going to produce with certainty, bound the deterministic run processes with the activity levels resulting from the optimal solution and then simulate this restricted model with the two-stage Monte Carlo simulation. This was done because management would not know all the actual values of the stochastic parameters ahead of time and the computer Monte Carlo simulation (RPM2) assumes that it does. The results of this simulation over the restricted bounded model after three simulations as shown in Table 5-6 yielded \$5,191,276.58 profit average objective function.

The difference between the objectives functions of the deterministic run and the bounded two-stage LP simulation with three simulations gave a "drop in profit." This can be stated as

Det. run - Bounded TS LP = Drop in profit

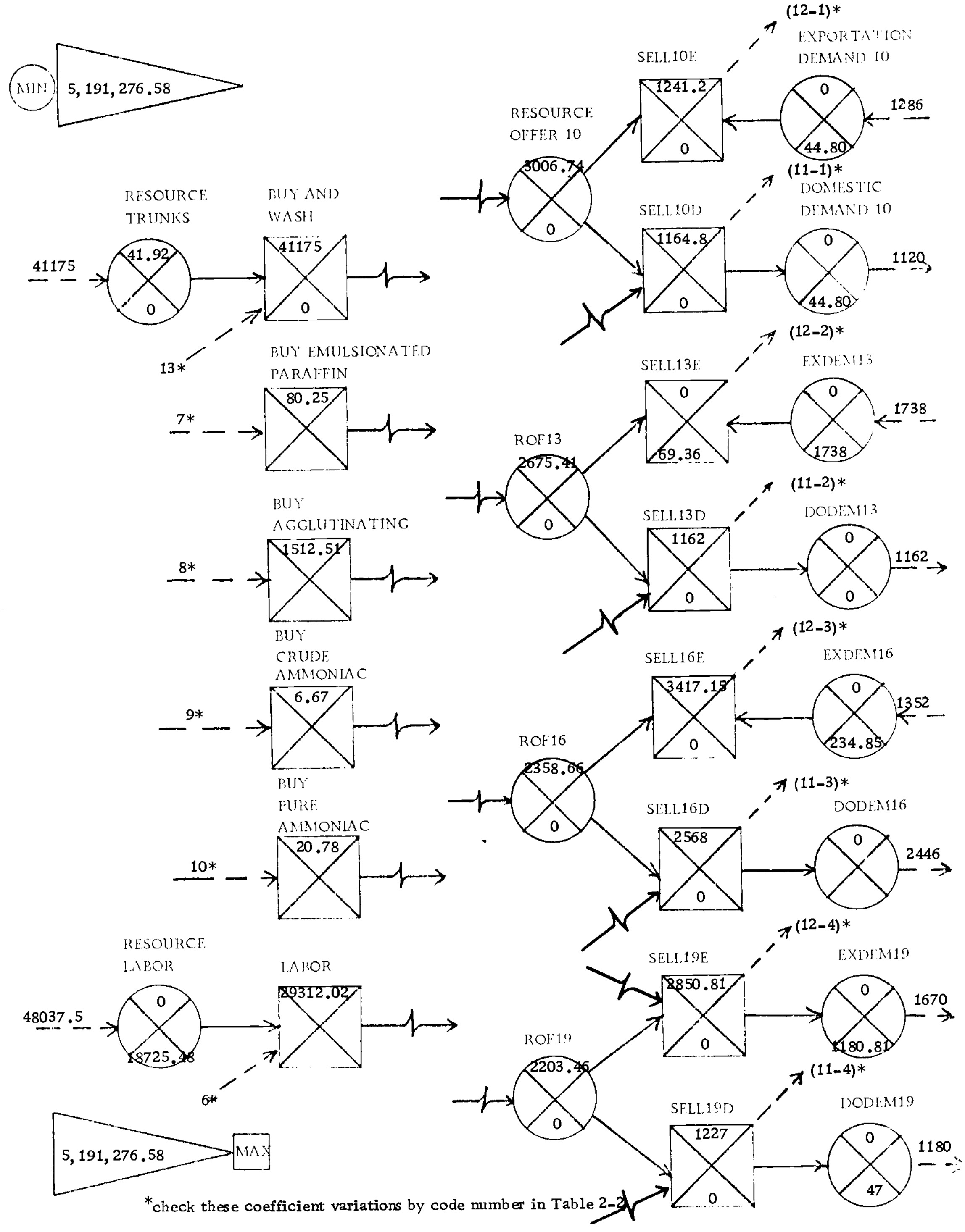
\$7,318,723.33 - \$5,191,276.58 = \$2,127,446.75

By comparing the results shown in Figure 5-1 with the ones in Figure 5-7, it can be noticed that management was going to sell 13,630.00 tons of particleboard in the case of the deterministic linear programming assumptions. That is essentially the same as 13,630.96 tons in the case of the bounded two-stage programming simulation. The random variations in the selling prices and costs of raw materials and labor explain this drop in profit. Notice also that the production plan is different in the case of bounded two-stage programming simulation where they would designate the production in

Table 5-6. Bounded two-stage programming simulation results and tests.

| Cost per run n sims ($) | Number of simulations (n) | Average max. objective ($) | Std. dev. (s) of the max. objective | $P_{calc} = \frac{(n-1)s^2}{\sigma_0^2}$ | Null hypothesis decision[a] |
|---|---|---|---|---|---|
| 1.843 | 2 | 5,275,262.29 | 506,801.73 | 4.46 | Reject |
| 2.525 | 3 | 5,191,276.58 | 386,761.93 | 5.19 | Accept |
| 3.163 | 4 | 5,187,124.89 | 315,898.94 | | |
| 4.001 | 5 | 5,216,098.83 | 281,143.29 | | |
| 4.446 | 6 | 5,368,173.91 | 449,437.67 | Stop simulating | Stop simulating |
| 5.111 | 7 | 5,416,586.08 | 429,807.73 | | |
| 6.507 | 8 | 5,459,023.92 | 473,559.60 | | |
| 5.739 | 9 | 5,510,553.89 | 478,522.31 | | |
| 7.148 | 10 | 5,463,598.67 | 446,710.58 | | |
| 10.427 | 15 | 5,472,533.38 | 415,252.41 | | |
| 13.640 | 20 | 5,464,795.21 | 512,791.56 | | |

[a]Significant at the $\alpha = .05$ level.




*check these coefficient variations by code number in Table 2-2

Figure 5-7. Bounded two-stage programming simulation results.

a different way where they would not designate any production to 13 mm WPB exportation purposes.

Methods for Solving the Drop in Profit

The proposed approach to handle the drop in profit if the major goal or major concern is to reach a certain level of profit, by increasing the number of tons to be offered to the market, and to do so, it would be necessary to increase the value of the resource availabilities up to the point that once is simulated with the random variations in costs and selling prices, the value of the new objective function would approximate the value of the objective function expected by management.

These resources increment can be found by the combination of "extrapolation" and "trial and error." We have made the following simple assumptions: the drop in profit was 29.07% due to the stochastic variations in resource availability. Therefore, an increase in the resources availability by the same percentage, i.e., an increase in the availability of trunks of up to 53,144.57 tons may compensate for this drop. This is approximately the capacity of adding another mill to our production process line, being able to process 54,900 tons of trunks. The simulation in this new model yielded $6,837,587.94 with five simulations, 6.58% less than the original objective profit. Details of this run can be found in the

Appendix, but the summary of results and the comparisons with the bounded with 41, 175 tons can be found in Table 5-7.

As a matter of illustrating the normality assumption of the random sample of 20 profit observations while simulating, we present the two-stage programming simulation and bounded two-stage simulation by means of histograms.

The comparison of the three models clearly shows the advantages of the fully stochastic model used in the last example. Monte Carlo simulation is the only reasonable approach for generating the distribution of variable values especially when all parameters are probabilistic. It would have been possible, but difficult to utilize a solution generated from either the two-stage or the chance-constrained programming and to second-guess the solution that would have resulted when all parameters are stochastically variable.

Table 5-7. Comparisons of two-stage bounded runs for solving drop in profit.

| | TS bounded with 41, 175 (tons) | TS bounded with 54, 900 (tons) | % Increment (33.33) |
|---|---|---|---|
| Trunks for processing | 41, 175 | 54, 900 | 33.33 |
| Emulsionated paraffin | 80.25 | 107.08 | 33.43 |
| Agglutinating | 1, 512.51 | 2, 016.80 | 33.34 |
| Crude ammoniac | 6.67 | 8.89 | 33.28 |
| Pure ammoniac | 20.78 | 27.75 | 33.54 |
| Labor | 29, 312.02 | 39, 083.10 | 33.33 |
| Sell 10 mm exportation | 1, 241.20 | 1, 286.00 | 3.61 |
| 13 mm | 0.00 | 1, 738.00 | - |
| 16 mm | 3, 417.15 | 3, 652.00 | 6.87 |
| 19 mm | 2, 850.81 | 5, 088.02 | 78.51 |
| 10 mm Mexico | 1, 164.80 | 1, 621.89 | 39.24 |
| 13 mm | 1, 162.00 | 1, 162.00 | 0.00 |
| 16 mm | 2, 568.00 | 2, 446.00 | - 4.75 |
| 19 mm | 1, 227.00 | 1, 180.00 | - 3.83 |
| $\Sigma$ of production | 13, 630.96 | 18, 174.81 | 33.33 |

Notice in this table that all the resources increased by 33 to 34%, but the disposition of the tons in the market is different but having also 33.33% increment.

## Two-stage Stochastic Programming Histogram

Number of simulations = 20
Avg. of max. obj. = \$5.98 MM
Std. dev. = 0.50 MM
(high) = 7.03 MM
(low) = 4.93 MM

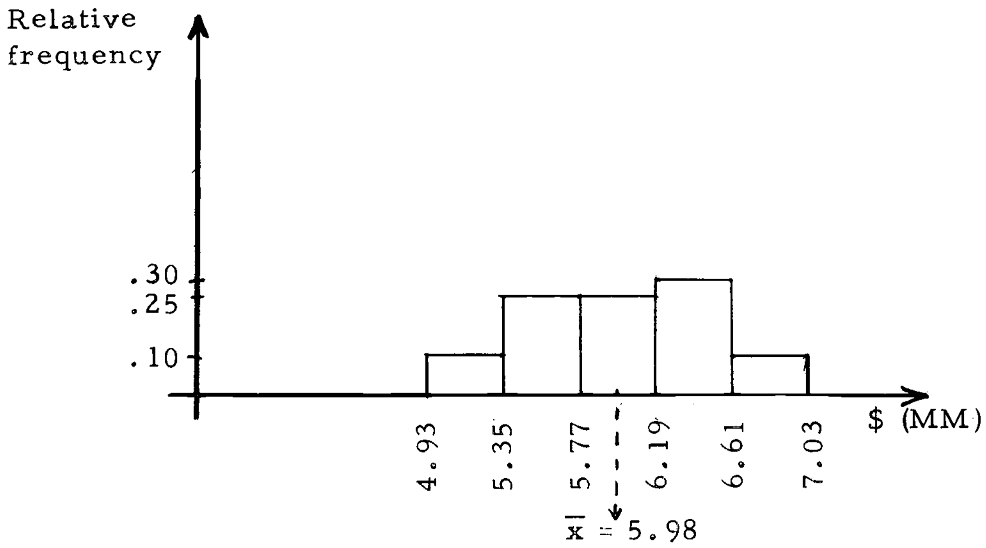
Observations:

| | | | |
|---|---|---|---|
| 5.40 | 6.53 | 6.44 | 5.52 |
| 6.16 | 6.50 | 6.02 | 6.56 |
| 5.24 | 6.62 | 6.26 | 5.77 |
| 5.69 | 5.53 | 5.37 | 4.93 |
| 5.78 | 6.19 | 6.24 | 7.03 |

Number of classes $= k = 2.33 \log_{10}(20) + 1 = 4.03 \hat{=} 5$ classes

$$\text{Class width} = \frac{\text{high - low}}{k} = \frac{7.03 - 4.93}{5} = 0.42$$

| Class limits | Tally | Absolute frequency | Relative frequency |
|---|---|---|---|
| 4.93 - 5.35 | 11 | 2 | 0.10 |
| 5.35 - 5.77 | 11111 | 5 | 0.25 |
| 5.77 - 6.19 | 11111 | 5 | 0.25 |
| 6.19 - 6.61 | 1111111 | 6 | 0.30 |
| 6.61 - 7.03 | 11 | 2 | 0.10 |
| | | 20 | 1.00 |



Bounded Two-stage Stochastic Programming Histogram

| | | |
|---|---|---|
| number of simulations | = | 20 |
| Avg. of max. obj. | = | \$5.46 MM |
| Std. dev. | = | 0.51 MM |
| (high) | = | 6.44 MM |
| (low) | = | 4.34 MM |

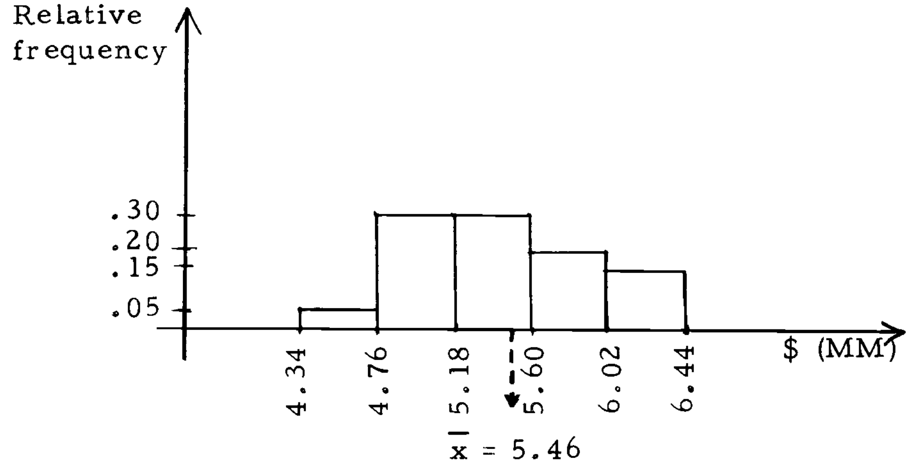
Observations:

| | | | |
|---|---|---|---|
| 4.92 | 6.13 | 5.85 | 5.17 |
| 5.63 | 5.71 | 5.56 | 5.97 |
| 5.02 | 6.17 | 5.66 | 5.28 |
| 5.17 | 5.05 | 4.82 | 4.34 |
| 5.33 | 5.50 | 5.56 | 6.44 |

Number of classes = k = 4.03 $\triangleq$ 5 classes

$$\text{Class width} = \frac{6.44 - 4.34}{5} = 0.42$$

| Class limits | Tally | Absolute frequency | Relative frequency |
|---|---|---|---|
| 4.34 - 4.76 | 1 | 1 | 0.05 |
| 4.76 - 5.18 | 111111 | 6 | 0.30 |
| 5.18 - 5.60 | 111111 | 6 | 0.30 |
| 5.60 - 6.02 | 1111 | 4 | 0.20 |
| 6.02 - 6.44 | 111 | 3 | 0.15 |
| | | 20 | 1.00 |



# VI. CONCLUSIONS

The main purpose of this thesis was to identify and evaluate practical methods to handle risk factors in optimization. We sought to find tools that adhere more closely to the reality, considering effects of risk over several of the factors that affected the particleboard production system. Though necessary for a proper planning, scheduling and control, this risk consideration had to depend heavily upon the availability of data and management readiness for dealing with the uncertain future.

Thus, this study was based upon data that represented the demands for particleboards in Mexico for typical years. The optimization models described in Chapters III and IV were supplemented by RPMS networks and Monte Carlo simulation of the affects of risk. The only data used for describing the probability distributions of the coefficients random variations, as discussed in Chapter IV, were the three estimators L, M, and U (Lower limit, Most likely and Upper limit). The example used throughout this thesis deals with one application of stochastic models subjected to simulation; another more basic and algorithmic study will be found in Chou (1977).

The following are some observations stemming from the results of this thesis:

1. It is no longer true that computer facilities for simulating the

effects of risk in linear programming are not existing, and even if there were, they would be very costly. Indeed, we did this entire computer study, funded with a research grant of $450.00.

2. The computer simulations showed that under considerations of risk in the model, we could expect a considerable under-achievement in profit and production yields as it is shown in Table 6-1. For example:

   a. In the case of two-stage programming simulation, a small variation in the costs and selling prices, say 9%, on the average, resulted in a drop in profit of 22.88%.

   b. In the case of chance-constrained programming simulation, a variation in availability of Henequen cactus trunks, labor, costs and selling prices, pay 6% on the average, gave as a result 22.36% drop in profit.

   c. Finally, in the stochastic programming simulation case, variations in processing transformations (technological coefficients), resources availability, selling prices and costs, say on the average 5%, resulted in 28.28% of drop in profit.

A 95% probability assurance of the expected profit, was given in terms of confidence intervals in Chapter V, for each of the programming simulation methods used.

Table 6-1. Comparison of yields under different methods of programming.

| Method approach | Yields: Profit ($) | % Decrease over DET (M) | Production (tons) | % Decrease over DET (M) |
|---|---|---|---|---|
| Deterministic using most likely value (M) | 7,318,723.33 | - | 13,621.00 | - |
| Deterministic using technological coefficients estimate (TCE) | 6,540,278.33 | 10.64 | 13,285.64 | 2.53 |
| Two-stage programming with 5 simulations | 5,644,545.16 | 22.88 | 13,631.09 | 0.00 |
| Chance-constrained programming with 8 simulations | 5,682,461.14 | 22.36 | 12,782.82 | 6.22 |
| Stochastic programming with 4 simulations | 5,248,906.42 | 28.28 | 12,652.11 | 7.18 |

While underachievement of profit was predicted on the average, the other major goal of satisfying the Mexican demands was always achieved, in spite of an expected decrease of the particleboard production yield, of as much as 7.18%, as in the case of stochastic programming simulation.

3. Our computer simulation outputs showed no discernible patterns in the behavior of the maximum effective function values. It is necessary, however, to comment that the restricted budget granted by Oregon State University did not permit a more extensive statistical analysis of these variances.

4. Even under the most pessimistic case as it is the stochastic programming simulation, the payback of 7.83 years is attractive, since this is a governmental managed industry (Altamirano, 1977).

5. If the major concern of management were the achievement of a reasonable level of profit over the model under stochiasticity, after some experimentation, a rule of thumb could be adopted. This rule would suggest to increase the value of the major resources (raw materials and labor) by approximately the same percentage that the bounded two-stage simulation profit would need to reach the desired profit level. In our case, we had increased the trunks raw material by 29%, up to 54, 900 tons. In the next step, the model was simulated again with the new

bounds, the result was judged to be an acceptable approximation of the desired profit (6.58% under the predicted level of profit).

Table 6-2 summarizes the relative advantages and disadvantages of the deterministic method, of the traditional stochastic approach and of the stochastic simulation method. These were proposed in this thesis and were earlier discussed throughout the thesis.

## A. Evaluation of the Study

Practical applications of the models used in this thesis are yet to be evaluated. The following comments on the utility of these models were made by Mr. Natal Altamirano of the "Department of Projects and Scientific Investigations" of CONACYT:

> The models through graphical representations of RPM networks provide information and visibility of production problem areas for planning decisions.
>
> The degree with which these models dealt with the reality, constitutes a very useful tool for the decision making task. The stochastic situations considered were appropriate especially for production yields, input expenses and availability of scarce resources. Rather than dealing with subjective deterministically known considerations, these models provided more accurate and reliable information for production planning, scheduling and control.
>
> The assumption of no limit in the Mexican demands is correct, because whatever quantity is left, is going to be used by the government in their mass construction of houses for the low income class in Mexico.
>
> Finally, simulation of the effects of risk and optimization properties of these models constitute a powerful service

Table 6-2. Advantages and disadvantages of methods.

| | Deterministic Linear Programming | Traditional Stochastic Programming | Stochastic Programming Simulation |
|---|---|---|---|
| Data needed | One value, easy to obtain | Probabilities for outcomes very difficult to obtain | 3 Parameters describing probability distributions difficult to obtain |
| Computer approach | Deterministic (most likely value) | Deterministic (expected values) | Monte Carlo simulation |
| Computer costs | Very low | Low | High but reasonable |
| Results | Crude approximation to the reality | Approximation to the reality | High approximation to the reality |
| Analysis of results | Easy to interpret | Easy to interpret | Harder to interpret (in terms of averages) |
| Planning tool | Not very realiable; high expected drop in profit | Reliable; medium expected drop in profit | Very reliable; low expected deviation in expected profit |

tool for promoting wider uses of systems analysis in industrial applications (Altamirano, 1977).

## B. Proposed Areas for Future Research

It is hoped that the conceptual framework, and ideas in this thesis, will stimulate additional research and investigations on topics of this nature. The following extensions of this thesis are recommended as promising areas of future study.

1. Incorporation of integer and goal programming methodologies into the systems approach developed.
2. Application of stochastic programming simulation to other systems.
3. A more extensive statistical analysis of the stochastic variations of the maximized objective function values during simulation.

Finally, it must be said that the proposed methods and techniques in this thesis are meant to supplement traditional procedures and management judgment and by no means intended to replace them. Hopefully further improvement of these methods will be made, and this thesis will be regarded as a positive contribution toward making operations research a "medium" rather than the goal.

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# APPENDIX

TITLE ADRI7 DETERMINISTIC RUN
OBJ =PROFIT
MAX. OBJECTIVE = 7318723.33038

RESOURCES (ROWS)

| | CONSTANT | P. RESIDUE | D. VALUE |
|---|---|---|---|
| RT | 41175.0000 | 0.0000 | 192.5650 |
| RM | 0.0000 | 0.0000 | 257.5650 |
| S1HF45 | 0.0000 | 0.0000 | 257.5650 |
| FFH | 0.0000 | 1556.9408 | 0.0000 |
| S2HF3 | 0.0000 | 0.0000 | 457.2431 |
| REP | 0.0000 | 0.0000 | 8150.0000 |
| RMWF | 0.0000 | 0.0000 | 457.2431 |
| RNF | 0.0000 | 0.0000 | 303.2051 |
| RFF | 0.0000 | 0.0000 | 611.2811 |
| FFF | 0.0000 | 0.0000 | 2442.1939 |
| BFR | 0.0000 | 2477.2323 | 0.0000 |
| RAG | 0.0000 | 0.0000 | 6206.2500 |
| RCA | 0.0000 | 0.0000 | 2750.0000 |
| RPA | 0.0000 | 0.0000 | 8300.0000 |
| RFS10 | 0.0000 | 0.0000 | 1771.1750 |
| RFS13 | 0.0000 | 0.0000 | 1432.9287 |
| RFS16 | 0.0000 | 0.0000 | 1220.6512 |
| RFS19 | 0.0000 | 0.0000 | 1075.9734 |
| ALABOR | 0.0000 | 0.0000 | 112.1941 |
| RSTO10 | 0.0000 | 0.0000 | 2094.5779 |
| RSTO13 | 0.0000 | 0.0000 | 1694.5704 |
| RSTO16 | 0.0000 | 0.0000 | 1443.5327 |
| RSTO19 | 0.0000 | 0.0000 | 1272.4378 |
| RP10 | 0.0000 | 0.0000 | 1771.1750 |
| RP13 | 0.0000 | 0.0000 | 1432.9287 |
| RP16 | 0.0000 | 0.0000 | 1220.6512 |
| RP19 | 0.0000 | 0.0000 | 1075.9734 |
| RLABOR | 48037.5000 | 19725.3083 | 0.0000 |
| ROF10 | 0.0000 | 0.0000 | 3085.4700 |
| EXDEM10 | 1286.0000 | 1286.0000 | 0.0000 |
| ODDEM10 | -1120.0000 | 2023.9346 | 0.0000 |
| ROF13 | 0.0000 | 0.0000 | 2695.4625 |
| EXDEM13 | 1738.0000 | 1738.0000 | 0.0000 |
| ODDEM13 | -1162.0000 | 0.0000 | 34.0625 |
| ROF16 | 0.0000 | 0.0000 | 2434.4249 |
| EXDEM16 | 3652.0000 | 3652.0000 | 0.0000 |
| ODDEM16 | -2446.0000 | 0.0000 | 13.3249 |
| ROF19 | 0.0000 | 0.0000 | 2263.3300 |
| EXDEM19 | -1670.0000 | 0.0000 | 75.5800 |
| ODDEM19 | -1180.0000 | 4029.1069 | 0.0000 |

PROCESSES (COLUMNS)

| | CONSTANT | P. VALUE | D. RESIDUE |
|---|---|---|---|
| RAW | -75.0000 | 41175.0000 | 0.0000 |
| MILL | 0.0000 | 41175.0000 | 0.0000 |
| DRYER | 0.0000 | 41175.0000 | 0.0000 |
| MWF | 0.0000 | 3429.0306 | 0.0000 |
| SFFP | 0.0000 | 11596.9389 | 0.0000 |
| BFP | -8150.0000 | 90.3792 | 0.0000 |
| SEPAR | 0.0000 | 26622.9081 | 0.0000 |
| BAG | -6206.2500 | 1512.6850 | 0.0000 |
| BCA | -2750.0000 | 6.5155 | 0.0000 |
| BPA | -8300.0000 | 20.7318 | 0.0000 |
| BLEND10 | 0.0000 | 3717.9927 | 0.0000 |
| BLEND13 | 0.0000 | 1374.1722 | 0.0000 |
| BLEND16 | 0.0000 | 2892.6206 | 0.0000 |
| BLEND19 | 0.0000 | 8135.1784 | 0.0000 |
| FORST10 | 0.0000 | 3717.9927 | 0.0000 |
| FORST13 | 0.0000 | 1374.1722 | 0.0000 |
| FORST16 | 0.0000 | 2892.6206 | 0.0000 |
| FORST19 | 0.0000 | 8135.1784 | 0.0000 |
| PRESS10 | 0.0000 | 3717.9927 | 0.0000 |
| PRESS13 | 0.0000 | 1374.1722 | 0.0000 |
| PRESS16 | 0.0000 | 2892.6206 | 0.0000 |
| PRESS19 | 0.0000 | 8135.1784 | 0.0000 |
| LABOR | -112.1941 | 29312.1917 | 0.0000 |
| STO10 | -749.6300 | 3143.9346 | 0.0000 |
| STO13 | -749.6300 | 1162.0000 | 0.0000 |
| STO16 | -749.6300 | 2446.0000 | 0.0000 |
| STO19 | -749.6300 | 6879.1069 | 0.0000 |
| SELL10E | 3009.8900 | 0.0000 | 75.5800 |
| SELL13E | 2575.8200 | 0.0000 | 109.6425 |
| SELL16E | 2345.5200 | 0.0000 | 88.9049 |
| SELL19E | 2187.7500 | 1670.0000 | 0.0000 |
| SELL10D | 3085.4700 | 3143.9346 | 0.0000 |
| SELL13D | 2651.4000 | 1162.0000 | 0.0000 |
| SELL16D | 2421.1000 | 2446.0000 | 0.0000 |
| SELL19D | 2263.3300 | 5209.1069 | 0.0000 |

TITLE ADRITCE TECHNOLOGICAL COEFF ESTIMATE DER RUN
OBJ =PROFIT
MAX. OBJECTIVE = 6540278.32697

RESOURCES (ROWS)

| | CONSTANT | P. RESIDUE | D. VALUE |
|---|---|---|---|
| RT | 40131.6700 | 0.0000 | 157.3682 |
| RM | 0.0000 | 0.0000 | 244.5382 |
| S1HF45 | 0.0000 | 0.0000 | 244.5382 |
| FFH | 0.0000 | 1517.4396 | 0.0000 |
| S2HF3 | 0.0000 | 0.0000 | 434.1172 |
| REP | 0.0000 | 0.0000 | 8448.8300 |
| RMWF | 0.0000 | 0.0000 | 434.1172 |
| RNF | 0.0000 | 0.0000 | 283.7737 |
| RFF | 0.0000 | 0.0000 | 584.4307 |
| FFF | 0.0000 | 0.0000 | 2335.1205 |
| BFR | 0.0000 | 2414.4619 | 0.0000 |
| RAG | 0.0000 | 0.0000 | 6370.2100 |
| RCA | 0.0000 | 0.0000 | 2850.8300 |
| RPA | 0.0000 | 0.0000 | 8604.3300 |
| RFS10 | 0.0000 | 0.0000 | 1756.8845 |
| RFS13 | 0.0000 | 0.0000 | 1417.4744 |
| RFS16 | 0.0000 | 0.0000 | 1204.4245 |
| RFS19 | 0.0000 | 0.0000 | 1059.2475 |
| ALABOR | 0.0000 | 0.0000 | 116.3100 |
| RSTO10 | 0.0000 | 0.0000 | 2377.5790 |
| RSTO13 | 0.0000 | 0.0000 | 1676.2942 |
| RSTO16 | 0.0000 | 0.0000 | 1424.3431 |
| RSTO19 | 0.0000 | 0.0000 | 1232.6580 |
| RP10 | 0.0000 | 0.0000 | 1756.8845 |
| RP13 | 0.0000 | 0.0000 | 1417.4744 |
| RP16 | 0.0000 | 0.0000 | 1204.4245 |
| RP19 | 0.0000 | 0.0000 | 1059.2475 |
| RLABOR | 45546.6700 | 16318.2456 | 0.0000 |
| ROF10 | 0.0000 | 0.0000 | 3093.5100 |
| EXDEM10 | 1286.0000 | 1286.0000 | 0.0000 |
| ODDEM10 | -1120.0000 | 1916.2502 | 0.0000 |
| ROF13 | 0.0000 | 0.0000 | 2692.1262 |
| EXDEM13 | 1738.0000 | 1738.0000 | 0.0000 |
| ODDEM13 | -1162.0000 | 0.0000 | 34.2362 |
| ROF16 | 0.0000 | 0.0000 | 2440.1751 |
| EXDEM16 | 3652.0000 | 3652.0000 | 0.0000 |
| ODDEM16 | -2446.0000 | 0.0000 | 13.3651 |
| ROF19 | 0.0000 | 0.0000 | 2268.4900 |
| EXDEM19 | -1670.0000 | 0.0000 | 74.3200 |
| ODDEM19 | -1180.0000 | 3791.3973 | 0.0000 |

PROCESSES (COLUMNS)

| | CONSTANT | P. VALUE | D. RESIDUE |
|---|---|---|---|
| RAW | -76.6700 | 40131.6700 | 0.0000 |
| MILL | 0.0000 | 40131.6700 | 0.0000 |
| DRYER | 0.0000 | 40131.6700 | 0.0000 |
| MWF | 0.0000 | 3342.1426 | 0.0000 |
| SFFP | 0.0000 | 11303.0849 | 0.0000 |
| BFP | -8448.8300 | 78.3405 | 0.0000 |
| SEPAR | 0.0000 | 25948.3123 | 0.0000 |
| BAG | -6370.2100 | 1474.3527 | 0.0000 |
| BCA | -2850.8300 | 6.3547 | 0.0000 |
| BPA | -8604.3300 | 20.2087 | 0.0000 |
| BLEND10 | 0.0000 | 3590.6459 | 0.0000 |
| BLEND13 | 0.0000 | 1374.1722 | 0.0000 |
| BLEND16 | 0.0000 | 2892.6206 | 0.0000 |
| BLEND19 | 0.0000 | 7854.0649 | 0.0000 |
| FORST10 | 0.0000 | 3590.6459 | 0.0000 |
| FORST13 | 0.0000 | 1374.1722 | 0.0000 |
| FORST16 | 0.0000 | 2892.6206 | 0.0000 |
| FORST19 | 0.0000 | 7854.0649 | 0.0000 |
| PRESS10 | 0.0000 | 3590.6459 | 0.0000 |
| PRESS13 | 0.0000 | 1374.1722 | 0.0000 |
| PRESS16 | 0.0000 | 2892.6206 | 0.0000 |
| PRESS19 | 0.0000 | 7854.0649 | 0.0000 |
| LABOR | -116.3100 | 29228.4244 | 0.0000 |
| STO10 | -759.9500 | 3036.2502 | 0.0000 |
| STO13 | -759.9500 | 1162.0000 | 0.0000 |
| STO16 | -759.9500 | 2446.0000 | 0.0000 |
| STO19 | -759.9500 | 6641.3973 | 0.0000 |
| SELL10E | 3019.1900 | 0.0000 | 74.3200 |
| SELL13E | 2583.6000 | 0.0000 | 108.5262 |
| SELL16E | 2352.4900 | 0.0000 | 87.6851 |
| SELL19E | 2194.1700 | 1670.0000 | 0.0000 |
| SELL10D | 3093.5100 | 3036.2502 | 0.0000 |
| SELL13D | 2657.9200 | 1162.0000 | 0.0000 |
| SELL16D | 2426.8100 | 2446.0000 | 0.0000 |
| SELL19D | 2268.4900 | 4971.3973 | 0.0000 |

Deterministic Run

```
       **  DATA FILE LISTING  **

ROWS
$PROFIT <RT <RM <S1HF45 <FFH <S2HF3 <REP <RMWF <RNF <RFF
<FFF <BFR <RAG <RCA <RPA <RFS10 <RFS13 <RFS16 <RFS19
<ALABOR <RSTO10 <RSTO13 <RSTO16 <RSTO19
<RP10 <RP13 <RP16 <RP19 <RLABOR
<ROF10 <EXDEM10 <DODEM10
<ROF13 <EXDEM13 <DODEM13
<ROF16 <EXDEM16 <DODEM16
<ROF19 <EXDEM19 <DODEM19
COLUMNS
BAW RT 1 RM -1 PROFIT -75
MILL RM 1 S1HF45 -1
DRYER S1HF45 1 FFH .09 S2HF3 -.5633
MWF RMWF 1 S2HF3 -1
SFFP RFF 1 FFH -.4538 FFF -.2503 BFR -.2959
BEP REP -1 PROFIT -8150
SEPAR S2HF3 1 RMWF -.1288 RNF -.4356 RFF -.4356
BAG RAG -1 PROFIT -6206.25
BCA RCA -1 PROFIT -2750
BPA RPA -1 PROFIT -5300
BLEND10 FFF .2672 RNF .5837 RAG .1392 REP .0074 RCA .0006
BLEND10 RPA .0019 RFS10 -1
BLEND13 FFF .2055 RNF .6797 RAG .1071 REP .0057 RCA .0005
BLEND13 RPA .0015 RFS13 -1
BLEND16 FFF .167 RNF .7398 RAG .087 REP .0046 RCA .0004
BLEND16 RPA .0012 RFS16 -1
BLEND19 FFF .1406 RNF .7909 RAG .0733 REP .0039 RCA .0003
BLEND19 RPA .001 RFS19 -1
FORST10 RFS10 1 RP10 -1
FORST13 RFS13 1 RP13 -1
FORST16 RFS16 1 RP16 -1
FORST19 RFS19 1 RP19 -1
PRESS10 BFR .0592 RP10 1 RSTO10 -.8456
PRESS13 BFR .0592 RP13 1 RSTO13 -.8456
PRESS16 BFR .0592 RP16 1 RSTO16 -.8456
PRESS19 BFR .0592 RP19 1 RSTO19 -.8456
LABOR RLABOR 1 ALABOR -1 PROFIT -112.1941
STO10 RSTO10 1 ALABOR 2.1504 ROF10 -1 PROFIT -749.63
STO13 RSTO13 1 ALABOR 2.1504 ROF13 -1 PROFIT -749.63
STO16 RSTO16 1 ALABOR 2.1504 ROF16 -1 PROFIT -749.63
STO19 RSTO19 1 ALABOR 2.1504 ROF19 -1 PROFIT -749.63
SELL10E ROF10 1 EXDEM10 1 PROFIT 3009.89
SELL13E ROF13 1 EXDEM13 1 PROFIT 2575.92
SELL16E ROF16 1 EXDEM16 1 PROFIT 2345.52
SELL19E ROF19 1 EXDEM19 -1 PROFIT 2187.75
SELL10D ROF10 1 DODEM10 -1 PROFIT 3085.47
SELL13D ROF13 1 DODEM13 -1 PROFIT 2051.4
SELL16D ROF16 1 DODEM16 -1 PROFIT 2421.1
SELL19D ROF19 1 DODEM19 -1 PROFIT 2263.33
RHS
RESOURCE RT 41175 RLABOR 48037.5 DODEM10 -1120 DODEM13 -1162
RESOURCE DODEM16 -2446 DODEM19 -1180
RESOURCE EXDEM10 1266 EXDEM13 1738 EXDEM16 3652 EXDEM19 -1670
EOF
```

Technological Coeff Estimate Det Run

```
       **  DATA FILE LISTING  **

ROWS
$PROFIT <RT <RM <S1HF45 <FFH <S2HF3 <REP <RMWF <RNF <RFF
<FFF <BFR <RAG <RCA <RPA <RFS10 <RFS13 <RFS16 <RFS19
<ALABOR <RSTO10 <RSTO13 <RSTO16 <RSTO19
<RP10 <RP13 <RP16 <RP19 <RLABOR
<ROF10 <EXDEM10 <DODEM10
<ROF13 <EXDEM13 <DODEM13
<ROF16 <EXDEM16 <DODEM16
<ROF19 <EXDEM19 <DODEM19
COLUMNS
BAW RT 1 RM -1 PROFIT -76.67
MILL RM 1 S1HF45 -1
DRYER S1HF45 1 FFH .09 S2HF3 -.5633
MWF RMWF 1 S2HF3 -1
SFFP RFF 1 FFH -.4538 FFF -.2503 BFR -.2959
BEP REP -1 PROFIT -8448.83
SEPAR S2HF3 1 RMWF -.1288 RNF -.4356 RFF -.4356
BAG RAG -1 PROFIT -6370.21
BCA RCA -1 PROFIT -2950.83
BPA RPA -1 PROFIT -8604.33
BLEND10 FFF .2672 RNF .5837 RAG .1392 REP .0074 RCA .0006
BLEND10 RPA .0019 RFS10 -1
BLEND13 FFF .2055 RNF .6797 RAG .1071 REP .0057 RCA .0005
BLEND13 RPA .0015 RFS13 -1
BLEND16 FFF .167 RNF .7398 RAG .087 REP .0046 RCA .0004
BLEND16 RPA .0012 RFS16 -1
BLEND19 FFF .1406 RNF .7809 RAG .0733 REP .0039 RCA .0003
BLEND19 RPA .001 RFS19 -1
FORST10 RFS10 1 RP10 -1
FORST13 RFS13 1 RP13 -1
FORST16 RFS16 1 RP16 -1
FORST19 RFS19 1 RP19 -1
PRESS10 BFR .0592 RP10 1 RSTO10 -.8456
PRESS13 BFR .0592 RP13 1 RSTO13 -.8456
PRESS16 BFR .0592 RP16 1 RSTO16 -.8456
PRESS19 BFR .0592 RP19 1 RSTO19 -.8456
LABOR RLABOR 1 ALABOR -1 PROFIT -116.31
STO10 RSTO10 1 ALABOR 2.2 ROF10 -1 PROFIT -759.95
STO13 RSTO13 1 ALABOR 2.2 ROF13 -1 PROFIT -759.95
STO16 RSTO16 1 ALABOR 2.2 ROF16 -1 PROFIT -759.95
STO19 RSTO19 1 ALABOR 2.2 ROF19 -1 PROFIT -759.95
SELL10E ROF10 1 EXDEM10 1 PROFIT 3019.19
SELL13E ROF13 1 EXDEM13 1 PROFIT 2583.6
SELL16E ROF16 1 EXDEM16 1 PROFIT 2352.49
SELL19E ROF19 1 EXDEM19 -1 PROFIT 2194.17
SELL10D ROF10 1 DODEM10 -1 PROFIT 3093.51
SELL13D ROF13 1 DODEM13 -1 PROFIT 2657.92
SELL16D ROF16 1 DODEM16 -1 PROFIT 2426.81
SELL19D ROF19 1 DODEM19 -1 PROFIT 2268.49
RHS
RESOURCE RT 40131.67 RLABOR 45546.67 DODEM10 -1120 DODEM13 -1162
RESOURCE DODEM16 -2446 DODEM19 -1180
RESOURCE EXDEM10 1266 EXDEM13 1738 EXDEM16 3652 EXDEM19 -1670
EOF
```

TITLE ADRITCE TECHNOLOGICAL COEFF ESTIMATE DER RUN

RANGING REPORT OF RESOURCES (ROWS)

| | CONSTANT | - RANGE - | |
|---|---|---|---|
| RT | 40131.6700 | 23490.8684 | 62537.3230 |
| RM | 0.0000 | -16640.8016 | 22405.6530 |
| S1HF45 | 0.0000 | -16640.8016 | 22405.6530 |
| FFH | 0.0000 | -1517.4896 | PINF |
| S2HF3 | 0.0000 | -6687.9225 | 12621.1043 |
| REP | 0.0000 | MINF | 78.3405 |
| RMWF | 0.0000 | -2911.6747 | 12621.1043 |
| RNF | 0.0000 | -2124.1799 | 2040.3062 |
| RFF | 0.0000 | -1467.6564 | 3884.8764 |
| FFF | 0.0000 | -367.3544 | 972.3846 |
| BFR | 0.0000 | -2414.4618 | PINF |
| RAG | 0.0000 | MINF | 1474.3527 |
| RCA | 0.0000 | MINF | 6.3547 |
| RPA | 0.0000 | MINF | 20.2087 |
| RFS10 | 0.0000 | -2266.1426 | 8771.7412 |
| RFS13 | 0.0000 | -4419.5177 | 1374.1722 |
| RFS16 | 0.0000 | -5664.6685 | 2892.6206 |
| RFS19 | 0.0000 | -4483.6770 | 8771.7412 |
| ALABOR | 0.0000 | -16318.2456 | 29228.4244 |
| RST010 | 0.0000 | -1916.2502 | 7417.3844 |
| RST013 | 0.0000 | -3737.1442 | 1162.0000 |
| RST016 | 0.0000 | -4790.0437 | 2446.0000 |
| RST019 | 0.0000 | -3791.3973 | 7417.3844 |
| RP10 | 0.0000 | -2266.1426 | 8771.7412 |
| RP13 | 0.0000 | -4419.5177 | 1374.1722 |
| RP16 | 0.0000 | -5664.6685 | 2892.6206 |
| RP19 | 0.0000 | -4483.6770 | 8771.7412 |
| RLABOR | 45546.6700 | 29228.4244 | PINF |
| ROF10 | 0.0000 | -1916.2502 | PINF |
| EXDEM10 | 1286.0000 | 0.0000 | PINF |
| DODEM10 | -1120.0000 | -3036.2502 | PINF |
| ROF13 | 0.0000 | -3737.1442 | 1162.0000 |
| EXDEM13 | 1738.0000 | 0.0000 | PINF |
| DODEM13 | -1162.0000 | -4899.1442 | -.0000 |
| ROF16 | 0.0000 | -4790.0437 | 2446.0000 |
| EXDEM16 | 3652.0000 | 0.0000 | PINF |
| DODEM16 | -2446.0000 | -7236.0437 | -.0000 |
| ROF19 | 0.0000 | -3791.3973 | 100000.0000 |
| EXDEM19 | -1670.0000 | -5461.3973 | 0.0000 |
| DODEM19 | -1180.0000 | -4971.3973 | PINF |

TITLE ADRITCE TECHNOLOGICAL COEFF ESTIMATE DER RUN

RANGING REPORT OF PROCESSES (COLUMNS)

| | CONSTANT | - RANGE - | |
|---|---|---|---|
| BAW | -76.6700 | -244.5382 | PINF |
| MILL | 0.0000 | -167.8682 | PINF |
| DRYER | 0.0000 | -167.8682 | PINF |
| MWF | 0.0000 | -378.2029 | PINF |
| SFFP | 0.0000 | -584.4807 | PINF |
| BEP | -8448.8300 | -92851.8635 | 0.0000 |
| SEPAR | 0.0000 | -259.6250 | PINF |
| BAG | -6370.2100 | -10854.6830 | 0.0000 |
| BCA | -2850.8300 | *********** | 0.0000 |
| BPA | -8604.3300 | *********** | 0.0000 |
| BLEND10 | 0.0000 | -54.2023 | 255.4757 |
| BLEND13 | 0.0000 | MINF | 28.9248 |
| BLEND16 | 0.0000 | MINF | 11.3015 |
| BLEND19 | 0.0000 | -14.2783 | 506.4225 |
| FORST10 | 0.0000 | -54.2023 | 255.4757 |
| FORST13 | 0.0000 | MINF | 28.9248 |
| FORST16 | 0.0000 | MINF | 11.3015 |
| FORST19 | 0.0000 | -14.2783 | 506.4225 |
| PRESS10 | 0.0000 | -54.2023 | 255.4757 |
| PRESS13 | 0.0000 | MINF | 28.9248 |
| PRESS16 | 0.0000 | MINF | 11.3015 |
| PRESS19 | 0.0000 | -14.2783 | 506.4225 |
| LABOR | -116.3100 | -268.8254 | -.0000 |
| ST010 | -759.9500 | -824.0493 | -457.8264 |
| ST013 | -759.9500 | MINF | -725.7438 |
| ST016 | -759.9500 | MINF | -746.5849 |
| ST019 | -759.9500 | -776.8354 | -161.8586 |
| SELL10E | 3019.1900 | MINF | 3093.5100 |
| SELL13E | 2583.6000 | MINF | 2692.1262 |
| SELL16E | 2352.4900 | MINF | 2440.1751 |
| SELL19E | 2194.1700 | MINF | 2268.4900 |
| SELL10D | 3093.5100 | 3029.4107 | 3395.6336 |
| SELL13D | 2657.9200 | MINF | 2692.1262 |
| SELL16D | 2426.8100 | MINF | 2440.1751 |
| SELL19D | 2268.4900 | 2251.6046 | 2867.3814 |

ADRI7 ADRITS WITH 5 SIMULATIONS

NUMBER OF SIMULATIONS = 5

AVERAGE MAX. OBJECT = 5644545.28

S.D. OF THE MAX. OBJECT = 351972.09

MAXIMUM OF THE MAX. OBJECT = 6155131.92

MINIMUM OF THE MAX. OBJECT = 5240613.82

** SOLUTION **

RESOURCES(ROWS)

| | AVE.P.RES. | S.D.P.RES. | AVE.D.VAL. | S.D.D.VAL. |
|---|---|---|---|---|
| RT | 0.00 | 0.00 | 141.56 | 10.13 |
| RM | 0.00 | 0.00 | 225.12 | 8.17 |
| S1HF+5 | 0.00 | 0.00 | 225.12 | 8.17 |
| FFH | 1556.94 | I | 0.00 | 0.00 |
| S2HF3 | 0.00 | 0.00 | 399.65 | 14.50 |
| REP | 0.00 | 0.00 | 8732.61 | 470.20 |
| RMWF | 0.00 | 0.00 | 399.65 | 14.50 |
| RNF | 0.00 | 0.00 | 229.75 | 62.39 |
| RFF | 0.00 | 0.00 | 569.55 | 92.21 |
| FFF | 0.00 | 0.00 | 2275.46 | 328.44 |
| BFR | 2477.23 | .02 | 0.00 | 0.00 |
| RAG | 0.00 | 0.00 | 6755.51 | 124.82 |
| RCA | 0.00 | 0.00 | 2844.94 | 52.02 |
| RPA | 0.00 | 0.00 | 8686.77 | 551.56 |
| RFS10 | 0.00 | 0.00 | 1765.31 | 37.55 |
| RFS13 | 0.00 | 0.00 | 1411.52 | 19.39 |
| RFS16 | 0.00 | 0.00 | 1199.44 | 14.81 |
| RFS19 | 0.00 | 0.00 | 1038.12 | 18.16 |
| ALABOR | 0.00 | 0.00 | 119.17 | 3.44 |
| RST010 | 0.00 | 0.00 | 2087.65 | 44.40 |
| RST013 | 0.00 | 0.00 | 1669.25 | 22.92 |
| RST016 | 0.00 | 0.00 | 1406.62 | 17.51 |
| RST019 | 0.00 | 0.00 | 1227.69 | 21.47 |
| RP10 | 0.00 | 0.00 | 1765.31 | 37.55 |
| RP13 | 0.00 | 0.00 | 1411.52 | 19.39 |
| RP16 | 0.00 | 0.00 | 1199.44 | 14.81 |
| RP19 | 0.00 | 0.00 | 1038.12 | 18.16 |
| RLABOR | 18725.22 | .49 | 0.00 | 0.00 |
| ROF10 | 0.00 | 0.00 | 3112.50 | 45.30 |
| EXDEM10 | 1286.00 | 0.00 | 0.00 | 0.00 |
| DODEM10 | 1194.60 | 852.99 | 23.03 | 51.62 |
| ROF13 | 0.00 | 0.00 | 2687.19 | 22.31 |
| EXDEM13 | 1738.00 | 0.00 | 0.00 | 0.00 |
| DODEM13 | 789.43 | 1765.22 | 20.63 | 16.69 |
| ROF16 | 0.00 | 0.00 | 2429.15 | 19.44 |
| EXDEM16 | 3652.00 | 0.00 | 0.00 | 0.00 |
| DODEM16 | 2036.15 | 2788.11 | 17.69 | 18.34 |
| ROF19 | 0.00 | 0.00 | 2261.25 | 16.29 |
| EXDEM19 | 0.00 | 0.00 | 53.63 | 17.52 |
| DODEM19 | 2032.91 | 2014.97 | 1.45 | 2.18 |

** SOLUTION **

PROCESSES(COLUMNS)

| | AVE.P.VAL. | S.D.P.VAL. | AVE.D.RES. | S.D.D.RES. |
|---|---|---|---|---|
| BAW | 41175.00 | 0.00 | 0.00 | 0.00 |
| MILL | 41175.00 | 0.00 | 0.00 | 0.00 |
| DRYER | 41175.00 | 0.00 | 0.00 | 0.00 |
| MWF | 3429.03 | .00 | 0.00 | 0.00 |
| SFFP | 11596.94 | .00 | 0.00 | 0.00 |
| BEP | 80.31 | .10 | 0.00 | 0.00 |
| SEPAR | 26622.91 | .00 | 0.00 | 0.00 |
| BAG | 1512.60 | .16 | 0.00 | 0.00 |
| BCA | 6.65 | .12 | 0.00 | 0.00 |
| BPA | 20.80 | .07 | 0.00 | 0.00 |
| BLEND10 | 2737.23 | 1008.74 | 0.00 | 0.00 |
| BLEND13 | 2307.75 | 2087.54 | 0.00 | 0.00 |
| BLEND16 | 5300.55 | 3297.19 | 0.00 | 0.00 |
| BLEND19 | 5774.49 | 2382.89 | 0.00 | 0.00 |
| FORST10 | 2737.23 | 1008.74 | 0.00 | 0.00 |
| FORST13 | 2307.75 | 2087.54 | 0.00 | 0.00 |
| FORST16 | 5300.55 | 3297.19 | 0.00 | 0.00 |
| FORST19 | 5774.49 | 2382.89 | 0.00 | 0.00 |
| PRESS10 | 2737.23 | 1008.74 | 0.00 | 0.00 |
| PRESS13 | 2307.75 | 2087.54 | 0.00 | 0.00 |
| PRESS16 | 5300.55 | 3297.19 | 0.00 | 0.00 |
| PRESS19 | 5774.49 | 2382.89 | 0.00 | 0.00 |
| LABOR | 29312.28 | .48 | 0.00 | 0.00 |
| ST010 | 2314.60 | 852.99 | 0.00 | 0.00 |
| ST013 | 1951.43 | 1765.22 | 0.00 | 0.00 |
| ST016 | 4482.15 | 2788.11 | 0.00 | 0.00 |
| ST019 | 4882.91 | 2014.97 | 0.00 | 0.00 |
| SELL10E | 0.00 | 0.00 | 95.96 | 57.25 |
| SELL13E | 0.00 | 0.00 | 99.05 | 34.60 |
| SELL16E | 0.00 | 0.00 | 71.04 | 32.24 |
| SELL19E | 1670.00 | 0.00 | 0.00 | 0.00 |
| SELL10D | 2314.60 | 852.99 | 0.00 | 0.00 |
| SELL13D | 1951.43 | 1765.22 | 0.00 | 0.00 |
| SELL16D | 4482.15 | 2788.11 | 0.00 | 0.00 |
| SELL19D | 3212.91 | 2014.97 | 0.00 | 0.00 |

ACRI7 ADRICC WITH 8 SIMULATICNS

NUMBER OF SIMULATIONS = 8

AVERAGE MAX. OBJECT = 5682461.14

S.D. OF THE MAX. OBJECT = 569999.29

MAXIMUM OF THE MAX. OBJECT = 6317893.71

MINIMUM OF THE MAX. OBJECT = 4820664.53

** SOLUTION **

RESOURCES(ROWS)

| | AVE.P.RES. | S.D.F.RES. | AVE.D.VAL. | S.O.D.VAL. |
|---|---|---|---|---|
| RT | 0.00 | 0.00 | 154.25 | 16.84 |
| RM | 0.00 | 0.00 | 234.93 | 13.03 |
| S1HF45 | 0.00 | 0.00 | 234.93 | 13.03 |
| FFH | 1460.05 | 81.28 | 0.00 | 0.00 |
| S2HF3 | 0.00 | 0.00 | 417.06 | 23.14 |
| REP | 0.00 | 0.00 | 8774.71 | 506.51 |
| RMWF | 0.00 | 0.00 | 417.06 | 23.14 |
| RNF | 0.00 | 0.00 | 279.33 | 65.68 |
| RFF | 0.00 | 0.00 | 554.79 | 85.71 |
| FFF | 0.00 | 0.00 | 2216.52 | 342.43 |
| BFR | 2323.08 | 129.32 | 0.00 | 0.00 |
| RAG | 0.00 | 0.00 | 6629.27 | 278.99 |
| RCA | 0.00 | 0.00 | 2959.32 | 120.82 |
| RPA | 0.00 | 0.00 | 8946.88 | 592.17 |
| RFS10 | 0.00 | 0.00 | 1761.80 | 31.35 |
| RFS13 | 0.00 | 0.00 | 1420.27 | 18.66 |
| RFS16 | 0.00 | 0.00 | 1205.84 | 19.48 |
| RFS19 | 0.00 | 0.00 | 1059.75 | 24.30 |
| ALABOR | 0.00 | 0.00 | 119.15 | 6.59 |
| RST010 | 0.00 | 0.00 | 2083.49 | 37.07 |
| RST013 | 0.00 | 0.00 | 1679.63 | 22.07 |
| RST016 | 0.00 | 0.00 | 1426.01 | 23.04 |
| RST019 | 0.00 | 0.00 | 1253.26 | 28.74 |
| RP10 | 0.00 | 0.00 | 1761.80 | 31.35 |
| RP13 | 0.00 | 0.00 | 1420.27 | 18.66 |
| RP16 | 0.00 | 0.00 | 1205.84 | 19.48 |
| RP19 | 0.00 | 0.00 | 1059.75 | 24.30 |
| RLABOR | 14706.56 | 3080.05 | 0.00 | 0.00 |
| ROF10 | 0.00 | 0.00 | 3108.33 | 27.11 |
| EXOEM10 | 1300.82 | 63.24 | 0.00 | 0.00 |
| DODEM10 | 1382.32 | 757.30 | 10.05 | 28.43 |
| ROF13 | 0.00 | 0.00 | 2701.49 | 7.44 |
| EXOEM13 | 1752.62 | 35.85 | 0.00 | 0.00 |
| DODEM13 | 332.11 | 939.35 | 34.11 | 26.36 |
| ROF16 | 0.00 | 0.00 | 2460.08 | 14.55 |
| EXOEM16 | 3651.93 | 57.80 | 0.00 | 0.00 |
| DODEM16 | 985.52 | 1825.03 | 24.90 | 22.14 |
| ROF19 | 0.00 | 0.00 | 2282.90 | 32.77 |
| EXOEM19 | 0.00 | 0.00 | 83.48 | 45.02 |
| DODEM19 | 2483.64 | 1768.75 | 8.93 | 24.29 |

** SOLUTION **

PROCESSES(COLUMNS)

| | AVE.P.VAL. | S.D.F.VAL. | AVE.O.RES. | S.D.O.RES. |
|---|---|---|---|---|
| SAW | 38612.72 | 2149.46 | 0.00 | 0.00 |
| MILL | 38612.72 | 2149.46 | 0.00 | 0.00 |
| DRYER | 38612.72 | 2149.46 | 0.00 | 0.00 |
| MWF | 3215.65 | 179.01 | 0.00 | 0.00 |
| SFFP | 10875.27 | 605.39 | 0.00 | 0.00 |
| BEP | 75.34 | 4.22 | 0.00 | 0.00 |
| SEPAR | 24966.19 | 1389.79 | 0.00 | 0.00 |
| BAG | 1418.50 | 79.00 | 0.00 | 0.00 |
| BCA | 6.18 | .27 | 0.00 | 0.00 |
| BPA | 19.48 | 1.04 | 0.00 | 0.00 |
| BLEND10 | 2948.81 | 886.20 | 0.00 | 0.00 |
| BLEND13 | 1781.68 | 1108.41 | 0.00 | 0.00 |
| BLEND16 | 4079.54 | 2197.32 | 0.00 | 0.00 |
| BLEND19 | 6306.82 | 2112.39 | 0.00 | 0.00 |
| FORST10 | 2948.81 | 886.20 | 0.00 | 0.00 |
| FORST13 | 1781.68 | 1108.41 | 0.00 | 0.00 |
| FORST16 | 4079.54 | 2197.32 | 0.00 | 0.00 |
| FORST19 | 6306.82 | 2112.39 | 0.00 | 0.00 |
| PRESS10 | 2948.81 | 886.20 | 0.00 | 0.00 |
| PRESS13 | 1781.68 | 1108.41 | 0.00 | 0.00 |
| PRESS16 | 4079.54 | 2197.32 | 0.00 | 0.00 |
| PRESS19 | 6306.82 | 2112.39 | 0.00 | 0.00 |
| LABOR | 27488.16 | 1530.06 | 0.00 | 0.00 |
| ST010 | 2493.52 | 749.37 | 0.00 | 0.00 |
| ST013 | 1506.59 | 937.28 | 0.00 | 0.00 |
| ST016 | 3449.66 | 1858.06 | 0.00 | 0.00 |
| ST019 | 5333.05 | 1786.24 | 0.00 | 0.00 |
| SELL10E | 0.00 | 0.00 | 72.50 | 33.06 |
| SELL13E | 0.00 | 0.00 | 109.57 | 22.26 |
| SELL16E | 0.00 | 0.00 | 96.02 | 26.11 |
| SELL19E | 1679.64 | 27.85 | 0.00 | 0.00 |
| SELL10D | 2493.52 | 749.37 | 0.00 | 0.00 |
| SELL13D | 1506.59 | 937.28 | 0.00 | 0.00 |
| SELL16D | 3449.66 | 1858.06 | 0.00 | 0.00 |
| SELL19D | 3653.41 | 1731.84 | 0.00 | 0.00 |

ADRI7 ADRIFS WITH 4 SIMULATIONS

NUMBER OF SIMULATIONS = 4

AVERAGE MAX. OBJECT = 5248906.42

S.D. OF THE MAX. OBJECT = 356952.25

MAXIMUM OF THE MAX. OBJECT = 5657860.61

MINIMUM OF THE MAX. OBJECT = 4887942.17

** SOLUTION **

RESOURCES(ROWS)

| | AVE.P.RES. | S.D.P.RES. | AVE.D.VAL. | S.D.D.VAL. |
|---|---|---|---|---|
| RT | 0.00 | 0.00 | 143.50 | 5.39 |
| RM | 0.00 | 0.00 | 225.45 | 8.20 |
| S1HF45 | 0.00 | 0.00 | 225.45 | 8.20 |
| FFH | 1505.97 | 219.09 | 0.00 | 0.00 |
| S2HF3 | 0.00 | 0.00 | 393.69 | 9.15 |
| REP | 0.00 | 0.00 | 8951.07 | 534.23 |
| RMWF | 0.00 | 0.00 | 393.63 | 9.15 |
| RNF | 0.00 | 0.00 | 221.19 | 47.66 |
| RFF | 0.00 | 0.00 | 559.42 | 50.40 |
| FFF | 0.00 | 0.00 | 2259.15 | 189.07 |
| BFR | 2399.25 | 190.56 | 0.00 | 0.00 |
| RAG | 0.00 | 0.00 | 6926.52 | 119.17 |
| RCA | 0.00 | 0.00 | 2811.51 | 24.96 |
| RPA | 0.00 | 0.00 | 8901.97 | 562.51 |
| RFS10 | 0.00 | 0.00 | 1782.57 | 29.52 |
| RFS13 | 0.00 | 0.00 | 1427.66 | 8.35 |
| RFS16 | 0.00 | 0.00 | 1196.41 | 13.19 |
| RFS19 | 0.00 | 0.00 | 1045.01 | 16.24 |
| ALABOR | 0.00 | 0.00 | 117.97 | 3.55 |
| RST010 | 0.00 | 0.00 | 2106.95 | 46.06 |
| RST013 | 0.00 | 0.00 | 1671.12 | 11.71 |
| RST016 | 0.00 | 0.00 | 1417.14 | 16.70 |
| RST019 | 0.00 | 0.00 | 1247.40 | 29.84 |
| RF10 | 0.00 | 0.00 | 1782.57 | 29.52 |
| RP13 | 0.00 | 0.00 | 1427.66 | 8.35 |
| RP16 | 0.00 | 0.00 | 1196.41 | 13.19 |
| RP19 | 0.00 | 0.00 | 1045.01 | 16.24 |
| RLABOR | 15187.01 | 4832.93 | 0.00 | 0.00 |
| ROF10 | 0.00 | 0.00 | 3134.26 | 42.26 |
| EXDEM10 | 1303.45 | 42.76 | 0.00 | 0.00 |
| DODEM10 | 930.64 | 797.14 | 16.85 | 33.70 |
| ROF13 | 0.00 | 0.00 | 2690.30 | 16.71 |
| EXDEM13 | 1726.15 | 26.22 | 0.00 | 0.00 |
| DODEM13 | 1077.01 | 2154.03 | 16.80 | 19.12 |
| ROF16 | 0.00 | 0.00 | 2440.82 | 15.01 |
| EXDEM16 | 3655.35 | 53.13 | 0.00 | 0.00 |
| DODEM16 | 1420.21 | 2798.95 | 8.71 | 13.02 |
| ROF19 | 0.00 | 0.00 | 2277.32 | 24.36 |
| EXDEM19 | 0.00 | 0.00 | 59.74 | 22.98 |
| DODEM19 | 1611.01 | 1995.33 | 13.09 | 21.07 |

** SOLUTION **

PROCESSES(COLUMNS)

| | AVE.P.VAL. | S.D.P.VAL. | AVE.D.RES. | S.D.D.RES. |
|---|---|---|---|---|
| SAW | 37804.83 | 2318.85 | 0.00 | 0.00 |
| MILL | 37804.83 | 2318.85 | 0.00 | 0.00 |
| DRYER | 37804.83 | 2318.85 | 0.00 | 0.00 |
| MWF | 3137.05 | 136.73 | 0.00 | 0.00 |
| SFFP | 10980.66 | 776.05 | 0.00 | 0.00 |
| SEP | 74.16 | 6.25 | 0.00 | 0.00 |
| SEPAR | 24778.39 | 1414.54 | 0.00 | 0.00 |
| BAG | 1414.16 | 114.50 | 0.00 | 0.00 |
| BCA | 6.25 | .50 | 0.00 | 0.00 |
| BPA | 20.01 | 1.11 | 0.00 | 0.00 |
| BLEND10 | 2445.75 | 858.94 | 0.00 | 0.00 |
| BLEND13 | 2621.45 | 2538.75 | 0.00 | 0.00 |
| BLEND16 | 4582.90 | 3329.97 | 0.00 | 0.00 |
| BLEND19 | 5336.98 | 2394.89 | 0.00 | 0.00 |
| FORST10 | 2445.75 | 858.94 | 0.00 | 0.00 |
| FORST13 | 2621.45 | 2538.75 | 0.00 | 0.00 |
| FORST16 | 4582.90 | 3329.97 | 0.00 | 0.00 |
| FORST19 | 5336.98 | 2394.89 | 0.00 | 0.00 |
| PRESS10 | 2445.75 | 858.94 | 0.00 | 0.00 |
| PRESS13 | 2621.45 | 2538.75 | 0.00 | 0.00 |
| PRESS16 | 4582.90 | 3329.97 | 0.00 | 0.00 |
| PRESS19 | 5336.98 | 2394.89 | 0.00 | 0.00 |
| LABOR | 27711.64 | 2327.17 | 0.00 | 0.00 |
| STO10 | 2073.28 | 745.63 | 0.00 | 0.00 |
| STO13 | 2239.97 | 2169.96 | 0.00 | 0.00 |
| STO16 | 3871.32 | 2818.78 | 0.00 | 0.00 |
| STO19 | 4467.53 | 1994.26 | 0.00 | 0.00 |
| SELL10E | 0.00 | 0.00 | 104.25 | 28.91 |
| SELL13E | 0.00 | 0.00 | 91.13 | 21.85 |
| SELL16E | 0.00 | 0.00 | 79.52 | 20.96 |
| SELL19E | 1673.99 | 18.42 | 0.00 | 0.00 |
| SELL10D | 2073.28 | 745.63 | 0.00 | 0.00 |
| SELL13D | 2239.97 | 2169.96 | 0.00 | 0.00 |
| SELL16D | 3871.32 | 2818.78 | 0.00 | 0.00 |
| SELL19D | 2793.55 | 1996.47 | 0.00 | 0.00 |

ADRI3 ADRIT3 WITH 3 SIMULATIONS

NUMBER OF SIMULATIONS = 3

AVERAGE MAX. OBJECT = 5191276.53

S.D. OF THE MAX. OBJECT = 336761.93

MAXIMUM OF THE MAX. OBJECT = 5633625.23

MINIMUM OF THE MAX. OBJECT = 4916393.75

** SOLUTION **

RESOURCES(ROWS)

| | AVE.P.RES. | S.D.P.RES. | AVE.D.VAL. | S.D.D.VAL. |
|---|---|---|---|---|
| RT | 0.00 | 0.00 | 41.92 | 72.61 |
| RM | 0.00 | 0.00 | 127.43 | 70.13 |
| S1HF45 | 0.00 | 0.00 | 160.14 | 64.73 |
| FFH | 1556.94 | I | 0.00 | 0.00 |
| S2HF3 | 0.00 | 0.00 | 350.45 | 20.54 |
| REP | 0.00 | 0.00 | 3574.23 | 327.14 |
| RMWF | 0.00 | 0.00 | 350.45 | 20.54 |
| RNF | 0.00 | 0.00 | 256.32 | 100.76 |
| RFF | 0.00 | 0.00 | 434.53 | 135.36 |
| FFF | 0.00 | 0.00 | 1736.24 | 540.80 |
| BFR | 2477.24 | 0.00 | 0.00 | 0.00 |
| RAG | 0.00 | 0.00 | 6720.37 | 158.63 |
| RCA | 0.00 | 0.00 | 2354.93 | 70.96 |
| RPA | 0.00 | 0.00 | 3351.03 | 38.17 |
| RFS10 | 0.00 | 0.00 | 1635.33 | 73.52 |
| RFS13 | 0.00 | 0.00 | 1320.40 | 33.65 |
| RFS16 | 0.00 | 0.00 | 1122.25 | 9.92 |
| RFS19 | 0.00 | 0.00 | 987.34 | 10.77 |
| RP10 | 0.00 | 0.00 | 1635.33 | 73.52 |
| RP13 | 0.00 | 0.00 | 1320.40 | 33.65 |
| RP16 | 0.00 | 0.00 | 1122.25 | 9.92 |
| RP19 | 0.00 | 0.00 | 987.34 | 10.77 |
| RLABOR | 18725.43 | .00 | 0.00 | 0.00 |
| ALABOR | 0.00 | 0.00 | 116.97 | 1.43 |
| RST010 | 0.00 | 0.00 | 1934.53 | 86.94 |
| RST013 | 0.00 | 0.00 | 1561.43 | 39.73 |
| RST016 | 0.00 | 0.00 | 1327.17 | 11.73 |
| RST019 | 0.00 | 0.00 | 1167.62 | 12.73 |
| ROF10 | 0.00 | 0.00 | 3005.74 | 4.36 |
| EXDEM10 | 44.90 | .00 | 0.00 | 0.00 |
| DODEM10 | 44.90 | I | 0.00 | 0.00 |
| ROF13 | 0.00 | 0.00 | 2675.41 | 38.65 |
| EXDEM13 | 1738.00 | 0.00 | 0.00 | 0.00 |
| DODEM13 | 0.00 | 0.00 | 0.00 | 0.00 |
| ROF16 | 0.00 | 0.00 | 2358.66 | 18.53 |
| EXDEM16 | 234.85 | .00 | 0.00 | 0.00 |
| DODEM16 | 122.00 | 0.00 | 0.00 | 0.00 |
| ROF19 | 0.00 | 0.00 | 2203.46 | 10.54 |
| EXDEM19 | 1180.31 | .00 | 0.00 | 0.00 |
| DODEM19 | 47.00 | 0.00 | 0.00 | 0.00 |
| LMILL | 0.00 | 0.00 | 32.66 | 56.57 |
| LDRYER | 0.00 | 0.00 | 37.26 | 64.54 |
| LMWF | .00 | .00 | 0.00 | 0.00 |
| LSFFP | .00 | .00 | 0.00 | 0.00 |
| LBEP | .00 | .00 | 0.00 | 0.00 |
| LSEPAR | .00 | .00 | 0.00 | 0.00 |
| LBAG | .00 | .00 | 0.00 | 0.00 |
| LBCA | .00 | .00 | 0.00 | 0.00 |
| LBPA | .00 | .00 | 0.00 | 0.00 |
| LBLEND10 | .00 | .00 | 0.00 | 0.00 |
| LBLEND13 | .00 | 0.00 | 0.00 | 0.00 |
| LBLEND16 | .00 | .00 | 0.00 | 0.00 |
| LBLEND19 | .00 | .00 | 0.00 | 0.00 |
| LFORST10 | .00 | .00 | 0.00 | 0.00 |
| LFORST13 | .00 | 0.00 | 0.00 | 0.00 |
| LFORST16 | .00 | .00 | 0.00 | 0.00 |
| LFORST19 | .00 | .00 | 0.00 | 0.00 |
| LPRESS10 | .00 | .00 | 0.00 | 0.00 |
| LPRESS13 | .00 | 0.00 | 0.00 | 0.00 |
| LPRESS16 | .00 | .00 | 0.00 | 0.00 |
| LPRESS19 | .00 | .00 | 0.00 | 0.00 |
| LST010 | .00 | .00 | 47.26 | 81.86 |
| LST013 | 0.00 | 0.00 | 95.00 | 46.17 |
| LST016 | .00 | .00 | 13.23 | 15.53 |
| LST019 | .00 | .00 | 0.00 | 0.00 |
| LSELL19E | 137.00 | .00 | 0.00 | 0.00 |
| LSELL10D | 0.00 | 0.00 | 78.73 | 12.17 |
| LSELL13D | 58.00 | 0.00 | 0.00 | 0.00 |
| LSELL16D | 0.00 | 0.00 | 52.46 | 34.23 |
| LSELL19D | 0.00 | 0.00 | 45.53 | 3.01 |

** SOLUTION **

PROCESSES(COLUMNS)

| | AVE.P.VAL. | S.D.P.VAL. | AVE.D.RES. | S.D.D.RES. |
|---|---|---|---|---|
| BAW | 41175.00 | 0.00 | 0.00 | 0.00 |
| MILL | 41175.00 | 0.00 | 0.00 | 0.00 |
| DRYER | 41175.00 | 0.00 | 0.00 | 0.00 |
| MWF | 3429.03 | .00 | 0.00 | 0.00 |
| SFFP | 11596.94 | .00 | 0.00 | 0.00 |
| BEP | 80.25 | .00 | 0.00 | 0.00 |
| SEPAR | 26622.91 | I | 0.00 | 0.00 |
| BAG | 1512.51 | .00 | 0.00 | 0.00 |
| BCA | 6.67 | .00 | 0.00 | 0.00 |
| BPA | 20.73 | .00 | 0.00 | 0.00 |
| BLEND10 | 2345.32 | I | 0.00 | 0.00 |
| BLEND13 | 1374.17 | .00 | 0.00 | 0.00 |
| BLEND16 | 7077.99 | .00 | 0.00 | 0.00 |
| BLEND19 | 4922.39 | .00 | 0.00 | 0.00 |
| FORST10 | 2345.32 | I | 0.00 | 0.00 |
| FORST13 | 1374.17 | .00 | 0.00 | 0.00 |
| FORST16 | 7077.99 | .00 | 0.00 | 0.00 |
| FORST19 | 4922.39 | .00 | 0.00 | 0.00 |
| PRESS10 | 2345.32 | I | 0.00 | 0.00 |
| PRESS13 | 1374.17 | .00 | 0.00 | 0.00 |
| PRESS16 | 7077.99 | .00 | 0.00 | 0.00 |
| PRESS19 | 4922.39 | .00 | 0.00 | 0.00 |
| LABOR | 29312.62 | .00 | 0.00 | 0.00 |
| ST010 | 2406.00 | .00 | 0.00 | 0.00 |
| ST013 | 1162.00 | 0.00 | 0.00 | 0.00 |
| ST016 | 5985.15 | I | 0.00 | 0.00 |
| ST019 | 4177.81 | .00 | 0.00 | 0.00 |
| SELL10E | 1241.20 | .00 | 0.00 | 0.00 |
| SELL13E | 0.00 | 0.00 | 69.36 | 44.52 |
| SELL16E | 3417.15 | 0.00 | 0.00 | 0.00 |
| SELL19E | 2350.31 | .00 | 0.00 | 0.00 |
| SELL10D | 1164.30 | 0.00 | 0.00 | 0.00 |
| SELL13D | 1162.00 | 0.00 | 0.00 | 0.00 |
| SELL16D | 2568.30 | 0.00 | 0.00 | 0.00 |
| SELL19D | 1227.00 | 0.00 | 0.00 | 0.00 |

AORI9 ADRITS WITH 5 SIMS FOR SOLVING DROP IN PROFIT

NUMBER OF SIMULATIONS = 5

AVERAGE MAX. OBJECT = 6837587.94

S.D. OF THE MAX. OBJECT = 354084.62

MAXIMUM OF THE MAX. OBJECT = 7385384.51

MINIMUM OF THE MAX. OBJECT = 6438578.74

** SOLUTION **

RESOURCES(ROWS)

| | AVE.P.RES. | S.D.P.RES. | AVE.D.VAL. | S.D.D.VAL. |
|---|---|---|---|---|
| RT | 0.00 | 0.00 | 45.89 | 63.29 |
| RM | 0.00 | 0.00 | 129.45 | 64.52 |
| S1HF45 | 0.00 | 0.00 | 167.39 | 47.28 |
| FFH | 2075.92 | .00 | 0.00 | 0.00 |
| S2HF3 | 0.00 | 0.00 | 337.73 | 24.59 |
| REP | 0.00 | 0.00 | 3732.61 | 470.20 |
| RMWF | 0.00 | 0.00 | 337.73 | 24.58 |
| RNF | .00 | .00 | 130.74 | 96.99 |
| RFF | 0.00 | 0.00 | 544.71 | 55.80 |
| FFF | 0.00 | 0.00 | 2176.23 | 222.92 |
| BFP | 3302.97 | .00 | 0.00 | 0.00 |
| RAG | 0.00 | 0.00 | 6755.51 | 124.82 |
| RCA | 0.00 | 0.00 | 2344.94 | 52.02 |
| RPA | 0.00 | 0.00 | 8686.77 | 551.56 |
| RFS10 | 0.00 | 0.00 | 1681.00 | 12.82 |
| RFS13 | 0.00 | 0.00 | 1323.82 | 16.90 |
| RFS16 | 0.00 | 0.00 | 1099.61 | 31.23 |
| RFS19 | 0.00 | 0.00 | 946.85 | 41.46 |
| RP10 | 0.00 | 0.00 | 1681.00 | 12.82 |
| RP13 | 0.00 | 0.00 | 1323.82 | 16.90 |
| RP16 | 0.00 | 0.00 | 1099.61 | 31.23 |
| RP19 | 0.00 | 0.00 | 946.85 | 41.46 |
| RLABOR | 8954.40 | .00 | 0.00 | 0.00 |
| ALABOR | 0.00 | 0.00 | 119.17 | 3.44 |
| RST010 | 0.00 | 0.00 | 1387.94 | 15.16 |
| RST013 | 0.00 | 0.00 | 1565.54 | 19.98 |
| RST016 | 0.00 | 0.00 | 1300.39 | 36.93 |
| RST019 | 0.00 | 0.00 | 1119.74 | 49.02 |
| ROF10 | 0.00 | 0.00 | 3016.54 | 22.16 |
| EXDEM10 | .00 | .00 | 0.00 | 0.00 |
| DODEM10 | 571.89 | 0.00 | 0.00 | 0.00 |
| ROF13 | 0.00 | 0.00 | 2548.14 | 27.31 |
| EXDEM13 | .00 | .00 | 0.00 | 0.00 |
| DODEM13 | 0.00 | 0.00 | 0.00 | 0.00 |
| ROF16 | 0.00 | 0.00 | 2331.14 | 40.40 |
| EXDEM16 | .00 | .00 | 26.94 | 41.55 |
| DODEM16 | 0.00 | 0.00 | 0.00 | 0.00 |
| ROF19 | 0.00 | 0.00 | 2153.31 | 51.56 |
| EXDEM19 | 3418.92 | .00 | 0.00 | 0.00 |
| DODEM19 | 0.00 | 0.00 | 0.00 | 0.00 |
| LMILL | 0.00 | 0.00 | 38.43 | 52.68 |
| LDRYER | 0.00 | 0.00 | 22.36 | 50.00 |
| LMWF | .01 | I | 0.00 | 0.00 |
| LSFFP | .00 | I | 0.00 | 0.00 |
| LBEP | .01 | .00 | 0.00 | 0.00 |
| LSEPAR | .01 | .00 | 0.00 | 0.00 |
| LBAG | .00 | .00 | 0.00 | 0.00 |
| LBCA | .00 | .00 | 0.00 | 0.00 |
| LBPA | .00 | .00 | 0.00 | 0.00 |
| LBLEND10 | .01 | .00 | 0.00 | 0.00 |
| LBLEND13 | .01 | .00 | 0.00 | 0.00 |
| LBLEND16 | .00 | .00 | 0.00 | 0.00 |
| LBLEND19 | .00 | .00 | 0.00 | 0.00 |
| LFORST10 | .01 | .00 | 0.00 | 0.00 |
| LFORST13 | .01 | .00 | 0.00 | 0.00 |
| LFORST16 | .00 | .00 | 0.00 | 0.00 |
| LFORST19 | .00 | .00 | 0.00 | 0.00 |
| LPRESS10 | .01 | .00 | 0.00 | 0.00 |
| LPRESS13 | .01 | .00 | 0.00 | 0.00 |
| LPRESS16 | .00 | .00 | 0.00 | 0.00 |
| LPRESS19 | .00 | .00 | 0.00 | 0.00 |
| LST010 | .00 | .00 | 3.74 | 8.37 |
| LST013 | .00 | .00 | 4.66 | 7.93 |
| LST016 | .00 | .00 | 8.25 | 13.26 |
| LST019 | .00 | .00 | 0.00 | 0.00 |
| LSELL19E | .00 | .00 | 54.31 | 61.55 |
| LSELL100 | 0.00 | 0.00 | 72.89 | 23.26 |
| LSELL130 | 0.00 | 0.00 | 78.42 | 33.86 |
| LSELL160 | 0.00 | 0.00 | 80.29 | 46.72 |
| LSELL190 | 0.00 | 0.00 | 106.43 | 67.78 |

** SOLUTION **

PROCESSES(COLUMNS)

| | AVE.P.VAL. | S.D.F.VAL. | AVE.D.RES. | S.D.D.RES. |
|---|---|---|---|---|
| SAW | 54900.00 | 0.00 | 0.00 | 0.00 |
| MILL | 54900.00 | 0.00 | 0.00 | 0.00 |
| DRYER | 54900.00 | 0.00 | 0.00 | 0.00 |
| MWF | -572.34 | .00 | 0.00 | 0.00 |
| SFFP | 15462.59 | .00 | 0.00 | 0.00 |
| BEP | 107.08 | .00 | 0.00 | 0.00 |
| SEPAR | 35497.21 | I | 0.00 | 0.00 |
| BAG | 2016.80 | .00 | 0.00 | 0.00 |
| BCA | 8.99 | .00 | 0.00 | 0.00 |
| BPA | 27.75 | .00 | 0.00 | 0.00 |
| BLEND10 | 3438.84 | .00 | 0.00 | 0.00 |
| BLEND13 | 3429.51 | .00 | 0.00 | 0.00 |
| BLEND16 | 7211.45 | .00 | 0.00 | 0.00 |
| BLEND19 | 7413.58 | .00 | 0.00 | 0.00 |
| FORST10 | 3438.84 | .00 | 0.00 | 0.00 |
| FORST13 | 3429.51 | .00 | 0.00 | 0.00 |
| FORST16 | 7211.45 | .00 | 0.00 | 0.00 |
| FORST19 | 7413.59 | .00 | 0.00 | 0.00 |
| PRESS10 | 3438.84 | .00 | 0.00 | 0.00 |
| PRESS13 | 3429.51 | .00 | 0.00 | 0.00 |
| PRESS16 | 7211.45 | .00 | 0.00 | 0.00 |
| PRESS19 | 7413.53 | .00 | 0.00 | 0.00 |
| LABOR | 39093.10 | .00 | 0.00 | 0.00 |
| ST010 | 2307.89 | .00 | 0.00 | 0.00 |
| ST013 | 2900.00 | .00 | 0.00 | 0.00 |
| ST016 | 6098.00 | .00 | 0.00 | 0.00 |
| ST019 | 6268.92 | .00 | 0.00 | 0.00 |
| SELL10E | 1296.00 | .00 | 0.00 | 0.00 |
| SELL13E | 1738.00 | .00 | 0.00 | 0.00 |
| SELL16E | 3652.00 | .00 | 0.00 | 0.00 |
| SELL19E | 5968.92 | .00 | 0.00 | 0.00 |
| SELL100 | 1671.89 | .00 | 0.00 | 0.00 |
| SELL130 | 1102.00 | 0.00 | 0.00 | 0.00 |
| SELL160 | 2446.00 | 0.00 | 0.00 | 0.00 |
| SELL190 | 1130.00 | 0.00 | 0.00 | 0.00 |