The Consistent Parameterization of the Effects of Cumulus Clouds on the Large-Scale Momentum and Vorticity Fields

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ABSTRACT

A physical and mathematical framework for the mutually consistent parameterization of the effects of cumulus convection on the large-scale momentum and vorticity fields is proposed. The key to achieving consistency is the understanding that the vorticity dynamics of the clouds below the spatial resolution of a large-scale dynamical model may be neglected in the vorticity budget when the clouds are considered to be independent buoyant elements sharing a common large-scale environment. This simplified approach is used to obtain a consistent pair of large-scale momentum and vorticity equations based on Ooyama's theory of cumulus parameterization. The results focus attention on the need to obtain a better understanding of the detrainment process and the pressure interactions between the clouds and their environment.

1. Introduction

Parameterization of the effects of cumulus convection on the momentum fields of a large-scale dynamical model is a practical problem requiring knowledge of the physical mechanisms of interaction and their spectral characteristics. Although the ultimate test of any cumulus parameterization scheme is the validity of the large-scale fields produced by the model, simple empiricism is not a satisfying or efficient approach. In practice, the modeler is not only unsure of the parameterization scheme, but has difficulty quantifying the extent of the agreement between the model and the real world. Physically based and internally consistent schemes provide a rational basis for evaluation and model improvement.

The major purpose of this paper is to provide a physical and mathematical framework for the mutually consistent parameterization of the effects of cumulus convection on the large-scale momentum and vorticity fields. Schemes derived in this manner insure that the parameterized effects of cumulus convection on the large-scale vorticity field are mathematically equivalent to the curl of the parameterized effects on the momentum field. Indeed, such mathematical consistency should be used as a design criterion. In practical application to a large-scale numerical model that utilizes momentum as its prognostic wind variable, exact mathematical equivalence is a direct consequence of the consistent application of the equations of motion.

Evidence for the practical importance of developing parameterizations for the large-scale momentum and vorticity budgets in a consistent manner has been provided by Tollerud and Esbensen (1983) and Sui and Yanai (1984). Tollerud and Esbensen (1983) found that a straight-forward application of Ooyama's (1971) parameterization of the effects of cumulus convection on the large-scale momentum fields appeared to be more appropriate for the interpretation of the observed large-scale upper-tropospheric vorticity budgets in the vicinity of GATE cloud clusters than previously proposed parameterizations based primarily on vorticity budget considerations. More recently, Sui and Yanai (1984) have quantitatively verified this suggestion and have found a clear association between the observed residuals in the large-scale vorticity budgets during GATE and the curl of parameterized effects of clouds on the momentum fields.

In this paper we formally develop the suggestion of Tollerud and Esbensen (1983) and present the physical basis for adopting this simplified approach to parameterizing the effects of clouds on the large-scale vorticity. Ooyama's parameterization theory is our point of departure. The theory explicitly represents only the transport effects of cumulus convection, but the scheme is quite flexible and can be easily modified to include other mechanisms of interaction.

Section 2 begins with Ooyama's expression for the cumulus effects on the large-scale momentum fields. We then formally derive the equation governing the evolution of the large-scale vorticity field. A physical discussion of the resulting large-scale vorticity equation follows in section 3. It is pointed out that our approach neglects the details of the vorticity dynamics of the clouds, and that there is a sound physical basis for doing so. The convective and nonconvective sources of vorticity are identified and briefly discussed. Section 4 concludes with a comparison of our parameterization scheme with previously proposed formulations and a discussion of the relevance of our work for large-scale dynamical modeling.

2. Formal development

Within the framework proposed by Ooyama (1971), individual convective-cloud elements share a common dynamical environment that can be described by three variables: the large-scale horizontal velocity vector $\bar{\mathbf{V}}$, the large-scale vertical "p-velocity" of the cloud environment ω_e, and the large-scale geopotential $\bar{\phi}$. No spatial averaging of the cloud environment and "in-cloud" properties is required to define the large-scale variables. This description is based on the fact that convective clouds are small in size and short in duration in comparison with the large-scale flow fields, and on the assumption that the convective clouds do not directly interact with each other.

Although Ooyama did not explicitly consider the problem of parameterizing the effects of cumulus convection on the large-scale vorticity fields, two approaches are immediately suggested. The first involves writing a conservation law for the vertical component of the large-scale vorticity and attempting to specify the convective sources and sinks through a combination of theory and empiricism. The difficulties presented by this method are formidable. Even if one were to succeed in determining the most important effects of the three-dimensional cloud motions on the evolution of $\bar{\zeta}$, the relationship between the resulting vorticity equation and its mutually consistent form of the momentum equation could not be specified by a consideration of vorticity dynamics alone.

A second approach is to obtain the effects of convection on the large-scale vorticity field by taking the curl of parameterized convective effects on the large-scale horizontal momentum. This approach has a number of practical advantages. Consistency is achieved by a simple mathematical operation. Furthermore, gradient quantities such as the horizontal wind shear are not formally required in defining the environment of the convection.

In the derivation that follows, we take the second approach. By limiting our description of the dynamical environment of the convection to the three variables, $\bar{\mathbf{V}}$, ω_e and $\bar{\phi}$, we implicitly adopt the simplifying assumption that large-scale horizontal wind shear plays a negligible role in determining the dynamical properties of the independent buoyant elements. Such simplification is a desirable goal for the parameterization, provided that the neglected dynamical processes are not of practical importance. The physical basis for the neglect of $\bar{\zeta}$ in defining the cloud environment will be presented in section 3a.

Following Ooyama, the general equation expressing the conservation of $\bar{\mathbf{V}}$ may be written as

$$\frac{\partial \bar{\mathbf{V}}}{\partial t} + \nabla \cdot \bar{\mathbf{V}}\bar{\mathbf{V}} + \frac{\partial}{\partial p}(\omega_e \bar{\mathbf{V}}) = \mathbf{C}_M + \mathbf{S}_M, \qquad (1)$$

where $\mathbf{C}_M$ and $\mathbf{S}_M$ denote convective and nonconvective sources of $\bar{\mathbf{V}}$, respectively. In general, $\mathbf{C}_M$ represents all mechanisms of interaction: effects of mass entrainment and detrainment, and pressure effects, including convectively generated gravity waves and organized mesoscale features, etc. For our purposes, the convective and nonconvective sources are most conveniently written as

$$\mathbf{C}_M = D(\mathbf{V}_D - \bar{\mathbf{V}}) + (D-E)\bar{\mathbf{V}} + \mathbf{F}_c,$$
$$\mathbf{S}_M = -\nabla\bar{\phi} - f\mathbf{k}\times\bar{\mathbf{V}}, \qquad (2)$$

where D and E are the total mass entrained and detrained by the convective elements, $\mathbf{V}_D$ is the value of the velocity detrained to the large-scale environment, and $\mathbf{F}_c$ represents all other convectively generated forces affecting the large-scale flow that are not explicitly represented as a cloud transport.

In Ooyama's (1971) theory, only the effects of mass entrainment and detrainment were considered explicitly. For clarity, we have represented the transport effects in a similar manner. The term $D(\bar{\mathbf{V}}_D - \bar{\mathbf{V}})$ in (2) is interpreted as the change in the momentum of the large-scale environment that results from a redistribution of cloud and environmental air through the process of detrainment. We include other effects in a symbolic manner through $\mathbf{F}_c$. For example, the dynamic pressure exerted by clouds on the large-scale flow when environmental air is forced around a strong updraft is included in $\mathbf{F}_c$. For notational convenience, we will use a "single-cloud-type" formulation to describe the bulk effects of the clouds. The explicit recognition of multiple cloud types can be restored by appropriate substitutions at any step of the mathematical development.

The mass conservation law for the large-scale environment is given by the expression

$$\nabla \cdot \bar{\mathbf{V}} + \frac{\partial}{\partial p}\omega_e = D - E. \qquad (3)$$

Note that the total mass flux is the sum of the convective and environmental values, i.e., $\bar{\omega} = -M_c + \omega_e$.

Combining Eqs. (1)–(3) we obtain the horizontal equation of motion for the large-scale flow,

$$\frac{\partial \bar{\mathbf{V}}}{\partial t} + \bar{\mathbf{V}} \cdot \nabla \bar{\mathbf{V}} + \omega_e \frac{\partial \bar{\mathbf{V}}}{\partial p} = D(\mathbf{V}_D - \bar{\mathbf{V}}) + \mathbf{F}_c - \nabla \bar{\phi} - f \mathbf{k} \times \bar{\mathbf{V}}. \quad (4)$$

Ignoring $\mathbf{F}_c$ for the moment, this remarkably simple form of the equation of motion states that the horizontal velocity in the large-scale environment of the convection changes as a result of direct mixing of cloud and environmental air through the process of detrainment, the large-scale pressure force, and the large-scale Coriolis force. Were we to use the definition of $\bar{\omega}$ to rewrite the vertical advection term as $\bar{\omega}\partial \bar{\mathbf{V}}/\partial p$, an additional virtual source of momentum (ref. Ooyama, 1971; the compensating subsidence term) would appear on the right-hand-side of (4). The equation governing the "total" large-scale momentum is then written as

$$\frac{\partial \bar{\mathbf{V}}}{\partial t} + \bar{\mathbf{V}} \cdot \nabla \bar{\mathbf{V}} + \bar{\omega} \frac{\partial \bar{\mathbf{V}}}{\partial p} = -M_c \frac{\partial \bar{\mathbf{V}}}{\partial p} + D(\mathbf{V}_D - \bar{\mathbf{V}}) + \mathbf{F}_c - \nabla \bar{\phi} - f \mathbf{k} \times \bar{\mathbf{V}}. \quad (5)$$

The determination of the strength of the convective activity as measured by the convective mass flux, M_c, and the cloud parameters needed to evaluate the detrainment and $\mathbf{F}_c$ terms, are beyond the scope of this paper. It is assumed that M_c can be determined from closure hypotheses that maintain certain equilibrium states (e.g., Arakawa and Schubert, 1974) or by relating the cumulus mass flux to the large-scale horizontal convergence of an explicit property of the large-scale flow. The mass detrainment is related to M_c through the cloud mass budget. Determination of $\mathbf{V}_D$ and $\mathbf{F}_c$ require a model of the momentum budget of the convective clouds.

Once (4) is adopted as the appropriate form of the equation of motion, the obvious method (and we would argue the only choice) for obtaining the large-scale vorticity equation, is to take the curl of (4). Thus,

$$\frac{\partial \bar{\zeta}}{\partial t} + \bar{\mathbf{V}} \cdot \nabla \bar{\zeta} + \omega_e \frac{\partial \bar{\zeta}}{\partial p} + \bar{\zeta}(\nabla \cdot \bar{\mathbf{V}}) + \mathbf{k} \cdot \nabla \omega_e \times \frac{\partial \bar{\mathbf{V}}}{\partial p} = \mathbf{k} \cdot \nabla \times [D(\mathbf{V}_D - \bar{\mathbf{V}})] + \mathbf{k} \cdot \nabla \times \mathbf{F}_c - \beta \bar{v} - f(\nabla \cdot \bar{\mathbf{V}}), \quad (6a)$$

or

$$\frac{\partial \bar{\zeta}}{\partial t} + \bar{\mathbf{V}} \cdot \nabla \bar{\zeta} + \bar{\omega} \frac{\partial \bar{\zeta}}{\partial p} + \bar{\zeta}(\nabla \cdot \bar{\mathbf{V}}) + \mathbf{k} \cdot \nabla \bar{\omega} \times \frac{\partial \bar{\mathbf{V}}}{\partial p} = -M_c \frac{\partial \bar{\zeta}}{\partial p} - \mathbf{k} \cdot \nabla M_c \times \frac{\partial \bar{\mathbf{V}}}{\partial p} + \mathbf{k} \cdot \nabla \times [D(\mathbf{V}_D - \bar{\mathbf{V}})] + \mathbf{k} \cdot \nabla \times \mathbf{F}_c - \beta \bar{v} - f(\nabla \cdot \bar{\mathbf{V}}). \quad (6b)$$

Consistent with (4), the only direct convective effect on the large-scale vorticity as represented by (6a) comes from spatial inhomogeneities of the detrainment effects on $\bar{\mathbf{V}}$ and of $\mathbf{F}_c$.

Although (6) is the most useful form for physical interpretation, it is helpful from a historical perspective to rewrite this equation in flux form. This facilitates the identification of convective and nonconvective sources of large-scale vorticity that are analogous to $\mathbf{C}_M$ and $\mathbf{S}_M$ in the momentum equation. Combining (3) and (6a) we have

$$\frac{\partial \bar{\zeta}}{\partial t} + \nabla \cdot \bar{\mathbf{V}} \bar{\zeta} + \frac{\partial}{\partial p}(\omega_e \bar{\zeta}) = \mathbf{k} \cdot \nabla \times [D(\mathbf{V}_D - \bar{\mathbf{V}})] + (D - E)\bar{\zeta} + \mathbf{k} \cdot \nabla \times \mathbf{F}_c - \beta \bar{v} - (\bar{\zeta} + f)\nabla \cdot \bar{\mathbf{V}} - \mathbf{k} \cdot \nabla \omega_e \times \frac{\partial \bar{\mathbf{V}}}{\partial p}. \quad (7)$$

We will discuss the physical meaning of the various source and sink terms in section 3b.

3. Physical interpretation of the large-scale vorticity equation

a. Relevance of vorticity dynamics in the cloud and its environment

The basic physical statement made by the large-scale vorticity equations derived in section 2 is that transport effects of clouds can change large-scale vorticity only by changing the momentum field that defines the large-scale environment of the clouds. Equations (6) and (7) completely neglect the vorticity dynamics of the clouds. All cloud parameters in (6) and (7) can be computed without knowing the value of the vorticity in the cloud or its environment. Thus, although (6) contains terms having the form of vertical vorticity advection and twisting, the effects of the three-dimensional advection and twisting of vortex tubes within the clouds and their surroundings are not explicitly represented. We believe that there is a sound physical basis for this approach.

First, from a local perspective, the vorticity dynamics associated with convective elements cannot cause a significant change in the value of the large-scale vorticity at a point when the vorticity "fluxes" associated with the clouds are locally homogeneous in the horizontal.[1] In a Boussinesq fluid, the vertical component of the baroclinicity vector is negligible, and one can express all of the mechanisms that cause local time changes of the vertical component of the vorticity as a horizontal convergence of vorticity fluxes. But local gradients of the vorticity fluxes due to clouds can occur only when a local gradient exists in the statistical prop-

[1] A rigorous demonstration of the fact for a Boussinesq fluid appears in an unpublished manuscript by R. Rotunno (personal communication).

erties of the clouds or in the variables that define the cloud environment. The simplest representation of clouds as independent buoyant elements sharing a common large-scale environment therefore precludes a direct connection between the vorticity dynamics of a given cloud ensemble and the local value of the large-scale vorticity.

From a large-scale perspective, however, clouds must be able to change the circulation over a large-scale neighborhood surrounding a given spatial point if they are to have any effect on the large-scale vorticity. For convection to change $\bar{\zeta}$, it must change $\bar{\mathbf{V}}$ in the surroundings. It is therefore obvious that a large-scale gradient of convective activity is necessary for a complete description of the physical mechanisms by which clouds can change the large-scale vorticity.

The framework proposed in section 2 resolves the apparent inconsistency between the local and large-scale perspectives in a physically reasonable manner. Large-scale variations in convective activity are explicitly represented in (6) and (7) and are able to change the large-scale vorticity, but only through their local effects on the large-scale momentum budget. The local homogeneity of the cloud properties and their environment simply implies that the vorticity dynamics of the interaction between a cloud and its surroundings are of secondary importance in changing the large-scale vorticity fields.

We can visualize the effects of the convection on the large-scale vorticity by considering the circulation around a horizontal area A. In the context of a gridpoint model, A would be the area of a grid box; in a spectral model, area A would be related to the smallest resolvable horizontal scales of the model.

Since it is possible to define a value of $\bar{\mathbf{V}}$ and an associated cloud ensemble at every point along the perimeter of A, we may approximate the true circulation around A in terms of the environmental values of the momentum. Specifically, we may write

$$C = \oint \bar{\mathbf{V}} \cdot d\mathbf{l}, \tag{8}$$

where $d\mathbf{l}$ is an incremental unit distance in the direction of the counterclockwise path around the perimeter of area A. The average circulation C/A is a valid approximation for the local value of the large-scale vorticity defined formally in section 2 as $\bar{\zeta} = \mathbf{k} \cdot \nabla \times \bar{\mathbf{V}}$.

At the same time, C/A is also a good approximation of the true areally-averaged, large-scale vorticity. Using Stokes theorem, we may write

$$C \approx \oint \mathbf{V} \cdot d\mathbf{l} = \iint_A \mathbf{k} \cdot \nabla \times \mathbf{V} dA \tag{9}$$

where $\mathbf{V}$ represents the total velocity field, including the details in $\mathbf{V}$ below the resolution of the model. In writing (9), we need only to make the reasonable assumptions that the time and space scales of the convective clouds are negligible in comparison with the corresponding scales of the large-scale model, and that the order of magnitude of the in-cloud momentum is on the same order as $\bar{\mathbf{V}}$.[2] These assumptions are consistent with the framework proposed by Ooyama (1971).

Changes in $\bar{\zeta}$ may therefore be viewed as the collective effects of processes that act independently on $\bar{\mathbf{V}}$ within each vertical column of the large-scale model. A large-scale gradient of convective activity is necessary for a complete description of the physical mechanisms by which clouds can change the vorticity. The gradients of convective activity need not, however, be considered locally within each vertical air column of the large-scale model.

Of course, it is possible to envision a consistent set of momentum and vorticity equations in which the parameterized effects of cumulus clouds include the local variation of both the cloud and environmental properties. But if these details are important to the dynamical phenomena of interest, the modeler should seriously consider increasing the model resolution. Inclusion of these subgrid-scale variations greatly increases the number of physical effects that must be addressed, possibly destroying the practical advantage of the parameterization scheme over the explicit representation of the convective processes.

b. *Convective and nonconvective sources of large-scale vorticity*

Proceeding strictly by analogy with Eqs. (1)–(3) for momentum, we may use (7) to identify the convective and non-convective sources of large-scale vorticity as:

$$C_\zeta = \mathbf{k} \cdot \nabla \times [D(\mathbf{V}_D - \bar{\mathbf{V}})] + (D-E)\bar{\zeta} + \mathbf{k} \cdot \nabla \times \mathbf{F}_c$$

$$S_\zeta = -\beta \bar{v} - (\bar{\zeta} + f)\nabla \cdot \bar{\mathbf{V}} - \mathbf{k} \cdot \nabla \omega_e \times \frac{\partial \bar{\mathbf{V}}}{\partial p}. \tag{10}$$

The formal justification for including $(D - E)\bar{\zeta}$ in the convective source term is by analogy with $\mathbf{C}_M$ in (2). The twisting term in S_ζ is nonconvective in the same sense as the term giving the vertical convergence of the vorticity flux on the left-hand-side of (7).

When applied to vorticity, however, the flux form of the equation given by (7) presents a curious mixture of advective and local source terms that are easily misinterpreted. Terminology borrowed from the world of conservative, scalar variables, such as detrainment of in-cloud vorticity and entrainment of environmental vorticity, must be used with caution when applied to the vertical component of the vector vorticity.

[2] Note that we have not assumed that the vorticity in the environment of the clouds, ζ_e, is approximately equal to $\bar{\zeta}$. Assumptions about the magnitude of the cloud and environmental vorticities are not needed in our approach.

We therefore prefer the advective form of the vorticity equation (6) for discussing the mechanisms of interaction. Each term is directly related to the curl of a real or apparent force in the momentum equation. Because the detailed vorticity dynamics of clouds can be ignored within the framework given in section 2, we are not required to account for the effects of the three-dimensional solonoidal circulations and turbulence as air moves in and out of the clouds. At best, the representation of the effects of vortex stretching and twisting with a simple mass-flux model of clouds and their local environment is a difficult and complex exercise.

As given by (6), the convective sources of vorticity are due to spatial inhomogeneities in the apparent drag force created by the redistribution of momentum between the cloud and its environment during the process of detrainment, represented by $\mathbf{k} \cdot \nabla \times [D(\mathbf{V}_D - \bar{\mathbf{V}})]$, and in the other convectively generated forces $\mathbf{F}_c$ that are not explicitly represented as a cloud transport of momentum. Since $\mathbf{k} \cdot \nabla \times \mathbf{F}_c$ includes the effect of dynamic pressure exerted by clouds on their environment, the appearance of this term in the large-scale vorticity equation requires some further explanation.

The appearance of pressure effects in the large-scale vorticity equations comes from viewing changes in $\bar{\mathbf{V}}$ from the point of view of the large-scale environment of the convection. Combining (4) with a proper budget of in-cloud momentum (e.g., Shapiro and Stevens, 1980) results in the cancellation of the excess pressure gradient forces in the clouds with those in their environment. Thus, from the point of view of the momentum budget averaged over both the cloud and its environment, the excess pressure forces play no direct role in the combined momentum budget. In other words, the appearance of pressure effects as a convective source term in the vorticity equation does not violate basic physical principles, provided that the in-cloud momentum budget is properly formulated.

4. Discussion

We have proposed a simple framework in which the effects of cumulus clouds on the large-scale vorticity equation are represented by the curl of an apparent drag force associated with the detrainment of momentum from clouds, and the torque due to cloud-environment interaction forces such as those caused by convective-scale or mesoscale pressure perturbations. We have demonstrated that this approach leads to a consistent and physically reasonable pair of large-scale momentum and vorticity equations.

Numerous other proposals for the parameterization of convective effects on the large-scale vorticity field have appeared in the literature. In most cases, the authors have adopted the approach of writing a conservation law for the vertical component of the large-scale vorticity and then attempting to specify the convective sources and sinks of vorticity through a combination of theory and empiricism. A convective ensemble model patterned after that of Yanai et al. (1973) for thermodynamic budgets was used by Reed and Johnson (1974) to interpret vorticity budgets over the tropical Pacific. A similar approach was used by Shapiro (1978) and Shapiro and Stevens (1980) to derive expressions for the apparent momentum and vorticity sources of the large-scale fields. Consideration of momentum as a quasi-conservative property led to a budget identical with (4), while consideration of vorticity led to an inconsistent form for the vorticity budget that differs from (6).[3] Yanai et al. (1982) independently derived a vorticity budget equivalent in form to that obtained by Shapiro and Stevens (1980). Cho and Cheng (1980), considering horizontal vorticity fluxes, also derived a formally equivalent result. In each of these cases, however, the proposed schemes require the explicit determination of cloud and environmental vorticities. Furthermore, the attempts have either led to inconsistent forms of the large-scale momentum and vorticity equations or the issue of consistency has been ignored altogether.

The simplicity and consistency of the present approach, as well as the observational support provided by the studies of Tollerud and Esbensen (1983) and Sui and Yanai (1984, 1986), suggest that our approach may be the most appropriate for application to a large-scale dynamical model. Adopting this point of view can have a profound effect on the search for improvements in cumulus parameterization schemes. The present formulation focuses attention on the redistribution of momentum during the detrainment process and pressure interactions between a cloud and its environment for a given vertical profile of large-scale wind velocity. The alternative formulations focus on the details of the vorticity budget of the clouds and their local environment, where both the horizontal and vertical distribution of the large-scale wind field must be considered.

One feature of our parameterization that appears at first glance to be less attractive than previous formulations is the need to consider the pressure terms in the convective source term for large-scale vorticity C_ζ. The attempt to directly evaluate cloud sources of vorticity, however, replaces the unattractive prospect of evaluating $\mathbf{F}_c$ with the necessity for evaluating subgrid-scale details of the cloud–environment vorticity dynamics.

Finally, we note that the logical extension of our approach to the divergence budget involves simply taking the divergence of (4). The observational and theoretical justification of this interpretation for the

[3] As shown in appendix A of Tollerud and Esbensen (1984), direct application of Arakawa and Schubert's (1974) formulation for the thermodynamic budgets leads to an equivalent result.

divergence budget is much more difficult than for the vorticity budget. On the other hand, the development of the divergence budget by a detailed consideration of the flow in the cloud and its surroundings would appear to be even less attractive. In any case, a large-scale model using vorticity and divergence as dynamical variables must have both a vorticity equation and a divergence equation. Mathematical consistency with the momentum equation should therefore be fully exploited as a tool for achieving physically realistic parameterizations of convective effects on the large-scale flow.

Note added in proof. The authors recently received a manuscript by P. H. Haynes and M. E. McIntyre (1986: On the evolution of vorticity and potential vorticity in the presence of diabatic heating and frictional or other forces. *J. Atmos. Sci.*, accepted for publication) which supports some of the key points in this paper. Haynes and McIntyre also reach the conclusion that a simple way to ensure consistency in a cumulus parameterization scheme is to formulate it in terms of momentum rather than vorticity.

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