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Title: MODEL STUDY OF DRAINAGE IN SHALLOW LAYERED SOILS

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Royal H. Brooks

The occurrence of relatively less permeable soil underlying a

relatively more permeable and shallow soil profile is widespread in

the Willamette Valley Region of Oregon. Recognizing that an under-

standing of the water transport process in such a situation would

provide reliable criteria for designing a drainage system has led to

this study.

A physical model of a drainage system was constructed and two

soil samples, one identified as a river bed sand and the other as

clayey loam, were used. The Brooks-Corey method of analysis was

used to identify the significant hydraulic parameters of the two sam-

ples from capillary pressure-saturation data, separately acquired

for the two samples.

Drains were installed at three different depths, 12.7 cm,

43.2 cm and 73.7 cm for two different spacings of 7.6 m and 15.2 m.

The data from the drainage experiments were extensively analyzed

in the light of drainage system geometry and the hydraulic soil

parameters. Degrees of saturation and drainage with respect to time and space indicated that drain spacing was not an important factor in the design of drainage systems for shallow layered soil profiles. The overriding factor was the depth placement of the drains and that the effectiveness of the drain was greatly enhanced by placing the drains at depths greater than the bubbling pressure heads of the soils involved.

Model Study of Drainage in Shallow Layered Soils

by

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Dedicated to the ever-living memory of my dear father,

Kwabena, whose silent whispers and invisible smiles

wiped the tears off my eyes and gave me the courage to

seek wisdom and truth even in those sorrowful moments.

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MODEL STUDY OF DRAINAGE IN SHALLOW LAYERED SOILS

I. INTRODUCTION

The importance of drainage in soil-water related problems and agriculture cannot be overemphasized. In many areas where high water tables and excess water used to hinder agricultural operations, we are now witnessing a booming agriculture. Not only that but also, diversification in crop practices can be achieved through drainage. Accumulation of certain salts can render an otherwise fertile agricultural land barren and inadequate aeration in a soil environment lead to the production of a variety of toxic chemicals and a shortage of necessary nutrients. However, to install drainage systems, the engineer has to understand the physical principles underlying the proper functioning of such systems and also the limiting factors inherent in the medium.

It has already been pointed out in classical drainage design theory the importance of factors as hydraulic conductivity, spacing of drains, depth of drains, location of the impermeable layer and the soil parameters. In most cases the values of the above mentioned factors have led to such qualitative classification of soils as "well drained," "fairly drained," "poorly drained" and to some extent "undrainable." Such characteristics present problems which sometimes seem to be unsolvable. The situations become more acute when

shallow drainable profiles are underlain by relatively deep and poorly drained or undrainable layers. Such is the situation in most if not all parts of the Willamette Valley Region in Oregon. This condition has rendered limited crop practices in the Valley with a serious repercussion of the odious field burning in the Valley. At the same time intense late fall and all winter rainfall in the area adds another dimension to the seriousness of the problem. It should be noted that this kind of situation is not pertinent to the Willamette Valley alone. Homogeneous soil profiles are usually not found in real world situations and therefore the aforesaid problem may plague other areas in the world.

To say that the problem of providing adequate drainage facilities to such cases is hard to come by would be a defeatist attitude. The present level of the sciences and engineering can be applied to handle the problem. At least some aspects of the problem have been attacked and approximate solutions have already been provided. However the author feels that a more accurate solution to the problem can be sought if the basic principles of the physics of water movement in porous media are analyzed through theoretical and experimental study. It is therefore the objective of this study to examine factors affecting drainage in layered soils through an experimental study and to propose a theoretical model based on the fundamental principles of fluid flow in porous media against which the experimental results could be tested.

The scope of the study involves the transient two-dimensional partly saturated and partly unsaturated flow situation. An attempt will be made to give a genuine comparison between the experimental results and other over-simplified published results in the literature. It is hoped that the limiting factors in the design of drainage systems for layered soil profiles would be alienated and recommendations could be made on them.

II. REVIEW OF LITERATURE

Numerous investigations have been made to develop equations from which drainage system geometry and general layout are derived. All these investigations started off from the basic Darcy Law and the continuity equations of fluid mechanics. However most of the results are based on oversimplified assumptions which make their validity as to application questionable. In cases where the underlying assumptions are not too simplified, the equations relating the drainage systems geometry to the specified drainage variable are too complicated to be used by most non-trained personnel. In spite of all this, drainage systems based on these earlier approximate solutions are functioning favorably in their particular situations. The author noticed that a bulk of the studies already made were limited to nonsteady one-dimensional flow situation in an isotropic homogeneous medium or steady two-dimensional potential flow. Most of the steady-state and transient drainage theories have arisen from mathematical investigations based on assumptions of horizontal flow towards ditch or tile drains, radial flow toward a tile drain, a combination of both horizontal and radial flow, or by use of more exact potential theory. Other investigators have used electric analogue studies, laboratory models, and field records to propound drainage theories.

Even though attempts have been made to improve upon drainage theories since the classical Darcy experiment in 1856, most of the more approximate and sophisticated works were not done until the end of the first quarter of this century. In this work the author would cite investigations as from the second quarter of this century and only those reviewed by the author are listed in the bibliography.

(i) General Drainage Theories

Most drainage theories describe the flow of water in an idealized soil-water system and the actual field problem is simplified to make it possible to get a mathematical solution. It should be borne in mind that an infinite variety of soil conditions exist in the field and therefore drainage theories only approximate field conditions. This situation requires some degree of caution on the part of any one applying a theory to a particular field problem and it is necessary to examine the assumptions made in its derivation and also to use judgement in applying the theory. It turns out that in some cases the theory works out quite well and often is suitable in determining the depth and spacing of the drains while in other cases the theory is useful only as a first approximation to the proper design. Nevertheless the drainage equations arising from the theories so far have given a great deal of aid in the solution of drainage problems.

Not only the practical aspects of the depth and spacing of drains can be derived from the theoretical examination of a problem but also conclusions that can be reached in no other way. These conclusions then can be used to reason about problems in a rational way even though the particular theory is not directly applicable.

(ii) Steady State Investigations

The steady state problem can be described exactly. Two types of approximate solutions have been proposed and are more or less widely used. The first is based on the horizontal flow assumption and the second, based on the radial flow assumption.

Van Schilfgaarde (1957) comments that the horizontal flow approximation theory of gravity-flow systems is based on assumptions which, if carried through consistently, lead to an absurdity. Its use, however, is widespread and, if the limitations of the underlying assumptions are thoroughly understood, it can in some cases lead to valuable solutions of far simpler form than would be obtained by a rigorous analysis based solely on Darcy's Law and the Laplace equation.

In this regard, two basic assumptions are due to Dupuit (1863). They are (i) that all streamlines in a system of gravity flow towards a shallow sink are horizontal and (ii) that the velocity along these streamlines is proportional to the slope of the free water surface, but

independent of the depth. Consideration of these assumptions show that, strictly, they imply that there be no flow. For, by means of definition of potential in terms of the vector velocity (i.e., $V = -k\nabla\psi$), it may be readily shown that

$$\frac{\partial v_x}{\partial z} = \frac{\partial v_z}{\partial x} , \qquad \frac{\partial v_y}{\partial z} = \frac{\partial v_z}{\partial y}$$

and since the velocity is assumed to be independent of depth, that

$$\frac{\partial v_x}{\partial z} = \frac{\partial v_y}{\partial z} = 0 = \frac{\partial v_z}{\partial x} = \frac{\partial v_z}{\partial y}$$

It follows that the vertical velocity must be constant in a horizontal plane. Since it will vanish along a vertical outflow surface, it will vanish everywhere. Hence there will be no vertical flow and, conse-quently, the slope of the free water surface must be zero so that there can be no horizontal flow either. Thus the approximate nature of the Dupuit assumptions is evident.

Based on the Dupuit assumptions, Forchheimer (1930) proposed a general equation for the free water surface, by considering a saturated soil column above an impervious layer. He incorporated into the Dupuit assumptions, the equation of continuity and obtained

$$\nabla^2 h = 0$$

where $h(x,y)$ is the height of the soil column conducting the flow.

If the Dupuit assumptions are accepted, Forchheimer's deviation enables the determination of the shape of the free water surface and the velocity at any point for a shallow gravity-flow system in equilibrium.

Many investigators have used the D-F theory of horizontal flow to analyze steady-state drainage problems. Colding (1872) appears to have been the first to employ it to determine ellipse equations for the free water surface over the drains. Many others have followed in this direction and among them are Rothe (1924), Kozeny (1932), Hooghoudt (1937), Aronovici and Donnan (1946), Gustafsson (1946) and Muskat (1946). Among these investigators only Muskat (1946) really examined the applicability and validity of the Dupuit-Forchheimer propositions. He showed that remarkably accurate results are obtained when the Dupuit-Forchheimer theory is used to determine the flux through a drain or towards a well, but that the shape of the free surface and the velocity distribution are generally greatly in error as determined by comparison with more exact theoretical solutions. Muskat however rejected the D-F theory entirely and credited the success of the flux determination to fortuitous coincidence rather than to reasonable approximations. Engelund (1951) on the other hand, showed that often the approximations are near enough to expect close agreement with exact calculations.

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The radial flow assumption takes care of the non-horizontal flow situations which occur at the immediate vicinities of seepage faces of drains. It has widely been applied in aquifer analyses and flow towards wells. In drainage investigations, the method of images technique, with the assumptions of radial flow towards the tile drains, have received considerable attention in the works of Hooghoudt (1940), Gustafsson (1946), Kirkham (1940), (1945), (1949), (1941), (1948) and (1951).

There is also a third class of investigation involving the steady-state drainage situation. The investigators in this group used a combination of horizontal flow assumptions at some distance from a tile drain with radial flow assumptions near the drain. Vedernikov (1939), and Hooghoudt (1940) applied this approach for the case of an impermeable layer at a finite distance below the tile. Also based on the combination of these assumptions, Van Deemter (1949), (1950) developed a hodographic solution for the drainage problem. For a steady-state drainage situation with a curved free water surface, Van Deemter (1949), (1950) employed the relaxation technique for solving problems based on potential theory.

(iii) Non-Steady State Investigations

It is felt that steady state formulations were based on assumptions which do not conform to the situation in the field. It is

an erroneous oversimplification to assume that the water table is in

equilibrium with rainfall or irrigation water. The general case is

that the water table is constantly moving through the soil. For a more

accurate method of design, formulations that result in the equation of

motion of water table through the soil would be required. This calls

for non-steady state or transient investigations.

In this state of affairs, the hydraulic head at any point in the soil

is not constant; it is changing with time.

It is encouraging to know that quite a large number of

investigators have considered the non-steady state situation.

Forchheimer (1930) was probably the first to investigate transient

drainage phenomena using Dupuit's approach. He developed a

differential equation analogous to the differential equation describing

heat flow. Kano (1940) and Visser (1953) each using a modified form

of the ellipse equation obtained a solution to the differential equation

(heat flow equation) for the transient case. Among the leaders in the

transient drainage investigations, R. D. Glover and his colleagues

working at the USBR need particular mention. As reported by Dumm

(1954), Glover obtained a solution on horizontal flow to tile drains

placed over an impermeable layer. Walker (1952) using radial flow

assumptions developed a descriptive solution for the falling water

table. Kirkham and Gaskell (1951) applied the method of relaxation

and followed the water table drawdown by solving a series of steady

state solutions at several times during a recession. Isherwood (1959) and Taylor and Luthin (1963) considered separate drainage problems but followed an approach similar to Kirkham and Gaskell. Van Schilfgaarde (1963) presented an analysis similar to that of Glover based on the Dupuit assumptions. In his case he used a variable, instead of a constant thickness for the water bearing stratum, introduced Hooghoudt's equivalent depth, and corrected for convergence of flow when the tile drain is not placed on an impermeable layer. Brutsaert, Taylor and Luthin (1961) used an electrical resistance network to describe a transient drainage situation. They used a more realistic approach by incorporating an "apparent drainable porosity" which varied linearly with depth, instead of remaining constant, in this way accounting for the capillary fringe that exists above the water table. Gardner (1962) gave an approximate solution of a non-steady state drainage problem. He accounted for the flow in the unsaturated zone by assuming the water content to be a linear function of the hydraulic head and treated the problem as one of unsaturated flow. Gardner's solution based on constant diffusivity led to the same mathematical relationship as those based on constant specific yield. This demonstrated that there is some confidence in the validity of solutions of drainage problems based on such assumptions. In an earlier study by Collis-George and Youngs (1958), the authors stated that,

a) The effects of the capillary fringe above the water table make

little change in the water table height at least for high water

tables.

b) In drainage design dimensional considerations indicate that it

is convenient to express lengths as a fraction of the half

spacing of the drain tubes and the flux cutting the water table

as a fraction of the saturated hydraulic conductivity of the

soil.

Thus the factors which are of interest in drainage problems are the

ratios h/L, D/L and q/K where h and D are respectively

the height of the water table midway between the drains and the depth

of the impermeable floor from the drain axis.

As can be deduced from the text so far, quite a number of

investigators have studied both the steady and non-steady state drain-

age problem under various conditions. Unfortunately, the studies

have either been the one-dimensional vertical drainage situation or

the horizontal flow towards drains. Surprisingly almost none of the

investigators made an attempt to study the case of layered soils. All

the same, their contributions to knowledge in the areas of drainage

and fluid flow in porous media cannot be overlooked. It will be a very

exhausting job to list all such investigators and review their works

in detail in this study, however the few whose works were reviewed

are listed in the bibliography.

Brooks (1961) analyzed the unsteady situation for the case of horizontal flow towards tile drains. His work was in fact an extension of Glover's work with some refinement. Employing perturbation method of analysis, Brooks showed that Glover's solution was in fact the zero order term of his solution and thus the first approximation. By considering the first order term, he improved the approximate solution to this particular drainage problem.

In most of the horizontal flow towards drains solutions, the initial water table configuration has been a matter of varied opinions. W. N. Tapp and W. T. Moody, (Luthin, 1973) engineers working for the U.S. Bureau of Reclamation developed drainage formulas for depth and spacing using a fourth-degree parabola to represent the initial water table conditions for the case when the drains are above the barrier. The study indicated that a fourth-degree parabola gave a relatively good reproduction of the initial shape, as well as of that maintained throughout the entire drawdown period. Dumm (1964) checked the formulas developed by Tapp and Moody against a wide range of field data and found out that the data checked well with the formulas. However a notable limitation as to the validity of the formulas was pointed out and it concerned the depth of the saturated layer below the drain. He pointed out that the spacing calculations were valid so long as the initial water table height above the drains at a point midway between the drains is small as compared to the saturated depth below the drain.

This limitation serves as a warning if one would like to employ those formulas to drainage in shallow profiles.

Van Schilfgaarde (1965) points out the limitations of the Dupuit-Forchheimer theory in drainage and the linearization techniques employed to achieve analytical solutions to the drainage into a parallel drain system. He made a comparison of several drawdown equations due to other investigators. He compared the solutions of Glover, Tapp and Moody, Guyon and his own solution, which like the Brooks (1961) solution, did not depend on linearization. Even though meaningful comparisons among the solutions are difficult because they are not all based on the same initial conditions, at least some points of interest were raised. These points concerned the configuration of the initial water table and the convergence of flow at the drains. The results indicated that the flat initial water table and the horizontal flow assumptions should be applied with caution. Also the error resulting from the linearization cannot be considered trivial in that the time predicted for a 1-ft drawdown was consistently higher than with two less approximate analysis. An additional point of caution, mentioned by the author concerned the fact that the linearized solution must only be applied to small increments in drawdown, which may work relatively well for shallow profiles.

Moody (1966) carried out a study for parallel drains which I consider to be closer to exactness than most of the work done before.

He obtained a time function of the water table height for parallel

drains by considering an intermediate case in which a finite depth of

soil exists between the elevation of the drains and the impermeable

barrier. His resulting model was a non-linear differential equation

which he solved numerically. The initial water table configuration

was represented by a fourth degree parabola. An important feature

of this investigation was the graphic solutions for the non-linear

partial differential equation relating spacing of parallel agricultural

drains, time, geometry of the configuration and the physical proper-

ties of the soil. Also a convenient approximate formula was derived

for computing Hooghoudt's equivalent depth for any size of drain thus

correcting for effect of losses due to convergence of flow to the drains,

an effect not accounted for by the basic Dupuit-Forchheimer assump-

tion.

Dass and Morel-Seytoux (1974) considered three different initial

water table profiles and presented a solution utilizing the increasingly

powerful Galerkin method. They compared their solutions with those

of Glover, Brooks and Boussinesq. Even though some other investi-

gators have stressed errors resulting from the linearization proce-

dures and the Dupuit-Forchheimer assumptions, Dass and Morel-

Seytoux found those solutions as valuable tools for drainage design.

McWhorter and Duke (1974) presented two solutions for the transient

drainage problem. In their analysis they accounted for capillary

storage and the non-linearity of flow. In the first analysis, they approximated the transient water content distribution in the partially saturated zone by a succession of equilibrium distributions. This quasi-linearization process led to an analytical solution which was in reasonably good agreement with experimental data. It should be pointed out that their analysis was for the water table response to parallel relief drains. They further asserted that when the drainage rate is small compared to the saturated conductivity, the drainage rate can be predicted quite accurately by the equilibrium state distribution of the water content. In the second analysis, the same problem was considered except that here they testified the relative importance of capillarity and non-linearity in drainage situations. Special features of much interest in this approach are that,

a) An appropriate simplified form of the solution can be selected based on the value of indices indicating the degree of importance and

b) The solutions are applicable to a wide range of conditions including drains placed at any elevation relative to the impervious substratum and for shallow as well as deep water tables.

Whereas some investigators have concerned themselves with the horizontal flow towards drains, others have carried out studies concerning solely vertical non-steady drainage. Notably among these

investigators are Gardner (1962), Jensen and Hanks (1967), Jackson

and Whisler (1970), Brooks et al. (1971), Dagan and Kroszynski, and

Kroszynski (1975). Rubin (1968) considered the unsaturated and

partially unsaturated case in both horizontal and vertical flow com-

bined. Almost every investigator concerned himself with the move-

ment of the free water surface in vertical drainage. However caution

should be exercized in the use of the position of the water table as a

measure of the amount of water in the soil profile and conversely.

Reason for this is that the movement of so little water is required in

order to change the water table position and as such the water table is

a rather unsatisfactory measure of the drainage status of shallow soil

profiles. Brooks et al. (1971) and Jensen and Hanks (1967) wrote the

differential equation of the vertical drainage with water content and

pressure head respectively as the dependent variable.

(iv) Layered Soils Investigations

The review of literature for this study will not be complete

without the inclusion of studies so far on layered soils. Kirkham

(1951) carried out a theoretical analysis of the seepage into drain

tubes in stratified soil. Scope of his analysis was a two-dimensional

steady state model, with ponding on the surface and the lower layer

extending to infinite depth. He examined both the cases where the drain tubes are located in either layer. Expressions for the drain flow, the surface inflow distribution and flow nets were derived by assuming that the potential function satisfies the complex hyperbolic functions and using the method of images. His results and discussion centered around the importance of the hydraulic conductivities and the thickness of the top layer. However the forms of the derived expressions are very complicated for a person without knowledge of rigorous mathematics to understand.

As an attempt to shed some light on the understanding of drainage in layered soils, Day and Luthin (1953) examined the pressure distribution in layered soils during continuous water flow for a vertical column. They constructed a single model based on Darcy's Law and found out that the Darcian approach is valid in analyzing the pressure distribution in such flow situations.

Eagleman and Jamison (1962) studied the effect of soil layering and compaction on unsaturated moisture movement. They concluded that a) The hydraulic conductivity values across the textured breaks indicated that the soil properties were favorable for moisture transfer from large pores to smaller pores but that a barrier existed for water movement from smaller pores to larger pores. b) The barrier developed as the suction increased in a coarse layer in contact with finer material. With water removal from the larger pores at

moderately low suctions, flow from the fine soil layer into the coarse material was reduced. c) They also found out that a silt loam soil underlain by sand drained to a water content of about 40% by volume as compared to a 1/3-bar value of 27% by volume.

Miller and Gardner (1962) carried out a laboratory investigation of the effects of textural and structural stratification within a profile on the rate of infiltration into soil. They found that the effects of strata within soil were related to the pore characteristic differences between the layering material and the surrounding soil. However they could not get their data to fit any of the suggested infiltration equations. Pressure distributions in layered soil columns have been studied for both the transient and steady state by Behnke and Bianchi (1965). Their study revealed the significant effects of entrapped air on the pressure distribution profiles. Dagan (1965) theoretically analyzed the steady state drainage of a two-layered soil by a combination of two approximations of the flow equation. First in a region close to the drain he linearized the flow equation and at a region starting from a distance equal to twice the depth of the water table to the impermeable datum, he employed the Dupuit assumption. Infiltration into layered soils has received some attention lately as shown by the works of Hill and Parlange (1972), Reichardt et al. (1972) and Aylor and Parlange (1973). In all these works the effect of the pore size differences at

the interface on the imbibition process became evident through the

plots of infiltration rate against depth, the motion of the wetting front

and the moisture content profiles.

III. BACKGROUND AND THEORY

(i) General

As mentioned earlier, heterogeneity in the soil medium is the general case and homogeneity is only an exception rather than the rule. As in the Willamette Valley Region of Oregon, according to some work done by investigators and more recently by Nibbler (1974), the profiles are characterized by stratification. He mentions that the Dayton and Concord soil series of the Willamette Catena have defini- tive restrictive clay layers in their subsoil profiles that have perme- abilities ranging from zero to 0.015 cm per hour. The effects of evaporation are minimal if not zero during the winter months and this coupled with the slowly permeable subsurface layers produce water tables near the surface. The slopes in the Valley are also relatively flat and thus runoff is small. Therefore, natural drainage seems to be negligible and the problem is more than compounded when the soil profiles necessary to cause any deep percolation are relatively shal- low. We thus see that, this in itself is a cause for drainage study.

It has been revealed in the section on the literature review that most of the work done in drainage are nearly concerned with steady, one dimensional homegeneous systems. The few which have touched on the heterogeneous or stratified case concentrated on either the

steady case or the one dimensional situation with the water table

height as a measure of drainage performance. However the water

table is not a good measure of drainage performance because only a

relatively small amount of deep seepage in shallow profiles is needed

to change the location of the water table.

As a preface to the development of the model, a discussion of

the theory of fluid flow in porous media and medium parameters will

be presented.

The Darcian approach for saturated flow forms the basis,

however modifications are required to take into account the effects of

unsaturation as saturated and unsaturated flow occurs concommit-

tently. This is usually the case in the field.

The Darcy equation for saturated flow is written in the form

$$\overline{q} = -K\nabla\psi \qquad\qquad (III-1)$$

Unsaturated Conductivity

As a first step towards the unsaturation modification for the

Darcian equation above, it should be recognized that the air filled

pores reduce the effective cross-section for liquid flow and at the

same time increase the tortuosity of the remaining liquid flow paths.

This situation increases the impedance to the liquid flow and thus the

value of the hydraulic conductivity for the unsaturated flow situation

is less than in the saturated case. Buckingham (1907) argued that the conductivity for the unsaturated case is a function of the volumetric water content and thus expressible as $K = K(\theta)$. This last expression $K(\theta)$ is sometimes called the unsaturated conductivity, however Buckingham called it capillary conductivity.

Many investigators have attempted to find the exact expression for this unsaturated conductivity function. Unfortunately they could not come out with a general expression and most of the expressions involve empirical constants. However Brooks and Corey (1964) and Laliberte, Brooks and Corey (1966) have come out with expressions which show great promise and have been tested to be valid to a great extent. Their contributions are unique in the sense that the expressions involved readily available quantities, measurable values and medium parameters which are derivable from them. They expressed the conductivity function as

$$K = K_{1.0} \left(\frac{P_b}{P_c}\right)^{2+3\lambda} , \quad P_c \geq P_b \qquad \text{(III-2)}$$

$$K = K_{1.0} , \quad P_e \leq P_b$$

Unsaturation Effects on the Hydraulic Head (ψ)

As a second step in the unsaturation modification of the saturated flow Darcian equation, we examine the hydraulic head function

$$\psi = \frac{P}{\gamma} + z \qquad\qquad\qquad (\text{III-3})$$

The gravitational function z should contribute in the same way as in the case of saturated flow but the component P/γ will be different. Thus the hydraulic head function will behave in a manner as reflected by the effects of saturation on the P/γ.

Soil-Water Diffusivity

It has been mentioned that the pressure head, P/γ, is a function of saturation (i.e., water content). Denoting P/γ by τ and using $K(\theta)$, $\tau(\theta)$ and $\psi = \tau(\theta) + z$ in Equation III-1, we can write the following

$$\overline{q} = -K(\theta)[\nabla \cdot \tau(\theta) + \nabla z] \qquad\qquad (\text{III-4})$$

Since $\dfrac{\partial \tau}{\partial x} = \dfrac{d\tau}{d\theta}\dfrac{\partial \theta}{\partial x}$ and $\dfrac{\partial \tau}{\partial z} = \dfrac{d\tau}{d\theta}\dfrac{\partial \theta}{\partial z}$ we can write

$$\overline{q} = -K(\theta)\nabla z - D(\theta)\nabla\theta \qquad\qquad (\text{III-5})$$

with

$$D(\theta) = K(\theta)\left(\frac{d\tau}{d\theta}\right)$$

This expression $D(\theta)$ was introduced by Buckingham (1907) but was identified as a diffusion type coefficient by Childs and

Collis-George (1948) and is now generally called the soil-water diffusivity with units of area per unit time. Swartzendruber mentions that the physical transport process is not one of diffusion in the classical sense, since the hydrodynamic basis of Equations III-1 and III-4 has not been changed in the process of arriving at Equation III-5. $D(\theta)$ is identified as diffusion coefficient because $\nabla\theta$ represents a concentration gradient.

Nature of Conductivity and Pressure Head Functions

From Equations III-4 and III-5 it is clear that K and τ are functions of very basic importance for describing the flow process. The general nature of these functions have the characteristics that K increases with the water content θ and reaches a maximum value called the saturated conductivity at a value of θ equal to the porosity. This saturated conductivity value is higher in coarse soils than in fine soils but the porosity of fine soils is higher than that of coarse soils. However for both soils, the change in K with θ is most pronounced over the higher water content range (Figure III-1).

The τ-function in general shows some marked contrast with the K-function. Since progressively more energy is required to extract water at low water content, τ increases with decreasing θ. In this case, it is evident that $d\tau/d\theta$ is inherently negative.
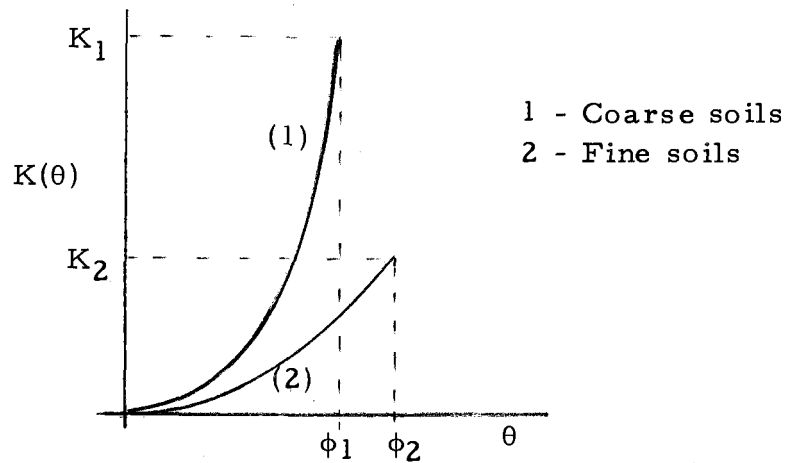
Figure III-1. General nature of the relationship between unsaturated conductivity and the water content.

Other important features of the τ-θ relationship is the generally less value of θ (for a given τ) for coarse soils as compared to fine soils. This can be explained as a consequence of large pores and smaller internal surface area in the coarse soils. This difference in θ for different soils at the same τ can also be used to demonstrate that Equation III-4 is a more general equation of flow than Equation III-5. If samples of both soils (coarse and fine) at the same τ value are brought into contact such that ∇z in Equations III-4 and III-5 are zero, Equation III-4 predicts that the flux is zero since $\nabla \tau = 0$ and as such no water transport takes place. Equation III-5 on the other hand gives the flux as $-D(\theta)\nabla \theta$. This product is not zero as neither of the values is zero, since the two soils have different θ values at the same τ and $D(\theta)$ is

not zero at the interface. We thus see that whereas Equation III-5

can hold only in a spacially uniform soil of given internal geometry,

Equation III-4 would hold even in stratified or non-uniform soils.

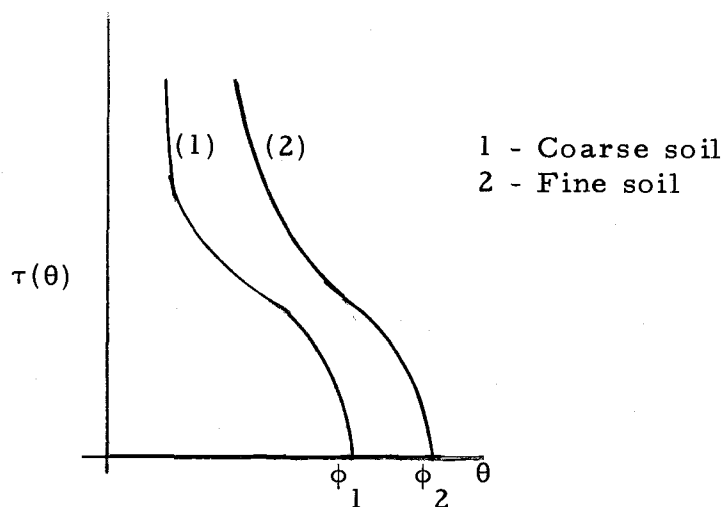Figure III-2 shows a general relationship between τ and θ.

Figure III-2. General nature of relationship between pressure head
and the water content.

Another important feature of the τ-θ relationship is a

phenomenon known as hysteresis. If the τ-θ relationship is plotted

for drying (drainage) and wetting (imbibition) processes for the same

soil sample, it is recognized that the τ-θ function is not unique for

some range of θ. This hysteresis phenomenon presents considerable

difficulty in the mathematical analysis of flow problems. This diffi-

culty is somewhat overcome by considering either imbibition or

drainage at a time and in this case the $\tau(\theta)$ function and all others
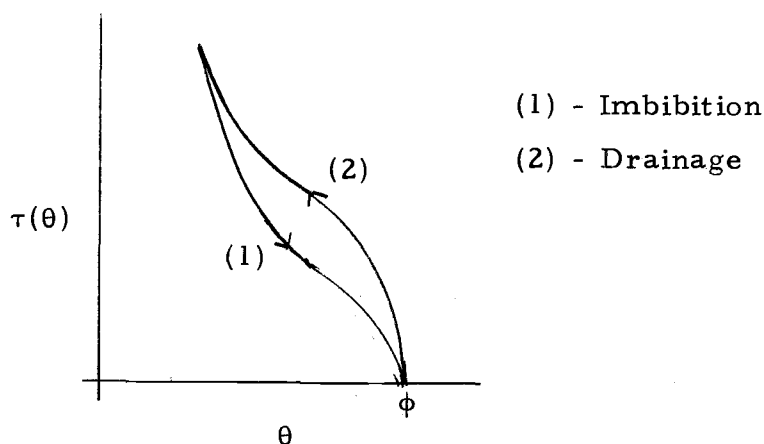
derived from it may be taken as unique.

Figure III-3. Hysteresis effect between drainage and imbibition for a
soil.

Brooks and Corey (1964) identified certain quantities as very
essential physical constants of porous media which can contribute a
great deal to the understanding of fluid flow in the media. They
identified what they called

   i) the bubbling pressure P_b of the medium

   ii) the pore-size distribution index λ

   iii) the residual saturation S_r

The bubbling pressure is a measure of the maximum pore-size
forming a continuous network of flow channels within the medium.
The residual saturation is that value of saturation at which the
pressure-saturation curve approaches a vertical asymptote. The
importance of λ and P_b were further demonstrated by functional

relationships developed for the soil-water characteristic curves and the conductivity and diffusivity functions. Defining S_e as

$$S_e = \frac{S - S_r}{1 - S_r},$$

Brooks and Corey (1964) found that the functional relationship between saturation and capillary pressure can be expressed as

$$S_e = (\frac{P_b}{P_c})^{\lambda} \qquad \text{for} \quad P_c \geq P_b$$

and

$$K = K_{1.0} (S_e)^{2/\lambda + 3}$$

These functional relationships make it possible to analyze some flow problems which would not have been otherwise and also they demonstrated the importance of $K_{1.0}$, P_b as characteristic for scaling of equations in similitude criteria for drainage systems.

Laliberte, Corey and Brooks (1966) developed an underlying theory for the significance of the pore-size distribution index and its usefulness in determining other fluid-media properties from readily available ones. The outcome provides a theoretical relationship among the parameters ϕ, S_r, $k_{1.0}$, P_b and λ and the readily available fluid property σ, the interfacial tension of the wetting and non-wetting fluids. The result which can be used to estimate the

permeability if data on pressure and saturation are available is of the form

$$k_{1.0} = \frac{\phi(1-S_r)\sigma^2}{5P_b^2} \left(\frac{\lambda}{\lambda+2}\right)$$

The importance of P_b and λ cannot be overlooked. However in some soils, P_b is very close to zero or even zero. In such a case the various expressions break down and scaling of equations for similitude criteria is out of the question. Su and Brooks (1975) have provided an answer to this by defining the capillary pressure at inflection point P_i as the most appropriate parameter and is always defined for any soil (unlike P_b). In this case, P_i plays the role of P_b in the functional relationships hitherto expressed in this work if the continuous function form is used. However if the step function form is used, we can retain P_b in the expressions. In this work, the step function form is used and since the P_b is defined for the soil samples used, this parameter will be retained in the functions defined by Brooks and Corey and Laliberte.

(ii) Assumptions for the Proposed Model

The flow of fluids in porous media, when formulated very rigorously, generate equations which are generally complex. Solutions to such equations become unthinkable even with the applications

of highly sophisticated mathematical techniques available. In the light of this, some assumptions have to be made. However the assumptions have to be reasonable enough so as not to oversimplify the model and render conclusions and recommendations drawn from it unquestionably applicable. With these aforementioned remarks in mind, the following assumptions are made concerning the model.

a) The conducting medium has a continuous pressure head and hydraulic head distribution throughout its length and thickness regardless of the layering sequence.

b) The hydraulic conductivity and the saturation functions may exhibit abrupt discontinuities at the interlayer boundaries.

c) Both vertical and horizontal components of the flux and hydraulic head are significant. Furthermore above the capillary fringe and throughout a particular medium, in the rather narrow range of heads with which we are concerned (< 1.5 m) the relation between the pressure head (and thus the hydraulic head) and the saturation may be approximated by a step function as defined by Brooks and Corey (1964).

d) Before the initiation of drainage, static conditions prevail, the medium is saturated and the hydraulic head at any height above the datum is given by the hydrostatic equation

$$\psi = \frac{P}{\gamma} + z$$

and the hydraulic head is constant.

e) It is further assumed that no biological or chemical
phenomenon change the fluid or the porous medium and that
the fluid transport in the medium occurs under isothermal
and salt-free conditions. This eliminates components of
vapor phase transport which can induce high suctions and
break down assumption c) and moisture diffusion due to salt
concentration gradients.

f) Each layer of the medium is considered homogeneous and
isotropic and that the Darcian approach and the continuity
equation are valid.

(iii) Mathematical Formulation of the Model

Figure III-4 shows the configuration for the geometry of the
physical model. A, B and C represent the drain locations, d is
the depth of the drain below the surface and D is the total thickness
of the profile. The thickness of the upper layer is a and kept fixed
for all the experiments. If we think of draining the profile to the level
of the drains, then the drainable depth of the lower layer is either
b_1 or b_2. The spacing between drains is 2L and the center line
is a line of mathematical symmetry. The physical model is set up in
such a way that, the material of the lower layer, has the lower
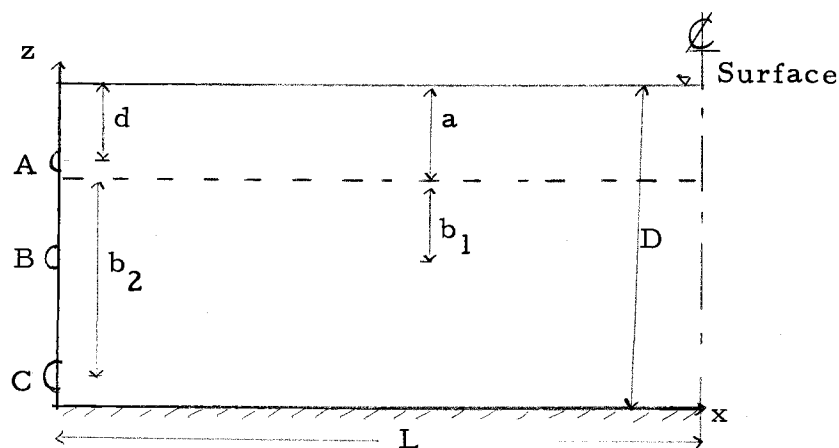hydraulic conductivity.

Figure III-4. Configuration for the geometry of the physical model.

Derivation of the Governing Differential Equation and Associated Initial and Boundary Conditions

Application of the Darcian approach and the equation of continuity for incompressible fluids yield

$$\overline{q} = -K\nabla \cdot \psi \tag{III-1}$$

$$\frac{\partial \theta}{\partial t} = -\nabla \cdot \overline{q} \tag{III-6}$$

where   K   is a function of ψ,   thus accounting for both saturated and unsaturated flow conditions which very often are concomitant conditions.

Combination of Equations III-1 and III-6 results in

$$\frac{\partial \theta}{\partial t} = \nabla \cdot (K \nabla \psi) \qquad (III-7)$$

Writing $\frac{\partial \theta}{\partial t}$ as $\frac{d\theta}{d\psi} \frac{\partial \psi}{\partial t}$ and substituting in III-7

$$\frac{d\theta}{d\psi} \frac{\partial \psi}{\partial t} = \nabla \cdot (K \nabla \cdot \psi)$$

Using the Brooks-Corey functions and noting that there is a linear transformation between P and ψ and λ being invariant (it is an intrinsic property of the medium) we can write the following,

$$S_e = \frac{\theta}{\phi_e} = (\frac{P_b}{\psi})^\lambda$$

and

$$\theta = \phi_e P_b^\lambda \psi^{-\lambda}$$

Therefore

$$\frac{d\theta}{d\psi} = \phi_e P_b^\lambda (-\lambda)\psi^{-\lambda-1} = -\phi_e \lambda P_b^\lambda \psi^{-(1+\lambda)}$$

Substituting for $\frac{d\theta}{d\psi}$ in Equation III-7,

$$\nabla \cdot (K\nabla \cdot \psi) = -\phi_e \lambda P_b^\lambda \psi^{-(1+\lambda)} \frac{\partial \psi}{\partial t}$$

$$\nabla \cdot (K\nabla \cdot \psi) = \beta(\psi) \frac{\partial \psi}{\partial t} \qquad (III-8)$$

with

$$K = K_{1.0} \left(\frac{\psi}{P_b} \right)^{-(2+3\lambda)}$$

$$\text{(III-9)}$$

$$\beta(\psi) = -\lambda \phi_e P_b^{\lambda} / \psi^{(1+\lambda)}$$

Equations III-8 and III-9 together with their associated initial
and boundary conditions defines the mathematical model for the
physical system under study.

The boundary and initial conditions to be satisfied are

$$\frac{\partial \psi}{\partial x}(0, z, t) = \frac{\partial \psi}{\partial x}(L, z, t) = 0$$

$$\frac{\partial \psi}{\partial z}(x, D, t) = \frac{\partial \psi}{\partial z}(x, 0, t) = 0 \qquad \text{(III-10)}$$

$$\psi(0, D\text{-}d, t) = D\text{-}d$$

$$\psi(x; z, 0) = \psi_0 \qquad \text{(III-11)}$$

$$\psi(x, (D\text{-}a)_-, t) = \psi(x, (D\text{-}a)_+, t) \qquad \text{(III-12)}$$

$$K \frac{\partial \psi}{\partial z} \bigg|_{x, (D\text{-}a)_-, t} = K \frac{\partial \psi}{\partial z} \bigg|_{x, (D\text{-}a)_+, t} \qquad \text{(III-13)}$$

It is clearly seen that III-10 are the no flow boundary conditions,
III-11 is the initial condition. Equations III-12 and III-13 take care
of the situation at the interlayer boundary.

IV. EQUIPMENT AND PROCEDURES

(i) Equipment and Setup

This study was carried out with the facilities in the Soil and Water Laboratory in the Gilmore Hall Annex. The drainage experiments were performed in a 7.62 m x 5.1 cm x 0.914 m plexiglass sand tank with a tensiometer grid of 30.48 cm x 15.24 cm to measure the hydraulic heads. The tensiometers which are made of porous material were connected to a NLS-S1 Portable Data Acquisition instrument by transparent plastic tubings through a Validyne model DP7 pressure transducers and a valve. A Teletype Corporation model 33TZ was hooked to the NLS-S1 instrument to print out the readings at the various tensiometer points. A 5.40 cm inside diameter reservoir was set up to measure the volume of water drained. Pace Engineering Pressure Transducers model P21G-5PS1-625-4 were connected to the reservoirs and then to the NLS-S1 to read the volume of water drained from the media. The reservoirs were calibrated against a pressure head reading on a manometer. Figures IV-1 and IV-2 show the instrumentation and the set up.

The properties of the porous media as determined from the drainage cycle of the retention curves were separately measured by a different set up in the same laboratory. This was done with the help
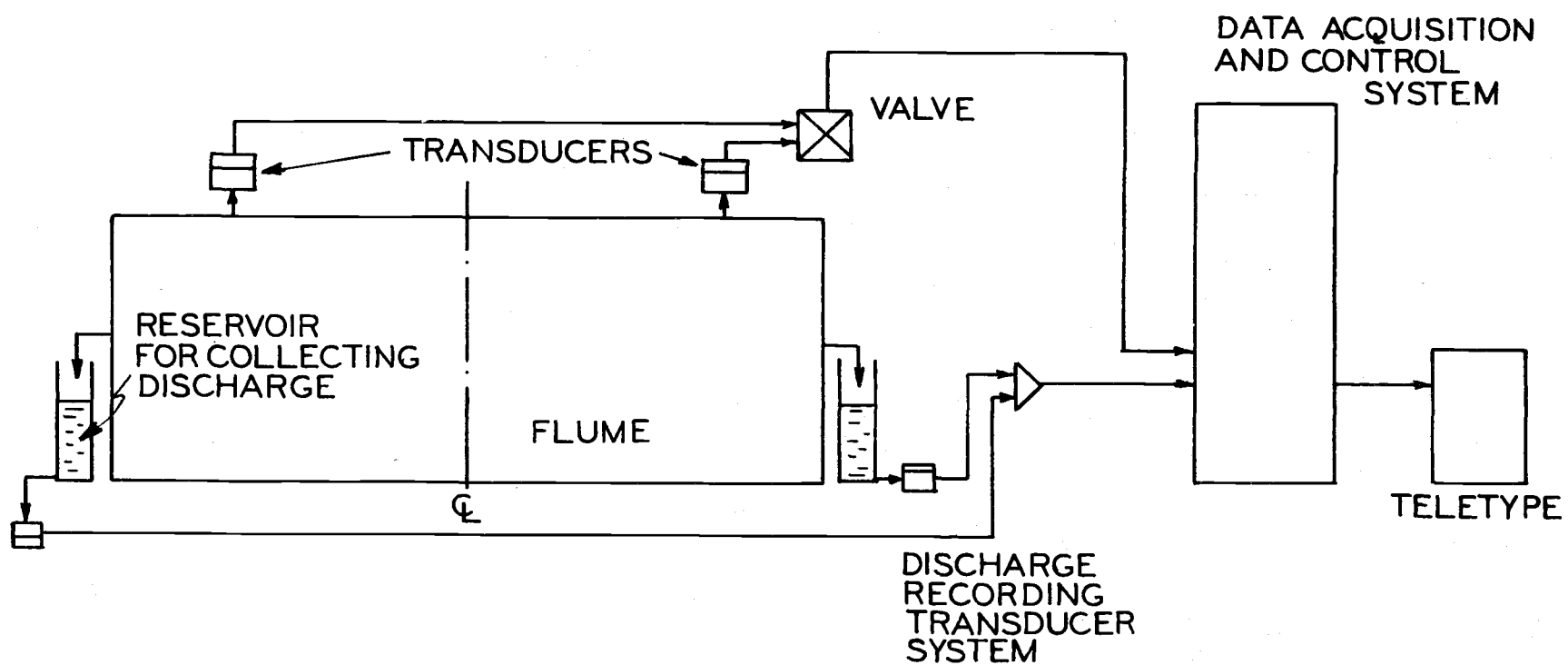
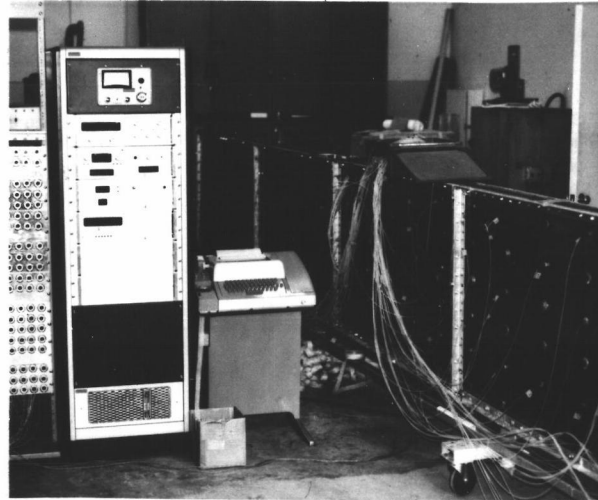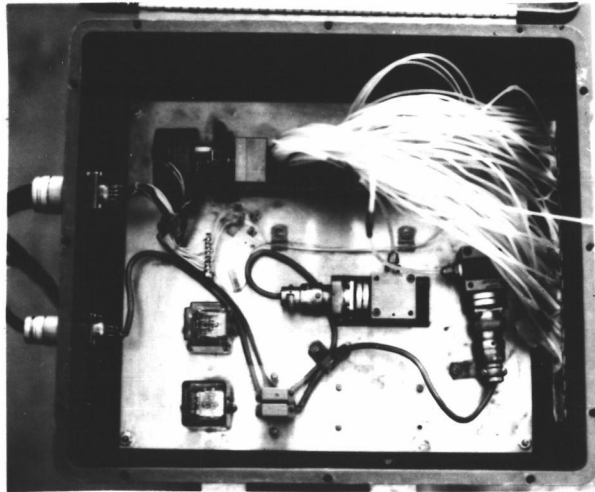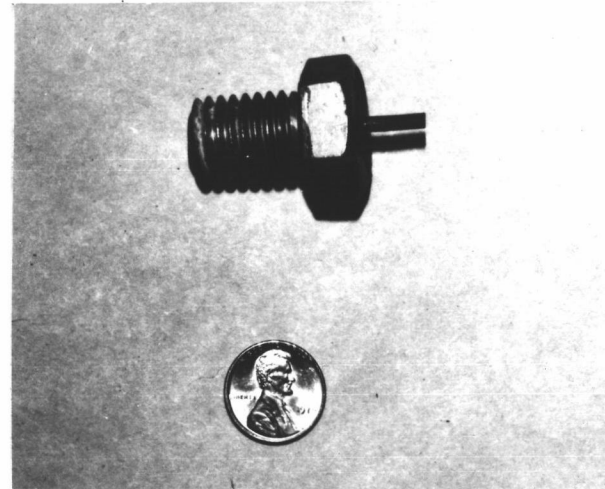Figure IV-1. Schematic diagram for the drainage experiments.

Figure IV-2. Instrumentation and equipment used to obtain data from drainage experiments.

i) Pressure transducer and scanivalve system.

ii) Tensiometer for measuring hydraulic head in the model.

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of manometers and a Nullamatic pressure regulator and porous plate apparatus.

Two different types of soil material were used in the experimental study. A more permeable material classified as a river bed sand and a less permeable material of very fine and higher percentage of clay acquired from Corvallis Sand and Gravel Company constitute the porous materials used.

The analysis of the data, the graphs and calculations were performed using HP 9810A calculator, HP 9860A marked card reader and HP 9862A calculator plotter.

(ii) Procedures

Drainage Experiments

The clay loam material was thoroughly dried in the sun and sieved to pass a #16 US sieve equivalent. This was done to avoid large aggregates from getting packed and creating large crevices in the porous matrix. The river bed sand was however not sieved.

Three sets of parallel drains were installed. When drains at both ends of the sand tank were in operation, the spacing for the drains was 7.62 m and when only the drains at one end of the sand tank were being used, the spacing was 15.24 m.

Drains were installed at 12.7 cm, 43.2 cm and 73.7 cm (all measured from the top of the sand to the middle of the drain) with sizes 2.03 cm, 6.35 cm and 11.43 cm respectively. The clay material was placed in the sand tank first. To ensure uniform compaction and avoiding large voids, a funnel was used in filling and the material was occasionally saturated. This was filled to a thickness of 67.31 cm and then the river bed sand material was added on top for another 16.51 cm.

Since any air in the porous plates of the tensiometers and the tubings would nullify the validity of the result, an attempt was made to remove as much air as possible. This was done by completely filling the tubings with water and fitting to the tensiometers when they were dripping. This was done in time to avoid the very fine materials of the porous medium from blocking the pores of the porous plate. Before actual experimental runs were made, 2.0 ppm $CuSO_4 \cdot 5H_2O$ solution was used to control the growth of algae at the south end of the sand tank. Two smaller reservoirs were installed one at the bottom and the other on top of the sand tank with a difference in elevation of 91.44 cms. These served as a means of calibrating the tensiometer point readings.

In all five experiments were ran and the procedure was the same for all. The only changes being the location of drains and the spacing. In all the experiments, the thickness of the river bed sand material

contributing to the drainage depth remained fixed while that due to the
clay varied according to as the depth of drain in question. This takes
care of the situation in classical drainage studies where the effects
of the proximity of the drains to the impermeable layer were studied.
Before taking data for any of the experiments, the whole profile is
saturated to the surface but with no ponding. With all the drains
plugged, a series of data are taken and analyzed to make sure the
system is in static equilibrium. Attaining perfect equilibrium is not
an easy thing as there might be slight leakages (and in this case we
could not get a perfect seal), a tolerant level of equilibrium was how-
ever achieved. Under this condition the hydraulic head is the same or
very nearly the same at all points in the porous matrix and this is
designated as the initial value condition for the analysis of the mathe-
matical model. The drains were then unplugged and the discharge
collected in a reservoir which was occasionally emptied and recorded.
The physical details of each experiment can be found in Table V-2.

Retention Curve Experiments

The experiments for the retention curves were performed on
the drainage cycle using a method suggested by Su (1976). The porous
plate apparatus was vacuum saturated and then the soil sample was
packed in it. The soil was then saturated as much as possible
without ponding. Various suctions were applied through the pressure

regulator and each time the volume drained at equilibrium was recorded. Volume of water remaining in the sample after the experiment was determined after oven drying. The curves for the soil samples are as shown in the Presentation and Discussion of Results. The hydraulic properties derived from the curves are tabulated in Table V-1.

V. PRESENTATION AND DISCUSSION OF RESULTS

In this chapter, the data from the two sets of experiments, namely a) drainage and b) retention curve experiments are analyzed. The data from the retention curve experiments were used to estimate the hydraulic parameters of the media employed in the drainage experiments and those of the drainage experiments to analyze the response and behavior of the system in the light of these hydraulic parameters and the geometry. Both of these analyses are imperative in this study for the reason that the knowledge of the water retention and conductivity curves provides sufficient means to characterize a porous material and its response to any water flow which in this case is a drainage study.

The hydraulic parameters obtained from the water retention curves by the Brooks-Corey functional relationships and the Laliberte equation are presented in Table V-1. It is clear that there are marked differences in the values of the parameters. It is interesting to note the large differences in the values of the pore-size distribution index, the bubbling pressure head and the hydraulic conductivity. The ratios of the values for the river bed sand to the clayey loam are 2.11, 0.29 and 19.8 for the pore-size distribution index, bubbling pressure head and hydraulic conductivity respectively.

Table V-1. Hydraulic parameters of porous media used.

Parameter	River Bed Sand	Clayey Loan
λ	3.240	1.534
$P_b/\rho g$	17.0 cm	58.6 cm
S_r	0.356	0.617
ϕ	0.375	0.539
ϕ_e	0.241	0.210
$K_{1.0}$	0.0516 cm sec^{-1}	0.0026 cm sec^{-1}

Table V-2. Parameters for drainage experiments.

Parameter	Expt. 1	Expt. 2	Expt. 3	Expt. 4	Expt. 5
a	12.7 cm	16.5 cm	16.5 cm	12.7 cm	16.5 cm
b	0	26.7 cm	57.2 cm	0	26.7 cm
d	12.7 cm	43.2 cm	73.7 cm	12.7 cm	43.2 cm
L	7.62 m	7.62 m	7.62 m	3.81 m	3.81 m
Q_∞	916 cm^3	9613 cm^3	16481 cm^3	458 cm^3	4806 cm^3
δ	2.03 cm	6.35 cm	11.43 cm	2.03 cm	6.35 cm

Since this study deals with drainage in shallow layered soils, it is important to examine the ratios of the actual thickness of the two soils to their respective bubbling pressure heads. In the case of the river bed sand this ratio is 0.971 and 1.149 for the clayey loam. The greater the value of this ratio, the less is the influence of capillarity on the total amount drained from a particular column. This is the same as saying that if the profile were deep enough, a large percentage of the drainable liquid would be removed from the

profile regardless of the value of λ for the particular medium. The influence of λ on the drainage process should be expected for shallow systems.

Large values of λ imply a rather uniform pore-size and for large λ, the effective permeability and thus hydraulic conductivity decrease tremendously with small increases in capillary pressure. The saturated pores of the medium with large λ (and thus $2+3\lambda$) empty rapidly until the capillary pressure reaches a value greater than the bubbling pressure after which the drainage process is abruptly slowed. The sudden drop in effective conductivity is associated with small capillary pressure increase above the bubbling pressure. After initial outflow, there is very little drainable liquid left because a medium with large λ must have rather uniform pore sizes. In fact as λ approaches infinity, the pore sizes all approach one particular size. The above mentioned differences can be seen in Figures V-1 and V-2 representing the water retention curves, pore size distribution and effective saturation as a function of capillary pressure.

Another point of interest concerning the hydraulic parameters and their subsequent effect on the drainage process involves ϕ, ϕ_e and S_r. It is found that even though the porosity of the clayey loam is higher than that of the river bed sand as expected, their effective porosities are in reverse order. This can be attributed to the high S_r for the clayey loam resulting from the capacity of very fine
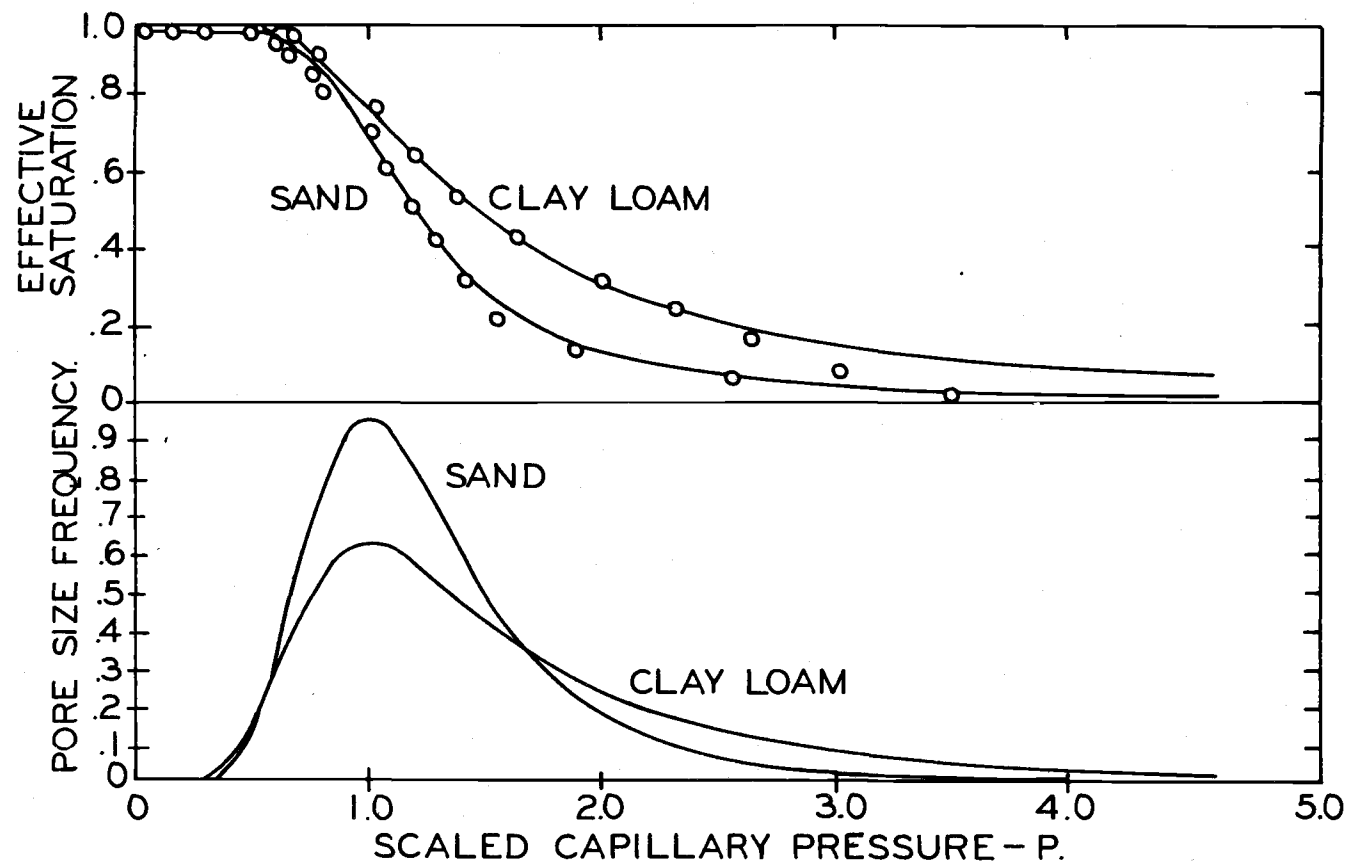
Figure V-1. i) Retention curves: effective saturation vs. scaled capillary pressure.
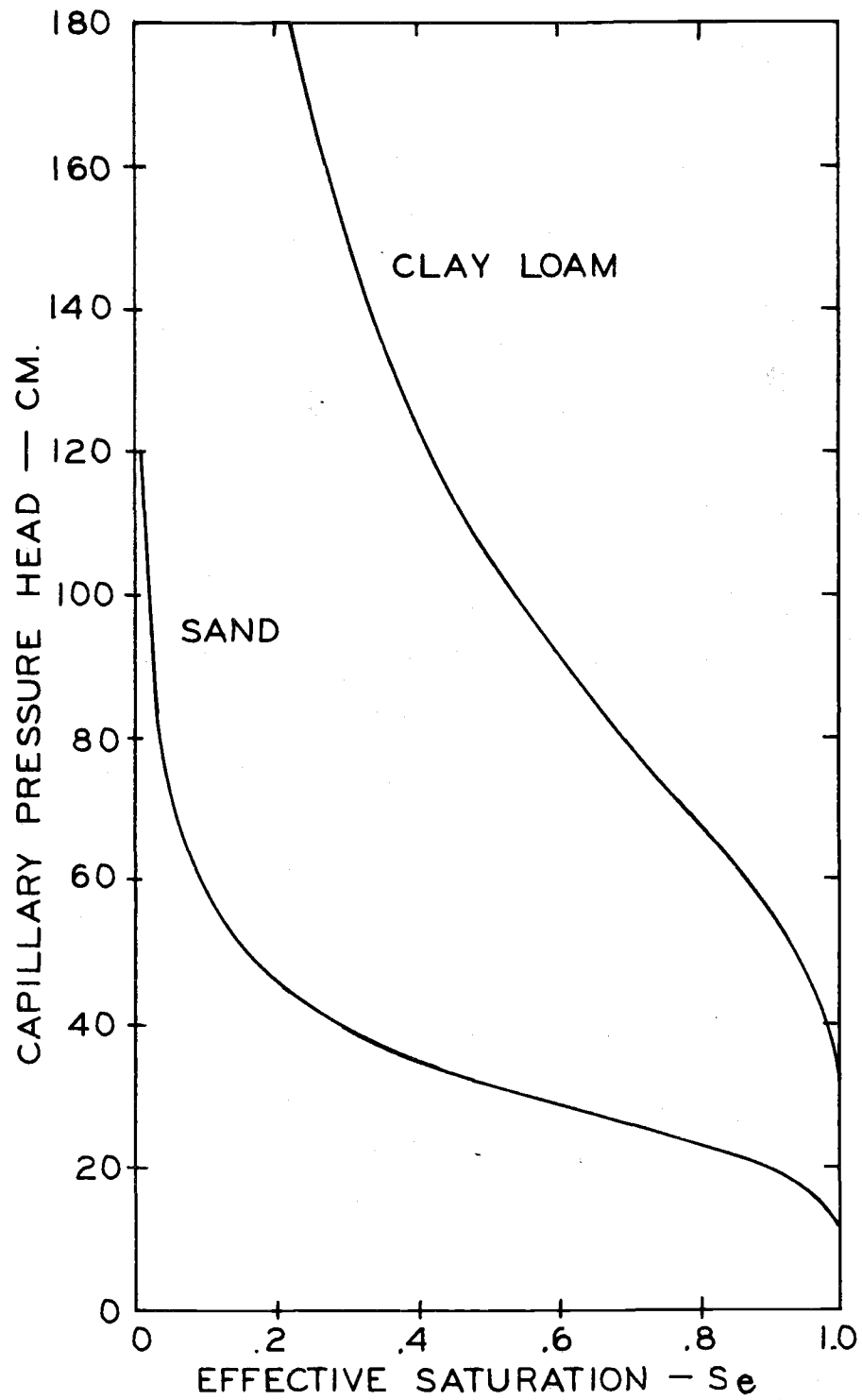ii) Pore size frequency distribution.

Figure V-2.   Retention curves: effective saturation vs. absolute
capillary pressure.

material in the clayey loam to block some of the very small pores.

The parameters for the drainage experiments are presented in Table V-2. These parameters only briefly describe the various geometrical arrangements of the physical model. The hydraulic head data obtained for the experiments were transformed into pressure head values through the relationship $\psi = P/\gamma + z$. The relationship between the pressure head and elevation was derived by graphical plots for both time and spatial variations. These plots served a dual purpose of (a) determining the location of the water table for different times and different geometries and (b) provide an indication of possible flow patterns and nature of the flow. Representative features of the families of curves are shown in Figure V-3 where the data is taken from Experiment 2. It is readily noted that the distribution of pressure head for early times and after a long time followed that for a static situation. In between these periods, there was still a linear relationship involving P/γ and z, yet the situation was not static. This linear relationship occurred throughout the soil below the water table (location of points with zero pressure with respect to atmospheric) and to a few centimeters above it. Above this almost static zone, the distribution became that for an almost constant pressure head indicating a near solely downward vertical flow and also contribution of drainage from above the water
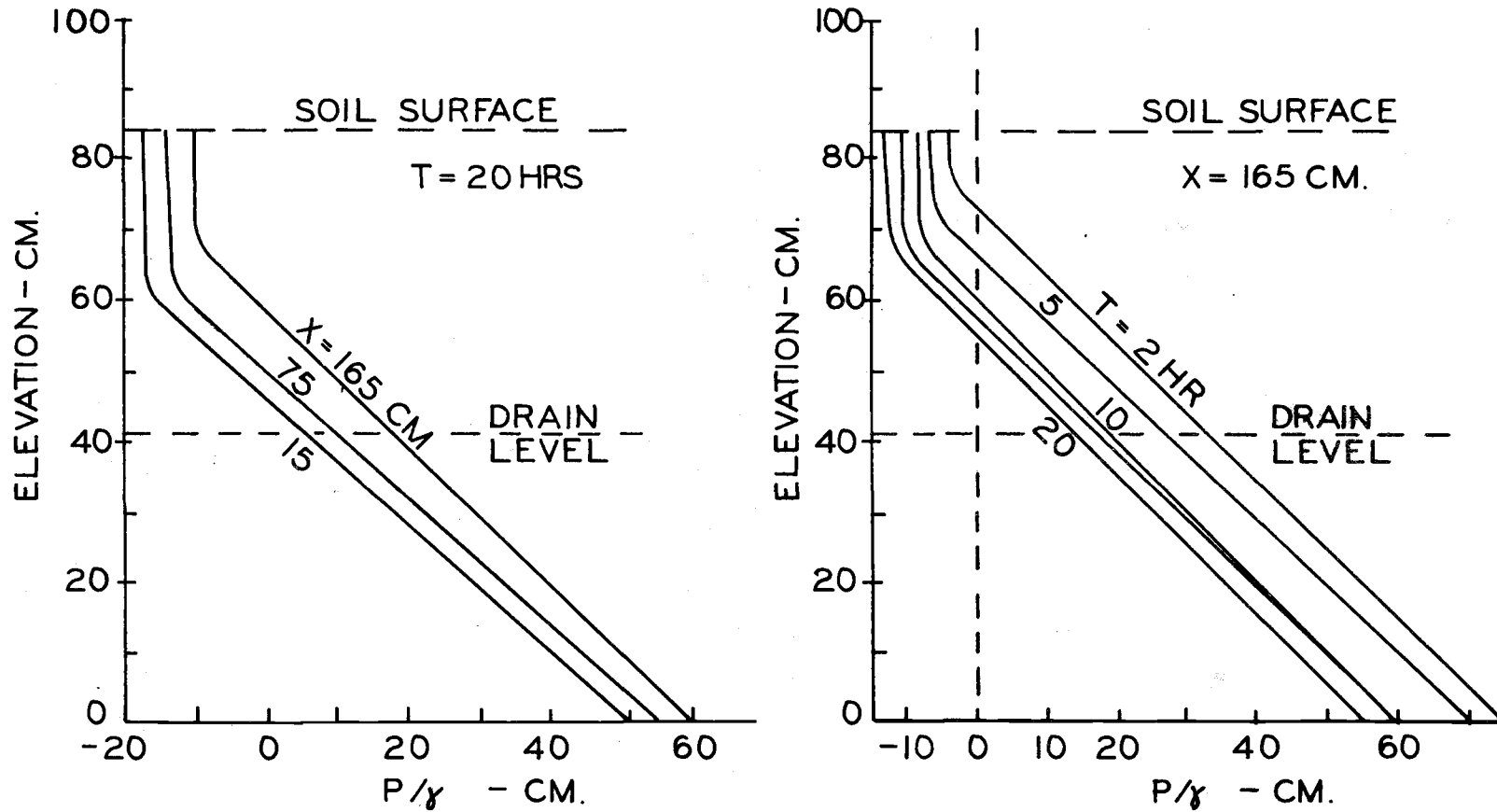
EXPERIMENT NO 2.

Figure V-3. Spatial and time pressure distributions.

table. This situation was more pronounced during the intermediate periods.

The spatial effects for a particular time indicated the same characteristics as for time variations. Deviation from the static situation was more pronounced at small horizontal distances from the drain and less pronounced for farther distances. In fact an almost perfect static situation is exhibited at locations close to the mid-point of the drain spacing. This effect could even be felt at distances as close as 0.5 L. The quasi-equilibrium states demonstrated in this study, add some justification and validity to the work of McWhorter and Duke (1974) involving transient vertical drainage. Their assumption of a succession of equilibrium states, capillary and non-linear effects have proved to be a very good approximation and valid for all practical purposes in a situation where they are expected to be less valid.

Equipotentials were also analyzed and representative samples for the cases of experiments 2, 3 and 4 are shown in Figures V-4 through V-6. It is interesting to note that they all exhibit a common feature. Higher hydraulic gradients existed in the sand and the flow pattern followed that of almost strictly vertical flow at distances closer to the drains. This pattern was more pronounced for the
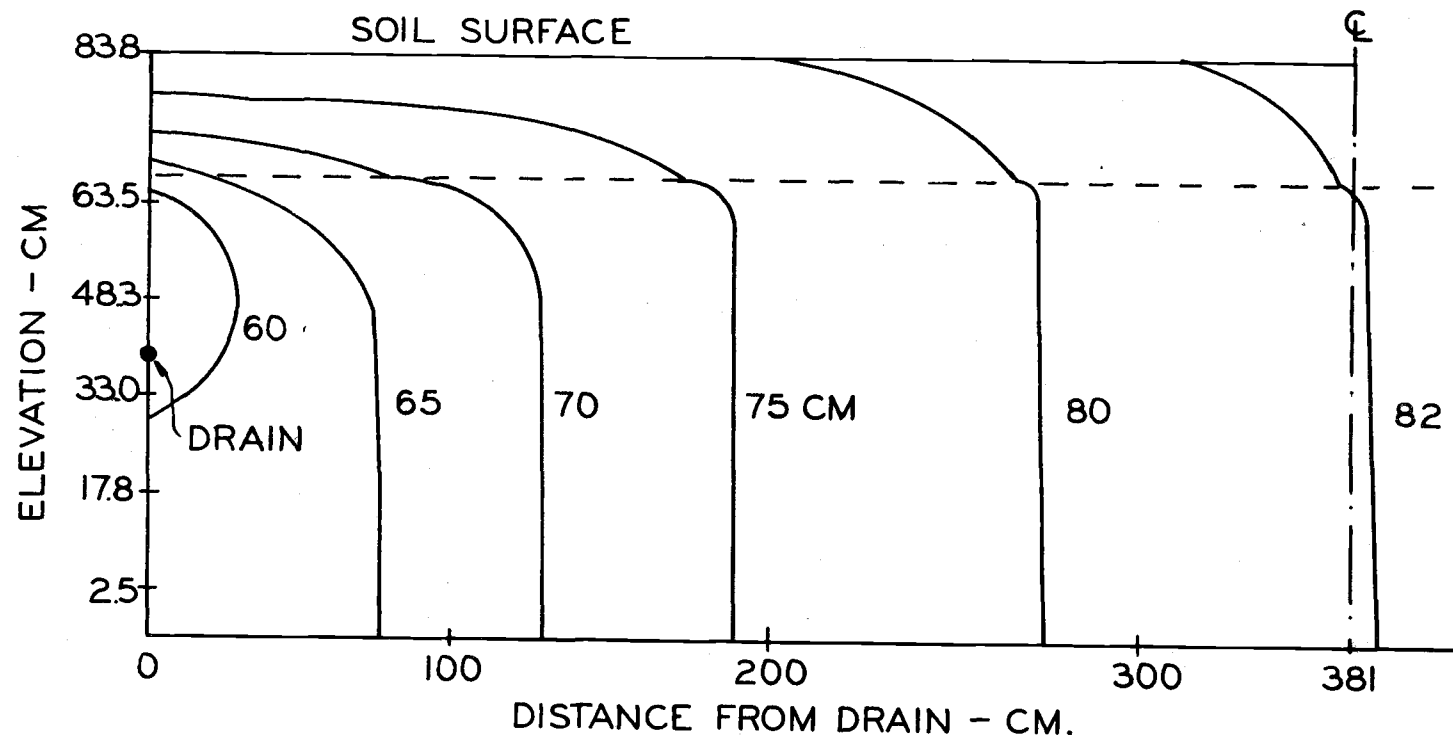
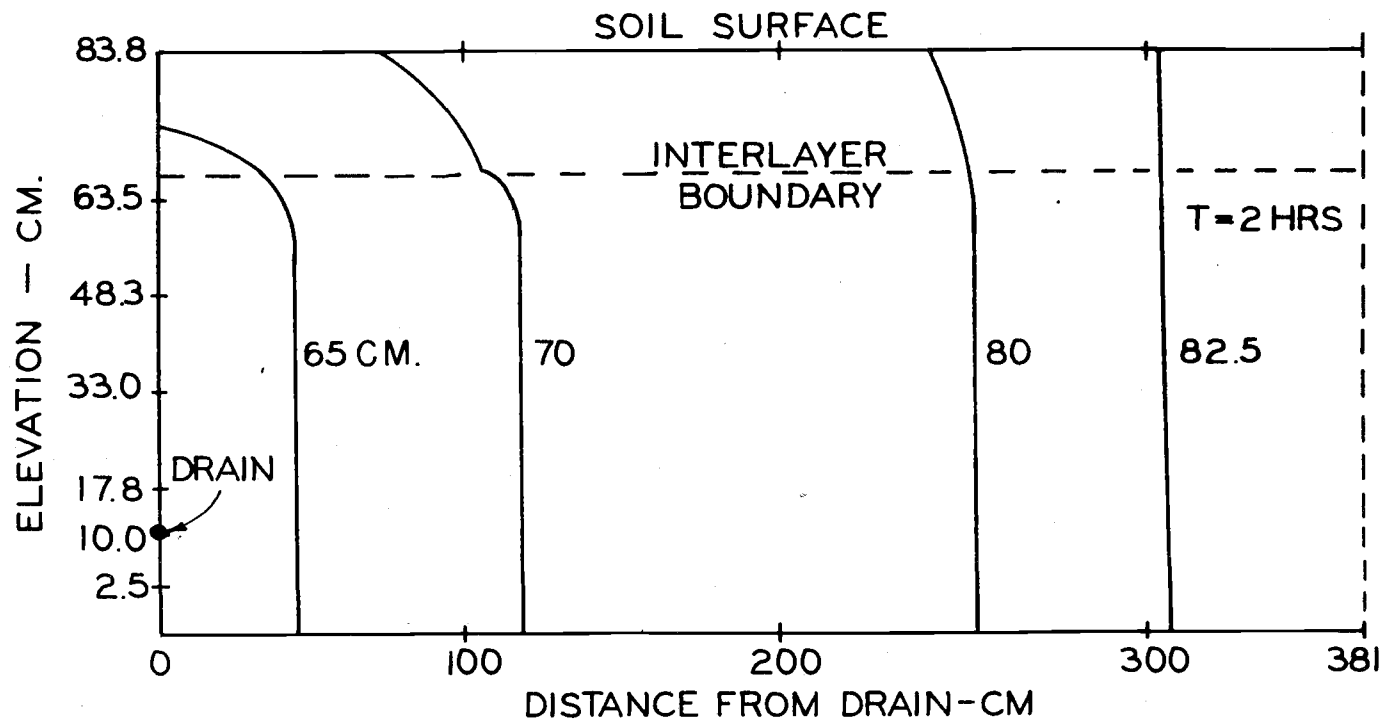Figure V-4.   Equipotential distribution for Experiment 2 at t = 2 hrs.

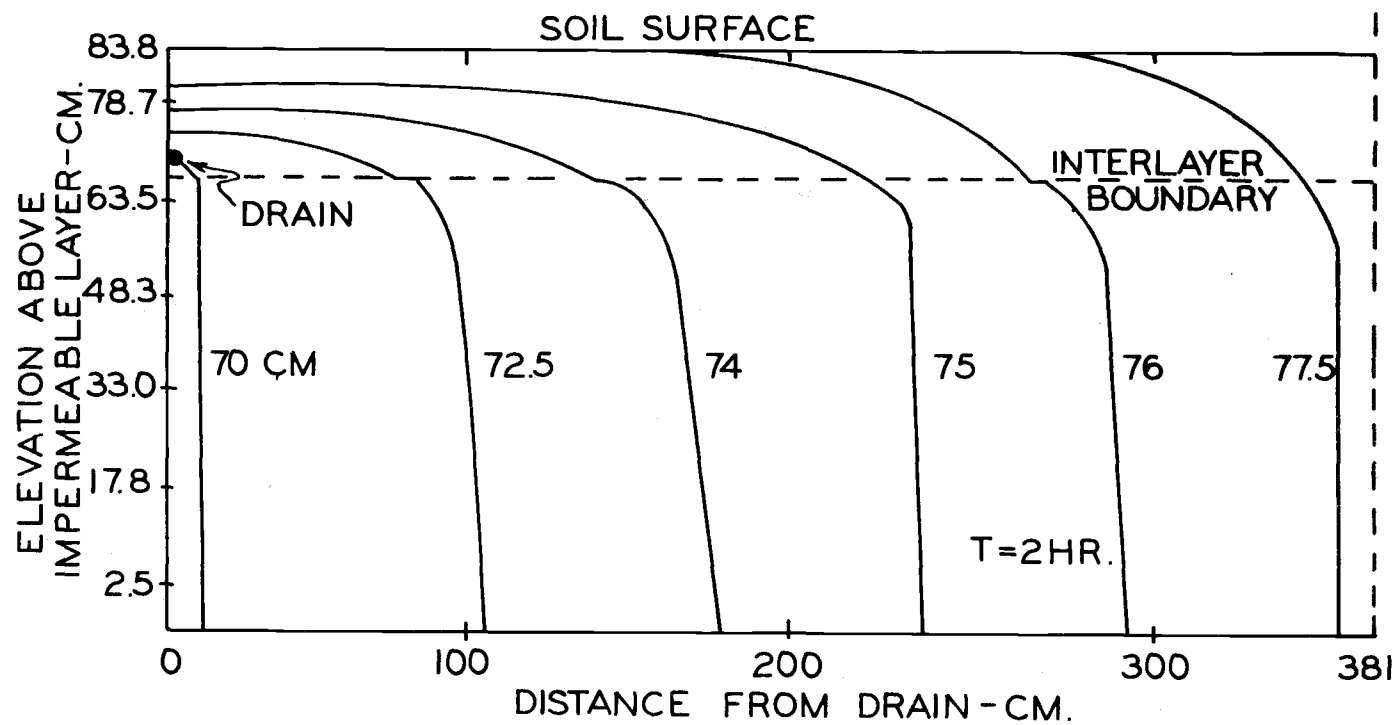Figure V-5.   Equipotential distribution for Experiment 3 at t = 2 hrs.

Figure V-6.   Equipotential distribution for Experiment 4 at t = 2 hrs.

situations where the drains were placed relatively shallow (i.e., Experiments 2 and 4). As the distances from the drain became larger, the flow pattern indicated contributions of both horizontal and vertical components (i.e., 2-dimensional flow). At still larger distances from approximately 0.5 L to L, the equipotentials tend to be almost or perfectly vertical indicating the dominance of horizontal flow. This may not be exactly true as the system is not steady. We can only infere that this is an indication of near steady state flow which can be expected to occur at distances farther away from the drains.

Another important characteristic of the equipotentials are the marked effect of the interlayer boundary, the very low hydraulic gradients in the clayey loam compared with those in the sand and the flow pattern in the less permeable clayey loam. It was estimated that the hydraulic gradient in the clayey loam for all the experiments ranged between 0.0047 and 0.128 compared with values in the sand which were at times as high as three order of magnitude of the smallest value in the clayey loam. Notably the flow patterns under these low hydraulic gradients were almost perfect horizontals.

The water table heights in relation to time at various distances from the drain and also the data for the water table profiles are shown in Figures V-7 through V-14. Again the effect of the distinct inter-layer boundary is pronounced for all situations in which the drain
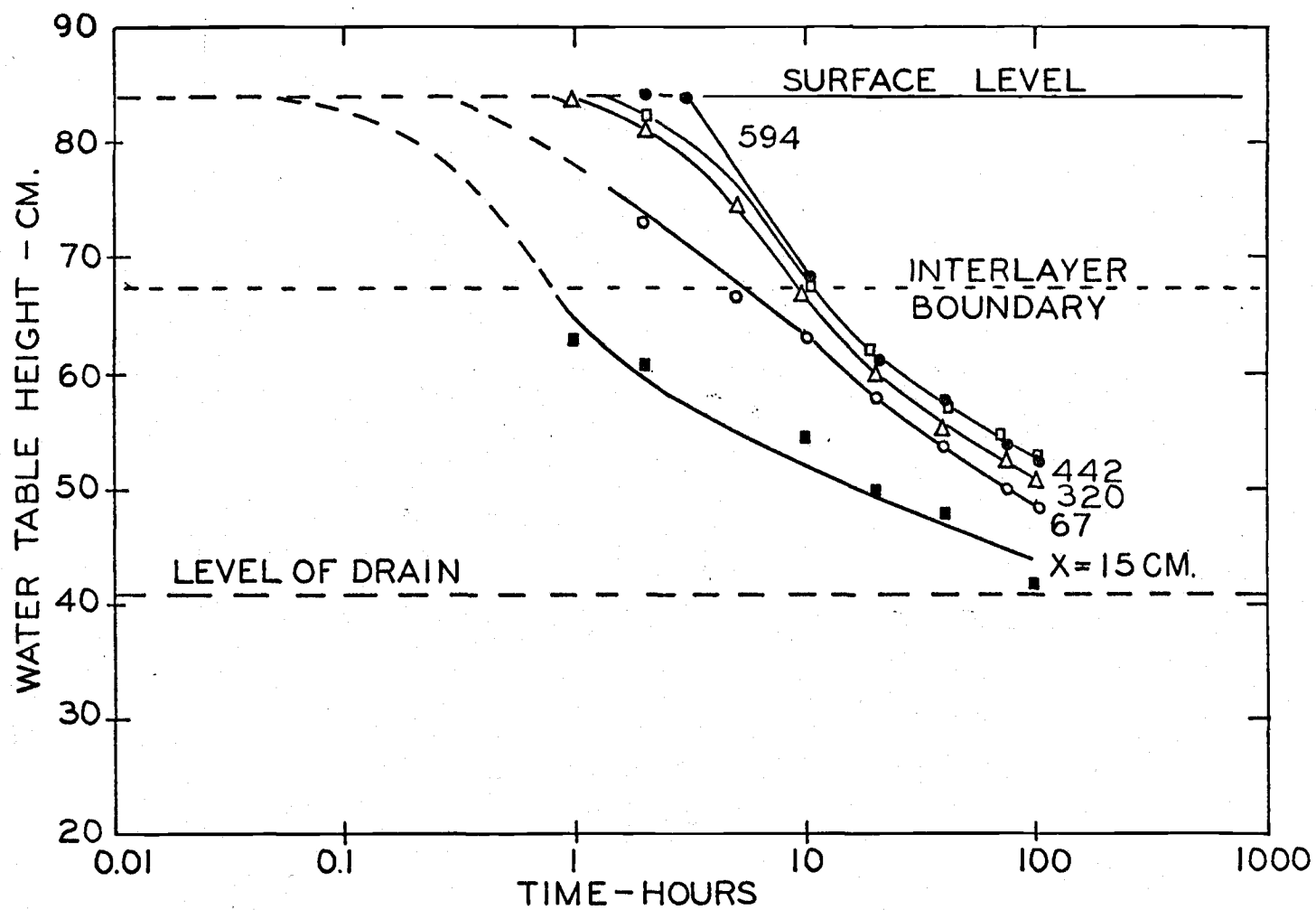
Figure V-7. Water table recession for fixed distances from drain (Experiment 2).
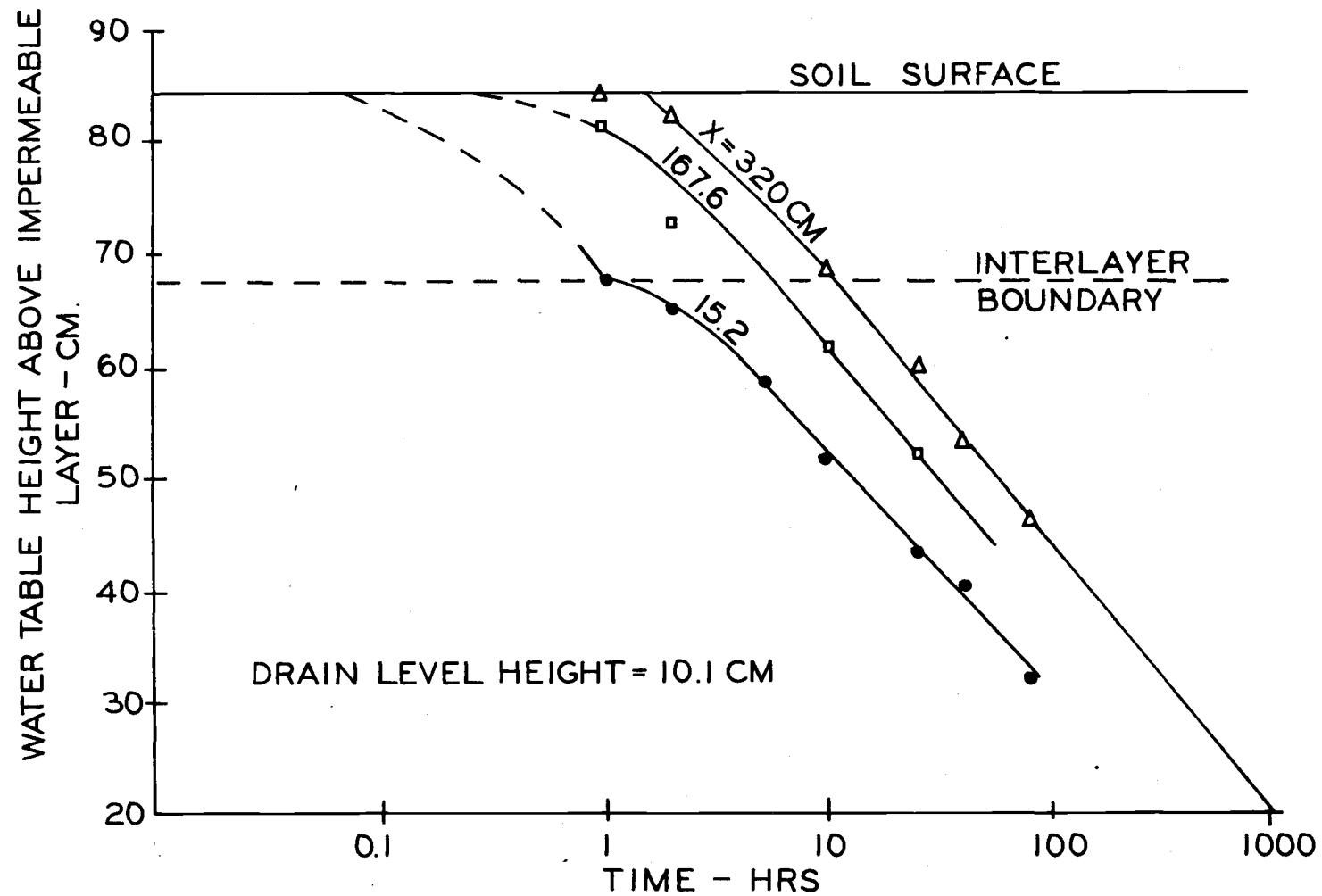
Figure V-8. Water table recession for fixed distances from drain (Experiment 3).

Figure V-9. Water table recession for fixed distances from drain (Experiment 4).

Figure V-10. Water table recession for fixed distances from drain (Experiment 5).

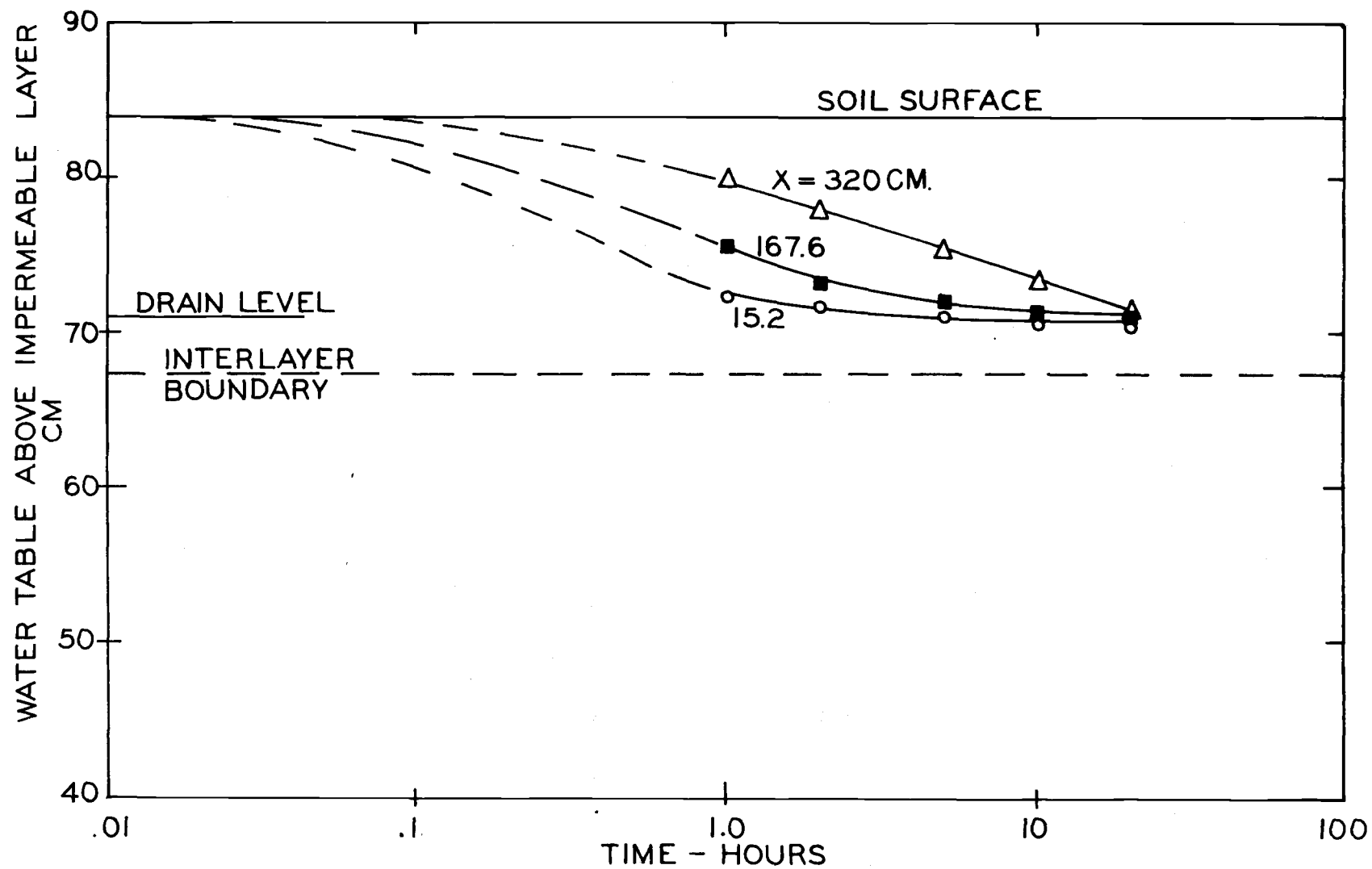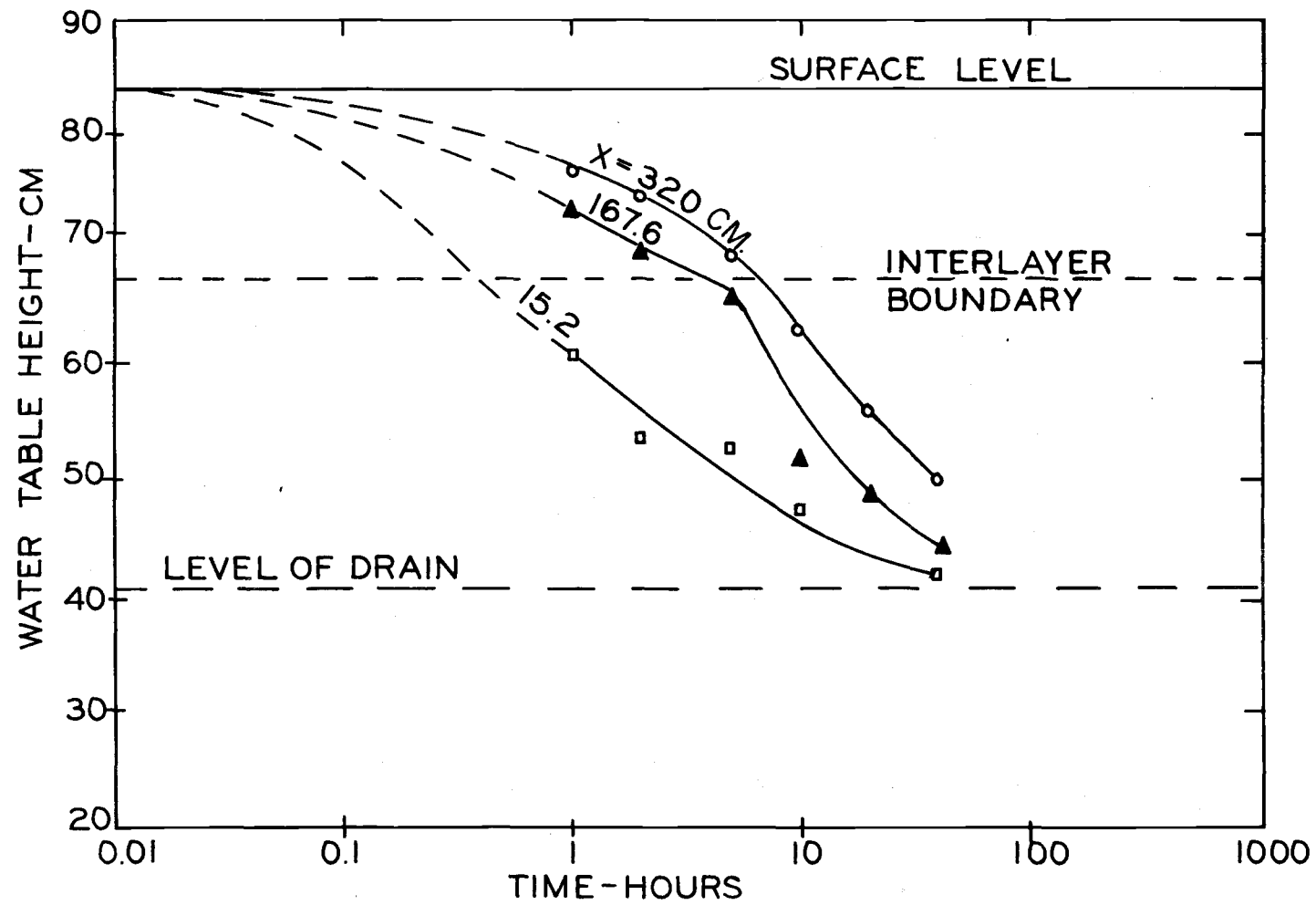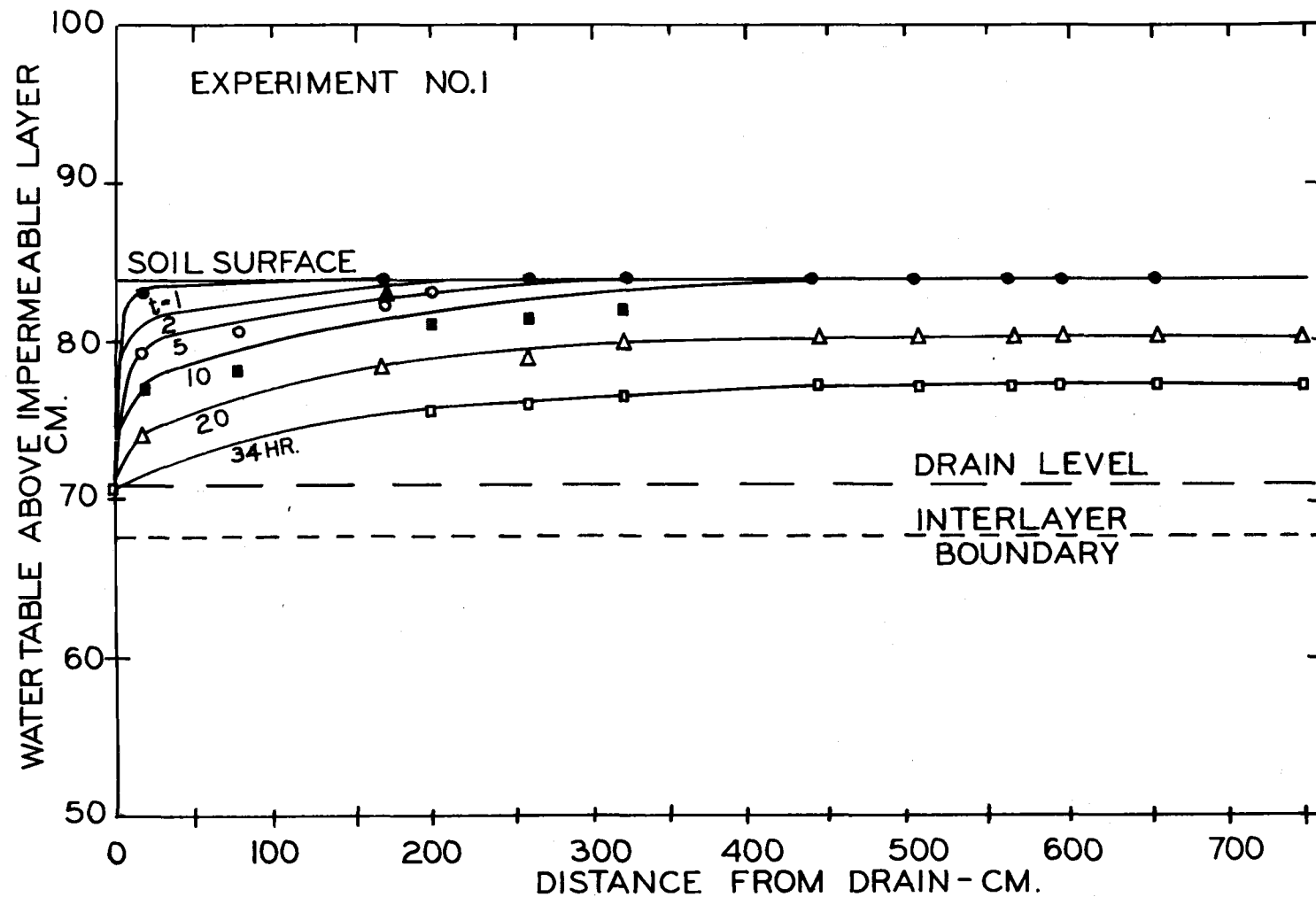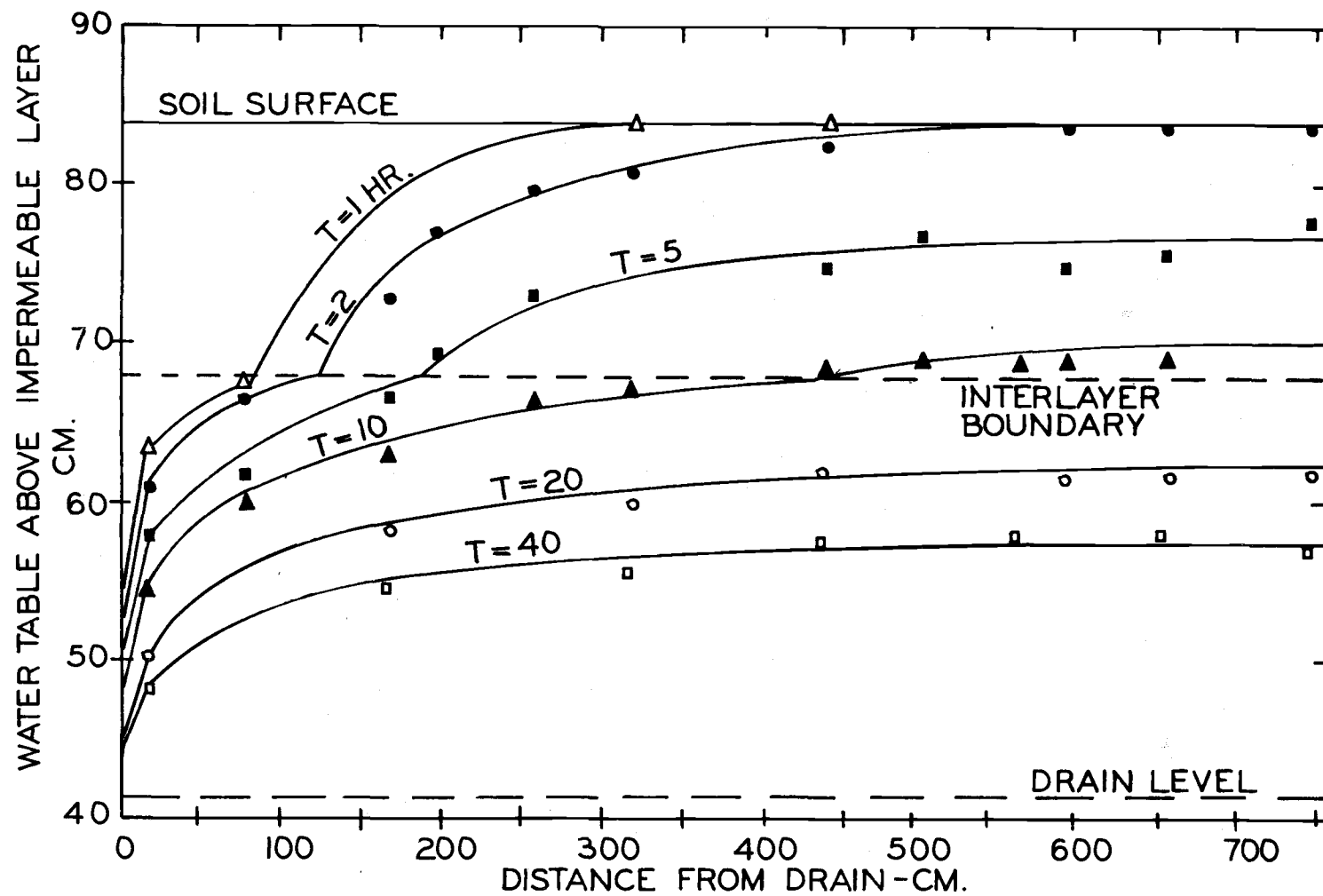Figure V-11. Water table profiles at various times (Experiment 1).

Figure V-12. Water table profiles at various times (Experiment 2).
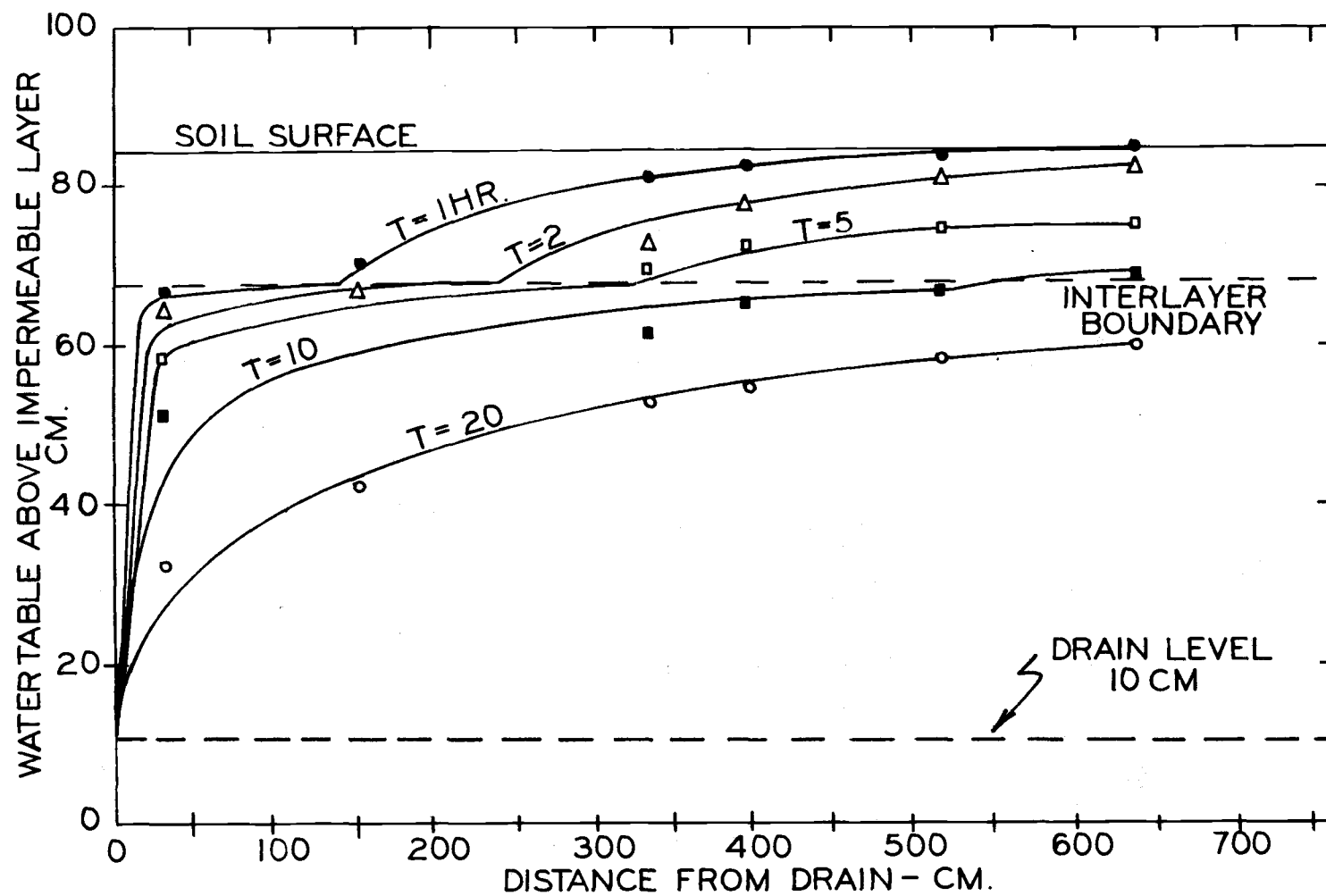
Figure V-13. Water table profiles at various times (Experiment 3).
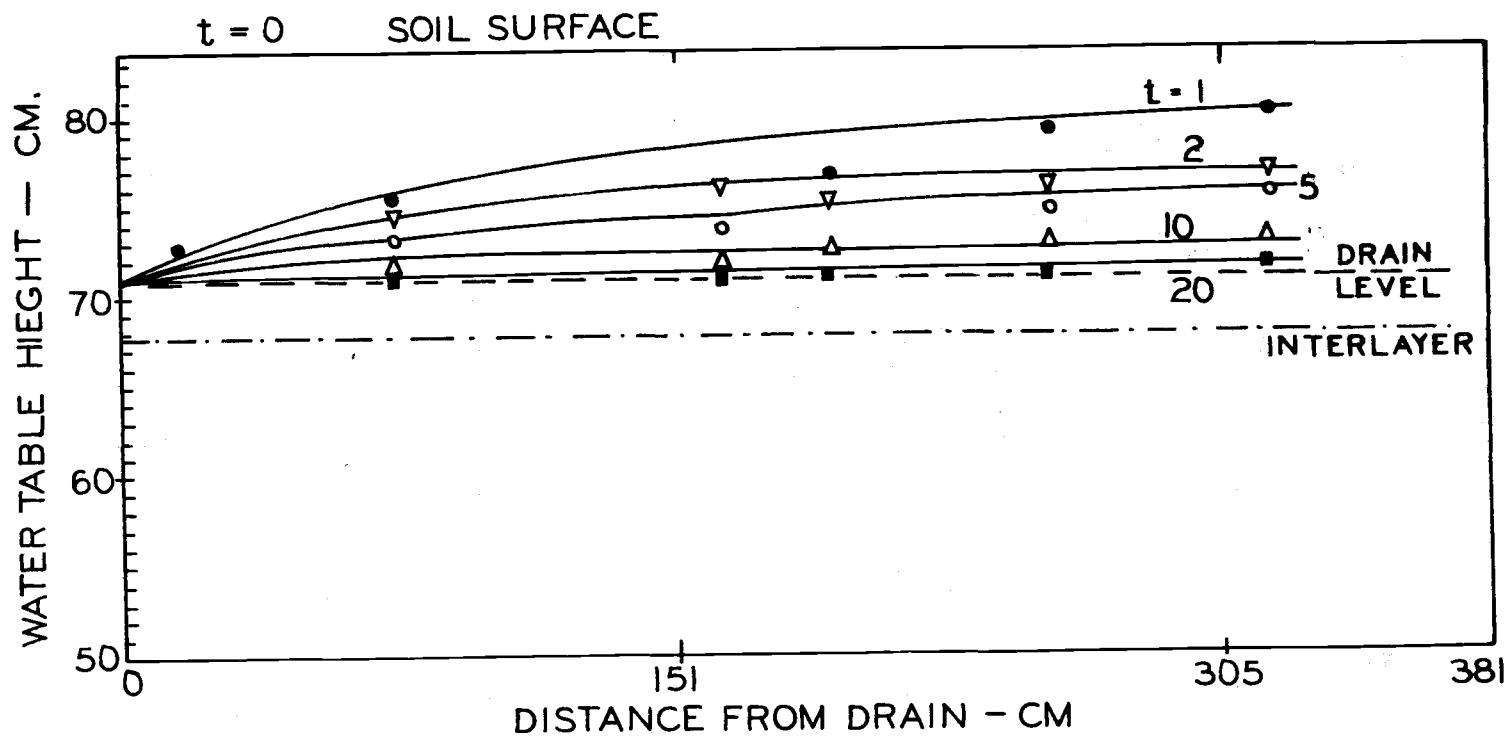
Figure V-14. Water table profiles at various times (Experiment 4).

is located below the boundary.

In both groups of figures, it is seen that the water table never

actually got to the level of the drain even after a long time. It was

interesting to observe that when drainage had apparently ceased,

the water table was still above the drain. This situation can be attri-

buted to prevailing very low hydraulic gradients. Even though there

is still some water to be drained, hydraulic gradients are so small

that discharge cannot be easily detected. Table V-4 summarizes the

above mentioned observation. It should be mentioned that it will be

very erroneous to draw concrete conclusions based on the water table

heights in profiles alone. The capacity of a drainage system to

remove enough drainable water so as to create a healthy environment

for viable production of crops should be the major consideration. In

this case, water table analyses can only be used to supplement drained

volume analyses. Under these circumstances, it is very imperative

to examine the drainage system performance from the point of view

of amount of water removed or can be removed at various times.

In order to make sure that all the drained amount collected in

the reservoir actually drained from the media, ponding on the surface

was avoided as much as possible. However in one case for the

shallowest drains at 15.24 m spacing, there was an initial ponding

of an average depth of 0.8 cm. The ponded volume was estimated

to be 3097 ml. The total volume collected after drainage had ceased

was 4013 ml. Theoretically the sand layer in which the drains were

installed should not have drained at all. This is because the thick-

ness of the drainable depth and even the whole sand layer is less

than the bubbling pressure head of 17 cm. Since field conditions do

not always adhere strictly to theoretical propositions, some amount

of drainage could be expected. After accounting for the ponded

volume, it was found that the actual maximum amount that was

drained was only 916 ml. Assuming this drainable depth was

saturated (which is the case because of the ponding and also the

imposed conditions), this 916 ml is only 7.73% of the total pore volume

of 11,848 ml. This 7.73% is low enough to validate the proposition

that the sand should not drain at all for this case especially since the

depth of the drain is less than the bubbling pressure head of the sand

layer.

The maximum possible drainable volume for other subsequent

drain depths were estimated by graphically integrating the water

retention curves for the stratified profile of the model. The equation

used in these estimates was

$$Q_\infty = Q_0 + \sum_j \Delta S_j \Delta z_j A \phi_{e_j}$$

Q_0 - is the maximum possible drainable volume estimated for

the sand using the uppermost drains,

ΔS - is the difference in saturation by moving from z_1 to z_2,

Δz - is $z_2 - z_1$,

A - surface area of the media in the flume,

ϕ_e - is effective porosity, and

Q_∞ - is the maximum theoretical drained volume that may be

found in Table V-2.

The results of drainage volume discharged as a function of

time is shown in figure V-15.

In the case of the drains installed in the sand layer, there was

a rapid response of the water table but the drainage was negligible

or relatively small. An interesting feature was the fact that the Q

versus t plot indicated a very high drainage rate for this case even
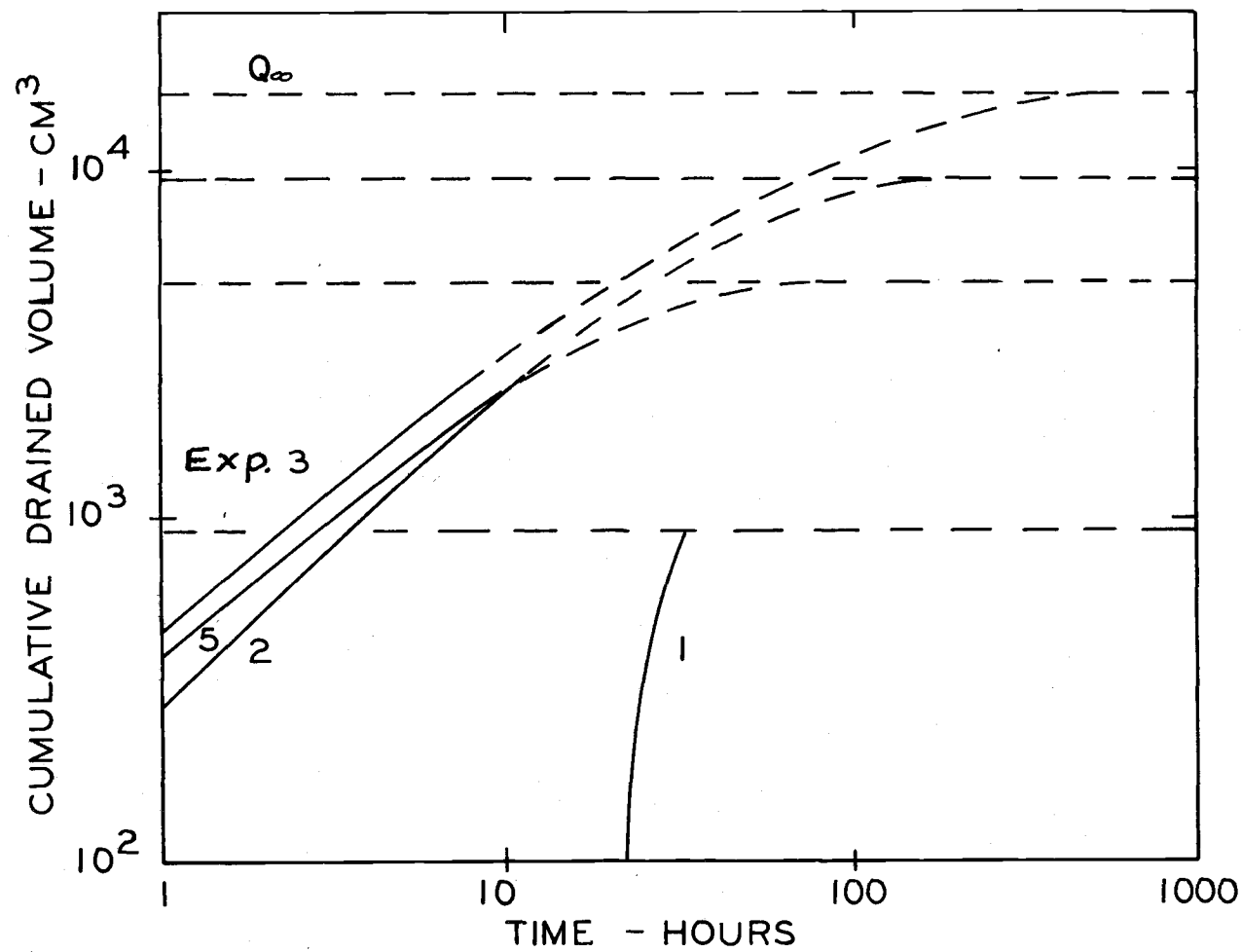
Figure V-15.   Cumulative drained volume as a function of time for drained
experiments.

though no significant amount of drainage was effected. This proves that drains in the sand layer is not an effective design for draining the sand. We can attribute this to relative magnitudes of the depth of the drains and the bubbling pressure head of the sand. Also the prevailing hydraulic gradients in such shallow profiles are too low to cause significant drainage.

For the drains in the clayey loam layer but closer to the interface, we also have a rapid water table response. Some degree of drainage of the sand layer was effected. The drainage rate was high but not as high as in the previous case. However this high drainage rate could not be sustained for a long time. Also notable was the fact that some degree of desaturation of the sand layer was achieved. This is illustrated in Figure V-16. The desaturation of the sand layer over a period of 10 hours was effected in areas close to the soil surface and within 240 cm from the drain. Therefore the importance of P_b/γ in relation to the depth placement of drains should be evident. With the drains at a depth of 43.2 cm

$$\left(\frac{P_b}{\gamma}\right)_s < 43.2 < \left(\frac{P_b}{\gamma}\right)_{cl} \, ,$$

At least we are bound to have some drainage from the sand.

Examining the case of the drains in the clayey loam layer at a depth of 73.7 cm, it is interesting to note that

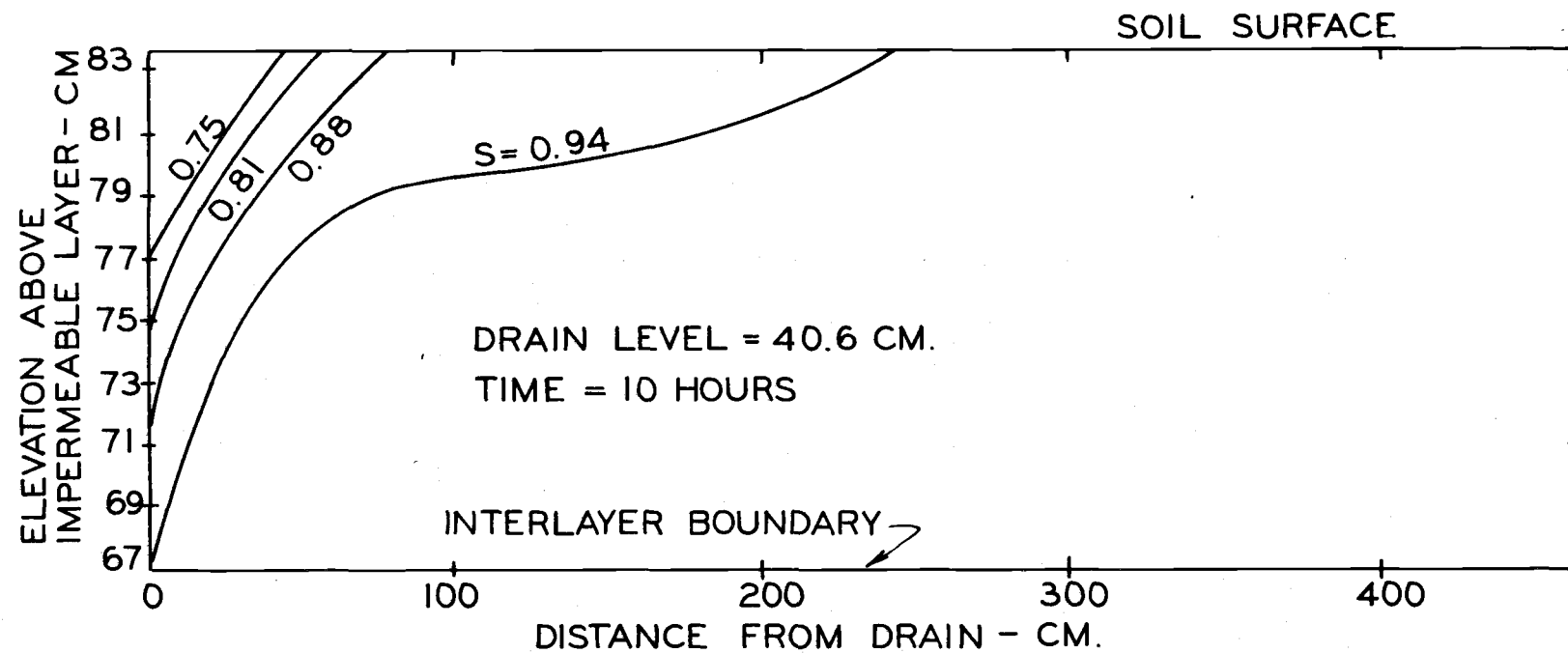Figure V-16.   Desaturation pattern of sand layer (Experiment 2) at t = 10 hours.
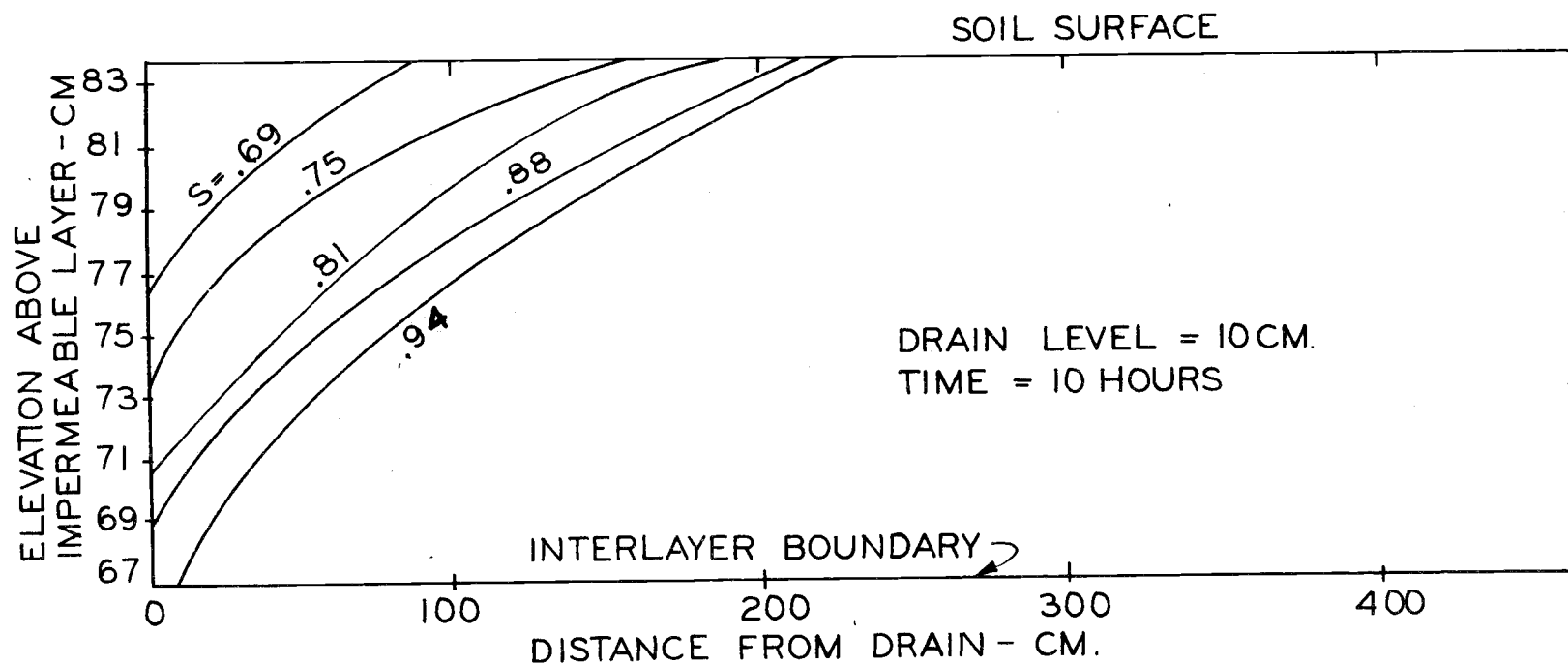
Figure V-17. Desaturation pattern of sand layer (Experiment 3) at t = 10 hours.

$$73.7 > \left(\frac{P_b}{\gamma}\right)_{cl} > \left(\frac{P_b}{\gamma}\right)_s .$$

Even though the drainage rate was lower than it was in any of the previous two experiments, on the average a higher drainage rate was sustained for a longer period of time than the two previous cases. For any given period of time, more water was drained from the sand than in any of the two cases already discussed. Greater desaturation of the sand layer was observed in the same 10 hour period as shown in figure V-17. The water table response was nearly the same as in the case where drains were at a shallow depth in the clayey loam. The relative performances of the various drainage systems are given in Tables V-3 through V-5.

Examination of table V-3 indicates that reducing the spacing from 15.2 meters to 7.6 meters improves the drainage performance. However the degree of importance of the reduction in spacing could not be measured because of lack of sufficient data.

Simultaneous examination of tables V-4 and V-5 reinforce the inadequacy of using the water table as an index of drainage system performance in shallow profiles. Table V-5 further proves that even though spacing may be a factor to be considered, the primary significant factor in the depth placement of the drains in relation to the P_b/γ of the soils used.

Table V-3. Times in hours for draining percentages of maximum drainage.

Percent Q_∞	Experiment				
	1	2	3	4*	5
25	24	10.5	16.5	--	4.1
50	27	27	54	--	10.5
60	29	38	80	--	15
75	30	62	140	--	25
90	32.5	110	270	--	42
100	34	270	640	--	80

*The transducer for Experiment 4 was accidentally unplugged and as such no discharge readings were obtained.

Table V-4. Mid-point water table drop expressed as a percent of maximum possible drained depth.

Time	Experiment				
	1	2	3	4	5
1	0	0	0	42.5	14.6
2	0	0	0	53.5	18.1
5	0	16.2	11.9	63.0	31.3
10	0	31.9	20.0	85.8	48.2
20	27.6	49.0	30.5	97.0	62.0

Table V-5. Percent of Q_∞ drained at various times.

Time	Experiment				
	1	2	3	4*	5
1	0	3.12	2.85	--	8.11
2	0	6.03	4.85	--	14.15
5	0	13.00	10.31	--	29.13
10	0	23.93	17.60	--	47.85
20	0	44.73	27.91	--	68.66

*The transducer for Experiment 4 was accidentally unplugged and as such no discharge readings were obtained.

VI. SUMMARY AND CONCLUSIONS

The findings and recommendations from the study will be presented in this chapter.

After a careful and length analysis of the results the following conclusions concerning drainage of shallow layered soils are drawn.

a) For shallow profiles, if the depth of the surface layer is equal to or less than P_b/γ of the layer, drainage cannot be effected by installing drains in this surface layer, even though the water table response is satisfactory.

b) Spacing of the drains is not an important factor when P_b/γ is greater than the drain depth and vice versa. Favorable water table response can be obtained from spacing but water table has no meaning in this case and has proved to be an erroneous index of estimating drainage performance.

c) Two important indices of drainage performance have been identified in this study. The first is the ratio of the depth of the drain below the interface to the whole thickness of the less permeable layer and the second is the relative values for P_b/γ and the depth placement of the drain. The greater the value of the former, the more effective is the system. The latter revealed that, the drainage performance is improved by installing drains at depths greater than either

of the P_b/γ values.

However how deep beyond the P_b/γ values was not analyzed. There is probably a limit for the depth placement or an optimum depth. It is recommended that these limit and optimum depth be quantified. In the light of this, as an extension to this study, the mathematical formulation presented in this text should be analyzed as an extension of this research project.

This study did not include the boundary conditions for the case of a homogeneous back filled trench above the drains. However, the conclusions mentioned in this work should be applicable for such cases.

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APPENDIX

NOTATION

$\bar{q}$	- flux or drainage rate	LT^{-1}
K	- hydraulic conductivity	LT^{-1}
$K_{1.0}$	- hydraulic conductivity at saturation	LT^{-1}
$k_{1.0}$	- permeability at saturation	L^{2}
∇	- vector differential operator	L^{-1}
ψ	- hydraulic head	L
P_b/γ	- bubbling pressure head	L
$P_c/\gamma, \tau$	- capillary pressure head	L
λ	- pore-size distribution index	dimensionless
γ	- specific weight of fluid	$ML^{-2}T^{-2}$
x	- horizontal coordinate	L
z	- vertical coordinate	L
$D(\theta)$	- soil moisture diffusivity function	$L^{2}T^{-1}$
θ	- volumetric water content	dimensionless
ϕ, ϕ_e	- porosity, effective porosity	dimensionless
S, S_r, S_e	- saturation, residual saturation, effective saturation	dimensionless
σ	- interfacial tension	MT^{-2}
$\beta(\psi)$	- soil-moisture capacity	$ML^{-2}T^{-2}$
δ	- drain size	L
μ	- dynamic viscosity	$ML^{3}T^{-3}$

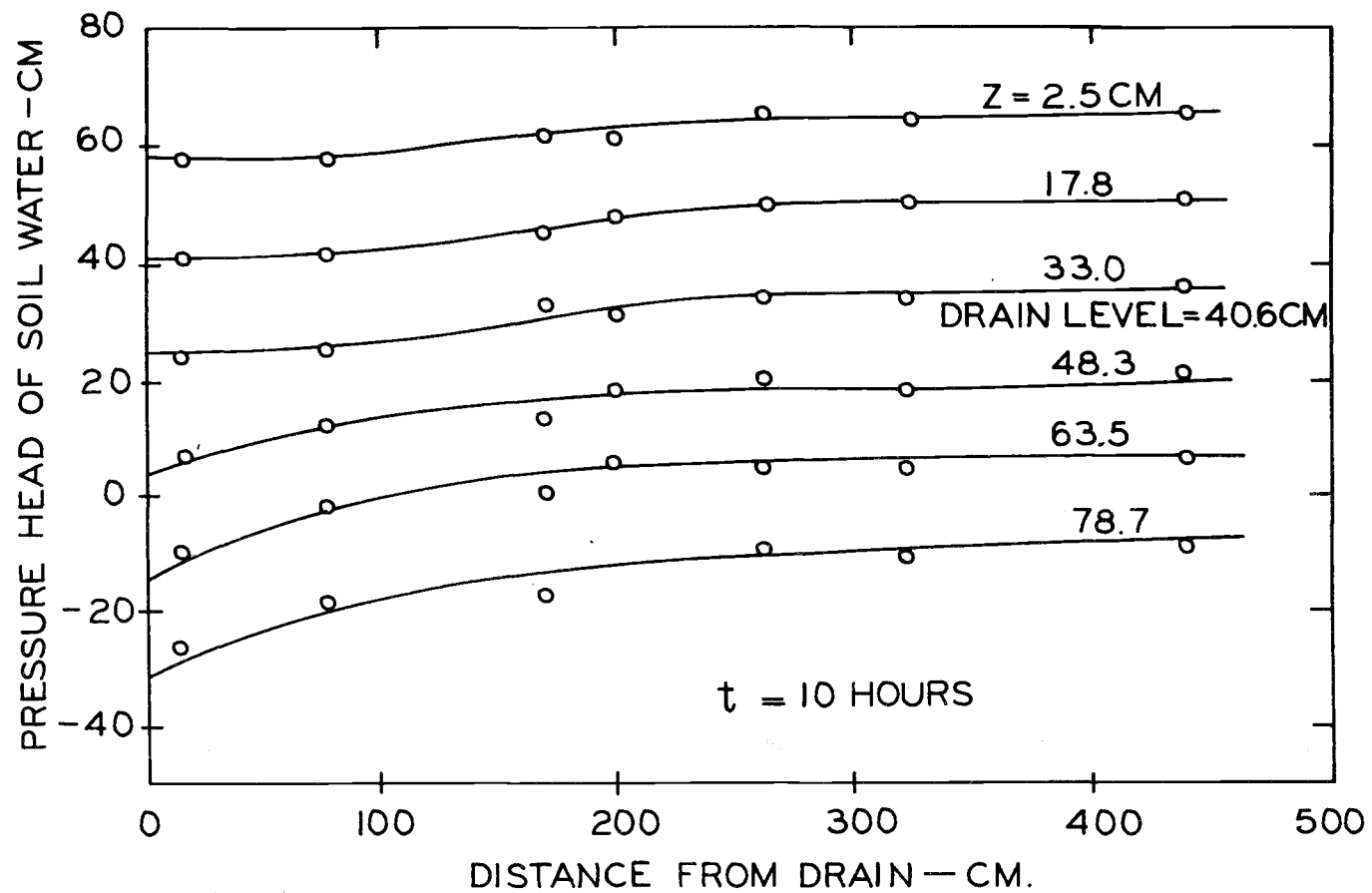|		
a	- depth of river bed sand above drain	L
b	- depth of clayey loam above drain	L
d	- depth of drain below surface	L
L	- half drain spacing	L
Q_∞	- maximum drainage volume	L^3
Q	- drainage volume	L^3

Figure A-1.   Pressure head variation at various horizontal levels for
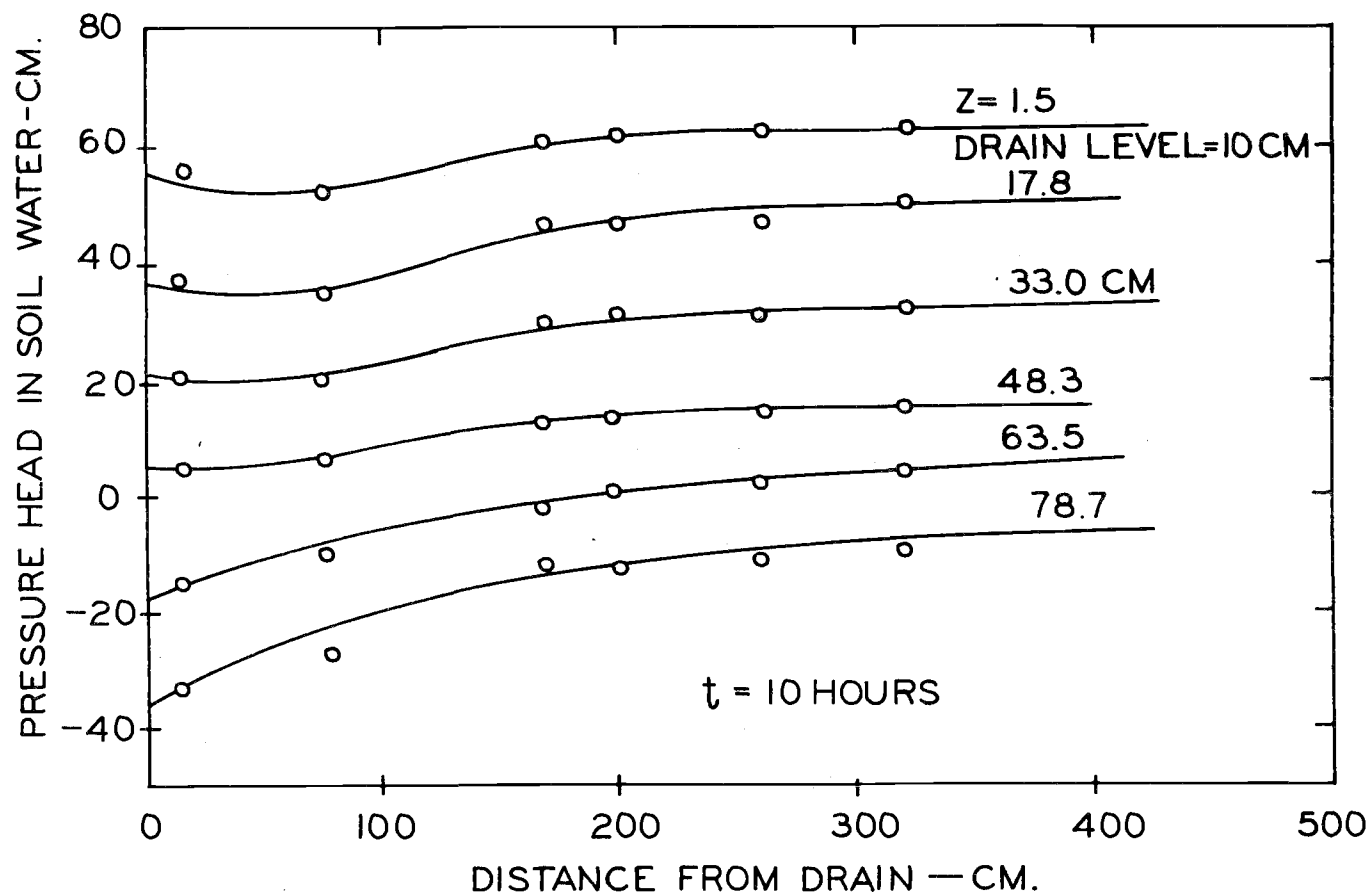t = 10 hours (Experiment 2).

Figure A-2.   Pressure head variation at various horizontal levels for
t = 10 hours (Experiment 3).