AN ABSTRACT OF THE THESIS OF

Jianqing Cui for the degree of Master of Science in Mechanical Engineering presented on December 9, 2014.

Title: Influence of Subset Size, Shape and Orientation on Texture Based Digital Image Correlation

Abstract approved: ________________________________________________

Brain K. Bay

Digital Image Correlation (DIC) is one of full-field strain measurement techniques developed in recent decades, which has been well-demonstrated to conduct the mechanical properties study of solid materials, by collecting data using digital cameras. X-ray tomography (CT) imaging helps the correlation techniques expanded to three dimensional investigation of materials, known as Digital Volume Correlation (DVC).

Adopting ideas from DIC, DVC is based on tracking the deformation of the material features within image volumes, by optimizing an objective function used to compare small subsets of image data from sequences of sample CT scan. However, speckling techniques generally used in DIC to create a distinguishable pattern for correlation are not applicable within an image volume of a material, so the natural texture of the materials becomes the correlation method relied on. Depending on the natural context, the DVC method cannot be adjusted in the sample images as DIC tuning speckle patterns, that natural texture does not always meet the requirements of digital correlation. This implies that
we have to adjust the DVC to fit the images[1], which leads the correlation algorithm encountering some challenges when the texture appears in oriented patterns.

In this thesis, a new approach is discussed to resolve the challenges by changing the subset both in target and reference images to impact the performance of the DVC and the optimization method applied, which refers to the feature of the sample texture. In order to show the influence of subset on the performance of DVC based on natural texture, these strategies of subset changes are used to generate the correlation result in the Region of Interest (ROI) with representative texture features in sample images. Different ROIs from two CT image volumes have been tested using an open source implement, which applies the objective function globally in the correlation region. By analyzing the results, a general relationship between subset adjusting and improvement of DVC performance on texture based samples is concluded. That is, a better performance of DVC can be achieved by adjusting subset to a shape similar with the feature of the natural texture in ROI.
Influence of Subset Size, Shape and Orientation on Texture Based Digital Image Correlation

by

Jianqing Cui

A THESIS

submitted to

Oregon State University

in partial fulfillment of
the requirements for the
degree of

Master of Science

Presented December 9, 2014
Commencement June 2015
Master of Science thesis of Jianqing Cui presented on December 9, 2014.

APPROVED:

________________________
Major Professor, representing Mechanical Engineering

________________________
Head of the School of Mechanical, Industrial and Manufacturing Engineering

________________________
Dean of the Graduate School

I understand that my thesis will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my thesis to any reader upon request.

________________________
Jianqing Cui, Author
ACKNOWLEDGEMENTS

Thanks for the help of Dr. Bay

Thanks for my parent’s supporting
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Abstract</td>
<td>1</td>
</tr>
<tr>
<td>2 Background</td>
<td>3</td>
</tr>
<tr>
<td>2.1 Digital Image Correlation and Digital Volume Correlation</td>
<td>3</td>
</tr>
<tr>
<td>2.1.1 Techniques used in Digital Image Correlation</td>
<td>4</td>
</tr>
<tr>
<td>2.1.2 Tracking Method for Correlation Method</td>
<td>8</td>
</tr>
<tr>
<td>2.1.3 Mechanism of Optimization Method</td>
<td>11</td>
</tr>
<tr>
<td>3 Method</td>
<td>17</td>
</tr>
<tr>
<td>3.1 Goal</td>
<td>17</td>
</tr>
<tr>
<td>3.2 Implement: a Plugin in ImageJ</td>
<td>18</td>
</tr>
<tr>
<td>3.3 Parameters Implemented</td>
<td>19</td>
</tr>
<tr>
<td>3.3.1 RepeatUnload Strategy and The Output Result</td>
<td>19</td>
</tr>
<tr>
<td>3.3.2 Objective Function</td>
<td>19</td>
</tr>
<tr>
<td>3.3.3 Data Collecting and Limit Searching Region</td>
<td>20</td>
</tr>
<tr>
<td>3.3.4 Strategies of the Subset Creation</td>
<td>21</td>
</tr>
<tr>
<td>4 Results and Discussion</td>
<td>28</td>
</tr>
<tr>
<td>4.1 Optimal 2D Correlation Result</td>
<td>28</td>
</tr>
<tr>
<td>4.1.1 Quadratic Fitting</td>
<td>29</td>
</tr>
<tr>
<td>4.1.2 Boxplot</td>
<td>32</td>
</tr>
<tr>
<td>4.2 Metal Texture</td>
<td>32</td>
</tr>
<tr>
<td>4.2.1 Single Spot</td>
<td>34</td>
</tr>
<tr>
<td>4.2.2 Edge</td>
<td>39</td>
</tr>
<tr>
<td>4.2.3 Discussion</td>
<td>55</td>
</tr>
<tr>
<td>4.3 Ceramic Texture</td>
<td>56</td>
</tr>
<tr>
<td>4.3.1 Light Strips</td>
<td>56</td>
</tr>
<tr>
<td>4.3.2 Other Three Set of Subsets</td>
<td>65</td>
</tr>
<tr>
<td>5 Conclusion and Future Work</td>
<td>76</td>
</tr>
<tr>
<td>5.1 Conclusion</td>
<td>76</td>
</tr>
<tr>
<td>5.2 Future Work</td>
<td>77</td>
</tr>
<tr>
<td>Bibliography</td>
<td>79</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Example of Digital Image Correlation with measurement points</td>
<td>5</td>
</tr>
<tr>
<td>2.2</td>
<td>General flow chart of DIC</td>
<td>7</td>
</tr>
<tr>
<td>2.3</td>
<td>Example of DIC on real material</td>
<td>9</td>
</tr>
<tr>
<td>2.4</td>
<td>Example of Coarse Searching Grid</td>
<td>12</td>
</tr>
<tr>
<td>2.5</td>
<td>Problems from Optimization Method</td>
<td>13</td>
</tr>
<tr>
<td>3.1</td>
<td>Implementation Parameter Window</td>
<td>17</td>
</tr>
<tr>
<td>3.2</td>
<td>Performing strategy in two dimensionals</td>
<td>21</td>
</tr>
<tr>
<td>3.3</td>
<td>Implemented Strategies</td>
<td>22</td>
</tr>
<tr>
<td>3.4</td>
<td>Example of Subsets</td>
<td>25</td>
</tr>
<tr>
<td>3.5</td>
<td>Example of Subset with Rotation</td>
<td>26</td>
</tr>
<tr>
<td>4.1</td>
<td>Optimal Result</td>
<td>30</td>
</tr>
<tr>
<td>4.2</td>
<td>The example of Quadratic Fitting</td>
<td>31</td>
</tr>
<tr>
<td>4.3</td>
<td>The Optimal Result Box plot</td>
<td>33</td>
</tr>
<tr>
<td>4.4</td>
<td>Objective Function Map of Single Spot</td>
<td>36</td>
</tr>
<tr>
<td>4.5</td>
<td>3D Image of Single Spot</td>
<td>37</td>
</tr>
<tr>
<td>4.6</td>
<td>Box Plot on Single Spot</td>
<td>38</td>
</tr>
<tr>
<td>4.7</td>
<td>Quadratic Fitting $R^2$ plots of Single Spot</td>
<td>39</td>
</tr>
<tr>
<td>4.8</td>
<td>Objective Function Map of Dark Edge I</td>
<td>41</td>
</tr>
<tr>
<td>4.9</td>
<td>3D Image of Dark Edge I</td>
<td>42</td>
</tr>
<tr>
<td>4.10</td>
<td>Boxplot on Dark Edge I</td>
<td>43</td>
</tr>
<tr>
<td>4.11</td>
<td>Quadratic Fitting $R^2$ plots of Dark Edge I</td>
<td>44</td>
</tr>
<tr>
<td>4.12</td>
<td>Objective Function Map of Dark Edge II</td>
<td>46</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4.13</td>
<td>3D Image of Dark Edge II</td>
<td>47</td>
</tr>
<tr>
<td>4.14</td>
<td>Boxplot on Dark Edge II</td>
<td>48</td>
</tr>
<tr>
<td>4.15</td>
<td>Quadratic Fitting $R^2$ plots of Dark Edge II</td>
<td>49</td>
</tr>
<tr>
<td>4.16</td>
<td>Objective Function Map of Sharp Corner</td>
<td>51</td>
</tr>
<tr>
<td>4.17</td>
<td>3D Image of Sharp Corner</td>
<td>52</td>
</tr>
<tr>
<td>4.18</td>
<td>Boxplot on Sharp Corner</td>
<td>53</td>
</tr>
<tr>
<td>4.19</td>
<td>Quadratic Fitting $R^2$ plots of Sharp Corner</td>
<td>54</td>
</tr>
<tr>
<td>4.20</td>
<td>Objective Function Map of Light Strips</td>
<td>58</td>
</tr>
<tr>
<td>4.21</td>
<td>Objective Function Map of Light Strips[continued]</td>
<td>59</td>
</tr>
<tr>
<td>4.22</td>
<td>3D Image of Light Strips</td>
<td>60</td>
</tr>
<tr>
<td>4.23</td>
<td>3D Fitting R-squared</td>
<td>61</td>
</tr>
<tr>
<td>4.24</td>
<td>Contour Plots of Light Strips</td>
<td>63</td>
</tr>
<tr>
<td>4.25</td>
<td>Box plots of Light Strips</td>
<td>64</td>
</tr>
<tr>
<td>4.26</td>
<td>Subset selections of Rotated Image samples</td>
<td>67</td>
</tr>
<tr>
<td>4.27</td>
<td>Compare between Rotated Images and Rotated Subset</td>
<td>68</td>
</tr>
<tr>
<td>4.28</td>
<td>3D Surface Plot of Stack 1</td>
<td>70</td>
</tr>
<tr>
<td>4.29</td>
<td>3D Surface Plot of Stack 2</td>
<td>71</td>
</tr>
<tr>
<td>4.30</td>
<td>3D Surface Plot of Stack 3</td>
<td>73</td>
</tr>
<tr>
<td>4.31</td>
<td>Quadratic Fitting $R^2$ plots of Three Subsets</td>
<td>74</td>
</tr>
<tr>
<td>4.32</td>
<td>Box plots of Three Subsets</td>
<td>75</td>
</tr>
</tbody>
</table>
Chapter 1: Abstract

Digital Image Correlation (DIC) is one of full-field strain measurement techniques developed in recent decades, which has been well-demonstrated to conduct the mechanical properties study of solid materials, by collecting data using digital cameras. X-ray tomography (CT) imaging helps the correlation techniques expanded to three dimensional investigation of materials, known as Digital Volume Correlation (DVC).

Adopting ideas from DIC, DVC is based on tracking the deformation of the material features within image volumes, by optimizing an objective function used to compare small subsets of image data from sequences of sample CT scan. However, speckling techniques generally used in DIC to create a distinguishable pattern for correlation are not applicable within an image volume of a material, so the natural texture of the materials becomes the correlation method relied on. Depending on the natural context, the DVC method cannot be adjusted in the sample images as DIC tuning speckle patterns, that natural texture does not always meet the requirements of digital correlation. This implies that we have to adjust the DVC to fit the images[1], which leads the correlation algorithm encountering some challenges when the texture appears in oriented patterns.

In this thesis, a new approach is discussed to resolve the challenges by changing the subset both in target and reference images to impact the performance of the DVC and the optimization method applied, which refers to the feature of the sample texture. In order to show the influence of subset on the performance of DVC based on natural texture, these strategies of subset changes are used to generate the correlation result in the Region of Interest (ROI) with representative texture features in sample images. Different ROIs
from two CT image volumes have been tested using an open source implement, which applies the objective function globally in the correlation region. By analyzing the results, a general relationship between subset adjusting and improvement of DVC performance on texture based samples is concluded. That is, a better performance of DVC can be achieved by adjusting subset to a shape similar with the feature of the natural texture in ROI.
Chapter 2: Background

Full-field displacement and strain measurement is an essential approach for studying the mechanical properties of solid materials. One of the techniques developed in recent decades is Digital Image and Volume Correlation. Digital Image Correlation (DIC) has been well demonstrated to quantify the properties of material, in both displacement and strain measurement. Traditional image samples are collected using digital cameras, which are able to gather images from planar samples or even revolved shape sample. Using X-ray tomography (CT) imaging, the correlation techniques has been expanded for three dimensional investigation of materials, through which the interior context of material is observed and analyzed. In order to acquire the material characteristics, the extension of Digital Image Correlation in three dimensional known as Digital Volume Correlation (DVC) has been developed. In this way, any material, which is able to present texture in volumetric image by tomography technique, can be studied inside and out by DVC, including many complex material subjected to mechanical or thermal loads[11], like cellular solids or synthetic materials[1] and composite materials.

2.1 Digital Image Correlation and Digital Volume Correlation

As the three dimensional extension of Digital Image Correlation (DIC), Digital Volume Correlation (DVC) inherits the general measurement techniques from DIC[1], which includes a subset strategy to acquire the data set to perform objective function, an objective function to evaluate the correlated relation between subparts of two locations in the de-
formed and reference image, and a search algorithm, such as coarse-fine search strategy, where an optimization method allow searching approach to the best result according to the objective function result. Sometime, a shape function will be introduced to map the function on the deformed image referring to reference image[11].

2.1.1 Techniques used in Digital Image Correlation

The fundamental principle of DIC is derived from the idea of comparing grayscale images of a sample captured before and after distortion, as reference and target images, respectively, based on some specific correlation criterion to get the information on displacements and strains of measurement points defined on the reference images[17]. Generally, there are several technique terms used frequently in DIC and DVC. The Figure.2.1 has been showed to illustrate these techniques.

- **Region of Interest**: Region of Interest (ROI) is defined the region of sample where we will perform measurement. In the Figure.2.1, the area with red dots is the ROI for this artificial sample.

- **Full-field Measurement**: In mechanical measurements, the full-field measurement indicates the Region of Interest (ROI) in analyzing throughout the whole sample. In the Figure.2.1, if the red dots are distributed into the whole sample region, it will be considered as a full-field measurement.

- **Subset**: In Figure.2.1 these red dots are representing the measurement points. The measurement points are used to track the subpart of ROI during measurement, which are defined by the software. The correlation software generates a subregion centered as each measurement point, also called subset. In the unloading image,
Figure 2.1: Example of Digital Image Correlation with measurement points
the subset will be the reference subset such as the Subset A in Figure.2.1 for measurement point A. The subset is compared with the subsets formed around the searching points in deformed images limited region, denoted as target subset. All the searching points is defined within a limited region shown as the yellow frame in Figure.2.1. Because of the limited deformation, the limited search region usually is defined referring to a nominal point (the cross point within yellow frame in Figure.2.1). The nominal point is calculated based on the location of reference subset.

The software extracts the subset data from each image unit in the subset image to track the location of measurement points, where the image unit is pixel for binary image. Every pixel has different intensity value for showing the different color of image by graphic software.

- Optimization method: The optimization method allows the DIC and DVC to save time from global search, by picking a sequence of measurement point to extract the subset data. Global Search means the program selects new measurement points in iterations by moving the subset from one pixel to next pixel continuously. Both of searching approaches assist the software decide the searching point for each measurement point. When the image gets larger, so does the subset size. The time costs of the search is getting huge due to the need of covering the growing set of measurement, which will be even exaggerated when the size of the data set grows from square of subset edge size in two dimensions to cubic of subset edge size in three dimensions.

There are various types of optimization methods available, but the challenges are always introduced by the unexpected texture, especially the oriented natural pat-
terns of material. We would like to address this part more specifically in the next section.

- Objective function: The objective function is designed to quantifying the correlation between two different data sets. It aims at returning a unique result for each measurement location, and present the properties of the data set. The result usually interprets how close the intensity data of reference and target subsets data are that the optimization method looks for the maximum of objective function result, or how different the two subsets that the minimum of objective function results will be searched by optimization method.

Using the techniques above, a general algorithm could be applied in DIC and DVC[13]. The flow chart Figure 2.2 also shows the general structure of the DIC.

1. Generating digital image samples in loaded and unloaded states.
2. Applying correlation procedure, the displacement field is measured by correlating the measurement points from unload to loaded status as Figure 2.2. The optimization method will be performed in taking the new measurement points, if the requirements are not full-filled.

Figure 2.2: General flow chart of DIC
3. Based on the displacement field, the strain tensor field can be calculated through gradient techniques.

Based on this structure, the correlation is performed by software. However, limits of the method still exists due to the constrain of material image and the mathematical optimizing. Thus, there are two part of the correlation need to explains more.

2.1.2 Tracking Method for Correlation Method

As introduced before, the DIC and DVC method usually rely on a set of measurement points on reference image ROI to track down the parts of materials. The set of measurement points are distributed through whole sample surface in full-field measurement, that each point represents a subset to track a part of sample. Figure 2.3a shows an example of the measurement points on real sample image, as the red dots, which are generated by correlation software virtually.

Each subset traditionally is generated by the software as a square crop of sample image centered as the measurement point. Here is an example of subset generated in the software for one measurement point, as Figure 2.3b. Using the subset, the surface image information is used to track the point locations during steps of the sample deforming.

Tracking the measurement point is making use of the pixel value of subset image unit, as intensity value, that objective function generate a unique scalar score representing this point, known as the correlation coefficient equation.

Considering the measurement points moving in a certain region, the software basically perform the searching within a limited range for each measurement point. For instance, if the first measurement point on the left top corner of sample is being searched, a limited
region will be defined in software execution as the yellow frame showed in Figure.2.3a.

![Example of Measurement points on DIC real material](image1)

(a) Example of Measurement points on DIC real material

![Example of Subset on DIC real material](image2)

(b) Example of Subset on DIC real material

Figure 2.3: Example of DIC on real material

Similarly, we visualized the full-field tracking points as the set of red dots in Figure.2.1, where the whole dark border frame represent the reference image sample. Another black frame represents the target image sample with load deformation. Assuming the correlation is conducting the tracking of the Subset A, that the subset is generated centering one measurement point in Figure.2.1, as Measurement Point A. Then the software conducts the searching in a limited region in target image, as the yellow border frame showing, using either global search or optimization searching. The limited search region is defined usually referring to a nominal point, related to the measurement point location in reference image. A set of searching point will be generated in searching, depends on different searching strategies. For example, the global search defines each pixel in limited search region as one search point, while the search points will be decided by
optimization method step by step, using a sequence of calculation. In the Figure.2.1, we plotted some representative searching points for the global search, where each step a target subset is extracted regarding to one search point to perform correlation calculation. The searching strategy will select the best objective function result from these searching point, either maximum or minimum from the result. The searching of software depends on the uniqueness of the subset image for each point, which can be interpreted to a distinguishable correlation objective function result. To be able to locate right location of measurement point, a good surface texture need to exist.

Basically, a set of fairly distributed, circle-like texture pattern with similar size can satisfy the requirements of correlation objective function, that artificial speckle patterns [7] is one kind of texture used, and sometimes the natural texture of material is good enough for correlation method, such as the sample of foam material. [3] Since DIC is only able to track the surface deformation of the material, the quality of the texture is able to be controlled by using spraying or digitally printing speckles on the sample surface.

On the other hand, DVC method keeps the essential idea about correlating set of pixel values to identify measurement points in sequences of images, but requires the set of measurement points throughout the whole volume of image, that the subset becomes a cubic image volume from a two dimensional area. X-Ray tomography images reflect the pixel intensity of material texture based on the density of material. Hence, the sample texture are not always perfect as the speckles patterns on two dimensional sample, which is also harder control the quality because of interior context involved. This implies that we have to adjust the DVC to fit the images,[1] which leads the correlation algorithm encountering some challenges when the texture appears in oriented patterns.
2.1.3 Mechanism of Optimization Method

The optimization method is applied in digital image and volume correlation, to enhance the efficiency of searching as opposed to using a global search. The efficiency and accuracy of the optimization function is essential to DIC and DVC, which sometimes varies based on different object function and image sample. [11] This displacement measurement is then used as an initial guess within an optimization algorithm, utilizing additional DOF for measuring sub-voxel displacements.

The correlation algorithm generally operates first by undertaking a coarse (integer voxels) search for subsets within the deformed image using only translational degree of freedom (DOF).[13] Most of the DVC and DIC methods are designed to perform these two runs of searching for each reference subset in a limited searching region. The coarse search helps save the optimization calculation cost, before approaching the minimal area enough.

1. Coarse Search: The software will generate a coarse searching grid, and calculate the objective function on each grid point. In this step, the software can select a starting point of the optimization method from these grid points, according to the objective function result on each point. The Figure.2.4 is the example of coarse searching grid on a global objective function map, where the grid point will get the value as the map result. The software usually pick the minimum point as the starting point. A similar example could be seen in Figure.2.1, where the grid points are the red dots in yellow limited region.

2. Fine Search: Using the starting point from coarse search result, the searching strategy of correlation is switched to optimization method, where the software adjust the searching direction and step size, based on the result of optimization
algorithm. The Figure 2.5 shows some example about fine search, where the black arrow line in Figure 2.5 represent the path of searching and the red crosses represent the possible starting points selected by coarse search.

The termination of the optimization method is decided by the convergent criterion, which is a tolerance relatively changed in each searching step. This is because the optimization method is designed to approach the minimum as much as possible, at the same time, balancing the searching speed by the limited iteration. The general optimization method structure will be introduced later.

As a well-developed mathematical technique, there are a large number of optimization methods available. There have been several methods widely used in DIC and DVC, such as the steepest descent (Zauel et al., 2006; Hardisty and Whyne, 2009), the Levenberg-Marquardt variation of the Gauss-Newton method (Bay et al., 1999) or the Broyden-Fletcher-Goldfarb-Shanno (BFGS) method (Smith et al., 2002; Verhulp et al., 2006)[11].
No matter which one, the basic idea of optimization remains same, sort from a initial guess or a starting point and assess the minimizing conditions on the candidate points, then set new assessment point. In the DIC method, the data assessed is the results of an objective function from a target subset from a deformed image and reference subset in the original image. The general structure of the optimization method looks like as below iteratively.[4]

1. Start the searching at the initial point $x^0$, and update the point as $x^k$ each iteration.

2. Assess the optimization conditions, as the convergent criterion. This part will be different respect to the method of optimization.

   - Some methods try to find the searching direction by assessing the optimization conditions, and picking the best choice from candidate points, like linear searching technique.
• Some of methods try to solve the assessment functions to decide the next step point, such as Newton method.

3. Update the point to $x^{k+1}$ if the condition holds.

As mentioned, the searching efficiency and accuracy of optimization on the same objective function could be influenced by the feature of the minimal areas[12] which leads the search in the wrong direction. It is hard to eliminate this deficiency completely because it is derived by the nature of the optimization. Generally, we can sort the deficiency into two major reason causing low efficiency or even failure of searching minimizer by optimization method. Also these two reasons might get exaggerated by the patterns of natural texture, especially features with similar orientation. Our subset selection strategies here will be mainly designed based on solving these following two problems.

1. Local Minimal Area nearby the Global Minimum Area.

The nature of optimization decides this method only looking for local minimizer or maximizer rather than global one, and some of the methods are aiming for the stationary points even not the minimal points.[4]. Since we are not only looking for the local minimum but also global minimum, sometimes the optimization method takes large cost to decided the real global one in whole region.

If the 2D result surface has two or more concave areas, the optimization method might stop at a local minimum (one of concave area) not the global minimal point. Even though the method could finally find the global minimum of line, the method still might be costly in the local minimal slope nearby the global minimum range. Same result will happen in the surface searching region into three dimensional.
More specific, the optimization method usually “guess” the next step search direction according to the current result and its deviation, that the searching will be decided towards to the direction where a smaller result most likely to appear.

Here is an example, Figure 2.5a, which shows the contour plot of a curvature surface, where the searching method suppose ending at the global minimum point, as the center of the figure. As we can see, depends on the different starting points of search, there are different possible destinations of the searching where two of them are the center of the local minimal areas, pointed by the arrows.

This situation mostly happens by picking the wrong starting point of finer step searching. A coarse search is responsible for picking a start point to conduct more complicated and finer searching. A coarse search grid is generated in the whole region of interest as the Figure 2.4 shown, where each grid point will be one search grid.

If one of the coarse searching points get to close to a local minimal area center, the result of this searching point might be more satisfying than rest of results. Thus the fine searching of correlation, shown as the Figure 2.5, will be performed in the limited region searching near the local minimal area, where the software is possibly taking the local minimum point as the best result.

2. Anisotropy Minimal Area.

Isotropic area means the magnitude of the area is growing radially symmetrically in every direction. Lacking an isotropic pattern, the shape of the result is not smoothly reducing to the minimum, where the stationary points appears.[4].

In this kind of objective function, the global minimum has the concave shape
that the similar small result points laying in one area near to the minimum one. Even though these similar results are much larger than the global minimum, the small difference between them still might satisfy the tolerance of the optimization search. The search will stop searching in the stationary area, since the difference between two step points is small enough within the tolerance defined by the method. The example is shown in the Figure.2.5b, where you can see multiple anisotropic areas, with shallow curved valley shapes. The large effort might be taken by the optimization method as the zigzag arrow line to finally reach right location, like the arrow line marked as $f$ in Figure.2.5b. Or even worse, like another arrow, the search just stop at the middle of the valley area.
Chapter 3: Method

3.1 Goal

The idea proposed in my thesis is adjusting the subset to influence the correlation performance positively, which is quite new for both DVC and DIC, especially changing the orientation and shape of the subset. The common software of DIC and DVC is designed using same square subset during the execution of all measurement points. We proposed that the subset shape orientation and size is adapted based on the feature of the reference subset. For demonstrating, we need to develop our own tool to perform the correlation using different types of the subset, which is able to generate the whole search region map of objective function result to assess the influence of subset adapting strategy. We evaluate the performance by examining the quality of these objective function map with an artificial optimal result. Quantitative methods are also used to explore a reliable measurement for the assessment.
3.2 Implement: a Plugin in ImageJ

In this work, the DVC method is implemented using the software ImageJ. ImageJ is an open source software coded in Java, which is a powerful tool for image processing and analysis. The software can extract the binary value of each pixel in an image, and a 3D image volume can be processed as slices of images.

We developed an ImageJ Digital Volume Correlation Plugin, based on the previous DVC algorithm description. The plugin is able to perform the correlation algorithm between the reference image stack and the target stack volume. It allows us selecting specific reference subset to correlate with specific target image in execution, with a set of choices about correlation parameters such as subset strategies and searching region. The Figure 3.1 shows the window of correlation parameters, which includes changing the subset shape, size and orientation, or changing the searching region size.

The plugin is implemented as global search rather than using optimization. In this way a full search region correlation result map can be generated, referring to each subset sample we selected. The plugin will conduct global search in a limited search region. This means the implementation will produce an objective function result map of single reference subset, where each pixel (point) in the map represents one result of objective function for a target subset from one searching point on the limited search region.

Based on the ImageJ processing mechanism, the plugin is able to implement the DVC algorithm as 2D DIC algorithm, so the subset data set is assessed slice by slice with the reference data set, and the correlation input is calculated pixel by pixel. In other words, when the number of slice in subset is one, the correlation will be similar as digital image correlation in two-dimensional.
3.3 Parameters Implemented

3.3.1 Repeat Unload Strategy and The Output Result

Since the subset adjusting is a fresh way to enhance the performance of correlation, as mentioned before we designed our implementation to generate the whole region objective function of each single subset selected. Using the example shown before Figure 2.1, we generate a result map for one measurement point in a limit search region as the yellow frame, where the searching points as the red dots are assigned as all pixels in yellow frame.

Meanwhile, we decided to use same image volume as reference image and the target image volume, so the minimum point can be precisely found at the center of the result map, as the repeat unload strategy.

3.3.2 Objective Function

The Normalized Sum of Squared Difference equation (NCCC)[1][13] is used to calculate the objective function in the thesis which is a commonly used image correlation matching formula. This formula is insensitive to the scale change of the intensity of the target subset, where the sum of squared difference is equivalent to the cross correlation coefficient, by minimizing the difference[9]. In our thesis, the plugin takes a measurement point for any target subset such as at pixel $S$, where $T$ is a target cube pixel value array that can be extracted and calculated with the template cube pixel array as $R$:

$$
\frac{\sum_{i=1}^{n}(T[i] - R[i])^2}{\sqrt{(\sum_{i=1}^{n} R[i]^2)(\sum_{i=1}^{n} T[i]^2)}}
$$
As we can see the scale of the objective function result is 0 to 1. If the two subsets are same as \( T = R \), we can get zero as the best result. In other words, using this formula the optimization method looks for the minimum in the region as the matching location.

### 3.3.3 Data Collecting and Limit Searching Region

The Region of Interest (ROI) is defined from the manual selection area on a slice of the reference image. Then the program will create the subset cube, automatically starting from the slice selected manually as middle of the subset volume. As a goal of the thesis, the subset cube will be re-defined by the program, based on the strategies of subset adjusting. In this thesis we use the same image volume to perform the correlation, so that the best matching is known, as the measurement point of interested. Hence, the global minimum limited of objective calculation, is the best match spot, and should be found within a Limit Searching Region (LSR). The nominal point of searching region in this work will be the same location of the reference subset center.

Meanwhile, the subset size usually is defined to be relatively large as length of shape due to the conventional experience that the large subset is helpful when identifying the measurement points in digital correlation either 2D or 3D.[15][10][7][14] This helps include more pixel or voxel data and also helps to enhance or reconstruct the digital images[6]. Considering the time cost caused by large scale calculation, the LSR seems suitable in our thesis. The searching region is defined in the target image, in the same location as the center of the subset cube in reference cube. The program could search \( 1 \times 1 \) and \( 2 \times 2 \), \( 3 \times 3 \) or \( 5 \times 5 \) of the subset size, in which the global search will be performed.
3.3.4 Strategies of the Subset Creation

Generally, DVC researchers are only able to change either the DVC mathematical assessment criteria or the subset data as correlation input rather than the sample quality. The latter approach has been demonstrated in some tests, where spatial resolution and subset size are the factors mainly interested.

As the thesis design, we would like to focus on adapting the subset without altering the quality of the image volume, where resolution of image has been shown to be able to affect result of changing subset size\cite{8}. The major goal of the thesis is developing and discussing the different strategies of creating proper subset of same region of interest, which are expected to improve the objective function from the perspective of optimization method. We categorized three types of subset strategies as in Figure.3.3: size of subset, shape of subset and orientation of subset. We will use the plugin to test each of these strategies on different features and texture. In this thesis, the demonstration is decided starting with changing the subset in two dimensional, rather than through 3D. For example, the elongating subset will be performed as the Figure.3.2.

![Figure 3.2: Performing strategy in two dimensionals](image)

1. Based on **Size**

As mentioned, previous researches have shown that the size of the subset has impacted on DIC and DVC performance. Most DIC researches concluded that
Figure 3.3: Implemented Strategies

generally the larger the subset size is, the smaller and sharper decreasing standard deviations in the measurements of displacement[10][16][14], as more accurate the measurement are[7]. For three dimensional researches, most tests conclude that large subsets give more voxel data and the correlation is therefore performed more accurately with less noise[8][12], while some shows the opposite that smaller subsets capture the solution more accurately but with higher noise[5].

We will start with the subset size whether enlarging the subset size would likely help to improve the 3D objective function density map. Enlarging the size of the subset in our project is defined as increasing the area of the subset in each slice of both of reference stack of interest and search target stack as many times of original as requested, that the number of pixels included in one subset will changed with the size change of the subset.

To avoid interpolating sub-pixel within extracting data during the changing subset, which introduce more cost of computation. Hence, when we enlarge the subset shape, or change the shape like elongation of the subset, we have to round up the
size of subset edges from the equation we used to the nearest integer.

For instance, if an enlargement of subset strategy to twice of original selection used, where original selection is a $w \times w$ pixels$^2$ square region, then the enlarged subset width of new subset will be $\sqrt{2} \times w^2 = \sqrt{2} \times w$ pixels$^2$. By generating the new subset, new side lengths need to be calculated without introducing extra pixel and avoid partial pixel which is meaningless. Thus we define one side of subset has the length as the ceil of $\sqrt{2} \times w$ noted as $\lceil \sqrt{2} \times w \rceil$ and the other side length is define to $\lfloor \frac{2 \times w^2}{\sqrt{2} \times w} \rfloor$ pixels to maintain the square shape as possible. Similarly if we use elongation strategy as discussed latter, we need change the square shape only without changing the size, we use the floor of the number to round the side size to an integer value and keep pixel number same.

2. Based on **Shape**

Again, one of the disadvantages of increasing the subset size in 3D X-Ray images is that the large area of image data introduces unwanted texture patterns, which is not helpful in DVC. Meanwhile, increasing the size of single subset means the loss of the independent displacement measurement points in total, that a full-field strain measurement is interested in.[8][14]. What’s more, the time cost of single measurement point is increased by the enlargement of subset. Hence exploring the relationship between the shape of subset and the result of material analysis is necessary as a fresh perspective. We would like to know the changes with respect to different shapes of subsets on the same samples, compared to the default square shape of the subset, in all other projects [2][16]. We also have an original expectation that the result will be best when the subset shape is similar with the texture pattern.
When defining the shape of the subset, we developed the most feasible change to a rectangular subset, which is the elongation of the original subset. In order to eliminate the affect of size changing, we fixed the subset size as the original manual selection. The specific implementation is elongating the subset, into a rectangle shape in 2D, which has the same size and center as the original selection.

3. Based on **Orientation of the Subset**

Since the elongation of subset has been included in this demonstration, the orientation of the subset especially elongated also need to be discussed. Hence we introduce the elongated subset with rotation as plan.

The demonstration of this strategy is performed by rotating the subset counterclockwise or clockwise in certain degree step by step. In all the demonstration of this thesis, we only use one rotation direction to avoid confusion. The rotation about the subset center \(C(Cx, Cy)\) is completed by performing the rotation matrix, that for each point of the subset, as \(P(x, y)\), a new location \((x', y')\) will be found as:

\[
x' = x \cdot \cos(\theta) - y \cdot \sin(\theta) + Cx
\]
\[
y' = x \cdot \sin(\theta) + y \cdot \cos(\theta) + Cy
\]

3.3.4.1 The Strategies Applied and the Abbreviations

Before presenting the results, a brief introduction of the strategies implemented will be presented, with the abbreviation of strategy names. First of all, in Figure 3.4, the general examples of the subsets are shown where only one orientation of elongation is included,
since the other orientation strategies are considered as rotated version. For example, the horizontally elongation subset is considered as the 90 degree rotation of vertically elongated subset. The Figure.3.5 presents the sample of subset rotated at a 45 degree angle, where the original subset could be adjusted into several types as shown. The most simple one is rotated directly from square subset. Or, the subset can be rotated after elongation. The rotation direction is also can be uniformed, that rotating Elongated 3 times counter clock wise 45 degree can also be rotated subset clock wise 135 as Figure.3.5 shown.

![Figure 3.4: Example of Subsets](image)

- Original Subset **SQ1**: A square subset of the same size as the original subset size is marked as the **Original** in the Figure.3.4a.

- Enlargement of Subset **SQ2**: An enlarged square template, twice size of original selection, centered coincident to the original selection as **2 times Square** in Figure.3.4a. Similarly, we have the **SQ3** with three times size of original selection showing as **3 times Square** in Figure.3.4a.
- Elongation of Subset **RY2**: An elongated subset, with overall size equal to the original selection and one edge twice the size length of original selection side marked as **2 Rect. Y** in Figure 3.4b, such that the elongation keeps the center of selection as the geometric center. Similar we also explore three times of original selection on the y axis orientation, with abbreviated name **RY3**, shown in Figure 3.4b as **3 Rect. Y**.

- Rotation of Original Subset **SQ Ro L deg**: A certain degree clockwise rotated square template, with same size of the original selection. For example **SQ Ro L 45** is the result square subset rotated clockwise 45 degrees as Figure 3.5.

- Rotation of Elongated Subset **RY2 Ro L deg**: A certain degree clockwise rotated elongated rectangle template, with same overall size of original selection and two times size of length of original selection side along the y-axis, such that the elongation keeps the center of selection as the geometric center, such that **RY2 Ro L 45**
is the elongated subset rotated 45 degrees in Figure.3.5 or rotated count clock wise
with three times elongation as **RY3 Ro R 45** in Figure.3.5. Meanwhile counter-
clockwise rotation is available, so is three time elongation. Usually we stick to one
wise rotation in one set of sample by calling **RY3 Ro R 45** as **RY3 RoL135**.

For using more simply, we keep all rotation in clock wise in the testing, that the
abbreviation of rotation will be shorter such as **RY2Ro45** for Figure.3.5
Chapter 4: Results and Discussion

Three sets of objective function result maps are generated in this session, that the maps are represent the full limited search region result respect to the reference subset. We will compared with our optimal result, in both eye examine and quantitative checking.

4.1 Optimal 2D Correlation Result

As introducing, the optimization method require single isotropic concave shape area in searching region, without large scale of stationary area near around the minimum point. A parabolic surface will be considered as an ideal objective function result surface. In following table of figures as Figure 4.1, we would like to exhibit the analysis of a simple parabolic surface, which will give us a general picture about the ideal result.

A simple version of the quadratic equation has been used: \( Z = X^2 + Y^2 \). In Figure 4.1a, the surface has plotted in ImageJ, where we can see isotropic change of the surface slope in every direction from minimum point. Each pixel or point of the surface is considered as the objective function result of one searching point with a reference subset.

In Figure 4.1b, the similar color scheme is performed into 2D map. Later the same color scale will be used in all real results, where the darker color suggests the lower objective function value of searching point. Each pixel will refer to the result of one searching point on target image. Hence the whole image map has the same locations as the limited search region of the reference subset.
In Figure 4.1c, the contour lines of the optimal surface are uniformly centered as the global minimum. Each of line represent one level of result points, which can help us observing the direction and speed of the result lowering. For optimal result, the evenly spreading contour lines suggests the direction of result change are uniformly, that no matter which direction the optimization start, it will find the minimum location in similar speed.

4.1.1 Quadratic Fitting

In the work, we selected the quadratic fitting to evaluate the surface quantitatively. The reason of evaluating the quadratic fitting of the result is that the most of optimization method, especially the Newton methods, works efficiently on the data with quadratic shape. If the quadratic shape can match a parabolic equation, it will be the optimal solution, that no matter where the searching start, the effort of search is close, that the starting point will be no impact on searching. The best strategy should return a surface with single global minimum, such that the surface raise uniformly around the minimum point which matches a quadratic module[4].

We performed the quadratic fitting of result using Matlab, where the function fit is able to generate a best fitting equation for the set of data and the assessment of fitting. A quadratic fit type is defined as the following equation, where $p_{ij}, i, j \in \{0, 1, 2\}$:

$$f(x, y) = p_{00} + p_{10} \times x + p_{01} \times y + p_{20} \times x^2 + p_{11} \times x \times y + p_{02} \times y^2$$

The $R^2$ of fitting is used to assess the goodness of fit statistically, where a higher value of $R^2$ represents better fit goodness. Considering the result surface changing commonly,
Figure 4.1a: This 3D surface is generated from Matlab using a parabolic function $Z = X^2 + Y^2$, that the surface is plotted here by the ImageJ 3D plug-in using same color scale. Each surface point is considered as objective function of one searching point.

Figure 4.1b: The color scale represent the magnitude of the objective function, where the X and Y pixel axes are showing the pixels distance of each searching point from the global minimum.

Figure 4.1c: The contour lines are generated by Matlab code, the different color are indicating different levels of magnitude of objective function result. Similarly, the center of the plot is the global minimum, as the matching point of one measurement point searching.

Figure 4.1: Optimal Result
we would like to perform the fitting function with growing fitting area step by step. The fitting area starts from a small area, where the global minimum area can be covered, which generally is fitted to a quadratic equation. Then we enlarge the fitting area till cover the range of result. An simple illustration is shown as the Figure.4.2a, that the fitting area of the result will start from $A_1$ and finally grow to $A_4$ as the whole result region. In each step, the Matlab function will generate a set of quadratic fitting assessment with fitted equation. Then the $R^2$ generated can be plotted as the scatter chart as Figure.4.2b, that each scatter point is one step quadratic fitting $R^2$ value for one fitting area $A_k, k \in \{1, 2, 3, 4\}$. Usually we start the fitting from a certain size of area to get reliable quadratic fitting, that is why the scatter chart does not start from the origin of the axis. As the example shown in Figures.4.2, most of result can generate a descending scatter line as the blue line in Figure.4.2b, while the optimal result is such perfect that the result line will be a flat line as the red one.
4.1.2 Boxplot

We also want to explore the statistic behavior of the objective function map, with the influence of the subset changes. The benefit of box plot is that the box plot emphasizes on the distribution of the subset rather than the specific values of the result and visualizes these distribution characteristics on one plot. Here is the optimal box plot with labels in Figure.4.3. The distribution of a box plot will follow the instruction shown in Figure.4.3. Matlab is able to conducting the box plot generation, where we can get the optimal box plot as showing in Figure.4.3. We can see that the box plot of this optimal example shows some unusual features. The majority of the optimal box plot in Figure.4.3 is really close to the lower whisker and the minimum of the data. At the same time, the length of the upper whisker is obviously longer than lower one. It suggest that the points of the isotropic surface changes slow near the minimum point while the speed of point magnitude growing increase with the distance from the minimum point.

4.2 Metal Texture

We will compare the objective function maps with the optimal result to evaluate the performance of strategies on different features of the metal crystal grains images in this section. We tried to test all different kinds of features. For each texture feature below, we used all kinds of subsets to find to match. The original reference subset size of this section sample is $12 \times 12$ pixels$^2$. In this section, we want to show that the changing subset does make difference in objective function maps. We started with covering different types of strategies, that 45 degrees and 90 degrees two angles have been used for rotation. Like the strategies, we named these reference subsets samples for simplifying description.
Figure 4.3: The Optimal Result Box plot
There is the list of reference subsets we used.

a). Single Spot: this subset has the feature close to a single speckle, which should yield a more consistent result as expect.

b). Edge: Unlike the spot texture, the edge patterns are more anisotropic which leads more problem during correlation.

- Dark Edge I: a subset with a horizontal edge across the subset where two sides of edge are different brightness.
- Dark Edge II: an edge across the subset with an angle around 45 degree.
- Sharp Corner: in the subset, we find a texture pattern has a sharp corner edge, like 90 degree.

The following is the comparison based on different strategies to these features.

4.2.1 Single Spot

Same as the optimal result, the objective function maps of the reference subset with single spot texture have been generated by implementation, where the center of the map is the global minimum as the location of measurement point. The color scale is marked for each result, where the magnitude of map is interpreted by different color. All the distances from the minimum point are measured using pixels.

All results of the reference subset are listed in the Figure 4.4. The comparison between results show that elongating the subset strategies does not enhance the map behavior positively. No matter elongating vertically as Figure 4.4f or horizontally as Figure 4.4g, the bright color area around the minimum point is only created along the
elongation direction which lead worse isotropic behavior of surface. Enlarging the subset, on the other hand, seems beneficial to increase bright color area encircling the global minimal area with dark color and the rest of area shows higher magnitude with lighter color in both Figure.4.4d and Figure.4.4c. In these two enlarging subset maps, larger isotropic area has been formed around minimum point, which is the goal of the result the subset strategies want to achieve.

Then the Figure.4.5 is used to provide a different way to present the data, where the magnitude change of objective function surface is shown more straightforward in three dimension. As we introduced before, a good surface should be close to a quadratic area. The best strategies by eyes examining for this feature above are selected. The 3D figures of the regular subset are presented in Figure.4.5a. Compared to the result from best subset strategy, as enlarging twice in Figure.4.5b and three times in Figure.4.5c. The enlarged subset clearly raises the overall magnitude of points around the global minimum and even creates a higher ridge which makes locating the global minimum easier through optimization method in Figure.4.5b. Meanwhile, we also can see the positive change from the original size subset in Figure.4.5a to enlarged subset in Figure.4.5. With the enlarging of the subset size, the more enlarging is performed, we can observe the overall surface has been raised to higher level, like Figure.4.5b with brighter color. Once the subset reach a size including enough information, a smoother surface of result, like Figure.4.5c. Not significantly, we can see the primal area with quadratic surface getting bigger in SQ3 in Figure.4.5c.

We also study our results using the box plot, as the Figure.4.6. Compared to the ideal result in Figure.4.3, we are expecting the majority of the result close to the minimum with shorter lower whisker. Looking at the first three strategies in Figure.4.6, we can observe the progress of the distribution, that the lower whiskers of the three square
Figure 4.4: Objective Function Map of Single Spot
Figure 4.5: 3D Image of Single Spot
subset strategies are getting close to the global minimum, and at same time the boxes are getting closer to the minimum too, like the ideal surface box plot. This gives the consistent conclusion of the color map, that enlarging the subset size helps the result surface to become a quadratic surface, which helps correlation. Checking the rest of box plots, we can observe that no boxes are lower than enlargement strategies. However, we can observe the lower whisker moving to global minimum in most strategies from SQ1, which can be considered as an improvement, despite the whisker length.

![Box Plot on Single Spot](image)

Figure 4.6: Box Plot on Single Spot

The quadratic fitting have also been tested like the optimal result in Figure.4.7, where the quadratic fitting result is not so good as expect. All $R^2$ lines have interesting behavior that the $R^2$ drops significantly when the fitting area grows to middle size, where in the objective function maps we can observe the bright color areas appearing in all figures. However, we still can see the enlargement of subset does raise the $R^2$ slightly over the rest of the $R^2$ data, shown as the green line in the Figure.4.7 for strategy SQ3,
as the best color map in both 2d Figure.4.4d and 3d Figure.4.5c

![Dark Spot Quadratic fitting](image)

Figure 4.7: Quadratic Fitting $R^2$ plots of Single Spot

4.2.2 Edge

4.2.2.1 Dark Edge I

Next, we turn our attention on some oriented patterns. The first edge pattern features horizontal orientation, where we expect more significant improvement on elongating subset. In Figure.4.8, the result maps suggest us that if we only change the shape of template to get close to the shape of texture pattern such as Figure.4.8f and Figure.4.8g, the result map will not reach to the best unless we get the correct orientation as Figure.4.8i. The
major improvement of RY2Ro90 in Figure.4.8h and RY3Ro90 in Figure.4.8i are shortening the local minimal area with light purple along the edge feature and reshaping the dark color area (primal minimal area) into a round shape. The rest of subset strategies cannot eliminate the anisotropy of global minimal area in dark color.

Meanwhile we also can observe the changes in figures with original subset strategy and the ones with better results in 3D as Figure.4.9. The 3D surfaces of elongating subset horizontally as RY2Ro90 in Figure.4.9b and RY3Ro90 in Figure.4.9c improve the result significantly. The 3D images show that the elongation subset with better angle can help the slope of the surface getting larger that the concave shape is more obvious both in Figure.4.9b and Figure.4.9c. With the elongation scale is getting large, the valley shape in original subset as Figure.4.9b is improved in Figure.4.9c, that we consider this is getting more close to our goal as isotropic shape. Clearly, we can find more consistent result with 2D maps in 3D Figure.4.8.

The box plots in Figure.4.10 exhibit the results in a statistic perspective regardless the shape of the surface. The left one is the original subset SQ1, as Figure.4.8b. Compared to it, all the strategies helps the result points gathering together. The enlarging subset box plots are showing long lower whiskers compared to other box plots. The elongating vertically subsets, especially RY3 with 2D map Figure.4.8g give the box plots shorter lower whiskers relatively. However, the best results by eyes in 2D maps, RY2Ro90 and RY3Ro90 do not have significant advantage in box plots, but the RY3Ro90 with 2D map Figure.4.8i box plot shows that population are highly accumulated. We have to say the improving pattern of this subset in box plot is not clear.

The quadratic fitting plots of selected result maps are showed in Figure.4.11, but the best strategy which is examined by eye RY3Ro90 as the Figure.4.8i seems not to have the most satisfying $R^2$ result. It might be explained using the 3D surface plotting in
Figure 4.8: Objective Function Map of Dark Edge I
Figure 4.9: 3D Image of Dark Edge I
4.2.2.2 Dark Edge II

Another dark edge feature has been tested for getting more details of rotation subset. This edge feature is close to 135 degree rotation (clock wise) rather than simply horizontally or vertically. In expectation, we believe that a simple elongated subset along the axises will not help the result but rotated into certain angle (clock wise 135 degrees). As expectation, we found the consistent result, that elongated subset with right orientation (clock wise 135 degrees) helps the primal minimal area (dark color global minimum area) reduce the size, shown in Figure.4.12k. Enlargement in Figure.4.12c, also influences the
Figure 4.11: Quadratic Fitting $R^2$ plots of Dark Edge I
map positively but not good enough as the elongation subset with a right angle, as the Figure.4.12k. Interestingly, the enlargement seems not always helpful that Figure.4.12d of SQ3 shows the anisotropy behavior of the surface exaggerated with longer curved primal minimal area with dark color. We think SQ3 includes too much other noise which compromise the enlargement improvement.

Looking at the figures of the raw result plotted in 3D as Figure4.13, we checked the strategies with the satisfying objective maps shown above, the RY2Ro135 as Figure.4.12k. The Figure.4.13c for RY2Ro135 3D surface and Figure.4.13b for SQ2 3D surface both show that changing the subset affects the size and the shape of minimum area positively. The overall surfaces in both figures change more sharply in the result region, compared to the original subset in Figure.4.13a. The shrinking of the minimum area makes the concave shape more isotropic, which as we discussed helps the optimization move to the minimum point effectively. The difference between the SQ2 as Figure.4.13b and RY2Ro135 as Figure.4.13c is the small ridge near the dark purple minimal area in SQ2, that the surface of RY2Ro135 change smoother around the minimal point than SQ2.

In box plots of this subset Figure.4.14, the largest median value belongs to the result of the strategy RY2Ro135, which is considered as the best result shown in objective map Figure.4.12. The high median of the data set indicates overall objective function result larger than the global minimum, so that the identification of minimum point can be easily made. Referring to the optimal result box plot in Figure.4.3, the strategies, RY2Ro135 and SQ2, are barely found with similar pattern except the median magnitude increasing. Both of the box plots show evenly whiskers while the SQ3 show slight improvement regardless of the 2D map Figure.4.12d, where the overall purple-dark color of SQ3 map might be the reason of the outstanding box plot. On the other hand, the RY3Ro90 and RY3 has similar shape of box plot, with shorter lower whisker. Tracing these two back
Figure 4.12: Objective Function Map of Dark Edge II
Figure 4.13: 3D Image of Dark Edge II

(a) SQ1

(b) SQ2

(c) RY2Ro135
to Figure.4.12, we will see generally light purple color in the Figure.4.12i, that the points of surface are more close to minimum.

![Figure 4.14: Boxplot on Dark Edge II](image)

The quadratic fitting is shown in Figure.4.15, where the orange color line is the best strategy examined by eyes in Figure.4.12. It has similar shape with the spot subset quadratic fitting in Figure.4.7 that the line has concave shape. The original subset as SQ1 with blue line and the RY2Ro90 $R^2$ line with light blue color have very high magnitude in large fitting area. In the objective function maps in Figure.4.12, the result maps of these two strategies are quite dark meaning that the overall magnitude is low, which can be fitted in a wide open quadratic surface.
Figure 4.15: Quadratic Fitting $R^2$ plots of Dark Edge II
4.2.2.3 Sharp Corner

The results of the last feature of this image volume is presented, that this feature consist of two orientations of the edges, with 90 degrees. Though we can barely predict the best strategy, we would like to test the feature to see the result. In Figure.4.16, because of combined orientations pattern, the elongated subset with any direction is only able to improve the performance of one orientation by rotation like Figure.4.16j, that the dark area along elongation orientation has been shortened while the normal direction otherwise shows bad influence with larger minimal area. Compared to the rest of the result, enlargement seems to return the best behavior that the dark area reshape to a round area showing in Figure.4.16c and Figure.4.16d, though the improvement is limited. The shrinkage of primal minimal area is not so obvious as the rest of tests.

There are also the 3D surface figures for the best result in Figure.4.17. From the 3D images of results, the advantage of SQ3 strategy can be observed again in Figure.4.17c that generally the distribution of SQ3 is less varied as SQ2 shown in Figure.4.17b, which gives higher isotropy by keeping more close values in same radius, regarding to the “valley” shape. We also can find the primal minimal area center changing faster than original subset in Figure.4.17a.

The box plots were drawn in Figure.4.18. As the maps showing in Figure.4.16, this corner texture pattern is hard to making progress as the rest of patterns, along single orientation of subset. The enlarged subset definitely improves the isotropic behavior of objective function result by introducing more pixels, that the box of SQ3 in Figure.4.18 is shorter, where the result accumulate together and the median is slightly moving to the minimum.

The whiskers change of plots shows the RY2 Figure.4.16f, RY3 Figure.4.16g, RY2Ro90
Figure 4.16: Objective Function Map of Sharp Corner
Figure 4.17: 3D Image of Sharp Corner
in Figure.4.16h and RY3Ro90 in Figure.4.16i have benefit with shorter lower whiskers relatively. The boxes of these plots (50% of points) are quite scattering in these two, indicating the most pixels in these two result magnitude are changing rapidly. Since the box represent fixed number of points, changing rapidly suggest less stationary area will be found in these two result maps. Conclusively, the trend of improving is highly unclear, due to the large scale area with smaller value as the light purple region in Figure.4.16, that we can not have consistent conclusion between statistical analysis and objective function map. Of course, the box plot does not consider the points’ location, so high speed change of majority of results might suggest un-smooth surface all over the searching region, which is one of disadvantage of box plot.

The quadratic fitting is showed in Figure.4.19, where the green color line is the best strategy examined by eye referring to the Figure.4.16. The figure shows that the best result as the orange flat line as RY2Ro90 has the highest magnitude of $R^2$ according
to our first theory, that the line with the highest magnitude is mostly like to the best strategy. In this figure, the best line actually has large anisotropic dark objective map in Figure 4.16h, suggesting that the objective function results of whole region form to a almost flat surface. Recalling the box plots of Figure 4.18, the box plot of RY2Ro90 is also very satisfying, which produce higher and stabler $R^2$ of quadratic fitting. Both methods show conflict with the conclusion in eye examine.

![Corner Shape Quadratic Fitting](image)

**Figure 4.19:** Quadratic Fitting $R^2$ plots of Sharp Corner
4.2.3 Discussion

Based on this first set of the result, we can observe the implementation works as plan, that the objective function result can be generated as expected. Basically, we can observe the advantage of changing the subset to enhance the map shape of objective function. Enlarging the subset generally can improve the result map, by raising the whole region result to a higher level of magnitude on different texture features. For the features with oriented edge, such as Edge I in Figure.4.8a and Edge II in Figure.4.12a, elongating the subset is a more effective way to improve correlation, especially elongation through the orientation of the texture, like Edge I RY2Ro90 in Figure.4.8h and Edge II RY2Ro135 in Figure.4.12k.

Although the improvement has been observed, there are still surprising behaviors existing. The first spot subset produces the objective function map with very dark edge, which can be explained by that high magnitude of maximum point of the result map compressed the low magnitude points color distribution.

Meanwhile, the quantification evaluation of most subset by box plot is barely following the expectation and the optimal result. The small number of result is hard to present the behavior as the optimal result. Besides, the box plot doesn’t consider the locations of points, so when the surface is not isotropic, the measurement of box plot might be confusing. Another approach of quadratic fitting also encounters the unexpected situation due to some “bad” result map with flat surface and some “good” result map only with improved global minimal area but not multiple local minimal area. It suggests using $R^2$ to evaluate the quadratic fitting of the result map not sufficient, but we will stick to $R^2$ in this thesis, since fitting to a quadratic surface is the first requirement of a good whole region objective function map for optimization method search.
Next, to show the advantage of rotation on the anisotropy texture, we tested more subsets in same way with repeated oriented texture.

4.3 Ceramic Texture

There is another set of demonstrations which are using a freeze cast ceramic image volume. The texture of ceramic is full of highly anisotropic oriented patterns, and barely have distinguishable speckle or dark color pattern to identify.

We are using the same implement to test the accuracy of these subset strategies and using correlate the same image volume as the reference and target subset. Thus the results are expected to return the exactly same location of selection. Because of image size, firstly we defined the manual selection for this set of images as 30 × 30 pixels², then we used 90 × 90 pixels² but single slice search range to save the time. Due to the larger area of strips feature, we would like to try more orientation strategies for each feature, such that we tried to rotate the subset 15 degrees every test.

4.3.1 Light Strips

The first set of data maps in Figure.4.20 and Figure.4.21 is generated from a region without significant change on brightness of strips repeatedly as Figure.4.20a. As the Figure.4.20b shows, the low value points distribution with darker color shapes as a long band like the texture in regular subset. Based on all result maps in Figure.4.20 and Figure.4.21, we should say the elongated 3 times subset with rotation 15 degrees (RY3RoL15) provides us the best result as Figure.4.21d, that the large white and yellow color area indicates the objective result increases fast around the minimum point.
times elongation with 15 degrees rotation (RY2RoL15) is showing similar performance in Figure.4.20g but with lower magnitude overall. Unfortunately, we still found slight improvement by enlarging the subset, as Figure.4.20c and Figure.4.20d shown.

Three dimensional surface figures are plotted in the Figure.4.22, so we can observe the slope changes based on the optimization requirement. As before, we only select the best results to present as 3D images, where we can still observe the valley like area along the minimum point exists, compromising the isotropic behavior. Even though the result has been refined by the subset strategy with less local minimal area, the valley shape completely cannot be eliminated completely in RY3RoL15 in Figure.4.22c and RY2RoL15 in Figure.4.22b.

1. Quadratic fitting – R-squared of three dimensional quadratic fitting

In order to quantify the strategies performance, the surface quadratic fitting has been used as usual in Figure.4.23. As expected, the purple color line as the best strategy (RY3RoL15) is getting larger than the other R-squared values of strategies when the fitting area becomes large. We also can observe that with the enlarging fitting area, all the $R^2$ magnitudes are decreasing, that the correlation surface shows isotropic in the small region close to the global minimum, but hard to keep the isotropic level generally, which is what we aimed to improve.

2. Contour Plot

Another approach for evaluating the performance of the strategies is similar to the function of color map by exploring the contour plot of the surface of results. In this method, we want to show that a good strategy should help the contour lines shape a circle around global minimum as the optimal result Figure.4.1c. The contour plot of the best match slice of the same strategies have been showed from the last part,
Figure 4.20: Objective Function Map of Light Strips
Figure 4.21: Objective Function Map of Light Strips [continued]
Figure 4.22: 3D Image of Light Strips
Figure 4.23: 3D Fitting R-squared
in Figure.4.24. As expected, the plot shows more encircling and uniformly growing lines when the strategy can fit the quadratic polynomials better, like the RY3RoL15 in Figure.4.24f especially when the values close to minimum point, as blue contour lines suggest. Similarly, we can find the original anisotropic primal minimal area as showing in Figure.4.24a and even Figure.4.24b reshaping into a rounder area, with connected local minimal region, as Figure.4.24e and Figure.4.24f. Comparing the RY2Ro15 and RY3Ro15, we can see the RY2Ro15 in Figure.4.24e has larger stationary area, and meantime the area normal to the orientation of the texture get higher magnitude with yellow color contouring lines.

3. Boxplot

The box plots of the result are also generated as before in Figure.4.25. However, the box plot of the best strategy from eye examine, RY3Ro15 as Figure.4.21d, is hard to conclude as a similar box plot as the ideal result box plot Figure.4.3, because of the anisotropic minimal area along the natural texture.

We can see that the higher whisker of RY3Ro15 reach to the global maximum, that the surface with high magnitude is more smooth. Besides, the RY3 has a lower whisker including minimum point. This helps the minimal area points growing more smoothly with the searching area enlarging. On the other hand, the highest value of median of data set can be noticed in RY3Ro15 strategy, which is shown the ability to help identifying the global minimal area, as the result in Dark Edge II in Figure.4.14.

4. Discussion

The result of this reference subset suggests the elongation subset with right orien-
Figure 4.24: Contour Plots of Light Strips
Figure 4.25: Box plots of Light Strips
tation is helpful to improve the objective function map. Actually multiple minimal areas have been eliminated especially the ones away from primal minimal area but not completely, that we suspect the comparatively small size of subset limits the sufficient information included once the shape elongated. However, the computation cost is quite large, that each map of objective function is taken around 5 hours to generate by the implementation, that we cannot run more tests with more angles nor larger size of subset. As we can see the angles of subset rotation as 15 degree in Figure.4.21d and Figure.4.20g is not completely consistent with the texture anisotropy angles( around 45 degree), where we suspect the narrow shape of the elongation subset does lose the integrity of the texture feature which also produces some local minimal area connected to the global minimum concave in Figure.4.21d.

Quantitative analysis are not consistent, that box plots are still hard to draw a clear conclusion. The most $R^2$ improvement of fitting evaluation appears as expected. The contouring plots are helpful for eye examining, but it does not give us scalar measurement related to subset strategy and result improvement.

In next testing, we changed some implementation function, which allows us to run a larger subset with lower time cost by limited the searching region that all the searching points locate in the single slice of the image volume.

4.3.2 Other Three Set of Subsets

Based on the results shown above, we see the power of rotating subsets. Considering the small size of previous subsets used, we selected three other subsets. The subsets are
featuring different angles: horizontal, vertical and 135 degree angle (clockwise) as shown in Figure 4.26.

In this set of samples, $80 \times 80$ pixels$^2$ subsets are used to perform global search using an updated . Then the Figure 4.27 shows the results surfaces, where we picked the best result surface in all strategies, for each of the patterns selected. All the result surfaces are shown in the same size $40 \times 40$ pixels$^2$ of area centered as the matching point since we focus on the global minimal area. Because of lacking repeating texture pattern, the multiple local minimal areas do not appear expect the part connected to the global minimum area.

The following Figures 4.27 are the three reference subsets objective function maps. Each column represents the results of one subset. Each row represent one type of subset strategy for each reference subset. For example, the third row lists the maps with best objective function maps, that the local minimal areas are reduced mostly where the stack 1 with flat anisotropy pattern has the best result at 120 degrees in Figure 4.27g, stack 2 with almost vertical texture has the best result at 75 degrees in Figure 4.27h, and stack 3 with 135 degree (clockwise) has its best improvement at 45 degrees in Figure 4.27i.

Notice, three samples show the consistent conclusion about the positive influence of orientation adjusting of subset, but all have different angles of subset orientation adjusting regarding to texture orientation. In other words, none of them get the best result at the same degree with the orientation of their texture pattern. The improvement are focusing on the elimination of local minimal areas, similar to the previous test. The orange color area away from the primal minimal concave are reducing into white color area. Also the anisotropic elongated areas are getting shallower, which are connected to primal minimal area, with dark color.

Three dimensions surface figures of first set of sample are listed in Figure 4.28. In the
Figure 4.26: Subset selections of Rotated Image samples
Figure 4.27: Compare between Rotated Images and Rotated Subset
original shape of subset results Figure.4.28a, we can see there is always a “valley” area lying in the surface, which is along the orientation of the texture pattern and through the center of image as the global minimum. Recall the two major reasons reducing the efficiency of searching, we know this situation will lead to the second type of optimization search risk. This “valley” shape can cause the software taking large time to search in this stationary area. Comparing to the best result surface as Figure.4.28c, when the subset is reformed to the similar shape of texture in right angle, data magnitude of the “valley” area on the surface can be increased significantly. At the same time, we also can observe the value of surrounding area around the minimum point has been raised higher than rest of result surface highest point. However, in Figure.4.28c, we still can notice the oriented minimal near the global minimum. The rotated elongating subset surface still have dent convex area between high value surface(bright color area). For Stack 2 in Figure.4.29 and Stack 3 in Figure.4.29, the results from SQ1 also presents highly oriented minimal area where the valley looks deeper than the Stack 1. Like the Stack 1, changing strategies are not able to improve the surrounding to isotropic surface completely in Figure.4.28c.

The Stack 2 results were plotted in three dimensions as Figure.4.29. Like the 2D maps, the surface of the result gets higher magnitude with right elongation orientation that more area becomes bright color in Figure.4.27e. The “valley” shape area also changed that the bottom of the “valley” is reshape from flat in Figure.4.29a to elevating slightly in Figure.4.29b. In best result the bottom get raised significantly where we even noticed the valley area get disconnected from the primal minimal area(global minimum area), in Figure.4.29c.

We also can observe the similar improvement in 3 dimensional plots of Stack 3 Figure.4.30. The best strategy is the rotating and elongating subset as Figure.4.30c,
Figure 4.28: 3D Surface Plot of Stack 1
Figure 4.29: 3D Surface Plot of Stack 2
but the edge of the searching region has more area of concave. The local minimal area in original subset in Figure.4.30a near the corner of region in orange color get raise into higher magnitude level. Meanwhile, the “valley” shape is almost eliminated from Figure.4.30a to Figure.4.30c. The unexpected is the anisotropy of the primal concave area, that the concave shape is shallow rather than round. However, due to the large slope of the concave surface, the searching effort is similar in the purple color area.

Next we performed the quadratic fitting for all results. However the incomplete of the isotropy reshaping leads the quadratic fitting fail to follow the expectation, that the best strategy result return the highest values of $R^2$ in Figure.4.31. None of the best strategies from eye examining get the highest $R^2$ after the fitting area getting large. The Stack 3 has a quite satisfying result before the fitting area getting largest, as Figure.4.31c. Similarly we can not have satisfying clue on box plots of result, as Figure.4.32, that all of the results shows outliers appears near the minimum, and with improvement showing in color map, the median of the best strategies have been increased in Stack 2, Figure.4.32b and Stack 3, Figure.4.32c.
Figure 4.30: 3D Surface Plot of Stack 3

(a) Stack 3 SQ1

(b) Stack 3 RY2Ro90

(c) Stack 3 RY2Ro45
Figure 4.31: Quadratic Fitting $R^2$ plots of Three Subsets

(a) Quadratic fitting of Stack1

(b) Quadratic fitting of Stack2

(c) Quadratic fitting of Stack3
Figure 4.32: Box plots of Three Subsets
Chapter 5: Conclusion and Future Work

5.1 Conclusion

Based on the results of two X-Ray tomography images, we demonstrated the existence of that performance of digital volume correlation on natural texture is able to be ameliorated, by adjusting the subset to have the shape with respect the texture shape and orientation.

The implementation we developed is working which conduct single measurement point volume correlation on the limit search region. The maps generated are revealing some interesting behaviors of the texture under the subset adjusting. Making use of subset strategies, we believe that considerable efforts of the complex correlation criteria calculation and optimization method search and even failure caused by texture feature can be improved. In most case, enlarging the subset can always provide certain enhancement of the quality of objective function map, while the elongation subset strategy is more helpful if the texture pattern is anisotropy, by rotating the elongated subset along the orientation of the subset.

However, the angle of subset rotation is not always through the texture patterns, where we suspect on of the causes is limit of subset size. Meanwhile, the quantification evaluation encounters surprising result that the advantage of improved result is not clear. As the discussion for first set result, we noticed the constrains of using $R^2$ and box plots to assess the quality of the result map. It will be improved in future work.

Our project should be considered as the starting of exploring the contribution of
subset strategies, since there still exists mysteries between the relationship of correlation method and subset strategy in our demonstration.

5.2 Future Work

1. Conduct more tomography data sets with larger overall size of the subsets:

   In the last chapter, we proved there is a relationship existed between the subset orientation and the performance of correlation method. However, we are only able to conclude that when the orientation of subset is rotated to an angle close to the orientation of texture pattern, the performance of correlation gets improved, rather than a specific quantitative relation between subset angle and the performance. It seem more factors other than simple orientation of texture affect this result, that more tests should be applied to unveil the relation. This includes using larger size of the subset to test.

   As mentioned, the time cost of global search is growing fast when the subset size and searching region get enlarged. The largest subset in our cases, $80 \times 80 \times 80$ pixels$^3$, takes over 3 hours to run an $80 \times 80 \times 1$ pixels$^3$ search region which still fairly small regarding to the scale of the image volume and the $90 \times 90 \times 60$ pixels$^3$ search region for $30 \times 30 \times 30$ pixels$^3$ takes over 5 hours.

   In the future, a better implement should be developed, probably with an proper optimization method. In this way, we can explore the impacting of adjusting subset shape on DVC in larger sample. With larger size of subset and limited searching region, we can include more types of texture, which helps us to confirm the conclusion.
In this thesis, the implementation we developed has certain constrain, that the computation cost is quite large, that days of execution were taken for one set of strategies test. In the future, a better implementation should be developed, with more powerful platform.

2. Investigate the influence of the subset on the performance of digital image and volume correlation:
   Considering the subset volume is in a coordinates system, that we only change the shape of the subset on the x-y plane, as the transverse plane of the subset, rather than the whole shape of volume. As we know, X-Ray tomography give the sample image texture through the whole volume. Every region of interest should have unique three dimensional context feature. In this case, we can expect that adjusting the orientation of subset volume in three dimensional might help the correlation method captures more useful data information just like how our subset strategies impacts planar orientated texture.

3. Investigate the evaluation method of the subset performance of digital image and volume correlation:
   In our project, we used several quantitative methods to assess the behavior of the correlation result with different subset strategies. As discussed before, the $R^2$ of the quadratic fitting and box plot have their own constrain to evaluate the quality of objective function map, that $R^2$ of quadratic fitting is not enough to reveal the shape of the quadratic behavior of the surface and box plot does not display the right distribution from a small scale number of data.

   In the future, a scalar measurement should be developed, which could more straightforwardly assess the quality of subset strategy. Using this scalar measurement, a
better implementation can be developed to automatically identify the best strategy from these adjusting of subset strategies.
Bibliography


