Spectral Analysis of Upper Ocean Surface Wave Structure using $\chi$SOLO Floats in the Bay of Bengal

By
Matthew Ball

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Surface waves play a vital role in air-sea interactions, and being able to easily measure them in-situ validates and improves predictive models. Here, we diagnose surface wave properties in the Bay of Bengal using modified vertical SOLO II profiling floats, which are regularly used as part of the Argo ocean float array. The modified χSOLO floats measure high frequency pressure, acceleration, velocity fluctuations, and turbulence data, in addition to traditional conductivity-temperature-depth data. The pressure and acceleration data are used to compute surface wave frequency and height for swells and surface wind waves. The frequency of the swells is computed using power spectral density of vertical acceleration at the bottom of a vertical dive, and surface wind wave frequency is computed using power spectral density of the pressure record while at the surface. To find amplitude, the observed frequencies are individually identified in depth-binned Fourier transforms of the net horizontal acceleration. The magnitudes of these Fourier components are fit to an exponential decay as a function of depth, and the terms of this fit give the amplitude of the wave component corresponding to that frequency. These methods unveil a series of swells of roughly half meter amplitude generated between 7000 km and 9500 km away and two short period wind wave bands of ~4 s and ~7 s with amplitudes 0.1 m and 0.3 m, respectively. The observed waves agree well with model predictions from the global WaveWatch III simulations.

Key Words: Ocean Waves, Spectral Analysis, Vertical Profiling, WaveWatch

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I understand that my project will become part of the permanent collection of Oregon State University Honors College. My signature below authorizes release of my project to any reader upon request.

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1 Introduction

The complex superposition of waves in the ocean can make understanding in situ data difficult due to the wide spectrum of ocean waves that exist at a point at any given time. For example, low frequency tides are caused by the gravitational pull of the sun and moon. Midrange frequencies are associated with tsunamis, storm surges, etc. The highest frequency waves are the distantly generated swells and short-period wind waves. A spectrum of the types of waves in the ocean is shown in Figure 1.1 [4].

Wind waves and swells are the most prominent (non-disaster related) waves in the ocean. Wind waves are driven by wind blowing along the surface of the water, where frictional stress between the two fluids causes turbulence which appears as waves. The height of the wind waves varies based on the speed of the wind. Generally these waves have wavelengths of a few meters, which translate to periods of a few seconds. Surface swells are generally formed via distant storms; the height of these waves varies based on how far away the wave was generated, but they have larger wavelengths of a few hundred meters, which translate to periods in the 20-s range. When traveling, surface waves disperse in a way that causes the longer period waves (i.e. swells) to arrive first [4, 12].

Figure 1.1: Ocean Wave Spectrum
The various waves found in the ocean with their usual sources. The region focused in this analysis are those waves with periods between 1 s and 30 s. Image credit: [4]
1.1 Motivation

Every year, millions of people in south/southeast Asia are affected by seasonal monsoons. While the approximate time of year that the monsoons arrive can be predicted, specific stretches of oscillating calm and heavy rain systems cannot \[2\]. The summer monsoon accounts for roughly 80\% of total rainfall in many parts of India. While beneficial to the subcontinent for agriculture, these rains can arrive in short intervals (as short as a few days) and bring large amounts of precipitation (up to 100 mm in under 24 hours), causing severe floods. These bursts cannot be predicted accurately using current weather models, which can be a major problem for residents whose lives can be drastically affected by them \[2\]. Many features of the monsoon atmospheric patterns (and many weather patterns in general) can be attributed to interactions with the top few meters of the ocean across the viscous boundary layer (air-sea interface) \[9\].

The Bay of Bengal is the primary source of monsoon growth due to a complex temperature structure in the upper ocean that is characterized by the layering of warm and cool water. Subsurface heat can impact the surface temperature by turbulent mixing and transport \[7, 13\]. A better understanding of the interactions between the ocean’s surface and the atmosphere could allow for more accurate predictions of the rainfall bursts within the monsoon.

The rate of temperature flux across the air-sea interface in the Bay of Bengal is affected by many factors, including temperature and salinity stratification, turbulence, and precipitation. One particularly prominent factor is breaking waves: when waves break, they also create turbulence and vorticity in the sub-surface area which acts to dissipate surface wave energy and mix the upper ocean \[7\]. Rough wave conditions contrast those with a smooth surface which allows only molecular diffusion to exchange heat and momentum between air and sea (as opposed to forced mixing) \[9\]. Understanding the development of the wave field in the Bay of Bengal would help to characterize one of the many influences of surface heat flux that contributes to the aforementioned unpredictable, oscillatory rainfall bursts.

1.2 Goal of the Project

This project marks the first deployment of a modified type of ocean profiler. These profilers combined the long-term vertical profiling capabilities of the SOLO floats used by the Argo ocean array with \(\chi\) pods developed at Oregon State University \[3, 8, 13\]. This project represents the first attempt to obtain wave data from these types of floats and presents methods to obtain the wave frequency spectra of the deployment as well as obtain the height of the waves using the simple deep water wave approximation as an analytical basis.
2 Background

Here, we diagnose surface wave conditions using the pressure and accelerometer records. We begin by presenting the mathematical theory behind surface gravity waves and their appropriate limits, followed by the methodology used to extract the desired wave data from the time records. We then discuss the results for the periods and heights of the waves identified and how these compare to data sets from different sources.

2.1 Traditional Wave Measurement Methodology

Traditional techniques for measuring ocean wave properties primarily consist of wave buoys. These are devices that sit at the surface of the ocean and via accelerometer data while tethered to the ocean floor, measure the wave amplitude over time. Accelerometer data is affordable to acquire and easily interpreted in the context of surface waves. A double integration in time of acceleration gives the displacement of the buoy. This relies on the large scale, physical movement of the device with the waves [14]. This is also largely their only purpose (i.e. no other function beyond wave measurements). These wave buoys can also be used in arrays to resolve the directional movement of the waves if used in arrays [16].

2.2 Mathematical Theory

The mathematical model used for this project is the surface wave problem for small amplitude waves in an incompressible, irrotational fluid. We also assume that the waves are not constrained by nearby land (that the ocean is infinite) and that the waves only move in one horizontal direction that we shall call the $x$ direction. We assume that the water has no viscosity, surface tension, or effect due to the Coriolis force. As derived in [5] and [6], we can express these sorts of waves via a potential function $\phi$:

$$\phi(x, z, t) = \frac{a \omega \cosh(k(z + H))}{k \sinh(kH)} \sinh(kH) \sin(kx - \omega t)$$  \hspace{1cm} (2.1)

where

$$\frac{\partial \phi}{\partial x} = v_x(x, z, t) = a \omega \frac{\cosh(k(z + H))}{\sinh(kH)} \cos(kx - \omega t)$$ \hspace{1cm} (2.2)

and

$$\frac{\partial \phi}{\partial z} = v_z(x, z, t) = a \omega \frac{\sinh(k(z + H))}{\sinh(kH)} \sin(kx - \omega t).$$ \hspace{1cm} (2.3)

$v_x$ and $v_z$ are velocity in the $x$ and $z$ directions, $z$ is the vertical depth (here
a negative value), $x$ is the horizontal displacement, $k$ is the wavenumber ($k = 2\pi/wavelength$), $g$ is acceleration due to gravity (9.81 m s$^{-2}$), $H$ is the depth of the water, $a$ is the amplitude of the wave relative to rest level, and $\omega$ is the angular frequency ($\omega = 2\pi/period$).

From the dispersion relation:

$$\omega = \sqrt{gk \tanh(kH)},$$

we can find the phase velocity of the wave.

$$c = \frac{\omega}{k} = \sqrt{\frac{g}{k} \tanh(kH)}. \quad (2.5)$$

Additionally, we can find the pressure $p$ at depth $z$, time $t$, displacement $x$, and water density $\rho$ from Bernoulli’s equation:

$$p = -\rho \frac{\partial \phi}{\partial t} - \rho g z. \quad (2.6)$$

From which the perturbation pressure $p'$ (pressure removing the hydrostatic pressure from depth, $(p' = p + \rho g z)$) can be written:

$$p' = -\rho \frac{\partial \phi}{\partial t} \quad (2.7)$$

### 2.2.1 Deep Water Limit

In water deep relative to the wavelength $kH \Rightarrow \infty$ the hyperbolic trigonometric terms approach exponentials, yielding:

$$\phi(x, z, t) = \frac{a\omega}{k} e^{kz} \sin(kx - \omega t), \quad (2.8)$$

$$v_x(x, z, t) = a\omega e^{kz} \cos(kx - \omega t), \quad (2.9)$$

and

$$v_z(x, z, t) = a\omega e^{kz} \sin(kx - \omega t). \quad (2.10)$$

We also can see that (since $\tanh(\infty) = 1$) our dispersion relation simplifies to:

$$\omega = \sqrt{\frac{g}{k}}. \quad (2.11)$$

This relation gives a phase velocity of $c = \omega/k = \sqrt{g/k}$ and a group velocity $c_g = d\omega/dk = 1/2 \sqrt{g/k}$. Since the wavenumber $k$ is proportional to the period, longer period waves travel faster than shorter period waves. We can find how the group velocity of the wave changes as we get further away from the source. Setting
the group velocity $c_g$ equal to $x/t$, the phase speed of a particular wavenumber $k_s$ varies according to:

$$c(k_s) = \frac{2(x - x_0)}{(t - t_0)} \quad (2.12)$$

where $x = x_0$ is the location of the origin of the wave, and $t = t_0$ corresponds to the time the wave was generated. This equation will allow us to calculate the propagation distance and formation time of distantly generated swells [12].

Additionally, we can use Equation 2.7 to find an expression for deep water pressure waves:

$$p'(x, z, t) = \rho gae^{kz} \cos(kx - \omega t). \quad (2.13)$$

Deep water waves have circular orbits that decay in depth with an e-folding scale of k. Because of this dependence, lower frequency (longer period) waves are relatively more energetic at depth compared to their higher frequency (shorter period) counterparts. Deep water waves are also dispersive, i.e., their phase speed depends on the period of the wave with longer period waves traveling faster than shorter period waves as discussed earlier.

Since the data for this project was taken in the Bay of Bengal with depths ranging in the thousands of meters, we can safely use the deep water approximations for our calculations. This approximation allows us to take advantage of the exponential decay feature for use in curve-fitting the data later on.
3 Methods

The goal of this project was to develop a process for characterizing the surface gravity wave field using data from the \( \chi \)SOLO floats collected in the summer of 2015. This included devising an algorithm to find the period, height, and direction of the surface waves. Analyses were restricted to the upper 50 meters of the ocean, as that is the range the units profiled.

3.1 Instrumentation

Two vertical profiling floats were deployed in the Bay of Bengal from August 29 to September 11, 2015. They were deployed roughly four days apart for six days apiece. They were deployed south of an ocean mooring operated by the Woods Hole Oceanographic Institute (WHOI) to monitor nearby atmospheric conditions. A map of the deployment area is shown in Figure 3.1.

![Map of Bay of Bengal with float trajectories](image)

Figure 3.1: Map of Bay of Bengal with float trajectories

The map displays the location of the floats and a nearby WHOI mooring and the sea surface height anomaly (SSHA) - deviation in wave height from average. Cutout shows velocity vectors of the floats and average salinity at each profile. Image credits: [13]

The two floats each consisted of two sensor modules: a \( \chi \)pod and a SOLO float. These two components each collected their own data sets. The \( \chi \)pod collected 50-Hz
measurements for pressure, acceleration, temperature, and velocity fluctuations. The SOLO float collected 1-Hz measurements of temperature, conductivity, and pressure. The SOLO float component is traditionally used to create "slices" of the ocean showing how the properties of the water column develop in time. An example of this for salinity and temperature is shown in Figure 3.2.

![Figure 3.2: Salinity and Temperature depth records from SOLO float](image)

Floats were equipped with a GPS to record the latitude and longitude of profiles. These floats were designed to collect data from the surface to a depth of 50 m in what are referred to as profiles or dives at a rate of about one profile every half hour. The floats continuously took data during the deployment, i.e., collecting data while profiling and when drifting at the surface between dives (profiles) [13]. The goal for this project was to quantify the wave field (i.e. the period and amplitude of the waves) in the Bay of Bengal during the deployment of these floats.

### 3.2 Finding the Period of the Waves

The periods of the waves were determined using spectral analysis. We considered two primary states: when the float was at the surface and when it was profiling. When considering the profiling portion of the record, we capitalize on the fact that in deep water, the exponential decay of the surface wave varies with the wavenumber of the wave. From Equation 2.9, 2.10, and the dispersion relation in 2.11, we can see that $k$ is proportional to $T^{-2}$. This means that the longer period waves should decay more slowly and have a more noticeable signal at depth. We broke up the dives by depth
and took the power spectral densities (PSD’s) of the pressure and vertical acceleration to detect how the spectral peaks changed with depth (see Figure 3.3).

![Figure 3.3: Decay of PSDs with depth](image)

PSD of the measured acceleration for depth ranges 0-18 m (left), 18-36 m (center), and 36-54 m (right). Notice how higher frequency peaks disappear with depth while lower frequency ones become more prominent. The vertical dotted lines are the frequencies of the expected wave bands, with red for waves of period 4 s, green for 7 s, and black for 20 s.

In Figure 3.3, we see the higher frequency peaks that appear in shallow regions decrease in magnitude with depth, whereas the lower frequency peaks become more prominent with greater depth. This effect was much more noticeable when performed on the vertical acceleration versus the pressure record, as the latter includes lower frequency signals that obscure the surface wave peaks. For this reason, we elected to use the vertical acceleration in the detection of wave peaks. The higher frequency peaks corresponded to surface seas or locally generated swells, whereas the lower frequency peaks correspond to remote swells formed by distant storms. Due to the exponential decay with period, we evaluate spectra of vertical acceleration - filtered using a fourth order, 3 vanishing moment, Daubechies wavelet transform - at depths of approximately 35 m to 60 m, in order to isolate peaks associated with longer period swells. To automate the process, a peak finding algorithm [15] was used to identify the frequencies of the prominent peaks in the spectra.

The χpod acquired data while sitting at the surface, acting as a surface buoy; spectra from the surface time series of both pressure and acceleration were dominated by shorter period seas. Here, the pressure record was used instead of the accelerometer data, as its spectra had easily detected peaks (see Figure 3.4). The pressure records from the time the float was at the surface were denoised using the wavelet denoiser.
used for the depth data. These cleaned records were then used to compute a Welch PSD estimate, and peaks of short period wind waves were detected following the same algorithm described above for swell detection.

**Figure 3.4: PSD of pressure at surface**

PSD of the pressure at the surface shows the prominence of the peak corresponding to high frequency seas. The vertical lines are periods of 20 s (black), 7 s (green), and 4 s (red).

### 3.3 Finding the Height of the Waves

The heights of the waves were determined using the exponential decay of the axial accelerations. Looking at the deep water limit for velocity in the $x$ direction (Equation 2.9), we can take the time derivative to get an approximation for acceleration in the $x$ direction:

$$\alpha_x(x, z, t) = -a\omega^2 e^{kz} \sin(kx - \omega t)$$

$$= -agke^{kz} \sin(kx - \omega t) \quad (3.1)$$

In our notation, the surface of the ocean is $z = 0$, and increasing depth causes a more negative $z$ value. The exponential decay in the magnitude of acceleration is clearly evident in Figure 3.5.

However, doing this with one exponential for the full decay ignores the fact that multiple significant wave periods were detected. To resolve the height of each wave type, we look to the exponential decay of the individual Fourier components. By
Figure 3.5: Exponential decay of magnitude of axial accelerations

Exponential Decay of $\sqrt{a_x^2 + a_y^2}$. Using the term that dictates the amplitude of the sinusoidal term in Equation 3.1, we can compute the average height of the wave. The value used in the computation was the same as is plotted here.

taking the Fourier transform of the data in binned depths much like what is shown in Figure 3.3 and following a particular frequency, we can create an exponential decay curve for that particular frequency. This method assumes, however, that the Fourier transform is properly normalized to have the same units as the original function, which for the fast Fourier transform (FFT), just requires dividing by the length of the signal.

Individual profiles were broken into 20 depth bin intervals over which normalized FFTs were computed. Transforms from surface data (prior to the dive) were included to provide an estimate of peak amplitude near the surface. Exponential curves were then fit to the amplitude decay of individual wave frequencies (i.e., either the 4-s, 7-s, or swell bands). Figure 3.6 shows an example of the FFT peak decay (4-s and 7-s...
wave) with depth and the fitted exponentials for one dive.

![Exponential Decay of FFT Component](image)

Figure 3.6: Exponential decay of FFT component
Example of decay of Fourier components of 4-s (left) and 7-s (right) sea bands. The different decay rates are due to the different periods of the waves. The fitting resulted in amplitudes of 0.11 m (left) and 0.31 m (right).

The observed amplitude decays for each wave frequency were fit to the equation $a g k e^{kz}$ using a nonlinear least squares fit to solve for $agk$ given a corresponding frequency (which would be converted to a wavenumber $k$ using the dispersion relation). Using the known value of $k$, we can solve for the amplitude $a$ of the wave.
4 Results

The primary results from the analysis were sets of peaks of defined periods and amplitudes in the wave spectra that were organized into time series for the full deployment.

4.1 Wave Period

Once all the depth and surface frequencies were identified, they were converted to periods (Figure 4.1). Throughout the deployment, two horizontal bands at periods of approximately 4 s and 7 s were regularly observed (Figure 4.1). Additionally, there are four clusters that appear at higher periods. These periods, and their characteristic “sweeping” pattern from high to low periods, match what is expected for distantly generated swells. Also noticeable are a series of dark red regions. These are simply artifacts from the coloring process, as this region is where the records of the two units deployed overlapped. It should be noted that one profilers’ accelerometer failed partway through the deployment causing the gap in the higher period band from roughly September 1st to 3rd. Additionally, as there was only one set of frequencies per profile, the horizontal position of these points are only accurate to within approximately 10-20 minutes.

![Figure 4.1: Density plots of dominant periods](image)

Time evolution of the dominant period bands measured using the spectral analysis methods. The points are colored by density (or proximity to each other) to highlight clusters of points (arbitrary units). Two low period bands are observed around periods of 4 s and 7 s as well as multiple diagonal bands in the 10/20-s region.
4.2 Wave Height

The height of the waves at each frequency were calculated using the exponential decay of the Fourier coefficients with depth. With this method, approximations for the height of each wave that had a period detected were found (Figure 4.2). The 4-s sea band had amplitudes of approximately 0.1 m compared to the 7-s sea band which had amplitudes of approximately 0.3 m to 0.4 m. The remote swell bands detected had amplitudes ranging from 0.3 m to 0.6 m.

![Time Evolution of Wave Amplitude](image)

**Figure 4.2: Amplitude of waves detected**

Time vs. period plot colored based on the calculated amplitudes of the waves. Most of the wave amplitudes detected are around 0.4 m. The 7-s band and the swells hover around this height, but the 4-s band is noticeably small, with amplitudes of less than 0.1 m. The peaks detected between the 7-s band and the swell bands are also associated with smaller amplitudes. The gap in the data is due to the failure of a sensor on one of the two floats.
5 Discussion

Here, the different wave structures identified earlier are explored for physical meaning. A method to track the origin of swells is discussed and compared with external data sets to confirm the method. Additionally, the reasoning for why certain data sets were more optimal for certain measurements than others is presented.

5.1 WaveWatch Model

To confirm the measured periods and amplitudes, an external data set was needed. The wave modeling software WaveWatch III, managed by the National Oceanic and Atmospheric Administration had modeled peak swell period during the time in question. The WaveWatch III model computes ocean properties such as significant wave height and dominant swell period based on measured wind conditions from NASA satellite data [10].

5.2 Understanding the Period of the Waves

The wave periods computed were similar to those expected. These are normal periods for ocean waves under normal conditions. What was odd were the parallel low period bands: the 7-s band and the 4-s band. It seems reasonable to expect waves of this period, but having both bands seemed odd, since one would expect them to normalize each other. Additionally, the 4-s band was extraordinary uniform in period and spread. It was upon examining the height of the waves did we realize that the 4-s band may not be a real band: Figure 4.2 shows the lowest frequency wave band with far smaller amplitude than the rest of the data.

5.3 Understanding the Height of the Waves

Observed wave heights were compared to the WaveWatch III model’s significant wave height in the Bay of Bengal. Choosing a location near to where the data set was actually collected gives the record of significant wave height shown in Figure 5.1.

The significant wave height computed by WaveWatch III (when adjusted to traditional amplitude instead of peak to peak height) gives waves with amplitudes of approximately 0.6 m to 1.2 m. This is close to the range that was computed using our curve fitting estimation (within an order of magnitude). If we take the significant wave height to be comparable to the sum of the individual wave heights we calculated, there is a stronger agreement with our results hovering closer to 0.8 m.

Additionally, of the wave heights computed, waves in the lowest period band have among the smallest amplitudes. This, combined with footage of the floats in the water (showing it bob with approximately the same period without correlating to a wave), implies that this band may not be a true wave. We assume this could be an artifact created by the bobbing of the float itself, not caused by the waves at all.
5.4 Origin of the Waves

In order to estimate origin location of the detected swell bands, we fit data to a theoretical model based on the propagation speed (Equation 2.12). The data points were converted from period to velocity using Equation 2.5. Due to the nature of the data set, not every point could be selected for each swell band, but multiple points were hand selected to fit the curves to, giving the curves plotted in Figure 5.2. From these fits, origin distances were computed ranging from 7000 km to 9500 km with temporal origins ranging from 4 to 6 days. These estimated propagation distances suggest a source region in the Southern Ocean.

5.4.1 Comparison to WaveWatch

The data sets generated by the WaveWatch III model were collected in the region closest to the average location of the floats (Figure 5.3). WaveWatch III data was collected from the NOAA data archive [11]. It should be noted that WaveWatch III only had the dominant period at any given time, so multiple swells were not recorded simultaneously by the model.

These swells were also compared to wave data taken off the coast of south Australia (data obtained from [1]). Through the WaveWatch III model, the swells that were detected in the Bay of Bengal on September 6th, 9th, and 11th were also detected off the coast of South Australia around September 4th, 7th, and 11th. While there was no spectral data of the same type we computed, there was a record of the height of the waves in the region. Plotting the wave height, we see distinct peaks on September
Figure 5.2: Curve fits of swell data

Curve fits of \(2(x - x_0)/(t - t_0)\) to the swell data. Hand selected points are colored to match the curve that fits them. These fits gave values for \(x_0\) (the distance to the source of the wave) ranging from 7000 km to 9500 km with \(t_0\) (the time since the wave formed) ranging from 4 to 6 days.

4th, 7th, and 11th (Figure 5.4) which agree with both WaveWatch III predictions and our data in the Bay of Bengal.

This additional data set showed the last wave detected from our data arriving in Australia approximately at the end of September 6. We then took the average speed of the wave that reached the Bay of Bengal (estimated as \(x_0/t_0\) with \(x_0 = 7136\) km and \(t_0 = 4.4\) d giving a speed of \(\sim 18.8 \text{ m s}^{-1}\)) and applied this to the wave that arrived in Australia. Since the wave we examined arrived in the Bay of Bengal halfway through September 8, that gave a 37 hour difference between the two sites. Taking into account the 4.4 day travel time for the Bay of Bengal site, we estimated the travel time to Australia to be 68 hours. Multiplying this by the average velocity estimated from the wave measured in the Bay, we get a travel distance of about 4600 km. These two range circles centered at their respective origins intersect in the Southern Ocean near a region of elevated wave height and winds from WaveWatch III (Figure 5.5).

If the period of the swell waves had also been recorded, we would have performed the same curve fit of Equation 2.12 to find the distance the waves had propagated and how long they had existed. However, this data was not available, so we looked at the last swell we detect and traced it through the WaveWatch III data to when it would arrive in Australia and cross referenced with the data there (Figure 5.4).
Figure 5.3: Measured swells compared to WaveWatch III model
The measured periods (black) compared to the dominant swell period computed by NOAA’s WaveWatch III model. It matches the swell bands measured extraordinarily well, implying that these are real swells rather than organized noise.

Figure 5.4: Wave heights on the southern coast of Australia
Three distinct peaks in wave height in South Australia. The wave height increased around September 4th, 7th, and 11th. These are the same dates that the WaveWatch III model predicted that the ocean swells that we detect in our data would arrive here.
Figure 5.5: WaveWatch III significant wave height at swell generation time
Worldwide significant wave height at the date calculated to be when the last swell observed in the Bay of Bengal was formed. The radii of the two circles are estimated based on swell arrival times. The map is colored by significant wave height in meters. There is a region of high wave activity near where these two circles overlap suggesting a storm in the region that caused the swells.

5.5 Directionality of the Waves

The direction of the waves was intended to be calculated; however, the process grew out of hand quickly. Just looking at a small section of a dive filtered around a swell frequency shows a signal with a clear preferred direction (Figure 5.6).

It was assumed that the data could be broken up into small sections to look for these correlations and map the direction. However, this only looks at the relation between the axial accelerations relative to the float, ignoring that the profiler may have rotated. Looking at the compass data, it is clear that the profiler was rotating enough such that it could not be ignored and at a rate that would make it impossible to just look at an average direction. The probe, due to the way it was modified, has a preferred orientation aerodynamically, causing it to rotate based on both ocean currents and wave motion.

This complicated system makes it very difficult to calculate the direction of a wave detected during a dive. However, it is a question that should be answered in future work.
Filtering the bottom 3 meters of a depth dive for signals with period $\sim 20$ s and plotting them against one another gives a plot with a preferred direction. Ignoring numbers, we see a linear relationship between the two directional accelerations.

5.6 The Usage of Particular Data Sets at the Surface and at Depth

One question remaining is why the pressure record had a greater signal-to-noise ratio at the surface (i.e. distinct peaks) than at depth, and why the vertical acceleration record had a greater signal-to-noise ratio at depth than at the surface. Both records should have had similar clarity at all depths, but when analyzing the data, it was found that certain data sets resolved wave information better at the surface and others resolved better at depth, as mentioned above. The working theory is that at the surface, the changes in pressure were larger when compared to the total pressure, and at depth, the changes in pressure were small compared to the total pressure. This is due to the amount of water already above the float and the decay of the amplitude according to Equation 2.13. The relative size of the pressure changes to the total pressure allowed for the greater resolution at the surface. The explanation for the acceleration resolution is similar: at depth, the prominent waves exerting influence on the probe were the lower frequency swells since their larger period caused them to decay slower (Equation 2.10 and 2.11). Thus, at greater depths, the swells caused the float to move more than other waves, so they were detected with greater resolution.
6 Conclusion

The goal of this project was to quantify the wave structure of the upper ocean to develop a method for obtaining this information from the high frequency data collected by the χSOLO floats. These data are from the first deployment of this particular combination of χpod and SOLO float. Through the vertical acceleration data taken at depth, the period of distantly generated swells was measured, resulting in finding four swell packets that propagated through the region. Using the pressure record while the float was at the surface, the period of the surface wind waves was measured, giving two wind wave bands of periods 4s and 7s. The horizontal accelerations taken during the descent and ascent of the float gave the amplitude of the wave via the exponential decay of the Fourier components for each wave band. Unfortunately, the direction of the waves was not easily computed, and it is known that pitch/roll buoys are intrinsically poor at measuring directional wave spectra [16]. Future work could aim to test using these floats as an array for measuring directional wave spectra to better build a directional wave field anywhere the floats are deployed.
References


