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Supplement to

# BUCKLING OF FLAT PLYWOOD PLATES IN COMPRESSION, SHEAR, OR COMBINED COMPRESSION AND SHEAR 

## Buckling Tests of Flat Plywood Plates in Compression With Face Graín at $150^{\circ}, 30^{\circ}, 45^{\circ}, 60^{\circ}$, and $75^{\circ}$, to Load

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UNITED STATES DEPARTMENT OF AGRICULTURE FOREST SERVICE FOREST PRODUCTS LABORATORY Madison 5 , Wisconsin In Cooperation with the University of Wisconsin

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This report presents test data in substantiation of formula (16), $P_{c r}=k_{c} E_{L} \frac{h^{2}}{a^{2}}$, for the buckling stress of plywood plates in compression as presented in Forest Products Laboratory Report No. 1316, "Buckling of Flat Plywood Plates in Compression, Shear, or Combined Compression and Shear." These data were obtained in the same manner as the data presented in Report No. 1316-D, "Buckling of Flat Plywood Plates in Compression with Face Grain at $0^{\circ}$ and $90^{\circ}$ to Load."

For a discussion of the problem, the materials used, the system of matching and marking specimens, and the method of test, see Report No. 1316-D.

Very few modifications of the procedure used in the compression tests of plywood at $0^{\circ}$ and $90^{\circ}$ to the direction of the face grain were necessary when the direction of the face grain was placed at other angles to
${ }^{\text {lhis report }}$ is one of a series of progress reports prepared by the Forest Products Laboratory relating to the use of wood in. aircraft. Results here reported are preliminary and may be revised as additional data become available.
${ }^{2}$ Original report dated November 1943.
${ }^{3}$ Maintained at Madison, Wis., In cooperation with the University of Wisconsin.
the load. Modifications were necessary, however, in the layout of specimens and plate sizes, and in the determination of the critical buckiing load of thick specimens.

A modification of specimen layout was necessary to obtain specimens and matched coupons for pack compression tests having the face grain at the same angles to their edges. The layouts were as shown in figure 91.4

The plate sizes were again computed from the elastic properties of the plywood to obtain the length-width ratio for the formation of a buckle in a single half wave. Dimensions of the plates from each panel were as shown in table 15 .

The second modification of the procedure used in the compression buckling tests at $0^{\circ}$ and $90^{\circ}$, for this series of tests at other angles, was an improvement in the technique of observing the critical bucking load of thick specimens when the critical buckling load was greater than 0.75 of the proportional-limit load. It was observed in the effective width tests (Report No. 1316-E) that the rate of increase of the average strain at the center of the plate decreases and that the rate of increase of the average strain at the transverse centerline near the edges increases at and above the critical buckling load. Therefore, in addition to the buckling deflection, strains were measured at three points on both faces of the thick plates, at the center and at $3 / 8$ inch from the edges on the transverse centerline. When the lateral, or buckiing, deflection curve gave no indication of a critical buckling load, the critical buckling load was taken from the strain curves as the load at which the distribution of strain at the center and the average strain at the edges started to change rapidly. A typical example of the method is presented in figure 92, in which the buckling deflection curve and the strain curves are plotted. 2 It may be noted that there is a sudden break in the "center" load-strain curve at a load of 2,800 pounds which is taken as the critical buckling load in this instance.

Except for these two nodifications, the same system of marking and matching of material, and the same buckling-test apparatus and procedure were used in this series of buckling tests in compression at $15^{\circ}, 30^{\circ}, 45^{\circ}$, $60^{\circ}$, and $75^{\circ}$ to the face grain direction as were used in the compression buckling tests at $0^{\circ}$ and $90^{\circ}$ to the face grain direction.

The figures and tables in this supplement are numbered consecutively with those of Report No. 1316 and its supplements, $1316-B,-C,-D$, $-E$, and $-F$.
${ }^{5}$ The Southwell method of determining the critical load is not applicable to flat plates in compression. (Report No. 1316, page 15, and Report No. $1316-\mathrm{D}$, page 4.)

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## Presentation of Data

Data obtained in this series of tests are presented in table 15, togeth er with various computed values and in figures 93 through 102, and 104 through 107. All test data from each panel are tabulated following the panel number given in column 1 of table 15. The test data for the plates are given in columns 2, 3, 4, 6, 11, 12, 13, and 15. When only one plate was obtained from a panel 24 by 24 inches, as for compression at $45^{\circ}$ to the face grain, columns 11 through 19 and column 20 are omitted. Data obtained from the tests of coupons are presented in columns 20 through 23.

In figures 93 through 97, and 104 through 107, are plotted the ratios of the observed critical buckling load to the proportional-limit load against the ratios of the computed buckling loads to the proportionallimit load. Two critical buckling loads were computed for each specimen by the method presented in Report No. 1316, first, (figs. 93-97) using $\frac{k_{c}}{k_{c_{\infty}}}$ assumed equal to $\frac{k_{s}}{k_{b_{\infty}}}$ as given in figure 12 of Report No. 1316, and $\frac{\mathrm{c}_{\mathrm{c} \infty}}{\sec 0 n d, ~(f i g s . ~ 104-107) ~ u s i n g ~ t h e ~ v a l u e s ~ o f ~} \frac{k_{c}}{k_{\mathrm{c}_{\infty}}}$ obtained from these tests as discussed later. Figures 93 and 104 show the results of tests in which the load was applied at $15^{\circ}$ to the face grain direction; figures 94 and 105 at $30^{\circ}$; figure 95 at $45^{\circ}$; figures 96 and 106 at $60^{\circ}$; and figures 97 and 107 at $75^{\circ}$. These figures present graphically an indication of the accuracy of the method used for computing the critical buckling load. Only the data included in the range for which the critical buckling load is from 0.1 to 0.9 of the proportional-limit load are significant in determining the accuracy of the formula (Report No. 1316-D, pages 5 and 6).

For tests within this range, the average ratios of the observed critical buckling load to the critical load computed using $\frac{k_{c}}{k_{c_{\infty}}}$ assumed equal to $\frac{k_{s}}{\underline{k_{s \infty}}}$ are:

Angle of compressive stress to direction of face grain

Observed critical buckling load expressed in percent of computed load

| $15^{\circ}$ | 68 |
| :--- | :--- |
| $30^{\circ}$ | 82 |
| $45^{\circ}$ | 99 |
| $60^{\circ}$ | 92 |
| $75^{\circ}$ | 88 |

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The curves in figures 93 through 97 and these percentages show that the assumption that $\frac{k_{c}}{k_{c_{\infty}}}$ equals $\frac{k_{s}}{k_{s \infty}}$ is correct only for the plates in which the face grain is at an angle of $45^{\circ}$ to the direction of the load, and gives high values of the buckling load for other angles.

The method of computing the critical buckling load is given in Report No. 1316 with the formula for the critical buckling stress

$$
p_{c r}=k_{c} E_{L} \frac{h^{2}}{a^{2}}
$$

In this formula, $h$ is the thickness and $a$ is the width of the plate. These quantities were measured and checked at the time of test and are not questioned. The value of the modulus of elasticity of the individual plies in a direction parallel to the grain ( $\mathrm{E}_{\mathrm{L}}$ ) was determined by static bending tests of coupons and the formula

$$
E_{L}=\frac{20}{21}\left(E_{1}+E_{2}\right)
$$

in which $E_{1}$ and $E_{2}$ are the bending moduli of elasticity of the plywood parallel $\frac{1}{\text { and }} \frac{2}{2}$ perpendicular to the face grain, respectively. This formula assumes that the ratio of the transverse to the longitudinal modulus of elasticity of the individual plies is 0.05 and has given satisfactory results previously. The value of $\mathrm{E}_{\mathrm{L}}$ is, therefore, believed reliable. The only other term in the formula is $k_{c}$. This term is made up of two factors, $k_{c_{\infty}}$ which is mathematically determined by the energy method and applies to panels infinitely long, and a correction factor for length. It is assumed that the correction factor for length is in error. The value of $k_{c}$ was determined for each specimen from the value of $k_{c \infty}$ for each specimen by the use of figure 12 of Report No. 1316. In that figure the ratios of these factors for panels subjected to shear stress applied parallel and perpendicular to the direction of the face grain $\left(\frac{\mathrm{k}_{\mathrm{s}}}{\mathrm{k}_{\mathrm{s} \infty}}\right)$ are plotted as ordinates and the ratio of the length of the panel to the length of a single ideal half wave $\left(\frac{b}{b^{\prime}}\right)$ as abscissas. The ordinates were assumed to be the same for $\frac{k_{c}}{k_{c \infty}}$ b

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This assumption was studied by computing an observed $\mathrm{k}_{\mathrm{c}}$ from the observed critical buckling load, the $E_{L}$, and the plate dimensions. These observed values of $k_{c}$ were divided $\frac{L}{b y}$ the values of $k_{c_{\infty}}$ obtained from Report No. 1316. These ratios were plotted as ordinates and the ratios of $\frac{b}{b^{i}}$ were plotted as abscissas in figures 98 through 102. Also shown in these figures by dashed lines is the curve for $\frac{k_{s}}{k_{\text {S } \infty}}$ from figure 12. It is apparent from a study of these figures (98 through 102) that the ratio of $\frac{k_{c} \text { observed }}{k_{c_{\infty}}}$ does not agree with $\frac{k_{s}}{k_{s_{\infty}}}$ when the load is applied at angles of $15^{\circ}, 30^{\circ}, 60^{\circ}$, or $75^{\circ}$ to the face grain direction.

It was assumed that the values of $k_{c_{\infty}}$ are correct; and that the ratios $\frac{k_{c}}{k_{c}}$ are not equal to $\frac{k_{s}}{k_{s}}$. Therefore a new group of curves similar to: $\frac{\mathrm{k}_{\mathrm{c} \infty}}{\text { figure } 12 \text { of Report } \frac{\mathrm{s}_{\mathrm{s}}}{\mathrm{No}} .1316 \text { were drawn for each angle between load and }}$ face grain. In drawing these curves, it was assumed that the form of the curve given in figure 12 of Report No. 1316 is correct. The new curves were, therefore, obtained from the old one by adjusting its ordinates so that the new curves passed through the average values of the test results and so that the value of the asymptote $\left(\frac{k_{S}}{k_{s \infty}}=1\right)$ was not changed.

This adjustment was made by determining a factor $F$ from each test using the formula

$$
F=\frac{\left(\frac{k_{c} \text { observed }}{k_{c_{\infty}}}\right)-1}{\left(\frac{k_{S}}{k_{S \infty}}\right)-1}
$$

and then finding the average factor for the tests at each grain angle. Using these average factors, the new curves were drawn for each angle with $\frac{k_{c}}{k_{c_{\infty}}}=1+F\left[\left(\frac{k_{s}}{k_{S \infty}}\right)-1\right]$ for all values of $\frac{b}{b^{\prime}}$.

Curves corrected in this way to agree with the observed data are presented in figure 103 as a family of curves for $\frac{k_{c}}{k_{c \infty}}$ at various angles to the face grain direction. The average correction factors, $F$, used are:

Angle of load to Correction factor face grain direction

| $15^{\circ}$ | 0.47 |
| :--- | ---: |
| $30^{\circ}$ | .58 |
| $45^{\circ}$ | 1.00 |
| $60^{\circ}$ | .86 |
| $75^{\circ}$ | .66 |

The critical buckling loads were computed using this method for the determination of $\frac{k_{c}}{k_{c \infty}}$ and are plotted in figures 104 through 107.

## Conclusions

The critical buckling stress of flat plywood plates in compression with the face grain at $15^{\circ}, 30^{\circ}, 45^{\circ}, 60^{\circ}$, and $75^{\circ}$ may be computed by formula (16) of Report No. 1316,

$$
p_{c r}=k_{c} E_{L} \frac{h^{2}}{a^{2}}
$$

if the curves of figure 103 of this report are used for determining $\mathrm{k}_{\mathrm{c}}$ instead of the curve of figure 12. The values obtained by this method agree with the average experimental results obtained.

The curves of figure 103 have not been experimentally determined throughout their entire lengths, but each curve agrees with the average experimental results. The curves were drawn according to the form of the curve given in figure 12 of Report No. 1316, and represent the best available information.

From consideration of the data in figures 98 to 102 , it is evident that values of $\frac{k_{c}}{k_{c \infty}}$ given in figure 103 are in best accord with the experimental data for $^{\text {cor }} \underline{\theta}=45^{\circ}$. The deviation of the data from the curves of figure 103 is greater for $\underline{\theta}=30^{\circ}$ and $\underline{\theta}=60^{\circ}$, and still greater for $\theta=15^{\circ}$ or $75^{\circ}$. For the grain inclinations of $15^{\circ}$ or $75^{\circ}$, the ratio $\bar{k}$ $\frac{\mathrm{k}_{\mathrm{c}}}{\mathrm{k}_{\mathrm{c}}}$ ranges approximately 20 percent on either side of the mean. There$\frac{\mathrm{c}_{\infty}}{\text { fore }}$, the lack of data for a wide range in values of $\frac{\mathrm{b}}{\mathrm{b}^{\prime}}$ and the deviation in the observed data at the tested values of $\frac{b}{b^{1}}$ should be given due consideration in using the curves of figure 103.







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a45 plate for gucklime test, face graim at $45^{\circ}$ to the loaded edges amd to the direction of the load.

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F AMD 6 COUPOHS FOR sTATIC EENDIMO TEST, FACE GRAIM PERPENDICULAR TO SPAN.
O AND P COUPONS FOR pACK cOMPRESSION TEST, fACE GRAIM AT $45^{\circ}$ TO LOAD.

A30 plate for auckling test, face grain at $30^{\circ}$ TO THE LOADED EDGES, OR $60^{\circ}$ TO THE DIRECTIOM OF LOAD.

A A $^{\circ}$ plate for bucklimg test, face grain at $60^{\circ}$ TO THE LOADED EDGES, OR $30^{\circ}$ TO THE DIRECTION OF THE LOAD.
D AMD E COUPONS FOR STATIC BENDING TEST, FACE GRAIM PaRallel to spak.

F AMD 6 COUPONS FOR static bemoimg test, face braly perpendicular to spall.

M AMD W COUPOMS FOR PACY COMPRESSION TEST, FACE GRAIM AT $60^{\circ}$ TO LOAD.

0 and $P$ COUPOMS FOR PACX COMPRESSIOM TEST, FACE graill at $30^{\circ}$ TO LOAD.

A15 PLate for bucklima test, face gainat at 15 ${ }^{\circ}$ TO THE LOADED EDGES, OR $75^{\circ}$ TO TME DIRECTIOM OF LOAD.
A75 PLATE FOR buckling test, face grain at $75^{\circ}$ to the loaded edges, on $15^{\circ}$ to the OIRECTION OF LOAO.

D amo e coupons for static bendine test, face oraill parallel to span.

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M AND $n$ coupons for pack compression test, pace gralk at $75^{\circ}$ TO LOAD.
O AMO P COUPOMS FOR PACK COMPRESSION TEST, FACE gmall at $15^{\circ}$ to load.

Figure 91.-Layout of plate specimens and coupons on panels.



Figure $98 .-$ Observed critical load plotted against the computed critical load, both expressed as ratios to computed proportional limit load. $k_{c} / \mathrm{k}_{\mathrm{c} \infty}$ assumed equal to $\mathrm{k}_{\mathrm{g}} / \mathrm{k}_{\mathrm{s} \infty}$ in computing critical load. Compression at $15^{\circ}$ to face grain.


Figure 94.--Observed critical load plotted against the computed criticel load, both expressed as ratios to computed proportional limit load. $k_{c} / k_{c \infty}$ assumed equal to $k_{g} / k_{s \infty}$ in computing critical load. Compression at $30^{\circ}$ to face grain.


Figure 85. - Obberved oritical load plotted againet the computed critical load, both expressed an ratios to conputed proportional IImit load. $r_{c} / K_{c \infty}$ aseuthed equal to $r_{d} / K_{0} \infty$ in conputing oritical load. Compression at $46^{\circ}$ to face grain.


Pigare 96.-Observed critical load plotted egeinet the computed critical load, both exprideti an ratioe to computed
 ( 18928 F $60^{\circ}$ to face grain.


Figure 97.-Observed eritical load plotted against the computed critical load, both exphemed as ratios to computed proportional limit load. $k_{c} / k_{o \infty}$ asesmed aqual to $k_{s} / k_{s \infty}$ in computing critical load. Compression at $75^{\circ}$ to face grain.

 for compranilom at 15 to face grain.


Figure 90,--observed values of $k_{c} / k_{0 \infty}$, recomended curve fir $k_{e} / k_{c \infty 0}$, and curve for $k_{g} / k_{8}$ for oompresilion at $30^{\circ}$ to fíco srain,


Figure 100.--Observed values of $k_{0} / k_{0 \infty}$, reoomended curve for $\mathbf{k}_{e} / k_{c \infty}$, and ourve for $k_{m} / k_{\infty}$, tor comprestion at $45^{\circ}$ to face grain.
Z Y 48930 F


Figure 101. --Observed values of $k_{c} / k_{c \infty}$, recomenended curve for $k_{0} / k_{c \infty}$, and curve for $k_{a} / k_{a \infty}$ for compresaion $60^{\circ}$ to face grain.


Figure 102. - Obeerved values of $k_{c} / k_{c \infty}$, recomended ourve for $k_{0} / k_{c \infty}$, and curve for $k_{0} / k_{\infty}$ for compression at $75^{\circ}$ to face grain.


Figure 103.-Walues of $k_{c} / k_{c \infty}$ corrected to agree with, observed data for compression at various angles to face grain.


Figure $104 .-k_{c} / k_{c}$ determined by correcting $k_{s} / k_{s}{ }_{\infty}$ in computing critical load for compression Z M 48962 F


Figure 105. $-k_{c} / k_{o \infty}$ determined by correcting $k_{g} / k_{5 \infty}$ in computing critical load for compression at $3 n^{\circ}$ to face grain.


Figure 106. $-k_{c} / k_{c} \infty$ determined by correcting $k_{s} / k_{B} \infty$ in computing critical load for compression at $60^{\circ}$ to face grain.
Z M 48933 F


