Grid-Averaged Surface Fluxes

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(Manuscript received 26 September 1986, in final form 29 December 1986)

ABSTRACT

This study examines the inadequacies of formulations for surface fluxes for use in numerical models of atmospheric flow. The difficulty is that numerical models imply spatial averaging over each grid area. Existing formulations are based on the relationship between local fluxes and local gradients and appear to describe the relationship between the grid-averaged flux and the grid-averaged gradient poorly. For example, area-averaging the bulk aerodynamic relationship reveals additional spatial correlation terms and a complex relationship between the grid-averaged exchange coefficient and the stability based on "model available" grid-averaged variables.

This problem is studied by assuming idealized spatial distributions of the Richardson number over a grid area. Some perspective is provided by consulting observed spatial distributions of the layer Richardson number at the surface. Various contributions to the area-averaged surface flux are studied by employing a small-scale numerical model as a grid box of a larger-scale numerical model. Based on these analyses, a new formulation is proposed for relating the area-averaged flux to the area-averaged gradient. However, this expression cannot be seriously tested with existing observations.

1. Introduction

Numerical models of geophysical flows require parameterization of the transport by all motions which are not resolved by the grid. The parameterization of such "subgrid-scale flux" at the surface is normally based on boundary-layer similarity theory and definition of a surface exchange coefficient.

Formulations of subgrid-scale flux suffer several major problems:

1) They do not explicitly include transport by motions which are larger than turbulent scale but are still small enough to be subgrid scale. These motions include nonlinear gravity waves, cloud-induced motions and flow responding to subgrid terrain and differential heating. Transport by such motions is poorly understood because they are usually observed with significant sampling problems. Since the smallest resolved motion and the largest subgrid-scale motions are of comparable scale and may be strongly interactive, the transport by the largest subgrid-scale motions cannot be simply related to the resolved gradient.

2) The surface is inhomogeneous on subgrid scales. Because the transport by the turbulence is related to gradients and stability in a nonlinear way, the area-averaged flux is not related to the area-averaged gradient in a simple manner. For example, the vertical gradient of the area-averaged potential temperature often corresponds to stable stratification even though the area-averaged heat flux is upward; that is, strong turbulence in small regions of unstable stratification can dominate the area-averaged heat flux which then becomes "counter" to the area-averaged vertical gradient of potential temperature. Small subregions of unstable stratification could result from local terrain elevation, dryer soil, or cloud-free pockets. As a result of similar averaging effects, the downward grid-averaged heat flux in the stable case may be substantially larger than predicted by the stability based on grid-averaged variables.

3) With stronger subgrid-scale inhomogeneity, the turbulence may not achieve equilibrium with the local surface in which case practical representations of turbulence are not applicable.

The above averaging problems are rarely formally recognized in modeling studies. Wyngaard (1982) and others have examined the mathematics of grid-volume averaging for cases where the grid volume is both larger and smaller than the characteristic scale of the turbulence. Sud and Smith (1984) simulate idealized grid-volume averaging by assuming that the surface bulk Richardson number varies within a grid box according to a Gaussian frequency distribution. The surface exchange coefficient for the bulk aerodynamic relationship, which depends on the Richardson number, is then averaged over the hypothetical grid volume. The resulting averaged exchange coefficient exhibits a much smoother dependence on stability which eliminates troublesome numerical oscillations. Further application of the smoothed exchange coefficient is found in Sud and Smith (1985).

Analogous problems have been studied with respect to longer term time averaging. Saltzman and Ashe (1976a,b) have considered contributions to the monthly averaged heat flux due to diurnal and synoptic variations and how such contributions relate to a local flux-gradient formulation. Mahrt et al. (1986) have studied certain averaging problems associated with use of the surface flux relationships with long time steps or omission of the diurnal variation. For the datasets examined, the actual long-term surface heat flux was upward and counter to the long-term vertical gradient of potential temperature.

In the present study, we examine the influence of horizontal averaging on the relationship between the flux and gradients. In particular, we horizontally average the local surface flux relationship to show how the nonlinear dependence of the exchange coefficient on local gradients generally leads to larger flux in the stable case than would be predicted by the usual neglect of spatial averaging.

The formal grid-area averaging of the flux-gradient relationship is developed in section 2. In section 3, the surface exchange coefficient is averaged for idealized distributions of the Richardson number. Low-level aircraft observations in the stable boundary layer are considered in section 4. In section 5, various averaging terms are evaluated by using a mesoscale model and viewing it as a grid box of a larger scale general circulation model. A modified relationship for the surface exchange coefficient is constructed in section 6.

2. Formulation

In numerical models, the flow is automatically divided into the resolved part and the unresolved or subgrid part. The flux divergence, due to the subgrid flow, influences the resolved flow and must be parameterized. In this study, we are concerned with fluxes from the earth's surface through the lowest atmospheric level in the model.

We partition the flow at the lowest model level as

$$\phi(x, y, t) = [\phi] + \hat{\phi}(x, y, t) \tag{1a}$$

where $[\phi]$ is the grid area-average of the local time-averaged part of the flow

$$[\phi] = \frac{1}{\tau A}\int_{dA}\int_{t}^{t+\tau} \phi(x, y, t)dtdA, \tag{1b}$$

and $\hat{\phi}$ is the deviation from the grid-averaged part. Here the independent variables x and y refer to position within the grid area, and t refers to a "fast" time for averaging the local flow. This averaging eliminates turbulent fluctuations corresponding to time scales smaller than τ. The grid-averaged flow is constant in terms of these independent variables, but of course varies on larger space and time scales.

In order to apply existing formulations for the surface fluxes, the subgrid-scale flow must be partitioned into a local time-averaged part $\phi^*(x, y)$ and fluctuations from this time average $\phi'(x, y, t)$ so that

$$\hat{\phi}(x, y, t) \equiv \phi^*(x, y) + \phi'(x, y, t) \tag{1c}$$

where

$$\phi^*(x, y) \equiv \frac{1}{\tau}\int_{t}^{t+\tau} \phi(x, y, t)dt - [\phi]. \tag{1d}$$

The part ϕ' is usually referred to as turbulent fluctuations although in practice ϕ' includes all motions whose time scales are smaller than the averaging time τ. Existing formulations for the surface fluxes are based on fluxes estimated from observations which use averaging times typically between 10 and 30 minutes. The flow component ϕ^* includes all motions on spatial scales larger than the "turbulent" scale (times scale τ) and smaller than the scale of the resolved flow. In global models with grid resolutions of 100 km or more, ϕ^* includes mesoscale motions. The particular partitioning of the flow selected above is not the only possibility but provides a useful framework for identifying the important grid-averaging problems.

The expression for the grid-averaged surface flux using the above decompositions becomes

$$[w\phi] = [w][\phi] + [w^*\phi^*] + [w'\phi'] + [[w]\phi^*] + [w^*[\phi]] + [[w]\phi'] + [w'[\phi]] + [w^*\phi'] + [w'\phi^*].$$

The last six terms on the right-hand side are cross terms, which vanish for simple unweighted time and space averaging; that is, $[w]$ can be pulled outside the space and time integrals defining the operator [ ], w^* can be pulled outside the time integral, the areal integral of w^* vanishes and the time integral of w' vanishes. With weighted averaging such as filtering, the cross terms would not normally vanish (e.g., Charnock, 1957). With simple unweighted averaging, the expression for the grid-averaged surface flux simplifies to

$$[w\phi] = [w][\phi] + [w^*\phi^*] + [w'\phi']. \tag{2}$$

The term $[w][\phi]$ is the resolved flux which is usually converted to advection format by applying incompressible mass continuity to the divergence of the flux. The term $[w^*\phi^*]$ represents the vertical flux due to the time-averaged flow, which varies spatially within the grid area. Near the surface, w^* is normally small except in terrain-related flows. Higher in the boundary layer, the transport term $[w^*\phi^*]$ can be large partly because the spatial (wavenumber) energy gap between the turbulent scales included in ϕ' and the resolved flow, $[w]$, often disappears. The subgrid-scale flux $[w^*\phi^*]$ is so situation dependent that there is no practical way to parameterize it in terms of the resolved flow. As a result, sophisticated formulations for the remaining flux are not justified for use in large-scale models.

To apply existing relationships for the turbulent surface fluxes in familiar form, we symbolize the time-averaging operator with an overbar and using (1d) define the total, local, time-averaged flow as

$$\bar{\phi}(x, y) \equiv \frac{1}{\tau}\int_{t}^{t+\tau} \phi(x, y, t)dt = [\phi] + \phi^*(x, y).$$

It will also be useful to express $[w'\phi']$ as

$$\frac{1}{\tau A}\int_{dA}\int_{t}^{t+\tau} w'\phi' dt dA = \frac{1}{A}\int_{dA}\overline{w'\phi'}dA = [\overline{w'\phi'}].$$

The use of the overbar is redundant but poses the grid-averaging problem in terms of the usual local operators.

With these expressions, we can now relate the surface turbulent flux to the local time-averaged flow using the usual bulk aerodynamic relationships for surface fluxes. The basic problem for numerical models is that such existing formulations relate the local turbulent flux to the local vertical gradient. In numerical models it has been necessary to use the identical relationship for relating the grid-averaged flux to the vertical gradient of the grid-averaged (resolved) flow even though there is no justification for such use. Except for the exploratory study of Sud and Smith (1984), there are no existing formulations for relating the area-averaged flux to the flow gradients resolved by the model. The following analyses will show that the difference between the local flux-gradient relationship and the grid-averaged one is more than a mathematical subtlety.

Although it is not possible to construct a relationship for $[w^*\phi^*]$, it is possible to examine plausible behavior of the area average of the flux due to the "turbulence" $[\overline{w'\phi'}]$. For example, consider the local surface flux formulated with the bulk aerodynamic relationship, which in present notation is of the form

$$(\overline{w'\phi'})_{\rm sfc} = \overline{C_\phi}\,\bar{V}(\bar{\phi}_{\rm sfc} - \bar{\phi})$$

or, in terms of the model resolved flow

$$(\overline{w'\phi'})_{\rm sfc} = ([C_\phi] + C_\phi^*)([V] + V^*) \times \{([\phi_{\rm sfc}] + \phi_{\rm sfc}^*) - ([\phi] + \phi^*)\} \quad (3)$$

where "sfc" refers to the surface value while other variables are defined with respect to the lowest atmospheric level, C_ϕ is the surface exchange coefficient, V is the surface wind speed, and ϕ represents the transported quantity such as heat, moisture or momentum. The surface exchange coefficient depends on the choice of the atmospheric level, such as 10 or 50 m, and is especially sensitive to the stability of the flow. The validity of this relationship (3) in evolving boundary layers is discussed in the next section.

Spatially averaging the surface flux relationship over the grid volume, we obtain

$$\begin{aligned}[(\overline{w'\phi'})_{\rm sfc}] &= [C_\phi][V]([\phi_{\rm sfc}] - [\phi]) \\ &+ [C_\phi^* V^*]([\phi_{\rm sfc}] - [\phi]) + [C_\phi][V^*(\phi_{\rm sfc}^* - \phi^*)] \\ &+ [V][C_\phi^*(\phi_{\rm sfc}^* - \phi^*)] + [C_\phi^* V^*\{\phi_{\rm sfc}^* - \phi^*\}] \quad (4)\end{aligned}$$

where, again, only area-averaged variables are resolved by numerical models. Substitution of this expression into (2) then defines the total vertical flux of ϕ within a grid box. However, only the first term in (2) can be computed in modeling studies and then only with the additional approximation that the spatially averaged exchange coefficient is related to that stability evaluated from spatially averaged variables. Formally,

$$[C_\phi] = \frac{1}{A}\int \bar{C}_\phi(S, z_0)dA, \quad (5)$$

whereas numerical models can evaluate only

$$\tilde{C}_\phi(\tilde{S}, [z_0]) \quad (6)$$

where S is the stability parameter and z_0 is the surface roughness parameter; dependencies on the boundary-layer depth and thermal wind are not considered here. The tilde signifies that the function is computed from variables that are already averaged over the entire grid box as opposed to averaging the function itself. Sud and Smith (1984) have examined the behavior of the area average of the surface exchange coefficient of Deardorff (1972) for a special case where the wind speed and roughness was constant over the grid area and the subgrid variation of vertical temperature gradient obeyed a Gaussian distribution. In their case, $[C_\phi] = \tilde{C}_\phi$. Except for their study, the dependence of $[C_\phi]$ on stability has received little formal attention.

In summary, the formulation of subgrid fluxes in a numerical model suffer three types of errors: (i) omission of the subgrid flux $[w^*\phi^*]$, (ii) omission of the various interaction terms in the expression for the averaged surface flux (4), and (iii) approximation of $[C_\phi]$ in terms of area-averaged variables instead of area-averaging the exchange coefficient. Analogous averaging problems occur with formulation of fluxes at model levels above the surface. In the next section, we examine the spatial averaging of the surface exchange coefficient.

3. Area-averaged exchange coefficient

The area-averaged exchange coefficient may be quite different from the exchange coefficient computed from the stability parameter based on area-averaged variables. This is because the turbulence and exchange coefficient depend on the stability in a nonlinear way. As one example, consider the case where the area-averaged stratification is stable but varies within the grid area. The exchange coefficient predicted by the area-averaged stability may be quite small in stable conditions. However due to the strong nonlinearity of the stability dependence, the area-average of the local exchange coefficient may be significantly larger due to subgrid areas where the stratification is near-neutral or unstable. In these subgrid areas, the local exchange coefficient may be one or more orders of magnitude larger than implied by the spatially averaged stability. Because of the nonlinear dependence of the exchange coefficient on stability, small subgrid regions could have a strong influence on the grid-averaged exchange coefficient and flux but little influence on the grid-averaged stability.

a. Relationship for exchange coefficient

To illustrate such averaging problems, we will adopt the formulation of Louis (1979) for the exchange coefficient for heat. This formulation closely approximates similarity theory but is considerably simpler and, consequently, has found considerable use in large scale models. The exact form of the relationship for the exchange coefficient is not too important provided that it includes the rapid decrease of the exchange coefficient with increasing stability which occurs at near-neutral values of stability. The behavior of the exchange coefficient at strong stability is quite uncertain partly due to flux sampling problems such as those discussed by Wyngaard (1973). The ability to assign roughness values and the occurrence of nonequilibrium conditions over realistic surfaces also reduces the importance of the details of the exchange relationship.

The Louis formulation relates the surface exchange coefficient to the surface bulk Richardson number[1]

$$\mathrm{Ri} = \frac{g}{\theta_0}[\bar{\theta}(z) - \bar{\theta}_{\mathrm{sfc}}]z/(\bar{V}(z))^2 \qquad (7)$$

where $\bar{\theta}_{\mathrm{sfc}}$ is the potential temperature corresponding to the surface temperature, z is the height of the first model level above ground and θ_0 is a basic state temperature scale. The dependence of the Louis exchange coefficient is plotted in Fig. 1 for three different values of z/z_0. The rapid variation of the coefficient at near-neutral stability will dominate the averaging effects.

b. Spatial variation and adjustment

The surface exchange coefficient varies primarily due to interrelated spatial changes of surface temperature, wind speed and surface roughness. Many land surfaces experience continuous changes of surface characteristics whereas previous studies have concentrated primarily on discontinuities of surface properties. Previous studies have also largely neglected variations of surface evapotranspiration which can occur over surfaces which otherwise appear to be homogeneous. This inhomogeneity is forced by variations of vegetation and variations of soil moisture. Soil moisture variations are forced by spatial changes of soil type and small scale precipitation patterns. Such inhomogeneity can force significant spatial variations of surface heat flux, atmospheric stability and boundary-layer development (Ookouchi et al., 1984; Pan and Mahrt, 1987). As a result, all surfaces are potentially inhomogeneous and the problem of adjustment to surface inhomogeneity must be considered before applying existing flux-gradient relationships.

Existing formulations for surface fluxes invoke a form of the flux-gradient relationship; nonequilibrium formulations do not exist. Even though the fluxes and other boundary-layer characteristics may change rapidly downstream from a change of surface properties, existing models assume that a local equilibrium is maintained between the local surface flux and the local vertical gradient. For example, in studies of neutral flow over a discontinuity of surface roughness, the flux-gradient relationship is assumed in the form of the logarithmic relationship as the lower boundary condition (Peterson, 1969; Panchev et al., 1971; Rao et al., 1974; and others).

Observational studies of flow over changes of surface heating and roughness have normally concentrated on the horizontal variation of the depth of the internal boundary layer. Little attention has been devoted to the degree of internal equilibrium between surface fluxes and local vertical gradients. Some studies of internal boundary layers generated by onshore flow have shown that turbulence statistics at relatively short distances from the shore show considerable agreement with similarity relationships based on observations over homogeneous terrain. Such similarity describes much of the data collected by Smedman and Högström (1983) 1500 m from the shore in weakly heated onshore flow. In a study of sea breeze flow 2 km inland, Mizuno

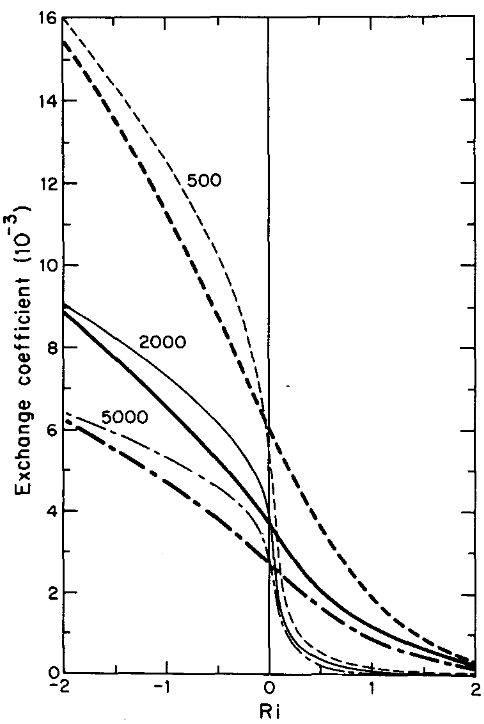

FIG. 1. Dependence of the averaged exchange coefficient on the averaged Richardson number for three different values of z/z_0 for standard deviation of the Richardson number equal to one (thick lines) and the original unaveraged exchange coefficient (thin lines).

[1] Although the Richardson number approaches infinity with free convection, the dependence of the Louis exchange coefficient on the Richardson number is such that the predicted heat flux remains well behaved in the free convection limit.

(1982) also found turbulence statistics to be described by local similarity theory, although the depth of the internal boundary layer exerted a greater influence compared to observations over homogeneous terrain. The local equilibrium implied by the success of similarity theory was argued by Mizuno in terms of small values of the turbulent adjustment time scale compared to the Lagrangian time scale of the mean flow between the coast and observation point.

For example the adjustment length scale for turbulence equilibrium can be posed as

$$L = Ul/u$$

where l is the length scale of the main eddies, probably some fraction of the depth of the internal boundary layer, u is the turbulence velocity scale which can be estimated as the square root of the turbulence kinetic energy and U is a scale value for the speed of the mean flow. For plausible values of l = 100 m, u = 1 m s^{-1}, and U = 5 m s^{-1}, the adjustment length scale is 500 m. For this particular example, the transition region where equilibrium conditions are not approximately valid would be narrow compared to the grid width of most larger scale models. However, it is not known how to apply this information to surfaces with continuous changes of surface conditions. Surface features which are smaller scale than the main transporting motions in the boundary layer will probably not exert an important influence on the overall boundary-layer flux. The influence of such surface variations will be integrated by the main boundary-layer eddies.

While it is not possible to consider realistic surfaces where distinct internal boundary layers are often prevented by complex continuous changes of surface conditions, it is necessary that the idealized subgrid variations are consistent with the application of the local flux–gradient approximation. More specifically, it must be assumed that changes in surface conditions are either sufficiently gradual that the flux–gradient relationship remains a useful approximation or that nonequilibrium regions occupy a small fraction of the grid area. Then the bulk aerodynamic relationship is a useful approximation if one can account for subgrid variations of the exchange coefficient due to horizontal variations of stability, roughness, and boundary layer depth. Here we address the variations of the exchange coefficient due to variations of stability which appears to be the most important influence.

c) Distribution and averaging

Variations within a given grid area depend on geographic location, time of day, season, and synoptic situation. It is not possible nor practical to consider such variations in large-scale models explicitly. Here we consider the probability distribution of a generic grid box. A Gaussian probability density of the Richardson number is suitable for demonstrating the potential importance of averaging errors.

Employing the Gaussian distribution as in Sud and Smith (1985), the averaged exchange coefficient becomes

$$[C_H] = \frac{1}{A}\int \bar{C}_H(\mathrm{Ri})dA = \int \bar{C}_H(\mathrm{Ri})f(\mathrm{Ri})d\mathrm{Ri} \quad (8)$$

where $f(\mathrm{Ri})$ is the assumed Gaussian probability density of the Richardson number over area A where Ri (7) is defined in terms of local time-averaged variables. Relationship (8) assumes nothing about the spatial coherence or pattern of the Richardson number but assumes that the overall distribution is Gaussian. The integral of the right-hand side of (8) was evaluated using Simpson's rule. It was found that using a step size of 0.1σ over a range of $\pm 8\sigma$ provided accurate results in that additional widening of the range or shortening the step size no longer significantly altered the results.

As an example, Fig. 1 shows the area-averaged surface exchange coefficient for a Gaussian distribution of the Richardson number with unity standard deviation and zero mean. The variation of the Richardson number together with the nonlinear dependence of the exchange coefficient cause the spatially averaged exchange coefficient to be significantly larger for stable conditions compared to the values for the original local relationship. In other words, when the area-averaged Richardson number is stable, the spatially averaged exchange coefficient may be dominated by the small part of the area corresponding to the unstable tail of the frequency distribution. In the subareas of near neutral or unstable stratification, the exchange coefficient is much larger than in stable areas and therefore has an important influence on the area-averaged value even if most of the area is stable.

The influence of averaging is minimal for near-neutral average stability where the dependence of the exchange coefficient on the Richardson number is characterized by an inflection point. The averaging influence on the exchange coefficient is percentage-wise small for large instability where the dependence of the exchange coefficient on the Richardson number becomes more linear. Considering the uncertainty of the original formulation for the exchange coefficient, the influence of averaging for the above conditions is probably important only for the stable case.

The enhancement of the exchange coefficient for stable conditions is even greater with larger standard deviation of the Richardson number since larger standard deviation implies that a greater portion of the area will be near neutral or unstable. This is indicated in Fig. 2, where the area-averaged exchange coefficient is shown for different values of the standard deviation.

When the standard deviation of the Richardson number exceeds about one-half, the averaging effect begins to change the behavior of the exchange coefficient completely. This change is partly due to extreme values at deviations greater than one or two standard deviations toward the unstable regime. When the standard deviation of the Richardson number becomes very

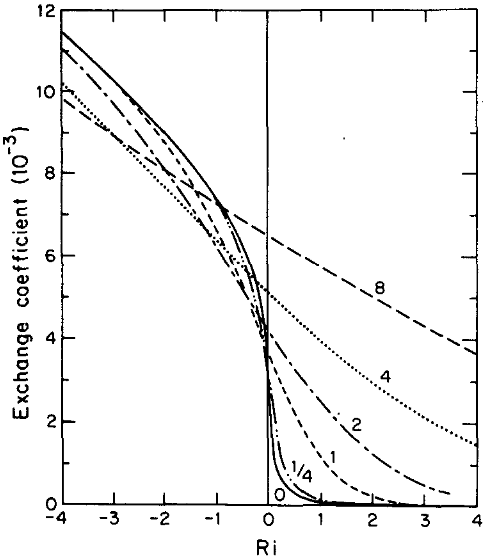

FIG. 2. Dependence of the averaged exchange coefficient on the standard deviation of the Richardson number for z/z_0 = 2000.

large, say greater than five, the dependence of the averaged exchange coefficient on the averaged Richardson number becomes almost linear and the rapid change of the exchange coefficient at near-neutral stability disappears.

The averaging effect could be even greater with significant skewness of the distribution of the Richardson number. However, typical distributions of the Richardson number over a given land area are not known. In the next section, limited information on frequency distributions of the Richardson number is computed from data collected with low-flying aircraft.

4. Observations

We have computed the Richardson number from data collected with low-flying aircraft in the stable nocturnal boundary layer over gently undulating terrain in south central Oklahoma, during quiet periods of the Severe Environmental Storms and Mesoscale Experiment (SESAME). The aircraft instrumentation is described in Mahrt (1985) and Wyngaard et al. (1978).

The Richardson number is computed between the aircraft level and the surface. The surface temperature is estimated from the radiation temperature inferred from the downward pointing radiometer mounted on the aircraft. The estimated error of the surface temperature is thought to be small compared to the temperature difference except for near-neutral conditions. Errors due to the assumption of unity surface emissivity are partially cancelled by the surface reflection of downward longwave radiation. More importantly, the difference between the surface land temperature and the air temperature at $z = z_0$ must be neglected, as in most modeling situations. Such differences may be several degrees with strong cooling or heating.

The surface-based layer Richardson number in analogy with (7) is

$$\mathrm{Ri} = \frac{(g/\theta_0)(\bar{\theta} - \bar{\theta}_{\mathrm{rad}})z}{\bar{V}^2} \tag{9}$$

where the potential temperature θ and wind speed V are averaged over segments of the aircraft record and θ_{rad} is the potential temperature corresponding to the surface radiation temperature. The level z of the horizontal flights ranges between 20 and 100 m for the various legs.

To estimate the "local" average $\bar{\phi}$ in lieu of a time average, the record is divided into 75 m segments (20 observational points) and the Richardson number was computed from variables averaged for each segment. This estimate is useful if the grid width of the intended model is large compared to 75 m. With increased averaging length, the standard deviation of this Richardson number for a given flight leg decreases but the relationship between the Richardson number, its standard deviation and the turbulence intensity does not change appreciably for the data analyzed here.

Statistics for the spatial distribution of the Richardson number were computed for each of the 37 aircraft legs which were typically 15–30 km long. The standard deviation of the Richardson number within a given flight leg increases significantly with increasing stability of the leg (Fig. 3a). As a result of this variability, some subregions of weak stability and significant turbulence are possible even when the averaged stability is large. As a further consequence, the turbulence and turbulence flux are unlikely to vanish even with large averaged stability. Note that in Fig. 3a, the standard deviation is plotted as a function of the mean of the Richardson numbers computed along the flight leg. This mean Richardson number was generally closely related to the mean Richardson number computed from variables averaged along the leg, although one can imagine realistic situations where this relationship would not hold.

The dependence of the standard deviation of the Richardson number on its mean value does not seem to be sensitive to the aircraft flight level although both the standard deviation and mean value tend to decrease with the height of the aircraft level. This decrease is due to the decrease of stratification with height. The correlation between the mean value and standard deviation of the Richardson number is partly due to the fact that the Richardson number is, with a few exceptions, bounded by zero since the surface is almost everywhere cooler than the overlying air for this data. As a result, greater standard deviation of the Richardson number leads to greater mean value and vice versa.

The Richardson number computed from the present data is often characterized by skewness towards large positive values, although this behavior is too erratic to incorporate into the analysis of section 3. On one of the days, the wind speed nearly vanishes leading to extremely large positive Richardson numbers which in

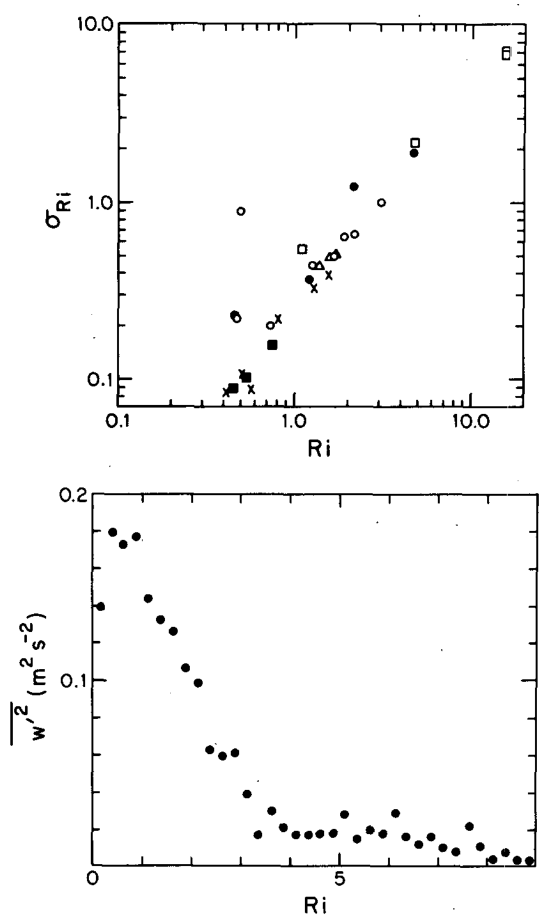

FIG. 3. (a) Standard deviation of the Richardson number for each aircraft leg as a function of the leg-averaged Richardson number for the evening of 4 May 1979 (open circles), early morning of 5 May (open squares), evening of 5 May (triangles), early morning of 6 May (solid circles), early morning of 7 May (crosses), and early morning of 9 May (solid squares). (b) Composited vertical velocity variance as a function of the Richardson number.

turn cause the standard deviation and mean value of the Richardson numbers to be "off scale" for Fig. 3. This occurred in three of the 37 aircraft legs. This behavior is one of the natural, but unfortunate, characteristics of the Richardson number which can lead to misleading statistics.

Since the standard deviation of the Richardson number increases with stability, the most appropriate averaging for the results reported in Fig. 2 would use larger standard deviations at larger Richardson numbers. This has the effect of slowing the decrease of the exchange coefficient with increasing stability as composited over many distributions with different means. The data cannot be used to study directly the relationship between the actual flux and the layer Richardson number. Fluxes computed at the flight level would be contaminated by large sampling problems. Since the fluxes are relatively weak and intermittent under very stable conditions, a much longer record over relatively homogeneous terrain would be needed.

However, it was possible to compute the variance of the vertical velocity as an indicator of turbulence strength for each record segment. The variance is less sensitive to sampling problems. The variance computed for each 75 m segment is due mainly to small-scale turbulence with less influence of gravity waves. The variances were then further averaged for different stability classes based on the value of the Richardson number; each class corresponds to a Richardson number interval of 0.25.

The dependence of the vertical velocity variance on the stability suggests three distinct regimes (Fig. 3b). For weak stability ($Ri < 1$), the strength of the turbulence does not appear to be sensitive to the value of the Richardson number although the class, $0 < Ri < 0.25$, contains fewer cases and may be subject to inadequate sampling. With moderate stability ($1 < Ri < 3$), the turbulence decreases linearly with increasing stability. In the very stable case ($Ri > 3$), the turbulence is quite weak and not very sensitive to the strength of the stability. Apparently, when the turbulence is sufficiently suppressed by the stratification, some residual weak turbulence remains regardless of the strength of the stability. Presumably, this weak turbulence occurs on scales smaller than that used to compute the Richardson number although the nature of motion was not studied.

The critical values of the Richardson number corresponding to the transitions, and the existence of the sharp transitions themselves, may depend upon the way in which the Richardson number is defined. It must be noted that the Richardson number used here is a layer Richardson number in contrast to the local gradient Richardson number where the turbulence is thought to be suppressed for values greater than about 0.25.

For the present data, the transition values did not depend on the choice of averaging length used to define the local average. Using only the lowest aircraft levels (20–35 m), the transitions are sharper and shifted toward slightly smaller values of the Richardson number. With higher flight levels, the transitions are less defined and shifted to significantly larger values of the Richardson number. That is, with stable stratification, the turbulence at higher levels becomes more determined by conditions at that level and less related to surface processes. Furthermore, Richardson numbers computed over thicker layers may be large but still allow for turbulence in thinner sublayers.

Although the relationship between the turbulence variance and the stability depends on the way in which the Richardson number is computed, the above turbulence–stability regimes are similar to those found by Kondo et al. (1978, Fig. 6). The main difference is that in the study of Kondo et al. (1978) the transition to the very stable regime of weak turbulence occurs at smaller Richardson numbers. This is probably related to the fact that in their study, the Richardson number was computed over thinner layers.

In conclusion, the surface-based Richardson number is a useful indicator of turbulence strength in spite of the fact that the Richardson number varies dramatically at low wind speeds. More sophisticated formulations of the averaging problem (8) might take advantage of the well-defined relationship between the mean value and standard deviation of the Richardson number. Even though the above data includes different synoptic conditions, the relationship between the turbulence and the Richardson number should be evaluated over other types of land surfaces.

5. Model evaluation

As an example of the behavior of the flux terms due to subgrid spatial correlations, we have iterated the three-dimensional, four layer, mesoscale model of Han et al. (1982; see, also, Deardorff et al., 1984) for an idealized diurnal cycle.

The entire mesoscale model is viewed as one grid box of a larger scale model so that $[\phi]$ is the average value over the entire mesoscale model, $\bar{\phi}$ is the local time-average evaluated at each grid point and ϕ' is the parameterized "turbulence." The surface turbulent transport is again formulated with the Louis relationship for the surface exchange coefficient. The model includes some surface terrain variations which lead to nocturnal drainage of cold air for cloudless cases of weak ambient flow. In the prototype numerical experiment, the incoming solar radiation varies diurnally leading to a surface heat flux which reaches a daytime maximum of about 0.2 K m s^{-1}. The geostrophic wind is specified to be constant with a nominal speed of 0.1 m s^{-1} to simulate conditions approaching free convection.

For the unstable daytime case, all of the spatial correlation terms in the expression for the grid-averaged flux (4) are small except for the contribution due to spatial correlation between the exchange coefficient and the surface wind speed [second term in Eq. (4)]. This term acts to reduce the total grid-averaged heat flux ($[C_H^* V^*] < 0$), in this case by 30%–40% (Fig. 4a). In other words, where the wind speed is stronger, the instability tends to be significantly less so that the exchange coefficient is significantly smaller.

However, the error due to the neglect of subgrid correlations between wind speed and the exchange coefficient is largely compensated by underestimation of the area-averaged exchange coefficient (Fig. 4b) which appears in the main contribution to the grid area-averaged heat flux [first term in Eq. (4)]. As a result, the model-estimated flux is close to the true grid-averaged flux. Recall that the large scale model can evaluate the exchange coefficient only in terms of the Richardson number based on grid area-averaged variables (6). That is, the exchange coefficient in the large-scale model must be computed as

$$\tilde{C}_H = f(\widetilde{\mathrm{Ri}})$$

$$\widetilde{\mathrm{Ri}} = (g/\theta_0)([\theta] - [\theta_{\mathrm{sfc}}])z/[V]^2 \qquad (10)$$

where f is the function for the dependence of the exchange coefficient on stability. Compared to $\tilde{C}_H$, the true area-average of the exchange coefficient $[C_H]$ is augmented by especially large values occurring at "hot spots" where the instability is enhanced due to larger vertical gradients of temperature and/or weaker winds. The near-cancellation of the important spatial correlation term with errors due to the underestimation of $[C_H]$ can be shown to occur with the Louis expression for the conditions $-\mathrm{Ri} > 10$ and $[\theta^{1/2}] \approx [\theta]^{1/2}$.

During the transitions between stable and unstable periods, the bulk aerodynamic relationship in the large-scale model can easily predict the wrong sign of the

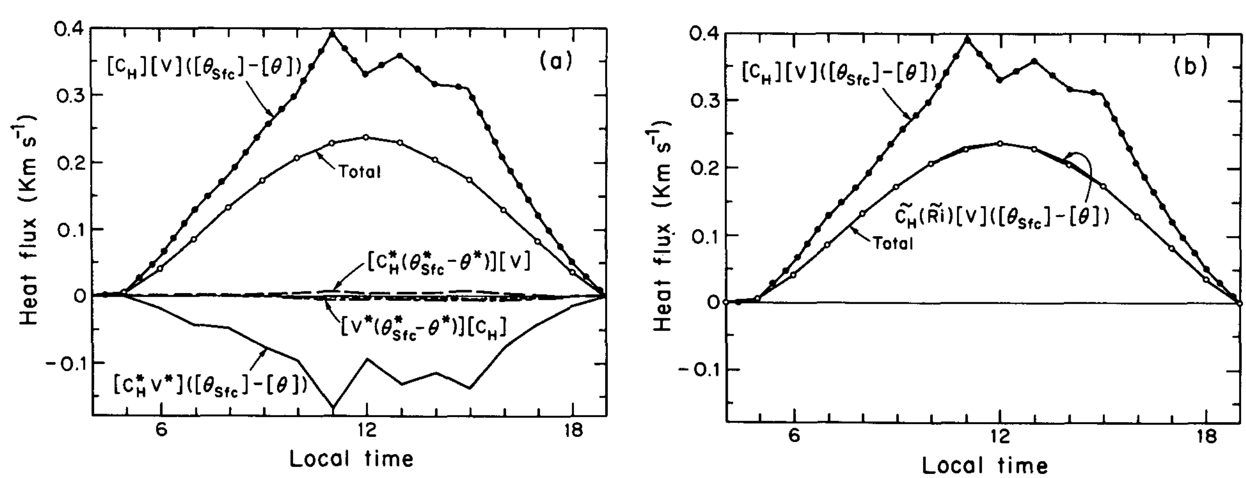

FIG. 4. (a) Various contributions to the grid-area surface heat flux [see Eq. (4)] for the daytime period. The triple correlation term is negligible. (b) The total heat flux computed from Eq. (4) (open circles) as compared to the flux computed from area-averaged variables and area-averaged exchange coefficient (solid circles) and the flux corresponding to the "model available" exchange coefficient computed from $\widetilde{\mathrm{Ri}}$ (solid line with no circles; essentially coincides with line with open circles for this case).

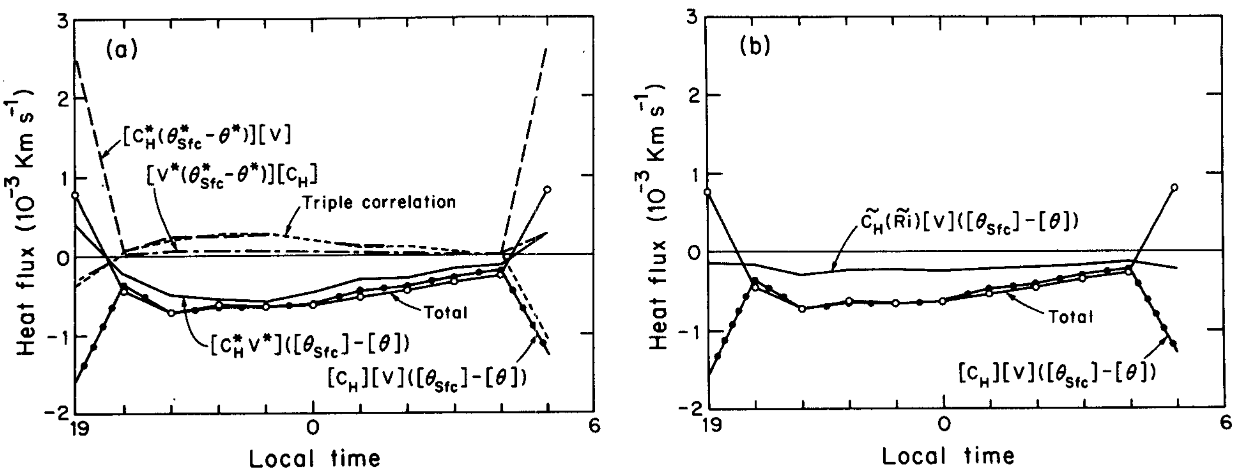

FIG. 5. (a) Various contributions to the grid-area surface heat flux for the nocturnal period. (b) The total heat flux computed from Eq. (4) (open circles) as compared to the flux computed from area-averaged variables and area-averaged exchange coefficient (solid circles) and the flux corresponding to the "model available" exchange coefficient computed from $\tilde{Ri}$ (solid line with no circles). For comparison with Fig. 4, note the factor of 10^3 scale shift for the ordinate.

grid area-averaged heat flux as occurs in Fig. 5b. This averaging problem results from the importance of subgrid correlations between the exchange coefficient and the temperature gradient [fourth term in Eq. (4)]. Large upward heat flux in regions where the vertical temperature gradient is unstable dominates the grid area-averaged heat flux even though the grid average of the vertical temperature gradient corresponds to stable stratification. This "countergradient" heat flux results from the fact that the exchange coefficient is much larger in the small part of the grid area which is unstably stratified. This particular averaging problem appears to be rather short-lived with diurnally varying flow. However, such an effect could exert a longer term influence in situations which are persistently characterized by near neutral stratification. This averaging problem is somewhat analogous to the countergradient heat flux resulting from time-averaging the vertical temperature gradient in the interior of the heated boundary layer (Deardorff, 1966) or time iteration which excludes diurnal variations (Mahrt et al., 1986).

For the strongly stratified nocturnal situation, the relative importance of the various subgrid correlation terms (Fig. 5a) is quite different from the unstable case. Now the wind speed and the exchange coefficient are positively correlated, leading to enhancement of the downward heat flux. That is, the stability is locally reduced in regions of strongest airflow which increases the exchange coefficient.

This increase of the grid-averaged heat flux is opposed by the negative correlation between spatial variations of the exchange coefficient and the vertical gradient of potential temperature. The exchange coefficient is small where the vertical gradient of potential temperature is large. The grid-averaged heat flux is also reduced by the triple correlation term [last term in Eq. (4)] which is physically quite complex.

The spatial correlation terms in (4) sum to near zero for the stable case so that the net modification of the area-averaged heat flux due to the subgrid correlation terms is small. That is, the total flux is close to that predicted by the main contribution to the area-averaged heat flux [first term in Eq. (4)]. However, this main term is underestimated by models due to the fact that the exchange coefficient $\tilde{C}_H$, which is based on the Richardson number computed from area-averaged variables, is considerably smaller than the true area-averaged exchange coefficient $[C_H]$ (Fig. 5b). This underestimation of the downward heat flux would be expected to lead to modeled surface temperatures which are too cold.

These results are for the case of strong radiational cooling at the surface. If the geostrophic wind speed is increased, subgrid variations are reduced. For winds on the order of 10 m s^{-1}, the averaging problems have become negligibly small at least based on the numerical experiments with the mesoscale model used in this study. The generality of these results is not known and Figs. 4–5 must be considered only as examples of potential averaging problems.

The transport by subgrid scale motions associated with local topography is not explicitly reported here because it involves correlations between temperature and downslope flow. The bulk aerodynamic relationship describes heat flux perpendicular to the local ground. In an absolute coordinate system where the vertical coordinate is parallel to the gravity vector and independent of local slope, downslope currents lead to an upward heat flux. Such a heat transport redistributes heat in response to cooling over sloped terrain. Since such differential heating is not included in numerical models on subgrid scales, the inclusion of such redistribution of heat would appear to be inappropriate. The same could be said of the influence of heat transport by daytime upslope currents. However, secondary effects could be important particularly if the upslope currents initiate moist convection.

The estimation of the area-averaged momentum flux is more difficult because of the influence of the terrain-induced pressure drag. This pressure drag is often incorporated by enhancing the drag coefficient or surface roughness length. However, the practice of using the subsequently enhanced surface friction velocity in the formulations for mixing in the boundary layer is not justified since the pressure drag of the topography does not translate into boundary-layer turbulence or at least not in a way which is described by existing boundary-layer theory.

6. Reformulation

Before reformulating the dependence of the exchange coefficient on the Richardson number, several additional complications must be noted. In theory, the turbulence vanishes as the gradient Richardson number exceeds a critical value. In numerical models, this asymptotic possibility should not be invoked, not only because of horizontal averaging problems but also because the Richardson number is computed over a finite depth determined by the model resolution. No matter how large the Richardson number for the grid layer, turbulence in actual atmospheric flows can always be generated over thinner layers where the Richardson number is locally small. The turbulence over such thin layers often occurs intermittently at changing levels so that flux over a deeper layer is established on a time scale which is longer than that of the intermittency.

In addition, momentum can be transported vertically by nonlinear gravity waves while significant vertical transport of heat may be generated by radiational flux divergence, especially with strong surface inversions. Both of these transport mechanisms are generally neglected in large-scale boundary-layer models. Because of these influences, and the significant spatial averaging effects in the stable case, any formulation of the surface flux may suffer large errors. For this reason, complicated schemes attempting to include the details of the influence of stability are not justified.

Both the idealized analysis in section 3 and the specific calculations in section 5 indicate that for the stable case, the exchange coefficient relating area-averaged fluxes to area-averaged gradients should be significantly larger than predicted by the usual expressions for the exchange coefficient. Furthermore, the exchange coefficient should not vanish for a large Richardson number as is implied by the observations reported in section 4, and as previously recommended by Kondo et al. (1978). The case of unstable stratification does not require systematic modification at least based on the above results, although the flux due to mesoscale subgrid motions could be important.

A slower decrease of the value of the exchange coefficient with increasing positive stability can be most simply formulated as

$$C_H(\mathrm{Ri}) = C_{H0}\exp(-m\mathrm{Ri}) \tag{11}$$

where C_{H0} is the value at neutral stability as predicted by the Louis formulation. The idealized averaging results in Fig. 1 suggest that m is a little greater than one for unity standard deviation of the Richardson number. Considering the dependence of the standard deviation on the Richardson number (section 4) and additional influences which enhance the area-averaged flux in stable conditions (section 5 and discussions above), $m = 1$ appears to be a suitable value.

Thus, expression (11) for the stable case and the usual Louis formulation for the unstable case form a tentative model of the surface exchange coefficient which attempts to include the most important qualitative aspects of grid-area averaging. With present lack of understanding, (11) could be applied to the transfer of other quantities in addition to heat. The performance of (11) in a given large-scale model would presumably depend on the details of the model, especially the height of the lowest model level. In the model of Troen and Mahrt (1986), relationship (11) applied to heat, momentum, and moisture leads to the expected enhancement of downward fluxes for the very stable cases although realistic representation requires several levels within the thin nocturnal boundary layer. In models which have been indirectly adjusted to compensate for the underestimation of downward heat fluxes and anomalous cooling, the use of (11) may not be beneficial.

7. Conclusions

The formulation of the subgrid scale flux in numerical models commits three types of errors related to the implied spatial averaging of the grid area. First, the flux due to subgrid motions larger than turbulence scales [see Eq. (2)] is not included. We did not examine this problem here since it is strongly dependent on situation. Second, extra flux terms result from the spatial averaging of the local flux-gradient relationship [see Eq. (4)]. These terms are due to spatial correlation between the locally averaged variables appearing in the flux-gradient relationship. Third, errors result from the necessity of relating the exchange coefficient to resolved grid area-averaged variables instead of spatially averaging the local exchange coefficient as required by the Reynolds averaging. Because these errors can be large, the use of sophisticated local relationships between fluxes and gradients does not appear to be justified for use in large-scale numerical models.

The particular modeling results of section 5 indicate that the important spatial correlation term for the unstable case approximately cancels errors due to the use of existing local formulations for the exchange coefficient. For the stable case, the spatial correlation terms approximately cancel each other while the required grid-averaged exchange coefficient is seriously underestimated. This underestimation is due to relatively large values of the exchange coefficient within the parts of the grid area where the stability is weakest. This problem is important because of the strong nonlinearity of the relation between the exchange coefficient and stability. Even though the absolute magnitude of the surface fluxes is small in the stable case, and probably exerts little influence on the overlying free atmosphere, such small fluxes become important in the surface energy balance and significantly influence the surface air temperature. The convergence of downward turbulent flux of heat occurs over a thin boundary layer in the strongly stratified case and therefore can be locally significant even if the flux magnitude is small. Then use of the usual local flux-gradient relationship and the associated underestimation of the grid-averaged downward heat flux will lead to unrealistically rapid surface cooling.

A revised formulation (11) for the dependence of the exchange coefficient on the Richardson number is constructed for the stable case. The revised formulation is thought to improve significantly the prediction of the grid-area-averaged flux. However, any formulation for the stable case remains tentative due to the incomplete understanding of turbulence with stable conditions and due to the lack of observations of spatial variations of surface fluxes.

Observational verification for the stable case is difficult since fluxes are weak and computed fluxes are often seriously contaminated by sampling problems. The observed relationship between small scale vertical velocity variance and the layer Richardson number indicates three distinct regimes as previously found in Kondo et al. (1978), although the values of the Richardson number at the transitions between regimes depend on the depth of the layer for the computation of the Richardson number. For the weakly stratified case ($Ri < 1$), the turbulence variance does not systematically vary with the Richardson number. For the moderately stratified case ($1 < Ri < 3$), the turbulence strength decreases linearly with increasing Richardson number. For strong stability ($Ri > 3$), the turbulence is weak but is not significantly reduced by further increases of the Richardson number. Even with a large layer Richardson number, turbulent transport may continue over thinner layers, at least intermittently. As a result, the exchange coefficient should not totally vanish with a large layer Richardson number.

Acknowledgments. The author gratefully acknowledges the computational assistance and comments of Paul Ruscher, Kyozo Ueyoshi, and the helpful comments of Michael Schlesinger. This material is based upon work supported by the Air Force Geophysics Laboratory, Cambridge, MA, under Contract F19628-81-K-0046.

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