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The literature cites several examples of investigations made concerning the size of bubbles rising in small particle fluidized beds at room temperature. In many practical applications, however, fluidized beds require the use of large particles at very high temperatures. Despite this fact, little information has been reported on bubble size involving parameters of this nature.

The object of this research was to develop a correlating equation, based on experimental data, which relates the significant parameters influencing bubble growth to the bubble diameter in large particle beds at high temperature.

Experiments were conducted in a rectangular, three dimensional fluidized bed. A stainless steel gas distributor plate was used. Tests
were run for different values of excess gas velocity, static bed height and bed temperature for a single particle size of 2230 μm. Static bed height varied from 0.254 m to 0.483 m and bed temperature was changed from 700 K to 1030 K. High speed cinematography was used to film the surface of the bed while fluidized for those varied conditions. The diameters of the erupting bubble, which showed on the film were then measured.

Computer techniques involving a Quasi-Newton method were employed to obtain the equation which best approximated the data. An experimental uncertainty of ten percent was associated with the data and the correlating equation reproduced the data under equivalent conditions within six percent for the worst case.

In an effort to establish the relevance of the equation to other research, it's predicted values of bubble diameter were compared to the only other known data available at similar conditions. The equation predicted much higher bubble diameters than the results presented by the other researchers. The only significant parameter different between the two studies was the particle size.

With that information, a nondimensional form of the equation was sought which related excess gas velocity, static bed height, bed temperature and particle size to bubble diameter. Such a form was established. A comparison with the other available data showed a substantial improvement in curve fit over the comparison which omitted particle size as a significant parameter. The experimental uncertainty for the nondimensional form of the equation was found to be about eight percent, while the nondimensional form reproduced the data available as
a results of other research within thirteen percent for the worst case.
A Study of Bubble Size in a Large Particle Fluidized Bed at Elevated Temperatures

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NOTATION

$a_{1,2,3}$  Arbitrary constants

$d_b$  Mean bubble diameter (cm)

$d_o$  Mean bubble eruption diameter (cm)

$d_p$  Surface mean particle size (μm)

$g$  Acceleration of gravity (9.81 m/s$^2$)

$H$  Static bed height (m)

$\Delta P$  Pressure drop across the bed (kPa)

$T$  Bed temperature (K)

$U$  Superficial gas velocity (m/s)

$U_{mf}$  Minimum fluidization velocity (m/s)

$\Delta U$  Excess gas velocity (m/s)

---

**Greek Symbols**

$\mu_f$  Absolute viscosity of gas (kg/m.s)

$\rho_f$  Gas density (kg/m$^3$)

$\rho_s$  Solid particle density (kg/m$^3$)

$\nu$  Kinematic viscosity of gas (m$^2$/s)
A STUDY OF BUBBLE SIZE IN A
LARGE PARTICLE FLUIDIZED BED AT
ELEVATED TEMPERATURES

Section I
Introduction
The direct-contact fluidized bed combustor involves the combustion of fuel in a bed of inert granular material such as limestone or dolomite, containing heat exchange tubes immersed in those particles which have been fluidized by the injection of a gas through a distributor system located at the base of the bed. Large particle fluidized beds have received considerable attention recently, especially in light of the developments of fluidized beds using high sulfur content coal as the combustion fuel. Conventional boilers using coal fired burners are currently experiencing difficulty in meeting the emission standards of $SO_x$, $NO_x$ and a number of other adverse gases.

When limestone is used as the bed material, the sulfur dioxide released by the combustion of the fuel undergoes a chemical reaction with the bed and forms a solid sulfate material which is easily disposed. Besides the advantage of reduced pollution problems, the fluidized bed combustor is also able to accept low grade fuel and offers cheaper and more thermally efficient steam generation and heating systems.

For fluidized bed design, many considerations must be explored. The tendency of solids to break or erode, the possibility of an explosion in the gas stream, operating temperature and heat exchanger size are but a few. Perhaps the single most important parameter in design, however, is the bubble size.

There are three different methods currently being used to determine bubble size; a probe submerged in the bed, x-ray techniques and high speed cinematography. A probe submerged in the bed can sense pressure changes over a period of time. The time period can then be related to
the velocity of the bubble by available correlations and yield the thickness of the bubble. With this method, allowances for the situation where the bubble doesn't intersect the probe at its thickest point must be observed.

Other investigators, such as Rowe and Everett [10], have carried out extensive measurements of bubble growth and size with x-ray techniques. In most of the cases using x-ray and probe techniques the bubble size measured at some height above the distributor has been quoted as an average value.

Methods using high speed cinematography (nominal speed 54 frames/second) also depend on an average bubble size. This approach obtains the bubble size by photographing the erupting bed surface, measuring the eruption diameters and then using the well established relationship: \( d_b = \frac{2}{3} d_o \). This technique was used for this study and has been used in the past by others [7]. The eruption diameter \( d_o \) was obtained by taking an average of the lengths of the major and minor axes for noncircular bubble circumferences as shown on the film.

Although the bubble size in fluidized beds has received considerable attention in the last few years, most experimental research has been done on beds using small particles \( d_p < 500 \mu \text{m} \) and temperatures near ambient.

Geldart and Kapoor [4] measured bubble sizes at temperatures of ambient, 473 K and 573 K in a 15 cm diameter tube using spherical steel shot of mean diameter 118 \( \mu \text{m} \), while burning a propane air mixture in the bed. They found that under equivalent conditions, the bubble sizes at 473 K were 75-85% of those obtained at ambient temperature, but no
significant change occurred between 473 K and 573 K. Yoshida et al. [14], using graphite particles ranging from 200 μm to 400 μm, also reported a substantial decrease in bubble size as bed temperature was increased from ambient to 673 K. The comparison of bubble size was made under identical conditions of excess gas velocity (U-U_{mf}) and static bed height.

In addition, Cranfield and Geldart [2] developed a correlation for bubble size in a bed of large particles using room temperature air as the fluidizing gas. As a result of this work, it was discovered that the correlations obtained from small particle bed tests proved to be quite unsatisfactory when applied to large particle behavior.

Fluidized bed combustors typically operate at temperatures near 1100 K and use large particles. In this paper the bubble size is examined in light of its relationship with bed height, bed temperature, and excess gas velocity. A correlation equation which relates these three independent parameters to bubble size for a large particle fluidized bed at high temperatures is presented. Apparently, no study of bubble size in large particle beds at elevated temperatures of this nature has previously been published.
SECTION II

Background
2.1 Effect of Distributor Design on Bubble Characteristics

Heat and mass transfer properties of gas fluidized beds are largely determined by bubble characteristics. In turn, bubble characteristics are largely affected by the design of the gas distributor. In spite of its importance, details of the effect of the distributor design on bubble characteristics remain largely unknown. This may be due in part to the lack of suitable methods for measurement of bubble activity in the region of the bed in close proximity to the distributor. There has been however, a significant amount of research performed concerning distributor plate design using miniaturized capacitance probes and based on statistical data processing. [13].

A multiholed distributor plate was the design used in this research. Theory suggests that gas issuing from the orifices of a distributor plate possess a high kinetic energy due to the relatively large jet velocities involved. The gas jets penetrate about 2 cm into the bed before the energy has dissipated enough to disperse laterally and form bubbles. After the gas has been distributed, the fluid dynamic drag on the localized particles becomes sufficient to counter the gravitational force and form a long narrow void in the bed. This process occurs at 2-1/2 to 3 cm above the distributor.

The void tends to rise slowly with much difficulty, due to its shape and the viscous nature of the bed. At the same time, additional gas is being forced into the void. The energy associated with that gas can not be dissipated sufficiently and is pushed to the side of the void causing a large portion of the bed to be supported entirely by the gas stream. Because this situation is unstable [8], the gas cavity tends to
be broken up and form more stable spherical bubbles which rise with
greater ease due to their streamlined shape. This phenomenon occurs in
the range of 3-5 cm above the distributor plate [2]. Beyond 5 cm bubble
growth is mainly attributed to coalescence.

2.2 Bubble Coalescence

Bubble concentration decreases with increasing bed height due
mainly to the interaction of adjacent bubbles. This interaction is
called coalescence and generally occurs in two different forms; vertical
coalescence and lateral coalescence or what Cranfield and Geldart termed
"cross absorption" [2].

Bubbles tend to rise in preferred paths. Coalescence occurs
between bubbles of neighboring streams and that the vertical distance
classified before interaction with an adjacent bubble depends on their
horizontal separation. It is well established that vertical coalescence
occurs when a "fast" bubble is introduced into the wake of a slower
bubble because of the difference in pressure. Except at the distributor
where vertical alignment of combining bubbles is common place, the
coalescing process generally involves some initial lateral movement.

Darton, et al., assumed that coalescence occurs in stages and
illustrated it in a diagram shown here as Figure 2.1. Between stages
the preferred vertical paths are separated by a distance taken as two
times the radius of the spherical region enclosing the bubble and its
wake [3].

The concept of "fast" and "slow" bubbles were explored by
Catipovic, Jovanovic and Fitzgerald in their study concerning the
Figure 2.1 Bubble paths during coalescence. The process should be visualized in three dimensions.
regimes of fluidization for large particle beds [1]. They defined a "fast" bubble as one where the interstitial gas velocity is moving upward at a rate slower than the bubble itself. Conversely, a "slow" bubble is one where the interstitial velocity of the gas exceeds the vertical velocity of the bubble. They suggested that because the interstitial velocity has to be high to achieve fluidization, slow bubbles tend to appear when large particles are used. Slow bubbles have much smaller wakes which causes less mixing of gas and solids than in small particle beds.

In large particle beds, once bubbles are formed, they tend to rise in horizontally aligned groups. However, if one of the bubbles undergoes some lateral deviation, the leading bubble will grow at the expense of the trailing bubble and in turn reduce the number of bubbles in that group. Cranfield and Geldart attribute this behavior to an interaction of the potential fields associated with each bubble [2]. A typical absorption sequence observed during their study is reproduced in this paper for convenience as Figure 2.2.

The interaction process can be described as follows: As two horizontally aligned bubbles rise, their potential fields are symmetrical and consequently they do not interact (Figure 2.2a). However, when one bubble lags behind due to a difference in diameter (and thus rise velocity) or a change in rise velocity, cross absorption occurs (Figure 2.2b). The absorption, probably due to the distorted potential field associated with the interacting bubbles causes the top of the trailing bubble to assume a blunt shape. As suggested before, bubbles of this shape experience difficulty in rising through the
Figure 2.2 Sketch of equipotential lines in the region of horizontally cross-absorbing bubbles.
viscous bed. This dilemma is alleviated by gas discharging from the region of low voidage to the region of high voidage (Figures 2.2 c-d). This process continues until the trailing bubble is completely absorbed in the leading bubble.

Film tests showed that the deeper the bed became the larger the bubbles grew and the more chance of vertical coalescence. They found that for bed depths of 25 to 30 cm at minimum fluidization velocity the rate of incidences of cross absorption to vertical coalescence was about 2:1. The bed depths used in this research ranged from 25 to 48 cm so that the expected ratio would be less.

2.3 Minimum Fluidization Velocity

As gas velocity is increased from the initial static condition of the bed, a point is reached where the bed media becomes suspended in the ascending gas stream. At this point the frictional force between a particle and the fluid offsets the weight of the particle. The pressure drop across any section of the bed is approximately equal to the weight of the particles and gas in that section. When this occurs, the bed is considered to be just fluidized. The fluid velocity which corresponds to that situation is generally referred to as the minimum fluidization velocity ($U_{mf}$).

The observation of change in pressure drop across the bed is the method that is predominantly being used today to determine $U_{mf}$. For beds with uniformly sized particles, plots of $\Delta p$ versus gas velocity show consistent behavior. For relatively low fluid velocities in packed beds the pressure drop is approximately proportional to the gas flow
Figure 2.3 Sketch of $\Delta P$ versus superficial gas velocity for the cases of uniformly sized particles and for a variance in particle size.
rate as illustrated in Figure 2.3. This occurs until a maximum value of $\Delta P$ is reached (usually slightly higher than $\Delta P$ across the static bed). As $U$ is increased beyond that point, the voidage increases which reduces $\Delta P$ to the pressure drop that occurred at static conditions. With fluid velocities above $U_{mf}$ the bed expands and gas bubbles are formed. Despite the rise in $U$, the bed is well entrained with gas, and is easily deformed without significant resistance.

Minimum fluidization is not as well defined for beds with a variance of particle sizes. Pressure drop stays proportional to fluid velocity until the smaller particles are fluidized; as $U$ is increased the larger particles also experience fluidization. Consequently, the precise change in pressure drop as occurs with uniformly sized particles, is not evident and causes the determination of $U_{mf}$ to be less exact.
SECTION III

Test Facility and Instrumentation
3.1 General Description

The experiment was conducted using the test facility as illustrated schematically in Figure 3.1. Air is compressed in a positive displacement, rotary blower. The compressed air is then directed through a venturi orifice and then on to a propane burner. The burner is attached at the inlet to the refractory lined combustion chamber. The high temperature combustion products are ducted through the inlet plenum and into the bed test section via a distributor plate. The fluidizing gas consisted only of the combustion products from the burner. No combustion takes place in the test section.

A large door to the lower test section allows for maintenance and inspection of any instrumentation which may be required. Due to the size and bulk of the door, it is mounted on a trolley system for easy manipulation. A drain plug is located just above the distributor plate in the test section to allow removal of bed media as required.

A disengaging zone is situated above the bed test section. The cross sectional area of the zone is twice that of the test section and is equipped with a view port which also accommodates the addition of bed media as necessary. The view port is cooled with air from the compressor discharge.

Exhaust gases which exit the disengaging zone are channeled through a 316 SST pipe to a cyclone separator. The cyclone removes particles entrained in the exhaust gas stream and deposits them in a collection bucket located at the base of the cyclone. The exhaust gases then are released to the atmosphere.
Figure 3.1 Schematic illustration of the test facility.
3.2 Design Parameters

The design parameters for the test facility included the following.

Cross Sectional Area of the Bed: 0.61 m x 0.30 m (2 ft. x 1 ft.)

Height of the Primary Test Section: 1.22 m (4 ft.)

Cross Sectional Area of the Bed Disengaging Zone: 0.61 m x 0.61 m (2 ft. x 2 ft.)

Height of the Disengaging Zone: 1.22 m (4 ft.)

Method of Heating the Bed: Propane combustion to preheat fluidizing gas stream.

Design Operating Temperature: 871 °C (1600 F)

Maximum Superficial Velocity: 4.57 m/s (15 ft/s)

Distributor Plate Design: Two perforated 310 SST plates separated by a #16 316 SST screen.

Details of distributor plate design are to follow.

Compressor Design: Positive displacement rotary blower.

3.3 Distributor Plate Design

The Distributor Plate Design consisted of two 310 SST perforated plates separated by a 316 SST mesh. The bottom plate (plate one) was 0.318 cm (1/8") thick and contained 0.635 cm (1/4") diameter holes drilled on 3.175 cm (1-1/4") centers in a square pattern. A detail drawing of plate one is shown in figure 3.2.

The top plate (plate two) was of the same material and design as plate one, however, the holes in the central portion of the plate were drilled with a 0.953 cm (3/8") diameter as opposed to the 0.635 cm diameter used in plate one.

Number sixteen mesh (0.046 cm) 316 SST screen was placed between the two plates. The purpose of the mesh was to prevent bed particles from falling down into the inlet plenum at slumped or low velocity conditions.

The plates and screen were held together by a bolted flange connection between the inlet plenum and the test section in such a way that holes in plate one lay directly beneath the corresponding hole in plate two.

3.4 Air Metering System

The compressed air was metered through a venturi orifice meter. The dimensions of the venturi meter are shown in figure 3.3. There were four upstream and four throat static pressure taps. These were manifolded with the appropriate copper tubing and fittings to enable the average pressure differential across the meter to be determined.

Pressure differentials across the venturi orifice were measured with a
Figure 3.2 Distributor plate design (plate 1) 310 SST sheet, 0.318 cm thick.
Figure 3.3 Schematic illustration of the design of the venturi orifice flow meter.
system of manometers. Located immediately downstream from the venturi orifice flow meter was a pipe spool with taps for the necessary temperature and pressure measurements.

Calibration of the venturi flow meter was accomplished with the aid of a calibrated laminar flow meter and a standard pitot tube. Mr. Clayton Gosmeyer, a graduate student in the Department of Chemical Engineering, Oregon State University, worked on the calibration and consequently developed relationships of flow for given pressure differentials, pressures and temperatures of the combustion gas. Gosmeyer developed the calibration in the form of a Fortran program.

Mr. Alan George, also of Oregon State University, subsequently took the Fortran program and converted it to run on a Hewlett Packard 41-C calculator. Both of these programs are listed in Appendix A.

3.5 Bed Media

Bed particles were ione grain with an average diameter \(d_p\) equal to 2230 µm and density of 2600 kg/m³. The mean particle diameter \(d_p\) was determined by the method illustrated in Kunii and Levenspiel [5], which involves a sieve analysis. A complete analysis of the determination of the mean particle diameter \(d_p\) may be found in Appendix B.
SECTION IV

Model Development
Since the relationship between the three independent variables $T$, $H$ and $\Delta U$ and the dependent variable $d_0$ involved an unexplored area, it was deemed best to establish the form of the equation by hand before employing computer techniques to be assured of a reasonable fit. Standard curve fitting methods were utilized.

Judging from the full logarithmic plots of $d_0$ versus excess gas velocity (Figure C1) for different bed heights and temperatures, it was observed that a change in height seemed to shift the curves vertically while a change in temperature appeared to shift the curves horizontally and that the excess gas velocity was responsible for the curvature. From that point is was necessary to find the dependence of bubble eruption diameter on excess gas velocity.

Various forms were tried. The form which seemed to provide the best reproduction of the curvature was,

$$d_0 = a_1 \left[ 1 + (a_2 \Delta U)^2 \right]^{1/2} \quad (4.1)$$

Where $a_1$ and $a_2$ are arbitrary constants. These constants were determined using a calculator for each of the nine data curves and are given in table C1. Plots were then drawn of bed height and temperature versus $a_1$ and $a_2$. This was done in order to establish any dependence of these two variables on the constants $a_1$ and $a_2$. The plots are shown in figure C2 and a direct dependence of $a_2$ on height is evident. Although temperature seemed to depend on $a_2$ as well, the nonuniformity allowed for no specific judgment of form at that point.

The next form that was tried was,

$$d_0 = a_1 \left[ 1 + (a_2 H \Delta U)^2 \right]^{1/2} \quad (4.2)$$
The same procedure was followed as before and a new set of $a_1$'s and $a_2$'s were developed (table C2). Height and temperature were plotted versus $a_1$ and $a_2$ (figure C3). The graph showed no substantial height dependence on $a_2$ but a significant temperature dependence instead. These observations led to the third form of,

$$d_0 = a_1 \left[ 1 + (a_2 HT \Delta U)^2 \right]^{1/2}$$  \hspace{1cm} (4.3)

After $a_1$ and $a_2$ had been determined to satisfy equation 4.3 (table C3), the necessary plots were drawn (figure C4). Both temperature and height were seen to be virtually independent of $a_2$ for the amount of data involved. There was no consistent pattern to suggest a form.

There was however, a slight dependence of both temperature and height on $a_1$ (as was also implied by the previous plots). To account for this the product of temperature and height was raised to the power of a new constant called $a_3$, and a new relationship was established.

$$d_0 = a_1 (HT)^{a_3} \left[ 1 + (a_2 TH \Delta U)^2 \right]^{1/2}$$  \hspace{1cm} (4.4)

At this point, computer techniques involving a quasi-newton method were applied to determine the unknown constants $a_1$, $a_2$ and $a_3$. The final correlating equation was determined to be,

$$d_0 = 5.92 \ (HT)^{-19} \left[ 1 + (.0031 HT \Delta U)^2 \right]^{1/2}$$  \hspace{1cm} (4.5)

Where bed height ($H$) is in meters, bed temperatures ($T$) is in degrees Kelvin, excess gas velocity ($\Delta U$) is in meters per second and eruption diameter ($d_0$) is in centimeters. Details of the routine are given in appendix D.

As previously mentioned, the literature suggests that the bubble
diameter can be approximated by two-thirds the bubble eruption
diameter. This led to another equation involving the actual bubble
diameter (\(d_b\)),

\[ d_b = 3.95 \cdot (HT) \cdot 19 \left[ 1 + (0.0031 HT \Delta U)^2 \right]^{1/2} \] (4.6)

The bubble diameter (\(d_b\)) is in centimeters and the other parameters
are in the units as mentioned above.
SECTION V

Experiment Results
5.1 Minimum Fluidization Velocity

Using the method described in Article 2.3, the minimum fluidization velocity was determined. A comparison of $U_{mf}$ between experimental results and a correlation presented by Wen and Yu [12] is shown in Figure 5.1. The minimum fluidizing velocity presented by Wen and Yu is given as,

\[ U_{mf} = \frac{\mu}{d_p \rho_g} \left[ (33.7)^2 + 0.0408 d_p^3 \rho_g (\rho_s - \rho_g) g \right]^{1/2} - 33.7 \]  

(5.1)

For nonspherical particles, the particle diameter $d_p$ was defined as the equivalent diameter of a spherical particle with the same volume. It was suggested however, that $d_p$ might be approximated as the mean diameter obtained as a result of a sieve analysis without introducing serious error.

The average particle diameter used for this research was 2230 microns. Wen and Yu submitted that for $d_p$ greater than 500 microns, equation 5.1 gives a standard deviation of 21.3 percent.

With the exception of one point, the results from this experiment seem to indicate that $U_{mf}$ increases with both temperature and static bed height. Although it is evident that gas properties would change with an increase in temperature, there is nothing in present literature which suggests that $U_{mf}$ is dependent on static bed height.

This trend might be explained by the observation that a certain amount of excess gas leakage occurred around the perimeter of the bed. As bed height increased the leaks offered less resistance to flow than did the bed in the emulsion phase. Consequently, more gas flowed
Figure 5.1 Comparison of $U_{mf}$ between the experimental results and the correlation presented by Wen and Yu (12).
through the leaks at the higher static bed height conditions than it did at lower heights, thus giving the "appearance" that $U_{mf}$ actually increased.

Beyond this explanation, it should be noted that all the points in Figure 5.1 lie within the standard deviation associated with equation 5.1 for similar conditions. That uncertainty, coupled with the uncertainty associated with $U_{mf}$ for this experiment (see appendix E), fair agreement with Wen and Yu is evident.

5.2 Experimental Data

A comparison between the measured bubble eruption diameter, as described by table 5.1, and the value given by equation 4.5 is presented in graphical form in Figures 5.2 (a-c) and 5.3 (a-c). Figure 5.2 shows the dependence of bubble eruption diameter ($d_o$) on temperature for various static bed heights and Figure 5.3 illustrates the dependence of $d_o$ on bed height for three different bed temperatures.

In all cases, good agreement is realized between the experimentally determined value and the corresponding value given by equation 4.5. Based on the worst case, equation 4.5 predicted the bubble eruption diameter within 5.2 percent of the experimentally determined value.

The only other known research of this nature was performed by Sitthiphong, George and Bushnell [11]. The results of which is pending publication. Their research was conducted using irregularly shaped particles with mean diameter of 3200 microns. The experiment was carried out over a range of excess gas velocity for four different temperatures and a single bed height of 0.51 m.
<table>
<thead>
<tr>
<th>$d_0$ (cm)</th>
<th>$H$ (m)</th>
<th>$T$ (K)</th>
<th>$\Delta U$ (m/s)</th>
<th>$U_{mf}$ (m/s)</th>
<th>$\mu_f$ (kg/m s)</th>
<th>$\rho_f$ (kg/m$^3$)</th>
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Table 5.1
Experimental Data
Figure 5.2 a-c Comparison of experimental data with equation 4.5 showing temperature dependence of $d_0$. Bubble eruption diameter versus excess gas velocity.
Figure 5.3 a-c  Comparison of experimental data with equation 4.5 showing static bed height dependence of $d_0$. Bubble eruption diameter versus excess gas velocity.
Figure 5.4 Comparison of $d_0$ between equation 4.5 and the experimental results presented by Sitthiphong, George and Bushnell.
In an effort to establish the relevance of the proposed equation, Figure 5.4 was drawn which compares the bubble eruption diameter predicted by equation 4.5 to \( d_0 \) as presented by Sithiphong, et al. at the same conditions. The plot seems to indicate that although equation 4.5 can predict bubble eruption diameter for an average particle size of 2230 microns, it is not valid for any other particle size. In light of this insight an effort was made to nondimensionalize equation 4.5 to include the particle size as a significant parameter.

5.3 Nondimensional Analysis

As has now been established, there are at least four significant parameters involved in determining bubble size: bed height, bed temperature, excess gas velocity and particle size.

The next step was to establish dimensionless groups which relate those particular parameters. The groups that seemed most appropriate were the dimensionless bubble diameter, the reciprocal Reynolds number based on \( U_{mf} \), the Reynolds number based on excess gas velocity (\( U - U_{mf} \)) and the Archimedes number, i.e.;

\[
\frac{d_0}{H} \quad \text{Bubble eruption diameter to bed height ratio} \quad (5.2)
\]

\[
\frac{\nu_f}{\rho_f U_{mf}} \quad \text{Reciprocal Reynolds number based on } U_{mf} \quad (5.3)
\]

\[
\frac{\Delta U d_p \rho_f}{\nu_f} \quad \text{Particle Reynolds number based on } \Delta U \quad (5.4)
\]

\[
gd_p^3 \left( \frac{\rho_s - \rho_f}{\rho_f} \right)^2 \quad \text{Archimedes number} \quad (5.5)
\]

The nondimensionalized form of equation 4.5 which gave the best fit to the experimental data was found to be,
\[ \frac{d_0}{H} = a_1 \left( \frac{\nu_f}{\rho_f} U_{mf} H \right)^{a_3} \left[ 1 + \left( a_2 \frac{\Delta U_{mf}}{g d_p^2 \rho_s} \right)^2 \right]^{1/2} \] (5.6)

Where the group \( \frac{\Delta U_{mf}}{g d_p^2 \rho_s} \) is the quotient of the particle Reynolds number divided by the Archimedes number after the difference of \( \rho_s - \rho_f \) was approximated as simply the density of the solid particles.

A review of Figure 5.4 seems to indicate that for gas velocities close to \( U_{mf} \) the particle size influence on bubble eruption diameter disappears. Equation 5.6 allows for this phenomenon. When \( U \) is equal to \( U_{mf} \) the term under the radical goes to one thus eliminating any particle size dependence. Temperature occurs implicitly in the equation through its effect on the gas properties.

After employing the use of the computer to determine the values for the arbitrary constants \( a_1, a_2 \) and \( a_3 \) the final form of the nondimensionalized equation was given as;

\[ \frac{d_0}{H} = 14.7 \left( \frac{\nu_f}{\rho_f} U_{mf} H \right)^{3.95} \left[ 1 + \left( 3480 \frac{\Delta U_{mf}}{g d_p^2 \rho_s} \right)^2 \right]^{1/2} \] (5.7)

Figure 5.7 shows a comparison of the bubble eruption diameter predicted by equation 5.7 to the values presented by Sitthiphong, et al. Although the slope of the curve has improved, a substantial variation in magnitude still exists between the two sets of curves.

The bubble eruption diameter can be replaced by the product of three-halves times the actual bubble diameter as before, to yield the relation,

\[ \frac{d_b}{H} = 9.8 \left( \frac{\nu_f}{\rho_f} U_{mf} H \right)^{3.95} \left[ 1 + \left( 3480 \frac{\Delta U_{mf}}{g d_p^2 \rho_s} \right)^2 \right]^{1/2} \] (5.8)
Figure 5.5 Comparison of experimental data with equation 5.7 showing temperature dependence of $d_0$. 

\[ \text{EQ. 5.7} \]

$T = 700 \text{K}$  
$T = 870 \text{K}$  
$T = 1030 \text{K}$
Figure 5.6 Comparison of experimental data with equation 5.7 showing bed height dependence of \( d_0 \).
Figure 5.7 Comparison of $d_0$ between equation 5.7 and the experimental results presented by Sitthiphong, George and Bushnell.
SECTION VI

Conclusions and Discussion
Completion of this research allows for a number of conclusions to be drawn.

1. In a large particle fluidized bed the bubble size is dependent on at least four different parameters. The bubble eruption diameter, and hence the bubble diameter, was found to depend directly on bed temperature (or implicitly through the gas properties as with equation 5.7), static bed height and excess gas velocity and inversely with particle size.

2. For the particles used in this research, $U_{mf}$ increased with bed temperature. There also was an apparent dependence of bubble size on static bed height. Although the literature supports a $U_{mf}$ dependence on temperature, it fails to comment on any bed height dependence. Considering the nature of the facility and the uncertainty in the method of determining $U_{mf}$, it would seem that further research is necessary before any conclusions concerning the dependence of $d_o$ on static bed height can be realized.

3. It is evident that the correlations now in existence for bubbles in small particle beds produce substantial error when applied to large particle beds. The present correlations for bubble size in small particle beds give negative curvature when plotted versus excess gas velocity [3, 9], while the results of this study show a positive curvature. This trend was also noted by others investigating large particles [2, 11].

Some of the bubble diameters presented in this report were as large
as 0.6 times the width of the bed (0.3 m). There is some uncertainty concerning the effects of the bed wall on bubble size as suggested by the literature. Mori and Wen [6] suggest that for bubble diameters greater than 0.3 times the bed diameter that wall effects become important, and that the bed is no longer freely bubbling. Rowe [9], however, assumed that bubble growth was not restricted for bubble sizes up to one half the diameter of the bed. In either case, the diameters of a few bubbles reported here exceeded those criteria but it is expected that those exceptions would not alter the results significantly.

Fluidized bed coal combustion will most likely take place at temperatures around 1100 K. The bed temperatures used here covered a range up to 1030 K. It seems a reasonable statement to suggest that the correlating equations developed here would be applicable to beds with temperatures at 1100 K, especially in view of the fact that equation 5.7 seemed to be more accurate for higher bed temperature when compared with available data.
BIBLIOGRAPHY


APPENDICES
Appendix A

PROGRAM UMO (INPUT, OUTPUT, DATAMO, LPM, TAPE4 = INPUT, TAPE5 = DATAMO,
* TAPE10 = LPM)

REAL MUG
G = 980.6
DS = 2.4414
FA = 1.000
F = 1.0342
C = 0.984
SY2 = 3.5
BETA = 0.50505051
READ (4,*) RHOS, PD
10 CONTINUE
READ (5,100) TB, T, DP, P
100 FORMAT (9X,F7.2, 3 (1X, F7.2))
IF (EOF (5) .NE.0.) GO TO 20
TFAC = (0.020*TB -1.29)/(1663.1-0.085*TB)
RHOG = 0.6297/(459.7 + TB)
MUG = 3.934E-4+1.376E-7* (TB -1200.0)
Al = MUG/(PD*RHOG)
A2 = PD* PD* PD* G* RHOG* (RHOS - RHOG) / (MUG * MUG)
A3 = SQRT (33.7* 33.7 + 0.0408* A2)
UMF = A1* (A3 - 33.7)
UMFF = UMF/30.48
P = P*0.4912
P1 = 14.697 + P
P2 = P1-0.03613 * DP
R = P2/P1
SY1 = R ** 1.429
SY3 = (1.-R ** 0.2857) / (1.-R)
SY4 = (1.0 - BETA **4) / (1.0 - (BETA **4) * (R**1.429))
YA = SQRT (SY1* SY2* SY3* SY4)
GA = 39.626/ (T + 459.7)
GZ = GA * P1* 0.068041
WM = 5.983 * C * F * DS * FA * YA * SQRT (DP * GZ)
ACFM = WM/GZ
SCFM = ACFM * 38.0414 * P1/ (T+459.7)
P = P/0.4912
B = SCFM * TFAC
CB = SCFM * 2.0* B) * (459.7 + TB) / 529.7
UO = CB/120.0
UOF = UO/UMFF
WRITE (10,200) RHOS, RHOG, MUG, UMF, UMFF, PD, P, T, TB, DP, WM,
ACFM,
* SDFM,B,DD,UOF

200 FORMAT (1X,F5.2,2 (1X,E12.4),1X,F7.2,1X,F7.3,2 (1X,F6.3),
* 1X,F7.1,1X,F7.0,1X,F6.2,1X,F7.3,2 (1X,F6.1),3(1X,F5.2))
GO TO 10

20 CONTINUE
CALL EXIT
END

Figure A1. Computer program for venturi data processing.
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<td>$T_{bed}$ ($^\circ$F)</td>
</tr>
<tr>
<td>02</td>
<td>$T_{inlet}$ ($^\circ$F)</td>
</tr>
<tr>
<td>03</td>
<td>$P_{inlet}$ (in. Hg)</td>
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<td>$P_{venturi}$ (in. $H_2O$)</td>
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## Program Listing

```
(1) (2) (3) (4)
LBL°VFLOW 0.2857 RCL 06 RCL 01
RCL 01 $y^x$ RCL 06 X X
0.02 + - 0.068041 +
X 1 RCL 07 X X
1.29 X - RCL 04 X
RCL 07 1/X
- STO 09 RCL 11 X
RCL 06 X
14.697 / RCL 09 X
RCL 00 X
STO 05 RCL 00 RCL 08 /
RCL 03 X 36.0414
X + RCL 06
RCL 12 X
RCL 09 X
RCL 04 X RCL 02
0.0361 RCL 08 459.7
X +
X 3.5 459.7
- X
RCL 06 SQRT RCL 10
/ STO 12 STOP
STO 07 RCL 10 RCL 05
1.429 RCL 02 X
$y^x$ 459.7 2
+ X
STO 08 1/X RCL 12
1 RCL 07 39.626 +
```
Appendix B

Ione Particle Sieve Analysis

The average diameter, \( d_p \), of the ione particles was found to be 2230 microns. The calculations were based on the statistical approach found in [5] by using the formula:

\[
d_p = \frac{1}{\sum i \left( \frac{x}{d_{pi}} \right)^i}
\]

Where

- \( d_p \) = Average particle size (\( \mu \)m)
- \( x_i \) = Weight fraction of material in size interval \( i \)
- \( d_{pi} \) = Average diameter of size interval
## TABLE B-1
Sieve Analysis

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<th>Weight in Sieve (g)</th>
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<tr>
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<td>4013.2</td>
<td>0.0079</td>
<td>1.969 x 10^{-6}</td>
</tr>
</tbody>
</table>

$$\sum (x/d_p)_i = 4.484 \times 10^{-4}$$

$$d_p = \frac{1}{\sum (x/d_p)_i} = \frac{1}{4.484 \times 10^{-4}} = 2230 \mu m$$
TABLE C1

List of $a_1$'s and $a_2$'s which satisfy equation 4.1.

<table>
<thead>
<tr>
<th>$H$ (m)</th>
<th>$T$ (K)</th>
<th>$a_1$</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.254</td>
<td>700</td>
<td>15.7</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>870</td>
<td>16.2</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>1030</td>
<td>17.2</td>
<td>0.80</td>
</tr>
<tr>
<td>0.381</td>
<td>700</td>
<td>17.3</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>870</td>
<td>17.9</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>1030</td>
<td>19.4</td>
<td>1.00</td>
</tr>
<tr>
<td>0.483</td>
<td>700</td>
<td>17.5</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>870</td>
<td>18.8</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>1030</td>
<td>20.4</td>
<td>1.25</td>
</tr>
</tbody>
</table>
Figure C1 a-c  Log-Log plot of $d_0$ versus $\Delta U$ for experimental data.
### TABLE C2

List of $a_1$'s and $a_2$'s which satisfy equation 4.2.

<table>
<thead>
<tr>
<th>$H$ (m)</th>
<th>$T$ (K)</th>
<th>$a_1$</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.254</td>
<td>700</td>
<td>15.6</td>
<td>2.53</td>
</tr>
<tr>
<td></td>
<td>870</td>
<td>16.4</td>
<td>2.85</td>
</tr>
<tr>
<td></td>
<td>1030</td>
<td>17.0</td>
<td>3.22</td>
</tr>
<tr>
<td>0.381</td>
<td>700</td>
<td>17.3</td>
<td>2.53</td>
</tr>
<tr>
<td></td>
<td>870</td>
<td>17.8</td>
<td>2.48</td>
</tr>
<tr>
<td></td>
<td>1030</td>
<td>19.4</td>
<td>2.81</td>
</tr>
<tr>
<td>0.483</td>
<td>700</td>
<td>17.0</td>
<td>2.50</td>
</tr>
<tr>
<td></td>
<td>870</td>
<td>18.7</td>
<td>2.39</td>
</tr>
<tr>
<td></td>
<td>1030</td>
<td>18.5</td>
<td>3.57</td>
</tr>
</tbody>
</table>
Figure C2 a  Log-Log plot of $H$ versus $a_{1,2}$ for equation 4.1.

Figure C2 b  Log-Log plot of $T$ versus $a_{1,2}$ for equation 4.1.
**TABLE C3**

List of $a_1$'s and $a_2$'s which satisfy equation 4.3.

<table>
<thead>
<tr>
<th>$H$ (m)</th>
<th>$T$ (K)</th>
<th>$a_1$</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.254</td>
<td>700</td>
<td>15.90</td>
<td>0.0033</td>
</tr>
<tr>
<td></td>
<td>870</td>
<td>16.05</td>
<td>0.0033</td>
</tr>
<tr>
<td></td>
<td>1030</td>
<td>16.90</td>
<td>0.0033</td>
</tr>
<tr>
<td>0.381</td>
<td>700</td>
<td>17.40</td>
<td>0.0036</td>
</tr>
<tr>
<td></td>
<td>870</td>
<td>17.80</td>
<td>0.0031</td>
</tr>
<tr>
<td></td>
<td>1030</td>
<td>19.25</td>
<td>0.0030</td>
</tr>
<tr>
<td>0.483</td>
<td>700</td>
<td>17.25</td>
<td>0.0033</td>
</tr>
<tr>
<td></td>
<td>870</td>
<td>18.65</td>
<td>0.0029</td>
</tr>
<tr>
<td></td>
<td>1030</td>
<td>18.90</td>
<td>0.0033</td>
</tr>
</tbody>
</table>
Figure C3 a  Log-Log plot of $H$ versus $a_{1,2}$ for equation 4.2.

Figure C3 b  Log-Log plot of $T$ versus $a_{1,2}$ for equation 4.2.
Figure C4 a  Log-Log plot of H versus \( a_{1,2} \) for equation 4.3.

Figure C4 b  Log-Log plot of T versus \( a_{1,2} \) for equation 4.3.
APPENDIX D

The following program is a subroutine written to utilize a routine named ZXMIN of the IMSL computer library on file at Oregon State University.

The purpose of ZXMIN is to find the minimum of a function of N variables using a Quasi-Newton method. See reference:


The subroutine LEAST simply calls ZXMIN and allows for the appropriate input and output.
PROGRAM LEAST
COMMON/DATA/M,E(40),D(40),HI(40),AV(40),DU(40),ROF(40),UMF(40)
EXTERNAL FUNCT
CHARACTER FName * 7
INTEGER N,NSIG,MAXFN,IOPT
REAL X(4), H(10), G(5), W(12), F
PRINT*, 'ENTER THE VALUE OF THE NUMBER OF DATA GROUPS, M'
READ*, M
PRINT*, 'ENTER THE VALUES FOR N,NSIG,MAXFN AND IOPT'
READ*, N, NSIG, MAXFN, IOPT
PRINT*, 'INPUT THE INITIAL GUESSES FOR THE UNKNOWN PARAMETERS, X(I)'
DO10 I=1,N
10 READ*, X(I)
PRINT*, 'ENTER THE DATA FILE NAME'
READ '(A)', FName
OPEN (1, FILE=FName)
OPEN (6, FILE='OUTPUT')
C . . .  INPUT THE DATA . .
READ (1, *) (D(I), HI(I), AV(I), DU(I), ROF(I), UMF(I), I=1,M)
C CALL ZXMIN (FUNCT,N,NSIG,MAXFN,IOPT,X,H,G,F,W,IER)
C CALL FUNCT (N,X,F)
WRITE (6,100) IER
K=1
DO 20 I=1,N
WRITE (6,101) K, X(I)
20 CONTINUE
WRITE (6,102) F, G(1), G(2), W(1), W(2), W(3)
C C
C 100 FORMAT ('//','IER = ',I5)
K=1
DO 20 I=1,N
WRITE (6,101) K, X(I)
101 FORMAT ('X(',I1,'= ',F8.3)
K=K+1
20 CONTINUE
WRITE (6,102) F, G(1), G(2), W(1), W(2), W(3)
C C
C 102 FORMAT ('F',4X,'<',E10.3, /,'G(1) <',E10.3, /,'G(2) <',E10.3, /,
2 'W(1) <',E10.3, /,'W(2) <',F10.1, /,'W(3) >',F5.0)
STOP
END
C SUBROUTINE FUNCT (N,X,F)
COMMON/DATA/M,E(40),D(40),HI(40),AV(40),DU(40),ROF(40),UMF(40)
INTEGER N
REAL X(N), F
F=0.0
DO 30 I=1,M
30 CONTINUE
RETURN
END

Figure D1 Subroutine LEAST
ARGUMENTS

Funct - A user supplied subroutine which calculates the function f for given parameter values \( x(1), x(2), \ldots, x(N) \). The calling sequence has the following form: call funct (n, x, f)

Where x is a vector of length n. Funct must appear in an external statement in the calling program. Funct must not alter the values of \( x(i), i=1, \ldots, N \).

N - The number of parameters (i.e., the length of x) (input)

NSIG - Convergence criterion. (input). The number of digits of accuracy required in the parameter estimates. This convergence condition is satisfied if on two successive iterations, the parameter estimates (i.e., x(i), i=1, \ldots, N) agree, component by component, to nsig digits.

MAXFN - Maximum number of function evaluations (i.e., calls to subroutine funct) allowed. (input)

IOPT - Options selector. (input)

IOPT = 0 causes zxmin to initialize the hessian matrix H to the identity matrix.

IOPT = 1 indicates that H has been initialized by the user to a positive definite matrix.

IOPT = 2 causes zxmin to compute the diagonal values of the hessian matrix and set H to a diagonal matrix containing these values.

IOPT = 3 causes zxmin to compute an estimate of the hessian in H.

X - Vector of length n containing parameter values.

On input, x must contain the initial parameter estimates.

On output, x contains the final parameter estimates as determined by zxmin.

H - Vector of length n* (n+1)/2 containing an estimate of the hessian matrix \( \frac{d^2f}{dx(i)dx(j)}, i,j=1, \ldots, n \). H is stored in symmetric storage mode.

On input, if IOPT = 0, 2 or 3 ZXMIN initializes H. An initial setting of H by the user is indicated by IOPT=1. H must be positive definite. If it is not, a terminal error occurs.

On output, H contains an estimate of the hessian at the final parameter estimates (i.e., at x(1), x(2), \ldots, x(N)).

G - A vector of length n containing an estimate of
THE GRADIENT $DF/DX(I), I=1, \ldots, N$ AT THE FINAL PARAMETER ESTIMATES. (OUTPUT)

$F$ - A SCALAR CONTAINING THE VALUE OF THE FUNCTION AT THE FINAL PARAMETER ESTIMATES. (OUTPUT)

$W$ - A VECTOR OF LENGTH $3*N$ USED AS WORKING SPACE. ON OUTPUT, $W(1)$, CONTAINS FOR
I = 1, THE NORM OF THE GRADIENT (I.E., SQRT $(G(1)**2+G(2)**2+\ldots+G(N)**2)$)
I = 2, THE NUMBER OF FUNCTION EVALUATIONS PERFORMED.
I = 3, AN ESTIMATE OF THE NUMBER OF SIGNIFICANT DIGITS IN THE FINAL PARAMETER ESTIMATES.

$IER$ - ERROR PARAMETER (OUTPUT)
IER = 0 IMPLIES THAT CONVERGENCE WAS ACHIEVED AND NO ERRORS OCCURED.
TERMINAL ERROR
IER = 129 IMPLIES THAT THE INITIAL HESSION USED BY ZXMIN IS NOT POSITIVE DEFINITE.
IER = 130 IMPLIES THAT THE ITERATION WAS TERMINATED DUE TO ROUNDING ERRORS BECOMING DOMINANT. THE PARAMETER ESTIMATES HAVE NOT BEEN DETERMINED TO NSIG DIGITS.
IER = 131 IMPLIES THAT THE ITERATION WAS TERMINATED BECAUSE MAXFN WAS EXCEEDED.

Figure D2 List of the variables contained in the Oregon State University IMSL library file ZXMIN.
OUTPUT:

IER = 0
X(1) = 1469.924
X(2) = 3482.041
X(3) = 395.255
F < .618E + 02
G(1) < .551E - 08
G(2) < -.116E - 08
W(1) < .180E - 07 (NORM OF GRADIENT)
W(2) < 323.0 (NUMBER OF EVALUATIONS)
W(3) > 7. (ESTIMATE OF SIGNIFICANT DIGITS IN X)

Figure D3 Output listing to determine the arbitrary constants associated with equation 5.7.
Appendix E

Uncertainty Analysis

An uncertainty analysis was performed on equation 4.5 with the worst case uncertainty of the various parameters as stated in table E1. The velocity measurement proved to be the largest source of error and gave a maximum bubble eruption diameter uncertainty of 9.2 percent. It should also be noted that a certain amount of error occurred in the measured values of \( d_0 \) which went into the formulation of equation 4.5. Based on random checks, the measured value of \( d_0 \) could vary up to about eight percent in addition to the uncertainty associated with the other parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Name</th>
<th>Mean Value</th>
<th>Units</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>Static Bed Height</td>
<td>37.3</td>
<td>cm</td>
<td>0.034</td>
</tr>
<tr>
<td>T</td>
<td>Bed Temperature</td>
<td>870</td>
<td>K</td>
<td>0.010</td>
</tr>
<tr>
<td>( \Delta U )</td>
<td>Excess Gas Velocity</td>
<td>.5</td>
<td>m/s</td>
<td>0.020</td>
</tr>
</tbody>
</table>

An uncertainty analysis was performed on equation 5.7 as well. The worst case uncertainty of the various parameters are listed in table E2. The analysis was based on the assumption that the tabulated gas properties have negligible uncertainty compared to the other measured quantities. The air properties change a negligible amount over the temperature uncertainty associated with this experiment. The analysis
gave a maximum bubble eruption diameter uncertainty of 7.7 percent. The same uncertainty associated with measuring $d_0$ as mentioned above should again be noted.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Name</th>
<th>Mean Value</th>
<th>Units</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>Static Bed Height</td>
<td>37.3</td>
<td>cm</td>
<td>0.034</td>
</tr>
<tr>
<td>$U_{mf}$</td>
<td>Min. Fluidizing Velocity</td>
<td>1.512</td>
<td>m/s</td>
<td>0.039</td>
</tr>
<tr>
<td>$\Delta U$</td>
<td>Excess Gas Velocity</td>
<td>.5</td>
<td>m/s</td>
<td>0.040</td>
</tr>
</tbody>
</table>