

AN ABSTRACT OF THE THESIS OF

David Leroy Goble for the degree of Master of Science in Mechanical Engineering presented on April 29, 1991.

Title: Strain Rate Sensitivity Index Of Thermoplastics
From Variable Strain Rate And Stress Relaxation Testing.

Redacted for Privacy

Abstract approved: _____

Dr. Ernest G. Wolff

Strain rate sensitivity index (m) values of several thermoplastics (HDPE, PP, PMMA, PS, PVC, PC, and PA) were determined at ambient temperature by variable strain rate and stress relaxation methods. Specimens were loaded in tension in the elastic portion of the stress-strain curve at various strain rates and the load was recorded as a function of elongation. Strain rate sensitivity index values were determined from the defining relation $m = [\partial \ln(\sigma)] / [\partial \ln(\dot{\epsilon})]_{\epsilon, T}$ for these data. Specimens were then loaded in tension at constant strain rate to the proportional limit, loading was halted, and load was recorded as a function of time at constant strain.

A numerical algorithm was implemented to minimize the root-mean-square difference between an empirical equation (the Kohlrausch function) and the relaxation response experimental data; i.e., $\Phi(n,\tau) = (1/N \sum_i \{P_o \exp[-(t_i/\tau)^n] - P(t_i)\}^2)^{1/2}$. The characteristic time parameter (τ) and the rate-of-decay parameter (n) were found when $\Phi(n,\tau)$ was minimized. Strain rate sensitivity index values were determined from the relation $m = [\partial \ln(P)]/[\partial \ln(-\dot{P})]_{\epsilon, T}$ for these data. A marked lack of correlation of strain rate sensitivity index values derived from the variable strain rate and stress relaxation methods for some of the thermoplastics tested indicate that different processes are operative during the implementation of each technique. Index values obtained by both experimental methods are explained in terms of the degree of hindrance offered to chain mobility. Index values are predicted based on cohesive energy density (for the variable strain rate technique) and side-chain group molar volume and main-chain group flexibility (for the stress relaxation technique).

**Strain Rate Sensitivity Index Of Thermoplastics
From Variable Strain Rate And Stress Relaxation Testing**

by

David Leroy Goble


**A THESIS
submitted to
Oregon State University**

**in partial fulfillment of
the requirements for the
degree of
Master of Science**

**Completed April 29, 1991
Commencement June, 1991**

APPROVED:

Redacted for Privacy



Associate Professor of Mechanical Engineering, in charge of major

Redacted for Privacy

Head of Department of Mechanical Engineering

Redacted for Privacy

Dean of Graduate School

Date thesis is presented _____ April 29, 1991

Manuscript typed by the author for David Leroy Goble

DEDICATION

This manuscript is dedicated to my father, Clyde Everett Goble (December 27, 1902 - December 23, 1961). Though his education ended after the fifth grade, his belief in the value of education was instilled in me and has sustained me through the rigors of the engineering education process.

ACKNOWLEDGEMENTS

I wish to thank Dr. Ernest Wolff for his guidance on the way to the completion of this manuscript. He was always there to point out my entrapment in the morass of the circular arguments to which I am prone. I will, no doubt, look back with good memories of our Thursday afternoons together. It should be noted that the stimulus for this work had as its origin a laboratory exercise developed by Dr. Peter Burke and Dr. Ernest Wolff.

I also wish to thank Dr. Joel Davis for his assistance in the field of nonlinear minimization. Though he had no hand in an algorithm that would probably not meet any standard for elegance, he is responsible for pointing me in the right direction.

I also wish to thank Dr. Timothy Kennedy, Dr. William McDougal, and Dr. William Warnes for their willingness to preside over my thesis defense. I can only hope that this document and my presentation of it is worthy of their interest.

Finally, I wish to thank the students and faculty with whom I have come into contact during my six years at Oregon State University. They have gotten me through and put me through some of the most interesting and difficult years of my life.

And you know who you are!

TABLE OF CONTENTS

1. Introduction	1
2. Theory Discussion	3
2.1 Strain Rate Sensitivity Index Development	4
2.2 Strain Rate Sensitivity Index Determination	5
2.3 Stress Relaxation Considerations	7
2.4 Stress Relaxation Modeling	9
2.5 Macromolecular Viscoelastic Mechanisms	15
3. Experimental Procedure	22
3.1 Specimen Characterization	22
3.2 System Characterization	23
3.3 Variable Strain Rate Testing Procedure	23
3.4 Stress Relaxation Testing Procedure	24
4. Data Analysis	25
4.1 Variable Strain Rate Data Analysis	25
4.2 Stress Relaxation Data Analysis	27
5. Experimental Results	30
5.1 Variable Strain Rate Testing Results	30
5.3 Stress Relaxation Testing Results	31
5.4 Testing Results Comparison	32

6. Results Discussion	60
6.1 Variable Strain Rate Results Discussion	60
6.2 Stress Relaxation Results Discussion	61
6.3 Strain Rate Sensitivity Index Comparison	66
6.4 Predictive Considerations	71
7. Conclusion	72
References	74
Appendix	77

LIST OF FIGURES

<u>FIGURE</u>	<u>TITLE</u>	<u>PAGE</u>
1.	Strain rate sensitivity index for representative thermoplastic specimens derived from variable strain rate testing.	37
2.	Strain rate sensitivity index for HDPE derived from variable strain rate testing average modulus and strain data.	38
3.	Strain rate sensitivity index for PP derived from variable strain rate testing average modulus and strain data.	39
4.	Strain rate sensitivity index for PMMA derived from variable strain rate testing average modulus and strain data.	40
5.	Strain rate sensitivity index for PS derived from variable strain rate testing average modulus and strain data.	41
6.	Strain rate sensitivity index for PVC derived from variable strain rate testing average modulus and strain data.	42
7.	Strain rate sensitivity index for PA derived from variable strain rate testing average modulus and strain data.	43
8.	Strain rate sensitivity index for PC derived from variable strain rate testing average modulus and strain data.	44

9.	Comparison of the relaxation response utilizing load versus time data with the Kohlrausch modeled response.	45
10.	Normalized relaxation response for PC, PMMA, PA, PVC, PP, PS, and HDPE derived from average parameter data.	46
11.	Strain rate sensitivity index for representative thermoplastic specimens derived from stress relaxation testing.	47
12.	Strain rate sensitivity index for HDPE derived from stress relaxation testing average parameter data.	48
13.	Strain rate sensitivity index for PP derived from stress relaxation testing average parameter data.	49
14.	Strain rate sensitivity index for PS derived from stress relaxation testing average parameter data.	50
15.	Strain rate sensitivity index for PA derived from stress relaxation testing average parameter data.	51
16.	Strain rate sensitivity index for PVC derived from stress relaxation testing average parameter data.	52
17.	Strain rate sensitivity index for PC derived from stress relaxation testing average parameter data.	53
18.	Strain rate sensitivity index for PMMA derived from stress relaxation testing average parameter data.	54

19.	Strain rate sensitivity index derived from variable strain rate testing as a function of cohesive energy density.	55
20.	Strain rate sensitivity index derived from stress relaxation testing as a function of cohesive energy density.	56
21.	Strain rate sensitivity index derived from variable strain rate testing as a function of molar volume.	57
22.	Strain rate sensitivity index derived from stress relaxation testing as a function of molar volume.	58
23.	Strain rate sensitivity index derived from stress relaxation testing as a function of characteristic time.	59

LIST OF TABLES

<u>TABLE</u>	<u>TITLE</u>	<u>PAGE</u>
1.	Mean and standard deviation strain rate sensitivity index values, variance values, and mean correlation coefficient values derived from variable strain rate testing.	34
2.	Mean and standard deviation elastic modulus values and strain rate values derived from variable strain rate testing and mean strain values derived from stress relaxation testing.	35
3.	Mean parameter values, mean and standard deviation strain rate sensitivity index values, variance values, and mean strain values derived from stress relaxation testing.	36

LIST OF APPENDICES

<u>APPENDIX</u>	<u>TITLE</u>	<u>PAGE</u>
1.	Physical, mechanical, and thermal properties of PP, HDPE, PS, PA, PMMA, PC, and PVC thermoplastics.	77
2.	Load versus elongation data of variable strain rate tested thermoplastic specimens.	78
3.	Elastic modulus, strain rate sensitivity index, and correlation coefficient of variable strain rate tested thermoplastic specimens.	103
4.	Load versus time data of stress relaxation tested thermoplastic specimens.	105
5.	Response function parameters, strain levels, and strain rate sensitivity index values of stress relaxation tested thermoplastics.	112
6.	RPL (Reverse Polish LISP) code listing of the program implemented for strain rate sensitivity index determination using data derived from variable strain rate testing.	114
7.	BASIC code listing of the program implemented for Kohlrausch parameter determination using data derived from stress relaxation testing.	115
8.	RPL (Reverse Polish LISP) code listing of the program implemented for strain rate sensitivity index determination using data derived from stress relaxation testing.	117

LIST OF SYMBOLS

<u>SYMBOL</u>	<u>DEFINITION</u>	<u>UNITS</u>
A_o	cross-sectional area	[m ²]
C	dynamic modulus	[Pa]
E	elastic modulus	[Pa]
ΔE	molar bond energy	[J/mole]
k	displacement-to-load conversion factor	[m]
l_o	gage length	[m]
m	strain rate sensitivity index	[numeric]
M	molecular weight	[kg/mole]
n	rate-of-decay parameter	[numeric]
N	degree of polymerization	[numeric]
P	tensile load	[N]
$-\dot{P}$	load relaxation rate	[N/s]
Q	activation energy	[J/mole]
q	variance	[numeric]
r	correlation coefficient	[numeric]
R	gas constant	[J/mole•K]
s	standard deviation	[numeric]
t	time	[s]
T	absolute temperature	[K]
u	chart velocity	[m/s]
U	cohesive energy density	[J/m ³]
v	crosshead velocity	[m/s]
V	sum of pendant group molar volumes	[m ³ /mole]
x	elongation chart displacement	[m]
y	load chart displacement	[m]

δ	elongation	[m]
Δ	percent change	[%]
ε	true strain	[numeric]
$\dot{\varepsilon}$	strain rate	[1/s]
η	viscosity coefficient	[Pa*s]
ν	Poisson's ratio	[numeric]
ρ	density	[kg/m ³]
σ	true stress	[Pa]
$-\dot{\sigma}$	relaxation rate	[Pa/s]
τ	characteristic time parameter	[s]
Θ	pendant group rotation angle	[rad]
Φ	error function	[N]
Ψ	relaxation response spectrum function	[Pa]

Strain Rate Sensitivity Index Of Thermoplastics From Variable Strain Rate And Stress Relaxation Testing

1. Introduction

Consideration of the viscoelastic phenomenon in polymers is of considerable importance to the design process in any application such as filaments in tension, pressure-sensitive adhesives in shear, and seals in compression where the material is subject to loading at a constant level of deformation for a prolonged period of time. In such a situation, the value of the strain rate sensitivity index (m) is a measure of the degree to which the load will have diminished over the time period and is dependent upon the molecular structure of the particular polymer and the environmental temperature. In this instance, the magnitude of a material's index value would be indicative of the performance characteristics of an in situ structure. Since the stress relaxation process involves motion of molecular chains throughout the material, the strain rate sensitivity index derived from this testing process is also indicative of the distribution of chain lengths in the

polymer which is, in turn, indicative of molecular weight distribution. Parenthetically, the degree of branching and the tacticity (isotactic, syndiotactic, atactic) of thermoplastics are also factors in molecular weight distributions.

Time dependent properties are also of interest in the forming of thermoplastics where the material is subject to incremental deformation for transient time periods. In this case, the value of the strain rate sensitivity index is a measure of the change in dynamic stiffness with variations in strain rate. In this instance, the magnitude of a material's index value would be indicative of the deformation characteristics of bulk material undergoing processing.

This treatise compares strain rate sensitivity behavior from variable strain rate testing with strain rate sensitivity behavior from stress relaxation testing for several thermoplastics. The results are evaluated in terms of cohesive energy density, side-chain group molar volume, main-chain group flexibility, and the characteristic time parameter among the seven thermoplastics tested: high density polyethylene (HDPE), polypropylene (PP), polymethylmethacrylate (PMMA), polystyrene (PS), polyvinylchloride (PVC), polycarbonate (PC), and polyhexamethylene apidamide (PA). Thus, a program of tests was undertaken to determine whether variable strain rate and stress relaxation data yield comparable index values. A further goal was the prediction of strain rate sensitivity index values on the basis of intermolecular bonding forces and conformational structure characteristics of the polymer chain.

2. Theory Discussion

The strain rate sensitivity effect can be understood in terms of plastic deformation processes in that the cold working of a strain rate sensitive material requires a higher magnitude of stress to maintain an equivalent strain rate than can be achieved by hot working the material. From this comparison it is evident that materials whose deformation stress requirement is temperature-dependent are also strain rate sensitive.

In thermoplastics, the strain rate sensitivity effect is manifested as the strain rate dependence of the elastic modulus of the material loaded in tension. When the material is loaded at a relatively low strain rate, the molecular chains have sufficient time to adjust to the imposed stress and the modulus value is thus lower than would be the case for the same material loaded at a higher strain rate.

Chanda and Roy [1] reported that virtually all thermoplastics exhibit some degree of room temperature strain rate sensitivity. Thus, since the stress-strain relationship is dependent on strain rate changes, the material response to deformation is characterized as viscoelastic. In this state, a part of the response is that of an elastic solid with a unique stress-strain relationship and no dissipation of deformational energy. The remainder of the response is that of a viscous fluid where the stress state is independent of the strain and dissipation of deformational energy through flow.

2.1 Strain Rate Sensitivity Index Development

The earliest description of the significance of the effect of strain rate sensitivity was that of Nadai and Manjoine [2] for polycrystalline copper. They reported that the logarithm of the tensile strength of the material was proportional to the logarithm of the rate at which the material was strained. They also demonstrated that the effect was heightened at elevated temperature; i.e., the slope to the plot became steeper with increased temperature.

The stress-strain rate behavior of materials at low temperatures and strain rates was reported by Backofen, Turner, and Avery [3] to obey the power law relation

$$\sigma = [C \cdot (d\varepsilon/dt)^m]_{\varepsilon, T} \quad (1)$$

where m is the strain rate sensitivity index ($0 < m < 1$) and C is a dynamic modulus that is a function of temperature, strain, and structure. In this form, linear viscous flow is the limiting case (C is the viscosity) where an index value of unity allows high levels of material deformation with a complete suppression of the necking phenomenon. From this relationship it can be seen that materials whose stress state is temperature dependent are also strain-rate sensitive in that the thermally activated mechanisms that promote extensive elongations are functions of time. Thus, for the low strain rates associated with most tensile testing applications and superplastic deformation ($d\varepsilon/dt \sim 10^{-5} \text{ s}^{-1}$ to 10^{-1} s^{-1}), the thermally activated processes will have sufficient time to operate

and a higher value of strain rate sensitivity index will result in a lower magnitude of stress required to produce an equivalent strain. For the high strain rates associated with most plastic deformation processes ($d\epsilon/dt \sim 10^0 \text{ s}^{-1}$ to 10^3 s^{-1}), a higher value of strain rate sensitivity index will result in a higher magnitude of stress required to produce an equivalent strain. Thus, an increase in strain rate is equivalent to a decrease in temperature for high strain rate processes ($d\epsilon/dt \geq 1$). A result of strain rate sensitivity is that, for materials loaded in tension at low strain rates ($d\epsilon/dt < 1$), a specimen with a higher value of strain rate sensitivity index will exhibit a higher amount of extension than a specimen with a lower index value. Another consequence of strain rate sensitivity is that, for materials loaded in tension to a constant strain, when the normalized evanescent responses as functions of time are compared, a specimen with a higher index value will exhibit a faster rate of relaxation than a specimen with a lower index.

2.2 Strain Rate Sensitivity Index Determination

The strain rate sensitivity index for viscoelastic materials loaded in tension was demonstrated by Hart [4] to be given as

$$m = [\partial \ln(\sigma) / \partial \ln(\dot{\epsilon})]_{\epsilon, T} \quad (2)$$

where it is assumed that the conditions approximate a steady state process. It has been reported [3,5] that, in general, the index is a function of temperature, strain, and strain rate. Backofen et. al. reported that the strain rate sensitivity index was found to be

independent of strain history. They also demonstrated that the strain rate sensitivity index varied directly with both temperature and strain rate below a certain critical temperature for a superplastic alloy. Leterrier and G'Sell [5] reported a similar relationship between the strain rate sensitivity index and temperature in thermosetting polyurethane resin (PUR). They found that the strain rate sensitivity index increases with increasing temperature below the glass transition temperature (T_g) at which point further temperature increase resulted in a decrease in the value of the index. For the relationship between the strain rate sensitivity index and strain rate, they found that, at constant temperature, the index decreased at an exponential rate with increasing strain rate. In addition, they found that, at constant temperature, an increase in the initially imposed strain caused a corresponding increase in the index (especially for strains on the order of 0.001) and that the effect was intensified by an increase in temperature for temperatures below the glass transition temperature.

Determination of the strain rate sensitivity index may, in principle, also be achieved by stress relaxation testing. Hart also demonstrated that if stress is proportional to strain, then stress rate is proportional to strain rate and the index could be determined by plotting $\ln(\sigma)$ as a function of $\ln(-\dot{\sigma})$

$$m = [\partial \ln(\sigma) / \partial \ln(-\dot{\sigma})]_{\epsilon, T} \quad (3)$$

where $\sigma = \sigma(t)$ is relaxed stress in the material as a function of time and $-\dot{\sigma} = -\sigma'(t)$ is the stress relaxation rate.

Though studies of superplastic alloys [6,7] have demonstrated that there is not much deviation between strain rate sensitivity index values obtained from variable strain rate and stress relaxation testing, a search of the literature has revealed no confirmation of the equivalence of Equations 2 and 3 for thermoplastics.

2.3 Stress Relaxation Considerations

In stress relaxation testing, when the material is subjected to a stress state maintained at constant strain, the strain has an elastic component and a viscous component where, in consideration of both the elastic behavior (a rapid initial relaxation response) and the viscous behavior (a slower terminal relaxation response), only the latter becomes more prevalent with increasing temperature. In addition, the relaxation rate can be dependent upon the level of applied deformation. Specifically, a high initial strain can result in a relatively faster decay rate, while a low initial strain can result in a relatively slower decay rate. If this is the case, Leterrier and G'Sell reported that the viscoelastic response is considered linear and the relaxation modulus (E_r) is independent of the imposed strain.

Though Aran [6] described numerous methods that have been utilized for the determination of the strain rate sensitivity index, Hedworth and Stowell [7] cautioned against some methods as exhibiting little correlation to actual physical processes.

Stress relaxation was generally regarded [5-7] as the preferred method for strain rate sensitivity index determination when the objective of the investigation was the correlation of mechanical properties and structural kinetic mechanisms. Leterrier and G'Sell suggested that stress relaxation testing would assess the viscoelastic behavior more appropriately than variable strain rate testing. They reasoned that, as the stress decays, the ratio of the viscous strain component to the elastic strain component increases and the viscoelastic response is enhanced. They also pointed out that, since loading is halted immediately after the proportional limit is reached, the total strain in the specimen is such that there is no significant plastic deformation to mask the viscoelastic response.

Though, in general, the stress relaxation method does yield meaningful results, it should be recognized that the technique is not without liabilities. Hedworth and Stowell have identified problems that exist with the stress relaxation technique which include the finite amount of time required to halt the crosshead, the time delay between the actual loading and the measurement of the loading, and that the halting of the crosshead at higher velocities causes a momentary reverse motion of the crosshead which results in the imposition of an initial compressive strain on the specimen. For these reasons they suggested that the initial data can be susceptible to error and should be weighted accordingly.

They also cautioned that since the strain rate sensitivity index is a function of strain rate, that data for long relaxation time durations not be used in the determination of strain rate sensitivity index values.

In regard to testing machine stiffness, Dieter [8] reported that the stress relaxation method requires that the stiffness of the testing device be much greater than the stiffness of the specimen for accurate results. Nielsen [9] has reported that it is important to compare stress relaxation and strain rate tests at the same strain level, since the stress relaxation modulus is highly dependent on the strain level (especially so in the case of polyhexamethylene apidamide and polyethylene). ASTM testing standards [10] indicate that the imposition of a state of constant strain is difficult to achieve in stress relaxation testing and, as a consequence, considerable care must be taken to maintain a constant strain level in the material being tested.

2.4 Stress Relaxation Modeling

The question as to what model should be employed to approximate the relaxation response is seen as the key issue in the resolution of the problem of the correlation of structural response to mechanical stimuli. In this regard, Halsey, White, and Eyring [11] suggested that though the fit of relaxation data to a general distribution function may provide the means to an end, the parameters derived for the approximation function are not likely to have any physical significance and cannot be viewed as an effective

model of internal processes. In addition, Kolb [12] has cautioned that, though the experimental data might be found to fit a particular distribution function with a high determination coefficient value, this alone is insufficient reason to ascribe a causal relationship between the derived regression coefficients and the mechanisms that produced the physical phenomenon. The objective then, is not only to successfully approximate the relaxation response, but also to employ a function that models the kinetic mechanisms within the material that effect the observed behavior.

Models for stress relaxation $\sigma(\epsilon, \dot{\epsilon}, t, T, \dots)$ have traditionally employed combinations of elastic and viscous elements. An early quantitative model of the viscoelastic behavior of a stressed material was that developed by Maxwell [13] in which the elastic component of the strain (modeled as a time- and temperature-independent linear/Hookean [14] spring: $\epsilon = \sigma/E$) is connected in series with the viscous component (modeled as a time- and temperature-dependent linear/Newtonian [15] frictional damper: $d\epsilon/dt = \sigma/\eta$).

In accordance with the fact that the total strain for this model is the sum of its component strains ($\epsilon_{\text{total}} = \epsilon_{\text{elastic}} + \epsilon_{\text{viscous}}$), Maxwell proposed a differential equation of the form

$$d\epsilon/dt = (d\sigma/dt)/E + \sigma/\eta \quad (4)$$

where E is the elastic modulus [Pa] and η is the viscosity coefficient [Pa*s]. The ratio $\tau = \eta/E$ is the relaxation time [s] and is

the time duration required for the stress to decay to approximately 0.37 (1/e) of the imposed value. The relaxation response of a Maxwell-modeled material to an imposed stress at constant strain (where $d\varepsilon/dt = 0$) and constant temperature was given by the decaying exponential function

$$\sigma(t) = \sigma_0 \cdot \exp[-t/\tau] \quad (5)$$

where σ_0 is the stress [Pa] initially imposed on the material and t is the decay response time of interest [s]. It was by the criterion of relaxation time that Maxwell classified material responses. Thus, for the Maxwell model a small value of relaxation time corresponds to a fast relaxation rate. Those processes which are completed in a short time compared with the relaxation time ($t \ll \tau$) are termed elastic, while those processes which are characterized by a long time compared with the relaxation time ($t \gg \tau$) are termed viscous, and those intermediate processes are appropriately termed viscoelastic. Though the simple Maxwell model provides a relatively fair approximation of viscoelastic relaxation behavior, it does not accurately represent the full spectrum of the relaxation response over time. Another limitation, as pointed out by Mascia [16], is that, since the viscous strain is not completely recovered when the material is unloaded, the Maxwell model cannot be appropriately used to model both stress relaxation and creep behavior in viscoelastic materials. Thus, the Maxwell model does not satisfy what Mascia termed the "material objectivity" criterion.

Another model for the approximation of the relaxation response was the generalized extension of Maxwell model proposed

by Wiechert [17], in which the material is modeled by a number of Maxwell elements coupled in parallel with a Hookean element. The response function of the Wiechert model was given in the summation form by

$$\sigma(t) = \sigma_e + \sum_i \sigma_i \exp[-t/\tau_i] \quad (6)$$

where $\sigma_e = E_e \cdot \epsilon_0$ is the equilibrium stress in the material when the relaxation response has terminated, $\sigma_i = E_i \cdot \epsilon_0$ is the partial stress in the i^{th} element, and τ_i is the relaxation time of the i^{th} element. Tobolsky [18] suggested that the Wiechert model is an adequate representation of the behavior of linear polymers in that, under stress relaxation conditions, the response function allows for the eventual decay of the imposed stress to an unstressed state. In addition, Rudra [19] demonstrated that coefficients for this model can be derived by means of the method of successive residuals and that, in general, three terms are sufficient to model the relaxation response of many materials (relaxation data from such diverse materials as grain dough, animal muscle, fruit flesh, and milk solids yielded a determination coefficient (r^2) on the order of 0.98). The Wiechert model is appealing in that it yields a close approximation to the entire spectrum of relaxation behavior of the material integrated over the entire duration of the response. Unfortunately, as Bates and Watts [20] have pointed out, the use of linear combinations of exponentials gives rise to parameter redundancy where a number of series with different σ_i and τ_i values could be found to represent virtually the same relaxation response.

Thus, the Wiechert model has associated with it a lack of identifiability which gives rise to what Bates and Watts term "bad ill-conditioning". In this regard, Struik [21] contends that "...the spectral representation of mechanical...response functions by a series of exponentials, is merely a mathematical formalism, without physical meaning." and further that "...we have no (molecular) theory of mechanical relaxation...".

Another model for the relaxation response was that introduced by Halsey et. al. (also known as the Zener model) in which a Maxwell element and a Hookean element are connected in parallel. The differential equation for this model is given by

$$d\sigma/dt \cdot \eta/E_s + \sigma \cdot (1 + E_p/E_s) = d\varepsilon/dt \cdot \eta - \varepsilon \cdot E_p \quad (7)$$

where E_s is the elastic modulus of the series spring and E_p is the elastic modulus of the parallel spring. Solution of this differential equation yields a relaxation response function of the form

$$\sigma(t) = \varepsilon_0 \cdot E_r + \varepsilon_0 \cdot E_0 \cdot \exp[-t/\tau_r] \quad (8)$$

where $E_r = E_s \cdot E_p / (E_s + E_p)$ is the relaxation modulus, $E_0 = E_s^2 / (E_s + E_p)$ is the instantaneous modulus, $\tau_r = \eta / (E_s + E_p)$ is the relaxation time parameter, and ε_0 is the strain which has been imposed in the material prior to the loading having been halted. Krausz and Eyring [22] reported that this relaxation response equation is effective in modeling the relaxation response of many polymers. In addition, Mascia reported that this model satisfied the "materials objectivity" criterion.

A contemporary of the Maxwell model was that proposed by Kohlrausch [23] in which an additional parameter is included within the exponential term. The relaxation response function of the Kohlrausch model was given by

$$\sigma(t) = \sigma_0 \cdot \exp[-(t/\tau)^n] \quad (9)$$

where τ is the characteristic time parameter and n is the rate-of-decay parameter. This extended exponential function was employed by Kohlrausch because it is a tractable approximation of the continuous series expansion

$$\sigma(t) = \int_0^{\infty} \Psi(\tau) \cdot \exp(-t/\tau) d\tau \quad (10)$$

where $\Psi(\tau)$ is a function representing the entire spectrum of the relaxation response. In general, the characteristic time parameter is a function of strain and temperature and its magnitude describes the position of the relaxation curve on the logarithmic time scale. The rate-of-decay parameter is, in general, a function of strain, temperature, and molecular weight (M) and its magnitude characterizes the distribution of active relaxation times. Thus, a decrease in the value of the rate-of-decay parameter will cause a corresponding increase in the width of the range of active relaxation times.

In addition to the derivation of the strain rate sensitivity index [4], Tobolsky reported that the results from stress relaxation testing provide data that can be used in the derivation of the relaxation modulus

$$E_r = \sigma(10)/\epsilon_0 \quad (11)$$

where $\sigma(10)$ is the stress in the specimen after ten seconds of relaxation type behavior have elapsed and ϵ_0 is the strain initially imposed in the material. The relaxation response also provides additional information in that the slope of the decay curve at any point is equivalent to the strain rate at that point. The results of stress relaxation testing can also be used to investigate the mechanisms of internal deformation in materials resulting from residual stresses where the thermal kinetics of the viscoelastic response are given by an Arrhenius [24] type relationship

$$\tau = \tau_0 \cdot \exp[Q/(R \cdot T)] \quad (12)$$

where τ_0 is a constant that represents the relaxation time at high temperature [s], Q is the activation energy [J/mole], R is the gas constant [8.3145 J/mole·K], and T is the absolute environmental temperature [K]. In practical terms, the test data can be used to determine the duration of annealing time and the temperature level required to thermally relieve any stresses brought about by material deformation.

2.5 Macromolecular Viscoelastic Mechanisms

The mechanism for the initial relaxation response in thermoplastics is the rotation and translation of the long-chain molecular bonds into the configurations that were their equilibrium positions prior to deformation. In general, the deformation associated with the elastic component of the relaxation

phenomenon is recoverable because the secondary van der Waals bonds that exist between the long-chain molecules have remained intact.

The mechanism for the subsequent relaxation response in thermoplastics is viscous flow in which there is molecular motion throughout the material. The viscous flow rate depends, in general, on the molecular structure, the strain history, the relative humidity, the environmental temperature, and the time duration.

In the unstressed state, the long polymer chains are entangled (a high probability configuration) to a degree dependent on molecular orientation and degree of polymerization. In this state the system is in a minimum free energy and maximum entropy equilibrium condition. When the material is subjected to an imposed stress, there is a general molecular motion which is expressed as chain stretching brought about by bond stretching and bond angle distortion (the elastic component), disentanglement and linearization of the polymer chains (a lower probability configuration), and the breaking and reforming of the secondary bonds between the molecular chains. The free energy of the system is increased, the entropy is decreased, and a non-equilibrium condition results. As reported by Chanda and Roy, the linearization of the chains can occur both with and without bond breaking. The portion that occurs without the secondary bonds being broken is elastic and recoverable, while the portion that results in the relative displacement of one chain with respect to another is plastic and permanent. In the stressed state at constant strain

(stress relaxation condition), there is a tendency for the polymer chain to return to the maximum entropy tangled orientation of the pre-stressed state. The result is the dissipation of the increased free energy in the form of heat and a relaxation of the imposed stress due to thermal motion of the polymer molecules.

Thus, the molecular motion responsible for the relaxation response is thought to be accomplished in an amorphous polymer by means of the linearized molecular chains recoiling and reentangling until the original configuration is realized. In theory, in an amorphous polymer, the imposed strain will eventually be reduced to a zero level, while in a crystalline polymer, some residual plastic stress will be retained.

In regard to addition type polymers, there are several factors which influence the degree to which movement of the molecular chains can occur. As Hertzberg [25] has described, pendant groups are conformally configured about the covalently bonded carbon-carbon primary chain in such a manner as to minimize the potential energy of the system. For the addition polymers, this requirement is achieved by the situation where the pendant groups are seen to be rotated relative to each other (the trans-configuration) when viewed on end. In this case, the rotation angle (Θ) varies as the sequence $0, 2\pi/3, 4\pi/3, 6\pi/3, \dots$ when proceeding along the chain and the interference of one side group with another is minimized. The potential energy of the system is maximized when the pendant groups do not alternate but instead, when viewed on end, are seen to eclipse each other (the cis-configuration). For this situation, the rotation angle follows

the sequence $\pi/3$, π , $5\pi/3$, $7\pi/3$, ... and the pendant groups are juxtaposed in such a manner as to provide more of an impediment to the motion of the molecular chain. A configuration of this type can be characterized as offering maximum steric hindrance to pendant group rotation. Thus, the facility with which the chains move relative to one another is governed by the magnitude of the potential energy barrier of the energetically unfavorable cis-configuration. Factors which influence the ease of rotational movement about the carbon-carbon bond are the size, complexity, and polarity of the pendant groups. Specifically, Hertzberg reported that, in general, it is expected that those molecular chains with smaller, less complex, and less polar side-chain constituents will exhibit greater main chain mobility and be able to move with greater ease relative to adjacent chains. Conversely, it is expected that those chains with larger, more complex, and more polar side-chain constituents will be more restricted in their movement.

From this general analysis of the factors which influence steric hindrance, it can be expected that an addition polymer with a smaller, less complex, and less polar side-chain constituent will relax at a faster rate than one with a larger, more complex, and more polar side-chain constituent and thus will have a relatively higher value of strain rate sensitivity index. As the data reported by Tobolsky suggests, for the addition polymers, there is a correspondence between side-chain constituent size and/or complexity and the relaxation modulus in that a thermoplastic with a small and/or less complex side-chain constituent will have a corresponding small relaxation modulus value.

Thus, theory predicts (in part) that the strain rate sensitivity index and the relaxation modulus will exhibit an inverse relationship.

In a condensation polymer, steric hindrance can also be enhanced by a different type of linear bond. The carbon-carbon bond of the addition polymer is replaced by a main-chain bond that can exhibit a greater or lesser degree of flexibility. Thus, in addition to the impediment to motion of pendant groups, the existence of more rigid main-chain molecules supports an argument for a relatively slower relaxation response in some condensation polymers compared with the response of a typical addition polymer. In addition, as Hertzberg has mentioned, condensation polymers exist (polyhexamethylene adipamide in particular) whose pendant groups are highly polar and thus retard chain motion by the formation of strong bonds between the pendant groups in adjacent chains.

Degree of polymerization (N) is directly proportional to the molecular weight of a polymer. Unfortunately, it is characteristic of polymers that there is always some variation in the molecular weight of a particular polymer type (dependent upon the monomer and polymerization conditions) so as to yield a distribution of molecular weight values. Such distributions of molecular weight in polymers are characterized as polydisperse. In regard to chain length and orientation, polymer processing is also to some degree a random process so as to yield a distribution of chain lengths and orientations.

Variations in density for a particular addition polymer are a function of pendant group location along the backbone of the carbon-carbon chain. Thus, an addition polymer whose side-group constituents are randomly arranged (atactic configuration) will, in general, have a lower packing efficiency and density than an addition polymer whose side-group constituents are symmetrically arranged (isotactic or syndiotactic configurations). In addition, density variations also occur due to the degree of main chain branching exhibited by a particular addition polymer. In general, extensive branching reduces the packing efficiency with a consequent density reduction in addition polymers. Thus, an addition polymer with symmetrically arranged pendant groups and a low degree of branching can be characterized as crystalline and will be expected to exhibit a corresponding high density. Conversely, an addition polymer with randomly arranged pendant groups and a high degree of branching can be characterized as amorphous and can be expected to exhibit a corresponding low density. Thus, it can be seen that density provides a measure of the degree of crystallinity for addition polymers.

As the data reported by Tobolsky suggests, the more dense and crystalline isotactic and syndiotactic forms of an addition polymer exhibit higher values of relaxation modulus than the less dense and amorphous atactic forms. Thus, it is also the case that density measurements are indicative of the stiffness that can be expected for addition polymers when subjected to stress relaxation testing conditions.

In regard to factors that influence the viscoelastic properties of thermoplastics, Billmeyer [26] has suggested that it is the magnitude of the cohesive energy density $U = \Delta E/V$ associated with a particular molecular structure that acts as the primary restraint on the free rotation of pendant groups about the carbon-carbon single bonds in the polymer chain and hence, the primary hindrance to long-chain flexibility. Since cohesive energy density (energy per unit molar volume required to disassociate a molecule) is a function of intermolecular bonding forces, it is the strength of the dipole, dispersion, and induction forces that most profoundly influence molecular mobility within a polymer. In addition, the size and complexity of the pendant groups are factors which influence the ease of rotational movement about carbon-carbon single bonds in the polymer chain. In this case, it is considered probable that the sum of pendant group molar volumes $V = \sum_i (V_p)_i$ [27] is a very strong contributory factor in the steric hindrance mechanisms involved in the rate at which molecular reorganization processes evolve. From this analysis of the factors which influence chain flexibility, it can be expected that a thermoplastic with smaller, less complex, less polar side-chain constituents and more flexible main-chain constituents will relax at a faster rate than one with larger, more complex, more polar side-chain constituents and less flexible main-chain constituents.

3. Experimental Procedure

3.1 Specimen Characterization

The thermoplastics tested were high density polyethylene (HDPE $-\text{CH}_2\text{CH}_2-$), polyvinylchloride (PVC $-\text{CH}_2\text{CHCl}-$), polystyrene (PS $-\text{CH}_2\text{CH}[\text{C}_6\text{H}_5]-$), polymethylmethacrylate (PMMA $-\text{CH}_2\text{C}[\text{CH}_3][\text{COOCH}_3]-$), polypropylene (PP $-\text{CH}_2\text{CH}[\text{CH}_3]-$), polyhexamethylene apidamide (PA $-\text{NH}[\text{CH}_2]_6\text{NHCO}[\text{CH}_2]_4\text{CO}-$), and polycarbonate (PC $-\text{C}_6\text{H}_4\text{C}[\text{CH}_3]_2\text{C}_6\text{H}_4\text{OCO}_2-$). Tensile specimens (eight of each for HDPE, PVC, PMMA, PA and six of each for PS, PP, PC) were machined from extruded rod stock in accordance with the ASTM standard [28]. The specimens were provided with threaded ends for gripping in the test instrument fixtures and were tested in an unmodified condition.

The HDPE, PVC, PS, PP, PA, and PC specimens were produced with a nominal gage length of 2.4 inches and a nominal gage diameter of 0.5 inches. The PMMA specimens were produced with a nominal gage length of 2.25 inches and a nominal gage diameter of 0.375 inches. Typical ranges of values for physical, mechanical, and thermal properties of the thermoplastics tested are shown in Appendix 1.

3.2 System Characterization

An Instron Model TTC was used for both variable strain rate and relaxation testing. Uncertainty within the testing system can be divided into three major categories. The first category is environmental which includes ambient temperature uncertainty (70 \pm 5 $^{\circ}$ F), relative humidity uncertainty (40 \pm 10 %RH), and test instrument vibration. Due to the nature of the local environment of the testing facility, these factors are largely uncontrollable and to a large extent beyond predictive characterization. The second category is mechanical which include slippage of the chart paper (which can be gross if sufficient care is not taken), random pen movement (\pm 0.02 in.), crosshead velocity variation (\pm 0.01 in./min.), and calibration drift (\pm 25 lbf). The third category is specimen physical properties which includes variation in degree of polymerization, molecular chain length and orientation, and density.

3.3 Variable Strain Rate Testing Procedure

The variable strain rate testing approach was to repeatedly load the specimen in the elastic stress-strain region at consecutively higher crosshead rates (0.02 in./min., 0.05 in/min., 0.1 in/min., 0.2 in/min., 0.5 in/min.). This procedure generated load versus elongation data plots with successively steeper slopes. The strain rate sensitivity index was then determined from the relation $m = \Delta \ln(\sigma) / \Delta \ln(\dot{\epsilon})$ from the load versus elongation data at strain levels identical to those achieved in stress relaxation testing.

Load versus elongation data at each crosshead velocity for all specimens that were variable strain rate tested are shown in Appendix 2. Elastic modulus values at each strain rate, the derived strain rate sensitivity index values, and the correlation coefficient values for all specimens that were variable strain rate tested are shown in Appendix 3.

3.4 Stress Relaxation Testing Procedure

In stress relaxation testing, the specimen was loaded at a constant rate (0.1 in./min.) to a load level immediately above the proportional limit at which point elongation was halted. This procedure resulted in a constant strain being maintained in the material. From then on until the strain was released, the specimen exhibited a decay response in which the load decreased as a function of time from the initially imposed load level to a lower load level according to some function $P = P(\text{time, temperature, structure, ...})$. The strain rate sensitivity index was then determined from the relation $m = \Delta \ln(P) / \Delta \ln(-\dot{P})$ from the load versus time data.

Load versus time data for all specimens that were stress relaxation tested are shown in Appendix 4. Response function parameters, strain levels, and derived strain rate sensitivity index values for all specimens that were stress relaxation tested are shown in Appendix 5.

4. Data Analysis

4.1 Variable Strain Rate Data Analysis

The data for load ($P = P_f \cdot y/k$) as a function of specimen elongation ($\delta = v \cdot x/u$) for each crosshead velocity (v) and chart velocity (u) combination were used to calculate stress and strain from the relations

$$\sigma = P \cdot (1 + \delta/l_0) / A_0 \quad (13)$$

$$\epsilon = \ln(1 + \delta/l_0) \quad (14)$$

where x is the elongation chart displacement [in.], y is the load chart displacement [in.], k is the displacement-to-load conversion factor [in.], P_f is the full scale load [lbf], l_0 is the gage length [in.], and A_0 is the cross-sectional area [in.²]. These data pairs were then used to form an array and the elastic modulus (E) was derived by linear regression from the relation $E = \Delta\sigma/\Delta\epsilon$. The derived modulus value and a constant strain value were then used to calculate $\ln(\sigma)$ and $\ln(\dot{\epsilon})$ for each crosshead velocity from the relations

$$\ln(\sigma) = \ln\{E \cdot \epsilon\} \quad (15)$$

$$\ln(\dot{\epsilon}) = \ln\{v/[l_0 \cdot \exp(\epsilon)]\} \quad (16)$$

where the strain value used in the calculation was identical to the strain value obtained in stress relaxation testing.

These data pairs were then used to form an array and the strain rate sensitivity index was derived by linear regression from the relation

$$m_{\dot{\epsilon}} = \Delta \ln(\sigma) / \Delta \ln(\dot{\epsilon}) \quad (17)$$

where $m_{\dot{\epsilon}}$ is the slope of $\ln(\sigma)$ data plotted as a function of $\ln(\dot{\epsilon})$ data and is given by

$$m_{\dot{\epsilon}} = a/b \quad (18)$$

$$a = n \cdot \Sigma [\ln(\dot{\epsilon}) \cdot \ln(\sigma)] - \Sigma [\ln(\dot{\epsilon})] \cdot \Sigma \ln(\sigma)$$

$$b = n \cdot \Sigma [\ln(\dot{\epsilon})]^2 - [\Sigma \ln(\dot{\epsilon})]^2$$

The program listing for the numerical determination of the strain rate sensitivity index by the variable strain rate testing technique is presented in Appendix 6.

In this analysis, the correlation coefficient ($r = \sqrt{r^2}$) represents a measure of the goodness-of-fit of the $\ln(\sigma)$ versus $\ln(\dot{\epsilon})$ data points to a straight line ($r = 1$ @ linear) and is given by

$$r = c/d \quad (19)$$

$$c = n \cdot \Sigma [\ln(\dot{\epsilon}) \cdot \ln(\sigma)] - \Sigma [\ln(\dot{\epsilon})] \cdot \Sigma \ln(\sigma)$$

$$d = \{n \cdot \Sigma [\ln(\dot{\epsilon})]^2 - [\Sigma \ln(\dot{\epsilon})]^2 \cdot n \cdot \Sigma [\ln(\sigma)]^2 - [\Sigma \ln(\sigma)]^2\}^{1/2}$$

As Kolb has indicated, the determination coefficient (r^2) is the accepted criterion by which a correlation can be established between the least-squares regression result and the transformed experimental data. The range of values that the determination coefficient can take are from zero (where the transformed values of the dependent and independent variables are totally unrelated) to

unity (where the transformed values of the variables have an exact linear relationship). The determination coefficient can also be interpreted as a measure of what proportion of variation in the dependent variable data is attributable to variation in the independent variable data. In the case of the present analysis, an average determination coefficient value of $r^2 = 0.99$ indicates that 99% of the variation in $\ln(\sigma)$ is attributable to variations in $\ln(\dot{\epsilon})$, while the other 1% of the variation in $\ln(\sigma)$ is due to factors unrelated to the independent variable. Though the determination coefficient is an appropriate means by which "goodness of fit" can be measured, Kolb has warned that a high determination coefficient value can be generated from low-noise data that does not necessarily exhibit a good fit to the transformed regression function.

4.2 Stress Relaxation Data Analysis

In view of its advantage as a good indicator of the physical mechanisms operative during the relaxation process [5,21], the Kohlrausch function was implemented to model the relaxation response of the thermoplastics tested. The data for load (P) as a function of time (t) was thus used to derive the parameters for the load relaxation equation

$$P(t) = P_0 \cdot \exp[-(t/\tau)^\eta] \quad (20)$$

where P_0 is the initially imposed load.

This initial load was typically at an elongation level a few percent above the elongation level at the proportional limit.

In order to determine the parameters n and τ an error function $\Phi(n, \tau)$ was defined

$$\Phi(n, \tau) = (1/N \cdot \sum_i \{P_o \cdot \exp[-(t_i/\tau)^n] - P(t_i)\}^2)^{1/2} \quad (21)$$

which is the root-mean-square difference between the empirical load relaxation equation and the experimental data. An exhaustive grid search algorithm was implemented to minimize the error function with the result that suitable parameters were found for each data set. The program listing for the determination of the Kohlrausch parameters from the stress relaxation data is shown in Appendix 7.

A number of equal-spaced time increments (whose last term was equal to the time duration of the relaxation test) were input into the response equation and the load relaxation rate equation

$$P'(t) = -(n \cdot P_o / \tau) \cdot (t/\tau)^{n-1} \cdot \exp[-(t/\tau)^n] \quad (22)$$

and the strain rate sensitivity index was derived by linear regression from the relation

$$m_{\dot{\epsilon}} = \Delta \ln(P) / \Delta \ln(-\dot{P}) \quad (23)$$

where $m_{\dot{\epsilon}}$ is the slope of $\ln(P)$ data plotted as a function of $\ln(-\dot{P})$ data and is given by

$$m_{\dot{\epsilon}} = a/b \quad (24)$$

$$a = n \cdot \sum [\ln(-\dot{P}) \cdot \ln(P)] - \sum [\ln(-\dot{P})] \cdot \sum \ln(P)$$

$$b = n \cdot \sum [\ln(-\dot{P})]^2 - [\sum \ln(-\dot{P})]^2$$

The program listing for the determination of the strain rate sensitivity index by the stress relaxation technique is shown in Appendix 8.

In this analysis, the correlation coefficient represents a measure of the goodness-of-fit of the $\ln(P)$ versus $\ln(-\dot{P})$ data points to a straight line and is given by

$$r = c/d \quad (25)$$

$$c = n \cdot \sum \ln(-\dot{P}) \cdot \ln(P) - \sum \ln(-\dot{P}) \cdot \sum \ln(P)$$

$$d = \{n \cdot \sum [\ln(-\dot{P})]^2 - [\sum \ln(-\dot{P})]^2 \cdot n \cdot \sum [\ln(P)]^2 - [\sum \ln(P)]^2\}^{1/2}$$

5. Experimental Results

5.1 Variable Strain Rate Testing Results

Mean and standard deviation strain rate sensitivity index values ($m_{\dot{\epsilon}}$), variance values (q), and mean correlation coefficient values (r) derived from variable strain rate testing are presented in Table 1. Linear regression analysis yielded an average correlation coefficient value on the order of 0.96 for all specimens that were strain rate tested. To demonstrate the variation in magnitude of the strain rate sensitivity index values ($m_{\dot{\epsilon}}$) generated by the variable strain rate method, Figure 1 has $\ln(\sigma)$ plotted as a function of $\ln(\dot{\epsilon})$ for representative specimens of each type of thermoplastic. To demonstrate the derivation of strain rate sensitivity index values ($m_{\dot{\epsilon}}$) generated by the variable strain rate method, Figure 2 through Figure 8 has $\ln(\sigma) = \ln\{E \cdot \epsilon\}$ derived from average strain data and average modulus data (as presented in Table 2) plotted as a function of $\ln(\dot{\epsilon}) = \ln\{v/[l_0 \cdot \exp(\epsilon)]\}$ derived from average strain data for each type of thermoplastic. The error bars on the plots are derived from the elastic modulus data standard deviation values for each strain rate.

5.2 Stress Relaxation Testing Results

Minimization of the error function $\Phi(n, \tau)$ by means of the numerical algorithm for each type of thermoplastic that was stress relaxation tested yielded mean parameter values, mean and standard deviation strain rate sensitivity index values ($m_{\dot{\epsilon}}$), variance values (q), and mean strain values (ϵ_0) as shown in Table 3. Figure 9 presents a comparison of the relaxation response curve utilizing load versus time data of a representative specimen (HDPE_3) with the modeled relaxation response curve using Kohlrausch parameters derived from numerical minimization. Figure 10 presents the normalized load response plotted as a function of time using the Kohlrausch function and average parameter data for each type of thermoplastic that was relaxation tested. To demonstrate the variation in magnitude of the strain rate sensitivity index values ($m_{\dot{\epsilon}}$) generated by the stress relaxation method, Figure 11 has $\ln(P)$ plotted as a function of $\ln(-\dot{P})$ for representative specimens of each type of thermoplastic. To demonstrate the derivation of strain rate sensitivity index values ($m_{\dot{\epsilon}}$) generated by the stress relaxation method, Figure 12 through Figure 18 has $\ln(P)$ plotted as a function of $\ln(-\dot{P})$ for each type of thermoplastic using mean parameter values as shown in Table 3. The error bars on the plots are derived from characteristic time parameter (τ) standard deviation values for each thermoplastic.

5.3 Testing Results Comparison

Mean strain rate sensitivity index values ($m_{\dot{\epsilon}}$) derived from variable strain rate data compared with mean strain rate sensitivity values (m_{σ}) derived from stress relaxation data (using the same strain in each case) agree quite closely in the cases of the thermoplastics HDPE ($\Delta = 2\%$) and PMMA ($\Delta = 2\%$) but differ by an increasing magnitude in the cases of PP ($\Delta = 41\%$), PC ($\Delta = 142\%$), PVC ($\Delta = 165\%$), PS ($\Delta = 217\%$), and PA ($\Delta = 339\%$) where the percent change parameter is given by $\Delta = [(m_{\sigma} - m_{\dot{\epsilon}})/m_{\dot{\epsilon}}] \cdot [100]$.

Strain rate sensitivity index values derived from variable strain rate testing are compared in Figure 19 where the strain rate sensitivity index ($m_{\dot{\epsilon}}$) is plotted as a function of the cohesive energy density (U) of each thermoplastic. Strain rate sensitivity index values derived from stress relaxation testing are compared in Figure 20 where the strain rate sensitivity index (m_{σ}) is plotted as a function of cohesive energy density (U) of each thermoplastic. Strain rate sensitivity index values derived from variable strain rate testing are compared in Figure 21 where the strain rate sensitivity index ($m_{\dot{\epsilon}}$) is plotted as a function of the sum of the pendant group molar volumes (V) of each thermoplastic.

Strain rate sensitivity index values derived from stress relaxation testing are compared in Figure 22 where the strain rate sensitivity index (m_g) is plotted as a function of the sum of the pendant group molar volumes (V) of each thermoplastic. Strain rate sensitivity index values derived from stress relaxation testing are also compared in Figure 23 where the strain rate sensitivity index (m_g) is plotted as a function of the characteristic time parameter (τ) of each thermoplastic.

Table 1. Mean and standard deviation strain rate sensitivity index values, variance values, and mean correlation coefficient values derived from variable strain rate testing.

TYPE	$m_{\dot{\epsilon}}$	s	q	r
HDPE	0.1291	0.0148	0.1146	0.9749
PP	0.0629	0.0102	0.1622	0.9785
PMMA	0.0418	0.0057	0.1364	0.9830
PS	0.0293	0.0182	0.6212	0.9485
PVC	0.0260	0.0086	0.3308	0.9283
PC	0.0191	0.0089	0.4660	0.9548
PA	0.0184	0.0054	0.2935	0.9408

Table 2. Mean and standard deviation elastic modulus values and strain rate values derived from variable strain rate testing and mean strain values derived from stress relaxation testing.

TYPE	ϵ [in./in]	E [psi]	s [psi]	$\ln(\dot{\epsilon})$ [1/s]
HDPE	0.0468	61,293	3,543	-8.93
		74,840	3,106	-8.01
		80,288	3,844	-7.32
		86,152	3,530	-6.63
		94,322	2,794	-5.71
PP	0.0445	70,741	3,849	-8.93
		76,417	3,156	-8.01
		80,455	2,643	-7.32
		83,322	2,493	-6.62
		86,548	2,453	-5.71
PMMA	0.0217	240,469	7,270	-8.84
		250,253	9,747	-7.92
		256,009	10,551	-7.23
		265,115	10,177	-6.54
		275,171	7,661	-5.62
PS	0.0222	117,244	14,450	-8.90
		122,769	12,578	-7.99
		124,467	11,179	-7.29
		125,877	10,772	-6.60
		129,047	8,904	-5.69
PVC	0.0398	239,691	17,867	-8.92
		251,113	15,742	-8.01
		255,349	14,516	-7.31
		257,697	15,210	-6.62
		261,402	13,882	-5.70
PC	0.0647	186,663	5,610	-8.95
		192,402	1,908	-8.03
		195,233	1,587	-7.34
		196,743	1,042	-6.64
		198,789	2,481	-5.73
PA	0.0344	278,900	15,958	-8.92
		288,082	11,178	-8.00
		291,524	11,596	-7.31
		294,067	12,113	-6.61
		296,314	12,401	-5.70

Table 3. Mean parameter values, mean and standard deviation strain rate sensitivity index values, variance values, and mean strain values derived from stress relaxation testing.

TYPE	P_o [lbf]	τ [s]	n	m_{σ}	s	q	ϵ_o [in./in.]
HDPE	432	13,290	0.279	0.1318	0.0052	0.0395	0.0468
PS	485	64,350	0.204	0.0915	0.0146	0.1596	0.0222
PP	482	67,320	0.271	0.0893	0.0063	0.0705	0.0445
PA	2001	109,000	0.297	0.0785	0.0076	0.0968	0.0344
PVC	1541	203,100	0.242	0.0687	0.0034	0.0501	0.0398
PC	1974	805,700	0.292	0.0460	0.0062	0.1348	0.0647
PMMA	501	1,523,500	0.253	0.0429	0.0031	0.0723	0.0217

Figure 1. Strain rate sensitivity index for representative thermoplastic specimens derived from variable strain rate testing.

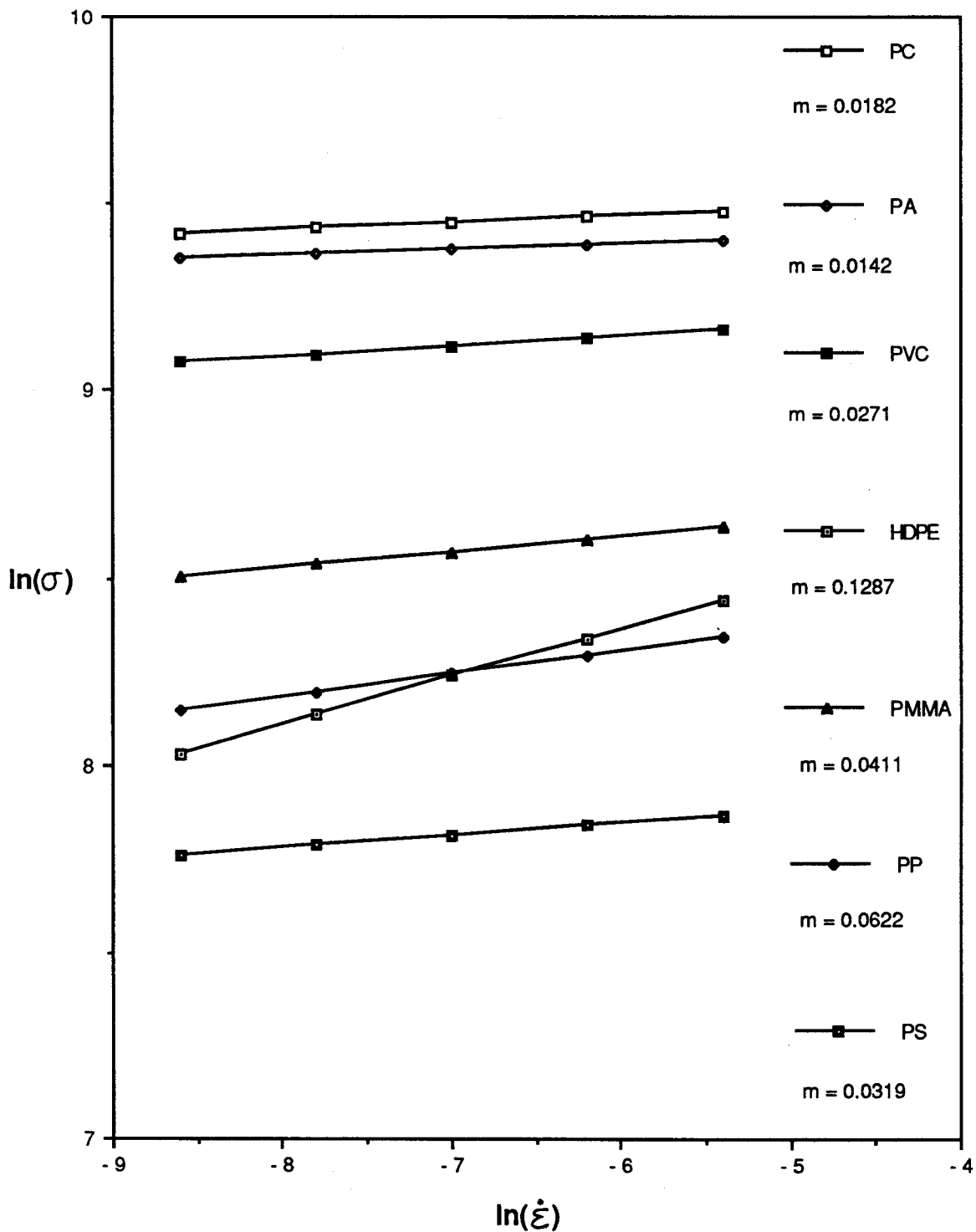


Figure 2. Strain rate sensitivity index for HDPE derived from variable strain rate testing average modulus and strain data.

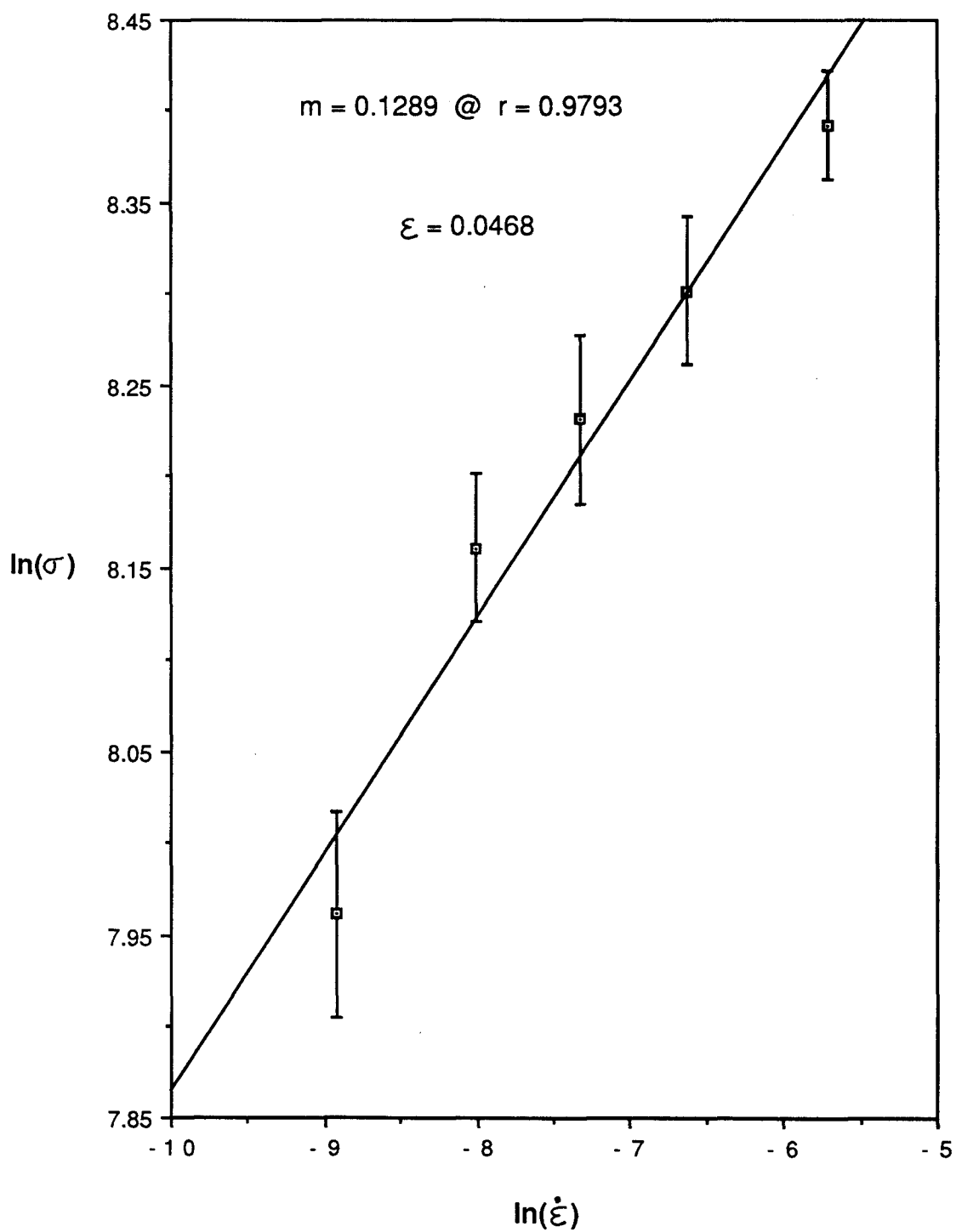


Figure 3. Strain rate sensitivity index for PP derived from variable strain rate testing average modulus and strain data.

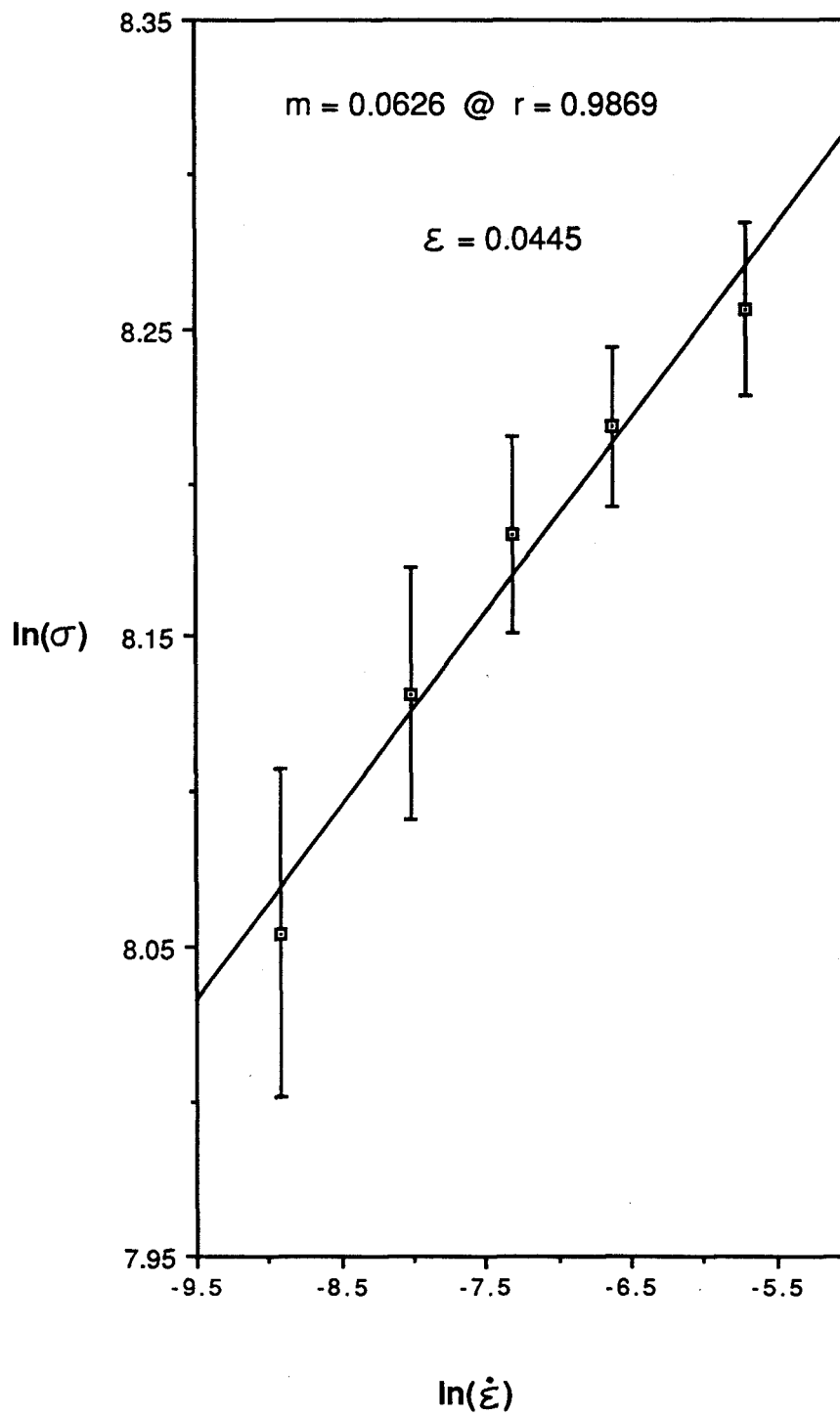


Figure 4. Strain rate sensitivity index for PMMA derived from variable strain rate testing average modulus and strain data.

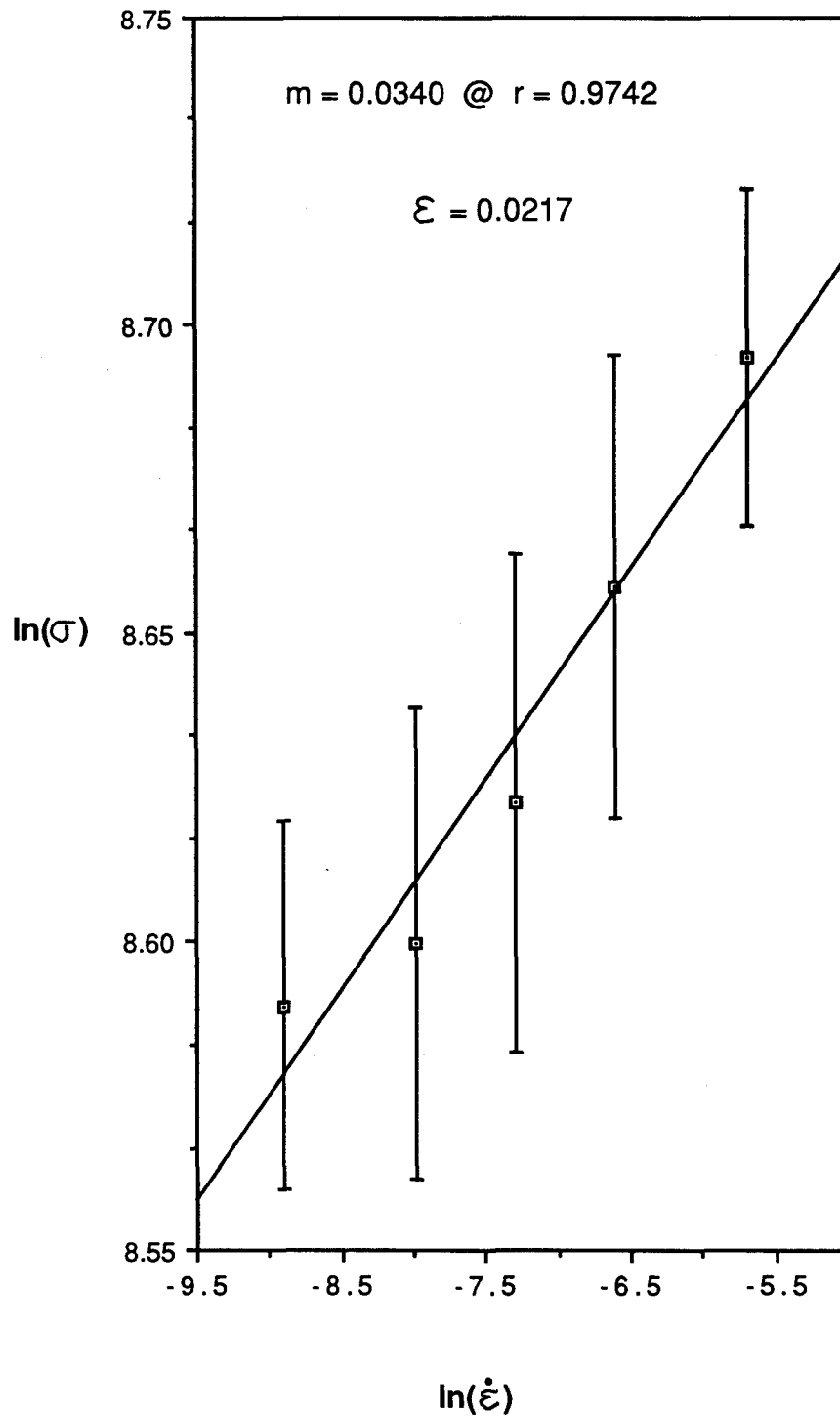


Figure 5. Strain rate sensitivity index for PS derived from variable strain rate testing average modulus and strain data.

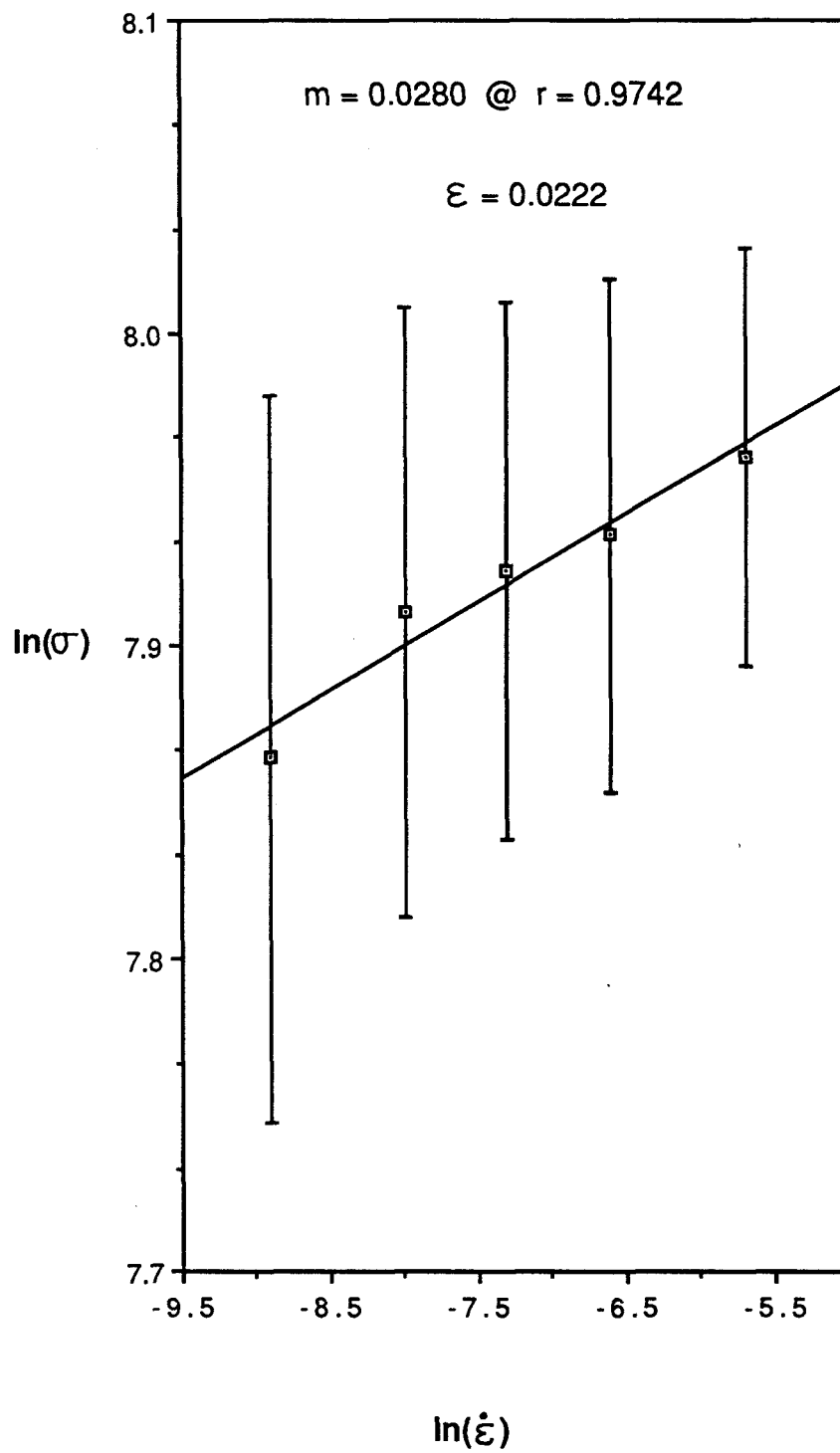


Figure 6. Strain rate sensitivity index for PVC derived from variable strain rate testing average modulus and strain data.

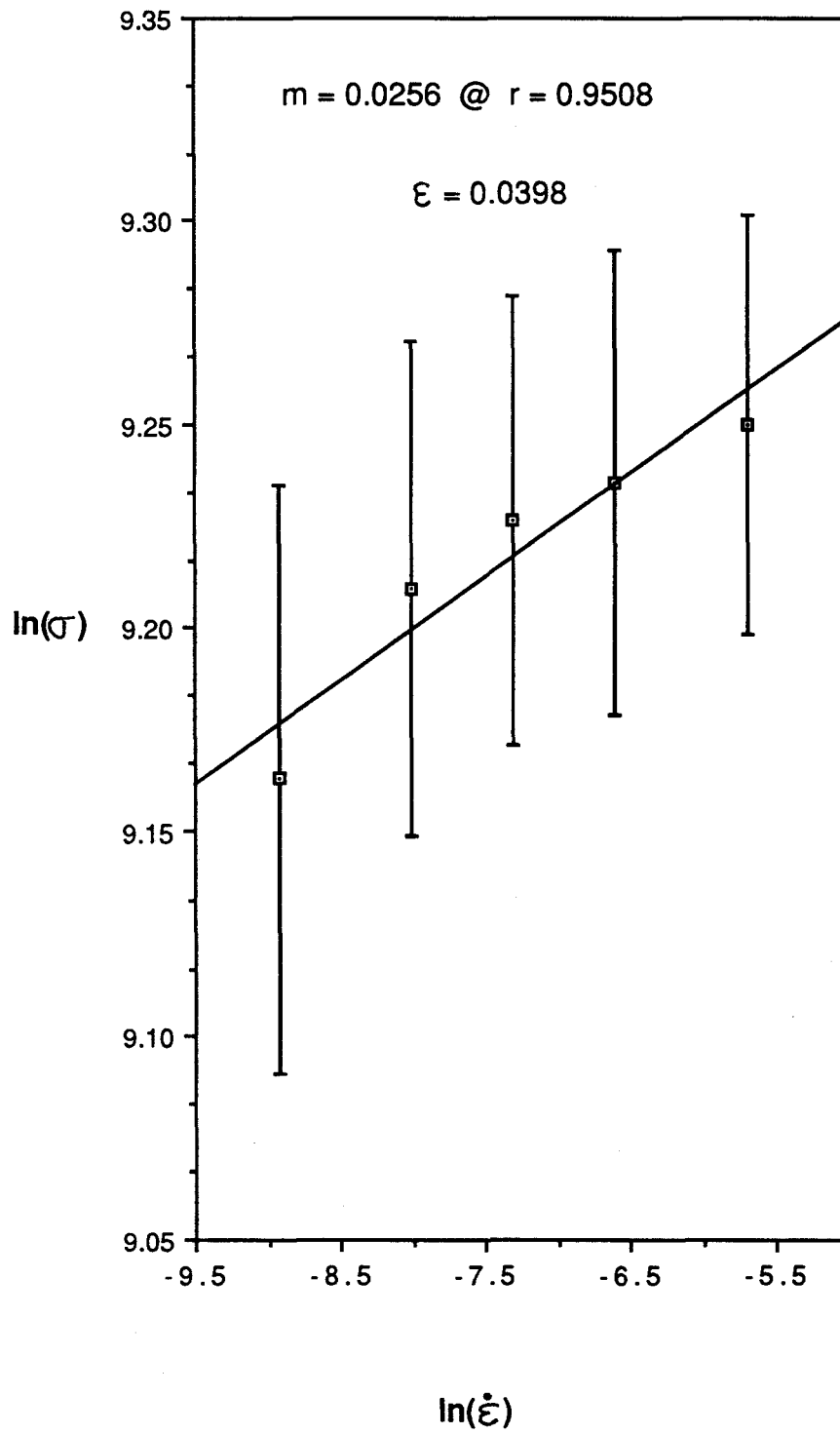


Figure 7. Strain rate sensitivity index for PA derived from variable strain rate testing average modulus and strain data.

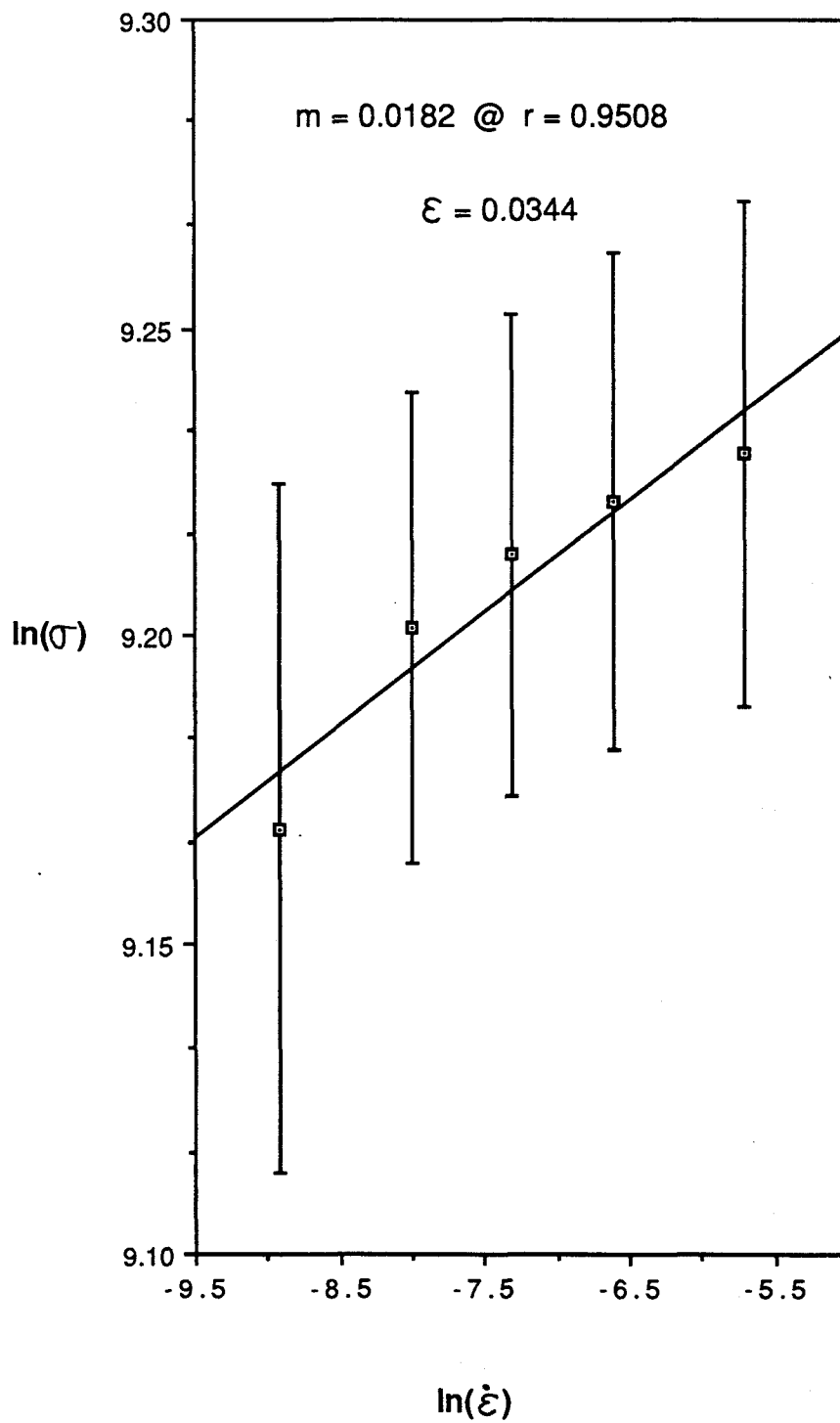


Figure 8. Strain rate sensitivity index for PC derived from variable strain rate testing average modulus and strain data.

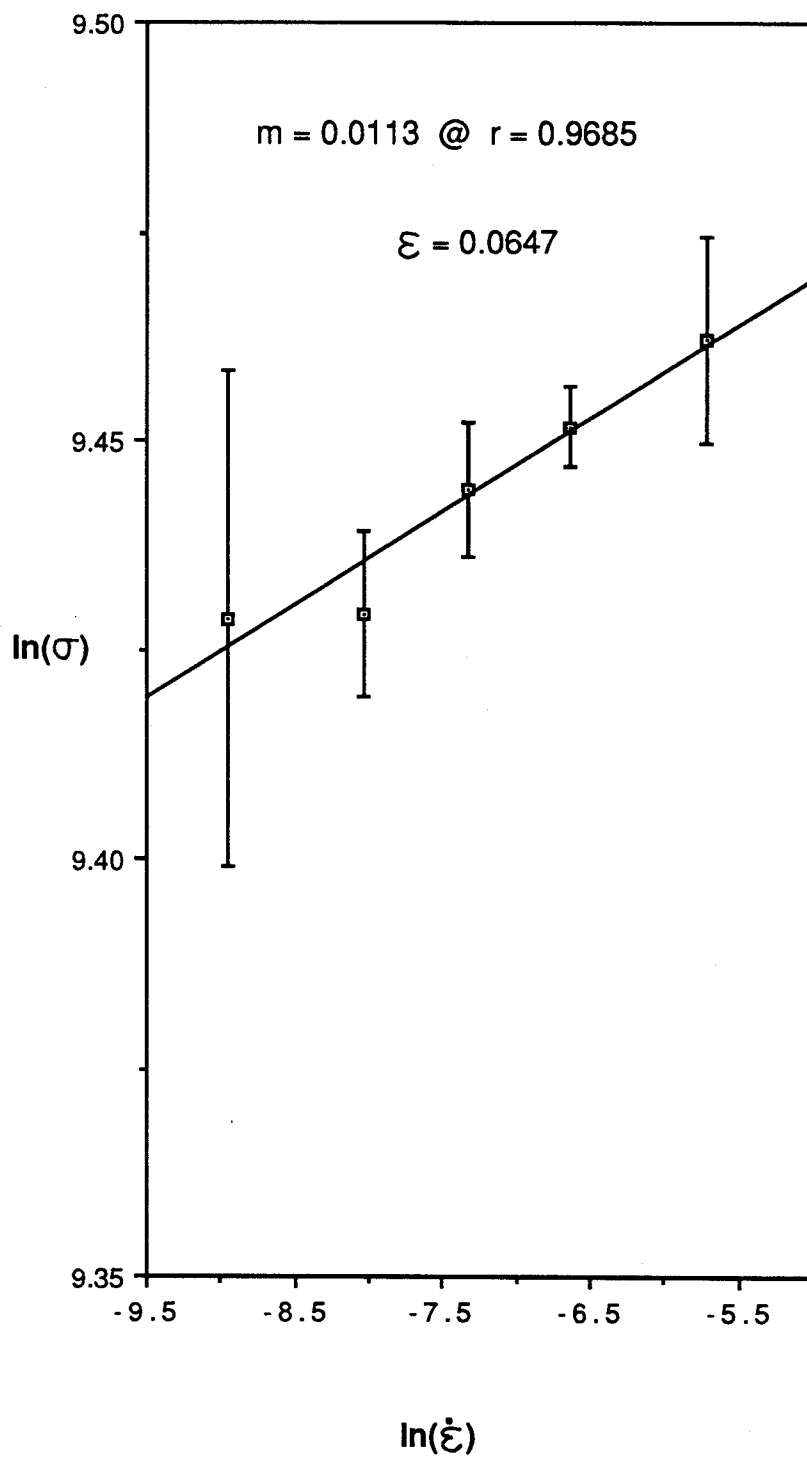


Figure 9. Comparison of the relaxation response utilizing load versus time data with the Kohlrausch modeled response.

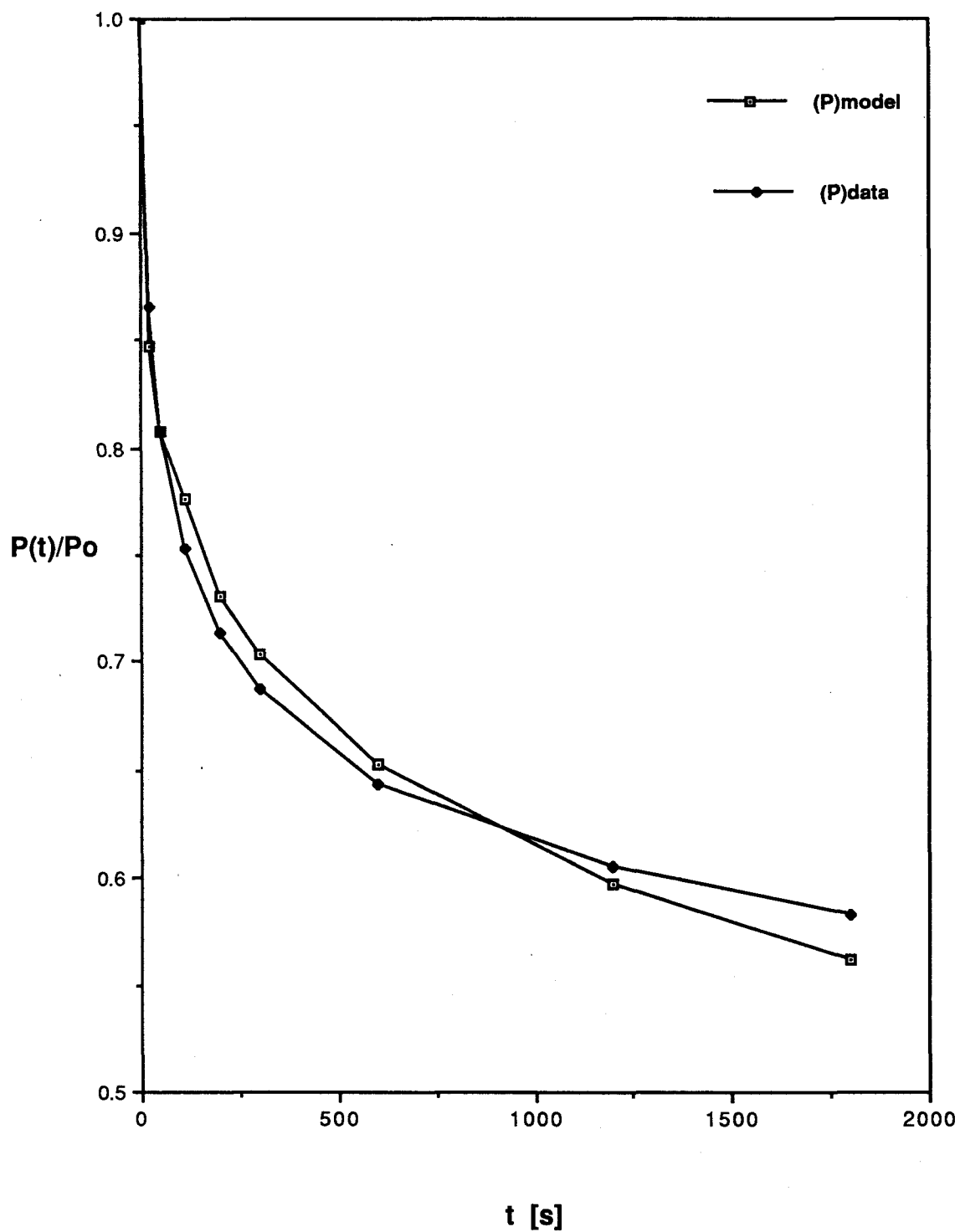


Figure 10. Normalized relaxation response for PC, PMMA, PA, PVC, PP, PS, and HDPE derived from average parameter data.

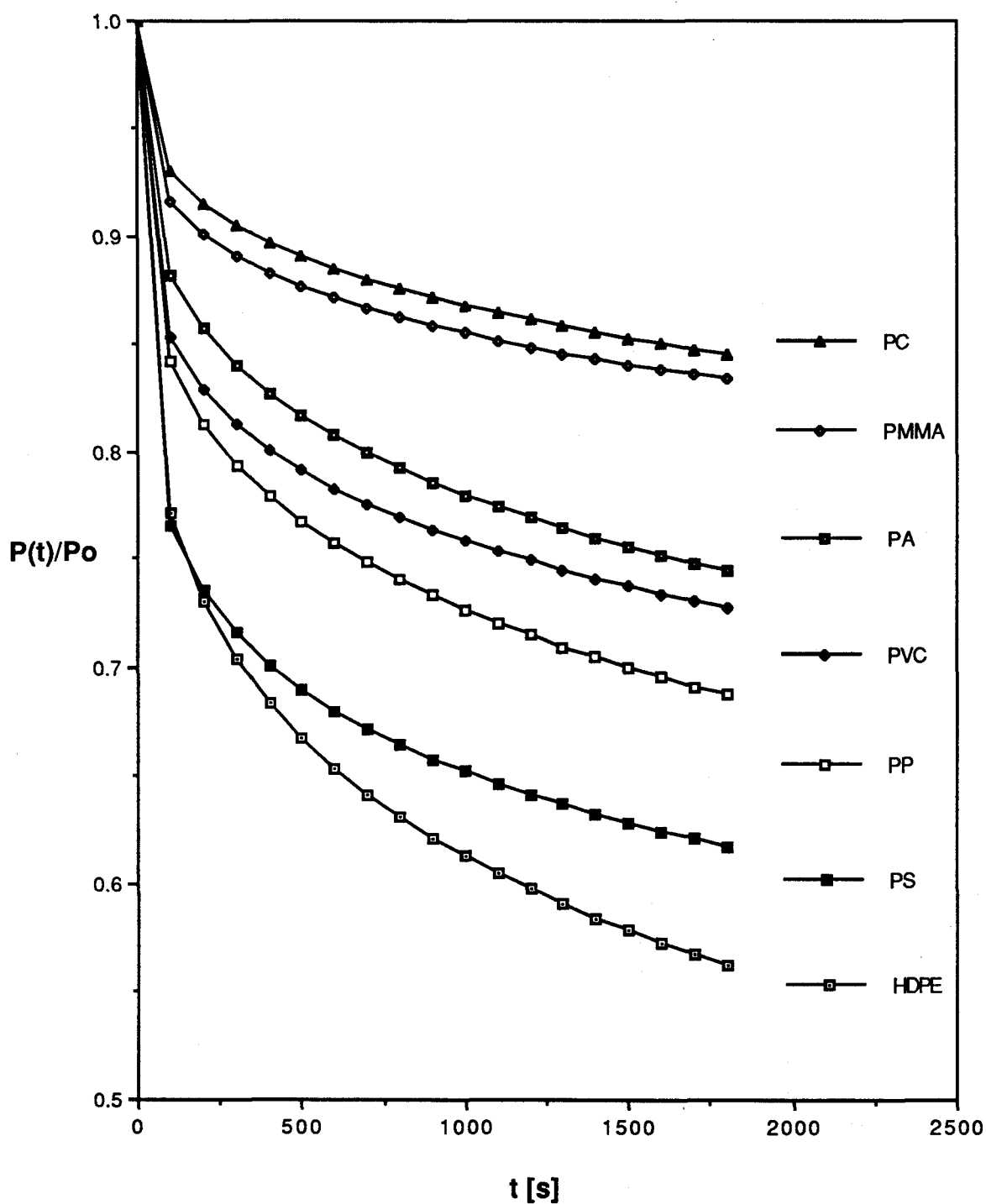


Figure 11. Strain rate sensitivity index for representative thermoplastic specimens derived from stress relaxation testing.

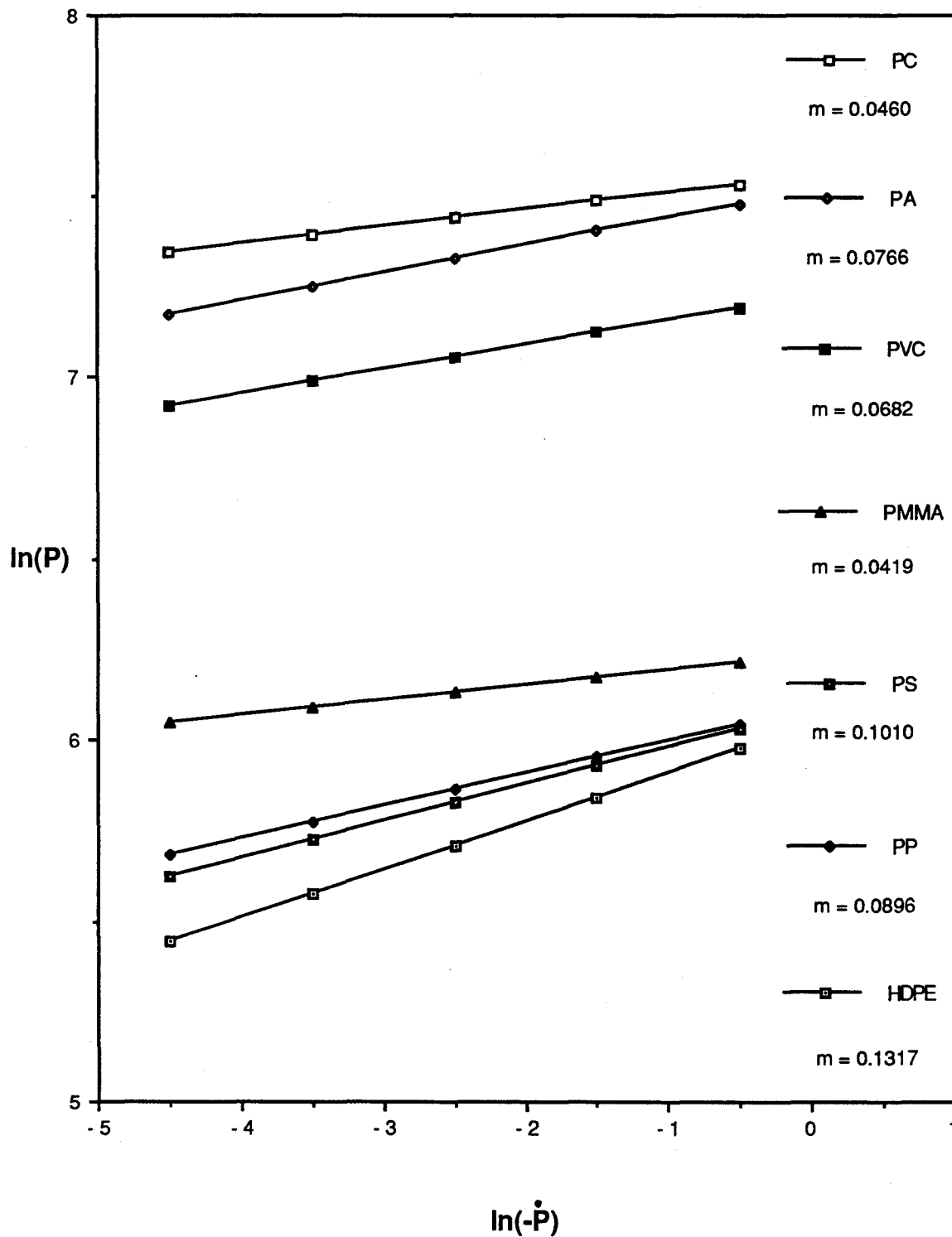


Figure 12. Strain rate sensitivity index for HDPE derived from stress relaxation testing average parameter data.

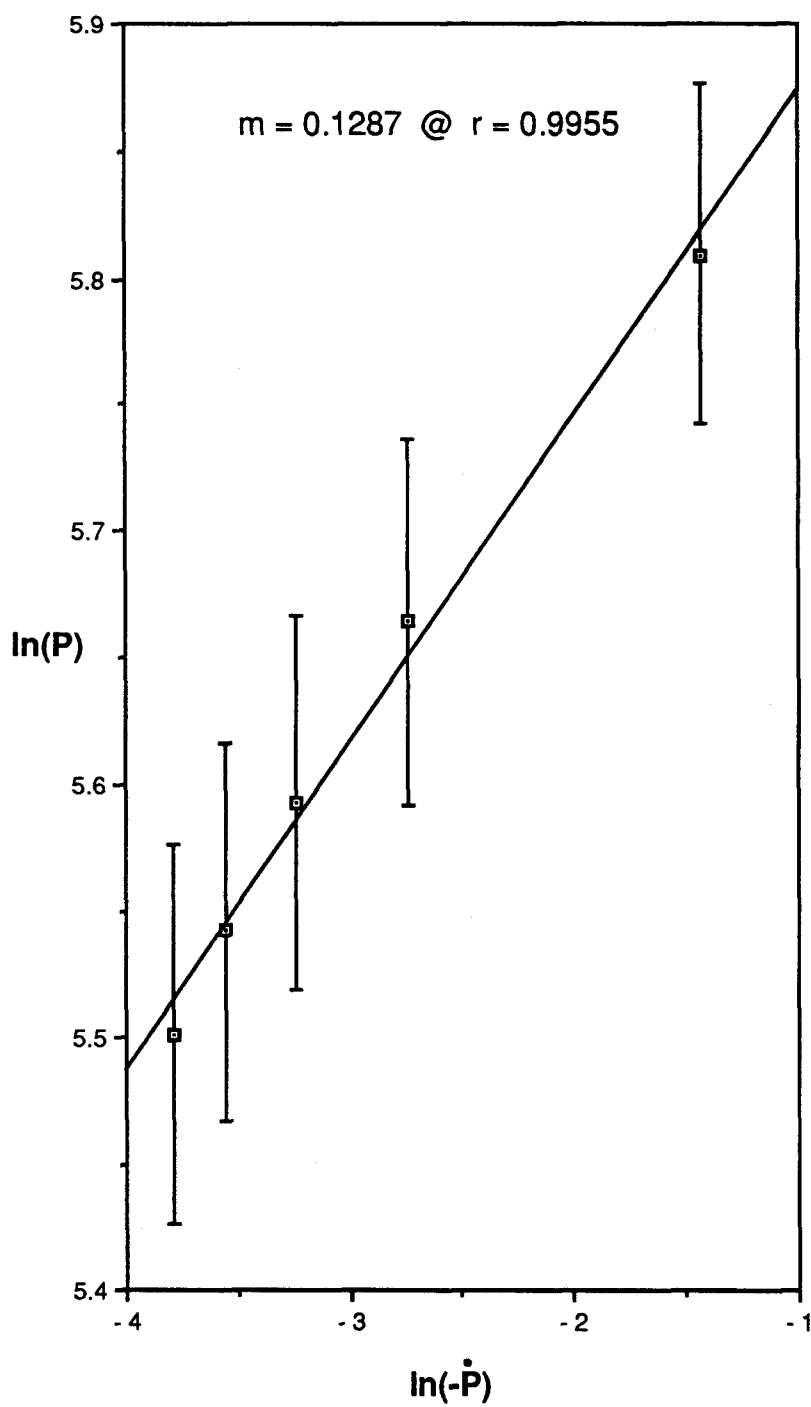


Figure 13. Strain rate sensitivity index for PP derived from stress relaxation testing average parameter data.

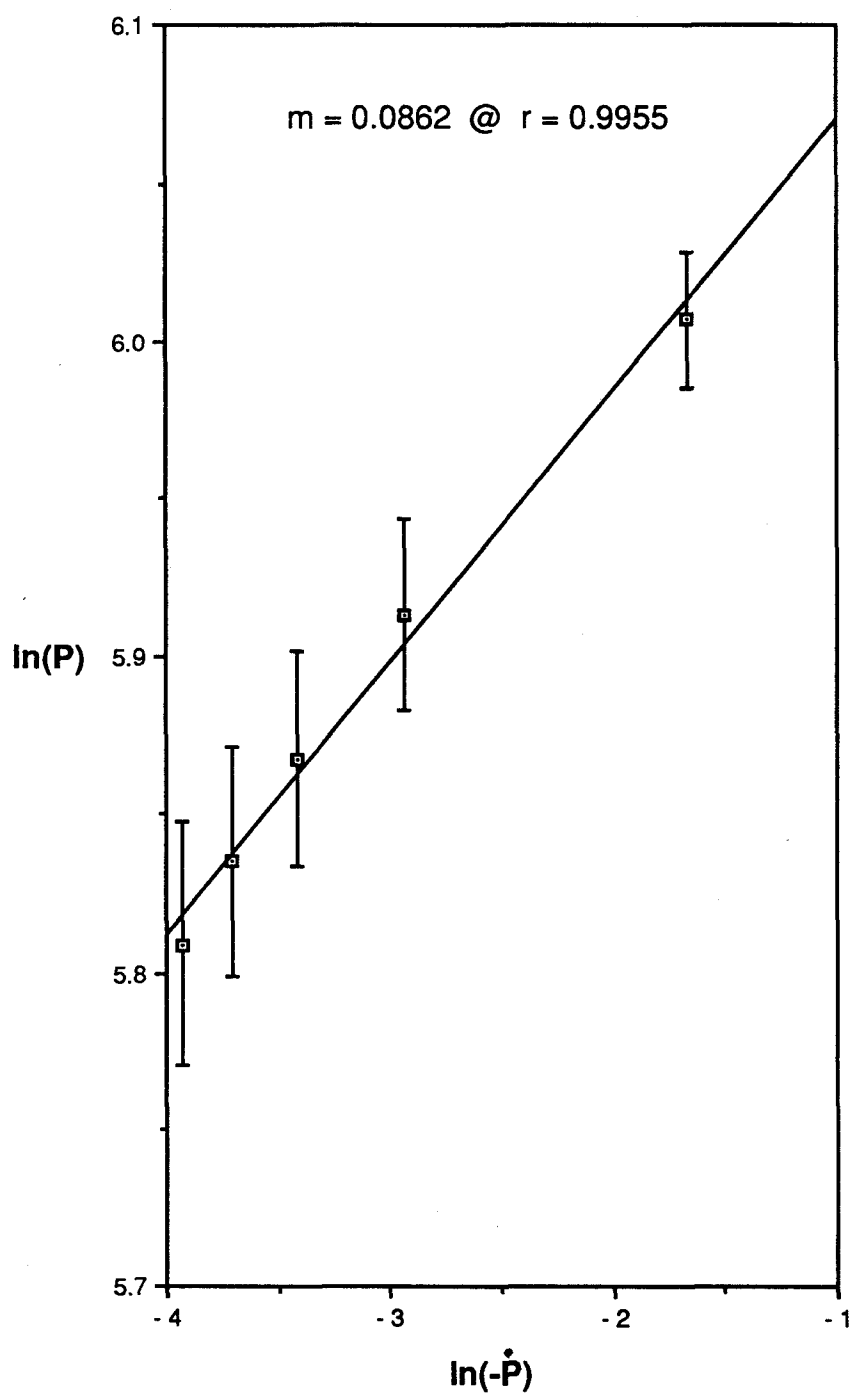


Figure 14. Strain rate sensitivity index for PS derived from stress relaxation testing average parameter data.

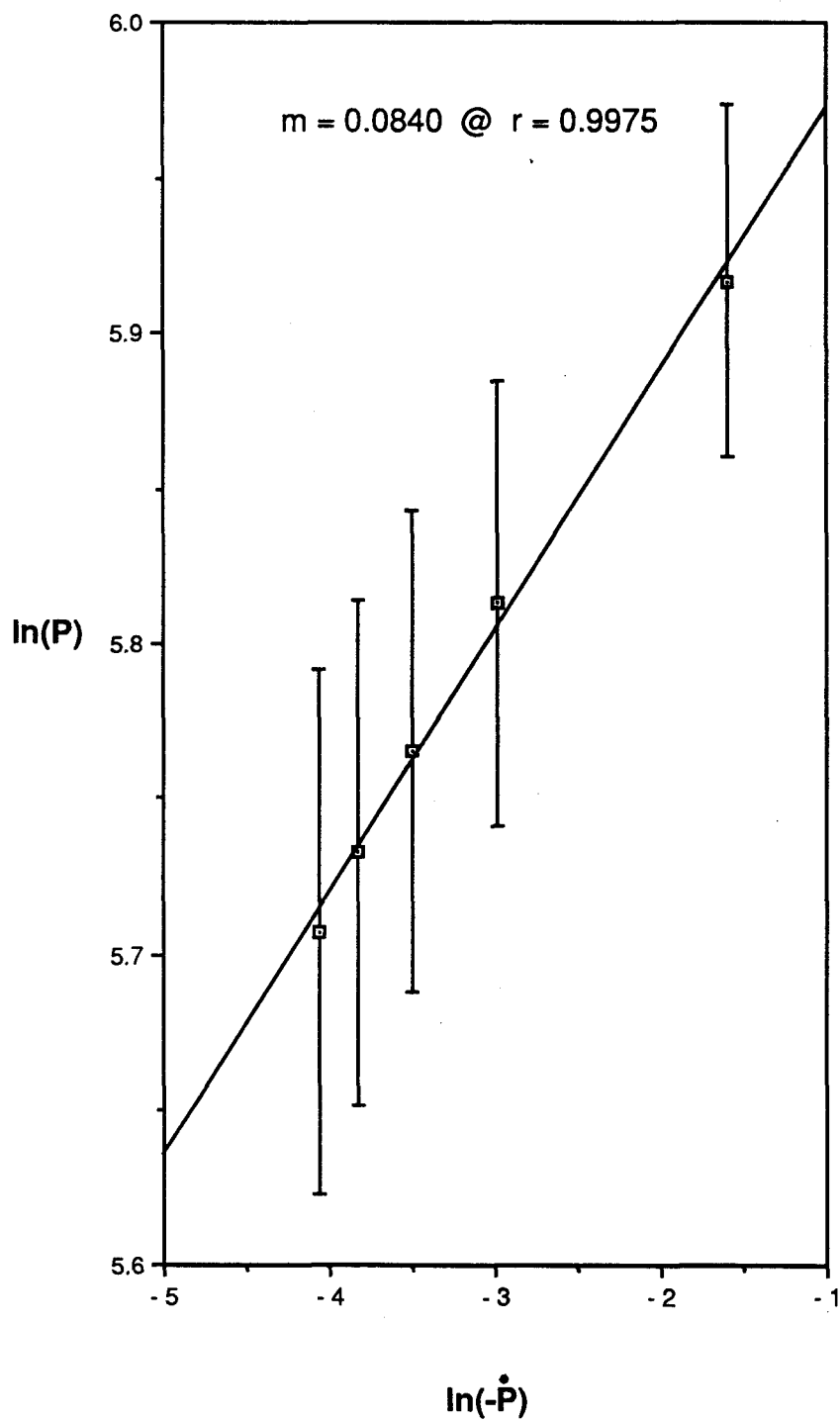


Figure 15. Strain rate sensitivity index for PA derived from stress relaxation testing average parameter data.

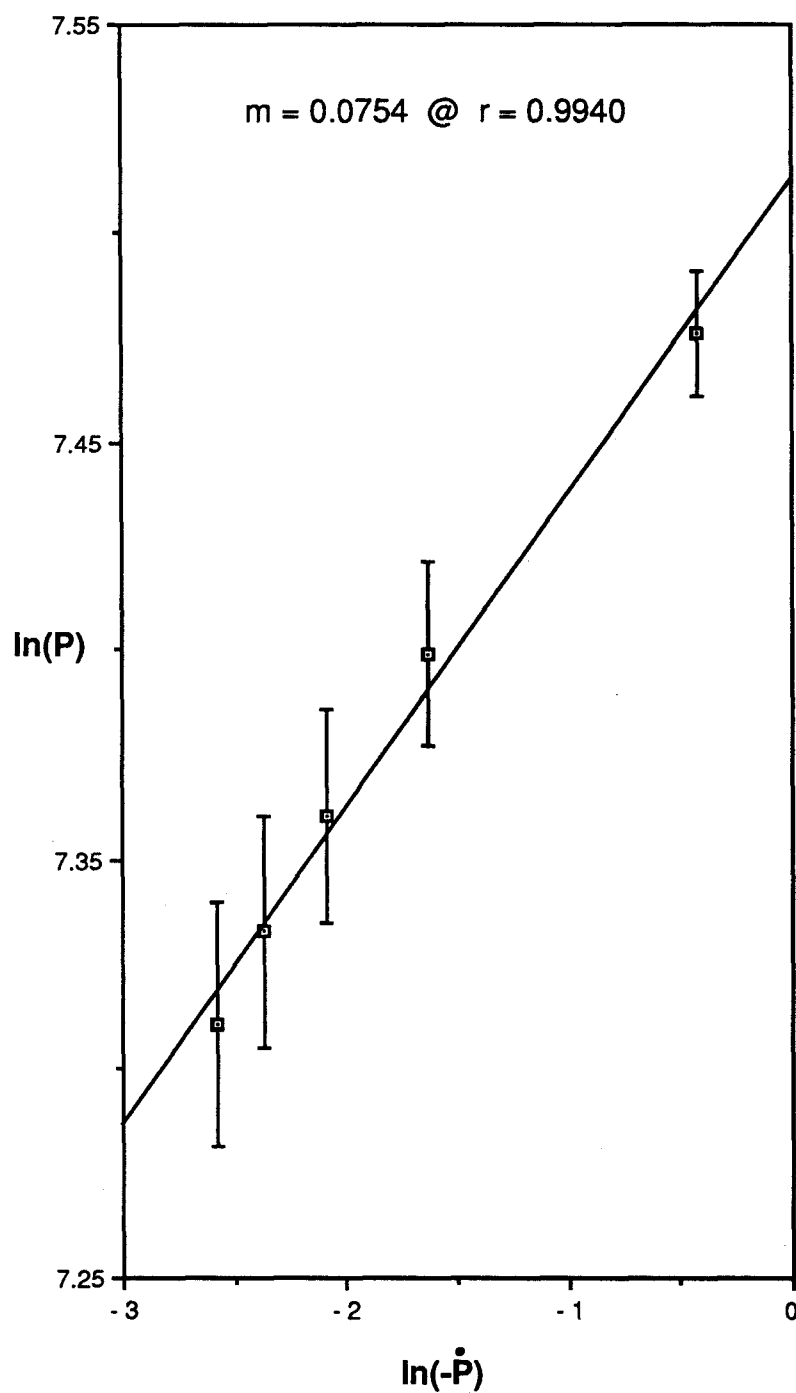


Figure 16. Strain rate sensitivity index for PVC derived from stress relaxation testing average parameter data.

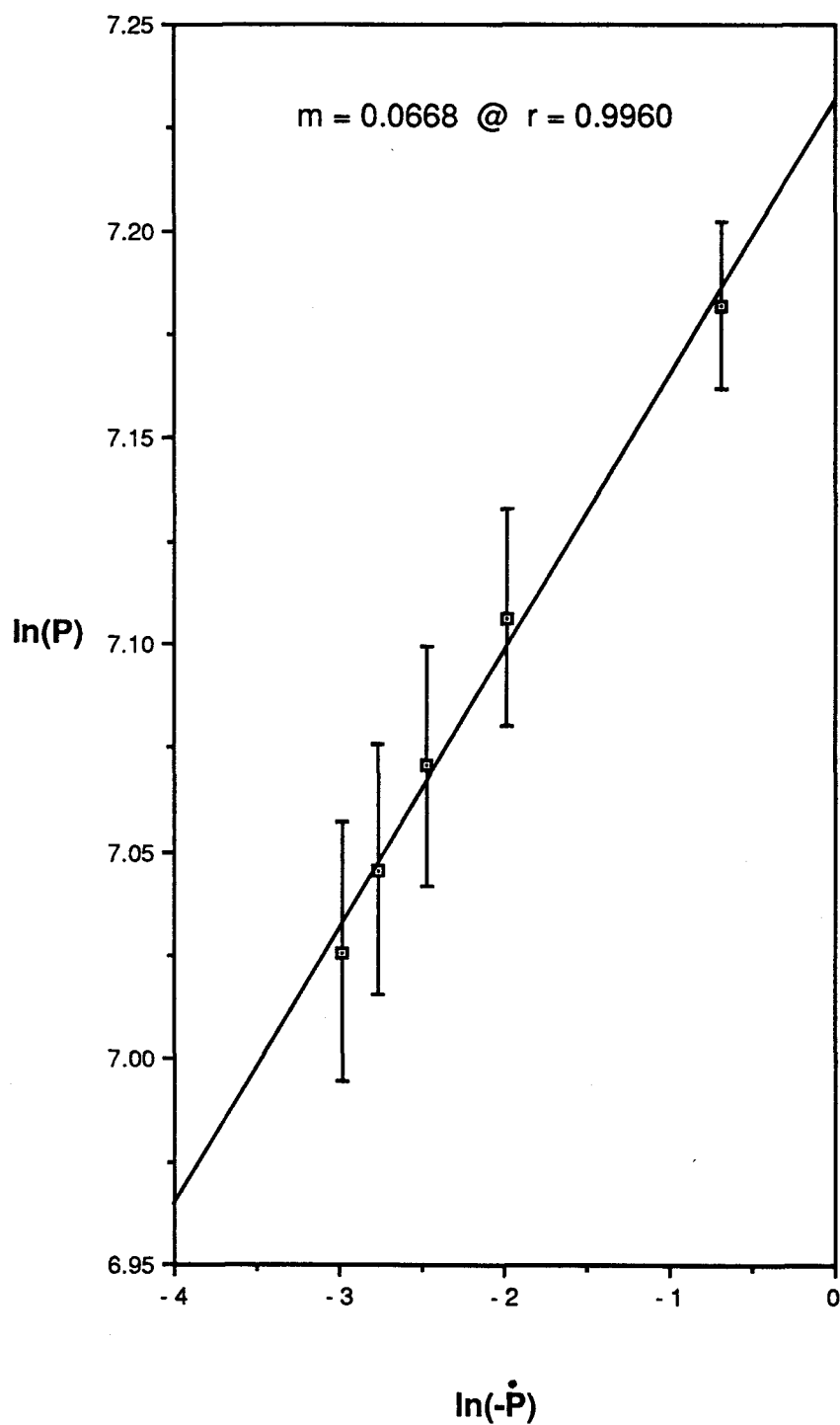


Figure 17. Strain rate sensitivity index for PC derived from stress relaxation testing average parameter data.

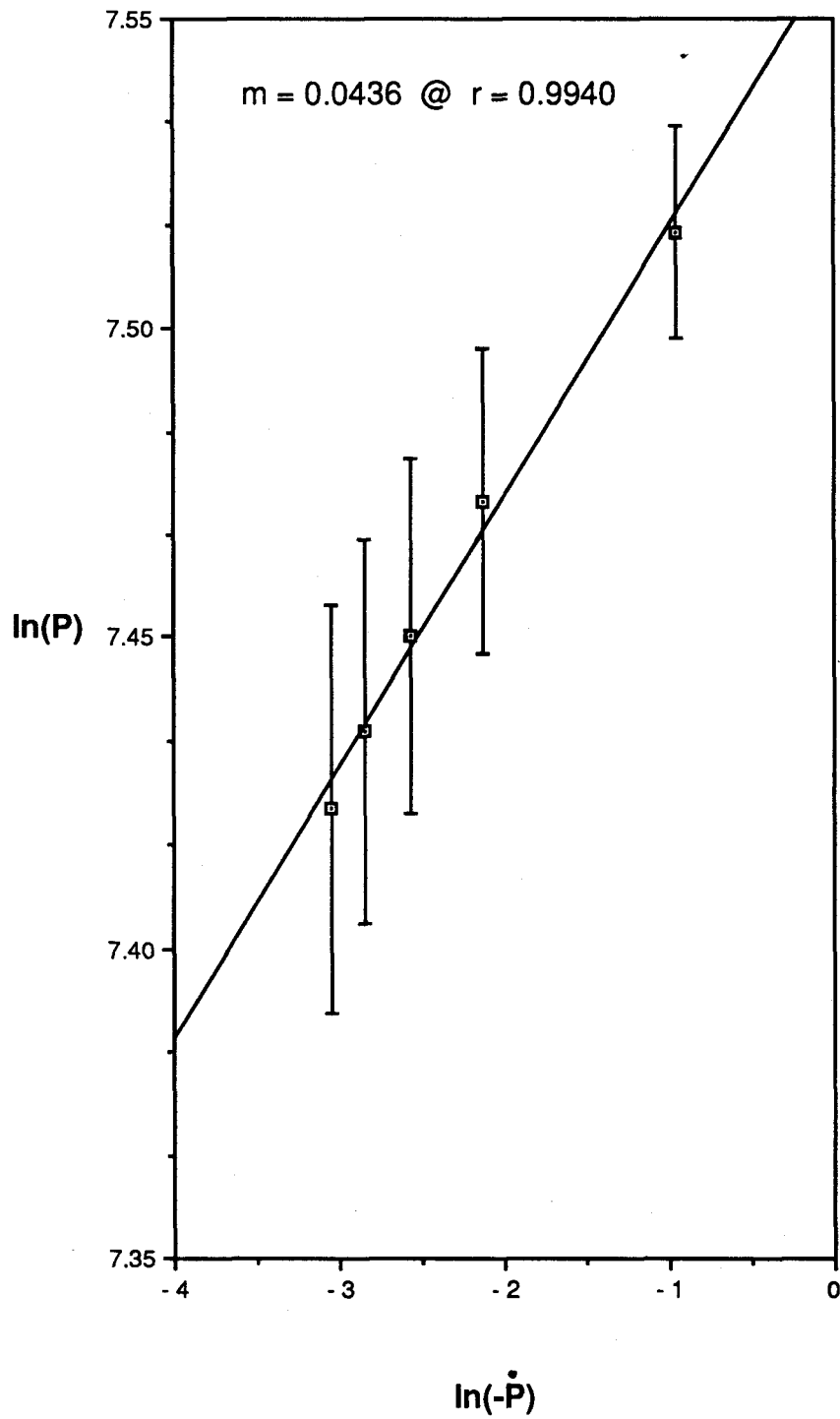


Figure 18. Strain rate sensitivity index for PMMA derived from stress relaxation testing average parameter data.

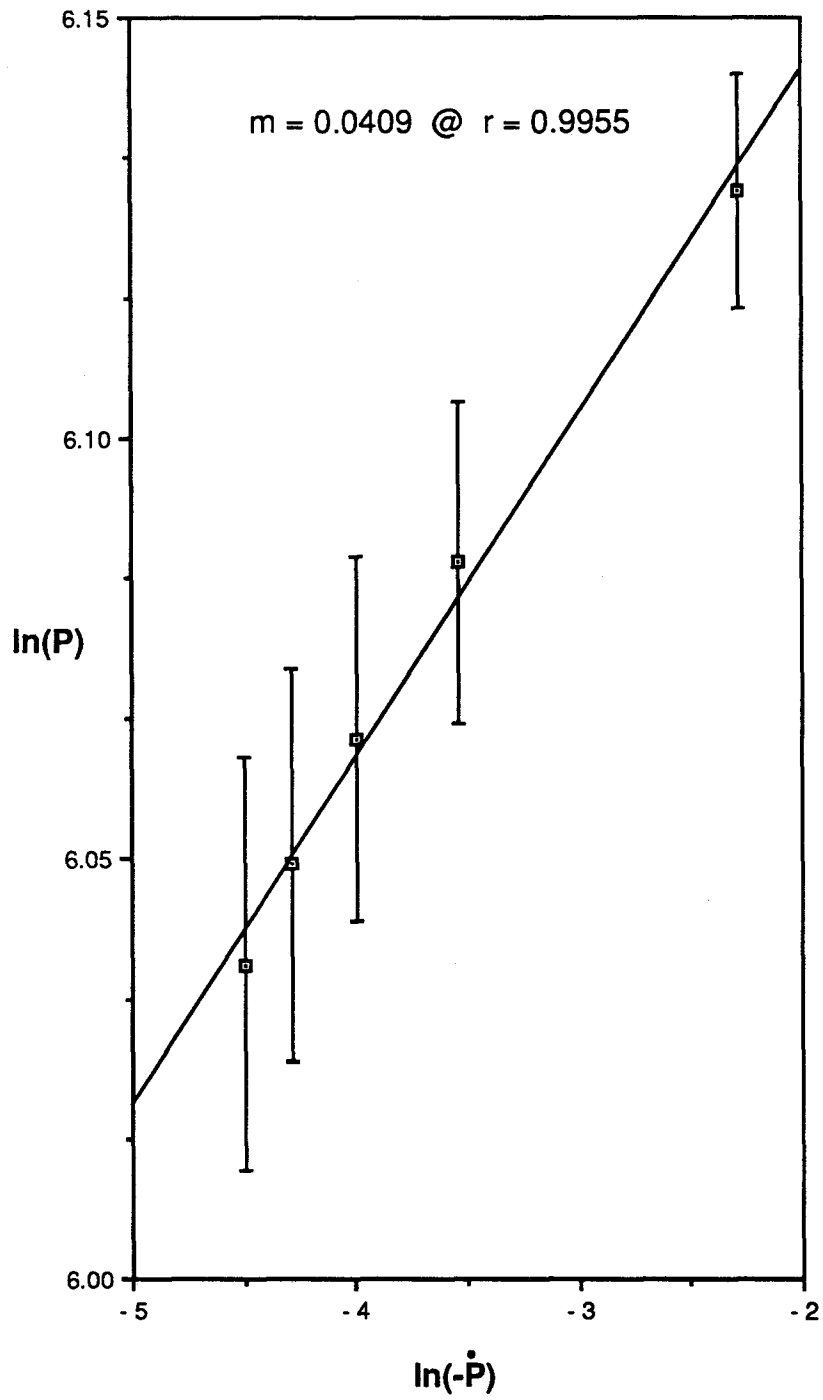


Figure 19. Strain rate sensitivity index derived from variable strain rate testing as a function of cohesive energy density.

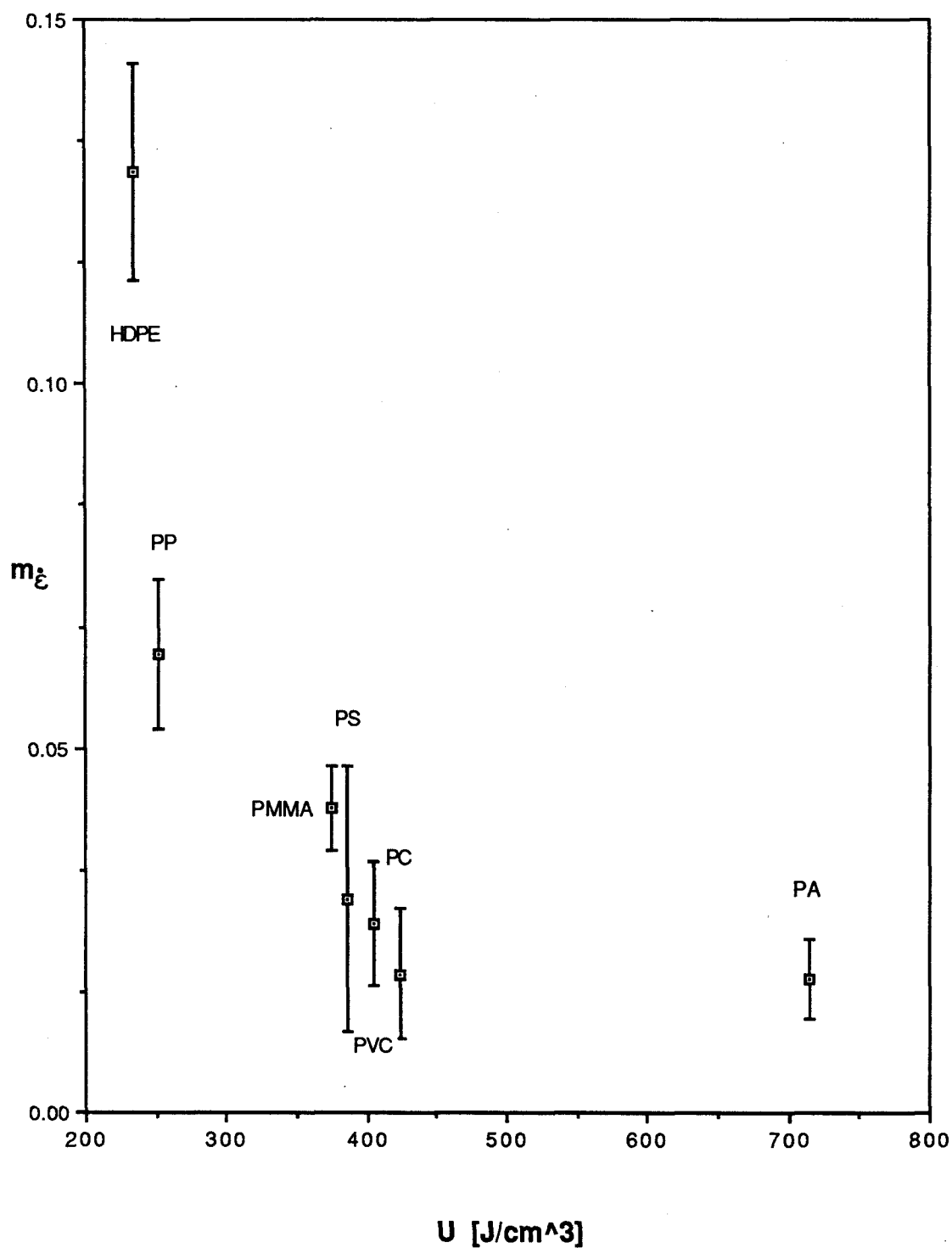


Figure 20. Strain rate sensitivity index derived from stress relaxation testing as a function of cohesive energy density.

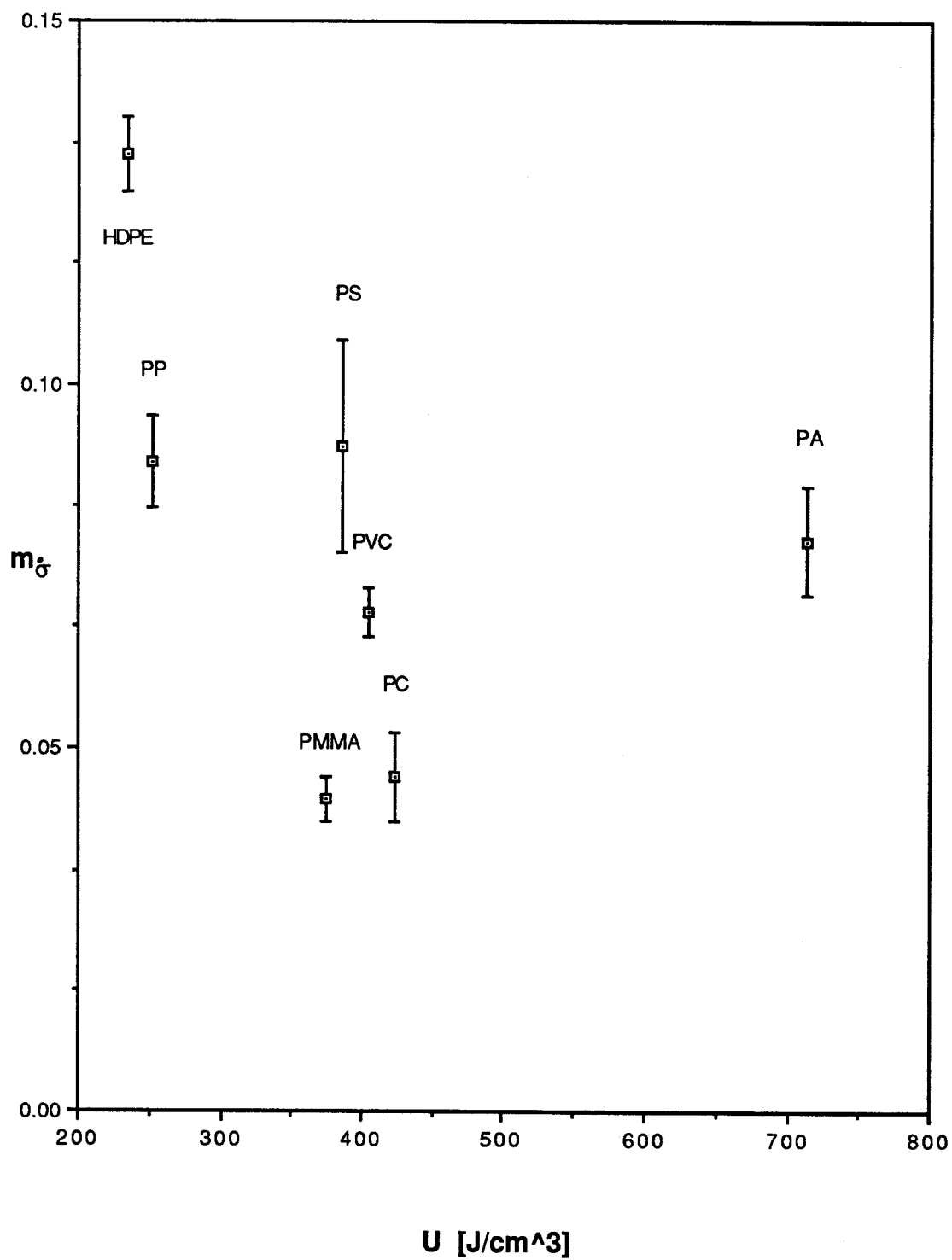


Figure 21. Strain rate sensitivity index derived from variable strain rate testing as a function of molar volume.

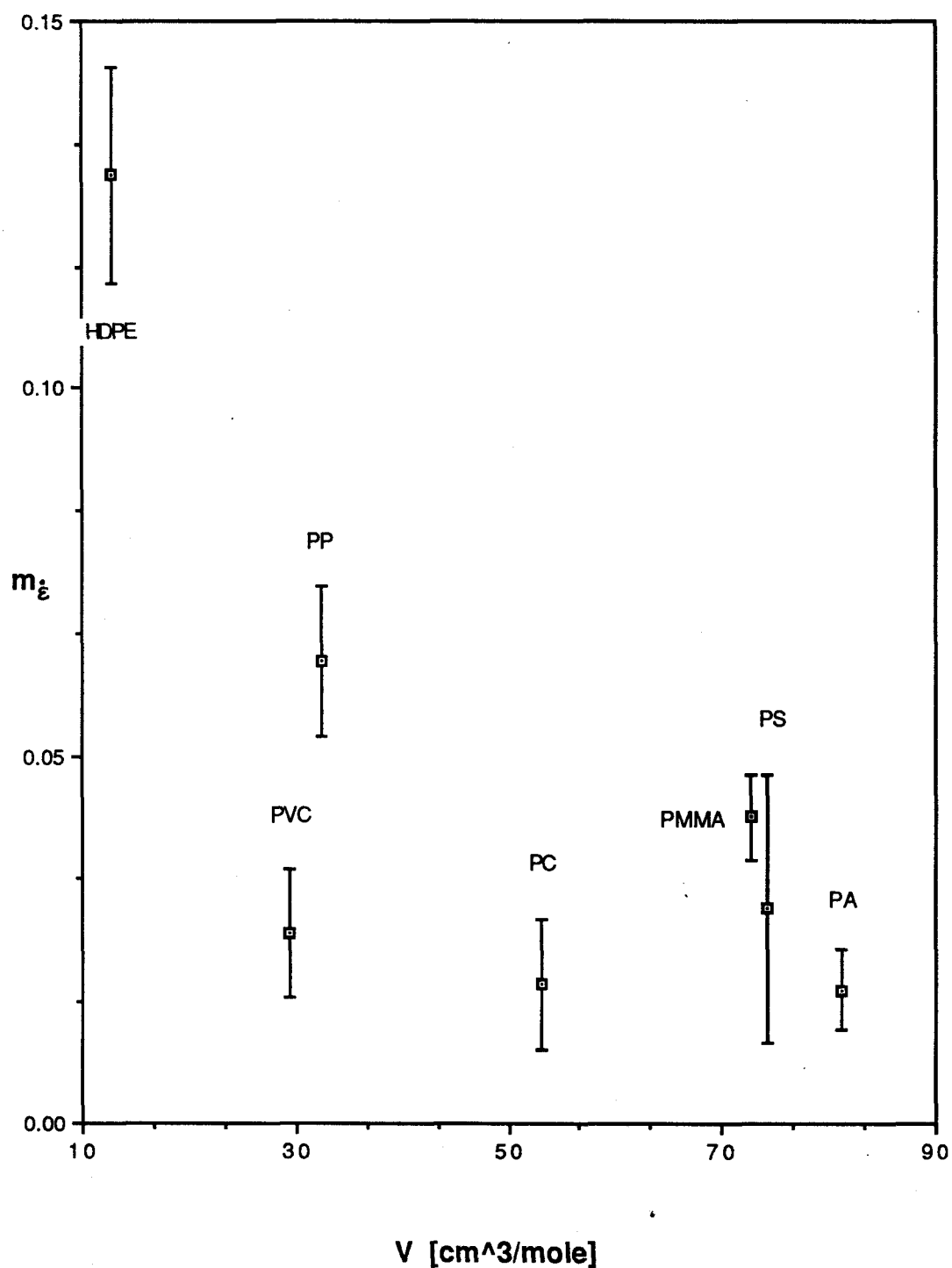


Figure 22. Strain rate sensitivity index derived from stress relaxation testing as a function of molar volume.

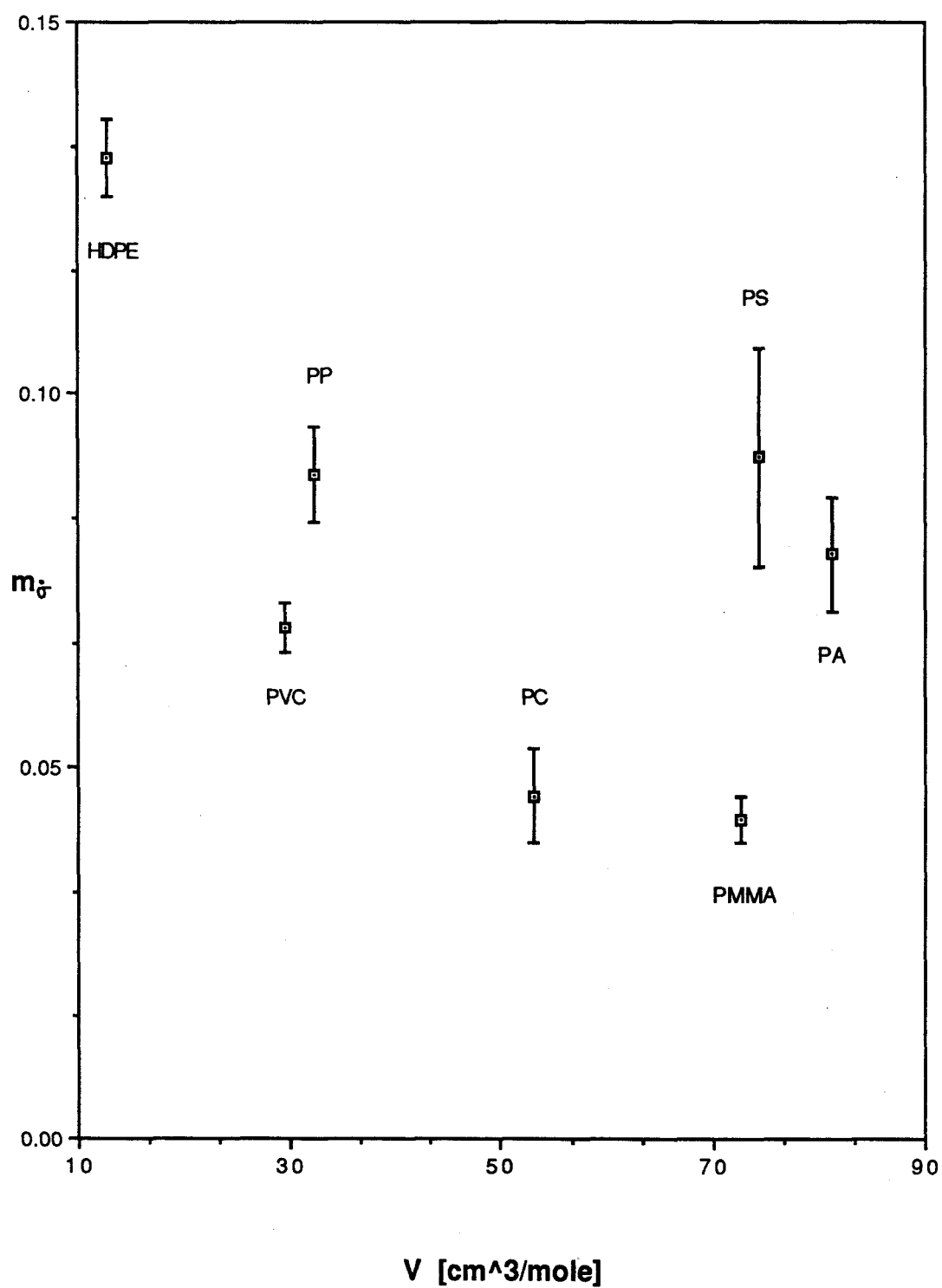
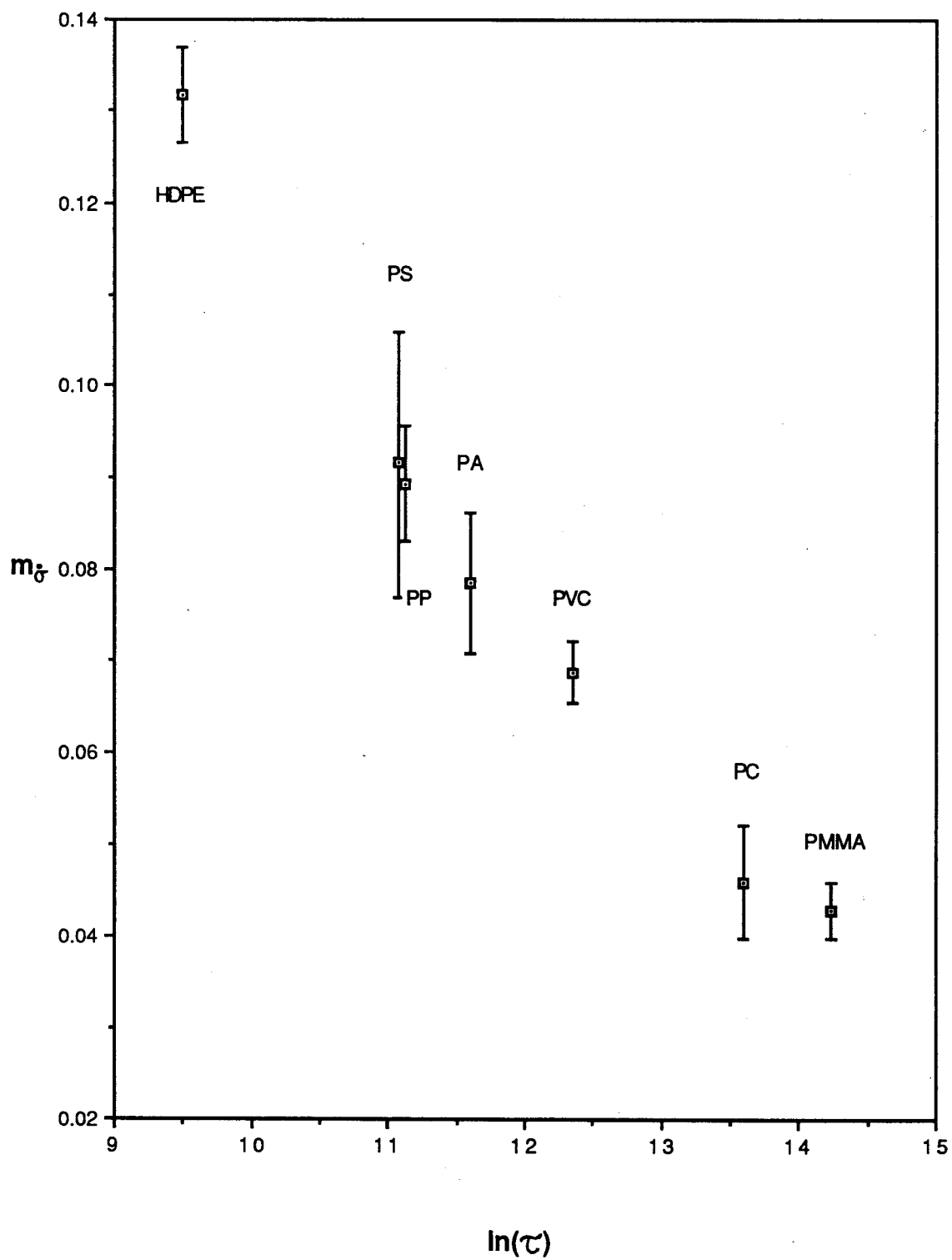


Figure 23. Strain rate sensitivity index derived from stress relaxation testing as a function of characteristic time.



6. Results Discussion

6.1 Variable Strain Rate Results Discussion

From the plots of $\ln(\sigma)$ as a function of $\ln(\dot{\epsilon})$ (Figure 1) it can be seen that HDPE is at the upper extreme of the spectrum with a steep slope and a high strain rate sensitivity index and that PA is at the lower extreme of the spectrum with a shallow slope and low strain rate sensitivity index. These results are as expected from theory in terms of the steric hindrance offered to chain mobility due to intermolecular bond energy density. It can also be seen from Figure 1 that for an equivalent change in $\ln(\dot{\epsilon})$, HDPE exhibits almost a order of magnitude larger change in $\ln(\sigma)$ than does PA. The result is that the strain rate sensitivity index value for HDPE is almost an order of magnitude larger than the strain rate sensitivity index value for PA.

It is apparent from the linearity of the plots of Figure 2 through Figure 8 that the variable strain rate method is only a moderately effective means of strain rate sensitivity index determination in that the scatter of the modulus values yield a relatively high variation in slope from point to point of the $\ln(\sigma)$ versus $\ln(\dot{\epsilon})$ data. In this regard, Table 1 is indicative of the reliability of the variable strain rate method in that the ratio of the standard deviation of the index values to the mean of the index values (the variance coefficient) is greater than 10% in the cases of all the thermoplastic specimens tested.

Though there is a relatively high variation in slope from point to point in these plots, in general, the least squares linear regression slope yields a relatively high correlation coefficient and in all cases is within the one standard deviation bounds for the modulus values. From this analysis it is concluded that the strain rate sensitivity index obtained by means of variable strain rate testing is not, in general, a function of strain rate for thermoplastics. Thus, while mean data from variable strain rate testing for HDPE, PP, PS, PVC, and PA all demonstrate some degree of convexity ($d^2\ln(\sigma)/d\ln(\dot{\epsilon})^2 < 0$), mean data from variable strain rate testing for PMMA and PC both demonstrate some degree of concavity ($d^2\ln(\sigma)/d\ln(\dot{\epsilon})^2 > 0$). That there is such a degree of variation in the modulus data is thought to be attributable to a relatively high degree of distribution in molecular weight values; i.e., that the thermoplastics are polydisperse results in a degree of variation in chain mobility which is expressed as a degree of variation in stiffness.

6.2 Stress Relaxation Results Discussion

From a comparison of the relaxation response curve utilizing load versus time data of a representative thermoplastic specimen with the modeled relaxation response curve using derived parameters (Figure 9), it is evident that, while the fit of the modeled curve to the experimental data is not exact, there is a relatively high degree of correspondence between the data sets.

In this regard, the percent change for the greatest overvalue deviation ($t = 300$ s) is $\Delta = +2.4\%$, while the percent change for the greatest undervalue deviation ($t = 1800$ s) is $\Delta = -3.7\%$. While not insignificant, this error is thought to be of a low enough magnitude to consider the Kohlrausch model an adequate representation of the relaxation response data.

From the plots of the normalized load response as a function of time (Figure 10) for the representative thermoplastic specimens, it can be seen that the relaxation response is in accordance with macroscopic-scale viscoelastic theory. Specifically, it is apparent from each plot that the stress decays rapidly in the early part of the response while the viscous component exhibits a more gradual decay that is still active in the latter part of the response. The plots of Figure 10 are also in accordance with accepted theory concerning internal mechanisms operating at the molecular level within the materials. Specifically, the plots of the thermoplastic responses illustrate the role of steric hindrance from both side-chain and main-chain groups in their relaxation behavior.

In the case of high density polyethylene, each pendant group consists of a small hydrogen atom ($V \sim 3$ cm³/mole). Thus, the high relaxation rate exhibited by HDPE is consistent with that expected for a small and non-complex side-chain constituent. In consideration of the bonding between the long-chain macromolecules, the instantaneous dipole-induced dipole bonds present are very weak ($\Delta E \sim 2$ kcal/mole) and thus contribute little hindrance to main-chain mobility.

In the case of polystyrene, every fourth side group consists of a large and complex benzene ($-\text{C}_6\text{H}_5$) molecule ($V \sim 65 \text{ cm}^3/\text{mole}$) in place of a hydrogen atom. It is also known that adjacent benzene rings prefer to be oriented with their major surfaces stacked parallel to one another. Though the initial high relaxation rate exhibited by PS is inconsistent with that expected for one large and complex side-chain constituent, evaluation of the relaxation response (Figure 10) suggests that the long-term relaxation rate would be consistent with those exhibited by PMMA and PC and the index value would be correspondingly lower.

In the case of polypropylene, every fourth pendant group consists of a moderately complex methyl ($-\text{CH}_3$) molecule ($V \sim 23 \text{ cm}^3/\text{mole}$). Thus, the moderately high relaxation rate exhibited by PP is consistent with that expected for a relatively large and moderately complex side-chain constituent.

In the case of polyvinylchloride, every fourth side group consists of a relatively large chlorine atom ($V \sim 20 \text{ cm}^3/\text{mole}$) in place of a hydrogen atom. Thus, the intermediate relaxation rate exhibited by PVC is consistent with that expected for one large and non-complex side-chain constituent. Also, since chlorine is relatively electronegative (E.N. ~ 3.0), it has a high affinity for hydrogen (E.N. ~ 2.1). Thus, the resultant dipole-dipole secondary bonds ($\Delta E \sim 6 \text{ kcal/mole}$) contribute to chain hindrance.

In the case of polymethylmethacrylate, every third pendant group consists of a relatively complex methyl molecule in place of a hydrogen atom and every fourth pendant group consists of a highly

complex methacrylate group ($-\text{COOCH}_3$) molecule ($V \sim 41 \text{ cm}^3/\text{mole}$) in place of a hydrogen atom. Thus, the slow relaxation rate exhibited by PMMA is consistent with that expected for the existence of two large and complex side-chain constituents. In addition, the existence of dipole-dipole secondary bonds between the chains offer considerable impediment to chain mobility.

In the case of the condensation polymer polyhexamethylene apidamide, the most significant factor appears to be the contribution of the large number of methylene (CH_2) groups to chain flexibility. Though the amide (NHCO) groups do provide some chain stiffening and though the sum of pendant group molar volume is high ($V \sim 81 \text{ cm}^3/\text{mole}$), the presence of ten flexible methylene groups along the macromolecular backbone allows an intermediate relaxation response. To compensate for methylene flexibility, the pendant groups of PA are highly polar and thus retard main-chain motion by the formation of strong hydrogen bonds ($\Delta E \sim 10 \text{ kcal/mole}$) between the oxygen and hydrogen atoms in adjacent chains.

In the case of the condensation polymer polycarbonate, the most significant factor appears to be the presence of two bulky methyl ($-\text{CH}_3$) molecules adjacent to each other across the chain and the presence of a carbonyl (CO) group and a pair of benzene (C_6H_4) groups in the chain to provide stiffening. Though there are ether (O) groups along the backbone to provide some flexibility, the aforementioned side-chain and main-chain groups have the predominant affect on the relaxation response of PC.

From the plots of $\ln(P)$ as a function of $\ln(-\dot{P})$ (Figure 11) it can be seen that HDPE at one extreme of the relaxation response spectrum with a fast relaxation rate has a high strain rate sensitivity index value compared with PMMA at the other extreme of the spectrum. This is in accordance with theory in terms of the steric hindrance offered to chain mobility due to side-chain molar volume and main-chain flexibility considerations. It can also be seen from Figure 11 that, for an equivalent change in $\ln(-\dot{P})$, HDPE exhibits almost an order of magnitude larger change in $\ln(P)$ than does PMMA. The result is that the strain rate sensitivity index value for HDPE is almost an order of magnitude larger than the index value for PMMA.

It is apparent from the relatively high degree of linearity of the plots of Figure 12 through Figure 18 that the stress relaxation method is an effective means of strain rate sensitivity index determination. In this regard, Table 3 is indicative of the reliability of the stress relaxation method in that the variance coefficient is less than 16% in the cases of all the thermoplastic specimens tested. In regard to the possibility of improvement in the linearity of plots derived by the stress relaxation method, it can be seen from the plots of Figure 12 through Figure 18 that the limiting factor in the derivation of strain rate sensitivity index values with high correlation coefficient values is the degree of accuracy achieved in the derivation of the characteristic time and rate-of-decay parameters.

Thus, if the numerical algorithm (see Appendix 7) were to be modified such that the value of the error function $\Phi(n, \tau)$ were to approach zero, the correlation coefficient of the slope of $\ln(P)$ versus $\ln(-\dot{P})$ would approach unity.

6.3 Strain Rate Sensitivity Index Comparison

From a comparison of the strain rate sensitivity index results derived from the variable strain rate and stress relaxation techniques, it is apparent that different hindrance mechanisms are operating preferentially during the implementation of each technique when applied to several of the thermoplastics tested. Since Figure 19 demonstrates that the strain rate sensitivity index (m_g) has a relatively high dependence on cohesive energy density (U) magnitude, it is concluded that, in the case of the variable strain rate testing technique, the chain reorientation process is primarily dependent on the strength of the intermolecular bonding forces involved; i.e., a thermoplastic with a high value of cohesive energy density has low chain mobility which is expressed as a higher modulus value for a given strain rate. In comparison, since Figure 20 demonstrates that the strain rate sensitivity index (m_g) derived from stress relaxation testing has a low correlation with cohesive energy density (U), the conclusion is that some other factor is operating preferentially to impede chain mobility during the relaxation process.

In the cases of the strain rate sensitivity index results from variable strain rate and stress relaxation testing plotted as a function of pendant group molar volume, the distinctions as to its importance as a chain mobility hindrance factor are not as clear. In the case of the strain rate sensitivity index (m_g) derived from variable strain rate testing plotted as a function of pendant group molar volume (V) (Figure 21), it appears that the correlative relation between them is relatively significant. It is therefore concluded that pendant group molar volume is also a factor of some importance to chain mobility under the variable strain rate testing regime.

Since Figure 22 demonstrates that the strain rate sensitivity index (m_g) derived from stress relaxation testing is (for some of the thermoplastics tested) a relatively strong function of pendant group molar volume (V), it is concluded that the chain reorientation process is (especially in the case of addition polymers) highly dependent on constituent pendant group considerations during the stress relaxation process. As previously indicated, the case of PA seems to be an exception to the relationship in that the large number of methylene groups in the main-chain backbone seem to enhance chain flexibility; i.e., in this case, not only the magnitude of the pendant group molar volume, but the character of the constituents as well, should be considered as a hindrance factor.

Though the strain rate sensitivity index derived from relaxation testing also appears to be a function of main-chain group flexibility in the case of condensation polymers, the lack of a quantitative measure of this phenomena prevents a graphical presentation of the relationship of strain rate sensitivity index as a function of group flexibility.

Thus, it is concluded that strain rate sensitivity, in the case of thermoplastics, is a measurement technique dependent parameter in that different experimental techniques lead to the prioritization of different hindrance mechanisms. From this conclusion, it seems appropriate to define different parameters to characterize related but inherently different phenomena that are made manifest (in thermoplastics) as a result of the particular testing technique employed. Thus, it is proposed that, for thermoplastics, m_{ϵ} retain its original meaning with respect to strain rate testing and that m_{σ} now refer to the parameter derived from stress rate testing. Furthermore, it is proposed that m_{σ} be referred to as the stress rate sensitivity index. In qualitative terms, in the case of the variable strain rate testing technique, it appears that the process is less random in nature in that the chains reorganize with emphasis on the precedence of energy density. Conversely, in the case of the stress rate testing technique it seems clear that the process is more random in nature in that the chains reorganize according to a precedence established by steric hindrance considerations based on side-chain and main-chain characteristics.

From the plot of average strain rate sensitivity index (m_g) as a function of the characteristic time parameter (Figure 23) for the representative thermoplastic specimens, it is apparent that the two material parameters exhibit a fairly linear inverse relationship. This behavior is consistent with previously outlined macromolecular mobility theory in that thermoplastics with side-chain and main-chain constituents that offer a high degree of steric hindrance to chain mobility exhibit a relatively slow relaxation rate with corresponding low strain rate sensitivity index values and high characteristic time parameter values. Conversely, molecular mobility theory predicts that thermoplastics with side-chain and main-chain constituents that do little to retard long-chain mobility will relax at a relatively fast rate and have high index and low characteristic time parameter values. Thus, the steric hindrance interpretation of macromolecular mobility is confirmed in the relaxation behavior of the thermoplastics tested.

Specifically, at the high end of the relaxation spectrum, HDPE (with small and non-complex pendant groups) is shown to have a high value of strain rate sensitivity index ($m_g \sim 0.132$) and a low value of characteristic time parameter ($\tau \sim 13,000$ s). Next (for addition polymers), with an intermediate position in the relaxation spectrum, PS (with one large pendant group) is shown to have an intermediate value of index ($m_g \sim 0.092$) and an intermediate value of characteristic time ($\tau \sim 60,000$ s).

Next (for addition polymers), with an intermediate position in the relaxation spectrum, PP (with one complex pendant group) is shown to have a intermediate value of index ($m_g \sim 0.089$) and a intermediate value of characteristic time ($\tau \sim 70,000$ s). Next (for addition polymers), with a intermediate position in the relaxation spectrum, PVC (with one large pendant group) is shown to have a intermediate value of index ($m_g \sim 0.069$) and a intermediate value of characteristic time ($\tau \sim 200,000$ s). Finally (again, for addition polymers), at the low end of the relaxation spectrum, PMMA (with two large and complex pendant groups) is shown to have a low value of index ($m_g \sim 0.043$) and a high value of characteristic time parameter ($\tau \sim 1,500,000$ s). In consideration of condensation polymers, PA, with an intermediate position in the relaxation spectrum consistent with main-chain flexibility considerations, is shown to have an intermediate value of index ($m_g \sim 0.079$) and a intermediate value of characteristic time ($\tau \sim 100,000$ s). Finally, for condensation polymers, PC, with a low position in the relaxation spectrum consistent with main-chain flexibility considerations, is shown to have a low value of index ($m_g \sim 0.046$) and a high value of characteristic time ($\tau \sim 800,000$ s). Thus, it is apparent that the characteristic time parameter is a good indicator of the molecular level processes going on during the stress relaxation phenomenon.

6.4 Predictive Considerations

In regard to future research in this area, it seems reasonable to expect that the viscoelastic material parameters of strain rate sensitivity index and characteristic time could be fairly accurately predicted in other thermoplastics from a knowledge of their molecular chemistry. A knowledge of pendant constituents alone would seem to be sufficient to yield a better than order of magnitude parameter values for the addition polymers, while parameter value prediction for condensation polymers would likely require a more extensive characterization of main-chain components and configurations. Though prediction of material parameters is more complex in the case of condensation polymers where large and/or complex molecules are an integral part of the long-chain molecular structure, flexibility considerations for main-chain groups are probably fairly indicative of the relaxation response that can be expected.

Thus, it can be seen that a knowledge of polymer chemistry is essential to the successful prediction of the mechanical behavior of the various thermoplastics in general engineering use. An understanding of the factors that determine the degree of steric hindrance and main-chain flexibility are therefore crucial to the successful selection and implementation of thermoplastics for specific design purposes.

7. Conclusion

From an examination of the results, the study demonstrated that the experimental data validated polymer molecular theory in that there was a correlation of the parameters of the viscoelastic materials in agreement with cohesive energy density values in the case of variable strain rate testing and the character of the side-chain and main-chain groups in the case of stress rate testing. Specifically, in the case of variable strain rate testing, the experimental work demonstrated an inverse relationship between the strain rate sensitivity index ($m_{\dot{\epsilon}}$) and cohesive energy density (U) dependent on intermolecular bond strength. In the case of stress rate testing, the experimental work demonstrated an inverse relationship between the stress rate sensitivity index ($m_{\dot{\sigma}}$) and the characteristic time parameter (τ) dependent on side-chain group size, complexity, and polarity and dependent on main-chain flexibility.

Another important result of the experimental work was the derivation of statistically reliable material parameter values from the relaxation responses of the specimens tested. Thus, it is concluded that the experimental data base was sufficient to generate statistically significant mean and standard deviation values of strain rate sensitivity index and characteristic time parameter for the different thermoplastics.

In view of the fact that the results are seen to be supportive of existing theory, it would seem appropriate to recommend that a more comprehensive study of this phenomenon be undertaken which would not only include more test specimens (to improve the statistical data base), but would be expanded to include a wider variety of thermoplastics. In response to the trend that the role of thermoplastics in engineering applications is an expanding one (significantly as matrix material in fiber-reinforced composites), it would appear that an expanding knowledge base of their mechanical properties is also in order.

In conclusion, it appears that the testing and analytic techniques employed in the study were adequate to achieve the correlation of the strain rate sensitivity index ($m_{\dot{\epsilon}}$) and cohesive energy density (U) for variable strain rate testing, the correlation of the stress rate sensitivity index ($m_{\dot{\sigma}}$) and the characteristic time parameter (τ) for stress rate testing, and the generation of statistically significant values for these parameters for various thermoplastic specimens at ambient temperature.

References

1. M. CHANDA and S. K. ROY, Plastics Technology Handbook, Dekker, New York, NY (1987).
2. A. NADAI and M. J. MANJOINE, "High-speed tension tests at elevated temperature", *Journal of Applied Mechanics* **8** (1941).
3. W. A. BACKOFEN, I. R. TURNER, and D. H. AVERY, "Superplasticity in an Al-Zn alloy", *Transactions of the American Society for Metals* **57** (1964) 980-990.
4. E. W. HART, "Theory of the tensile test", *Acta Metallurgica* **15** (1967) 351-355.
5. Y. LETERRIER and C. G'SELL, "Viscoelastic analysis of a polyurethane thermosetting resin under relaxation and at constant compression rate", *Journal Of Materials Science* **23** (1988) 4209-4216.
6. A. ARAN, "An experimental comparison of different methods to determine the strain rate sensitivity index of superplastic materials" *Scripta Metallurgica* **13** (1979) 843-846.
7. J. HEDWORTH, and M. J. STOWELL, "The measurement to strain-rate sensitivity in superplastic alloys", *Journal of Materials Science* **6** (1971) 1061-1069.
8. G. E. DIETER, Mechanical Metallurgy, 3rd edition, McGraw-Hill, New York, NY (1986).
9. L. E. NIELSEN, Mechanical Properties of Polymers and Composites, Dekker, New York, NY (1974).

10. ASTM Standard E328-86, "Standard Methods for Stress Relaxation Tests for Materials and Structures", Annual Book of ASTM Standards 08.01 (1989).
11. G. HALSEY, H. J. WHITE, and H. EYRING, "Mechanical properties of textiles", *Textile Research Journal* **15** (1945) 295-311.
12. W. M. KOLB, Curve Fitting For Programmable Calculators, 3rd edition, Syntec, Bowie, MD (1984).
13. J. C. MAXWELL, "On the dynamical theory of gases", *Philosophical Transactions* **35** (1868) 129-145, 185-219.
14. R. HOOKE, De Potentiva restitutiva, London (1678).
15. I. NEWTON, Philosophiae Naturalis Principia Mathematica, London (1686).
16. L. MASCIA, Thermoplastics: Materials Engineering, 2nd edition, Elsevier, London (1989).
17. E. WIECHERT, "Gesetze der elastischen nachwirkung fur constante temperatur", *Annalen Der Physik Und Chemie* **50** (1893) 546-570.
18. A. V. TOBOLSKY, Properties and Structures of Polymers, Wiley, New York, NY (1960).
19. R. P. RUDRA, "A curve-fitting program to stress-relaxation data", *Canadian Agricultural Engineering* **29** (1987) 209-211.
20. D. M. BATES and D. G. WATTS, Nonlinear Regression and Its Applications, Wiley, New York, NY (1988).

21. L. C. E. STRUIK, "Physical Aging: Influence on the Deformation Behavior of Amorphous Polymers" in Failure in Plastics, W. BROSTOW and R. D. CORNELIUSSEN, Editors, Hanser, Munich (1986) 218-220.
22. A. S. KRAUSZ and H. EYRING, Deformation Kinetics, Wiley, New York, NY (1975).
23. R. KOHLRAUSCH, *Annalen Physikalische Leipzig* **12** (1847) 393.
24. S. ARRHENIUS, *Zeitschrift Fur Physikalische Chemie* **4** (1889) 226-248.
25. R. W. HERTZBERG, Deformation and Fracture Mechanics of Engineering Material, Wiley, New York, NY (1983).
26. F. W. BILLMEYER, Textbook of Polymer Science, Wiley, New York, NY (1984).
27. D. W. VAN KREVELEN, Properties of Polymers, Elsevier, New York, NY (1976).
28. ASTM Standard D638-89, "Standard Test Method for Tensile Properties of Plastics", Annual Book of ASTM Standards 08.01 (1989).

APPENDIX

Appendix 1. Physical, mechanical, and thermal properties of PP, HDPE, PS, PA, PMMA, PC, and PVC thermoplastics.

TYPE	ρ [g/cm ³]	V [cm ³ /mole]	U [J/mole]	E [10 ⁹ Pa]	ν [numeric]	T _g [K]	T _m [K]
PP	0.90...0.91	32.4	251...254	0.6...1.6	0.43	238...299	385...481
HDPE	0.95...0.97	12.8	187...281	1.0...1.1	0.47	143...250	368...414
PS	1.04...1.05	74.3	302...470	2.3...3.4	0.38	353...373	498...523
PA	1.13...1.15	81.2	654...774	1.9...2.8	0.44	318...330	523...545
PMMA	1.17...1.20	72.7	332...417	2.2...3.2	0.40	266...399	433...473
PC	1.19...1.21	53.2	378...470	2.3...2.5	0.42	393...420	513...573
PVC	1.30...1.58	29.5	302...507	2.4...4.1	0.42	247...354	485...583

Note: ρ = density, V = pendant group molar volume, U = cohesive energy density

E = elastic modulus ν = Poisson's ratio, T_g = glass transition temperature

T_m = melting temperature

Note: all values derived from D. W. VAN KREVELEN, Properties of Polymers, Elsevier (1976).

Note: cohesive energy density values derived from

1. P. C. HIEMENZ, Polymer Chemistry, Dekker (1984).
2. R. B. SEYMOUR and C. E. CARRAHER, Polymer Chemistry, 2nd Edition, Dekker (1988).
3. H. R. ALLCOCK and F. W. LAMPE, Contemporary Polymer Chemistry, Prentice-Hall (1980).
4. D. H. KAEUBLE, Computer-Aided Design of Polymers and Composites, Dekker (1985).

**Appendix 2. Load versus elongation data of
variable strain rate tested
thermoplastic specimens.**

SPECIMEN	v [in./min.]	u [in./min.]	δ [10^{-2} in.]	P [lbf]
HDPE1	0.02	2	0.8359	44.3
			1.5625	88.6
			2.3750	133.0
			3.2813	177.9
	0.05	5	0.6250	44.3
			1.2969	88.6
			2.0000	133.0
			2.7969	177.9
	0.10	10	0.5938	44.3
			1.1875	88.6
			1.8594	133.0
			2.5938	177.9
	0.20	20	0.5703	44.3
			1.1406	88.6
			1.7500	133.0
			2.4375	177.9
	0.50	50	0.5156	44.3
			1.0547	88.6
			1.6250	133.0
			2.2656	177.9
HDPE2	0.02	2	1.0156	44.3
			1.8438	88.6
			2.7500	133.0
			3.7578	177.9
	0.05	5	0.8047	44.3
			1.5000	88.6
			2.2266	133.0
			3.0391	177.9
	0.10	10	0.6172	44.3
			1.2344	88.6
			1.9063	133.0
			2.6563	177.9
	0.20	20	0.6016	44.3
			1.1875	88.6
			1.8125	133.0
			2.5156	177.9
	0.50	50	0.5156	44.3
			1.0625	88.6
			1.6484	133.0
			2.2813	177.9

SPECIMEN	v [in./min.]	u [in./min.]	δ [10^{-2} in.]	P [lbf]
HDPE3	0.02	2	0.7734	44.3
			1.5859	88.6
			2.4766	133.0
			3.4688	177.9
	0.05	5	0.7031	44.3
			1.3906	88.6
			2.1016	133.0
			2.8359	177.9
	0.10	10	0.7188	44.3
			1.3672	88.6
			2.0313	133.0
			2.7344	177.9
	0.20	20	0.6328	44.3
			1.2188	88.6
			1.8281	133.0
			2.4922	177.9
	0.50	50	0.6250	44.3
			1.1875	88.6
			1.7344	133.0
			2.3438	177.9
HDPE4	0.02	2	1.2891	44.3
			2.1172	88.6
			3.0000	133.0
			3.9375	177.9
	0.05	5	0.7656	44.3
			1.4609	88.6
			2.1563	133.0
			2.9453	177.9
	0.10	10	0.7656	44.3
			1.3984	88.6
			2.0391	133.0
			2.7500	177.9
	0.20	20	0.6875	44.3
			1.2969	88.6
			1.9063	133.0
			2.6953	177.9
	0.50	50	0.6250	44.3
			1.1719	88.6
			1.7344	133.0
			2.3438	177.9

SPECIMEN	v [in./min.]	u [in./min.]	δ [10^{-2} in.]	P [lbf]
HDPE5	0.02	2	1.1797	44.3
			2.0078	88.6
			2.9063	133.0
			3.8984	177.9
	0.05	5	0.8047	44.3
			1.5078	88.6
			2.2500	133.0
			3.0547	177.9
	0.10	10	0.7891	44.3
			1.4609	88.6
			2.1172	133.0
			2.8438	177.9
	0.20	20	0.7422	44.3
			1.3594	88.6
			1.9766	133.0
			2.6406	177.9
	0.50	50	0.6563	44.3
			1.2266	88.6
			1.8047	133.0
			2.4219	177.9
HDPE6	0.02	2	0.9766	44.3
			1.7734	88.6
			2.6641	133.0
			3.6875	177.9
	0.05	5	0.7500	44.3
			1.4609	88.6
			2.1797	133.0
			2.9531	177.9
	0.10	10	0.6875	44.3
			1.3516	88.6
			2.0234	133.0
			2.7422	177.9
	0.20	20	0.6250	44.3
			1.2500	88.6
			1.8672	133.0
			2.5469	177.9
	0.50	50	0.6094	44.3
			1.1719	88.6
			1.7344	133.0
			2.3672	177.9

SPECIMEN	v [in./min.]	u [in./min.]	δ [10^{-2} in.]	P [lbf]
HDPE7	0.02	2	1.1875	44.3
			2.0078	88.6
			2.8750	133.0
			3.8594	177.9
	0.05	5	0.8828	44.3
			1.6172	88.6
			2.2625	133.0
			3.0547	177.9
	0.10	10	0.8203	44.3
			1.4766	88.6
			2.1641	133.0
			2.8750	177.9
	0.20	20	0.6953	44.3
			1.3125	88.6
			1.9375	133.0
			2.6016	177.9
	0.50	50	0.5625	44.3
			1.1250	88.6
			1.7031	133.0
			2.3125	177.9
HDPE8	0.02	2	0.9609	44.3
			1.7734	88.6
			2.6641	133.0
			3.6406	177.9
	0.05	5	1.0469	44.3
			1.7500	88.6
			2.4688	133.0
			3.2656	177.9
	0.10	10	1.0469	44.3
			1.7031	88.6
			2.3750	133.0
			3.1016	177.9
	0.20	20	0.8672	44.3
			1.5000	88.6
			2.1484	133.0
			2.8125	177.9
	0.50	50	0.7656	44.3
			1.3438	88.6
			1.9141	133.0
			2.5234	177.9

SPECIMEN	v [in./min.]	u [in./min.]	δ [10^{-2} in.]	P [lbf]
PP1	0.02	2	1.1406	44.3
			1.9375	88.6
			2.7188	133.0
			3.9375	177.9
	0.05	5	1.0000	44.3
			1.7188	88.6
			2.4375	133.0
			3.1719	177.9
	0.10	10	0.8438	44.3
			1.5313	88.6
			2.2188	133.0
			2.9063	177.9
	0.20	20	0.7656	44.3
			1.4351	88.6
			2.1250	133.0
			2.8125	177.9
	0.50	50	0.6953	44.3
			1.3750	88.6
			2.0000	133.0
			2.6563	177.9
PP2	0.02	2	1.0078	44.3
			1.7500	88.6
			2.4688	133.0
			3.2422	177.9
	0.05	5	0.8594	44.3
			1.5313	88.6
			2.2031	133.0
			2.8906	177.9
	0.10	10	0.7813	44.3
			1.4375	88.6
			2.0703	133.0
			2.7188	177.9
	0.20	20	0.7500	44.3
			1.4063	88.6
			2.0313	133.0
			2.6563	177.9
	0.50	50	0.7500	44.3
			1.4063	88.6
			2.0078	133.0
			2.6250	177.9

SPECIMEN	v [in./min.]	u [in./min.]	δ [10^{-2} in.]	P [lbf]
PP3	0.02	2	0.8281	44.3
			1.5781	88.6
			2.3125	133.0
			3.0625	177.9
	0.05	5	0.8125	44.3
			1.5000	88.6
			2.1875	133.0
			2.8750	177.9
	0.10	10	0.7500	44.3
			1.4219	88.6
			2.0625	133.0
			2.7188	177.9
	0.20	20	0.7344	44.3
			1.3750	88.6
			2.0000	133.0
			2.6563	177.9
	0.50	50	0.6563	44.3
			1.2656	88.6
			1.8750	133.0
			2.5000	177.9
PP4	0.02	2	0.8438	44.3
			1.6250	88.6
			2.3906	133.0
			3.1641	177.9
	0.05	5	0.7891	44.3
			1.5000	88.6
			2.2344	133.0
			2.9688	177.9
	0.10	10	0.7656	44.3
			1.4688	88.6
			2.1563	133.0
			2.8594	177.9
	0.20	20	0.7031	44.3
			1.3750	88.6
			2.0313	133.0
			2.7031	177.9
	0.50	50	0.6563	44.3
			1.3125	88.6
			1.9375	133.0
			2.5781	177.9

SPECIMEN	v [in./min.]	u [in./min.]	δ [10^{-2} in.]	P [lbf]
PP5	0.02	2	0.8125	44.3
			1.5313	88.6
			2.2500	133.0
			3.0000	177.9
	0.05	5	0.7500	44.3
			1.4063	88.6
			2.0938	133.0
			2.7891	177.9
	0.10	10	0.7109	44.3
			1.3494	88.6
			2.0000	133.0
			2.6563	177.9
	0.20	20	0.6875	44.3
			1.3125	88.6
			1.9531	133.0
			2.5781	177.9
	0.50	50	0.6094	44.3
			1.2188	88.6
			1.8359	133.0
			2.4531	177.9
PP6	0.02	2	0.8750	44.3
			1.5781	88.6
			2.2969	133.0
			3.0469	177.9
	0.05	5	0.8125	44.3
			1.4922	88.6
			2.1719	133.0
			2.8594	177.9
	0.10	10	0.7813	44.3
			1.4297	88.6
			2.0938	133.0
			2.7578	177.9
	0.20	20	0.7109	44.3
			1.3359	88.6
			1.9688	133.0
			2.6016	177.9
	0.50	50	0.7031	44.3
			1.3281	88.6
			1.9453	133.0
			2.5547	177.9

SPECIMEN	v [in./min.]	u [in./min.]	δ [10^{-2} in.]	P [lbf]
PS1	0.02	2	0.7500	44.3
			1.0000	66.6
			1.2344	88.8
			1.4609	111.1
	0.05	5	0.6250	44.3
			0.7969	66.6
			1.0156	88.8
			1.3594	111.1
	0.10	10	0.6172	44.3
			0.7891	66.6
			1.0078	88.8
			1.2656	111.1
	0.20	20	0.6016	44.3
			0.7656	66.6
			1.0000	88.8
			1.2344	111.1
	0.50	50	0.4688	44.3
			0.6875	66.6
			0.9219	88.8
			1.1406	111.1
PS2	0.02	2	0.4063	44.3
			0.7813	88.6
			1.1719	133.0
			1.5625	177.9
	0.05	5	0.3828	44.3
			0.7656	88.6
			1.1719	133.0
			1.5469	177.9
	0.10	10	0.3750	44.3
			0.7578	88.6
			1.1563	133.0
			1.5391	177.9
	0.20	20	0.3672	44.3
			0.7500	88.6
			1.1484	133.0
			1.5313	177.9
	0.50	50	0.3594	44.3
			0.7422	88.6
			1.1406	133.0
			1.5156	177.9

SPECIMEN	v [in./min.]	u [in./min.]	δ [10^{-2} in.]	P [lbf]
PS3	0.02	2	0.4375	44.3
			0.8750	88.6
			1.3281	133.0
			1.7656	177.9
	0.05	5	0.4063	44.3
			0.8359	88.6
			1.2656	133.0
			1.6719	177.9
	0.10	10	0.3984	44.3
			0.8203	88.6
			1.2500	133.0
			1.6563	177.9
	0.20	20	0.3906	44.3
			0.8125	88.6
			1.2188	133.0
			1.6406	177.9
	0.50	50	0.3750	44.3
			0.7813	88.6
			1.2031	133.0
			1.6094	177.9
PS4	0.02	2	0.5938	44.3
			1.0625	88.6
			1.5000	133.0
			1.9297	177.9
	0.05	5	0.5391	44.3
			0.9922	88.6
			1.4297	133.0
			1.8359	177.9
	0.10	10	0.5313	44.3
			0.9766	88.6
			1.4141	133.0
			1.8203	177.9
	0.20	20	0.5234	44.3
			0.9609	88.6
			1.3828	133.0
			1.7891	177.9
	0.50	50	0.4688	44.3
			0.9141	88.6
			1.3359	133.0
			1.7422	177.9

SPECIMEN	v [in./min.]	u [in./min.]	δ [10^{-2} in.]	P [lbf]
PS5	0.02	2	0.5859	44.3
			1.0547	88.6
			1.5000	133.0
			1.9297	177.9
	0.05	5	0.5234	44.3
			0.9609	88.6
			1.3906	133.0
			1.8125	177.9
	0.10	10	0.5156	44.3
			0.9531	88.6
			1.3828	133.0
			1.8047	177.9
	0.20	20	0.4922	44.3
			0.9375	88.6
			1.3672	133.0
			1.7891	177.9
	0.50	50	0.4688	44.3
			0.9219	88.6
			1.3516	133.0
			1.7656	177.9
PS6	0.02	2	0.6484	44.3
			1.1250	88.6
			1.5625	133.0
			2.0000	177.9
	0.05	5	0.5547	44.3
			1.0078	88.6
			1.4375	133.0
			1.8594	177.9
	0.10	10	0.5391	44.3
			1.0000	88.6
			1.4297	133.0
			1.8438	177.9
	0.20	20	0.5234	44.3
			0.9844	88.6
			1.4141	133.0
			1.8281	177.9
	0.50	50	0.4766	44.3
			0.9375	88.6
			1.3438	133.0
			1.7656	177.9

SPECIMEN	v [in./min.]	u [in./min.]	δ [10^{-2} in.]	P [lbf]
PC1	0.02	2	1.8438	221.4
			3.2656	442.9
			4.6094	665.2
			6.0000	889.3
	0.05	5	1.7500	221.4
			3.1563	442.9
			4.4844	665.2
			5.8281	889.3
	0.10	10	1.7188	221.4
			3.1094	442.9
			4.4219	665.2
			5.7656	889.3
	0.20	20	1.7031	221.4
			3.0781	442.9
			4.4063	665.2
			5.7500	889.3
	0.50	50	1.5938	221.4
			2.9688	442.9
			4.2656	665.2
			5.5938	889.3
PC2	0.02	2	2.1563	221.4
			3.5781	442.9
			4.9375	665.2
			6.2813	889.3
	0.05	5	1.9375	221.4
			3.3281	442.9
			4.6875	665.2
			6.0000	889.3
	0.10	10	1.7813	221.4
			3.1563	442.9
			4.4844	665.2
			5.7813	889.3
	0.20	20	1.7188	221.4
			3.0938	442.9
			4.4375	665.2
			5.7188	889.3
	0.50	50	1.6563	221.4
			3.0313	442.9
			4.3438	665.2
			5.6250	889.3

SPECIMEN	v [in./min.]	u [in./min.]	δ [10^{-2} in.]	P [lbf]
PC3	0.02	2	2.2344	221.4
			3.7188	442.9
			5.0625	665.2
	0.05	5	6.4375	889.3
			1.9063	221.4
			3.3125	442.9
			4.6250	665.2
			5.9688	889.3
	0.10	10	1.8594	221.4
			3.2656	442.9
			4.5625	665.2
			5.9063	889.3
	0.20	20	1.7969	221.4
			3.2031	442.9
			4.4844	665.2
			5.8125	889.3
	0.50	50	1.7813	221.4
			3.1875	442.9
			4.4688	665.2
			5.7969	889.3
PC4	0.02	2	1.7031	221.4
			3.2031	442.9
			4.5625	665.2
			5.9375	889.3
	0.05	5	1.6875	221.4
			3.1719	442.9
			4.5313	665.2
			5.8750	889.3
	0.10	10	1.6719	221.4
			3.1563	442.9
			4.5000	665.2
			5.8125	889.3
	0.20	20	1.6250	221.4
			3.0938	442.9
			4.4375	665.2
			5.7500	889.3
	0.50	50	1.5781	221.4
			3.0469	442.9
			4.3594	665.2
			5.6875	889.3

SPECIMEN	v [in./min.]	u [in./min.]	δ [10^{-2} in.]	P [lbf]
PC5	0.02	2	1.7969	221.4
			3.2188	442.9
			4.6094	665.2
			5.9844	889.3
	0.05	5	1.7813	221.4
			3.1875	442.9
			4.5625	665.2
			5.8906	889.3
	0.10	10	1.7344	221.4
			3.1406	442.9
			4.4844	665.2
			5.8281	889.3
	0.20	20	1.7031	221.4
			3.1094	442.9
			4.4531	665.2
			5.7813	889.3
	0.50	50	1.6875	221.4
			3.0938	442.9
			4.4375	665.2
			5.7500	889.3
PC6	0.02	2	1.8125	221.4
			3.2344	442.9
			4.6094	665.2
			5.9844	889.3
	0.05	5	1.7656	221.4
			3.1563	442.9
			4.5156	665.2
			5.8594	889.3
	0.10	10	1.7031	221.4
			3.0938	442.9
			4.4375	665.2
			5.7813	889.3
	0.20	20	1.6875	221.4
			3.0625	442.9
			4.4219	665.2
			5.7500	889.3
	0.50	50	1.6406	221.4
			3.0313	442.9
			4.3906	665.2
			5.7188	889.3

SPECIMEN	v [in./min.]	u [in./min.]	δ [10^{-2} in.]	P [lbf]
PA1	0.02	2	0.9688	221.4
			1.9375	442.9
			2.9063	665.2
			3.8750	889.3
	0.05	5	0.9375	221.4
			1.8828	442.9
			2.8438	665.2
			3.7813	889.3
	0.10	10	0.9219	221.4
			1.8513	442.9
			2.7891	665.2
			3.7188	889.3
	0.20	20	0.9141	221.4
			1.8438	442.9
			2.7734	665.2
			3.6875	889.3
	0.50	50	0.9063	221.4
			1.8359	442.9
			2.7500	665.2
			3.6484	889.3
PA2	0.02	2	0.9844	221.4
			1.9688	442.9
			2.9531	665.2
			3.9453	889.3
	0.05	5	0.9453	221.4
			1.8906	442.9
			2.8438	665.2
			3.7891	889.3
	0.10	10	0.9297	221.4
			1.8594	442.9
			2.7969	665.2
			3.7188	889.3
	0.20	20	0.9219	221.4
			1.8438	442.9
			2.7578	665.2
			3.6875	889.3
	0.50	50	0.9063	221.4
			1.8125	442.9
			2.7266	665.2
			3.6484	889.3

SPECIMEN	v [in./min.]	u [in./min.]	δ [10^{-2} in.]	P [lbf]
PA3	0.02	2	0.9531	221.4
			1.9219	442.9
			2.8750	665.2
			3.8516	889.3
	0.05	5	0.9375	221.4
			1.8828	442.9
			2.8203	665.2
			3.7656	889.3
	0.10	10	0.9297	221.4
			1.8594	442.9
			2.7891	665.2
			3.7266	889.3
	0.20	20	0.9219	221.4
			1.8516	442.9
			2.7813	665.2
			3.7188	889.3
	0.50	50	0.9141	221.4
			1.8438	442.9
			2.7656	665.2
			3.6875	889.3
PA4	0.02	2	0.9531	221.4
			1.9063	442.9
			2.8594	665.2
			3.8125	889.3
	0.05	5	0.9297	221.4
			1.8516	442.9
			2.7891	665.2
			3.7109	889.3
	0.10	10	0.9219	221.4
			1.8438	442.9
			2.7813	665.2
			3.7031	889.3
	0.20	20	0.8974	221.4
			1.8047	442.9
			2.7109	665.2
			3.6250	889.3
	0.50	50	0.8906	221.4
			1.7969	442.9
			2.7031	665.2
			3.6172	889.3

SPECIMEN	v [in./min.]	u [in./min.]	δ [10^{-2} in.]	P [lbf]
PA5	0.02	2	0.9844	221.4
			1.9766	442.9
			2.9453	665.2
			3.9453	889.3
	0.05	5	0.9766	221.4
			1.9531	442.9
			2.9141	665.2
			3.8906	889.3
	0.10	10	0.9688	221.4
			1.9297	442.9
			2.8750	665.2
			3.8438	889.3
	0.20	20	0.9609	221.4
			1.8906	442.9
			2.8359	665.2
			3.8047	889.3
	0.50	50	0.9531	221.4
			1.8672	442.9
			2.7969	665.2
			3.7969	889.3
PA6	0.02	2	0.9531	221.4
			1.9141	442.9
			2.8594	665.2
			3.8281	889.3
	0.05	5	0.9453	221.4
			1.8906	442.9
			2.8281	665.2
			3.7813	889.3
	0.10	10	0.9375	221.4
			1.8516	442.9
			2.7891	665.2
			3.7188	889.3
	0.20	20	0.9219	221.4
			1.8438	442.9
			2.7734	665.2
			3.7031	889.3
	0.50	50	0.9063	221.4
			1.8203	442.9
			2.7344	665.2
			3.6484	889.3

SPECIMEN	v [in./min.]	u [in./min.]	δ [10^{-2} in.]	P [lbf]
PA7	0.02	2	1.4688	221.4
			2.4922	442.9
			3.4688	665.2
			4.4531	889.3
	0.05	5	1.2578	221.4
			2.2500	442.9
			3.1953	665.2
			4.1563	889.3
	0.10	10	1.2344	221.4
			2.2031	442.9
			3.1563	665.2
			4.0938	889.3
	0.20	20	1.1875	221.4
			2.1563	442.9
			3.1016	665.2
			4.0625	889.3
	0.50	50	1.1719	221.4
			2.1484	442.9
			3.0938	665.2
			4.0313	889.3
PA8	0.02	2	1.4688	221.4
			2.4766	442.9
			3.4688	665.2
			4.4609	889.3
	0.05	5	1.2813	221.4
			2.2500	442.9
			3.2031	665.2
			4.1563	889.3
	0.10	10	1.2813	221.4
			2.2500	442.9
			3.1953	665.2
			4.1328	889.3
	0.20	20	1.2188	221.4
			2.2031	442.9
			3.1641	665.2
			4.0938	889.3
	0.50	50	1.2109	221.4
			2.1875	442.9
			3.1406	665.2
			4.0625	889.3

SPECIMEN	v [in./min.]	u [in./min.]	δ [10^{-2} in.]	P [lbf]
PVC1	0.02	2	1.6172	221.4
			2.6641	442.9
			3.6719	665.2
			4.7813	889.3
	0.05	5	1.5938	221.4
			2.6172	442.9
			3.6406	665.2
			4.7188	889.3
	0.10	10	1.4219	221.4
			2.4375	442.9
			3.4531	665.2
			4.5000	889.3
	0.20	20	1.3750	221.4
			2.3750	442.9
			3.3906	665.2
			4.4219	889.3
	0.50	50	1.2891	221.4
			2.2813	442.9
			3.2813	665.2
			4.2969	889.3
PVC2	0.02	2	1.4453	221.4
			2.4531	442.9
			3.4531	665.2
			4.5469	889.3
	0.05	5	1.2031	221.4
			2.1875	442.9
			3.1797	665.2
			4.2344	889.3
	0.10	10	1.1406	221.4
			2.1406	442.9
			3.1406	665.2
			4.1563	889.3
	0.20	20	1.1250	221.4
			2.1094	442.9
			3.1172	665.2
			4.1406	889.3
	0.50	50	1.0938	221.4
			2.0859	442.9
			3.0938	665.2
			4.1172	889.3

SPECIMEN	v [in./min.]	u [in./min.]	δ [10^{-2} in.]	P [lbf]
PVC3	0.02	2	0.9844	221.4
			1.9766	442.9
			2.9688	665.2
			4.0547	889.3
	0.05	5	0.9688	221.4
			1.9688	442.9
			2.9531	665.2
			4.0000	889.3
	0.10	10	0.9609	221.4
			1.9375	442.9
			2.9375	665.2
			3.9922	889.3
	0.20	20	0.9531	221.4
			1.9219	442.9
			2.9063	665.2
			3.9531	889.3
	0.50	50	0.9453	221.4
			1.9141	442.9
			2.8984	665.2
			3.9375	889.3
PVC4	0.02	2	1.5078	221.4
			2.5781	442.9
			3.6250	665.2
			4.7656	889.3
	0.05	5	1.4453	221.4
			2.4766	442.9
			3.5156	665.2
			4.6094	889.3
	0.10	10	1.4375	221.4
			2.4688	442.9
			3.5078	665.2
			4.5625	889.3
	0.20	20	1.4297	221.4
			2.4609	442.9
			3.5000	665.2
			4.5547	889.3
	0.50	50	1.2969	221.4
			2.3125	442.9
			3.3438	665.2
			4.3906	889.3

SPECIMEN	v [in./min.]	u [in./min.]	δ [10^{-2} in.]	P [lbf]
PVC5	0.02	2	0.7969	110.7
			1.3516	221.4
			1.8672	332.6
			2.3750	444.6
	0.05	5	0.7109	110.7
			1.2422	221.4
			1.7422	332.6
			2.2500	444.6
	0.10	10	0.6875	110.7
			1.2109	221.4
			1.6953	332.6
			2.1875	444.6
	0.20	20	0.6563	110.7
			1.1797	221.4
			1.6797	332.6
			2.1719	444.6
	0.50	50	0.6250	110.7
			1.1484	221.4
			1.6406	332.6
			2.1406	444.6
PVC6	0.02	2	0.8438	110.7
			1.3594	221.4
			1.8672	332.6
			2.3750	444.6
	0.05	5	0.6953	110.7
			1.2109	221.4
			1.7109	332.6
			2.2031	444.6
	0.10	10	0.6641	110.7
			1.1875	221.4
			1.6797	332.6
			2.1797	444.6
	0.20	20	0.6172	110.7
			1.1250	221.4
			1.6250	332.6
			2.1094	444.6
	0.50	50	0.5938	110.7
			1.1094	221.4
			1.5938	332.6
			2.0938	444.6

SPECIMEN	v [in./min.]	u [in./min.]	δ [10^{-2} in.]	P [lbf]
PVC7	0.02	2	0.8828	110.7
			1.4141	221.4
			1.9141	332.6
			2.4141	444.6
	0.05	5	0.7031	110.7
			1.2266	221.4
			1.6953	332.6
			2.1953	444.6
	0.10	10	0.6797	110.7
			1.2109	221.4
			1.6953	332.6
			2.1953	444.6
	0.20	20	0.6641	110.7
			1.1953	221.4
			1.6875	332.6
			2.1797	444.6
	0.50	50	0.6484	110.7
			1.1641	221.4
			1.6641	332.6
			2.1563	444.6
PVC8	0.02	2	0.9844	110.7
			1.5625	221.4
			2.0938	332.6
			2.6094	444.6
	0.05	5	0.8203	110.7
			1.3984	221.4
			1.9297	332.6
			2.4531	444.6
	0.10	10	0.8125	110.7
			1.3750	221.4
			1.8984	332.6
			2.4141	444.6
	0.20	20	0.7734	110.7
			1.3516	221.4
			1.8828	332.6
			2.4063	444.6
	0.50	50	0.7656	110.7
			1.3281	221.4
			1.8438	332.6
			2.3750	444.6

SPECIMEN	v [in./min.]	u [in./min.]	δ [10^{-2} in.]	P [lbf]
PMMA1	0.02	5	0.3920	44.3
			0.7640	88.6
			1.1560	133.0
			1.5200	177.9
	0.05	5	0.3594	44.3
			0.7188	88.6
			1.0469	133.0
			1.4375	177.9
	0.10	10	0.3516	44.3
			0.7109	88.6
			1.0625	133.0
			1.4219	177.9
	0.20	10	0.3126	44.3
			0.6250	88.6
			1.0000	133.0
			1.3126	177.9
	0.50	20	0.3125	44.3
			0.6250	88.6
			0.9570	133.0
			1.2890	177.9
PMMA2	0.02	2	0.9531	110.7
			1.9219	221.4
			2.8672	332.6
			3.8281	444.6
	0.05	5	0.9063	110.7
			1.8203	221.4
			2.7344	332.6
			3.6563	444.6
	0.10	10	0.8906	110.7
			1.7813	221.4
			2.6719	332.6
			3.5781	444.6
	0.20	10	0.8282	110.7
			1.6876	221.4
			2.5468	332.6
			3.4062	444.6
	0.50	20	0.8203	110.7
			1.6408	221.4
			2.4610	332.6
			3.3203	444.6

SPECIMEN	v [in./min.]	u [in./min.]	δ [10^{-2} in.]	P [lbf]
PMMA3	0.02	2	0.4063	44.3
			0.8047	88.6
			1.2188	133.0
			1.6094	177.9
	0.05	5	0.3906	44.3
			0.7813	88.6
			1.2031	133.0
			1.5781	177.9
	0.10	10	0.3984	44.3
			0.7813	88.6
			1.1719	133.0
			1.5625	177.9
	0.20	10	0.3592	44.3
			0.7500	88.6
			1.1250	133.0
			1.4688	177.9
	0.50	20	0.3515	44.3
			0.7423	88.6
			1.0548	133.0
			1.4063	177.9
PMMA4	0.02	2	0.9219	110.7
			1.8438	221.4
			2.7813	332.6
			3.7422	444.6
	0.05	5	0.8906	110.7
			1.7813	221.4
			2.6641	332.6
			3.5938	444.6
	0.10	10	0.8438	110.7
			1.7188	221.4
			2.5859	332.6
			3.4688	444.6
	0.20	20	0.8359	110.7
			1.7031	221.4
			2.5625	332.6
			3.4219	444.6
	0.50	50	0.8203	110.7
			1.6875	221.4
			2.5156	332.6
			3.3516	444.6

SPECIMEN	v [in./min.]	u [in./min.]	δ [10^{-2} in.]	P [lbf]
PMMA5	0.02	2	0.9219	110.7
			1.8750	221.4
			2.7969	332.6
			3.7500	444.6
	0.05	5	0.8984	110.7
			1.8281	221.4
			2.7266	332.6
			3.6563	444.6
	0.10	10	0.8750	110.7
			1.7734	221.4
			2.6563	332.6
			3.5703	444.6
	0.20	20	0.8594	110.7
			1.7031	221.4
			2.5625	332.6
			3.4609	444.6
	0.50	50	0.8281	110.7
			1.6563	221.4
			2.4844	332.6
			3.3438	444.6
PMMA6	0.02	2	0.9375	110.7
			1.8750	221.4
			2.8203	332.6
			3.8594	444.6
	0.05	5	0.8906	110.7
			1.7813	221.4
			2.6797	332.6
			3.6250	444.6
	0.10	10	0.8672	110.7
			1.7344	221.4
			2.5938	332.6
			3.5000	444.6
	0.20	20	0.8438	110.7
			1.7109	221.4
			2.5781	332.6
			3.4453	444.6
	0.50	50	0.8125	110.7
			1.6563	221.4
			2.5313	332.6
			3.3125	444.6

SPECIMEN	v [in./min.]	u [in./min.]	δ [10^{-2} in.]	P [lbf]
PMMA7	0.02	2	1.0000	110.7
			1.9844	221.4
			2.9766	332.6
			4.0078	444.6
	0.05	5	0.9688	110.7
			1.9219	221.4
			2.8828	332.6
			3.8750	444.6
	0.10	10	0.9375	110.7
			1.8750	221.4
			2.8125	332.6
			3.7656	444.6
	0.20	10	0.9062	110.7
			1.8438	221.4
			2.7500	332.6
			3.6875	444.6
	0.50	20	0.8360	110.7
			1.6993	221.4
			2.5625	332.6
			3.4533	444.6
PMMA8	0.02	2	0.9563	110.7
			1.9000	221.4
			2.8438	332.6
			3.8219	444.6
	0.05	5	0.9141	110.7
			1.8438	221.4
			2.7500	332.6
			3.6813	444.6
	0.10	10	0.8906	110.7
			1.7938	221.4
			2.6828	332.6
			3.5938	444.6
	0.20	10	0.8438	110.7
			1.7000	221.4
			2.5626	332.6
			3.4532	444.6
	0.50	20	0.8008	110.7
			1.6408	221.4
			2.4453	332.6
			3.3008	444.6

Appendix 3. Elastic modulus, strain rate sensitivity index, and correlation coefficient of variable strain rate tested thermoplastic specimens.

SPECIMEN	$E_{0.02}$ [psi]	$E_{0.05}$ [psi]	$E_{0.1}$ [psi]	$E_{0.2}$ [psi]	$E_{0.5}$ [psi]	$m_{\dot{\epsilon}}$	r
HDPE ₁	68,360	79,178	85,381	90,969	97,523	0.1088	0.9859
HDPE ₂	60,049	73,762	83,532	88,305	96,615	0.1449	0.9746
HDPE ₃	64,099	78,198	81,461	89,252	95,038	0.1181	0.9706
HDPE ₄	57,660	75,948	81,490	83,408	95,044	0.1415	0.9464
HDPE ₅	58,237	73,250	78,726	84,650	91,980	0.1361	0.9684
HDPE ₆	61,241	75,413	81,012	87,091	94,052	0.1287	0.9724
HDPE ₇	58,903	73,692	77,848	85,624	95,571	0.1438	0.9818
HDPE ₈	61,797	69,282	72,856	79,920	88,755	0.1107	0.9975
PP ₁	63,823	70,822	76,796	79,094	83,517	0.0829	0.9794
PP ₂	69,676	77,408	81,975	83,617	84,557	0.0594	0.9251
PP ₃	72,717	77,510	81,763	83,876	88,707	0.0610	0.9949
PP ₄	70,320	74,931	77,744	82,020	85,741	0.0622	0.9976
PP ₅	74,372	79,822	83,624	85,930	89,947	0.0581	0.9908
PP ₆	73,539	78,008	80,827	85,395	86,817	0.0537	0.9834
PMMA ₁	240,402	256,466	257,135	275,467	283,344	0.0511	0.9762
PMMA ₂	242,580	253,765	259,338	271,388	279,402	0.0446	0.9948
PMMA ₃	227,005	230,502	234,672	246,794	259,674	0.0429	0.9638
PMMA ₄	248,323	258,668	266,846	270,169	275,552	0.0322	0.9784
PMMA ₅	247,507	253,692	259,773	268,511	277,353	0.0362	0.9954
PMMA ₆	241,764	256,480	265,454	268,505	277,244	0.0411	0.9757
PMMA ₇	232,609	240,303	246,716	251,592	268,252	0.0425	0.9839
PMMA ₈	243,563	252,144	258,140	268,497	280,550	0.0441	0.9956

PS ₁	97,728	103,637	109,583	111,809	119,764	0.0619	0.9943
PS ₂	140,984	141,257	142,115	142,676	143,819	0.0063	0.9801
PS ₃	124,216	130,592	131,804	133,545	135,554	0.0254	0.9483
PS ₄	114,797	120,062	121,134	123,454	126,084	0.0277	0.9787
PS ₅	114,687	122,050	122,506	123,228	124,342	0.0223	0.8575
PS ₆	111,050	119,016	119,659	120,548	124,718	0.0319	0.9320

PVC ₁	238,380	241,148	251,770	255,862	262,658	0.0321	0.9801
PVC ₂	250,167	266,542	270,037	271,101	272,172	0.0240	0.8514
PVC ₃	275,883	278,860	279,473	282,284	283,158	0.0082	0.9787
PVC ₄	238,771	246,302	248,217	248,600	257,083	0.0204	0.9595
PVC ₅	234,399	247,580	254,247	255,479	259,146	0.0299	0.9332
PVC ₆	235,404	252,482	255,032	262,614	264,771	0.0352	0.9382
PVC ₇	231,161	249,078	253,323	254,593	257,417	0.0307	0.8822
PVC ₈	213,362	226,910	230,689	231,043	234,811	0.0271	0.9029

PC ₁	189,959	194,886	197,017	197,516	202,444	0.0182	0.9766
PC ₂	181,785	189,656	196,326	197,984	201,113	0.0313	0.9616
PC ₃	177,500	190,911	192,828	195,610	196,078	0.0288	0.8741
PC ₄	190,686	192,440	194,184	196,150	198,324	0.0124	0.9981
PC ₅	189,962	192,627	194,757	196,040	196,877	0.0114	0.9742
PC ₆	190,083	193,890	196,287	197,159	197,896	0.0124	0.9443

PA ₁	287,220	293,690	298,760	300,970	303,970	0.0176	0.9775
PA ₂	282,340	293,470	298,750	301,730	304,770	0.0323	0.9554
PA ₃	289,050	295,370	298,520	299,010	301,230	0.0122	0.9423
PA ₄	291,820	299,500	299,990	306,560	307,080	0.0160	0.9525
PA ₅	282,560	286,320	289,960	293,240	294,640	0.0137	0.9836
PA ₆	290,890	294,420	299,260	300,240	304,510	0.0142	0.9881
PA ₇	253,774	270,814	274,564	276,762	278,230	0.0266	0.8786
PA ₈	253,542	271,075	272,387	274,020	276,083	0.0235	0.8483

**Appendix 4. Load versus time data of stress
relaxation tested thermoplastic
specimens**

HDPE₁ @ $\epsilon = 0.0539$

P [lbf]	t [s]
460.5	0.1
435.9	3.8
384.9	33.8
354.1	93.8
333.0	183.8
317.2	300.0
298.8	600.0
279.4	1200.0
270.7	1800.0

HDPE₂ @ $\epsilon = 0.0421$

P [lbf]	t [s]
416.5	0.1
362.0	16.9
335.7	46.9
312.8	106.9
296.1	195.9
284.7	300.0
266.3	600.0
249.6	1200.0
240.3	1800.0

HDPE₃ @ $\epsilon = 0.0508$

P [lbf]	t [s]
444.6	0.1
384.9	19.7
359.0	49.7
334.8	109.7
317.2	198.7
305.4	300.0
286.0	600.0
268.9	1200.0
259.2	1800.0

HDPE₄ @ $\epsilon = 0.0408$

P [lbf]	t [s]
409.5	0.1
397.2	1.9
374.8	5.6
341.8	30.0
311.1	90.0
293.1	180.0
280.8	300.0
264.1	600.0
247.8	1200.0
239.0	1800.0

HDPE₅ @ $\epsilon = 0.0471$

P [lbf]	t [s]
416.5	0.1
365.6	15.0
337.4	45.0
314.1	105.0
297.0	195.0
285.1	300.0
267.6	600.0
256.2	900.0
249.1	1200.0
242.1	1800.0

HDPE₆ @ $\epsilon = 0.0458$

P [lbf]	t [s]
444.5	0.1
388.2	15.9
359.6	45.9
334.5	105.9
316.9	195.9
305.9	300.0
287.4	600.0
277.7	900.0
270.7	1200.0
260.6	1800.0

PS₁ @ $\varepsilon = 0.0289$

\underline{P} [lbf]	\underline{t} [s]
421.8	0.1
386.6	4.7
349.3	34.7
332.6	94.7
323.4	184.7
317.2	300.0
309.3	600.0
304.0	900.0
300.5	1200.0
297.0	1800.0

PS₂ @ $\varepsilon = 0.0193$

\underline{P} [lbf]	\underline{t} [s]
494.3	0.1
419.6	3.8
365.6	33.8
346.2	93.8
335.2	183.8
328.2	300.0
318.5	600.0
314.6	900.0
309.8	1200.0
303.6	1800.0

PS₃ @ $\varepsilon = 0.0206$

\underline{P} [lbf]	\underline{t} [s]
500.0	0.1
430.6	9.4
377.0	39.4
351.9	99.4
338.3	189.4
329.1	300.0
318.1	600.0
313.3	900.0
308.4	1200.0
301.8	1800.0

PS₄ @ $\varepsilon = 0.0213$

\underline{P} [lbf]	\underline{t} [s]
500.0	0.1
426.9	13.6
377.2	43.6
351.7	103.6
336.3	193.6
327.9	300.0
316.5	600.0
309.9	900.0
306.3	1200.0
299.7	1800.0

PS₅ @ $\varepsilon = 0.0225$

\underline{P} [lbf]	\underline{t} [s]
489.4	0.1
389.1	17.8
368.0	47.8
352.1	107.8
341.5	197.8
335.4	300.0
325.7	600.0
320.4	900.0
316.5	1200.0
310.3	1800.0

PS₆ @ $\varepsilon = 0.0206$

\underline{P} [lbf]	\underline{t} [s]
501.8	0.1
408.5	25.8
376.8	55.8
355.2	115.8
341.5	205.8
334.1	300.0
321.7	600.0
315.1	900.0
309.9	1200.0
303.7	1800.0

PP₁ @ $\varepsilon = 0.0508$

P [lbf]	t [s]
500.0	0.1
471.9	3.8
441.6	33.8
417.8	93.8
402.9	183.8
392.4	300.0
379.6	600.0
372.1	900.0
366.4	1200.0
358.1	1800.0

PP₂ @ $\varepsilon = 0.0458$

P [lbf]	t [s]
500.0	0.1
463.1	15.0
439.4	45.0
420.0	105.0
405.5	195.0
395.5	300.0
380.5	600.0
371.7	900.0
365.6	1200.0
356.8	1800.0

PP₃ @ $\varepsilon = 0.0458$

P [lbf]	t [s]
500.0	0.1
451.7	21.6
430.1	51.6
410.4	111.1
395.4	199.7
384.9	300.0
367.8	600.0
358.5	900.0
352.4	1200.0
343.6	1800.0

PP₄ @ $\varepsilon = 0.0477$

P [lbf]	t [s]
500.0	0.1
472.7	7.0
438.4	37.0
414.6	97.0
397.9	187.0
385.6	300.0
369.7	600.0
360.0	900.0
353.9	1200.0
345.1	1800.0

PP₅ @ $\varepsilon = 0.0383$

P [lbf]	t [s]
445.4	0.1
403.2	20.6
384.2	50.6
366.6	110.6
353.9	200.6
345.1	300.0
331.0	600.0
322.6	900.0
317.3	1200.0
310.3	1800.0

PP₆ @ $\varepsilon = 0.0383$

P [lbf]	t [s]
445.0	0.1
411.5	13.1
387.8	43.1
368.8	103.1
354.3	193.1
345.1	300.0
332.3	600.0
323.9	900.0
318.2	1200.0
310.3	1800.0

PA₁ @ $\varepsilon = 0.0396$

\underline{P} [lbf]	\underline{t} [s]
2000.7	0.1
1956.1	4.7
1885.8	19.7
1824.3	49.7
1761.0	109.7
1711.8	199.7
1678.4	300.0
1623.9	600.0
1588.8	900.0
1565.9	1200.0
1546.6	1500.0
1532.5	1800.0

PA₂ @ $\varepsilon = 0.0213$

\underline{P} [lbf]	\underline{t} [s]
2000.0	0.1
1903.3	12.9
1857.6	27.9
1804.9	57.9
1745.2	117.9
1701.2	207.2
1669.6	300.0
1609.8	600.0
1574.7	900.0
1551.8	1200.0
1532.5	1500.0
1514.9	1800.0

PA₃ @ $\varepsilon = 0.0213$

\underline{P} [lbf]	\underline{t} [s]
2000.0	0.1
1884.0	16.6
1812.0	46.6
1746.9	106.1
1701.2	196.7
1662.6	300.0
1606.3	600.0
1574.7	900.0
1553.6	1200.0
1536.0	1500.0
1518.5	1800.0

PA₄ @ $\varepsilon = 0.0408$

\underline{P} [lbf]	\underline{t} [s]
2000.0	0.1
1927.9	7.5
1827.8	37.5
1752.2	97.5
1699.5	187.5
1660.8	300.0
1602.8	600.0
1569.4	900.0
1546.6	1200.0
1525.5	1500.0
1511.4	1800.0

PA₅ @ $\varepsilon = 0.0436$

\underline{P} [lbf]	\underline{t} [s]
2000.0	0.1
1829.5	23.9
1757.5	53.9
1685.4	113.9
1627.4	203.9
1588.8	300.0
1514.9	600.0
1472.8	900.0
1442.9	1200.0
1423.6	1500.0
1406.0	1800.0

PA₆ @ $\varepsilon = 0.0396$

\underline{P} [lbf]	\underline{t} [s]
2000.0	0.1
1952.5	4.7
1847.1	34.7
1771.5	94.7
1720.6	184.0
1680.1	300.0
1623.9	600.0
1592.3	900.0
1567.7	1200.0
1548.3	1500.0
1532.5	1800.0

PVC₁ @ $\varepsilon = 0.0371$

P [lbf]	t [s]
1511.4	0.1
1421.8	8.4
1344.5	38.4
1290.0	98.4
1251.3	188.4
1225.0	300.0
1188.1	600.0
1149.4	1200.0
1126.5	1800.0

PVC₂ @ $\varepsilon = 0.0421$

P [lbf]	t [s]
1581.7	0.1
1408.4	29.1
1339.2	88.1
1295.3	178.1
1265.4	300.0
1221.4	600.0
1196.1	900.0
1179.3	1200.0
1165.9	1500.0
1154.0	1800.0

PVC₃ @ $\varepsilon = 0.0408$

P [lbf]	t [s]
1557.1	0.1
1377.9	30.0
1273.5	99.4
1262.9	188.0
1231.3	300.0
1190.5	600.0
1167.0	900.0
1151.8	1200.0
1138.8	1500.0
1128.3	1800.0

PVC₄ @ $\varepsilon = 0.0421$

P [lbf]	t [s]
1536.0	0.1
1372.6	30.5
1305.8	90.5
1265.4	180.5
1233.7	300.0
1195.1	600.0
1175.0	900.0
1158.9	1200.0
1146.9	1500.0
1136.4	1800.0

PVC₅ @ $\varepsilon = 0.0408$

P [lbf]	t [s]
1549.3	0.1
1380.3	27.2
1331.0	57.2
1283.5	117.2
1246.5	207.2
1223.6	300.0
1183.1	600.0
1160.2	900.0
1142.6	1200.0
1116.2	1800.0

PVC₆ @ $\varepsilon = 0.0358$

P [lbf]	t [s]
1510.6	0.1
1371.5	27.2
1331.0	57.2
1288.7	117.2
1260.6	207.2
1237.7	300.0
1200.7	600.0
1176.1	900.0
1162.0	1200.0
1139.1	1800.0

PC₁ @ $\varepsilon = 0.0643$

\underline{P} [lbf]	\underline{t} [s]
1996.5	0.1
1966.6	2.8
1882.2	32.8
1834.8	92.8
1803.2	182.8
1780.3	300.0
1752.2	600.0
1734.6	900.0
1715.3	1200.0
1697.7	1800.0

PC₂ @ $\varepsilon = 0.0667$

\underline{P} [lbf]	\underline{t} [s]
1978.9	0.1
1848.9	30.0
1792.6	90.0
1757.5	180.0
1729.3	300.0
1710.0	600.0
1674.9	900.0
1660.8	1200.0
1641.5	1800.0

PC₃ @ $\varepsilon = 0.0673$

\underline{P} [lbf]	\underline{t} [s]
2000.0	0.1
1873.5	31.9
1820.7	91.9
1785.6	181.9
1759.2	300.0
1724.1	600.0
1703.0	900.0
1688.9	1200.0
1669.6	1800.0

PC₄ @ $\varepsilon = 0.0661$

\underline{P} [lbf]	\underline{t} [s]
1978.9	0.1
1901.4	11.7
1850.4	41.7
1809.9	101.7
1779.9	191.7
1760.6	300.0
1727.1	600.0
1707.7	900.0
1691.9	1200.0
1672.5	1800.0

PC₅ @ $\varepsilon = 0.0655$

\underline{P} [lbf]	\underline{t} [s]
2000.0	0.1
1920.8	14.1
1875.0	44.1
1838.0	104.1
1811.6	194.1
1794.0	300.0
1771.1	600.0
1753.5	900.0
1739.4	1200.0
1713.0	1800.0

PC₆ @ $\varepsilon = 0.0582$

\underline{P} [lbf]	\underline{t} [s]
1890.8	0.1
1852.1	9.8
1816.9	39.8
1788.7	99.8
1769.4	189.8
1757.0	300.0
1737.7	600.0
1725.4	900.0
1714.8	1200.0
1698.9	1800.0

PMMA₁ @ $\varepsilon = 0.0193$

P [lbf]	t [s]
500.0	0.1
480.0	9.4
464.0	39.4
457.0	69.4
449.0	129.4
441.0	249.5
434.0	428.5
430.0	600.0
425.0	900.0
421.0	1200.0
419.0	1500.0
417.0	1800.0

PMMA₂ @ $\varepsilon = 0.0193$

P [lbf]	t [s]
500.9	0.1
478.5	14.5
465.3	44.5
455.2	104.5
447.3	224.5
441.1	404.5
436.3	300.0
432.1	600.0
428.8	900.0
425.7	1200.0
423.6	1800.0

PMMA₃ @ $\varepsilon = 0.0206$

P [lbf]	t [s]
500.0	0.1
487.3	5.1
466.6	35.1
453.9	95.1
443.3	215.1
435.9	393.8
431.0	600.0
425.7	900.0
422.2	1200.0
419.6	1500.0
417.4	1800.0

PMMA₄ @ $\varepsilon = 0.0199$

P [lbf]	t [s]
501.8	0.1
475.0	20.6
464.0	50.6
454.3	110.6
446.4	200.6
440.2	320.6
436.7	440.1
432.3	600.0
427.1	900.0
424.0	1200.0

PMMA₅ @ $\varepsilon = 0.0203$

P [lbf]	t [s]
502.6	0.1
474.5	27.2
465.7	57.2
456.5	117.2
449.0	206.7
444.2	300.0
435.9	600.0
430.6	900.0
426.6	1200.0
424.0	1500.0
421.4	1800.0

PMMA₆ @ $\varepsilon = 0.0199$

P [lbf]	t [s]
500.9	0.1
468.4	34.2
455.6	94.2
447.3	183.2
441.1	300.0
432.3	600.0
427.1	900.0
424.0	1200.0
420.9	1500.0
419.2	1800.0

Appendix 5. Response function parameters, strain levels, and strain rate sensitivity index values of stress relaxation tested thermoplastic specimens.

SPECIMEN	P_0 [lbf]	τ [s]	n	Φ [lbf]	ϵ_0 [in./in.]	m_σ
HDPE ₁	461	11,730	0.293	10.2	0.0539	0.1391
HDPE ₂	417	12,450	0.276	7.7	0.0421	0.1336
HDPE ₃	445	13,210	0.276	8.3	0.0508	0.1317
HDPE ₄	410	14,770	0.265	7.9	0.0408	0.1262
HDPE ₅	417	12,270	0.279	7.5	0.0471	0.1347
HDPE ₆	445	15,300	0.268	8.4	0.0458	0.1257
PS ₁	422	173,520	0.209	9.4	0.0289	0.0710
PS ₂	494	51,370	0.186	16.8	0.0193	0.0867
PS ₃	500	27,770	0.212	16.0	0.0206	0.1014
PS ₄	500	23,860	0.218	15.8	0.0213	0.1055
PS ₅	489	80,850	0.184	14.8	0.0225	0.0800
PS ₆	502	28,730	0.214	14.1	0.0206	0.1010
PP ₁	500	105,850	0.255	6.8	0.0508	0.0790
PP ₂	500	75,580	0.275	6.2	0.0458	0.0857
PP ₃	500	51,940	0.273	6.6	0.0458	0.0941
PP ₄	500	47,500	0.282	7.5	0.0477	0.0965
PP ₅	445	62,890	0.268	5.9	0.0383	0.0896
PP ₆	445	60,150	0.273	6.2	0.0383	0.0907

PA ₁	2007	113,200	0.304	20.5	0.0396	0.0762
PA ₂	2000	108,800	0.302	17.7	0.0213	0.0772
PA ₃	2000	144,100	0.285	18.3	0.0213	0.0723
PA ₄	2000	113,000	0.296	20.6	0.0408	0.0766
PA ₅	2000	54,300	0.293	19.8	0.0436	0.0935
PA ₆	2000	120,800	0.304	19.9	0.0396	0.0749
PVC ₁	1511	165,080	0.258	19.2	0.0371	0.0710
PVC ₂	1582	185,030	0.241	18.2	0.0421	0.0697
PVC ₃	1557	208,760	0.229	22.9	0.0408	0.0682
PVC ₄	1536	275,850	0.232	18.4	0.0421	0.0642
PVC ₅	1549	148,010	0.242	19.4	0.0408	0.0733
PVC ₆	1511	235,810	0.249	16.2	0.0358	0.0657
PC ₁	1997	474,150	0.308	18.3	0.0643	0.0503
PC ₂	1979	549,150	0.278	17.5	0.0667	0.0512
PC ₃	2000	619,150	0.278	16.0	0.0673	0.0496
PC ₄	1979	819,150	0.279	13.6	0.0661	0.0460
PC ₅	2000	884,150	0.288	15.6	0.0655	0.0441
PC ₆	1891	1,488,300	0.318	11.3	0.0582	0.0347
PMMA ₁	500	1,322,000	0.250	3.8	0.0193	0.0436
PMMA ₂	501	2,433,500	0.242	3.5	0.0193	0.0386
PMMA ₃	500	1,058,400	0.260	4.2	0.0206	0.0450
PMMA ₄	502	839,200	0.264	3.2	0.0199	0.0472
PMMA ₅	503	1,528,800	0.252	3.1	0.0203	0.0419
PMMA ₆	501	1,948,800	0.241	3.4	0.0199	0.0408

Appendix 6. RPL (Reverse Polish LISP) code listing of the program implemented for strain rate sensitivity index determination using data derived from variable strain rate testing.

```

<< rates? HALT SWAP DROP 'M' STO
strain? HALT SWAP DROP 'S' STO
length? HALT SWAP DROP 'L' STO
radius? HALT SWAP DROP SQ  $\Pi$  >NUM * 'A' STO 'M' RCL 2 2 >LIST
0 CON 'ARR' STO I M
FOR i
  pairs? HALT SWAP DROP 'N' STO
  crosshead? HALT SWAP DROP 'V' STO
  chart? HALT SWAP DROP 'U' STO
  FOR j
    deflection? HALT SWAP DROP 'D' STO
    load? HALT SWAP DROP 'P' STO
    'D' RCL 'V' RCL * 'L' RCL 'U' RCL * / 1 + LN 'D' RCL
    'V' RCL * 'U' RCL / 'L' RCL + 'P' RCL * 'A' RCL 'L' RCL * /
    2 >ARRY  $\Sigma$ +
  NEXT
  'ARR' RCL i 1 2 >LIST 'V' RCL 60 / 'S' RCL EXP 'L' RCL * / LN
  PUT 'ARR' STO CL $\Sigma$ 
NEXT
'ARR' RCL STO $\Sigma$  LR SWAP DROP 'm' STO CORR 'r' STO { A D L M N P
S U V ARR  $\Sigma$ DAT  $\Sigma$ PAR } PURGE
CLLCD "m = " 'm' RCL 1 DISP "r = " 'r' RCL 3 DISP >>

```

NOTE: The code is written in the language (RPL) implemented by the HP 28S and HP 48SX calculators manufactured by Hewlett-Packard.

Appendix 7. BASIC code listing of the program implemented for Kohlrausch parameter determination using data derived from stress relaxation testing.

```

100 ! The program implements an exhaustive grid search technique
110 ! to minimize the root-mean-square difference between the
120 ! empirical load relaxation equation (the Kohlrausch function)
130 ! and the experimental data of load as a function of time; i.e.,
140 !  $\Phi(n,\tau)$ . The characteristic time parameter ( $\tau$ ) and the
150 ! rate-of-decay parameter ( $n$ ) are found when  $\Phi(n,\tau)$  is a
160 ! minimum.
170 CLEAR
180 DISP "data_pairs?"
190 INPUT D
200 DISP "n_lower? & n_upper"
210 INPUT N0,N9
220 DISP " $\tau$ _lower? &  $\tau$ _upper"
230 INPUT T0,T9
240 DISP "n_increment?"
250 INPUT I1
260 DISP " $\tau$ _increment?"
270 INPUT I2
280 CLEAR
290 DIM A(10,2)
300 FOR I = 1 TO D
310     FOR J = 1 TO 2
320         READ A(I,J)
330     NEXT J
340 NEXT I
350 P0 = A(1,2)
360 FOR H = 1 TO 4
370     F = 0 @ I = N0
380     FOR J = T0 TO T9 STEP I2
390         P = 0
400         FOR K = 1 TO D
410             X = A(K,1)
420             Y = A(K,2)

```

```

430          U = (P0*EXP(-(X/J)^I)) - Y)^2
440          P = P + U
450      NEXT K
460      P = SQR(P/D)
470      IF H = 1 AND I = N0 AND J = T0 THEN E = P @ N = N0 @
          T = T0
480      DISP "(Φ)curr. = " ; P
490      DISP "(n)curr. = " ; I
500      DISP "(τ)curr. = " ; J
510      DISP "(Φ)min. = " ; E
520      DISP "(n)min. = " ; N
530      DISP "(τ)min. = " ; T
540      DISP
550  NEXT J
560      M = N + 2*I1
570      IF M < I THEN F = 1
580      IF I = N9 THEN F = 1
590      I = I + I1
600      IF F = 0 THEN 390
610      RESTORE
620      N1 = N - 3*I1/2 @ N2 = N + 3*I1/2
630      N3 = N - 5*I1 @ N4 = N + 5*I1
640      T1 = T - 3*I2/2 @ T2 = T + 3*I2/2
650      IF H = 1 AND (N = N0 OR N = N9 OR T = T0 OR T = T9)
          THEN GOTO 700
660      IF H = 1 THEN N0 = N1 @ T0 = T1 @ N9 = N2 @ T9 = T2 @
          I1 = I1/10 @ I2 = I2/5
670      IF H = 2 THEN N0 = N3 @ T0 = T1 @ N9 = N4 @ T9 = T2 @
          I2 = I2/10
680      IF H = 3 THEN N0 = N3 @ T0 = T1 @ N9 = N4 @ T9 = T2 @
          I2 = I2/10
690  NEXT H
700      DATA .1,501,34,468,94,456,183,447,300,441,600,432,900,
          427,1200,424,1500,421,1800,419
710      CLEAR
720      DISP "n = " ; N @ DISP "τ = " ; T @ DISP "Φ = " ; E
730      END

```

NOTE: The code is written in the language (BASIC) implemented by the HP 85 computer/controller manufactured by Hewlett-Packard.

Appendix 8. RPL (Reverse Polish LISP) code listing of the program implemented for strain rate sensitivity index determination using data derived from stress relaxation testing.

```

<< initial_time? HALT SWAP DROP 'I' STO
    final_time? HALT SWAP DROP 'F' STO
    time_increment? HALT SWAP DROP 'J' STO
    Po? HALT SWAP DROP 'P' STO
    n? HALT SWAP DROP 'N' STO
    tau? HALT SWAP DROP 'T' STO
    'F' RCL 'I' RCL - 'J' RCL / 2 + 2 2 >LIST CON 'ARR' STO 1 'M' STO
    I F
    FOR t
        'ARR' 'M' RCL t 'T' RCL / 'N' RCL ^ -1 * EXP t 'T' RCL / 'N' RCL
        1 - ^ * 'N' RCL * 'P' RCL * 'T' RCL / LN PUT 1 'M' STO+ 'ARR'
        'M' RCL t 'T' RCL / 'N' RCL ^ -1 * EXP 'P' RCL * LN PUT 1 'M'
        STO+ J
    STEP
    'ARR' RCL STO Σ LR SWAP DROP 'm' STO CORR 'r' STO { ARR F I J M
    N P T ΣDAT ΣPAR } PURGE
    CLLCD "m = " 'm' RCL 1 DISP "r = " 'r' RCL 3 DISP >>

```

NOTE: The code is written in the language (RPL) implemented by the HP 28S and HP 48SX calculators manufactured by Hewlett-Packard.