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Title: OPTIMAL GOVERNOR AND EXCITATION CONTROL OF POWER SYSTEMS WITH DAMPING

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The work in this thesis solves the problem of restoring a power system to its operating stable equilibrium point, following a transient disturbance, in minimum time without any oscillation. The idealized power system considered consists of cylindrical rotor generator connected to an infinite busbar with the following cases of control:

i) Governor Control

ii) Excitation Control

iii) Combined Governor and Excitation Control.

In each case, the system is proved to be controllable, at least locally, in an open region around the stable equilibrium point. Also, the optimal control is proved to exist and is unique. The maximum principle of Pontryagin is used to determine the optimal control law. The condition for no oscillation is determined and the system is shown to be stable. The method of solving the system back in time

is used to construct the switching locii, and the optimal control law was synthesized.

An example is given and is simulated on an analog computer to illustrate the theory.

Optimal Governor and Excitation Control of Power Systems with Damping

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I. INTRODUCTION

A Power System Model

A synchronous machine working in parallel with other synchronous machines supplying a variety of loads forms a power system. Under steady state conditions, it rotates with constant angular speed $\omega = 2\pi f/p$, where f is the system frequency and p is the number of the machine pole-pairs. Further, it has a constant load angle between the electromagnetic fields of its stator and rotor. Any sudden change in this relative steady state position will set up transient oscillations superposed on this position. The behavior of the machine after a sudden disturbance is exactly described by its dynamic equation of motion, which is called, generally, the swing equation.

The power system considered in this thesis consists of one machine system connected to an infinite busbar (constant voltage and frequency network) as shown in Figure 1.

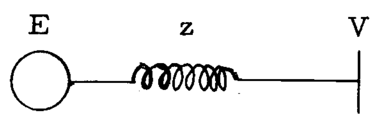


Figure 1. Power system model.

Where:

E is the excitation voltage

V is the busbar voltage

z is the sum of armature and line reactances.

(The reactance is denoted by z since the general reactance notation x will be considered as the load angle.)

The following assumptions are satisfied by the system:

1. The angular momentum of the synchronous machine is constant.
2. The armature and line resistances are neglected.
3. The machine has a cylindrical rotor.
4. The disturbance to the system is removed after its occurrence. This means that this disturbance is represented by initial conditions.
5. The system has a linear damping coefficient.

Under these above assumptions the swing equation of the system is given by

$$\ddot{x} + D\dot{x} + \frac{EV}{z} \sin x = P \qquad (1)$$

where:

x is the load angle

D is the damping coefficient

P is the power input to the system and controlled by the governor

E, V and z as above.

$$\dot{x} = \frac{dx}{dt}, \qquad \ddot{x} = \frac{d^2x}{dt^2}.$$

The state equations of (1) are:

$$x = x_1$$
$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -Dx_2 - \frac{EV}{z}\sin x_1 + P \tag{2}$$

At steady state, we have:

$$\dot{x}_{e_1} = x_{e_2} = 0$$

$$\dot{x}_{e_2} = -\frac{EV}{z}\sin x_{e_1} + P = 0$$

where e means equlibrium.

$$x_{e_1} = \sin^{-1}(Pz/EV)$$

$$x_{e_2} = 0$$

Denote x_{e_1} by x_e.

The last two equations give two types of equilibrium points for the power system considered. The first is a focal point and the second is a saddle point. The focal point $(x_e, 0)$ is a stable equilibrium point. If the following change of variable is made;

$$x = x_1 - x_e$$

Equations (2) become:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -Dx_2 - \frac{EV}{z} \sin(x_1 + x_e) + P \qquad (3)$$

Equations (3) have the origin $(0, 0)$ as a stable equilibrium point.

Statement of the Problem

The system state equations (3) may be written in the compact form:

$$\dot{x} = f(x, E, P) \qquad (4)$$

where x and f are vector valued functions.

$$x = (x_1, x_2)'$$

$$f = (f_1, f_2)'$$

where

' denotes the transpose

E and P are the control functions.

We wish to investigate the controllability and existence of optimal control of the system (4). Also, we wish to determine and synthesize the optimal control that transfers the system from some initial condition $x(t_0)$ at time t_0 to the stable equilibrium point (the origin) in minimum time with no oscillations considering the following three cases of control:

i) Governor Control P, the excitation Control E is held constant and 1 is taken, arbitrarily, as an upper bound on P, i.e., $|P| \leq 1$.

ii) Excitation Control E, the Governor Control is held constant and $|E| \leq 1$.

iii) Combined Governor and Excitation Control $(P, E)'$, $|P| \leq 1$, $|E| \leq 1$.

The General Optimal Control Problem (Pontryagin's)

Most of the control systems are described by a set of first order differential equations:

$$\dot{x}_i = f_i(x, u, t) \qquad i = 1, 2, \ldots, n \tag{5}$$

where $x = (x_1, x_2, \ldots, x_n)'$ is a state vector of the system and $u = (u_1, u_2, \ldots, u_n)$ is the control vector. The functions f_i will be considered twice continuously differentiable in its arguments x and u. The control functions u_i are chosen from a class of all the piece-wise continuous functions having discontinuities of the jump type. This class is called the class of admissible controls. The admissible controls lie on an arbitrary segment of the time axis and, at every instant, take their values from a closed region with a piece-wise smooth boundary U (See [10, p. 262]).

The optimal control problem is formulated in the following way. We wish to find an admissible control $u(t)$ such that the system (5) is taken from some given initial state $x(t_0)$ to some fixed final state $x(t_1)$, and such that the system performance is minimum.

The index or cost functional used to evaluate the system performance will be taken to have the form

$$J = \int_{t_0}^{t_1} f_0(x, u)dt \tag{6}$$

where $f_0(x, u)$ is a functional satisfying the same conditions as the functions $f_i(x, u)$, $i = 1, \ldots, n$.

All admissible controls satisfying this formulation are called optimal controls; the corresponding trajectory of (5) is called the optimal trajectory. (See [10, p. 263]).

Depending on the choice of the functional $f_0(x, u)$, the integral (6) can indicate the expenditure of time, energy, etc., during the course of process under consideration. For example, if $f_0(x,u) = 1$, then the integral (6) is equal $(t_1 - t_0)$, i.e., the process occurs in minimum time.

The necessary conditions which satisfy every optimal control and its corresponding optimal trajectory are given by the maximum principle of Pontryagin.

The Maximum Principle of Pontryagin

If $u^*(t)$, $t_0 \leq t \leq t_1$, is an optimal control (for the problem stated above), and $x^*(t)$ is the corresponding optimal trajectory; then there exists a vector valued function $p(t)$, called the adjoint, defined, continuous, differentiable and nonidentically zero over $t_0 \leq t \leq t_1$ such that:

i) $$\dot{x}(t) = \left. \frac{\partial H(x, u, p)}{\partial p} \right|_{\substack{u=u^* \\ x=x^*}} \tag{7}$$

ii) $$\dot{p}(t) = \left. \frac{-\partial H(x, u, p)}{\partial x} \right|_{\substack{u=u^* \\ x=x^*}} \tag{8}$$

iii) $$H(x^*, u^*, p) = \sup_{u \in U} H(x^*, u, p) \tag{9}$$

iv) $$H(x^*, u^*, p) = 0 \tag{10}$$

v) $p_0(t) = \text{constant} \leq 0$ for all $t_0 \leq t \leq t_1$

where

$H(x, u, p)$ is the Hamiltonian function defined by:

$$H(x, u, p) = p \cdot f(x, u) = \sum_{i=0}^{n} p_i f_i(x, u)$$

Main Results of the Thesis

The power system was considered from the point of view of the mathematical theory of optimal control. The system was proved to be, at least locally, controllable around the stable equilibrium point of the system (the origin). Also, it was proved that a time optimal control, that transfers the system from some initial point to the origin, exists and is unique. The Maximum Principle of Pontryagin was used to determine the optimal control law. The optimal controller resulting was a mechanism that switches abruptly from one extreme value to another, and is represented by an ideal relay. A method was given to determine the switching locus of this relay in the (x_1, x_2) phase plane. The condition for taking the system from some initial point to the origin without any oscillation, using this optimal control, was determined. Then a feedback optimal controller was synthesized and an engineering realization of the optimal control system was accomplished. An example of a power system was given and simulated by the analog computer to illustrate the results obtained.

II. OPTIMAL GOVERNOR CONTROL

Statement of the Problem

Consider the power system described by the nonlinear differential equations

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -Dx_2 - \frac{EV}{z} \sin(x_1 + x_e) + P \qquad (11)$$

It was established that this system has an equilibrium point at the origin, where $(x_1, x_2)'$ is the state vector whose components are the load angle and load angle rate of change.

P is the power input, regulated by the governor and will be considered as the direct control of the system (11). This control will be bounded due to engineering considerations, and a bound of 1 is assumed arbitrarily, i.e.,

$$|P| \leq 1 \qquad (12)$$

P which satisfies (12) is called an admissible control. E is the excitation voltage and considered constant.

We wish to determine the admissible control P^*, which steers the system (11) from some initial state (x_1^0, x_2^0) at time $t = t_0$ to the origin and minimizes the performance index J,

$$J = \int_{t_0}^{t} (1)dt = t - t_0 \tag{13}$$

and such that the trajectory, due to this control, makes no oscillations.

The admissible control P^* that satisfies the above stated problem is called time-optimal governor control.

Controllability

Consider the linear approximation of the power system (11), near the origin, in the vector form:

$$\dot{x} = Ax + bP \tag{14}$$

$$x = (x_1, x_2)'$$

A is 2×2 matrix defined by:

$$A = [\frac{\partial f(x, P)}{\partial x}](0, 0) \tag{15}$$

where $f(x, P)$ is a vector $(f_1, f_2)'$ such that

$$f_1(x, P) = x_2$$

$$f_2(x, P) = -Dx_2 - \frac{EV}{z} \sin (x_1 + x_e) + P$$

and b is a 2×1 constant vector defined by

$$b = [\frac{\partial f}{\partial P}](0, 0) \qquad (16)$$

If A and b are evaluated by (15) and (16) we have:

$$A = \begin{bmatrix} 0 & 1 \\ \frac{-EV\cos(x_e)}{z} & -D \end{bmatrix}$$

$$b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Also, if the matrix $[b \vdots Ab]$, which is called the controllability matrix, is evaluated and the theorem (1) in Appendix A is applied we get:

$$[b \vdots Ab] = \begin{bmatrix} 0 & 1 \\ 1 & -D \end{bmatrix} \qquad (17)$$

which obviously has rank 2. Therefore, the domain of null controllability of system (11) is open in the phase plane R^2. This means that every initial point (x_1^0, x_2^0) in the neighborhood of the origin can be steered to the origin in a finite time. Thus, system (11) is locally controllable in an open region around the origin.

<u>The Existence of Governor Optimal Control</u>

The power system, given in (11) can be written in the compact form:

$$\dot{x} = f(x, P) \tag{18}$$

with the constraint:

$$|P| \leq 1$$

where:

f(x, P) is the vector $(f_1, f_2)'$

and

$$f_1 = x_2$$

$$f_2 = -Dx_2 - \frac{EV}{z} \sin (x_1+x_e) + P$$

It is well known (Shwartz inequality) that,

$$\|f\| \leq \|f_1\| + \|f_2\| \tag{19}$$

where $\| \ \|$ denotes the Euclidean norm.

$$\|f_1\| = \|x_2\|$$

$$\|f_2\| = \|-Dx_2 - \frac{EV}{z} \sin (x_1+x_e) + P\|$$

$$\|f_2\| \leq D\|x_2\| + \|\frac{EV}{z}\| + 1$$

Therefore,

$$\|f\| \leq \|x_2\| + D\|x_2\| + \| \frac{EV}{z} \| + 1$$

$$= D[\|x_2\|(\frac{D+1}{D}) + (\|\frac{EV}{z}\| + 1)/D]$$

denote $(\frac{D+1}{D})$ by N, and ND by M, then

$$\|f\| \leq ND[\ \|x_2\| + (\|\frac{EV}{z}\| + 1)/ND]$$

$$= M[\ \|x_2\| + (\|\frac{EV}{z}\| + 1)/M] \qquad (20)$$

By Theorem (2) in Appendix A, the solution $x(P, t)$ of (18) exists, is unique and is uniformly bounded. Consequently, and by Theorem (3) in Appendix A; there exists an optimal control $P*$ for the system (18).

Computing the Optimal Control Law

The method of the maximum principle is used to determine the optimal $P*$.

The Hamiltonian H for system (11) is given by:

$$H(x, P, p) = -1 + p_1x_2 + p_2[-Dx_2 - \frac{EV}{z} \sin (x_1+x_e) + P]$$

$$= -1 + (p_1 - p_2D)x_2 - p_2 \frac{EV}{z} \sin (x_1+x_e) + p_2P \qquad (21)$$

$p(t) = (p_1(t)p_2(t))'$ is the ajoint vector and it is the solution of the following differential equations, which are called the adjoint equations:

$$\dot{p}_1 = -\frac{\partial H}{\partial x_1} = p_2 \frac{EV}{z} \cos (x_1+x_e) \qquad (22)$$

$$\dot{p}_2 = -\frac{\partial H}{\partial x_2} = -p_1 + p_2D \qquad (23)$$

The optimal control $P*$ should satisfy:

$$H(x, P*, p) = \sup_{|P| \leq 1} [-1+(p_1-p_2D)x_2-p_2\frac{EV}{z}\sin(x_1+x_e)+p_2P] \quad (24)$$

It is obvious that (24) is satisfied when:

$$P* = \text{sgn } p_2(t) \quad (25)$$

where

$$\text{sgn } p_2 = \begin{cases} +1 & \text{if } p_2 > 0 \\ -1 & \text{if } p_2 < 0 \end{cases} \quad (26)$$

Therefore, the optimal governor control $P*$ is a switch that switches abruptly from one extreme value of P to another. This switch is represented, in electrical engineering, by an ideal relay. The function $p_2(t)$ is called the switching function of this relay.

<u>Uniqueness of the Optimal Governor Control</u>

The optimal control $P*$ is unique if the switching function p_2 does not vanish on any subinterval $[t_1, t_2] \subset [0, t*]$, where $t*$ is the minimum time. In this case the optimal control system is called normal system. If there exists a subinterval $[t_1,t_2] \subset [0,t*]$ on which the function $p_2(t)$ vanishes identically, the optimal control system is called singular system and in this case the optimal control is not defined in $[t_1, t_2]$. Therefore, to prove that the optimal governor control system is normal, it is necessary to prove first that the switching function $p_2(t)$ does not vanish identically in a subinterval $[t_1, t_2] \subset [0, t^*]$. This is established by contradiction. Suppose that $p_2(t)$ vanishes identically on some subinterval $[t_1, t_2]$, i.e.,

$$p_2(t) = 0 \quad \text{when} \quad t \in [t_1, t_2]$$

Also $$\dot{p}_2(t) = 0 \quad \text{when} \quad t \in [t_1, t_2]$$

By (23), $$p_1(t) = 0 \quad \text{when} \quad t \in [t_1, t_2]$$

This means that the adjoint vector $p(t) = (p_1(t), p_2(t))'$ vanishes identically on $[t_1, t_2] \subset [0, t^*]$ which contradicts the maximum principle of Pontryagin. Therefore $p_2(t)$ does not vanish on $[t_1, t_2] \subset [0, t^*]$, and consequently the optimal governor control system is normal.

Synthesizing the Optimal Governor Control

It was proved that the solution of the power system given in Equations (11) exists and is unique for each value of the control P. The synthesizing of the optimal control $P^*(t) = \operatorname{sgn} p_2(t)$, as an optimal feedback control $P^*(x)$, depends mainly on this fact. First, we find the optimal control that steers some initial point (x_1^0, x_2^0) to the origin (0, 0), along a unique optimal trajectory $(x_1^*(t), x_2^*(t))$ corresponding to this optimal control. Then, the feedback controller

$P^*(x)$ is established by following back in time along this trajectory curve starting from the origin and marking the points where the optimal control P^* changes sign. The switching locus $S(x)$, with respect to the optimal control $P^* = \text{sgn}\, p_2(t)$ is the set of all such points in the domain of null controllability where the optimal control changes sign. If it is proved that the time optimal control can switch at most once, then the switching locus $S(x)$ consists of two unique trajectories S_+ and S_- correspond to controls $+1$ and -1 respectively. It is clear that S_+ is the set of states that can be forced to the origin by $P(t) = +1$, and $S_-(x)$ is the set of states that can be forced to the origin by $P(t) = -1$. The switching locus divides the phase plane into two regions R_+ and R_-. R_+ is the set of states to the left of $S(x)$ i.e.,

$$R_+ = \{(x_1, x_2) \mid \text{ if } (x_1, x_2') \in S(x), \text{ then } x_2 < x_2'\}$$

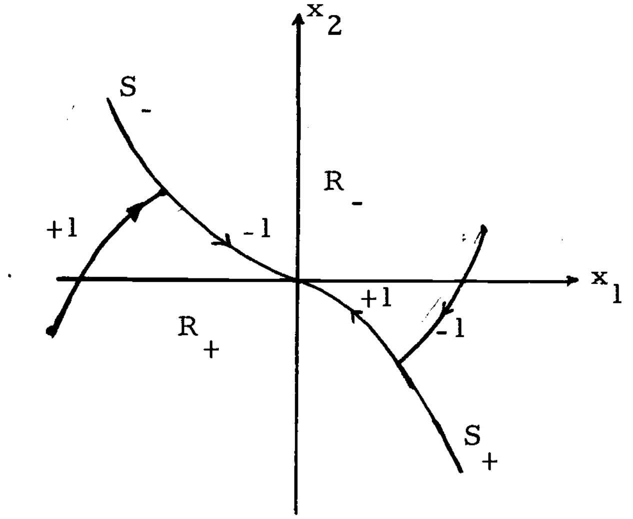
Similarly, R_- is the set of states to the right of $S(x)$ i.e.,

$$R_- = \{(x_1, x_2) \mid \text{if } (x_1, x_2') \in S(x), \text{ then } x_2 > x_2'\}$$

Then the optimal feedback control $P^*(x)$ is given by:

$$\begin{aligned} P^*(x) &= +1 \quad \text{for all} \quad (x_1, x_2) \in R_+ \cup S_+ \\ P^*(x) &= -1 \quad \text{for all} \quad (x_1, x_2) \in R_- \cup S_{-1} \end{aligned} \qquad (27)$$

This is shown in Figure 2. (See [1, p. 513; 5, p. 136, 426]).



$$S(x) = S_+ \cup S_-$$

$$P^*(x) = \{+1, -1\}$$

Figure 2. Optimal feedback governor control trajectory.

Generation of the Switching Curve by Analog Computer

The switching curve $S(x)$ can be generated on the analog computer by the following steps: (See [1, p. 619]).

i) Simulate the power system (11) on the analog computer with time reversed. The power system (11) with the time reversed will be,

$$\begin{aligned} \dot{x}_1 &= -x_2 \\ \dot{x}_2 &= Dx_2 + \frac{EV}{z} \sin (x_1 + x_e) - P \end{aligned} \tag{28}$$

ii) Start with initial conditions (0, 0) and using $P = +1$, plot the generated trajectory by x - y plotter; this trajectory is S_+ curve.

iii) Repeat (ii), using $P = -1$, this will yield to S_-. Then, the switching locus $S(x)$ is

$$S(x) = S_+ \cup S_-$$

An example was given, with $D = 2.5$ and $\frac{EV}{z} = 1.44$. The switching curve for this example is shown in Figure 3.

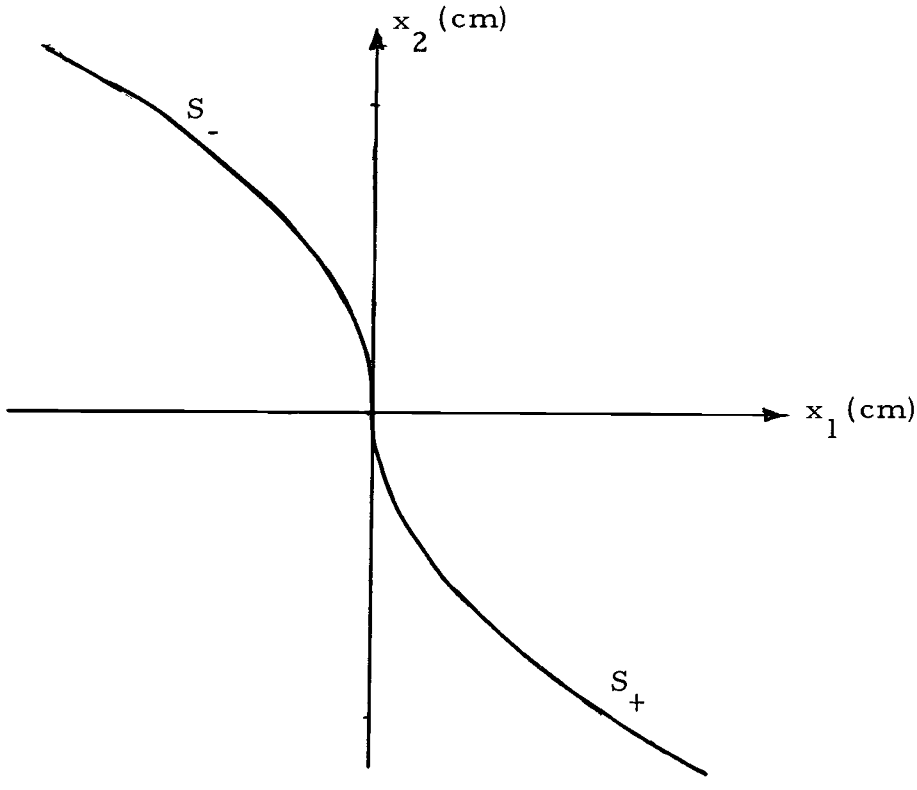


Figure 3. Switching curve for $D = 2.5$ and $\frac{EV}{z} = 1.44$.
Scale x_1:1v/cm; x_2:1v/cm

Condition of No Oscillation

The condition to steer any initial point (x_1^0, x_2^0) optimally to the origin $(0, 0)$ without any oscillation, is that the optimal control switches, at most, once along the corresponding optimal trajectory. The optimal control law is:

$$P^*(t) = \operatorname{sgn} p_2(t)$$

Therefore, the condition that $P^*(t)$ switches, at most, once, along the optimal trajectory is equivalent to the condition that the function $p_2(t)$ has at most one zero on $(0, t^*)$; where t^* is the minimum time spent to steer the system, optimally, to the origin.

Differentiating Equation (23) we get:

$$\ddot{p}_2 = -\dot{p}_1 + \dot{p}_2 D \tag{29}$$

substitute (22) in (29) and we get

$$\ddot{p}_2 - \dot{p}_2 D + [\frac{EV}{z} \cos (x_1 + x_e)] p_2 = 0 \tag{30}$$

If the transformation:

$$p_2 = e^{\frac{1}{2}\int_0^t D dt} y = e^{\frac{1}{2} Dt} y \tag{31}$$

with $y = 0$ if and only if $p_2 = 0$, is applied to (30) we get:

$$\ddot{y} + y[-\frac{D^2}{4} + \frac{EV}{z}\cos(x_1+x_e)] = 0 \qquad (32)$$

This is Hill's equation of the type given in Appendix B. If

$$D^2 \geq 4\frac{EV}{z} \qquad (33)$$

the solution of Equations (32) is non oscillatory and has, at most, one zero on the interval $[-\infty, \infty]$.

Engineering Realization of Optimal Governor Control System

The engineering realization is a design of a nonlinear feedback control system for the power system (11). This feedback control system has two state variables x_1 and x_2 and delivers the correct optimal control P^*. This engineering realization is shown in Figure 4. The ideal relay accomplishes the signum operation. The switching curve S is constructed by a diode function generator. The state variables $x_1(t)$ (load angle) and $x_2(t)$ (lead angle rate) are observed at each instant of time. $x_1(t)$ is introduced to S and the difference m between the output of S and $x_2(t)$ is applied to the actuate the relay.. The relay output is the governor optimal control $P^*(t)$. The reason is that (See [1, p. 513])

if $(x_1, x_2) \in R_-$, then $m < 0$,

if $(x_1, x_2) \in R_+$, then $m > 0$.

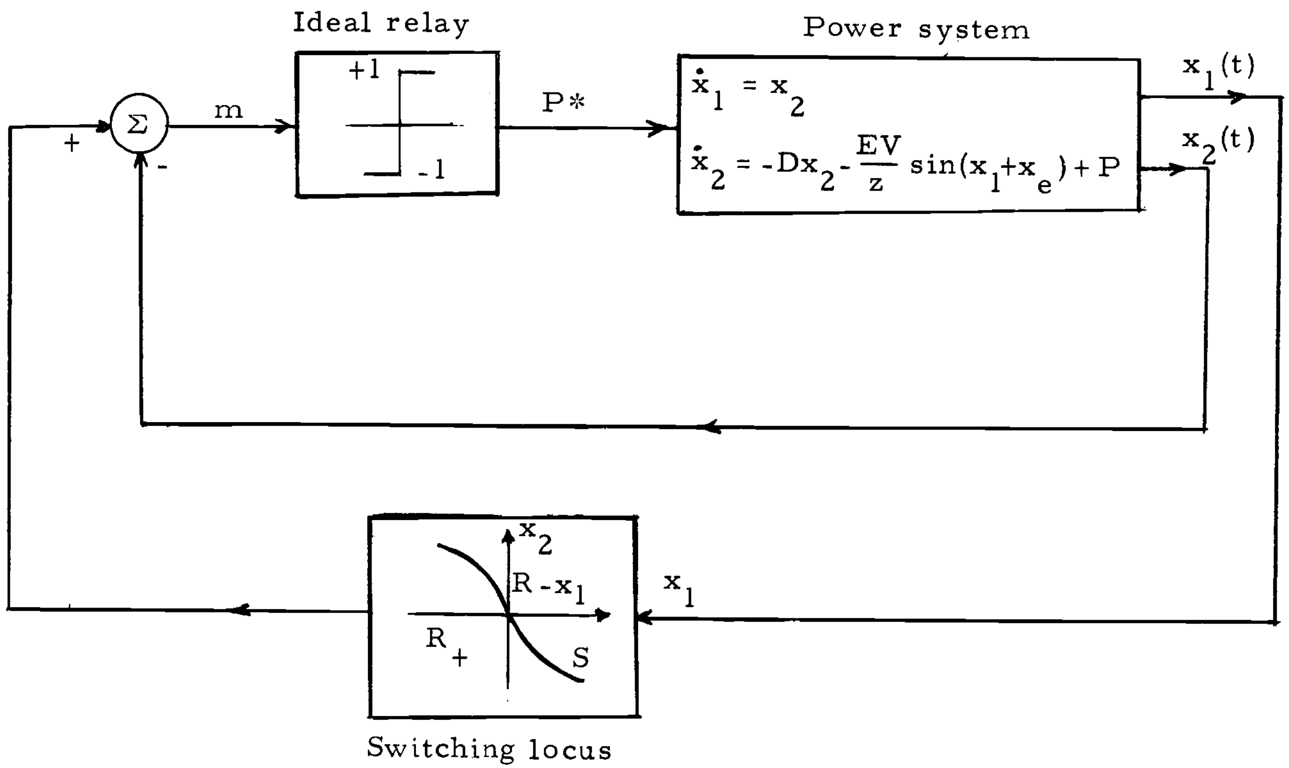


Figure 4. Engineering realization of optimal governor control system.

III. OPTIMAL EXCITATION CONTROL

Statement of the Problem

Consider the power system described by the following first order nonlinear differential equations:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -Dx_2 - [\frac{V}{z} \sin (x_1+x_e)]E + P \qquad (34)$$

It was shown that the origin (0, 0) is the system equilibrium point.

$(x_1, x_2)'$ is the state vector of the system. The entries x_1 and x_2 are the load angle and load angle rate respectively.

P is the power input and considered constant in this case.

E is the excitation voltage which will be the direct control of the system (34). This control is bounded due to saturation. The bound 1 is taken arbitrarily, i.e.,

$$|E| \leq 1 \qquad (35)$$

The control E which satisfies (35) is called admissible control.

We wish to determine an admissible control E*, which steers the system (34) from some initial state (x_1^0, x_2^0) at time

$t = t_0 =$ zero to the origin, and minimizes the performance index J

$$J = \int_{t_0}^{t} (1)dt = t - t_0 \tag{36}$$

and such that the corresponding optimal trajectory makes no oscillations.

Controllability of the System

The nonlinear approximation of system (34) near the origin is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{EV}{z}\cos x_e & -D \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} 0 \\ \frac{V}{z}\sin x_e \end{bmatrix} E \tag{37}$$

which is in the form

$$x = Ax + bu . \tag{38}$$

The controllability matrix $[b : Ab]$ is evaluated and

$$[b : Ab] = \begin{bmatrix} 0 & -\frac{V}{z}\sin x_e \\ -\frac{V}{z}\sin x_e & \frac{DV}{z}\sin x_e \end{bmatrix} \tag{39}$$

It is clear that matrix (39) has rank 2.

Therefore, the domain of null controllability of the system (11) is open in the phase plane R^2. Consequently, the system (34) is controllable in an open region around the origin.

Existence of Optimal Excitation Control

The power system (34) can be written in the compact form:

$$\dot{x} = f(x, E)$$

with the constraint:

$$|E| \leq 1$$

where $f(x, E)$ is a vector $(f_1, f_2)'$ such that:

$$f_1 = x_2$$

$$f_2 = -Dx_2 - [\frac{V}{z} \sin (x_1+x_e)]E + P$$

and

$$\|f\| \leq \|f_1\| + \|f_2\|.$$

If we take $P = 1$, and use the same techniques used in Chapter II we get the inequality

$$\|f\| \leq M[\|x_2\| + (\|\frac{EV}{z}\| + 1)/M] \tag{40}$$

where

$$M = D + 1$$

But since $|E| \leq 1$, the inequality (40) becomes:

$$\|f\| \leq M[\|x_2\| + (\|\frac{V}{z}\| + 1)/M] \tag{41}$$

By Theorem (2) in Appendix A, the solution $x(E, t)$ of (11) exists, is unique and is uniformly bounded. Consequently and by Theorem (3) in Appendix A; there exists an optimal excitation control $E*$ which takes the system (34) from some initial point, in the domain of null controllability, to the origin in minimum time.

Determination of the Optimal Control Law

The maximum principle of Pontryagin will be used to determine the optimal control law $E*$.

The Hamiltonian, $H(x, E, p)$ for the system (34) is

$$H(x, E, p) = -1 + p_1 x_2 + p_2[-Dx_2 - (\frac{V}{z} \sin (x_1 + x_e))E + P] \tag{42}$$

where $p = (p_1, p_2)'$ is the adjoint vector which is the solution of the adjoint equations given by:

$$\dot{p}_1 = -\frac{\partial H}{\partial x_1} = [p_2 \frac{V}{z} \cos (x_1 (x_e)]E \tag{43}$$

$$\dot{p}_2 = -\frac{\partial H}{\partial x_2} = -p_1 + p_2 D \tag{44}$$

The optimal control $E*$ should satisfy:

$$H(x, E^*, p) = \sup_{|E| \leq 1} [-1 + (p_1 - p_2 D)x_2 - (p_2 \frac{V}{z} \sin (x_1 + x_e)E + p_2 P] \tag{45}$$

where sup means the supremum.

It is clear Equation (45) is satisfied when:

$$E^* = - \operatorname{sgn} p_2 \frac{V}{z} \sin (x_1 + x_e) \tag{46}$$

Since $\frac{V}{z}$ is constant, Equation (46) becomes

$$E^* = - \operatorname{sgn} p_2 \sin (x_1 + x_e)$$

where E^* is the optimal excitation control and

$$\operatorname{sgn} p_2 \sin (x_1 + x_e) = \begin{cases} +1 & \text{if} \quad p_2 \sin (x_1 + x_e) > 0 \\ -1 & \text{if} \quad p_2 \sin (x_1 + x_e) < 0 \end{cases} \tag{47}$$

Therefore, the optimal controller is an ideal relay and the function

$$p_2 \sin (x_1 + x_e) \tag{48}$$

is the switching function of this relay.

<u>Uniqueness of the Optimal Control</u>

The optimal control E^* is unique if the power system (34) is normal. The system (34) is normal when the switching function

$p_2 \sin(x_1+x_e)$ does not vanish identically on any subinterval $[t_1, t_2] \subset [0, t^*]$.

Suppose that the function $p_2 \sin(x_1+x_e)$ vanishes identically on any interval $[t_1, t_2] \subset [0, t^*]$. Since $\sin(x_1+x_e)$ does not vanish identically on this interval then, only, $p_2 = 0$ on $[t_1, t_2]$. But

$$\dot{p}_2 = -p_1 + p_2 D$$

Then p_1 will equal zero on this interval. This means that the adjoint vector $(p_1, p_2)'$ will vanish identically on some subinterval in $[0, t^*]$, which contradicts the maximum principle of Pontryagin. Consequently the optimal excitation control system is normal and the optimal excitation control E^* is unique.

<u>Condition of No Oscillations</u>

The optimal excitation control was given by:

$$E^* = -\operatorname{sgn} p_2 \sin(x_1+x_e)$$

If the optimal control system makes no oscillations on $(0, t^*)$, $\sin(x_1+x_e)$ has no zeros there, and it will equal zero at $t = t^*$ only. Therefore, the function $p_2(t)$ should have at most one zero on $(0, t^*)$ for the optimal system to have no oscillations there. From the adjoint equations (43) and (44) we have,

$$\ddot{p}_2 - D\dot{p}_2 + [\frac{EV}{z} \cos (x_1+x_e)] p_2 = 0 \qquad (49)$$

Equation (49) is the same as Equation (30), in Chapter II, and the condition that the solution $p_2(t)$ has at most one zero, on $(0, t^*)$, is given by:

$$D^2 \geq \frac{4EV}{z}$$

Synthesizing the Optimal Excitation Control

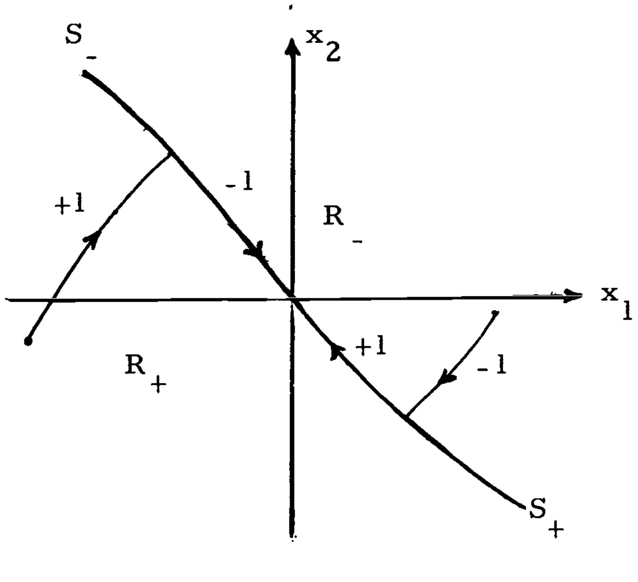
It was proved that the excitation control power system was controllable, at least, locally. Also, it was shown that the optimal control exists and is unique, and in the last section the condition of no oscillation was found.

To synthesize the optimal excitation control E^*, we use the same method used to synthesize the optimal governor control. The switching locus $S(x)$ is obtained by integrating, the power system (34), backward in time, starting from the origin and using the optimal control E^*. We obtain two unique optimal trajectories S_+ and S_-. The trajectory S_+ is obtained when $E^* = +1$ and S_- is obtained when $E^* = -1$. Then the switching curve $S(x)$ will equal $S_+ \cup S_-$. $S(x)$ will divide the phase plane into two separate regions R_+ and R_-, where R_+ and R_- are defined as in the case of optimal governor control. Then, the optimal feedback excitation control $E^*(x)$ is given by:

$$E^*(x) = +1 \quad \text{for all} \quad (x_1, x_2) \in R_+ \cup S_+$$

$$E^*(x) = -1 \quad \text{for all} \quad (x_1, x_2) \in R_- \cup S_-$$

This is illustrated in Figure 5.



$S(x) = S_+ \cup S_-$ is the switching curve

$E^*(x) = \{+1, -1\}$

Figure 5. Optimal feedback excitation control trajectory.

An engineering realization of the optimal excitation control system will be similar to that used for optimal feedback governor control but with the switching curve given in Figure 5, and the relay output is E^*.

IV. OPTIMAL GOVERNOR AND EXCITATION CONTROL

In a practical power system, both the governor and excitation act simultaneously to restore the system to equilibrium after the occurrence of a disturbance. Therefore, the combined optimal governor and excitation control will be investigated in this chapter.

The power system in Equation (11), is written in the form,

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -Dx_2 - [\frac{V}{z}\sin(x_1+x_e)]E + P \tag{50}$$

The control pair (P, E) is bounded due to engineering considerations. The bound on P and E is taken to be arbitrarily 1, i.e.,

$$|P| \leq 1, \quad |E| \leq 1 \tag{51}$$

The control pair which, (P, E), satisfies (5) is called admissible.

We wish to find an admissible control pair (P*, E*) which steers the system (50) from some initial state (x_1^0, x_2^0) at time $t = t_0$ = zero to the origin in minimum time, and such that the corresponding trajectory makes no oscillations.

The system (50) is locally controllable in a neighborhood around the origin, with respect to the pair (P, E). This is because it is locally controllable with respect to each of P and E acting alone.

The solution x(E, P, t) of (50) exists and, is unique and uniformly bounded, since the inequality (41) still holds, i.e.,

$$\|f\| \leq M[\|x_2\| + (\|\frac{V}{z}\| + 1)/M] \tag{53}$$

Therefore, there exists an optimal control pair (P*, E*) which takes the system (50) from some initial state, in the domain of null controllability, to the origin in minimum time.

<u>Determination of the Optimal Control Pair</u>

The maximum principle of Pontryagin is used to obtain the optimal control pair (P*, E*). The Hamiltonian H(x, E, P, p) for the power system (50) is:

$$H(x, E, P, p) = -1 + p_1 x_2 + p_2[-Dx_2 - (\frac{V}{z} \sin (x_1 + x_e))E + P] \tag{54}$$

where $p = (p_1, p_2)'$ is the adjoint vector which is the solution of the adjoint equations given by:

$$\dot{p}_1 = -\frac{\partial H}{\partial x_1} = [p_2 \frac{V}{z} \cos (x_1 + x_e)] E \tag{55}$$

$$\dot{p}_2 = -\frac{\partial H}{\partial x_2} = -p_1 + p_2 D \tag{56}$$

The optimal control pair (P^*, E^*) must satisfy

$$H(x, E^*, P^*, p) = \sup_{|P| \leq 1, |E| \leq 1} [-1+(p_1 - p_2 D)x_2 - (p_2 \frac{V}{z} \sin (x_1 + x_e) E + p_2 P] \tag{57}$$

The control pair (P^*, E^*) that satisfy (47) are given by:

$$P^* = \operatorname{sgn} p_2 \tag{58}$$

$$E^* = -\operatorname{sgn} p_2 \sin (x_1 + x_e) \tag{59}$$

i. e.,

$$(P^*, E^*) = \operatorname{sgn} [p_2, -p_2 \sin (x_1 + x_e)] \tag{60}$$

where,

$$[p_2, -p_2 \sin (x_1 + x_e)] \tag{61}$$

is called the switching function pair.

It was proved, previously, that the pair $[p_2, -p_2 \sin (x_1+x_e)]$ does not vanish on any subinterval $[t_1, t_2] \subset [0, t^*]$, where t^* is the minimum time to steer the system (50) to the origin by the pair (P^*, E^*). Therefore, the optimal control pair (P^*, E^*) is unique, and the following four control pairs,

$$(1, 1), \quad (1, -1), \quad (-1, 1), \quad (-1, -1) \tag{62}$$

are the only candidates for the optimal control.

The Condition for No Oscillation

The optimal control pair is given by,

$$(P^*, E^*) = \text{sgn}\,[p_2, -p_2 \sin(x_1+x_e)]$$

If the optimal control system given in (50) makes no oscillation on $(0, t^*)$, (x_1+x_e) does not change sign there. This means that (x_1+x_e) has no zeros on $(0, t^*)$. Therefore, the function p_2 should have at most one zero on $(0, t^*)$. This was found to be the case when,

$$D^2 \geq \frac{4EV}{z}$$

and consequently the optimal control pair (P^*, E^*) switch at most once on $(0, t^*)$. Then, the only candidate pairs for the optimal control are,

$$(+1, -1) \quad \text{and} \quad (-1, +1) \tag{63}$$

Synthesizing the Optimal Control

The method of synthesizing the optimal control pair,

$$(P^*, E^*) = \text{sgn}\,[p_2, -p_2 \sin(x_1+x_e)]$$

is the same as that used in Chapters II and III.

The switching locus $S(x)$ is obtained by integrating (50) backward in time, starting from the origin optimal trajectories S_+ and S_- are obtained. The trajectory S_+ is obtained when $(P^*, E^*) = (+1, -1)$ and S_- when $(P^*, E^*) = (-1, +1)$. The switching locus $S(x)$ will equal $S_+ \cup S_-$, and $S(x)$ will divide the phase plane into two separate regions R_+ and R_-, defined by:

$$R_+ = \{(x_1, x_2) | \text{ if } (x_1, x_2') \in S(x), \text{ then } x_2 < x_2'\}$$

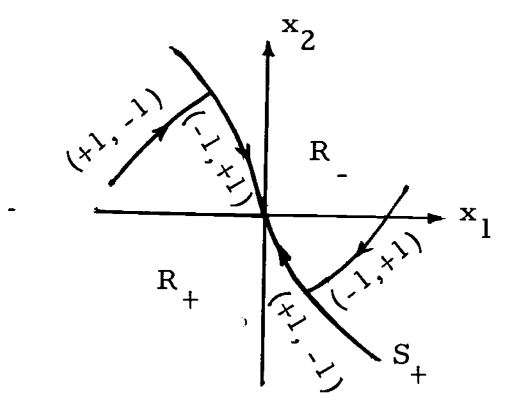
$$R_- = \{(x_1, x_2) | \text{ if } (x_1, x_2') \in S(x), \text{ then } x_2 > x_2'\} .$$

The feedback optimal control pairs are given by:

$$(P^*, E^*) = (+1, -1) \quad \text{for all} \quad (x_1, x_2) \in R_+ \cup S_+$$

$$(P^*, E^*) = (-1, +1) \quad \text{for all} \quad (x_1, x_2) \in R_- \cup S_0$$

This is illustrated in Figure 6.



$S(x) = S_+ \cup S_-$

$(P^*, E^*) = (+1, -1)$ or $(-1, +1)$

Figure 6. Optimal feedback governor and excitation control trajectory.

An engineering realization of the above optimal system is shown in Figure 7, where the state variables are x_1 (load angle) and the x_2 (load angle rate) are measured at each instant of time. $x_1(t)$ is introduced to the realized switching locus K and the difference, m, between the output of K and x_2 is applied to actuate the ideal relays A and B. The output of the relay A and B are P* and E* respectively. The reason for that is, (See [1, p. 513])

$$\text{if } (x_1, x_2) \in R_+, \text{ then } m > 0 \text{ and } (P^*, E^*) = (+1, -1)$$

$$\text{if } (x_1, x_2) \in R_-, \text{ then } m < 0 \text{ and } (P^*, E^*) = (-1, +1).$$

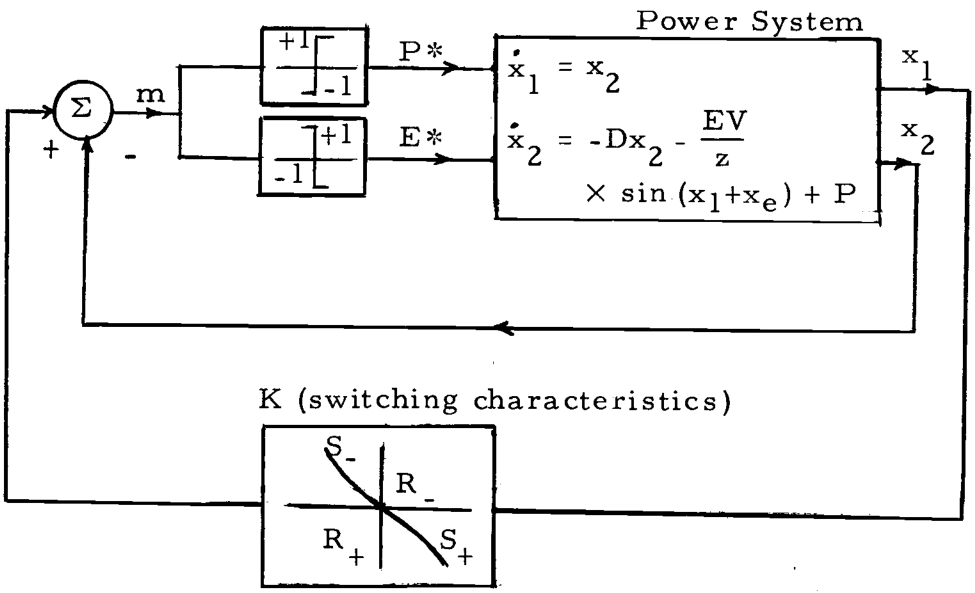


Figure 7. Engineering realization of optimal governor and excitation control system.

Example

Consider a power system with

$$x_e = \frac{\pi}{6}, \quad D = 1.4, \quad \frac{V}{z} = 0.42$$

$$|P| \leq 0.2, \quad |E| \leq 1$$

The time-optimal control pair (P^*, E^*) is given by:

$$P^* = \mp 0.2$$

$$E^* = \mp 1$$

The candidate pairs for optimal control are,

$$(+0.2, -1), \quad (-0.2, +1).$$

The optimal system with the time reversed is

$$x_1 = -x_2$$

$$x_2 = 1.4x_2 + [0.42 \sin (x_1 + \frac{\pi}{6})] E^* - P^*$$

The above system is simulated on the analog computer. Starting from the origin $(0, 0)$, two trajectories S_+ and S_- for $(P^*, E^*) = (+0.2, -1)$ and $(P^*, E^*) = (-0.2, +1)$ respectively. The switching curve $S(x)$ is,

$$S(x) = S_+ \cup S_-$$

and shown in Figure 8. Then an engineering realization of the optimal system,

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -1.4x_2 - [0.42 \sin (x_1 + \frac{\pi}{6})] E^* + P^*$$

was simulated on the analog computer using the above switching curve which was realized by a variable function generator. The optimal control transfers some initial point in the phase plane to the origin. The optimal trajectory is shown in Figures 9 and 10.

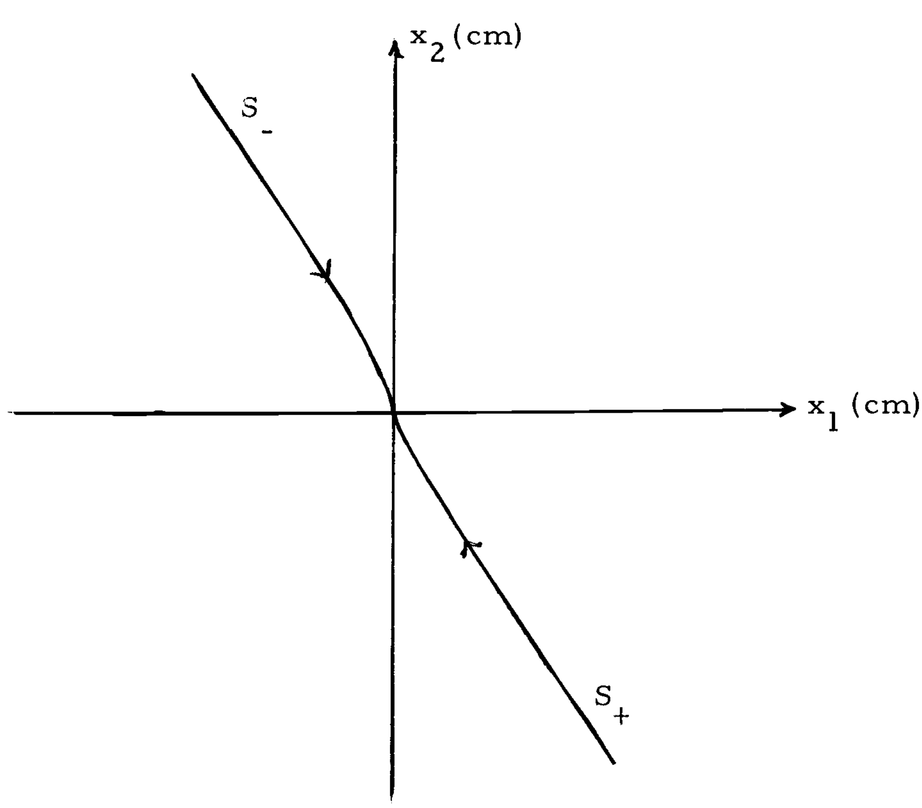


Figure 8. Switching curve of the example. Scale x_1: 1v/cm; x_2: 1v/cm.

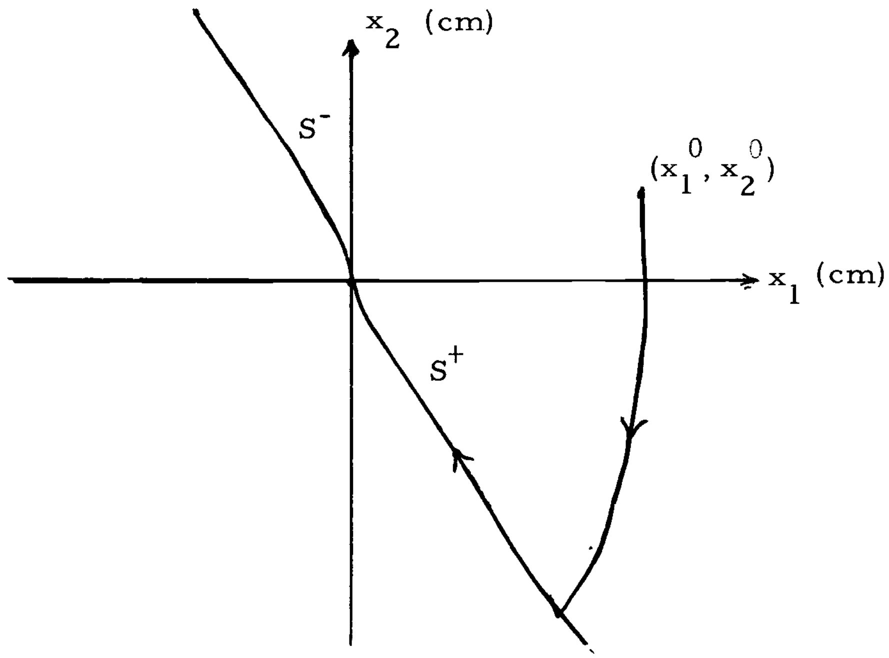


Figure 9. Optimal governor and excitation feedback control. Scale x_1:1v(cm; x_2:1v/cm.

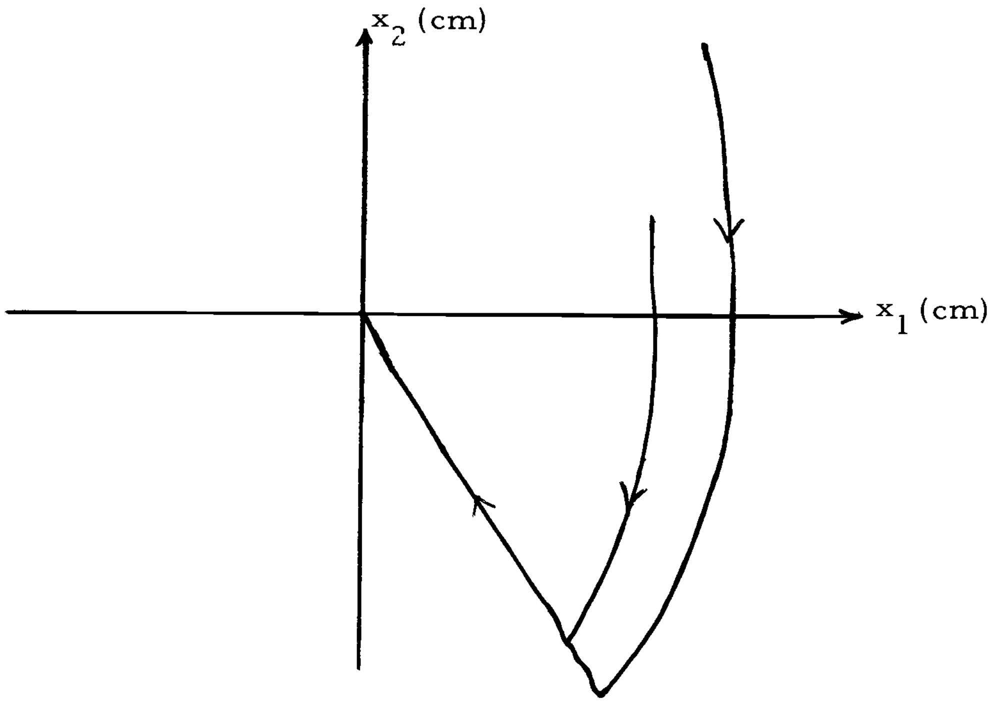


Figure 10. Optimal trajectories for two initial conditions. Scale x_1:1v/cm; x_2: 1v/cm.

V. CONCLUSION AND DISCUSSION

The optimal control of a power system was considered from the point of view of optimal control theory. It was shown that the governor and excitation can steer the power system, after a disturbance, from some initial point in the controllability domain to the stable equilibrium point in minimum time. A large amount of damping was needed to prevent oscillations. This was given in the inequality,

$$D^2 \geq \frac{4EV}{z} \tag{64}$$

Electrical power engineers, in trying to stabilize the power system, introduced several methods of damping, instantly after occurrence of a disturbance. But this is the first time, the exact amount of damping was determined to prevent oscillations and stabilize the system in minimum time. Some of the many methods of damping introduced are load or dynamic braking, machine input derivative control, controlled capacitor switching and application of a real damping to the generator.

The stability of the system is established by the existence of time optimal control that restores the system to its stable equilibrium point in minimum time.

The method of time-optimal control of a power system implies fast responses of the governor and excitation. Modern governors and excitation systems are designed to have a short time delay and, consequently, a fast response. Besides that, the small delay can be taken into consideration in synthesizing the time-optimal control. Suppose the time delays of governor and excitation are τ_1 and τ_2 respectively. Then the power system of equations (11) are written in the form,

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -Dx_2 - [\frac{V}{z} \sin (x_1+x_e)]E(t-\tau_2) + P(t-\tau_1) \quad (64)$$

$$|E(t-\tau_2)| \leq 1, \qquad |P(t-\tau_1)| \leq 1 \quad (65)$$

The performance index,

$$J = \int_0^t (1)dt = t.$$

By the maximum principle, the condition for optimality is,

$$H(x, P*, E*, p) = \sup_{|E| \leq 1, |P| \leq 1} [-1+p_1x_2-p_2Dx_2-p_2[\frac{V}{z} \sin (x_1+x_e)] \times E(t-\tau_2)+p_2P(t-\tau_1)] \quad (66)$$

which gives,

$$P*(t-\tau_1) = \text{sgn } p_2 \quad (67)$$

$$E*(t-\tau_2) = -\text{sgn } p_2 \sin (x_1+x_e) \quad (68)$$

and the adjoint equations are,

$$\dot{p}_1 = p_2 \frac{V}{z} [\cos (x_1 + x_e)] E(t - \tau_2)$$

$$\dot{p}_2 = -p_1 - p_2 D.$$

Therefore, the optimal control laws were not changed, but the optimal controllers P* and E* were delayed by τ_1 and τ_2 respectively. These delays will shift the switching locus, S(x), to the left a distance proportional to them.

BIBLIOGRAPHY

1. Athans, M. and P. Falb. Optimal control: An introduction to the theory and its application. New York, McGraw Hill, 1966. 310p.

2. Balakirev, V. S. The principle of masimum in the theory of second order optimal systems. Automation and Remote Control 23: 948-956. 1962.

3. Gless, G. E. Direct method of Liapunov applied to transient power system stability. IEEE Transaction on Power Apparatus and Systems 85:159-168. 1966.

4. Hulkin, Hubert. On the necessary condition for optimal control of nonlinear system. Journal d'Analyse Mathematique 12:1-82. 1962.

5. Lee, E. B. and L. Markus. Foundations of optimal control theory. New York, John Wiley, 1967. 576 p.

6. Leitman, George. Introduction to optimal control. New York, McGraw-Hill, 1966. 163 p.

7. Leitman, George (ed.). Topics in optimization. New York, Academic, 1967. 469 p.

8. Mittlestadt, W.A. Four methods of power system damping. IEEE Transactions on Power Apparatus and Systems 87:1323-1329. 1968.

9. Moore, R.A. The behaviour of solutions of linear differential equations of second order. Pacific Journal of Mathematics 5:125-145. 1955.

10. Oldenburger, Rufus (ed.). Optimal and self-optimizing control. Cambridge, Massachussets Institute of Technology, 1966. 497 p.

11. Reiten, D.K. and N. Ramaroa. Pontrygin's maximum principle aids transient stability: Bang-bang control of reactance. IEEE Proceedings 56:1734. 1968.

12. Say, M. G. The performance and design of alternating current machines. London, Pitman, 1961. 664 p.

APPENDIX A

Controllability of Nonlinear System

Definition. Domain of null controllability: Consider the control system given by:

$$\dot{x} = f(x, u)$$

where, the vector valued function $f(x, u)$ in C' in $R^n \times \Omega$, x is $n \times 1$ state vector, u is $m \times 1$ control vector in Ω the set of all piece wise continuous control functions. Ω is compact in R^m.

The domain C of null controllability is defined as the set of all initial states which can be steered to the origin by some bounded controller $u(t)$ in Ω on finite time duration. If C contains an open neighborhood of the origin, then the system (1) is said to be locally controllable near the origin. (See [5, p. 364]).

Theorem 1. Consider the control system in R^n, given by,

$$\dot{x} = f(x, u)$$

in C' in R^{n+m}, with $u = 0$ interior to the restraint set $\Omega \subset R^m$. Assume:

i) $f(0, 0) = 0$

ii) rank $[B, AB, A^2B, \ldots, A^{n-1}B] = n$ where

$$A = (\frac{\partial f}{\partial x})(0, 0) \quad \text{and} \quad B = (\frac{\partial f}{\partial u})(0, 0)$$

i.e., the derivatives are evaluated at (0, 0). Then the domain of null controllability is open in R^n. (See [5, p. 366]).

Existence of Optimal Control

Theorem 2. Consider the control system given by:

$$\dot{x} = f(x, u)$$

with all the above assumptions concerning the vector valued function $f(x, u)$ are valid. Assume there exists a constant $M < \infty$ such that:

$$|f(x, u, t)| \leq M(|x|+1)$$

for all x in R^n, all u in Ω and all $t_0 \leq t \leq t_1$ where $|x|$ denotes the euclidean norm of the vector x, i.e.,

$$|x| = (\sum_{i=1}^{n} x_i^2)^{1/2}$$

The function $x(u, t)$ exists, is unique and uniformly bounded over $t_0 \leq t \leq t_1$. (See [7, p. 204]).

Theorem 3. Consider the nonlinear control process in R^n, given by (See [5, p. 259])

$$\dot{x} = f(x, u) \quad \text{in} \quad C' \quad \text{in} \quad R^{n+m}$$

$$J = \int_{t_0}^{t_1} (1)dt$$

with all the assumptions concerning the vector valued functions $f(x, u)$ and $u(t)$ are valid. If there exists a uniform bound

$$|x(t)| \leq a \quad \text{on} \quad t_0 \leq t \leq t_1$$

Then there exists a time-optimal control $u^*(t)$ on $t_0 \leq t \leq t_1$ in Ω minimizing J.

APPENDIX B

Zeros of the Nonoscillatory Solution of Hill's Equation

Consider Hill's equation of the type:

$$y'' + (-a+bg(x))y = 0 \tag{B-1}$$

where

$$y'' = \frac{d^2y}{dx^2}$$

a and b are real parameters

$g(x)$ is real valued continuous periodic function.

Theorem 1. If $a \geq b$ the solutions of (B-1) are nonoscillatory. If $a < b$ all solutions of (B-1) are oscillatory.

Theorem 2. If a nontrivial solution of Equation (B-1) is nonoscillatory, it has at most one zero on $(-\infty, \infty)$. (See [9])

APPENDIX C

Bibliography Index

The numbers in parentheses are the bibliography reference numbers which refer to an author's work. The numbers before these numbers indicate the pages of the thesis where an author's work is referred to and the numbers after then indicate the pages referred to in the author's work.

Author's name	Thesis Pages		Author's Pages
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