TRANSIENT BEHAVIOR OF TRANSMISSION CIRCUITS WITH SERIES-REACTOR COMPENSATION

by

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INTRODUCTION

The necessity for long distance transmission of large blocks of power has grown rapidly during the last few years. Some of the large scale hydro-electric developments in the Western States are located in relatively remote districts and have created transmission problems involving distances of the order of 250 miles and blocks of power of the order of 300,000 kilowatts. One of the most important considerations of a power development is to deliver the power to a natural load center with a minimum annual cost, and that this cost not exceed that of another source of available power.

The problem discussed in this thesis has arisen as a result of some speculations regarding a transmission tie-line between the Bonneville Dam power generating station and the Grand Coulee Dam generating station. Since a normal 230-kilovolt transmission line of that length (240 miles) has a normal power limit of approximately130,000 kilowatts, a number of lines may have to be constructed in order to provide sufficient power transfer. The power limit of a transmission system is determined largely by its impedance, most of which is reactance.

The high voltage used on a power transmission line requires that there be a large spacing between the conductors; then, each conductor is surrounded by a large magnetic field resulting in a high reactance. The most economical point at which to operate a transmission line may be at two to three times its ordinary power limit. The transmission line proper does not contain all of the reactance; the reactance in the generators themselves is quite large. If, however, the reactance in the transformers and the transmission line could be compensated, then the most economical operating point may be approached.

Because of recent manufacturing trends, it is now possible to construct capacitors of sufficient voltage and current rating to be used for compensating the line reactance. The conjecture is that a capacitor may be placed in series with each conductor in a transmission system to compensate for all or part of the inductive reactance.

The behavior of a power system as a whole is quite complex. This study has been narrowed down to a few of the more pertinent points in connection with the steady-state behavior and a few of the possible types of transients that would occur on the transmission line. This thesis was developed from the standpoint of considering each effect in its individuality so that for any total effect the proper components could be put together. The idea being that since this development is not carried through to its completion, a complete correlation of the results would not be of much value at this point.

There have been a number of instances where a series of capacitors have been used in small feeder lines for the purpose of voltage regulation, which is of no consequence in the problem; but to date, there has been no installation for the purpose of increasing the power limit of a transmission system. Hence, the literature on the background of this investigation is limited. The bibliography will consist of material relevant to the nature of behavior of transmission systems; but, it will not pretend to be comprehensive.

There have been new trends in the notation of engineering functions and quantities. These new conventions have been adhered to whenever possible. For a complete list of the notation and symbols, consult Appendix I.

Chapter I

PROBLEMS OF POWER TRANSFER IN A FOUR TERMINAL NETWORK AS A FUNCTION OF VOLTAGE ANGLE AND TERMINAL VOLTAGES

Since a transmission line may be considered as a four terminal network, the performance may be represented by the matrix equation

$$\left\| \begin{matrix} E_1 \\ I_1 \end{matrix} \right\| = \left\| \begin{matrix} A & B \\ C & A \end{matrix} \right\| \cdot \left\| \begin{matrix} E_2 \\ I_2 \end{matrix} \right\|$$

where the subscript 1 indicates sending end quantities, and the subscript 2 indicates receiving end quantities. Then

$$E_1 = AE_2 + BI_2$$

$$I_1 = CE_2 + AI_2$$

but the power received

$$P_2 = I_2\hat{E}_2$$

Then

$$E_1 = AE_2 + B\frac{P_2}{E_2}$$

solving for P_2

$$P_2 = \frac{E_1\hat{E}_2 - AE_2\hat{E}_2}{B}$$

Let

$$E_1 = |E_1|$$

$$B = |B| \operatorname{cis}(\psi)$$

$$A = |A| \operatorname{cis}(\gamma)$$

$$E_2 = |E_2| \operatorname{cis}(\delta + \phi)$$

ϕ and δ are to be determined later

$$P_2 = \frac{|E_1||E_2| \operatorname{cis}(-\delta - \phi) - |A||E_2|^2 \operatorname{cis}\alpha}{|B| \operatorname{cis}(\psi)}$$

$$= \frac{|E_1||E_2|}{|B|} \operatorname{cis}(-\delta - \phi - \psi) - \frac{|A||E_2|^2}{|B|} \operatorname{cis}(\alpha - \psi) \qquad (1)$$

$$P_2 = P_{2r} + P_{2J}$$

$$P_{2r} = \frac{|E_1||E_2|}{|B|} \cos(\delta + \phi + \psi) - \frac{|A||E_2|^2}{|B|} \cos(\gamma - \psi) \qquad (2)$$

If δ is chosen so that it is the displacement from the angle of zero power, then

$$\phi = \cos^{-1}\left[|A| \frac{|E_2|}{|E_1|} \cos(\gamma - \psi) \right] - \psi$$

Equation (2) represents, then, the equation of power transfer for a four terminal network. It is noted that if an angle ϕ cannot be found that, under those conditions, there can be no power transfer. Consider now a special case of the above equation. In the case of a smooth transmission line used for power transmission, the constants A, B, and C have the following values

$$A = \cosh \theta$$

$$B = Z_0 \sinh \theta$$

$$C = \frac{1}{Z_0} \sinh \theta$$

Consider the case where A is real and B is pure reactance. This condition is approached very closely in power transmission circuits. Then

$$A = |A| \qquad , \alpha = 0$$

$$B = |B| \operatorname{cis}\left(\frac{\pi}{2}\right), \ \psi = \frac{\pi}{2}$$

$$\phi = 0$$

$$P_{2r} = \frac{|E_1||E_2|}{|B|} \sin \delta$$

From Equation (1)

$$P_{2J} = \frac{|E_1||E_2|}{|B|} \cos \delta$$

If the line is compensated for by a series capacitor at any point, the angle of A, B, and C is not materially effected; hence, the above expressions will apply approximately.

For the nominal pi consideration of a transmission line, the quantity B in the above equations is the series reactance of the section. The nominal pi calculation of power for a given angle gives a smaller amount of power than does the same calculation on the basis of equations above.

The equation for power transfer in a four-terminal network is of the same type as that for a simple series impedance except that the series impedance is effected by the shunt admittance and an equivalent impedance must be used.

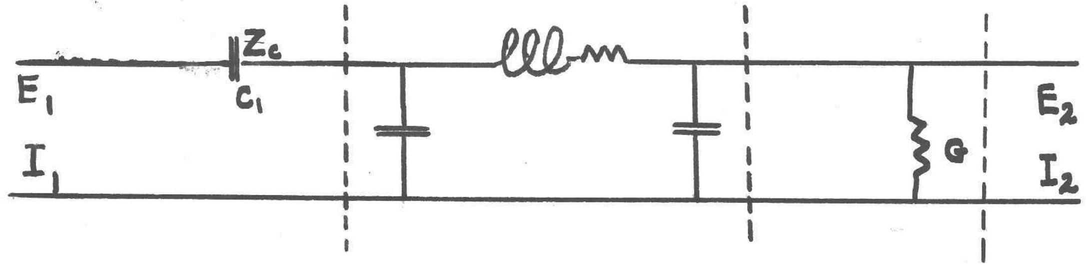
Chapter II

STEADY STATE BEHAVIOR OF SERIES CAPACITORS

The general problem of determining the size of a series capacitor to neutralize a given length of line has several subdivisions, and the logical one to discuss first is the problem of the series capacitor being located at the generator end of the transmission line. The problem is to find a value of series capacitance such that the line behaves as pure resistance. This same condition is satisfied if a value of capacitance is found such that the angle between the sending end voltage vector and the receiving end voltage vector is independent of the load provided, the load is of pure resistance.

The problem is attacked in the following manner: The load is introduced into the network representing the transmission line as an element, and the receiving end current will be considered zero. The receiving end current in this case is not the load current. The load is considered as a conductance G.

A. The Case of the Series Capacitor Located at the Sending End of the Line



$$\begin{Vmatrix} E \\ I \end{Vmatrix} = \begin{Vmatrix} 1 & Z_c \\ 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} A & B \\ C & A \end{Vmatrix} \cdot \begin{Vmatrix} 1 & 0 \\ G & 1 \end{Vmatrix} \cdot \begin{Vmatrix} E_2 \\ I_2 \end{Vmatrix}$$

$$= \begin{Vmatrix} A + Z_c C + G(B + Z_c A) & B + Z_c A \\ C + AG & A \end{Vmatrix} \cdot \begin{Vmatrix} E_2 \\ I_2 \end{Vmatrix}$$

Since $I_2 = 0$

$$\frac{E_1}{E_2} = \left[A + Z_c C + G(B + AZ_c)\right] = \left|\frac{E_1}{E_2}\right| \text{ cis } (\phi)$$

Where ϕ is to be a constant.

Let

$$Z_c = -JX_c, \qquad A = A_r + JA_J$$

$$B = B_r + JB_J, \quad C = C_r + JC_J$$

Then

$$\frac{E_1}{E_2} = \left[A_r + C_J X_c + G(B_r + A_J X_c)\right]$$

$$+J\left[A_J - C_r X_c + G(B_J - A_r X_c)\right]$$

$$\tan \phi = \frac{A_r + C_J X_c + G(B_r + A_J X_c)}{A_J - C_r X_c + G(B_J - A_r X_c)}$$

If

$$\frac{A_J - C_r X_c}{B_J - A_r X_c} = \frac{A_r + C_J X_c}{B_r + A_J X_c}$$

then tan ϕ is independent of G. X_c must satisfy the following relation.

$$X_c^2\left[A_r C_J - A_J C_r\right] + X_c\left[A_J^2 + A_r^2 - C_r B_r - C_J B_J\right] + \left[A_J B_r - A_r B_J\right] = 0 \qquad (3)$$

There will be two solutions to this equation. One solution will make ϕ equal to 90° and the other one will make ϕ equal to 0°, which is the value which is wanted. The latter one may be recognized easily since it will be the smaller value. The other value may be negative indicating that it would take inductive reactance to accomplish that result. The solution equation (3) may be written down with the aid of the Hindu formula. The general method of solution was carried to completion in this case to show the method of attack. For the succeeding cases some assumptions will be made in order to simplify the equations. These assumptions introduce an error of very small magnitude for power transmission circuits. They are:

$$A = A_r \,, \quad A_J = 0$$

$$B = JB_J \,, \quad B_r = 0$$

$$C = JC_J \,, \quad C_r = 0$$

These assumptions are equivalent to saying that the line consists of pure inductance and capacitance.

The solutions to equation (3) under these conditions are

$$X_c = \frac{B_J}{A_r} \,, \quad \frac{-A_r}{C_J}$$

If the long line equations are considered,

$$B = Z_o \sinh\theta \,, \quad A = \cosh\theta$$

For the previous assumptions θ is pure imaginary $\theta = J\beta$. Let L and C be the inductance and capacitance per unit length and l the length of the line.

$$\theta = l\sqrt{ZY} = Jl\sqrt{LC}\omega = J\beta$$

$$Z_o = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{L}{C}}$$

Therefore

$$X_c = Z_o \tan\beta \qquad (4)$$

In terms of the nominal pi representation of the transmission line

$$B = JlL\omega \,, \quad A = 1 - l^2 LC\omega^2$$

In terms of the nominal pi representation of the transmission line

$$X_c = \frac{\ell L \omega}{1 - \ell^2 L C \omega^2} \tag{5}$$

B. The Case Where the Series Capacitor is to be Placed on the Receiving End of the Transmission Line.

$$\begin{Vmatrix} E_1 \\ I_2 \end{Vmatrix} = \begin{Vmatrix} A & B \\ C & A \end{Vmatrix} \cdot \begin{Vmatrix} 1 & Z_c \\ 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} 1 & 0 \\ G & 1 \end{Vmatrix} \cdot \begin{Vmatrix} E_2 \\ I_2 \end{Vmatrix}$$

$$\begin{Vmatrix} E_1 \\ I_1 \end{Vmatrix} = \begin{Vmatrix} A + G(B + Z_c A) & B + Z_o A \\ C + G(A + Z_c C) & A + Z_c C \end{Vmatrix} \cdot \begin{Vmatrix} E_2 \\ I_2 \end{Vmatrix}$$

Since $I_2 = 0$, then

$$\frac{E_1}{E_2} = A + G(B + Z_c A)$$

By the previous assumptions $\frac{E_1}{E_2}$ is real, then Z_c must be chosen so that the coefficient of G is zero.

$$0 = B + Z_c A$$

Then

$$X_c = Z_o \tan\beta$$ for the equivalent pi line

or

$$X_c = \frac{\ell L\omega}{1-\ell^2 LC\omega^2} \quad \text{for the nominal pi line.}$$

It is noted that these are the same as for the previous case.

C. The Case Where the Series Capacitor is in the Center of the Line

If a series capacitor were to be placed in the center of a transmission line to compensate for the reactance of the entire line, it may be considered to compensate for the line in each direction, and its reactance, then, would equal twice the reactance of that necessary to compensate one-half the line.

Then

$$X_{c_1} = 2 Z_o \tan\beta$$

where β is the phase constant for one-half the line, and X_{c_1} is the reactance necessary to compensate for the whole line.

Then the decrease in reactance necessary to compensate for the entire line when the capacitor is placed at the center of the line is

$$\Delta X_c = Z_o\left[\tan 2\beta - 2\tan\beta\right] \tag{6}$$

where β is the phase constant for one-half the line.

In terms of the nominal pi constants where ℓ is the total length of the line and L and C are the inductance and capacity per unit length

$$\Delta X_c = \frac{3\ell^3 L^3 C \omega^3}{(1-\ell^2 LC\omega^2)(4-\ell^2 LC\omega^2)}$$

For a 300 mile line, where Z_o is 400 and β is 19.6° (for one-half the line) then

X_c installed at one end is 326 ohm.

X_c installed in the center is 286 ohm.

From observing the results of the three cases, the question might be asked; is there a position in the transmission line where a given series capacitor may be installed so that its effect may be a maximum or a minimum?

The problem now will be to find the position in the transmission line for a given series capacitor and a given load on the line such that the angle between the sending end voltage vector and the receiving end voltage vector is a minimum or maximum.

Let the line be of length l, the distance from the sending end of the series capacitor x, and G the load.

Then

$$\begin{Vmatrix} E_1 \\ I_1 \end{Vmatrix} = \begin{Vmatrix} A_1 & B_1 \\ C_1 & A_1 \end{Vmatrix} \cdot \begin{Vmatrix} 1 & Z_c \\ 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} A_2 & B_2 \\ C_2 & A_2 \end{Vmatrix} \cdot \begin{Vmatrix} 1 & 0 \\ G & 1 \end{Vmatrix} \cdot \begin{Vmatrix} E_2 \\ I_2 \end{Vmatrix}$$

$$\begin{Vmatrix} E_1 \\ I_1 \end{Vmatrix} = \begin{Vmatrix} \left\{A_1[A_2+C_2Z_c+G(B_2+A_2Z_c)]+B_1(C_2+GA_2)\right\} & \left\{A_1(B_2+A_2Z_c)+B_1A_2\right\} \\ \left\{C_1[A_2+C_2Z_c+G(B_2+A_2Z_c)]+A_1(C_2+GA_2)\right\} & \left\{C_1(B_2+A_2Z_c)+A_1A_2\right\} \end{Vmatrix} \cdot \begin{Vmatrix} E_2 \\ I_2 \end{Vmatrix}$$

Let $I_2 = 0$

$$\frac{E_1}{E_2} = \left\{A_1A_2 + A_1C_2Z_c + B_1C_2 + B_1A_2G + A_1B_2G + A_1A_2Z_cG\right\}$$

For this problem the long line equations will be more convenient. For this case, let θ be the propagation constant length and $x\theta$ be the propagation constant for the length x.

$$A_1 = \cosh x\theta \qquad B_2 = Z_0 \sinh(\ell - x)\theta$$

$$A_2 = \cosh(\ell - x)\theta \qquad C_1 = \frac{1}{Z_0} \sinh x\theta$$

$$B_1 = Z_0 \sinh x\theta \qquad C_2 = \frac{1}{Z_0} \sinh(\ell - x)\theta$$

By substitution and combining multiple angle formulas

$$\frac{E_1}{E_2} = \frac{1}{2}\left\{2\cosh\theta + Z_c\left[\frac{1}{Z_0}\sinh\ell\theta - \frac{1}{Z_0}\sinh(2x-\ell)\theta\right] + 2GZ_0\sinh\ell\theta + GZ_c\left[\cosh\ell\theta + \cosh(2x-\ell)\theta\right]\right\}$$

In order to idealize the solution, the restrictions imposed by the lossless line are assumed. No loss of generality is suffered, however.

$$Z_c = -JX_c$$

$$\cosh l\theta = \cos l\beta$$

$$\sinh l\theta = J\sin l\beta$$

Then

$$\frac{E_1}{E_2} = \frac{1}{2}\Big\{2\cos l\beta + \frac{X_c}{Z_0}\big[\sin l\beta - \sin(2x-l)\beta\big]$$

$$+ JG\big[2Z_0 - X_c\cos l\beta - X_c\cos(2x-l\beta)\big]\Big\}$$

$$= R + JQ$$

$$= \sqrt{R^2+Q^2}\quad \operatorname{cis}\left[\tan^{-1}\frac{Q}{R}\right]$$

Since Z_0 will normally be relatively large as compared to X_c, then R is not effected to a large extent by a variation in x, so that the maximum and minimum of $\tan^{-1}\frac{Q}{R}$ may be coincident with the maximum and minimum of Q.

$$\frac{\partial Q}{\partial X} = + X_c G\beta \sin(2x-l)\beta$$

$$= 0 \quad \text{when}$$

$$(2x-l)\beta = 0, \pi$$

It is assumed that $X_c, G, \beta \neq 0$

$$x_1 = \frac{\pi + l\beta}{2\beta}$$

$$x_2 = \frac{l}{2}$$

These values form a maximum or a minimum if the second derivative of Q is negative or positive.

$$\frac{\partial^2 Q}{\partial x^2} = -2X_c G\beta^2 \cos(2x - \ell)\beta$$

at x_1

$$= -2X_c G\beta^2$$

at x_2

$$\frac{\partial^2 Q}{\partial x^2} = +2X_c G\beta^2$$

Therefore, Q is a minimum at x_2 and a maximum at x_1.

Since, in the ordinary power transmission line, the line angle $\ell\beta$ is less than 90°, the point x_1 does not lie in the line. Hence, it is concluded that the position of the capcitor for the minimum effect is at either end of the line. (The maximum effect is considered to be the minimum voltage angle displacement.) A capacitor at the center of the line will provide a maximum effect.

COMPENSATION FOR TRANSFORMER REACTANCE

It may be desirous to compensate for the sending end transformer reactance when the series capacitor is at the receiving end of the line. For this problem the usual procedure is followed.

$$\begin{Vmatrix} E_1 \\ I_1 \end{Vmatrix} = \begin{Vmatrix} 1 & Z_t \\ 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} A & B \\ 0 & A \end{Vmatrix} \cdot \begin{Vmatrix} 1 & Z_c \\ 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} 1 & 0 \\ G & 1 \end{Vmatrix} \cdot \begin{Vmatrix} E_2 \\ I_2 \end{Vmatrix}$$

$$\begin{Vmatrix} E_1 \\ I_1 \end{Vmatrix} = \begin{Vmatrix} \{A+Z_tC+G[A(Z_t+Z_c)+CZ_tZ_c+B]\} & \{A(Z_t+Z_c)+CZ_tZ_c+B\} \\ C+G(A+CZ_c) & A+CZ_c \end{Vmatrix} \cdot \begin{Vmatrix} E_2 \\ I_2 \end{Vmatrix}$$

Then

$$\frac{E_1}{E_2} = A + Z_tC + G[A(Z_t+Z_c) + CZ_tZ_c + B]$$

and Z_c must be such that

$$A(Z_t+Z_c) + CZ_tZ_c + B = 0$$

$$Z_c = \frac{B - AZ_t}{A + CZ_t}$$

Using the equivalent pi relationships

$$X_c = \frac{X_t \cos B - Z_o \sin\beta}{\cos\beta - \frac{X_t}{Z_o}\sin\beta} \tag{7}$$

The increase in reactance of the capacitor due to the presence of the transformer is ΔX_c

$$\Delta X_c = \left[\frac{X_t \cos\beta - Z_o \sin\beta}{\cos\beta - \frac{X_t}{Z_o}\sin\beta}\right] - Z_o \tan\beta$$

$$\Delta X_c = \frac{X_t}{\cos\beta\left(\cos\beta - \frac{X_t}{Z_0}\sin\beta\right)} \tag{8}$$

From these results it appears that the series capacitor will compensate for only a limited amount of transformer reactance. The limiting condition being when the denominator of the above expression becomes zero.

For the nominal pi consideration

$$B = J\ell L\omega, \quad A = 1 - \ell^2 LC\omega^2$$

$$C = J\ell C\omega(2 - \ell^2 LC\omega^2)$$

$$X_c = \frac{X_t(1-\ell^2 LC\omega^2) - \ell L\omega}{(1-\ell^2 LC\omega^2) - \ell C\omega(2-\ell^2 LC\omega^2)X_t} \tag{9}$$

or

$$X_c = \frac{X_t A - |B|}{A - X_t|C|} \tag{10}$$

VOLTAGE AND CURRENT RELATIONS IN THE SERIES CAPACITOR

From the standpoint of this investigation, there were only two possible locations for a series capacitor in a transmission line. In consideration of faults on the transmission line, if the series capacitor were at the sending end of the line, and a fault occurred near that end, then the series capacitor would tend to neutralize the part of the line out to the fault and the remaining capacitive reactance would neutralize machine reactance. The resulting condition would produce very high short-circuit currents. Synchronous machines for power generation are constructed with a low short-circuit current ratio so that a short-circuit on the terminals of the machine will not produce excessive currents because of the machine reactance. If, however, part of the machine reactance were neutralized with a series capacitor, then the short-circuit current would become excessive.

With this in mind, then it would seem reasonable that the best place to install a series capacitor would be in such a position that if a fault occurred in the worst possible manner that the effect on the machine would not be greater than if a fault occurred on the terminals of the machine. If a generator is supplying a load, then the proper location would be at the receiving end of the line with the series capacitor compensating for everything back to the terminals of the generator. If, however, the series capacitor is to be used to reduce the line reactance between two generating stations, then one solution would be to have the capacitor in the center of the line compensating for one-half the line. If further compensation is desired, the line may be broken into a number of sections each of which is compensated. It must be necessary to arrange the sectioning such that for the worst possible fault condition that severe conditions will not be placed on either machine.

In calculating the voltage and current relations and line loss, it will be necessary to consider the resistance of the line. The value of series capacitors may be calculated by the approximate formulas, and then used in the equations for the exact behavior of the line. The exact value of series capacitors, found by considering resistance, does not differ within slide rule accuracy with the value found by the approximate formulas.

The matrix equations of voltage and current for the three cases mentioned will be calculated explicitly. For this work the long line formulas greatly expedite the process.

CASE I Series capacitor at the receiving end of the transmission line.

$$\begin{Vmatrix} E_1 \\ I_2 \end{Vmatrix} = \begin{Vmatrix} A & B \\ C & A \end{Vmatrix} \cdot \begin{Vmatrix} 1 & Z_c \\ 0 & I \end{Vmatrix} \cdot \begin{Vmatrix} E_2 \\ I_2 \end{Vmatrix}$$

$$= \begin{Vmatrix} A & \{B + AZ_c\} \\ C & \{A + CZ_c\} \end{Vmatrix} \begin{Vmatrix} E_2 \\ I_2 \end{Vmatrix}$$

$$A = \cosh\theta \quad = A_r + JA_J$$

$$B = Z_0 \sinh\theta \quad = B_r + JB_J$$

$$C = \frac{1}{Z_0}\sinh\theta \quad = C_r + JC_J$$

$$Z_c = -JX_c$$

Then

$$E_1 = (A_r + JA_J)E_2 + \left[(B_r + A_J X_c) + J(B_J - X_c A_r)\right] I_2 \tag{11}$$

$$I_1 = (C_r + JC_J)E_2 + \left[(A_r + C_J X_c) + J(A_J - X_c C_r)\right] I_2 \tag{12}$$

If a fault occurred at the receiving end, the steady state current at the sending end would be

$$I = E_1\left[\frac{(A_r + C_J X_c) + J(A_J - X_c C_r)}{(B_r + A_J X_c) + J(B_J - X_c A_r)}\right] \tag{13}$$

If $X_c = Z_0 \tan\beta$, then

$$I_1 \cong E_1 \left[\frac{A_r}{B_r}\right]$$

For the ordinary case, A_r is nearly unity and B_r is the architrave resistance of the equivalent pi. For the nominal pi consideration, it is the line resistance.

The voltage across the series capacitor as a function of power

$$P_2 = I_2 \hat{E}_2$$

$$|E_c| \quad \frac{|P_2|}{|E_2|} X_c \qquad (15)$$

The effect of the sending-end and the receiving-end transformer may easily be included in the above equations.

CASE II For a line with the series capacitor in the exact center, where A, B, and C are the line constants for one-half the line

$$\begin{Vmatrix} E_1 \\ I_1 \end{Vmatrix} = \begin{Vmatrix} A & B \\ C & A \end{Vmatrix} \cdot \begin{Vmatrix} 1 & Z_c \\ 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} A & B \\ C & A \end{Vmatrix} \cdot \begin{Vmatrix} E_2 \\ I_2 \end{Vmatrix}$$

$$= \begin{Vmatrix} A' & B' \\ C' & A' \end{Vmatrix} \cdot \begin{Vmatrix} E_2 \\ I_2 \end{Vmatrix}$$

Where

$$A' = \cosh 2\theta + \frac{1}{2}\frac{Z_c}{Z_0} \sinh 2\theta$$

$$B' = Z_0 \sinh 2\theta + \frac{1}{2} Z_c(\cosh 2\theta + 1)$$

$$C' = \frac{1}{Z_0} \sinh 2\theta + \frac{1}{2}\frac{Z_c}{Z_0^2}(\cosh 2\theta - 1)$$

Let E_3 be the voltage from the end of the series capacitor nearest the receiving end, and I_3 be the current, then

$$\begin{Vmatrix} E_3 \\ I_3 \end{Vmatrix} = \begin{Vmatrix} A & B \\ C & A \end{Vmatrix} \cdot \begin{Vmatrix} E_2 \\ I_2 \end{Vmatrix}$$

$$E_c = Z_c I_3$$

$$= Z_c\left[C E_2 + A\frac{P_2}{\hat{E}_2}\right] \qquad (16)$$

$$E_3 = AE_2 + B\frac{P_2}{\hat{E}_2} \qquad (17)$$

Assume that the line is free from resistance and that the series capacitor is compensating for one half the line.

$$A' = \frac{1}{2}\left[\cosh 2\theta + 1\right] = \frac{1}{2}\left[\cos 2\beta + 1\right] \qquad (18)$$

$$B' = \frac{1}{2} Z_0 \sinh 2\theta = J\frac{1}{2} Z_0 \sin 2\beta \qquad (19)$$

$$C' = \frac{1}{Z_0}\left[\frac{1}{2}\sinh 2\theta + \tanh\theta\right] = \frac{J}{Z_0}\left[\frac{1}{2}\sin 2\beta + \tan\beta\right] \qquad (20)$$

It is seen from equations 18, 19, and 20 that the resulting line behaves as a line of slightly less than one half the total length.

CASE III If it is desirous to break the line into a number of smaller sections and compensate for each section, the problem may be handled in the following manner: Assume that there are n sections all identical. Let A, B, and C be the line constants of a section.

$$\begin{Vmatrix} E_1 \\ I_1 \end{Vmatrix} = \begin{Vmatrix} A & B \\ C & A \end{Vmatrix} \cdot \begin{Vmatrix} 1 & Z_c \\ 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} A & B \\ C & A \end{Vmatrix} \cdot \begin{Vmatrix} 1 & Z_c \\ 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} A & B \\ C & A \end{Vmatrix} \cdot \begin{Vmatrix} E_2 \\ I_2 \end{Vmatrix}$$

If the number of sections is large, they may be grouped and multiplied by Sylvester's theorem or by the following theorem.

$$\begin{Vmatrix} r\cosh\phi & \sinh\phi \\ r^2\sinh\phi & r\cosh\phi \end{Vmatrix}^q = \begin{Vmatrix} r^q\cosh q\phi & r^{q-1}\sinh q\phi \\ r^{q+1}\sinh q\phi & r^q\cosh q\phi \end{Vmatrix}$$

EFFECT OF CHANGE OF FREQUENCY ON VOLTAGE DISPLACEMENT ANGLE

If a transmission line is used as a tie-line between two generating stations, and a fault occurs on the system, the disturbance may be so great as to cause a reduction of the machine speed, and hence, a reduction in frequency. Since the capacitive reactance increases and the inductive reactance decreases with a decrease in frequency, then the line angle may decrease and the impedance connection between the two stations may behave as pure capacitive reactance. Under this condition, the system will be very unstable, as will be shown later. The problem is now to discover the change in line angle as a function of the change in frequency. For a given application to a line, then, knowledge of the possible change in frequency is necessary so that the capacitor may be chosen of such a value that under faulty conditions, the shift of line angle is of no serious consequence.

Let the change in frequency be designated by $\Delta\omega$ and the change in phase position of the sending end voltage with respect to the receiving end voltage be $\Delta\phi$.

Then

$$\begin{Vmatrix} E_1 \\ I_1 \end{Vmatrix} = \begin{Vmatrix} A & B \\ C & A \end{Vmatrix} \cdot \begin{Vmatrix} E_2 \\ I_2 \end{Vmatrix}$$

$$\Delta \begin{Vmatrix} E_1 \\ I_1 \end{Vmatrix} = \Delta w \begin{Vmatrix} \frac{dA}{dw} & \frac{dB}{dw} \\ \frac{dc}{dw} & \frac{dA}{dw} \end{Vmatrix} \cdot \begin{Vmatrix} E_2 \\ I_2 \end{Vmatrix}$$

$$\Delta E_1 = \Delta w \frac{dA}{dw} E_2 + \Delta w \frac{dB}{dw} I_2$$

Where ΔE_1 is the change in sending end voltage to maintain E_2 and I_2 with the given change in frequency.

$$\frac{\Delta E_1}{\Delta w} = \left| \frac{\Delta E_1}{\Delta w} \right| \operatorname{cis} \left\{ \tan^{-1} \frac{\operatorname{Imag}\left[\frac{dA}{dw} E_2 + \frac{dB}{dw} I_2 \right]}{\operatorname{real}\left[\frac{dA}{dw} E_2 + \frac{dB}{dw} I_2 \right]} \right\} \quad (21)$$

The change in voltage $\left| \frac{\Delta E_1}{\Delta w} \right|$ is of little consequence. The change in phase angle is

$$\Delta \phi = \tan^{-1} \left\{ \Delta_w \frac{\operatorname{Imag}\left[\frac{dA}{dw} E_2 + \frac{dB}{dw} I_2 \right]}{\operatorname{real}\left[\frac{dA}{dw} E_2 + \frac{dB}{dw} I_2 \right]} \right\} \quad (22)$$

and if Δw is small

$$\Delta \phi \cong \Delta w \frac{\operatorname{Imag}\left[\frac{dA}{dw} E_2 + \frac{dB}{dw} I_2 \right]}{\operatorname{real}\left[\frac{dA}{dw} E_2 + \frac{dB}{dw} I_2 \right]} \quad (23)$$

As an example, consider a tie-line which has a series capacitor in the exact center, compensating for one-half the line in one direction. Assume that the load is of unity power factor, and the line resistance is neglected.

Then

$$A = \frac{1}{2}(\cos 2\alpha\omega + 1)$$

$$B = \frac{JZ_0}{2} \sin 2\alpha\omega$$

$$C = J\frac{1}{Z_0}\left[\frac{1}{2}\sin 2\omega + \tan\alpha\omega\right]$$

Where $\alpha = l\sqrt{LC}$, l being one-half the length of the line, L and C being the inductance and capacitance per unit length.

$$\frac{dA}{d\omega} = -\left[\alpha \sin 2\alpha\omega + \frac{1}{2\omega}(1-\cos 2\alpha\omega) + \alpha \tan\alpha\omega\right]$$

$$\frac{dB}{d\omega} = J\left[Z_0\alpha\cos 2\alpha\omega + \frac{1}{2\omega}\sin 2\alpha\omega + \lambda Z_0\right]$$

$$\frac{dC}{d\omega} = J\left[\frac{\alpha}{Z_0}(\cos 2\alpha\omega + 1) + \frac{1}{Z_0\omega}\left(\frac{1}{2}\sin 2\alpha\omega + \tan\alpha\omega\right)\right]$$

$$\Delta\phi \cong +\Delta\omega \frac{\frac{dB}{d\omega} I_2}{\frac{dA}{d\omega} E_2}$$

$$\Delta\phi \cong -\Delta_w \frac{\left[Z_o \alpha \cos 2\alpha w + \frac{1}{2w} \sin 2\alpha w + 2 Z_o\right] I_2}{\left[\alpha \sin 2\alpha w + \frac{1}{2w}(1-\cos 2\alpha w) + \alpha \tan \alpha w\right] E_2} \tag{24}$$

And very generally

$$\Delta\phi \cong -\Delta w \, Z_o \frac{I_2}{E_2} \tag{25}$$

It was hoped that from this study that a value of series capacitive reactance could be found such that the phase shift could be minimized, but the resulting function permitted a minimum for inductive reactance only.

Chapter III

TRANSIENT BEHAVIOR OF SERIES CAPACITORS

For the consideration of this part of the problem only a few of the special cases will be considered. There are innumerable possibilities which can be considered, but this thesis is very limited in scope.

The first type of problem to be considered is that of charging an infinite line with a series capacitor. There are two extreme possibilities; closing the breaker at the zero point of the voltage wave and at the maximum point of the voltage wave. Each of these are introduced into the problem simultaneously.

Notation:

$$G(p) = \mathcal{M}\left[f(t)\right], \quad \text{(Read "mate of")}$$

$$f(t) = \mathcal{M}\left[G(p)\right]$$

where

$$G(p) = \int_{\infty}^{\infty} f(t)\ \exp(-pt)\ dt$$

$$f(t) = \frac{1}{2\pi J}\int_{h-J\infty}^{h+J\infty} G(p)\ \exp(pt)\ dp$$

h must be chosen so that the path of integration lies to the right of the singularities of G (p).

For any physical system where the principle of superposition hold, then; where C(t) is the cause, E(t) is the effect, Z(p) is the steady state impedance, and $P_o = J\omega$ = the imaginary radian frequency,

$$E(t) = \partial\eta \left\{ \frac{\partial\eta[C(t)]}{Z(p)} \right\}$$

$S_o(t)$ is the unit impulse function

$S_{-1}(t)$ is the unit step, equal to Heaviside's unit function

$\mathrm{Cis}(\omega_o t)\ S_{-1}$ equals $\cos(\omega_o t)\ S_{-1}(t)$ $J \sin(\omega_o t)\ S_{-1}(t)$

$$\partial\eta \quad S_o(t) = 1$$

$$\partial\eta \quad S_{-1}(t) = \frac{1}{P}$$

$$\partial\eta \quad \mathrm{cis}(\omega_o t)\ S_{-1}(t) = \frac{1}{P-P_o}$$

Consider a capacitor C_1 in a series with an infinite transmission line consisting of only inductance and capacitance per unit length of characteristic impedance Z_o (which is pure resistance).

Then for steady state

$$E_{c_1} = \frac{E_1}{Z_o CP + 1}$$

For E_c as a function of time where $E_1 = \text{cis}\,(p_o t)\, S_{-1}(t)$

$$E_{c_1} = \frac{1}{Z_o C_1}\, 2\eta\, \frac{1}{(p-p_o)(p-\frac{1}{Z_o C_1})}$$

$$= \frac{1}{(1+C_1 Z_o p_o)}\left[\text{cis}\,(\omega_o t) - \exp\,(\frac{t}{Z_o C_1})\right] \tag{26}$$

The real part of this expression is the response to $\cos(\omega_o t)\, S_{-1}(t)$ and the imaginary part is the response to $\sin(\omega_o t)\, S_{-1}(t)$.

$$\text{Real}\,(E_c) = \frac{\cos(\omega t - \phi)}{\sqrt{1+(C_1\omega_o Z_o)^2}} - \frac{\exp(-\frac{t}{Z_o C_1})}{\left[1+(C_1\omega_o Z_o)^2\right]} \tag{27}$$

$$\phi = \tan^{-1}(C_1 Z_o \omega_o)$$

$$\text{Imag}\,(E_c) = \frac{\sin(\omega t - \phi)}{\sqrt{1+(C_1\omega_o Z_o)^2}} + \frac{\exp(-\frac{t}{Z_o C_1})}{\left[1+(C_1\omega_o Z_o)^2\right]} \tag{28}$$

It would be desirous to have an explicit value of the maximum value of this function as a function of C_1 , but this is impossible since the derivative set equal to zero is a transcendental equation. For C_1 equal to 20 mfd, Z_o 400 ohm, and ω_o 377 (60 cycles), the maximum value of Imag(E_c) is 0.4 volts, and the steady state value is 0.3 volts.

THE RESPONSE OF A FINITE LINE TO A UNIT IMPULSE

The problem to be investigated next is that of the voltage across a series capacitor when the line is charged with the unit impulse, $S_o(t)$. The direct application of the result to be obtained from this solution to a transmission line is rather obscure, but the knowledge of the physical system and of the result may be used to interpret the results of succeeding problems.

Consider a capacitance C_1 in series with a transmission line consisting of inductance and capacitance per unit length. Assume that the line is open on the receiving end.

Let: (The notation in this and succeeding parts may be different than in preceding chapters.)

Z_o be the characteristic impedance $= \sqrt{\frac{L}{C}}$

α be the length of time for a wave to travel the length of the line $= \ell\sqrt{LC}$

p be the imaginary radian frequency.

Then, for steady state conditions

$$E_c = \frac{\sinh \alpha p E_1}{\sinh \alpha p + Z_o C_1 p \cosh \alpha p} \tag{29}$$

The response to the unit impulse $S_o(t)$ is

$$E_c = 2\eta \frac{\sinh \alpha p}{\sinh \alpha p + Z_o C_1 p \cosh \alpha p} \tag{30}$$

Let $\gamma = \frac{1}{Z_o C_1}$ and

$$\Gamma = \frac{\sinh \alpha p}{\sinh \alpha p + \frac{p}{\gamma} \cosh \alpha p} \tag{31}$$

By long division Γ may be expanded into the following series which is seen to be convergent for all values of p to the right of the imaginary axis in the complex plane. (The value of γ is always real and positive.)

$$\Gamma = \gamma \left[\frac{1}{p+\gamma} + 2 \sum_{n=1}^{\infty} \frac{(-1)^n p (p-\gamma)^{n-1} \exp(-2n\alpha p)}{(p+\gamma)^{n+1}} \right] \tag{32}$$

In order to find the mate of Γ it will be necessary to find the value of an integral of the type

$$\phi = \frac{1}{2\pi J} \int_{Br} \frac{p (p-\gamma)^{n-1} \exp(pt)}{(p+\gamma)^{n+1}} dp \tag{33}$$

= the sum of residues of ϕ in the finite plane. It may be shown easily by Cauchy's residue theorem that

$$\frac{1}{2\pi J} \int_{Br} \frac{\exp(pt)}{(p+\gamma)^k} dp = \frac{t^{k-1} \exp(-\gamma t)}{(k-1)!} S_{-1}(t) \tag{34}$$

By expanding (30) in a Laurent series and applying (31)

$$\phi = \sum_{q=1}^{n} \frac{(n-1)!\,(-2r)^{q-1}\,(q-rt)\,t^{q-1} \exp(-rt)}{(q-1)!\,(n-q)!\,q!}\, S_{-1}(t) \tag{35}$$

It may also be shown by Cauchy's residue theorem that if

$$f(t) = 2\eta\ G(p)$$

then (36)

$$f(t-g) = 2\eta\ G(p) \exp(-gp)$$

This is the integral equation form of Heaviside's shifting theorem.

Denoting

$$M\left[n, r, (t)\right] = \phi$$

then by (30), (31), (32), (34), (35), and (36)

$$E_c = \frac{1}{Z_0 C_1} \exp\left(-\frac{t}{Z_0 C_1}\right) + \frac{2}{Z_0 C_1} \sum_{n=1}^{k} (-1)^n M\left[n, \frac{1}{Z_0 C}, (t - 2n\alpha)\right] \tag{37}$$

for $2kd \leqq t < 2(k+1)\alpha$

Since $I_1 = \frac{d}{dt} E_c$ and $\frac{d}{dt} S_{-1}(t) = S_0(t)$ and referring to (35)

$$I_1 = \frac{1}{Z_0} S_0(t) + \frac{2(-1)^n}{Z_0} S_0(t - 2n\alpha)$$

$$+ \frac{1}{Z_0} e^{-\frac{t}{RC_1}} + 2 \sum_{n=1}^{k} (-1)^n N\left[n, r, (t - 2n\alpha)\right] \tag{38}$$

for

$$2k\alpha \leqq t < 2(k+1)\alpha$$

where

$$N\left[n,r,t\right] = S_{-1}(t)\sum_{q=1}^{n}\frac{(n-1)!(-2r)^{q-1}\exp(-\gamma t)\left[q(q-1)t^{q-2}-2q r t^{q-1}+r^{2}t^{q}\right]}{(q-1)!\,(n-q)!\,q!} \tag{39}$$

The line under question is a distortionless line, since RC = LG, meaning that any wave is propagated along the line without distortion. The magnitude of the current impulse is changed by the reflection from the closed end, which is the sending end. The change in magnitude of the voltage across the capacitor does not influence the order of magnitude of the current pulse, $S_0(t-2n\alpha)$. The first current pulse is unity, and each succeeding pulse is twice unity because of the reflection from the closed end. Each pulse that is returned to the sending end is reversed in sign over the preceeding one. There are discontinuities in the voltage across the capacitor since

$$E_c = \frac{1}{C}\int I_1\,dt$$

and

$$\int_{-\infty}^{\infty} S_0(t)\,dt = 1$$

But each time the unit impulse is reversed in sign changing the sign of the voltage across the capacitor, there are other voltages across the capacitor because of reflections of finite magnitude, but these voltages are continuous. Therefore, from the consideration of the physics of this problem, without going into the mathematical analysis, E_c is a bounded function with a maximum bound of $\frac{2}{Z_0 C}$ and the maximum bound of

$$\sum_{n=1}^{k} (-1)^n M\left[n, r, (t - 2n\alpha)\right] \text{ is } 1. \tag{40}$$

By the integration of equation (40) with respect to time from zero to t, the response of the line to the unit step may be found.

In a general way, equation (40) indicates the response to a lightning impulse on the line. For the purposes of analysis, the lightning impulse may be thought of as of very short duration, but of high value.

THE RESPONSE OF A SERIES CAPACITOR TO A FAULT ON THE SYSTEM

The problem to be considered now is that of determining the voltage across a series capacitor which is on the receiving end of a transmission line as a function of time immediately after a fault occurs on the load side of the capacitor. By applying the theorem of superposition, a voltage is applied at the point of short circuit equal and opposite to the voltage that existed before the short circuit. The total effect on the capacitor is the sum of the voltage caused by the application of the new voltage and the voltage that existed before the short circuit. In consideration of the transient response it will be assumed that the line is being supplied by an infinite bus, so that this part of the analysis narrows to the transient solution of a line shorted at the receiving end and being charged with a unit cissoid, cis($w_o t$), in series with a capacitor C_1.

For steady state conditions

$$E_c = \frac{\cosh \alpha p E_1}{Z_o C_p \sinh \alpha p + \cosh \alpha p} \tag{40}$$

and for E_c as a function of time

$$E_c = \frac{\cosh \alpha p}{(Z_o C_p \sinh \alpha p + \cosh \alpha p)(p - p_o)} \tag{41}$$

Let

$$\Gamma = \frac{\cosh \alpha p}{Z_o C_p \sinh \alpha p + \cosh \alpha p} \tag{42}$$

$$= \gamma \left[\frac{1}{p+\gamma} + 2 \sum_{n=1}^{\infty} \frac{p\,(p-\gamma)^{n-1}}{(p-\gamma)^{n+1}} \quad \exp(-2n\gamma p) \right] \tag{43}$$

$$E_c = 2\eta \frac{\Gamma}{p-p_o}$$

Consider E_c as made up of two parts, an oscillating component and a component due to the reflection of the transient waves. The mate of the oscillating component may be obtained by expanding each term of Γ in a Laurent series. The oscillating terms, then, are the residues of the pole p_o.

$$\frac{p\ (p-\gamma)^{n-1}}{(p-p_o)(p+\gamma)^{n+1}} = \frac{p_o\ (p_o-\gamma)^{n-1}}{(p-p_o)(p_o+\gamma)^{n+1}} + \left\{ \begin{array}{l} \text{other terms} \\ \text{which are ana-} \\ \text{lytic at } p_o. \end{array} \right\}$$

$$\frac{(p_o-\gamma)^{n-1}}{(p_o+\gamma)^{n+1}} = \frac{(-1)^n \exp(-J\,2n\phi)}{p_o^{\,2} - \gamma^2}$$

where

$$\phi = \tan \frac{w_o}{\gamma}$$

The oscillating component of $E_c = E_{cp_o}$ is then,

$$E_{cP_0} = \frac{\gamma e^{P_0 t}}{P_0+\gamma} + 2n\left[\frac{2\gamma}{P_0^2-\gamma^2}\sum_{n=1}^{\infty}\frac{(-1)^n \exp(-J2n\phi)\exp(-2n\alpha)}{P-P_0}\right] \tag{44}$$

Since the result is a geometric series, it may be written in the form

$$E_{cP_0} = \frac{\gamma e^{P_0 t}}{P_0+\gamma} + \frac{2P_0\,\gamma \exp[P_0 t - 2(J\phi + P_0\alpha)]}{P_0^2-\gamma^2} \times$$

$$\left[\frac{1-(-1)^k \exp[-2k(J\phi+P_0\alpha)]}{1+\exp[-2(J\phi+P_0\alpha)]}\right] \tag{45}$$

for $2k\alpha \leqq t < 2(k+1)\alpha$

It is noted that the denominator of the quantity in brackets is zero when $\phi + \omega_0\alpha = \frac{\pi}{2}$. By the theory of indeterminate forms, when this is true, the value of the quantity in brackets is equal to n. Although this function appears to be discontinuous, the total voltage across the capacitor is continuous since this is only part of the function. It is noted that when the size of a series capacitor is determined to compensate for the entire line, that the voltage under short circuit rises approximately linearly with time. For this case

$$\phi = \tan^{-1} \frac{\omega_0}{\gamma} = \frac{\pi}{2} - \omega_0 \alpha$$

or

$$X_c = \frac{1}{\omega_0 C_1} = Z_0 \tan \omega_0 \alpha \qquad (46)$$

and $\omega_0 \alpha$ is equal β in the previous discussion of steady state behavior.

It is not necessary that $\phi + \omega_0 \alpha$ be limited to $\frac{\pi}{2}$, for if it were $\frac{\pi}{2}(m+1)$, (m an integer), then the same condition would exist. This discussion would also apply to high frequency transmission lines and radiators. It may be observed from these derivations the reason that a series capacitor in a radiating antenna shortens the antenna, and by a similar derivation why an inductance lengthens it.

Although the other part of E_c is not pertinent to this discussion, it is of interest to record it.

Let

$$Q\left[n, \gamma, p_0, t\right] = \sum_{q=0}^{n} \frac{(n-1)!\, t^{q-1} (q - \gamma t) \exp(-\gamma t)}{q!} \times$$

$$\sum_{J=0}^{n-q} \frac{(-1)^{q-J} (2\gamma)^{q+J-1}}{(n-q-J)!\,(q+J-1)!\,(p_0+\gamma)^{J+1}} \qquad (47)$$

$$\text{Then } E_{cr} = 2r \sum_{n=1}^{k} Q\left[n, r, p_0 (t - 2n\alpha)\right] - \frac{r \exp(-rt)}{p_0 + r} \tag{48}$$

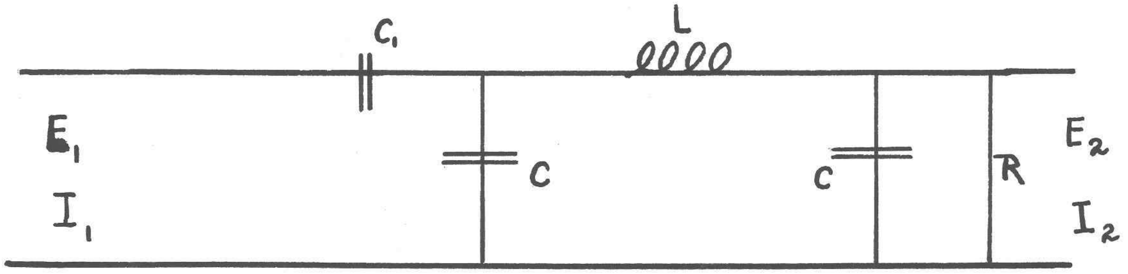
By applying a similar type of reasoning to this problem as to the previous problem, it is seen that this function must be bounded for $\phi + \omega\alpha \neq \frac{\pi}{2}$ since, in this case, the current pulses are of finite value, and hence, the voltage across the series capacitor must be continuous. It is noted from (43), since it does not possess the alternating sign, that if a unit impulse had been impressed on a line which was closed at the receiving end, that the voltage across the series capacitor would have increased indefinitely with time. In the case of an ordinary transmission line, the resistance would be sufficient to provide dissipation for these transient exponential waves. To the knowledge of the writer very little work has been done on the mathematical behavior of the M, N, and Q functions. Functions of this nature occur frequently in transient studies.

It is noted that the initial transient before the return of the first reflection is the same as that of an infinite line.

Chapter IV

TRANSIENT BEHAVIOR OF THE LUMPED CONSTANT EQUIVALENT OF A TRANSMISSION LINE AND SERIES CAPACITOR

From the standpoint of interest, it may be well to develop the transient solution for the lumped constant representation of the transmission line. Assume that a fault is to occur on the load side of the series capacitor. Let it be required to find the voltage across the series capacitor as a function of time. By the theorem of superposition, the effect will be the same as to consider the response of the system to an applied unit cissoid. Part of the notation in this section is different from that of preceding sections.



The resistance of the short circuit is considered for the purpose of analysis, but for the final equation, R is placed equal to zero.

$$\begin{Vmatrix} E_1 \\ I_1 \end{Vmatrix} = \begin{Vmatrix} 1 & \frac{1}{C_1 p} \\ 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} 1 & 0 \\ Cp & 1 \end{Vmatrix} \cdot \begin{Vmatrix} 1 & Lp \\ 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} 1 & 0 \\ \frac{1}{R} & 1 \end{Vmatrix} \cdot \begin{Vmatrix} E_2 \\ I_2 \end{Vmatrix}$$

$$\begin{Vmatrix} \left\{1 + \frac{Lp}{R} + \frac{1}{C_1 p}\left[\frac{1}{R} + Cp\left(1+\frac{Lp}{R}\right)\right]\right\} & \left\{Lp + \frac{1}{C_1 p}\left[1 + LCp^2\right]\right\} \\ \left\{\frac{1}{R} + Cp\left(1 + \frac{Lp}{R}\right)\right\} & \left\{1 + LCp^2\right\} \end{Vmatrix} \quad (49)$$

For R equals zero for steady state behavior.

$$E_{c_1} = \frac{(LCp^2 + 1)\ E_1}{L(C_1 + C)p^2 + 1} \quad (50)$$

Then for E_{c_1} as a function of time in response to the unit cissoid

$$E_{c_1} = \frac{1}{L(C+C_1)} \quad 2\eta \frac{1}{(p-p_0)(p-J\lambda)(p+J\lambda)} \quad (51)$$

where

$$\lambda = \sqrt{\frac{1}{L(C+C_1)}}$$

$$E_{c_1} = \frac{\lambda^2}{(p_0^2+\lambda^2)}\Bigg[(1 + LCp_0^2)\exp(p_0 t)$$

$$+ (LC\lambda^2 - 1)\cos\lambda t$$

$$\frac{+ J(1-LC\lambda^2)\,p_0 \sin\lambda t}{\lambda}\Bigg] \quad (52)$$

For a value of λ such that $\lambda = \omega_0$ (since $p_0 = J\omega_0$), the denominator of the coefficient of the above expression will approach zero, and the oscillations will build up indefinitely of the form $\exp(p_0 t)$ (as can be shown from the theory of indeterminant forms).

$$\lambda^2 = \frac{1}{L(C + C_1)}$$

or

$$X_{c_1} = \frac{L\omega_0}{1 - LC\omega_0^2} \tag{53}$$

Therefore, for a line which is compensated for entirely by a series capcitor, the transient analysis on the basis of a lumped constant network shows that the voltage across the series capacitor approaches an indefinitely high value. This may also have been noticed from the steady state equations, since the input admittance is infinite.

It is noted from the equations that the transient behavior of a lumped constant network is very different than that of a distributed constant network. In the analysis of a lumped constant network, the finite velocity of propagation is neglected. However, from the standpoint of a steady state analysis, the lumped constant network is quite adequate.

Chapter V

MACHINE TRANSIENTS

In connection with the transients of the transmission line, there must be some consideration given to the machine transients. While the machine transients are too slow to effect the line in general, particularly with heavy machines, the effect of a purely capacity reactance coupling between machines may produce violent machine transients. This situation may come about if a fault occurred in such a manner as to eliminate part of the system, and the remaining part was left to carry the load; hence, a reduction in frequency. If a series capacitor were in the line, and for this situation it was overcompensating for the line and machine impedances, then the restoring torque on the machine for a displacement is the negative of the displacement. The differential equation of the machine is represented as follows, where δ is the displacement for the position of equilibrium and k^2 is a constant representing the moment of inertia, the total impedance, the terminal and excitation voltage.

$$\frac{d^2\delta}{dt^2} - k^2 \sin\delta = 0 \qquad (54)$$

This equation neglects a number of factors, one of them being the non-saliency of the machine, but that effect is small at small angles. The other effect is the damping due to the damper winding and other effects. The solution to this equation will merely indicate a tendency. There has been considerable work done on the numerical integration of the damped equation. That equation has no explicit solution in terms of elementary functions. (Refer to the bibliography.)

Equation (54) admits a solution with the following terminal conditions.

$$\left.\begin{array}{l} \delta = \delta_0 \\ \dfrac{d\delta}{dt} = 0 \end{array}\right\} t = 0$$

This is the same as saying that δ_0 is the displacement from the normal and that its velocity at that point is zero at time equal to zero.

$$t = \frac{-\sin^2 \frac{\delta_0}{2}}{2k} \int_0^{2\delta} \frac{d\phi}{\sqrt{1 - \cos^2 \frac{\delta}{2} \sin^2 \phi}} \qquad (55)$$

This is an elliptic integral of the first kind, known as Legendre's normal form.

Rewriting, where sn is the abbreviation for sine-amplitude

$$sn\left[2k \csc 2 \frac{\delta_0}{2} t\right] = -\sin 2\delta \tag{56}$$

The sine-amplitude function is a periodic function, the maximum value of which (as a function of a real variable) is a function of the modulus which is in this case $\cos \frac{2\delta_0}{2}$. When the modulus is equal to unity, the sine-amplitude function is the sine function. When the modulus is less than unity, the maximum value of the oscillation is less than unity. It is easily seen, then, for δ as a function of t will be an oscillating function if S_0 does not equal zero or π. The mechanical analogy of this problem is an inverted pendulum with δ measured from the vertical upward position.

If the connection to the machine had been inductive reactance or resistance, the sign in the original differential equation would have been reversed, and the mechanical analogy would then be an ordinary pendulum.

SUMMARY AND CONCLUSION

It has been shown that it is possible to find a size of series capacitor to compensate for a given length of transmission line so that the effect under steady state conditions is that of a pure resistance coupling between the sending and the receiving end. A series capacitor located at the exact center of a transmission line will provide greater compensation than at any other position. It is possible for a series capacitor located at one end of a transmission line to compensate for the inductive reactance of a transformer located at the other end. There is a limited amount of reactance which may be compensated for, but this condition is never approached in ordinary power transmission circuits.

The effect of placing the series capacitor at the receiving end of the line is just the same as that of placing it at the sending end so far as the phase angle of the sending and receiving end voltage is concerned. There is a slight difference as to the magnitudes of the terminal voltages, and the behavior of the system during fault conditions. It is not advisable to place a series capacitor at the sending end of the line, for, if a short-circuit fault occurred near the sending end, the capcitor would compensate for some of the machine reactance, causing very high fault currents. The main purpose of series capacitors is to reduce the phase angle between the sending and receiving end voltages so that the power transmitted over the system may be increased to a more economic value.

The effect of frequency has been pointed out and applied to a particular example. It is necessary in a particular installation to determine the possible reduction of frequency of the system during fault conditions, since, with a series capacitor in the line, the phase of the receiving ~~sending~~ end voltage will advance with respect to the sending ~~receiving~~ end voltage with a reduction in frequency.

A method of analysis has been set up so that if a line is to be sectionalized and a number of series capacitors distributed throughout the length of the line, the calculations may be carried out.

The voltage across the series capacitor has been derived as a function of time for the application of a unit impulse to the transmission line. This analysis may be used for the treatment of a lightning impulse on the line. The voltage across the series capacitor has been derived as a function of time after a short-circuit fault has occurred on the load side of the capacitor. This result is valuable in determining the voltage rating of a capacitor is certain switching operations are to occur on the line. If a line is sectionalized and a fault occurs in one section, and it is necessary for the capacitors to remain in the circuit until the breakers have had time to clear the fault, then it is necessary to know the voltage across the capacitor as a function of time.

In the analysis of the transmission line, ideal conditions were assumed, such as, e.g., the omission of resistance in the line. In the first place, the effect of the resistance is quite small, resulting in only a minor distortion of the transient wave. In the second place, the inclusion of resistance would have produced an answer too unwieldy to be of much value. The voltage across the series capacitor is a pessimistic value, since the analysis assumes that there is no energy loss of the reflected wave at the terminals of the line.

The behavior of a lumped constant system and a distributed constant system exhibit a noticeable difference. This difference is revealed in the analysis of the lumped constant network.

An analysis of the machine differential equation was made with a pure capacitive reactance coupling between a synchronous generator and an infinite bus to show the effect of an over-compensated line on the machine. It is noted from the resulting equation that there is only one point of equilibrium and that the entire system is unstable. If the rotor of the machine is displaced from the position of equilibrium, it will oscillate with respect to a rotating axis. Since the original differential equation omitted the effect of damper windings on the machine, the resulting solution will indicate only a tendency.

This thesis has covered only a few out of a multitude of problems that may confront an engineer in the installation of a series capacitor in a transmission line.

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APPENDIX I

NOTATION AND SYMBOLS

$$A = \cosh\theta = A_r + JA_J$$

$$B = Z_0 \sinh\theta = B_r + JB_J$$

$$C = \frac{1}{Z_0}\sinh\theta = C_r + JC_J$$

$\theta = l\sqrt{ZY}$ = Propagation constant

l = Length of line

$Z = R + Lp$ per unit length

$Y = G + Cp$ per unit length

$Z_0 = \sqrt{\frac{Z}{Y}}$ = Characteristic impedance

$p = J\omega$ = Imaginary radian frequency

$p_0 = J377$ for a 60 cycle per second line

$\omega = 2\pi f$

$P = P_r + JP_J$ = Complex power

δ = Voltage displacement angle

$\beta = \mathrm{Imag}\ (\theta)$ = Phase constant

$\alpha = l\sqrt{LC}$ = Time for a wave group to the length of the line (for a line of pure L and C).

$\left.\begin{matrix} E_1 \\ I_1 \\ P_1 \end{matrix}\right\}$ = Sending end quantities

$\left.\begin{matrix} E_2 \\ I_2 \\ P_2 \end{matrix}\right\}$ = Receiving end quantities

$\hat{E}$ = Complex conjugate of E

$\mathrm{Imag}\,[p]$ = Imaginary part of P

$\mathrm{Real}\,[p]$ = Real part of P

$|P|$ = Absolute magnitude of P

C_1 = Capacitance of series capacitor

$$Z_c = JX_c$$

$$X_c = \frac{1}{wC_1}$$

$$\exp(x) = \varepsilon^{x}$$

$$\mathrm{cis}\,(x) = \varepsilon^{Jx}$$

$\mathfrak{M}[\ \ ]$ is read "mate of" if two functions satisfy the following integral equation:

$$f(t) = \frac{1}{2\pi J} \int_{B_r} G(p) \operatorname{cis}(pt)\, dp$$

and inverting

$$G(p) = \int_{-\infty}^{\infty} f(t) \operatorname{cis}(-pt)\, dt$$

then

$$f(t) = \mathfrak{M}\left[G(p)\right]$$

and

$$G(p) = \mathfrak{M}\left[f(t)\right]$$

Br is the Bromwich contour, which is a path parallel to the imaginary axis extending from $-J\infty$ to $+J\infty$ passing the singularities of G(p) to the right.

$$S_{-1}(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geqq 0 \end{cases}$$ Unit step, or Heaviside's unit function.

$$S_0(t) = \begin{cases} 0, & t \neq 0 \\ \infty & t = 0 \end{cases}$$ Unit impulse