

AN ABSTRACT OF THE THESIS OF

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Abstract approved :

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The moulding and millwork industry consumes large quantities of shop grade lumber which is becoming less plentiful and of a lower quality. This fact has compelled industry to explore the use of new technology to increase product yield. One prospect for increasing yield is the application of thin-kerf sawing technology. This research investigated the effect of rip-kerf width on 5/4 Ponderosa pine cutting stock yield by calculating the yields of two hundred and fifty boards in each of three lumber grades. Each board's yield was calculated using computer sawing simulation for each combination of nine rip-kerf widths, from 0.050 to 0.250 inches, and four cutting bills. Analysis of variance and regression techniques were used to determine the relationships between rip-kerf widths, cutting bills and percent yield. The results showed that reducing rip-kerf width by 0.050 inches will increase yield from a half to one percent, depending on the cutting bill, and that the narrower the maximum cutting bill width, and the more

numerous and varied the other widths, the greater the yield increase. The results also indicated that the rate at which yield increases with reductions in rip-kerf width is the same for a specific cutting bill regardless which shop lumber grade is being sawn.

The Effect of Rip-Kerf Width
on 5/4 Ponderosa Pine Cutting Stock Yield

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**THE EFFECT OF RIP-KERF WIDTH ON
5/4 PONDEROSA PINE CUTTING STOCK YIELD**

1. INTRODUCTION

A. Background

In secondary lumber manufacturing processes such as those found in the moulding and millwork industry, boards are cut into smaller pieces for use as parts in door and window units, as well as in mouldings. The moulding and millwork industry consumes nearly 1.3 billion board feet of Ponderosa pine (Pinus ponderosa Laws.) annually (McDonald et al.; 1981, 1983). Since the Ponderosa pine resource is changing to smaller trees, the industry faces the problem of maintaining production from smaller and generally lower quality lumber. This situation makes lumber purchasing and lumber processing decisions extremely critical. This has encouraged studies of the factors influencing a rough mill's efficiency and profitability, such as raw material quality, sawing methods, and kerf width.

B. Objective

This study was conducted to determine the effect of varying rip-kerf width on cutting stock yield for different grades of 5/4 Ponderosa pine lumber. It was also designed to explore the relationship between cutting stock yield and cutting bill structure.

2. LITERATURE REVIEW

A. The Moulding and Millwork Industry

The moulding and millwork industry remanufactures lumber into a wide range of products. Millwork is a generic term for any building material consisting of finished wood manufactured in millwork plants. It comprises such items as doors and windows, as well as their frames, blinds, porch work, mantels, panel work, stair parts, and mouldings which are wood strips, commonly with a decorative cross-sectional profile, used as trim for concealing construction joints (William, 1984). The millwork process generally starts with shop grade lumber of ponderosa pine, sugar pine, white fir, douglas fir, or western hemlock. However, Gevaart (1961) indicates that the millwork industry prefers ponderosa pine when producing products with one or more clear faces and edges.

The industry consumes approximately 1.3 billion board feet of ponderosa pine annually (McDonald et al.; 1981, 1983). The Western Wood Product Association (WWPA) reports that in 1985 by percentage of species volume, ponderosa pine was the major species used for factory lumber with 35.63 percent of its volume being used for this industry (Sherrill, 1987).

Because the supply of large old-growth ponderosa pine trees is dwindling, smaller trees that characteristically produce lower quality lumber will be supplying the moulding

and millwork industry in the future. This factor, coupled with the fact that raw material cost is such a significant factor in millwork manufacturing (Sherrill, 1987), makes recovering maximum cutting stock yield an industry necessity.

The millwork industry typically uses a three-staged, rip-first operation (McDonald et al.; 1981, 1983). The sawing process begins with a two-stage (multiple-rip followed by a multiple-crosscut), fixed-dimension sequence followed by a fixed-width single-rerip operation for recovering random-and-fixed-length salvage. The random-length pieces are finger-jointed into usable material. Occasionally, a fourth operation (crosscut) will be implemented to recover additional salvage pieces.

B. Saw Kerf Width

Kerf width, or simply kerf, is defined as that portion of wood removed from a log or board by the saw blade. Kerf width will vary depending on a saw's design and condition. Kerf width is a function of blade thickness which varies with the type of saw. Campbell (1973) and Steele et al. (1987) have discussed kerf width and machine type and explain that band saws have the narrowest kerf width and large-diameter circle saws the widest, while small-diameter circle saws have kerfs ranging between the two. Saw maintenance affects kerf width because dull or ill-tensioned saws create wider kerfs than saws that are in excellent

condition. Kerf width is also a function of the type of operation and the depth of cut. Crosscutting requires a stronger, i.e. thicker, blade than ripping, while both operations require heavier blades as the depth of cut increases.

The relationship between kerf width and percent yield is intuitive -- as kerf width decreases, percent yield should increase. However, the effect may not be easily quantified, especially when sawing natural materials such as logs or lumber. The reported research in this area considers almost exclusively the sawing of softwood logs into dimension lumber. This primarily occurs because the process involves a limited number of standard sized products sawn only with regard to log dimensions; whereas, the sawing of lumber into millwork parts involves products of many different sizes sawn with regard to removing defects, as well as board dimensions, which makes analysis more difficult.

The effect of kerf width and/or target size on lumber yield from logs has been examined in empirical studies of actual mills and in analytical studies using methods such as geometric mapping (Hallock; 1962, 1964; Wang, 1983), or computer-simulated sawing (Giese and McDonald, 1982; Lewis, 1985; McDonald et al; 1981, 1983; Usenius; 1982, 1984). The analytical studies model logs as cylinders or truncated cones in order to simplify the investigation.

Hallock (1964) reported that reducing kerf width from 12/32 to 9/32 inches will increase lumber yield 2.9 to 33.5

percent from logs 5.5 to 12 inches in diameter. The average increase was determined to be 7.31 percent.

Campbell (1973) stated that reducing kerf width by 1/1000 inch will increase profits by forty dollars per million board feet of lumber produced. The dollar increase is undoubtedly greater today, not only because of inflation but also because logs now constitute a greater portion of manufacturing costs.

Usenius (1982, 1984) developed a simulation program to study the significance of saw kerf and dimensional accuracy on sawing yield; this program is being used at several Finnish sawmills for production planning by determining the appropriate blade settings. He reported an average seven-tenths of a percent reduction in sales income for each millimeter (0.038") increase in saw kerf for logs having 5.8 to 11.8 inch top diameters , 15 foot average length and 0.1 inch/foot average taper. In this simulation, the saw kerfs were 0.0, 0.016, 0.157, and 0.236 inches. Lumber and price were included as program inputs.

3. PROCEDURE

A. Computer Sawing Simulation

Computer simulation was selected to generate the yield data for this research. An empirical study would have been more expensive and required finding a cooperating mill willing to change its saw kerf sizes while filling a single cutting bill. Simulation also has the advantage of repeatedly "sawing" a board with different kerf widths thereby removing the effect of board variability. The experimental results should be applicable if the board sample is large enough to represent the board populations found in actual moulding and millwork roughmills.

A.1. Simulation programs

There are a number of computer programs that simulate the cut-up of a board - YIELD (Wodzinski and Hahm, 1966), MULRIP (Stern, 1978), RIPPYIELD (Stern and McDonald, 1978), OPTYLD (Giese and McDonald, 1982), CROMAX (Giese and Danielson, 1983), and CORY (Brunner et al., 1989). Of these computer programs only OPTYLD and CORY are designed to model the sawing process used in the moulding and millwork industry.

OPTYLD was developed by Giese and McDonald (1982) and was used to generate yield tables for 5/4 and 6/4 ponderosa pine lumber (McDonald et al.; 1981, 1983). Unlike its predecessors, OPTYLD has the advantage of using specified values for sawn parts instead of area yield for its

objective function. However, its fixed dimension data structure can only model one-quarter inch kerf widths. Program modification to permit smaller kerf widths would require excessive amounts of memory and execution times.

CORY (computerized optimization of recoverable yield) was originally developed by Brunner (1984) to model infinite stage, random-width, fixed-length sawing processes.

Subsequent versions have been developed that execute on IBM compatible microcomputers in reasonable times using less than five-hundred kilobytes of memory. One such version models the moulding and millwork sawing process described earlier with kerf widths specified to 1/1000 of an inch. Like OPTYLD, it also uses specified values for sawn parts in its objective function. Therefore, this version of the CORY program was used to generate yield data for this study.

A.2. Cutting bill and kerf width information

To calculate the desired yield data, the program requires both sawn part and board information. The information concerning the parts, or cuttings, to be produced are provided in the program's input options file where the relevant cutting bill information specifies the length, width, and associated value for the fixed-dimension parts and a minimum length for salvage cuttings. The fixed-dimension sizes used in this study are a collection of eight widths in three length that are typical of those produced in industry (WWPA, 1988). The sizes and their values are

shown in Table 3.1. The values were chosen to reflect the industry practice of favoring longer lengths over shorter

Table 3.1. The cutting sizes and their values

Cutting Length (Inches)	Cutting Width (Inches)							
	2.063	2.381	2.870	3.129	3.630	4.378	4.874	5.517
36	5852	7310	10712	12656	17030	24806	30800	39204
48	10404	12996	19044	22500	30276	44100	54756	69696
84	31862	39800	58322	68906	92720	135056	167690	213444

ones and wider widths over narrower ones if the cuttings' lengths are equal. This was implemented by assigning a value equal to the cutting's length squared, which favors longer lengths, times its width squared, which should favor wider widths, but only for the same length.

Table 3.1 was used to formulate the four cutting bills presented in Table 3.2. These cutting bills were used to investigate the interaction of rip-kerf and cutting width on percent yield. Only the included widths were varied -- each width included all three cutting lengths.

Table 3.2. Sizes used in the four cutting bills

Cutting Bill Description	Cutting Bill Widths (inches)							
	2.063	2.381	2.780	3.129	3.630	4.378	4.874	5.517
Full Range	x	x	x	x	x	x	x	x
Narrow Range	x	x	x					
Middle Range			x	x	x	x		
Wide Range						x	x	x

Kerf widths are also specified in the options file. A typical 0.190 inch crosscut kerf was used for all simulation experiments because preliminary studies showed that crosscut kerf has a negligible effect on percent yield. Nine rip-kerf widths from 0.050 to 0.250 of an inch in increments of 0.025 inch were used. Consequently, for each cutting bill and grade combination, yield results were developed for each of the nine rip kerfs.

A.3. Board data

The board data was provided for the simulation program in three data files of two-hundred and fifty boards each. They represent a total of 14,567 bd.ft. of 5/4 ponderosa pine lumber in three shop lumber grades, 5,866 bd.ft. in grade no.1, 4,786 bd.ft. in grade no.2, and 3,915 bd.ft. in grade no.3. The three board samples' size information is presented in Table 3.3.

Table 3.3. Board size information for each lumber grade

Board Measure	Board Grade	Average	Minimum	Maximum
Length (feet)	no.1	15.28	6.00	16.22
	no.2	13.86	7.97	16.30
	no.3	13.91	6.00	16.10
Width (inches)	no.1	14.64	5.88	23.76
	no.2	13.08	5.64	21.84
	no.3	10.68	5.76	20.88
Volume (bd.ft.)	no.1	23.46	4.73	39.81
	no.2	19.14	4.70	36.48
	no.3	15.66	3.98	34.86

The board data numerically "maps" a board's overall dimensions and its defects' sizes and locations thus permitting the computer simulation to determine a sawing solution and the board's yield. The original board data was obtained from the United States Department of Agriculture's Forest Product Laboratory in Madison, Wisconsin and is the same data used to develop the 5/4 ponderosa pine yield tables found in publication FPL 394 (McDonald et al., 1981).

Modification of the data was necessary because it was originally recorded in 0.250 inch Cartesian (x-y) coordinate units which are larger than the saw kerfs currently found in industry. Accordingly, the board data was transformed using a pseudo-random number generator and uniform distribution function to create board and defect coordinates of 0.025 inches. The procedure's underlying assumption, that the actual dimensions are uniformly distributed within the original 0.250 inch coordinate, seems reasonable given that defect dimensions are continuous and that the original measurements were inclusive of a defect as described in Publication FPL 394 (McDonald et al., 1981).

A.4. Summary of the simulation experiments

After calculating a board's sawing solution, the simulation program reports production for total, fixed dimension, and for aggregate random-length salvage percent yields. This data was generated thirty-six times for each board, once for each cutting bill and kerf width

combination. When lumber grade is included, a total of 27,000 board analyses were performed to produce this study's basic data. One caveat for the reader: while percent yield is a commonly used and understood term employed in the wood products industry, its maximization was not the goal of the sawing simulation's objective function.

B. Statistical Analysis

The practical goal of this study was to develop a model for predicting cutting yield increases when rip kerf is reduced. It would be especially useful if an easily calculated measure for the cutting bill could be included. To explore these issues, a number of statistical tools, including regression methods, were required. The statistical procedures described in this section were implemented using Statistical Analysis System, or SAS, software available for use on MS-DOS compatible microcomputers (Statistical Analysis System, 1982).

B.1. Effect of rip-kerf width and cutting bills

A separate two-factor analysis of variance (ANOVA) procedure was used with each lumber grade to determine if the rip-kerf widths or the cutting bills affected the grade's percent yield and, if they do, to determine if there is an interaction between them. The presence of any interaction is important because it determines the type of procedures required for any further analysis.

The ANOVA used a full fixed-effect model because each grade's boards were "sawn" at each factor's level permitting the boards to constitute a block with repeated measures (Cody and Smith, 1987). This design eliminated the random effect attributable to between board variation. The ANOVA model's components and the applicable terms for calculating their F statistics are presented in Table 3.4.

Table 3.4. Format for the fixed-effect, two-factor ANOVA

Source	DF	SS	MS	F
1. Board	249	SS _R	MS _R	F _R = MS _R /MS _{RAB}
2. Kerf	8	SS _A	MS _A	F _A = MS _A /MS _{RA}
3. Bill	3	SS _B	MS _B	F _B = MS _B /MS _{RB}
4. Kerf x Bill	24	SS _{AB}	MS _{AB}	F _{AB} = MS _{AB} /MS _{RAB}
5. Board x Kerf	1992	SS _{RA}	MS _{RA}	-----
6. Board x Bill	747	SS _{RB}	MS _{RB}	-----
7. Board x Rip Kerf x Cutting Bill	5976	SS _{RAB}	MS _{RAB}	-----
Total	8999	SS _{Total}		

The model and its definition is as follows:

$$Y_{ijk} = \mu + \rho_i + \alpha_j + \beta_k + (\rho\alpha)_{ij} + (\rho\beta)_{ik} + (\alpha\beta)_{jk} + (\rho\alpha\beta)_{ijk} \quad (3.1)$$

where :

Y_{ijk} : yield of the j th level of factor A (rip kerf width), k th level of factor B (cutting bill) in the i th block (board number)

μ : overall mean yield

ρ_i : effect of the i th board

- α_j : added effect of the j th level of rip kerf width
- β_k : added effect of the k th level of cutting bill
- $(\rho\alpha)_{ij}$: added effect of the combination of the i th level of board number with j th level of rip kerf width
- $(\rho\beta)_{ik}$: added effect of the combination of the i th level of board number with k th level of cutting bill
- $(\alpha\beta)_{jk}$: added effect of the combination of the j th level of rip-kerf width with k th level of cutting bill, the $A_j \times B_k$ interaction effect
- $(\rho\alpha\beta)_{ijk}$: added effect of the combination of the i th level of board number with j th level of rip kerf width and k th level of cutting bill

The F-test statistic is $F^* = \text{Mean Square for each source of interest} / \text{appropriate Mean Square}$ indicated in the F statistic column in Table 3.4. F_A is a test statistic for the hypothesis that the main effect of factor A is zero. That is, it tests $H_0: \mu_1 = \mu_2 \dots \mu_a = 0$. If this hypothesis is rejected the conclusion is that there are significant differences among the means of the A factor levels. F_B is a test statistic for the hypothesis that the main effect of factor B is zero. F_{AB} is a test for the hypothesis that there is no A x B interaction. F_R is a test statistic for the significance of differences among blocks.

If the F_r is larger than the F critical value, the conclusion is that the block effectively reduced experimental error.

If there is no significant interaction between rip-kerf widths and cutting bills then their data may be pooled and a single regression model (discussed below) developed. If there is interaction, then a separate regression model will need to be developed for each cutting bill.

B.2. Rip-kerf width vs percent yield

Although individual board values are interesting, this study is concerned with the effect of rip-kerf widths when a large volume of lumber is sawn. Therefore, it was desirable to find an analysis method that could minimize the effect of board variation. Steel and Torrie (1980) suggest that orthogonal polynomial regression or second-order modeling, is more convenient to use than other regression methods and is, at the same time, a more accurate estimator of model significance when using ill-conditioned data. This last attribute is important because, while past research has shown board yield data is sufficiently normal for statistical testing (Brunner, 1984), percentage data often tends to be non-normal.

The orthogonal polynomial is generated as:

$$\hat{Y} = \bar{Y} + \beta_1 \lambda_1 \xi_1 + \beta_2 \lambda_2 \xi_2 \quad (3.1)$$

where :

\bar{Y} : average estimated yield

\bar{y} : average yield

λ_1, λ_2 : coefficient of orthogonal polynomial is 1 and 1 respectively

β_1, β_2 : are defined as

$$\beta_1 = \frac{\sum (X - \bar{X}) / (Y - \bar{Y})}{\sum (X - \bar{X})^2} \quad (3.2)$$

$$\beta_2 = \frac{\sum (X - \bar{X}) / (Y - \bar{Y})}{\sum (X - \bar{X})^2} \quad (3.3)$$

where:

X : rip-kerf width

\bar{X} : average rip-kerf width

Y : observed percent yield

\bar{Y} : average percent yield.

ξ_1, ξ_2 are defined as :

$$\xi_o = 1, \text{ all } X; \quad (3.4)$$

$$\xi_1 = \frac{(X_i - \bar{X})}{(d)} \quad (3.5)$$

$$\xi_2 = \left[\frac{(X_i - \bar{X})}{d} \right]^2 - \left[\frac{n^2 - 1}{12} \right] \quad (3.6)$$

$$\xi_{k+1} = \xi_1 \xi_k - \frac{k^2(n^2 - k^2)}{4(nk^2 - 1)} \xi_{k-1} \quad (3.7)$$

where d is the spacing between consecutive rip kerfs, that is 0.025 inch, k is the degree of the polynomial, and n is the number of rip-kerf widths, that is 9.

The first polynomial term is of degree zero and represents the mean yield. The second is of the first degree and measures each X (kerf width) from the mean in units of the X spacing (0.025 inches) as a linear relationship. The values of the polynomials, found by substituting values of X , are multiplied by the coefficient λ to give the integers found in Table 3.5.

The Q value is obtained by summation of each treatment total times each orthogonal coefficient for each polynomial effect. Sum of squares for each polynomial effect is obtained by dividing each Q square by the total number of boards times the divisor.

The F test statistic, $F^* = MSR/MSE$, is used to test the significance of the linear, quadratic, cubic terms, etc.

Table 3.5. Coefficients and divisors for the first five terms of an orthogonal-regression model with nine equally spaced treatments.

Degree of Polynomial	Treatment totals									Divisor $\Sigma(c_i)^2$	λ	Q	SS	F
	T ₁	T ₂	T ₃	T ₄	T ₅	T ₆	T ₇	T ₈	T ₉					
1	- 4	- 3	- 2	- 1	0	+ 1	+ 2	+ 3	+ 4	60	1	Q ₁	SS ₁	F ₁
2	+28	+ 7	- 8	-17	-20	-17	- 8	+ 7	+28	2772	3	Q ₂	SS ₂	F ₂
3	-14	+ 7	+13	+ 9	0	- 9	-13	- 7	+14	990	5/6	Q ₃	SS ₃	F ₃
4	+14	-21	-11	+ 9	+18	+ 9	-11	-21	+14	2002	7/12	Q ₄	SS ₄	F ₄
5	- 4	+11	- 4	- 9	0	+ 9	+ 4	-11	+ 4	468	3/20	Q ₅	SS ₅	F ₅

After Steel and Torrie (1980).

Because each sum of squares has only one degree of freedom, it is also the mean square. The F values are obtained by dividing each mean square by the error mean square. The error mean square is obtained from the two-way anova described above.

If only the linear term is significant, a single-term linear regression model will be sufficient to describe the relationship between rip-kerf width and percent yield. Otherwise, a multiple-linear regression model using terms of the appropriate degree will be required. In either case the correlation between rip-kerf width and percent yield will be tested using an F test to choose between the following alternatives:

H_0 : $\beta_1 = 0$ there is a correlation

H_1 : $\beta_1 \neq 0$ there is no correlation

The decision rule to control the Type I error with an alpha level equal to 0.001 is:

if $F^* \leq F(1-0.001; 1; 7)$, conclude H_0

if $F^* \geq F(1-0.001; 1; 7)$, conclude H_1

The r^2 statistic is used to determine the actual degree of correlation between rip-kerf width and percent yield and is calculated as follows:

$$r^2 = 1 - \frac{(SS \text{ Regression})}{(SS \text{ Total})} \quad (3.8)$$

The r^2 value falls between 0 and 1. The larger the value, the greater the correlation between the dependent and

independent variables. Also, confidence intervals can be calculated for the model's polynomial terms. Only one regression model must be computed for each lumber grade if there is no significant interaction between rip-kerf widths and cutting bills. However, if there is a significant interaction, then a model for each cutting bill must be computed and pair-wise comparisons performed among their slopes. These comparisons are performed by subtracting each board's observed yield at a given kerf width and cutting bill from the same board's yield at the same kerf width, but for another cutting bill. A linear regression model of the resulting observations can then be created and its slope tested for a significant difference from zero. If the test indicates a nonzero slope, then the rates of change in yield are different for the cutting-bill pair. This test also permits the relative rankings of the model slopes to be determined and presented the same as other multiple-means tests such as Duncan's range test (Steel and Torrie, 1980).

B.3. Lumber grade vs percent yield

The differences between lumber grade and their effects on percent yield are obviously significant given that the better lumber grades have fewer defects and larger board clear areas. This produces large differences in average percent yield but is not important to this study's purpose, which is the effect of changing rip-kerf width on percent area or volume yield. Accordingly, the variables of

interest are the slopes of the cutting-bill regression models and not their intercepts. The differences between regression models' slopes can be tested by determining if they are parallel.

To test the parallelism of slopes, Seber (1977) proposes the hypothesis $H_1 : \beta_1 = \beta_2 = \beta_3 = \dots = \beta_k$. The test statistic for H_1 is

$$F = \frac{(RSS_H - RSS) / (K-1)}{RSS / (N-2K)} \quad (3.9)$$

where RSS is simply the sum of the residual sums of squares for each regression, namely,

$$RSS = \sum_{k=1}^K \left[\sum_{i=1}^{n_k} (Y_{ki} - \bar{Y}_k)^2 - \beta_k^2 \sum_{i=1}^{n_k} (X_{ki} - \bar{X}_k)^2 \right] \quad (3.10)$$

$$RSS_H = \sum_k \sum_i (Y_{ki} - \bar{\alpha}_k - \bar{\beta} X_{ki})^2 \quad (3.11)$$

$$\beta_k = \frac{\sum_i (Y_{ki} - \bar{Y}_k) (X_{ki} - \bar{X}_k)}{\sum_i (X_{ki} - \bar{X}_k)^2} \quad (3.12)$$

$$\hat{\alpha}_k = \bar{Y}_{ki} - \beta_k \bar{X}_{ki} \quad (3.13)$$

$$RSS_H = \sum_{k=1} \sum_{i=1} (Y_{ki} - \bar{\alpha}_k - \bar{\beta} X_{ki})^2 \quad (3.14)$$

where the common slope is

$$\bar{\beta} = \frac{\sum_k \sum_i (Y_{ki} - \bar{Y}_k) (X_{ki} - \bar{X}_{ki})}{\sum_k \sum_i (X_{ki} - \bar{X}_{ki})^2} \quad (3.15)$$

The decision rule to control the Type I error with an alpha level of 0.01 is:

if $F^* \leq F(1-\alpha; 2; 19)$, conclude H_1

if $F^* \geq F(1-\alpha; 2; 19)$, does not conclude H_1

If H_1 is true than there are no significant differences among the slopes. In this case for each cutting bill, a single regression equation's slope can be developed to predict yield changes with kerf-width reductions without regard for the lumber grade.

4. RESULTS AND DISCUSSION

Lumber shop grades were devised so that each produces significantly different yield from the other. Consequently, each grade was analyzed separately before investigating any relationships between them.

A. Shop Grade No.1

The two-way ANOVA results for shop grade no.1 are shown in Table 4.1. As anticipated, they indicate highly

Table 4.1. Anova results of the sawing simulations with four cutting bills and nine rip-kerf widths as factors and two hundred and fifty ponderosa pine grade no.1 boards as experimental units

Source	DF	Mean Square	Error Term	F value	Probability
1. Board	249	1849.48	-----	-----	-----
2. Rip Kerf	8	1137.32	8.57	132.75	0.0001
3. Cutting Bill	3	22742.72	280.87	80.97	0.0001
4. Rip Kerf x Cutting Bill	24	51.77	8.73	5.93	0.0001
5. Board x Rip Kerf	1992	8.57	-----	-----	-----
6. Board x Cutting Bill	747	208.87	-----	-----	-----
7. Board x Rip Kerf x Cutting Bill	5976	8.73	-----	-----	-----
Total	8999	-----	-----	-----	-----

significant differences among the four cutting bills and nine kerf widths. The interaction term's probability value indicates that changing kerf width does not affect yield the same in each cutting bill. This meant that separate orthogonal regression models had to be developed and tested for each cutting bill. These models' test results are shown in Table 4.2.

Table 4.2. The F and probability values for the four cutting bill orthogonal models' terms for shop grade no.1

Cutting Bill	Degree of Polynomial Term				
	Linear	Quadratic	Cubic	Quartic	Quintic
	F value (P value)	F value (P value)	F value (P value)	F value (P value)	F value (P value)
Full-Range	233.978 (0.0000)	1.07782 (0.3002)	0.000116 (0.9914)	0.28805 (0.5919)	1.72401 (0.19039)
Narrow-Range	672.903 (0.0000)	0.18380 (0.6685)	0.28336 (0.5950)	0.015343 (0.9015)	0.004477 (0.94671)
Middle-Range	204.479 (0.0000)	0.55565 (0.4567)	0.01409 (0.9056)	0.072835 (0.7875)	0.16730 (0.68288)
Wide-Range	99.424 (0.0000)	0.90966 (0.3411)	0.47914 (0.4895)	0.012854 (0.9098)	0.17569 (0.67547)

They indicate that the relationship between kerf width and cutting yield is linear. The parameters for the linear models are shown in Table 4.3 and their graphical representations in Figure 4.1. The models fit the data well with r^2 's greater than 0.98 denoting that kerf-width

Table 4.3. The r^2 's, means, and confidence intervals for the intercept and slope of the four cutting bills' linear regression models for shop grade no.1

Cutting Bill (R-Square)	Intercept		Slope	
	Estimate (Standard error)	Minimum (Maximum)	Estimate (Standard Error)	Confidence Interval Min.(Max)
Full-Range (-0.9869)	71.028847 (0.415341)	70.613506 (71.444188)	-0.012118 (0.000527)	-0.0121707 (-0.011591)
Narrow-Range (-0.9987)	75.756512 (0.392471)	75.364041 (76.148983)	-0.024771 (0.000343)	-0.025114 (-0.024428)
Middle-Range (-0.9837)	71.297581 (0.477541)	70.820040 (71.775122)	-0.015179 (0.000738)	-0.015917 (-0.014441)
Wide-Range (-0.9548)	65.889818 (0.617506)	65.272312 (66.507324)	-0.010175 (0.000541)	-0.010716 (-0.009634)

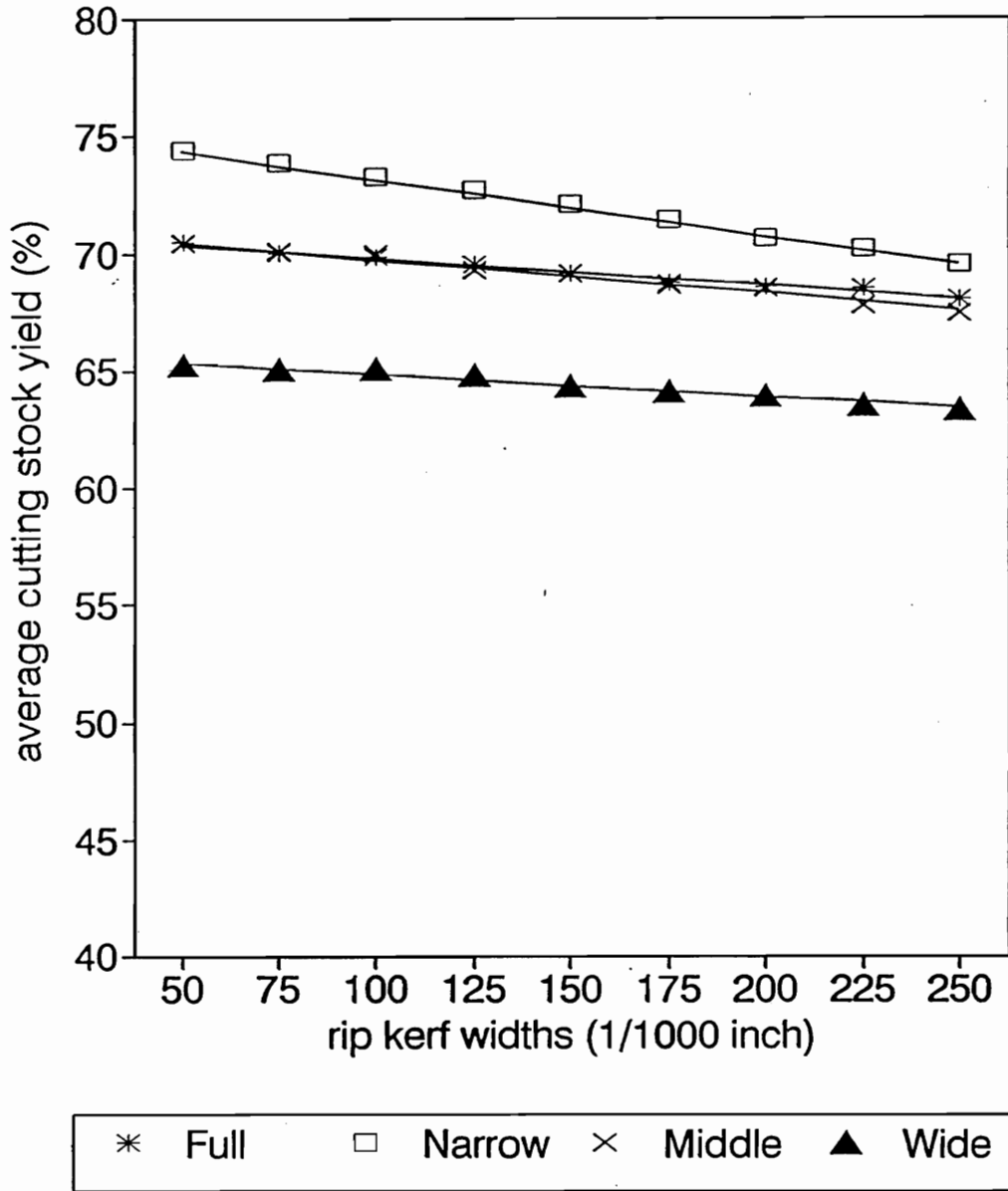


Figure 4.1. Linear Relationship between Rip Kerf Width and Percent Yield for Four Cutting Bills in Shop Grade No.1

reduction accounts for more than 98 percent of the cutting yield increase. Although a test for parallelism with the cutting-bill models indicates that their slopes are different, the graph in Figure 4.1 suggests that the effect of reducing kerf width (model slope) may be the same for the full-, mid-, and wide-range cutting bills while the slope for the narrow-range bill is greater. A pair-wise comparison of the different models' slopes, see Figure 4.2, indicated that this was indeed the case. This finding could

Narrow-Range	Middle-Range	Full-Range	Wide-Range
-0.0248	-0.0152	-0.0121	-0.0102

Figure 4.2. Pair-Wise Comparison of the Four Cutting Bill slopes for Shop Grade No.1

have practical importance because a single coefficient would be sufficient for any cutting bill employing at least three of the eight study widths and a widest width that exceeds approximately 2.8 inches. A preliminary examination of this concept was tested with five additional cutting bills, each containing the two narrowest widths and, as its third and widest width, one of the five widths not included in the original narrow-range cutting bill. See Table 4.4. This strategy was chosen because narrow widths should permit the sawing algorithm maximum flexibility when calculating sawing decisions and, therefore, accentuate the effect of the

Table 4.4. Cutting widths forming the five alternate cutting bills

Additional Cutting Bills Descriptions	Cutting Bill Widths (inches)							
	2.063	2.381	2.780	3.129	3.630	4.378	4.874	5.517
Alternate Bill A	x	x		x				
Alternate Bill B	x	x			x			
Alternate Bill C	x	x				x		
Alternate Bill D	x	x					x	
Alternate Bill E	x	x						x

widest cutting width. Pair-wise comparison results for the different model's slopes are presented in Figure 4.3.

Statistically the hypothesis that the alternate cutting bills have an identical effect on yield when changing kerf width must be rejected. However, the results do demonstrate how including the narrowest widths enhances the effect of reducing rip-kerf widths.

Alt.A	Alt.D	Alt.B	Alt.C	Alt.E
-0.02165	-0.01853	-0.01845	-0.01613	-0.01343

Figure 4.3. Pair-Wise Comparison of the Five Alternate Cutting Bills Slopes in Shop grade No.1

The significance of the two-way anova's interaction term given in Table 4.1 also means that no statistical statements can be made concerning the relationships between cutting bills, except at a given rip-kerf width. However,

inspection of the curves in Figure 4.1 show that they generally lay, as would be expected, one above the other which permits some speculation concerning this matter. It appears that the mid- and full-range bills produced nearly identical yields while the narrow-range bill produced significantly greater yields and the wide-range bill significantly smaller yields. This seems reasonable because the sawing algorithm places the widest possible cutting in a given board clear area and the narrower a cutting bill's maximum and minimum widths the greater the opportunity for fitting additional cuttings into the remainder of the area.

B. Shop Grade No.2

Table 4.5. represents the sawing simulation results for shop grade no.2. They show significant differences among

Table 4.5. Anova results of the sawing simulations with four cutting bills and nine kerf widths as factors and two hundred and fifty shop grade no.2 boards as experimental units

Source	DF	Mean Square	Error Term	F value	Probability
1. Board	249	4112.89	-----	-----	-----
2. Rip kerf	8	837.78	8.44	132.75	0.0001
3. Cutting Bill	3	22615.33	345.73	80.97	0.0001
4. Rip kerf x Cutting bill	24	58.86	8.40	5.93	0.0001
5. Board x Rip Kerf	1992	8.44	-----	-----	-----
6. Board x Cutting Bill	747	345.73	-----	-----	-----
7. Board x Rip Kerf x Cutting Bill	5976	8.40	-----	-----	-----
Total	8999				

the four cutting bills and nine kerf widths similar to those for shop grade no.1. As with shop grade no.1, interaction

Table 4.6. The F and probability values for the four cutting-bill orthogonal models' terms for shop grade no.2

Cutting Bill	Degree of Polynomial term				
	Linear	Quadratic	Cubic	Quartic	Quintic
	F value (P value)	F value (P value)	F value (P value)	F value (P value)	F value (P value)
Full-Range	168.327 (0.0000)	0.37537 (0.5406)	1.27149 (0.2606)	0.11861 (0.7308)	2.34381 (0.1270)
Narrow-Range	576.755 (0.0000)	2.74887 (0.0986)	2.03261 (0.1552)	5.71351 (0.01758)	0.080102 (0.7774)
Middle-Range	142.391 (0.0000)	0.60559 (0.4372)	0.029219 (0.8644)	2.37412 (0.1246)	1.36341 (0.2441)
Wide-Range	68.4988 (0.0000)	2.05080 (0.1534)	2.22590 (0.1370)	0.23262 (0.6300)	0.88744 (0.3471)

between the ANOVA model's two main factors required each cutting bill have its own orthogonal regression model. The results for the orthogonal models, Table 4.6, indicated that the relationship between kerf width and cutting yield is also linear for shop grade no.2. The parameters for the linear models are shown in Table 4.7 and their graphical

Table 4.7. The r^2 's, means, and confidence intervals for the intercept and slope of the linear regression models for shop grade no.2

Cutting Bill (R-Square)	Intercept		Slope	
	Estimate (Standard Error)	Confidence Interval Min.(Max)	Estimate (Standard Error)	Confidence Interval Min.(Max)
Full-Range (-0.9735)	62.654734 (0.596358)	62.058376 (63.251092)	-0.012102 (0.000755)	-0.012857 (-0.011347)
Narrow-Range (-0.9751)	66.008339 (0.587696)	65.420643 (66.596035)	-0.022998 (0.001037)	-0.024035 (-0.021961)
Middle-Range (-0.9669)	60.398425 (0.662015)	59.736410 (61.060440)	-0.010801 (0.000569)	-0.018579 (-0.010232)
Wide-Range (-0.91.83)	56.300465 (0.753864)	55.546601 (57.054329)	-0.007409 (0.000849)	-0.008258 (-0.00656)

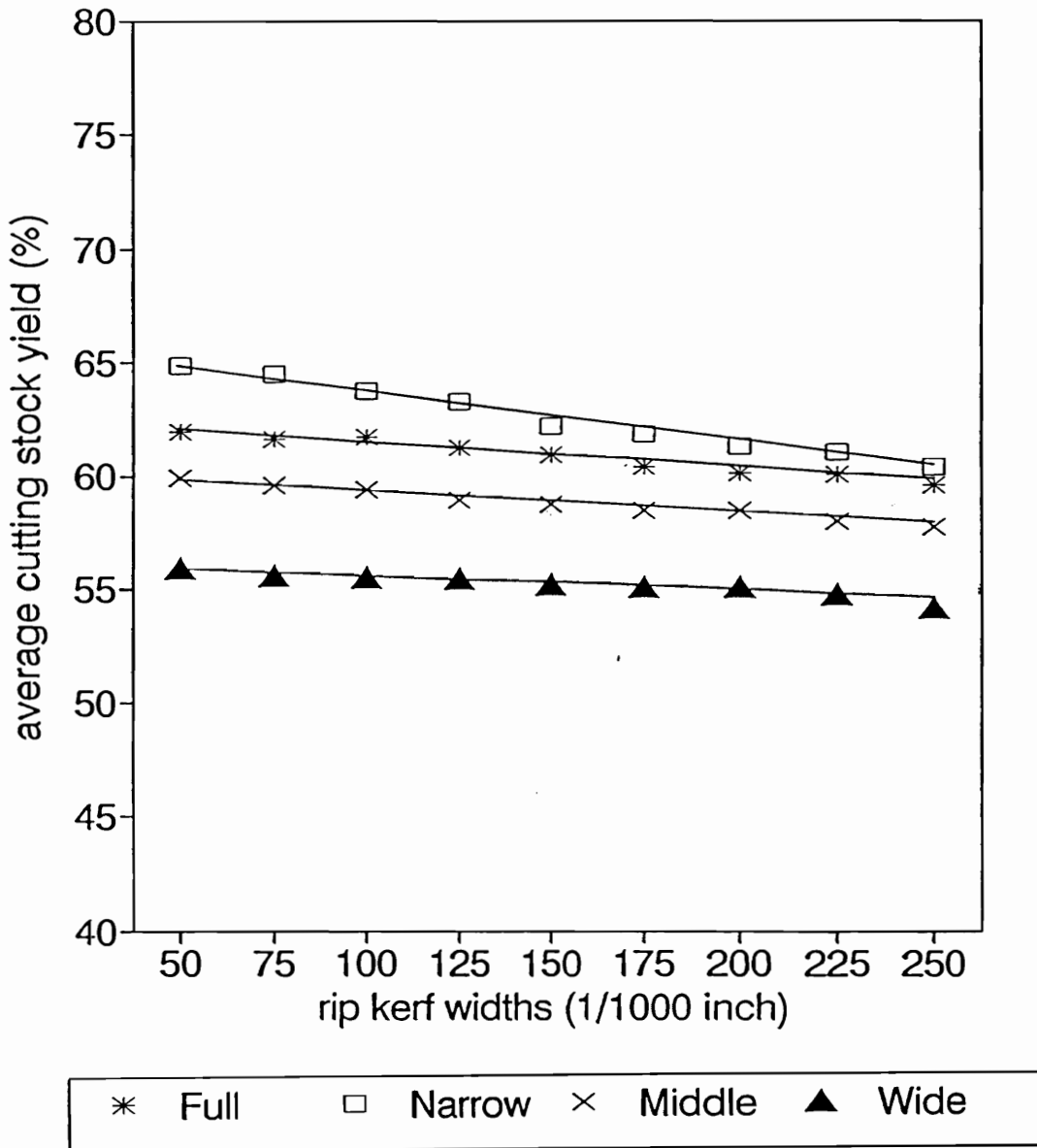


Figure 4.4. Linear Relation between Rip Kerf Width and Percent Yield Four Cutting Bills in Shop Grade No.2

representations in Figure 4.4. The model r^2 's are comparable to those for shop grade no.1, except for the wide-range cutting bill, which is only slightly lower at 0.91.

Pair-wise comparisons of the models' slopes were also performed to determine any differences. See Figure 4.5. The results show that, unlike shop grade no.1, only the full- and middle-range cutting bills produce similar effects when reducing kerf width while the wide-range cutting bill

<u>Narrow-Range</u>	<u>Full-Range</u>	<u>Middle-Range</u>	<u>Wide-Range</u>
-0.0283	-0.0121	-0.0115	-0.0074

Figure 4.5. Pair-wise Comparisons of the Four Cutting Bills for Shop Grade No.2

produces significantly less. This appears to be reasonable given the lower average clear-area width found in no.2 grade shop lumber which reduces the opportunities for the sawing algorithm to fit wider cuttings into the sawing solution.

Inspection of the cutting-bill yield curves (Figure 4.4) shows that each is well separated from the other with the wide-, middle-, full- and narrow-range cutting bills producing the least to the most yield, respectively. This contrasts somewhat from the shop grade no.1 results where the full- and middle-range cutting bills appeared to produce similar yields. The probable basis for the differences between the full- and middle-range bills is that shop grade

no.1 has an average clear-area width of 5.61 inches while shop grade no.2's is only 4.73. In this situation, a greater number of cutting bill widths permits more combinations of widths and, consequently, greater yield from a lower grade material.

C. Shop Grade No.3

Table 4.8 presents the sawing simulation results for shop grade no.3 and shows that there are significant differences among the four cutting bills and nine kerf widths as there were for the other two shop grades. As with the other

Table 4.8. Anova results of the sawing simulations with four cutting bills and nine kerf widths as factors and two hundred and fifty shop grade no.3 boards as experimental units.

Source	DF	Mean Square	Error Term	F value	Probability
1. Board	249	4474.80	-----	-----	-----
2. Rip kerf	8	419.31	7.64	54.88	0.0001
3. Cutting Bill	4	39657.52	366.36	108.25	0.0001
4. Rip kerf x Cutting bill	24	48.39	7.82	6.19	0.0001
5. Board x Rip Kerf	1992	7.64	-----	-----	-----
6. Board x Cutting Bill	747	366.36	-----	-----	-----
7. Board x Rip Kerf x Cutting Bill	5976	7.82	-----	-----	-----
Total	8999				

grades, the interaction term is significant indicating the need for separate orthogonal-regression models. These results are shown in Table 4.9 and again show a linear relationship between kerf width and cutting yield. The parameters for the linear models are shown in Table 4.10 and their graphical representations in Figure 4.6.

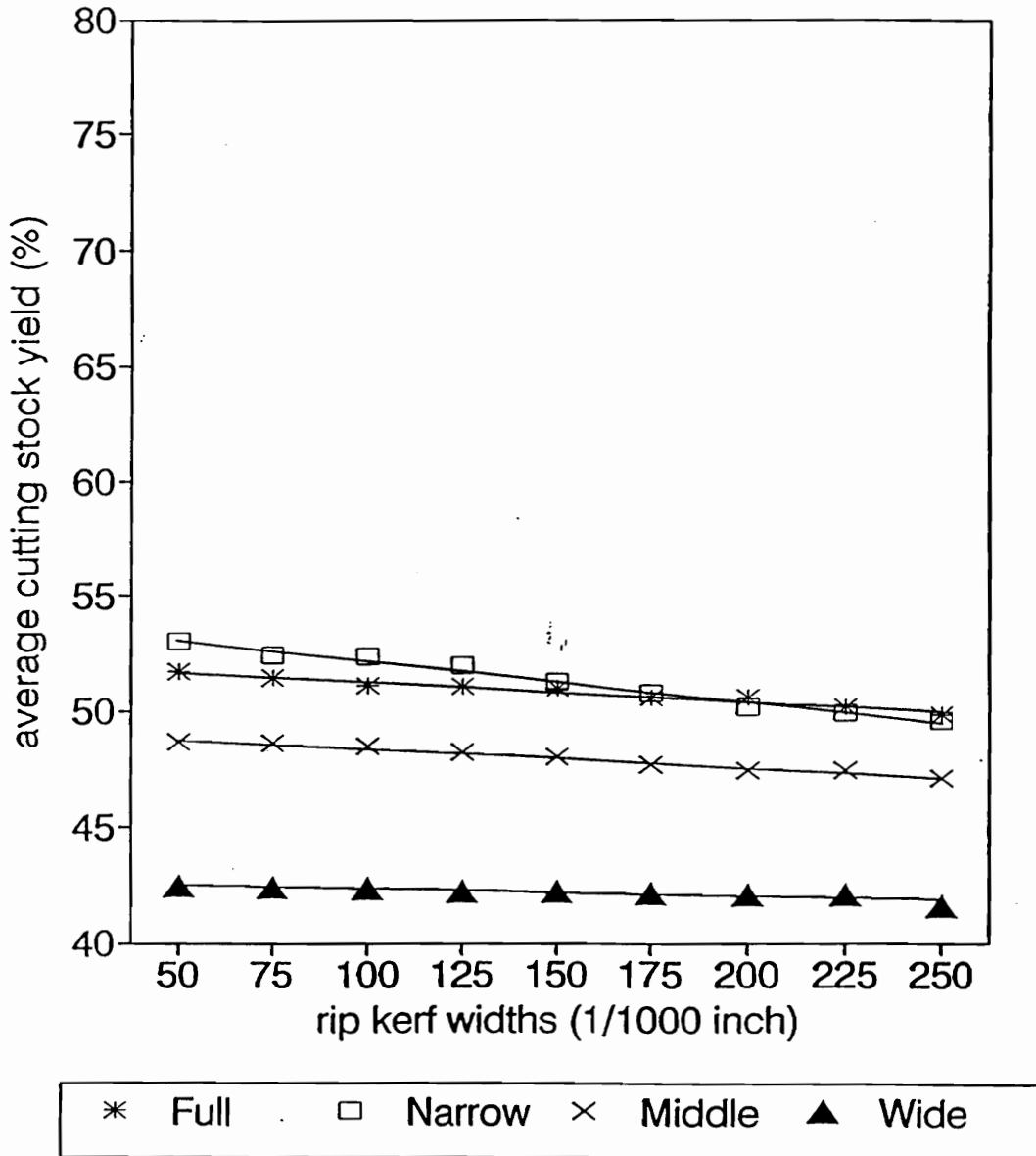


Figure 4.6. Linear Relationship Between Rip-Kerf Width and Percent Yield for Four Cutting Bills in Shop Grade No.3

Table 4.9. The F and probability values for the four cutting bill orthogonal models' terms for shop grade no.3

Cutting Bill	Degree of Polynomial Term				
	Linear	Quadratic	Cubic	Quartic	Quintic
	F value (P value)	F value (P value)	F value (P value)	F value (P value)	F value (P value)
Full-Range	119.456 (0.0000)	0.28473 (0.5941)	0.60622 (0.4369)	0.12026 (0.7290)	1.09947 (0.2954)
Narrow-Range	333.917 (0.0000)	0.10909 (0.7415)	3.79624 (0.0525)	0.74042 (0.3903)	0.032983 (0.8560)
Middle-Range	73.204 (0.0000)	0.09748 (0.7551)	0.78935 (0.3751)	0.19119 (0.6623)	0.10074 (0.7512)
Wide-Range	17.695 (0.0000)	0.73498 (0.3921)	1.65593 (0.1993)	0.83197 (0.3626)	0.30424 (0.5817)

Table 4.10. The r^2 's, means, and confidence intervals for the intercept and slope of the four cutting bills' linear regression models for shop grade no.3

Cutting Bill (R-Square)	Intercept		Slope	
	Estimate (Standard Error)	Confidence Interval Min.(Max)	Estimate (Standard Error)	Confidence Interval Min.(Max)
Full-Range (-0.9571)	52.129736 (0.594226)	51.53551 (52.723962)	-0.008475 (0.000613)	-0.009088 (-0.007862)
Narrow-Range (-0.9832)	53.981468 (0.670407)	53.311061 (54.651875)	-0.017863 (0.000832)	-0.018695 (-0.017031)
Middle-Range (-0.9768)	49.211977 (0.696702)	48.515275 (49.908679)	-0.008331 (0.000466)	-0.008797 (-0.007865)
Wide-Range (-0.8277)	42.664630 (0.739385)	41.925245 (43.404015)	-0.003203 (0.000651)	-0.003854 (-0.002552)

The r^2 's are comparable to those for the other grades except that the wide-range cutting bill's is only 0.76 which is probably due to the narrowest width in this cutting bill being greater than the average clear-area width of 3.76 inches found in this sample of no.3 shop grade lumber. This

situation creates greater variability because there are many boards that produce little or no yield and do not benefit from a reduced kerf width.

Pair wise comparisons of the different models' slopes produce a similar ranking to the shop grade no.2 results, see Figure 4.7. However, the effect of reducing rip-kerf

<u>Narrow-Range</u>	<u>Full-Range</u>	<u>Middle-Range</u>	<u>Wide-Range</u>
-0.0190	-0.0090	-0.0084	-0.0032
Figure 4.7. Pair-Wise Comparison of the Four Cutting Bill Slopes for Shop Grade No.3			

is considerably less with mean slope values being 20 to 60 percent less than those for the same cutting bill in shop grade no.2.

The cutting bill yield curves have the same ranking as the no.2 shop grade. However, one obvious difference is the relatively lower yield of the wide-range bill compared to the next highest yield curve (middle-range). The probable cause is the same for the wide-range bill having a lower r^2 , namely, that the cutting bill's narrowest width is greater than the average clear-area width.

D. Differences between Lumber Grade

Some general discussion of the different lumber grades was included in the previous material. However, this

section is intended to answer specific questions of potential practical importance, such as, does the reduction of rip-kerf width affect yield identically for each lumber grade when sawing to a given cutting bill.

Table 4.11 presents the slopes for the various linear models by cutting bill and lumber grade. Figures 4.8 through 4.11 show the yield curves for each of the four cutting bills.

Table 4.11. The slopes for the various linear models by cutting bill and lumber grade

Cutting Bill	Lumber Grade		
	No.1	No.2	No.3
Full-Range	-0.012118	-0.012102	-0.008475
Narrow-Range	-0.024770	-0.022998	-0.017863
Middle-Range	-0.015179	-0.010801	-0.008331
Wide-Range	-0.010175	-0.007409	-0.003203

One probable trend appearing in Table 4.11 is the decreasing effect of changing rip-kerf width with lowering lumber grade for a given cutting bill. The decrease appears to be greater between lumber grades no.2 and no.3 than between no.1 and no.2. In fact, the differences between grades no.1 and no.2 appear to be identical for the full- and narrow-range cutting bills. To determine if this is the case, a test for parallelism was performed on the three grade's yield curves for each of the four cutting bills. The results established that the relationship between rip-

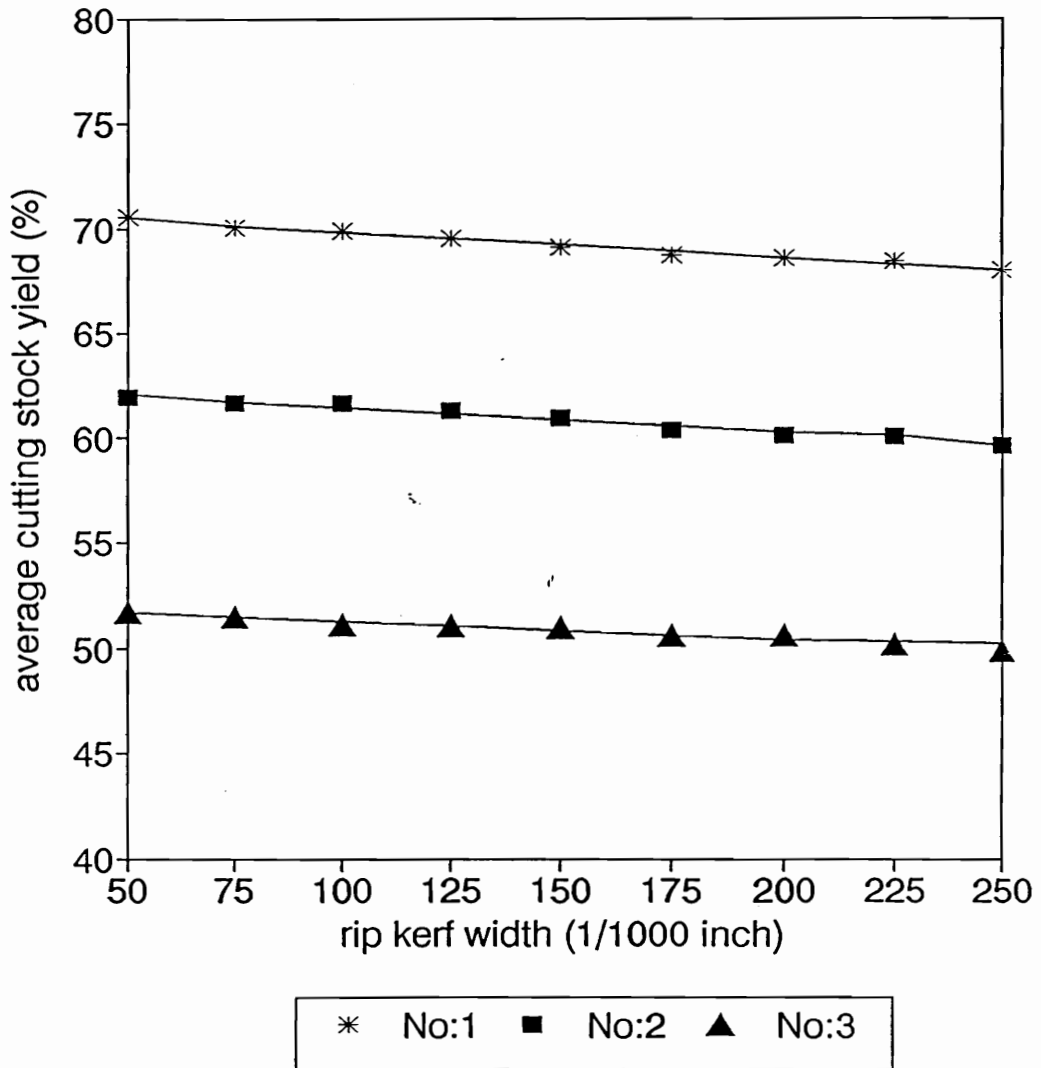


Figure 4.8. Yield Curves of the Three Shop Grades for the Full-Range Cutting Bill

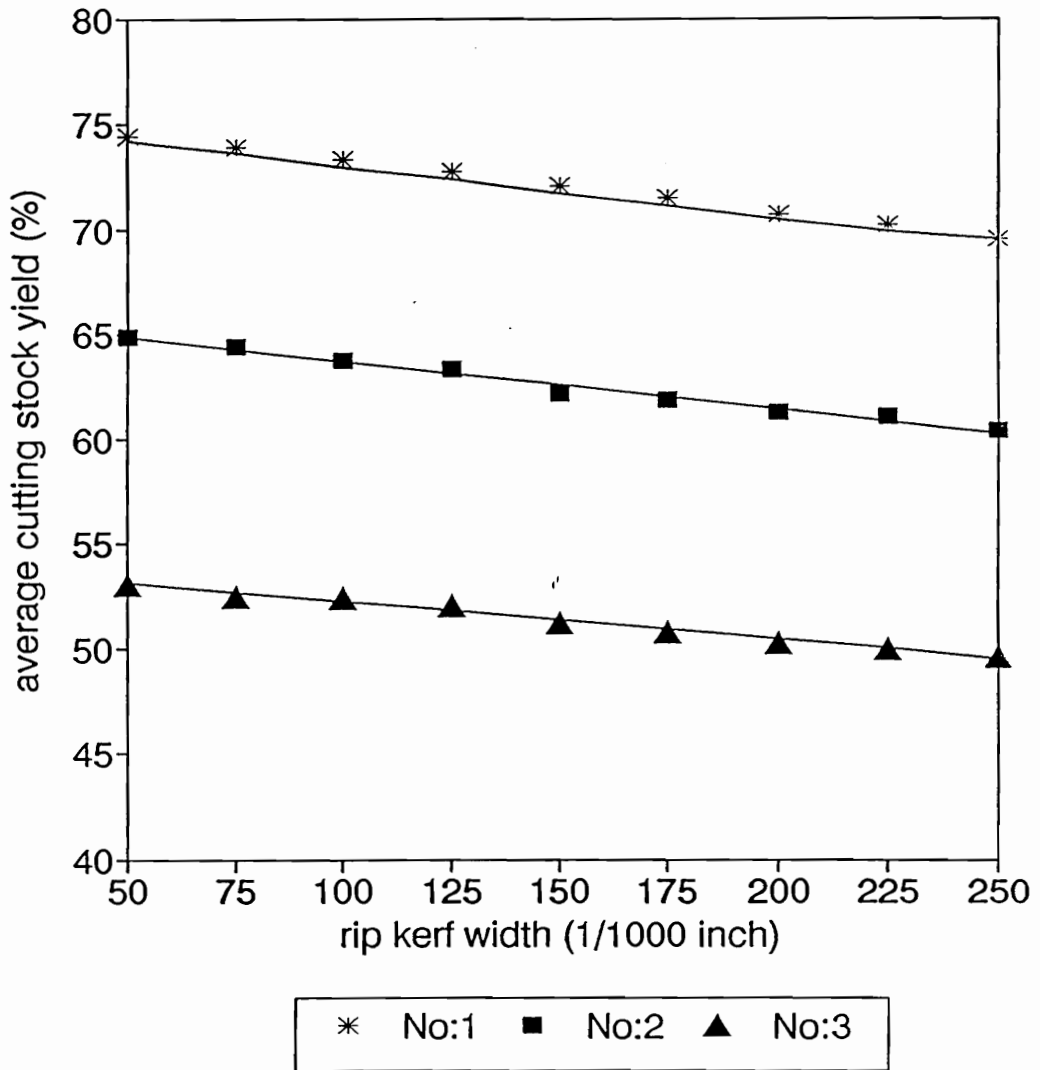


Figure 4.9. Yield Curves of the Three Shop Grades for the Narrow-Range Cutting Bill

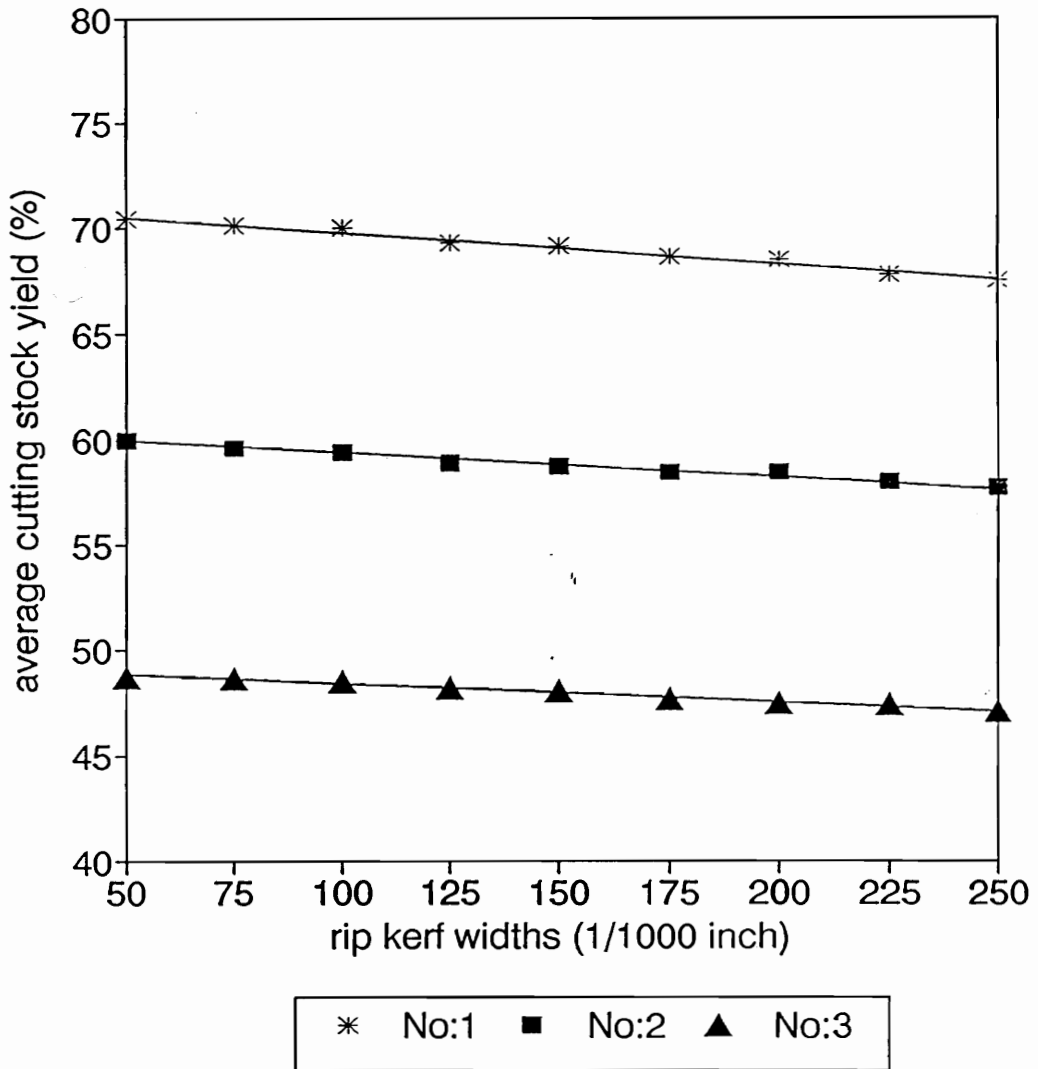


Figure 4.10. Yield Curves of the Three Shop Grades for the Middle-Range Cutting Bill

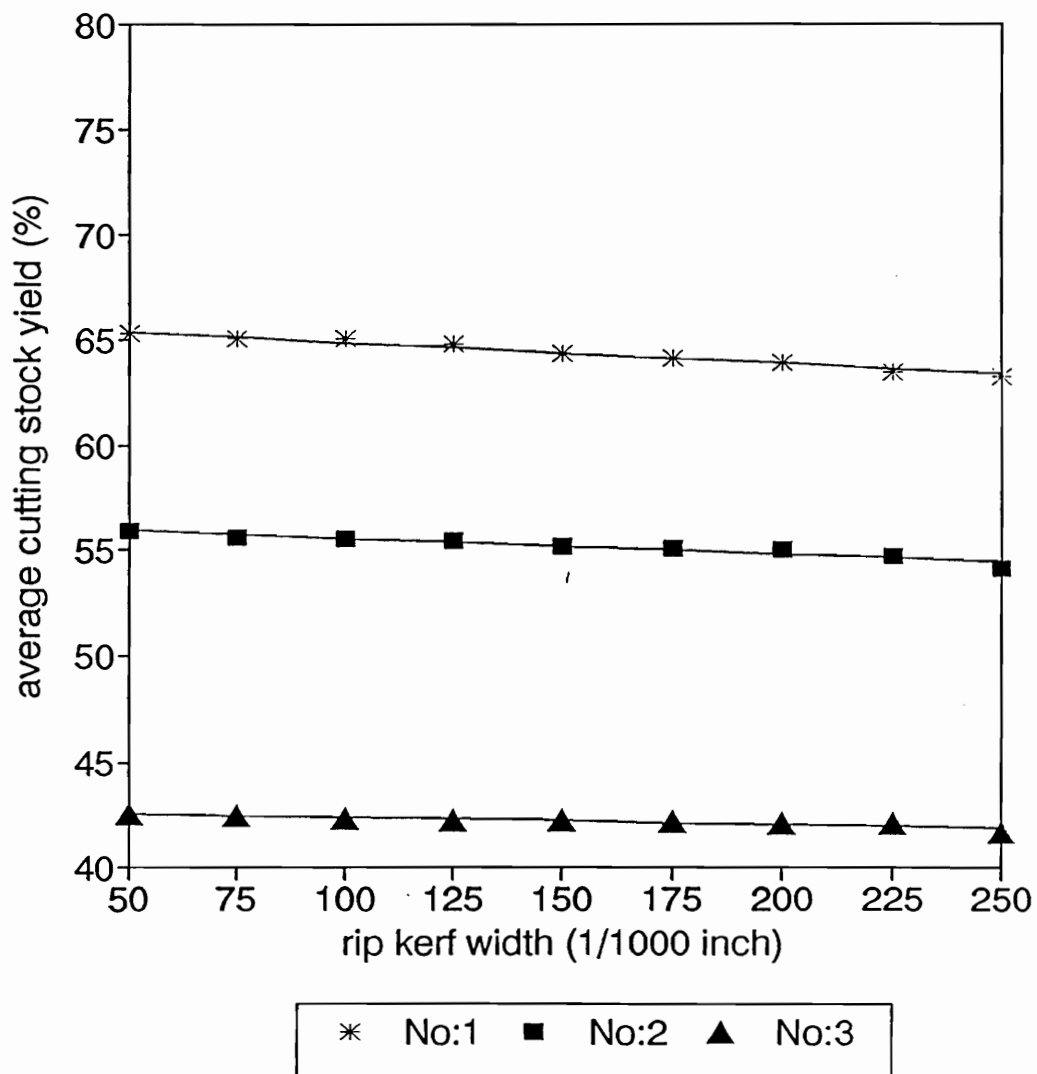


Figure 4.11. Yield Curves of the Three Shop Grades for the Wide-Range Cutting Bill

kerf width and cutting yield is statistically the same for each cutting bill regardless of the lumber grade. Consequently by ignoring lumber grade and aggregating their yield data, the number of models required per cutting bill can be reduced to one. The slopes of the four aggregated cutting-bill regression models are presented in Table 4.12.

Table 4.12. The average of the three shop grades' slopes for each of the cutting bills

Cutting Bill	Average Slope
Full-Range	- 0.01090
Narrow-Range	- 0.02188
Middle Range	- 0.01144
Wide-Range	- 0.00702

5. CONCLUSIONS AND RECOMMENDATIONS

As anticipated, this research found a statistically significant inverse relationship between rip-kerf width and product output as measured by percent volume yield. Even though the effect is relatively small, it should be of practical significance because a yield increase of one-tenth percent can increase a typical moulding and millwork operation's revenues upwards of a hundred-thousand dollars annually.

The particular results apply only to this study's cutting bill, but, if it is assumed that its sizes fairly represent the range and number of those found in the moulding and millwork industry, some general conclusions concerning industry practice can be offered. First, the results show that reducing rip-kerf width by 0.050 inches will increase yield from a half to one percent depending on the cutting bill and, second, that the narrower the maximum cutting bill width and the more numerous and varied the other widths, the greater the yield increase.

There are a number of additional questions that this study did not answer and should be addressed by further research. For instance, what is the effect of using different objective functions, such as one that maximizes area yield, on the increases predicted in percent yield from decreasing rip-kerf width? Another area of interest would be the effect of reduced rip-kerf on individual cutting bill widths.

Bibliography

- Brunner, Charles C. CORY -- A Computer Program to Determine Furniture Cutting Yields for Both Rip-First and Crosscut-First Sawing Sequences. 1984. Unpublished Ph.D. diss. Virginia Polytechnic Institute and State Univ., Blacksburg, Va.
- Brunner, Charles C., M.S. White, F.M. Lamb, J.G. Schroeder. 1989. Cory: A Computer Program for Determining Dimension Stock Yield. For. Prod. J. 39(2):23-24.
- Campbell, Richard D. 1973. High Speed Thin Kerf Sawing. Modern Sawmill Techniques Vol:2 Proceeding of the Second Sawmill Clinic. New Orleans. Louisiana. Miller Freeman Publication. San Francisco, California.
- Cody, Ronald P., and J.K. Smith. 1987. Applied Statistics and the SAS Programming Language. Second Edition. Elsevier Science Publishing Co., Inc. New York.
- Gevaart, T.C. 1961. The Cost and Pricing of Ponderosa Pine Cut Stock. E.L. Jackson Publ. Portland, Oregon.
- Giese, Pamela J., and J.D. Danielson. 1983. CROMAX: A Crosscut-First Computer Simulation Program to Determine Cutting Yield. USDA. For. Serv. Research Paper. FPL 38, 40 pp.
- Giese, Pamela J., and K.A. McDonald. 1982. Optyld: A Multiple Rip-First Computer Program to Maximize Cutting Yields. USDA. For. Serv. Research Paper FPL 412, 33 pp.
- Hallock, Hiram. 1962. A Mathematical Analysis of the Effect of Kerf Width on Lumber Yield from Small Log. USDA. For. Serv. FPL 254, 22 pp.
- Hallock, Hiram. 1964. Kerf Width and Lumber Yield. For. Prod. J. 14(2):80-85.
- Kirbach, Eb. 1989. Thin-Kerf Sawing: How to make it work. For. Industries. 116(5):6-10.

- Kirchmeler, S. 1976. Thin Kerf Sawing, Abrasive Planing Boost Production. Wood and Wood Product 81(9):35-36.
- Lewis, David W. 1985. Best Opening Face System for Sweezy Eccentric Logs: A User's Guide. USDA. For. Serv. FPL 49, 12 pp.
- McDonald, Kent A., P.J. Giese, and R.O. Woodfin. 1981. 5/4 Ponderosa Pine Shop Grade Cutting Yields. USDA. For. Serv. Research Paper. FPL 394, 12 pp.
- McDonald, Kent A., P.J. Giese, and R.O. Woodfin. 1983. Maximum Cutting Yields for 6/4 Ponderosa Pine Shop Lumber. USDA. For. Serv. Research Paper FPL 438, 5 pp.
- Seber, GAF. 1977. Linear Regression Analysis. John Wiley & Sons. New York.
- Sherrill, Sam. 1987. A Perspective on : Western Factory Lumber. Special Report. Crows Digest. C.C. Crow Publication. Portland, Oregon.
- Statistical Analysis System. 1982. SAS users Guide. SAS Inst., Cary. North Carolina.
- Steele, Philip H. 1984. Factors Determining Lumber Recovery in Sawmilling. USDA. For. Serv. Research Paper. FPL 39, 8 pp.
- Steele, Philip H., F.G. Wagner., R.D. Seale., F.W. Taylor, and R. Bennet. 1987. Kerf Width by Machine Type. For. Prod. J. 37(3):35-37.
- Steel, Robert G.D., Torrie, James H. 1980. Principles and Procedures of Statistics. A Biometrical Approach. Second Edition. McGraw-Hill, New York.
- Stern, Abigail R. 1978. Unpublished computer program, 1978, USDA. U.S. Forest Products Laboratory. Madison, Wisconsin.

Stern, A.R., McDonald, K.A. 1978. Computer Optimization of Cutting Yield from Multiple-ripped Boards. USDA. For. Serv. Research Paper. FPL 318. Madison, Wisconsin.

Usenius, Arto. 1982. Computer Study of the Volume Part of Sawing Kerf when Chipping with A Center-Chipper and Chipper-Edger. Paperi Ja Puu 64(2): 67-70.

Usenius, Arto. 1984. Effect of Saw Kerf and Accuracy of Dimensions on sawing Yield. Paperi Ja Puu 66(3):181-183.

Wang, Steve W. 1983. An Analytic Approach to Estimating the Increase in Lumber Recovery due to Reduced Target Sizes and Saw Kerfs. For. Prod. J. 33(11/12):29-32.

William, Dean. 1984. Terms of the Trade. Second Edition. Random Lengths Publication, Inc. Portland, Oregon.

Wodzinski, Claudia and Eldona Hahm. 1966. A Computer Program to Determine Yields of Lumber. USDA. For. Serv. FPL Unnumbered Pbl., For. Prod. Lab., Madison, Wisconsin.

WWPA. 1988. Western lumber grading rules 88. Western Wood Products Association. Portland, Oregon.