Nonlocal theories of continuum mechanics are used to model the fracture behavior of particleboard in tension perpendicular to the plane of the board. Classical theories of fracture are examined in comparison to a nonlocal crack solution which has removed the stress singularity at the crack tip. With the removal of the classical stress singularity a fracture criterion based on a maximum stress hypothesis results. The nonlocal fracture equations are in terms of a material's internal characteristics such as the dimensions of internal structures (characteristic length) and the bonding between them (intrinsic strength). Examination of the fracture phenomena using the nonlocal equations provides an insight of the actual mechanism of failure.

Many materials, including wood, consist of internal structures which are, themselves, composites of smaller units of structure. The level of internal structure that is important to the fracture process is a function of the relative size of internal structures in comparison to the flaw where fracture initiates and the bond strength at the various levels of internal structure.

In order to test the nonlocal fracture model it was essential to work with a composite material for which the dimensions of its internal
structures may be varied. This was possible with particleboard, a composite formed of wood particles and resin. If the nonlocal model is able to accurately predict the fracture for particleboard, it should be able to describe the manner in which a change in particle size affects fracture resistance. Experimental results in this study give good correlation with the nonlocal prediction.

For the particleboard formed in this study the characteristic length of the nonlocal relationship is shown to correspond to particle thickness at low resin contents (5% for a high efficiency laboratory blender). Fracture toughness increases with the square root of particle thickness as predicted by the nonlocal solution. There should be a limit in which an increase in particle thickness can no longer increase fracture resistance. This limit will depend on the relative intrinsic bond strength between particles and within particles. By increasing resin content, which increases the number of bonds between particles, the limiting process was shown to exist.

This study showed that a particleboard formed from relatively long thin particles has an average intrinsic flaw size of approximately 0.34 inch. Because this intrinsic flaw size changed only slightly with change in resin content it was postulated that the resin-particle bond causes stress concentration due to a dissimilar response to load. Hence, the compatibility of the resin and wood particles found in particleboard could be as important to the final board properties as was the amount of resin used.

If the orientation of substructures in a material can be expected to change the properties of the material, the material is best described
as a micropolar continuum. The micropolar crack solution predicts couple stress (rotational stress) in the vicinity of a flaw. Conceivably, particles in the vicinity of a flaw could rotate to align their strongest axis with the direction of maximum stress. This interpretation may explain the initiation of the crazing phenomena in amorphous glassy polymers. For particleboard a flexible resin and short particles may allow such rotation and a large increase in strength could be expected. Caution should be taken in using the approach until the combined nonlocal micropolar crack problem is solved.
ON THE PHENOMENA OF FRACTURE IN PARTICLEBOARD

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Larry Bert Ilciewicz

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COMMITTEE MEMBERS

Redacted for privacy

J. B. Wilson

Redacted for privacy

M. N. L. Narasimhan

Redacted for privacy

H. Resch

Redacted for privacy

R. Schmitt
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>I. Introduction</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>II. Objectives</td>
<td>4</td>
</tr>
<tr>
<td>III. Literature Review</td>
<td>5</td>
</tr>
<tr>
<td>Classical Methods of Fracture Analysis</td>
<td>5</td>
</tr>
<tr>
<td>Stress Analysis in the Vicinity of Flaws</td>
<td>6</td>
</tr>
<tr>
<td>Brittle Fracture Criteria</td>
<td>10</td>
</tr>
<tr>
<td>Fracture Mechanics</td>
<td>13</td>
</tr>
<tr>
<td>Anisotropic Effects</td>
<td>17</td>
</tr>
<tr>
<td>Application of Classical Fracture Mechanics</td>
<td>18</td>
</tr>
<tr>
<td>Structural Considerations</td>
<td>24</td>
</tr>
<tr>
<td>Intrinsic Strength</td>
<td>25</td>
</tr>
<tr>
<td>Empirical Relations</td>
<td>29</td>
</tr>
<tr>
<td>Alternative Theories</td>
<td>32</td>
</tr>
<tr>
<td>Micropolar Theory</td>
<td>34</td>
</tr>
<tr>
<td>Nonlocal Theory</td>
<td>41</td>
</tr>
<tr>
<td>IV. Analytical Model</td>
<td>49</td>
</tr>
<tr>
<td>Composite Levels of Fracture and Characteristic</td>
<td>51</td>
</tr>
<tr>
<td>Dimensions</td>
<td>54</td>
</tr>
<tr>
<td>Plastic versus Elastic Behavior</td>
<td>58</td>
</tr>
<tr>
<td>Characteristic Dimensions of Glassy Polymers</td>
<td>62</td>
</tr>
<tr>
<td>Particleboard</td>
<td>68</td>
</tr>
<tr>
<td>V. Experimental Procedure</td>
<td>68</td>
</tr>
<tr>
<td>Particle Production</td>
<td>70</td>
</tr>
<tr>
<td>Board Production</td>
<td>71</td>
</tr>
<tr>
<td>Specimen Preparation and Testing</td>
<td>80</td>
</tr>
<tr>
<td>Board Types</td>
<td>82</td>
</tr>
<tr>
<td>VI. Results</td>
<td>83</td>
</tr>
<tr>
<td>VII. Discussion</td>
<td>83</td>
</tr>
<tr>
<td>IL versus K\text{ic}</td>
<td>88</td>
</tr>
<tr>
<td>Fracture Toughness versus Resin Content</td>
<td>90</td>
</tr>
<tr>
<td>Fracture Toughness versus Particle Thickness</td>
<td>96</td>
</tr>
<tr>
<td>VIII. Conclusions</td>
<td>98</td>
</tr>
<tr>
<td>IX. Recommendations</td>
<td>106</td>
</tr>
<tr>
<td>X. Bibliography</td>
<td>106</td>
</tr>
<tr>
<td>Figure</td>
<td>Illustration</td>
</tr>
<tr>
<td>--------</td>
<td>--------------</td>
</tr>
<tr>
<td>1</td>
<td>Stress in the vicinity of an elliptic hole</td>
</tr>
<tr>
<td>2</td>
<td>The three modes of brittle fracture</td>
</tr>
<tr>
<td>3</td>
<td>The brittle fracture of glassy polymers</td>
</tr>
<tr>
<td>4</td>
<td>Mass density versus volume size</td>
</tr>
<tr>
<td>5</td>
<td>Mappings of classical and micropolar continua</td>
</tr>
<tr>
<td>6</td>
<td>Micropolar crack tip solution</td>
</tr>
<tr>
<td>7</td>
<td>Classical versus nonlocal constitutive equations and balance laws</td>
</tr>
<tr>
<td>8</td>
<td>Particleboard formed of relatively long (as compared to thickness) particles</td>
</tr>
<tr>
<td>9</td>
<td>Particle production procedure</td>
</tr>
<tr>
<td>10</td>
<td>Specimen cutting pattern</td>
</tr>
<tr>
<td>11</td>
<td>Fracture toughness specimen</td>
</tr>
<tr>
<td>12</td>
<td>$K_{IC}$ versus crack length using equations V-1, V-3 and V-4</td>
</tr>
<tr>
<td>13</td>
<td>$IB$ versus $K_{IC}$</td>
</tr>
<tr>
<td>14</td>
<td>$K_{IC}$ versus crack length--convergence to intrinsic flaw sizes</td>
</tr>
<tr>
<td>15</td>
<td>$K_{IC}$ versus resin content</td>
</tr>
<tr>
<td>16</td>
<td>$K_{IC}$ versus particle thickness (5% resin content)</td>
</tr>
<tr>
<td>17</td>
<td>$K_{IC}$ versus particle thickness (7.3% and 11.4% resin content)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Intrinsic strength of crystalline materials</td>
<td>47</td>
</tr>
<tr>
<td>2</td>
<td>Characteristic dimensions of PMMA and PS</td>
<td>59</td>
</tr>
<tr>
<td>3</td>
<td>Characteristic dimensions of particleboard</td>
<td>81</td>
</tr>
<tr>
<td>4</td>
<td>Average values of $IB$ and $K_{IC}$</td>
<td>82</td>
</tr>
</tbody>
</table>
ON THE PHENOMENA OF FRACTURE IN PARTICLEBOARD

I. INTRODUCTION

One purpose of engineering analysis is to provide information on the stress and strain that results from various loads that a given structural element is expected to face in its proposed lifetime. Design procedures are then provided to safely utilize a given material. Safety factors of design are a function of the reliability of relationships established between a material and its response as determined by the engineering analysis. In many cases these relationships are uncertain and safety factors are increased accordingly. Thus, a major objective of engineering science is to provide a better understanding of the material's behavior to improve utilization.

The strength of a given material is directly related to its internal constitution. This is true for all materials. Different size levels of internal structure can be expected to be important for different materials and phenomena. The important level of internal structure may depend on the particular mode of failure, whether it be yielding or actual fracture. If the mechanism of failure for a given mode takes place over areas which are much greater than the largest internal element, then the analysis becomes accurate without considering the anatomy of the material (treatment as a classical continuum).

Classical theories of mechanics have no intrinsic mechanism to account for the innerstructures of a material. Hence, relationships between the internal constitution of a body and its response to given stimuli are provided empirically by experimental analysis. Quantum and
statistical mechanic approaches provide an allowance for atomic effects but they remain non-tractable in many applications to real materials. Recently, alternative theories have arisen which allow for the basic mathematical treatment of continuum principles while incorporating the effect of innerstructures (Micropolar Theory) and their interactions (Nonlocal Theory) (Eringen, 1976).

The main objective of this study is to ascertain the use of a non-local fracture model for particleboard, a wood composite whose inner-structural dimensions have been shown to be empirically related to its strength (Maloney, 1977).

Particleboard has a tensile strength for loading perpendicular to the plane of the board which is far less (1/3 to 1/2) than the tensile strength perpendicular to the grain of clear wood (Wood Handbook, 1974). This remains true even though particleboard has a greater density. The strength reductions are a result of stress concentrations that arise in the composite due to two forms of discontinuity in structure. First, the stress distribution is unfavorably changed by voids and nonbonded regions that tend to concentrate the stress at their sharpest corners. Second, a combination of dissimilar materials, resin and wood particles, is expected to cause further stress concentrations at interfaces due to a difference in response to load.

The former effect lends itself to treatment via a fracture mechanics approach. The inherent strength of the structural elements of the board, the wood particles, is expected to be approximately the same as the wood from which they were formed, depending on the extent of damage done to the particles during processing. Yet, the resulting strength
of the board made from the particles does not approach the strength of solid wood. Hence, a model that represents the mechanism by which flaws in a board promote the propagation of cracks that eventually lead to catastrophic fracture would be important for predicting the behavior of the material to use in design procedures. The model may also introduce possible crack arrest hypotheses.

Questions arise as to where the bulk of the fracture occurs. We may conjecture that fracture initiates in voids or nonbonded regions of the board, but in what manner and where does the crack propagate? The alternatives are obvious: the wood particles, the wood-resin bond, or the resin. We may then ask what property of the resin would allow for an increase in fracture strength? Also, how can a given particle geometry promote or arrest fracture? These are all questions which, if answered, would allow for safer design, greater product utilization, and possible increases in product quality.
II. OBJECTIVES

A review of fracture mechanics will be presented with special emphasis on a nonlocal theory that removes the stress singularity at the crack tip. A nonlocal crack solution that has terms pertaining to the internal constitution of the material will be qualitatively evaluated for selected materials.

With the use of the nonlocal fracture model, a hypothesis will be presented describing the behavior of particleboard when subjected to tension perpendicular to the plane of the board. The model will relate the board's strength to its constitutive properties such as particle dimensions and strength of the composite elements (wood, wood-resin bond, resin).

Experimentally verify the proposed fracture model by studying the effect of particle dimension and resin content on the fracture toughness and internal bond (tension perpendicular to the plane of the board) strength of the board.
III. LITERATURE REVIEW

Classical Methods of Fracture Analysis

The first step of the researcher is to choose a mathematical model which best represents the problem in question. All methods of mathematical analysis are based on certain underlying simplifications that may or may not allow application to a particular behavior of a given material. All theories based on the classical continuum hypothesis stem from an assumption that the mass of a body is smoothly distributed in every infinitesimal element of volume (all geometric points of a body). In real materials the mass is distributed in discrete particles ( atoms, molecules, grains, etc.). As phenomena reaches the level of these particles, theories based on classical continuum will be less accurate.

The time scale is equally important. A material may behave elas-
tically and then slowly enter a plastic state over the time interval of a particular loading. During this period neither an elastic or plastic analysis will individually describe the materials response for the entire time interval. A combination of the two theories will give a better model, but it will not describe the zone between the two states in which the material contains both viscous and elastic elements under-
going transition.

The behavior of wood based materials is expected to be dependent on internal structures that may also be composites of smaller units. Hence, the use of classical continuum theories may not provide an accurate pre-
diction of a phenomena that originates at levels near the dimensions of
these units. The behavior of wood is also strongly time related and a model used to predict fracture of wood for short term loading is not expected to be accurate for long term loading unless the time effects have been incorporated into the model. With these important concepts in mind the models of classical fracture analysis will be reviewed.

Stress Analysis in the Vicinity of Flaws

The stress of an element is best defined as a force per unit area due to an influx of linear momentum. This definition is derivable from the equation of motion (force equals mass times acceleration for one-dimensional Newtonian physics). The calculation of stress requires the designation of an area that resists the applied loads. In many applications the stress is calculated as a function of the applied loads over the cross-sectional area of the element. This procedure yields an average value based on the assumption of a uniform stress distribution. In real materials stress distributions are not, in general, uniform. Nonhomogeneities (flaws) such as voids, non-bonded regions, or dissimilar materials will redistribute stresses in their vicinity, thus, resulting in points of high and low stress relative to the average value obtained by physical measurement and calculations based on the entire area. The average stress remains the important measurement for safe utilization of a material. However, for an understanding of the actual fracture mechanism, the distribution of stress in the vicinity of a flaw becomes essential.

Postulating that fracture initiates in the vicinity of the worst flaw (the flaw which results in the highest stress concentration), it
becomes evident that as the size of the element is reduced the statistical chance of the occurrence of a detrimental flaw at a given stress level is decreased. Hence, smaller elements will have greater strengths. This behavior, which is known as the size effect, is well documented and can be found in most design codes for materials, including wood.

If flaws were eliminated the overall strength of the material would approach the theoretical bond strengths. Flaws may arise in a material in a variety of ways. In wood flaws may occur naturally, such as voids in structure, or occur due to processing, such as drying checks. Discontinuities such as voids and nonbonded regions, or residual stresses at particle-adhesive interfaces are characteristic flaws of all composites including wood-based particleboard. Flaws may also appear during service due to surface scratches or degradation of material by the chemical environment. It remains the job of the material scientist to determine how defects in structure may be eliminated. In order to guide his research, the analysis of how a particular defect will affect the strength of a given material must be known.

The stress distribution in the vicinity of a flaw has been examined by many authors using methods of classical elasticity (Liebowitz, 1968, Vol. II). The problem of an elliptic hole in a two dimensional plate with a tensile stress ($P$) applied at infinity (Figure 1a) was first formulated by Inglis (1913). His solution for the maximum stress ($\sigma_y$)$_{\text{max}}$ that occurs at the tip of the major axis of the ellipse is given by

\[
(\sigma_y)_{\text{max}} = P(1 + 2 \frac{a}{b})
\]

where $a$ and $b$ correspond to the major and minor semiaxes respectively. If the ellipse is allowed to shrink into a crack then the solution may
Figure 1. Stress in the vicinity of an ellipic hole. A) The geometry of Inglis' problem. B) Stress concentration near the tip of a sharp crack (stress concentration = 1 corresponds to a stress equivalent to the applied stress, P).
be approximated by

\[(\sigma_y)_{\text{max}} = 2P \left(\frac{a}{\rho}\right)^{1/2}\]

where \(\rho\) represents the radius of the tip of the crack. From this equation it becomes apparent that as the radius becomes smaller the stress is more highly concentrated at the crack tip. It follows that as the crack becomes sharp \((\rho \to 0)\) the stress approaches infinity. Hence, the classical elasticity solution contains a singularity.* Figure 1b shows the distribution of stress in the vicinity of the crack tip. Note that the stress near the crack tip is many times greater than the average value.

Since the solution by Inglis, similar problems have been solved for different geometry and boundary conditions. Sneddon (1951), using the theory of Hankel transforms, gave solutions for the stresses and displacements in the vicinity of a penny-shaped (circular) crack for both two- and three-dimensional cases. In the three-dimensional case he obtained a state of triaxial stress in the vicinity of the crack. He noted that the two-dimensional solution contains a biaxial stress field corresponding to the plane of the plate.

Irwin (1957), using the semi-inverse method of Westergaard, solved the two-dimensional problem for both plane stress and strain. He pointed out that Sneddon's calculation of a biaxial stress field for the two-dimensional case was valid only for the plane stress problem and that the plane strain problem was actually one of triaxial stress in the vicinity of the crack. Irwin further showed for points near the

*Singularity is a term used to denote the breakdown of the mathematics for a portion of the complete solution.
outer boundary of a penny-shaped crack that the stresses and strains will correspond to a state of locally plane strain. Irwin's work led directly to many of the concepts of fracture mechanics as applied today.

Classical solutions also exist for many other problems including various crack geometries and orientations, different modes of loading, multiple rows of cracks, cracks between dissimilar materials, and others. All these solutions for a sharp crack retain the stress singularity at the crack tip.

**Brittle Fracture Criteria**

There are many modes of failure that a structural element may encounter during service. It may fail by actual fracture, in which case the fracture may be classified as brittle or ductile. Brittle fracture represents a situation in which the plastic deformation at failure is small. Brittle fracture is synonymous with the formation and propagation of brittle cracks. Ductile fracture, however, involves large amounts of plastic flow at failure. A structural component may also fail to perform in service even though the actual fracture of the element has not occurred. This would be the case if a loaded member undergoes excessive displacement and in the process interferes with other elements or perhaps extends past its elastic limit causing a permanent undesirable change of shape. There are other modes of failure. The mode that dominates for a given material is a function of the loading, the environmental effects such as temperature, and the duration of service.
In order to obtain a failure criteria that serves to predict a particular mode of failure, the response of the element to a system of loads in terms of stresses or displacements must be known. In many instances once this problem is solved, giving the stresses and displacements as a function of loads, structural geometries, and material constants, the failure criteria follows directly. For example, if the stress on a given element is required to remain below the yield stress, then the failure criteria arises by substitution of the material's yield stress into the equation for stress followed by rearrangement to give a maximum allowable load in terms of material constants. Hence, in many instances the important step in the formation of a failure criteria is the stress analysis. This is not the case in the study of classical brittle fracture. As seen in the past section the classical solution contains a singularity at the point of maximum stress, subsequently the fracture criteria cannot immediately follow.

Griffith (1921) realized the drawback of the classical solution and had to look elsewhere for the formulation of a brittle fracture criteria. He found an alternative approach using a direct consequence of the first law of thermodynamics which was termed, in his day, the "theorem of minimum energy." Simply stated, the theorem represents the condition that the decrease in potential energy of the system is a minimum if the stressed elastic body is in an equilibrium state. Griffith postulated that the equilibrium position, if such a position is possible under a given stage of loading, is one in which rupture of the solid has occurred. With this in mind he had to take into account the increase in potential energy due to the formation of new surfaces (fracture
surfaces). Using molecular considerations he stated that the potential energy, $U$, has the form

$$U = 4 \frac{1}{\gamma}$$

III-3

where $\gamma$ is the half crack length and $\gamma$ is the "surface tension" of the material.

The decrease in potential energy as the body attains equilibrium is equal to twice the strain energy, $W$. The strain energy was obtained by Griffith using Inglis' solution and may be expressed

$$W = \frac{(3 - \gamma) \pi r^2 p^2}{8 \mu}$$

III-4

where $\mu$ is the modulus of rigidity of the material, $P$ is the applied stress, and $\gamma$ is a function of Poisson's ratio, which takes on a different form depending on whether the condition is either plane stress or plane strain.

The total decrease in potential energy of the system becomes

$$W - U = \frac{(3 - \gamma) \pi r^2 p^2}{8 \mu} - 4 \frac{1}{\gamma}$$

III-5

The condition of minimum decrease in potential energy, which represents fracture, arises from the condition

$$\frac{\partial (W - U)}{\partial r} = 0$$

III-6

By combining Equations III-5 and III-6 a fracture criteria may be obtained which, as presented in 1921, was in error due to a slight mismatch of the solution for stress in Inglis' equation and the boundary conditions. Griffith (1924) later corrected the stress criteria to read

$$P^2 r = \frac{2 \gamma E}{(1 - \nu^2) \pi}$$

PLANE STRAIN

III-7
where $E$ is modulus of elasticity and $\nu$ is Poisson's ratio.

Griffith's work became the basis for subsequent research in the field of brittle fracture. The criteria as presented had one shortcoming when applied to real materials. The trouble comes in assigning a value to the surface energy term ($\gamma$). Surface tension is most easily measured for a material in the liquid state, however, in the case of many solids it becomes difficult to obtain an accurate value. Furthermore, Griffith's idealization will work only if the material retains a brittle nature even in the vicinity of the crack. If this is the case the use of the surface energy term in the equation is correct. Griffith showed this to be accurate for glass. If a material shows some plastic deformation in the vicinity of the crack, plastic work will add to the surface energy term making it larger.

Any material which can be cleaved has the capability of propagating brittle cracks for some combination of time and temperature effects. Ductility becomes dominant under conditions where cracks are prevented from forming or propagating (Kelly et al., 1966). There is no clear boundary separating the two phenomena and it is apparent for many cases that the fracture process involved a combination of the two.

Fracture Mechanics

Fracture mechanics may be termed as a discipline designed to give suitable fracture criteria for materials that fail due to the propagation of cracks. Many materials contain differences in behavior not initially accounted for in the formulation of Griffith's problem.
Plastic flow in the vicinity of the crack tip, as well as changes in geometry and loading, are among these differences.

By way of atomic consideration, Orowan (1955) postulated that the Griffith criteria was not only necessary but also a sufficient condition for completely brittle fracture. His major contribution however, involved modifications of Griffith's work for application to low carbon steels that show plastic deformation in a small region near the crack tip. These steels, although classified ductile, apparently still experienced "quite brittle fracture" under certain conditions. Orowan replaced the surface energy term found in Griffith's equation by a plastic work term that corresponds to behavior at the crack tip. He further showed that the energy concept used by Griffith is only applicable when the plastic flow is confined to a thin layer of material at the fracture surface and cannot be used for a description of ductile fracture.

G. R. Irwin (1948, 1957) was also responsible for modifications to Griffith's theory for brittle fractures that occur in the presence of energy release from the surrounding stress field (plastic work). He was responsible for the introduction of two related quantities: the strain energy release rate (\( G \)) and the stress intensity factor (\( K \)). Both were useful in providing measures of brittle fracture resistance in the presence of localized plastic effects because they are easily obtained by experiment. The strain energy release rate and stress intensity factor are functions of the test configuration, loads, crack length, and material constants. They may be measured for three separate modes of loading as shown in Figure 2. Of these the opening mode
Figure 2. The three modes of brittle fracture: A) opening mode (mode I), B) sliding mode (mode II), and C) tearing mode (mode III).
for plane strain. The critical stress intensity factor for the opening mode (fracture toughness, $K_{Ic}$) is defined as

$$K_{Ic} = \frac{P_{\text{max}}}{Y}$$

for the case of a crack in an isotropic medium with loads applied at infinity. For realistic loading conditions $K_{Ic}$ may be expressed as

$$K_{Ic} = P_{\text{max}} \sqrt{\pi I Y}$$

where $Y$ is a constant that is a function of the specimen's geometry. $Y$ values are used to give consistent measures of the stress intensity factor for varying experiment techniques and will be discussed further in later sections. Fracture toughness is a material property, a fact experimentally verified for many materials, and physically represents a measure of a materials resistance to crack extension. Fracture toughness for an isotropic material is related to the critical strain energy release rate by

$$K_{Ic} = \frac{\sqrt{E}}{(1 - \nu^2)}$$
for plane strain. In many instances critical strain energy release rates and fracture toughness values have been used as a measure of fracture in place of equations of stress which still retain the singularity.

Anisotropic Effects

A material is termed anisotropic for a given property if its behavior is directionally dependent. With the use of a complex variable approach the general crack tip stress field for an anisotropic body may be obtained (Sih, Paris and Irwin, 1965). For a rectilinear anisotropic body the stress singularity is of the same order \( r^{-1/2} \), where \( r \) is the distance from the crack tip) as the isotropic case.

If an anisotropic material has orthotropic (three mutual perpendicular planes of symmetry) or transversely isotropic (isotropic in one plane) symmetries and the crack naturally aligns itself in one of these planes, then it is possible to have three independent modes of deformation corresponding to the isotropic case. If this is the case the definition for the stress intensity factor remains the same as for the isotropic case. The strain energy release rate will differ because it is dependent on elastic moduli, and anisotropy introduces more material constants. However, if the crack does not lie in a plane of symmetry then the problem becomes one of general anisotropy. The stress intensity factor in this instance will no longer correspond to the isotropic case and instead takes the form

\[
K_I = \frac{P}{b} r^{1/2} \sin^2 \alpha
\]

where \( \alpha \) represents the angle of the crack to the direction of load (Sih et al., 1965).
Applications of Classical Fracture Mechanics

In the past 20 years the field of fracture mechanics has expanded to include many materials. Concepts of Griffith and Irwin, which were originally applied to glass and metal respectively, have grown to include application to such solids as polymers, ceramics, composites, rock, ice and bone. Theories of crack propagation from an inherent flaw are the basis for numerous accounts of the fracture mechanism in a given material. Many differences of opinion have raised questions that remain unanswered. It has become the problem of the newcomer in the field to decide which theory best serves his purpose. Liebowitz (1968-1972), in his seven volumes of Fracture, has provided an excellent overview.

The study of the strength of a material requires some relative measure of a material's resistance to stress. For fracture the stress intensity factor and strain energy release rate provide such a measure. Metals have shown the most application of the concepts of fracture mechanics and they have a standardized testing procedure for measurement of $K_{IC}$ (ASTM E399-74). Because many extensive studies have been made with metals they will not be reviewed at this time. For an overview of metals the interested reader is referred to volume VI of Liebowitz's series (1970).

Glassy polymers such as polystyrene (PS) and poly methyl methacrylate (PMMA) fracture in a brittle manner. Figure 3a shows the basic monomer unit of these two plastics. Berry (1961a, 1961b) was able to show that both PMMA and PS follow the Griffith fracture criteria. By introducing cracks at various depths and measuring the ultimate
Figure 3. The brittle fracture of glassy polymers: A) the monomer units of polystyrene and poly(methyl methacrylate); B) breaking strength versus crack length (Berry, 1961b).
strength of each cracked specimen he was able to plot the breaking strength versus the crack length (Figure 3b). Because Griffith's relation predicts an inverse square root relationship of stress versus crack length, a theoretical line was best fit to the data (the solid line in Figure 3b). With this plot he was able to obtain two important material parameters, fracture surface energy and intrinsic flaw size.

At the point of deviation from the Griffith's relation (see arrow in Figure 3b), it may be postulated, because the experimental data flattens out, that the cracks that were induced into the plastic were smaller than the intrinsic flaws of the material. This was further substantiated by the fact that fracture did not always occur at the induced crack for small crack sizes. The size of the intrinsic flaw was found to be 0.043 and 0.004 inch for PS and PMMA respectively.

After the Griffith relation had been fit to the data, a value for the surface energy term may be obtained by direct calculation. This value was many times greater (two to three orders of magnitude) than would be expected based on theoretical considerations. The reason for this discrepancy is due to the plastic flow in the neighborhood of the crack (this was pointed out earlier in the discussion or Orowan's work on metals). In polymers the plastic flow is readily seen on the fractured surface. Although PS and PMMA are both amorphous polymers, the fracture surface shows evidence of orientation and hence, flow has occurred. The craze phenomena, which will be discussed later in detail, is further evidence of a plastic phenomena in the vicinity of flaws.
From Berry's work it was shown that some amorphous polymers behave in a brittle manner even in the presence of isolated plastic flow. Because polymers behave viscoelastically under long term loading the factor of time is expected to further affect the amount of plasticity. More extensive accounts of the plastic nature of the brittle fracture were undertaken in order to better describe the fracture mechanism of glassy polymers (Rosenfield and Kanninen, 1973; Marshall et al., 1970; Liebowitz, Vol. VII, Ch. 2, 1972). These works considered both the effects of time until fracture and the crack tip plasticity in an attempt to establish a more general fracture criteria for amorphous glassy polymers.

Wood, an anisotropic material, may be termed orthotropic if the geometric axes of a given problem coincide with the orthotropic axes (Jayne and Suddarth, 1966). For crack propagation this occurs when the crack lies in one of the planes of orthotropic symmetry (radial, tangential, or longitudinal). For a general orthotropic material, this would result in six different directional systems of crack propagation along axes of symmetry. In wood these may be designated by two indices, the first indicating the normal to the crack plane and the second referring to the direction of crack propagation (e.g. RL designates a crack perpendicular to the radial plane which travels in the longitudinal direction). Of these six systems, four (RL, RT, TL, TR) propagate cracks resulting in relatively planar fracture surfaces and the remaining two (LR, LT) give very rough fracture surfaces which include crack propagation normal to the original crack (parallel to the grain). Because of the rough fracture surface as well as high values of tensile
strength compared to other planes the application of fracture mechanics to the LR and LT directions is limited and usually ignored (Schniewind and Pozniak, 1971). Evaluation of $K_{IC}$ for different systems of crack propagation in wood showed that the TR and RT values are approximately equivalent and the RL was higher than the TL quantity. This is attributed to the rays in the RL system which act as crack arrestors (Schniewind and Centeno, 1973). Because $K_{IC}$ depends on the direction of propagation, the natural origin of flaws in wood becomes important. Checks, initiated by drying stresses, are commonly found in the RL plane (Panshin and deZeeuw, 1970).

The application of fracture mechanic concepts to wood may be traced back to the early 60s. Atack et al. (1961), using the Griffith equation as modified by Orowan, found an effective surface energy for green wood parallel to the grain. This effective surface energy was equal to the sum of the surface free energy (Griffith's surface tension) and plastic work (Orowan's modification). In this initial study by Atack, scanning electronmicrographs were taken that showed the roughness of the fracture surface. The actual fracture surface area was observed to be greater than the planar cross-sectional area. Conclusions pointed out that absolute values of effective surface energy are not obtainable from this method until an accurate way of measuring the fractured surface area is developed.

As stated in an earlier section (Anisotropic Effects), the isotropic stress intensity factor and the strain energy release rates have analogous definitions when applied to orthotropic materials as long as the crack is in a plane of symmetry. In fact the stress intensity
factor remains unchanged and the strain energy release rate changes only because more elastic constants exist for an orthotropic material. Wu (1967) experimentally verified this fact for balsa wood. He also obtained a fracture criteria for combined mode I and II loading by means of an empirical method.

Stress intensity factors for wood are dependent on methods of measurement (Schniewind et al., 1971, 1973) and, in some instances, on the specimen geometry (Barrett, 1976). There has been some discussion of the reliability of stress intensity factors taken for a specimen with an artificially induced crack versus the values obtained from analysis of real flaws in wood (Schniewind and Lyon, 1973). $K_{IC}$ has been shown to be statistically dependent on crack length in some investigations (Schniewind and Lyon, 1973) but not in others (Porter, 1974; Johnson, 1973). Fracture toughness values, however, are generally accepted to be material parameters and, hence, should not depend on crack length. When the field of fracture mechanics for wood expands and absolute values are required for design, as is the case for metals, then these discrepancies will require a closer look.

Because wood is viscoelastic its strength properties are expected to depend on temperature, moisture content and time. An increase in temperature or moisture content (up to about 18%) will tend to increase the critical strain energy release rate (Porter, 1964). This is to be expected because plastic flow, a competing phenomena to brittle fracture, becomes more prevalent at higher temperatures and moisture contents. This results in a more favorable redistribution of stresses in the vicinity of a flaw. The effect of load duration on fracture
resistance is best explained by a description of the associated crack growth kinetics (Mindess et al., 1975, 1976). Because the growth of cracks is time dependent the chances of a microcrack growing to critical dimensions will increase with time. This phenomena, termed "static fatigue," allows application of brittle fracture mechanics to time dependent behavior.

Calculation of an inherent flaw size for wood has been obtained by a number of investigators using various methods. Values of approximately 0.10 to 0.15 inch were obtained for Douglas-fir wood (Schniewind et al., 1971, 1973; Mindess et al., 1975; Lei, 1978). This quantity has no relation with the wood structure, hence, defects which arise during the process of machining or drying may be the critical flaws. This agrees with the findings of Schniewind and Lyon (1973) who stated that checks were the most important flaw among other considerations that included knots, resin streaks, pitch pockets and pith.

Application of fracture mechanics to wood based composites has been limited. There has been some successful work with wood adhesive bonds (White, 1977; Mijovic and Koutsky, 1977a) and oriented flakeboard (Lei, 1978). These studies have shown that fracture mechanics may provide a useful tool for the description of the tensile strength perpendicular to the plane of the board.

**Structural Considerations**

In the past sections we dealt with the application of classical elasticity concepts that were based on the continuum hypothesis. These methods provided good insight of the macroscopic response of materials
under the influence of local stress concentrators. By use of thermodynamic considerations a failure criteria as well as measures of the fracture resistance were established. The study of fracture has progressed past its measurement and prediction to a stage where further advancement in the field requires methods of prevention. The question now arises of whether or not the classical methods of analysis will provide answers to these new challenges.

The starting point for the prevention of a phenomena such as fracture must begin in the understanding of the actual mechanism involved. How a crack spreads through a material and what controls the size of the initiating flaw is no doubt a function of the microstructure. The continuum hypothesis erases the effect of innerstructures; hence, any incorporation of their effect must arise from other considerations or alternative theories. This section will present various methods of accounting for the effect of the internal characteristics of a material on its macro response. The question arises as to whether or not these considerations evolve at the atomic, molecular or higher level of structure. The answer to this question will vary among materials and insight may be obtained on the basis of the bond strengths at these levels.

**Intrinsic Strength**

The intrinsic strength is traditionally defined as the cohesive bond strength between the subunits of structure found in the material. If these subunits are different in chemical nature, as is the case in composites, the bond between them is termed adhesive. A composite consisting of material A and material B has two cohesive bonds (A-A and B-B)
as well as an adhesive bond \((A-B)\). If rupture is given an equivalent chance to start between any of these three bonds intuition tells us it will proceed through the weakest bond. In this study the term intrinsic strength will be synonymous with the strength between subunits which are broken at the onset of critical crack growth.

Because flaws cause a reduction in strength, the overall strength of an element is lower than the intrinsic value. As the size of an element is shrunk, resulting in removal of some flaws, its strength approaches the intrinsic strength. However, because it would be very difficult to remove all the flaws of a material, direct measurement of this strength will not evolve from experiment alone.

If the stress at the tip of a crack were known as a function of the applied loads and material parameters, it would be possible to obtain values of the intrinsic strength from this relationship. Simple substitution of the ultimate strength of the element for the applied stress term results in a stress at the tip of the crack equal to the intrinsic strength. In the classical elasticity solution the stress singularity blocks the attainment of the intrinsic strength. As pointed out by Griffith (1921, p. 166), regarding a cracklike flaw in a material,

\[
\text{The molecular attractions across such a crack must be small except very near its ends, it may therefore be said that the application of the mathematical theory of elasticity on the basis that the crack is assumed to be a traction-free surface, must give the stresses correctly at all points of the body, with the exception of those near the ends of the crack.}
\]

Griffith realized that the sharpness of the crack was a mathematical idealization and in reality there was a finite radius associated with it. With this concept in mind he presented a first order approximation for the intrinsic strength \((t_c)\) of a material as
where $P$ is the overall applied stress on the body, $I$ is the half crack length and $\rho$ is the radius of curvature of the crack tip. Calculation of this value is difficult since the radius of curvature of the crack tip in a material is not easily obtained.

Examination of interatomic force and potential curves can result in calculation of the theoretical cohesive strength at the atomic level. The shapes of these curves are dependent on the bond type (covalent, ionic, metallic, or secondary). Orowan (see Lawn and Wilshaw, 1975) obtained an order of magnitude estimate for the theoretical cohesive strength of materials whose force separation curve may be approximated by a half sine curve. The relationship obtained was

$$\sigma_{\text{max}} \approx \left( \frac{E \gamma}{a} \right)^{1/2}$$

where $E$ represents the modulus of elasticity, $\gamma$ the surface energy, and $a$ the spacing between atomic planes. This theoretical strength ($\sigma_{\text{max}}$) is synonymous with the intrinsic strength ($t_c$) only if fracture occurs between the atomic units of the material. This may be the case in perfect crystals. Relationship III-15 can be further reduced to

$$t_c \approx 0.1 E$$

which represents a crude approximation for any solid (Bartenev and Zuyev, 1968). More exact values for the theoretical cleavage strength as well as values for the theoretical shear stress are given by Kelly (1966).

At this point it is should be reemphasized that a material under stress may flow apart rather than cleave; therefore, the intrinsic
strength in tension is not the only important consideration for the general fracture of materials. In the process of flow in crystals the theoretical shear stress, $\tau_{\text{max}}$, becomes the important quantity. Ductile and brittle behavior are competing phenomena and in a given fracture process one may dominate in the presence of the other. The most common example is the plastic zone at the crack tip in brittle cleavage. In general, however, materials tend to fracture on the bulk level in either a ductile or brittle manner. Kelly et al. (1966, 1967) proposed that whether or not brittle instead of ductile fracture occurs is a function of the ratio of theoretical tensile to shear stresses ($\sigma_{\text{max}}/\tau_{\text{max}}$). In an admittedly "crude criterion" they state that if $\sigma_{\text{max}}/\tau_{\text{max}}$ is large (greater than 10) then these materials will fail with a large amount of plastic flow and they are considered ductile. If the ratio is close to one they postulate that the fracture will be primarily brittle, at least at low temperature. At best, Kelly's criterion provides a rough approximation for the prediction of the expected fracture behavior in perfect crystals but other factors should be considered. As pointed out by Tyson (1966) the $\tau_{\text{max}}$ is expected to decrease under the influence of a normal stress. Rice and Thomson (1974) pointed out that even though the shear stress at the tip of the crack exceeds its theoretical maximum, bulk flow will not occur because $\tau_{\text{max}}$ is not exceeded in surrounding areas. Furthermore, if $\tau_{\text{max}}$ is exceeded only in local areas, flow may not occur, or may occur only to a limited extent, even in these areas because the atoms at the flaw are trapped from flow by atoms outside the influence of the stress concentration. Hence, it is postulated that $\tau_{\text{max}}$ must be exceeded in a larger area in order to
result in "blunting" of the crack tip (e.g. transition to ductile fracture).

The calculation of theoretical strengths in polymers becomes more difficult because of the lack of a perfect crystal structure. It has been generally accepted that $t_c$ does not reach the value of the strength of a carbon-carbon bond due to considerations of weaker secondary bonds found between the molecules. In fact, many real materials consist of higher units of structure and may not be bonded together at strengths attainable at the atomic levels. Hence, it is reemphasized that the intrinsic strength, as used in this study, is the bond strength between units that are severed during crack propagation. This may be interchangeable with the atomic theoretical strength only in the case of perfect crystals.

**Empirical Relations**

Because most innerstructural considerations are absent in the application of a classical continuum, researchers using such analysis are forced to look to experiments for answers rather than verification. Experiments become the critical step in the study of materials. Empirical equations are formed to describe the behavior as a function of structural variables. These relationships must be looked at with caution because physical insight is not inherent in their formulations and their usefulness is dependent on the ability of the researcher to incorporate the physical meaning. This is not to say that the researcher is unable to interpret the results but in many instances behavior which can be predicted by an organized theory is not intuitively obvious.
Description of past studies of empirical relations will be limited at this time and only a qualitative discussion of the effects of microstructure on the behavior of polymers including wood will be given. The process of fracture for all real materials is expected to be dependent on the size of internal structure because rupture travels between these structural units.

Crazing, a phenomena associated with brittle fracture in PS and PMMA, appears to be a function of supermolecular units, termed fibrils, or domains, found in these plastics (Hull, 1972). It has long been known that the molecular motion of subunits or particles in polymers have an effect on their brittle behavior (Saver, 1970). For epoxy resin the fracture surface is dimpled or nodular leading to the conclusion that the size of the supermolecular structure controls the fracture mechanism (Mijovic and Koutsky, 1979a and b). The fractured surface of nylon 66 shows units of supermolecular structure on the order of one micron (Kireenko et al., 1972).

The dependence on the size of the structural units in polymers is not restricted to brittle fracture. There is evidence that the process of melt flow of polyvinyl chloride (PVC) involves the slippage of resin particles past one another (Collins and Metzger, 1970). Melt flow rate was shown related to the diameters of these particles (Berens and Folt, 1968). Studies of fracture surfaces of cured resins showed particles that were formed during polymerization retained their identity after such melt flow processes as milling and extrusion. The list of relationships between subunits and strength behavior in polymers is by no means complete and further examples will be given in later sections.
Wood has many levels of structure beginning with the basic monomer unit in cellulose, anhydro-D-glucopyranose. In the crystalline regions of cellulose the polymer chains align forming theoretical groupings called unit cells. The unit cell is not an independent level of structure and the smallest aggregation of cellulose molecules is termed the elementary fibril. Elementary fibrils bond together to form units known as microfibrils that are observable by an electron microscope. Microfibrils are the building blocks for layers of the cell wall of fibers, tracheids, and other types of cells found in wood. Therefore, wood itself is a composite at many different levels of study and it becomes obvious that the fracture of the cellulose chain is not necessary for the bulk fracture of wood.

The strength of wood has been shown to be experimentally related to many variables including specific gravity, moisture content, grain orientation, time, temperature and type of defect structure to name a few. Many studies of these effects have strong theoretical backing stemming from continuum theories. The effect of microstructures on bulk strength, however, requires a stronger theory. Experiment cannot be expected to provide quantitative data in this instance because the variance of the structure of a natural product becomes difficult if not impossible.

Wood composites such as particleboard have undergone vast amounts of experimental analysis to relate their structure to resulting strength properties. In a manufactured composite such as particleboard the particle geometry, resin level, and other intrinsic characteristics can be varied in controlled experiments and their effects empirically
determined. Maloney (1977) presents a good review of such studies. The property most important to this study, tensile strength perpendicular to the board (internal bond strength), shows an increase when thick short particles and higher resin percentages are used.

**Alternative Theories**

Quantum mechanics, lattice dynamics, finite element and statistical methods are all theories that have been adopted by researchers to consider the effect of the structure of materials in an organized mathematical analysis. Many problems incorporating quantum effects become non-tractable in the description of macro response. The level of study in these instances has been so reduced that numerous interactions will not allow combination to give prediction of bulk behavior.

Applications of lattice dynamics requires a lattice structure. Real materials, unlike perfect crystals, contain too many impurities, voids, and imperfections to allow accurate solutions from lattice considerations. Lattice dynamics also requires knowledge of the forces between subunits as well as their range. In many cases these force distributions become too complicated. Finite element analysis provides more tractable problems but there is a question of whether it physically represents the structure of a material.

Statistical methods have found some applications in the study of fracture. Statistical theories must be distinguished from statistical mechanic approaches. Statistical theories are able to explain the scatter of results, characteristic of fracture measurements, as well as
the size effects involved in going from test specimens to actual materials in service. This may be done by invoking a flaw hypothesis in which a given form of statistical distribution of flaws is prescribed. A fracture criteria may evolve through one of three statistical models (uniform defect, weakest link, or classical bundle) based on distribution functions (Liebowitz, Vol. II, 1968). The form of the function used must arise from physical considerations instead of experiment. These statistical theories are limited in application due to a lack of understanding of the flaw distribution in most materials.

Statistical mechanic approaches are based on plausible molecular considerations coupled with experimental results. For this reason they are sometimes labeled semi-empirical methods. They have been successful in predicting time and temperature effects but fall short in the description of stress distributions near failure because they are unable to couple the atomic force considerations with bulk phenomena. Bueche (1957, 1958) has done some work in this area to predict time to failure for plastics.

From the above discussion it becomes obvious that classical continuum approaches dominate the study of fracture. Continuum methods involve much simpler analysis and in many instances the bulk behavior is accurately described without structural considerations. However, one must realize that in order to prevent or detour fracture the actual mechanism must be described in terms of material characteristics.

Further insight into the study of fracture can be given by incorporation of certain physical considerations at the tip of a crack in a continuum model. Barrenblatt (1962), Khristianovich (1955), and
Dougdale (1960) introduced cohesive force functions at the crack tip to eliminate the classical singularity (see Eringen, 1977). Their efforts were successful to an extent. However, the incorporation of these cohesive effects arrived after years of frustrating attempts to understand fracture mechanisms because the mathematical theories used did not contain structural considerations.

**Micropolar Theory**

Classical continuum theories immediately erase the effect of microstructure with the application of its fundamental hypothesis

\[
\rho = \lim_{\Delta V \to 0} \frac{\Delta M}{\Delta V} \tag{III-17}
\]

where \(\rho\) is the mass density and \(\Delta M\) the total mass contained in the volume \(\Delta V\). From this relationship it is postulated that \(\rho\) is independent of the size of \(\Delta V\). The value of \(\rho\) is allowed to vary as a function of position (i.e. nonhomogeneous material) in a continuum but as a volume element of given density is shrunk to an infinitesimal point \((\Delta V \to 0)\), its density must remain constant. Figure 4 shows the accuracy of this restriction for a general homogeneous material. The classical theory is accurate for phenomena occurring at a level of \(\Delta V^*\) or higher. The classical singularity as well as the dependence on microstructure shows that fracture must occur at levels below this \(\Delta V^*\). Micropolar theory makes certain microstructural allowances to extend the applicability of continuum theory.

The question arises as to how one may incorporate the effects of microstructure and still have the mathematical simplicity of a continuum.
Figure 4. Mass density versus volume size
In a classical continuum the geometric points of the body as they move from an undeformed \((\mathbf{X}, \text{reference frame})\) state to a deformed state \((\mathbf{x})\) are described by a mapping

\[
\mathbf{x} = \mathbf{x}(\mathbf{X}, t)
\]

where \(t\) is time. In this mapping the geometric points are allowed to translate causing the body as a whole to translate, rotate, and deform. However, in many real materials the points (microstructures) have directional properties associated with them (e.g. a wood fiber is strongest along its longitudinal axis), hence, the rotation as well as the translation of these points becomes important. As stated above, the continuum body as a whole is allowed to translate, rotate and deform due to geometric point translations and within this concept lies the basis for the incorporation of point rotations. Restricting the fundamental hypotheses of a continuum to microstructures allows them to undergo translations, rotations, and deformations independent of the body as a whole. The resulting body which is comprised of such continuum units may be referred to as a microcontinuum (see Figure 5).

This idea of small continuum units within the body is used as a visualization to relate continuum and microcontinuum ideas. Actually the geometric points remain part of the microcontinuum theories but added degrees of freedom are incorporated that allow the behavior of microstructures to be added. Thus the geometric points of classical continuum mechanics, which have the physical property of mass, will be distinguished from microelements (the geometric points of microcontinuum theories) that have additional physical properties (inertia, etc.).
Figure 5. Mappings in classical and micropolar* continua. (→ represents a vector notation).

*If the microcontinuum units support only translations and rigid rotations then the body is referred to as a micropolar continuum. Most materials including wood, granular, and fibrous materials are known to consist of "rigid" molecules which possess micromoment of inertia.
With these new degrees of freedom given to the micropolar body many revisions that incorporate microstructural effects will follow. Solution of the micropolar field equations involve determination of an orientational field as well as a displacement field. A new balance law analogous to the conservation of mass, involving the property of inertia, known as the conservation of microinertia is obtained. Couple stress within a body, which disappears at a point in classical continuum theories because the moment arm of the force couple is reduced to zero under the continuum hypothesis (as ΔV → 0), is retained in micropolar theory. With this added allowance for couple stress the balance law of moment of momentum no longer states that the stress tensor is symmetric. There are many other differences between the classical and micropolar theories but since the micropolar theory is more general it will reduce to the classical form if the microstructure is unimportant to the problem. A complete description of the mathematics involved in the micropolar theory is found in Continuum Physics Volume IV as edited by Eringen (1976).

Micropolar theories have found vast application since the mid-60s and should be distinguished from the indeterminate couple stress theory of Mindlin and others (Perkins, 1972). Materials which may be considered micropolar include composites, suspensions, polymers, blood, liquid crystals, wood, and porous media to name a few. However, an absolute classification of micropolar materials should be avoided since the phenomena under study becomes equally important. For instance if the fluid flow through a large diameter pipe is well described as a classical continuum it does not mean the same fluid will behave this way
under capillary flow. As the pipe diameter shrinks whatever structural units exist in the fluid become more important to its flow. Even a fluid such as water may become micropolar under certain flow conditions.

Inglis' (1913) original problem of an elliptic hole in an infinite medium has been solved by Kim and Eringen (1973) for a micropolar elastic plate. The solution for the stress at the tip of a crack retains the classical singularity. Hence, even with certain structural considerations, micropolar theory has not incorporated effects which would erase the prediction of infinite stress at the crack tip. However, some very important features that were not present in the classical solution should be noted. Figure 6a shows the micropolar solution as compared to the classical equivalent for the stress distribution near the crack tip. Note the dip in $\sigma_{yy}$ away from the crack tip for the micropolar solution. This is accompanied by an increase in $\sigma_{xx}$ and couple stress (the couple stress distribution is not shown on the graph). In other words when at a point near a flaw in a micropolar media, a couple stress is present which serves to release stress in one direction and redistribute it in another.

Let us examine the above results in the presence of a micropolar media. Polystyrene (PS) and polymethyl methacrylate (PMMA), as discussed earlier, are glassy, amorphous polymers that remain isotropic on the bulk level. The units at the microscopic dimensions are anisotropic whether these units are the molecules themselves or aggregates of molecules termed domains and are, therefore, best represented as micropolar media. In these plastics, crazes will form at the tip of a flaw (Hsiao and Saver, 1950; Menges, 1972). A craze is a localized
Figure 6. Micropolar crack tip solution: A) the stress distribution in the vicinity of a crack for a micropolar elastic medium; B) the craze phenomena in amorphous thermoplastics.
region of plastic deformation in which the polymer chains align in the
direction of maximum tensile stress (see Figure 6b) (Hull, 1972).
Crazing is a stress activated process in which localized molecular
motion aligns the polymer units in their strongest direction. In the
process of alignment voids form between the polymer units making their
presence obvious.

The question of the mechanism of craze formation may be explained
in part by the micropolar solution just discussed. In the presence of
a couple stress the molecules may conceivably begin rotation to align-
ment. The stress concentration in the weak direction (perpendicular to
their axis, $\sigma_{xx}$) may explain void formation. Crazes are not restricted
to the tips of cracks in these polymers; however, couple stresses may
be present allowing redistribution of stress at other flaws. Crazing
involves plastic deformation and an elastic solution cannot be expected
to explain it entirely. However, in order for plastic flow to end in
rotational alignment the yield stress in this rotational direction
must be exceeded and the micropolar elastic solution can be used to
explain the initiation process.

Nonlocal Theory

With the advent of electromicroscopy and a greater understanding
of the anatomy of materials, physical insight of material behavior has
expanded requiring more powerful mathematical models. Micropolar
theory has provided the first step in this direction. However, refer-
ing back to Figure 4 we see that micropolar theory is not accurate
below a certain volume limit. Below this limit there is the factor of
molecular and atomic interactions. Neither classical or micropolar theories account for these effects because their stresses, strains, or other physical measures at a point are independent of neighboring points.

The sub-bodies of a material, whether they are atoms, molecules, grains or fibers, are attracted to one another by some form of interatomic force. An internal characteristic length, $\lambda$, may be associated with the dimensions of these sub-bodies (e.g. grain diameter, atomic distance, pore size, etc.). There is also an external characteristic length, $l$, associated with external stimuli (e.g. stress waves) or geometric discontinuities (e.g. cracks). Whether or not the interatomic forces become important is a function of the ratio of $\lambda/l$. If the ratio is much less than one then the internal sub-bodies are excited as a group under the influence of an external stimuli. If this is the case the individual characteristics of the sub-bodies become unimportant and their average effect, which arrives through classical concepts, is sufficient (Eringen, 1977).*

Nonlocal continuum mechanical theories have arrived within the last decade and have provided a mathematical idealization of neighboring effects (atomic or grain interactions). The term nonlocal may appear to be a contradiction since all balance laws must undergo a localization process (making them valid for every point of the body) which provides forms suitable for application. The term nonlocal in this context is synonymous with integro-differential field equations. The integrals arrive through the constitutive equation due to the

*The internal and external time scales of a body are as important as the characteristic dimensions and may give further enlightenment.
summation of neighboring effects.

Consider the isotropic forms of Hooke's law as compared to the non-local counterpart (see Figure 7a). In the nonlocal constitutive equation the dependence on other points of the body will be brought in by an influence function (Eringen, 1976). Because points nearer the point in question are more influential the influence function should have an attenuating nature. For instance a Dirac delta function provides such a form (Sneddon, 1951).

Nonlocal and classical continuum theories differ in the localization of their balance laws. As mentioned above all balance laws must be localized in order to allow substitution of the constitutive equations, hence, formation of field equations. In classical continuum mechanics this localization follows from the postulate that the global law, which is valid for the body as a whole, is valid at every point and thus the integral may be dropped. Nonlocal continua have the same global balance laws but the localization allows for effects which vanish for the body as a whole but may be present locally at a point. Figure 7b shows an example using the conservation of mass (Eringen, 1976). By incorporation of a "localization residual" we see that mass is conserved on a global scale but possibly not at a point, as is the case in many chemically reactive processes. From this example one can see that nonlocal theory provides a mathematical tool capable of handling more complex physical problems.

Application of nonlocal field theories has been limited due to their recent arrival. The complexity of problems that have been solved by this new approach provide an indication of its strong potential. A
Figure 7a. Stress Constitutive Equations

**Classical**

\[ t_{ij} = \lambda_1 \delta_{ij} + 2\mu l_{ij} \]

where \( i \) and \( j \) can take on the values 1 to 3 and repeated indices involve a summation of 1 to 3

\( t_{ij} \) is the stress tensor

\( l_{ij} \) is the strain tensor

\( \lambda \) and \( \mu \) are material constants (Lame's coefficients)

\( \delta_{ij} \) is the Kronecker delta which = 0 if \( i \neq j \)

= 1 if \( i = j \)

**Nonlocal**

\[ t_{ij} = \lambda_1 \delta_{ij} + 2\mu l_{ij} + \int_V (\lambda' l_{ij} + 2\mu' l_{ij}) \, dv' \]

where ' denotes dependence on other points of the body.

---

Figure 7b. Localization of balance laws

**Global Form - Nonlocal and Classical**

\[ \int_V \left( \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) \right) \, dv = 0 \]

where \( \rho \) is the mass density and \( \mathbf{v} \) is the velocity vector

\( v \) represents the volume of the body

\( t \) is time

\( \nabla \) is the gradient operator

**Local Forms**

**Classical**

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \]

**Nonlocal**

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = \hat{\rho} \]

where \( \hat{\rho} \) is a localization residual in which \( \int_V \hat{\rho} \, dv = 0 \)
recent review article by Eringen (1977) presents solutions to a number of problems by nonlocal methods.

Eringen et al. (1977) obtained the solution to the nonlocal equivalent of the two-dimensional crack problem. With the use of nonlocal theories the classical stress singularity at the crack tip disappears. This opened the door to a fracture criterion based on a maximum stress hypothesis that is given in terms of material parameters such as intrinsic strength and characteristic dimensions.

The stress at the tip of a crack \( t \) in a material whose characteristic dimension may be given by an atomic distance, \( a \), is given by

\[
t = C \left( \frac{t}{a} \right)^{\frac{1}{2}} t_0
\]

where \( t_0 \) represents the applied uniform tension, \( I \) is the half crack length, and \( C \) is a stress concentration factor equal to 0.73. From this relationship it becomes obvious that the stress at the crack tip is infinite only when \( a \) approaches zero, which is the case with the classical continuum limit (\( \Delta V \rightarrow 0 \)). When the applied stress, \( t_0 \), reaches the ultimate strength \( (P_{\text{max}}) \) of the material a natural fracture criterion (based on a maximum stress hypothesis) results in which the stress at the tip of the crack equals the intrinsic strength \( (t_c) \). The resulting criterion is given by

\[
P_{\text{max}}^2 \frac{I}{t_c^2} = \left( \frac{a}{2C^2} \right) t_c^2
\]

The intrinsic strength would be equivalent to the theoretical cohesive strength, \( \sigma_{\text{max}} \), for those materials (perfect crystals) where the characteristic length (the important structural dimension) is the atomic distance.
Griffith's relationship, as stated previously and rewritten for convenience, is

\[ \frac{P^2}{I_{\text{max}}} = \frac{2 E}{(1 - \nu^2)} \gamma \]

where \( \gamma \) is surface energy, \( E \) is the modulus of elasticity, and \( \nu \) is Poisson's ratio. If equations III-20 and III-21 are set equal to each other a relationship for \( t_c \) is given by

\[ t_c = 0.82 \left( \frac{E \gamma}{a(1 - \nu^2)} \right)^{\frac{1}{2}} \]

Now compare equation III-22 to Orowan's equation for cohesive stress which, as stated previously, is

\[ \sigma_{\text{max}} = \left( \frac{E \gamma}{a} \right)^{\frac{1}{2}} \]

Eringen's value for cohesive stress would be less than Orowan's for most Poisson ratios. Since it is generally accepted that Orowan's relation is an overestimate (Kelly et al., 1967), Eringen's solution may provide a better value.

If the classical definition of fracture toughness (III-10) is used with equation III-20 the nonlocal equivalent,

\[ K_{Ic} = 1.72 \, t_c a^{\frac{1}{2}} \]

is formed. Clearly the nonlocal equation for \( K_{Ic} \) defines it as a material property, therefore independent of crack length. Fracture toughness has been experimentally accepted as a material property but the classical equation itself which contains crack length gives no indication that this is true.

Eringen's criteria of fracture can be verified for some crystals by comparing the calculated value of \( t_c \) from equation III-22 against atomic data (Table 1).
Table 2. Intrinsic strength of crystalline materials.

<table>
<thead>
<tr>
<th>Crystal</th>
<th>$\gamma$ CGS</th>
<th>$\mu$ CGS</th>
<th>$\nu$</th>
<th>$a$ $\AA^3$</th>
<th>$t_c$ (x10^{-11})</th>
<th>Atomic Models $t_c$ (x10^{-11})</th>
</tr>
</thead>
<tbody>
<tr>
<td>C$^5$ (diamond)</td>
<td>5400</td>
<td>50.9</td>
<td>0.068</td>
<td>1.54</td>
<td>16.12</td>
<td>20.5</td>
</tr>
<tr>
<td>MgO</td>
<td>1200</td>
<td>11.57</td>
<td>0.173</td>
<td>4.20</td>
<td>2.33</td>
<td>3.7</td>
</tr>
<tr>
<td>NaCl</td>
<td>250</td>
<td>1.63</td>
<td>0.248</td>
<td>5.63</td>
<td>0.36</td>
<td>0.43</td>
</tr>
<tr>
<td>W</td>
<td>1700</td>
<td>16.0</td>
<td>0.278</td>
<td>2.73</td>
<td>4.37</td>
<td>8.6</td>
</tr>
<tr>
<td>$\alpha$-iron</td>
<td>1975</td>
<td>6.92</td>
<td>0.291</td>
<td>2.48</td>
<td>3.25</td>
<td>3.0</td>
</tr>
<tr>
<td>Zn$^5$</td>
<td>575</td>
<td>3.83</td>
<td>0.333</td>
<td>2.66</td>
<td>1.30</td>
<td>0.38</td>
</tr>
<tr>
<td>Cu</td>
<td>1688</td>
<td>4.05</td>
<td>0.324</td>
<td>2.55</td>
<td>2.32</td>
<td>3.9</td>
</tr>
<tr>
<td>Ag</td>
<td>1135</td>
<td>2.56</td>
<td>0.354</td>
<td>2.88</td>
<td>1.46</td>
<td>2.4</td>
</tr>
</tbody>
</table>

$^1\gamma$, $\mu$ and $\nu$ are taken from Rice and Thomson (Table I, 1974).

$^2\mu = E/2(1+\nu)$.

$^3$From Kittel (Table V, 1971).

$^4$t$_c$ from Lawn and Wilshaw (Table 7.1, p. 160, 1975).

$^5$As reported in Eringen (1977).
Nonlocal theory shows its superiority to classical treatises in that the solution of the stress field in the vicinity of the crack gives a finite stress at the tip. Not all problems in engineering analysis may require a nonlocal solution but fracture is one phenomena that does. Like micropolar theory, nonlocal theories represent a generalized continuum and equations will revert back to the classical forms if the added considerations become unimportant.

As expressed in Liebowitz's Fracture (1968, Vol. II, p. 303), "Ultimately a fracture criteria in terms of local average stress and strain history over a small region with characteristic microstructural dimensions is desired." Due to nonlocal continuum mechanics the equations of fracture are now in forms dependent on the microstructure and are suitable for application to real materials.
Application of the nonlocal model of fracture to real materials that are devoid of a perfect lattice structure requires some modification. When a crack propagates through a material it is expected to do so along the surface offering the least thermodynamic resistance. Obviously the resistance of atomic bonds that are present in all materials is greater than weaker secondary forces that may link groups of atoms. In the study of the actual kinematics of crack growth the process is seen to be intermittent. Atomic spacing provides the propagation steps in crystal structures whereas much larger steps (grains, molecular spacings, etc.) control crack propagation in other materials (polycrystalline metals, polymers, etc.). If the crack is to traverse larger units of structure more surface area is created than if the crack propagates in a relatively straight line between atomic units. Therefore, the difference between the fracture strengths of crystals and materials held together by weaker forces may not be as great as the difference in the two bond strengths.

The question has arisen as to what characteristic dimension may be used to replace the atomic distance found in the nonlocal solution for crystals. The atomic distance has originated in this problem as the limit of the influence function present in the equations for nonlocal moduli. This means that neighboring effects for the moduli are considered only within the radius of an atomic distance away from a point of interest. The reason for this limit comes from the nature of stress waves propagating through the crystal. When a stress is applied
to a perfect crystal structure it will travel as a wave. As the wavelength becomes small (i.e. comparable to the atomic distance) the waves become dispersive (i.e. phase velocity depends on wavelength) and non-local effects become important (Eringen, 1974). For fracture in crystals we have seen that nonlocal effects are important and hence, wavelengths associated with this phenomena must approach atomic dimensions. However, for real materials, which may be granular or fibrous, dispersion will begin when the wavelength becomes comparable to larger characteristic dimensions (i.e. grain or fiber diameter). This means that nonlocal effects cover a larger range and the atomic distance will be replaced by $\lambda$.

Equation III-20 shows that an increase in atomic distance, $a$, would result in an increase in the ultimate strength, $P_{\text{max}}$. However, for perfect crystals the intrinsic strength, $t_c$, may decrease with an increase of atomic distance (Table 1) and because $P_{\text{max}}$ is also dependent on $t_c$ the overall effect of a larger atomic spacing might be a decrease in $P_{\text{max}}$. It may be postulated that atomic distance appears in the equation because it represents the separation between possible fracture sites. If $t_c$ remained constant with an increase in $a$ the material would become stronger. This occurs because a crack that appears in the vicinity of the lattice point and displaced from the fracture site will traverse the point for a greater distance if these fracture sites are more widely spaced.

Modifications to the nonlocal equations of the past section for materials which may not fracture between atomic units will be made by the replacement of atomic distance $a$ by the characteristic dimension, $\lambda$. 
It then becomes necessary to establish \( \lambda \) values for different materials and conditions. The modified nonlocal fracture equations for a general material appear as

\[
p^2_{\text{max}} \frac{1}{(2C^2)} t_c^2 \quad \text{FRACTURE CRITERIA} \quad \text{IV-1}
\]

\[
K_{\text{Ic}} = 1.72 t_c(\lambda)^{\frac{1}{2}} \quad \text{FRACTURE TOUGHNESS} \quad \text{IV-2}
\]

Composite Levels of Fracture and Characteristic Dimensions

As evident from equation IV-2 the nonlocal fracture toughness is a function of \( t_c \) and \( \lambda \). This concept, however, may be traced as far back as Griffith (1921). In an earlier section it was shown that a first order approximation of the intrinsic strength of a material, as given by Griffith, can be represented by

\[
t_c = 2 P_{\text{max}} \left( \frac{1}{\rho} \right)^{\frac{1}{2}} \quad \text{IV-3}
\]

where \( \rho \) is the radius of curvature of the crack tip. Combining the classical definition of \( K_{\text{Ic}} \) \( [K_{\text{Ic}} = P_{\text{max}} (\pi \rho)^{\frac{1}{2}}] \) with equation IV-3 we obtain

\[
t_c = 2 K_{\text{Ic}}/(\pi \rho)^{\frac{1}{2}} \quad \text{IV-4}
\]

Rearrangement results in

\[
K_{\text{Ic}} = 0.89 t_c(\rho)^{\frac{1}{2}} \quad \text{IV-5}
\]

which is in a form analogous to equation IV-2. Because it seems reasonable to expect that the radius of a flaw tip in a material to be some function of \( \lambda \), a \( K_{\text{Ic}} \) formed by Griffith's relation equation (IV-3) becomes

\[
K_{\text{Ic}} = \text{CONSTANT } t_c[f(\lambda)]^{\frac{1}{2}} \quad \text{IV-6}
\]
Hence, the effect of the intrinsic strength and a characteristic dimension may be traced back to Griffith.

A similar result is obtained if Barrenblatt's solution for the crack tip stress is combined with the definition of fracture toughness (Johnson, 1973). The resulting equation takes the form

\[ K_{IC} = S(\lambda)^{\frac{1}{2}} \]  

where \( S \) represents a cohesive stress distribution over the microstructure whose characteristic length is \( \lambda \). The definition of \( S \) is analogous to \( t_c \); hence, this relationship is also in the form of equation IV-2. This comes as no surprise since Barrenblatt incorporated non-local effects in the vicinity of the crack tip.

All material bodies consist of smaller units of structure which will be referred to as "particles." These particles are attracted to one another by a force that must be exceeded in the process of fracture. This force over the area on which it acts has been referred to as the intrinsic or flawless strength. Particles that make up a material are, in many instances, composites of smaller particles. Even the atom has been shown to consist of subatomic particles.

Examine the form of equation IV-2. For a given material we must determine what value should be assigned to \( \lambda \). Obviously this depends on the level of fracture. If fracture occurs at the molecular level then \( \lambda \) should correspond to molecular dimensions. Suppose, however, that fracture occurs at a given "particle" level in general and the dimensions of the particle in question are important. Equation IV-2 shows that as \( \lambda \) increases the fracture toughness would increase if it were possible to keep the strength between particles, \( t_c \), constant.
during an increase in $\lambda$. As the particle size increases, resulting in a raise in $K_{ic}$, a limit may be reached in which the fracture process will switch from between to within the particles.

The factor controlling the limit on the size of $\lambda$ is found by again considering equation IV-2. Assume that the characteristic dimension of the original particles of the material is given by $\lambda_A$ and that the size of the next smaller important units that make up these particles are designated by $\lambda_B$. There then exists an intrinsic strength associated with each of these levels which we will denote by $t_{cA}$ and $t_{cB}$. The resulting $K_{ic}$ values for the material (A) and the particle (B) become

$$K_{icA} = 1.72 \ t_{cA} \left(\lambda_A\right)^{\frac{1}{2}}$$

and

$$K_{icB} = 1.72 \ t_{cB} \left(\lambda_B\right)^{\frac{1}{2}}$$

respectively. As $\lambda_A$ is increased $K_{icA}$ will approach $K_{icB}$ and when they become equivalent $\lambda_A$ will no longer be the characteristic dimension of A. Instead $\lambda_B$ becomes the important parameter because fracture will switch to the smaller level. This is obvious since a further increase in $\lambda_A$ could not raise the fracture toughness of a material greater than the $K_{ic}$ of particles from which it was formed.

A physical explanation of this switch in fracture level resulting in a change in $\lambda$ may be given by a discussion of nonlocal effects. As stated earlier the size of $\lambda$ controls at what wavelength the stress waves traveling through the material become dispersive and hence, nonlocal effects become important. Substructures of a material cause dispersion of waves because their boundaries represent inhomogeneities in
structure. In some instances the size of a substructure is much larger than the flaw that causes the critical stress intensity. Nonlocal effects that occur at the boundary of this substructure will not be a major influence on behaviors that occur in the vicinity of the much smaller flaw. Instead the nonlocal effects that occur at the smaller level of structure which has dimensions closer to the flaw size are of interest. In some instances the discontinuous boundary between the larger particles represents the flaw and as the particles increase in size it becomes increasingly likely that the crack will split the particles rather than traverse them. The transition from one level of structure to another is probably not abrupt. This may be shown in some materials by the crack branching that often occurs as the flaw at a grain boundary enters the grain.

Plastic versus Elastic Behavior

Plastic flow and brittle fracture may be said to be competing processes for the relief of stress concentrations. If this is true then factors that inhibit flow should promote brittle rupture. Decreased temperatures and increased strain rate are both examples of factors that inhibit plastic deformation and thus result in brittle behavior. Because the fracture strength of a material, as given by nonlocal equations IV-1 and IV-2, is directly related to $\lambda$ we would expect the competing phenomena to be inversely related to some characteristic dimension. If $\lambda$ could be increased resulting in a raise in brittle strength we would expect some measure of the plastic strength to decrease and flow to become more evident.
Elastic limit or yield stress are measures of the onset of plastic flow. The grain diameter of polycrystalline aggregates including iron have been inversely related to the yield stress \( \sigma_{y.s.} \) by
\[
\sigma_{y.s.} = \sigma_0 + k(\lambda_p)^{-\frac{1}{2}}
\]
where \( \sigma_0 \) and \( k \) are constants and \( \lambda_p \) (plastic characteristic length) is the grain diameter (Armstrong et al., 1962; Petch, 1958). This relationship shows that as grain diameter increases the yield stress will decrease causing the onset of plastic flow at lower stress levels. The question arises as to whether the grain diameter, which proves to be a characteristic dimension for yield stress, will in turn be the \( \lambda \) for brittle fracture.

In the preceding section it was suggested that there is a natural limit placed on any process tending to increase the brittle \( \lambda \) because eventually the fracture will switch to within \( \lambda \) causing dependence on a different characteristic length. Perhaps this may be further substantiated if a mechanism were given in which a crack is allowed to leave the particle interface and enter the particle.

It is well known that dislocation movement, resulting in plastic flow, is impeded at a grain boundary in polycrystalline metals (Liebowitz, 1968, Vol. I). If the grain diameter is large more dislocations will build up at the boundary because, as seen in equation IV-8, the yield stress is lower. As these dislocations build up the stress intensity at this point increases and eventually a micro crack will form in the adjacent grain to relieve the stress. Many references (Liebowitz, 1968, Vol. II) have suggested that this is the mechanism by which cracks nucleate and eventually result in brittle behavior of
otherwise ductile metals. The grain diameter of these polycrystalline materials is not the characteristic dimension of equations IV-1 and IV-2 since fracture resistance is inversely proportional to grain diameter. Because the cracks of these metals travel through the grains instead of around them we would not expect the grain diameter to represent the $\lambda$. However, the grain diameter does indirectly affect the fracture resistance simply because it affects $\sigma_{y.s.}$ (equation IV-8), and hence, the nucleation of cracks.

A hypothesis may be given based on the nonlocal equations and the above physical evidence to explain the fracture behavior of these metals. Perhaps at very small grain sizes the brittle strength of these metals is directly dependent on grain diameter and the resultant propagating cracks travel between the grain boundaries. We may then represent the characteristic dimension of equations IV-1 and IV-2 by $\lambda'$, which will be the grain diameter. But as $\lambda'$, which is also $\lambda$ in equation IV-8, is increased the yield stress is lowered. Eventually a limit will be reached in which the yield stress is much lower than the fracture strength and cracks will no longer propagate around grain boundaries resulting in bulk brittle fracture. Instead dislocation movement will dominate. However, the grain boundaries stop the movement of these dislocations until a stress intensity level is reached in which a crack is formed in the adjacent grain. It may be postulated that this isolated plastic flow is the mechanism by which brittle fracture switches levels as discussed in the preceding section. Once the level has switched the new characteristic dimension, $\lambda$, for brittle fracture will be a length corresponding to the subgranular "particles."
The above discussion has proposed an explanation of fracture processes in polycrystalline metals. There are many other variables that affect elastic and plastic behavior which should be considered. An important concept that must be realized at this time, however, is the effect of microstructural dimensions at various levels of the material that control the cleavage or flow process. In the case of these metals a restriction placed on flow by a grain boundary can cause formation of a brittle crack in an otherwise ductile material. Hence, the plastic behavior of a material is also strongly influenced by the microstructural effects present at a surface of discontinuity such as a grain boundary.

One question which may arise in the discussion of plastic phenomena as related to innerstructures is the actual size of the units that flow past one another. For metals it is not the grains that flow as units. Any tendency for these grains to flow is restricted by adjacent grains. Instead plastic deformation originates at a smaller level as a result of crystallographic plane slippage.

In many polymers the flow units are supermolecular structures. As discussed earlier the flow evident in the formation of crazes is due to groups of polymer chains slipping past one another under the influence of shear (Mooney and Wolsteinholme, 1954). Polyvinyl chloride melts exhibit particle slippage and the shape and size of the particle is important to this flow (Hattori et al., 1972; Collins and Metzger, 1970; Berrens and Folt, 1967, 1968). At higher temperatures (200°C) these units break down and flow supposedly occurs by molecular slippage.
Characteristic Dimensions of Glassy Polymers

As discussed in previous sections there is strong evidence of the existence of supermolecular structures in glassy polymers. The flow behavior of uncured resins show dependence on these microstructures. Even after the polymer has attained a solid state the supermolecular units control strength behavior as evidenced by fracture surface studies. These innerstructures, which have been called many different names in the past, will be referred to as "domains."

Domains may exist due to inhomogeneities that occur at regular intervals and cause the material between them to behave as a unit (Nielson, 1959). Domains could also be bundles of oriented molecules (Hsiao and Saver, 1950). These substructures have stronger internal bonds than they do with their neighbors. The oriented units (micro-anisotropic units) randomly distribute themselves resulting in amorphous (macro-isotropic) materials. Support showing that these domains are bundles of oriented molecules has been substantiated by strong physical evidence (Kargin, 1958), as well as mathematical treatment (Robertson, 1965).

We have shown how the micro-orientation of these domains affect fracture and how micropolar theory can give some theoretical explanation of the mechanism. However, the behavior of these domains under fracture conditions may be further explained by the application of nonlocal theory.

Referring back to equation IV-2 fracture toughness may be expressed as

$$K_{IC} = 1.72 \ t_c \ (\lambda)^{\frac{\lambda}{2}}$$

IV-10
Rearrangement of this equation can result in a relation for $\lambda$ which is given by

$$\lambda = \left(\frac{K_{IC}}{1.72 \ t_c}\right)^2$$

IV-11

Thus, if $K_{IC}$ and $t_c$ are known, the $\lambda$ can be determined. Values of $K_{IC}$ for both PMMA and PS have been obtained (Marshal et al., 1973a, 1973b). The question arises as to what equation should be used to calculate intrinsic strength, $t_c$. Griffith's relation for $t_c$ (equation III-1) cannot be used since the crack tip radius is not known. Orowan's relation (equation III-2) is for atomic bonding and is not expected to apply for polymer fracture. However, equation III-3 can provide a rough estimate for $t_c$. For convenience the equation is given here by

$$t_c = 0.1 \ E$$

IV-12

where $E$ is modulus of elasticity. Hence, an estimate of $\lambda$ may be obtained (Table 2). The values calculated for $\lambda$ by this procedure do not seem to correspond with any structural dimensions reported for PMMA or PS. Values for craze thickness near failure have been reported for PMMA and PS to be about 0.30 μm (Hull, 1972). The reported spacing between microflaws averaged to be about 0.25 μm in PMMA (Marshall et al., 1970). Table 2 gives $\lambda$ values that conceivably represent the dimensions of the domain structures of PMMA and PS. However, conclusions should

<table>
<thead>
<tr>
<th>Material</th>
<th>$K_{IC}$ (ksi (in)$^{1/2}$)</th>
<th>$E$ (psi)</th>
<th>$t_c$ (psi)</th>
<th>$\lambda$ (x10^{-4} in.) / (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMMA</td>
<td>1.3</td>
<td>425000</td>
<td>42500</td>
<td>3.16 / 8.03</td>
</tr>
<tr>
<td>PS</td>
<td>.85</td>
<td>450000</td>
<td>45000</td>
<td>1.51 / 3.84</td>
</tr>
</tbody>
</table>
be avoided since relationship IV-12 is a crude approximation and may be totally invalid for polymers. The elastic modulus used in the calculation arrives from bulk measurements, while the material in the vicinity of a flaw, where the important \( t_c \) values is associated, has changed compared to the rest of the material due to orientation.

Because \( t_c \) is not easily obtained for polymers perhaps equation IV-10 should be restructured to give values for it. A relationship for intrinsic strength follows as

\[
t_c = \frac{K_{Ic}}{1.72} (\lambda)^{\frac{3}{2}}
\]  

Characteristic length is now the value that must be found by another method or relationship. The topography of fracture surfaces should provide some insight.

For epoxy resin the diameter of nodules on the fracture surface are 250-300 Å (10^{-10} m) (Mijovic and Koutsky, 1979b). Polyvinyl chloride (PVC) shows two domain structures, one is a particle of 0.5 to 1.5 mm in diameter and the other a building block for this particle which is 300 Å in diameter (Hattori et al., 1972). The larger particle of PVC is evident on the fracture surface. Nylon 66 shows domain type structures of the order of 1 µm in diameter (Kikeenko et al., 1972).

A \( t_c \) value for epoxy can be calculated from equation IV-13 using the values of \( K_{Ic} = 550 \) psi in.\(^{\frac{3}{2}}\) (derived from Mijovic and Koutsky, 1979b) and the \( \lambda = 300 \) Å (Mijovic and Koutsky, 1979b). This gives

\[
t_c = 3.0 \times 10^5 \text{ psi}
\]  

from relationship IV-13. A value for the \( t_c \) of a carbon-carbon bond is approximately 5x10^6 psi (Kelly, 1966). Because it is generally accepted that the intrinsic strength of a polymer is below this value relationship
IV-14 may provide a good approximation. The value calculated is somewhat higher than the theoretical strength of a hydrogen bond ($6 \times 10^4$ psi) (Weiss, 1961).

There is a problem associated with using the fracture surface topography to obtain values of $\lambda$. As stated earlier fracture in many materials is associated with plastic deformation at the crack tip. The fracture surface must then contain areas of rearranged material. However, if the crack propagates quickly thereby allowing only a small amount of plastic flow, we can expect a good approximation of $\lambda$ to result from observation of the fractured surface. Of the polymers discussed, epoxy and PVC would allow such treatment but PS and PMMA have complicated fracture surfaces. In conclusion, more accurate values for $\lambda$ are required before nonlocal equations may be used to correctly predict phenomena like fracture in glassy polymers. A combination of micropolar and nonlocal theories would also produce a more accurate description of polymer behavior due to the orientational abilities of the microstructures. Later a discussion will be given on alternative methods of obtaining $\lambda$. These methods will be developed through mathematical and physical considerations.

Before leaving the discussion of amorphous polymers a theoretical article by Menges (1972) that is closely associated with the nonlocal model used in this paper will be reviewed. Menges reached the conclusion that crazes in thermoplastics are adhesion fractures at particle boundaries. These particles are the domain structures that we have previously referred to. Crazes in polymers always form at a threshold value of strain and it was postulated that this was due to the power
law dependence of secondary valence forces on distance of separation. The only factor that will change this critical value of strain appeared to be a change of state.

The adhesion cracks that occur at domain boundaries are arrested when they reach a particle whose interface is perpendicular to the direction of propagation. For further crack movement either the critical strain must be exceeded in directions different from the axis of the arrested crack resulting in more particle separation or the crack must proceed through the particles. There is a higher critical strain associated with the separation of units within the particles. In the last paragraph it was suggested that the critical strain in a local area will change due only to a change of state. This gives further evidence that plastic flow, which can be associated with a transition of state, is the process by which elastic phenomena like brittle fracture switches levels.

Particleboard

If the size of the substructures that are found in a material could be varied while leaving other factors constant the dependence on λ found in the nonlocal equations could be verified. Obviously for this to be true the dimensions of these subunits must correspond to λ. This correspondence becomes more likely with weaker bonds between these "particles." Hence, the ability to control the intrinsic strength between particles is also important.

Particleboard, a wood composite consisting of wood particles and resin, can provide a material that the nonlocal model may be tested.
The dimensions of the wood particles can be experimentally controlled and the bonding strength between particles varied by changes in resin content. Because a wood particle is three dimensional we must determine the dimension that becomes the important $\lambda$. The answer depends on a number of variables including particle orientation and the direction of load application. In this study we are interested in the tensile strength perpendicular to the plane of the board; this has been traditionally called the internal bond (IB) strength.

Wood particles, like the "domains" of polymers, are anisotropic with an axis of orthotropic symmetry. In particleboard, if there is random orientation of the particles during processing, the resulting product will remain isotropic in the plane of the board. The three dimensions of the particles may be varied and conceivably a cubical or round unit could be formed in which the characteristic length is expressible as a diameter. This may present a problem because, as was the case with glassy polymers, a micropolar element may rotate in the vicinity of a flaw causing orientational effects. Nonlocal theory does not account for these orientational effects. In this instance nonlocal-micropolar combined effects are present and a theory incorporating both is expected to give a better comparison with physical behavior. In order to use nonlocal theory alone it becomes essential to remove the ability of the particles to orient. Conceivably if the length of a particle was much greater than the other dimensions it would not be able to rotate to any extent.

Consider a particle which has length, width and thickness dimensions with relative values decreasing respectively. The important
characteristic dimension would be the one found perpendicular to the crack's path. In tension perpendicular to the board for a particleboard formed from relatively long particles the important dimension becomes the thickness because invariably the length and width of the particles remain in the plane of the board (Figure 8). Hence, a measure of the thickness of the particles in this study will be the \( \lambda \). Length and width will be held constant and the manufactured dimensions will be large enough relative to thickness so they will remain in the plane of the board.

From equation IV-2 we see that the fracture toughness of a material is related to \( t_c \) and \( \lambda \) by

\[
K_{IC} = 1.72 \ t_c (\lambda)^{\frac{1}{2}}
\]

If the intrinsic strength, \( t_c \), of particleboard is constant at a given resin content the particle thickness may be varied and \( K_{IC} \) should be proportional to \( \lambda^{\frac{1}{2}} \). The square root proportionality will hold as long as \( t_c \) remains constant and particle thickness is the characteristic dimension.

The specific gravity of particleboard is generally greater than that of the solid wood. A ratio may be formed called the compression ratio which is expressed as

\[
\text{Board Specific Gravity} / \text{Wood Specific Gravity}
\]

As the specific gravity of the board increases the particles become more compressed and their thickness in the board reduces. To adjust for this reduction in thickness we divided the manufactured thickness of the particles by the compression ratio to obtain a \( \lambda \) value for the substructures within the board. This does not affect the analysis since
Figure 8. Particleboard formed of relatively long (as compared to thickness) particles.
the $\lambda$ is changed by a constant, namely the inverse of the compression ratio. For example, if a proportionality exists between $\lambda$ and $K_{IC}$ it will hold whether or not $\lambda$ is divided by the compression ratio. The only factor that will change is the proportionality constant. Since this proportionality constant contains $t_c$ (which we plan to calculate) it is postulated that a more accurate value of $t_c$ will result if the $\lambda$ used is closer to the actual $\lambda$ found in the board. This we conjecture to be true if we divide the manufactured $\lambda$ by the compression ratio.

Values of $K_{IC}$ must be obtained by the classical experimental procedure. The classical and nonlocal definitions of $K_{IC}$ are equivalent since the nonlocal equation results from the substitution of the classical relation for $K_{IC}$ into the nonlocal fracture criteria. If the values obtained for $K_{IC}$ are proportional to $\lambda^{3/2}$ the proportionality constant equals $1.72 t_c$ and values of intrinsic strength will follow.

The nonlocal fracture criteria, equation IV-1, repeated for convenience, is given by

$$\frac{P_{\text{max}}^2}{T_o} = \left(\frac{\lambda}{2c^2}\right) t_c^2$$

IV-17

The value of $P_{\text{max}}$ is the measured ultimate tensile strength in the direction perpendicular to crack propagation and corresponds to internal bond strength (IB) for particleboard. Substituting IV-15 into IV-17 we obtain a relationship between fracture toughness and internal bond given by

$$K_{IC} = (\pi T_o)^{1/3} \text{(IB)}$$

IV-18

where $T_o$ represents half the intrinsic flaw length. Relationship IV-18 is actually the classical definition of $K_{IC}$ and again shows the equivalence between classical and nonlocal $K_{IC}$. Many authors (e.g. Schniewind
and Lyon, 1973) have used relationship IV-18 to obtain values of intrinsic flaw sizes and it will again be used in this study.

Three particle thicknesses as well as three resin contents were used in the experimental verification of equation IV-16. The three resin percents used were 5, 7.3 and 11.4 which will be referred to as low, medium and high, respectively, throughout the remaining discussion. The two sets of three variables resulted in a 3x3 factorial statistical design. Two samples of each type will be made resulting in a total of 18 boards. There will also be one board formed from a combination of the three thickness classes for each resin level to see if the \( \lambda \) can result from an average value.
V. EXPERIMENTAL PROCEDURE

Particle Production

In order to control the size and nature of the particles it was essential to start with solid wood. We obtained approximately 40 planks of slow-growth green Douglas-fir (nominal dimensions 2x6 in. by 8, 10, 12 or 14 ft) donated by Bohemia Inc., Culp Creek, Oregon. The wood was relatively flat grained and was of grade two or better. The 2x6 in. planks were cut into approximately 4x2x6 in. blocks (Figure 9). A random sample of these blocks were used to obtain an average oven-dry specific gravity of 0.48.

The wood blocks were soaked for approximately four days in large barrels. This was done to yield high quality flakes during flaking. The flaker knives were kept sharp. Knife cuts were made along the radial-longitudinal plane of the block (Figure 9). By varying the position of the knives we were able to obtain three thickness classes. A statistical sample of each thickness class was taken for dried flakes resulting in

<table>
<thead>
<tr>
<th>Particle class</th>
<th>Average thickness (in.)</th>
<th>± Standard deviation (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.013</td>
<td>± 0.0026</td>
</tr>
<tr>
<td>II</td>
<td>0.019</td>
<td>± 0.0038</td>
</tr>
<tr>
<td>III</td>
<td>0.029</td>
<td>± 0.0054</td>
</tr>
</tbody>
</table>

The wet flakes were immediately reduced to particles by hammermilling. The thickness dimension did not change in this process and the width and length of particles were much greater than the thickness. These
Figure 9. Particle production procedure.
particles were screened to remove any fine material. They were then
cold stored in large plastic bags. Particles were placed in wire-
screen baskets and dried in an oven at 150°C. It took about two days
for the particles to reach the desired moisture content of approximately
3% (OD weight basis).

**Board Production**

Particleboards were produced with a specific gravity of 0.70 and a
board size of 5/8x18x18 in. The properties of particleboard were easily
varied by any one of a number of factors in the production process
(Maloney, 1977). Because we are only interested in the effect of
particle thickness and resin content other process parameters must be
controlled. The process parameters kept constant included:

- **Resin type:** urea-formaldehyde liquid resin (65.5% solids)
- **Resin temperature at application:** 68°F
- **Wax type and percentage:** Bordens 403H-49% solids, 0.5% (OD basis)
- **Mat moisture content into press:** 9.7% ± 0.4%
- **Press temperature:** 325°F
- **Press closing time:** 30 sec ± 7 sec
- **Pressure:**
  1. hold 30 seconds after press is closed at 720 psi
  2. slowly released during next 30 seconds
  3. constant pressure for 5.0 minutes at 249 psi
  4. relieve pressure for 30 seconds
- **Total press cycle:** 7 minutes

Resin was applied by spraying with a single nozzle air gun at an
air pressure of 60 psi. The air gun was located at the center of a
large rotating drum (4 ft in diameter) that contained the particles. The blending time varied with the different resin classes and came to about 5.5, 6.5 and 8.0 minutes for low, medium and high resin contents, respectively.

Mat formation was achieved by hand felting in a 18x18 in. forming box in order to control an even distribution of particles. This is an important step because density variation across the finished board will obviously be a function of the density variation across the mat. The particles within the plane of the board were randomly oriented.

The pressing procedure began immediately after the mat was placed in the press. The most critical step of the board formation, closing time, determines the density profile through the thickness of the finished board and must be controlled carefully. Not only the closing time but also the rate of closure is important to the final product. Because all boards formed in this study were of the same specific gravity we were able to accurately control the closing procedure.

Upon removal from the press the boards were stacked, stickered, and cooled. The boards were then moved to a standard room (72°C, 65% R.H.) where they were allowed to reach equilibrium over a two week conditioning period.

Specimen Preparation and Testing

From the boards we needed both internal bond and fracture toughness specimens. These were obtained using the cutting procedure shown in Figure 10a. Samples marked with a K designate $K_{IC}$ specimens and those labeled with an I are internal bond specimens. Figure 10b is the
Figure 10. Specimen cutting pattern: A) test specimen; B) test specimen for $K_{IC}$ versus crack length.
cutup procedure for two boards in which a crack length versus $K_{IC}$ study was made. All samples were measured for width, length and thickness (to accuracy of 0.001 in.) to be used in the calculation of IB and $K_{IC}$, and weighed to obtain density. Excess board material was used for determination of moisture content.

Internal bond specimen size was specified by ASTM standards (ASTM D1037-72) to be approximately 2x2 in. blocks. A hot melt glue was used to secure the IB specimens to aluminum loading blocks which allowed application of a uniform load. The testing speed and procedure of measurement is outlined in the standards.

The dimension of fracture toughness samples have not been standardized for wood or wood composites. ASTM Designation E399 (1974) outlines the procedure used for plane strain testing of metals. The classical equation for fracture toughness is as stated earlier (equation III-10)

$$K_{IC} = \frac{P_{max}}{\sqrt{\pi} I}$$

where $P_{max}$ is the applied ultimate stress and $I$ is the half crack length. This equation holds for a loading at infinite boundaries (a mathematical idealization), a condition which is not easy to experimentally simulate. In attempts to obtain an absolute value of $K_{IC}$ for different load applications and specimen dimensions, various methods of calibration have been used. These methods are well outlined in a number of ASTM publications (ASTM 381, 1965; Brown and Strawley, 1966). In general the form of equation V-1 becomes

$$K_{IC} = P_{max} \sqrt{\pi} f \text{ (specimen geometry)}$$

For fracture toughness testing of wood a number of different specimen configurations have been used (Johnson, 1973; Schniewind et al.,
1973; Barrett, 1976; Tomin, 1972; Lei, 1978). In this study the calculation of $K_{IC}$ will be obtained by

$$K_{IC} = P_{max} (1/w)^{3/2} Y(I/w)$$

(for $I/w < 0.6$)

where

$$Y(I/w) = 1.99 - 0.41 (I/w) + 18.70 (I/w)^2 - 38.48 (I/w)^3 + 53.85 (I/w)^4$$

$$P_{max} = \text{max force/w*t}$$

$w = \text{specimen width (= 2 in. for this study)}$

$t = \text{specimen thickness (= 1 in. for this study)}$

$I = \frac{1}{2} \text{crack length (= 3/4 in. for this study)}$

which is an equation used for single-edge cracked metallic plates in tension (Brown and Strawley, 1966). A number of authors have used this equation for wood based materials including solid wood (Schniewind and Lyon, 1973) and oriented flakeboard (Lei, 1978).

The specimen geometry used for the testing of $K_{IC}$ in this study is pictured in Figure 11a. Note that the board thickness is not the specimen thickness desired in equation V-3. A critical step in fracture toughness testing involves introduction of a crack of known length into the specimen. In the case of metals the crack is induced by a fatigue method. This would not be possible in wood composites due to the difficulty of measuring the length of the induced crack. An alternative method of introducing a crack with a razor blade has been used for wood. A razor blade cannot be expected to produce an accurate cut in a specimen as thick as the one in this study (1 in.) for a depth as great as 3/4 in. A method of producing a crack in an analogous specimen of flakeboard was first adopted by Lei (1978). He used a band saw with a
Figure 11. Fracture toughness specimen: A) specimen dimensions; B) specimen, buffer blocks, and loading blocks.
blade 0.1 cm thick to produce a notch 0.68 in. long. The remaining crack length (up to 0.75 in. deep) was produced with a band saw blade 0.05 cm thick with its teeth ground down to a sharp edge. This procedure produced a fairly sharp crack tip and thus has been adopted for this study.

The loading specification for the use of equation V-3 for metals requires a uniform load distribution at a distance from the crack not less than the width of the specimen. Because of this requirement, buffer-blocks of particleboard (1.0 in. thick) with approximately the same specific gravity as the boards to be tested were glued to the faces of each $K_{IC}$ specimen. This provided a distance of about 1 3/4 in. from the point of load application to the crack plane. In metals the stress is usually applied by a point load but in particleboard the load may be applied to the specimen uniformly. In conditions analogous to internal bond testing aluminum blocks were secured to the faces of the buffer-blocks. Figure 11b shows the $K_{IC}$ specimens glued to the loading blocks.

The loading speed used to test $K_{IC}$ was 0.04 in./min. This value was obtained by considering standard procedures of $K_{IC}$ testing for metals and IB testing for particleboard. The load displacement curve obtained at this speed is essentially linearly elastic to the point of failure. This is a requirement of $K_{IC}$ testing in general since it is based on the postulate of bulk elastic behavior.

As stated earlier the purpose of an equation of the type V-2 is to provide a measure of $K_{IC}$ which is a material property. Hence, for different methods of measurement and specimen geometries we can use
different forms of equation V-2, such as equation V-3, to obtain the same value of $K_{IC}$. Relation V-3 has been adopted for this study because the specimen and loading procedure used is analogous to the restraints placed on metal using the same equation. Perhaps the equation is not completely accurate for wood. In fact, many of the restrictions placed on metals are due to their ductile nature.

At this point we will reemphasize that fracture mechanics in wood has not reached the stage where absolute measurements are required, as would be the case for design. Instead fracture studies in wood have progressed to where it is important to have a relative measure of fracture resistance. Any of the equations of fracture toughness, including equation V-1, can give relative measures for a given specimen geometry. This can be simply shown by examining both equations V-1 and V-3. Say that equation V-3 is able to give absolute values for $K_{IC}$ of a particular metal specimen. For a given specimen geometry, crack length, and loading procedure equations V-1 and V-3 will give different values of $K_{IC}$ but they will vary by a given constant. This constant will be the $Y(1/w)$ value. If we are only interested in relative values, such as one metal compared to another, $K_{IC}$'s obtained from either equation will show how fracture resistance changes from one metal to the other. With this being the case it becomes essential to use the same equation and specimen for $K_{IC}$ measurement in a given study. In this way we are able to see how a change in a given variable will affect the measure of fracture resistance which we have adopted.

We may test if a given equation of fracture resistance can be used as an absolute value. Even though at this stage a relative value is
sufficient we would like to use the measure which is closest to the absolute value. If \( K_{IC} \) given by an equation of the form of equation V-2 is a material property it should not vary with a change in crack length.

A test was conducted to see how equations V-1, and V-3 and

\[
K_{IC} = P_{max} (1) \frac{1}{2} \left[ 1.15 - 0.24 \frac{1}{\omega} + 1.87 \left( \frac{1}{\omega} \right)^2 \right]
\]

V-4

which was used by Johnson (1973) are affected by a change in induced crack length. Figure 12 shows how \( K_{IC} \) as measured by these three equations varies with crack length. Similar results for equation V-3 were shown by Schniewind and Lyon (1973) for solid wood. The absolute value of \( K_{IC} \) would come from an equation which would produce a constant for varying crack lengths and hence, a horizontal (\( K_{IC} = \text{constant} \)) line plot. Equation V-3 gives a plot which is closest to a horizontal line of the three evaluated. From the slopes of the three plots it may be postulated that an equation which predicted \( K_{IC} \) values somewhere between V-3 and V-4 will plot a horizontal line for varying crack lengths and may be considered the absolute value. In absence of this equation for wood, small crack lengths should be used since they give values for \( K_{IC} \) closer to such an absolute value.

Equation V-3 which will be used in this study conceivably represents a slight overestimate of the actual \( K_{IC} \) and hence, is not the absolute measure which would be necessary for design standards. It has been concluded that if we are to use equation V-3 as a relative measure of fracture resistance we must hold specimen geometry and crack length constant for all samples. With this in mind we may proceed with the verification of our nonlocal model.
Figure 12. $K_{lc}$ versus crack length using equations V-1, V-3 and V-4.
If our measure of $K_{ic}$ is proportional to $\sqrt{\lambda}$ we know that the absolute measure of $K_{ic}$ will show that same proportionality. This is because the $K_{ic}$ we used differs from the absolute $K_{ic}$ by a constant for a given specimen geometry. When it comes time to predict $t_c$ values from the relationship between $K_{ic}$ and $\sqrt{\lambda}$ we will not be able to obtain an absolute measure. However, because our $K_{ic}$ appears close to the actual quantity, as seen in Figure 12 (plot of equation V-3 is close to a horizontal line) we will be able to report good approximate values of intrinsic strength.

**Board Types**

Twelve different types of boards were produced in this study. For each of the three particle classes there were three resin content levels (5, 7.3, 11.4%). In addition there were also boards consisting of a combination of the thickness classes. These combination boards were formed by adding equal weights of each particle thickness, and provided a test of the hypothesis that $\lambda$ may arise as an average.

A measure of $\lambda$ as it would appear in the board was calculated by the relation

$$\lambda_{Board} = \frac{\lambda_{Manufactured}}{Compression\ Ratio}$$

where the compression ratio was given in equation IV-16 as

$$Compression\ Ratio = \frac{Board\ Specific\ Gravity\ O.D.}{Wood\ Specific\ Gravity\ O.D.} = \frac{0.70}{0.48} = 1.46$$

The resulting thickness classes are reported in Table 3.
Table 3. Characteristic dimensions of particleboard.

<table>
<thead>
<tr>
<th>Class</th>
<th>( \lambda ) Manufactured (in.)</th>
<th>( \lambda ) Board (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.013</td>
<td>0.009</td>
</tr>
<tr>
<td>II</td>
<td>0.019</td>
<td>0.013</td>
</tr>
<tr>
<td>III</td>
<td>0.029</td>
<td>0.020</td>
</tr>
</tbody>
</table>
VI. RESULTS

The average values for fracture toughness and internal bond for the different boards are reported in Table 4. The following data trends are apparent:

1. IB and $K_{IC}$ are increasing functions of resin content and particle thickness.

2. IB and $K_{IC}$ show a greater dependence on particle thickness at low resin contents than at high resin contents.

3. There appears to be a direct relationship between IB and $K_{IC}$.

4. The boards formed from combined particle thicknesses have properties closest to the 0.013 thickness class.

Table 4. Average values of IB and $K_{IC}$.

<table>
<thead>
<tr>
<th>Specimen particle thickness/resin content (in./%)</th>
<th>Density (lb/ft$^3$)</th>
<th>M.C. (%)</th>
<th>I.B. (psi)</th>
<th>$K_{IC}$ (psi/in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.009/5</td>
<td>47.2</td>
<td>8.25</td>
<td>98.7</td>
<td>99.6</td>
</tr>
<tr>
<td>0.009/7.3</td>
<td>47.4</td>
<td>7.98</td>
<td>150.8</td>
<td>155.4</td>
</tr>
<tr>
<td>0.009/11.4</td>
<td>47.2</td>
<td>8.01</td>
<td>200.8</td>
<td>198.3</td>
</tr>
<tr>
<td>0.013/5</td>
<td>47.1</td>
<td>7.72</td>
<td>132.4</td>
<td>136.9</td>
</tr>
<tr>
<td>0.013/7.3</td>
<td>47.3</td>
<td>8.50</td>
<td>171.8</td>
<td>167.9</td>
</tr>
<tr>
<td>0.013/11.4</td>
<td>47.1</td>
<td>7.52</td>
<td>208.6</td>
<td>206.8</td>
</tr>
<tr>
<td>0.020/5</td>
<td>46.7</td>
<td>8.27</td>
<td>137.3</td>
<td>151.8</td>
</tr>
<tr>
<td>0.020/7.3</td>
<td>47.0</td>
<td>8.18</td>
<td>175.9</td>
<td>186.3</td>
</tr>
<tr>
<td>0.020/11.4</td>
<td>46.9</td>
<td>7.99</td>
<td>224.3</td>
<td>224.2</td>
</tr>
<tr>
<td>$C^a/5$</td>
<td>47.5</td>
<td>7.74</td>
<td>129.3</td>
<td>127.8</td>
</tr>
<tr>
<td>$C/7.3$</td>
<td>47.7</td>
<td>8.16</td>
<td>185.6</td>
<td>177.3</td>
</tr>
<tr>
<td>$C/11.4$</td>
<td>47.0</td>
<td>7.80</td>
<td>196.4</td>
<td>215.3</td>
</tr>
</tbody>
</table>

$C$ represents a combination of the three thickness classes (on the basis of weight).
VII. DISCUSSION

IB versus $K_{IC}$

A strong linear relationship ($r^2 = 0.98$) exists between average values of IB and $K_{IC}$ (Figure 13). A negligible $y$-intercept value (-10) is natural since zero fracture toughness should correspond to zero IB. If we assume that the graph represents a direct proportionality ($y$-intercept = 0) then $K_{IC}$ is related to IB by a constant.

Equation V-3 shows that $K_{IC}$ is related to $P_{max}$ by

$$K_{IC} = P_{max} (\bar{I})^{\frac{3}{2}} Y(\bar{I}/w)$$

VII-1

where

$$Y(\bar{I}/w) = 1.99 - 0.041 (\bar{I}/w) + 18.70 (\bar{I}/w)^2 - 38.48 (\bar{I}/w)^3 + 53.85 (\bar{I}/w)^4$$

If $P_{max}$ is the internal bond then half the crack length, $\bar{I}$, becomes half the intrinsic flaw size, $\bar{T}_o$, and the quantity

$$(\bar{T}_o)^{\frac{3}{2}} Y(\bar{T}_o/w)$$

VII-2

must be a constant equal to the inverse of the proportionality constant between IB and $K_{IC}$ as given in Figure 13. The intrinsic flaw is then found to be a constant given by

Intrinsic Flaw = 2 $\bar{T}_o = 0.42$ in.

VII-3

A study by Schniewind and Lyon (1973) used equation VII-1 and with a similar procedure to ours they obtained values for the intrinsic flaw size of solid wood ($2 \bar{T}_o = 0.10 - 0.15$ in.).

The linear nature of Figure 13 results in an unexpected constant intrinsic flaw size. We would intuitively expect the intrinsic flaw size to be a function of resin content and particle size. These results
Figure 13. $K_{IC}$ versus $IB$. Note: Each point on the graph represents a different bond type.

The equation $IB = -10.05 + 1.04K_{IC}$ holds with a goodness of fit $r^2 = 0.98$. 
do not show such a dependence. We should, however, be careful before
drawing conclusions from this trend.

All $K_{ic}$ samples had induced side cracks 0.75 in. long. In order
to use the $K_{ic}$ values obtained by extrapolation for other crack lengths,
as would be the case in using equation V-3 to obtain an intrinsic flaw
size, we must postulate that $K_{ic}$ remains constant at all crack lengths.
The validity of such an assumption is as accurate as stating that the
plot of equation V-3 versus crack length (Figure 12) is a horizontal
line. Therefore, the absolute values of intrinsic flaw size are not
attainable. Thus, relationship VII-3 gives an estimate. It does, how-
ever, seem to provide a good estimate because the plot of equation V-3
is nearly horizontal (Figure 12).

Caution should be taken before assuming the estimate of intrinsic
flaw size to be a constant, as suggested by Figure 13, because $K_{ic}$ is a
function of crack lengths and it is impossible to draw such a conclusion.
The value of $K_{ic}$ at the 0.75 in. crack length can only give an indication
of the $K_{ic}$ at intrinsic flaw sizes. We postulate that the intrinsic
flaw size reported in relationship VII-3 is an overestimate of the
average value of all of the particleboards in this study. We arrived
at this conclusion because the $K_{ic}$ at intrinsic flaw sizes would be
less than the $K_{ic}$ at $T = 3/4$ in. if half the intrinsic flaw size is
less than 3/4 in. (see slope of equation V-3 in Figure 12).

Examine Figure 14 (reproduction of Figure 12). The fracture tough-
ness corresponding to the intrinsic flaw size ($T_o$) is $145 \text{ psi} \sqrt{\text{in.}}$.
This differs from the $K_{ic}$ value used in the calculation of $T_o$ by 19 psi
$\sqrt{\text{in.}}$. Using a $K_{ic}$ of $145 \text{ psi} \sqrt{\text{in.}}$ we can gain a more accurate value of
Figure 14. $K_{IC}$ versus crack length - convergence to intrinsic flaw sizes.
the intrinsic flaw size for the board used in the crack length plot (0.013 in. particle thickness and 7.3% resin content). The new predicted flaw size, \( 2T'_o \), is given as

\[
2T'_o = 0.34 \text{ in.}
\]

This provides a better estimate because, as seen in Figure 14, the \( K_{ic} \) associated with it is closer to the value used in the calculation (within 1 psi \( \sqrt{\text{in.}} \)). This method of convergence gives a more exact value of \( 2T'_o \). However, the method should be approached with caution because it is strongly dependent on average experimental measurements. The standard deviation of \( K_{ic} \) values are large enough that any exact calculation resulting from average values is not critical. Instead an estimate and expected range of the intrinsic flaw size serves the purpose.

We cannot ignore the linear relationship between \( K_{ic} \) and \( IB \) (Figure 13). Nor can we conclude that intrinsic flaw size is a constant for all particleboards. The flaw size probably does depend on the resin content and particle thickness but the dependence is limited (i.e. a two-fold increase in resin content cannot result in a twofold decrease in \( 2T'_o \)). Most likely there is a small range of intrinsic flaw sizes associated with particleboard for which the average value is near that of relationship VII-4. We would expect increased resin content to decrease the nonbonded regions found in the board by a proportionate amount. This does not mean that flaws in general will be reduced by that much.

There is a tendency to think of flaws as cracks. For composites the nonbonded regions correspond to such an interpretation. However,
the crack is a mathematical idealization and the intrinsic flaw that is identified as a result of calculations using our mathematical model may be a function of a combination of effects. Flaws could include a coalescence of voids or nonbonded regions and boundaries of dissimilar material, all of which cause stress intensity. Hence, as resin content increases, one type of flaw (nonbonded regions) decreases and another type of flaw (dissimilar material interaction) increases. Calculations using the mathematical model can then conceivably result in similar \( \Gamma_0 \) sizes at different resin contents. Therefore, it becomes equally important to control the compatibility of various components found in a composite and decrease the size of voids due to a lack of bonding.

Fracture Toughness versus Resin Content

Fracture toughness is directly dependent on resin content, as seen in Figure 15. The data points found on the graph are average values for fracture toughness at the various resin contents; however, the regression equations and curves are functions of all data points. We cannot expect a higher correlation \( (r^2 \) values) between fracture toughness values and any other variable. This is due to the inherent scatter associated with any \( K_{IC} \) testing (ASTM, 1965). Thus, we conclude that \( K_{IC} \) is linearly dependent upon resin content for all three thickness classes. Correlations of internal bond with resin content have reportedly shown similar dependence (Lehmann, 1974; Maloney, 1977).

Because \( K_{IC} \) represents a measure of the resistance of a material to brittle fracture, we may conclude that this resistance increases with resin content. By examining Figure 15 we see that the particle
Figure 15. $K_{IC}$ versus resin content.
thickness plays an equally important role in increasing fracture resistance. For instance, a board made of the thickest particles (0.020 in.) and 5% resin content is as resistant to fracture as a board formed from the thinnest particles (0.009 in.) and 8.2% resin content (Figure 15). With increasing resin costs a process that controls the particle dimensions could conceivably reduce resin cost by a considerable amount.

Fracture Toughness versus Particle Thickness

We postulated that there is a square root proportionality between $K_{IC}$ and a controllable characteristic dimension, $\lambda$. Particle thickness was controlled and resin content varied in anticipation that thickness was the characteristic dimension. As explained earlier there is a natural limit placed on the ability of $\lambda$ to increase fracture resistance. This limit is a function of the size of $\lambda$ and the amount of bonding between these subunits. In this study there was an experimental restriction placed on increasing particle thickness because the flaker could only cut flakes in the range of 0.01 in. to 0.03 in. It then became essential to use higher resin contents if we were to gain an idea of the limit of applicability of particle thickness as the important $\lambda$ value. As resin content increases and the number of bonds between particles increases, eventually the discussed level switch should occur. When the level switch happens the fracture toughness becomes dependent on subunits of particles. Experimentally a complete level switch would be synonymous with zero dependence on particle thickness because the subunits found within particles of any thickness are the same. This switch cannot be expected to occur immediately.
At 5% resin content the linear relationship between the square root of particle thickness and fracture toughness confirms the proposed nonlocal model (Figure 16). The regression equation (Figure 16), which is a function of all $K_{Ic}$ samples at 5% resin content, was obtained by a curve fitting procedure. The curve shown gave the best correlation (highest $r^2$) with the experimental data points out of all the attempted curve fits (exponential, power, logarithmic, and linear regression with $\lambda$). By neglecting the insignificant y-intercept value (-0.7) we have an equation that is in the form of the nonlocal fracture toughness (equation IV-2). We concluded that at 5% resin content the particle thickness is the characteristic dimension.

By dividing the proportionality constant between $(\lambda)^{\frac{1}{2}}$ and $K_{Ic}$ by 1.72 we have an estimate of the intrinsic strength, $t_c$. This is calculated to be

\[ t_c \approx 650 \text{ psi} \quad \text{VII-5} \]

This value is higher than the strength of Douglas-fir wood in tension perpendicular to the grain as given by the Wood Handbook (340-390 psi) (1974). The particles found in the board have specific gravities that are higher than the original wood. It is conceivable that their strength has increased beyond 650 psi. If so the cohesive strength (equation VII-5) may be an estimate of the strength of the particle-resin bond. Microscopic fracture observations for particleboard have shown that at 6% resin content there is some fracture within the particle as well as at the particle-resin interface (Wilson and Krahmer, 1976). We postulated earlier that the $K_{Ic}$ measured in this study is a slight overestimate of the absolute value. This would mean that
Figure 16. $K_{IC}$ versus particle thickness at 5% resin content. Note: Regression analysis includes all data points except those from combination board (C).
relationship VII-5 is a slight overestimate of $t_c$. 

At higher resin contents (7.3% and 11.4%) the dependence on $(\lambda)^{1/2}$, where $\lambda$ represents particle thickness, disappears (Figure 17). The $r^2$ values ($r^2 = 0.55$ for 7.3% and $r^2 = 0.28$ for 11.4%) show that fracture resistance becomes less related to particle thickness at higher resin contents. This is because the values of $K_{IC}$ for different thickness classes tend to overlap at higher resin contents (see standard deviations in Figure 17.). This agrees with the suggested level switch. Once fracture toughness becomes independent of particle thickness all boards at a given resin content will have statistically identical fracture resistance. We expect this because the new $\lambda$ is a function of the anatomy of the particles which remains the same regardless of their thickness. The level switch cannot be expected to be immediate and some dependence on particle thickness will remain during the transition. Because there is still some dependence on particle thickness even at 11.4% ($r^2 = 0.28$) we have not attained the complete level switch in this study.

Past studies (Brumbaugh, 1960; Maloney, 1977) have shown an increase in internal bond strength for both an increase in particle thickness and a decrease in particle length. We used relatively long (as compared to thickness) particles in order to limit the rotation of particles in the vicinity of a flaw as was predicted by the micropolar crack tip solution. In this instance the nonlocal fracture criterion appears accurate. The fact that the internal bond increases with decreasing particle length suggests that shorter particles may rotate as suggested for a micropolar media. Maximum rotation would allow
Figure 17. $K_{IC}$ versus particle thickness (1.3% and 11.4% resin content).
alignment of the strong axis of the particles in the direction of maximum stress as was the case for glassy polymers. In particleboard it is conjectured that maximum rotation (alignment with maximum stress) of even small particles is restricted by the nature of the resin. A more flexible resin matrix would allow more rotation. It may be postulated that this rotation is favorable to the fracture resistance and more flexible resins could result in a large increase in internal bond strength of particleboard.

The above suggestions arise from studying the nonlocal and micropolar theories separately. Their combined effects have been incorporated in the nonlocal polar theory (Eringen, 1972, 1976). The solution of the nonlocal polar crack problem has not been obtained. Once the solution is found we can expect greater insight into the fracture processes of glassy polymers and wood-based materials.

The value of \( \lambda \) can result from an average of different particle sizes. Both Figures 16 and 17 show that boards formed with a combination of particle thicknesses give fracture toughness values that fall near the curves.
VIII. CONCLUSIONS

The intrinsic flaw size of a particleboard formed from relatively long thin particles may be estimated as 0.34 in. This value is expected to decrease with increasing resin content but not to a great extent. The compatibility of resin and wood particles in the board may be as important as the previously documented relationship of resin content to final board properties.

At low resin contents (≤ 5% for a high efficiency laboratory blender) the fracture resistance of the board will be strongly dependent on particle dimensions (an increased particle thickness results in increased $K_{IC}$). Any process that controls particle geometry in a favorable manner can be expected to reduce resin cost.

In this study it was shown that a combination of particle thicknesses gives board properties equivalent to those of boards made of a particle thickness corresponding to the statistical average of the combined particle thicknesses. Hence, a strict control of uniform particle size is probably not necessary since an averaging effect appears present (within range of thicknesses studied).

Fracture resistance is also dependent on resin content (an increase in resin content results in an increase of $K_{IC}$). At higher resin contents (7.3% and 11.4% in this study) fracture resistance is less dependent on particle thickness ($r^2 = 0.55$ and $r^2 = 0.28$, respectively). It was postulated that this is due to a "level switch" in which the phenomena of fracture becomes dependent on substructures of particles which are of the same dimensions regardless of particle thickness.
The nonlocal fracture model used in this study has been shown to be accurate to predict the relationship between the dimensions of the substructures (particle thickness) and the resulting fracture resistance of a composite formed of these particles. Hence, the strength of nonlocal theory in application to phenomena dependent on the interactions of substructures has been substantiated. A combination of nonlocal and micropolar theories is expected to provide greater insight into fracture processes of materials like particleboard.
IX. RECOMMENDATIONS

As suggested in the section on glassy polymers there is a need for a method of obtaining more accurate $\lambda$ values. This method may arrive through the study of wave mechanics. As stress waves travel through a material they become dispersive when their wavelength approaches the dimensions of $\lambda$. If we could accurately detect this dispersion then a prediction of $\lambda$ would follow.

An alternative method of obtaining $\lambda$ is suggested by equating the classical and nonlocal fracture criteria. They are repeated here for convenience as

\begin{align*}
P_2^1 &= 2E/(1 - \nu^2) \gamma \quad \text{IX-1} \\
&= (\lambda/2c^2) \tau_c^2 \quad \text{IX-2}
\end{align*}

we see that

$$\lambda = f(\tau_c, \gamma, E, \nu) \quad \text{IX-3}$$

Hence, $\lambda$ is a function of material constants.

We suggested previously that both elastic and plastic phenomena are dependent on characteristic dimensions. Possibly a $\lambda$ value will follow from examination of the respective equations of stress.

From the theory of elasticity the first stress invariant (summation of the normal stress components), $\sigma_{ii}$, is related to the cubical dilatation, $\tilde{D}$ (a measure of volume expansion) by the relation

$$\sigma_{ii} = (3\delta + 2\mu) \tilde{D} \quad \text{IX-4}$$

where $\delta$ and $\mu$ are elastic moduli (classical Lame's coefficients). The bulk modulus, $B$, is defined as

$$B = (3\delta + 2\mu) \quad \text{IX-5}$$
The dimensions of $B$ are
\[ \text{dim} (B) = \frac{M}{t^2L} \] IX-6
where $M$ is mass, $L$ is length and $t$ is time.

For a fluid state the analogous equation to IX-4 is given by
\[ \tau_{ii} = (3\dot{\theta} + 2\ddot{\theta}) \varepsilon_{ii} \] IX-7
where $\tau_{ii}$ and $\varepsilon_{ii}$ are the sums of the normal components of the viscous stress and velocity strain tensors, respectively. The constants $\ddot{\theta}$ and $\dot{\theta}$ correspond to the first and second coefficients of viscosity. The bulk coefficient of viscosity, $\overline{B}$, is defined as
\[ \overline{B} = 3\dot{\theta} + 2\ddot{\theta} \] IX-8
The dimensions of $\overline{B}$ are
\[ \text{dim} (\overline{B}) = \frac{M}{tL} \] IX-9
If we multiply a ratio between $B$ and $\overline{B}^2$ by mass density, $\rho$, we have
\[ \rho B / \overline{B}^2 \] IX-10
The dimensions of this ratio are
\[ \text{dim} (\rho B / \overline{B}^2) = L^{-2} \] IX-11
Perhaps ratio IX-10 represents a measure of $\lambda$ given after rearrangement as
\[ \lambda = (\overline{B}^2 / \rho B)^{\frac{1}{3}} \] IX-12
This relationship shows a dependence on material constants (refer back to equation IX-3). We may test this relationship with materials for which experimental data of $\overline{B}$ and $B$ exist (Mantell, 1958). Using equation IX-12 plus the elastic and viscous constants of zinc (Zn) and silver (Ag) we obtain
\[ \lambda_{\text{Zn}} = 1.12 \text{ Å} \] IX-13
\[ \lambda_{\text{Ag}} = 1.03 \text{ Å} \]
The respective atomic distances for zinc and silver are given as

\[
a_{Zn} = 2.66 \, \text{Å} \quad \text{IX-14} \\
a_{Ag} = 2.88 \, \text{Å}
\]

(Kittel, 1971).

Equations analogous to IX-4 and IX-7 can be found in plasticity, viscoelasticity, micropolar, and nonlocal theories. Other ratios may be formed for characteristic lengths pertaining to various phenomena.

If we divide the internal characteristic length, \( \lambda \), by an external characteristic length, 1 (e.g. crack dimension) we have a dimensionless number. For solids the ratio of \( \lambda/1 \) may be analogous to the Reynolds number for fluids. The Reynolds number can characterize the flow of a fluid, and \( \lambda/1 \) values may predict strength phenomena for solids. Many nonlocal phenomena that occur in materials are expected to be related to \( \lambda/1 \), therefore, its measurement may lead to nondestructive testing methods.

Nonlocal and micropolar theories can be expected to provide better mathematical models for the prediction of the behavior of real materials. There are many phenomena that classical continuum theories cannot adequately describe. These phenomena originate at the level of the internal structures where it becomes important to account for such factors as micro orientation and "particle" interactions.

Many challenging questions remain unanswered in the fields of wood physics. Moisture and heat movement in wood are dependent on nonlocal and micropolar effects. Impregnation of preservatives into wood is expected to fall under the category of capillary flow of a micropolar fluid. Non-destructive testing, which has its roots in wave mechanics,
is expected to be enlightened with the application of nonlocal effects (wave dispersion). Nonlocal continuum is the only theory that allows for surface tension effects in its field equations. There are numerous examples of the potential of these new theories and there is a strong need for workers in this field.
BIBLIOGRAPHY


