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A STUDY OF SOME OBSERVED VARIATIONS IN
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Raindrop-size distributions taken with methylene blue dyed filter paper recorders produced significantly different values of $A$ in the "Z-R relationship", $Z=A R^{B}$, at various station locations in the OSU Mesometeorological Network for given types of storms. However, the raindrop-size distributions did not produce significantly different values of $B$ of the $Z-R$ equation at stations. For certain storms, calculation of a correlation coefficient revealed a moderate correlation between the values of the coefficient $A$ and the elevations at stations. For any given value of $R$, the average $Z-R$ equation of windward stations yielded a higher value of $Z$ than the average $Z-R$ equation of leeward stations. At stations located in valleys, there occurred the highest average $Z$ values for given $R$ values followed in order by the $Z$ values for stations located on hillsides and those for hilltop stations in three of the four storms
analyzed. The analysis of covariance revealed no significant difference in the constants $A$ and $B$ at stations located in valleys during given storms. However, significant differences do appear between the values of $A$ for hillside stations and those for hilltop stations. The highest coefficient of the $Z-R$ equation occurred at different stations during different storms.

# A STUDY OF SOME OBSERVED VARIATIONS IN RADAR Z-R RELATIONSHIPS WITHIN STORMS 

by
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# A STUDY OF SOME OBSERVED VARIATIONS IN RADAR Z-R RELATIONSHIPS WITHIN STORMS 

## I. INTRODUCTION

When an electromagnetic wave transmission from a radar encounters an area of rainfall, scattering occurs, and a very small percentage of the electromagnetic radiation returns toward the radar. If the cross-sectional area of the radar beam at the point of contact intercepts an area of rainfall having a cross-section equal to or greater than the radar beam, the average power received by the radar antenna $\overline{\mathrm{P}}_{\mathrm{r}}$ appears in the equation as (2, p. 30):

$$
\begin{equation*}
\bar{P}_{r}=\frac{\pi^{5} P_{t} A_{p}^{2} \theta \phi h|k|^{2}}{72 \lambda^{6}} \frac{Z}{r^{2}} \tag{1}
\end{equation*}
$$

where
$P_{t}=$ The power transmitted by the radar, watts.
$A_{p}=$ The apertural area of the radar antenna, $m^{2}$.
$\theta=$ The horizontal width of the radar beam, rad.
$\phi=$ The vertical width of the radar beam, rad.
$h=$ The pulse length, m.
$\lambda=$ The wave length of the radar, mm .
$|k|^{2}=$ The complex index of refraction, no units.
$Z=$ The reflectivity factor, $\mathrm{mm}^{6} \mathrm{~m}^{-3}$
$r=$ The range of the meteorological target, m.

Now $P_{t}, A_{p}, \theta, \phi$, and $\lambda$ relate to the properties of the radar set and remain constant, while $|k|^{2}$ remains constant for a given type of precipitation, and is close to unity for radar wavelengths greater than approximately 3 cm . Therefore, equation (1) expressed as

$$
\begin{equation*}
\bar{P}_{r}=C_{1} \quad \frac{Z}{r^{2}} \tag{2}
\end{equation*}
$$

remains valid for a given radar where $C_{1}$ indicates a constant. The reflectivity factor $Z$ depends on the number and size of the raindrops in the radar beam, and appears as

$$
\begin{equation*}
Z=\sum_{\text {vol }} N_{i} D_{i}^{6} \tag{3}
\end{equation*}
$$

where $N_{i}$ represents the number of spherical raindrops of diameter $D_{i}$, and $\sum_{\text {vol }}$ represents summation over a unit volume (2, p. 30).

From empirical studies made by others, a relation between $Z$ and rainfall intensity $R$ appears as

$$
\begin{equation*}
\mathrm{Z}=\mathrm{A}\left(\frac{\mathrm{R}}{\frac{\mathrm{~mm}}{\mathrm{hr}}}\right)^{\mathrm{B}} \tag{4}
\end{equation*}
$$

where $R$ has the units of $\mathrm{mm} \mathrm{hr}^{-1}$, and $A$ and $B$ represent constants (7, p. 25). The coefficient $A$ has the units of $\mathrm{mm}^{6} \mathrm{~m}^{-3}$, and the exponent $B$ has no units. This equation will hereinafter be referred to the $Z-R$ equation. Substituting equation (4) for $Z$ into
equation (2) yields

$$
\begin{equation*}
\bar{P}_{r}=\frac{C_{2} R^{B}}{r^{2}} \tag{5}
\end{equation*}
$$

where $C_{2}$ represents the product $C_{1} A$. Thus one can make quantitative precipitation measurements at a point by knowing the constants $A, B$, and $C_{1}$, and the average power received by a radar from range $r$

$$
\begin{equation*}
R=\left(\frac{P_{r} r^{2}}{C_{2}}\right)^{\frac{1}{B}} \tag{6}
\end{equation*}
$$

Investigators ( $8,12,13,23$ ) early found that the constants $A$ and $B$ varied from one storm to the next. However, they did not examine the variations in the values of $A$ and $B$ for a given storm at different locations. Since $C_{1}$ remains constant for any given storm, and since one could determine $\overline{\mathrm{P}}_{\mathrm{r}}$ and r from the radar, one can better estimate rainfall intensity at a point with radar if one knows how $A$ and $B$ vary with station location. This study seeks to determine any significant differences in $A$ and $B$ at various locations in the Oregon State University Mesometeorological Network for a given storm. The study also compares the differences in $A$ and $B$ at various locations for a given storm with other storms. Data collected in the Oregon State University Mesometeorological Network
for the following dates provided the only data used:
29 December 1962
2 February 1963
2 March 1963
27 March 1963
28 March 1963

## II. DATA COLLECTION AND REDUC TION

This study makes use of raindrop-size distributions collected at field stations in the OSU Mesometeorological Network. The majority of the field stations in the network lie to the west of Philomath on U. S. Highway 20 toward Newport and Oregon Highway 34 toward Waldport (Plate I). The observers did not operate at all of the same stations during the five storms. Observers equipped with altimeters, battery-operated fan psychrometers, rain gauges, filter papers, and hail recording panels took meteorological data while the Weather Radar Set AN/CPS-9 operated at McCulloch Peak. The observers exposed Whatman No. 4 filter papers rubbed with methylene blue dye to the rain for a period of a few seconds to collect raindropsize distributions (7, p. 11). The observers recorded the exposure duration of the filter paper to the rain, time of day, filter paper number, station number, and other pertinent data on the back of each exposed filter paper. Drop-size sampling occurred at ten minute intervals and more often when rainfall intensities changed noticeably.

With the aid of calibrated circular templates, workers then measured the diameters of raindrop stains on each filter paper and tabulated the numbers of raindrop stains in each 0.2 mm diameter interval.


## III. CALCULATIONS OF $Z$ AND $R$ VALUES FROM DROP-SIZE DISTRIBUTIONS

## A. THEORY

The numbers of raindrop blots of different sizes on a sheet of filter paper exposed for a given time will provide data on the dropsize distribution of drops reaching the surface. However, this does not provide directly the distribution of different sized drops in a given volume at any instant. In order to determine the number of drops having diameters $D_{i}$ per unit volume, one must make allowance for the different rates of fall of drops of given diameters because the depth of fall of raindrops varies with the size of the drops. A raindrop falling at its terminal velocity $v_{i}$ descends a depth d

$$
\begin{equation*}
\mathrm{d}=\mathrm{v}_{\mathrm{i}} \mathrm{t} \tag{7}
\end{equation*}
$$

during time $t$. If only one drop falls on a given horizontal area $A_{f}$ in time $t$, the volume $V$ enclosing that one drop appears as

$$
\begin{equation*}
\mathrm{V}=\mathrm{A}_{\mathrm{f}} \mathrm{~d}=\mathrm{A}_{\mathrm{f}} \mathrm{v}_{\mathrm{i}} \mathrm{t} \tag{8}
\end{equation*}
$$

If $\mathrm{N}_{\mathrm{i}}$ number of drops of the same diameter fell on a given area in time $t$, the number of drops per unit volume appears as

$$
\begin{equation*}
N_{i} / V=\frac{N_{i}}{A_{f} v_{i} t} \tag{9}
\end{equation*}
$$

Thus, one can make approximations of the reflectivity factor $Z$ defined as the sixth power of the diameters of all raindrops in a unit volume by using the numbers $C_{j}$ of raindrop blots having average diameters $D_{\bar{j}}$ in each 0.2 mm diameter range, the filter paper area $A_{f}$, the terminal velocities $v_{\bar{j}}$ of the raindrops having average diameters $D_{\bar{j}}$, and the exposure duration $t$ of the filter paper.

$$
\begin{equation*}
z \doteq \sum_{j=0.2} \frac{C_{j} D_{j}^{6}}{A_{f} v_{i} t} \tag{10}
\end{equation*}
$$

or

$$
\begin{equation*}
z \doteq \frac{\sum_{j=0.2}\left(C_{j} K_{j}^{\prime}\right)}{t} \tag{11}
\end{equation*}
$$

where

$$
\begin{aligned}
& \sum_{j=0.2}=\text { Summation over all } 0.2 \mathrm{~mm} \text { diameter ranges. } \\
& K_{j}^{\prime}=\text { The constants }\left(D_{j}^{6} A_{f}^{-1} v_{j}^{-1}\right) .
\end{aligned}
$$

The constants $\mathrm{K}_{\mathrm{j}}$ ' appear in Plate II.
The numbers of raindrop blots of different sizes on a sheet of filter paper exposed for a given time will also provide data for calculations of rainfall intensity. The volume of rain falling on a given area per unit time defines the rainfall intensity $R$.

$$
\begin{equation*}
R=\frac{\text { Volume of Rain }}{A_{f}} \tag{12}
\end{equation*}
$$

If one assumes spherical raindrops, the volume of a raindrop of diameter $D_{i}$ equals

$$
\begin{equation*}
\text { Volume of Rain }=\frac{\pi D_{i}^{3}}{6} \tag{13}
\end{equation*}
$$

Thus one can approximate the rainfall intensity by using the numbers $C_{j}$ of raindrop blots having average diameters $D_{\bar{j}}$ in each 0.2 mm diameter range, the filter paper area $A_{f}$, and the exposure duration of the filter paper.
or

$$
\begin{equation*}
R \doteq \sum_{j=0.2} \frac{\pi D_{j}^{3} C_{j}}{6 A_{f}^{t}} \tag{14}
\end{equation*}
$$

where

$$
\begin{aligned}
& \sum_{j=0.2}=\text { Summation over all } 0.2 \mathrm{~mm} \text { diameter ranges. } \\
& K_{j}^{\prime \prime}=\text { The constants }\left(D_{j}^{3} A_{f}^{-1} \pi 6^{-1}\right) .
\end{aligned}
$$

The constants $\mathrm{K}_{\mathrm{j}}$ " appear in Plate II.

PLATE II


## B. DISCUSSION

One can obtain values for Z from either the radar or dropsize distributions. This study did not make use of radar for determining $Z$ values at stations because the radar beam did not approach low enough to the surface to permit assuming no changes in the rain-drop-size distributions from the radar beam to the surface. When the radar beam aims low the radar cannot distinguish between ground echoes and precipitation echoes. In order to get the rainfall intensity most representative of the calculated $Z$ value, one must use the same drop-size distributions for calculating both $Z$ and $R$ values. Thus, this study made use of drop-size distributions collected at field stations to determine both the $Z$ and $R$ values (Table A-I to A-XV).

## C. SOURCES OF ERROR

Splashing of relatively large raindrops upon contact with the exposed filter papers increases the numbers of smaller raindrop blots. Mason and Andrews (20, p. 349-353) found that a drop of 3.5 mm diameter produced approximately 57 smaller raindrops of 0.5 mm diameter or less. They also reported that raindrops having diameters of 2.5 mm or less produced very few drops by splashing.

Upon referring to the constants in Plate IJ., the percentage error in the $Z$ value and $R$ value caused by one drop of 3.5 mm diameter splashing 57 drops having the same diameter of 0.5 mm results in an error of $0.2 \%$ for $Z$ and $16.3 \%$ for R. However, the actual percentage error in $Z$ and $R$ remains smaller since the 57 drops can also be smaller than 0.5 mm diameter. Also, the majority of observed raindrops had diameters smaller than 3.5 mm .

The author believes that the greatest error lies in the recorded exposure duration of sampling. A probable error of plus or minus one second can cause a percentage error in $Z$ and $R$ as high as $200 \%$ for recorded exposure durations of 0.5 seconds. This error increases with the increase in rainfall intensity because the observer must obtain drop-size distributions on the filter papers which can be measured. If the observer exposes the filter paper too long, one cannot distinguish between two different blots on the filter due to an over saturation of blots on the filter paper.

Filter papers exposed perpendicularly to the fall trajectories of the raindrops instead of horizontally to the ground cause errors in estimating the numbers of drops per unit volume. This error in sampling increases the numbers of drops per unit volume, and hence, overestimates the values of $Z$.

Conditions of strong downdrafts cause overestimations of $Z$ values, and conditions of strong updrafts cause underestimations of $Z$ values. This error results from the fact that $Z$ relates inversely to the actual fall velocity of the raindrops (Equation 10), and that the constants used the terminal velocities of the raindrops.

# IV. A COMPARISON OF CHANGES IN RADAR DATA OF $\bar{P}_{r}$ OVER STATIONS WITH CHANGES IN Z VALUES OBTAINED AT THE STATIONS 

The Weather Radar Set AN/CPS-9 at McCulloch Peak provides data in decibels, db , which permit relating the actual signal strength $\bar{P}_{r}$ received compared to the signal $P_{o}$ needed to produce a barely detectable indication on the RHI-Scope and PPI-Scope indicators with the receiver at maximum gain.

$$
\mathrm{db}=10 \log \frac{\overline{\mathrm{P}}_{\mathrm{r}}}{\mathrm{P}_{\mathrm{o}}}
$$

The camera records precipitation echoes on the PPI-Scope and RHIScope indicators automatically at pre-set receiver gain settings below the maximum receiver gain setting labeled "O db". For instance, if an echo begins to disappear at a gain setting labeled " -6 db ", one knows that $\bar{P}_{r}$ exceeds $P_{o}$ by +6 db .

Changes in $\overline{\mathrm{P}}_{\mathrm{r}}$ from a beam-filling target at a given range depend primarily on changes in $Z$ at the given range

$$
\begin{equation*}
\bar{P}_{r}=C_{3} Z \tag{16}
\end{equation*}
$$

where $C_{3}$ indicates the product $C_{1} \mathrm{r}^{-2}$. The following equations show how changes in $Z$ in the radar beam at a given range from the radar correspond to changes in $\bar{P}_{r}$ in $d b$.

$$
\begin{equation*}
\mathrm{db}_{1}=10 \log \left(\frac{\overline{\mathrm{P}}_{\mathrm{r}_{1}}}{\mathrm{P}_{\mathrm{o}}}\right) \tag{17}
\end{equation*}
$$

$$
\begin{align*}
& \mathrm{db}_{2}=10 \log \left(\frac{\overline{\mathrm{P}}_{\mathrm{r}_{2}}}{\mathrm{P}_{\mathrm{o}}}\right)  \tag{18}\\
& d b_{2}-d b_{1}=10 \log \left(\frac{\bar{P}_{r_{2}}}{P_{o}}\right)-10 \log \left(\frac{\bar{P}_{r_{1}}}{P_{o}}\right)  \tag{19}\\
& d b_{2}-d b_{1}=10\left[\log \left(\frac{\bar{P}_{r_{2}}}{P_{o}}\right)-\log \left(\frac{\bar{P}_{r_{1}}}{\mathrm{~F}_{\mathrm{o}}}\right)\right]  \tag{20}\\
& d b_{2}=d b_{1}=10 \log \left(\frac{\bar{P}_{r_{2}}}{\frac{P_{o}}{P_{r_{1}}}} \frac{\bar{P}_{o}}{P_{0}}\right)  \tag{21}\\
& \mathrm{db}_{2}-\mathrm{db}_{1}=10 \log \left(\frac{\overline{\mathrm{P}}_{\mathrm{r}_{2}}}{\overline{\mathrm{P}}_{\mathrm{r}_{1}}}\right)  \tag{22}\\
& d b_{2}-d b_{1}=10 \log \left(\frac{C_{3} Z_{2}}{C_{3} Z_{1}}\right)  \tag{23}\\
& d b_{2}-d b_{1}=10\left[\log Z_{2}-\log Z_{1}\right]
\end{align*}
$$

Thus, changes in $\overline{\mathrm{P}}_{\mathrm{r}}$ in db from targets at a given range should equal ten times the changes in the common logarithm values of $Z$.

For the storms on 2 March and 27 March, the author compared changes in $\bar{P}_{r}$ in db over two stations with changes in calculated $Z$ values obtained at those stations. The expression on the right of equation (24) determined the changes in calculated $Z$ values obtained
at the stations. Since the camera recorded precipitation echoes on the RHI-Scope and PPI-Scope indicators automatically at fixed receiver gain settings, and not necessarily when the echoes over the station disappeared, the data provided by the radar represent limits of $\overline{\mathrm{P}}_{\mathrm{r}}$ in db . For example, if the camera records no echo over a station at a receiver gain setting labeled " -12 db " and on the succeeding gain setting labeled "-9 db" records an echo, one only knows that $\overline{\mathrm{P}}_{\mathrm{r}}$ ranges between +9 db and +12 db greater than $\mathrm{P}_{\mathrm{O}}$. Therefore, in calculating changes in $\overline{\mathrm{P}}_{\mathrm{r}}$ over stations, one can compute only the maximum possible change in $\overline{\mathrm{P}}_{\mathrm{r}}$. For example, if an initial echo disappeared between receiver gain settings labeled " -9 db " and " -3 $d b "$, and after a time disappeared between receiver gain settings labeled " -12 db " and " -9 db ", the maximum possible change between successive values of $\bar{P}_{r}$ equals +9 db . If an echo appears at the lowest receiver gain setting, one does not know the limits of $\overline{\mathrm{P}}_{\mathrm{r}}$ and thus cannot calculate the maximum possible change of $\overline{\mathrm{P}}_{\mathrm{r}}$. A simple numerical coefficient $R R$ determined how well the changes in $Z$ values obtained at the stations compare with the changes in $\overline{\mathrm{P}}_{\mathrm{r}}$ over the stations. The following rules state the criteria for solving $R R$.

$$
\begin{equation*}
R R=\frac{\sum_{N}(P Q)}{N} \tag{25}
\end{equation*}
$$

1. If changes in $\bar{P}_{r}$ and $Z$ values obtained at the ground have the same sign, let $P=2.0$.
2. If changes in $\bar{P}_{r}$ and $Z$ values obtained at the ground have opposite signs, let $P=0.0$.
3. If the magnitude of change in $Z$ in $d b$ exceeds the maximum possible change in $\bar{P}_{r}$, let $Q=1.0$.
4. If the magnitude of change of $Z$ in $d b$ remains smaller or equals the maximum possible change in $\bar{P}_{r}$, let $Q=2.0$.
5. N represents the number of changes compared.

In determining how well changes in $\overline{\mathrm{P}}_{\mathrm{r}}$ over stations compare with changes in $Z$ values in $d b$ obtained at the stations, $R R=4.0$ indicates the best possible relation, and $R R=0.0$ indicates no relation between $\bar{P}_{r}$ and $Z$ since $Z$ relates directly to $\bar{P}_{r}$ (Equation 24). A computed RR coefficient of 3.6 for a given station indicates a better relation between $\bar{P}_{r}$ and $Z$ at the given station than a computed RR coefficient of 3.0 for another station.

The results (Table I) indicate a better relation between changes in both $\overline{\mathrm{P}}_{\mathrm{r}}$ and Z in db obtained at the ground for Station 18 than for Station 1. The results seem reasonable because the radar beam approached closer over Station 18 than over Station 1 by 300 feet, and therefore evaporation, coalescence, and wind shear could have affected the raindrop-size distribution from the radar beam less at Station 18 than at Station 1.

TABLE I

|  |  | Maximum | Z at | , | Z |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { Time } \\ & \text { (PST) } \\ & \hline \end{aligned}$ | $\overline{\mathrm{P}}_{\mathrm{r}} \text { over Station } 18$ | possible change $\overline{\mathrm{P}}_{\mathrm{r}}(\mathrm{db})$ | $\begin{aligned} & \text { Station } 18 \\ & \left(\mathrm{~mm}^{6} \mathrm{~m}^{-3}\right) \end{aligned}$ | $\log \mathrm{Z}$ | change <br> (db) | $(\mathrm{P} \times \mathrm{Q})$ |
| 1550 | 9 to 12 | 3 | 39.9 | 1. 600 | 3,30 | 2 |
| 1601 | 9 to 12 . | + 3.0 | 85.2 | 1.930 |  |  |
| 1610 | 12 to 18 | + 9.0 | 500 | 2. 699 | 7.69 | 4 |
| 1618 | Greater than 18 | Unknown | 3390 | 3.530 | + 8.31 | 4 |
| 1628 | 12 to 18 | Unknown | 1000 | 3. 000 | - 5.30 | 4 |
| 1640 | 9 to 12 | - 90 | 14.7 | 1. 167 | -18.33 | 2 |
| 1702 | 12 to 18 | - 9.0 | 2280 | 3.356 | +21.89 | 2 |
| 1711 | 12 to 18 | - 6.0 | 1990 | 3.299 | -0.56 | 4 |
| 1721 | 12 to 18 | - 6.0 | 1070 | 3.030 | - 2.69 | 4 |
| 1731 | 12 to 18 | - 6.0 | 25.71 | 1. 410 | -16. 20 | $\frac{2}{28}$ |
|  | - |  | S |  |  | $\mathrm{RR}=3.11$ |

RR Calculation for Station 18 on 27 March 1963

| RR Calculation for Station 18 on 27 March 1963 |  |  |  |  |  |  |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| 1339 | 13.02 to 14.86 |  | 525 | 2.720 | +2.88 | 2 |
| 1349 | 13.02 to 14.86 | +1.84 | 1020 | 3.008 | +4.90 | 4 |
| 1359 | Greater than 19.48 | Unknown | 3150 | 3.498 | +0.31 | 0 |
| 1408 | 14.86 to 19.48 | Unknown | 3380 | 3.529 | -1.07 | 4 |
| 1418 | 14.86 to 19.48 | -4.62 | 2640 | 3.422 | +1.02 | 4 |
| 1429 | 14.86 to 19.48 | +4.62 | 3340 | 3.524 | -2.23 | 4 |
| 1438 | 14.86 to 19.48 | -4.62 | 2000 | 3.301 | -2.40 | 4 |
| 1448 | 14.86 to 19.48 | -4.62 | 1150 | 3.061 | -12.98 | 2 |
| 1500 | 13.02 to 14.86 | -6.46 | 57.9 | 1.763 |  | $\frac{24}{24}$ |

TABLE I (continued)

| $\begin{aligned} & \text { Time } \\ & \text { (PST) } \end{aligned}$ | $\mathrm{P}_{\mathrm{r}}$ over Station 1 (db) | $\begin{gathered} \text { Maximum } \\ \text { possible change } \\ \mathbb{P}_{\mathrm{I}}(\mathrm{db}) \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{Z} \text { at } \\ \text { Station } 1 \\ \left(\mathrm{~mm}^{6} \mathrm{~m}^{-3}\right) \end{gathered}$ | $\log \mathrm{Z}$ | Z change (db) | ( $\mathrm{P} \times \mathrm{Q}$ ) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1313 | 3.28 to 13.02 |  | 209 | 2.318 |  |  |
| 1333 | 0.00 to 3.28 |  | 64.2 | 1.808 |  | 0 |
| 1343 | 3.28 to 13.02 | +13.02 | 21.9 | 1.336 | - 4.72 | 0 |
| 1353 | 0.00 to 3.28 | -13.02 | 63.5 | 1.803 | + 4.67 | 0 |
| 1411 | 14.86 to 19.48 | +19.48 | 11.1 | 1.045 | -7.58 | 0 |
| 1422 | 14.86 to 19.48 | +4.62 +4.62 | 304 | 2. 483 | +14.38 $+\quad 358$ | 4 |
| 1432 | 14.86 to 19.48 | + 4.62 | 694 | 2. 841 | + 3.58 | 4 |
| 1442 | 14.86 to 19.48 | + 4.62 | 842 | 2.925 | +0.84 +4.33 | 4 |
| 1453 | Geater than 19.48 | Unknown | 2280 | 3. 358 | + 4.33 | 4 |

$R \mathrm{R}=2.25$

|  | RR Calculation for Station 1 on 2 March 1963 |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| 1437 | 12 to 18 |  | 890 | 2.949 | -3.68 | 0 |
| 1447 | Greater than 18 | Unknown | 381 | 2.581 | -10.37 | 4 |
| 1617 | 12 to 18 | $"$ | 35.1 | 1.544 | -9.47 | 0 |
| 1626 | Greater than 18 | $"$ | 3.95 | 0.597 | +22.64 | 4 |
| 1638 | Greater than 18 | $"$ | 726 | 2.861 | +6.78 | 4 |
| 1648 | Greater than 18 | $"$ | 3470 | 3.539 | -3.63 | 4 |
| 1658 | Greater than 18 | $"$ | 1500 | 3.176 | +0.57 | 4 |
| 1708 | Greater than 18 | $"$ | 3090 | 3.490 | -30.29 | $\frac{4}{4}$ |
| 1717 | Greater than 18 |  | 2.89 | 0.461 |  | 24 |

## V. DETERMINATION AND COMPARISON OF CONSTANTS IN THE Z-R EQUATION

## A. THEORY

In order to determine and to compare the values of the constants $A$ and $B$ of the $Z-R$ equationatstations, one can resort to the techniques of statistics summarized below.

Equation (4) written in the linear form appears as
or

$$
\begin{align*}
\log Z & =\log A+B(\log R)  \tag{26}\\
Y & =C+B x \tag{27}
\end{align*}
$$

where $Y$ represents $\log Z, C$ denotes $\log A$, and $x$ signifies $\log R$. The method of least squares (18, Chapter 16) estimates C and $B$ by minimizing the residual $S S$ defined as the sum of squares of the deviations of the experimental $\log Z$ values, $y$, from the estimate of $Y, \bar{y}_{x^{\prime}}$ (18, p. 253), giving estimates of $C$ and $B$ designated $c$ and $b$, respectively.

$$
\begin{equation*}
\sum\left(y-\bar{y}_{x}\right)^{2}=\text { Minimum }=\text { Residual } S S \tag{28}
\end{equation*}
$$

Although one can obtain different estimates of $C$ and $B$ (Equation 27) from data collected at stations during a given storm, this does not necessarily indicate the constants $A$ and $B$ (equation 4) differ significantly between stations because of inherent errors caused by sampling only part of the raindrops falling at each of the
stations. Thus, one determines the equivalence of $C$ and $B$ at different stations by using the analysis of covariance (18, Chapter 19) which examines the equivalence of the estimates $c$ and $b$ at the stations under the following assumptions (18, p. 248): (A) The y values for any given $R$ follow a normal distribution, (B) Equations (26) and (27) represent correct relationships, and (C) The variance of $y$ for all values of $R$ for any given storm remains equivalent at all stations. In general, the analysis of covariance using Snedecor's F-test measures the ratio of means of sums of squared deviations of $c$ or $b$ at each station from an average $c$ or $b$ from all stations divided by a pooled residual SS from all the stations.

$$
\begin{align*}
& F=\frac{\text { Mean } \sum(b-\bar{b})^{2}}{\text { Mean (Pooled Residual SS) }}  \tag{29}\\
& F=\frac{\text { Mean } \sum(c-\bar{c})^{2}}{\text { Mean (Pooled Residual SS })} \tag{30}
\end{align*}
$$

One accepts the hypothesis that all values of $B$ or $C$ have single, identical values at all stations if the F -value approaches close to or equals zero.

The level of significance states the probability of rejecting a true hypothesis and the number of degrees of freedom DF defines the particular distribution the F -value follows if the hypothesis remains true. The variance of the array, $s^{2}$, gives a measure of scatter of the points from the best fitting line which represents the

Z-R equation at each station. Increasing scatter results in increasing $s^{2}$.
B. RESULTS

Calculated $Z$ and $R$ values from drop-size distributions collected at each of the operating stations during the entire observation period on a given date provided data for determining the constants A and B. The method of least squares yielded estimates of the constant $A$ and the constant $B$ at each of the operating stations for each of the five storms (Table II). The first analysis (18, p. 349) examined the equivalence of $B$ at the operating stations for a given storm (Table III). Since the results of the tests revealed that $B$ has a single, identical value at the stations, an average weighted exponent $\bar{b}$ represented the exponent $B$ at each station for a given storm (18, p. 345). The average weighted exponent then yielded a different estimate of $C, c^{\prime}$, at each of the operating stations because the method of least squares uses the exponent in determining the value of c. The antilogarithm of the different estimates $c^{\prime}$ then estimates the coefficient A at each station (Table IV).

The F-test (18, p. 364) then examined the equivalence of $A$ by examining the equivalence of the common logarithm value of $Z$ at each station during a given storm (Table V). The Z-R equation
(using the average $R$ observed at all stations during a given storm) with the estimates $c^{\prime}$ and $\bar{b}$ solved for the $Z$ value at each station during a given storm. Since the exponent at all of the stations on a given date has a single, identical value, the lines representing the Z-R equation at each station on a logarithmic graph parallel each other. Therefore, this test also determined the equivalence of the coefficient A at stations for a given storm. The results revealed that all stations did not have a single value of this coefficient at a significance level of $5 \%$ in each storm except the storm on 29

December.

Since the $F$-test revealed that both the exponent and coefficient at all operating stations on 29 December had identical values, the author combined the data from all the stations and obtained the constants of the $Z-R$ equation applicable to all the stations.

$$
\mathrm{Z}=192\left(\mathrm{R} / \mathrm{mm}-\mathrm{hr}^{-1}\right)^{1.13}
$$

To determine whether any significant difference existed in the exponent at stations during different storms, the F-test examined the equivalence of $B$ at ten stations, two stations from each storm. The results (Table VI) revealed that significant differences existed in the exponent at the stations during different storms.

Computing weighted averages from all the data obtained from all stations reporting on a given date yielded a single average $Z-R$

TABLE II

| Z-R Relationships by Method of Least Squares |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 29 December 1962 |  |  |  | 27 March 1963 |  |  |  |
| Station | $\mathrm{Z}=\mathrm{A}$ | $\left.-h r^{-1}\right)^{B}$ | $s^{2}$ | Station | $\mathrm{Z}=\mathrm{A}\left(\mathrm{R} / \mathrm{mm}-\mathrm{hr}^{-1}\right)^{\mathrm{B}}$ |  | $s^{2}$ |
|  | A | B |  |  | A | B |  |
| 2 | 213 | 1.18 | 0.0043 | 1 | 289 | 1.38 | 0.0590 |
| 15 | 339 | 1.28 | 0.0166 | 2 | 276 | 1.47 | 0.0487 |
| 17 | 235 | 1.13 | 0.1390 | 5 | 274 | 1.50 | 0.0363 |
| 22 | 94.3 | 0.97 | 0.0795 | 8 | 737 | 1.49 | 0.0562 |
|  |  |  |  | 9 | 456 | 1.41 | 0.0760 |
|  |  |  |  | 10 | 159 | 1.24 | 0.0358 |
| 2 February 1963 |  |  |  | 11 | 364 | 1.44 | 0.0333 |
| Station | $\mathrm{Z}=\mathrm{A}\left(\mathrm{R} / \mathrm{mm}-\mathrm{hr}{ }^{-1}\right)^{B}$ |  | $s^{2}$ | 12 | 294 | 1.68 | 0.0244 |
|  | A | B |  | 15 | 306 | 1.43 | 0.0205 |
| 1 | 219 | 1.44 | 0.0635 | 17 | $\begin{aligned} & 312 \\ & \\ & \hline 77 \end{aligned}$ |  | 0.0277 0.0254 |
| 2 | 233 | 1.42 | 0.0878 | 21 | 320 | 1.34 | 0.0254 |
| 4 | 243 | 1.22 | 0.1095 |  |  |  |  |
| 6 | 230 | 1.49 | 0.0392 |  |  |  |  |
| 8 | 200 | 1.36 | 0.0781 | 28 March 1963 |  |  |  |
| 9 | 221 | 1.57 | $\begin{aligned} & 0.0505 \\ & 0.0689 \end{aligned}$ | Station | $\mathrm{Z}=\mathrm{A}\left(\mathrm{R} / \mathrm{mm}-\mathrm{hr}{ }^{-1}\right)^{\mathrm{B}}$ |  | $s^{2}$ |
| 10 | 175 | 1.52 |  |  | A | B |  |
| 15 | 187 | 1.50 | 0.0530 |  |  |  |  |
| 17 | 270 | 1.51 | 0.1159 | 1 | 404 | 1.41 | 0.1399 |
| 18 | 257 | 1.30 | 0.0909 | 2 | 398 | 1.34 | 0.0148 |
| 20 | 151 | 1. 30 | 0.0608 | 4 | 286 | 1.35 | 0.0152 |
|  |  |  |  | 6 | 337 | 1.37 | 0.0175 |
|  |  |  |  | 15 | 236 | 1. 30 | 0.0094 |
|  |  | 1963 |  | 17 | 244 | 1.41 | 0.0213 |
| Station | $\underline{Z}=\mathrm{A}($ | $\left.-h r^{-1}\right)^{\text {B }}$ | $s^{2}$ | 18 | 373 | 1.43 | 0.0338 |
| Station | A | B |  |  |  |  |  |
| 1 | 365 | 1.30 | 0.0236 |  |  |  |  |
| SLE | 215 | 1.25 | 0.0489 |  |  |  |  |
| 15 | 406 | 1. 29 | 0.0744 |  |  |  |  |
| 16 | 307 | 1.25 | 0.0428 |  |  |  |  |
| 17 | 214 | 1.04 | 0.2015 |  |  |  |  |
| 18 | 377 | 1.32 | 0.0213 |  |  |  |  |

TABLE III

| Summary of the Analysis of Covariance |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Date | Source of variation | Sum of squares | $\begin{gathered} \text { Degrees } \\ \text { of } \\ \text { freedom } \end{gathered}$ | Mean square | F |
| 29 December | Variation among b's | 0.11797 | 3 | 0.03932 | 1. 1606 |
|  | Pooled Residual | 0.84701 | 25 | 0.03388 |  |
| 2 February | Variation among b's | 0.83807 | 10 | 0.08381 | 1. 0925 |
|  | Pooled Residual | 18.41130 | 240 | 0.07671 |  |
| 2 March | Variation among b's | 0.08656 | 5 | 0.01731 | 0.3944 |
|  | Pooled Residual | 2. 85299 | 65 | 0.04389 |  |
| 27 March | Variation among b's | 0.7166 | 11 | 0.06515 | 1. 8057 |
|  | Pooled Residual | 10.96848 | 304 | 0.03608 |  |
| 28 March | Variation among b's | 0.08139 | 6 | 0.01356 | 0.3711 |
|  | Pooled Residual | 3.69082 | 101 | 0.03654 |  |

REMARKS: The exponent $B$ of the Z-R equation at all stations during a given storm has a single, identical value at the $5 \%$ level of significance.

TABLE IV

| Coefficients of the Z-R Relationship at Stations Having the Exponent $\overline{\mathrm{b}}$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 February | $\overline{\mathrm{b}}=1.41$ | 2 March | $\overline{\mathrm{b}}=1.27$ | 27 March | $\overline{\mathrm{b}}=1.40$ | 28 March | $\mathrm{B}=1.36$ |
| Station | A | Station | A | Station | A | Station | A |
| 18 | 261 | 15 | 399 | 8 | 617 | 2 | 402 |
| 6 | 242 | 18 | 381 | 9 | 453 | 1 | 392 |
| 9 | 238 | 1 | 357 | 11 | 364 | 18 | 359 |
| 2 | 234 | 16 | 315 | 12 | 332 | 6 | 340 |
| 4 | 225 | SLE | 207 | 17 | 317 | 4 | 287 |
| 1 | 220 | 17 | 205 | 21 | 316 | 15 | 262 |
| 17 | 217 |  |  | 15 | 311 | 17 | 238 |
| 10 | 194 |  |  | 1 | 294 |  |  |
| 15 | 190 |  |  | 5 | 283 |  |  |
| 8 | 188 |  |  | 2 | 274 |  |  |
| 20 | 150 |  |  | 18 | 260 |  |  |
|  |  |  |  | 10 | 131 |  |  |

TABLE V

| Date |  |  |  | Summary | th | nalysis | Cov |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Source of variation | $\mathrm{SS}_{\mathrm{x}}$ | SP | $\mathrm{SS}_{\mathrm{y}}$ | DF | Residual |  |  |  | Remarks |
|  |  |  |  |  |  | SS | DF | MS | F |  |
| 29December | Among sample | 0.68116 | 0.91618 | 1. 31080 | 3 | 0.11307 | 3 | 0.0377 | 1. 094 | The coefficients at Stations 17, 15,22 , and 2 do not differ significantly from each other at the 5\% level of significance with 3 and 28 degrees of freedom |
|  | Within sample | 10.90671 | 12. 13771 | 14.47258 | 29 | 0.96493 | 28 | 0.0345 |  |  |
|  | Total | 11. 58787 | 13. 05389 | 15.78338 | 32 | 1.07800 |  |  |  |  |
| $2$ <br> February |  |  |  |  |  |  |  |  | 2. 588 | The coefficients at Stations 4,6 , $2,15,18$, and 1 differ significantly from each other at the $0.5 \%$ level of significance with 5 and 138 degrees of freedom |
|  | Among sample | 2. 00261 | 2. 39750 | 3.90767 | 5 | 1. 10640 | 5 | 0. 2221 |  |  |
|  | Within sample | 38.95371 | 54.27272 | 87.46055 | 139 | 11. 84440 | 138 | 0.0858 |  |  |
|  | Total | 40.95632 | 56.67022 | 91.36822 | 144 | 12.95510 |  |  |  |  |
| 2 | Among sample | 8, 09452 | 8.91218 | 10.44740 | 5 | 0.83140 | 5 | 0. 1663 | 3.960 | The coefficients at Stations 1, $18,16,17$, SLE, and 15 differ significantly from each other at the $0.5 \%$ level of significance with 5 and 157 degrees of freedom |
| March | Within sample | 47.89026 | 60.79438 | 80.11509 | 71 | 2. 93960 | 70 | 0.0420 |  |  |
|  | Total | 55.98478 | 69.70656 | 90.56249 | 76 | 3.77096 |  |  |  |  |
| 27 | Among sample | 8.92594 | 9.78236 | 10.78477 | 5 | 0.75108 | 5 | 0.1502 | 4.521 | The coefficients at Stations 8,9, $11,21,17$, and 15 differ significantly from each other at the $0.5 \%$ level of significance with 5 and 157 degrees of freedom |
| March | Within sample | 61.93613 | 86.26163 | 125.35817 | 158 | 5.21717 | 157 | 0.0332 |  |  |
|  | Total | 70. 86207 | 96.04399 | 136. 14294 | 163 | 5.96825 |  |  |  |  |
| 28 | Among sample | 14. 13614 | 20.44875 | 30.23409 | 6 | 0.72730 | 6 | 0. 1212 | 3.349 | The coefficients at Stations 1, 2, $6,4,18,17$, and 15 differ significantly from each other at the $0.5 \%$ level of significance with 6 and 107 degrees of freedom |
| March | Within sample | 37. 04369 | 50.46724 | 72.62730 | 108 | 3. 87221 | 107 | 0.0362 |  |  |
|  | Total | 51.17983 | 70.91599 | 102.86239 | 114 | 4. 59951 |  |  |  |  |

## TABLE VI

|  | Summary of the Analysis of Covariance |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Source of variation | SS | DF | MS | F |
| Variation among b's | 1.46009 | 9 | 0.16223 | 3.038 |
| Pooled residual | 8.11594 | 152 | 0.05339 |  |

REMARKS: The exponents of the Z-R equations at Stations 17 and 22 on 29 December, 9 and 4 on 2 February, 18 and 16 on 2 March, 12 and 10 on 27 March, and 1 and 15 on 28 March differs significantly from each other at the $1.0 \%$ level of significance.

TABLE VII

| Date | Average Z-R Relationships | $s^{2}$ |
| :---: | :---: | :---: |
| 2 February | $\mathrm{Z}=214\left(\mathrm{R} / \mathrm{mm}-\mathrm{hr}^{-1}\right)^{1.41}$ | 0.0767 |
| 2 March | $\mathrm{Z}=313\left(\mathrm{R} / \mathrm{mm}-\mathrm{hr}{ }^{-1}\right)^{1.27}$ | 0.0439 |
| 27 March | $\mathrm{Z}=309\left(\mathrm{R} / \mathrm{mm}-\mathrm{hr}{ }^{-1}\right)^{1.40}$ | 0.0361 |
| 28 March | $\mathrm{Z}=309\left(\mathrm{R} / \mathrm{mm}-\mathrm{hr}{ }^{-1}\right)^{1.36}$ | 0.0365 |

equation valid for that body of data. Such an equation appears in Table VII for each of the following dates: 2 February, 2 March, 27 March, and 28 March.
C. DISCUSSION OF RESULTS

The results of the analysis of covariance reveal that the exponent $B$ at all stations has a single, identical value but that the coefficient A does not have a single value for a given storm. Hence, curves for the $Z-R$ equation at stations parallel each other but have different intercepts. One can only hypothesize as to why the exponent $B$ has the same value for a given storm at stations. Since the exponent $B$ has a single, identical value for a given storm, but changes from storm to storm, this value might depend upon the same factors which uniquely determine the different synoptic situations in those storms. With regard to the varying values of the coefficient $A$, one might also hypothesize that coalescence, evaporation, accretion, and wind shear (all of which can determine in part the value of A) depend on the synoptic situations.

Figures I to $V$ show the highest and lowest value of the coefficient at the operating stations during each of the five storms. For given values of $R$, the difference in $Z$ values between the two stations during a given storm exceeds the difference in average $Z$


Figure I. Widest Variation in the Coefficient of Z-R Relationship for Stations on 29 December 1962.


Figure II. Widest Variation in the Coefficient of Z-R Relationship for Stations on 2 February 1963.


Figure III. Widest Variation in the Coefficient of Z-R Relationship for Stations on 2 March 1963.


Figure IV. Widest Variation in the Coefficient of Z-R Relationship for Stations on 27 March 1963.


Figure V. Widest Variation in the Coefficient of Z-R Relationship for Stations on 28 March 1963.
values at all stations between storms (Table VIII).
Table IX summarizes the synoptic conditions of the five storms. This study did not examine the variations in A with the synoptic conditions associated with the type of storm, height of freezing level, stability index, and relative humidity because of inadequate data. Statistical study requires $Z-R$ relationships at stations for many different storms having the same synoptic conditions in order to eliminate the random variations caused by sampling. For example, one may find A increasing with station elevation for two storms having the same synoptic condition. However, A at stations may then decrease with station elevation during the next two storms having the same synoptic condition as the two previous storms. Thus, if one concludes that the particular synoptic condition causes $A$ to increase with station elevation from data collected during the first two storms, one makes a false conclusion.

To study the variations in the constant $A$ at stations for a given storm, the author omitted the storm on 29 December because the constants $A$ and $B$ had single, identical values at all of the operating stations.

TABLE VIII


TABLE IX
Summary of Synoptic Conditions

| Date | Type of storm | Summary of Synoptic Conditions <br> freezing of level | Stability <br> index | Height of <br> echoes | Relative <br> humidity |
| :---: | :--- | :---: | :---: | :---: | :---: |
| 29 December | Cold front | 4300 ft | +12.5 | $20,000 \mathrm{ft}$ | $73 \%$ |
| 2 February | Warm front | 6500 ft | +3.0 | $25,000 \mathrm{ft}$ | $86 \%$ |
| 2 March | Instability shower | 3900 ft | +3.0 | $17,000 \mathrm{ft}$ | $83 \%$ |
| 27 March | Warm front | 4400 ft | +2.5 | $20,000 \mathrm{ft}$ | $87 \%$ |
| 28 March | Cold front | 2800 ft | +5.5 | $20,000 \mathrm{ft}$ | $71 \%$ |

## VI. OBSERVED VARIATIONS IN THE COEFFICIENT A WITHIN STOR MS

## A. INTRODUCTION

As mentioned earlier, this study seeks to determine how the constants of the $Z-R$ equation vary with station location during a given storm. At the individual stations, the method of least squares yielded different values of the exponent $B$, but the differences resulted from sampling errors. Therefore, the exponent at all stations during a given storm has a single, identical value. Thus, one can better estimate rainfall intensity at a point with radar by determining how the values of $A$ vary with station locations. The following sections examine how the coefficient A (from Table IV) varies with station elevation, windward or leeward location of stations, station topography, and time.

## B. VARIATION WITH STATION ELEVATION

Several investigators $(3,4,6,12,14,16)$ studied the effect of accretion, coalescence, and evaporation on the constants $A$ and $B$ of the $Z-R$ equation. Accretion and coalescence will increase the size and number of the larger drops while reducing the number of smaller drops. Evaporation decreases the size of all drops.

However, the ratio of surface area to mass of small drops exceeds the same ratio for large drops. Hence, starting with a given dropsize distribution, we can expect evaporation to reduce more rapidly the number of a given small diameter drop than of any larger size drop. Since a typical distribution has a peak in the number of the smallest diameter of drop, we must also expect proportionally greatest reduction in the number of the smallest drops than of larger drops. Investigators ( 1,12 ) found (by computing the changes in given hypothetical drop-size distributions) that accretion and coalescence decrease the value of the coefficient $A$ and increase the value of the exponent B. However, they found that evaporation increases the value of the coefficient and decreases the value of the exponent. Thus, one could expect A to vary with station elevation if either of the processes dominated during the observation period of a given storm since both coalescence and evaporation increase with the depth of fall of raindrops. One must assume that the raindrops form at a constant elevation above mean sea level, regardless of station elevation.

If accretion and coalescence affect the drop-size distribution more than evaporation during the observation period of a given storm, the value of $A$ increases with station elevation. If evaporation dominates, the value of $A$ decreases with station elevation. A correlation coefficient, $r$, determines whether a significant relation
exists between $A$ and station elevation (18, p. 267). A positive $r$ indicates that the value of A relates directly to station elevation, while a negative $r$ indicates that the value of $A$ relates inversely to station elevation. A correlation coefficient close to or equal to plus or minus one indicates the existence of a significant correlation, and an $r$ value equal to zero indicates the absence of a relationship.

Results

2 February
A calculated $r$ value of -0.30 indicates a very weak tendency, if any, for A to vary inversely with station elevation.

2 March

The $r$ value of +0.07 indicates hardly any tendency for $A$ to vary directly with station elevation.

27 March
The $r$ value of +0.58 indicates a rather weak direct dependency of A to station elevation, possibly the effect of accretion and coalescence.

28 March

The $r$ value of -0.67 indicates a moderate tendency for $A$ to vary inversely with elevation and thus to show that evaporation may have dominated during the observation period.

## C. WINDWARD AND LEEWARD VARIATIONS

This section examines whether $Z-R$ equations have higher values of $A$ at stations located on the windward side of mountains than at stations located on the leeward side. The surface synoptic maps for the day yielded the wind direction, and the locations of stations on a topographic relief map determined whether to classify a station location as the windward or leeward side. Table X shows average weighted (18, p. 355) coefficients $\bar{A}$ found by averaging log A values at stations located on windward and leeward side of mountains, and weighted in proportion to the number of data points determining each value of $A$.

Windward stations have higher values of $A$ than leeward stations for the four storms analyzed. This result suggests that windward stations experience different drop-size distributions from leeward stations. One can only hypothesize the reason for experiencing a different drop-size distribution on the windward side from that on the leeward side. The wind can lift the small drops from the windward side over the mountain to the leeward side. During ascent, the small drops may coalesce and fall on the windward side. The result of lifting then decreases the number of small drops and increases the number of large drops at the windward stations.

In order to determine whether this hypothesis results in a higher value of $A$ at the windward side from that on the leeward side, one must compute the values of the coefficient A at two stations; one for the windward side and the other for the leeward side. To do this, first assume a hypothetical drop-size distribution for the leeward side. Then, eliminate the drops in the three smallest intervals of that assumed distribution and assume this modified distribution for the windward side. Such a procedure resulted in the following $\mathrm{Z}-\mathrm{R}$ equations:

$$
\begin{array}{cc}
\mathrm{Z}=220(\mathrm{R} / \mathrm{mm}-\mathrm{hr} \\
\text { Modified Distribution } & \mathrm{Z}=215\left(\mathrm{R} / \mathrm{mm}-\mathrm{hr} \mathrm{r}^{-1}\right)^{1.2} \\
\text { Hypothetical Distribution } \\
\text { Windward Side } & \text { Leeward Side }
\end{array}
$$

Although the windward distribution of raindrops did not have more large drops, the results of the computation still yielded a higher coefficient A for the windward side. Adding more large drops to the windward distribution increases the calculated $Z$ value more than the $R$ value, thus resulting in a still higher value of $A$. The value of $Z$ increases more than $R$ because $Z$ depends on the sixth power of the diameter of the raindrops while $R$ depends on the third power of the diameter.

TABLE X

| Date | Wind direction | Average <br> Windward stations | onships for Windwa <br> Leeward stations | d and Leeward Stations Average Z-R Relationship of windward stations | Average Z-R Relationship of leeward stations |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 February | SW | 18, 8, 9, 17, 10, 1 | 15, 2, 4, 20, 6 | $\mathrm{Z}=218\left(\mathrm{R} / \mathrm{mm}-\mathrm{hr}{ }^{-1}\right)^{1.41}$ | $\mathrm{Z}=205\left(\mathrm{R} / \mathrm{mm}-\mathrm{hr}^{-1}\right)^{1.41}$ |
| 2 March | NW | 18, 15 | 1, 16, 17, SLE | $\mathrm{Z}=390\left(\mathrm{R} / \mathrm{mm}-\mathrm{hr}{ }^{-1}\right)^{1.26}$ | $\mathrm{Z}=263\left(\mathrm{R} / \mathrm{mm}-\mathrm{hr}^{-1}\right)^{1.26}$ |
| 27 March | SE | 11, 12, 1, 2, 9 | 15, 17, 21, 18, 5, 10 | $\mathrm{Z}=374\left(\mathrm{R} / \mathrm{mm}-\mathrm{hr}{ }^{-1}\right)^{1.40}$ | $\mathrm{Z}=296\left(\mathrm{R} / \mathrm{mm}-\mathrm{hr}^{-1}\right)^{1.40}$ |
| 28 March | SW | 18, 17, 1 | 4, 6, 2, 15 | $\mathrm{Z}=322\left(\mathrm{R} / \mathrm{mm}-\mathrm{hr}{ }^{-1}\right)^{1.36}$ | $\mathrm{Z}=318\left(\mathrm{R} / \mathrm{mm}-\mathrm{hr}^{-1}\right)^{1.36}$ |

TABLE XI

| Average Coefficients at Topographic Regions |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Date | Hilltop stations | Hillside stations | Valley stations | Hilltops A | Hillsides $\bar{A}$ | Valleys A |
| 2 February | 10 | 8,9,17, 20 | $4,6,15,18,2,1$ | 194 | 195 | 227 |
| 2 March | None | 17, 16 | 1, 15, 18, SLE | --- | 254 | 390 |
| 27 March | 11, 12, 10 | 8, 9, 17, 21 | $5,15,18,1,2$ | 251 | 409 | 284 |
| 28 March | None | 17 | 4, 6, 1, 2, 15, 18 | --- | 238 | 351 |

## D. VARIA TION WITH STATION TOPOGRAPHY

In order to study the effects of topography, consider the $Z-R$ relationship grouped for stations on hillsides, hilltops, and valleys. Regions of relatively flat land surrounded by regions of relatively steep positive slope on at least two sides describe valley stations. One-tenth of the altitude from the floor of the valley to the lowest peak around the valley represents the upper limit of valley stations. Regions of relatively flat land surrounded by regions of relatively steep negative slope describe hilltop stations. Dropping one-tenth of the altitude difference from the peak to the floor of the most elevated valley on any side determines the lower limit of hilltop stations. All other regions represent hillside stations. For each of the four storms, Table XI gives the average coefficient $\bar{A}$ found by averaging $\log \mathrm{A}$ values at stations from each topographic region.

In all of the four storms except the storm on 27 March, valley regions have the highest value of $\overline{\mathrm{A}}$ followed in order by hillside and hilltop regions. In the case of 27 March hillside regions have the highest value of $\bar{A}$ followed in order by valley and hilltop regions.

Can the $Z-R$ equation at one station from a topographic region represent the $Z-R$ equation at other stations in the same topographic region? The analysis of covariance (18, p. 364) gives an
answer by examining the equivalence of $A$ at all stations in a given topographic region for each of the four storms (Table XII). The results of the analysis of covariance reveal no significant difference in A from one station to another at hillside regions on 2 March, and at valley regions on 2 February, 27 March, and 28 March at the $5 \%$ level of significance. The results indicate drop-size distributions at different valley stations yield single, identical values for the constants A and B during a given storm. Therefore, a radar could reliably approximate rainfall intensities in valley regions with just one rain gauge located in a valley. However, a radar with one rain gauge on a hillside or hilltop cannot approximate the rainfall intensities at other stations located on hillsides or hilltops.

## E. VARIATION WITH TIME AND LOCATION OF STATIONS

Does the $Z-R$ equation at a particular station as compared to another station always have a higher coefficient $A$ during different storms? Consider the values of $A$ at stations for given dates appearing in Table IV. The highest coefficient occurred at different stations during different storms suggesting that the constant $A$ does not relate only to station location but also to other factors which change from one storm to the next. Interestingly, the rankings of A at stations on 27 March reverse on 28 March. The synoptic

TABLE XII

| Summary of the Analysis of Covariance |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Date | Source of variation | $\mathrm{SS}_{\mathrm{x}}$ | SP | $\mathrm{SS}_{\mathrm{y}}$ | DF | Residual |  |  |  |
|  |  |  |  |  |  | SS | DF | MS | F |
| 2 February | Among sample | 3.03736 | 3.85586 | 5. 82181 | 3 | 1.00062 | 3 | 0.3335 | 4.034 |
| Hillside stations | Within sample | 27.52031 | 39.45474 | 63.75783 | 88 | 7.19319 | 87 | 0.0826 |  |
|  | Total | 30.55767 | 43.31060 | 69.57964 | 91 | 8.19381 |  |  |  |
| 2 February | Among sample | 1. 33628 | 1. 77484 | 2. 65933 | 5 | 0.30669 | 5 | 0.0613 | 0.818 |
| Valley stations | Within sample | 57.50186 | 80.61235 | 124. 33599 | 152 | 11.32485 | 151 | 0.0750 |  |
|  | Total | 58.83814 | 82.38719 | 126.99292 | 157 | 11.63154 |  |  |  |
| 2 March | Among sample | 0.99643 | 0.92448 | 0.85771 | 1 | 0.08912 | 1 | 0.0891 | 1. 548 |
| Hillside stations | Within sample | 17.32091 | 21. 39885 | 27.64298 | 22 | 1.20882 | 21 | 0.0575 |  |
|  | Total | 18.31734 | 22.32223 | 28.50069 | 23 | 1. 29794 |  |  |  |
| 2 March | Among sample | 4.64311 | 4. 59999 | 4.91488 | 3 | 0.71569 | 3 | 0.2386 | 6.739 |
| Valley stations | Within sample | 30. 56935 | 39. 39663 | 52.47211 | 49 | 1.69921 | 48 | 0.0354 |  |
|  | Total | 35.21246 | 43.99662 | 57.38699 | 52 | 2.41490 |  |  |  |
| 27 March | Among sample | 2.34199 | 1. 35609 | 0.95918 | 2 | 1.68298 | 2 | 0.8414 | 23.650 |
| Hilltop stations | Within sample | 12.79023 | 18. 57321 | 28.89238 | 55 | 1.92147 | 54 | 0. 0356 |  |
|  | Total | 15.13222 | 19.92930 | 29.85156 | 57 | 3. 60455 |  |  |  |
| 27 March | Among sample | 7.96119 | 8.71787 | 9.60836 | 3 | 0.61953 | 3 | 0. 2065 | 5.517 |
| Hillside stations | Within sample | 48.31066 | 66.70161 | 95.94894 | 105 | 3.85529 | 104 | 0. 0374 |  |
|  | Total | 56.27185 | 75. 41948 | 105. 55730 | 108 | 4.47482 |  |  |  |
| 27 March | Among sample | 14.49137 | 19.64740 | 26. 73709 | 4 | 0.12642 | 4 | 0.0316 | 0.837 |
| Valley stations | Within sample | 71. 12161 | 99.81417 | 145.93690 | 156 | 5.85474 | 155 | 0.0378 |  |
|  | Total | 85.61298 | 119.46157 | 172.67399 | 160 | 5.98120 |  |  |  |
| 28 March | Among sample | 14.13533 | 20.46378 | 29.92940 | 5 | 0.38608 | 5 | 0.0772 | 2. 020 |
| Valley stations | Within sample | 29.58377 | 39.92428 | 57.24280 | 89 | 3. 36366 | 88 | 0.0382 |  |
|  | Total | 43.71910 | 60.38806 | 87.16220 | 94 | 3. 74974 |  |  |  |

conditions on 27 March show a warm front passage, and the synoptic condition on 28 March show a cold front passage over the stations. This reversal in A suggests that different synoptic conditions could also affect the coefficient $A$ at stations.

Using the best-fitting coefficient A at Station 17 on 27 March ('Table II), the author found the exponent $B$ of the $Z-R$ equation for the raindrop population on each filter paper collected at Station 17 on 27 March. He then plotted B of each filter paper against time and rainfall intensity. The time cross-section shows wide variations in the exponent $B$ from one filter paper to the next (Plate III).

## F. DISCUSSION

The following discussion interprets the observed variations in the values of the coefficient A for given storms in estimating the rainfall amounts at stations using weather radar.

Equation (6), p. 3, shows that range-normalized radar signals $\bar{P}_{r n}$ of the same intensity from targets over stations can indicate different amounts of rainfall at those stations if they have different values for the coefficient $A$.

$$
\begin{equation*}
R=\left(\frac{\bar{P}_{\mathrm{rn}}}{\mathrm{C}_{1} \mathrm{~A}}\right) \frac{1}{\mathrm{~B}} \tag{31}
\end{equation*}
$$



Plate III. Time Cross-section of Exponents and Rainfall Intensity for Station 17 on 27 March 1963 Obtained from Each Filter Paper.
(Lines connecting points are for visual aids only.)

Here $\overline{\mathrm{P}}_{\mathrm{rn}}$ indicates the product $\overline{\mathrm{P}}_{\mathrm{r}} \mathrm{r}^{2}$ in Equation (6). This of course assumes that the drop-size distribution does not change while falling from the radar beam to the station. For example, if the $\mathrm{Z}-\mathrm{R}$ equation at a given station has a higher value of $A$ than that of another station, all range-normalized radar signals of the same intensity over the two stations indicate the station with the higher value of A experiences a smaller amount of rainfall. Thus, for rangenormalized radar signals having the same intensity from targets over stations, this study concludes that:

1. The amount of rainfall at leeward stations exceeds the amount of rainfall at windward stations.
2. The amount of rainfall on hilltops exceeds the amount of rainfall on hillsides and in valleys.

## VII. CONCLUSIONS

This study concludes that:

1. Significant differences exist in the coefficient A but not in the exponent $B$ of the $Z-R$ equation at various locations in the OSU Mesometeorological Network during given storms.
2. For given values of $R$, the difference in values of Z between stations during a given storm exceeds the difference in average values of Z for all stations between storms.
3. The value of the coefficient A correlates moderately with station elevation during some storms.
4. The average $\mathrm{Z}-\mathrm{R}$ equation has a higher coefficient $\overline{\mathrm{A}}$ at windward stations than at leeward stations.
5. The average $Z-R$ equation at valley stations has the highest coefficient $\bar{A}$ followed, in order, by the average values of A at hillside and at hilltop stations.
6. In three of the four storms analyzed, no significant differences existed in the coefficient $A$ at valley stations.
7. The highest coefficient $A$ occurs at different stations during different storms.

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## APPENDIX

TABLE A-I

| Station $17 \quad n=10$ |  |  | - Station $15 \quad \mathrm{n}=7$ |  |  | Station $22 \quad \mathrm{n}=10$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time | Z | R | Time | Z | R | Time | Z | R |
| 1506 | 0.103 | 1.221 | 1507 | 0.159 | 3.129 | 1500 | 1.075 | 13.100 |
| 1520 | 6.921 | 34.760 | 1522 | 2.485 | 18.210 | 1510 | 0. 334 | 2.668 |
| 1530 | 2.880 | 25.930 | 1550 | 12.590 | 76.540 | 1515 | 1. 648 | 5.391 |
| 1540 | 1. 245 | 9.569 | 1600 | 19.930 | 117.900 | 1520 | 4.887 | 32.860 |
| 1550 | 39.630 | 176.900 | 1610 | 2.355 | 13.680 | 1525 | 0.477 | 7.115 |
| 1600 | 6.739 | 27.770 | 1620 | 5.556 | 42.630 | 1530 | 1. 131 | 14.260 |
| 1610 | 14.000 | 81.870 | 1630 | 3.632 | 143.500 | 1535 | 5.137 | 115.100 |
| 1620 | 30.070 | 170.100 |  |  |  | 1540 | 0.786 | 15.380 |
| 1640 | 20.480 | 154.800 |  |  |  | 1545 | 28.320 | 171.500 |
| 1650 | 23.570 | 143.500 |  |  |  | 1550 | 14.700 | 81.283 |
|  |  |  |  | ation 2 |  |  |  |  |
|  |  |  | 1348 | 5.504 | 41.310 |  |  |  |
|  |  |  | 1520 | 50.130 | 298.700 |  |  |  |
|  |  |  | 1547 | 20.570 | 134.000 |  |  |  |
|  |  |  | 1553 | 5.086 | 50.480 |  |  |  |
|  |  |  | 1628 | 2. 170 | 17.950 |  |  |  |
|  |  |  | 1745 | 0.549 | 6.976 |  |  |  |

TABLE A-II

|  | Station 4 | $n=23$ |  | Station 9 | $\mathrm{n}=18$ |  | Station 6 | $\mathrm{n}=20$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time | Z | R | Time | Z | R | Time | Z | R |
| 0950 | 4.814 | 15.900 | 1005 | 374.500 | 1431.000 | 1000 | 322. 800 | 1382.000 |
| 1000 | 69.610 | 382.500 | 1020 | 539.400 | 1002.000 | 1010 | 1837.000 | 2284.000 |
| 1010 | 488.300 | 1408.000 | 1030 | 885.400 | 1921.000 | 1020 | 89.390 | 308.900 |
| 1020 | 486.100 | 1371.000 | 1040 | 737.000 | 1508.000 | 1030 | 1056. 000 | 2480.000 |
| 1030 | 2204.000 | 3325.000 | 1050 | 4232.000 | 6428.000 | 1040 | 1344. 000 | 2659.000 |
| 1040 | 544.900 | 1673.000 | 1100 | 4243.000 | 7468.000 | 1050 | 5114.000 | 6595.000 |
| 1050 | 2217.000 | 3500.000 | 1110 | 535.600 | 2253.000 | 1100 | 3760.000 | 7343.000 |
| 1100 | 3968.000 | 4437.000 | 1120 | 3350.000 | 4626.000 | 1110 | 1721.000 | 4284.000 |
| 1110 | 1792.000 | 3051.000 | 1130 | 1039.000 | 2804.000 | 1120 | 1119.000 | 4448.000 |
| 1310 | 43.400 | 592.800 | 1140 | 145.500 | 514.900 | 1130 | 1595.000 | 5160.000 |
| 1320 | 161.700 | 2020.000 | 1150 | 3660.000 | 7318.000 | 1140 | 1.346 | 43.970 |
| 1330 | 11.330 | 242.300 | 1200 | 1428.000 | 2045.000 | 1150 | 571.800 | 2625.000 |
| 1340 | 103.500 | 529.200 | 1210 | 4.517 | 130.800 | 1200 | 2458.000 | 4367.000 |
| 1410 | 389.500 | 3139.000 | 1220 | 907.300 | 4024.000 | 1210 | 129.500 | 1016.000 |
| 1415 | 551.800 | 3506.000 | 1300 | 237.800 | 1322.000 | 1220 | 150.800 | 555.900 |
| 1420 | 6464.000 | 10380.000 | 1310 | 75.320 | 784.000 | 1230 | 1077.000 | 3170.000 |
| 1430 | 412.500 | 2701.000 | 1320 | 114.600 | 623.100 | 1240 | 710.900 | 1912.000 |
| 1440 | 4882,000 | 9509.000 | 1330 | 57.620 | 438.644 | 1250 | 390.500 | 1345.000 |
| 1450 | 9502.000 | 13330.000 |  |  |  | 1300 | 3352.000 | 4644.000 |
| 1500 | 1178.000 | 4655.000 |  |  |  | 1310 | 103. 200 | 633.900 |

$1510 \quad 231.300 \quad 677.700$
$1520 \quad 64.210 \quad 408.400$
$1530 \quad 131.200 \quad 371.500$

|  | Station 20 | $\mathrm{n}=20$ |  | Station 8 | $\mathrm{n}=22$ |  | Station 18 | $\mathrm{n}=23$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1000 | 10.780 | 118.900 | 0950 | 42, 150 | 233.900 | 1020 | 43,990 | 345.900 |
| 1010 | 332.700 | 1601.000 | 1000 | 306.700 | 1206.000 | 1030 | 977.800 | 2122.000 |
| 1020 | 231.100 | 1069.000 | 1010 | 1961.000 | 2521.000 | 1040 | 445.600 | 1335.000 |
| 1030 | 861.700 | 1429.000 | 1020 | 3254.000 | 4250.000 | 1045 | 2377.000 | 2861.000 |
| 1040 | 532.400 | 2296.000 | 1030 | 1769.000 | 3520.000 | 1055 | 1752.000 | 4088.000 |
| 1050 | 1388.000 | 3127.000 | 1040 | 31810.000 | 29880.000 | 1110 | 75. 140 | 658.300 |
| 1100 | 427.900 | 2305.000 | 1100 | 1855.000 | 2824.000 | 1120 | 224.100 | 625.300 |
| 1110 | 99.710 | 730.300 | 1115 | 3074.000 | 9758.000 | 1130 | 811.800 | 2157.000 |
| 1120 | 289.100 | 1811.000 | 1150 | 700.400 | 3124.000 | 1140 | 68.790 | 297.100 |
| 1130 | 89.990 | 1043.000 | 1200 | 1425.000 | 5913.000 | 1150 | 629.800 | 1669.000 |
| 1230 | 33.940 | 348.900 | 1240 | 1122.000 | 2149.000 | 1200 | 2055.000 | 2996.000 |
| 1240 | 67.770 | 479.600 | 1330 | 18.370 | 236.200 | 1210 | 842.700 | 3653.000 |
| 1250 | 112.300 | 653.000 | 1350 | 425.500 | 4092.000 | 1220 | 214.600 | 1053.000 |
| 1300 | 189.900 | 811.800 | 1400 | 13.950 | 273.900 | 1230 | 156.400 | 162. 500 |
| 1310 | 187.700 | 1354.000 | 1410 | 342.900 | 2581.000 | 1240 | 13.700 | 147,600 |
| 1350 | 24.360 | 533.300 | 1411 | 15310.000 | 36270, 000 | 1245 | 84.940 | 391.400 |
| 1400 | 209. 100 | 3084.000 | 1420 | 669.200 | 2847.000 | 1250 | 76.690 | 412.500 |
| 1410 | 46.560 | 547.000 | 1430 | 4807.000 | 16910.000 | 1300 | 273.900 | 805.700 |
| 1450 | 852.100 | 5086.000 | 1440 | 22350.000 | 21990.000 | 1310 | 99.130 | 758.800 |
| 1500 | 460.200 | 1941.000 | 1450 | 6869.000 | 0282.000 | 1400 | 195. 700 | 2097.000 |
|  |  |  | 1510 | 2943.000 | 9344.000 | 1410 | 2. 455 | 50.710 |
|  |  |  | 1511 | 3. 274 | 42.550 | 1440 | 493. 200 | 1250.000 |
|  |  |  |  |  |  | 1450 | 235.400 | 2076.000 |

TABLE A-II (continued)

|  | Station 17 | $\mathrm{n}=32$ |  | Station 2 | $\mathrm{n}=33$ |  | Station 15 | $\mathrm{n}=35$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time | 2 | R | Time | Z | R | Time | Z | R |
| 1004 | 106.800 | 572.800 | 1008 | 716.700 | 2665,000 | 1002 | 37.670 | 346.400 |
| 1010 | 32.590 | 307.400 | 1021 | 508.500 | 2122.000 | 1010 | 938.400 | 1864.000 |
| 1015 | 1036.000 | 1397.000 | 1029 | 222. 100 | 1071.000 | 1020 | 451, 300 | 1622.000 |
| 1020 | 320.600 | 1436.000 | 1034 | 778.900 | 2508.000 | 1030 | 287.400 | 1282.000 |
| 1030 | 243.100 | 838.400 | 1039 | 186.800 | 830.100 | 1040 | 1185.000 | 3242.000 |
| 1040 | 171.400 | 909.200 | 1047 | 2698.000 | 3304.000 | 1050 | 2341.000 | 3895.000 |
| 1045 | 1560.000 | 2114.000 | 1051 | 1871.000 | 3809.000 | 1100 | 2069.000 | 4639.000 |
| 1050 | 8412.000 | 7252.000 | 1109 | 1849.000 | 4967.000 | 1110 | 1744.000 | 4349.000 |
| 1100 | 147.300 | 1199.000 | 1059 | 821.400 | 1885.000 | 1115 | 661.500 | 3412.000 |
| 1110 | 272.200 | 1168.000 | 1119 | 29.270 | 298. 100 | 1120 | 61.300 | 650.600 |
| 1120 | 301.900 | 630.200 | 1126 | 616.600 | 2557.000 | 1130 | 876.400 | 2728.000 |
| 1130 | 2562.000 | 2948.000 | 1127 | 3206. 000 | 5519.000 | 1140 | 370.100 | 2164,000 |
| 1135 | 5714.000 | 7051.000 | 1129 | 26.950 | 90.960 | 1150 | 38,480 | 426.300 |
| 1140 | 215. 100 | 637.500 | 1139 | 235. 500 | 1206.000 | 1200 | 767.700 | 1945.000 |
| 1150 | 138.500 | 481. 100 | 1153 | 240.400 | 1151.000 | 1210 | 2371.000 | 3324.000 |
| 1200 | 1325.000 | 2314.000 | 1157 | 4269.000 | 3773.000 | 1220 | 4861.000 | 8204.000 |
| 1201 | 3684.000 | 3122.000 | 1159 | 6524.000 | 14119.000 | 1230 | 116.700 | 566.500 |
| 1202 | 9.957 | 128.800 | 1207 | 6425.000 | 8095.000 | 1240 | 19. 200 | 176.600 |
| 1210 | 740.500 | 2672.000 | 1210 | 936.800 | 1735.000 | 1250 | 8.538 | 144.600 |
| 1212 | 5566.000 | 9781.000 | 1215 | 2. 237 | 53.730 | 1300 | 206. 100 | 894.400 |
| 1220 | 475.000 | 1442.000 | 1220 | 1606.000 | 5320.000 | 1310 | 1838.000 | 3432.000 |
| 1230 | 1155.000 | 1155.000 | 1229 | 7.989 | 129.600 | 1315 | 1081.000 | 1848.000 |
| 1240 | 58.910 | 305.800 | 1237 | 1075.000 | 5524.000 | 1320 | 2251.000 | 4726.000 |
| 1250 | 949.800 | 1962.000 | 1241 | 147.100 | 704.300 | 1330 | 178.100 | 833.700 |
| 1252 | 2038.000 | 3211.000 | 1249 | 578.600 | 1360.000 | 1340 | 29.740 | 126. 100 |
| 1300 | 3373.000 | 4870.000 | 1259 | 978.800 | 3074.000 | 1350 | 174, 300 | 579.400 |
| 1310 | 222.200 | 857.900 | 1309 | 1768.000 | 2579.000 | 1400 | 3,909 | 125.200 |
| 1320 | 210.400 | 435,000 | 1320 | 54.310 | 665.900 | 1410 | 20.400 | 349.300 |
| 1330 | 2.239 | 52.530 | 1329 | 261.900 | 283.300 | 1420 | 15,900 | 230.900 |
| 1340 | 36.860 | 342.700 | 1330 | 0.633 | 23.640 | 1423 | 1176.000 | 5070.000 |
| 1350 | 12.510 | 205.500 | 1349 | 20.180 | 100.500 | 1440 | 156.300 | 1531.000 |
| 1410 | 8.481 | 162.400 | 1359 | 6.857 | 220.200 | 1450 | 1983.000 | 8066.000 |
|  | Station 10 | $\mathrm{n}=12$ | 1400 | 143.600 | 1157.000 | 1500 | 609.100 | 3061.000 |
| 1032 | 1246.000 | 2355.000 | Station $1 \quad \mathrm{n}=24$ |  |  | 1510 | 21.790 | 301.600 |
| 1050 | 2494.000 | 5519.000 | 1030 | 1226.000 | 2477.000 | 1520 | 57.400 | 717.100 |
| 1110 | 1005.000 | 1742.000 | 1050 | 1355.000 | 3408.000 | Station $1 \mathrm{n}=24$ (continued) |  |  |
| 1130 | 292.200 | 1839,000 | 1100 | 5448.000 | 6831.000 | 1350 | 0.631 | 17.130 |
| 1140 | 6921.000 | 8245.000 | 1110 | 7355.000 | 7392.000 | 1410 | 0.606 | 12.810 |
| 1210 | 405.700 | 2474.000 | 1120 | 2206.000 | 4914.000 | 1420 | 2,329 | 78.330 |
| 1230 | 15. 670 | 228. 100 | 1130 | 222.500 | 562.000 | 1430 | 212.600 | 1560.000 |
| 1250 | 2536.000 | 6221.000 | 1140 | 597.700 | 1952.000 | 1440 | 293.500 | 1696.000 |
| 1400 | 275.700 | 2464.000 | 1150 | 85.960 | 321.700 | 1450 | 1120.000 | 5250.000 |
| 1410 | 3055.000 | 5082.000 | 1250 | 2439.000 | 3877.000 | 1500 | 1906.000 | 3669.000 |
| 1430 | 1025.000 | 5138.000 | 1300 | 513.000 | 1975.000 | 1510 | 2949.000 | 7228.000 |
| 1500 | 244.300 | 949.300 | 1310 | 527.600 | 897.700 | 1530 | 157.200 | 640.800 |
|  |  |  | 1320 | 243.900 | 1415.000 | 1540 | 111.500 | 1056.000 |
|  |  |  | 1330 | 41.370 | 618.500 | 1542 | 664.400 | 2875.000 |

TABLE A-III

|  | Station 18 | $\mathrm{n}=20$ |  | Station 16 | $\mathrm{n}=20$ |  | Station SLE | $\mathrm{n}=14$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time | Z | R | Time | Z | R | Time | Z | R |
| 1550 | 39.940 | 129.900 | 1543 | 126.700 | 375.200 | 1310 | 11.710 | 73.870 |
| 1600 | 85.200 | 303.700 | 1545 | 32.730 | 251.100 | 1320 | 73.240 | 296.000 |
| 1604 | 927.000 | 1542.000 | 1550 | 4.510 | 37.700 | 1330 | 3.679 | 68.930 |
| 1605 | 1219.000 | 2674.000 | 1555 | 53.870 | 147.400 | 1420 | 1208.000 | 4934.000 |
| 1606 | 2977.000 | 4802.000 | 1600 | 2166.000 | 6956.000 | 1425 | 6910.000 | 16780.000 |
| 1610 | 500.800 | 1319.000 | 1605 | 8032.000 | 10740.000 | 1430 | 1343.000 | 2876.000 |
| 1615 | 649.600 | 1664.000 | 1610 | 983.700 | 3934.000 | 1435 | 18010.000 | 20010.000 |
| 1616 | 5655.000 | 8822.000 | 1615 | 0.150 | 2.004 | 1440 | 152.500 | 880.700 |
| 1620 | 1080, 000 | 3960.000 | 1620 | 1105.000 | 2438.000 | 1445 | 640.300 | 1544.000 |
| 1625 | 1771.000 | 3712.000 | 1625 | 1053.000 | 1757.000 | 1450 | 1560.000 | 9047.000 |
| 1630 | 415.600 | 1047.000 | 1630 | 44.840 | 258.700 | 1455 | 588.300 | 3587.000 |
| 1635 | 44.490 | 350.400 | 1635 | 2. 320 | 21.390 | 1500 | 3944.000 | 11630.000 |
| 1640 | 14.720 | 88.660 | 1640 | 61.540 | 204.800 | 1505 | 2327.000 | 5472.000 |
| 1701 | 2276.000 | 3084.000 | 1645 | 424.700 | 1436.000 | 1510 | 1180.000 | 5042.000 |
| 1705 | 272.100 | 3746.000 | 1650 | 427.800 | 910.100 |  |  |  |
| 1710 | 6989.000 | 7368.000 | 1700 | 8.270 | 33.540 |  |  |  |
| 1715 | 6407.000 | 6778.000 | 1705 | 93.810 | 358.500 |  |  |  |
| 1720 | 1076.000 | 1968.000 | 1710 | 4.150 | 64.490 |  |  |  |
| 1725 | 41.700 | 144.700 | 1715 | 18.820 | 205.900 |  |  |  |
| 1731 | 25.690 | 99.320 | 1720 | 604.100 | 1507.000 |  |  |  |
| Station $1 \quad n=10$ |  |  | Station 17 |  | $\mathrm{n}=4$ |  | Station 15 | $\mathrm{n}=9$ |
| 1420 | 6.929 | 47.020 | 1630 | 157.500 | 1853.000 | 1516 | 125.900 | 466.900 |
| 1440 | 890.300 | 2752.000 | 1700 | 1071.000 | 1625.000 | 1600 | 0.542 | 6.186 |
| 1450 | 391.100 | 1157.000 | 1720 | 717.500 | 3460.000 | 1606 | 163.900 | 265.800 |
| 1620 | 35.410 | 218.500 | 1730 | 36.980 | 200.600 | 1610 | 2611.000 | 2346.000 |
| 1630 | 3.946 | 23.770 |  |  |  | 1620 | 516.300 | 1383.000 |
| 1640 | 726.100 | 2334.000 |  |  |  | 1640 | 31.720 | 86.490 |
| 1650 | 3495.000 | 3934.000 |  |  |  | 1650 | 3621.000 | 7051.000 |
| 1700 | 1907.000 | 3190.000 |  |  |  | 1710 | 36.490 | 297.200 |
| 1710 | 3114.000 | 3878.000 |  |  |  | 1720 | 23. 440 | 165.200 |
| 1720 | 2.888 | 26.010 |  |  |  |  |  |  |

TABLE A-IV

| Station $9 \quad \mathrm{n}=21$ |  |  | Station $5 \quad n=24$ |  |  | Station $12 \quad \mathrm{n}=22$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time | Z | R | Time | Z | R | Time | Z | R |
| 1240 | 461.900 | 578.900 | 1220 | 1641.000 | 3706.000 | 1230 | 2078.000 | 3156.000 |
| 1250 | 282.500 | 578.000 | 1230 | 1182.000 | 3360.000 | 1240 | 1246.000 | 2614.000 |
| 1300 | 6.819 | 68.920 | 1240 | 2387.000 | 4253.000 | 1250 | 1488.000 | 2740,000 |
| 1321 | 268. 200 | 648.000 | 1250 | 772.200 | 2730.000 | 1300 | 4189.000 | 4484.000 |
| 1330 | 140.100 | 501.600 | 1300 | 234.300 | 1187.000 | 1303 | 82.170 | 555. |
| 1337 | 306.900 | 526.300 | 1310 | 48.810 | 306.700 | 1310 | 202.900 | 718.900 |
| 1342 | 1523.000 | 1925.000 | 1320 | 36.020 | 226.600 | 1320 | 319.600 | 1265.000 |
| 1350 | 1676,000 | 1215.000 | 1330 | 6.585 | 67.280 | 1330 | 33.490 | 272.400 |
| 1400 | 347.600 | 692.600 | 1340 | 323.300 | 624.800 | 1340 | 129.100 | 499.900 |
| 1420 | 1209.000 | 1861.000 | 1400 | 31.980 | 256.800 | 1350 | 2813.000 | 2452.000 |
| 1430 | 1408.000 | 2830.000 | 1410 | 1381.000 | 2199.000 | 1400 | 513.800 | 1334.000 |
| 1440 | 1001.000 | 2182.000 | 1420 | 1770.000 | 2309.000 | 1410 | 998.300 | 2182.000 |
| 1450 | 5577.000 | 4650.000 | 1430 | 2528.000 | 2840.000 | 1420 | 1213.000 | 1880.000 |
| 1600 | 317.200 | 757.900 | 1440 | 9183.000 | 7685.000 | 1430 | 4166.000 | 3612.000 |
| 1610 | 31.040 | 242.000 | 1450 | 7269.000 | 7081.000 | 1440 | 2367.000 | 3571.000 |
| 1640 | 1268.000 | 3918.000 | 1500 | 1577.000 | 3270.000 | 1450 | 3045.000 | 3616.000 |
| 1650 | 507.900 | 1257.000 | 1610 | 783.900 | 2756.000 | 1500 | 3882.000 | 4785.000 |
| 1700 | 5.465 | 66.670 | 1620 | 904.700 | 3828.000 | 1610 | 169.200 | 590.600 |
| 1710 | 10.820 | 124.700 | 1630 | 33,230 | 305.600 | 1630 | 56.160 | 460.100 |
| 1720 | 11.040 | 25.130 | 1640 | 290. 300 | 1295.000 | 1640 | 29.940 | 276.000 |
| 1730 | 4.998 | 64.200 | 1650 | 1351.000 | 3294.000 | 1650 | 1275.000 | 3712.000 |
|  |  |  | 1700 | 1214.000 | 2197.000 | 1700 | 726.300 | 2530.000 |
|  |  |  | 1710 | 227.300 | 970.900 |  |  |  |
|  |  |  | 1720 | 14.760 | 190.400 |  |  |  |


| Station $1 \quad n=35$ |  |  | Station $1 \mathrm{n}=35$ |  | (continued) | Station 2 | $2 \mathrm{n}=29$ | (continued) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1100 | 299.300 | 754.800 | 1523 | 0.001 | 0.569 | 1330 | 10.640 | 117.400 |
| 1110 | 186.900 | 869.300 | 1535 | 5.093 | 45,690 | 1340 | 53.920 | 310.300 |
| 1118 | 219.400 | 1060.000 | 1543 | 0.303 | 2. 171 | 1341 | 6.726 | 64.110 |
| 1130 | 393.100 | 1170.000 | 1620 | 357.500 | 1119.000 | 1400 | 258. 400 | 548.900 |
| 1142 | 3217.000 | 5067.000 | 1633 | 18.830 | 158.200 | 1401 | 537.500 | 1199.00 |
| 1152 | 1896.000 | 3111.000 | 1640 | 0.436 | 4.283 | 1410 | 100.700 | 460.100 |
| 1200 | 4381.000 | 6489.000 | 1650 | 0.370 | 5. 501 | 1413 | 791.300 | 1228.000 |
| 1210 | 4493.000 | 6132.000 | 1700 | 107.800 | 681,900 | 1420 | 1068.000 | 2138.000 |
| 1220 | 291.400 | 1274.000 | 1710 | 0.936 | 23.580 | 1430 | 1336.000 | 3140.000 |
| 1230 | 3653.000 | 5612.000 | 1725 | 97.430 | 284.000 | 1435 | 2211.000 | 4411.000 |
| 1240 | 683.500 | 2387.000 | 1730 | 14.520 | 143.800 | 1440 | 1783.000 | 2888.000 |
| 1317 | 443.100 | 1018.000 | 1740 | 9.853 | 98.470 | 1500 | 5700.000 | 8256.000 |
| 1332 | 64.210 | 491.800 | 1803 | 80.500 | 478.200 | 1510 | 2.649 | 33.640 |
| 1340 | 21.770 | 186.600 |  | tation 2 | $\mathrm{n}=29$ | 1520 | 4.376 | 105. 100 |
| 1348 | 28.560 | 255.300 | 1210 | 1396.000 | 2720.000 | 1610 | 96.090 | 642.000 |
| 1355 | 63.500 | 314.400 | 1220 | 941.800 | 2244.000 | 1615 | 150.200 | 903.200 |
| 1410 | 11.060 | 79.920 | 1230 | 1246.000 | 3638.000 | 1620 | 267.400 | 2120.000 |
| 1420 | 302.100 | 1091.000 | 1240 | 566.400 | 2087.000 | 1630 | 36.540 | 435.400 |
| 1430 | 694.400 | 1908.000 | 1250 | 671.200 | 1756.000 | 1640 | 7.709 | 81.060 |
| 1440 | 842.300 | 2544.000 | 1300 | 1323.000 | 2463.000 | 1650 | 4862.000 | 3926.000 |
| 1502 | 1107.000 | 2122.000 | 1310 | 30.610 | 260.800 | 1700 | 767.600 | 3017.000 |
| 1510 | 1176,000 | 2557.000 | 1320 | 554.500 | 943.300 |  |  |  |

TABLE A-IV (continued)
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| Station $21 \quad \mathrm{n}=50$ |  |  | Station $17 \quad \mathrm{n}=32$ |  |  | Station 15 |  | $=31$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time | Z | R | Time | Z | R |  |  | R |
| 1240 | 2526.000 | 4690.000 | 1220 | 593.900 | 2352.000 | 1040 | 624.500 | 1788.000 |
| 1245 | 2238.000 | 3400.000 | 1230 | 3365.000 | 4453.000 | 1050 | 2403.000 | 3861.000 |
| 1250 | 1101.000 | 2091.000 | 1235 | 3241.000 | 4545.000 | 1100 | 1421.000 | 2385,000 |
| 1255 | 1937.000 | 4421.000 | 1240 | 5372.000 | 6303.000 | 1110 | 865.200 | 2228.000 |
| 1300 | 499.600 | 1283.000 | 1245 | 2186.000 | 4291.000 | 1120 | 1212,000 | 2288.000 |
| 1305 | 1951.000 | 3884.000 | 1250 | 1899.000 | 4109.000 | 1130 | 1762.000 | 3850.000 |
| 1310 | 272.700 | 1188.000 | 1300 | 1649.000 | 4172.000 | 1140 | 1342.000 | 3215.000 |
| 1315 | 493.600 | 1690.000 | 1305 | 4863.000 | 6743.000 | 1150 | 4856,000 | 6625.000 |
| 1320 | 72. 160 | 423.800 | 1310 | 686.300 | 1749.000 | 1200 | 818.400 | 2250.000 |
| 1325 | 291.900 | 1024.000 | 1315 | 469600 | 1478.000 | 1210 | 4677.000 | 3843.000 |
| 1330 | 8,056 | 63.570 | 1320 | 413.800 | 1022.000 | 1220 | 168.800 | 1041.000 |
| 1335 | 117.600 | 383,700 | 1330 | 389.600 | 871.800 | 1230 | 486.900 | 1688.000 |
| 1340 | 700.500 | 1620.000 | 1335 | 66.700 | 298.000 | 1240 | 1162.000 | 2223.000 |
| 1345 | 1238.000 | 1689.000 | 1340 | 465.200 | 1556.000 | 1250 | 1121.000 | 2441.000 |
| 1350 | 184.900 | 922.600 | 1345 | 400. 200 | 1346.000 | 1255 | 2346.000 | 4640.000 |
| 1400 | 3389.000 | 3465.000 | 1350 | 1843,000 | 2629.000 | 1300 | 2627.000 | 3819.000 |
| 1410 | 2557.000 | 5433.000 | 1400 | 4791.000 | 5384.000 | 1310 | 38,670 | 211.500 |
| 1415 | 1983.000 | 3782.000 | 1410 | 3231.000 | 3853.000 | 1320 | 174.400 | 883.300 |
| 1420 | 4521.000 | 6883.000 | 1413 | 3783.000 | 3961.000 | 1330 | 37.030 | 267.800 |
| 1425 | 3485. 000 | 4767.000 | 1420 | 1345.000 | 3651.000 | 1340 | 52.810 | 279.500 |
| 1430 | 3181.000 | 5887.000 | 1425 | 1603.000 | 3121.000 | 1347 | 458.600 | 881.800 |
| 1435 | 1196,000 | 2731.000 | 1440 | 1128.000 | 2618.000 | 1350 | 587.400 | 1891.000 |
| 1440 | 893.200 | 1932,000 | 1452 | 1868.000 | 3196.000 | 1410 | 1156.000 | 2962.000 |
| 1445 | 1512.000 | 1981.000 | 1500 | 6623,000 | 7798.000 | 1420 | 2886.000 | 3791,000 |
| 1450 | 795,800 | 2213.000 | 1510 | 2.076 | 42.810 | 1430 | 1015.000 | 3373.000 |
| 1455 | 524.600 | 1562.000 | 1558 | 22.710 | 114.800 | 1440 | 2456.000 | 5889.000 |
| 1500 | 207. 100 | 784.700 | 1605 | 258.100 | 831,100 | 1455 | 6830.000 | 7962.000 |
| 1505 | 1406.000 | 3754.000 | 1620 | 101.200 | 878.700 | 1500 | 255,200 | 695.400 |
| 1521 | 1.778 | 24.970 | 1629 | 20,330 | 153.300 | 1510 | 2.306 | 34.160 |
| 1545 | 0.199 | 3.912 | 1643 | 765.200 | 2362,000 | 1519 | 129.000 | 553.900 |
| 1555 | 271.300 | 416.200 | 1649 | 1662,000 | 3772.000 | 1602 | 354.400 | 817.900 |
| 1600 | 120.600 | 471.200 | Station $11 \quad n=24$ |  |  | Station 11 |  | ntinued) |
| 1605 | 29.160 | 156.700 | 1225 | 1419.000 | 3188.000 | 1611 | 51.440 | 267.900 |
| 1610 | 563.200 | 881.000 | 1231 | 1124.000 | 2984.000 | 1621 | 112.800 | 564.700 |
| 1615 | 339.800 | 949.500 | 1240 | 1406.000 | 3078.000 | 1633 | 10.770 | 85.690 |
| 1620 | 5.638 | 88,420 | 1250 | 659.200 | 2121.000 | 1640 | 566.300 | 629.700 |
| 1625 | 2. 837 | 26.830 | 1300 | 1571.000 | 2469.000 | 1650 | 1092,000 | 1732.000 |
| 1630 | 41.780 | 377.600 | 1310 | 119.000 | 519,000 | 1700 | 199.400 | 1093.000 |
| 1635 | 33,830 | 157.400 | 1321 | 380.900 | 1216,000 |  | ation 8 n |  |
| 1640 | 901.700 | 2385,000 | 1330 | 31.370 | 193.300 | 1250 | 552.800 | 1240,000 |
| 1645 | 1693.000 | 5120.000 | 1340 | 69,320 | 392.700 | 1300 | 3.103 | 31.590 |
| 1650 | 2539.000 | 4994.000 | 1350 | 7.640 | 88.240 | 1320 | 0, 144 | 3.019 |
| 1655 | 9979.000 | 13650.000 | 1400 | 784.200 | 1696.000 | 1340 | 243.500 | 433.900 |
| 1700 | 191.900 | 1079.000 | 1413 | 352.300 | 744.300 | 1350 | 126.900 | 332.900 |
| 1705 | 2270.000 | 3919.000 | 1420 | 720.500 | 1115. 000 | 1400 | 748.000 | 582,100 |
| 1710 | 1894.000 | 4701.000 | 1430 | 1981.000 | 2256.000 |  |  |  |
| 1717 | 3859.000 | 6871.000 | 1440 | 3097.000 | 3309.000 |  |  |  |
| 1720 | 1815.000 | 2957.000 | 1450 | 4272.000 | 6039.000 |  |  |  |
| 1725 | 4169.000 | 7134.000 | 1500 | 1410.000 | 2641,000 |  |  |  |
| 1730 | 538,100 | 1670.000 | 1607 | 331.200 | 783.000 |  |  |  |

TABLE A-IV (continued)

|  | Station 18 | 42 |  | tion 10 n | $=12$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Time | Z | R | Time | Z | R |
| 1220 | 1261.000 | 4467.000 | 1530 | 102.900 | 641.400 |
| 1230 | 1255. 000 | 3297.000 | 1540 | 28.880 | 313.900 |
| 1235 | 4302.000 | 5882.000 | 1610 | 174.400 | 1297.000 |
| 1240 | 2035.000 | 4878.000 | 1620 | 1817.000 | 6025.000 |
| 1243 | 3060.000 | 6903.000 | 1630 | 717.600 | 2889.000 |
| 1247 | 2716.000 | 7673.000 | 1640 | 2300.000 | 7446.000 |
| 1251 | 2712.000 | 6193.000 | 1642 | 900.200 | 5638.000 |
| 1255 | 4450.000 | 6610.000 | 1645 | 3181.000 | 13000.000 |
| 1300 | 2672.000 | 5052.000 | 1655 | 2236.000 | 5009.000 |
| 1303 | 2555.000 | 5099.000 | 1707 | 8334.000 | 22740.000 |
| 1306 | 2618.000 | 6430.000 | 1710 | 2276.000 | 12210.000 |
| 1314 | 624.700 | 2380.000 | 1720 | 181.300 | 1246.000 |
| 1321 | 1116.000 | 4082.000 |  |  |  |
| 1327 | 445.600 | 2289.000 |  |  |  |
| 1328 | 1632.000 | 4390.000 |  |  |  |
| 1342 | 524.900 | 1436.000 |  |  |  |
| 1353 | 1024.000 | 1316.000 |  |  |  |
| 1356 | 4566.000 | 6292.000 |  |  |  |
| 1359 | 3147.000 | 4783.000 |  |  |  |
| 1407 | 3385.000 | 4370.000 |  |  |  |
| 1415 | 1419.000 | 3955.000 |  |  |  |
| 1417 | 2639.000 | 4882.000 |  |  |  |
| 1425 | 3338.000 | 6681.000 |  |  |  |
| 1433 | 1966.000 | 5456.000 |  |  |  |
| 1443 | 2246.000 | 5456.000 |  |  |  |
| 1445 | 1150.000 | 2162.000 |  |  |  |
| 1455 | 113.500 | 624.900 |  |  |  |
| 1501 | 57.910 | 460.300 |  |  |  |
| 1556 | 70.410 | 182.100 |  |  |  |
| 1600 | 47.260 | 217.500 |  |  |  |
| 1610 | 4.905 | 59.920 |  |  |  |
| 1633 | 39.870 | 300.700 |  |  |  |
| 1638 | 166.700 | 766.300 |  |  |  |
| 1645 | 1016.000 | 2483.000 |  |  |  |
| 1646 | 1114.000 | 3345.000 |  |  |  |
| 1650 | 6473.000 | 7611.000 |  |  |  |
| 1655 | 2599.000 | 4836.000 |  |  |  |
| 1657 | 3095.000 | 5135.000 |  |  |  |
| 1705 | 490.300 | 1404.000 |  |  |  |
| 1710 | 2714.000 | 6069.000 |  |  |  |
| 1721 | 439.900 | 2093.000 |  |  |  |
| 1725 | 285.900 | 1109.000 |  |  |  |

TABLE A-V

|  | Station $6 \quad \mathrm{n}=20$ |  | Station $1 \quad n=17$ |  |  | Station 18 |  | =14 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time | Z | R | Time | Z | R |  |  | R |
| 1100 | 3389.000 | 5808.000 | 1400 | 39.880 | 123.200 | 1107 | 134.900 | 359.200 |
| 1115 | 3915.000 | 5808.000 | 1510 | 3853.000 | 3226.000 | 1120 | 109.800 | 487.400 |
| 1140 | 925.600 | 2876.000 | 1520 | 3152.000 | 2586.000 | 1130 | 426.600 | 957.200 |
| 1150 | 27.580 | 187.800 | 1530 | 1118.000 | 1488.000 | 1145 | 134.600 | 375.100 |
| 1210 | 79.400 | 428.400 | 1540 | 34.880 | 150.600 | 1204 | 112.600 | 728.300 |
| 1220 | 2116.000 | 3322.000 | 1550 | 302.200 | 639.100 | 1217 | 69.240 | 269.100 |
| 1230 | 2921.000 | 4995.000 | 1600 | 72.910 | 173.500 | 1224 | 61.230 | 430.700 |
| 1235 | 1174.000 | 2289.000 | 1854 | 6.376 | 22.010 | 1234 | 125.800 | 458.900 |
| 1240 | 3290.000 | 3723.000 | 1911 | 199,300 | 596.100 | 1237 | 715.600 | 1096.000 |
| 1300 | 1846.000 | 4209.000 | 1920 | 77.750 | 657.800 | 1243 | 1464.000 | 2041.000 |
| 1310 | 4029,000 | 5683.000 | 1931 | 1925.000 | 2341.000 | 1251 | 64.440 | 247.300 |
| 1320 | 685.900 | 2059.000 | 1940 | 774,000 | 1189,000 | 1302 | 504.700 | 1290.000 |
| 1330 | 673,400 | 1083.000 | 1951 | 805.200 | 1739.000 | 1330 | 234.700 | 1034.000 |
| 1340 | 374.900 | 842.300 | 2000 | 8.270 | 146.800 | 1422 | 57.690 | 334.500 |
| 1350 | 1307.000 | 1900.000 | 2013 | 15.180 | 313.800 |  |  |  |
| 1400 | 229.900 | 853.800 | 2032 | 1456.000 | 3720.000 |  |  |  |
| 1440 | 3371.000 | 5203.000 | 2041 | 9.388 | 165.900 |  |  |  |
| 1450 | 4523.000 | 8159.000 |  |  |  |  |  |  |
| 1452 | 12650.000 | 12500.000 |  |  |  |  |  |  |
| 1700 | 1819.000 | 4652.000 |  |  |  |  |  |  |


| Station $2 \mathrm{n}=8$ |  |  | Station 15. $\mathrm{n}=20$ |  |  | Station $17 \mathrm{n}=20$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1050 | 329.800 | 806.200 | 1130 | 456.200 | 1759.000 | 1105 | 2081.000 | 4986.000 |
| 1100 | 5582.000 | 5384.000 | 1140 | 52.220 | 527.700 | 1118 | 270.500 | 1082.000 |
| 1110 | 240.000 | 895.000 | 1150 | 57.670 | 366.800 | 1127 | 378.400 | 1447.000 |
| 1150 | 78.650 | 262.200 | 1205 | 3.791 | 35.030 | 1139 | 167.500 | 944.200 |
| 1200 | 6.962 | 41.590 | 1210 | 6.337 | 60.190 | 1149 | 78.220 | 282. 200 |
| 1220 | 46.150 | 237.900 | 1220 | 6.764 | 69.500 | 1209 | 85.460 | 586.600 |
| 1230 | 373.900 | 1129.000 | 1230 | 79.640 | 377.100 | 1221 | 120.700 | 883.600 |
| 1240 | 1436.000 | 2733.000 | 1240 | 224.900 | 859.000 | 1226 | 98.220 | 604.700 |
| Station $4 \mathrm{n}=16$ |  |  | 1246 | 1858,000 | 4249.000 | 1235 | 101.500 | 542.500 |
| 1052 | 1.718 | 27.280 | 1250 | 1214.000 | 2098.000 | 1239 | 853.000 | 1970.000 |
| 1100 | 151,600 | 725.800 | 1300 | 1.645 | 17.870 | 1242 | 1606.000 | 2905,000 |
| 1110 | 716.800 | 1697.000 | 1310 | 381.900 | 1919.000 | 1251 | 3526.000 | 5554,000 |
| 1120 | 255.700 | 1622.000 | 1320 | 25.040 | 207.500 | 1259 | 95.160 | 610.700 |
| 1130 | 511.800 | 1359.000 | 1330 | 1.537 | 19.910 | 1309 | 1775.000 | 3191.000 |
| 1140 | 90.020 | 431. 400 | 1340 | 4.731 | 51.170 | 1321 | 8.589 | 74.280 |
| 1150 | 260. 200 | 900.700 | 1350 | 7.014 | 74.880 | 1336 | 91.240 | 421.100 |
| 1220 | 19.110 | 129.600 | 1400 | 4.516 | 44.140 | 1349 | 1.821 | 26.260 |
| 1250 | 691.600 | 2419.000 | 1410 | 0.793 | 14.560 | 1418 | 5.723 | 85.500 |
| 1300 | 2256.000 | 4486.000 | 1420 | 5. 464 | 53.000 | 1428 | 91.800 | 743.800 |
| 1310 | 222. 400 | 811.900 | 1430 | 17.650 | 152.600 | 1443 | 8.030 | 97.450 |
| 1320 | 63.700 | 318.200 |  |  |  |  |  |  |
| 1330 | 412.400 | 965.100 |  |  |  |  |  |  |
| 1340 | - 65.160 | 295.700 |  |  |  |  |  |  |
| 1420 | 124.400 | 477.000 |  |  |  |  |  |  |
| 1450 | 136.900 | 574. 200 |  |  |  |  |  |  |

A. FORTRAN PROGRAMS FOR SOLVING $Z$ AND $R$ VALUES AND THE CONSTANTS OF THE Z-R EQUATIONS AT STATIONS

For future studies, one can save valuable time in computing the constants $A$ and $B$ at stations by using the following computer programs written in the FORTRAN language.

The data required for use of the first program includes the date, the station number, the time of the filter paper exposure, the exposure duration of each filter paper in seconds, and the numbers of raindrops in each 0.2 mm diameter range from each filter paper. This program solves for the $Z$ and $R$ value of each filter paper. For convenience, the first six columns of the punched data card indicate the year, month, and day. The data card punched as 630327 indicates 27 March 1963. The next two columns of the data card present the station number. Columns 9 to 12 show the time of exposure in PST followed by columns 13 to 15 which show the exposure duration. The next 44 columns present the numbers of raindrops in the first 110.2 mm diameter ranges of four columns per diameter range. The last 21 columns present the numbers of raindrops in the last seven diameter ranges of three columns per diameter range. For example, if one counted 2120 raindrops in the 0.2 to 0.4 mm diameter range and two raindrops in the 2.6 to 2.8 mm diameter range, columns 20 to 23 of the punched card and column 65 read 2120 and 2 respectively. This program will not solve the correct $Z$ and
$R$ value from a filter paper if raindrops have diameters larger than 3.8 mm . After entering the data cards into the computer, the computer punches the solutions on cards which include the date, time of exposure, station number, a value of $Z$ in $\mathrm{mm}^{6} / \mathrm{m}^{3}$, and a value of $R$ in $\mathrm{mm} / \mathrm{hr}$. The number of solution cards punched equals the number of data cards entered into the computer.

After sorting and grouping the solution cards of the first program into dates and stations, the solution cards serve as data cards for the program which solves for the constants of the $Z-R$ equation at stations. Following a set of data cards for a station, the computer pauses and lights a panel to indicate to the operator the addition of a different set of data cards from a different station. The computer punches one solution card for each set of data cards. The solution card presents the constants $A$ and $B$ of the $Z-R$ equation at each station, the station number, the date, and the variance of the array. The two programs appear in Table A-XVI and Table XVII. The statements R1 to R18 follow after Z18 but appear on the same lines as $Z 1$ to $Z 18$ to save space in this paper.

TABLE A-VI

## FORTRAN Program for Finding $Z$ and $R$ Values

*0806


An example of a punched solution.
DATE $=63027$ STATION $=8$ TIME PST $=1250 \mathrm{Z}=552.8014 \quad \mathrm{R}=1.2312$
SHCULD PE: "630327"
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| FORTRAN Progzam for Finding Z-R Relationships |  |
| :---: | :---: |
| *0806 |  |
| 1 | $\mathrm{C}=0$. |
|  | $\mathrm{SR}=0$. |
|  | $\mathrm{SZ}=0$. |
|  | $\mathrm{SZR}=0$. |
|  | $S Q Z=0$. |
|  | $S Q R=0$. |
|  | $\mathrm{D}=.43429488$ |
| 10 | READ 100, NDA, NSTA, $\mathrm{Z}, \mathrm{R}$ |
| 100 | FORMAT ( $7 \mathrm{X}, 16,11 \mathrm{X}, \mathrm{I} 2,21 \mathrm{X}, \mathrm{F} 10.4,5 \mathrm{X}, \mathrm{F10.4)}$ |
|  | $\mathrm{C}=\mathrm{C}+1$ |
|  | SR=SR+D*(LOGF(R)) |
|  | SZ $=$ SZ + D*(LOGF(Z) ) |
|  | SQR=SQR+(D*(LOGF(R)) ) **2 |
|  | SQZ=SQZ+(D*(LOGF(Z)) ) ${ }^{*}$ (2 |
|  |  |
|  | IF (SENSE SWITCH 9)20,10 |
| 20 | $\mathrm{B}=(\mathrm{SZR}-((\mathrm{SR} * \mathrm{SZ}) / \mathrm{C})) /(\mathrm{SQR}-((\mathrm{SR} *$ SR)/C)) |
|  | $\mathrm{A}=\operatorname{EXPF}((\mathrm{SZ} / \mathrm{C}-((\mathrm{B} * \mathrm{SR}) / \mathrm{C})) / \mathrm{D})$ |
|  | VAR=((SQZ-((SZ*SZ)/C))-B*(SZR-((SZ*SR)/C) $)$ )/( $\mathrm{C}-2$. |
|  | PUNCH 200, NDA, NSTA, B, A, VAR |
| 200 | FORMAT ( 5 HDATE $=16,4 \mathrm{X}, 8 \mathrm{HSTATION}=12,4 \mathrm{X}, 2 \mathrm{HB}=\mathrm{F7} .4,4 \mathrm{X}$, |
| 1 | $2 \mathrm{HA}=$ F7. 2 , 4X, 9HVARIANCE $=$ F7.5) |
|  | PAUSE 13431 |
|  | GO TO 1 |
|  | END |
|  | An example of a punched solution |
| DATE=63 | 30327 STATION $=8 \mathrm{~B}=1.4881 \quad \mathrm{~A}=$ 737.43 VARIANCE $=.05622$ |

