AN ABSTRACT OF THE THESIS OF

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The purpose of this study was to investigate teaching models that may aid in improving student's understanding of how to interpret graphs of kinematic variables. The teaching models that were considered incorporated the use of cooperative groups and calculator-based laboratory instruments, alone and in concert. The subjects used in this study were college students in integral calculus courses intended for students with a science or engineering background.

Roschelle's Theory for Convergent Conceptual Change was used as a framework for describing the dynamics by which student discourse could result in improving student understanding of graphs of kinematic variables. A portion of Beichner's Test for Understanding Kinematics (TUG-K) was used as the pre- and post -test. The laboratory assignments used in this study were designed using a general treatment strategy for reconceptualization outlined by Dykstra et al.

The investigation consisted of two parts: (1) student performance on the pre- and post-test were analyzed to compare the effectiveness of the teaching models incorporating the use of cooperative groups and calculator-based laboratory instruments, separately and in concert, and (2) analysis of student discourse when working in cooperative groups in calculator-based laboratory settings and traditional

settings was performed to see if this type of discourse was a factor in changing in students' conceptions, and to examine any differences in student discourse in the two settings.

Results of this study indicate that student-student discourse has a significant impact on student's conceptions of graphs of kinematic variables. Evidence was found that student's misconceptions were not only repaired but were also reinforced, and possibly built through student-student discourse. The results indicate that since student discourse can build misconceptions, this teaching strategy was less effective for improving student understanding of graphs of kinematic variables when it was not followed by a well organized classroom discussion led by the instructor giving students the opportunity to have misconceptions repaired.

Building Conceptions and Repairing Misconceptions In Student Understanding of Kinematic Graphs — Using Student Discourse in Calculator Based Laboratories

by

Patricia L. Hale

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Patricia L. Hale

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CHAPTER I

INTRODUCTION

The relationships between a function and its derivative and its antiderivatives are predominant themes of most college calculus courses. For a physical interpretation of these relationships, closely related topics from kinematics are often included in the calculus course curriculum. Some personal experience with kinematics can be considered to be common to virtually all students, and thus, students come to the calculus classroom with some understanding (perhaps incomplete or erroneous) of the properties of distance, velocity and acceleration. Educators often refer to kinematics when introducing topics in calculus. Clement states (1989, p. 1), "We assume that it is desirable to be able to ground new material in that portion of the student's intuition which is in agreement with accepted theory. When this is possible, it should help students to understand and believe physical principles at a 'make sense' level instead of only at a more formal ."

The desire to build conceptual understanding of functions, derivatives, and accumulation using knowledge that students already have is consistent with widely accepted constructivist principles. Constructivist ideas suggest that a sound strategy to teach students the relationships among functions, derivatives, and antiderivatives is to build on their experiences with distance, velocity, and acceleration. Unfortunately, the belief that students' understanding of kinematic variables is in agreement with accepted physics principles, or that their understanding is in need of only minor refinement and/or more systematic articulation through algebra, has been shown to be naive.

Many studies have shown that students' understanding of kinematics is based on misconceptions and properties that hold true in some, but not all, circumstances. Student conceptions are based on practical experiences, which do not always generalize to a scientific or theoretical setting. When trying to solve a complex problem, students often over-generalize a particular principle in an attempt to solve the problem (Monk, 1990). Overgeneralization is a problem for calculus students working in a context based on kinematics, and also when they try to use their understanding of kinematics to solve other problems involving functions, derivatives, and antiderivatives. Students' difficulties and misunderstandings are often brought to the surface when they attempt to use the abstract and powerful language of graphs. A sound understanding of functions, derivatives, and antiderivatives is not indicated merely in a student's ability to manipulate algebraic formulas, but also in the ability to correctly interpret information provided by graphs. Students who may be able to do algebraic manipulations may not be able to identify an antiderivative with the area under a graph or the derivative with the slope of the graph.

The calculus reform movement of recent years has brought an increased emphasis on graphical representations of functions. The call for this increased emphasis is not restricted to the calculus reform movement. Bell and Janvier (1981) state:

> We felt that the treatment of this topic (graphical representation) in the mathematics curriculum was generally underdeveloped, and related too much to specialized mathematical techniques, such as the solution of equations by reading off points of intersection of two graphs. (p. 34)
>
> We believe that this (instruction in graphs) should mainly come in mathematics rather than science lessons. For instance, graph reading techniques could be more seriously developed in mathematics. Also, we think that complex graphs should be introduced and analyzed in graphical terms without reference to situations. (p. 41)

McDermott (1986) states, "It has been our experience that literacy in graphical representations often does not develop spontaneously and that intervention in the form of direct instruction is needed" (p. 513).

Thus, there appear to be cogent reasons for instructing students in kinematics concepts in general, and the interpretation of graphs of kinematic variables in particular, in the calculus classroom.

The traditional model of instruction for mathematics has been a lecture/homework format. The instructor lectures, assigns homework problems related to the lecture, and the student completes the homework assignment and turns it in individually. This traditional format may not be effective for developing understanding of graphs of kinematic variables: "Teachers cannot simply tell students what the graphs' appearance should be. It is apparent from the testing results this traditional style of instruction does not work well for imparting knowledge when of kinematics graphs" (Beichner, 1994, p. 755).

The calculus reform movement has led many instructors to experiment with changes to this traditional format, including having students working in groups instead of individually, and attempts at implementing discovery learning through laboratory activities as opposed to lectures. These alternative techniques may be more effective in helping students understand and interpret kinematics graphs.

There is evidence that cooperative group structures in the mathematics classroom can be effective in improving achievement on certain types of problems (Slavin, 1980; Dees, 1991). The research indicates that generally, for more difficult tasks requiring analysis and other problem solving skills, cooperative groups tend to be a beneficial teaching tool. The interpretation of graphs of kinematic variables is not a simple computational problem, but involves understanding of several concepts. Student discourse may help students confront and repair misconceptions. Researchers report that student discourse may assist students understanding with kinematic graphs (Beichner, 1994; Monk, 1994; Dykstra et al., 1992). Cooperative groups may give students greater opportunity for such discourse and may be a beneficial alternative to individual instruction in graphs of kinematic variables.

One instructional model that has shown promise in improving student understanding of graphs of kinematic variables is the Microcomputer Based Laboratory (MBL). One drawback with this instructional setting is that it can be costly in terms of the microcomputers themselves. Also, space to accommodate both the computers and students while working on an activity with moving objects can be a difficulty. With recent developments in technology, there are now Calculator Based Laboratories (CBL). The CBL instruments function in a similar manner to the MBL instruments, but are considerably less expensive and more portable, being easily moved from classroom to classroom without the need for special facilities.

The study described in this dissertation was motivated by the goal of better understanding the dynamics by which these models (alone and in concert) could be expected to improve student understanding of graphs of kinematic variables. The purpose of this study is to investigate the benefits of two instructional models: (1) The use of CBL instruments in a laboratory context, and (2) the use of cooperative groups in promoting student discourse.

In Chapter II we review the relevant background literature. This review examines the literature on student difficulties interpreting graphs of kinematic variables, using a physical activity as an instructional method to overcome those difficulties, and using a cooperative group structure to improve student understanding. The review also establishes the theoretical framework for this study based on the work of Beichner, Dykstra, and Roschelle.

In Chapter III we detail the specific research methodology. This includes descriptions of the development of the laboratory assignments, pre-test, and post-test that were used in the study. The laboratory assignments were developed using models to promote conceptual change described by Dykstra et al. (1992) and Roschelle (1992). The pre- and post-test were developed based on the Test for Understanding Kinematics (TUG-K) developed by Beichner, 1994.

In Chapter IV the results are reported. The results include quantitative and qualitative data analyses. In Chapter V we discuss the implications of these results for teachers, curriculum developers, and mathematics education researchers as well as the limitations of the study.

CHAPTER II

THEORETICAL PERSPECTIVE

Our purpose in this chapter is to review the research on student difficulties with kinematic graphs and the efforts to overcome those difficulties through Calculator Based Laboratories and student discourse.

First research in the area of students' difficulties with kinematic graphs is reviewed. Much of this work involves the use of Microcomputer Based Laboratories (MBL's) and microworlds. MBL's and microworlds use computer software to create learning activities. The MBL makes use of various probes, attached to a micro-computer, that enable the computer to produce real-time graphs of such variables as position, velocity, acceleration, light intensity or temperature. A *microworld* is software that produces a system composed of objects, relationships among objects, and operations that transform objects and relationships (Thompson, 1987). For example, The Envisioning Machine (EM) is a specific example of a microworld in which the objects are a particle, a velocity vector, and an acceleration vector that are related in that they each represent the motion of a ball. The motion of a ball can be modeled by changing the initial position of the particle, the velocity vector, or the acceleration vector.

The research using MBL tools and microworlds has indicated that their use can improve students' understanding of kinematic graphs (Brasell, 1987; Dykstra et al., 1992; Nemirovsky et al., 1992; Thornton et al., 1990). This may indicate that similar, but more affordable and accessible tools such as the Calculator Based Laboratory, may also be used to improve student understanding. Also, the use of these tools has aided researchers in better understanding the nature of students' difficulties with kinematic graphs and how these difficulties might be overcome. In particular, some researchers have indicated that student discourse may improve students' understanding of kinematic graphs (Monk, 1994, Dykstra et al., 1992).

Before proceeding, a framework is needed to clarify what is meant by student understanding of kinematic graphs. Graphs are mathematical tools and the interpretation of information provided by graphs requires knowledge of several mathematical concepts and conventions, as well as procedural skills. These skills include reading points off a graph, computing the slope of the graph, and finding the area under the curve of a graph. In this discussion, I will adopt an important distinction in the meanings for the words *concept* and *conception* due to Sfard (1991):

> ...the word concept (sometimes replaced by notion) will be mentioned whenever a mathematical idea is concerned in its official form - as a theoretical construct within the formal universe of ideal knowledge; the whole cluster of internal representations and associations evoked by the concept - the concept's counterpart in the internal, subjective universe of human knowing - will be referred to as a conception. (p. 3)

Using Sfard's definitions, student understanding can be discussed in terms of the student's conceptions and misconceptions. A misconception is a flawed conception, incorporating incorrect or incomplete information and/or having connections that need repair or replacement. A student's understanding can be said to have improved if a misconception is corrected, or if the student's conception is strengthened so that it converges to the accepted "community" conception, i.e., the one commonly held by mathematicians and scientists.

A review of the research related to misconceptions in students' understanding of graphs leads to examination of the following areas:

- Identification of students' difficulties with mathematical concepts related to interpreting graphs of functions, and in particular, graphs relating kinematic variables. Included in the review of research on student difficulties will be description of the misconceptions related to the difficulty, theories on the nature of the misconceptions, and suggested treatments for the misconceptions.

- Student discourse as a suggested treatment for student misconceptions. This leads to , a review of research on the use of cooperative group structure as a teaching tool in the mathematics classroom (since cooperative groups are designed to give students the opportunity for discourse and collaboration).
- Theories of how student discourse and collaboration work to improve student understanding.

Student Difficulties in Interpreting Graphs of Kinematic Variables

McDermott, Rosenquist and van Zee (1987) found that students had difficulty interpreting kinematic graphs and that these "difficulties with graphing have been identified even among students in the honors section of a calculus-based university physics course" (p. 504). This implied that either the students had not been given the opportunity to learn the concepts of graphical representations, or they had taken math courses in which graphical representations are taught, but had not learned the material. This latter possibility is particularly discouraging when considering that these students appear very capable since they were in honors courses.

The following are some of the common difficulties that researchers report students have in interpreting graphical representations:

- Discriminating between the slope and height of a graph (McDermott et al., 1987, Bell et al., 1981, Clement, 1989)
- Relating one type of graph to another (Brasell, 1987, McDermott et al., 1987, Monk & Nemirovsky)
- Interpreting the area under a graph (McDermott et al., 1987).

- Separating the shape of a graph from the path of the motion (McDermott et al., 1987, Bell et al., 1981, Clement, 1989, Nemirovsky and Rubin, 1992, Monk, 1990).

Each of these difficulties will be discussed separately and will include examples of how the difficulty is expressed, researchers' explanation of why the difficulty exists, and suggested remedies.

Discriminating Between the Slope and Height of a Graph

Researchers have observed that many students will incorrectly respond to a question by giving the height of a graph at a point when the slope of the graph at that point is the correct response. Such an error is often considered a "simple mistake," i.e., an error which is not the result of a misconception, but of a misreading or a clerical error. For example, McDermott (1986) gave students a graph similar to the one shown in Figure 1 and asked which object had the greater velocity at time t=2. Many students incorrectly chose object B.

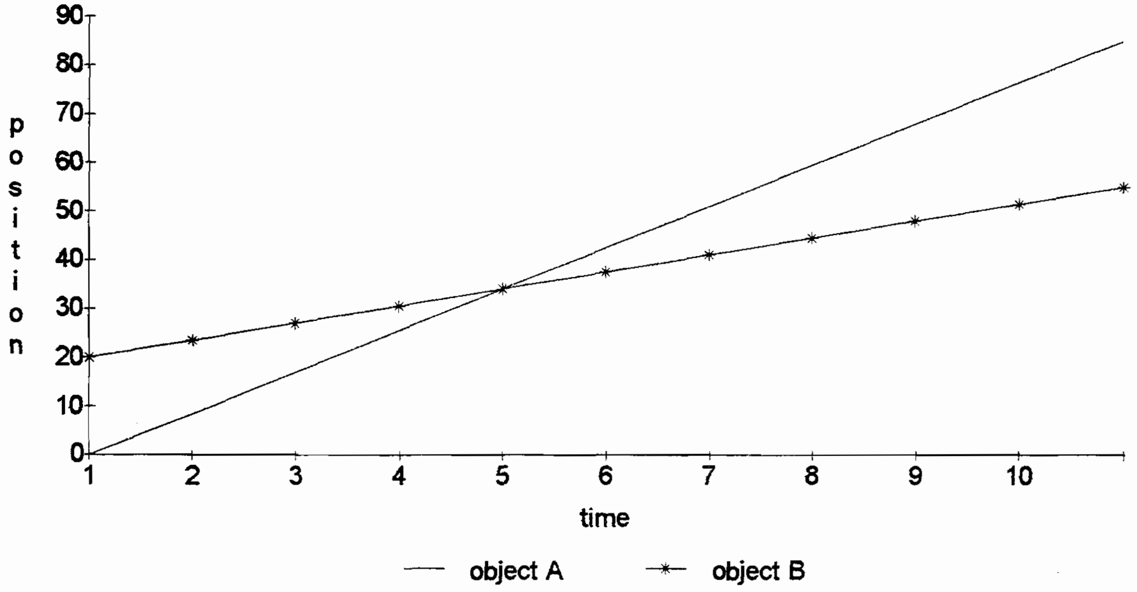


Figure 1. Position vs. time graph from McDermott

Such an error could be due to a misreading of either axis. In that case, this type of error would not indicate that the student's conception was necessarily faulty. However, in one study Monk (1994) reported on a student (Carl) who made this type of error, but did read the axes correctly. Carl was a college student enrolled in first term calculus and had completed the section on differentiation. He received an "A" in this course as well as in the subsequent calculus course. Carl used principles from his own experience, and the visual qualities of the graph which supported his principles, to incorrectly respond to a similar question. The question involved the graph in Figure 2, which is a graph of velocity vs. time for two cars, a red one and a blue one. Carl was asked to describe the distance between the two cars after time t=5.

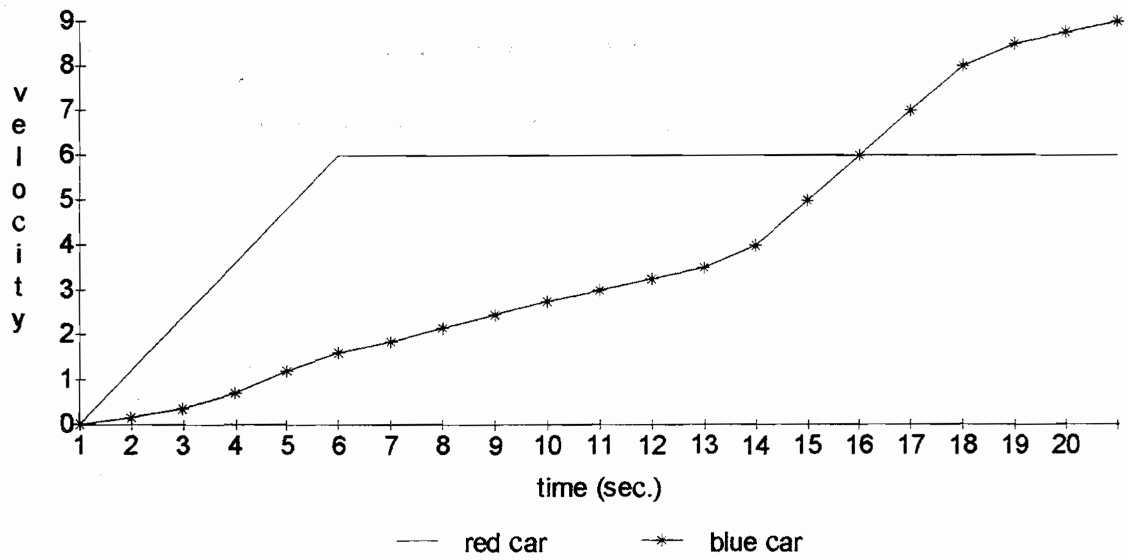


Figure 2. Velocity vs. time graph from Monk

Some of the "principles" that Monk reported Carl using are as follows:

- the acceleration principle: If car B is behind car A and is accelerating very rapidly while car A is not accelerating at all, then car B will get closer to car A.

- the speed principle: If car A is ahead of car B at time t_0, and car A is going faster than car B, then car A will stay ahead of car B and, in fact, will get further ahead of car B.

(Monk, 1994, p. 5)

Carl's principles are reasonable and based on practical experience, but do not generalize to the given situation. Through Monk's discussions with Carl it is clear that Carl uses visual aspects of the graph to support his principles. In a series of conversations, Monk is able to observe the complex processes through which Carl finally comes to the correct conclusion. Monk generalizes his observations as follows:

- Carl's understanding is robust, rich, complex, and grounded in his experience.

- It cannot be understood by focusing on his errors nor what he lacks. It has to be understood within itself.

- In its coherence and completeness, it has within it the capacity to strongly resist change. But it can change, of course, although this only happens in relatively rare moments of creativity and reflection. (p. 16)

Monk further claims that the implication of this study for changes in the way calculus should be taught include:

- An emphasis on conceptual vs. procedural learning - on understanding the ideas as opposed to knowing how to do the procedures.

- An emphasis on relating the mathematical ideas to real situations.

- Classroom formats that encourage discussion, especially among students, in contrast to lecturing and telling by the teacher. What we saw is that the change in Carl's understanding took place as he was trying to explain his own ideas (pp. 16-17).

From Monk's study we see that student difficulty in correctly reading the slope of the graph can be rooted in the student's conceptions of kinematic variables, which are in turn influenced by personal experience. Monk suggests that the student needs to be made aware of misconceptions through exercises that relate mathematical concepts with practical experience. Monk accomplishes this through the use of a Microcomputer Based Laboratory (MBL) and student/interviewer discussion.

Relating One Type of Graph to Another

Students often expect the position graph of an object to be similar to the velocity graph of that object (Nemirovsky et al., 1992, Brasell, 1987). Nemirovsky and Rubin (1992) found that students expected the following types of resemblance between a function and its derivative:

1. Simple replication (the predicted graph is identical to original graph)
2. Same direction of change (e.g., increasing derivatives correspond to increasing functions, and decreasing derivatives correspond to decreasing functions)
3. Same shape (e.g., straight lines correspond to straight lines)
4. Same sign (graphs above the x-axis generate graphs above the x-axis and vice versa)
5. Same geometrical transformation.

(Nemirovsky and Rubin, 1992, pp. 6-7)

Nemirovsky and Rubin state, "Resemblances give students tools for making sense of a complex situation. Students probably do not adopt resemblances because they have solid reasons to believe the tools are appropriate, but rather because the tools enable them to organize and solve a bewildering domain of problems" (p. 9). They go on to say that students may establish their own set of principles, which may be incorrect but are supported by other facts. This was the case with Carl in the study by Monk, when he used visual features of the graph to support incorrect general assumptions based on practical knowledge. Nemirovsky claims the principle that a function and its derivative resemble each other is supported by the fact that "They both describe the behavior of the same object over the same time period" (Nemirovsky et al., 1992, p. 9). They also found other cues that support the principle of resemblances:

1. *Syntactic* cues are distinguished by the fact that they are based on graphical features, unrelated to the student's knowledge of motion or air flow. For example, given the position functions of two objects, the student draws the graphs of the velocity functions for the two objects as a geometric transformation of position.
2. *Semantic* cues elicit students' ideas that the function and its derivative behave similarly on the basis of real-world knowledge. For example, the common experience that going faster implies traveling further sometimes results in the overgeneralization that velocity and position always move in the same direction, either both increasing or both decreasing.
3. *Linguistic* cues are ambiguities of language that support resemblances between a function and its derivative or a function and its indefinite integral. Words such as more and less, or up and down can have ambiguous meanings. For example, it is true that less velocity for car A than for car B means less distance traveled for car A. But, if we consider a single car A, it is not true that less velocity now than earlier implies that we have traveled less distance now than earlier.

Nemirovsky and Rubin performed an observational study (1992) involving a student (Dan) in an MBL setting. The observations took place over several learning episodes during which Dan was using MBL instruments and was being interviewed/instructed by Nemirovsky. The results of their study gave Nemirovsky and Rubin reason to state that "It is clear that students' learning, at least in our teaching interviews, is not a progressive sequence of "getting" (or "not getting") one idea after another. We do not consider the use of resemblances a matter of "confusion" in the sense that students cannot discriminate between volume and flow rate" (p. 15). They go on to conclude, "The case study also supports the importance of our technique (MBL) of using physical contexts to provide students with tools to explore mathematical ideas from a variety of directions, and gives us insight into how these tools help frame the interviewer/student discourse through which learning occurs" (p. 33).

Interpreting Area Under the Graph

The problem of students' difficulty interpreting area under a graph is not as well researched as other difficulties previously discussed. This may be because students generally gain experience with interpreting area under a graph in integral calculus, while most of the students in the observational studies already reported have been high school students. In the case of Carl (a college student), Monk claims Carl had difficulty interpreting the slope versus the height of a graph. It could be argued that Carl also was having difficulty interpreting the area under the graphs. Clearly, the distance between the two cars is represented by the difference between the areas under each graph. This argument is probably not made since Carl had not yet had integral calculus and so was probably unaware of this concept in interpreting information provided by the graph.

McDermott et al. (1986) found that students had the following difficulties interpreting area under a curve:

- Students often find it difficult to envision a quantity that they associate with square units as representing a quantity with linear units.
- Students do not recognize the area below the horizontal axis has negative value.
- Most of the difficulties the students have with this type of problem is directly related to an inability to visualize the motion that is depicted in a velocity vs. time graph.

(McDermott et al., 1986, p. 506)

McDermott et al. do not give any specific recommendations for teaching that would decrease students' difficulty in this area.

Separating the Shape of a Graph From the Path of Motion

The final student difficulty to be discussed here is in separating the shape of a graph from the path of motion. For example, consider the physical situation of a bicycle traveling over a hill. When asked to draw a speed vs. time graph, students may simply draw a hill (Clement, 1989), such as that shown in Figure 3.

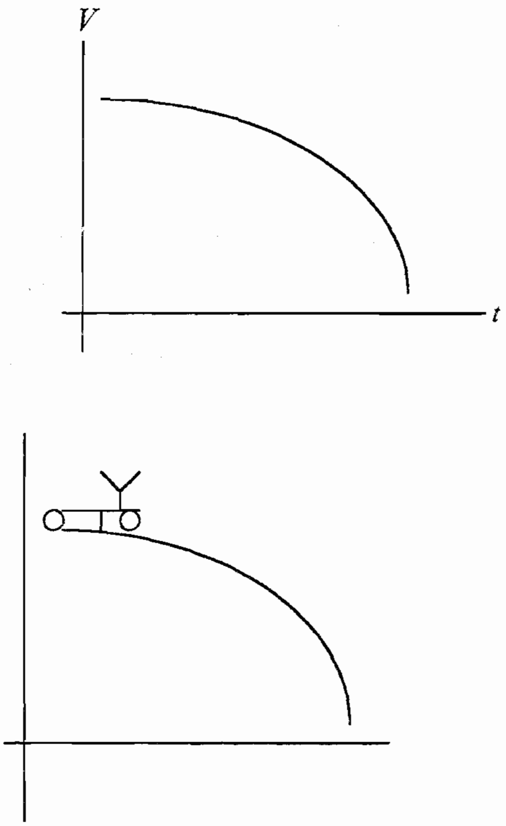


Figure 3. Example of Graph Resembling Path of Motion

Monk (1990) defined this Beichner's type of error as Iconic Translation and he found that this is not a simple mistake. He found that students use Iconic Translation for the following reasons:

- They seem to have an impulse to include in their graph a visual aspect of the situation. However, in this study, the students are seen to introduce inappropriate visual features of the situation quite spontaneously, at a time at which there are other options available to them for answering the question. This seems a firmer basis for suggesting that the impulse to Iconic Translation is a genuine one within the students' own conceptualization and is not in some way induced (p. 18).

- Students want to invoke a global rule that simplifies the problem. ...more than half of these students spontaneously conjectured (more or less explicitly) a particular general principle that they believed governed the variables of the problem... What would seem significant, though, is how quick these students were to make such statements and how strong their commitments were to them (p. 13).

Monk (1994) also stated that he observed students interpreting a graph as a literal picture because this may support other conceptions the student had of the physical situation.

An Instrument for Measuring Student Understanding of Kinematic Graphs

There has been a great deal of research documenting students' difficulties with kinematics graphs. Due to the importance of students' ability to interpret these graphs, Beichner (1994) recognized the need for further research and sought to develop a valid and reliable instrument to assess these skills. He conducted informal interviews with teachers, examined texts, and reviewed test banks to identify objectives held by most educators. Eight objectives emerged from this process. Beichner then performed a study to investigate student's competence with these objectives. As a result of the study, Beichner identified seven objectives that students commonly have difficulties with (Beichner, 1994, p. 752):

	Given	The student will
1.	Position-Time Graph	Determine Velocity
2.	Velocity-Time Graph	Determine Acceleration
3.	Velocity-Time Graph	Determine Displacement
4.	Acceleration-Time Graph	Determine Change in Velocity
5.	A Kinematics Graph	Select Another Corresponding Graph

6. A Kinematics Graph — Select Textual Description
7. Textual Motion Description — Select Corresponding Graph

That students have these difficulties confirms the work of Monk, Nemirovsky and Rubin. To facilitate future research, Beichner developed a Test for Understanding Kinematics (TUG-K) which assessed student's abilities on these seven objectives. studies using the TUG-K again confirmed previous research regarding student's difficulties. The mean test score was 40% for high school and college students who had already been given some traditional instruction in kinematics. As a result of his study, Beichner concluded:

> The students must be given (1) the opportunity to consider their own ideas about kinematics graphs and then (2) encouragement to help them modify those ideas when necessary. Teachers cannot simply tell students what the graphs' appearance should be. It is apparent from the testing results that this traditional style of instruction does not work well for imparting knowledge of kinematics graphs (p. 755).

Interpretation of kinematic graphs is a complex process. Students come to the physics and calculus classrooms with their own understanding of velocity, acceleration and distance based on personal physical experiences (Nemirovsky et al., 1992). We cannot simply ask students to abandon their concepts and replace them with ours. Many students may be similar to Dan in the study by Nemirovsky et al. (1992): "Dan does not seem to be in a process of overcoming a misunderstanding, or of replacing one understanding of this situation with another. As was true then, Dan's learning is better described in terms of a process of refining his initial understanding into one that is somewhat more adequate" (p. 163).

The researchers do not mention student/student discourse as a possible aid in instruction, but it may be conjectured that since discourse between the interviewer and student was beneficial, that similar discourse between students may also be beneficial. Dan's change in understanding took place as he was explaining his own ideas. Similarly, in the case of Monk's student, Carl, Monk (1994) said, "I don't see how Carl could possibly change his mind if my ideas or another expert's ideas were what was being discussed" (p. 17). This would imply that the discussion being about the student's ideas is as important as with whom the discussion is taking place.

Nemirovsky and Rubin put the most emphasis on the use of a physical activity such as occurs in the MBL setting. Since a CBL uses similar physical activities, it is possible that they could also be beneficial in changing student conceptions of resemblances between graphs.

Student Observation of Physical Motion

Microcomputer Based Laboratory (MBL) instruments create different learning opportunities than traditional instruction that may be more effective in repairing students' misconceptions. The use of MBL instruments in the classroom has been shown to improve student understanding of kinematic graphs for a wide range of students (Thornton, 1987). Thornton and Sokoloff (1990) found in a study of university physics students that those who worked with an MBL-based curriculum showed considerable improvement in understanding velocity and acceleration. In this study, students worked together in the classroom using the MBL instruments and completed assignments at home.

Rosenquist et al. (1986) found instruction based simply on observation of physical motion, without the use of MBL instruments, could help students' conceptions concerning kinematic variables including graphical representation. They observed students were improperly using a position criterion when comparing the velocities of two objects. The instructors demonstrated for the whole class several speed comparisons using balls. This gave the students the opportunity to observe the physical phenomena in a setting that confronted their own inconsistencies. Subsequently, almost all of the students were able to determine that position would not be used to determine a speed comparison.

Dykstra et al. (1992) state that the MBL activities are "...the most effective approach of which we are aware, this differentiated view arises as students come to view graphs as realistic representations of motion. The graphs are then used to help students confront paradoxes that arise from their new view of motion being in conflict with their previous undifferentiated view of motion" (p. 638). Dykstra found that the MBL gave students the opportunity to be confronted by discrepancies in conceptions. These findings imply that similar CBL activities may also be effective in restructuring and repairing students' conceptions.

Dykstra also found it important to establish "...a discussion to develop and test new ideas in order to resolve perceived discrepancies" (p. 639). He further states that changes in conception " ... can be observed to occur in student-student discussions" (p. 641).

Cooperative Groups

Monk and Nemirovsky have repeatedly emphasized the role that discourse plays in changing student understanding.

> We find more change in classroom formats that encourage discussion, especially among students, in contrast to lecturing and telling by the teacher.
>
> (Monk, 1994, p. 17)

> ...students who display overly simple Global Rules should not be expected to jettison all such rules, it seems unwise to propose that students should not act at all upon their picturing impulse. Rather, ways should be found to help articulate and refine this impulse in the experience of expressing it.
>
> (Monk, 1990, p. 21)

> Our analysis suggests an alternative approach to teaching graphing based on activities in which students explore graphing situations using all they know about graphs, playing with visual attributes, making predictions, and communicating about relationships with curves.
>
> (Nemirovsky and Monk, 1994, p. 167)

Much of the literature on cooperative groups focuses on the use of cooperative groups vs. lecturing strategies, and the effect these strategies have on student achievement in a particular area of mathematics. Slavin (1980) has examined several projects involving elementary school students studying mathematics. He reviewed 28 research studies on the use of cooperative groups and reported, "The achievement results, though usually positive, seem to depend on the particular techniques, settings, measures, experimental designs, or other characteristics" (p.333). Slavin found that, at that time, the research indicated:

> 1. For academic achievement, cooperative learning techniques are no worse than traditional techniques, and in most cases they are significantly better.
>
> 2. For low level learning outcomes, such as knowledge, calculation and application of principles, cooperative learning techniques appear to be more effective than traditional techniques to the degree that they use:
> (a) A structured, focused, schedule of instruction;
> (b) Individual accountability for performance among team members;
> (c) A well-defined group reward system, including rewards or recognition for successful groups.
>
> 3. For high level cognitive learning outcomes, such as identifying concepts, analysis of problems, judgment, and evaluation, less structured cooperative techniques that involve high student autonomy and participation in decision-making may be more effective than traditional individualistic techniques.
>
> (Slavin, 1980, p. 337)

In another meta-analysis of research concerned with the relationship between verbal interaction and learning in small groups in the mathematics classroom Webb (1991) found varying levels of improvement based on the nature of that interaction.

> In conclusion, although some questions remain unanswered, it is clear that the experiences of students in small groups can influence their learning. From the research described here, the optimum small group setting is one in which students freely admit what they do and do not

> understand, consistently give each other detailed explanations about how to solve the problems, and give each other opportunities to demonstrate their level of understanding. (p. 386)

In studies reporting the use of cooperative groups with college students, one tends to find less structure than described in item two given by Slavin above. However, studies involving college students do appear to substantiate the findings that cooperative groups are more effective in improving achievement for higher level learning outcomes, but less effective in improving achievement for lower level outcomes. We summarize the findings of two of these studies below.

Gentry (1991) found that using cooperative groups in a Beginning College Algebra course did not have a statistically significant effect on student achievement. Student achievement was measured by the results of a pre- and post-test given to students. The students were divided into a control group that received individualized instruction and a treatment group in which the students were assigned to groups of 2 or 3 students. Gentry did not report the nature of the problems on the pre and post-test. Thus, we do not know how many of the problems were computational, and how many required analysis or other higher-order problem-solving skills. An analysis by type of problems may have given different results. For example, it is possible that a comparison of control vs. treatment groups may not have shown a difference in achievement for computational problems or for problems overall, but still have shown a statistical difference in achievement for more difficult problems.

A study done by Dees (1991) illustrated that students working in groups gained in achievement when working on difficult problems. Her study included students in a college remedial course designed to remove students' deficiencies in high school algebra and geometry. The sample included 77 students who were divided into 4 recitation classes, all of which were assigned to one lecture section of the remedial course. In the two treatment sections of recitation classes students were actively encouraged to participate in groups and were occasionally given assignments that they had to complete in groups. In the other two control sections the students were not encouraged or discouraged to work in groups. Results of the study indicated that the treatment group did work together cooperatively in groups significantly more than the control group.

In Dee's study eight measures were used to test student achievement as follows:

1. Total score on a standardized algebra test which was given as a pre-test and again at mid-term.

2. Average percentage score on eight weekly tests given in algebra.

3. Total score on the final examination in algebra.

4. Total score on the word problem section of the algebra final exam.

5. Average percentage score on six weekly tests given in geometry.

6. Score on the definitions-and-applications section of the geometry final exam.

7. Score on the proof-writing section of the geometry final exam.

8. Total score on the geometry final exam.

The treatment group scored significantly higher than the control group on items #3, #4, and #7. For the other five items, any differences were not statistically significant. Note that although the cooperative group classes scored significantly better on the proof section of the geometry final, their score on the final overall did not show a statistically significant difference from the control classes. The statistical significance of the difference in mean scores on the word problems section of the algebra final was greater than the significance for the algebra final overall. This indicated that the main differences between the groups appeared in ability to do algebra word problems and geometry proofs.

Geometry proofs and algebra word problems require higher order skills than definitions and standard algebra problems. Thus, Dees' study substantiates Slavin's findings and gives evidence that cooperative groups may be effectively used to improve students' achievement on problems requiring higher-order skills.

The research examining student understanding of kinematic graphs indicates that student discourse (discourse between students) may help correct students' misconceptions concerning graph interpretation. Research on the effect of student discourse through the use of cooperative groups indicates that it could have a positive effect on student understanding particularly for problems requiring higher-order skills. Problems involving kinematic graphs often require such higher-order skills to solve.

The studies by Monk and Nemirovsky suggest that students' conceptions of graphs of kinematic variables are robust and complex, but that students' misconceptions can be deeply rooted and difficult to correct. The important role that discourse can play in correcting misconceptions may be realized through the use of cooperative groups.

A Framework for Studying Collaboration and Conceptual Change

Student conceptions of kinematic graphs can be significantly different from those of mathematicians and scientists, and a student's difficulties may lie in the misconceptions held by the student. Researchers of students' misconceptions have observed that student discourse may be beneficial in removing misconceptions, and cooperative groups give the opportunity for student discourse. Student discourse occurs as students collaborate on a problem. To discuss the actual process by which student discourse removes misconceptions, we adopt a framework posed by Roschelle (1992), who sets forth a theory of how collaboration can affect a conceptual change.

- Collaboration is analyzed as a process that gradually can lead to convergence of meaning (p. 235).

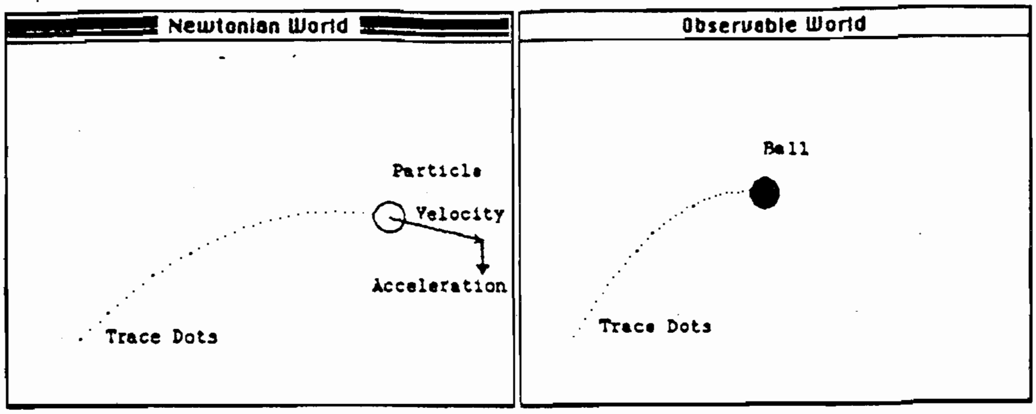
- Specifically, it is argued that conversational interaction provides a means for students to construct increasingly sophisticated approximations to scientific concepts collaboratively, through gradual refinement of ambiguous, figurative, partial meanings (p. 237).

Roschelle states that the problem is one of *convergence*: "How can two (or more) students construct shared meanings for conversations, concepts, and meanings? Few theories account for the achievement of convergence in the face of tendencies for meanings to diverge" (p.236). This is particularly true for problems which involve acceleration, velocity, and distance as research indicates that students are particularly prone to have divergent conceptions from those of the scientific community (McDermott, 1986; Monk, 1990; Dykstra, 1992). In considering how students' collaboration can accomplish convergent conceptions, Roschelle discusses how scientists collaborate:

> To converge on meaningful new theories, scientists collaborate. Ethnographic and sociological analyses of scientific theory construction argue that scientific collaboration shares most of the features of everyday, informal interaction, including the use of conversational turn-taking structures to negotiate meaning. Investigators of scientific conceptual change emphasize two differences that distinguish scientific work: (a) the production of visible displays that represent features of the world at an intermediate level of abstraction and (b) the interplay and recombination of metaphors drawn from experience to construct explanations. Similar findings have emerged in studies of science learning. Conceptual change is seen as a process of learning to register "deep features" of situations and restructuring systems of physical metaphors.
>
> (Roschelle, 1992, p. 236)

To understand how student collaboration leads to conceptual change, Roschelle examined a case study involving two students (Carol and Dana) working at an envisioning machine (EM). The EM is a computer with software which allows direct manipulation for graphical simulation of the concepts of velocity and acceleration.

The computer screen is split in two sections called the observable world and the Newtonian world. In the observable world, the student sees the path of motion for a ball. In the Newtonian world, the students sees a ball with two arrows protruding from it, as seen in Figure 4.



The Envisioning Machine (labels added).

Figure 4. Screens from the Envisioning Machine

The student is not given the labeling of the arrows as velocity and acceleration. The students' task is to adjust these arrows and the initial position so that the path of motion in the Newtonian world matches that in the observable world.

Through a process of problem-solving tasks involving constant velocity or constant acceleration, Dana and Carol converge to an understanding of the arrows which is consistent with the scientific conception of velocity and acceleration vectors.

> The analysis showed that Carol and Dana cooperatively constructed an understanding of acceleration that constituted (a) a large conceptual change from their previous concept, (b) a qualitative approximation to the scientific meaning of acceleration, and (c) a closely shared meaning between one another.
>
> (Roschelle, 1992, p. 238)

Roschelle states that this shows that Carol and Dana achieved *convergent conceptual change*. Moreover, the process which took place for the convergent conceptual change was characterized by:

1. The construction of a "deep-featured" situation, at an intermediate level of abstraction from literal features of the world.

2. The interplay of metaphors in relation to each other and to the constructed situation.

3. An iterative cycle of displaying, confirming, and repairing situated actions.

4. The application of progressively higher standards of evidence for convergence.

The first two features capture the main thrust of conceptual change as involving the construction of metaphoric explanations in relation to appropriate deep-featured situations. The second two features draw in the analysis of conversational interaction emerging from ethnomethodology. Linking conceptual change with the more recent research on convergence in conversation analysis suggests a pragmatic process by which convergent conceptual change can occur incrementally, interactively, and socially.

(Roschelle, 1992, p.268)

A diagram of this process is shown in Figure 5.

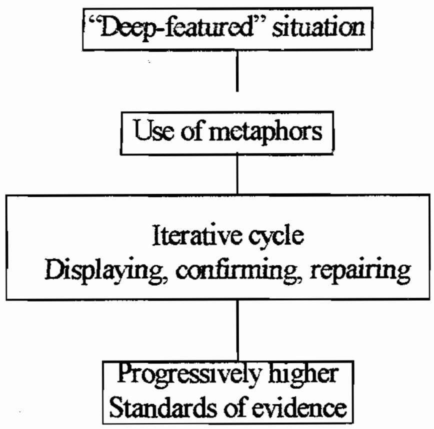


Figure 5. Model of the process of conceptual convergence

It should be noted that conceptual convergence was not an inevitable consequence of using of the Envisioning Machine. Of 14 students who participated in this study, only six students converged to concepts similar to those of mathematicians. The other eight students either never converged, or converged to concepts that were incompatible with conventional understanding. Further, although all 14 students worked in pairs, the individuals in the pairs did not always have the same conceptions in the end. Some students' conceptions concerning acceleration diverged from each other and from the conceptions of the mathematical community.

Roschelle emphasizes the need for a physical activity which will provide the basis for student discourse. The physical activity provides the deep-featured situation about which students can use metaphor and other forms of communication to display, confirm, and repair their conceptions. In a similar study, he finds that, "...they use perception, language, and gesture to construct a shared understanding of what the notation on the computer screen means" (Roschelle, 1991, p. 1).

Roschelle's work implies that a study of change in student conceptions must provide students with a "deep-featured" situation which is an abstraction of the physical world. If this situation is provided in the form of an in-class, laboratory assignment we would expect to see this process of using metaphors, displaying, confirming, and repairing conceptions, to continue to occur iteratively, at higher standards of understanding.

Dykstra et al. (1992) sets forth the following strategies for conceptual change:

1. Use and develop trust in tools that extend the senses. It is preferable to use a phenomenon (a) that is going to have widely differing explanations by students, or (b) a phenomenon whose outcome students feel confident predicting but whose outcome differs with their predictions.
2. Have students predict the outcome or explain the phenomenon.
3. Focus on inducing disequilibration by having students test their predictions or explanations.
4. Establish a "town meeting" to discuss, develop and test new ideas in order to resolve perceived discrepancies and differences in explanations.

(Dykstra et al., 1992, p. 642)

Motivation For the Present Study

Roschelle's framework supports the conjectures of researchers in the field of student understanding of kinematic graphs that both discourse and a physical activity (actual or simulated) may be essential in removing student misconceptions. The work of Roschelle and Dykstra suggests guidelines for developing laboratory activities to promote conceptual change in student understanding of graphs of kinematic variables. Furthermore, Roschelle's framework for how students' conceptions change through collaboration gives a perspective to analyze the effectiveness of such activities on students' conceptions of interpretation of kinematic graphs.

Beichner's development of the TUG-K provides a quantitative tool for measuring student understanding of kinematic graphs and identifying their difficulties. To investigate student understanding of graphs of kinematic variables, qualitative techniques such as detailed classroom observations and student interviews are tools that may also be employed. Beichner states, "The ideal course of action is probably found in the combination of the strengths of both these research methodologies" (1994, p. 1).

The present study adopts the theoretical framework of Roschelle as a process by which student discourse can affect a conceptual change. Following the guidelines suggested by the work of Roschelle and Dykstra, we develop treatments in various teaching environments to improve student understanding of graphs of kinematic variables. We seek not only to analyze the effectiveness of these various teaching strategies through the use of the TUG-K developed by Beichner (1994), but we also hope to gain a better understanding of how conceptual change occurs.

CHAPTER III

RESEARCH METHODS

Research Questions

The purpose of this study was to investigate alternative models of instruction that may improve student understanding of graphs of kinematic variables in the context of integral calculus. The review of research has led us to investigate the benefits of two instructional models: (1) The use of CBL (Calculator Based Laboratory) instruments in a laboratory context, and (2) the use of cooperative groups in promoting student discourse. Our primary goal is to better understand the dynamics by which these models (alone and in concert) can be expected to improve student understanding of graphs of kinematic variables. To serve this goal, the following research questions were posed:

1. What specific difficulties with interpreting graphs of kinematic variables do students bring to the integral calculus classroom?
2. What misconceptions, or lack of conception, are indicated by the difficulties which students bring to the integral calculus classroom?
3. What is the relative effectiveness of the traditional, cooperative group, and CBL models of instruction for building conceptions, repairing misconceptions and removing difficulties with interpretation of graphs of kinematic variables? In particular, are certain types of difficulties more readily removed by one of these instructional models?
4. How does the process of student discourse aid in repairing misconceptions? In particular, are there meaningful differences in the student discourse generated by a laboratory setting using CBL-

instruments and laboratory setting using the more abstract tool of algebraic formulas? Can we find confirmation of Roschelle's Theory of conceptual convergence?

Both quantitative and qualitative methods were employed in the study. Questions (1) and (2) were investigated by analysis of a test of understanding graphs of kinematics (a pre-test based on Beichner's TUG-K) that students were given on the first day of class. Question (3) was investigated by comparative analysis of the pre-test with an identical test (post-test) given following the use of a variety of instructional models, including both cooperative groups and/or CBL model(s). Corroboration of this analysis was sought through interviews with students who showed improvement in their understanding of graphs of kinematic variables. Question (4) was investigated by analyzing videotapes made of student groups as they work on a laboratory assignment about kinematics.

Setting

The study took place at a medium-sized, public university on the west coast of the United States. The main calculus sequence at the university serves primarily the science and engineering colleges. The differential and integral calculus courses are usually taught in lecture sections of approximately 60 to 100 students, meeting for three 50-minute periods per week (Monday, Wednesday and Friday). In addition, students meet in recitation classes of 25 to 35 students for 80 minutes, one time each week (Thursday). A professor is the lecturer and a graduate student teaching assistant instructs the recitation class. The recitation classes provide opportunities for students to work on assignments, which are given as "labs" similar to projects given in a science laboratory class. In addition, recitation classes provide the opportunity for discussion of the material presented in lecture.

Subjects

The sample was drawn from students in two lecture sections of integral calculus, Winter Term, 1996. One lecture section was taught in the morning and one in the afternoon. Both lecture sections were taught by the same professor (Logan). The students were enrolled in four recitation sections, all taught by the same graduate student teaching assistant (Diane). (The names "Logan" and "Diane" are pseudonyms used to protect anonymity). Two of the recitation sections were made up of students from the morning lecture and two from the afternoon lecture. Hence, the teaching personnel were identical for all the students involved in the study.

Development of Pre- and Post-Tests

The pre-test and post-test for this study were adapted from the Test of Understanding Graphs in Kinematics (TUG-K) developed by Beichner (1994) (see Appendix B for a copy). In his study, Beichner reported the TUG-K had Kuder-Richardson reliability coefficient of 0.83 and a Ferguson's Delta score of 0.98 (indicating that the test does discriminate student ability by spreading the distribution of scores). Beichner established content validity through examination of the items by 15 science educators including high school, community college, four year college, and university faculty. Beichner had similar versions of the TUG-K given as a pre- and post-test to 165 students in high school or college. The Pearson product-moment correlation between the pre-and post-test scores was 0.79, indicating the two versions of the test were similar. After these students had taken the pre-test they were given instruction in kinematics. One week following instruction they were given the post-test. A paired samples t-test revealed a significant increase in mean scores ($p<0.01$), further indicating that the TUG-K is a valid instrument for measuring student understanding of kinematic graphs.

The TUG-K was designed to test for seven objectives that relate to students' difficulties with kinematic graphs. It consists of 21 questions, three questions for each of the seven objectives. Table 1 lists the seven objectives, and the average percentage correct for the three questions testing that objective. The percentages are based on administration of a final version of the TUG-K given to 524 high school and college students across the country.

Table 1. Objectives of the TUG-K

Given	The student will	Percent Correct	Item Number
1. Position-Time Graph	Determine Velocity	51	5, 13, 17
2. Velocity-Time Graph	Determine Acceleration	40	2, 6, 7
3. Velocity-Time Graph	Determine Displacement	49	4, 18, 20
4. Acceleration-Time Graph	Determine Change in Velocity	23	1, 10, 16
5. A Kinematics Graph	Select Another corresponding Graph	38	11, 14, 15
6. A Kinematics Graph	Select Textual Description	39	3, 8, 21
7. Textural Motion Description	Select Corresponding Graph	43	9, 12, 19

Due to time constraints on the administration of the pre- and post-test in the present study, it was necessary to reduce the number of items from 21 questions. During the Fall of 1995 a pilot study was done. For the pilot study, the pre- and post-tests were made of 10 questions and 11 questions respectively, together comprising the 21 questions on Beichner's TUG-K. The breakdown of how many questions on each of the pre- and post-test addressed each of the seven objectives is given in Table 2.

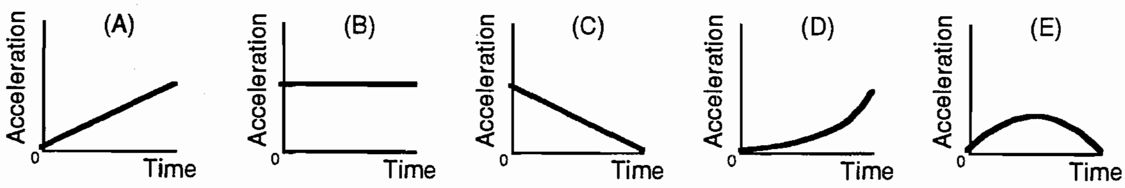
The pre-and post-test used for the present study each consisted of 14 identical items from Beichner's TUG-K, two questions for each objective. Items were initially chosen on the basis of the point biserial coefficient, and similarity to other questions. Of the 3 items for each objective, the two with the highest point biserial coefficient were first considered. Item responses were then examined. In cases where the correct response could plausibly be obtained through incorrect analysis the item was replaced

Table 2. Beichner's TUG-K items used on Pre- and Post-Test
Pilot Study

Objective Given	Objective The Student Will	Pre-Test Beichner Item Number	Post-Test Beichner Item Number
1. Position-Time Graph	Determine Velcocity	5, 17	13
2. Velocity-Time Graph	Determine Acceleration	6, 7	2
3. Velocity-Time Graph	Determine Displacement	18	4, 20
4. Acceleration-Time Graph	Determine Change In Velocity	10	1, 16
5. A Kinematics Graph	Select Another Corresponding Graph	14	11, 15
6. A Kinematics Graph	Select Textual Description	3	8, 21
7. Textual Motion Description	Select Corresponding Graph	9, 19	12

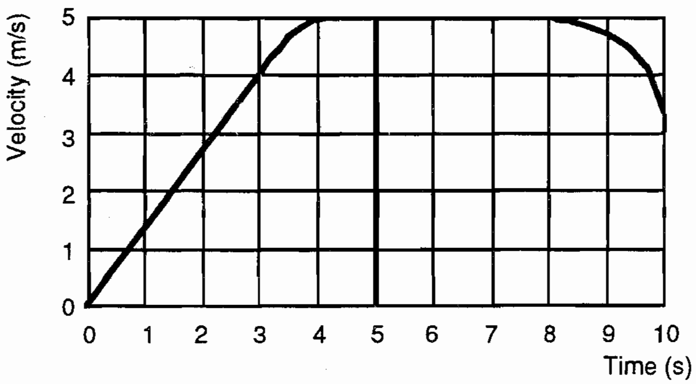
with the third item for that objective. All items were then examined for diversity in question format and final adjustments were made. For example, TUG-K items 4 and 20 are very similar in that they both give a velocity graph and ask the student to compute change in distance for a specific time interval. Thus, only one of these items was included on the test for this study (item 4). The pre- and post-test consisted of Beichner's items #1, #4, #6, #7, #8, #9, #10, #11, #13, #15, #17, #18, #19 and #21. This 14-item instrument has a Kuder Richardson-21 reliability coefficient of 0.74. The test items for this instrument follow.

1. Acceleration versus time graphs for five objects are shown below. All axes have the same scale. Which object had the greatest change in velocity during the interval?

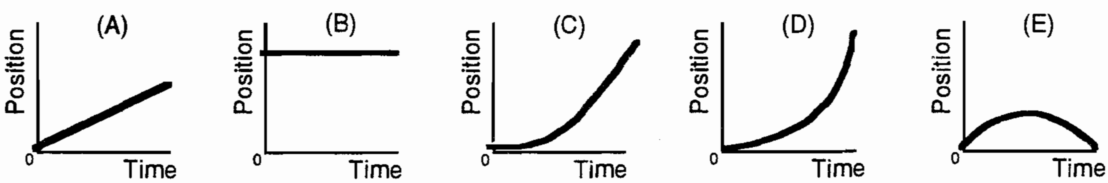


2. An elevator moves from the basement to the tenth floor of a building. The mass of the elevator is 1000 kg and it moves as shown in the velocity-time graph below. How far does it move during the first three seconds of motion?

(A) 0.75 m
(B) 1.33 m
(C) 4.0 m
(D) 6.0 m
(E) 12.0 m

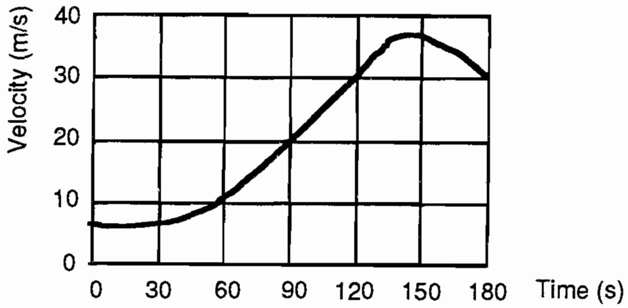


3. Position versus time graphs for five objects are shown below. All axes have the same scale. Which object had the highest instantaneous velocity during the interval?



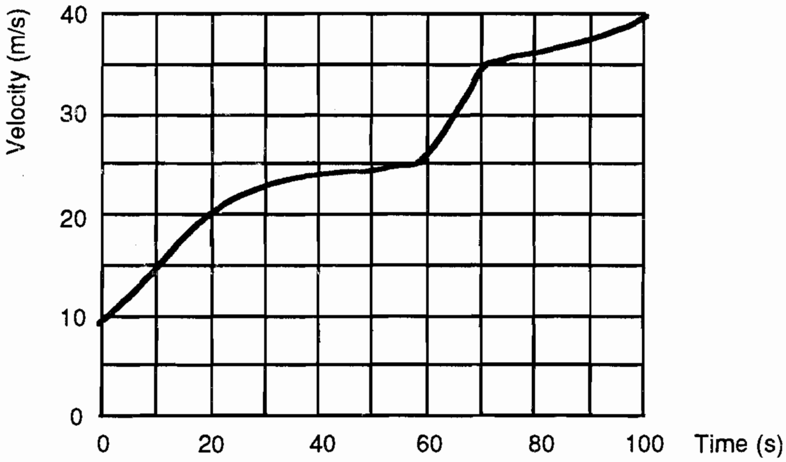
4. This graph shows velocity as a function of time for a car of mass 1.5 x 10^3 kg. What was the acceleration at the 90 s mark?

(A) 0.22 m/s^2
(B) 0.33 m/s^2
(C) 1.0 m/s^2
(D) 9.8 m/s^2
(E) 20 m/s^2



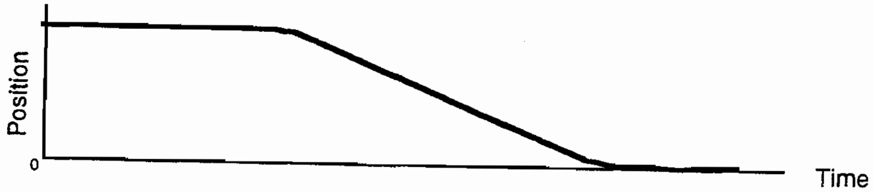
5. The motion of an object traveling in a straight line is represented by the following graph. At time t = 65 s, the magnitude of the instantaneous acceleration of the object was most nearly:

(A) 1 m/s^2
(B) 2 m/s^2
(C) +9.8 m/s^2
(D) +30 m/s^2
(E) +34 m/s^2

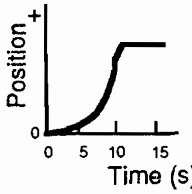


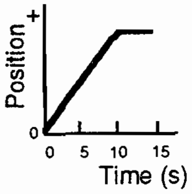
6. Here is a graph of an object's motion. Which sentence is a correct interpretation?

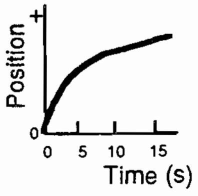
(A) The object rolls along a flat surface. Then it rolls forward down a hill, and then finally stops.
(B) The object doesn't move at first. Then it rolls forward down a hill and finally stops.
(C) The object is moving at a constant velocity. Then it slows down and stops.
(D) The object doesn't move at first. Then it moves backwards and then finally stops.
(E) The object moves along a flat area, moves backwards down a hill, and then it keeps moving.

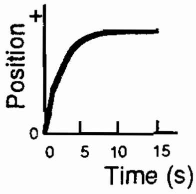


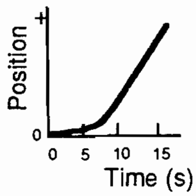
7. An object starts from rest and undergoes a positive, constant acceleration for ten seconds. It then continues on with constant velocity. Which of the following graphs correctly describes this situation?

(A)


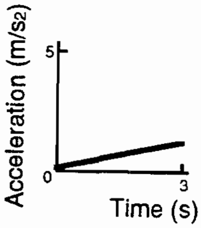
(B)


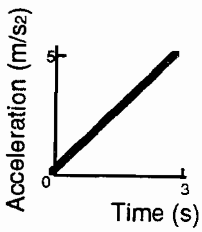
(C)


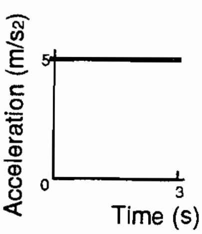
(D)


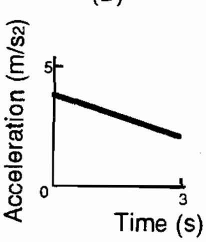
(E)


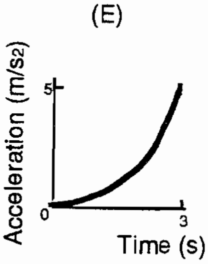
8. Five objects move according to the following acceleration versus time graphs. Which has the smallest change in velocity during the three second interval?

(A)


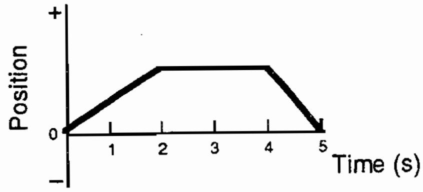
(B)


(C)


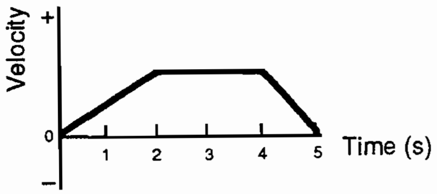
(D)


(E)


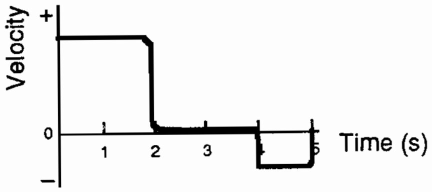
9. The following is a position-time graph for an object during a 5 s time interval. Which one of the following graphs of velocity versus time would best represent the object's motion during the same time interval?



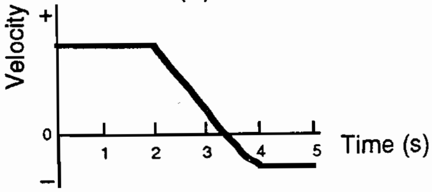
(A)



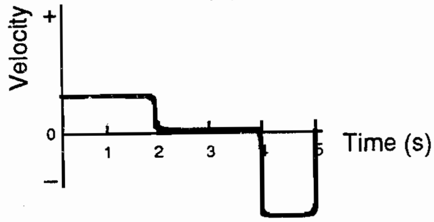
(B)



(C)

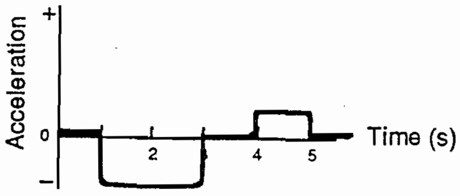


(D)

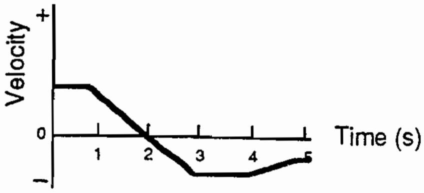


(E)

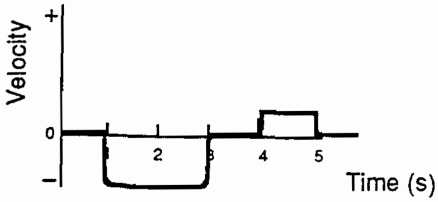
10. The following represents an acceleration graph for an object during a 5 s time interval. Which one of the following graphs of velocity versus time would best represent the object's motion during the same time interval?



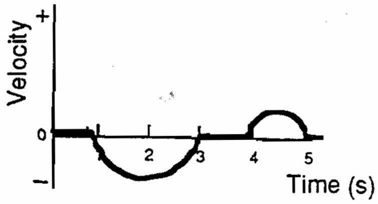
(A)



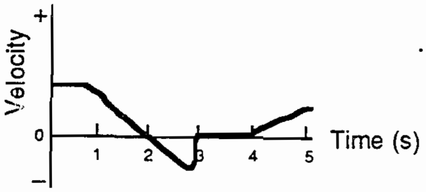
(B)



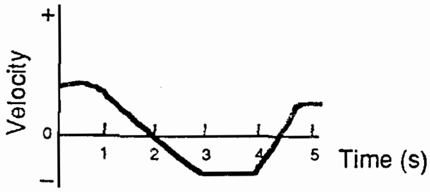
(C)



(D)

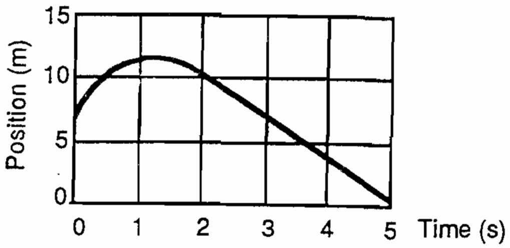


(E)



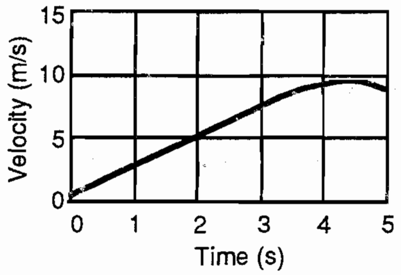
11. The velocity at the 3 second point is about:

(A) –3.3 m/s
(B) –2.0 m/s
(C) –.67 m/s
(D) 5.0 m/s
(E) 7.0 m/s

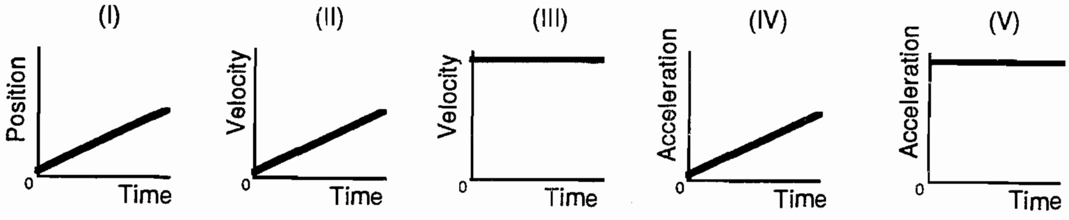


12. If you wanted to know the distance covered during the interval from t = 0 s to t = 2 s, from the graph below you would:

(A) read 5 directly off the vertical axis.
(B) find the area between that line segment and the time axis by calculating (5x2)/2.
(C) find the slope of the line segment by dividing 5 by 2.
(D) find the slope of that line segment by dividing 15 by 5.
(E) Not enough information to answer.

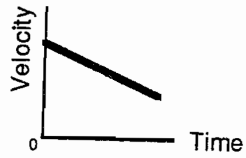


13. Consider the following graphs, noting the different axes: Which of these represent(s) motion at constant, non-zero acceleration?



(A) I, II, and IV (B) I and III (C) II and V (D) IV only (E) V only

14. To the right is a graph of an object's motion. Which sentence is the best interpretation?



(A) The object is moving with a constant acceleration.
(B) The object is moving with a uniformly decreasing acceleration.
(C) The object is moving with a uniformly increasing velocity.
(D) The object is moving at a constant velocity.
(E) The object does not move.

Diagnosing Student Kinematic Graph Conceptions

To investigate the question of what specific difficulties with interpreting graphs of kinematic variables students bring to the integral calculus classroom, the pre-test was analyzed. To compare the effectiveness of the various teaching strategies, the post-test was analyzed in the same manner.

The analysis of the pre- and post-tests consisted of examining each student's incorrect responses. If a student responded incorrectly to a question it was examined to determine if the response given was possibly due to a particular misconception, or lack of conception. For example, question (11) asks the student to find the velocity for an object at the 3 second point. Response (B) could be found by dividing the height of the graph by 3 which could indicate that although the student had difficulty with this problem, he may have had a conception for velocity as the slope of a position graph. Response (E) gives the height of the graph. This response, in conjunction with other similar incorrect responses, may indicate that the student has a lack of conception for velocity as the slope of a position graph.

An instrument was developed that identified incorrect responses (distracters) that may indicate a particular misconception or lack of conception if the response is part of a pattern of responses given by the student. This instrument is given in Table 3. A 'C' in the table represents a correct response, a blank represents that the response is incorrect but is not associated with a particular misconception or lack of conception. A description of the particular misconception or lack of conception indicated by a given type of distracter follows:

Distracters:

Velocity: The student choice reflects that they were unaware that the solution was based on velocity is the slope of the position graph. A student giving two of these responses is diagnosed with a lack of conception for velocity as the slope of a position graph.

Accel.: The student choice reflects that they were unaware that the solution was based on acceleration is the slope of the velocity graph. Instead of responding with the slope, the student may have responded with the height of the graph or a constant for acceleration due to gravity. A student giving three of these responses is diagnosed with a lack of conception for acceleration as the slope of a velocity graph.

Area: The student choice reflects that they were unaware that the solution was based on the area under the curve. Instead of giving the area under the graph, the student may have responded with the height of the graph at the point, the slope of the graph at the point, the time divided by the height of the graph at the point, or the time multiplied by height of the graph at the point ($D=V \times T$). A student giving three of these responses is diagnosed with a lack of conception for interpreting the area under a graph.

Syntactic: The student choice is based on using syntactic cue. The response is either an exact duplicate of the given graph, or has the same sign/shape as the given graph. A student giving two of these responses is diagnosed with a misconception based on use of syntactic cues.

Linguistic: The student responds to a linguistic cue such as "greatest", "highest", "smallest" or "constant" to respond with a graph that looks like it is constant, has the greatest or smallest change or has the highest values. A student giving three of these responses is diagnosed with a misconception based on use of linguistic cues.

Iconic: The student choice is based on iconic translation. A student giving two of these responses is diagnosed with a misconception based on use of iconic translation.

Table 3. Instrument For Diagnosing Student Conceptions

Response:	(a)	(b)	(c)	(d)	(e)
Question Number					
1	Area	C	Area	Area	Area
	Linguistic			Linguistic	
2	Area	Area	Area	C	Area
3	Velocity	Velocity		C	Velocity
		Linguistic			
4		C		Accel.	Accel.
5	C		Accel.	Accel.	Accel.
6	Iconic		Linguistic	C	Iconic
7	Linguistic	Linguistic		Linguistic	C
8	C	Area	Area	Area	Area
			Linguistic		
9	Syntactic			C	Syntactic
10	C	Syntactic	Syntactic		
11	C			Velocity	Velocity
12	Area	C	Area	Area	Area
13	Syntactic	Accel.	C	Iconic	Accel.
	Iconic				
14	C	Iconic			

Table 4 summarizes the questions on the pre- and post-tests, the corresponding objectives, and possible sources of difficulty.

Table 4. Pre- and Post-Test Objectives and Difficulties

Question #	Objective	Misconception Lack of Conception
1	Given an acceleration-time graph determine velocity	Area under the graph Linquistic cue
2	Given a velocity-time graph determine dispacement	Area under the graph
3	Given a position-time graph determine velocity	Velocity as slope of position
4	Given velocity-time graph determine acceleration	Acceleration as slope of velocity
5	Given velocity-time graph determine acceleration	Acceleration as slope of velocity
6	Given a kinematics graph select textual description	Linquistic cue Iconic translation
7	Given textual motion description select corresponding graph	Linquistic cue Iconic translation
8	Given an acceleration-time graph determine velocity	Area under the graph Linquistic cue
9	Given a kinematics graph select another corresponding graph	Syntactic cue
10	Given a kinematics graph select another corresponding graph	Syntactic cue
11	Given a position-time graph determine velocity	Velocity as slope of position
12	Given a velocity-time graph determine dispacement	Area under the graph
13	Given textual motion description select corresponding graph	Acceleration as slope of velocity Iconic translation
14	Given a kinematics graph select textual description	Iconic translation

Lack of conception for graphs of kinematic variables have two categories for slope and one category for area. In the pilot study it was noted that more students had difficulty computing acceleration given a graph of velocity than had difficulty computing velocity given a graph of distance. However, students had similar difficulty computing change in velocity given acceleration and displacement given velocity. Students had much less difficulty with area problems where the given graph was constant than when the graph had non-zero slope. Thus, problems that involved computing area under a constant graph were not included on the instrument for this study.

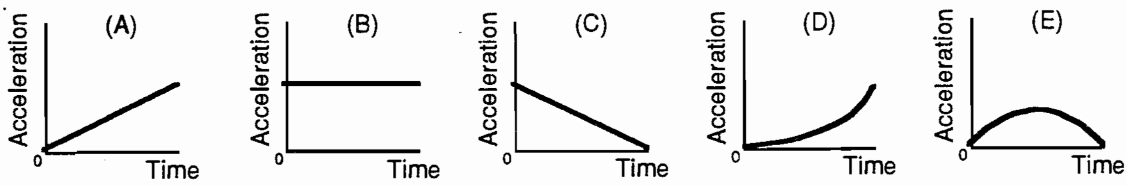
A lack of conception was suggested by a student's consistent failure to use a particular mathematical concept to interpret the graphs of kinematic variables. For example, a student who responded with the height of the position graph instead of the slope to answer a question about velocity may or may not have a lack of conception of velocity as the slope of the position graph. If the student displays this type of error consistently, then the student is said to have this lack of conception. A student who gives a response where local properties instead of global properties are used to find the slope of a distance graph in order to compute velocity would not be said to have a lack of conception of velocity as the slope of a position graph.

Misconception assumes that the student has a conception of the mathematical notion. The student's conception may be correct in some ways , but incorrect in others, making it difficult to recognize the misconception. For the purpose of this study, misconceptions were associated with students' use of linguistic cues, syntactic cues, and iconic translations based on the research indicating students use these tools more frequently when a misconception exists. As noted by Monk (1990), "...those who appeared to have clearer and firmer concepts involving the variables in the functional situation were able to more quickly override their tendency to iconic translation by being able to correctly interpret the graph they had constructed" (p. 18). Nemirovsky, et al. (1992) stated when discussing students' use of syntactic, linguistic, and semantic cues, "Students often cling to this approach in the face of contradictory evidence because they lack a coherent alternative approach to figuring out a function from its derivative or vice versa" (p. 13).

If a response was not indicated for the last two or more problems it was assumed the student was not able to complete the test in the given amount of time. Such tests were considered incomplete and were treated as if the student had not taken the pre-test.

The following is a discussion of the distracter analysis of the 14 questions on the pre- and post-test. Following each test item is the objective and misconception/lack of conception associated with that question. The number preceded by a B in parenthesis indicates what number the question is on Beichner's TUG-K.

1. Acceleration versus time graphs for five objects are shown below. All axes have the same scale. Which object had the greatest change in velocity during the interval?

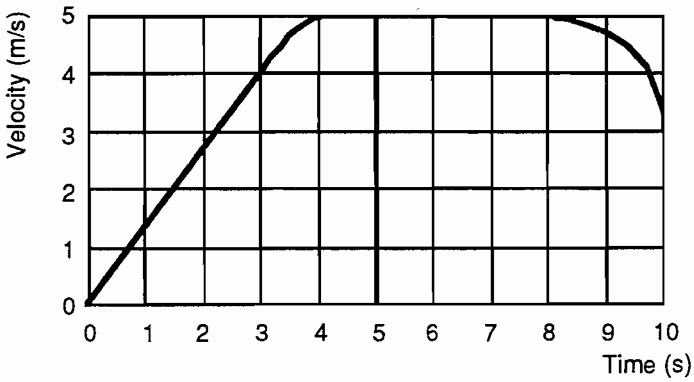


(B1) Objective #4, given an acceleration-time graph, determine change in velocity.

Students fail to recognize that *velocity is the area under the acceleration graph.* All incorrect responses could indicate such a problem. Responding (A) or (D) may be due to the student responding to the *linguistic cue* of "greatest change", to choose a graph that is perceived to have the greatest change in height.

2. An elevator moves from he basement tot he tenth floor of a building. The mass of the elevator is 1000 kg and it moves as shown in the velocity-time graph below. How far does it move during the first three seconds of motion?

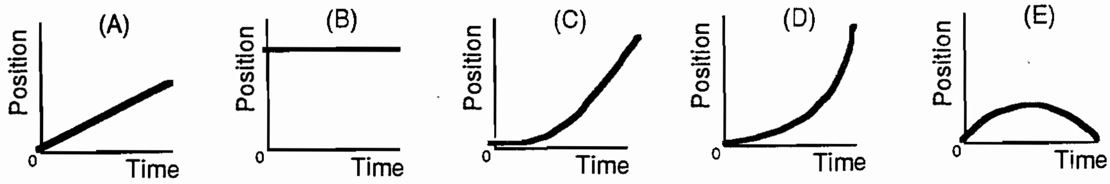
(A) 0.75 m
(B) 1.33 m
(C) 4.0 m
(D) 6.0 m
(E) 12.0 m



(B4) Objective #3, given a velocity-time graph, determine displacement.

Students fail to recognize that the answer is the *area under the graph.* Any incorrect response could indicate such a problem.

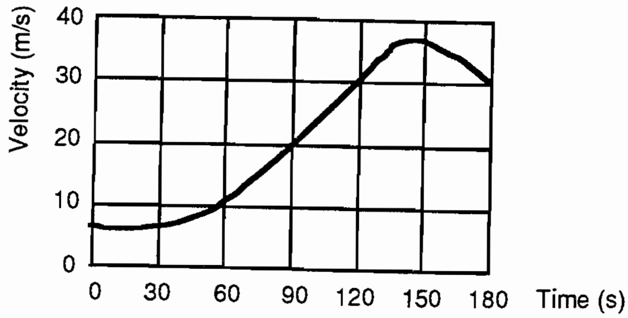
3. Position versus time graphs for five objects are shown below. All axes have the same scale. Which object had the highest instantaneous velocity during the interval?



(B13) Objective #1, given a position-time graph, determine velocity. Students may fail to recognize that the correct answer is the graph with the steepest slope. Responses (A), (B) or (E) could indicate a failure to recognize that *velocity is the slope of the position graph.* Response (C) may not indicate such a failure since it could be misinterpreted to have the steepest slope. Response (B) may be due to the student responding to the *linguistic cue* of "highest ...velocity", to choose a graph that is perceived to have the greatest height.

4. This graph shows velocity as a function of time for a car of mass 1.5 x 10^3 kg. What was the acceleration at the 90 s mark?

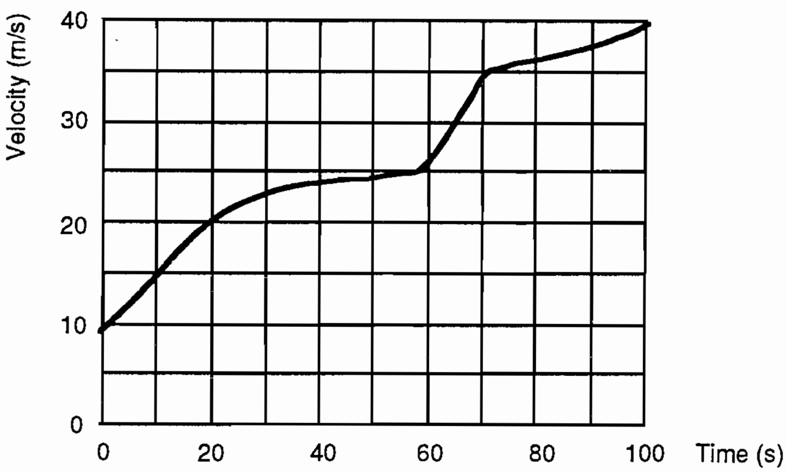
(A) 0.22 m/s^2
(B) 0.33 m/s^2
(C) 1.0 m/s^2
(D) 9.8 m/s^2
(E) 20 m/s^2



(B6) Objective #2, given a velocity-time graph, determine acceleration. Students may fail to recognize that the correct answer is due to *acceleration being the slope of the velocity graph.* Responses (D) and (E) could indicate such a failure. Response (C) may not indicate such a failure since it could be due to the graph looking like it has a slope of one due to scaling. Response (A) may not indicate such a failure since the student may have computed the slope based on local instead of global properties.

5. The motion of an object traveling in a straight line is represented by the following graph. At time $t = 65$, the magnitude of the instantaneous acceleration of the object was most nearly:

(A) 1 m/s^2
(B) 2 m/s^2
(C) +9.8 m/s^2
(D) +30 m/s^2
(E) +34 m/s^2

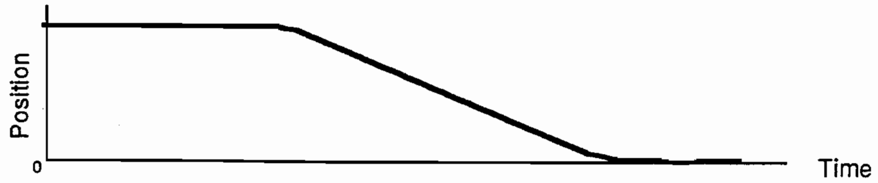


(B7) Objective #2, given a velocity-time graph, determine acceleration.

Students may fail to recognize that the correct answer is due to *acceleration being the slope of the velocity graph.* Responses (C), (D) and (E) could indicate such a failure. Response (B) may not indicate such a failure since it could be due to the graph looking like it has a slope of two due to scaling.

6. Here is a graph of an object's motion. Which sentence is a correct interpretation?

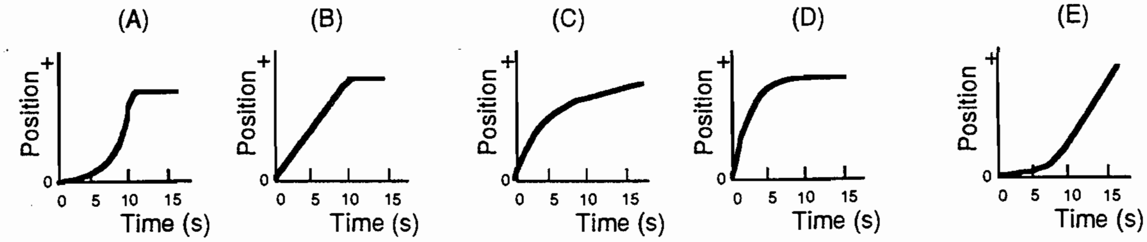
   (A) The object rolls along a flat surface. Then it rolls forward down a hill, and then finally stops.
   (B) The object doesn't move at first. Then it rolls forward down a hill and finally stops.
   (C) The object is moving at a constant velocity. Then it slows down and stops.
   (D) The object doesn't move at first. Then it moves backwards and then finally stops.
   (E) The object moves along a flat area, moves backwards down a hill, and then it keeps moving.



(B8) Objective #6, given a kinematics graph, select textual description

Students responding (A) or (E) may be using *iconic translation*. Students responding (C) may be responding to the *linguistic cues* of "constant velocity" and "slows down", to correspond to a graph which is initially constant and then "drops down". Response (B) could be correct if the student assumed a coordinate system measuring vertical distance of the object.

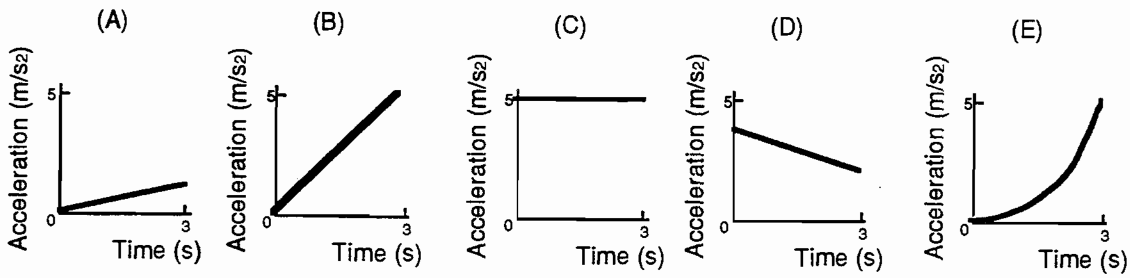
7. An object starts from rest and undergoes a positive, constant acceleration for ten seconds. It then continues on with constant velocity. Which of the following graphs correctly describes this situation?



(B9) Objective #7, given textural motion description, select corresponding graph.

Responses (A), (B), or (D) may indicate use of *linguistic cue* from the words "positive" and "constant" to give a graph that is increasing and then constant. Responses (C) may indicate the student was unaware the graph should be concave up.

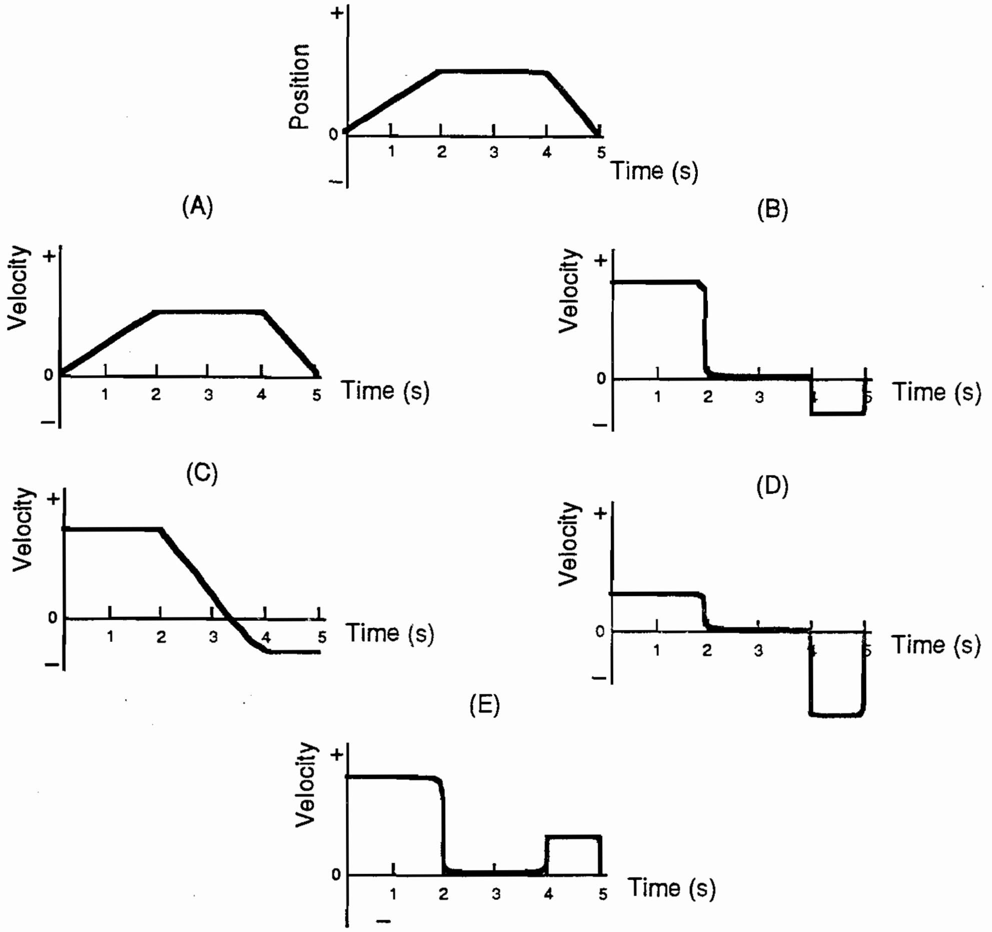
8. Five objects move according to the following acceleration versus time graphs. Which has the smallest change in velocity during the three second interval?



(B10) Objective #4, given a kinematics graph, select textual description.

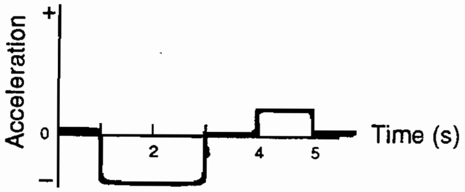
Students fail to recognize that *velocity is the area under the acceleration graph.* All incorrect responses could indicate such a problem. Responding (C) may be due to the student using the *linguistic cue* of "smallest change", to choose a constant graph.

9. The following is a position-time graph for an object during a 5 s time interval. Which one of the following graphs of velocity versus time would best represent the object's motion during the same time interval?

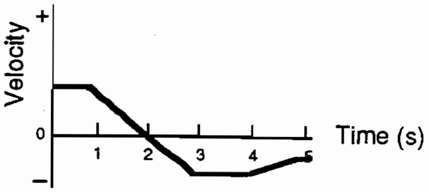


(B11) Objective #5, given a kinematics graph, select another corresponding graph. Response (A) indicates use of *syntactic cue*. Response (E) may indicate use of *syntactic cue* in that the graph (E) is always positive as is the given graph. Responses (B) and (C) may indicate the student has scaling difficulties, but not necessarily any misconceptions.

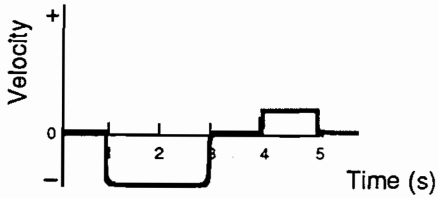
10. The following represents an acceleration graph for an object during a 5 s time interval. Which one of the following graphs of velocity versus time would best represent the object's motion during the same time interval?



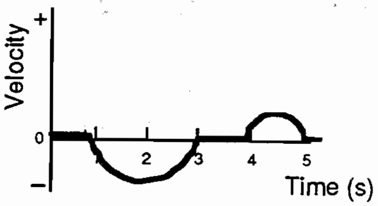
(A)



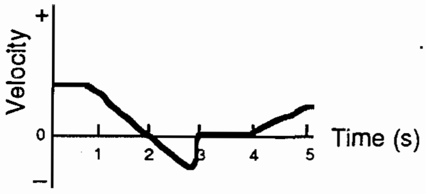
(B)



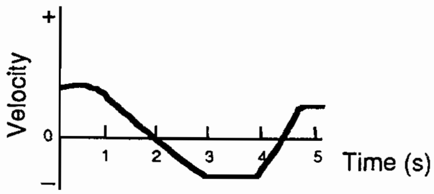
(C)



(D)



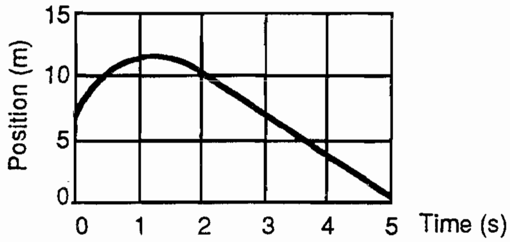
(E)



(B15) Objective #5, given a kinematics graph, select another corresponding graph. Response (B) indicates use of *syntactic cue*. Response (C) may indicate use of *syntactic cue* in that the graph (C) has a shape similar to the given graph. Responses (D) and (E) may indicate the student has scaling difficulties, but not necessarily any misconceptions.

11. The velocity at the 3 second point is about:

(A) –3.3 m/s
(B) –2.0 m/s
(C) –.67 m/s
(D) 5.0 m/s
(E) 7.0 m/s

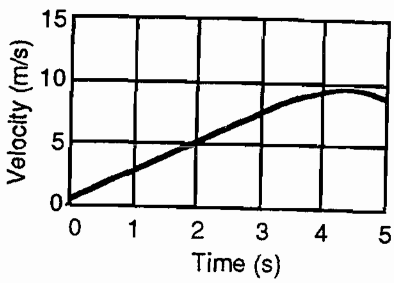


(B17) Objective #1, given a position-time graph, determine velocity.

Responses (D) or (E) could indicate a failure to recognize that *velocity is the slope of the position graph*. Response (C) may not indicate such a failure since it could be misinterpreted to have the steepest slope. Responses (B) and (C) may indicate the student has computational difficulties, but not necessarily a lack of conception.

12. If you wanted to know the distance covered during the interval from t = 0 s to t = 2 s, from the graph below you would:

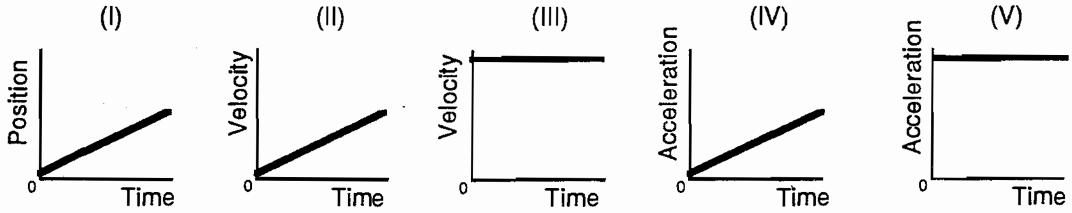
(A) read 5 directly off the vertical axis.
(B) find the area between that line segment and the time axis by calculating (5x2)/2.
(C) find the slope of the line segment by dividing 5 by 2.
(D) find the slope of that line segment by dividing 15 by 5.
(E) Not enough information to answer.



(B18) Objective #3, given a velocity-time graph, determine displacement.

Students fail to recognize that the answer is the *area under the graph*. Any incorrect response could indicate such a problem.

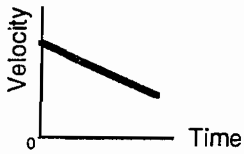
13. Consider the following graphs, noting the different axes: Which of these represent(s) motion at constant, non-zero acceleration?



(A) I, II, and IV (B) I and III (C) II and V (D) IV only (E) V only

(B19) Objective #7, given textual motion description, select corresponding graph. Students responding (A) or (D) may be using *iconic translation* and mentally picturing constant acceleration as a line with positive slope. Students responding (A) may also be using syntactic cue in that the graphs of distance, velocity and acceleration are all the same. Responses (B) and (E) may indicate a failure to recognize *acceleration as the slope of the velocity graph.*

14. To the right is a graph of an object's motion. Which sentence is the best interpretation?



(A) The object is moving with a constant acceleration.
(B) The object is moving with a uniformly decreasing acceleration.
(C) The object is moving with a uniformly increasing velocity.
(D) The object is moving at a constant velocity.
(E) The object does not move.

(B21) Objective #6, given a kinematic graph, select textual description. Students responding (B) may be using *iconic translation.*

Development of Laboratory Activities

Laboratory activities on kinematics were developed to give students the opportunity to repair and refine their conceptions concerning graphs of these variables in four different instructional environments.

To promote conceptual change, Dykstra's strategies (Dykstra et al., 1992) were implemented by using labs that have the following elements:

1) A phenomenon that either is an actual physical situation arising in the classroom, or is an occurrence known to the students. This phenomenon must also have an abstract representation. The focus of the lab should be on the object and the two representations.

2) Student prediction and/or explanation concerning the physical phenomenon, and its abstract representation, which will bring forth the student's misconceptions.

3) Observation of the actual explanation and/or physical situation which may cause disequilibrium for the student.

4) Opportunity for the student to be confronted by, and resolve, any discrepancies, or disequilibrations.

Three labs were designed for this study (see appendix A for the actual handouts used in each lab). Two labs incorporated use of Calculator Based Laboratory (CBL) instruments. One CBL-lab was designed for students who worked in groups, and the other CBL-lab was designed for students who worked individually.

In a Calculator Based Laboratory (CBL) students are working on activities that require the use of a graphing calculator to graph physical phenomena that they are observing in class. The purpose of such a laboratory is to engage students actively in the learning process, to promote understanding of the relationships between the physical variables they are graphing. A CBL system consists of a graphing calculator, a probe or detector, and a CBL device. The CBL device transforms data from the detector (or probe) to the calculator. The detector may detect heat, level of pH, motion or other variables.

This study is concerned with kinematics, and thus, the CBL system was used with a motion detector (see figure 6).

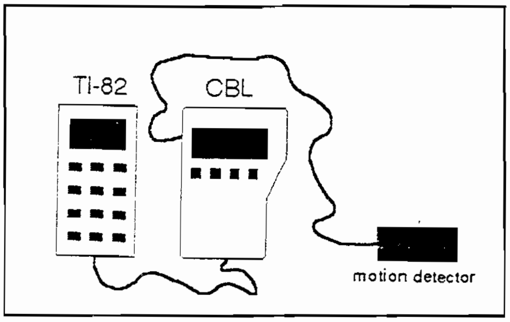


Figure 6. Illustration of the CBL System

The motion detector uses reflection of ultrasonic sound waves to detect motion of objects in a cone-shaped region in front of it. It detects the object closest to it in a 15° to 20° cone, and has a range of about 0.5 to 6.0 meters. The motion detector sends out pulses over a specified period of time, and at a specified rate. The time period and rate are determined by the program on the calculator. Each time the program is called up, a time period is requested and the rate is determined by that time period divided by 99 (the maximum number of data points this system can manage is 99). The program calculates the position, velocity, and acceleration of the object using the data from the motion detector and CBL device. The program plots data points collected by the motion detector and CBL device to graph distance. To graph velocity the program compiles the difference between distance data points, then utilizes a 29-point Savitzky-Golay smoothing function before plotting the velocity. The filtering process will have an "end effect," that is the first and last 14 data points for the velocity graph will not be accurate. The program graphs acceleration by compiling the difference between the velocity data points and then uses a 29-point Savitzky-Golay smoothing function on this data.

The graph of any one of these variables is available after the measurements are completed. The data are stored in the calculator until measurements on a different object are taken, or the data is cleared through some other means. Thus, students can view the graph of velocity for an object and subsequently view the graph for distance or acceleration. The use of the CBL system is menu driven and does not require any programming abilities by the students. See Appendix A for a detailed description of the software interface of the CBL program on the calculator.

The CBL system is very similar to the microcomputer-based laboratory (MBL) instruments. MBL instruments use the same motion detector, a computer, and computer software. One difference is that the MBL system will display the data and graph for one of the kinematic variables, as the data is being collected. In other respects, the systems are the same.

CBL-Laboratory Activity For Cooperative Groups

The CBL-activity for students in groups incorporates the four elements for a lab to promote conceptual change as follows:

1) The physical phenomenon is the motion of several objects: a person walking; a toy car moving with constant deceleration; a toy car moving on a flat surface, then up a ramp until its motion reverses and it moves back down the ramp toward its original position. The abstraction of these physical situations are their respective graphs of acceleration, velocity and distance.
2) Given the graphs of acceleration, students were to predict which physical phenomenon would produce such an acceleration graph. They were to explain, in detail, the physical motion that would

produce such a graph and predict the velocity and distance graphs for this motion.

3) Students were to then create the physical motion which they described and monitor that motion with the CBL system. They were then to produce the corresponding graphs for distance, velocity and acceleration.

4) Students were asked to explain any discrepancies between their prediction in (2) and the actual graph in (3). Students were asked to give a velocity graph based on velocity as the area under the acceleration graph. They were asked to describe how this differed (if at all) from the velocity graph of the physical situation.

CBL-Laboratory Activity For Students Working Individually

The lab for the students working individually was different only in item (3) described as follows:

3) The instructor asked students for their predicted graphs and the physical situation they describe. The instructor then created that situation and monitored it with the CBL-system for the class to observe. The instructor then used the calculator to show the class the velocity, distance, and acceleration graphs of the physical situation. A class discussion followed the demonstration.

Laboratory Activity Not Using the CBL-Tools

The third lab did not use CBL-tools. This non-CBL lab was used both in a recitation where students were assigned to work in groups and in a recitation in which students worked individually. This lab incorporated the four elements for conceptual change as follows:

1) Physical motion is an occurrence the students are familiar with. The students were given the abstraction of motion in the form of graphs of acceleration.

2) Given the graphs of acceleration, students were to sketch graphs of velocity and distance. Further, they were to describe the physical situations these might describe.

3) Students were then given the algebraic formula for the acceleration graphs, and were asked to compute the formulas and sketch graphs for velocity and acceleration, based on given initial conditions.

4) Students were asked to explain any discrepancies between their graphs in (2) and (3). They were asked how the velocity graphs differ from, or are the same as, the graph for the area under the acceleration curve.

The lab addressed the four student difficulties in the same manner as the CBL-system based labs. The principal difference between the CBL and non-CBL lab was that students observed the actual physical situations creating the graph in the CBL-lab, while in the non-CBL lab they were given the algebraic formulas generating the graphs. Whether algebraic formulas create the same opportunity for student discourse, and disequilibrium, as a CBL-based activity was a question investigated in this study.

Summary

These labs address student misconceptions and lack of conceptions in the following ways:

1) Lack of conception of velocity as slope of position. Students were given a graph of constant, negative acceleration and were asked to produce two situations for this graph, with positive velocity (the object is moving away from the motion detector), and with negative velocity (the object is moving

towards the motion detector). For each of these situations they were asked to produce a position graph. Also, students were given an acceleration graph that was not constant (problem #3). From this graph students were to produce a velocity graph and then a position graph. To correctly find the position graph, students had to recognize that velocity was the slope of position or that position was the area under the velocity graph. For all three problems students had the opportunity to confront a lack of conception for slope of the distance graph corresponding to the velocity.

2) Lack of conception of acceleration as slope of velocity. Students were given a graph of constant, negative acceleration and were asked to produce two situations for this graph, one with positive velocity (the object is moving away from the motion detector), and one with negative velocity (the object is moving towards the motion detector). In the pilot study, it was observed that students would often assume a resemblance of same sign for a function and its antiderivative. Thus, if students were not specifically requested to create a graph with positive velocity, they often assumed that the solution must have negative velocity, since acceleration was negative. In being confronted with the fact that the velocity could be either positive or negative, students may realize that height and slope are completely unrelated. Students were also given the opportunity to be confronted by the general slope of the velocity graph corresponding to the sign of the acceleration graph.

3) Lack of conception concerning area under the graph. The opportunity to confront this lack of conception is provided in question (5) of , the lab. Students were to produce the graph of the area under the acceleration

curve and account for any differences between this graph for velocity and their other graphs for velocity.

4) Misconception based on use of syntactic cues. Students were given the opportunity to see that graphs of a function and its derivative (antiderivative) do not necessarily have the same sign, shape, or direction of change. In the pilot study, students would make incorrect predictions based on *syntactic cues*. This was expressed predominantly in students assuming the graph of velocity, for acceleration graph (3) would be a simple replication, and then creating a physical situation based on that velocity. Students were then confronted with graphs for the physical situation they created that were not consistent with their predictions. Students also had difficulty producing a graph with positive velocity from a graph of negative acceleration, believing that the graphs should have the same sign (problem (2)).

5.) Misconception based on use of linguistic cues. Students were given the opportunity to confront misconceptions based on linguistic cues in problems (2) and (3). In problem (2) they could see that a decreasing graph for velocity could represent an object with increasing speed in the negative direction. In problem (3) they again had to work with the ideas of velocity in the negative direction and whether the objects speed was increasing and decreasing. Further, in graphs (1) and (2) they were able to see that constant acceleration was related to decreasing graphs for velocity, and both increasing and decreasing curves for distance.

6.) Misconception based on use of iconic translation. The opportunity to confront this misconception is provided in part (c) of problems (1), (2) and (3). In the pilot study, students assumed the graphs gave a literal picture of the motion described. This was evidenced by some responses where the graphs were a picture of the description in part (c) but were not related to the graphs in (a) and (b). In this case, it was conjectured that students described the physical situation and made the graph a picture of that situation. Conversely, students would derive the graphs appropriately and then describe the physical situation as if the graph were a picture of it. In either event, creating the physical situation and seeing the actual graphs confronted this problem.

Procedures

Assignment of Recitations to Treatments

One of the two lecture sections was randomly selected to assign students in that section to cooperative groups for lab activities in the two recitations corresponding to that lecture section. The other two recitations (corresponding to the other lecture section) had students work individually on lab activities. One cooperative group recitation and one individual activity recitation were randomly selected to receive the CBL-system, kinematics lab assignment. (See Table 5 for a summary of the assignment of students to recitation classes).

There were 121 students enrolled in the two lecture sections 114 of whom agreed to have data collected about them recorded for purposes of this study. Of those 114 students, 98 took the pre- and post-tests which are a part of this study. Of those 98 students, 86 were in attendance on the day of treatment. These 86 students were the sample for the quantitative data analysis. Qualitative data included some of the 114 students who had either not taken the pre-test, the post-test, or both, or who had not attended the recitation in which the kinematics laboratory assignment was given. The reason some students were included in the qualitative analysis, that were not included in the quantitative analysis is as follows:

1. In several of the student groups that were videotaped, one of the students was absent on the day of the pre-test or the post-test or both. Quantitative data is complete for other group members. Thus, the student for whom the quantitative data is incomplete is included in the qualitative analysis of the videotaped observation of that group.
2. Two students were interviewed that were absent on the day of the kinematics lab. These students' interviews were analyzed to investigate their perception of what had helped them improve their understanding of kinematic graphs.

Table 5. Assignment of Students to Recitations

	Recitation #1	Recitation #2
Morning	Cooperative Group	Cooperative Group
	CBL	Non-CBL
Enrolled	n = 26	n = 22
Included in Study	n = 15	n = 16
Afternoon	Individual	Individual
	CBL	Non-CBL
Enrolled	n = 39	n = 34
Included in Study	n = 27	n = 28

<u>Formation of Cooperative Groups</u>

Prior to the first recitation classes, a list of the students registered for the cooperative group recitation sections was compiled which indicated the student's grade in differential calculus. Students were classified as "high-ability" if they received a B+ to an A, "medium-ability" if they received a C to a B, and "low-ability" if they received a C- or less in the previous differential calculus course.

From this list students were assigned to a group of three during the first recitation class. Each group was *mixed-ability with a narrow-range*, i.e., all groups had students that were a mix of high-ability and medium-ability only, or medium-ability and low-ability only. Webb (1991) found, "All students in these groups (mixed-ability, narrow-range) tended to be active participants, with questions eliciting help more frequently than in mixed-ability groups with a wider range of ability" (p. 379). Webb also found that high-ability and low-ability students performed better in these types of groups than in homogeneous groups (all the same ability). Medium-ability students performed better in this type of group than in wide-range, mixed-ability groups.

Assigning students to a group of three was based on the advice of a consultant experienced in college math instruction using cooperative groups, (Elizabeth Lundy, personal communication, September 18, 1995) whose observations of student groups had indicated that groups of size three were optimal for promoting student interaction, with groups of size two or four considered acceptable. Since the integral calculus course has a large turnover of students in the first weeks of the course, it was expected that there would be early fluctuation in group membership. Assigning students to groups of three allowed for groups of two if one student dropped out or the remaining students could join with another group of two to form a group of four if they wished. A new student to the class was added to a group of two or three based on the new student's ability matching the range of an existing group's abilities. The configuration of the cooperative groups by the fifth week of class was nine three-member groups and five four-member groups.

Laboratory Assignment Procedures

In the weekly recitations, students were usually given an assignment to complete before the end of class. In the cooperative group sections, each group turned in only one completed assignment. This allowed the students to establish a "group rapport" prior to their group assignment on graphs of kinematic variables. The in-class, group lab on kinematics was given in the fifth recitation class.

The students in the individual recitation sections were not assigned to groups, but were given the same homework assignments and labs to complete in-class as the cooperative group recitation sections. They were allowed to talk to other students while working on assignments and labs, but each student had to turn in their own individual in-class work.

When given the in-class assignment on graphs of kinematic variables, the cooperative group sections turned in one assignment for each group and in the individual sections each student turned in an assignment.

Pre-test Administration

In the first recitation class for both the cooperative group sections and the individual sections the students were asked to complete the pre-test. They were told that it would not affect their grade, but that they would be given a similar test later in the term that would count towards their course grade. Table 5 shows a breakdown of the number of students taking the pre-test.

Table 6. Students Taking Pre-Test

	Recitation #1	Recitation #2
Morning	Cooperative Group	Cooperative Group
	CBL	Non-CBL
	n = 19	n = 17
Afternoon	Individual	Individual
	CBL	Non-CBL
	n = 38	n = 32

The pre-test was analyzed for student difficulties, misconceptions and lack of conception. To investigate what difficulties students have, percentages of students missing both test items related to a particular Beichner objective were determined. To investigate what misconceptions or lack of conception may be related to the students' difficulties, percentages of students with a particular misconception/lack of conception were determined.

Videotaping

During the second, third and fourth weeks student groups in the cooperative group recitation classes were videotaped during the time they spent working on in-class assignments. The purpose of these tapes was two-fold. The first purpose was to acclimate the students to having video equipment recording their conversations in the classroom. The second purpose was to review these tapes to determine the nature of the student discourse occurring in each of the groups. All student groups were videotaped at least once prior to the lab on kinematics. Notes were made concerning the amount of participation by all group members, and whether the discourse pertained to mathematics or activities outside the classroom. The videotape recordings made during the lab on kinematics in the fifth week of class were used as data in this study.

Instructional Treatments

In the fifth week of class the students were given the in-class laboratory assignment on graphs of kinematic variables developed for their particular recitation.

Post-test Administration and Pre/Post Analysis

During the sixth week of class the students in all recitation sections were given a 14-item post-test identical to the pre-test given the first week of class (items derived from Beichner's TUG-K). Student responses from the pre- and post-test were compiled.

Responses on individual test items were examined for patterns that could indicate a particular lack of conception or misconception for the student (see page 43 of this study for a detailed description of this analysis). For each student, their misconceptions and/or lack of conception at the time of the pre-test were compared to their misconceptions/lack of conception at the time of the post-test. This information was used to determine if the student had possibly repaired a misconception, built a conception, retained a misconception or lack of conception or built a misconception. Percentages of students in each recitation indicating they had a misconception or lack of conception at the time of the pre-test were compared to percentages of students in the same recitation who indicated a particular misconception or lack of conception at the time of post-test. Odds ratios for improvement were computed for each of the six misconceptions/lack of conception and statistically analyzed using logistic regression.

Students' score of 0, 1, or 2 for each objective on the post-test was compared to the corresponding objective score on the pre-test. Odds were computed using the students' difference in score for each objective and was statistically analyzed using logistic regression.

A two by two, factorial design with two fixed factors was used to statistically analyze the effectiveness of the various teaching models. Factor I represents the use of CBL-tools in the classroom during the laboratory assignment on graphs of kinematic variables. Factor II represents the requirement that students work in assigned cooperative groups on the laboratory assignment on graphs of kinematic variables.

The dependent variables were:

1.) The mean difference in total score on the pre- and post-test.

2.) The mean difference in number of student misconceptions/lack of conception on the pre-test, with number of student misconceptions/lack of conceptions on the post-test.

Regression analysis was done to determine the most significant factor in predicting the outcome of the dependent variables. For this analysis, in addition to the factors included in the design, two other independent variables were considered: The student's concurrent enrollment in a physics course and the student's grade in integral calculus. These variables were considered as most likely to affect student outcomes on the dependent variables.

Interviews

Comparisons made on the pre-and post-test on particular items were suggestive that 30 students had repaired misconceptions or built conceptions concerning graphs of kinematic variables significantly. Significant in this case was defined to be a decrease of two or more of the misconceptions and/or lack of conceptions listed on page 44.

Attempts were made to contact these 30 students either in the classroom or by telephone to request an interview. Requests for interviews were made in writing to the 30 students identified in their recitations or lecture section. Students receiving the requests were asked to turn them in to the professor for the lecture or to the teaching assistant. Students could use this form to notify the researcher that they did not want to be interviewed or to give a telephone number where they could be reached to set up an interview time. For any of the 30 students who had not returned a form by the second week following the post-test, the researcher requested a local phone number from the registrar's office. During the next four weeks the researcher attempted to contact these students. Students contacted by telephone were advised that they would be paid $5.00 for the interview and that there would be a raffle amongst all students participating in the interviews (the raffle was for a Hewlett Packard, HP-38 calculator). Students who had been contacted through the recitation or lecture classes were advised about payment and the raffle when the interview was set up or at the time of the interview. Some students declined payment.

Sixteen of the students agreed to be interviewed. Six of the students declined to be interviewed due to time constraints resulting from finals, other responsibilities causing time constraints, and for reasons they did not give. Four students did not list a local telephone number with the registrar and were not in attendance on days that the researcher attempted to contact them in the classroom. Four students did not reply to telephone messages left by the researcher.

During the two months following the post-test, the researcher interviewed the sixteen students to obtain their perspective on how their understanding of graphs of kinematic variables changed. The protocol for these interviews was as follows:

1.) On a blank test, the student's responses on the pre- and post-test were both indicated. There were four possibilities for the responses given: (a) wrong/wrong; (b) wrong/right; (c) right/wrong; (d) right/right.

2.) At the beginning of the interview the student would be advised by the interviewer:

> "I have circled your responses for the first and second tests. On questions which you gave two different responses, I will ask you if you know which response is correct, or if both are wrong, if you can determine which is the correct answer now. This is not meant to be an oral exam. If you do not know which is the correct response, have any questions, or would just prefer that I explain the question and answer to you, please just say so. When looking at these questions, if you can remember when your understanding changed, such as something that you worked on in the homework, something from Logan or Diane's lectures, or from the lab work in recitation classes, I would like to know that. I will ask you about this again when we have gone through the whole test".

3.) Each test item was reviewed by the interviewer and student. The interviewer probed, based on the student response as follows:

> wrong/wrong: "This one you had some trouble with both times - can you see what the correct answer should be"?

> wrong/right or right/wrong: "Your answer changed on this one. Do you know which response is correct? Why"?
>
> right/right: "You answered this one correctly both times. Good job"!

4.) After all 14 items had been gone through, the interviewer probed with questions similar to the following:

> "Do you think your understanding of the graphs changed as a result of this course? If so, what do you think was most helpful, Logan's lectures, Diane's lectures, the homework, or the lab work done in recitation? Did you work with other students? Was this helpful? Were you in class for the lab assignment on kinematics? Was it helpful"?

The interviews were audio taped and transcribed for analysis. The tapes were analyzed for student's beliefs concerning their improvement or decline in understanding. Another purpose of the audio tapes was to look for patterns of responses for students who had something in common, particularly conceptions which the student's pre- and post-test indicated may have been built, and the misconceptions which the student's pre- and post-test indicated may have been repaired.

Observations

Videotapes were made of the students as they worked in the cooperative group recitation classes on the laboratory assignment on graphs of kinematic variables. The videotapes were made in the two recitation classes in which students were assigned to work in groups. Three of the seven student groups in each recitation class were videotaped. The groups that were videotaped on the day of the kinematics lab were selected by the following criterion:

1. Prior observation that most group members usually participated in group activities.
2. Prior observation of an adequate level of discourse between group member that concerned mathematics.

3. Ability to position the video equipment to record the student group. The three cameras were placed so that they would be unobtrusive for the instructor and were dispersed throughout the room. Thus, in each area of the room only one group in that area could be videotaped.
4. Evidence that group members had an opportunity for building conceptions or repairing misconceptions as indicated by their pre-test.

Videotapes were made of the students working in cooperative groups on the laboratory assignment of graphs of kinematic variables. The tapes were analyzed for evidence of how student discourse was an aid in building conceptions or repairing misconceptions and how well this evidence supported Roshcelle's model for convergent conceptual change. That is, evidence was sought for student display of knowledge, student use of metaphor, confirmation or repair of the displayed knowledge, and that this (use of metaphor; display, confirmation, and repair of knowledge) occurred on progressively higher levels of understanding. Absence of all or most of the characterizations of Roshcelle's model (use of metaphor; display, confirmation, and repair of knowledge) were also looked for to see if this corresponded to poor performance by the students or increased misunderstanding.

Evidence of peer-tutoring or other types of student discourse that would improve a student's understanding was also sought as well as possible sources of student confusion. Student confusion could be caused by student discourse, difficulty understanding the assignment as it was written, or difficulty with CBL-equipment.

Analysis of the videotapes consisted of examining and categorizing student discourse following the model of Roschelle:

a) A student display of knowledge, such as a student making the statement, "the graph should be increasing." Evidence that the display of knowledge was confirmed, repaired or ignored was sought.

b) Student use of metaphor. If students used metaphors in their discourse concerning the lab assignment on graphs of kinematic variables, the discourse was examined for other evidence of Roschelle's Theory of Convergent Conceptual Change.

c) A student asked a question. It was noted if the question was answered correctly and by whom. Specifically, it was noted if students were answering each others' questions, or the instructor was usually called upon to answer questions.

CHAPTER IV

RESULTS

Pre-Test

The pre-test was used to investigate difficulties and misconceptions students bring to the integral calculus classroom.

Table 7 gives the percentage of students who missed both test items related to a particular objective.

Table 7. Percentage of Students Having Difficulty With a Specific Objective

Objective	Total	Cooperative Group CBL	Cooperative Group Non-CBL	Individual CBL	Individual Non-CBL
	n = 86	n = 15	n = 16	n = 27	n = 28
1. Given a Position-Time Graph Determine Velcocity	24%	33%	6%	30%	25%
2. Given a Velocity-Time Graph Determine Acceleration	41%	53%	25%	48%	36%
3. Given a Velocity-Time Graph Determine Displacement	48%	47%	44%	56%	43%
4. Given an Acceleration-Time Graph Determine Change In Velocity	55%	80%	31%	44%	64%
5. Given a Kinematics Graph Select Another Corresponding Graph	30%	47%	19%	15%	43%
6. Given a Kinematics Graph Select Textual Description	24%	13%	25%	37%	18%
7. Given textual Motion Description Select Corresponding Graph	52%	60%	50%	52%	50%

Table 8. Percentage of Students Indicating a Specific Misconception/Lack of Conception

Misconception/ Lack of Conception	Total	Cooperative Group CBL	Cooperative Group Non-CBL	Individual CBL	Individual Non-CBL
	n = 86	n = 15	n = 16	n = 27	n = 28
1. Lack of conception Veclocity as slope of a position graph	9%	20%	0%	11%	7%
2. Lack of conception Acceleration as slope of a velocity graph	17%	33%	6%	22%	11%
3. Lack of conception Concerning area under graph	64%	60%	44%	70%	71%
4. Misconception Syntactic cue	6%	7%	0%	11%	4%
5. Misconception Linquistic cue	30%	33%	6%	37%	36%
6. Misconception Iconic translation	19%	13%	13%	30%	11%

Table 8 suggests students bring a lack of conception for interpreting area under the curve to the integral calculus classroom. To a lesser extent misconceptions are indicated by students' misuse of linguistic cues and iconic translations.

Post-Test

The post-test was used for comparison to the pre-test to investigate the relative effectiveness of the four teaching models, Cooperative Group, CBL; Cooperative Group, Non-CBL; Individual, CBL; and Individual, Non-CBL. Results from the post-test will be given and then compared to the pre-test.

Table 9 gives a breakdown by recitation section of the mean score for students on each objective. A student could score 0, 1 or 2 for each objective and the sum of the scores for each objective are the students total score.

Table 9. Breakdown of Mean Score by Objective

Objective		Total	Cooperative Group CBL	Cooperative Group Non-CBL	Individual CBL	Individual Non-CBL
	n =	86	15	16	27	28
1. Given a Position-Time Graph	Pre-Test	1.01	0.73	1.44	1.00	0.93
Determine Velocity	Post-Test	1.23	0.87	1.75	1.26	1.11
	Difference	0.22	0.13	0.31	0.26	0.18
	Std. Dev.	0.73	0.64	0.58	0.81	0.77
2. Given a Velocity-Time Graph	Pre-Test	0.91	0.73	1.19	0.74	1.00
Determine Acceleration	Post-Test	1.15	1.00	1.38	1.04	1.21
	Difference	0.24	0.27	0.19	0.30	0.21
	Std. Dev.	0.96	1.16	0.81	0.99	0.92
3. Given a Velocity-Time Graph	Pre-Test	0.73	0.73	0.94	0.59	0.75
Determine Displacement	Post-Test	1.35	0.73	1.56	1.44	1.46
	Difference	0.62	0.00	0.63	0.85	0.71
	Std. Dev.	0.84	0.76	0.78	0.86	0.76
4. Given an Acceleration-Time Graph	Pre-Test	0.64	0.27	1.00	0.74	0.54
Determine Change In Velocity	Post-Test	1.01	0.73	1.44	0.85	1.07
	Difference	0.37	0.47	0.44	0.11	0.54
	Std. Dev.	0.83	0.92	0.70	0.64	0.96
5. Given a Kinematics Graph	Pre-Test	0.87	0.73	1.06	1.11	0.61
Select Another Corresponding Graph	Post-Test	1.16	0.80	1.44	1.26	1.11
	Difference	0.29	0.07	0.38	0.15	0.50
	Std. Dev.	0.75	0.80	0.60	0.77	0.75
6. Given a Kinematics Graph	Pre-Test	0.93	0.93	1.13	0.74	1.00
Select Textual Description	Post-Test	1.27	1.13	1.38	1.44	1.11
	Difference	0.34	0.20	0.25	0.70	0.11
	Std. Dev.	0.79	0.68	0.56	0.91	0.74
7. Given textual Motion Description	Pre-Test	0.69	0.53	0.75	0.70	0.71
Select Corresponding Graph	Post-Test	1.08	0.87	1.31	1.15	1.00
	Difference	0.40	0.33	0.56	0.44	0.29
	Std. Dev.	0.91	0.98	0.86	0.97	0.85

Odds ratios were computed for each of the seven objectives. The ratios compared the odds of improving to the odds of not improving. Table 10 shows the breakdown of how pre- and post-test scores were used to identify students who improved and those that did not improve.

Table 10. Pre- And Post-Test Scores That Indicate Improvement and No Improvement On Objectives

	Pre-Test Score	Post-Test Score
Improved	2	2
Improved	1	2
Improved	0	1
Improved	0	2
No Improvement	2	0
No Improvement	2	1
No Improvement	1	0
No Improvement	1	1
No Improvement	0	0

The following are the results of logistic regression analysis of the odds ratio for each objective.

Objective 1, Given a position-time graph, determine velocity: There was not strong evidence that the odds ratio was related to the CBL or Cooperative Group treatment alone but there was suggestive evidence ($p<0.027$) that there was an interactive effect.

a. Individual, Non-CBL treatment. The odds of improving are 0.75 times that of not improving.
b. Individual, CBL treatment. The odds of improving are 1.24 times that of not improving.
c. Cooperative Group, Non-CBL treatment. The odds of improving are 4 times that of not improving.
d. Cooperative Group, CBL treatment. The odds of improving are 0.50 times that of not improving.

Objective 2, Given a velocity-time graph, determine acceleration: There was no evidence that the odds ratio for the CBL or Cooperative Group treatments were related to this objective ($p>0.86$).

Objective 3, Given a velocity-time graph, determine displacement: There is evidence that the interactive effect of the Cooperative Group and CBL treatments are related to this objective ($p< 0.015$).

a. Individual, Non-CBL treatment. The odds of improving are 2.5 times that of not improving.
b. Individual, CBL treatment. The odds of improving are 3.5 that of not improving.
c. Cooperative Group, Non-CBL treatment. The odds of improving are 4.3 times that of not improving.
d. Cooperative Group, CBL treatment. The odds of improving are 0.50 that of not improving.

Objective 4, Given an acceleration-time graph, determine change in velocity: There was suggestive evidence of an effect due to the CBL treatment ($p <0.084$), but not due to the Cooperative Group treatment ($p > 0.11$).

a. Non-CBL treatment (Individual and Cooperative Group) the odds of improving are 1.44 times that of not improving.
b. CBL treatment (Individual and Cooperative Group) the odds of improving are 0.68 that of not improving.

Objective 5, Given a kinematics graph, select another corresponding graph: There was no evidence that the odds ratio for the CBL or Cooperative Group treatments were related this objective ($p>0.65$).

Objective 6, Given a kinematics graph, select textual description: There was strong evidence that the odds ratio was related to each of the CBL or Cooperative Group treatments alone but the effect of CBL depended on the Cooperative Group Treatment ($p<0.042$).

a. Individual, Non-CBL treatment the odds of improving are 0.65 times that of not improving.

b. Individual, CBL treatment the odds of improving are 1.7 times that of not improving.
c. Cooperative Group, Non-CBL treatment the odds of improving are 1.7 times that of not improving.
d. Cooperative Group, CBL treatment the odds of improving are 0.67 times that of not improving.

Objective 7, Given textual motion description, select corresponding graph: There was no evidence that the odds ratio for the CBL or Cooperative Group treatments were related this objective ($p>0.85$).

Student difficulties may be caused by underlying student misconceptions or lack of conception. Table 11 gives a breakdown by recitation section of the percentage of students whose pre- and post-test indicated they had a particular misconception/lack of conception.

Table 11. Percentages of Students With a Specific Misconception/Lack of Conception

Misconception/ Lack of Conception		Total	Cooperative Group CBL	Cooperative Group Non-CBL	Individual CBL	Individual Non-CBL
	n =	86	15	16	27	28
1. Lack of conception	Pre-Test	9%	20%	0%	11%	7%
Velocity as slope of a	Post-Test	2%	7%	0%	0%	4%
2. Lack of conception	Pre-Test	17%	33%	6%	22%	11%
Acceleration as slope of a	Post-Test	5%	7%	0%	7%	4%
3. Lack of conception	Pre-Test	64%	60%	44%	70%	71%
Concerning area under graph	Post-Test	29%	53%	13%	30%	25%
4. Misconception	Pre-Test	6%	7%	0%	11%	4%
Syntactic cue	Post-Test	2%	7%	0%	0%	4%
5. Misconception	Pre-Test	30%	33%	6%	37%	36%
Linquistic cue	Post-Test	16%	33%	0%	19%	14%
6. Misconception	Pre-Test	17%	13%	13%	30%	11%
Iconic translation	Post-Test	8%	7%	13%	7%	7%

Odds ratios were computed for each of the six misconceptions/lack of conception. The ratios compared the odds of improving to the odds of not improving. See the breakdown of pre- and post-test indications in Table 12 which identify students who improved and those that did not improve.

Table 12. Pre- And Post-Test Indications
Improvement and No Improvement
On Misconceptions/Lack of Conception

	Pre-Test	Post-Test
	Misconception	Misconception
	Lack of Conception	Lack of Conception
Improved	indicated	not indicated
Improved	not indicated	not indicated
No Improvement	indicated	indicated
No Improvement	not indicated	indicated

The following are the results of logistic regression analysis of the odds ratio for each misconception/lack of conception.

Lack of Conception for velocity as the slope of a position graph: There were only 2 students in the sample who were identified as making no improvement, and thus, logistic regression analysis was not valid for this variable.

Lack of Conception for acceleration as the slope of a velocity graph: There were only 5 students in the sample who were identified as making no improvement, and thus, logistic analysis was not valid for this variable.

Lack of Conception for area under a graph: There was suggestive but inconclusive evidence that this response depended on the CBL and Cooperative Group treatments ($p < 0.086$), and the effect of CBL depends on whether it was combined with the Cooperative Group treatment.

a. Individual, Non-CBL treatment. The odds of improving are 3 times that of not improving.

b. Individual, CBL treatment. The odds of improving are 2.4 times that of not improving.
c. Cooperative Group, Non-CBL treatment. The odds of improving are 7 times that of not improving.
d. Cooperative Group, CBL treatment. The odds of improving are 0.87 times that of not improving.

Misconception based on use of syntactic cue: There were only 2 students in the sample who were identified as making no improvement, and thus, logistic regression analysis was not valid for this variable.

Misconception based on use of linguistic cue: There was suggestive evidence that this response was related to the CBL treatment ($p < 0.074$) but was not related to the Cooperative Group treatment ($p > 0.97$).

a. Non-CBL treatment (Individual and Cooperative Group). The odds of improving are 10 times that of not improving.
b. CBL treatment (Individual and Cooperative Group). The odds of improving are 3.19 that of not improving.

Misconception based on use of iconic translation: There were only 7 students in the sample who were identified as making no improvement, and thus, logistic regression analysis was not valid for this variable.

To further investigate the comparative effectiveness of these teaching strategies the difference between the pre- and post-test on the variables of total score and number of misconceptions/lack of conception indicated were analyzed. See Table 13 for a breakdown of these differences.

Table 13. Difference Pre- and Post-Test
Total Score and Number of Misconceptions/Lack of Conceptions

Variable		Total	Cooperative Group CBL	Cooperative Group Non-CBL	Individual CBL	Individual Non-CBL
		n = 86	n = 15	n = 16	n = 27	n = 28
Total Score	mean	2.48	1.47	2.75	2.82	2.54
	standard deviation	2.37	2.70	1.35	2.70	2.25
Number of Misconceptions/ Lack of Conception	mean	0.84	0.60	0.44	1.22	0.82
	standard deviation	1.35	1.55	0.79	1.60	1.19

Analysis of variance techniques were applied to determine if any of the instructional treatment variables helped explain the variation of the response. The basic design for this analysis was a 2x2 factorial design since each of the categorical measures of Cooperative Group and CBL have two levels, with a covariate included for the Differential Calculus grade and student's concurrent enrollment in a Physics course. Extra sum of squares procedures were used to assess the statistical significance of the independent variables and covariates included in the model.

1) Difference in total score: There was convincing evidence that none of the independent variables measured were associated with the difference in scores ($F(5, 80)=1.11$, $p>0.36$). These results remained consistent if we did not account for the Differential Calculus grade or students' enrollment in a Physics course ($F=1.18$, $p>0.32$). The interaction term for CBL and Cooperative Group did not contribute to the explanation of the response ($F(1, 82)=2.17$, $p>0.14$). The main effects of Cooperative Group and CBL were not associated with the response, assuming there was no effect due to Differential Calculus grade, enrollment in a Physics course or an interactive effect ($F(2, 83)=0.67$, $p>0.51$).

2) Difference in number of misconceptions/lack of conception: There is moderate but strongly suggestive evidence that the Cooperative Group term is associated with the response after accounting for the students' Differential Calculus grade and enrollment in a Physics course ($F(3, 82)=3.25$, $p<0.08$). There was no evidence that either the Differential Calculus grade or enrollment in a Physics course were significant factors ($F(2, 82)=1.11$, $p>0.33$). If we do not account for the students' Differential Calculus grade or enrollment in a Physics course, there is still evidence that the Cooperative Group term is significant ($F(1, 84)=2.78$, $p<0.10$). There was no evidence to suggest the interaction between the Cooperative Group and CBL instructional treatments was associated with the response ($F(2,80)=0.19$, $p>0.66$). The means and standard deviations for the two Cooperative Group levels are shown below in Table 14:

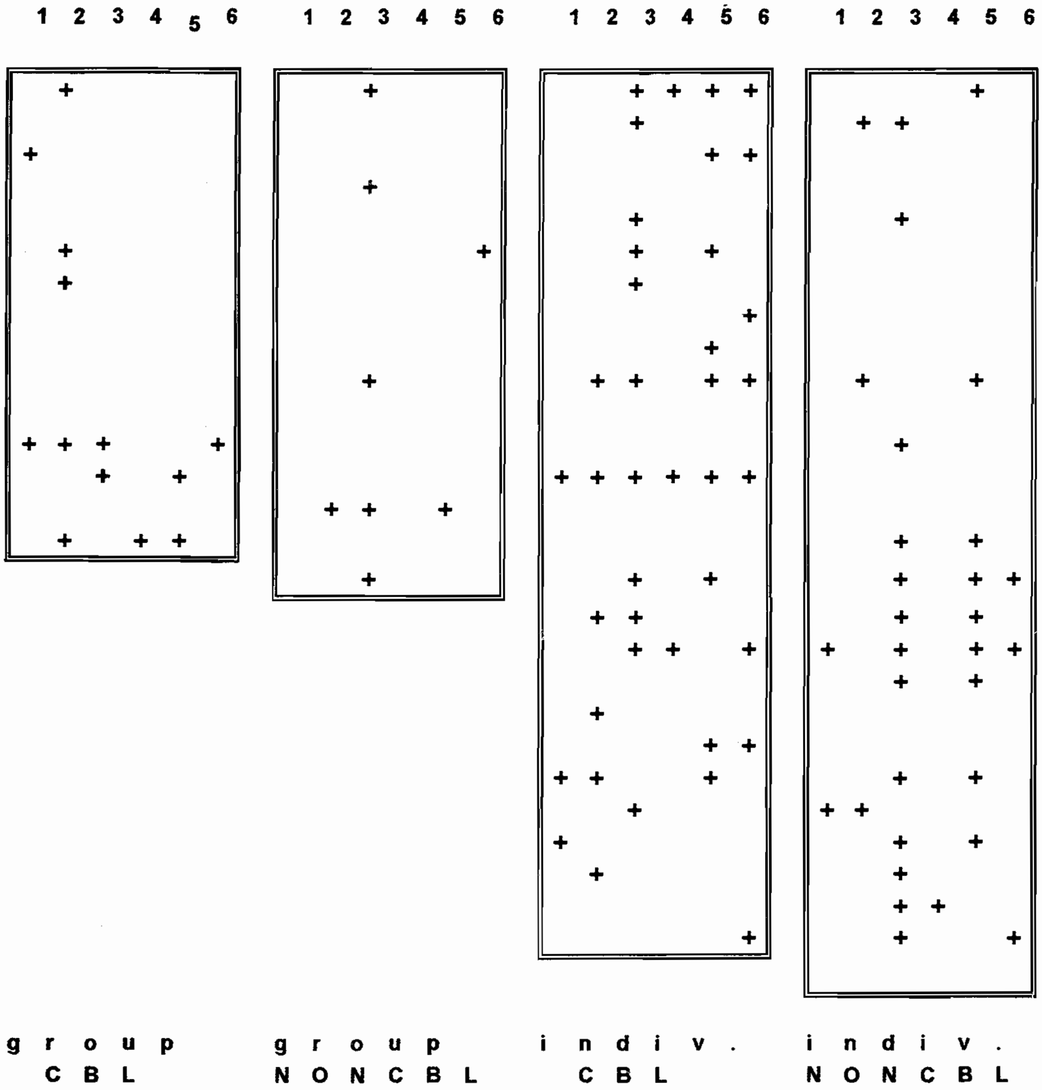
Table 14. Difference Pre- and Post-Test
Number of Misconceptions/Lack of Conception
Individual/Cooperative Group

	Total	Individual	Cooperative Group
Misconcecptions			
Lack of Conception			
	n = 86	n = 55	n = 31
Mean	0.84	1.02	0.52
Std. Dev.	1.35	1.41	1.21

This suggests that the Individual treatment was more successful at repairing misconceptions.

Tables 15, 16, and 17 are charts of the student misconceptions/lack of conception that appeared to be repaired (built), retained or gained during the five weeks between the pre-and post-test. This breakdown is by recitation section, for each student in the section. A misconception was defined as repaired if it was indicated on the student's pre-test and not the post-test. A conception was said to be built if a lack of conception was indicated on the student's pre-test, but not on the student's post-test. A misconception or lack of conception that was indicated on both the student's pre- and post-test was said to be retained. A misconception/lack of conception was said to be gained if it was not indicated on the student's pre-test, but was indicated on the student's post-test.

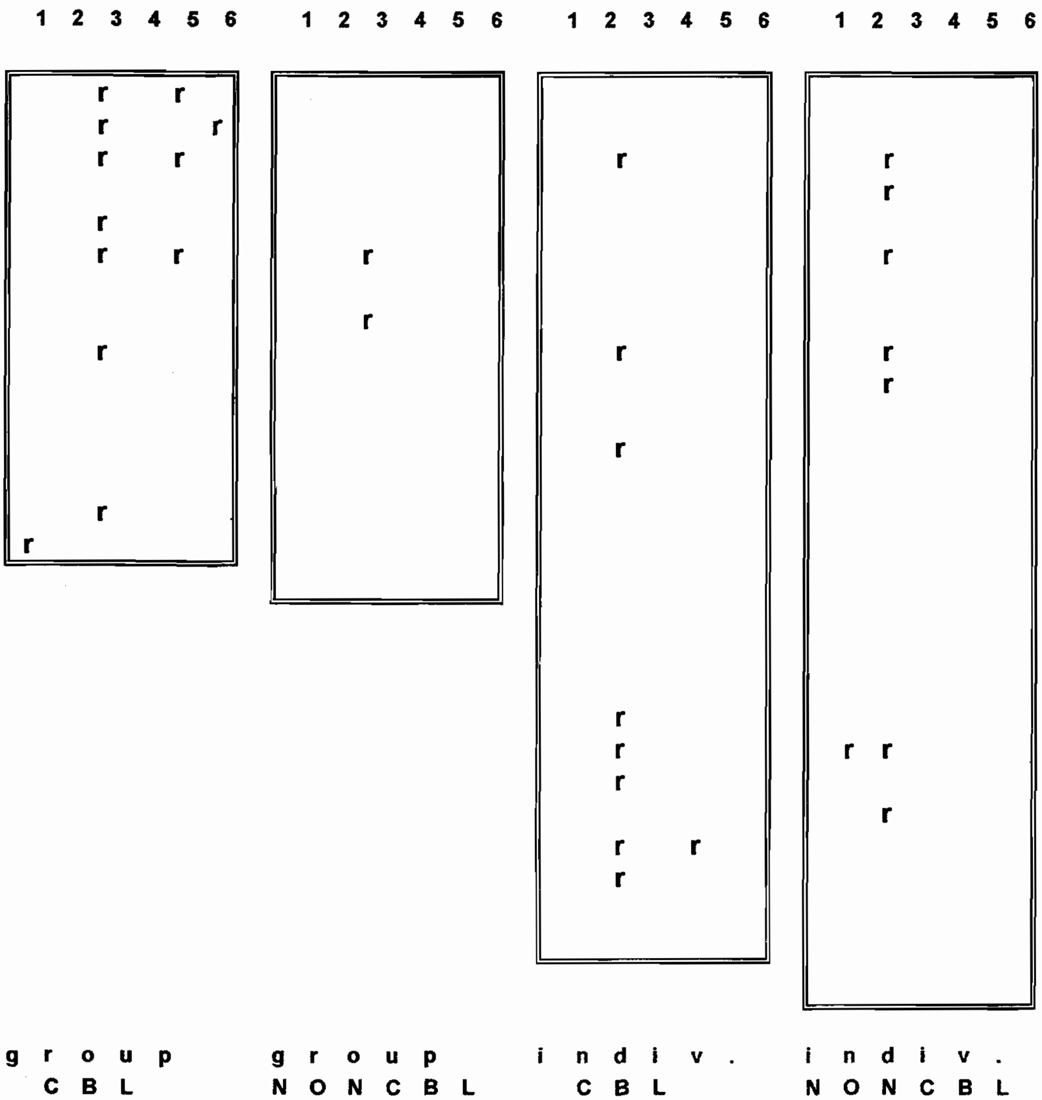
Table 15. Misconceptions/Lack of Conception Repaired or Built



Key: + Indicates the student built a conception or repaired a misconception.
r Indicates the student retained a misconception or lack of conception.
G Indicates the student gained a misconception or lack of conception.

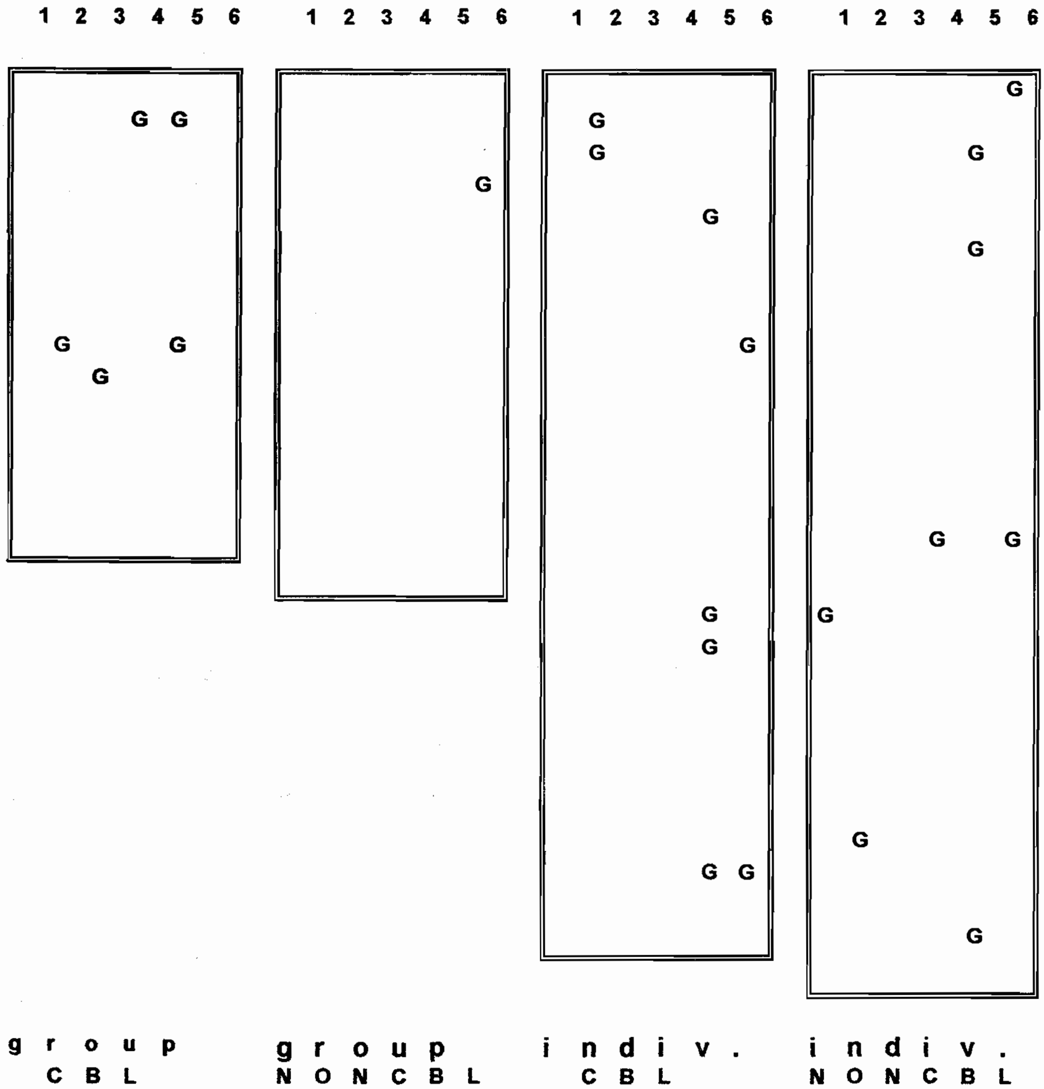
1 Lack of conception for velocity as the slope of a position graph
2 Lack of conception for acceleration as the slope of a velocity graph
3 Lack of conception for interpreting area under a graph
4 Misconception based on use of syntactic cues
5 Misconception based on use of linguistic cues
6 Misconception based on use of iconic translation

Table 16. Misconceptions/Lack of Conception Retained



Key: + Indicates the student built a conception or repaired a misconception.
r Indicates the student retained a misconception or lack of conception.
G Indicates the student gained a misconception or lack of conception.
1 Lack of conception for velocity as the slope of a position graph
2 Lack of conception for acceleration as the slope of a velocity graph
3 Lack of conception for interpreting area under a graph
4 Misconception based on use of syntactic cues
5 Misconception based on use of linguistic cues
6 Misconception based on use of iconic translation

Table 17. Misconceptions/Lack of Conception Gained



Key: + Indicates the student built a conception or repaired a misconception.
r Indicates the student retained a misconception or lack of conception.
G Indicates the student gained a misconception or lack of conception.
1 Lack of conception for velocity as the slope of a position graph
2 Lack of conception for acceleration as the slope of a velocity graph
3 Lack of conception for interpreting area under a graph
4 Misconception based on use of syntactic cues
5 Misconception based on use of linguistic cues
6 Misconception based on use of iconic translation

Interviews

A summary is given for each student interviewed that includes which treatment the student received, what they thought was most helpful in improving their understanding of graphs of kinematic variables, their pre- and post- test scores for each of Beichner's seven objectives, and the results indicated on their pre- and post-test concerning misconceptions/lack of conceptions. Excerpts from the interviews are included that provide evidence of how misconceptions may have been repaired or conceptions built. In the summaries, fictitious names are assigned to each student Statements made by the Interviewer are identified with an I, the graduate student teaching assistant's name during the interviews has been changed to Diane and the professor's name to Logan.

Dave

Dave was in the Individual, Non-CBL recitation, but was not in class on the day of the kinematics lab treatment. Dave did not find anything helpful for improving his understanding of graphs of kinematic variables.

Table 18. Results of Pre- and Post-Test, Dave

	Obj. 1	Obj. 2	Obj. 3	Obj. 4	Obj. 5	Obj. 6	Obj. 7	Total
DAVE								score
Post-Test	2	0	2	2	0	2	2	10
Pre-Test	1	0	0	0	0	0	1	2
Difference	1	0	2	2	0	2	1	8

	Lack of	Lack of	Lack of	Mis-	Mis-	Mis-
	Conception	Conception	Conception	conception	conception	conception
	Velocity	Accel.	Area	Syntactic	Linguistic	Iconic
Dave						
Post-Test						
Pre-Test			Indicated			indicated

The following are excerpts from the interview with Dave:

DAVE: I don't know how I improved on this. When we did the lab that was related to this was when the flooding was going on and I didn't make it, and I hadn't done the lab when I took this test so I didn't really know what I was doing, so a lot of it I guessed on.

I: Okay. But you guessed a lot better.

DAVE: Okay. Yeah, I knew the relationships after that, you know, but on the first one I didn't know that they were all related by a derivative.

I: On number two, you gave two different responses and one of them is correct.

DAVE: Hmm...I must have guessed on that one, actually. I know that the position is the anti-derivative of velocity.

I: And so it's the area under the curve?

DAVE: Right.

I: Okay, so what we'd be looking for then, in the first 3 seconds, is we'd be looking for that.

DAVE: Oh! Okay. Yeah.

I: On number six you answered two different things and one of them is correct.

DAVE: Okay. Let's see. Well the position doesn't change, at first. And then it goes negative, so I imagine it's gone backwards. And then it stops.

I: Right. Do you remember as far as at all what you were thinking there - while the object moves along a flat area and moves backwards down the hill and then it keeps moving?

DAVE: Umm, I don't know that. I mean here it shows that it's not changing positions, so I don't really know what I was thinking.

I: Number eight, two different responses and one of them is correct.

DAVE: Okay. Would it be this one?

I: Uh uh.

DAVE: No?

I: No. So we're looking at acceleration and they're asking about velocity.

DAVE: Right.

I: So velocity's the anti-derivative of acceleration. So again we're talking about area under the curve.

DAVE: Oh, okay. That would be a bigger area.

I: Yeah, this is a larger area than this one, so this has got the smallest area.

DAVE: Okay.

I: So, why were you thinking that this was the correct answer?
DAVE: Umm, I don't know, it just seemed like, I don't know, I was just thinking that if the acceleration was going up that it would change - the velocity would change more?

DAVE: Yeah, I don't know, I was just kind of confused, I guess, when I was taking it. Just kind of trying to think things out, but you only have like 15 minutes to do it.
I: You did real well!
DAVE: I guessed really well, I guess.

DAVE: I mean, I still knew the relationships, so I was kind of able to figure it out on some, but I got on those ones where they were constantly changing and - I kind of wish I would have known the slope relationship. That helps a lot.
I: So you kind of had the area under the curve part down, but not necessarily that the slope was the one that...
DAVE: Yeah, the slope part was the one that I missed on the test. I didn't know that you were supposed to find the constant on the mid-term, yeah.
I: Okay. So you missed the lab, so as far as with these relationships you didn't find much else that was going on in the class helpful? Or did you find other things helpful as far as working with Diane, or things out of the homework?
DAVE: For this?
I: Yeah.
DAVE: Or just the class in particular?
I: Yeah, well for understanding this. Anything within the framework of the class - working with Logan, or Logan's lectures, or lectures Diane gave, or working with Diane, or working on your homework, working with other people?
DAVE: Yeah, I'm not really sure where I picked it up, but before I came into the class I didn't know that they were all related by derivatives or anything because I hadn't taken 251 yet.
I: Right.
DAVE: I don't know exactly who I learned that from. But I guess that was the most helpful thing that I got out of this.
I: And you don't remember as far as anything specific?
DAVE: No, I think I learned it in recitation. I don't know if we even covered this in lecture or not.

Bob

Bob in the Cooperative Group, CBL recitation, but was not in class on the day of the kinematics lab. He thought the homework was helpful in improving his understanding of graphs of kinematic variables. He worked with others in class (by requirement) and outside of class (by choice).

Table 19. Results of Pre- and Post-Test, Bob

	Obj. 1	Obj. 2	Obj. 3	Obj. 4	Obj. 5	Obj. 6	Obj. 7	Total
BOB								score
Post-Test	1	0	0	0	0	1	2	4
Pre-Test	0	0	1	0	1	0	1	3
Difference	1	0	-1	0	-1	1	1	1

	Lack of	Lack of	Lack of	Mis-	Mis-	Mis-
	Conception	Conception	Conception	conception	conception	conception
	Velocity	Accel.	Area	Syntactic	Linguistic	Iconic
BOB						
Post-Test			indicated			
Pre-Test	indicated	indicated	indicated			

The following are excerpts from the interview with Bob:

I: Okay. So on the next one, so for number three, there's two different answers and one of them is correct.
BOB: So, let's see... the derivative of position is what we're looking for.
I: Uh huh.
BOB: So we're looking for the greatest slope at the start.
I: Uh huh. Not necessarily at the start I don't think.
BOB: No. Oh, during the interval. Okay.
I: Right. So at any time.
BOB: That slopes, so it's that one.
I: Yeah. Yep, it's D.
BOB: It's probably not what I was thinking of when I did that.
I: It was on the second exam, so actually that may be.
BOB: I definitely wasn't thinking like that, though, I'm sure.
I: Yeah.
BOB: Because you just showed me how to think like that.
I: So, during this test you weren't thinking of that at all?

BOB: Not in those terms, no.

I: So you were just trying to relate it to your own practical experience.

BOB: Uh huh. Uh huh.

I: And nothing about slope, or area under the curve, or any of that stuff, just flat trying to reason it out.

BOB: Uh huh. Yeah.

BOB: Okay. It was one section we did in the homework. It was one night's worth of homework, and, you know, we messed around with it a little bit, but even so, it never stuck. In fact, I was feeling pretty strong. The first test, I really messed up. Because I haven't taken Math in a long time, and I just kind of jumped back into it, and I have to finish this course, or I'm going to be doing it again. But, anyway, when I got on the exam I was feeling pretty good, and able, and I remembered that there was a relationship like this, so I guessed, and got it right, and got the problem right on the test. Because I remembered that we were supposed to do that, but I certainly didn't remember for sure how it was supposed to be. The little section in the homework was -- I didn't feel like that was really, really - no, I didn't think that was enough. I had a friend helping me with that, and he was going through it, and maybe there were only three or four examples that were actually of this type - like, just evaluate what's going on in this graph - and my friend and I were going through that and he just asked me and said, "well, you know, what's happening here?", and I could tell him and he said, "well, so which is your answer?", and say, "oh, this is it!" And he said, "yeah, that's right." And so we just kept going through and I was getting them right. And he was kind of asking questions to me on my terms and trying to make me make the middle conversion over what he was asking.

Sue

Sue was in the Cooperative Group, CBL treatment. She thought the lab on kinematics was helpful. Sue worked with others in class (by requirement) and outside of class (by choice).

Table 20. Results Pre- and Post-Test, Sue

	Obj. 1	Obj. 2	Obj. 3	Obj. 4	Obj. 5	Obj. 6	Obj. 7	Total
SUE								score
Post-Test	1	1	1	2	1	1	2	9
Pre-Test	0	0	0	0	2	0	0	2
Difference	1	1	1	2	-1	1	2	7

	Lack of Conception Velocity	Lack of Conception Accel.	Lack of Conception Area	Mis-conception Syntactic	Mis-conception Linguistic	Mis-conception Iconic
SUE						
Post-Test						
Pre-Test	indicated	indicated	indicated			indicated

The following are excerpts from the interview with Sue:

I: Okay, so number one.
SUE: I came back to it again. I don't know why. I mean, but I did come back to it.
I: Oh, okay.
SUE: It just threw me off, I think.
I: Okay, so can you tell me now as far as what the correct answer is?
SUE: I don’t know.

I: On number four, two different responses and one of them is correct.
SUE: That's another number - I just don't deal with them well.
I: So this one we're given velocity, and it wants acceleration. So we're looking for derivative, so we're looking for the slope. When we go for slope we do that 2 point thing, rise over run, so we could go 10...
SUE: I think that's what I did on the last one.
I: Right, and that's why you got it right!
SUE: Okay. So I was kind of guessing.

I: On number six you gave two different answers.
SUE: So is either one correct?
I: Yeah. One of them is correct.
SUE: Okay. Well, at first I just thought that it was, you know, it was just really flat! But with the second one, then, what we learned

was that being constant I thought that that meant that it didn't move. And then if it went down then it was going backwards.

I: Right.

SUE: Like we did in that lab. And that's why I pretty much came up with that.

I: Okay. Seven - one of them is correct.

SUE: I think on that one I just made the graphs, like above - I don't know if I left them or not, but I made it what it would look like.

I: So (E) is the one that's actually correct. And, so you're saying like you did something along the lines: if an object starts from rest and undergoes a positive, constant acceleration, so you'd maybe start off with let's say it start with 3, then the velocity would be 3x, and the distance would be 3/2 X^2, and this looked like an X^2 function?

SUE: Yeah, more or less.

I: Okay. And maybe in this first one then just going ...

SUE: And the first one I just saw a constant.

I: Right.

I: So you said at one point that you were answering a question and you just sort of thought about the lab that you'd done the week before.

SUE: That really helped a lot. And it being fresh in my mind helped.

I: Were there other things that you found particularly beneficial as far as about the class, as far as comparing Logan's lectures, Diane's lectures - I don't know if you worked with Diane in here - if you worked with the group in lab? Outside of class, did you work with other people? Was there anything that you found to be particularly helpful?

SUE: I got together with some people outside of class.

Ed

Ed was in the Cooperative Group, Non-CBL treatment. Ed found the lab on kinematics helpful. Ed worked with others in class (by requirement) and outside of class.

Table 21. Results of Pre- and Post-Test, Ed

	Obj. 1	Obj. 2	Obj. 3	Obj. 4	Obj. 5	Obj. 6	Obj. 7	Total
ED								score
Post-Test	2	2	2	1	1	1	1	10
Pre-Test	2	1	0	0	1	1	0	5
Difference	0	1	2	1	0	0	1	5

	Lack of	Lack of	Lack of	Mis-	Mis-	Mis-
	Conception	Conception	Conception	conception	conception	conception
	Velocity	Accel.	Area	Syntactic	Linguistic	Iconic
ED						
Post-Test						
Pre-Test		indicated	indicated		indicated	

The following are excerpts from the interview with Ed:

Ed: (item #1) Exactly. Yeah, 'cause I know that with the first time that I took this test, then as it was with some of the people that I talked to, we just thought it was something that it was kind of like, you know, we just take it, and you know, it was no big deal, that it wasn't affecting our grade, but the second time it was supposed to be kind of a lab grade so we all kind of concentrated a bit more on it, but I don't know why I missed this one twice. Kind of boggles my mind. But third time's a charm, though.

I: Yeah. Okay, the next one - one of them is correct.

Ed: Okay. Let's see here. I believe... velocity, I believe, is the derivative of position, and that's what I did is that I just took the area underneath the curve, from 0 to 3 seconds, and I got the answer of I think 6.

I: Right.

Ed: And my reasoning for picking 12, possibly, the first time, probably was because... hmm, I don't really know. I'm not real sure.

I: Three times four is 12?

Ed: Yeah, probably something like that. Yeah.

I: One of them is correct - number seven.

Ed: I would have to say this one is the one I got correct. I mean, if it was moving with constant velocity it would be continuing forward, so as time progressed then ... I don't know why I would choose that one. I was probably just thinking about a constant deal, I mean, because all this other stuff wasn't constant, it was all running in a different type of curve. Okay. So number eight, did I get one of those two right, also?

I: Yes, one of them is right.

Ed: Find the 3 second interval... it would be that one then.

I: Yeah.

Ed: Yeah, because I know that during the term we -- I believe it was in the later stages of Chapter 4 we learned about displacement and stuff, and I just remember like a little thing in my head it was AVP, acceleration, velocity and position, or velocity is the acceleration of that - derivative is velocity and velocity is derivation position, and that way you can just go through.

I: Right. Yeah, I just kind of think, well, down goes this way and you know, back up. This one you got correct both times.

Ed: Okay. Because if you know how to -- like, I just picture anti-derivatives and derivatives in my head is not very hard, it just takes a little while, but then you kind of just draw it out, you know, mentally in your head, and if you know which order everything goes in it's pretty easy, so it's not too bad.

Ann

Ann was in the Individual, CBL treatment. Ann worked alone in class and out of class. Ann thought she was as confused about kinematics at the end of the course as she was at the beginning.

Table 22. Results of Pre- and Post-Test, Ann

ANN	Obj. 1	Obj. 2	Obj. 3	Obj. 4	Obj. 5	Obj. 6	Obj. 7	Total score
Post-Test	0	0	1	0	1	1	1	4
Pre-Test	0	0	1	0	1	1	0	3
Difference	0	0	0	0	0	0	1	1

ANN	Lack of Conception Velocity	Lack of Conception Accel.	Lack of Conception Area	Mis-conception Syntactic	Mis-conception Linguistic	Mis-conception Iconic
Post-Test		indicated	indicated			
Pre-Test			indicated		indicated	indicated

The following are excerpts from the interview with Ann:

I: So that's about it. Okay. So, on number one, you gave two different responses and neither one of them were correct.
ANN: I know.
I: You know that!
ANN: That's what's weird, though, because I seemed to do decent in the class but it didn't seem to help me figure out -- on the test, I don't know why, I just kind of confused. Because it was given' different - I don't know, the way they were given was different than the way we were taught, in class, so it didn't seem to click.

I: So, did you feel that anything helped you as far as during the time of the course with understanding these graphs any better? Or, do you feel like you were just as confused by them at the end as you were at the beginning?
ANN: Probably pretty much.

ANN: More like we dealt with numbers, so you didn't really relate them as much. I mean there were a couple of things, but that was more towards the beginning, right after we did it, and then we didn't really deal with it towards the end. And so you just kind of forgot about those and moved on and did things just dealing with numbers and derivatives.
I: Right. So the homework wasn't terribly helpful.
ANN: No.
I: Were you there as far as for the lab that was going on? So you didn't find that helpful as far as looking at those graphs, or...?
ANN: I probably did, but then I... I don't know, it didn't help. It didn't help me on this.
I: But you were there?
ANN: Yeah - oh yeah, I was there. And I started understanding more when we were doing it, it made sense.
I: Right, at that particular time, but there wasn't enough of it.
ANN: Yeah. Yeah.

<u>Andy</u>

Andy was in the Individual, CBL treatment. Andy worked with others in class but not outside of class. Andy thought the homework was the most helpful and that the lab on kinematics and working with a partner were somewhat helpful.

Table 23. Results of Pre- and Post-Test, Andy

ANDY	Obj. 1	Obj. 2	Obj. 3	Obj. 4	Obj. 5	Obj. 6	Obj. 7	Total score
Post-Test	2	1	1	0	1	2	2	9
Pre-Test	2	0	0	0	1	0	0	3
Difference	0	1	1	0	0	2	2	6

ANDY	Lack of Conception Velocity	Lack of Conception Accel.	Lack of Conception Area	Mis-conception Syntactic	Mis-conception Linguistic	Mis-conception Iconic
Post-Test			indicated			
Pre-Test			indicated		indicated	indicated

The following are excerpts from the interview with Andy:

I: Okay. Number six, you gave two different responses and one of them was correct. Do you want to take a look at that for a second?
ANDY: (D) is correct?
I: Yeah, (D) is correct. What about your answer if the object doesn't move at first, then it rolls forward down a hill and finally stops, any idea?
ANDY: Well, that would mean that this is getting greater that way.
I: Right.

I: Okay. And number 13, one of them is correct.
ANDY: (C) is correct?
I: Yeah, (C) is correct.
ANDY: It can't be (A) because the acceleration is not constant.

Ben

Ben was in the Individual, CBL treatment. Ben thought the lab on kinematics was helpful. Ben worked with others in class.

Table 24. Results of Pre- and Post-Test, Ben

	Obj. 1	Obj. 2	Obj. 3	Obj. 4	Obj. 5	Obj. 6	Obj. 7	Total
BEN								score
Post-Test	1	0	2	0	1	0	1	5
Pre-Test	1	1	0	1	1	1	0	5
Difference	0	-1	2	-1	0	-1	1	0

	Lack of	Lack of	Lack of	Mis-	Mis-	Mis-
	Conception	Conception	Conception	conception	conception	conception
	Velocity	Accel.	Area	Syntactic	Linguistic	Iconic
BEN						
Post-Test					indicated	
Pre-Test			indicated	indicated		indicated

The following are excerpts from the interview with Ben:

I: On number two, you gave two different responses and one of them is correct.
BEN: Let's see, I don't remember all these. I guess I just said that velocity times time... I just took the area for one of them - under the graph.
I: Right.
BEN: And for the other one I just times the two because the velocity times time gives the distance...
I: Right.
BEN: And, that's how I figured that one.

BEN: I think doing that one lab where we actually had to come up with the scenarios and then kind of play them out to see if they worked - that helped out the most, I think. Other than just repetition of just going through them.

<u>Mike</u>

Mike was in the Individual, CBL treatment. Mike thought the lab on kinematics was helpful. Mike worked with others in and out of class.

Table 25. Results of Pre- and Post-Test, Mike

	Obj. 1	Obj. 2	Obj. 3	Obj. 4	Obj. 5	Obj. 6	Obj. 7	Total
MIKE								score
Post-Test	0	0	2	2	0	2	1	7
Pre-Test	0	0	0	1	0	0	0	1
Difference	0	0	2	1	0	2	1	6

	Lack of	Lack of	Lack of	Mis-	Mis-	Mis-
	Conception	Conception	Conception	conception	conception	conception
	Velocity	Accel.	Area	Syntactic	Linguistic	Iconic
MIKE						
Post-Test						
Pre-Test	indicated	indicated	indicated	indicated	indicated	indicated

The following are excerpts from the interview with Mike:

I: Okay, so number two is the same thing, but switched answers. Any idea as far as which one was correct? Question number 2?
MIKE: Well the mass doesn't have anything to do with it, I don't think.
I: Uh uh.
MIKE: So it has to do with... with the area under the graph.
I: Yeah.
MIKE: The first answer, again, I thought it was the point on the graph, at three seconds.

I: Okay, number six, you got two different answers, so, can you tell which one of those is right?
MIKE: Yeah, it would be this one - number B, because I remember this - this is a position graph, and it's not moving, and then it's moving backwards, or it's moving downhill. Yes, this is coming back down to... I guess it would make more sense if it was coming back towards the beginning, or something like that. Because the bottom of the hill is the beginning, I guess, and then it stops again.
I: Okay, so it's D? Or B?
MIKE: Actually it's D.
I: Because the object doesn't move at first.
MIKE: Because then it moves backwards, and finally stops. Because it's going back towards zero.
I: Right. Okay. And I guess that makes sense that if you were looking at the distance...
MIKE: Up and down.

I: Uh huh, as opposed to sideways, that then B would be a reasonable answer.

I: Okay. And so number seven you answer two different things and one of them is right. Here again, can you see which one it is?
MIKE: One of them is right?
I: Uh huh.
MIKE: Okay. It's going to have to be this one (response E), because this one (response B) is stopping up here (points to flat portion of position graph).
I: Right.
MIKE: So it's going the same way, or something. This one (response E) is on the freeway, or whatever.
I: Right. Okay. Can you remember what you were thinking as far as when you answered that one?
MIKE: Umm, yeah, I just misunderstood what happened. The graph is of position where it had a zero slope. The velocity was zero, so it's not going anywhere.
I: Okay, as opposed to constant velocity.
MIKE: Right.

I: Do you remember anything such as homework where you were doing a problem or anything else that you have any recollection of a light bulb going on?
MIKE: Yeah, the lab where we did this, in the class where we did the computer simulation. That helped out a lot. Just because at that point, you know, you're actually doing this stuff here, and I don't think this stuff is really in the homework, that I recall.

Ken

Ken was in the Individual, CBL treatment. Ken thought the homework was helpful and that the lab was confusing. Ken worked alone in and out of class.

Table 26. Results of Pre- and Post-Test, Ken

KEN	Obj. 1	Obj. 2	Obj. 3	Obj. 4	Obj. 5	Obj. 6	Obj. 7	Total score
Post-Test	1	2	2	0	2	2	1	10
Pre-Test	2	0	1	0	1	0	0	4
Difference		2	1	0	1	2	1	6

KEN	Lack of Conception Velocity	Lack of Conception Accel.	Lack of Conception Area	Mis-conception Syntactic	Mis-conception Linguistic	Mis-conception Iconic
Post-Test						
Pre-Test		indicated	indicated		indicated	indicated

The following are excerpts from the interview with Ken:

I: Okay, on number 2 you gave two different responses and one of them is correct.

KEN: Oh, this one, actually I know how to do it but I didn't really understand why it worked, but I just knew how to do it. So, I know that (D) is right, but (C) - I may have just gone like three seconds and it's gone four meters per second.

I: Right. Uh huh.

KEN: You know, for 4 seconds, so I think that's probably what I was thinking.

I: So you just went ahead and took the height off the graph - the Y value off of the graph at that point.

KEN: Yeah.

I: Okay, number 4. You gave two different responses and one of them is correct.

KEN: Well (B) is right, and (E) ... I may have done something with these two heights.

I: Or did you maybe just go and grab the same thing as far as just pick the heights off of this? Oh, I guess you're saying that you could have gone 30 minus 10, or something like that.

KEN: Yeah. Or, I mean, this one I took like 30 minus 10 divided by 120 minus 60. Yeah, it's probably close, but I'm not really sure how I got 20.

I: Number 5, two different responses and one of them is correct.

KEN: Well, I don't really remember, but what's the deal with these pluses?
I: I don't know why these would say plus and these two wouldn't because they need a plus, also.
KEN: So (B)s the right answer, right?
I: No, (A) is the right answer. This one. Taking like those two points.
KEN: Oh. Oh, okay.

I: And on number 14 one of them is correct. And this one's a little tricky..
KEN: Okay, it's (A).
I: Right.
KEN: Um, well it says - well, like it says the derivative, acceleration, of velocity.
I: Uh huh.
KEN: And it says it's moving with constant acceleration, which means the slope is the same.
I: Right.
KEN: And so this is the same.
I: But I was wondering as far as when you gave the answer (B), was it just easier to associate the decreasing with this graph?
KEN: Yeah, it could have been.
I: Who knows?
KEN: It seems like when I didn't know what it was I just like circled what this was instead of the answer.

I: So you get more out of the homework. Do you work with anybody as far as outside of class?
KEN: No, not really.
I: And, it wasn't necessarily in these things in lab, or did you work with other people when you worked in lab, or did you pretty much work by yourself?
KEN: Yeah.
I: Okay, and the working with the graphs wasn't necessarily of any particular help? That day where you worked with those little cars and stuff in class, were you there that day?
KEN: Yeah. Yeah. That was kind of like, "huh? I don't get it!" That was confusing.

Tom

Tom was in the Individual, CBL treatment. Tom thought the homework was helpful. Tom worked with others outside of class.

Table 27. Results of Pre- and Post-Test, Tom

	Obj. 1	Obj. 2	Obj. 3	Obj. 4	Obj. 5	Obj. 6	Obj. 7	Total
TOM								score
Post-Test	0	1	2	1	1	2	0	7
Pre-Test	0	1	0	0	1	1	0	3
Difference	0	0	2	1	0	1	0	4

	Lack of	Lack of	Lack of	Mis-	Mis-	Mis-
	Conception	Conception	Conception	conception	conception	conception
	Velocity	Accel.	Area	Syntactic	Linguistic	Iconic
TOM						
Post-Test					indicated	
Pre-Test		indicated	indicated			

The following are excerpts from the interview with Tom:

I: Okay, number two - one of those answers is correct.
TOM: Is it (D)?
I: Yeah.
TOM: It's just two times three. Because, okay, I looked at the velocity, and it's like meters per second, and if I multiply that by the seconds then the seconds cancel out and you've got meters the distance.
I: I'm sorry, say again how you did that?
TOM: Okay. Like 3 meters per second times the 2 seconds, and then if you look at these - the fractions - mean over s times s, then the s's cancel out and you've got m - that's how I know. Does that make sense?
I: Okay, so where does the 2 come into it?
TOM: Well at the -- it says after...
I: The first three seconds.
TOM: Okay. Okay, I was looking at the 2 times.
I: Okay, so that's maybe how... yeah, by your reasoning it would have been 3 x 4 is 12, right? (this was not an answer he gave on either test)
TOM: Yeah.
I: Okay, and that would work if this were a straight line. This is, again, the area under the curve.
TOM: Then you have to divide it by two.

I: Right. Okay. And your answer (response A) of 0.75? Maybe for 0.75 you just took 3 and divided it by 4, or something?
TOM: Maybe.
I: Who knows?
TOM: Probably (A) was the first one?
I: Uh huh.
TOM: Yeah.
I: Yeah.
TOM: Probably the first time through I just was... did I put (A) on most of them?
I: Nope.
TOM: So I was putting some effort towards it.

I: Uh huh. And, number five - one of them is correct. So we're given a velocity graph.
TOM: Is it the slope right here?
I: Yeah, so it's the slope right there.
TOM: So it's five up then - it would probably be (A).
I: Yeah, it'd be (A).
TOM: Yeah, that's what I was trying - I was trying to see how much it went up and went over.

I: Number seven, neither one of them are correct, and you can take a look at it...
TOM: Is it (E)?
I: Yeah.
TOM: That was just a lucky guess.
I: Okay. So, (C) is what I'd guess that you got on the second one. The whole thing is an object starts from rest and undergoes a positive, constant acceleration for ten seconds. So, if it's positive then it's going to be concave up, because this is the second derivative, so it's going to be concave up.
TOM: And then it continues on then.
I: This one, so what do you think was going on with that one, when you answered (A)?
TOM: Probably I was thinking it was more of an acceleration graph, and it just accelerated up.
I: Uh huh.
TOM: I mean, just the wording in the question, you look at this and it looks like this would be -- I mean, it accelerates and then it levels off.
I: And then it's constant.
TOM: And it just would make the most sense to me.
I: Right.
TOM: If I didn't know anything.

I: Okay, number eight, one of them is correct.
TOM: (A)?
I: Yeah.
TOM: The smallest area.

I: Right. Okay, and 14 - one of them is correct.
TOM: (B)?
I: No! I knew you were going to do that...
TOM: So why isn't it uniformly... oh, it's uniformly decreasing - it's less and less.
I: Decreasing, right. So if this were an acceleration graph it would be (B).
TOM: Okay.
I: Uniformly decreasing means like a line.
TOM: Yeah.

Deb

Deb was in the Individual, Non-CBL treatment. Deb thought the homework, the kinematics lab, and the lectures were all helpful. Deb worked alone inside and outside of class.

Table 28. Results of Pre- and Post-Test, Deb

	Obj. 1	Obj. 2	Obj. 3	Obj. 4	Obj. 5	Obj. 6	Obj. 7	Total
DEB								score
Post-Test	1	0	1	2	0	1	0	5
Pre-Test	0	0	1	0	0	0	0	1
Difference	1	0	0	2	0	1	0	4

	Lack of Conception Velocity	Lack of Conception Accel.	Lack of Conception Area	Mis-conception Syntactic	Mis-conception Linguistic	Mis-conception Iconic
DEB						
Post-Test						
Pre-Test	indicated		indicated		indicated	indicated

The following are excerpts from the interview with Deb:

I: So, on number one, you did give two different responses and one of them is correct.

DEB: This one.

I: Nope! Okay, so I'll just start with this little diagram that I tend to use.

DEB: Uh huh.

I: Now when we're going back up, from acceleration to velocity ...

DEB: The area.

I: Right, or the area under the curve.

DEB: Uh huh.

I: Okay? The same thing. Back from velocity, area under the curve. This little diagram says, "well, I'm going to look for the largest area under the curve."

DEB: And that was the larger area under the curve?

I: Right.

DEB: Got it.

I: On number 11, one of the responses is correct, and I don't know if you can figure it out, or just tell me how to go about it.

DEB: I have no idea how I came up with those answers.

I: Okay, so you weren't thinking slope at all as far as... and didn't compute the slope?

DEB: Uh huh.

I: You weren't thinking that velocity is the derivative of position?

DEB: Yeah, but I didn't think about it as slope. I didn't have the formula.

I: Okay. So maybe you would have known that it would have been a negative value?

DEB: Yeah.

DEB: I probably just guessed.

I: Right. Okay. Number 12 - one of them is correct.

DEB: Is it (B)?

I: Yes.

DEB: The area under the curve?

Josh

Josh was in the Individual, Non-CBL treatment. Josh thought the homework, the labs, and the lectures were all helpful. Josh worked with others outside of class.

Table 29. Results of Pre- and Post-Test, Josh

	Obj. 1	Obj. 2	Obj. 3	Obj. 4	Obj. 5	Obj. 6	Obj. 7	Total
JOSH								score
Post-Test	1	2	2	2	1	2	1	11
Pre-Test	1	2	1	0	1	1	0	6
Difference	0	0	1	2	0	1	1	5

	Lack of Conception Velocity	Lack of Conception Accel.	Lack of Conception Area	Mis-conception Syntactic	Mis-conception Linguistic	Mis-conception Iconic
JOSH						
Post-Test						
Pre-Test			indicated		indicated	

The following are excerpts from the interview with Josh:

I: On number one. So, can you just tell which one is correct?
JOSH: Which one?
I: Number one. So you answered (A) on one and (B) on another test.
JOSH: I'd say (A). No... (B). Is it (B)?
I: Right. Right, it's (B). And, that's to a certain extent because the... so what is your process as far as with answering that?
JOSH: I was thinking if this was 5 right here, then it would start at 5 and it'd go up, over 3 seconds or whatever, go up to whatever, 8? And this one if it was 0 and just went to 5 - okay, so if it were 5 seconds, this one would go 5 to 25, this one from 0 to 5, it seems like it was a greater steepness.

I: Okay. And so then on to number two, there again you gave two different responses, one of them is correct.
JOSH: I think it's six (as counting square units).
I: Yeah, it's six. And that's because that it's the number of squares you were counting.
JOSH: Yeah, I think maybe I just miscounted or something, 'cause it seems like I've done this in classes.
I: Okay.
JOSH: And we used this same process. Because we used -- in our high school Physics we used the velocity and time stuff a lot.
I: And so you came to this class knowing that it was area under the curve.
JOSH: Yeah, yeah.

I: Okay. And that was from Physics.
JOSH: Yeah, and I didn't know anything about integrals or anything like that, because I didn't have Calculus in high school, but because of Physics I knew the area under the curve.

I: So, number seven you gave two different responses and one of them was correct.
JOSH: Is it (E)?
I: Uh huh.
JOSH: I don't know what I was thinking over here. I might have just been thinking that this is like a velocity versus time graph, instead of position.
I: Uh huh.
JOSH: This would make more sense.

Joe

Joe was in the Individual, Non-CBL treatment. Joe thought kinematics lab was helpful. Joe worked with others inside and outside of class.

Table 30. Results of Pre- and Post-Test, Joe

	Obj. 1	Obj. 2	Obj. 3	Obj. 4	Obj. 5	Obj. 6	Obj. 7	Total
JOE								score
Post-Test	2	2	2	0	1	0	1	8
Pre-Test	1	1	1	0	1	1	1	6
Difference	1	1	1	0	0	-1	0	2

	Lack of Conception Velocity	Lack of Conception Accel.	Lack of Conception Area	Mis-conception Syntactic	Mis-conception Linguistic	Mis-conception Iconic
JOE						
Post-Test					indicated	
Pre-Test			indicated			indicated

The following are excerpts from the interview with Joe:

I: Okay, so number two, as we said before, there's two responses and one of them is correct.
JOE: Well you have the velocity, so you want to find the total distance, which is the area of the velocity curve. So you would have 3 x 4, so 6.

I: Number five, two different responses and one of them is correct.

JOE: Okay. Let's see, instantaneous acceleration, times 65. So, basically I think what this is the velocity graph and if you want the instantaneous acceleration you want the slope at the point 65.

I: Right.

JOE: So, that looks like it's about, oh, 10 over 10, it's about one meter per second square.

I: Right. So, do you have any idea what you were possibly doing on the two meters?

JOE: Umm, I don't know. I'm not really sure, actually. I think maybe for some reason I just -- because I think I would have known that -- I think maybe just for some reason I counted like maybe half of this, or ...

I: Over one, up two?

JOE: Yeah, for some reason, yeah.

I: Okay, on number six, two different responses and one of them is correct.

JOE: Okay. Well it would be this one, or part (D), because the object - this is a position graph, and over time the position isn't increasing for like the first whatever seconds. And then you've got a downward slope which is negative velocity, or decreasing velocity.

I: Right.

JOE: And then it levels off. So it stops again.

I: Yeah, so like you say, (C) would be wrong because it's not moving at a constant velocity.

JOE: Yeah. If this were a velocity graph then yeah.

Kate

Kate was in the Individual, Non-CBL treatment. Kate thought the homework, the kinematics lab, and Diane were all helpful. Kate worked with others inside and outside of class.

Table 31. Results of Pre- and Post-Test, Kate

	Obj. 1	Obj. 2	Obj. 3	Obj. 4	Obj. 5	Obj. 6	Obj. 7	Total
KATE								score
Post-Test	1	2	2	1	2	1	2	11
Pre-Test	1	2	1	0	0	1	0	5
Difference	0	0	1	1	2	0	2	6

	Lack of Conception Velocity	Lack of Conception Accel.	Lack of Conception Area	Mis-conception Syntactic	Mis-conception Linguistic	Mis-conception Iconic
KATE						
Post-Test						
Pre-Test			indicated	indicated		

The following are excerpts from the interview with Kate:

I: Okay, so this one (number 7) you gave two different responses and one of them is correct.
KATE: Okay.
I: And so again, can you see which one it is? This is number seven.
KATE: Okay, so it starts from rest and undergoes positive, constant acceleration for ten seconds, which means that it should be a straight line. Is that right? Position times - and it's accelerating - oh no, that's not necessarily true. I'm thinking this was velocity, it would be a straight line, but that's not - that's position. So... okay, then it continues at a constant velocity - it's this one.
I: Yeah. Because this one actually has zero velocity at that point.
KATE: Right.
I: So it's not continuing on.
KATE: Was this my first choice?
I: Right.
KATE: Okay.
I: So do you know as far as with this one what you were possibly thinking at the time was?
KATE: I think I was probably thinking that this was velocity instead of position.

I: So now we're on number eight, and one of them is correct.
KATE: This one, obviously, because it's the same as that first problem.
I: Right. So again we're looking at the area.

KATE: It's the area under the graph because it's constant acceleration.
I: Good.
KATE: I'll see what I was thinking. I don't know, let's see. Acceleration... was this my second answer?
I: Uh huh.
KATE: Okay. 'Cause that didn't click, I mean when we started talking about areas under the graph - that's probably why I ...
I: Right, so you just hadn't had any of that yet.
KATE: I don't think I connected that there. I can't really tell you what else I was thinking.

I: This one again, one of them is correct.
KATE: Number nine?
I: Yeah, number nine.
KATE: Okay, so this is the position, it's going from zero to 2, and... it has a constant velocity. Well, they're both constant there, so that tells me a lot! And then from 2 to 4 it doesn't move, so the velocity is zero, and they're both the same there. And then from 4 to 5 it is a negative acceleration -- no! It's going backwards, so it's decelerating. Can we say that?
I: Yeah, or actually, you know you were saying this is constant and this is positive, right?
KATE: Right.
I: So this is going to be exactly the same except for that what's the slope of this line, is it positive or negative?
KATE: Negative.
I: Okay.
KATE: So that's why this one's right.
I: Yeah.
KATE: Well it was good that I got up to here, on both of them.
I: Right, right.
KATE: Yeah, so this one is just a little bit steeper than this one.
I: Right.
KATE: And I remember thinking that, right, because this is less steep, so this one is larger.
I: Right.
KATE: Right. Okay. Yeah, I'm not quite sure what I was thinking then (pre-test), probably just... I don't know.
I: Was it maybe just not even thinking about that this is negative here?
KATE: Yeah, that could be. Just thinking that the velocity was - or it was moving again, so it had a positive velocity. I imagine, yeah. It's kind of good for me to see why I think I chose what I chose. It's hard to remember, sometimes, what you were

thinking, but, like I really can remember what I was thinking on that one, you know.

KATE: Okay, number ten. Was one of these right?
I: Yep.
KATE: Okay, I didn't think this then, but now I should probably think about the space under the graph, huh?
I: Yeah, that's one way. The only problem with that is if we were going to go space in the graph this would be zero - each one of these would start at zero. If they started at zero then we could do just strictly that. So, kind of the way you might want to think of it is, okay, so here's the derivative, which one of these functions gets this for a derivative?
KATE: So this is the anti-derivative of this.
I: Right.
KATE: Okay. Right, so this is the slope of this graph.
I: Yeah.
KATE: Which is zero, and negative something - whatever.
I: Right.
KATE: Which would account for both of those. Oh wait, what am I doing?
I: Right - so does this one, does the slope really ever change?
KATE: No.
I: The slope was zero there, the slope is zero there.
KATE: Right. So this one's just wrong.
I: Right.
KATE: I don't know what - that was the same as that one. I hope this was my first test.
I: Right.
KATE: Okay. Yeah, because that's exactly the same. What was I thinking with that one? Okay, let me just go through this in my head. Wait, why is it zero here?
I: What's the slope of that, between 3 and 4?
KATE: Zero.
I: Right.
KATE: But that doesn't make sense in my head.
I: Okay, so the velocity is 3 ...
KATE: Oh, it's zero!
I: Right.
KATE: Because the velocity's constant so there is direct acceleration. And then it's slightly increasing. Or a small acceleration.
I: Right.
KATE: From here to here, and then it pretty much stops at 5. Okay. Was there something I should have been able to spot about this 2, though, crossing here?

I: Not necessarily, uh uh..

KATE: I don't know if I can pinpoint anything better than that. It did help when I kind of caught on to the area under the graph.
I: And so was that in the homework?
KATE: Yeah, I suppose... yeah, we did quite a bit of that, I guess. But it was more like in the Riemann sum kind of a context.
I: Right.
KATE: Which I don't know if I really related to this.
I: Right.
KATE: You know? I don't really think I did. I mean I know that's where maybe I got that concept, but I don't that I put these in the same kind of categories. We spent a lot of time on that Riemann sum bit, but I don't know that I ...
I: But once the Riemann sums the area under the curve concept sort of ...
KATE: Yeah, I don't think I related that a whole lot to this.
I: That's interesting for me, as far as pinpointing it. I think that might be true, that the course tends to emphasize area under the curve - at that point - before you're really doing a lot of integration, and then that kind of starts to take a background position where it's not really brought up again.
KATE: Right. Like now.
I: Right.
KATE: I wish I could give you more reasons why, but I don't really know why.
I: Somehow or another the concept just sort of got there.
KATE: Yeah. I think probably a lot of it was Diane and the lab and probably homework. Sadly enough, probably not Logan a whole lot. But that's just me, like I said, 'cause I go kind of slow. And I do go over things myself. It may not be even the homework problems, but going through the book or going back over a lab or something.

Ted

Ted was in the Individual, Non-CBL treatment. Ted thought the lectures were helpful and that working with someone on the lab was the most helpful. Ted worked with others in class.

Table 32. Results of Pre- and Post-Test, Ted

	Obj. 1	Obj. 2	Obj. 3	Obj. 4	Obj. 5	Obj. 6	Obj. 7	Total
TED								score
Post-Test	1	2	1	0	1	1	1	7
Pre-Test	0	0	0	0	1	1	1	3
Difference	1	2	1	0	0	0	0	4

	Lack of	Lack of	Lack of	Mis-	Mis-	Mis-
	Conception	Conception	Conception	conception	conception	conception
	Velocity	Accel.	Area	Syntactic	Linguistic	Iconic
TED						
Post-Test			indicated			
Pre-Test		Indicated	indicated		indicated	

The following are excerpts from the interview with Ted:

I: And for number three, two different responses and one of them is correct.
TED: Umm... position is constant. The instantaneous velocity would be zero.
I: Uh huh.
TED: So it's wrong.
I: Right.
TED: It would have to be that one.
I: Yeah. And this one actually has the steepest slope of any of them.
TED: Right.

I: Number four, two different responses, one of them is correct. And you might even want to just say, as far as how you go about computing this.
TED: Well, I... hmm... I looked at the slope as being about 10, is what I ended up doing. The slope of the line, from 60 to 90, as being about 10. And also from 90 to ...
I: Oh, okay, so 10 over 30.
TED: Is what I got for this answer.
I: Right. Right. That's fine. And that's correct. Do you know what you were thinking at all, as far as when you answered (D) to number four?
TED: I probably just looked at the change in the velocity.
I: Ah! Okay. So, it was just like it went from 10 to 20, so you just said 9.8?
TED: Yeah, I think that's probably what I figured out.

I: Twelve, one of those responses is correct.
TED: It's (B).

I: Yeah.
TED: I probably thought (E), without thinking the first time.
I: And then actually, when you took this test initially, I mean, you hadn't learned anything about area under the curve yet.
TED: Right. Well I had, but I didn't think about that.
I: Oh, okay. So where had you heard that before?
TED: I took Calculus in high school, about four years ago.

I: And were you there when they did the lab on kinematics?
TED: Yeah, yeah I was. That was a long lab. It helped, actually, it helped a lot because the person I worked with understood it really well, and so it was coming to her fairly fast. I started picking up the material better.
I: Okay, so you were able to work with someone else?
TED: Yeah, and we kind of checked each other's work as we went through it and made little mistakes, and I think that was the biggest help.

Dan

Dan was in the Individual, Non-CBL treatment. Ted thought working with the graduate teaching assistant Diane was helpful. Dan worked with another student in class and alone out of class.

Table 33. Results of Pre- and Post-Test, Dan

	Obj. 1	Obj. 2	Obj. 3	Obj. 4	Obj. 5	Obj. 6	Obj. 7	Total
DAN								score
Post-Test	0	0	2	1	0	1	1	5
Pre-Test	0	0	0	0	0	0	0	0
Difference	0	0	2	1	0	1	1	5

	Lack of	Lack of	Lack of	Mis-	Mis-	Mis-
	Conception	Conception	Conception	conception	conception	conception
	Velocity	Accel.	Area	Syntactic	Linguistic	Iconic
DAN						
Post-Test	indicated					
Pre-Test			indicated		indicated	

The following are excerpts from the interview with Dan:

I: So here on number 1 you gave two different responses on the two different tests.
DAN: Yeah.
I: So, do you know which one is correct?
DAN: I think (B) is correct.
I: Uh huh. And how do you go about figuring that out?
DAN: Actually, the first time I just think about the parts being the time, length of time, so I just it like this.
I: Oh, okay.
DAN: Yeah, but the constant variation from the time, so I just figured out number (B) is correct.

I: Okay, on number (2) you answered two different things, and one of them is correct. So, do you know which one that is and how you go about figuring it out?
DAN: I'm not sure about number (2), actually. But... I think I just 3 x 4 is the work, or something, so... because ...
I: So that's how you came up with the answer of 12.
DAN: Yeah, yeah.
I: And how did you come up with the answer of 6, do you remember that?
DAN: I think I was just computed about the ... the time between the distance and velocity, so I just make...
I: Okay, so velocity, so the relationship between velocity and position is... velocity is the derivative, right. So, position would be the anti-derivative of velocity, or the area under the curve?
DAN: Yeah, that's right. So, I just computed the velocities there.
I: Yeah. Actually, what you were saying as far as 3 x 4, when you have constant velocity - that's actually correct. But here the velocity's changing, so that's why the 12 actually doesn't work in this case, because it's changing all the time. If it was a constant velocity... so for the first three seconds it was always going 4, right. So if this was a constant across there, then that would have been the correct answer. But since it's not constant we just count up the squares underneath.
DAN: Yeah, yeah, yeah.

I: On number 6.
DAN: I think that D is correct.
I: Yeah.
DAN: Yeah. Because I just figured out that this is negative sign, and this one is zero, and this is the other from on one place and two at the other place. I guess, so that make them moving from here to here. So I just think about increase go forward, like this, but this makes a different sign like this.
I: So that's why it's going backwards?
DAN: Yeah, yeah.

I: Okay.
DAN: Actually, I'm taking Math 251 (differential calculus) and I just used...
I: Oh, you're taking 251 at the same time?
DAN: Yeah, yes.

I: So, anything that you can remember at all, that was the most beneficial with helping with any of this?
DAN: Actually, I usually used to meet with Diane, but kind of just homework stuff, and so I don't know, but...
I: So working with Diane was more beneficial than working with other students, or...
DAN: Yeah, yeah - that's right.
I: Okay. Okay, so you used Diane a lot as far as to explain the homework and stuff, and that was the most beneficial?
DAN: Yeah, yeah.

Observations of Cooperative Group Dynamics

Six groups of students were observed and videotaped, three groups from the Cooperative Group, CBL recitation and three groups from the Cooperative Group, Non-CBL recitation. In each report of the dynamics of the group interaction, we provide a summary of the pre- and post-test results for each member of the group. Excerpts from the group's discourse are presented that provide characteristic evidence of Roschelle's Theory of conceptual change.

Cooperative Group, CBL

Group A

This was a group of three students Jack, Mary and Adam.

Table 34. Results of Pre- and Post-Test, Group A

Group A		Obj. 1	Obj. 2	Obj. 3	Obj. 4	Obj. 5	Obj. 6	Obj. 7	Total score
Jack	Post-Test	0	2	2	1	1	0	0	6
	Pre-Test	0	1	1	1	0	1	0	4
	Difference	0	1	1	0	1	-1	0	2
Mary	Post-Test	0	2	1	1	1	1	1	7
	Pre-Test	1	2	2	0	1	1	0	7
	Difference	-1	0	-1	1	0	0	1	0
Adam	Post-Test	1	0	1	0	1	0	1	4
	Pre-Test	1	1	0	0	0	1	0	3
	Difference	0	-1	1	0	1	-1	1	1

Group A		Lack of Concept. Velocity	Lack of Concept. Accel.	Lack of Concept. Area	Mis-concept. Syntactic	Mis-concept. Linguistic	Mis-concept. Iconic
Jack	Post-Test	indicated					
	Pre-Test	indicated	indicated		indicated	indicated	
Mary	Post-Test						
	Pre-Test						
Adam	Post-Test			indicated		indicated	
	Pre-Test			indicated		indicated	

In discussing problem 1 of the lab the members of the group exhibited the characteristics of Roschelle's Theory for Convergent Conceptual Change. They used metaphor:

Mary: So all of a sudden a car...how about this - a car is slowing down - at a constant rate.

Jack: A car is braking slowly.

Mary: A car is slowing down to make a right hand turn.

They displayed knowledge and that display was either confirmed or repaired. However we noted that most of the displays of knowledge were given by Mary and that almost all of the discourse was between Mary and Jack:

Jack: So it's slowing down constantly right? So it'll gain less each time.
Mary: Oh! It has a constant acceleration of <u>negative</u> 0.5. I didn't see that. Our velocity graph is wrong.
Jack: It would go down right?
Mary: Yeah.
Jack: To what?
Adam: Well it doesn't matter what it is. It's constant isn't it?
Mary: No it's going down. It has negative acceleration so its decreasing. I don't think we need exact values.
Jack: Straight across or down?
Mary: Down! Like this...the down is probably...
Jack: Is this too harsh?
Mary: You say he stopped moving.
Jack: So too harsh?
Mary: I don't know. We need to know what V is right?

Subsequently, Mary does problem 2 and then shows her work to Jack to confirm that it is correct. They can't decide what the negative acceleration in the two situations means. They ask Diane for clarification:

Mary: That one is right (referring to 2). It's coming back down a hill . We were wrong on that one (referring to 1).
Diane: What was wrong.
Mary: It's moving away with constant acceleration and we had said it was slowing down.
Diane: Okay.

So they now incorrectly believe that in both problems 1 and 2 the object is moving with a constant acceleration, that the object is speeding up in both cases.

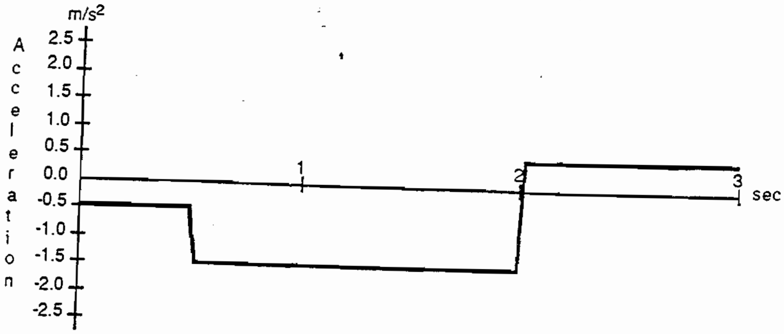
For Problem 3 they get help from Diane on how to compute the velocity graph. The discourse is similar to that above. It mostly concerns computation and most of the dialogue is between Mary and Jack. The velocity graph is drawn very well. They were able to accurately plot points. Jack draws the position graph. It appears that they were not able to accurately determine points for this graph. The position graph crosses the x-axis at the same point that the acceleration graph does for no apparent reason, but this is also where the position graph correctly changes concavity. The position graph has a shape similar to the velocity graph (see their graph in Figure 7).

It appears that Jack had an impulse towards relying on a syntactic cue, but also used his knowledge of when a graph is increasing, decreasing, and where inflection points are to draw this graph.

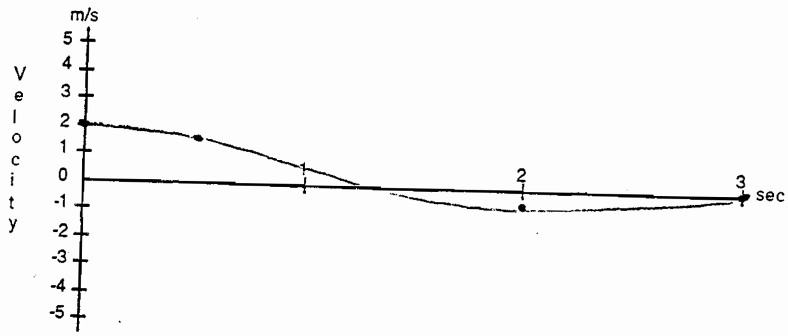
Jack and Adam work on the graphs created by the CBL-instruments while Mary works on problem 5 (which asks the group to create graphs using the area under the graph). Jack and Adam initially create a situation where one of them is walking away from the motion detector at an increasing velocity. Diane talks to them about that the person should be slowing down. They are still having difficulty and ask the researcher for assistance in creating the situations they are to monitor. They are unable to explain to the researcher how the situations would be different given negative velocity with negative acceleration and given positive velocity and negative acceleration. They spend most of their time discussing how to achieve the correct initial velocity. See their graph for problem 4-1.c) in Figure 8.

They do not pay attention when Diane attempts to have a classroom discussion towards the end of class.

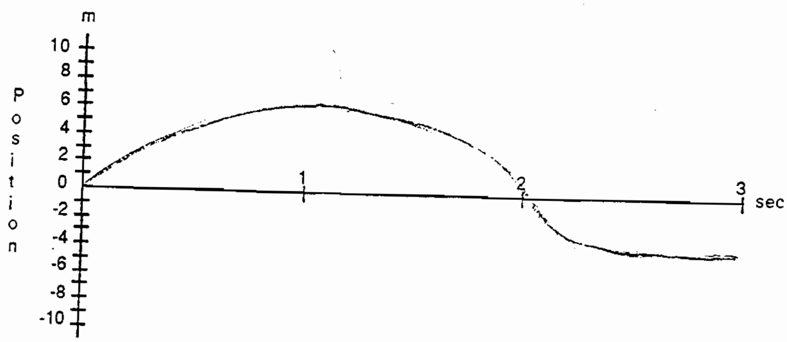
3. Here is the graph of another object's acceleration over time for $0 \leq t \leq 3$ seconds:



a) Assume that this object has an initial velocity of 2 m/s. Sketch a graph of the object's velocity for $0 \leq t \leq 3$ seconds:



b) Assume that this object starts at the position 0 m. Sketch a graph of the object's position for $0 \leq t \leq 3$ seconds:

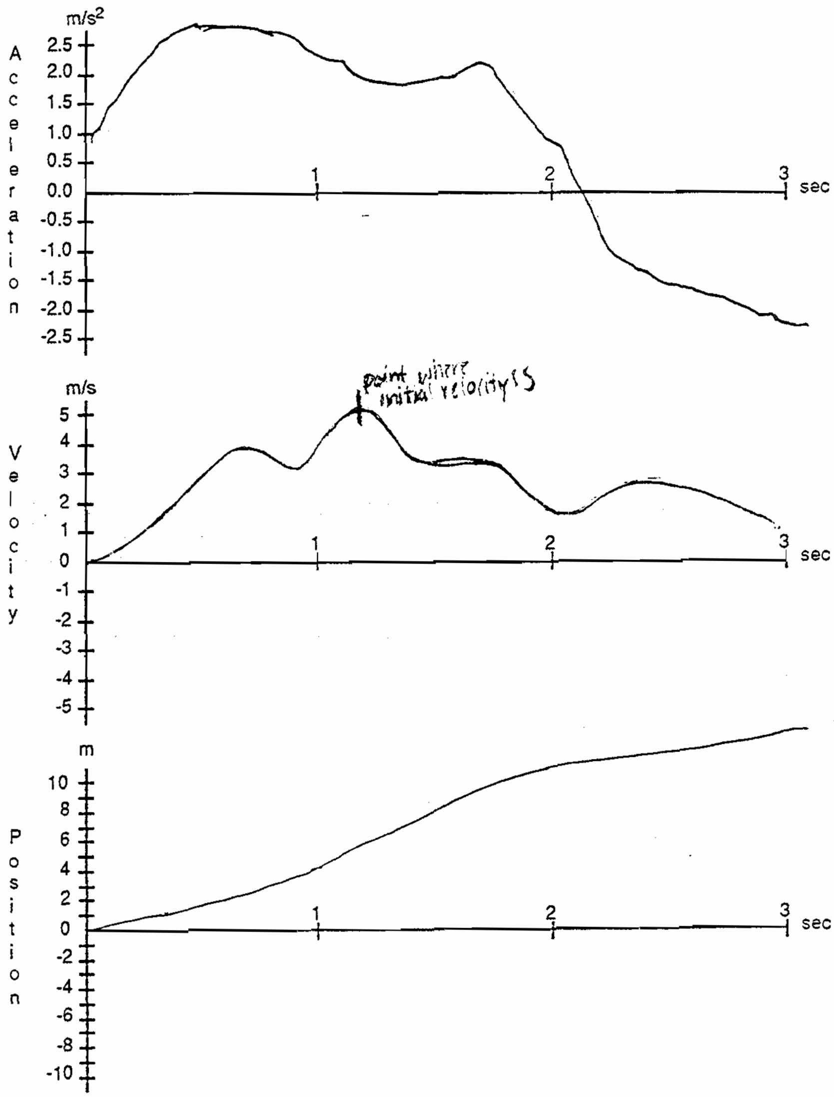


c) Suppose that these graphs describe the motion of a toy car (or a person walking) being monitored by a CBL system. Describe in words how the car is moving and describe a situation which might account for this motion. *This toy car goes past a point in a positive direction, then goes back past the same point in a negative direction. The car starts and stops at zero.*

Figure 7. Kinematics Lab, Problem 3, Group A

4. Your group will use the CBL system to create situations similar to those you described in 1.c), 2.c). and 3.c). Sketch the acceleration, velocity, position graphs produced by the CBL system in each case, and explain any differences you see between the graphs you predicted and the CBL graphs.

Graphs produced by the CBL for situation 1.c)



Explanation of differences between your predicted graphs and these:

Figure 8. Kinematics Lab, Problem 4, Group A

Group B

This group consists of four students: Sara, Amy, Todd, and Brad.

Table 35. Results of Pre- and Post-Test, Group B

Group B		Obj. 1	Obj. 2	Obj. 3	Obj. 4	Obj. 5	Obj. 6	Obj. 7	Total score
Sara	Post-Test	2	0	0	1	1	2	2	8
	Pre-Test	2	2	0	2	2	1	2	11
	Difference	0	-2	0	-1	-1	1	0	-3
Amy	Post-Test	1	0	0	0	0	1	0	2
	Pre-Test	0	1	0	0	0	1	1	3
	Difference	1	-1	0	0	0	0	-1	-1
Todd	Post-Test	1	0	1	0	0	1	0	3
	Pre-Test	0	1	1	0	0	0	0	2
	Difference	1	-1	0	0	0	1	0	1
Brad	Post-Test	1	2	1	1	1	1	1	8
	Pre-Test	1	0	1	1	0	1	1	5
	Difference	0	2	0	0	1	0	0	3

Group B		Lack of Concept. Velocity	Lack of Concept. Accel.	Lack of Concept. Area	Mis-concept. Syntactic	Mis-concept. Linguistic	Mis-concept. Iconic
Sara	Post-Test			indicated			
	Pre-Test						
Amy	Post-Test			indicated	indicated	indicated	indicated
	Pre-Test			indicated			indicated
Todd	Post-Test			indicated		indicated	
	Pre-Test	indicated		indicated		indicated	
Brad	Post-Test						
	Pre-Test		indicated				

This group displayed characteristics of Roschelle's Theory for convergent conceptual change. They converged to a misconception that negative acceleration always implies an object is slowing down, even when the velocity is negative. During their discussion of question 2 they never come to realize that negative velocity and negative acceleration indicate that the object's speed is increasing, not that the object is slowing down. The misconception is displayed and never repaired. This misconception would be evidenced by reliance on a linguistic cue. The following is an excerpt of their discussion of problem 2.

Knowledge displayed:

Brad: It's at negative.
Todd: And it's still decelerating so...
Brad: It's going to go like that...okay, not quite like that (pointing at drawing on lab).
Amy: Not with those ups and downs there.
Todd: If velocity is negative than its going in reverse.
Sara: It's slowing down and going in reverse.
Brad: What do you call that?

Confirmed:

Todd: That's right.
Sara: What? This is right - it just shifted.
Brad: The best fit line, that's the one that you take there.
Todd: No! If velocity is negative, then it's going in reverse.
Sara: It's slowing down and going in reverse - both.
Todd: Okay.
Sara: Okay with you? Negative 2. The distance starts at zero.

Metaphor (hand motion):

Amy: It's going to swing up...yeah, it's going to go like that (swings her hand up like half of a parabola). Are you just guesstimating or do you know?
Sara: You can't do it...
Amy: That's cool.
Todd: Is that position?
Sara: Well wait.

Metaphor (going in reverse):

Todd: Yeah, it's going in reverse than the position is going to be negative from where it started.

Figure 9 shows their graphs, indicating that when they completed the problem they still thought the car was slowing down.

As they move on to problem 3, Brad dominates the displays of knowledge. He does all the computations to get the velocity graph. Amy does ask Brad to explain how he is getting the graph and Brad explains what he is doing to Amy, but she does not appear to fully understand his explanation. She does not question him further. After Brad has worked through problem 3 algebraically, they go back and redo problems 1 and 2 algebraically, but never change their view that the car in problem 2 is slowing down. Brad continues working on problems 2 and 3 while Sara, Amy and Todd start working with the CBL-instruments on problem 4.

They create the situation which they monitor on top of their table. The table top does not give a large enough range for the motion detector to collect data on the toy car that they are working with. They are not sure why the graphs they are getting on the TI-82 calculator look the way they do. They are unsure whether or not they are supposed to get graphs using the CBL-instruments that match their predictions exactly, but decide that if they have to, they can report that the graphs matched exactly whether they do or not:

Sara: Are we supposed to get the graphs to match exactly?
Todd: We can always draw the graph like we got it exactly.

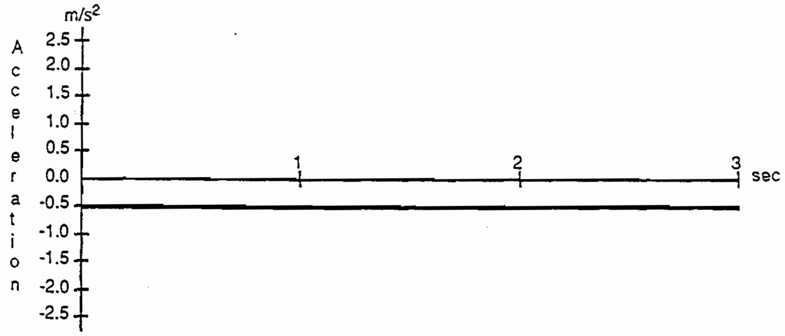
They get assistance from Diane and from the researcher on how to operate the CBL-instruments. They move to the floor so that they have a wide enough range for the motion detector to collect data on the moving toy car. After they have set the instruments up, they begin to have trouble with the CBL-instruments not working correctly. All connections are checked by the researcher but the instruments do not seem to be working. They are given a new set of instruments. They push the toy car away from the motion detector and after five trials, they achieve the graphs they want for problem 4-1.c.

The situation they set up for problem 4-2.c has the toy car being pushed towards the motion detector. It takes them numerous attempts to get graphs that they consider satisfactory. Once they have graphs that are satisfactory, they do not recognize that the velocity graph is increasing instead of decreasing and that the acceleration graph is positive. This is possibly due to the window they used on the TI-82 for these graphs, and also the difficulty of getting smooth, accurate graphs with the CBL-instruments.

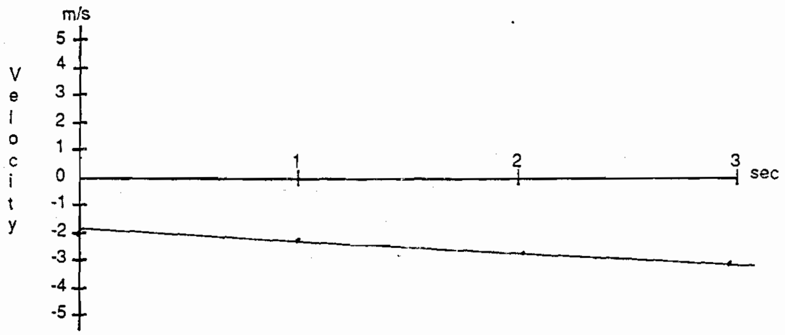
The situation they set up for problem #4-3.c is a person walking away from the motion detector, and then back towards it. They seem to be doing this appropriately, but the microphone did not pick up their dialogue. It again takes them numerous tries and they have problems with operating the CBL-instruments. The connections become loose and must be constantly checked.

Most of the CBL simulations are done by Sara, Amy, and a student from another group. Todd and Brad do very little with any of the simulations. As stated before, Brad finishes problem 3 when the other group members are first running the simulations and he may also be the group member who completed problem 5.a) for interpreting area under the graph. This group did not start problem 5.b). This group spent the last 10-15 minutes of class talking about subjects other than math. They do not stop talking or pay attention when Diane attempts to have a classroom discussion concerning the graphs.

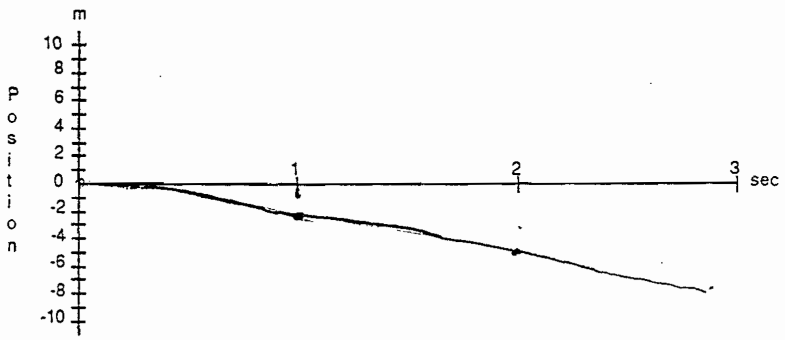
2. Another object also moves with a constant acceleration of $-0.5\ m/s^2$ for $0 \le t \le 3$ seconds as shown in the graph below.



a) Assume that this object has an initial velocity of -2 m/s. Sketch a graph of the object's velocity for $0 \le t \le 3$ seconds:



b) Now assume that the object starts at the position 0 m. Sketch a graph of the object's position for $0 \le t \le 3$ seconds:



c) Suppose that these graphs describe the motion of a toy car (or a person walking) being monitored by a CBL system. Describe in words how the car is moving and describe a situation which might account for this motion.

The car is moving backwards and slowing down

Figure 9. Kinematics Lab, Problem 2, Group B

Group C

There were four people in this group: Nick, John, Max and Tina.

Table 36. Results of Pre- and Post-Test, Group C

Group C		Obj. 1	Obj. 2	Obj. 3	Obj. 4	Obj. 5	Obj. 6	Obj. 7	Total score
Nick	Post-Test	*	*	*	*	*	*	*	*
	Pre-Test	*	*	*	*	*	*	*	*
	Difference								
John	Post-Test	1	0	0	0	1	2	0	4
	Pre-Test	*	*	*	*	*	*	*	*
	Difference								
Max	Post-Test	1	0	1	0	0	1	0	3
	Pre-Test	1	0	0	0	0	1	1	3
	Difference	0	0	1	0	0	0	-1	0
Tina	Post-Test	1	0	0	0	1	2	1	5
	Pre-Test	1	0	0	0	1	1	0	3
	Difference	0	0	0	0	0	1	1	2

Group C		Lack of Concept. Velocity	Lack of Concept. Accel.	Lack of Concept. Area	Mis-concept. Syntactic	Mis-concept. Linguistic	Mis-concept. Iconic
Nick	Post-Test	*	*	*	*	*	*
	Pre-Test	*	*	*	*	*	*
John	Post-Test		indicated	indicated			
	Pre-Test	*	*	*	*	*	*
Max	Post-Test		indicated	indicated		indicated	
	Pre-Test			indicated			
Tina	Post-Test			indicated			
	Pre-Test			indicated			

* Test was not taken

Nick did not take the pre- or post-test. John took the post-test only and the results indicated that he had a lack of conception of acceleration as the slope of a velocity graph and a lack of conception for interpreting area under a graph. Max's pre-test indicated that he had a lack of conception for interpreting area under a graph. His post-test indicated he still had this lack of conception as well as a lack of conception of acceleration as the slope of a velocity graph and a misconception based on his use of linguistic cues. Max scored three points on both the pre- and post-test and answered seven of the fourteen questions exactly the same. Tina's pre- and post-test both indicated she had a lack of conception for interpreting area under a graph but no other misconceptions or lack of conception.

Nick displayed use of iconic translation on two occasions, first when discussing problem 1:

John: It's accelerating at negative 0.5. I don't know how to do this.
Max: It's decelerating.
Nick: It starts at four, so it's going downhill.
Max: Goes down from four to three.

Nick: Now for the position graph. It starts at zero and goes up at 4 m/s.
John: It doesn't make any sense to me.
Nick: It starts at four and goes down 0.5 per second so from four to 3.5...now for position it starts at zero and goes up four per second...We should probably ask her.
Max: Which one you having trouble with - did you get to the bottom? It could be friction or something that's slowing it back down.
Nick: Or gravity. Like you could push it up a hill and then it could come back down or something with the graphs that we're drawing.

The next time Nick appears to use iconic translation is when the group is discussing problem 3:

John: This should never go past the x-axis because you have the acceleration is like, you have a car and it's slowing down and it's speeding up, it's never going to come back.
Nick: This is just the way they're doing it. It's going forward and coming back and going forward (has car in hand).
John: Yeah but the acceleration would...

Nick: I'm not putting anything in there about how...it could be like a bouncing ball.

This group converges to a misconception that negative acceleration implies the object is slowing down, even when the velocity is also negative. The following is an excerpt from the group's discussion of problem 2:

Metaphor (motion described in terms of the detector):

John: So here's your detector so doesn't that just mean that the car is here, going here?
Nick: It's decelerating.
John: It's going in this direction.

Knowledge displayed:

Max: It's decelerating.
Nick: I guess we start it negative, I guess that's what they want. I don't think it'll come back, I think it'll just keep going down.
John: Will it stop?
Nick: Well no, if its going in a negative direction it won't come to a stop.
Max: Well it's just for three seconds.
John: Well if it's got negative acceleration its got to come to a stop sometime.
Nick: Well I don't know if they're considering...
Max: You don't know how fast it's going in the first place.
John: This has negative velocity of 2 m/s.
Max: Plus the negative acceleration, so it's negative 2.5.

Knowledge Confirmed:

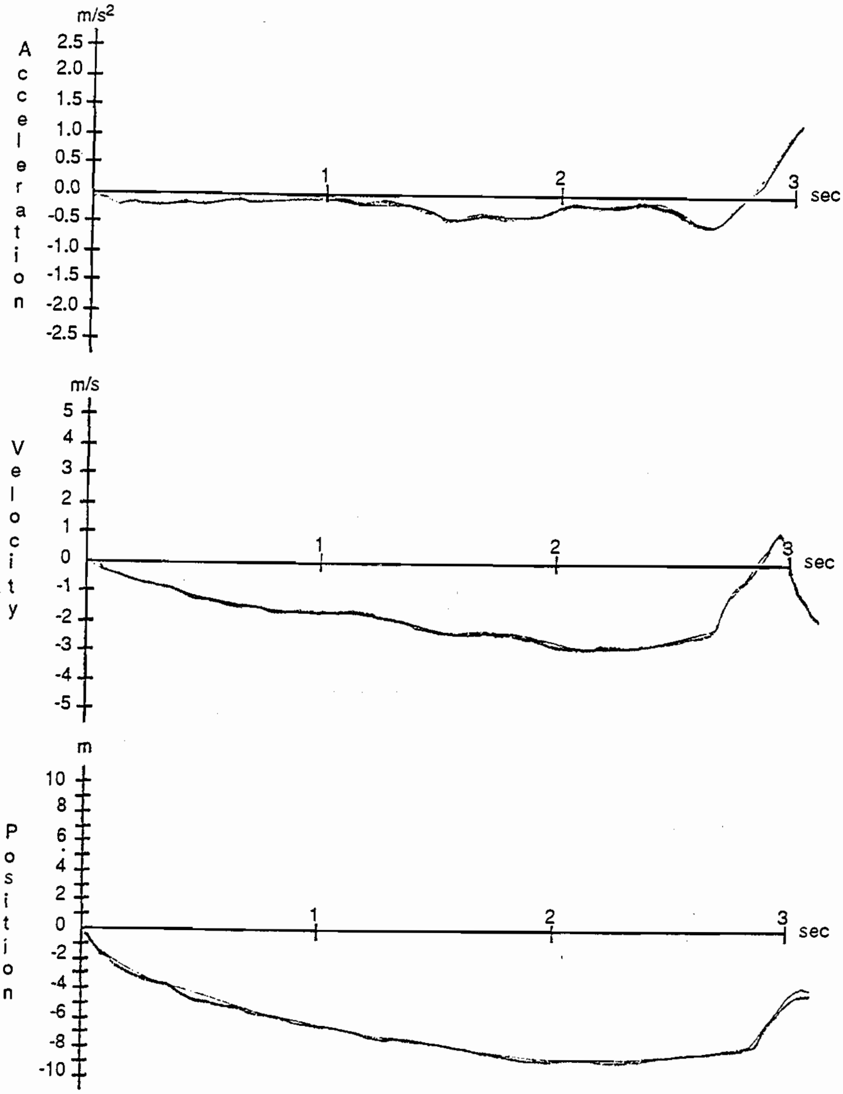
Nick: That's what I'm say. So it's accelerating like slower in way. It'll go form negative 2.5 to negative 3.

The above shows evidence of the characteristics of Roshcelle's Theory for convergent conceptual change, but again the convergence is to a misconception. This group continues to maintain that this is a graph of an object going in a negative direction and slowing down, even though Max and Nick have clearly stated that the velocity is going from negative 2 to negative 2.5 to negative 3. The situation that they create with the CBL-instruments is pushing a car towards the motion detector. The graphs they claim to have created with the CBL-instruments are given below. The acceleration and velocity graphs do not reflect the situation this group monitored with the CBL-tools. At one point Max states, "We'll just use the best graph from each trial."

This group does not attempt to do problems 5.a) and 5.b) as they run out of time. When Diane attempts a classroom discussion about the graphs, this group is working with a toy car going up a ramp in an attempt to complete problem 4-3.c). They continue to work with the CBL-instruments and do not pay attention to Diane.

4. (continued)

Graphs produced by the CBL for situation 2.c)



Explanation of differences between your predicted graphs and these:

Figure 10. Kinematics Lab, Problem 4, Group C

Cooperative Group, Non-CBL

Group D

There were three people in this group: Ryan, Rose, and Jim.

Table 37. Results of Pre- and Post-Test, Group D

Group D		Obj. 1	Obj. 2	Obj. 3	Obj. 4	Obj. 5	Obj. 6	Obj. 7	Total score
Ryan	Post-Test	2	2	2	2	2	2	2	14
	Pre-Test	2	0	2	2	2	2	2	12
	Difference	0	2	0	0	0	0	0	2
Rose	Post-Test	1	1	0	1	0	1	1	5
	Pre-Test	1	1	0	0	0	0	0	2
	Difference	0	0	0	1	0	1	1	3
Jim	Post-Test	2	2	1	2	2	1	0	10
	Pre-Test	2	2	0	1	2	0	0	7
	Difference	0	0	1	1	0	1	0	3

Group D		Lack of Concept. Velocity	Lack of Concept. Accel.	Lack of Concept. Area	Mis-concept. Syntactic	Mis-concept. Linguistic	Mis-concept. Iconic
Ryan	Post-Test						
	Pre-Test						
Rose	Post-Test			indicated			
	Pre-Test			indicated			indicated
Jim	Post-Test						
	Pre-Test			indicated			

This group moves pretty quickly through the lab. They realize that the object in problem 2 has increasing speed, even though the graph is decreasing. They show evidence of characteristics of Roschelle's Theory for convergent conceptual change in that they display knowledge which is either confirmed or repaired. The use of the words "speeding up" may be considered use of metaphor. The following are excerpts from their discussions on problems 1 and 2:

Jim: It (velocity graph 1) would be a straight line, not straight horizontally, but linear. Right? Because it's always decreasing.
Ryan: No. All this says is that it has an initial velocity of 4 m/s, sketch the graph. Wouldn't it be straight because it's the velocity graph?
Jim: No, because this is negative acceleration.
Rose: Doesn't it mean...
Ryan: Is it the same object?
Jim: Yeah.
Rose: Doesn't this mean that it's constantly decelerating?
Jim: Yeah.
Ryan: So it's decelerating at 0.5 seconds.

Ryan: (on problem 2) I think it will start out at negative two and end up at negative 5 slope. It'll get steeper.
Rose: Okay, this time it'll start at negative two and it increases.
Jim: Negatively.
Rose: Negatively.
Ryan: It's going in a negative direction.
Rose: It's going in a negative direction right?
Ryan: Yeah, it's speeding up.
Jim: It's speeding up.
Rose: Okay.

They work through problem 3 in a similar fashion, using metaphor (going back up), displaying knowledge, and having that knowledge confirmed or repaired appropriately.

Jim: But then it's going to go back up right?
Rose: Yeah.
Jim: After two seconds.
Ryan: After two seconds it's going to start going back up.
Jim: It'll have positive slope.
Ryan: But it's not going to go all the way back up to zero 'cause look at the area under this and the area under this. So it'll still be negative.

The group quickly completes problems 4.a) and 4.b). Ryan does most of the work on these with Jim confirming his computations. They recognize quickly the similarity between these problems and problems 1 and 2.

The group has difficulty with problem 4.c). They are not sure what to do with the three equations. They think that possibly they should add them. They are unsure what to do and then begin talking about subjects unrelated to math. This discussion goes on for about five minutes. They then get the teaching assistant's help. After receiving help they return to their discussion of the non-math topic. They finish problem 4.c) during Diane's classroom discussion of the graphs. Following completion of problem 4.c), Ryan begins work on problem 5. Jim leaves because he has another class. Ryan explains problem 5 to Rose.

Group E

There were three students in this group: Ian, Jane, and Sam.

Table 38. Results of Pre- and Post-Test, Group E

		Obj.	Obj.	Obj.	Obj.	Obj.	Obj.	Obj.	Total
Group E		1	2	3	4	5	6	7	score
Ian	Post-Test	1	0	1	1	0	1	0	4
	Pre-Test	1	0	0	0	0	0	0	1
	Difference	0	0	1	1	0	1	0	3
Jane	Post-Test	1	2	0	2	2	2	2	11
	Pre-Test	2	2	0	1	1	2	0	8
	Difference	-1	0	0	1	1	0	2	3
Sam	Post-Test	2	2	2	1	2	2	2	13
	Pre-Test	2	2	2	2	1	1	1	11
	Difference	0	0	0	-1	1	1	1	2

		Lack of	Lack of	Lack of	Mis-	Mis-	Mis-
		Concept.	Concept.	Concept.	concept.	concept.	concept.
Group E		Velocity	Accel.	Area	Syntactic	Linguistic	Iconic
Ian	Post-Test						indicated
	Pre-Test			indicated			
Jane	Post-Test			indicated			
	Pre-Test			indicated			
Sam	Post-Test						
	Pre-Test						

Ian's pre-test indicated he had a lack of conception for interpreting area under a graph. His post-test indicated that he had built this conception but had a misconception based on his use of iconic translation. Jane's pre-test indicated she had a lack of conception for interpreting area under a graph. Her post-test indicated she still had this lack of conception. Sam's pre- and post-test did not indicate any misconceptions or lack of conception.

This group also converges to the misconception that negative acceleration implies the object is slowing down:

Jane: So it's moving towards the motion detector this time. That's why the velocity is negative.

Sam: Yeah.
Ian: Is it?
Jane: Yeah.
Sam: Well eventually it would move back towards it cause acceleration is decreasing but...
Jane: The other one didn't move back towards it.
Sam: Eventually.
Jane: No it wouldn't - this is the motion detector - the one was moving away and was slowing down, this one is just moving towards it and slowing down.
Sam: Once the velocity get below zero then it will start moving back unless it just stops once it got to zero.
Jane: It could move back because its on a ramp. It wouldn't just automatically come back.
Sam: That's true.
Jane: Okay. The velocity is - so it's just going to go like this.
Sam: I don't think velocity would be increasing.
Jane: It's not. It's decreasing - or I mean it's getting to zero.
Ian: So once it gets there it'd be zero.
Jane: Actually no. The acceleration is negative so the slope should be negative on the velocity graph.
Sam: I think it's the exact same as last time. It's just down to negative two.
Jane: Going what?
Sam: It's the same as last time, going down at negative two - it starts at negative two and goes down.
Jane: Like this? But isn't it slowing down at a rate of negative 0.5?
Sam: It was last time too.
Jane: Shouldn't it be? So why would the velocity be getting greater? Oh it's getting less. But that zero though. I'm confused about that.
Ian: It kind of doesn't make sense. It seems kind of weird.
Jane: The velocity is never going to get to zero? That doesn't make sense.
Sam: The velocity is negative which means its going backwards.
Ian: The velocity is just getting more negative.
Sam: The velocity is negative then it is slowing down and it's slowing down even more cause the acceleration is negative and so...
Ian: It's just decelerating.
Jane: If the velocity is negative it doesn't mean its slowing down, it means it's moving towards the motion detector. Acceleration is negative it means it's slowing down, and if it's positive it's speeding up. And it's slowing down...
Sam: If the velocity is negative it means it's moving towards...
Jane: Yeah.

Ian: Yeah.
Jane: But the acceleration is negative so it should be a negative slope. I don't understand this.

This group gets Diane's help and they are able to move on to the next problem. We see in the above their use of metaphor in that all motion is described as moving towards, or away from the motion detector. Knowledge is displayed ("So it's moving towards the motion detector this time"), confirmed ("Yeah"), or displayed ("The velocity is so - it's just going like this") and repaired ("I don't think velocity would be increasing").

This group uses their new knowledge that negative acceleration does not necessarily imply that the object is slowing down and works through problem 3. This problem requires work at a higher level as exemplified when they realize the velocity graph crosses the x-axis:

Jane: So this is going to be an undefined kind of thing.
Sam: No. It just goes from two to 1.5.
Jane: So the slope is gonna get greater like that?
Sam: It's constant to two - negative 1.5 slope.
Jane: And then it goes - all of a sudden at two it goes to positive. I don't know if it's going to go above the x-axis.
Ian: I don't think it's going to go above. It just means that it turned around so it's just going to be like a parabola and at the bottom is where it turned around right? The minimum point is actually where it switched directions.
Sam: (nodding his head in agreement)
Ian: I think it's going to switch directions.
Jane: I think it goes like that. This means it's concave down to two and then concave up like that. so this means it's positive, so means increasing so at that point it...and then it's decreasing...this is so confusing.
Sam: If you use exact slope...

We see that this discourse is on a higher level. We also see a misconception displayed by Ian. He knows that the velocity graph will have a parabolic shape, with a minimum, but he states that this implies that the object will change direction at the minimum. What this actually implies is that the object 's velocity will change from a decreasing function to an increasing one at the minimum point. This is a display of iconic translation. This misconception is confirmed by Sam nodding his head. When stated again the misconception is not repaired by Jane. This may have contributed to Ian's display of iconic translation on the post-test.

This group works in a similar manner through problem 4 of the lab. They have difficulty with 4.c) where the constant changes for the three different equations. They appear to be bored with the lab.

All members of the group appear to pay attention while Diane leads a classroom discussion about the graphs. Jane participates in the discussion.

This group appears weary by the time they start problem 5. They get help from a student in another group who advises them that it will just be the same graphs as on previous problems. However they are still unsure as to how to do the problem and get help from Diane. They do problem 5.a) but not 5.b).

Group F

There were four students in this group: Joy, Lucy, Jeff, and Kyle.

Table 39. Results of Pre- and Post-Test, Group F

		Obj. 1	Obj. 2	Obj. 3	Obj. 4	Obj. 5	Obj. 6	Obj. 7	Total score
Group F									
Lucy	Post-Test	*	*	*	*	*	*	*	*
	Pre-Test	1	1	0	0	0	0	1	3
	Difference								
Joy	Post-Test	*	*	*	*	*	*	*	*
	Pre-Test	*	*	*	*	*	*	*	*
	Difference								
Jeff	Post-Test	2	0	2	1	2	1	1	9
	Pre-Test	1	0	1	0	1	1	1	5
	Difference	1	0	1	1	1	0	0	4
Kyle	Post-Test	2	2	2	2	1	2	2	13
	Pre-Test	**	**	**	**	**	**	**	**
	Difference								

		Lack of Concept. Velocity	Lack of Concept. Accel.	Lack of Concept. Area	Mis-concept. Syntactic	Mis-concept. Linguistic	Mis-concept. Iconic
Group F							
Lucy	Post-Test	*	*	*	*	*	*
	Pre-Test			indicated		indicated	
Joy	Post-Test	*	*	*	*	*	*
	Pre-Test	*	*	*	*	*	*
Jeff	Post-Test						
	Pre-Test			indicated			
Kyle	Post-Test						
	Pre-Test	**	**	indicated	**	**	**

* Test was not taken
** Test was not completed

Joy did not take the pre- or post-test. Lucy did not take the pre-test. Her post-test indicated she had a lack of conception for interpreting area under a graph and a misconception indicated by use of linguistic cue. Jeff's pre-test indicated he had a lack of conception for interpreting area under a graph. His post-test indicated he had built a conception for interpreting area under a graph and did not indicate any other lack of conception or misconceptions. Kyle only completed nine of the fourteen problems on the pre-test so there was not enough information to indicate all the misconceptions and lack of conception being considered in this study. His pre-test did indicate that he had a lack of conception for interpreting area under a graph. Kyle received a perfect score on his post-test.

This group uses metaphor not only verbally, but with hand motions as well:

Lucy: So how do you find the distance it's gone?
Kyle: So how can those be two and all that?
Jeff: This number here tells you the slope on this line.
Lucy: This is like the second derivative, this the derivative and this the function. Just think of it that way.
Kyle: I have been but...
Jeff: So the slope at zero is four, soooo, this is like weeeeeek (hand motion in the air like a straight line). That kind of thing.
Lucy: Like this? (makes a hand motion in the air like a curly cue)
Jeff: Well...
Kyle: But how do you know it started at 4?
Jeff: (pointing at paper) Right here.
Kyle: Okay, I see, I wasn't reading that. I just looked at the top part. Okay.

While working on problem 2 this group displayed the misconception that negative acceleration implies the object is slowing down. They also display their use of metaphor (car is going backwards):

Jeff: Car is going backwards.
Lucy: It can have a negative velocity?
Jeff: Well it can but he had this frame of reference.
Kyle: It all depends on how you define it.
Lucy: I know I just want you to explain why.
Jeff: Okay, well its uh, slowing down, like your car.
Lucy: Okay.
Jeff: And you're slowing down because a stop light is coming but you're going backwards.
(Lucy looks confused, Jeff tries again)
Jeff: So you're going backwards and you see a kid behind you so you slow down.

Lucy: Oh - okay. He's in reverse and he's slowing down.

This group's discourse consists of a lot of questions and answers as above. When working on part (c) of problems 1, 2 and 3 they continue to give colorful descriptions for the motion these graphs depict. The following is an excerpt from their discussion of problem 3.c):

Lucy: It just says in words describe how this could happen. Just say he's slowing down.
Jeff: Yeah.
Kyle: Hit black ice on...
Lucy: If he starts from zero...
Kyle: And slows down with constant acceleration...
Jeff: Well he is slowing down cause this is the velocity.
Kyle: Right.
Lucy: So all of a sudden he just starts off at...
Kyle: In the car...
Lucy: He has to speed up somehow right?
Joy: Well he starts off - but then when it starts to get flatter he's slowing down. This is his position, like the slope of this since he's accelerating more here or going faster here.
Jeff: Maybe he's going backwards.
Kyle: Oh no. It's just he's just decelerating.
Joy: He just goes a little bit and slows down.
Jeff: It's true he has to accelerate to start, so maybe he just maybe, he was going wooooo (hand motions in air).
Kyle: At a constant velocity.
Jeff: And then as he crosses your camera he starts slowing down.
Lucy: Okay.
Jeff: Right. Does that make sense?
Lucy: He was already going and then when he hit zero then...
Kyle: What happens is there is this huge rubber band and he's going at a constant velocity and the he hits it! he decelerates and then it stops and he goes back. Let's make up a story!
Lucy: Make it interesting.

This group has errors on their computations which are corrected during the classroom discussion led by Diane. Initially they do not pay attention to the classroom discussion until Lucy advises the group that Diane is doing the problems on the board. At that time they start to pay attention and make corrections appropriately. They do not work on problem 5 which asks the student to compute graphs using the area under the graph. Diane explains to them how to do problem 5, but it is past the time that the class is over and so she collects the work they have completed.

CHAPTER V

DISCUSSION

The relationships between a function and its derivative and its antiderivatives are predominant themes of most college calculus courses. A physical representation of these relationships exists in the form of the velocity, acceleration, and position of a moving object. All college students have some knowledge of these relationships through personal physical experience. The purpose of this study was to determine the extent of integral calculus students' understanding of kinematics and to investigate the most effective ways to build on it.

In this chapter, results from this study will be considered in the context of our research questions and what conclusions might be drawn from those results. Implications of the study for teachers and curriculum developers will be discussed. Finally, limitations of the study will be discussed followed by recommendations for future research.

Our primary goal was to better understand the dynamics by which various teaching models (alone and in concert) could be expected to improve student understanding of graphs of kinematic variables. To serve this goal, the following research questions were posed:

1. What specific difficulties with interpreting graphs of kinematic variables do students bring to the integral calculus classroom?
2. What misconceptions, or lack of conception, are indicated by the difficulties which students bring to the integral calculus classroom?
3. What is the relative effectiveness of the traditional, cooperative group, and CBL models of instruction for building conceptions, repairing misconceptions and removing difficulties with

interpretation of graphs of kinematic variables? In particular, are certain types of difficulties more readily removed by one of these instructional models?

4. How does the process of student discourse aid in repairing misconceptions? In particular, are there meaningful differences in the student discourse generated by a laboratory setting using CBL-instruments and laboratory setting using the more abstract tool of algebraic formulas? Can we find confirmation of Roschelle's Theory of conceptual convergence?

A discussion of each of the research questions follows.

Difficulties, Misconceptions, and Lack of Conceptions Brought to the Classroom

Students in this sample had particular difficulty with determining acceleration given a velocity graph, determining displacement given a graph of velocity, determining change in velocity given a graph of acceleration, and selecting a graph corresponding to a given textual description. Determining displacement given velocity and determining change in velocity given acceleration, both concern interpreting area under the curve.

Many students are first instructed how to interpret the area under a curve in the integral calculus classroom, so it is not surprising that many students might have difficulty with problems based on using this concept early in the course. These difficulties may be due to a lack of conception concerning area under the curve. Results of the pre-test indicate that 64% of the students had this lack of conception. Students' lack of conception for interpreting area under a graph was further evidenced by the absence of reference to this concept in the student discourse observed. Only one group actually made reference to this concept while working on problems 1 through 4 of the lab on kinematics. For problem 5 of the lab which asked students to sketch graphs using the concept of area under the graph, there was very little student discourse. Three of the groups (D, E and F) needed assistance from the teaching assistant to start this problem and two of these groups still did not complete it. In two groups (A and B) one group member alone did the work on problem 5. Group C did not attempt problem 5.

That students' pre-tests indicated they had problems interpreting the slope of a velocity graph is unsettling because differential calculus is a prerequisite for integral calculus so all students should have received instruction on this topic in a mathematics classroom. It is reassuring that only 17% of the students' pre-tests indicated they had a lack of conception for acceleration as the slope of a velocity graph. Some of the students having difficulty with this objective may have been trying to find a solution based on their own experience (instead of using the concept of slope which their pre-tests indicated they did have a conception of) and since students are generally less familiar with the concept of acceleration than with velocity, more difficulties were indicated. In the interviews we saw that students sometimes reasoned solutions out without using the concepts of slope, even when they understood what the slope represented. In particular, we saw students' difficulties with understanding the physical properties of acceleration represented by a graph in the videotaped observations of students working in cooperative groups.

Difficulty with objective 7 (given textual motion description, select a corresponding graph), indicates that students have problems relating a physical situation to the mathematical concepts incorporated in a graph. It would be expected that when this difficulty is due to a student's misconceptions, the student may be using iconic translation and/or linguistic cues in an inappropriate manner. We saw that more students indicated misconceptions using linguistic cues (30%) than iconic translation (19%). In the videotaped observations we also saw more misconceptions where students were using linguistic cues than we did using iconic translation.

Relative Effectiveness of Teaching Strategies

Student performance on objective 1 (given a graph of position, determine velocity) improved for students in the Individual/CBL treatment. However, for students in the Cooperative Group/CBL treatment, their performance went down on this objective. These results were duplicated for objective 3 (given a velocity graph, determine displacement) and objective 6 (given a kinematics graph, select a corresponding graph). Students who were in the Non-CBL treatments (both individual and cooperative group) performed better on objective 4 (given an acceleration graph, determine change in velocity) than students in the CBL treatments. There were no differences in student performance on the other objectives.

These results could indicate that the CBL-tools are better used by an instructor giving a demonstration than by students working in groups. However, during treatment was the first opportunity that these students had to work with CBL-instruments and the results may differ when students are given the opportunity to become more adept at using these tools.

What the results might also indicate is the importance of a discussion led by the instructor following laboratory exercises. We saw that Group F (Cooperative Group, Non-CBL recitation) converged on the misconception that the car was slowing down when it had negative velocity and negative acceleration. During the classroom discussion they listened to the teaching assistant and realized their error. Two of the three groups in the Cooperative Group, Non-CBL section listened and/or participated during the classroom discussion. None of the groups in the Cooperative Group, CBL section listened or participated during the classroom discussion. The level of activity in the room during the time the teaching assistant began this discussion made it difficult to get, and hold, the students' attention. On the other hand, when the teaching assistant began the classroom discussion using the CBL-instruments at the front of the class, she had almost all of the students' attention. Further, students were asked to volunteer to help with moving the objects or to have their motion monitored in the Individual, CBL treatment. Again, this seemed to aid in keeping students' attention. The classroom discussion held in the Individual, Non-CBL recitation was not as captivating. However, as there was very little activity other than the teaching assistant speaking at the front of the room, it was conjectured that more students were paying attention than in the Cooperative Group recitations, especially the Cooperative Group, CBL section. The researcher questioned the teaching assistant about this. To the best of her recollection, the teaching assistant thought that she had more students' attention in the Individual recitation sections than she did in the Cooperative Group, CBL section. She thought that the number of students participating in the discussion in the Individual, CBL section was the greatest and that the Non-CBL sections were about the same. In the Cooperative Group, CBL recitation she stated that she felt like she was "Talking to thin air."

The statistical analysis of students' misconceptions/lack of conception on the pre- and post-test gave similar results. For building a conception for area under a graph or repairing misconceptions related to use of linguistic cue, students in Individual/CBL treatments did better while those in the Cooperative Group/CBL treatments did worse. Again, this may indicate that CBL tools are better used by an instructor for classroom demonstrations or the importance of a classroom discussion following lab activities.

The result that students in the Individual treatment were somewhat more successful at repairing misconceptions and building conceptions than the students in the Cooperative Group treatments should also be considered with caution. The researcher made two brief observations in each of the Individual recitations which were not videotaped. The purpose of these observations was to see how many students worked alone on the laboratory assignments in these recitations. In the Individual, CBL recitation there were eighteen of thirty-nine students working alone on one occasion and ten of thirty-six students were working alone on another occasion. In the Individual, Non-CBL recitation there were eight of thirty-three students and five of thirty students working alone on the two occasions. These informal observations indicate that more than half of the students in the "so-called" Individual sections were actually working in pairs or larger groups. One conjecture may be that for college students, cooperative groups are more effective when students choose their group members as opposed to being assigned to a group. That many students find it beneficial to work with others was evidenced by the student interviews. Ted was in an Individual treatment section and found working with others very helpful in improving his understanding of kinematic graphs.

Student Discourse

The student discourse that occurred while students worked on problems 1 through 3 of the lab was not substantially different in the setting using CBL-instruments from the setting using algebraic formulas. This would be expected in that the labs in these settings were essentially identical for these problems (all students had been exposed to the CBL-instruments in lecture class).

The labs were different for problem 4. In the discourse for the groups of students that were observed, it was not seen that students successfully confronted their misconceptions in either setting. To the contrary, in the CBL settings many students completed problem 4 and retained their misconceptions. For example, in the CBL setting, Group B created a situation in which a car was going in a negative direction (towards the motion detector) and slowing down (due to friction). This would create a negative, decreasing position graph, a negative, increasing velocity graph and a positive, constant acceleration graph. Yet, the students claim to get a negative, decreasing position graph, a negative, decreasing velocity graph and a negative, constant acceleration graph. They did not confront their misconception that an object with negative acceleration is always slowing down, even when the velocity is also negative. In the Non-CBL section Group F had not started on problem 4 when the classroom discussion began. They confronted their misconception during the classroom discussion. Group E realized they had a conflict in their conceptions while working on problem 1 and 2 and got help from the teaching assistant then. Both Groups E and F were frustrated by the tedious aspects of problem 4.c), as stated by Lucy in reference to problem 4, "I'm tired. I hate this. You are way too precise, this takes way too long". The students in the CBL section got frustrated with the CBL-instruments when they did not work as expected.

Our analysis of student discourse showed evidence of many of the characteristics of Roschelle's Theory for Convergent Conceptual Change. Unfortunately, as Roschelle (1992) noted in his own work, the convergence at times was to a misconception. In cases where the misconception was displayed and confirmed, we saw evidence for it in the post-test as well. For example, in group B, two group members displayed a misconception using linguistic cues that negative acceleration means the object is slowing down. This group did not confront their misconception in problem 4 or through assistance from the teaching assistant. Two of the group members showed a misconception based on use of linguistic cue on the post-test. Further, in Group E, Ian displayed use of iconic translation which was confirmed by a group member. Ian showed further evidence of a misconception based on use of iconic translation on his post-test. The impact of student discourse is seen again in Group A, where Adam did not participate in the discourse to a great extent. Adam retained a misconception and lack of conception while Jack, who did participate in the discourse built a conception for acceleration as the slope of a velocity graph and repaired misconceptions evidenced by use of syntactic cue and linguistic cue.

Implications for Teachers and Curriculum Developers

This study has several implications for those educators wishing to use constructivist principles and tie abstract concepts concerning a function, its derivative, and antiderivative to students' knowledge of kinematics.

We saw in the observations that student discourse could be a powerful (although not always beneficial) tool in promoting conceptual change, and that this change often followed the model outlined by Roschelle's Theory for Convergent Conceptual Change. Student discourse sometimes resulted in conceptual change that was a misconception instead of a change to the concept held by mathematicians and scientists. This is one reason that classroom discussion may be one of the most influential factors in determining effectiveness of a teaching strategy for building conceptions and repairing misconceptions, supporting Dykstra's model for promoting conceptual change.

Student discourse gives students the opportunity to articulate and repair their misconceptions. Cooperative groups give students the opportunity for discourse and may be a preferred teaching tool when building on students' understanding, especially when the students' understanding include misconceptions. However, student discourse can also result in students' conceptions converging to a misconception. We saw that this could be repaired through a "whole" classroom discussion. Although the statistical results in this study might imply that "Individual" treatment was a more effective teaching strategy for building conceptions and repairing misconceptions, we cannot ignore the observed effects of student discourse and the fact that many students in the "Individual" treatment were actually working in groups and found the groups beneficial.

Student discourse gives students the opportunity to articulate their misconceptions. A misconception may be repaired through student discourse, but should definitely be confronted (and repaired) through a classroom discussion.

Articulation of the misconception, immediately followed by an instructor-led classroom discussion, gives students the opportunity to realize the need to correct their own ideas.

CBL tools may be most beneficial when used by an instructor in a classroom demonstration. We saw that students working in groups (Group B, Group C) did not realize that the results of their simulations using CBL-instruments contradicted their predicted results. This could have been due to inability to properly use the CBL-tools, a lack of trust in the results given by the CBL-instruments, or frustration with using the instruments. When using CBL-instruments in cooperative group settings, it may be desirable to give students the opportunity for instruction and practice in how to use these tools separately from the time they are to spend on laboratory activities.

This study suggests that one of the students' most strongly held misconceptions is that of negative acceleration implies that an object is slowing down, regardless of the objects' velocity (positive or negative). The reasons for this misconception may be due to students associating negative acceleration with deceleration which means "slowing down", while positive acceleration means "speeding up". We saw evidence of this in the video tape of Jane. Jane has a conception that acceleration is the slope of a velocity graph, she understands that the object is not slowing down unless the graph is approaching zero, both of which contradict that the object in problem 2 is slowing down. However, she continues maintains, in the face of contradictory evidence, that negative acceleration means the object is slowing down. This misconception should be addressed by all instructors in kinematic topics.

To repair this misconception it appears to be of benefit to develop laboratory activities that promote student discourse on this topic. After giving students the opportunity to reach disequalibrium, a classroom discussion is essential. In the discussion the instructor may want to not only correct any misconceptions, but reinforce this with a physical demonstration. Students found it helpful to have a physical occurrence to relate to the concepts.

Students in this sample also had difficulty with the interpreting the area under a curve. We saw evidence that some students had developed a concept for interpreting area under a graph, but that many others had not. We know that the students had received instruction on this concept prior to the time they were observed. Students' lack of conception for interpreting area under a graph at the time of observation may be due to it being emphasized early during discussions of Riemann sums during the course, but not during subsequent discussions on integration and antiderivatives. Teachers and curriculum developers may take the advice of several students in this study who stated they needed to hear concepts several times before they really incorporated them into their personal understanding. It is also recommended that concepts be presented in several formats, lectures, homework and laboratory activities. This is due to the result that students responded with a variety of items that they found "most helpful" for improving their understanding of graphs of kinematic variables. Also, several students stated that they needed to see things in several settings, homework, labs, and lecture before they really understood the concept.

In terms of specific improvements that could be made in the lab activities used in this study, we can make several suggestions. The lab activity used in this study asked students to calculate graphs using area under the curve on the final question. Many students did not have time to work on this problem. Since difficulty with this concept is pervasive, more student time should have been devoted to the problem. Problem 3, which gave students a changing acceleration graph, could be simplified to give students more time to concentrate on interpretation of area under a graph. Another recommendation would be to give students a laboratory activity that confronted misconceptions using linguistic cue, iconic translation, and syntactic cue in one class session and follow it with an entire class devoted to a laboratory activity interpreting area under graphs.

Finally, we saw that Group D was able to complete the lab activity rapidly and did not display any difficulty with linguistic cue on problem 2. These group members scored very high on the pre-test. This suggests the validity of the testing instrument used in this study which is a subset of test items from Beichner's TUG-K. Instructors wishing to assess their students may find either of these instruments useful.

Limitations of the Study and Recommendations for Future Research

We saw in tables 7 and 8 of the pre-test results that the Cooperative Group, non-CBL recitation section indicated fewer difficulties and misconceptions prior to any instructional treatments than the other recitations. On the pre-test measures of total score and number of misconceptions a statistically significant difference was found between the four treatment groups. See tables in Appendix D for a breakdown of these measures on the pre-test.

The initial differences between students in the four recitation sections may have affected the statistical results. The results that the students in the Cooperative Group, Non-CBL recitation performed better on Beichner's 7 objectives on the pre-test, building conceptions and repairing misconceptions could be due to they had fewer misconceptions/lack of conception to begin with. Recall that a score of 2 on the pre-test and a score of 2 (for each objective) on the post-test was counted the same as a student who actually improved their score. Students who had no difficulty to begin with were counted the same as students who improved was a choice made by the researcher as it seemed more appropriate than counting these students as having shown no improvement. This was particularly of concern on analysis of the odds ratio where not having a misconception indicated on either the pre- or post-test is considered the same as repairing a misconception (misconception indicated on pre-test, not indicated on post-test). It would be of benefit to statistically analyze treatment groups where the initial differences in the groups was not significant.

Another limitation of this study is the method by which students were diagnosed as having a misconception or lack of conception. The researcher did not notice during the interviews or observations any instances where a student was diagnosed as having a misconception or lack of conception where the diagnosis was incorrect. However, it was seen repeatedly that students had a lack of conception which was not diagnosed. This shows how difficult it to diagnose student misconceptions and lack of conceptions with only a written assessment instrument. Student interviews were a crucial triangulation measurement instrument.

Finally, the treatment category labeled "Individual" did not mean that students only worked individually in those recitation sections. One should not therefore conclude that the benefits using cooperative groups are minimal. The statistical analysis may have indicated the difference between the recitation sections in the classroom discussions and not the treatment effect of cooperative groups. The label of "Group" or "Individual" treatment may be better served by allowing students to self select into these categories. Students may self select into a category that is best for their learning style or it may be found that one treatment is actually better than the other. Future research on the use of cooperative groups in the college classroom may want to concentrate on students' learning style and preference for working in cooperative groups. Further, are cooperative groups an effective teaching strategy for students who prefer this teaching model, all students (those who prefer it and those who don't), or is it no more effective than traditional models? Research in this area could be of benefit to instructors want to know how much to encourage students to work together outside of the classroom, as well as how much to use cooperative group teaching strategies in the classroom.

In the student interviews we saw a wide range of responses as to what students found the most helpful in improving their understanding of kinematic graphs. It was interesting that one student found the professor's lectures the most helpful while several other students did not believe the material was covered in lecture. Similarly, many found the lab on kinematics helpful while one student thought it was confusing. Future research may investigate the relationship between students' learning style and teaching strategies that improve students' understanding of kinematics. This study raises the question of whether students who find a particular teaching strategy most helpful are those students who traditionally have difficulty with kinematic concepts or are those students who generally succeed at building and repairing these concepts in calculus.

Closing Comments

Student difficulty in calculus courses is well known. College graduates will frequently wince at recollections from their calculus class or openly discuss how they changed majors because they were unable to complete this course. That many college students find the concepts in a calculus course inaccessible is a situation many educators would like to remedy. Math education researchers have identified, defined, categorized and investigated pedagogical strategies to reduce student difficulties for a variety of calculus topics.

One refrain that is often heard in calculus reform is the need to better ground the abstract concepts of mathematics to real-world experiences. Building conceptual understanding on knowledge that students already have is a widely accepted constructivist principle. This principle was described well by Krussell (1994, p. 2):

> I am convinced that no one learns mathematics in a vacuum and that mathematical knowledge is best understood and retained by connecting it to previously existing knowledge. It is only when a new mathematical idea or concept is connected to an individual's existing mathematical

> knowledge - either about that concept or about related concepts - that it begins to be understood.

Moreover, a common theme to many calculus reform projects is a move to greater emphasis of graphical interpretations. The context of kinematics provides a particularly rich base of physical examples in calculus and is rooted in experiences that are shared by all students. We maintain that this research study supports the use of kinematics graph activities as a powerful setting for exposing student misconceptions and for discourse (both student-student and student-teacher) as a means of confronting and repairing those misconceptions.

BIBLIOGRAPHY

Beichner, R.J., (1994). Testing student interpretation of kinematic graphs. American Journal of Physics, 62 (8), 750-762.

Bell, A.; Janvier, C. (1981). The interpretation of graphs representing situations. For the Learning of Mathematics, 2 (1), 34-42.

Brasell, H. (1987). The effect of real-time laboratory graphing on learning graphic representations of distance and velocity. Journal of Research in Science Teaching, 24, 385-395.

Clement, J. (1989). Not all preconceptions are misconceptions: Finding "Anchoring Conceptions" for grounding instruction on students' intuitions. Paper presented at the Annual Meeting of the American Educational Research Association at San Francisco, March, 1989.

Clement, J. (1989). The concept of variation and misconceptions in Cartesian graphing. Focus on Learning Problems in Mathematics, 11 (2), 77-87.

Dees, R.L. (1991). The role of cooperative learning in increasing problem-solving ability in a college remedial course. Journal for Research in Mathematics Education, 22 (5), 409-421.

Dykstra, D.I.; Boyle, C.F.; Monarch, I.A. (1992). Studying conceptual change in learning physics. Science Education, 76 (6), 615-652.

Gentry, B.C. (1991). The effects of cooperative learning groups on mathematics achievement and attitude in pre-college algebra classes (Doctoral dissertation University of Illinois at Urbana-Champaign, 1991). Dissertation Abstracts International, 52, 07/A.

Krussel, C. (1994). Visualization and reification in advanced mathematical thinking. (From Dissertation Abstracts International, 1994, 56,01/A, University Microfilms no. AAC 9517901)

McDermott, L.C.; Rosenquist, M.L., van Zee, E.H. (1986). Student difficulties in connecting graphs and physics: Examples from kinematics. American Journal of Physics, 55 (6), 503-513.

Monk,.S. (1990). Students' understanding of a function given by a physical model. Paper presented at the conference on the learning and teaching of the concept of function at Purdue University, October, 1990.

Monk, S. (1994). How students and scientists change their minds. MAA invited address, Joint Mathematics Meeting , Cincinnati, Ohio, January, 1994.

Nemirovsky, R.; Monk, S., G (1994). The case of Dan: Student construction of a functional situation through visual attributes. Research in Collegiate Mathematics Education, 4, 139-168.

Nemirovsky, R.; Rubin, A., (1992). Students' tendency to assume resemblances between a function and its derivative. TERC working paper.

Roschelle, J; Clancey, W.J. (1991). Learning as Social and Neural. Paper presented at the AERA symposium, implications of cognitive theories of how the nervous system functions for research and practice in education, Chicago, Il, April, 1991.

Roschelle, J. (1992). Learning by collaborating: Convergent conceptual change. The Journal of the Learning Sciences, 2 (3), 235-276.

Rosenquist, M.L.; McDermott, L.C. (1986). A conceptual approach to teaching kinematics. American Journal of Physics, 55 (5), 407-415.

Sfard, A. (1991). On The dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. Educational Studies in Mathematics, 22, 1-36.

Slavin, R.E. (1980). Cooperative learning. Review of Educational Research, 50 (2), 315-342.

Slavin, R.E.; Leavey, M.B.; Madden, N.A. (1984). Combining cooperative learning and individualized instruction: Effects on student mathematics achievement, attitudes, and behaviors. Elementary School Journal, 84 (4), 408-422.

Smith III, J.P.; DiSessa, A.A.; Roschelle, J. (1993). Misconceptions reconceived: A constructivist analysis of knowledge in transition. The Journal of the Learning Sciences, 3 (2), 115-163.

Thompson, P.W. (1987). Artificial Intelligence and Instruction: Applications and Methods. Reading, MA: Addison-Wesley.

Thornton, R.K. (1987). Tools for scientific thinking - microcomputer-based laboratories or physics teaching. Physics Education, 22, 230-238.

Thornton, R.K.; Sokoloff, D.R. (1990). Learning motion concepts using real-time microcomputer-based laboratory tools. American Journal of Physics, 58 (9), 858-867.

Webb, N.M. (1991). Task-related verbal interaction and mathematics learning in small groups. Journal for Research in Mathematics Education, 22 (5), 366-389.

APPENDICES

Appendix A

CBL Software Interface

Software Interface of the CBL

To use the CBL system the student makes choices from a menu driven system. The steps are as follows:

- Push the PRGM. The calculator will display a list of programs available on that calculator. Select CBL.
- The following menu will appear: (1) motion, (2) temperature, (3) sound , (4) force, (5) plots, (6) quit.
  Select MOTION.
- The following menu will appear: (1) motion, (2) distance-rt, (3) d-t match, (4) bouncing ball, (5) quit.
  Select MOTION.
- The calculator will display: "hit enter to zero". The student will hit the ENTER key. The motion detector will make a clicking noise.
- The calculator will display: "enter collection time in seconds". The student will enter the time, generally 2 to 5 seconds, depending on the physical action.
- The calculator will display: "hit enter to start collecting data". The timing of hitting the ENTER key, and beginning the physical action is important. The students may need several opportunities to time this correctly. The motion detector will make a clicking noise while it is collecting the data.
- Once the data has been collected, there will be a lag time of about 5 seconds and then the following screen will appear: (1) distance-time, (2) velocity-time, (3) d-t and v-t, (4) accelrtn-time, (5) force-time, (6) quit. The student can select a distance graph, velocity graph, both or an acceleration graph.
- The selected graph will appear on the screen. The student can analyze the graph further by using the TRACE, ZOOM, and WINDOW keys on the calculator.

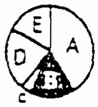
- To get back to the home screen, the student will hit the QUIT key. At this time the student can view the graphs of the motion most recently completed by again hitting the PRGM key, selecting PLOTS from the list of programs, and then selecting the graph she wishes to view.

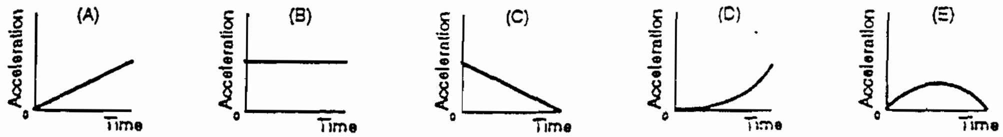
The student can continue to view graphs in this manner until the CBL system is used to detect motion of a new physical situation, or the data stored in the calculator is replaced by some other means.

Appendix B

Tests

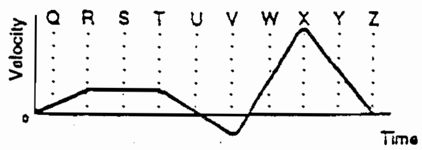
❶ Acceleration versus time graphs for five objects are shown below. All axes have the same scale. Which object had the greatest change in velocity during the interval?

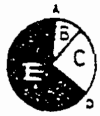




❷ When is the acceleration the most negative?

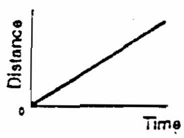
(A) R to T
(B) T to V
(C) V
(D) X
(E) X to Z





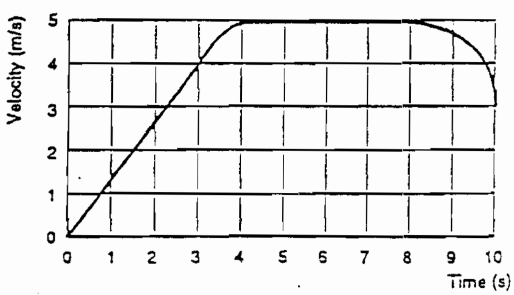
❸ To the right is a graph of an object's motion. Which sentence is the best interpretation?

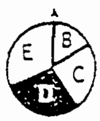
(A) The object is moving with a constant, non-zero acceleration.
(B) The object does not move.
(C) The object is moving with a uniformly increasing velocity.
(D) The object is moving at a constant velocity.
(E) The object is moving with a uniformly increasing acceleration.



❹ An elevator moves from the basement to the tenth floor of a building. The mass of the elevator is 1000 kg and it moves as shown in the velocity-time graph below. How far does it move during the first three seconds of motion?

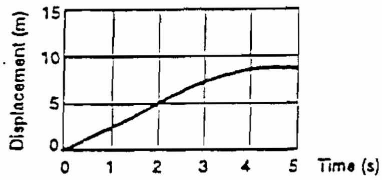
(A) 0.75 m
(B) 1.33 m
(C) 4.0 m
(D) 6.0 m
(E) 12.0 m





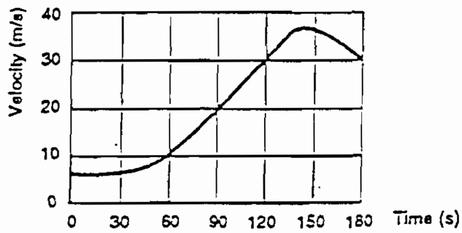
❺ The velocity at the 2 second point is:

(A) 0.4 m/s
(B) 2.0 m/s
(C) 2.5 m/s
(D) 5.0 m/s
(E) 10.0 m/s



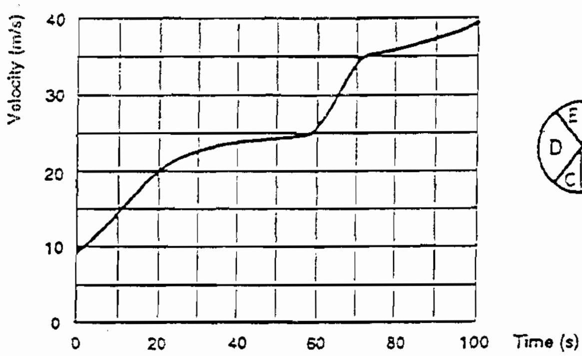
❻ This graph shows velocity as a function of time for a car of mass 1.5×10^3 kg. What was the acceleration at the end of 90 s ?

(A) 0.22 m/s²
(B) 0.33 m/s²
(C) 1.0 m/s²
(D) 9.8 m/s²
(E) 20 m/s²

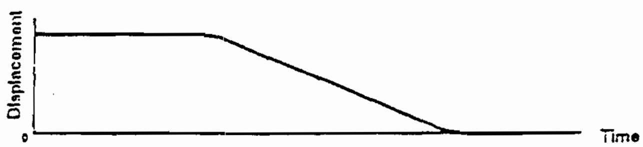


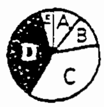
❼ The motion of an object traveling in a straight line is represented by the following graph. At time = 65 s, the magnitude of the instantaneous acceleration of the object was most nearly:

(A) 1 m/s²
(B) 2 m/s²
(C) +9.8 m/s²
(D) +30 m/s²
(E) +34 m/s²



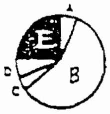
❽ Here is a graph of an object's motion. Which sentence is a correct interpretation?

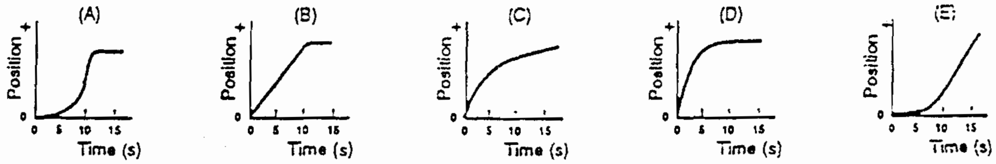




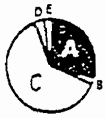
(A) The object rolls along a flat surface. Then it rolls forward down a hill, and then finally stops.
(B) The object doesn't move at first. Then it rolls forward down a hill and finally stops.
(C) The object is moving at a constant velocity. Then it slows down and stops.
(D) The object doesn't move at first. Then it moves backwards and then finally stops.
(E) The object moves along a flat area, moves backwards down a hill, and then it keeps moving.

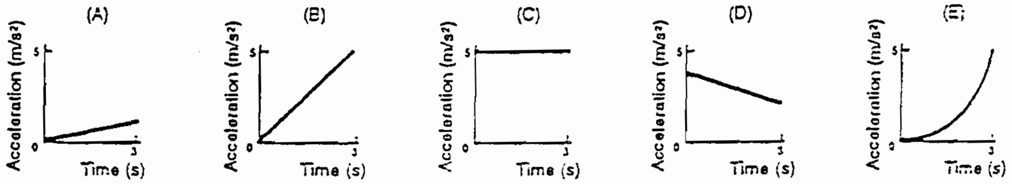
9. An object starts from rest and undergoes a positive, constant acceleration for ten seconds. It then continues on with constant velocity. Which of the following graphs correctly describes this situation?



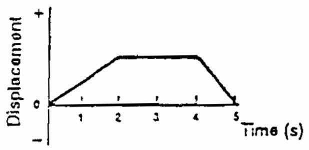


10. Five objects move according to the following acceleration versus time graphs. Which has the smallest change in velocity during the three second interval?

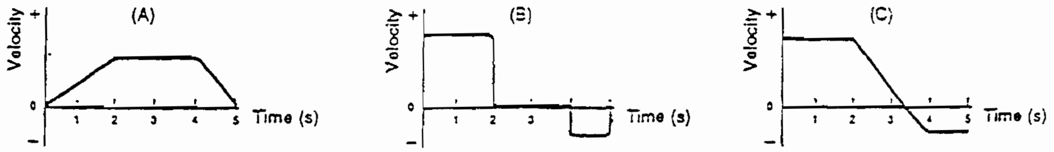


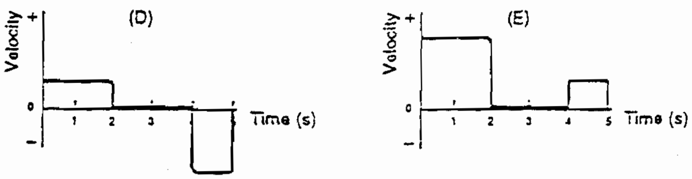


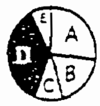
11. The following is a displacement-time graph for an object during a 5 s time interval.



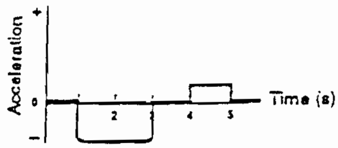
Which one of the following graphs of velocity versus time would best represent the object's motion during the same time interval?

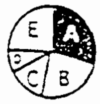




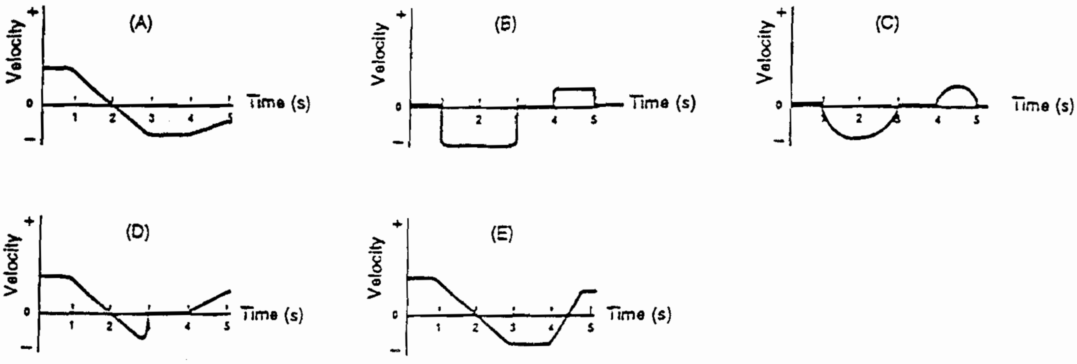


15 The following represents an acceleration graph for an object during a 5 s time interval.

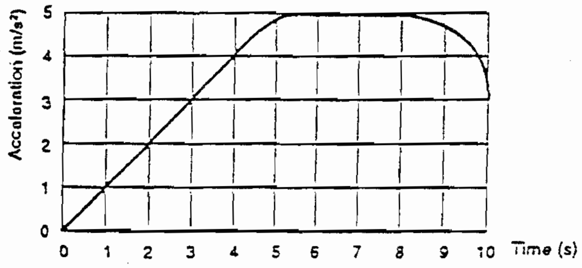


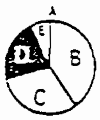


Which one of the following graphs of velocity versus time would best represent the object's motion during the same time interval?



16 An object moves according to the graph below:



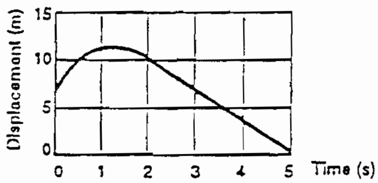


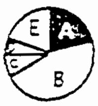
The object's change in velocity during the first three seconds of motion was:

(A) 0.66 m/s (B) 1.0 m/s (C) 3.0 m/s (D) 4.5 m/s (E) 9.8 m/s

17 The velocity at the 3 second point is about:

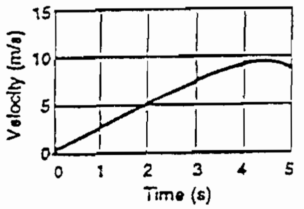
(A) −3.3 m/s
(B) −2.0 m/s
(C) −.67 m/s
(D) 5.0 m/s
(E) 7.0 m/s

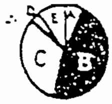




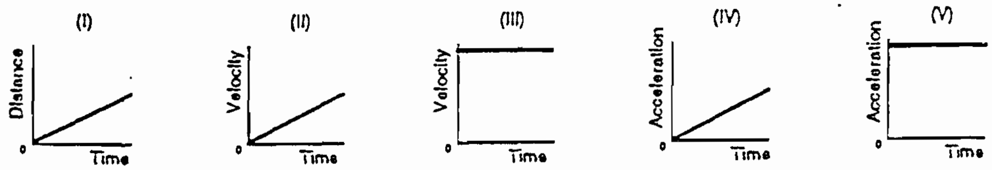
18 If you wanted to know the distance covered during the interval from t = 0 s to t = 2 s, from the graph below you would:

(A) read 5 directly off the vertical axis.
(B) find the area between that line segment and the time axis by calculating (5x2)/2.
(C) find the slope of that line segment by dividing 5 by 2.
(D) find the slope of that line segment by dividing 15 by 5.
(E) Not enough information to answer.





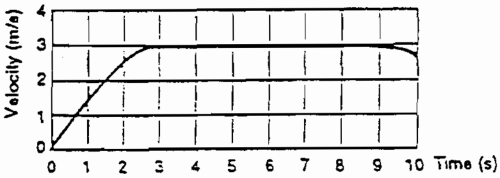
19 Consider the following graphs, noting the different axes:



Which of these represent(s) motion at constant, non-zero acceleration?

(A) I, II, and IV
(B) I and III
(C) II and V
(D) IV only
(E) V only

20 An object moves according to the graph below:

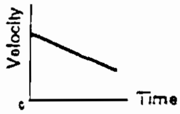


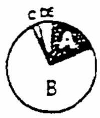
How far does it move during the interval from t = 4 s to t = 8 s ?

(A) 0.75 m (B) 3.0 m (C) 4.0 m (D) 8.0 m (E) 12.0 m

21 To the right is a graph of an object's motion. Which sentence is the best interpretation?

(A) The object is moving with a constant acceleration.
(B) The object is moving with a uniformly decreasing acceleration.
(C) The object is moving with a uniformly increasing velocity.
(D) The object is moving at a constant velocity.
(E) The object does not move.





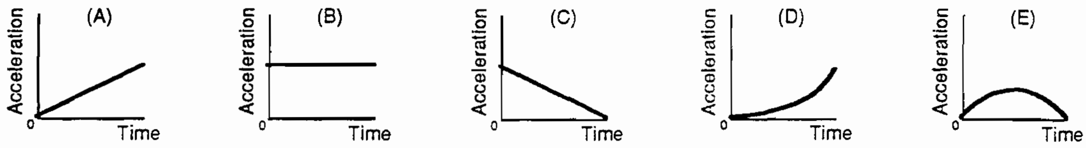
MTH 252 Assessment of understanding of position/velocity/acceleration graphs

Name:______________________________ **SS#**____________________

Recitation Section #____________

My score may be recorded for the purpose of the research project described in class: ________Yes ________No

1. Acceleration versus time graphs for five objects are shown below. All axes have the same scale. Which object had the greatest change in velocity during the interval?



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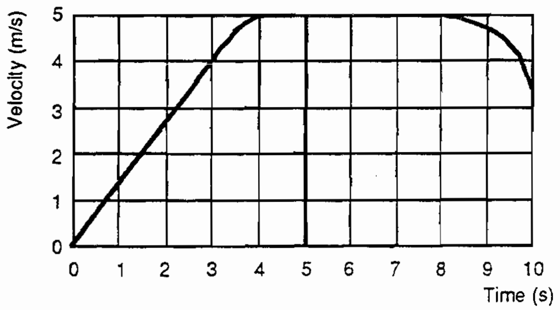
2. An elevator moves from the basement to the tenth floor of a building. The mass of the elevator is 1000 kg and it moves as shown in the velocity-time graph below. How far does it move during the first three seconds of motion?

(A) 0.75 m
(B) 1.33 m
(C) 4.0 m
(D) 6.0 m
(E) 12.0 m



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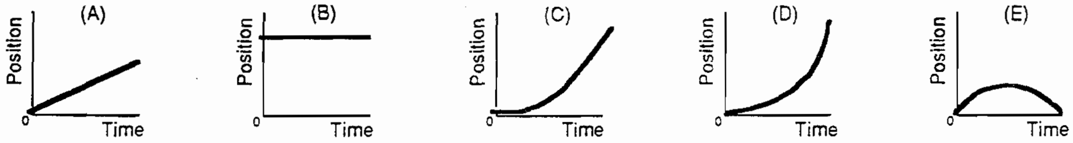
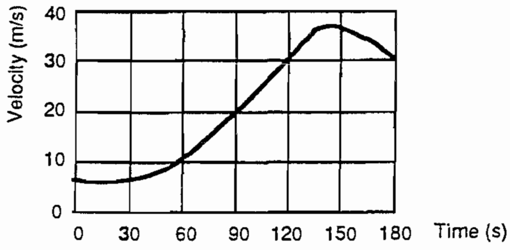
3. Position versus time graphs for five objects are shown below. All axes have the same scale. Which object had the highest instantaneous velocity during the interval?



4. This graph shows velocity as a function of time for a car of mass 1.5×10^3 kg.
What was the acceleration at the 90 s mark?

(A) 0.22 m/s^2
(B) 0.33 m/s^2
(C) 1.0 m/s^2
(D) 9.8 m/s^2
(E) 20 m/s^2



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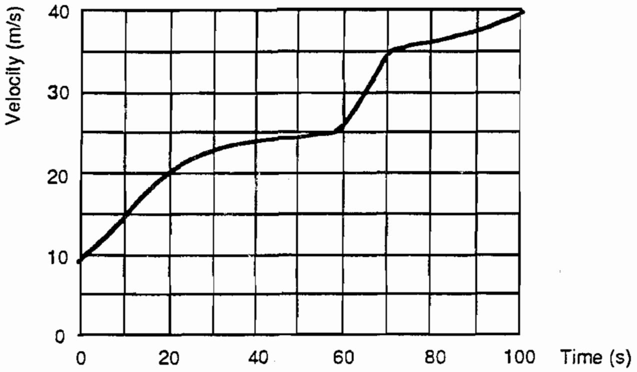
5. The motion of an object traveling in a straight line is represented by the following graph.
At time t = 65 s, the magnitude of the instantaneous acceleration of the object was most nearly:

(A) 1 m/s^2
(B) 2 m/s^2
(C) +9.8 m/s^2
(D) +30 m/s^2
(E) +34 m/s^2



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6. Here is a graph of an object's motion. Which sentence is a correct interpretation?

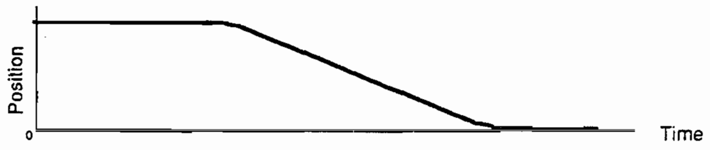
(A) The object rolls along a flat surface. Then it rolls forward down a hill, and then finally stops.

(B) The object doesn't move at first. Then it rolls forward down a hill and finally stops.

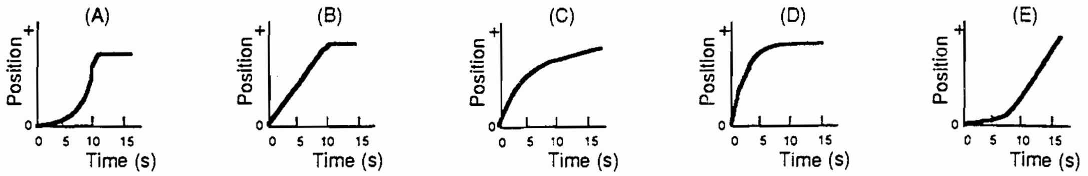
(C) The object is moving at a constant velocity. Then it slows down and stops.

(D) The object doesn't move at first. Then it moves backwards and then finally stops.

(E) The object moves along a flat area, moves backwards down a hill, and then it keeps moving.



7. An object starts from rest and undergoes a positive, constant acceleration for ten seconds. It then continues on with constant velocity. Which of the following graphs correctly describes this situation?



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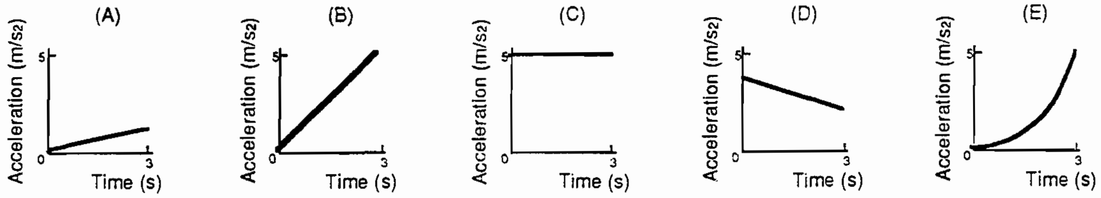
8. Five objects move according to the following acceleration versus time graphs. Which has the smallest change in velocity during the three second interval?



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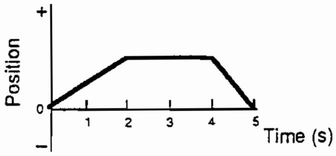
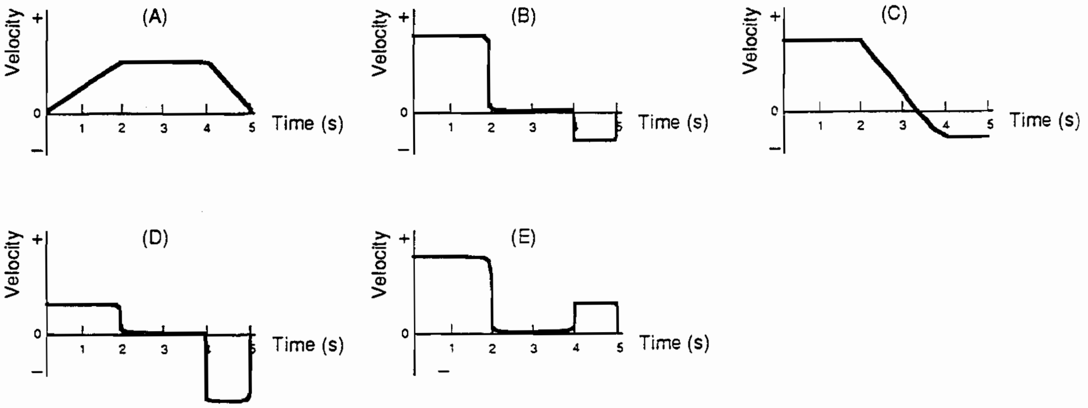
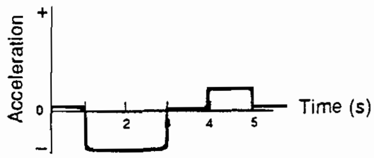
9. The following is a position-time graph for an object during a 5 s time interval. Which one of the following graphs of velocity versus time would best represent the object's motion during the same time interval?

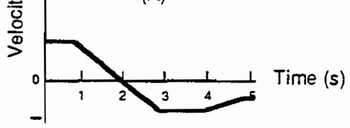




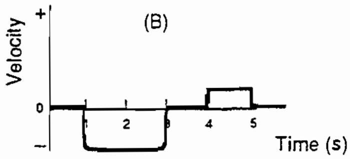
10. The following represents an acceleration graph for an object during a 5 s time interval. Which one of the following graphs of velocity versus time would best represent the object's motion during the same time interval?



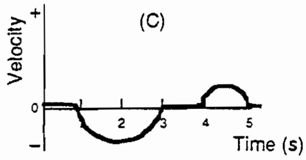
(A)



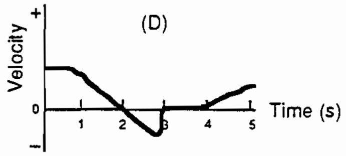
(B)



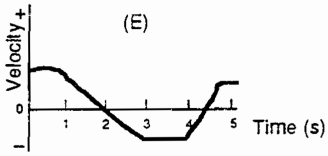
(C)



(D)



(E)



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11. The velocity at the 3 second point is about:

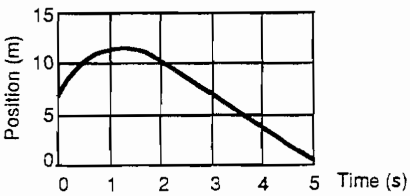
(A) – 3.3 m/s

(B) – 2.0 m/s

(C) – .67 m/s

(D) 5.0 m/s

(E) 7.0 m/s



12. If you wanted to know the distance covered during the interval from t = 0 s to t = 2 s, from the graph below you would:

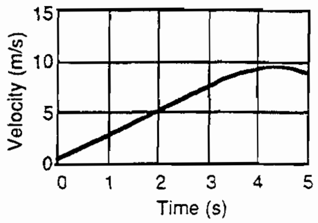
(A) read 5 directly off the vertical axis.

(B) find the area between that line segment and the time axis by calculating (5X2)/2.

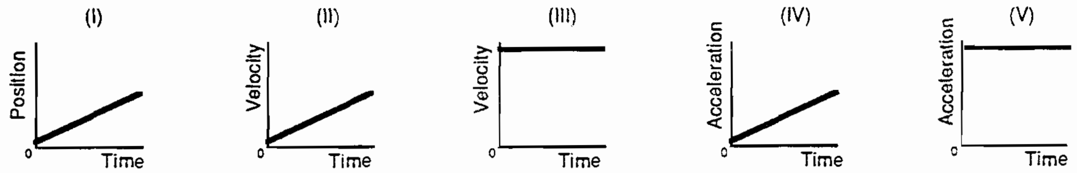
(C) find the slope of that line segment by dividing 5 by 2.

(D) find the slope of that line segment by dividing 15 by 5.

(E) Not enough information to answer.

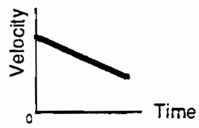


13. Consider the following graphs, noting the different axes:
Which of these represent(s) motion at constant, non-zero acceleration?



(A) I, II, and IV (B) I and III (C) II and V (D) IV only (E) V only

14. To the right is a graph of an object's motion.
Which sentence is the best interpretation?



(A) The object is moving with a constant acceleration.

(B) The object is moving with a uniformly decreasing acceleration.

(C) The object is moving with a uniformly increasing velocity.

(D) The object is moving at a constant velocity.

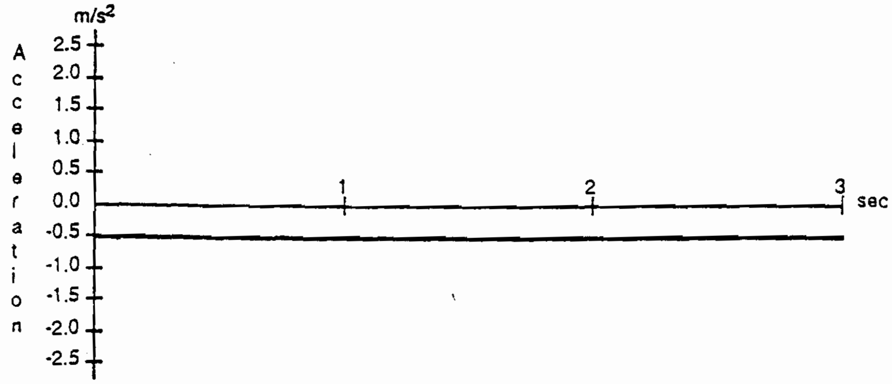
(E) The object does not move.

Appendix C

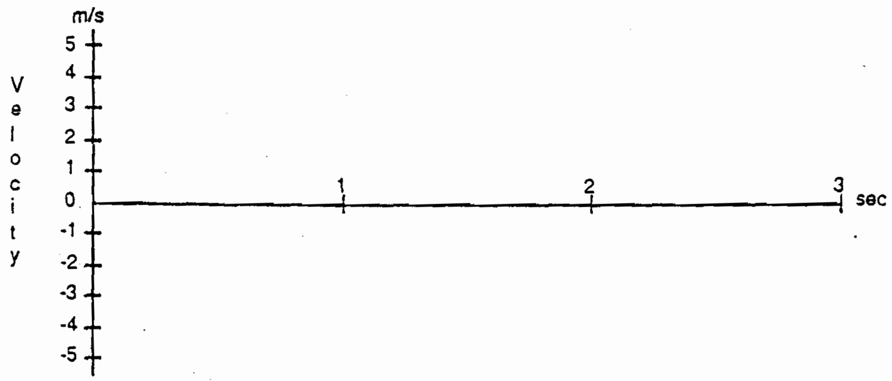
Laboratory Assignments

MTH 252 Laboratory Name ______________________________

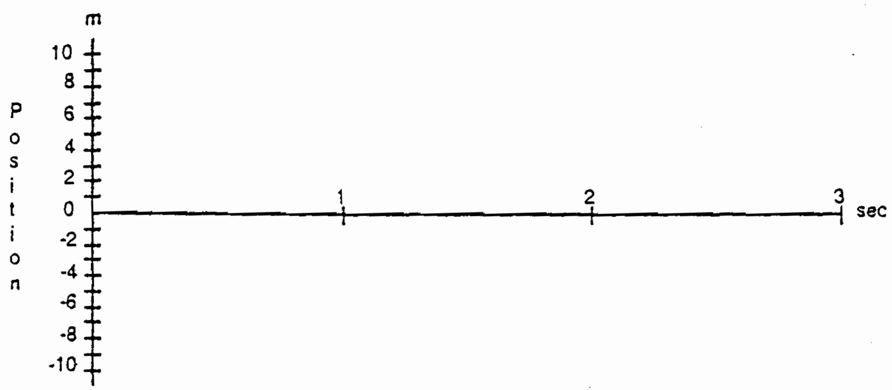
1. An object moves with a constant acceleration of $-0.5\ m/s^2$. A graph of the object's acceleration is shown below for $0 \leq t \leq 3$ seconds.



a) Assume that the object has an initial velocity of 4 m/s. Sketch a graph of the object's velocity for $0 \leq t \leq 3$ seconds:

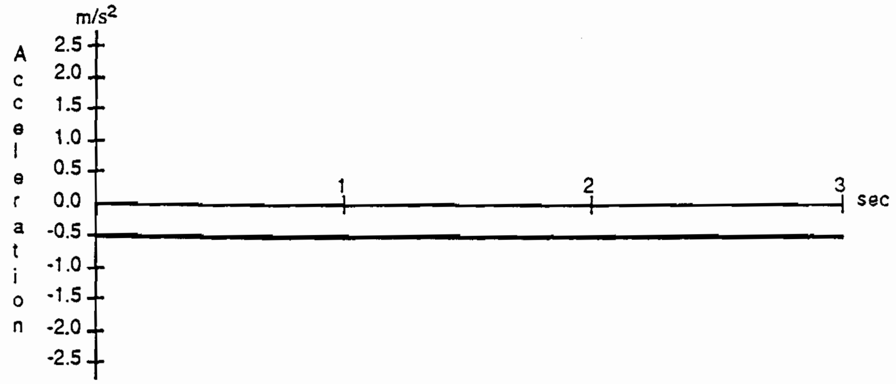


b) Now assume that the object starts at the position 0 m. Sketch a graph of the object's position for $0 \leq t \leq 3$ seconds:

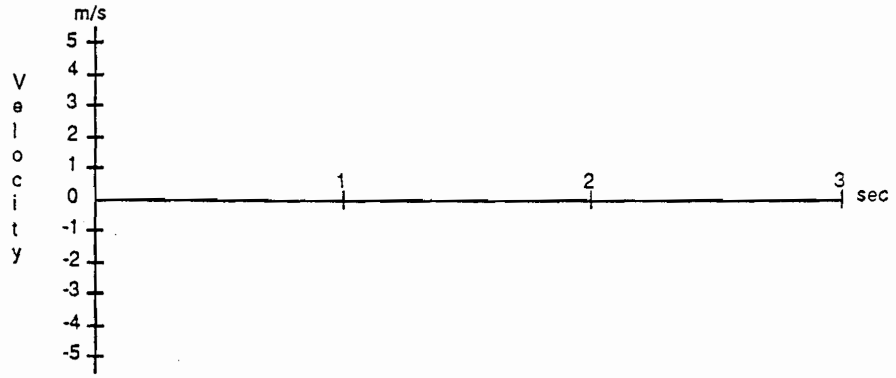


c) Suppose that these graphs describe the motion of a toy car (or a person walking) being monitored by a CBL system. Describe in words how the car is moving and describe a situation which might account for this motion.

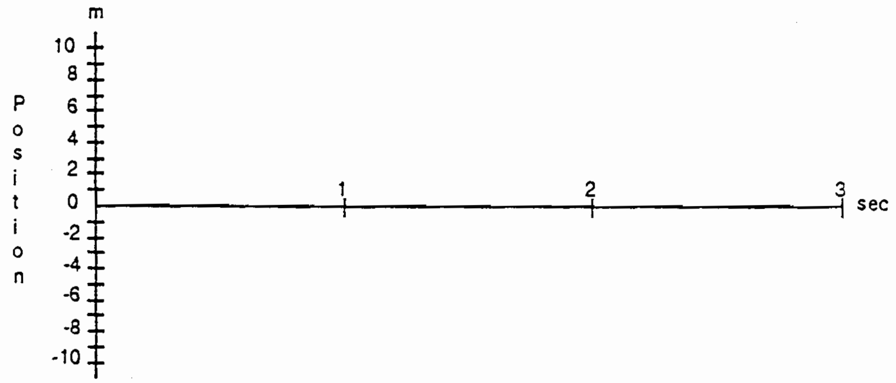
2. Another object also moves with a constant acceleration of -0.5 m/s^2 for $0 \leq t \leq 3$ seconds as shown in the graph below.



a) Assume that this object has an initial velocity of -2 m/s. Sketch a graph of the object's velocity for $0 \leq t \leq 3$ seconds:

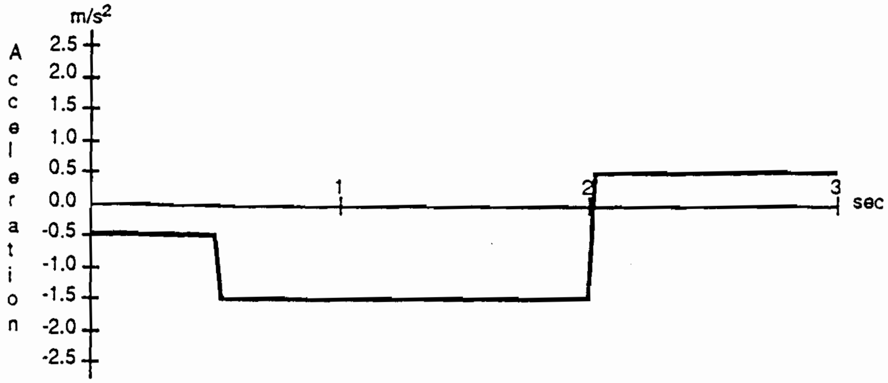


b) Now assume that the object starts at the position 0 m. Sketch a graph of the object's position for $0 \leq t \leq 3$ seconds:

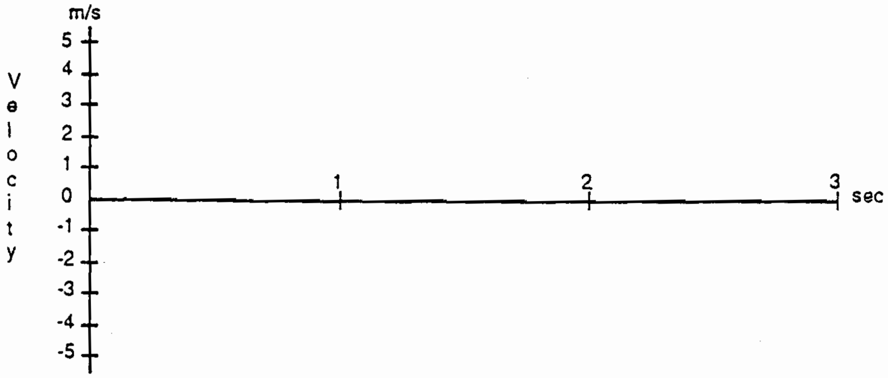


c) Suppose that these graphs describe the motion of a toy car (or a person walking) being monitored by a CBL system. Describe in words how the car is moving and describe a situation which might account for this motion.

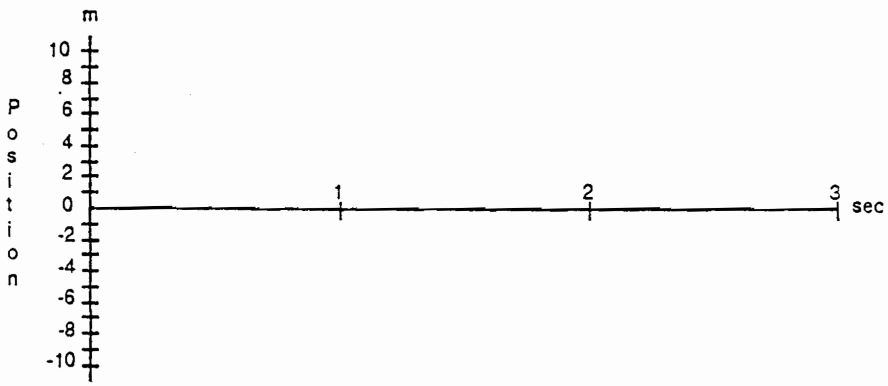
3. Here is the graph of another object's acceleration over time for $0 \leq t \leq 3$ seconds:



a) Assume that this object has an initial velocity of 2 m/s. Sketch a graph of the object's velocity for $0 \leq t \leq 3$ seconds:



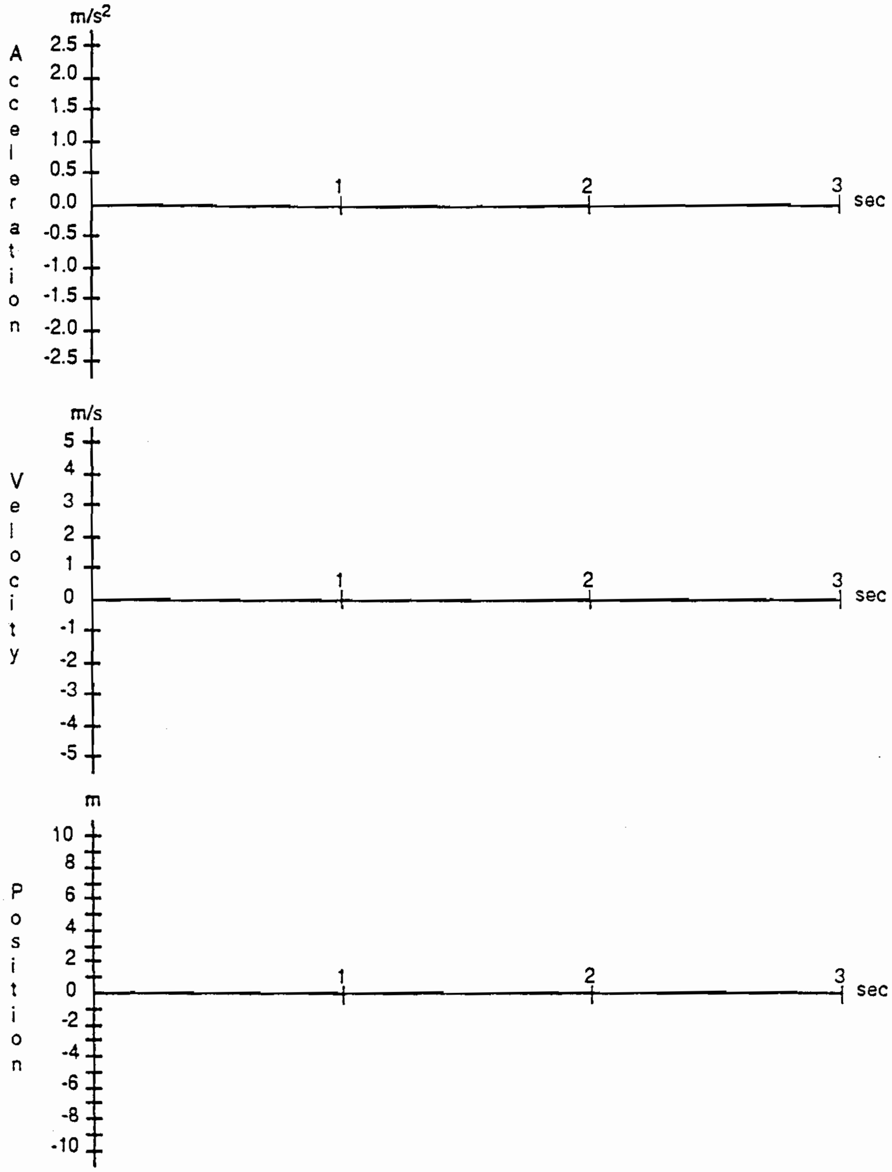
b) Assume that this object starts at the position 0 m. Sketch a graph of the object's position for $0 \leq t \leq 3$ seconds:



c) Suppose that these graphs describe the motion of a toy car (or a person walking) being monitored by a CBL system. Describe in words how the car is moving and describe a situation which might account for this motion.

4. Your group will use the CBL system to create situations similar to those you described in 1.c), 2.c), and 3.c). Sketch the acceleration, velocity, position graphs produced by the CBL system in each case, and explain any differences you see between the graphs you predicted and the CBL graphs.

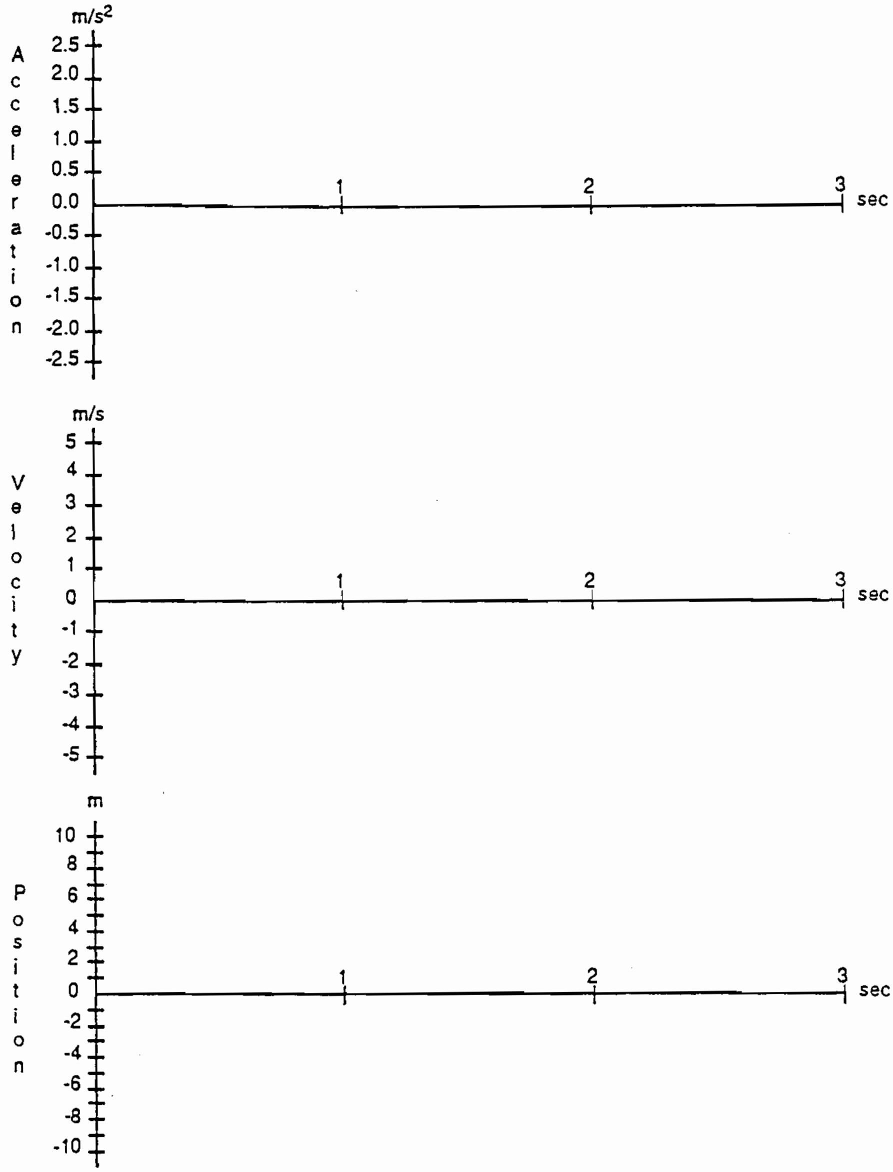
Graphs produced by the CBL for situation 1.c)



Explanation of differences between your predicted graphs and these:

4. (continued)

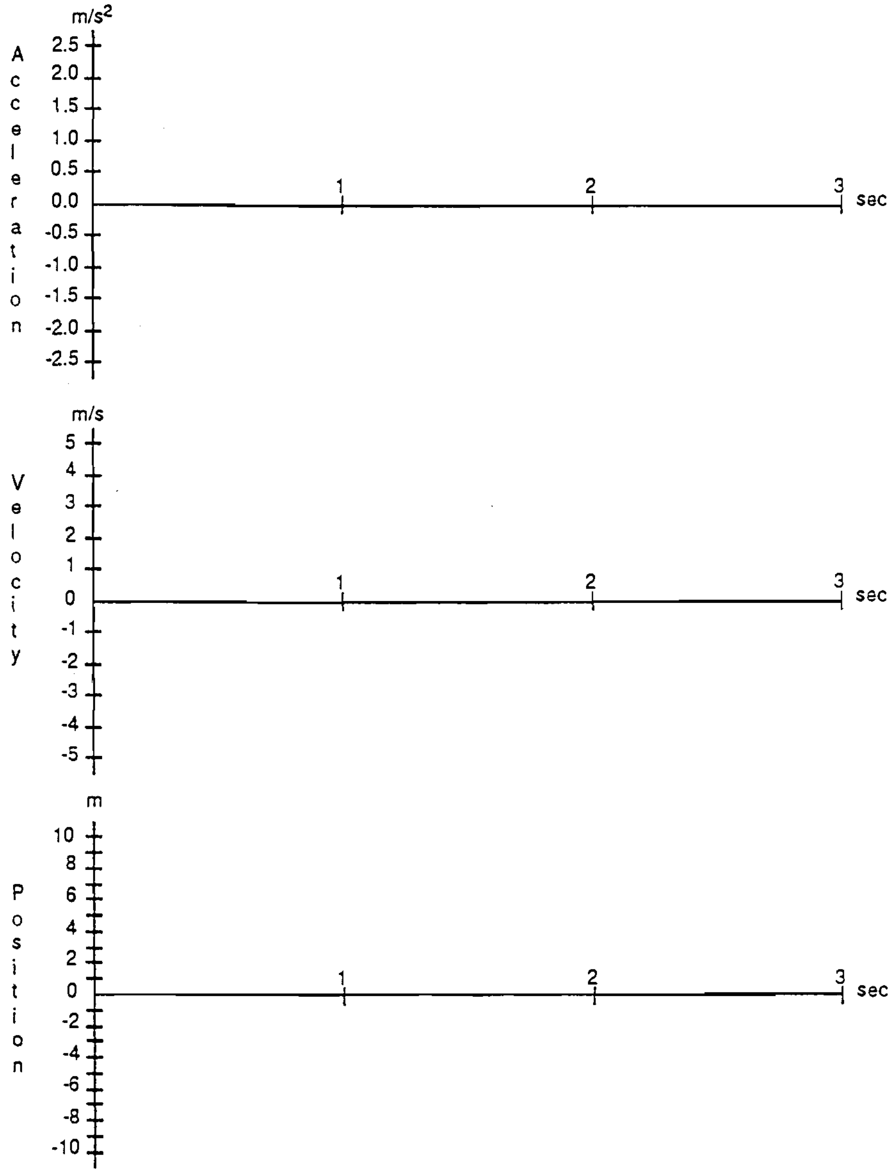
Graphs produced by the CBL for situation 2.c)



Explanation of differences between your predicted graphs and these:

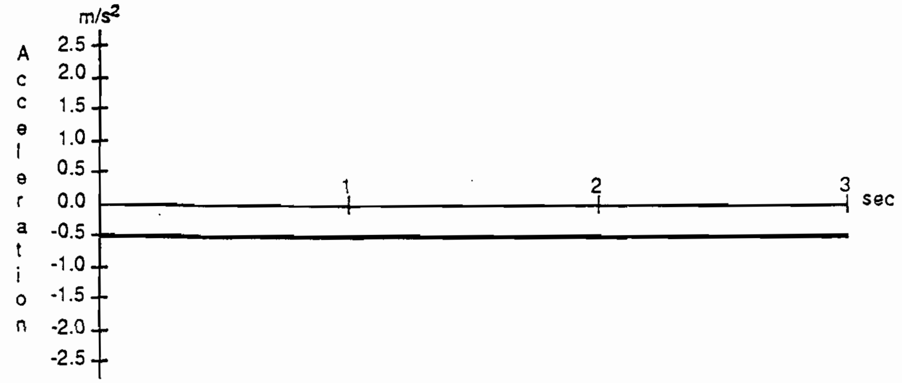
4. (continued)

Graphs produced by the CBL for situation 3.c)

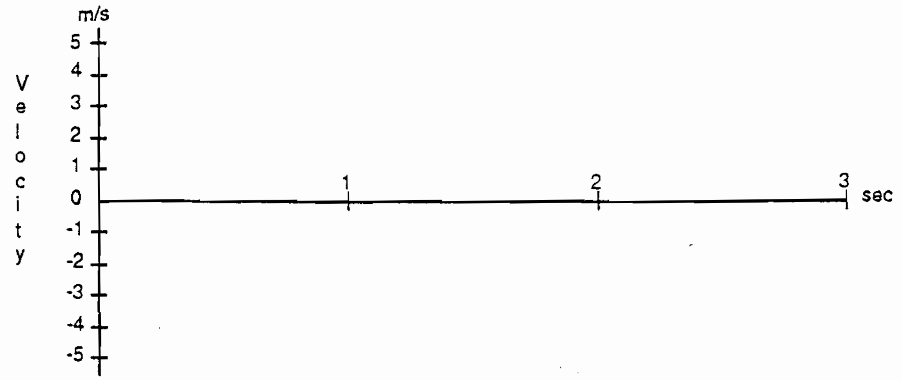


Explanation of differences between your predicted graphs and these:

5. An acceleration graph is given below (the same as in problem 1).

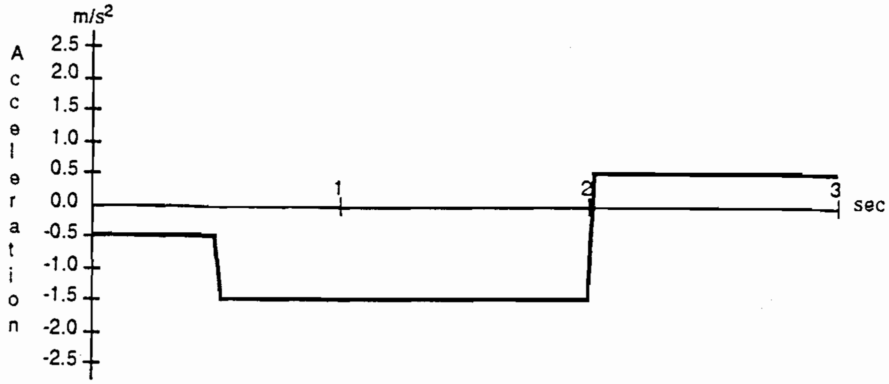


a) Sketch the graph of $A(x) = \int_0^x a(t)\,dt$, where $a(t)$ is the acceleration function described by the graph above.

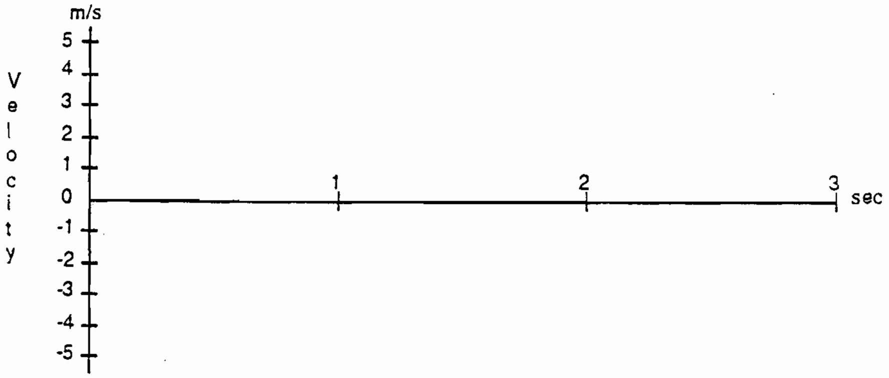


The graph you just sketched should describe the velocity of an object having the given acceleration. What accounts for any difference between this graph, the one you predicted in 1.a) and the one produced in 4.a?

5. **(continued)** An acceleration graph is given below (the same as in problem **3**).



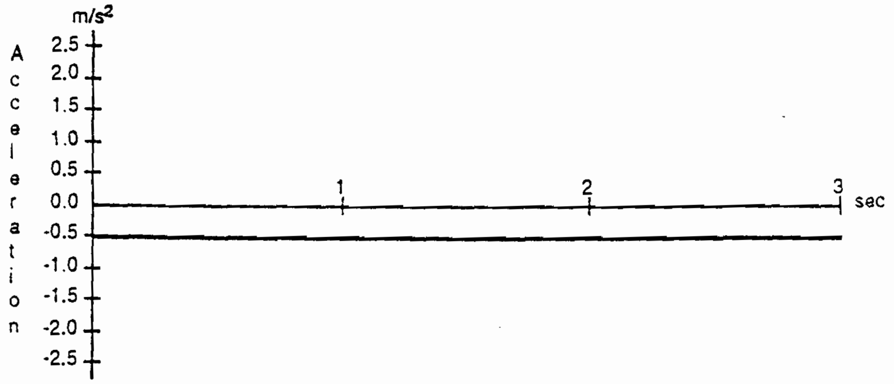
b) Sketch the graph of $A(x) = \int_0^x a(t)\,dt$, where $a(t)$ is the acceleration function described by the graph above.



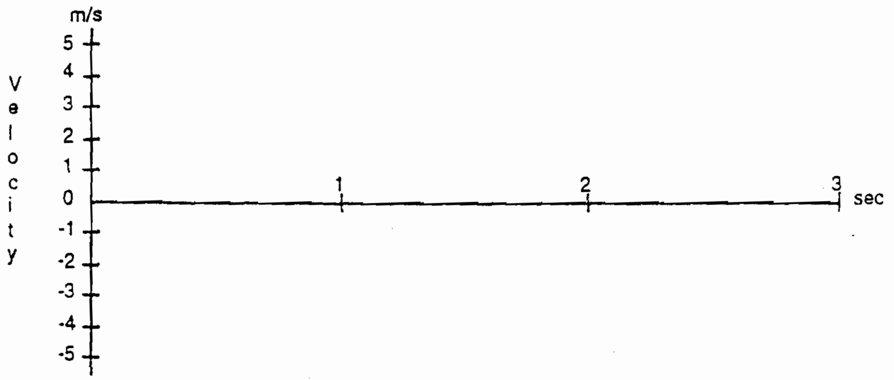
The graph you just sketched should describe the velocity of an object having the given acceleration. What accounts for any difference between this graph, the one you predicted in 3.a) and the one produced in **4.c**?

MTH 252 Laboratory Name ____________________

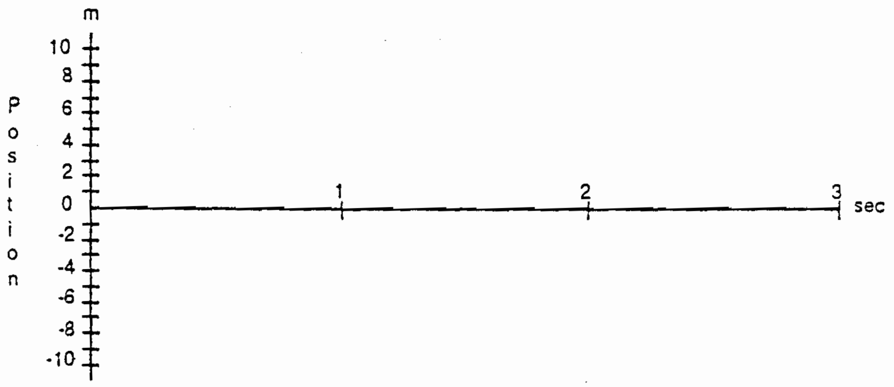
1. An object moves with a constant acceleration of $-0.5\ m/s^2$. A graph of the object's acceleration is shown below for $0 \le t \le 3$ seconds.



a) Assume that the object has an initial velocity of 4 m/s. Sketch a graph of the object's velocity for $0 \le t \le 3$ seconds:

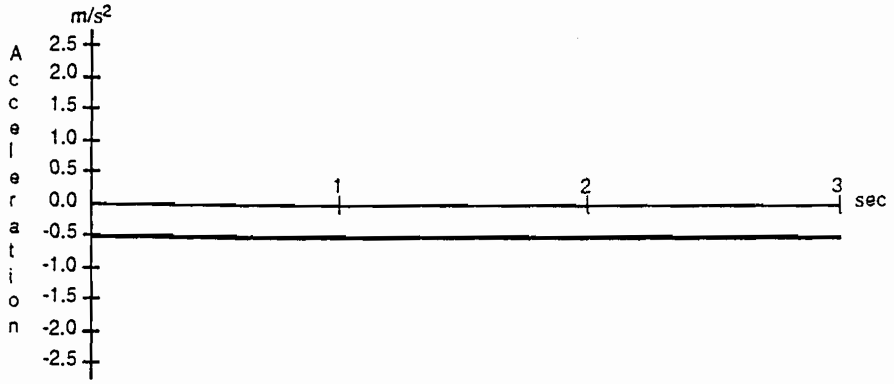


b) Now assume that the object starts at the position 0 m. Sketch a graph of the object's position for $0 \le t \le 3$ seconds:

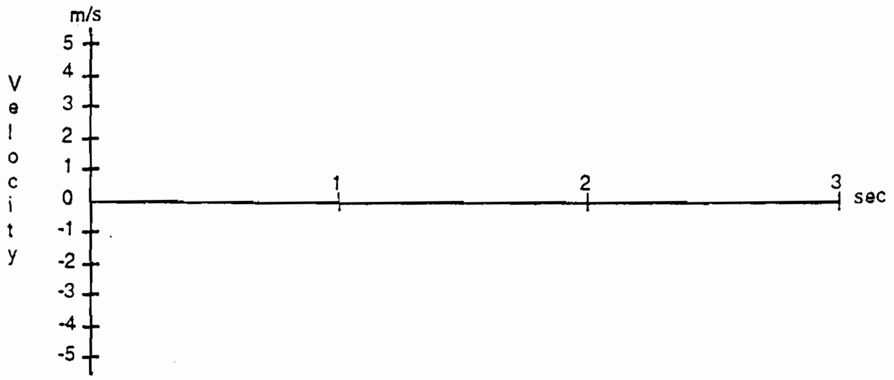


c) Suppose that these graphs describe the motion of a toy car (or a person walking) being monitored by a CBL system. Describe in words how the car is moving and describe a situation which might account for this motion.

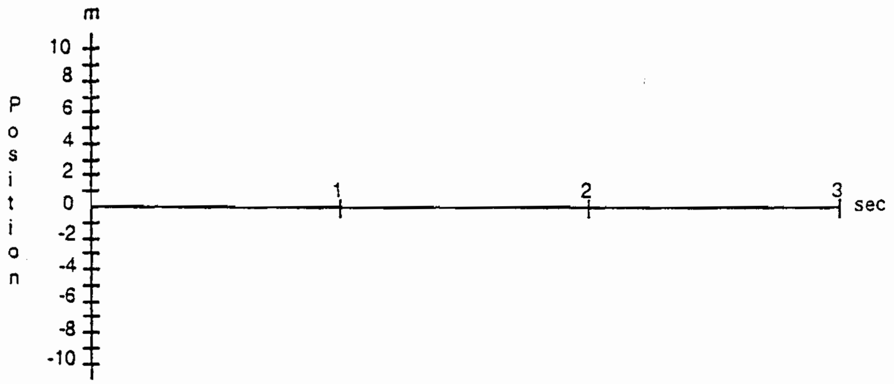
2. Another object also moves with a constant acceleration of $-0.5\ m/s^2$ for $0 \le t \le 3$ seconds as shown in the graph below.



a) Assume that this object has an initial velocity of -2 m/s. Sketch a graph of the object's velocity for $0 \le t \le 3$ seconds:

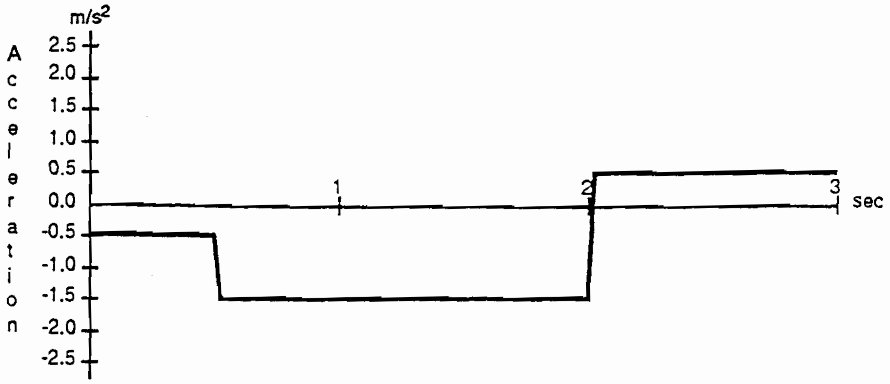


b) Now assume that the object starts at the position 0 m. Sketch a graph of the object's position for $0 \le t \le 3$ seconds:

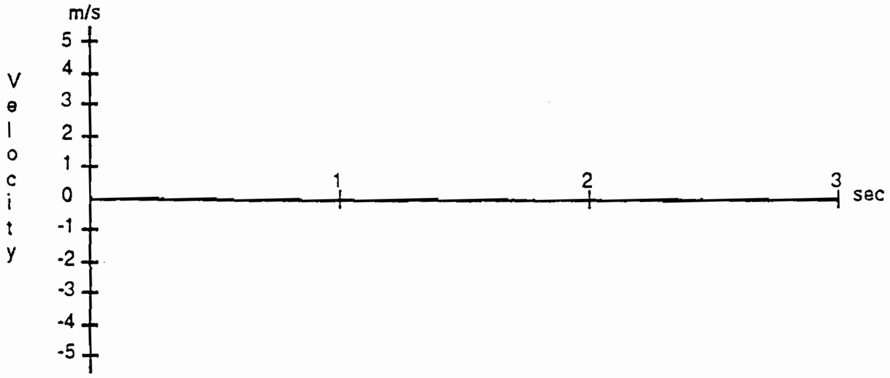


c) Suppose that these graphs describe the motion of a toy car (or a person walking) being monitored by a CBL system. Describe in words how the car is moving and describe a situation which might account for this motion.

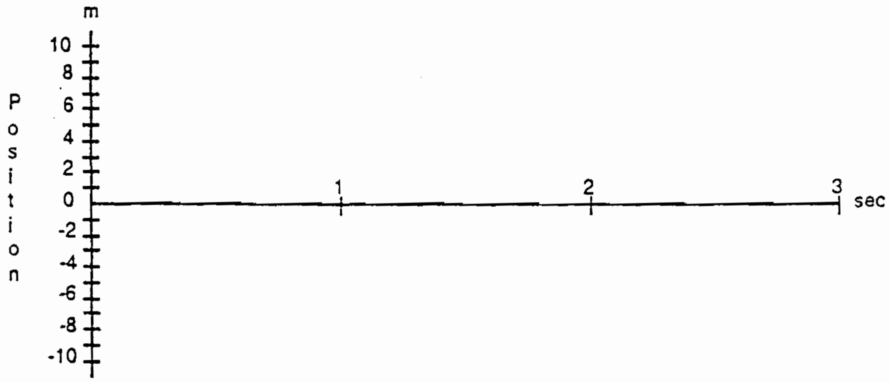
3. Here is the graph of another object's acceleration over time for $0 \leq t \leq 3$ seconds:



a) Assume that this object has an initial velocity of 2 m/s. Sketch a graph of the object's velocity for $0 \leq t \leq 3$ seconds:



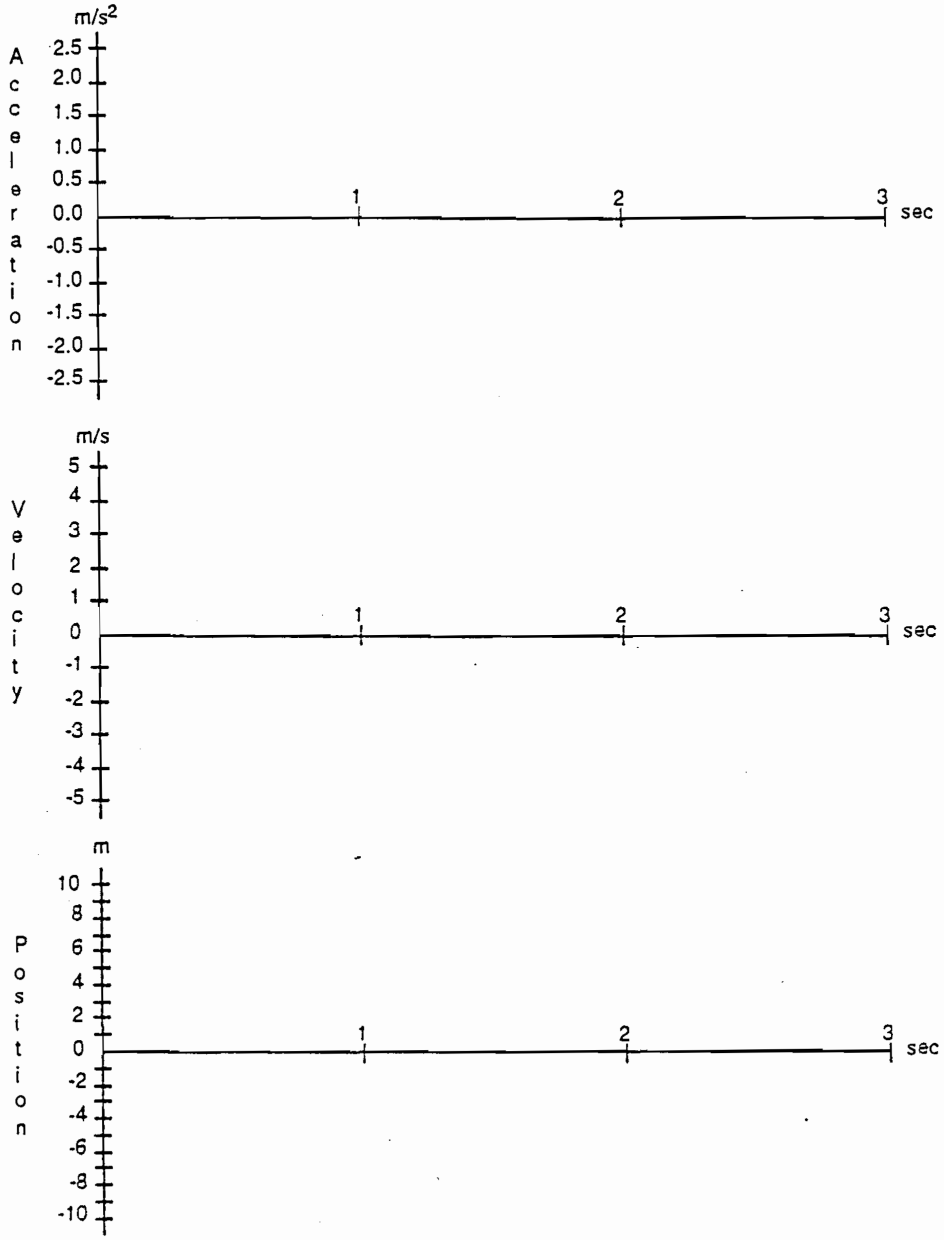
b) Assume that this object starts at the position 0 m. Sketch a graph of the object's position for $0 \leq t \leq 3$ seconds:



c) Suppose that these graphs describe the motion of a toy car (or a person walking) being monitored by a CBL system. Describe in words how the car is moving and describe a situation which might account for this motion.

4. Your instructor will use the CBL system to monitor situations similar to those you described in 1.c), 2.c), and 3.c). Sketch the acceleration, velocity, position graphs produced by the CBL system in each case, and explain any differences you see between the graphs you predicted and the CBL graphs.

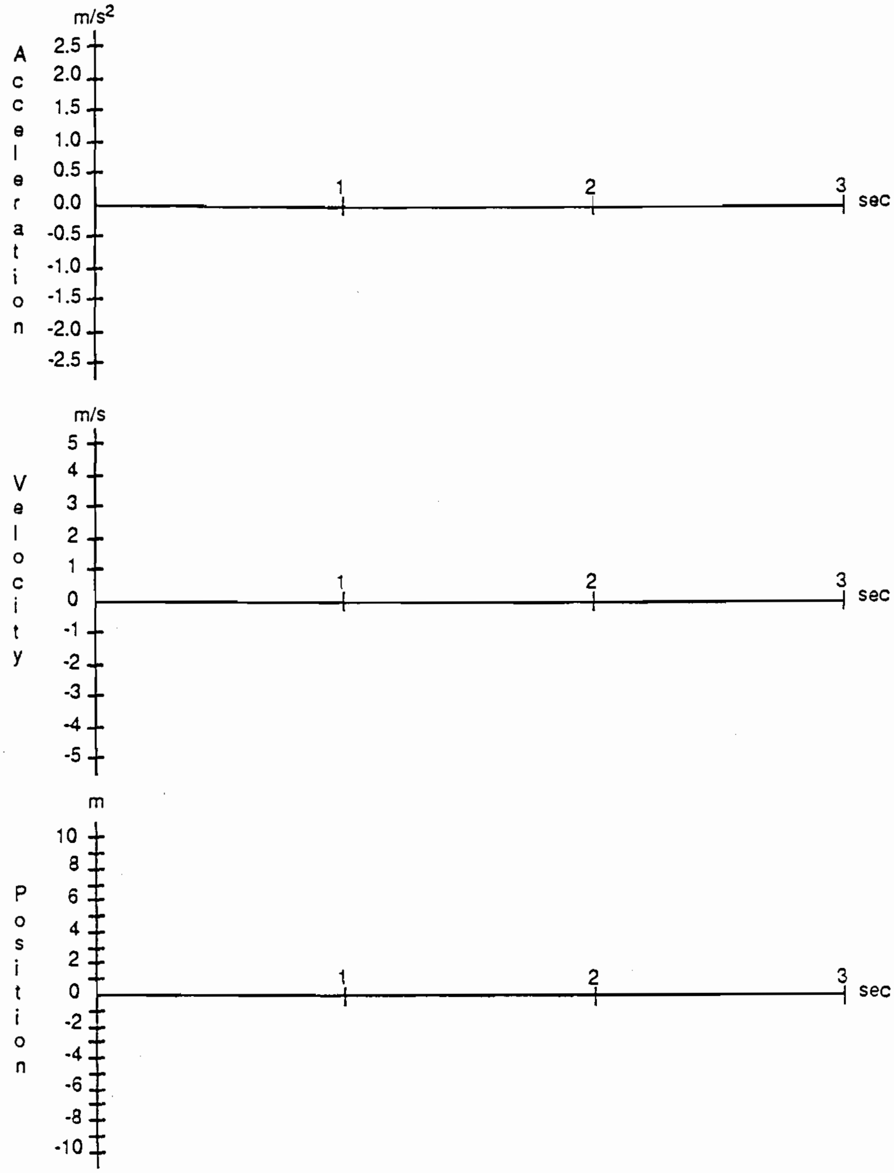
Graphs produced by the CBL for situation 1.c)



Explanation of differences between your predicted graphs and these:

4. (continued)

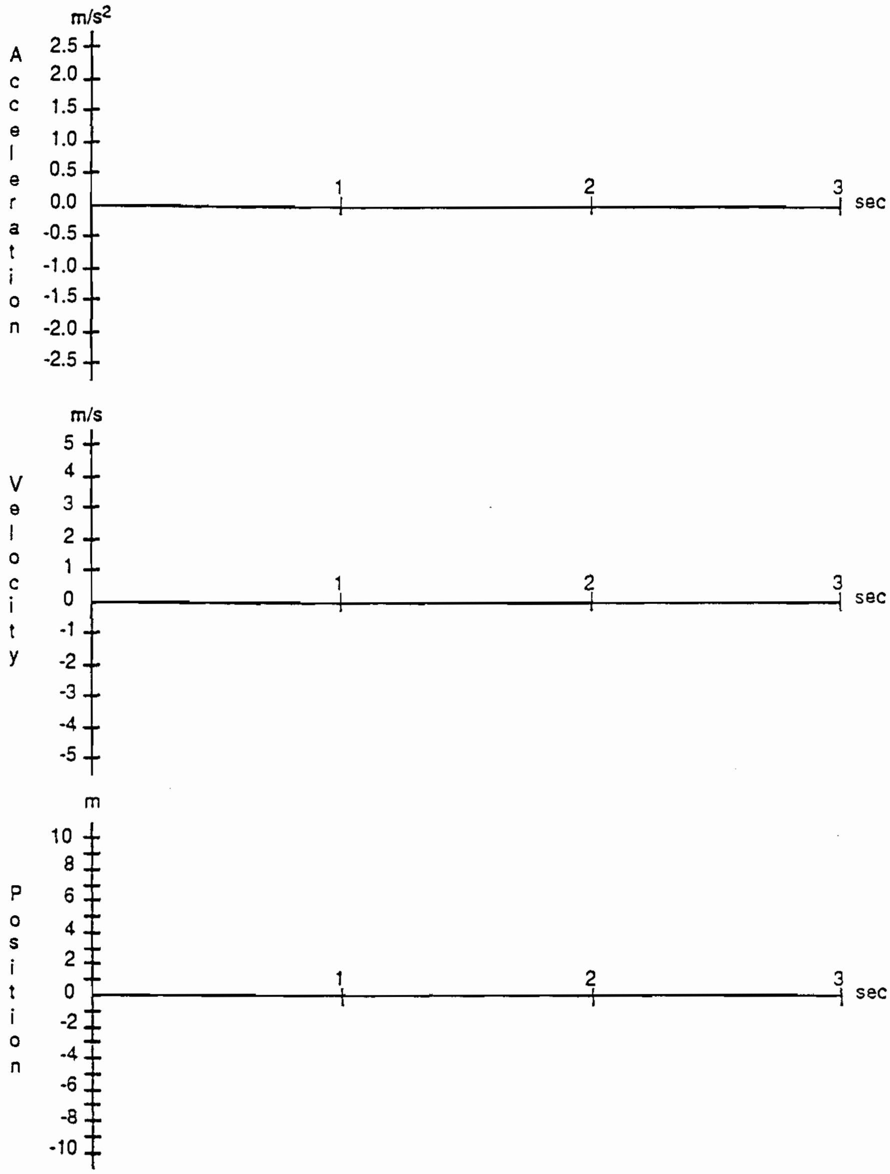
Graphs produced by the CBL for situation 2.c)



Explanation of differences between your predicted graphs and these:

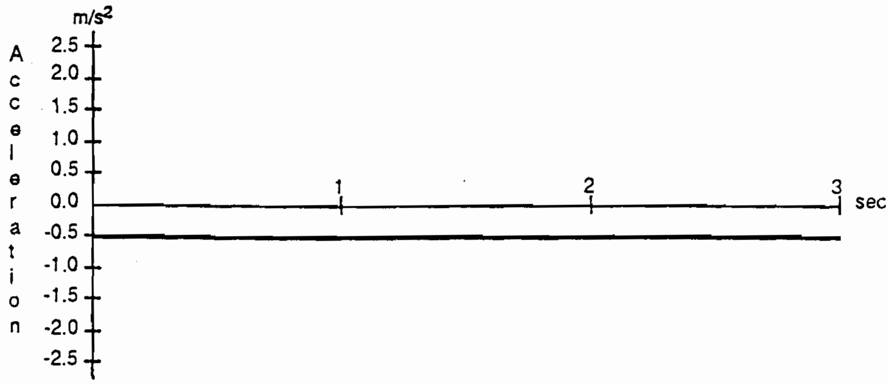
4. (continued)

Graphs produced by the CBL for situation 3.c)

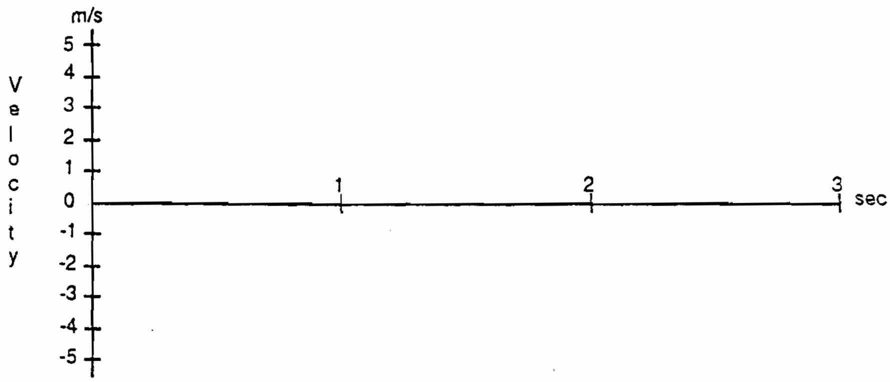


Explanation of differences between your predicted graphs and these:

5. An acceleration graph is given below (the same as in problem 1).

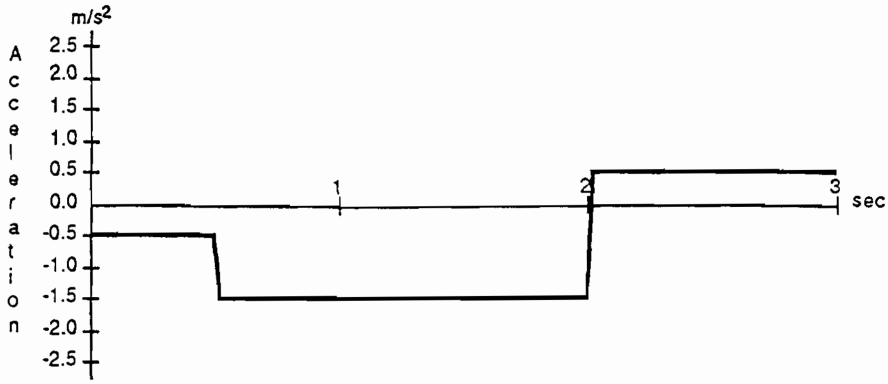


a) Sketch the graph of $A(x) = \int_0^x a(t)\,dt$, where $a(t)$ is the acceleration function described by the graph above.

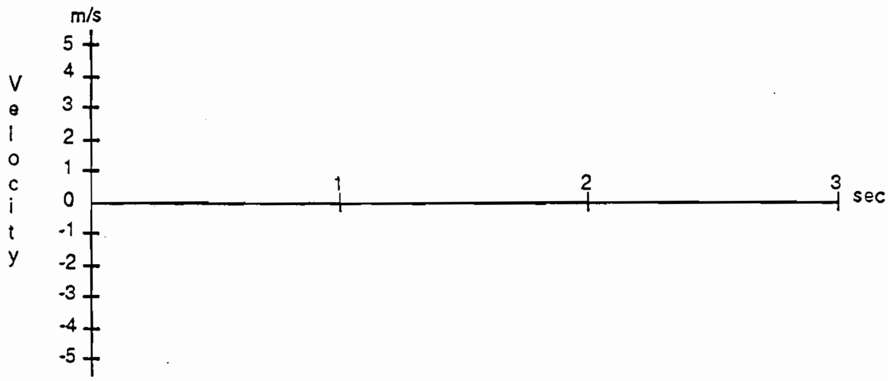


The graph you just sketched should describe the velocity of an object having the given acceleration. What accounts for any difference between this graph, the one you predicted in 1.a) and the one produced in 4.a?

5. **(continued)** An acceleration graph is given below (the same as in problem 3).



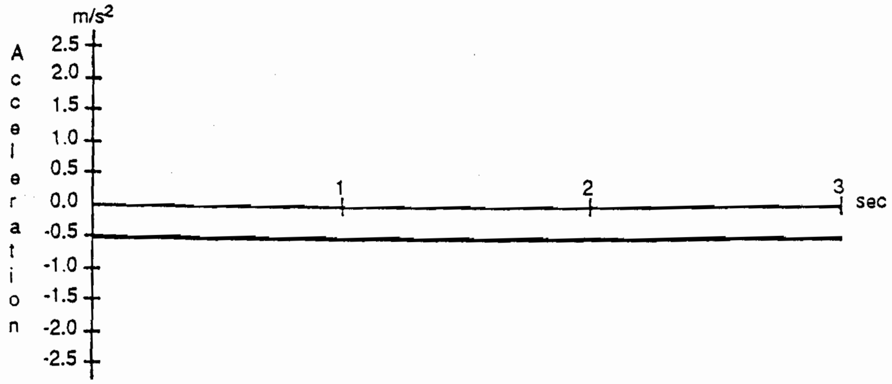
b) Sketch the graph of $A(x) = \int_0^x a(t)\,dt$, where $a(t)$ is the acceleration function described by the graph above.



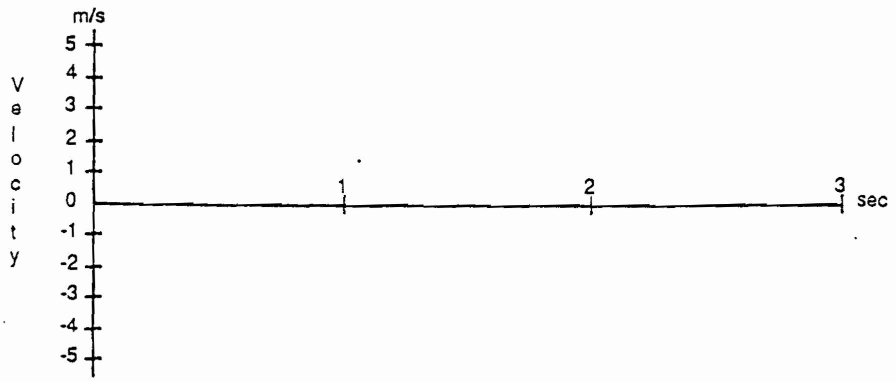
The graph you just sketched should describe the velocity of an object having the given acceleration. What accounts for any difference between this graph, the one you predicted in 3.a) and the one produced in 4.c?

MTH 252 Laboratory Name ________________________

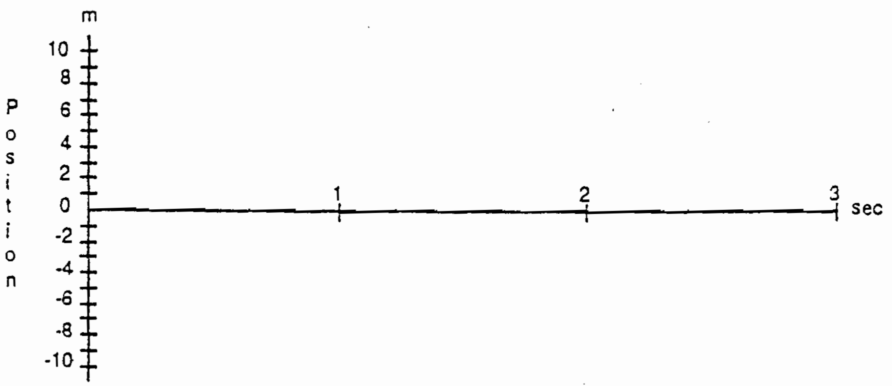
1. An object moves with a constant acceleration of -0.5 m/s^2. A graph of the object's acceleration is shown below for $0 \le t \le 3$ seconds.



a) Assume that the object has an initial velocity of 4 m/s. Sketch a graph of the object's velocity for $0 \le t \le 3$ seconds:

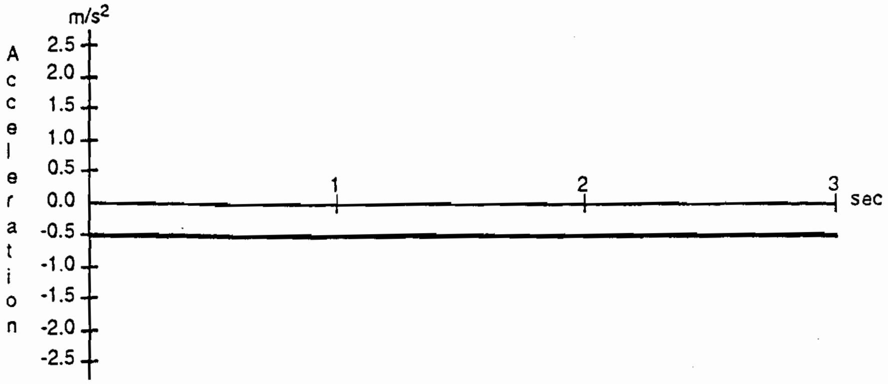


b) Now assume that the object starts at the position 0 m. Sketch a graph of the object's position for $0 \le t \le 3$ seconds:

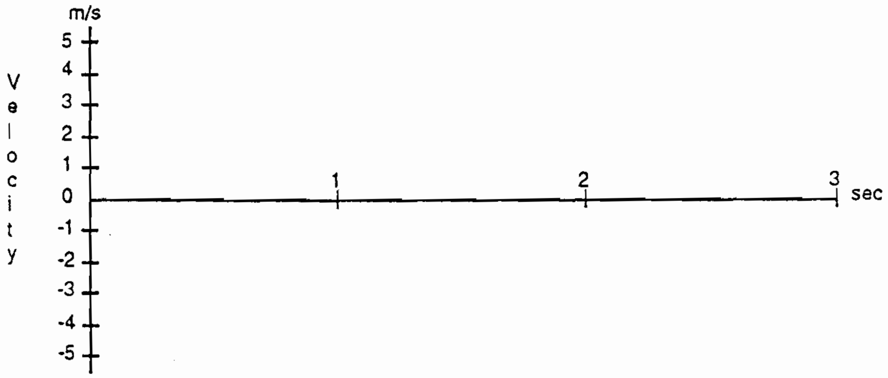


c) Suppose that these graphs describe the motion of a toy car (or a person walking) being monitored by a CBL system. Describe in words how the car is moving and describe a situation which might account for this motion.

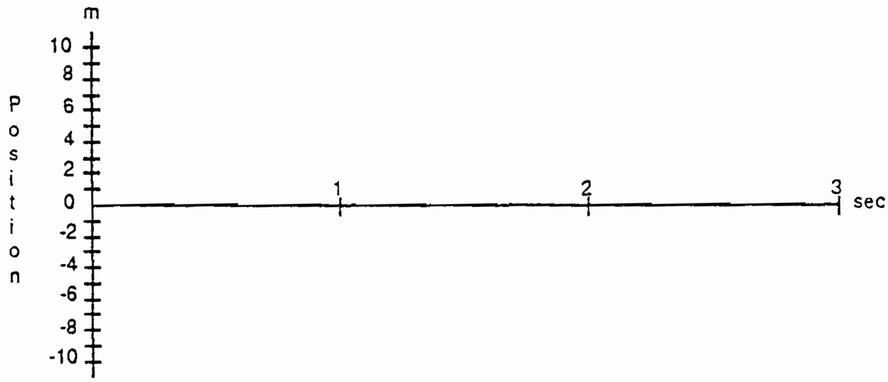
2. Another object also moves with a constant acceleration of -0.5 m/s^2 for $0 \le t \le 3$ seconds as shown in the graph below.



a) Assume that this object has an initial velocity of -2 m/s. Sketch a graph of the object's velocity for $0 \le t \le 3$ seconds:

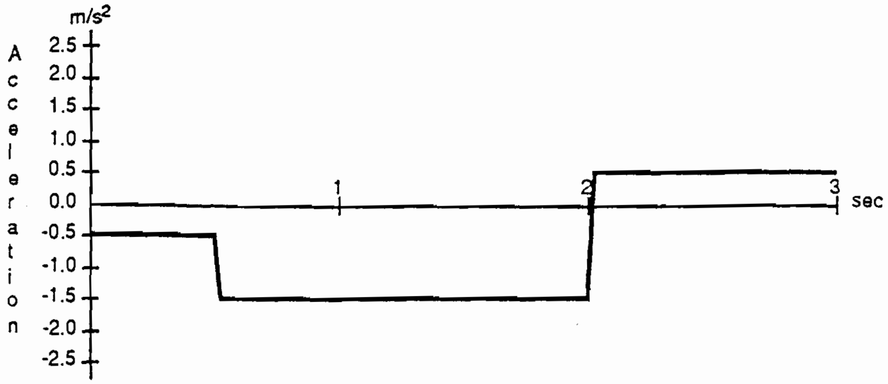


b) Now assume that the object starts at the position 0 m. Sketch a graph of the object's position for $0 \le t \le 3$ seconds:

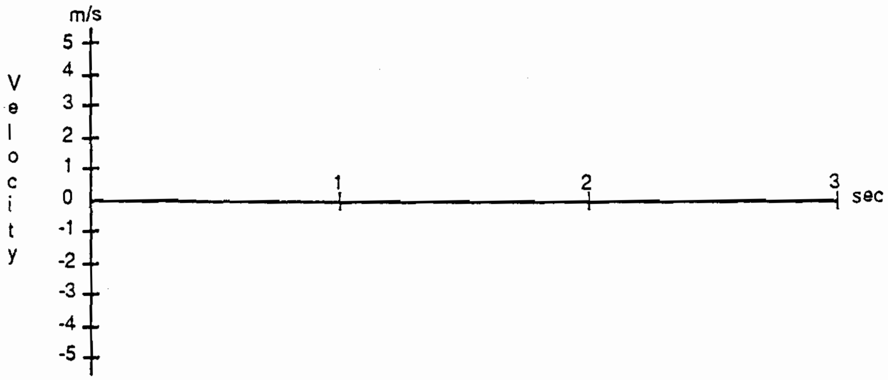


c) Suppose that these graphs describe the motion of a toy car (or a person walking) being monitored by a CBL system. Describe in words how the car is moving and describe a situation which might account for this motion.

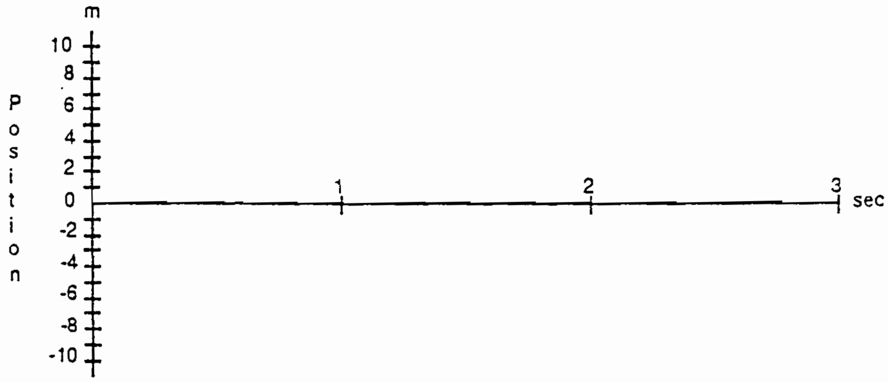
3. Here is the graph of another object's acceleration over time for $0 \leq t \leq 3$ seconds:



a) Assume that this object has an initial velocity of 2 m/s. Sketch a graph of the object's velocity for $0 \leq t \leq 3$ seconds:



b) Assume that this object starts at the position 0 m. Sketch a graph of the object's position for $0 \leq t \leq 3$ seconds:



c) Suppose that these graphs describe the motion of a toy car (or a person walking) being monitored by a CBL system. Describe in words how the car is moving and describe a situation which might account for this motion.

4.

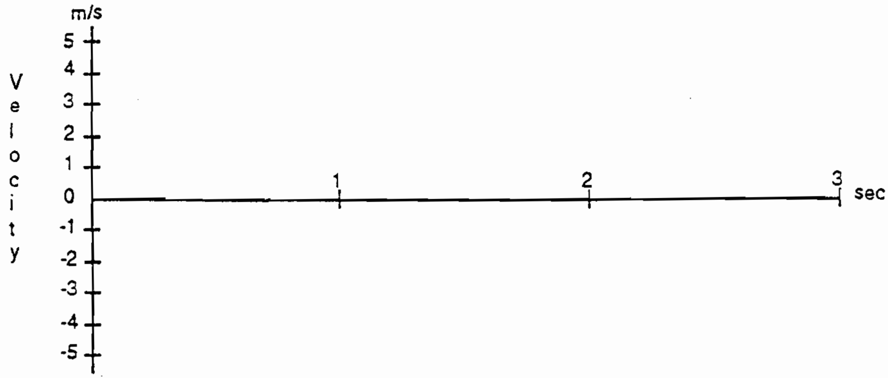
a) The motion of an object is described by an acceleration and an initial velocity and position as follows:

$a(t) = -0.5 \text{ m/s}^2$, for $0 \leq t \leq 3$, $v(0) = 4$ m/s, $s(0) = 0$ m.

Find expressions in terms of t for the velocity $v(t)$ and the position $s(t)$. Sketch their graphs and explain any differences you see between the graphs you predicted in problem 1.

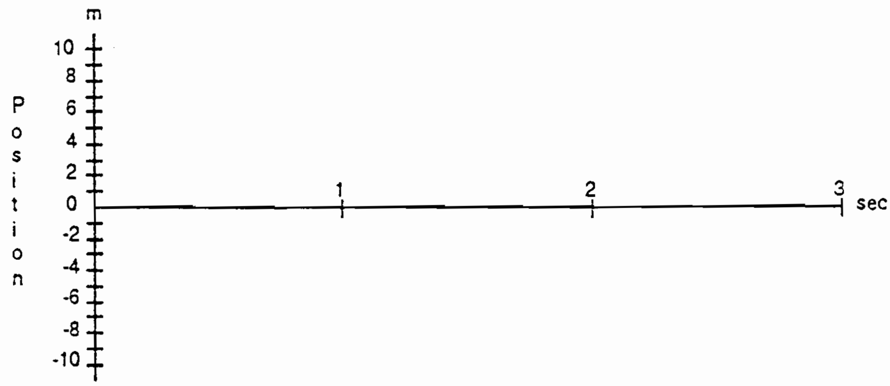
$v(t) =$ ____________________ m/s for $0 \leq t \leq 3$.

Graph of $v(t)$



$s(t) =$ ____________________ m for $0 \leq t \leq 3$.

Graph of $s(t)$



Explanation of differences between your predicted graphs and these:

4. **(continued)**

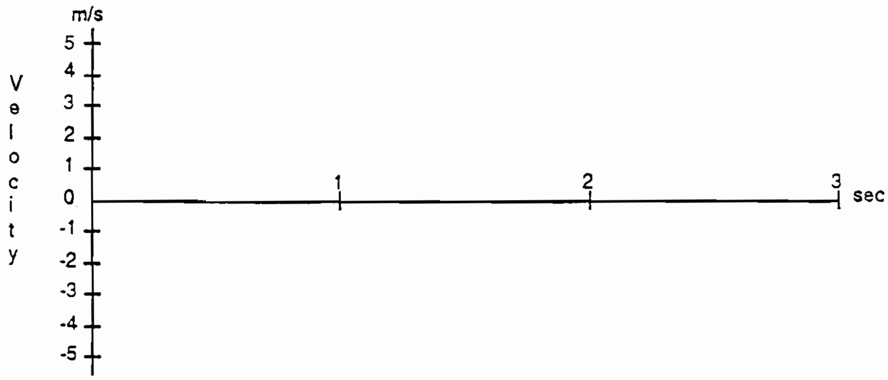
b) The motion of an object is described by an acceleration function with an initial velocity and position as follows:

$a(t) = -0.5$ m/s^2, for $0 \le t \le 3$, $v(0) = -2$ m/s, $s(0) = 0$ m.

Find expressions in terms of t for the velocity $v(t)$ and the position $s(t)$. Sketch their graphs and explain any differences you see between the graphs you predicted in problem **2**.

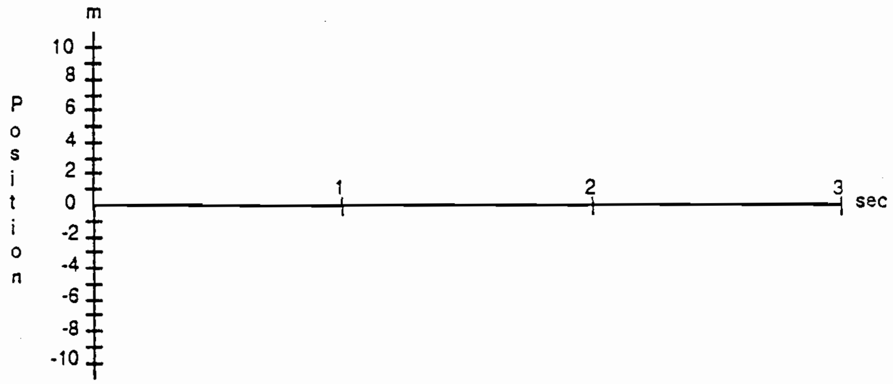
$v(t) =$ ____________________ m/s for $0 \le t \le 3$.

Graph of $v(t)$



$s(t) =$ ____________________ m for $0 \le t \le 3$.

Graph of $s(t)$



Explanation of differences between your predicted graphs and these:

4. (continued)

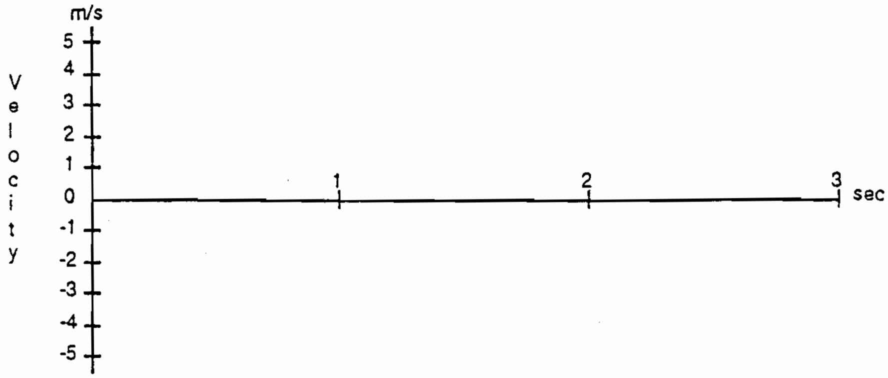
c) The motion of an object is described by an acceleration function with an initial velocity and position as follows:

$-0.5m/s^2, \quad 0 \leq t \leq 0.5a(t) = \quad -1.25m/s^2, \quad 0.5 < t \leq 2 \quad v(0) = 2m/s, \quad s(0) = 0m. \quad 0.$:

Find expressions in terms of t for the velocity $v(t)$ and the position $s(t)$. Sketch their graphs and explain any differences you see between the graphs you predicted in problem **3**.

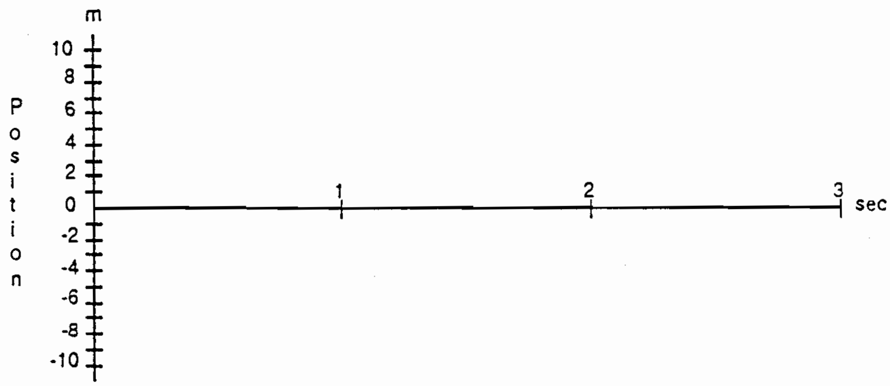
$v(t) =$ ____________________ m/s for $0 \leq t \leq 3$.

Graph of $v(t)$



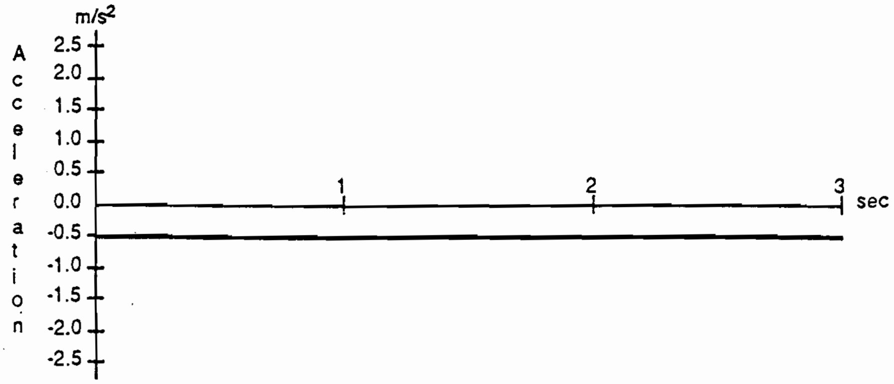
$s(t) =$ ____________________ m for $0 \leq t \leq 3$.

Graph of $s(t)$

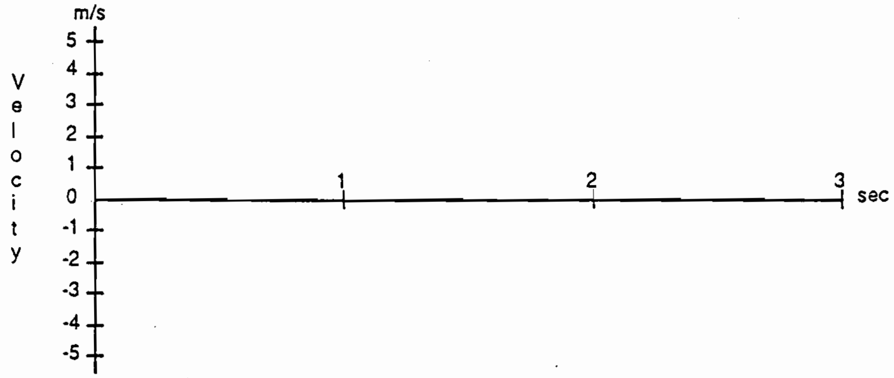


Explanation of differences between your predicted graphs and these:

5. An acceleration graph is given below (the same as in problem 1).

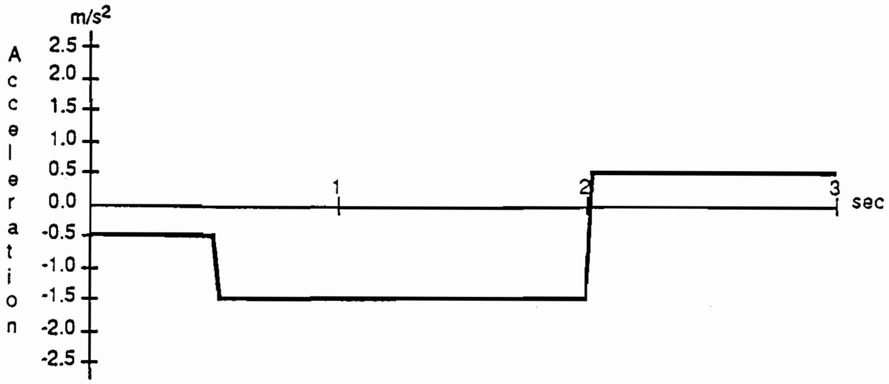


a) Sketch the graph of $A(x) = \int_0^x a(t)\,dt$, where $a(t)$ is the acceleration function described by the graph above.

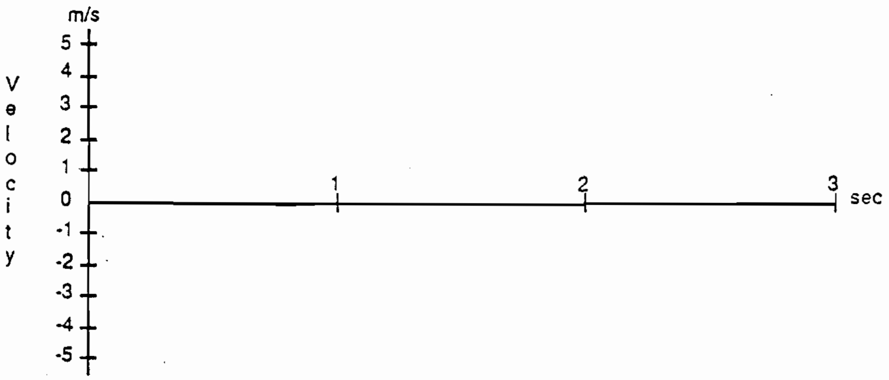


The graph you just sketched should describe the velocity of an object having the given acceleration. What accounts for any difference between this graph, the one you predicted in 1.a) and the one produced in 4.a?

5. (continued) An acceleration graph is given below (the same as in problem **3**).



b) Sketch the graph of $A(x) = \int_0^x a(t)\,dt$, where $a(t)$ is the acceleration function described by the graph above.



The graph you just sketched should describe the velocity of an object having the given acceleration. What accounts for any difference between this graph, the one you predicted in **3**.a) and the one produced in **4**.c?

Appendix D

Tables of Pre-Test Measures

Table D1. Breakdown of Means on Pre-Test

Variable	Total	Cooperative Group CBL	Cooperative Group Non-CBL	Individual CBL	Individual Non-CBL	F-Statistic p-value
n =	86	15	16	27	28	
Total Score on Pre-test mean	5.78	4.67	7.50	5.63	5.54	F(3,82)=2.64
standard deviation	3.24	2.77	3.26	3.34	3.10	p<0.06
Number of misconceptions/						
lack of conceptions as indicated						
by pre-test mean	1.48	1.73	0.69	1.85	1.43	F(3,82)=3.03
standard deviation	1.36	1.44	0.92	1.59	1.14	p<0.04
Differential calculus grade mean	2.59	2.51	2.73	2.56	2.60	F(3, 82)=0.45
standard deviation	0.84	0.85	0.79	0.91	0.83	p>0.71

Table D2. Breakdown of Means on Pre-Test
Cooperative Group/Individual Recitations

Variable	Total	Cooperative Group	Individual	t-statistic p-value
n =	86	31	55	
Total Score on Pre-test mean	5.78	6.13	5.58	t=0.75
standard deviation	3.24	3.31	3.19	p>0.20
Number of misconceptions/				
lack of conceptions as indicated				
by pre-test mean	1.48	1.19	1.64	t=1.47
standard deviation	1.36	1.28	1.38	p<0.10

Table D3. Breakdown of Means on Pre-Test
CBL/Non-CBL Recitations

Variable	Total	CBL	Non-CBL	t-statistic
				p-value
n =	86	42	44	
Total Score on Pre-test mean	5.78	5.29	6.25	t=1.39
standard deviation	3.24	3.11	3.3	p<0.10
Number of misconceptions/				
lack of conceptions as indicated				
by pre-test mean	1.48	1.81	1.16	t=2.28
standard deviation	1.36	1.5	1.12	p<0.02