

ECONOMICS OF INCREASED HAY PRODUCTION BY USE OF NITROGEN
FERTILIZER ON MOUNTAIN MEADOWS IN THE HARNEY BASIN OREGON

by

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CHAPTER I

INTRODUCTION

The Problem Situation

The problem which initiated this study arose out of interpretation of data from fertilizer experiments in hay production (50). The Department of Agricultural Economics of Oregon State College was approached by the staff of the Squaw Butte - Harney Range and Livestock Experiment Station, who asked for an economic interpretation of their experimental data.

Their interest in such an interpretation was directed towards formulating recommendations of an economic as well as technical nature, on the quantities of fertilizer to apply to native flood meadows producing hay. Specifically the economic aspect of their problem was to establish the rates of fertilizer use which give the greatest profit.

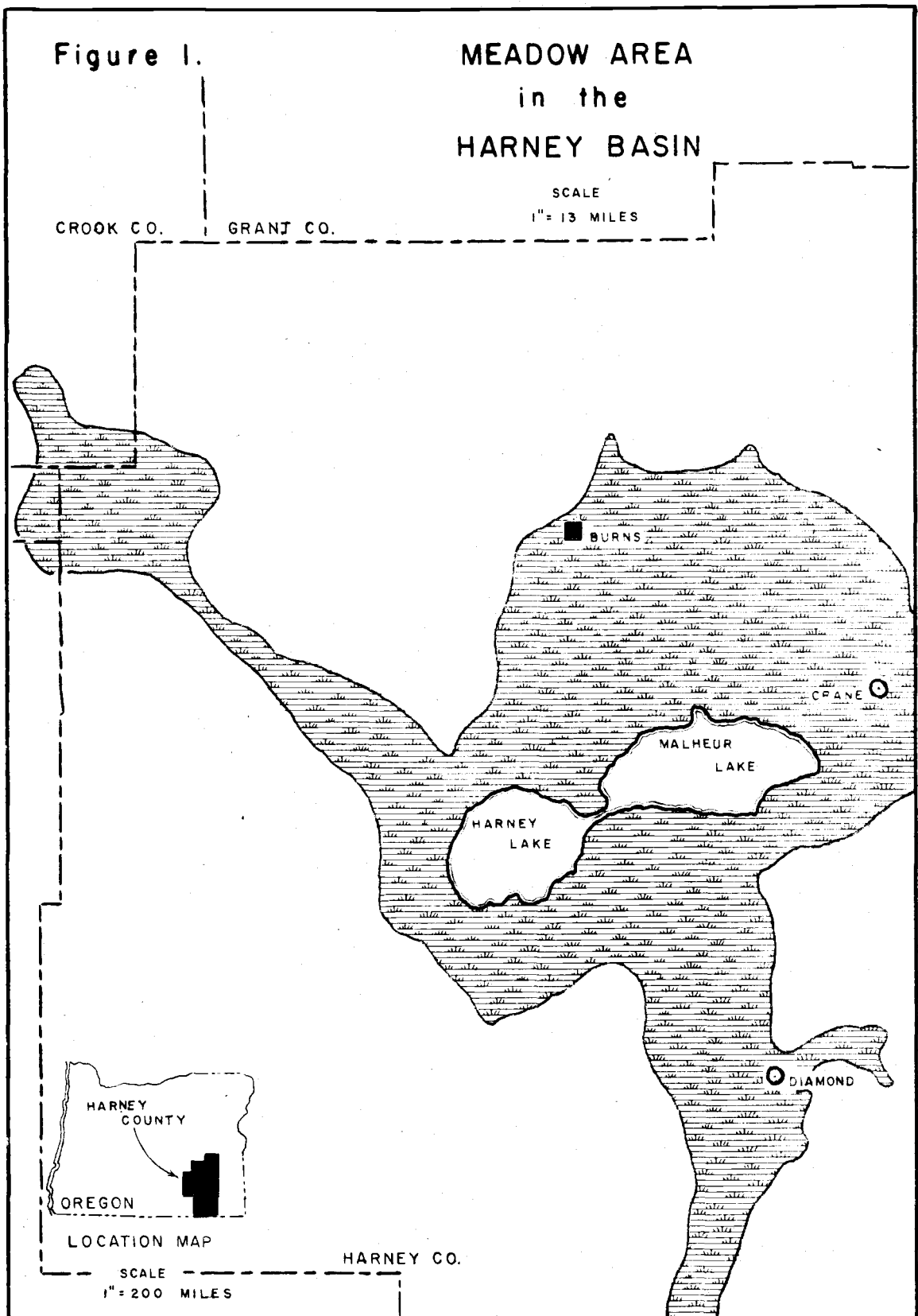
Native flooded meadows occupy nearly one half million acres in eastern Oregon. These lands serve as wintering grounds for cattle in the sagebrush-bunchgrass country and provide the major portion of hay for winter feeding. Hay production from these lands has an important influence on all phases of the cattle operation, through its direct effects on management of rangeland and livestock nutrition.

It is believed by those familiar with this area that an increase in cattle production would bring the ranchers a larger net income than they now enjoy.

Hay production from meadows is an intermediate activity in the ranch operation, the final product being beef. Thus an increase in meadow forage production through the use of fertilizers may require adjustment of the whole ranch organization. The purpose of this study is to investigate the economic aspects, i.e. the costs and returns, associated with maximizing profits by expanding the beef enterprise through fertilizer use on the meadows. The primary aim is to integrate experimental fertilizer-hay response data with the economics of expanding beef production in such a way as to provide an estimate of the most profitable rate of fertilization.

Description of the Area

This study is concerned with ranches in the northern half of Harney County (see Figure 1). All these ranches have a combination of summer range and meadowland, on which is grown wild hay, and in some cases, alfalfa, grain and improved pasture. A number of physical factors cause variations in productivity of the meadows. The most important of these is the amount, time, distribution and depth of the spring run-off, which is governed by the altitude of the watershed, the winter snow fall, the nature of the spring thaw, and the topography of the meadow. Drainage also varies considerably; in some areas the slope is so slight that drainage can only be made



effective by pumping. In other areas the water table is low enough, or can be forced down sufficiently by ditching, to allow growth of alfalfa. Variations in alkalinity are also related to drainage; in the poorly drained areas the soil is excessively alkaline. These variations in water supply, drainage and alkalinity have given rise to three main types of vegetation. (i) The Nevada bluegrass type - areas containing almost pure stands of Nevada blue grass. These areas are characterized by short periods of early spring flooding, and are generally well drained. (ii) The rush-sedge-grass type - areas containing a mixture of rushes, sedges, and water-loving grasses with some native clovers. These areas are flooded for 6 to 12 weeks in the spring to a depth of 1 to 6 inches. (iii) The rush type - areas which are almost pure baltic rush (wire grass). These areas are alkaline, poorly drained, and flood to a depth of 6 inches or more for three months in the spring. Of these three types the rush-sedge-grass is dominant. The Nevada bluegrass areas occur only in the narrow and better drained mountain valleys, and the rush type is restricted to the low-lying area which surrounds Malheur and Harney Lakes.

The meadow soils are medium or fine textured, and are generally mildly calcareous and slightly to moderately alkaline.

Most of the sedge and rush species are of a rhizomatous nature and form a compact sod with root penetration seldom exceeding 12 inches. This sod in combination with variations in the nature of the subsoil, determines those areas which can withstand cattle grazing

during the period of flooding without serious damage to the pasture. In those areas where the pasture breaks up under cattle trampling, not only are hay yields impaired but it becomes much more difficult to harvest because of the rough surface.

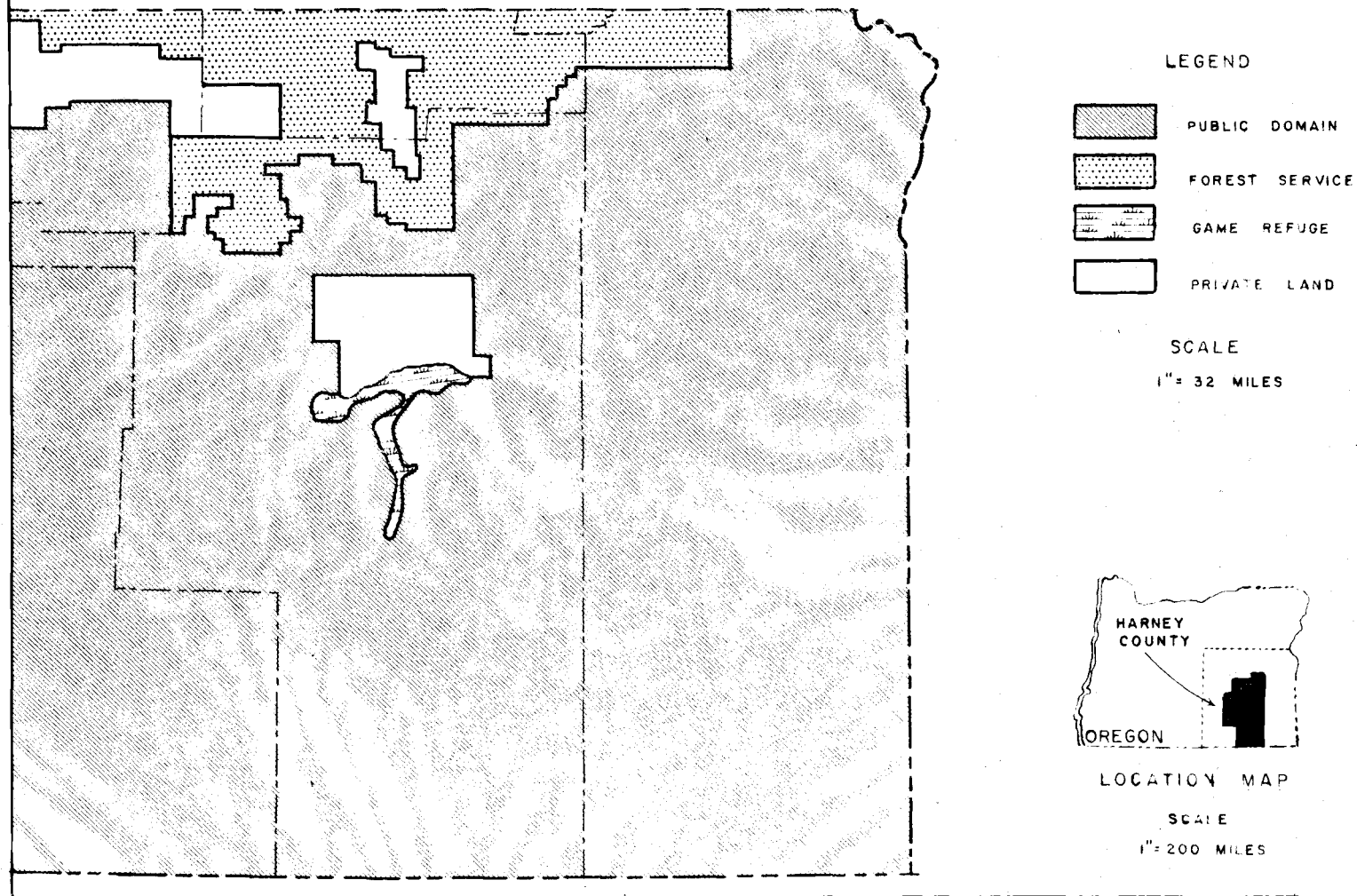
The climate also varies somewhat between meadow areas; the average annual rainfall is 8 to 10 inches, but there are differences of up to 30 days in the length of the growing season due to location and altitude. Most of the hay meadow land lies between 4,000 and 4,500 feet. In some low-lying parts frost has caused heaving of the meadows, which has seriously impaired the hay yields due to uneven spring flooding.

The range may be classified into two types, mountain and desert range. In general the mountain range has sufficient water for cattle, and the best feed is found at higher altitudes. The desert range varies according to its altitude. The lower desert is drier, with less growth and less winter snow, hence water can become critical except where it is feasible to drill wells. The high desert receives more rain and snow, hence there is more feed and a better water supply, but growth is later, and it is often not economically feasible to drill wells because of the depth of the ground water. Some areas of the desert have been ploughed out of sagebrush, fenced, and sown to crested wheat grass, and in a few cases it has been irrigated and farmed to produce pasture, rye and alfalfa.

Institutional factors, land tenure in particular, play an important part in ranch operation in this area (see Figure 2). Much

Figure 2.

PUBLIC LANDS IN THE HARNEY BASIN AREA



of the desert and mountain range is owned and administered under the Taylor Grazing Act by the Bureau of Land Management or the Forest Service, who annually regulate the number of animals permitted to graze and the grazing period. The greater portion of this range is run "in common", that is up to 4 or 5 ranchers will graze their cattle over the same area. In some cases the range is fenced and leased in private allotments. In addition to Federal leases, most ranchers own limited areas of range. Some of these only include valleys and water holes and were originally purchased to gain access to and control of large areas. The grazing permits are issued on the basis of "animal unit months" (AUMs) which is one cattle beast over 6 months old for one months grazing, at the rate of 15 cents to 44 cents per AUM. These permits are issued on the basis of periodical surveys which determine the grazing capacity of the range. The permits are allocated among ranchers, mainly according to base-property, which is the capacity of the ranchers' meadows to winter cattle. This situation gives rise to the problem of establishing and maintaining the balance between summer and winter feed supplies on range and meadow.

The tenure of the meadows also varies. Most of it is privately owned, but some, the Malheur Game Refuge, is administered by the Wild Life Service, and is leased to ranchers during specified months at the rate of \$1 per head per month (see Figure 2). This land is not included as base-property in the allocation of range permits.

Objectives and Scope of the Study

From the above section it can be seen that a wide variety of factors have bearing on the basic problem. In this section it will be shown why only certain aspects are abstracted to limit the scope of the study.

Firstly, all those aspects which cannot be empirically measured must be either assumed constant or allowed to vary only within specifically defined limits. It is necessary to assume that all the physical, biological, social, institutional and human aspects of the problem will remain unchanged. Before considering the economic aspects, it is necessary to justify their consideration by defining the basic hypothesis of the study - that some adjustment or expansion of current resource use, organization or technique will give rise to increased profits.

The problem now focuses on the economic factors governing the expansion of the ranch operation. In order to reduce the problem to manageable proportions, and at the same time lay the foundations for further investigations in this field, it is necessary to assume that some of these factors remain constant.

As already stated, the initial impulse for this study came from the objective of evaluating fertilizer use on meadow lands, and associated with this the implication of meadow improvement with regard to the ranch operation as a whole. It is therefore proposed to orient and limit the study along these lines. There are

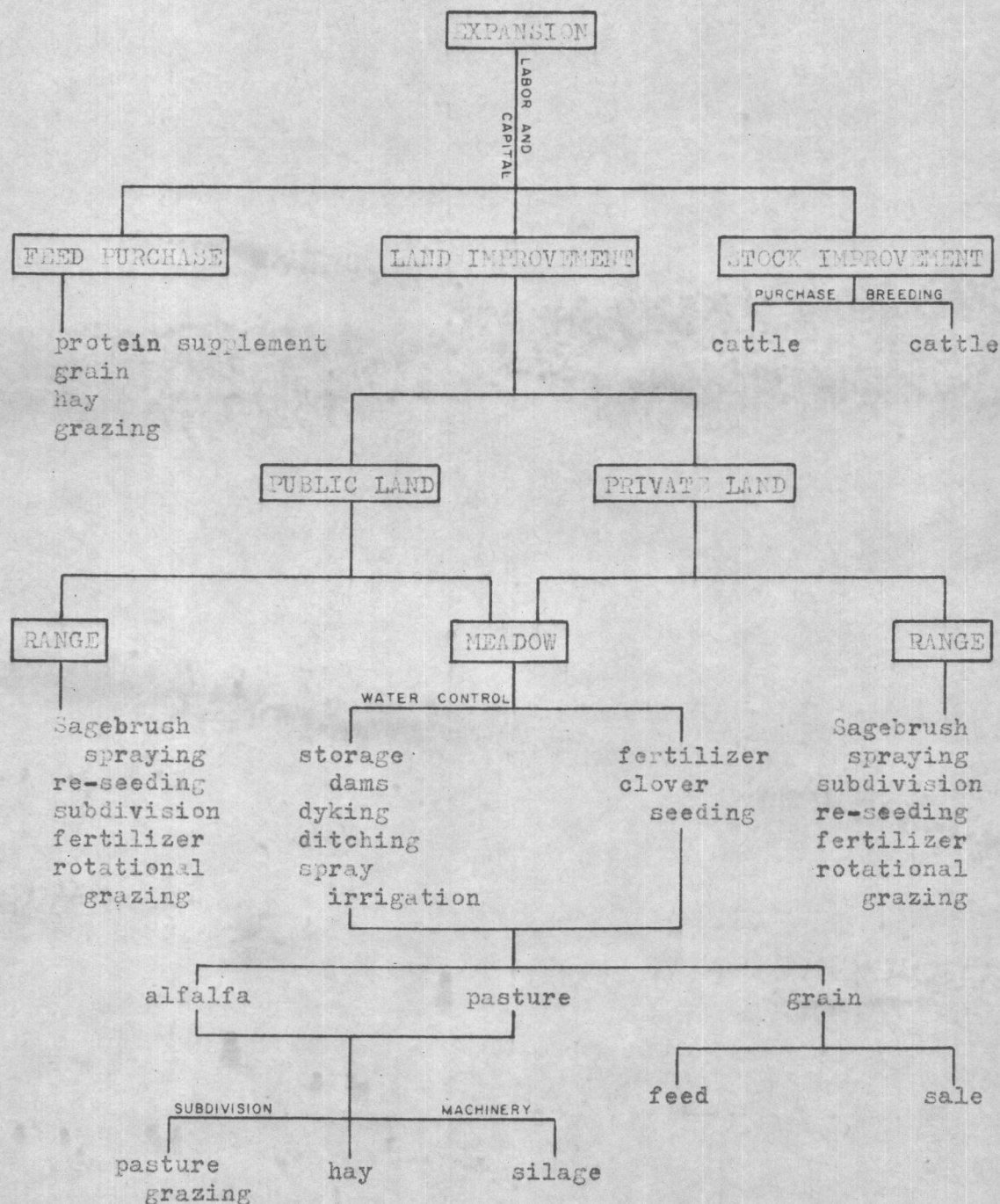
possibilities for ranch expansion other than through improvement of meadows (see the diagram on the following page). However, for the purposes of this study it is considered that investigation of meadows gives good possibility of success. It is therefore proposed to eliminate any consideration of range land improvement. (48 and 51). However, this study yields results which have implications for range improvement. Thus the scope of the study is restricted to the consideration of those economic factors involved in the expansion of the ranch operation through increased production from meadow land.

Within the limits outlined above, and with the basic assumption that some adjustment or expansion of current resource use, organization or techniques will give rise to increased profits on the ranches in the Harney Basin, the hypothesis which forms the basis of this study may be stated as follows: It is economically feasible to increase the ranch operation in the Harney basin through increased forage production on the meadows by use of fertilizer.

Source of Data and Method of Analysis

The primary data on hay yield response to nitrogen fertilizer was supplied by the Squaw Butte - Harney Experiment Station (see Appendix I). Before it is possible to make an economic analysis of the experiments, it is necessary to consider the various resource situations in which the fertilizer may be used and those factors which influence a rancher's decision on whether or not to use fertilizer. This information was obtained from Federal and State agencies operating

Methods of Expanding the Ranch Operation
Where Land Area is Limited



in the area and from a survey of ranchers. There are approximately 60 ranches in the Harney Basin, Silver Creek and Diamond areas.

Because of the size of the population and the nature of the study it was decided that a selected sample of 20 ranchers would be sufficient to provide information on the various conditions and problems found in the area. The ranchers were selected on the recommendation of the county agent.

From the section describing the area it is readily apparent that the resource situation and management problems of each ranch are unique. Because of this no attempt has been made in the study to present average results or recommendations.

The procedure followed was first to analyse the major factors in decision making on ranches in the area. Ranch operations are all interrelated in yielding one final product, beef; thus, in considering factors governing a decision on use of fertilizer it is necessary to take into account the other operating and developmental decisions which must be made.

Having established the decision making framework for the whole ranch operation, the next step is the economic interpretation of the fertilizer experiments. To permit such an interpretation to be made, it is first necessary that the hay yield may be estimated for any given level of nitrogen (not just at the 5 levels of nitrogen used in the trials). This is obtained by formulation of an estimating equation and a hay production function from the experimental data by use of curvilinear regression techniques.

The object of an economic analysis is to estimate the maximum profit combination of the scarce resources available. In this case the particular resource we are interested in evaluating is nitrogen fertilizer, and the medium by which we measure its economic usefulness is increased forage production and consequent increased beef output. There are a number of techniques available by which such an analysis could be made, notably budgeting, Cobb-Douglas type analysis, and linear programming. The latter method was selected for the analysis used in this study because it permits the simultaneous consideration of a larger range of alternatives in obtaining the optimum solution than would have been possible in the time available had budgeting been used. Further, experience has shown that the Cobb-Douglas function is not a satisfactory tool for this type of intra farm analysis.

The object of the linear programming analysis is to estimate optimum input-output relationships in beef production under various selected resource and price situations. The production and cost data used to calculate the coefficients used in the programming models were obtained from the survey, market reports and experimental results on hay and livestock production on meadows.

CHAPTER II

DECISION MAKING

In each resource situation the rancher must consider 5 areas of imperfect knowledge (35) which influence his decisions on operating and developing his ranch. The 5 categories of imperfect knowledge are:

(i) Price structures and price changes of all factors and products with which he deals.

(ii) Production methods: this involves the carrying out of technical operations on the ranch.

(iii) Development methods: this involves appraisal of technological advances, in such things as strains of crops, grasses and clovers, use of fertilizer, improved machinery, methods of controlling irrigation water, as well as any unforeseen developments which may appear as possibilities.

(iv) Human aspects: this includes the value which the rancher and his family place on leisure, security, risk, uncertainty, community service or social prestige.

(v) Institutional setting: this includes such items as availability of credit, and government policy as it effects credit, insurance, farm prices, land tenure, land development, rural electrification and taxation.

The above framework is used as a basis for the following analysis of the more important factors which influence the rancher's major

decisions.

Operating Decisions

Nine important decisions have been isolated for consideration. These are decisions which a manager must make in handling the month to month operation, and year to year organization of the ranch.

1. How many cattle should he run? Apart from the physical capacity of the range and meadow the rancher must weigh his decision in terms of risk, security and long-run profit. In unfavorable years he may not want to overgraze, which may lead to permanent damage, or buy additional feed. On the other hand, he may not wish to sell cattle because of the losses involved. Under these conditions he may decide to sacrifice efficiency and opportunity of higher profits by carrying fewer cattle, and producing more beef per head, to obtain flexibility and reduce risk. Another factor the manager must consider is the numbers and age of cattle which will best utilize the feed available through his range permit.

2. What should be the calving date? The two major factors in this decision are the weather, as it effects feed conditions on range and flood conditions on the meadow, and the turn-out date permitted by the Bureau of Land Management. If calving can be practically completed before turning-out there is a better chance of a higher calving percentage due to better husbandry.

3. When should cattle be turned out on range? In general this is determined by the turn-out date set by the Bureau of Land Management.

However under certain conditions the rancher may request permission to turn-out early. Such conditions would arise if the meadows became flooded early with the possibility of damage to pastures through cattle trampling, or an acute shortage of hay. On the other hand the manager may delay turning out to allow range growth to get started, and so increase the total summer feed available. On the desert range he may have to delay turning out if there is no water, as it is often not feasible to haul water.

4. How much hay should be cut? The major factor in this decision is the balance between meadow and range. In most cases all available meadow is cut for hay and either bunched or stacked. One of the main problems facing ranchers is variation in hay production from year to year due to climate. To overcome uncertainty managers have resorted to carrying over reserves of hay varying from one half to a complete season's requirements. The extent of this carry-over depends on the variability of hay yields and how close the ranch is to its productive limit in terms of cattle. The amount of bunched hay cut relative to stacked hay is usually determined by the stacked hay requirements. When these have been met the rest of the meadow is bunch raked. Accessibility and size of some meadow areas is a factor where it may not pay to take in the necessary hay-making machinery.

5. How should range be best utilized? A major consideration in this decision is the composition of the cattle herd which will make best use of the range permit. Cattle management while on range involves such factors as disease, maintenance of water supply,

"salting out" to encourage cattle away from water holes, and avoiding use of range where larkspur is prevalent until July, after which time it is no longer poisonous. The distance from the range to meadow may also be a factor. In some cases this distance is as great as five days drive.

6. When should cattle be brought in off range? This decision is normally determined by the date set by the Bureau of Land Management and Forest Service. However the estimated price of yearlings and weaners in the fall may influence this decision. In a case where yearlings are run the rancher generally aims to bring in his cattle before the feed is reduced sufficiently to cause the weight gain to fall to zero. In some cases deer limit the range feed available in the fall, and in these areas it may be prudent to take out the cattle before the hunting season opens.

7. How should fall feeding be organized? The main factor in feeding during this period is to avoid the expense and labor involved in feeding hay any earlier than is absolutely necessary. The amount of aftermath available is important; although it is dry and low in feed value, it is cheaper than hay and sufficient to sustain cows. Where insufficient aftermath is available, bunch-raked hay is the cheapest alternative. The possibility of early winter snow, necessitating early feeding of stacked hay and loss of bunched hay, must also be taken into account.

8. At what age should young cattle be sold? This decision is whether to sell weaners or yearlings, and is determined mainly by the

resource situation. On poor quality range weight gains in yearlings are often unsatisfactory relative to gains made by calves. Also calves usually come off this class of range in better condition than yearlings. When the range is of this type it is likely that it can be best utilized by carrying cows and calves only and selling weaners. In those cases where winter feed is a limiting factor the tendency is to equate the cattle numbers to the available feed supply by selling as many weaners as necessary in the fall. A variable calving date is a factor affecting age of sale, as younger calves are often too small to sell as weaners in the fall. Where grain production is feasible an alternative is to develop a limited feed-lot operation, to allow more flexibility in the age and finish at which cattle may be marketed.

Another factor involved is the range permit, which is set up in such a way that only cattle six months of age or more are designated as animal units. Thus the same charge is made for a yearling as for a cow and calf. In this case the best utilization of range might be accomplished by stocking completely with cows and calves, and selling weaners, rather than restricting cows in favor of yearlings.

9. How should winter feeding be organized? Decisions on winter feeding depend mainly on the prices and rates of substitution for the various feeds available, plus consideration of the rate of gain desired in weaners throughout the feeding period. The composition of the herd also governs the feed operation. In most cases the winter feed program is designed to fit a herd which can best utilize summer range.

Development Decisions

Development is usually undertaken for two reasons: The first is to bring the strong and weak points of the operation more nearly into balance, thus allowing a more efficient use of available resources by increasing production. The second is to reduce the uncertainty associated with operation of a ranch. In some cases the uncertainty and risk associated with the balance of feeding between seasons and between range and meadow, has forced ranchers to operate below their capacity. Thus a reduction of uncertainty through development becomes a significant item in the efficiency of utilization of resources.

The decisions of a ranch manager are primarily directed towards providing an adequate feed supply for his cattle throughout the year. The annual feed requirements may be divided into spring, summer, fall and winter. Ranch development will involve increasing the feed supplies for one or more of the periods.

1. Spring feed: On many ranches this is a problem period for feed. With the tendency on the part of the Bureau of Land Management to further delay range turn-out dates, ranchers must provide more hay or an alternative source of grazing, preferably not from the flooded meadows. Where suitable land is available one of the most promising avenues of development seems to be ploughing of sagebrush, fencing and sowing crested wheat grass to provide the necessary grazing (51, pp.19-20). Another alternative is to provide additional hay by using fertilizer on the meadows (15). If private range is available, this

may be developed to provide some spring feed (48).

2. Summer feed: On many ranches this is the most limiting factor in the cattle operation. Development to overcome this limitation may follow two lines - first, development of meadow to provide summer pasture, or second, the development of range. At present the only economically feasible methods of range development are extension of water holes, subdivision, reseeding and sagebrush spraying (51). The major factor effecting any decision on range development is the policy of the Bureau of Land Management and Forest Service. Both of these agencies have been tending towards establishment of private allotments by fencing, which would encourage ranchers to improve their range. In some areas the authorities themselves have improved range in other ways, such as ploughing, sowing, reseeding, spraying or expanding water holes. The Bureau of Land Management and the Forest Service are prepared to subsidize development by providing the machinery, materials and seed required.

The tenure of range is an important factor in development decisions. Ranchers are more likely to improve their own range. However there are a number of reasons why more range is not privately owned. (a) Under the present permit system the rent per AUM is extremely cheap relative to the value of the land. (b) In some areas the range is deteriorating, and it is better from the rancher's point of view that the government be responsible for taking the risk of further deterioration, or of restoring productivity. (c) Taxation on land is a deterrent to private ownership. (d) To warrant development,

a rancher would need to own large areas of contiguous range; under present regulations he is only permitted to buy a section of Federal land at a time. Thus it is difficult to obtain sufficient area to warrant fencing and developing. In those cases where ranchers do own range and run it on an "exchange of use" permit, an alternative is fencing and development. Before embarking on any project of range development, the costs must be weighed against the estimated increase in returns resulting either from increased cattle weights or increased numbers. Another important factor is that to be effective this type of improvement must be carried out on a large scale, requiring heavy capital outlay.

3. Fall feed: This is the feeding period between the time cattle come off range in October until the start of hay feeding in December. In general, provision of feed at this time of year is not a serious problem. The Malheur Game Refuge provides an important source of cheap fall feed for many ranches located in the center of the county. The possible loss of grazing rights on this area due to homesteading would mean the ranchers would have to develop alternative sources of fall feed from their own meadows.

4. Winter feed: The problem of increasing winter feed supplies centers on development of the meadows. The prime factors in a decision on meadow improvement are the costs involved and the response to the various programs which may be undertaken, such as fertilizer use, diking, ditching, irrigation storage dams, pump drainage, pump irrigation, (64) leveling, weed spraying, ploughing, reseeding or

oversowing (50). The relative importance of increased winter feed depends on the balance between range rights and the wintering capacity. On some ranches, "voluntary non-use" is being taken on part of the range permit; however, after three years there is a risk that the permit will be permanently reduced. In such cases the shortage of winter feed is critical. On the other hand, where range is limiting if additional cattle are carried through the winter they would either have to be sold in spring or pastured on the meadow in summer. Thus, the first factor to be considered in a decision on improvement of meadow is how the additional forage will be utilized (73). If summer grazing is considered, the factors are provision of adequate water supply, how well the meadow will withstand trampling in June and July, and the possibility that flies may worry the cattle excessively. If sale of hay is an alternative, the factors to consider are the prospective market for wild hay and the possibility of growing alfalfa as a more saleable product. Another factor here is that the Bureau of Land Management and Forest Service do not favor the sale of hay, in view of their use of the base-property capacity as an index for allocation of range permits.

If additional meadow production can be efficiently used, then the decision becomes that of selecting the best method of improvement. The two methods available are use of fertilizer and controlled irrigation. There are a number of institutional factors influencing a rancher's decision to adopt one or other of the above practices. The government policy of subsidizing water control work up to a

maximum of \$20 per acre, with no subsidy for fertilizer, is an important consideration. Water rights on some properties are poor, thereby providing a greater incentive to develop those with good rights. Taxation being levied at a fixed rate per acre regardless of productivity is a further incentive to develop.

Besides the institutional factors, two other considerations, the cost and response of the two methods, must be taken into account.

In a decision on whether to use fertilizer, the main factors to consider are: (a) The response obtained, which is limited by the type of vegetation and by alkalinity (51, pp.1-8). In this area the limiting factor in growth is alkali rather than fertility, and because of this the effect of the fertilizer may be completely nullified by forming compounds which are not available to the plant. Depth of flooding also limits the response to fertilizer; where flooding is over 4 inches there is little or no response. Availability of flood water is also limiting. Fertilizer will show its best absolute response in years when water is well controlled and plentiful. Fertilizer will offset to some extent the effect of a poor water supply on hay yields, but its most effective use would be to enable a build-up of suitable hay reserves in good years. Ranchers are in a good position to know 2 or 3 months in advance whether there is likely to be adequate water or not through snow surveys carried out by the Soil Conservation Service, but temperature is still a limiting and unpredictable factor in growth. (b) The flexibility and relative costs; fertilizer is a flexible method of building up hay reserves.

In a decision on whether to develop controlled irrigation or not, the main factors to consider are: (a) The yield response due to the additional control, either in the form of higher yields or more reliable yields. (b) The time period involved in the development during which the land is out of production. (c) The possibility of a drainage problem. Where better utilization is made of water, and it is made available over longer periods, drainage could become a problem. (d) The possibility of increased alkalinity: if water is not allowed to flow over the land and wash off the alkali or carry it down, e.g. where sub-irrigation is practiced, the alkali may come to the surface in sufficient quantity to damage pastures. (e) The importance of conservation of water resources: the primary aim of development of this nature would be to enable the limited water to be applied to a greater area over a longer time. (f) The possibility of ploughing and planting improved grasses: this requires that flood water be well controlled, otherwise it is likely that the pasture will revert to the original species within three years. However, if the improved species can be maintained, this type of hay requires only about one third of the water needed by rush-sedge type meadow, hence, where water is short this would reduce variability of yields. This type of hay may not yield any more than the wild hay, and the protein content is lower. (g) The possibility of ploughing and sowing alfalfa: for this the land must be suitable for sub-irrigation, and frost is a limiting factor in some areas.

The factors which have been discussed are those areas of

imperfect knowledge a rancher must study. It is obvious there are interrelationships among these areas. A careful consideration of all of the ranch business is necessary when a fundamental decision is faced. Increasing beef production by the production of additional hay is an example of a decision that affects many parts of the ranch business. This chapter was designed to provide the framework within which this decision must be made.

CHAPTER III

REVIEW OF LITERATURE

The determination of profitability of using fertilizer requires knowledge of the physical response to fertilizer, prices of factors and products, and comparative returns to all factors when devoted to alternative enterprises. This chapter presents a brief summary of some of the research findings and theories on plant response to nutrients, statistical interpretation of these responses, evaluation of resources, and methods of analysis by which profitability of fertilizer use may be estimated.

Fitting a Production Function to Fertilizer Data

Ibach and Mendum (34, pp.1-3) state the first problem in an economic interpretation of crop response data to fertilizer application is to describe the response curve from the data available. Hutton (30, pp.14-16) sets down two criteria which may be used in choosing one function that "best" explains the situation, from the infinite number of unique functions which may be fitted. These criteria are: (a) that the function not violate biological laws insofar as they are understood; (b) that the function pass certain tests of statistical logic. On the problem of what violates biological laws there is a wide variety of opinion among soil scientists. Bray (40, pp.53-54) defines five major concepts on the relationship between plant growth and soil nutrients. (i) The availability concept

which recognises that different forms of nutrients in soils vary in their availability and that it is often the relatively small amount of a rather highly available form which has the most influence on plant growth. (ii) Liebig's law of the minimum, which states that the yield is limited by that factor which is at the minimum. (iii) The law of diminishing returns, as formulated by W. J. Spillman (57, pp.75-77), which states in effect that with each additional increment of fertilizer the increase in yield diminishes. (iv) The Baule percentage yield concept which states that the final yield is the product of all the factors in yield, and not a result of a minimum factor as asserted by Liebig. Each nutrient is expressed in terms of its ability to produce a certain percentage yield. (v) The elasticity concept, developed by Bray himself (11) which states that the available soil nutrients have a variable availability which depends on the mobility of the nutrients in the soil and on the nature of the plant.

In addition to these concepts Willcox (71, pp.527-530 and 72, pp.38-39) has advanced two propositions. (i) The inverse nitrogen law which states that the yields of all agrotypes are inversely proportional to the percentage of nitrogen in their whole, dry, above ground substance. (ii) The concept of the nitrogen constant (68, pp.36-48) which states that when plants are "normally" grown under optimum conditions of all growth factors, they absorb a constant 318 pounds of nitrogen per acre in a single growth cycle. Black and Kempthorne (7, pp.303-309) and (66, pp.310-314) disagreed with

Willcox (69, pp.315-328 and 70, pp.499-502) over some specific relationships of growth factors in plant yields and particularly over certain applications of the Mitscherlich equation (8, pp.497-498). Throughout the literature on this subject the only generally accepted concept appears to be that of diminishing marginal returns to nutrient inputs over all but the lowest range of nutrient uptake. Thus from this it might be concluded that from a biological standpoint all statistical functions are equally good if they permit decreasing marginal returns. However there are exceptions to this conclusion for statistical reasons. The second criteria, stated by Hutton (30, pp.14-16) in selecting a function is that it should pass certain tests of statistical logic. Such tests may show that a function which does not have diminishing marginal returns, such as a linear equation or a Cobb-Douglas function, may give the line of best fit. These functions would not necessarily be in conflict with biological theory in those cases where the range of experimental data is relatively narrow, so that the difference in marginal returns is so slight as to be readily explained by experimental error.

Hutton (30, pp.14-16) in his appraisal of statistical measures of a function, such as correlation coefficient, or standard deviation of actual from estimated values, points out that by increasing the degree of a functional polynomial equation it is possible to eliminate the deviations of actual from observed values. However the desire to obtain a relatively wide application of the experimental results would discourage the choice of such a function. Also equations above third

degree would be difficult to justify in terms of biological law. The conclusion from this is that statistical tests limit the number of alternative equations to be considered, but having obtained these alternatives there are no tests which allow one to assert conclusively that any one function is the best for interpreting the experimental data, unless a large amount of data is available.

Functions which have been used in analysis of fertilizer data fall into three main classes, the power or logarithmic type, the polynomial, and the exponential.

(a) The power function: Tintner (62, p.51) states that one of the pioneers in econometrics, Paul H. Douglas, first applied the power function to production data. The equation most often used is of the form $\hat{y} = ax^b$, i.e. a function which is linear in the logarithms, commonly known as the Cobb-Douglas function (19, pp.139-145). Here \hat{y} is the yield, x is the fertilizer input and a and b are parameters. In his appraisal of this type of function Tintner (62, p.54) states that a major advantage of this function is its ease of fitting by the classical method of least squares. Johnson (37, p.520) points out that a disadvantage of this function is when $b > 0$ the equation implies a continually increasing yield without limit. Such an implication is not in accord with biological logic at high levels of nutrient uptake by plants. A variation of the Cobb-Douglas function has been investigated by W. G. Brown¹. This variation takes the form

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$\hat{y} = ab^x$ and has the advantage that it may be fitted to the declining phase of input-output data. Johnson's conclusion (37, p.528) on the conventional power function is that it seems to give a poor fit especially in the upper range of fertilizer application.

(b) The polynomial function: There are two forms of this type of function which have been applied to fertilizer data. One is the regular quadratic equation with the form $\hat{y} = a + bx + cx^2$. Johnson (37, pp.528-529) concludes that this is one of the simplest forms to fit and for purposes of interpolation it gives results in many cases equally as satisfactory as other more complicated expressions. However in experiments reported by Heady, Pesek and Brown, (28, p.43, 73 and 95), the square-root equation, with the form $\hat{y} = a + bx + c\sqrt{x}$ was found to give better results, particularly in cases of multiple inputs. French (20) also indicated that the square-root form was the most generally applicable of the two. Johnson (37, p.519) is of the opinion that the disadvantage of the polynomial expressions is imputing any biological significance to either the squared or square-root term. However Black (6) has stated that such terms are not contrary to biological theory. Hutton (31, p.17) concludes that for the range of data normally covered by fertilizer experiments, it would be difficult to justify biologically the use of equations greater than of the second degree, except where experiments are run under unusual conditions where the nutrient content of the soil is so low that increasing returns to inputs may be possible over a limited range of application.

(c) The exponential function: This function is based on the general principle of decreasing increments. The development and application of this function is attributed largely to the work of Mitscherlich (42, pp.413-428), Spillman (50), Baule (4, pp.363-385) and Hartley (25, pp.32-45). Applied to fertilizer, this principle states that as fertilizer is added in units of uniform size, with other factors unchanged, yields increase at a decreasing rate in such a way that each increment in yield throughout the series is a constant percentage of the one which precedes it. Exponential equations have been applied to fertilizer - yield data in a number of different forms, the most commonly known being the Spillman function. This has the form $\hat{y} = M - AR^x$ where \hat{y} is the calculated yield, M is the theoretical maximum yield obtainable through use of fertilizer, for conditions accompanying the reported yields, R is the ratio of successive increments in yield (a constant having a value between 1 and 0), x is the quantity of fertilizer, and A is the difference between the yield M and the check-plot yield. This procedure provides least squares estimates of the constants in the exponential equation, but estimates cannot be made of the standard errors of these constants. To overcome this limitation Hartley (25) developed a variation which may be solved by use of internal least squares. The form of his equation is $\hat{y} = y(1 - Ce^{Kx})$ where y is the limiting response to fertilizer, C is a constant, e is the base of natural logarithms, K is a constant and x is the fertilizer input. Paschal and French (46, pp.9-11) have used an iterative procedure, developed by Stevens (58), for obtaining

a least-squares solution which provides estimates of the standard errors. French (20) has used another exponential expression, with the form $\hat{y} = e(M - AR^x)$. He found that in some cases this gave a better fit than the Spillman function.

Hutton (30, p.17) points out that the exponential function makes no allowance for a declining phase. However, Iback and Mendum (34, p.2) consider that over the economically useful range of fertilizer inputs the exponential function is as logical and satisfactory as any other; Paschal and French (46, p.1) also hold this view.

There are gaps in both biological and statistical theory without which it is impossible to set down comprehensive rules on selection of a function. The best that can be done is to use what criteria are available to narrow down the choice, and the final selection will probably require judgement.

Resource Valuation

The problem of resource valuation as stated by Heady (26, pp.402-403) is one of allocating or imputing the total product forthcoming in a single production process to each of several resources involved.

The application of formal economic theory to evaluation of inputs in agriculture has received detailed investigation since 1946.

Johnson and Hardin (36) have applied this type of analysis to forage evaluation. They list three ways of pricing forage to livestock as a feed input. 1. acquisition cost. 2. salvage value. 3. marginal

value productivity or use value.

1. Acquisition cost: This is the cost of acquiring by the most economical means available the same quantity of feed units, or their equivalent, as would be produced and consumed on the farm. This would include both off-farm and on-farm acquisition.

2. Salvage value: This is the highest net price realizable through off-farm disposal.

3. Marginal productivity value: As indicated by Heady, Olson and Scholl (29), forage and especially pasture, often has no direct market value; their values must be assessed in terms of their values as livestock feed. In making a decision on whether or not to increase forage production, the marginal productivity value of this forage is the one most relevant to the problem.

There are three ways by which the marginal value product of forage may be obtained.

(a) Residual imputational procedures, as discussed by Heady (26, pp.403-408) which revolve around imputation of a total physical or value product. This procedure applied to the valuation of forage is described by Johnson and Hardin (36, pp.12-13) as the use of accounting procedures over a twelve month period to calculate costs of other inputs and returns, the residual being imputed to forage. The assumptions of this procedure stated by Heady (26, p.407) are that there is constant returns to scale, the market price equals the marginal value product, and no residual can remain when each factor is imputed its exact reward expressed in terms of market prices.

This procedure has two disadvantages - first, that the marginal value product does not always equal the market price, and second, the problem, discussed by Scott (52), of imputing a return to management and unpaid family labor.

(b) Multiple regression analysis: the method is described by Tintner (62, pp.51-57) and the function most commonly applied is the Cobb-Douglas. This procedure yields the elasticities of the various factors of production, and from these the marginal productivities may be calculated. The method assumes that the products can be aggregated into a single dependent variable expressed in money terms, and the inputs can be aggregated into different independent variables. The disadvantages and problems associated with these assumptions have been pointed out by Plaxico (49, pp.664-666) and seriously limit the usefulness of Cobb-Douglas estimates as guides for intra-farm decisions.

(c) Linear programming: Heady (26, p.407) points out that in contrast to residual theories of valuation more recent production principles state that each resource may receive its marginal product as a reward. Euler's theorem (26, p.408) states this more precisely - if each factor is imputed its marginal product, the total product will be exactly exhausted if the condition of constant returns to scale is fulfilled. Where the production function is linear there is constant returns to scale at all points on the function, but in the case of a curvilinear function this condition holds only if we are producing at the point on the function where the elasticity of

production = 1, hence the marginal and average products are equal.

Euler's theorem forms the basis of one of the assumptions of linear programming, that of the linear relationship or constant returns to scale. The other assumptions listed by Dorfman (18, p.27 and p.81) are that the processes available are finite, and that the processes are independent, additive and divisible. Dorfman (18, p.45) points out that linear programming is directed towards allocation of scarce resources, and in this the problem of resource evaluation is implicit. Charnes, Cooper and Henderson (13, pp.25-29) have described the logic of the procedures used to obtain the marginal value product by this method. An application of these procedures to an agricultural problem has been made by Boles (9, pp.25-29).

Linear programming and multiple regression analysis both permit the rewards to each factor to be determined simultaneously. These models are totally different from the budget approach involving the residual imputation procedure, in which all but one of the factors must be already valued in order to obtain a solution. As stated above the Cobb-Douglas function is not satisfactory for intra-farm analysis and resource valuation. Compared to linear programming budgeting does allow a more complete treatment of realistic subjective factors in management. However, a considerable amount of subjectivity can be incorporated in the linear programming model, if desired, through the limitations imposed by the manager. In those cases where the problem and data available are such as to permit the use of the linear programming technique, it would seem that simultaneous

solution of rewards to factors of production is more logical than the residual imputation procedure.

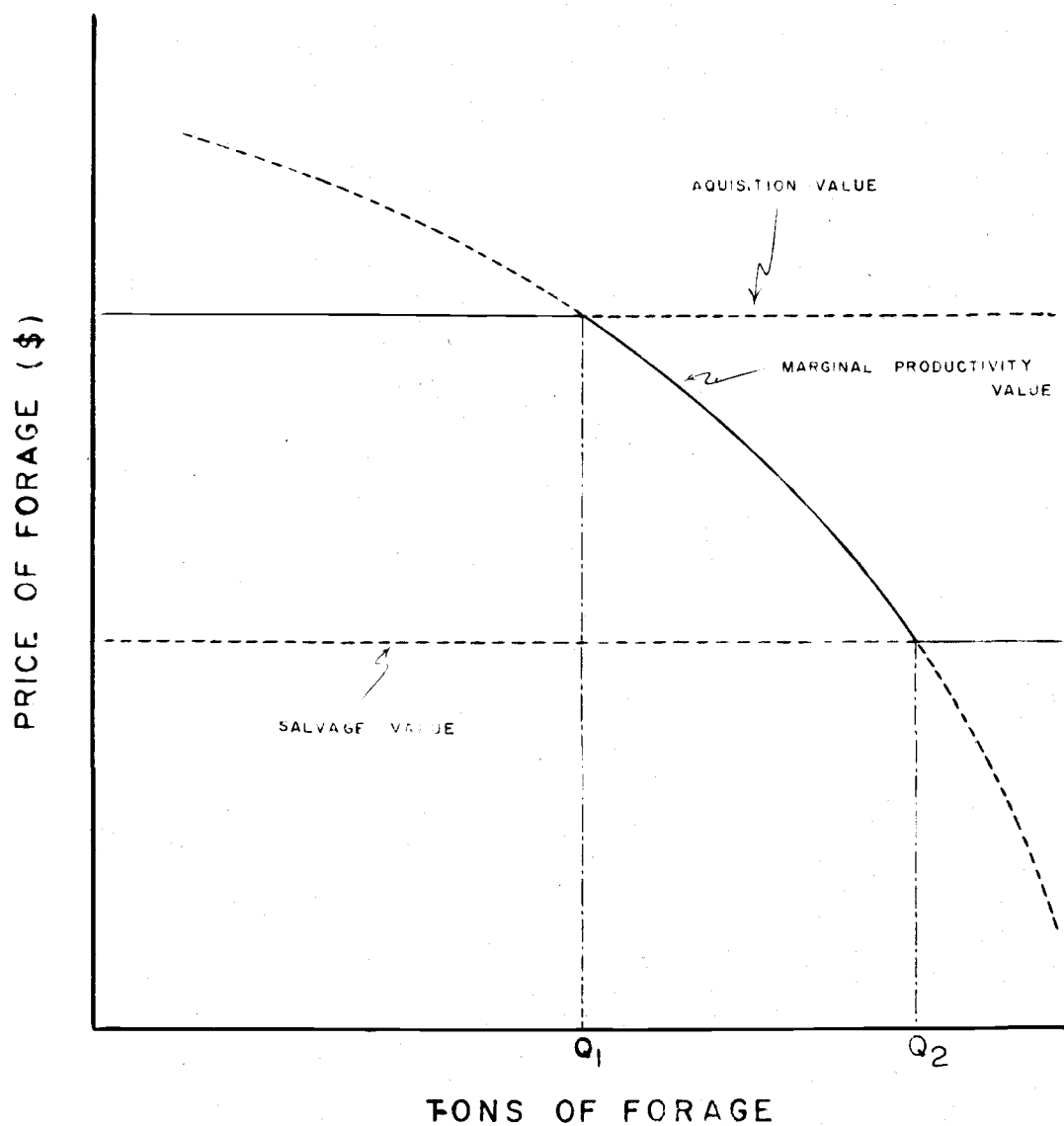
Time is also a problem in resource evaluation. Heady's analysis (26, p.382-394) of this problem is in terms of discounting expected future revenue and compounding expected future costs. The weakness is that there is no satisfactory empirical data which might provide a basis for estimating both the rate of interest and the time period which should be used. The usual procedure is to apply the current rate of interest and use a time span of 20 to 25 years. Another aspect of this problem is that of adjusting values for risk and uncertainty. Allowance for risk may be taken as the cost of insurance, but again there is no satisfactory standard from which the discount rate for uncertainty may be established.

By using any of these procedures a schedule of marginal productivity values may be drawn for various quantities of hay, given prices and costs of other factors and products.

The following graphical comparison of acquisition, salvage and marginal productivity values was made by Johnson and Hardin (36, p.7). See Figure 3. At production levels less than Q_1 the acquisition value is the value of the forage, for decision making purposes. At production levels greater than Q_2 the salvage value is the relevant value of the forage. However before Q_1 and Q_2 can be defined, it is necessary to calculate the marginal value product curve. At production levels between Q_1 and Q_2 the decision on whether to expand or contract forage production depends on the marginal cost of forage. The equation

Figure 3.

RELATION OF AQUISITION AND SALVAGE VALUE
TO
MARGINAL PRODUCTIVITY VALUE



of the marginal cost with the discounted marginal value product enables the optimum level of forage production to be determined. This model isolates the relevant variables involved in the problem of establishing optimum input-output relationships in forage production.

CHAPTER IV

THE HAY PRODUCTION FUNCTION

Fertilizer Experiments

The experiments analysed in this chapter were carried out by Squaw-Butte Harney Experiment Station, Burns, Oregon and were designed to test the response of wild hay to nitrogen and phosphorus applications, on the flood meadows growing rush-sedge type grasses (see Appendix I). The aim of this analysis is to evaluate the nitrogen response only, as the responses to phosphorus so far have been inconclusive.

The experiments were run in two distinct groups, station trials and off-station trials (16). The station trials were carried out at the Experiment Station's Section 5. Nitrogen was applied as ammonium nitrate and ammonium sulphate at 5 rates of application - 0, 50, 100, 150 and 200 pounds of elemental nitrogen. There were two dates of application, spring and fall. The experiment was conducted as a complete 5 x 2 x 2 factorial in a randomized block design of 4 replications. A uniform application of 80 pounds of P_2O_5 per acre was applied to all plots in the fall sowings. Altogether there were three trials, one run in 1954 and two in 1955, one of which was on the same location as the 1954 trial.

The off-station trials in 1954 were carried out at 21 locations. Of these one was improved flood meadow and one Nevada blue grass

meadow, leaving 19 trials on rush-sedge type meadow on 14 different ranches. In these trials only three nitrogen levels were used - 0, 60 and 120 pounds per acre, and two levels of phosphorus - 0 and 80 pounds per acre. In the 1955 off-station trials, nitrogen was applied at the same rates, but phosphorus was applied at three levels, 0, 40 and 80 pounds per acre. The trials in 1955 were carried out at 9 locations, of which two were improved flood meadow, leaving 7 on rush-sedge type meadow on 6 ranches.

Table 1 below gives a comparison between the station and off-station results.

Table 1. Comparison of Hay Yield Response to Nitrogen Fertilizer in Station and Off-station Trials.

Results	Number of Off-station trials	Pounds of Nitrogen		
		0	60 ^{/1}	120 ^{/2}
1954				
Average station yield	19	1.78 ^{/2}	2.69	3.14
Average off-station yield		1.58	2.30	2.74
Off-station as a % of station yield		88.76	85.50	87.26
1955				
Average station yield	7	1.86	2.77	3.22
Average off-station yield		1.76	2.52	2.86
Off-station as a % of station yield		94.62	90.97	88.81

^{/1} These yields are not adjusted for the phosphorus response, and hence are about .50 tons per acre more than the check-plot value used in the linear programming analysis.

^{/2} As the 60 and 120 pound levels of nitrogen were not used in the station trials the comparison here is based on the estimated values obtained from the Spillman function derived later in this chapter.

Table 1 above shows that in both 1954 and 1955 the off-station trials gave an average response to nitrogen that was within 15% of the average results of the three station trials. It therefore seems reasonable to assume that the response curve obtained from the more complete station trials, and used in the linear programming analysis, gives a good approximation of the response which may be expected on ranches in the area.

Analysis of Experimental Data

Analysis of variance of each station trial was done separately. The results are given in Table 2 below.

Table 2. Summary of F Values Obtained from Analysis of Variance of the Data from the Three Station Trials.

Source of Variation	Trial Number					
	1		2		3	
	d.f.	F value	d.f.	F value	d.f.	F Value
Replication	3	5.37*	3	3.07*	3	4.82*
Date of application	1	.56	1	2.15	1	1.67
Source of nitrogen	1	.06	1	4.71*	1	10.88**
Rate of nitrogen	4	55.85**	4	52.87**	4	35.59**
Interaction						
Date x source	1	1.35	1	2.30	1	.0005
Date x rate	4	2.27	4	1.48	4	.75
Rate x source	4	3.24*	4	1.87	4	1.29
Source x rate x date	4	.89	4	.97	4	.45
Error	57	-	57	-	57	-
Total	79	-	79	-	79	-

*significant at the 5% level.

**significant at the .1% level.

The ultimate aim was to fit a regression line to the data. We are therefore interested in pooling the data from the above three trials in order to obtain a greater number of points in fitting this line.

In Table 2 above it is seen that the only source of variation significant at the .1% level, other than rate of N (nitrogen), is source of N in trial 3; however, this would not affect the fit of any equation used to predict the relationship between nitrogen level and yield in the pooled data. Replication is significant at 5% level in all trials, but has no effect in pooling. The rate by source interaction in Trial 1 indicates a possible difference in response to N level from one source to the other in this trial, but this may have occurred through random error (it is expected that this will happen 5 times in 100 when there is no interaction) especially when the F is so small (3.24) relative to the F of rate (55.85). Therefore this interaction may be justifiably ignored.

Analysis of variance (14, pp.394-396) was used on the pooled data to determine whether treatment by trial interaction was significant. Results of this test are given in Table 3 below.

Table 3. Results of Analysis of Variance of the
Combined Data from the Three Station Trials.

Source of variation	d.f.	SS	MS	F
Trial	2	104.3801	52.1901	3.13*
N treatment	4	385.8963	96.4741	5.79*
Treatment x trial	8	4.6912	.5864	.04
Pooled error	171		16.6485	
Total	239	494.9676		

*significant at the 5% level.

The F value of .04 for treatment by trial interaction indicates that it is unlikely that the treatment effects in the three trials are significantly different; so the results may be pooled. The pooled results are shown in Table 4 below.

Table 4. Pooled Results of the Fertilizer-hay Response
Data from the Three Station Trials.

Rate of Nitrogen Application (pounds per acre)	Hay Yield per Acre	
	Pounds	Tons
0	3664	1.83
50	5243	2.62
100	6102	3.05
150	6681	3.34
200	7316	3.66

Fitting a Production Function

The discussion of the various regression equations which may be used was presented in the review of literature. This section is devoted to consideration of the procedures used in estimating and selecting the regression equation which will best fit the above data from both the statistical and biological points of view.

As already stated in Chapter III it is possible by increasing the degree of the functional equation to obtain a perfect fit statistically. The following comparison, given in Table 5, shows the variation explained by equations of different degree (55, p.410).

Table 5. Components of the Treatment SS from Analysis of Variance of the Station Fertilizer Yield Data (see Table 3).

Degree of Polynomial	Type of Equation	SS and MS	F Value
1	Linear	366.8703	22.04**
2	Quadratic	16.1324	.97
3	Cubic	2.8861	.18
4	Quartic	.0075	-
Total	-	385.8963	-

**significant at the .1% level.

From Table 5 it can be seen that the additional variation explained by the quadratic, cubic or quartic equations, over the linear form, is not significant. However the linear assumption is not closely in accord with biological logic over the entire growth range, so the problem becomes that of selecting a second degree equation.

Five expressions, representing the three different types of function discussed in Chapter II are applied to the data. The results of this comparison are given in Table 6.

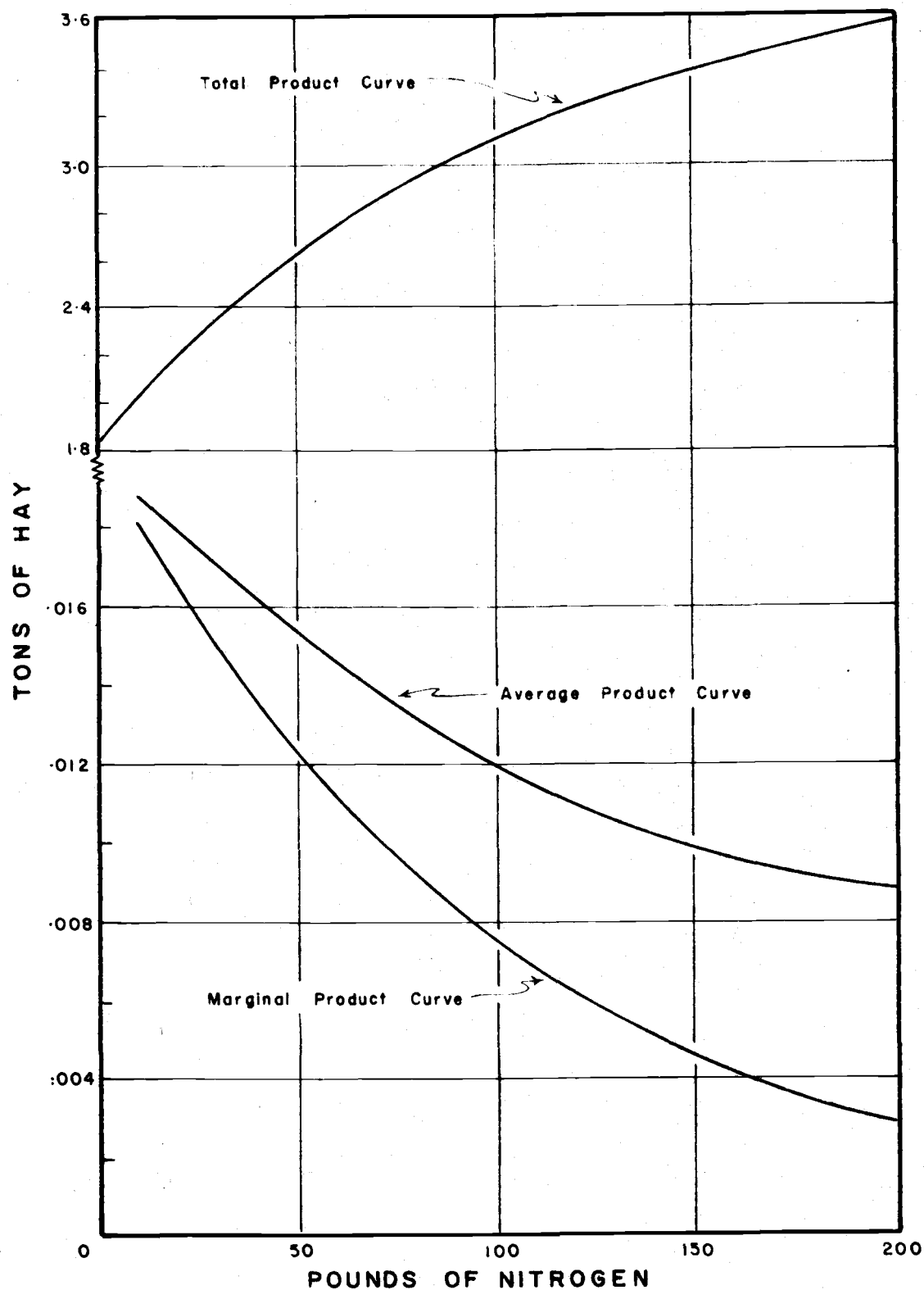
In absence of any strong statistical reasons for selection of one expression, biological theories would tend to support the selection of either the exponential or the polynomial form. On this basis, therefore, the two power functions may be eliminated. At the present time there is no satisfactory statistical test which may be used as a criterion for choosing between the three remaining expressions. However, in this case the minimum sum of squares is used as the basis for selection in absence of anything better. From Table 6 it will be seen that the polynomial with the form $\hat{y} = a + b_1x + b_2\sqrt{x}$ gives the lowest sum of squares, .000953. But both the b coefficients in this expression are positive, thus giving a continuously increasing function which cannot be justified on the grounds of biological theory, over the entire growth range. Of the two remaining expressions the exponential, $\hat{y} = M(1 - R^x)$ has the lowest sum of squares, .012712 and consequently is the one selected for use in this analysis.

Table 6. Comparison of Goodness of Fit Obtained by Use of Different Prediction Equations.

Type of Function	Expression	$\sum(y-\hat{y})$	$\sum(y-\hat{y})^2$	Estimating equation
Exponential	$\hat{y} = M(1 - R^x)$	-.021667	.012712	$Y = 3.851755 (1-.8^{2.892581} .044435x)$
Power	(1) $\hat{y} = ab^x x$.015370	.013241	$Y = 0.018244 (.99635)^x x$
	(2) $\hat{y} = ab_1^x b_2$	-.000370	.000967	$Y = 0.071025 (.99981)^x x^{0.618845}$
Polynomial	(1) $\hat{y} = a + b_1 x + b_2 \sqrt{x}$.000001	.000953	$Y = 1.830943 + .002205x + .097691 \sqrt{x}$
	(2) $\hat{y} = a + b_1 x - b_2 x^2$	-.000090	.015054	$Y = 1.871226 + .014946x - .000031016x^2$

Figure 4 shows the total product, marginal product and average product curves obtained from the exponential equation and its derivative.

Figure 4. TOTAL, AVERAGE AND MARGINAL HAY YIELD RESPONSE TO NITROGEN



CHAPTER V

SELECTION OF A MAXIMUM PROFIT COMBINATION OF FERTILIZER INPUT
AND CATTLE ENTERPRISE UNDER VARIOUS RESOURCE AND PRICE SITUATIONSIntroduction

The objective of this chapter is to integrate the fertilizer data into an entire ranch analysis for the purpose of evaluating the intermediate product, hay. Once this value has been obtained the problem of selecting the appropriate amount of fertilizer to apply is relatively simple. It is obvious however that in practice these quantities are not determined in isolation but hay production and cattle production are being carried on simultaneously. It becomes necessary therefore to turn to a method of solving simultaneous relationships.

Linear programming as reviewed in a previous chapter treats various relationships simultaneously. As a by-product the valuation of the resources used is also determined. The method will maximize returns to the fixed factors of production, meadow land, capital, labor and management.

The resource situation considered was determined by the survey of ranchers mentioned previously. The situation selected was a two man unit with a range allotment of 3025 AUMs. The base property is 750 acres of flood meadow all of which may be cut for hay. The meadow may be used for stacked hay, bunched hay or pasture.

Of the 750 acres it is assumed that 35% or 260 acres gives an unsatisfactory response to nitrogen because of deep swales or excess alkalinity of the soil. This is classed as meadow II giving a hay yield of one ton per acre. The remaining 490 acres are classed as meadow I, giving an average yield of 1.2 tons of hay per acre without fertilizer. For purposes of analysis it is assumed that all additional capital, necessary for the operation of a system using nitrogen fertilizer and running additional cattle, will be available at 7% interest.

The capital requirements used in the following analysis were estimated from survey data, U.S. Department of Agriculture prices (65) and Ontario stock market prices. Labor costs were obtained from interviews with ranchers and hay contractors. A hired man is assumed to receive \$200 plus an allowance of \$1.25 per day for keep, a total of \$2850.25 per year. Wages for hay making crews are estimated at \$10 per day, plus \$1 for two meals. Freight costs and costs of operating machinery and water pumps were obtained from U.S. Department of Agriculture reports (See Appendix 3). Prices of protein supplements and fertilizer were obtained directly from ranchers. The cattle prices used are based on 12 auctions at the Ontario market between September and November 1955. The prices were derived by taking the weighted average of 4 grades of slaughter cows, 3 grades of bulls, and 2 grades of feeder steers and heifers. The average prices, per 100 pounds, used prior to weighting were - fat cows, \$11.10; cull cows, \$7.30; bulls, \$10.30; feeder steers, \$15.70 and feeder heifers, \$13.20.

The specifications with respect to capital, stacked hay, bunched hay and pasture requirements, and limited range and meadow provide a set of relations that limit the choice of the profit maximizing combination of cattle enterprise and nitrogen fertilizer input for production of hay and pasture.

Model for Situation I

The initial problem is to obtain an estimate of the effect fertilization will have on the ranch organization and returns under a situation where range is limited.

The first analysis, in Table 7, is set up to cover a wide range of nitrogen application, from 0 to 160 pounds per acre. It also includes four levels of beef production (see Appendices II and III).

Table 7 shows the four main activities considered, production of stacked hay, bunched hay, pasture, and beef. The first three of these are each broken down into seven levels according to the rate nitrogen is applied - 0, 40, 60, 80, 100, 120 and 160 pounds per acre. These activities are specified in columns A_1 to A_{21} . The four levels of beef production are specified in columns A_{23} to A_{26} .

The first of these, P_1 , is the situation described above, grazing 300 cows, using no fertilizer, and running all stock on range through the summer. Cattle are sold in October and November, the sales being 12 fat cows at 975 pounds, 3 cull bulls at 1400 pounds, 80 spayed yearling heifers at 625 pounds, and 118 yearling steers at 700 pounds. Total beef production is 167,925 pounds, giving a gross

income of \$22,750.

The second beef production process, P_2 , is the same unit as above in terms of land and labor, but has 314 cows and grazes 25 yearlings on meadow through the summer, while all available range is utilized by the remaining cattle. Cattle sales under this system are 13 fat cows, 20 aged cows, 3 cull bulls, 84 spayed yearling heifers, 98 yearling steers off range at 700 pounds, and 25 yearling steers off meadow at 776 pounds average weight. Total beef production is 177,850 pounds, giving a gross return of \$24,117.

Beef process three, P_3 , is also the same unit in terms of the land and labor resources, but 327 cows are grazed and 50 yearlings are pastured on the meadow through the summer. Cattle sold are 13 fat cows, 21 aged cows, 4 cull bulls, 87 spayed yearling heifers, 79 yearling steers off range at 700 pounds, and 50 yearling steers off meadow at 776 pounds. Total beef produced is 188,200 pounds, giving a gross income of \$25,538.

Under beef production process P_4 , 355 cows are carried, and 100 yearling steers pastured on meadows through the summer. The additional cattle are assumed to be handled by the same permanent labor as is required for the other processes. Annual sales are 14 fat cows, 23 aged cows, 4 cull bulls, 95 spayed yearling heifers, 40 yearling steers off range, and 99 yearling steers off meadow pasture. Total beef production is 206,925 pounds, giving a gross return of \$28,140.

The coefficients in columns A_1 to A_{21} specify, in row 1 the variable capital requirements per acre for the various activity levels;

in row 2 the unit acreage requirement of meadow I, in row 3, the yield of stacked hay in tons per acre, obtained from the production function derived in the previous chapter; in row 4, the fall feeding capacity of bunched hay and aftermath in AUMs per acre; and in row 5, the summer feeding capacity of meadow pasture in AUMs per acre. The coefficients in columns A_{23} to A_{26} represent the requirements of variable operating capital, stacked hay, bunched hay, pasture and range per 100 pounds of beef produced under the four processes. The coefficients for stacked and bunched hay in columns A_{23} to A_{26} are adjusted for the fixed production from meadow II, it being assumed that 65% of this area will be utilized for stacked hay and the remainder for bunched hay. This assumption is made because the meadow II is so widely distributed throughout the whole meadow area that it is not feasible to harvest it separately, although selective fertilization is feasible.

The C values for A_1 to A_{21} are zero as it is assumed that the products of these activities will be fully utilized by cattle, thus their price will be reflected in the price of beef, in columns A_{23} to A_{26} . This beef price is a weighted average of all beef sold. The capital buying activity A_{22} has a C value of 1.07, representing a 7% interest rate on additional capital requirements.

Equation 1 assures that the quantity of capital obtained will just equal the additional capital required under any system of fertilization selected. This requires that a capital buying activity, A_{22} , be included in the system. The inclusion of the capital buying activity is not essential to the solution of the model. Although it

does not limit production, information is yielded on capital requirements that otherwise would not be available unless calculated separately. However, capital would be limiting if its productivity fell below \$1.07 before some other resource was exhausted. Equations 3, 4 and 5 state that the stacked hay, bunched hay and pasture consumed by cattle must be produced from the meadow available. It is assumed that no hay or pasture will be purchased; if such an assumption were not made it would be necessary to introduce a hay-buying activity into the system. It is further assumed that no hay will be sold as such. Without this assumption it would be necessary to introduce a hay-selling activity. Relationships 2 and 6 state that 490 acres of meadow I and 3025 AUMs of range are available for beef production.

Plan 10 in Table 7 is the final plan giving the maximum profit combination of the specified activities and activity levels, and obtained after nine iterations of the matrix. This shows that cattle plan 4 is selected, which involves running 355 cows, or 55 more cows than plan 1 using no nitrogen. Stacked hay is cut off 290 acres of meadow I fertilized at 40 pounds of nitrogen; 19 acres receive no nitrogen and are cut for bunched hay, and 97 acres receive 40 pounds of nitrogen and are cut for bunched hay. The same yield could be obtained from the 116 acres of bunched hay if it were all fertilized at 30 pounds of nitrogen per acre, and because of the diminishing returns to nitrogen this would be a lower cost practice than using differential rates of fertilization to obtain the same product.

Eighty four acres of meadow receive 40 pounds of nitrogen and are used for pasturing 100 yearling steers through the summer.

The annual variable operating expense for this system is \$5725 or \$4835 more than the outlay required without fertilization. Row 16 of the A_0 column shows the net return over fixed costs to be \$22,014, which is an increase of \$2045 over the original system.

The model shown in Table 7 does not have sufficient range of cattle production levels to enable the selection of an optimum size of cattle enterprise with fertilization. It is therefore necessary to consider an additional model in Table 8 in which the same activities are included at different levels. In this model P_5 , a fifth beef production process is introduced. The level of production in this process is set such that it would be well above the expected optimum for the range of beef and nitrogen prices considered in this study to ensure full utilization of resources. The meadow and range resources are the same as the other processes, and three permanent men are required. The cow herd numbers 550. All 440 yearlings are pastured on the meadow through the summer. Cattle sales are 22 fat cows, 37 aged cows, 6 cull bulls, 146 spayed yearling heifers at 700 pounds, and 215 yearling steers at 776 pounds. Annual beef production is 336,760 pounds, giving a gross income of \$45,766.

Plan 9 in Table 8 shows the maximum profit combination of activities in beef production to be 123,830 pounds of beef produced under process 1, and 88,190 pounds under process 5. Expressing this in terms of one production process, this would be a 360 cow herd,

Table 8. Linear Programming Solution for Optimum Level of Beef Production with Three Feed Producing Activities from Meadow, Two Levels of Nitrogen Application on Each, and Five Limitational Resources (Capital Unlimited).

C value (\$)			0		0	0	0	0	0	0	0	0	0	0	0	1.07	13.549	13.590	Check	R		
Relation number	Resources	Unit	Quantity	Relation-ship	Activity and Activity Levels																	
					Disposal					Stacked hay		Bunched hay		Pasture		Capital buying	Beef producing processes					
					A ₁₀ Range	A ₁₁ Meadow	A ₁₂ Stacked hay	A ₁₃ Bunched hay	A ₁₄ Pasture	A ₁ N ₅₀	A ₂ N ₆₀	A ₃ N ₄₀	A ₄ N ₅₀	A ₅ N ₅₀	A ₆ N ₆₀	A ₇	A ₈ P ₁	A ₉ P ₅				
(1)	Plan 1																					
(1)	A ₇ Capital	\$	0	=	0	0	0	0	0	12.305	14.129	8.443	9.913	8.480	10.131	1	.530266	.672518	66.603784	0		
(2)	A ₁₀ Range	A.U.Ms	3025	≥	1	0	0	0	0	0	0	0	0	0	0	0	1.803134	.898263	3028.701397	∞		
(3)	A ₁₁ Meadow	acres	490	≥	0	1	0	0	0	1	1	1	1	1	1	0	0	0	497.000000	490		
(4)	A ₁₂ Stacked hay	tons	0	=	0	0	1	0	0	1.989606	2.105721	0	0	0	0	0	-.253685	-.280938	4.560704	0		
(5)	A ₁₃ Bunched hay	A.U.Ms	0	=	0	0	0	1	0	0	0	4.101342	4.354605	0	0	0	-.221747	-.238194	8.996006	0		
(6)	A ₁₄ Pasture	A.U.Ms	0	=	0	0	0	0	1	0	0	0	0	7.610515	8.054671	0	0	-.770503	15.894683	0		
(7)	Z		0		0	0	0	0	0	13.166350	15.118030	9.034010	10.606910	9.073600	10.840170	1.07	.567385	1.789594				
(8)	Z - C		0		0	0	0	0	0	13.166350	15.118030	9.034010	10.606910	9.073600	10.840170	0	-12.981315	-11.800606				
(9)	Plan 9																					
(9)	A ₇ Capital	\$	-6294.691268	=	.250785	-16.565998	2.141619	1.980693	1.069916	0	2.072689	0	1.959126	0	2.182934	1	0	0	-6298.599504			
(10)	A ₈ Beef, P ₁	100 pounds	1238.298378	=	.793224	-2.369806	1.191093	.577820	.311836	0	.138304	0	.146292	0	.141933	0	1	0	1240.229074			
(11)	A ₉ Beef, P ₅	100 pounds	881.903509	=	-.479023	4.757048	-2.390950	-1.159889	-.625965	0	-.277627	0	-.293658	0	-.284908	0	0	1	882.148537			
(12)	A ₁ Stacked hay at N ₅₀	tons	282.416451	=	.033501	.369544	.316874	-.091104	-.051236	1	1.036793	0	-.022813	0	-.022132	0	0	0	284.985878			
(13)	A ₃ Bunched hay at N ₄₀	A.U.Ms	118.168081	=	.015065	.148149	-.074462	.207700	-.019496	0	-.008645	1	1.052496	0	-.008873	0	0	0	120.480015			
(14)	A ₅ Pasture at N ₅₀	A.U.Ms	90.227359	=	-.049008	.486687	-.244615	-.118644	.068741	0	.028403	0	-.030044	1	1.040369	0	0	0	92.409248			
(15)	Z		22,027.258645		4.505475	14.815825	-14.064195	-5.815072	-3.056141	0	.318610	0	.087460	0	.386790	1.07	13.549	13.590				
(16)	Z - C		22,027.258645		4.505475	14.815825	-14.064195	-5.815072	-3.056141	0	.318610	0	.087460	0	.386790	0	0	0				

running 111 yearling steers on the meadow through the summer, and producing 212,000 pounds of beef. To obtain this production it is necessary to fertilize 282 acres at 50 pounds of nitrogen for stacked hay, 118 acres at 40 pounds of nitrogen for bunched hay and 90 acres at 50 pounds of nitrogen for pasture. These quantities are shown in the A_0 column in Table 8.

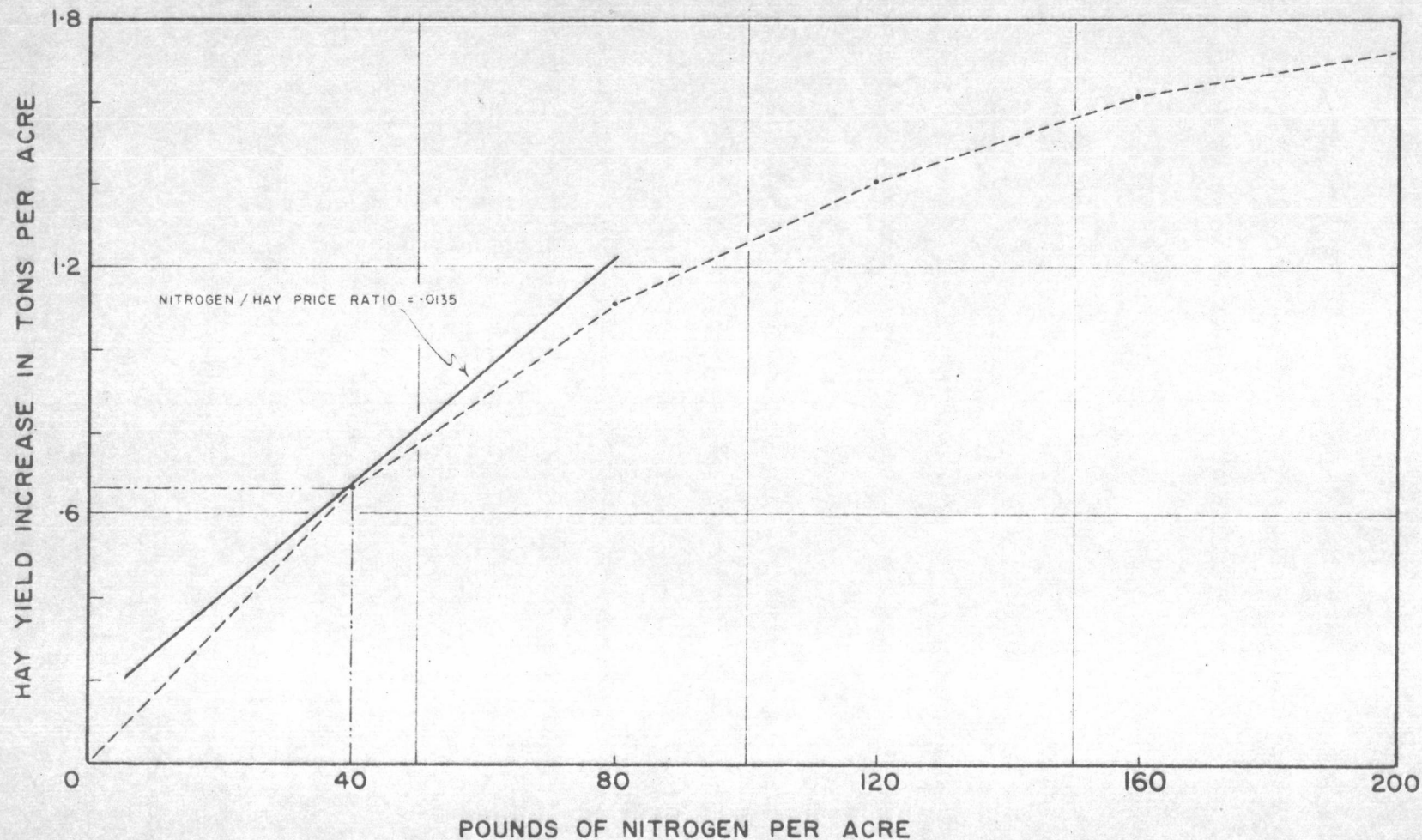
The relationship between the solution for the optimum fertilizer rate by linear programming, in Table 7, and the solution by marginal analysis is shown graphically in Figure 5. The dashed line joins five of the seven discrete points on the hay production function which were selected for use in the programming model (see Table 7). The solution from Table 7 showed the optimum application of fertilizer to be 40 pounds per acre. In marginal analysis the solution is given by the point of tangency between the nitrogen-hay price ratio line and the production function. In this model tangency can only occur at one of the discrete points considered. Further, when only discrete points are considered the fertilizer rate remains constant over a range of price ratios. The optimum input of fertilizer is obtained when:

$$MPP_{\text{hay}} = \frac{P_N}{P_{\text{hay}}} .$$

The MPP_{hay} (marginal physical product) is derived from the production function given in Chapter 4 (see Figure 4). The MPP of stacked hay at 40 pounds of nitrogen is given in Table 11, and is .0135 tons.

Thus if 40 pounds is the optimum rate then the nitrogen-hay price ratio

58 Figure 5. RELATIONSHIP OF DISCRETE POINTS ON THE PRODUCTION FUNCTION
USED IN THE LINEAR PROGRAMMING MODEL,
AND THE NITROGEN-HAY PRICE RATIO LINE DERIVED IN MARGINAL ANALYSIS



of .0135 should be tangent at this point. It will be seen in Figure 5 that this is the case.

Table 8 gives a more realistic solution than Table 7 as fertilizer is fully utilized. In this case the optimum rate of fertilizer use is 50 pounds of nitrogen. Thus tangency between the nitrogen-hay price ratio, .0122 (the MPP_{hay} at N_{50} , from Table 11) and the production function should occur at 50 pounds of nitrogen.

Resource Valuation

Linear programming gives as a by-product the marginal value products of the resources considered. The following is an interpretation of the final $Z_j - C_j$ row, which gives these values.

In Table 7, row 16 in column A_{28} shows that the marginal value product of stacked hay is \$14.68 per ton, so that at hay prices below this it would pay to buy additional hay rather than fertilize, if other prices remain the same. If beef prices fell 18% it would pay to reduce fertilization and adopt beef plan 3, which would mean reducing the cow herd from 355 to 337. If beef prices fell by more than 21% it would pay to eliminate fertilization and revert to the original 300 cow herd (see Table 12).

Row 16 of column A_{11} in Table 8 shows the marginal value product of meadow to be \$14.82. Using a time preference limit of 25 years and discounting at 5% for the time lapse, the total discounted return over this period may be calculated as follows:

$$V_{25 \text{ years}} = \frac{14.82}{(1+.05)} + \frac{14.82}{(1+.05)^2} \dots\dots + \frac{14.82}{(1+.05)^{25}} = \$185$$

Thus, the estimated value of the meadow land is \$185 per acre. In practice this figure would have to be discounted further for risk and uncertainty.

If meadow could be rented for less than \$14 per acre on a yearly basis, it would pay to rent and reduce fertilization.

Row 16 of column A₁₂ shows the marginal value product of hay to be \$14.06 per ton, so at prices below this figure it would pay to buy rather than fertilize. Column A₁₃ shows the marginal value product of bunched hay to be \$5.82 per AUM. Expressing this on a per acre basis when 40 pounds of nitrogen are applied, the marginal value product from one acre of bunched hay is \$23.85, or \$12.81 per ton where the yield is 1.86 tons per acre. The marginal value product of pasture is shown in column A₁₄, row 16, as \$3.06 per AUM. At 50 pounds of nitrogen, this represents a marginal value product per acre of \$23.26 or the equivalent of \$14.03 per acre without nitrogen. Thus if meadow can be rented for pasturing at less than \$14, or less than \$2.34 per AUM, it would pay to do so. Column A₁₀, row 16, shows the marginal value product of range as \$4.51 per AUM. This is the amount a rancher could afford to pay for range in the short run. In the long run this price would have to be discounted for risk and uncertainty. At the present time the Bureau of Land Management charges 15 cents per AUM, and the Forest Service 44 cents for somewhat better

quality range. Thus the undiscounted quasi-rent accruing to the rancher in this situation amounts to between \$4.07 and \$4.36 per AUM.

The use of linear programming as a tool solely for resource valuation is demonstrated in Table 9. The object is to assess the marginal value product of meadow land and range for a situation in which no fertilizer is used, i.e. under the situation where P_1 is in operation. The table is set up in exactly the same manner as the others, except for elimination of all activities involving use of fertilizer. The resources considered are again capital, meadow, stacked hay, bunched hay and range, with the limitations the same as before. In order to obtain a finite solution it is necessary that a substitution be made for each of the four disposal activities, A_6 to A_9 . It is, therefore, necessary to include an additional activity in the form of another beef process, P_{1a} , which will absorb any surplus resource not taken up by P_1 . P_{1a} is a process which has the same cattle numbers and requirements as P_2 except no fertilizer is used. The deficiency in hay for winter feed is made up by renting 25 acres of hay land, and the cattle numbers in summer are equated to the range permit by selling 25 short yearling heifers in the spring.

In Plan 4 (Table 9) it will be seen that if only P_1 is considered, 2.29 acres of meadow remain idle with the result that the marginal value product for meadow is shown to be zero in Row 14. Plan 5 shows the solution where all resources are completely utilized.

Row 21, columns A_6 to A_9 , shows the marginal value products of the four resources. The marginal productivity value of meadow is \$10.08;

Table 9. Linear Programming Solution for Marginal Value Product of Range and Meadow with Two Feed Producing Activities from Meadow without Nitrogen, and Four Limitational Resources (Capital Unlimited).

C value			0		0	0	0	0	0	0	1.07	13.549	13.709	Check	R
Relation number	Resources	Unit	Quantity A ₀	Relation-ship	Activity and Activity Levels										
					Disposal				Stacked hay	Bunched hay	Capital buying	Beef	Beef		
					A ₆ Meadow	A ₇ Stacked hay	A ₈ Bunched hay	A ₉ Range	A ₁ N ₀	A ₂ N ₀	A ₃	A ₄ P ₁	A ₅ P _{2a}		
(1)	Plan 1 A ₃ Capital	\$	0	=	0	0	0	0	2.694	1.390	1	.530266	.687131	6.301397	0
(2)	A ₆ Meadow	acres	490	≥	1	0	0	0	1	1	0	0	0	493	490
(3)	A ₇ Stacked hay	tons	0	=	0	1	0	0	1.2	0	0	-.253685	-.265662	1.680653	0
(4)	A ₈ Bunched hay	A.U.Ms	0	=	0	0	1	0	0	2.796053	0	-.221747	-.230588	3.343718	0
(5)	A ₉ Range	A.U.Ms	3025	≥	0	0	0	1	0	0	0	1.803134	1.769523	3029.572657	∞
(6)	Z		0		0	0	0	0	2.882580	1.487300	1.07	.567385	.614115		
(7)	Z - C		0		0	0	0	0	2.882580	1.487300	0	-12.981315	-12.985885		
(8)	Plan 4 A ₃ Capital	\$	-2029.980700	=	0	-2.244999	-.497129	-.671068	0	0	1	0	.210704	-2032.165192	20,817.752600
(9)	A ₆ Meadow	acres	2.294375	=	1	-.833333	-.357647	-.161225	0	0	0	0	.018563	1.960733	123.599364
(10)	A ₁ Stacked hay at N ₀	tons	354.660075	=	0	.833333	0	.117243	1	0	0	0	-.013921	356.596730	2547.667193
(11)	A ₂ Bunched hay at N ₀	A.U.Ms	133.048575	=	0	0	.357647	.043983	0	1	0	0	-.004640	134.445565	2867.426180
(12)	A ₄ Beef, P ₁	100 pounds	1677.634750	=	0	0	0	.554590	0	0	0	1	.981360	1680.170700	1709.499820
(13)	Z		20,557.690588		0	-2.402148	-.531928	7.341463	0	0	1.07	13.549	13.400490		
(14)	Z - C		20,557.690588		0	-2.402148	-.531928	7.341463	0	0	0	0	-.308510		
(15)	Plan 5 A ₃ Capital	\$	-2062.947537	=	-11.350751	7.213956	3.562433	1.158957	0	0	1	0	0	-2061.362942	
(16)	A ₅ Beef, P _{2a}	100 pounds	123.599365	=	53.870603	-44.892151	-19.266660	-8.685288	0	0	0	0	1	105.625869	
(17)	A ₁ Stacked hay at N ₀	tons	356.380703	=	.749933	.208389	-.268211	-.003665	1	0	0	0	0	358.067148	
(18)	A ₂ Bunched hay at N ₀	A.U.Ms	133.622077	=	.249960	-.208389	.268250	.003683	0	1	0	0	0	134.935670	
(19)	A ₄ Beef, P ₁	100 pounds	1556.339277	=	-52.866455	44.055362	18.907529	9.077984	0	0	0	1	0	1576.513697	
(20)	Z		20,573.419073		10.084278	-10.805704	-4.138547	5.170090	0	0	1.07	13.549	13.709		
(21)	Z - C		20,573.419073		10.084278	-10.805704	-4.138547	5.170090	0	0	0	0	0		

capitalizing this over a 25 year period allowing a 5% discount rate for time preference, the value of the meadow would be \$143 per acre. The marginal value product of stacked hay is \$10.80 per ton and of bunched hay \$4.14 per AUM or \$11.57 per acre, which is \$9.64 per ton if the yield is 1.2 tons per acre.

The valuations obtained for range in the three matrices are given in Table 10 below.

Table 10. Marginal Productivity Values of Range Land.

	Table Number		
	8	7	9
(Rate of N used on meadow land)	(50)	(40)	(0)
Value of range per AUM	\$4.51	\$4.90	\$5.17

Two relationships are apparent from Table 9. First, there is an inverse relationship between the value of range and intensity of production from meadows in terms of fertilizer use. The reason for this is that as the meadows become developed as an alternative source of summer feed the range becomes relatively less important to the whole operation. But it would be profitable to develop meadow production only as long as the marginal cost of the meadow pasture is less than the marginal value product of range.

The second relationship is the relative consistency of the estimates of the marginal value product of range. Although the data

on which these valuations are based is partially incomplete (see Appendix 2), this consistency does allow an estimate of \$4 to \$5 per AUM to be made with a reasonable degree of confidence. The very considerable discrepancy between this estimate and the rent, \$.15 to \$.44 per AUM actually charged by Federal agencies, needs explanation. In the first place the estimate of \$4 to \$5 per AUM should be discounted for risk and uncertainty. Because of the high degree of risk and uncertainty associated with the desert cattle operation, the discount rate could well be as high as 50%. If this were the case the estimate would be reduced to \$2 to \$2.50. The historical aspect is also a factor in explaining this discrepancy. Originally the range land was free and when it came under the administration of the Forest Service and Bureau of Land Management, neither agency felt justified in charging the full market price for grazing rights. This feeling still prevails.

Effect of Price Changes

For this analysis it is assumed that the price of stacked hay is a direct function of the price of beef, if all other prices remain the same. Table 11 shows the marginal value products of hay at different levels of nitrogen fertilization under three different prices of nitrogen.

Table 11. Marginal Cost of Stacked Hay at 20 levels
of Nitrogen and 3 Price Levels of Nitrogen.

Pounds of Nitrogen per Acre	MPP of Stacked Hay in Tons	Marginal Cost of Stacked Hay in \$ per Ton			
		Prices of Nitrogen in Cents per Pound			
		15.5¢	16.5¢		17.5¢
			From Formula	From Matrix	
10	.01814	9.143	9.733	-	10.322
20	.01643	10.094	10.746	-	11.397
30	.01487	11.153	11.873	-	12.592
40	.01347	12.313	13.107	13.902	13.901
50	.01220	13.594	14.475	14.064	15.348
60	.01104	15.022	15.996	-	16.961
70	.01001	16.568	17.642	-	18.706
80	.00906	18.305	19.492	-	20.667
90	.00821	20.200	21.510	-	22.807
100	.00743	22.321	23.768	24.132	25.201
110	.00672	24.680	26.279	-	27.864
120	.00608	27.277	29.046	-	30.797
130	.00551	30.100	32.050	-	33.983
140	.00499	33.236	35.390	36.663 ¹	37.525
150	.00451	36.773	39.157	-	41.518
160	.00410	40.451	43.073	-	45.670
170	.00371	44.703	47.601	-	50.471
180	.00336	49.360	52.559	-	55.729
190	.00304	54.555	58.092	-	61.595
200	.00275	60.309	64.218	-	68.090

¹ Obtained from a matrix in which capital, range and meadow II were unlimited resources.

From Table 11 it can be seen that in situation I if the beef price falls by 10% it would pay to decrease fertilization from 50 pounds to 40 pounds, and decrease cattle production in proportion. Table 12 below shows the manner in which the optimum rate of fertilization changes with changes in the price of beef and nitrogen.

Table 12. Relationship Between Price Changes in Beef and Nitrogen and the Optimum Rate of Fertilization.

Price of N Price of Beef	Pounds of Nitrogen Applied per Acre											
	Cents per pound											
	13.5	14	14.5	15	15.5	16	16.5	17	17.5	18	18.5	
\$7 per 100 lb.	0	0	0	0	0	0	0	0	0	0	0	0
\$8	10	10	0	0	0	0	0	0	0	0	0	0
\$9	20	20	20	10	10	10	0	0	0	0	0	0
\$10	30	30	30	20	20	20	10	10	10	10	0	
\$11	40	40	40	30	30	30	20	20	20	20	10	
\$12	50	50	50	40	40	40	30	30	30	20	20	
\$13	60	60	50	50	50	40	40	40	30	30	30	
\$14	70	60	60	60	50	50	50	50	40	40	40	
\$15	80	70	70	60	60	60	50	50	50	50	40	
\$16	80	80	70	70	70	60	60	60	60	50	50	
\$17	90	80	80	80	70	70	70	60	60	60	60	
\$18	90	90	90	80	80	80	70	70	70	60	60	
\$19	100	100	90	90	90	80	80	80	70	70	70	
\$20	100	100	100	90	90	90	80	80	80	80	70	
\$30	150	140	140	130	130	130	120	120	120	120	110	
\$40	180	170	170	160	160	160	150	150	150	140	140	
\$50	200	190	190	190	180	180	180	170	170	170	160	

From Table 12 it can be seen that under the currently feasible price range for beef, up to \$20 per 100 pounds, the highest optimum rate of fertilization is 100 pounds per acre at the lowest nitrogen price. At the nitrogen prices above 16 cents per pound beef must be worth \$10 or more per 100 pounds before any fertilization is profitable.

Model of Situation II

In this model the situation is assumed to have the same meadow property as situation I, but has unlimited range. The objective of this analysis is to show the economic potential of the meadow under nitrogen fertilization if the range capacity could be expanded either by development or purchase of additional grazing rights.

Column A_0 in Table 13 shows the initial resource limitations. Capital is unlimited but must be purchased as required at 7% interest. The same situation applies to range, row 2, all range requirements being purchased at 33 cents per AUM. This price is the average paid over the five and one half month range grazing period, if two months are on Bureau of Land Management range at 15 cents per AUM, and three and one half months are on Forest Service range at 44 cents per AUM. Rows 3 and 4 in the A_0 column show the acreage limitations of Meadow I and II. Columns A_1 to A_9 show four levels of nitrogen on stacked hay and five levels on bunched hay. It is assumed that with unlimited range no meadow will be used for pasture, as Table 7 and 8 show the marginal value product of range to be \$1.44 and \$1.80 per AUM, more than fertilized pasture. The coefficients are set up in the same manner as in Table 7, and the equalities and inequalities take the same form.

Plan 7 in Table 13 shows the maximum profit combination under this resource situation. The optimum production level is shown to be 280,000 pounds of beef, which is given by an operation running a 500

cow herd, wintering 921 cattle, and selling yearlings. The resource requirements for this production are given in rows 9 to 14 of the A_0 column in Table 13. Row 9 shows the capital requirement for this meadow production to be \$11,142. Row 10 shows that 5053 AUMs of range are required, or 2028 AUMs (67%) more than the range requirement without fertilizer. Charging the additional range at an average of \$.33 per AUM the annual capital requirement becomes \$11,868, or \$9909 more than Situation I without fertilizer. Row 11 shows that all of Meadow II will be used for stacked hay without fertilizer. Rows 13 and 14 show 313 acres of Meadow I to be fertilized at 100 pounds of nitrogen and cut for stacked hay, and 176 acres receive 90 pounds of nitrogen and are cut for bunched hay.

The marginal value products of the resources are shown in row 16. The A_0 column shows the net return to fixed factors as \$24,356. The marginal value product of range is zero as this resource is in excess in this situation. The marginal value product of meadow II is \$24.13. If this is discounted at 5% for time preference over a period of 25 years, the discounted value of meadow II is \$300 per acre with unlimited grazing rights. The marginal value product of meadow I is \$36.89 per acre. Allowing a discount of 5% for time preference over 25 years, for a long term investment, the discounted value per acre of meadow I is \$460. The average value of the 750 acres of meadow, where range is unlimited, is then \$405 per acre without discounting for risk and uncertainty.

Table 13. Linear Programming Solution for Optimum Level of Beef Production with Two Feed Producing Activities from Meadow, Five Levels of Nitrogen Application, and Four Limitational Resources (Capital and Range Unlimited).

C value			0		0	0	0	0	0	0	0	0	0	0	0	0	0	0	.335	1.07	13.549	Check	R	
Relation number	Resources	Unit	Quantity	Relation-ship	Activity and Activity Levels																			
					Disposal				Stacked hay				Bunched hay					Stacked hay off Meadow II	Range buying	Capital buying	Beef Production			
			A ₀		A ₁₄ Meadow II	A ₁₅ Meadow I	A ₁₆ Stacked hay	A ₁₇ Bunched hay	A ₁ N ₈₀	A ₂ N ₁₀₀	A ₃ N ₁₂₀	A ₄ N ₁₄₀	A ₅ N ₈₀	A ₆ N ₉₀	A ₇ N ₁₀₀	A ₈ N ₁₂₀	A ₉ N ₁₄₀	A ₁₀ N ₀	A ₁₁	A ₁₂	A ₁₃ P ₁			
Plan 1																								
(1)	A ₁₂ Capital	\$	0	=	0	0	0	0	17.735	21.293	24.742	28.293	15.229	16.920	18.608	21.956	25.342	0	0	1	.530266	191.648266	0	
(2)	A ₁₁ Range	A.U.Ms	0	=	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1.803134	7002.803134	0	
(3)	A ₁₄ Meadow II	acres	260	≥	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	262	∞	
(4)	A ₁₅ Meadow I	acres	490	≥	0	1	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	500	490	
(5)	A ₁₆ Stacked hay	tons	0	=	0	0	1	0	2.306154	2.470536	2.613208	2.715904	0	0	0	0	0	1	0	0	-.370010	11.685792	0	
(6)	A ₁₇ Bunched hay	A.U.Ms	0	=	0	0	0	1	0	0	0	0	4.979250	5.201000	5.303690	5.569763	5.787967	0	0	0	-.325862	29.916124	0	
(7)	Z		0		0	0	0	0	18.976450	22.783510	26.473940	30.273510	16.295030	18.104400	19.910560	23.492920	27.115940	0	.335	1.07	13.549			
(8)	Z - C		0		0	0	0	0	18.976450	22.783510	26.473940	30.273510	16.295030	18.104400	19.910560	23.492920	27.115940	0	0	0	0			
Plan 7																								
(9)	A ₁₂ Capital	\$	-11,141.523639	=	-.481416	-22.482357	.481416	1.080671	-3.637127	0	3.493622	7.118126	-1.872417	0	1.857313	5.492721	9.114535	0	0	1	0	11,140.358566		
(10)	A ₁₁ Range	A.U.Ms	-5052.880736	=	-3.499518	-8.488808	3.499518	1.631910	-.564817	0	.318425	.843089	-.363118	0	.166380	.600541	.956633	0	1	0	0	5056.780501		
(11)	A ₁₀ Stacked hay	tons	260	=	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	262		
(12)	A ₁₃ Beef, P ₁	100 pounds	2802.276716	=	1.905581	4.707807	-1.905581	-.905041	.312242	0	-.176595	-.467569	.201382	0	-.092273	-.333055	-.550539	0	0	0	1	2805.994075		
(13)	A ₂ Stacked hay at N ₁₀₀	tons	312.805088	=	-.120494	.702313	.120494	-.136820	.980193	1	1.011167	1.029565	.021053	0	-.023340	-.059740	-.089595	0	0	0	0	317.239887		
(14)	A ₆ Bunched hay at N ₉₀	A.U.Ms	175.606055	=	.119414	.295017	-.119414	.135593	.019629	0	-.011067	-.029300	.970169	1	1.014164	1.050238	1.079825	0	0	0	0	181.130317		
(15)	Z		24,355.547750		24.132441	36.880937	-24.132441	-10.512236	.149816	0	1.452056	1.563456	.603518	0	.792800	1.562996	2.613459	0	.335	1.07	13.549			
(16)	Z - C		24,355.547750		24.132441	36.880937	-24.132441	-10.512236	.149816	0	1.452056	1.563456	.603518	0	.792800	1.562996	2.613459	0	0	0	0			

The results of this model show that if rangeland can be developed to carry up to 66% more cattle through the summer months, the fall and winter feed for the increased numbers can be profitably provided by the meadows under a system of fertilization. It is obvious, therefore, that any likely increase in range production can be matched by increased meadow production through fertilization.

Redistribution of Range Grazing under Fertilization

In recent years ranchers in the Harney area have had to face the possibility of delayed range turn-out dates in the spring. Already in many range areas the Bureau of Land Management has changed the turn-out date from April 1 to April 15, and it is probable that in some cases the date set may be May 1. If a rancher in situation I were faced with a reduction in range rights of 275 AUMs, resulting from a two week delay in turn out date, he would have two alternatives - one, to restrict his cattle numbers to the point where he could feed them for two weeks longer with his current hay supply. This would involve a voluntary reduction of his remaining grazing rights. Secondly, he may maintain his herd and increase his hay supply by purchase, renting or fertilization. For the 15 day additional feeding period he needs 70 tons of hay, which could be obtained by fertilizing 355 acres of meadow I at ten to fifteen pounds of nitrogen per acre. From Table II it can be seen that the marginal value product of stacked hay when 10 to 15 pounds of nitrogen are applied would be about \$10 per ton. So unless the rancher can buy hay for less than

\$10 per ton, or rent unfertilized meadow for less than \$8.85 per acre, it would pay him to maintain production by fertilizer, rather than cut back his herd.

Another problem faced by ranchers is the prolonged hay feeding season, especially in those cases where after April 1 the cattle must be held and fed on meadows already flooded. To offset this a rancher in situation I who used only Bureau of Land Management range may consider trading 550 AUMs of fall feed, for 275 AUMs of spring feed, by bringing in cattle on August 31 and putting out on April 1. In this way some of the range regrowth in the fall would be saved and carried over to allow light grazing in the early spring. If such a system were adopted he could readjust his feed supply by using fertilizer. He would need 70 tons less stacked hay and 550 AUMs more fall feed in the form of bunched hay and aftermath. The readjustment would require that 225 acres of meadow I be fertilized at 20 to 25 pounds of nitrogen to give 360 tons of stacked hay, and 265 acres would be fertilized at 20 pounds of nitrogen for bunched hay. The total added cost due to this adjustment would be \$2206 or \$4.01 per AUM. Under this resource and price situation it would pay the rancher to maintain his herd by using nitrogen, unless he could buy bunched hay in the field for \$11.21 per acre or rent meadow for less than \$9.82 per acre. Unless the price of beef fell by more than 15% (see Table 12) it would pay him to maintain his herd, rather than cut it back to the point where the original meadow production would cover the requirements.

Another aspect of this problem is the possibility of a complete redistribution of Taylor Grazing Rights. This could be effected in cases where ranchers graze Bureau of Land Management range until June 15, and then transfer their stock to Forest Service range. At the present time the permits for Forest Service and Bureau of Land Management range are generally in balance. However, if summer pasture were available, it would no longer be necessary for a rancher to have a Forest Service permit for the same number of animal units per month as his Bureau of Land Management permit. When he transfers his cattle from the Public Domain on June 15, he could put some back onto the meadows and the remainder onto Forest Service range. It would be difficult to put into effect such a redistribution of range permits, but it could lead to a more efficient use of range land, which is at present the main limiting factor in the cattle operation. Such a redistribution might also be considered where there is a differential improvement of range by the Bureau of Land Management or the Forest Service, but not in cases where ranchers do their own development.

CHAPTER VI

SUMMARY AND CONCLUSIONS

Cattlemen in Harney county and in many other desert areas of eastern Oregon have been faced with the possibility of serious adjustment problems due to two factors. 1. The early spring turn-out date on range, forced on ranchers by hay shortages and the flooded condition of the meadows, has contributed to some deterioration of the sagebrush-bunchgrass vegetation on rangelands. As a result of this the Bureau of Land Management in recent years has applied pressure to delay the official turn-out date by as much as four to six weeks. 2. Ranchers have become increasingly concerned over the apparent decline in hay production from meadows.

Because of the importance of native hay meadows in the above problem and in the range cattle operations as a whole, the Squaw Butte-Harney Experiment Station conducted a series of trials to test the effect of fertilizer applications on yield and quality of hay and regrowth forage. The main purpose of this study is to provide an economic interpretation of the fertilizer data available after the experiment had been running for two years, 1954 and 1955.

The basic hypothesis which the study is designed to test is: It is economically feasible to expand the ranch operation in the Harney basin through increased forage production on meadows by use of fertilizer. The major questions considered were: 1. What is the

optimum economic rate of fertilizer application? 2. How is this rate effected by different resource conditions and situations? 3. How is this rate effected by factor and product price changes? 4. What are the policy implications of increased forage production from meadow lands?

The study area comprised that part of Harney county which lies to the north and west of the Steens Mountains. A survey was made of 20 ranchers to gain information on resource situations, production methods and responses, and problems in the area. The information in this survey was used to describe the decision-making framework within which a rancher must operate. In this way the relevant factors involved in a decision to use fertilizer were isolated. These factors were as follows: the resource situation in terms of land, labor and capital, the price of nitrogen and beef, the cost and requirements of stacked hay, bunched hay and pasture.

In order to obtain a solution to the problem it was apparent that a method of analysis was required which would allow the simultaneous selection of the level of beef production, areas of meadow to be fertilized for stacked hay, bunched hay and pasture, and the rate at which these should be fertilized in order to maximize returns to the fixed factors of production, land, labor, capital and management. The technique selected for the analysis was linear programming. In order to calculate the forage production coefficients used in the matrices, it was necessary to derive a hay production function. This production function was estimated from the experimental data by using

an exponential regression equation of the form $\hat{y} = M(1 - R^X)$. All other coefficients were calculated from ranch survey data, experiment station results, and market reports.

The initial model was set up for a two man unit producing 167,900 pounds of beef and running 300 cows, with six limitational resources, 3025 AUMs of range, 260 acres of meadow II cut for stacked hay and giving an unsatisfactory response to fertilizer, 490 acres of meadow I which may be fertilized to produce only stacked hay, bunched hay or pasture. This model allowed for four levels of beef production and seven levels of nitrogen on each of the three forage activities. The solution selected the highest level of beef production considered, i.e. 206,900 pounds of beef from a herd of 355 cows. It also showed that 40 pounds of nitrogen should be applied to meadow I. As the highest level of beef production had been selected in this model, a second matrix was set up to determine the optimum level of beef production and the fertilizer requirements. A fifth beef production process was included with a production level well beyond the feasible limits of the resources available, to ensure full utilization of resources. The solution showed that the optimum nitrogen application was 50 pounds on 282 acres for stacked hay, 40 pounds on 118 acres for bunched hay, and 50 pounds on 90 acres for pasture. The level of beef production which this forage output would support is 212,000 pounds from a herd of 360 cows. This is a 26% increase over the production without fertilizer, and gives a net increase of \$2058 in the return to fixed factors.

A third model was set up to take account of any possible expansion in range grazing through development or purchase. Here there were four limitational resources, meadow I and II, stacked hay and bunched hay, and four levels of nitrogen on each of the two forage activities. The results of this analysis showed that the optimum production level would be 280,000 pounds of beef given by an operation running 500 cows. The range requirement for this operation is 5053 AUMs or 67% more than the requirement without fertilization of meadow. The nitrogen application required to support this production would be 100 pounds on 313 acres for stacked hay and 90 pounds on 177 acres for bunched hay.

From the results of these models it is obvious that any increase in range capacity can readily and profitably be matched by meadow output under a system of fertilization. However, the second model does show that without some development of range, the expansion through fertilization of meadow alone is limited to about 25%.

A by-product obtained in the linear programming procedure is the MVPs (Marginal value products) of the resources considered. In order to allow a comparison between the values obtained under fertilization and without fertilization a fourth matrix was set up. The sole purpose of this model was to obtain the MVPs under the original resource situation where no fertilizer was used. The results, for comparison, are given in Table 14 below.

Table 14. Marginal Productivity Values of Resources
at Various Levels of Nitrogen Use.

Rate of Nitrogen Use. Pounds per Acre.	Marginal Productivity Value (\$)			
	Per ton of stacked hay	Per ton of bunched hay	Per AUM of pasture	Per AUM of range
0	10.81	9.64	-	5.17
40	13.90	12.74	3.09	4.90
50	14.06	12.81	3.06	4.51
100	24.13	22.77	-	-
140 ^{/1}	36.63	34.71	-	-

^{/1} Obtained from a matrix in which meadow II was unlimited.

It is of interest to note in regard to the MVP of hay that in 1955 stacked hay was selling in the area for \$20 to \$27 per ton. Range values show a relative consistency and indicate that prices charged by Federal agencies are significantly lower than the productive capacity of range.

An analysis of the effect price changes in beef and nitrogen would have on the fertilization rate showed the range of possibilities to be as follows: If nitrogen costs 18.5 cents per pound it would not pay to use it if beef prices were below \$12 per 100 pounds; if nitrogen were 13.5 cents per pound it would still pay to use it if beef went as low as \$8. However, even at 13.5 cents per pound, beef

would have to be \$50 per 100 pounds before it would pay to use 200 pounds of nitrogen.

The policy implications of meadow improvement are only indirectly related to fertilizer, but are nevertheless of importance in appraising fertilizer as it effects ranch management problems. Fertilizer does provide a relatively flexible method of increasing hay production and reserves. In this way it acts as a form of insurance and reduces the uncertainty in the operation. Where this is true the rancher can increase production, but summer range is still the most limiting factor. The administrators of public lands are, therefore, faced with the problem of obtaining the best utilization of range, and at the same time allowing the best use to be made of the meadows. There are two courses of action available to them. One is to develop rangeland, either themselves, or by financial assistance to ranchers; the second is to redistribute the range grazing currently available in light of meadow potential. In some cases it is impossible for the rancher to hold cattle on meadows in April and May due to pasture damage. However, he may well be able to pasture them from July onwards. Other ranchers may be able to hold some cattle on pasture throughout the spring and summer. Thus, under a situation of meadow development a redistribution of range permits, in terms of absolute number of AUMs and in terms of month to month distribution of AUMs, could lead to a more efficient utilization of resources. At present the administrative feasibility of such a recommendation is questionable.

Of necessity the problem covered in this study is broad and the answers, therefore, are somewhat approximate. The problem of establishing the most economic method of expanding the ranch operation requires considerably more physical input-output data than is currently available. Physical experiments are needed to determine the following relationships. 1. The fall feeding capacity of bunched hay and aftermath under different fertilization rates. 2. The carrying capacity of pasture from May to September under different rates of fertilizer. 3. The weight gains of calves and yearlings on meadow pasture from May to September. In addition to these, more complete empirical data is needed in the whole field of range development methods and responses. Several projects are at present in progress which will provide some of the data required.

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APPENDICES

APPENDIX I

EXPERIMENTAL DATA ON HAY RESPONSE TO NITROGEN FERTILIZER

Table 15. Hay Yields from Station Trials on Site 1, 1954.

Replication Number	Time of Application	Source of Nitrogen	Hay Yields in Tons per Acre/ ¹				
			Nitrogen Treatment in Pounds				
			0	50	100	150	200
1.	Spring	Nitrate N	1.79	2.57	3.57	3.90	3.31
		Ammonium N	2.50	3.21	2.76	2.74	3.08
	Fall	Nitrate N	1.75	2.35	2.92	3.44	4.00
		Ammonium N	2.02	2.60	2.87	3.37	3.48
	Spring	Nitrate N	1.91	2.78	3.62	3.63	3.69
		Ammonium N	1.94	2.93	3.58	3.35	3.89
2.	Fall	Nitrate N	1.86	2.75	2.97	3.17	4.34
		Ammonium N	2.15	2.65	2.77	3.09	2.87
	Spring	Nitrate N	1.58	2.09	2.49	3.80	2.38
		Ammonium N	1.79	2.51	3.36	3.22	3.48
	Fall	Nitrate N	1.83	2.35	2.73	3.81	3.69
		Ammonium N	1.28	2.41	3.36	2.40	3.71
3.	Spring	Nitrate N	1.66	2.09	2.41	3.09	3.57
		Ammonium N	1.12	2.99	2.57	3.48	2.90
	Fall	Nitrate N	1.73	1.80	2.66	2.66	3.78
		Ammonium N	1.63	2.61	3.24	2.51	3.34
	Spring	Nitrate N	1.66	2.09	2.41	3.09	3.57
		Ammonium N	1.12	2.99	2.57	3.48	2.90
4.	Fall	Nitrate N	1.73	1.80	2.66	2.66	3.78
		Ammonium N	1.63	2.61	3.24	2.51	3.34
	Spring	Nitrate N	1.66	2.09	2.41	3.09	3.57
		Ammonium N	1.12	2.99	2.57	3.48	2.90
	Fall	Nitrate N	1.73	1.80	2.66	2.66	3.78
		Ammonium N	1.63	2.61	3.24	2.51	3.34

¹ In the yields reported from station trials, no correction has been made for the increment in yield due to phosphorus.

Table 16. Hay yields from Station Trials on Site 1, 1955.

Replication Number	Time of Application	Source of Nitrogen	Hay Yields in Tons per Acre				
			Nitrogen Treatments in Pounds				
			0	50	100	150	200
1	Spring	Nitrate N	1.75	3.07	3.99	4.88	3.53
		Ammonium N	2.51	2.97	3.40	3.05	3.20
	Fall	Nitrate N	2.21	2.72	3.99	4.09	4.47
		Ammonium N	2.29	3.00	3.51	3.45	3.38
2	Spring	Nitrate N	2.24	2.84	3.15	4.29	3.78
		Ammonium N	2.08	2.87	3.51	3.28	4.22
	Fall	Nitrate N	2.01	3.15	3.23	3.91	4.27
		Ammonium N	2.49	2.56	2.67	3.78	3.35
3	Spring	Nitrate N	1.96	3.00	3.71	4.67	3.66
		Ammonium N	2.16	3.40	3.71	4.52	3.91
	Fall	Nitrate N	2.13	2.98	3.68	3.43	4.88
		Ammonium N	1.47	2.79	3.23	3.48	4.11
4	Spring	Nitrate N	2.26	3.12	3.84	3.81	5.36
		Ammonium N	1.75	4.01	3.89	4.42	4.80
	Fall	Nitrate N	3.00	2.44	4.11	3.56	4.75
		Ammonium N	2.06	3.66	3.56	3.12	3.99

Table 17. Hay Yields from Station Trials on Site 2, 1955.

Replication Number	Time of Application	Source of Nitrogen	Hay Yields in Tons per Acre				
			Nitrogen Treatment in Pounds				
			0	50	100	150	200
1	Spring	Nitrate N	1.37	1.65	2.13	2.03	2.95
		Ammonium N	1.47	2.64	2.64	3.56	3.71
	Fall	Nitrate N	1.47	1.70	2.21	2.57	2.62
		Ammonium N	1.40	1.75	2.64	2.18	3.51
2	Spring	Nitrate N	1.45	2.11	2.57	3.66	3.00
		Ammonium N	1.52	2.11	2.74	3.38	3.63
	Fall	Nitrate N	1.66	1.93	2.39	3.76	3.05
		Ammonium N	2.03	2.69	2.84	3.20	3.25
3	Spring	Nitrate N	1.96	1.55	3.40	1.85	3.94
		Ammonium N	1.85	2.41	2.72	3.02	3.99
	Fall	Nitrate N	1.32	1.78	1.68	1.40	2.64
		Ammonium N	1.37	2.51	2.84	2.74	3.71
4	Spring	Nitrate N	1.32	1.96	2.79	3.84	3.48
		Ammonium N	1.40	3.40	3.07	2.97	3.61
	Fall	Nitrate N	2.11	2.84	2.24	3.18	3.76
		Ammonium N	1.32	3.53	2.49	3.61	3.56

Table 18. Hay Yields from Off-station Trials
at 19 Locations, 1954.

Location Number	Hay Yield in Tons per Acre		
	Nitrogen Treatment in Pounds		
	0	60	120
1	2.19	3.01	3.64
2	2.62	3.43	3.92
3	2.38	3.16	3.03
4	1.39	2.48	2.89
5	2.00	2.42	3.26
6	1.40	2.32	2.34
7	1.79	2.62	2.72
8	.46	1.04	1.74
9	1.93	2.84	3.18
10	1.44	2.12	2.42
11	1.50	2.09	2.82
12	1.99	2.89	3.53
13	.47	.80	.95
14	1.70	2.70	3.66
15	1.41	2.48	3.35
16	1.06	1.20	1.93
17	.25	.62	.77
18	2.71	3.51	3.79
19	1.29	1.95	2.12

Table 19. Hay Yields from Off-station Trials
at 7 Locations, 1955.

Location Number	Hay Yields in Tons per Acre		
	Nitrogen Treatment in Pounds		
	0	60	120
1	2.10	2.94	3.39
2	1.41	2.38	3.08
3	2.32	3.22	3.53
4	1.47	1.98	2.33
5	1.98	2.66	2.72
6	1.54	2.48	2.92
7	1.54	2.05	2.08

APPENDIX II

NOTES ON THE PROCEDURE USED IN SETTING UP AND
SOLVING THE MATRIX IN TABLE 7

The computation used in this matrix follows the conventional simplex method, but as the initial tableau in Table 7, contains both equalities and inequalities the solution requires some adjustment in procedure (47).

Columns A_{23} to A_{26} in Table 7 show the coefficients for bunched hay, stacked hay and pasture to have negative signs. This is because these represent consumption of products which have already been produced positively in the system, in columns A_1 to A_{21} . The coefficients for range and capital are positive as neither of these was produced by another activity considered in the analysis.

From Table 7 it can be seen that in the A_0 column the four equalities have zero values; it follows that in the first tableau, regardless of the incoming row, the R values will be zero for these four equations, thus giving no basis for selection by the usual procedure of taking the lowest R value (17). Instead the procedure is to choose the lowest R other than zero. This step can only be taken when sufficient is known about the problem situation, to know that the out-going resource will in fact be limiting in the production process. If this is not known with certainty then it is necessary to proceed by selecting one of the resources with a zero R value, which it is reasonable to expect would be limiting. Capital should never

be selected in any case where there is a capital buying activity included in the system. The same thing applies to range in Table 13, where there is a range buying activity.

This form of analysis is only as accurate as the coefficients in the initial tableau. A different figure in the second decimal place of one of the yield coefficients may change the final solution by as much as 10 or even 20 pounds of nitrogen per acre. Many of the coefficients used in this matrix were computed on an arbitrary basis because of lack of complete experimental data on the feeding capacity of bunched hay and pasture at different levels of nitrogen fertilization (3).

This form of analysis of farm data is sometimes criticised because it gives what may be termed "pseudo-accuracy". In a case such as this, where the data on the major input-output relationships are not all complete, such a charge may be warranted. However, the method is no more inaccurate than other forms of analysis, and has the advantage that more alternatives may be considered per unit of time. With more complete experimental data the linear programming procedure has distinct advantages over the budgeting technique. Less field data is required for the analysis; the strategic variables are isolated more decisively; it is time saving; and it is also suitable for programming on electronic computers.

APPENDIX III

RESOURCE, COST AND PRICE DATA USED IN THE LINEAR PROGRAMMING MODELS^{/1}

Table 20. Details of the Beef Producing Processes Used in the Matrices^{/1}

	Beef Process					
	P ₁	P _{1a}	P ₂	P ₃	P ₄	P ₅
Permanent Labor	2	2	2	2	2	3
Cattle:						
breeding cows	300	314	314	327	355	550
calves weaned (80%)	240	251	251	262	284	440
bulls	13	13	13	14	15	25
weaner heifer replacements wintered	38	40	40	42	45	71
spayed weaner heifers wintered	82	86	86	89	97	149
weaner steers wintered	120	125	125	131	142	220
Total cattle wintered	553	578	578	603	654	1015
Cattle sales:						
fat cows	12	13	13	13	14	22
aged cows	19	20	20	21	23	37
cull bulls	3	3	3	4	4	6
yearling heifers	80	59	84	87	95	146
short yearling heifers	-	25	-	-	-	-
yearling steers (off pasture)	-	-	25	50	99	215
yearling steers (off range)	118	123	98	79	40	-
Total cattle sold	232	243	243	254	275	426
Total pounds of beef	167,925	170,950	177,850	188,200	206,924	336,761
Gross income from beef (\$)	22,751.73	23,435.20	24,117.25	25,537.83	28,140.39	45,766.33
Average price per 100 pounds beef (\$)	13.5487	13.7088	13.5604	13.5695	13.5994	13.5902

Table 20. (continued)

	Beef Process					
	P ₁	P _{1a}	P ₂	P ₃	P ₄	P ₅
Permanent Labor	2	2	2	2	2	3
Feed requirements:						
Fall feed in AUMs	527	548	548	565	603	1203
Winter feed - tons of stacked hay	622	650	650	678	735	1142
- tons of cottonseed cake	8.40	8.75	8.75	9.17	9.94	15.40
Summer feed - AUMs of range	3025	3025	3025	3025	3025	3025
- AUMs of pasture	-	-	125	250	500	2610
Variable operating expenses (\$)						
Winter supplement	805.80	840.00	840.00	880.32	954.24	1474.40
Freight on stock	84.65	84.65	84.65	101.57	101.57	169.29
Pumping water to pasture	-	-	40.15	80.25	160.50	698.18
Fencing maintenance	-	-	16.37	43.34	58.14	116.27
Other expenses	-	250.00	-	-	-	-
Total variable expenses (\$)	890.45	1174.65	981.17	1105.48	1274.45	2458.14

¹ Sources of data: survey of ranchers, hay contractors, experimental data from the Squaw Butte-Harney Experiment station, U. S. Department of Agriculture market reports, and Ontario stock market reports (1, 15, 22 and 65).

Table 21. Costs of Making Stacked Hay at Yields Obtained from Various Rates of Nitrogen Application¹

Operation	Pounds of Nitrogen Applied per Acre									
	0	10	40	50	60	80	100	120	140	160
Hay yield - tons	1.20	1.38	1.86	1.99	2.11	2.31	2.47	2.61	2.72	2.81
Mowing - acres/hour	2.850	2.710	2.354	2.258	2.171	2.020	1.897	8.828	1.713	1.645
cost/acre	.611	.642	.739	.771	.801	.861	.917	.952	1.016	1.058
Raking - acres/hour	2.780	2.780	2.780	2.780	2.780	2.780	2.780	2.780	2.780	2.780
cost/acre	.608	.608	.608	.608	.608	.608	.608	.608	.608	.608
Bucking and yarding - acres/hour	2.780	2.404	1.789	1.674	1.581	1.444	1.348	1.299	1.276	1.186
cost/acre ²	.205	.237	.319	.341	.361	.395	.423	.439	.465	.481
Yarding to stack - acres/hour	5.000	5.000	5.000	5.000	5.000	5.000	5.000	5.000	5.000	5.000
cost/acre ²	.085	.085	.085	.085	.085	.085	.085	.085	.085	.085
Stacking - tons/hour	6.660	6.660	6.660	6.660	6.660	6.660	6.660	6.660	6.660	6.660
cost/ton	1.058	1.058	1.058	1.058	1.058	1.058	1.058	1.058	1.058	1.058
cost/acre	1.270	1.382	1.969	2.020	2.228	2.440	2.614	2.712	2.873	2.969
Total cost of hay making per acre	2.694	2.954	3.635	3.825	3.998	4.304	4.562	4.711	4.962	5.116
Nitrogen - cost/pound	-	.165	.165	.165	.165	.165	.165	.165	.165	.165
cost/acre	-	1.650	6.600	8.250	9.900	13.200	16.500	19.800	23.100	26.400
Fertilizer spreading - acres/hour	-	4.222	4.222	4.222	4.222	4.222	4.222	4.222	4.222	4.222
cost/acre	-	.231	.231	.231	.231	.231	.231	.231	.231	.231
Total cost per acre of applying nitrogen	-	1.881	6.831	8.481	10.131	13.431	16.731	20.031	23.331	26.631
Total cost of hay - per acre	2.694	4.835	10.466	12.306	14.129	17.735	21.293	24.742	28.293	31.747
- per ton	2.245	3.503	5.627	6.184	6.696	7.677	8.621	9.665	10.402	11.298

¹ Sources of data: survey of ranchers, hay contractors, experimental data from the Squaw Butte-Harney Experiment Station, and U.S. Department of Agriculture studies (22 and 65).

² The driver of the buck-rake and tractor is assumed to be a permanent ranch hand and his labor is not charged against the hay making operation.

Table 22. Costs of Making Bunched Hay at Yields Obtained from Various Rates of Nitrogen Application (1 and 22).

Operation	Pounds of Nitrogen Applied per Acre										
	0	40	50	60	80	90	100	120	130	140	160
Feed in AUMs	2.796	4.101	4.355	4.584	4.979	5.201	5.304	5.570	5.684	5.788	5.967
Mowing and raking											
- cost/acre (\$)	1.219	1.347	1.379	1.409	1.469	1.498	1.525	1.560	1.601	1.624	1.666
Bunching											
- acres/hour (\$)	3.333	2.149	2.010	1.900	1.734	1.671	1.619	1.561	1.502	1.473	1.425
- cost/acre (\$)	.171	.265	.284	.300	.329	.341	.352	.365	.379	.387	.400
N application											
- cost/acre (\$)	-	6.831	8.250	10.131	13.431	15.081	16.731	20.031	21.681	23.331	26.631
Total cost											
- per acre (\$)	1.390	8.443	9.913	11.840	15.229	16.920	18.608	21.956	23.661	25.342	28.697
- per AUM (\$)	.497	2.059	2.276	2.583	3.059	3.253	3.508	3.942	4.163	4.378	4.809

Table 23. Cost of Meadow Pasture at Responses Obtained from Various Rates of Nitrogen Application (1 and 22).

Operation	Pounds of Nitrogen Applied per Acre							
	0	40	50	60	80	100	120	160
Feed in AUMs	4.590	7.120	7.611	8.055	8.821	9.450	9.966	10.736
Cost of fertilizer per acre	-	6.831	8.250	10.131	13.431	16.731	20.031	26.631
Total cost per AUM	-	.959	1.084	1.258	1.523	1.770	2.010	2.481

Table 24. Weight and Prices of Cattle Used
in the Matrices.^{/1}

Type and Grade of Animal	Average Weight (pounds)	Average Price (\$ per 100 pounds)
Slaughter cows		
Fat: Commercial	-	12.20
Utility	-	9.94
Weighted average of fat cows	1050	11.10
Aged: Cutter	-	8.10
Canner	-	6.60
Weighted average of aged cows	975	7.30
Bulls		
Commercial	-	12.80
Utility	-	10.05
Cutter	-	8.50
Weighted average of bulls	1400	10.30
Feeder steers		
Good heavy	-	16.83
Good	-	17.18
Medium	-	14.18
Weighted average of feeder steers	700	15.70
Feeder heifers		
Good	-	14.20
Medium	-	12.12
Weighted average of feeder heifers	625	13.20

^{/1} Sources of data: survey of ranchers, and Ontario market reports.