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Title: Stabilizing Control Strategies for the Doubly-Excited Machine

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Mathematical models for the doubly-fed machine are presented. The properties of the machine with rotor excitation as supplied by a power converter with voltage source character are reviewed. The peculiar torque-speed characteristic of the machine in this operating mode and its effect on the stability of the machine are presented. A strategy to stabilize the machine by speed feedback is discussed.

With rotor excitation supplied by a converter in current source mode, the developed torque is found to be independent of slip. Conditions for stable operation are given. The goal of this thesis is to find a control strategy which effectively stabilizes the machine by simple and practical means. Two stabilizing feedback control strategies are investigated. An analysis is presented, showing that the machine can be stabilized through either of these control strategies. They lead to the possibility of securing stability through a practical and convenient feedback of the machine speed. This feedback only involves an amplifier with fixed gain, independent of load conditions and machine slip.

Stabilizing Control Strategies for the Doubly-Excited Machine

by

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STABILIZING CONTROL STRATEGIES FOR THE DOUBLY-EXCITED MACHINE

PART I : INTRODUCTION

The doubly-excited machine is basically a wound-rotor induction machine. In induction machine operation, rotor windings are either short-circuited or connected to each other by variable resistors that allow to change the slip-torque characteristic of the induction machine.

In the doubly-excited machine, the rotor windings are supplied with an excitation current of a fixed frequency. In steady operation, stator and rotor fields must rotate at the same speed. Hence, the frequency of the stator current must be equal to the sum of the rotor mechanical speed, expressed in electrical degrees, and the frequency of the rotor excitation current, when stator and rotor phase sequences are identical. If these phase sequences are opposite, the stator frequency must equal the difference of rotor mechanical speed and rotor excitation current frequency.

Usually, the stator windings of the machine are directly connected to the utility grid, which can be considered to supply three-phase source voltages with constant amplitude and frequency, the so-called "infinite bus". By varying rotor excitation frequency between zero and grid frequency, the doubly-fed machine can rotate at speeds between zero and synchronous speed. Moreover, for reversed rotor phase sequence, it can also operate between synchronous and twice synchronous speed. In terms of slip, defined as the relative deviation from synchronous speed, the doubly-fed machine can operate at slips between one and minus one.

This possibility of operation at various speeds makes the doubly-excited machine a highly suitable device for application to sophisticated electro-mechanical energy conversion systems. It can be used as a variable speed driving motor or as a variable speed generator. The generator should be particularly useful for the generation of electric power from erratic energy sources such as water or wind. In such a case, the power input to the prime mover will vary; so will the speed of the turbine at which maximum efficiency could occur. With a doubly-excited generator, it would be possible to electrically control the turbine speed for maintaining maximum efficiency operation.

The basic reason for the doubly-fed machine not to have found widespread application in hydroelectric and wind power plants is the fact that, until quite recently, it was not possible to make use of power electronic converters capable of supplying adequate excitation power at varying frequency to the rotor windings. Another hurdle to overcome was the observation that with excitation as supplied by a voltage source, such as a synchronous generator or a conventional power electronic converter, the doubly-fed machine is dynamically unstable for all but very small slips.

Recent developments in solid state technology, however, have shown the design of a suitable power converter to be quite feasible. The promising potential of the Schwarz converter in this respect is worth to be mentioned (Ref. 5). This converter type has the capability to provide fast controllable current sources, which can be used to directly exploit the magnetic coupling effect of the machine.

Part II of this thesis presents the mathematical models used to investigate the steady state and dynamic properties of the doubly-fed machine.

In Part III, the properties of the machine with rotor excitation provided by a voltage source are derived. A recent publication on a strategy to stabilize the machine over its entire speed range is presented and discussed.

With the mentioned possibility and anticipated benefits of current source excitation of the rotor windings in view, a rigorous analysis of this mode of operation is presented in Part IV. Machine properties with this kind of excitation are shown to be much more favorable. For operating conditions where the machine will not operate stably, two convenient ways of stabilizing it by feedback control are presented and analysed.

PART II : MATHEMATICAL MODEL OF THE DOUBLY-FED MACHINE

II.1 The Machine Configuration

In its construction, the doubly-fed machine is a three-phase induction machine with wound rotor. Thus it has three phase windings on its stator as well as its rotor. The rotor windings are provided with terminals which can be connected to external circuits through slip rings. The schematic configuration is shown in Fig. II.1.

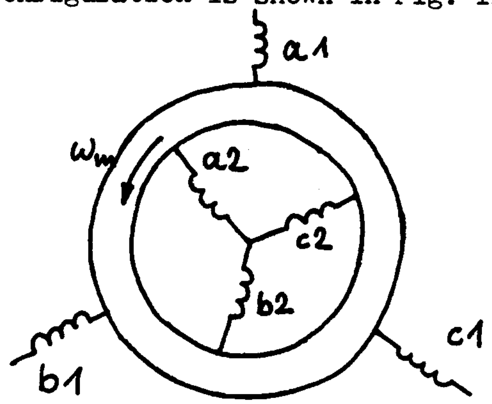

Fig. II.1 Configuration of windings on stator & rotor

II.2 System Configuration

The stator windings are connected to a power grid with three-phase symmetrical voltages of constant frequency ω_1 and constant amplitude, or equivalently, rms value V_1. Such a power grid is called an "infinite bus" and is a reasonable representation of a power grid connecting a great number of generators and consumers. To the single machine, its voltage and frequency will appear constant since they cannot be significantly effected by the characteristics of one consumer or generator.

The doubly-fed machine, like any other type of electromechanical power converter for that matter, can work either as a motor, drawing power from the grid and producing mechanical torque, or as a generator, converting mechanical power that is supplied to it by a prime mover to electrical power which is fed into the grid.

On the rotor side, the doubly-fed machine is supplied with excitation power from a power electronic converter. The converter produces a set of symmetric three-phase source voltages of variable rms value V_2 and frequency ω_2. Certain converter types can also operate as current sources rather than voltage sources. Converter output frequency is taken to be smaller or equal to grid frequency.
Under certain operating conditions, power flow on the rotor side of the doubly-fed machine is inverted: power is generated by the machine and supplied to the converter. Thus, in order to have an efficient system, the converter needs to have the capability of passing power from its secondary, low-frequency side back into the grid.

Fig. II.2 shows the system configuration with the infinite bus, the power converter and the doubly-fed machine.

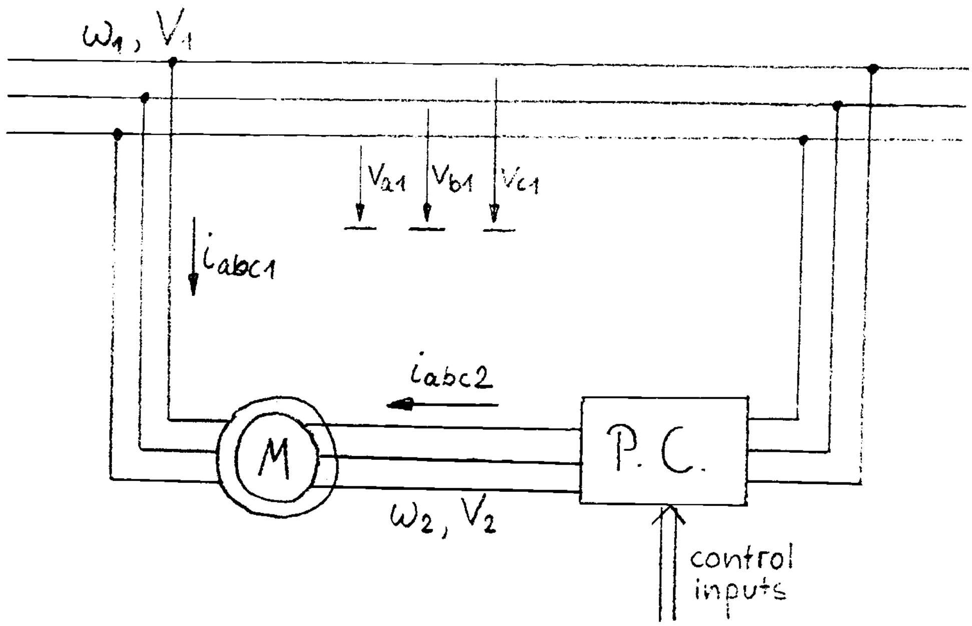

Fig. II.2 System configuration

II.3 Stable Operating Conditions

The currents in the three-phase windings of stator and rotor each produce a magnetic field rotating with the frequency of these currents. Thus,the stator field rotates with ω_1 (or in the actual machine, with ω_1/P , as we would have P pole pairs rather than one). The rotor field rotates within the machine air gap with the sum of rotor current frequency ω_2 and the rotational speed of the rotor body ω_m, measured in electrical degrees. To produce a steady torque, both field waves must travel at the same speed. In steady operating conditions,

$$\omega_1 = \omega_m + \omega_2 \qquad \text{(II.1)}$$

must hold if phase sequences of stator and rotor are identical. If the phase sequence of the rotor voltages (or currents) is reversed,

$$\omega_1 = \omega_m - \omega_2 \qquad \text{(II.2)}$$

holds instead.

II.4 Park's Transformation

For expediency of the analysis, all stator and rotor quantities (voltages, currents, flux linkages) are transformed into the so-called Park domain: a reference frame of two orthogonal axes rotating with stator frequency.

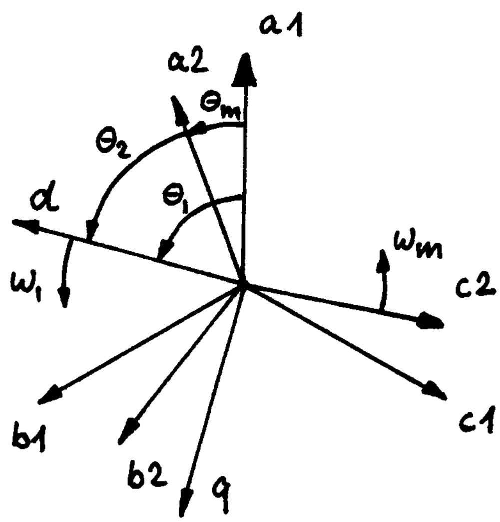

Fig. II.3 d,q-reference frame and angles definitions

Fig. II.3 shows the definition of the angles used in Park's transformation:

- θ_m is the position angle (in electrical degrees) of the rotor a-axis with respect to the stator a-axis
- θ_1 is the angle between the stator a-axis and the direct axis of the reference frame rotating with
- θ_2 is the difference between these two: $\theta_2 = \theta_1 - \theta_m$.

Without loss of generality it can be assumed that stator and rotor a-axes are aligned at t=0 and the following set of equations can be written:

$$\frac{d\Theta_m}{dt} = \omega_m ,$$

$$\Theta_1 = \omega_1 t ,$$

$$\Theta_2 = \omega_1 t - \Theta_m . \qquad \text{(II.3)}$$

In steady state,

$$\Theta_m = \omega_m t .$$

The transformation matrix for stator quantities is given by ref. 1 as

$$T_1 = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\Theta_1 & \cos(\Theta_1 - \frac{2\pi}{3}) & \cos(\Theta_1 + \frac{2\pi}{3}) \\ -\sin\Theta_1 & -\sin(\Theta_1 - \frac{2\pi}{3}) & -\sin(\Theta_1 + \frac{2\pi}{3}) \end{bmatrix} \qquad \text{(II.4a)}$$

and for rotor quantities as

$$T_2 = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\Theta_2 & \cos(\Theta_2 - \frac{2\pi}{3}) & \cos(\Theta_2 + \frac{2\pi}{3}) \\ -\sin\Theta_2 & -\sin(\Theta_2 - \frac{2\pi}{3}) & -\sin(\Theta_2 + \frac{2\pi}{3}) \end{bmatrix} . \qquad \text{(II.4b)}$$

The quantities in the Park domain are now obtained by

$$\begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} = [\ T_1\ ] \begin{bmatrix} i_{a1} \\ i_{b1} \\ i_{c1} \end{bmatrix} ; \begin{bmatrix} V_{d1} \\ V_{q1} \end{bmatrix} = [\ T_1\ ] \begin{bmatrix} V_{a1} \\ V_{b1} \\ V_{c1} \end{bmatrix} ;$$

$$\begin{bmatrix} i_{d2} \\ i_{q2} \end{bmatrix} = [\ T_2\ ] \begin{bmatrix} i_{a2} \\ i_{b2} \\ i_{c2} \end{bmatrix} ; \begin{bmatrix} V_{d2} \\ V_{q2} \end{bmatrix} = [\ T_2\ ] \begin{bmatrix} V_{a2} \\ V_{b2} \\ V_{c2} \end{bmatrix} . \tag{II.5}$$

According to Kirchhoff's law, the voltage across every one of the actual machine windings is

$$v(t) = R\ i(t) + \frac{d}{dt}\ L(t)\ i(t). \tag{II.6}$$

All the voltages and currents can be combined to vectors, and all the resistances and inductances to matrices to obtain

$$\underline{V} = [R]\ \underline{I} + \frac{d}{dt}\left([L(t)]\ \underline{I}\right) \tag{II.7}$$

where

$$\underline{V} = \begin{bmatrix} V_{a1} \\ V_{b1} \\ V_{c1} \\ V_{a2} \\ V_{b2} \\ V_{c2} \end{bmatrix} , \qquad \underline{I} = \begin{bmatrix} i_{a1} \\ i_{b1} \\ i_{c1} \\ i_{a2} \\ i_{b2} \\ i_{c2} \end{bmatrix} , \tag{II.8a,b}$$

$$[R] = \begin{bmatrix} r_1 & & & & & \\ & r_1 & & & \emptyset & \\ & & r_1 & & & \\ & & & r_2 & & \\ & \emptyset & & & r_2 & \\ & & & & & r_2 \end{bmatrix}, \qquad \text{(II.8c)}$$

$$[L] = \begin{bmatrix} l_1 & -m_1 & -m_1 & m\cos\theta_m & m\cos(\theta_m-\frac{2\pi}{3}) & m\cos(\theta_m+\frac{2\pi}{3}) \\ & l_1 & -m_1 & m\cos(\theta_m+\frac{2\pi}{3}) & m\cos\theta_m & m\cos(\theta_m-\frac{2\pi}{3}) \\ & & l_1 & m\cos(\theta_m-\frac{2\pi}{3}) & m\cos(\theta_m+\frac{2\pi}{3}) & m\cos\theta_m \\ & & & l_2 & -m_2 & -m_2 \\ & \% & & & l_2 & -m_2 \\ & & & & & l_2 \end{bmatrix}.$$

(L is symmetric.) (II.8d)

The machine parameters are defined in Appendix 1.

Combining transformation matrices T_1 and T_2 to

$$T = \left[\begin{array}{c|c} T_1 & \emptyset \\ \hline \emptyset & T_2 \end{array}\right] \qquad \text{(II.9)}$$

the voltage equations for all six windings can be transformed into Park's domain:

$$TV = T(R)T^{-1}TI + T\frac{d}{dt}(LT^{-1}TI)$$

$$TV = T(R)T^{-1}TI + TLT^{-1}\frac{d}{dt}(TI) + T\frac{d}{dt}(LT^{-1})\,TI \qquad \text{(II.10)}$$

which is subsequently written as

$$V_{dq} = (R_{dq})\, I_{dq} + (L_{dq}) \frac{d}{dt} I_{dq} + (G)\, I_{dq}\,. \qquad \text{(II.11)}$$

The term $L_{dq} \frac{d}{dt} I_{dq}$ results from changes in the d and q currents. The term $(G)\, I_{dq}$, also known as the speed voltage, results from the relative motion of stator and rotor windings.

The matrices R_{dq} , L_{dq} and G are evaluated as

$$[R_{dq}] = \begin{bmatrix} r_1 & & & \emptyset \\ & r_1 & & \\ & & r_2 & \\ \emptyset & & & r_2 \end{bmatrix},$$

$$[L_{dq}] = \begin{bmatrix} L_1 & 0 & M & 0 \\ 0 & L_1 & 0 & M \\ M & 0 & L_2 & 0 \\ 0 & M & 0 & L_2 \end{bmatrix}, \qquad \text{(II.12)}$$

$$[G] = \begin{bmatrix} 0 & -\omega_1 L_1 & 0 & -\omega_1 M \\ \omega_1 L_1 & 0 & \omega_1 M & 0 \\ 0 & -s\omega_1 M & 0 & -s\omega_1 L_2 \\ s\omega_1 M & 0 & s\omega_1 L_2 & 0 \end{bmatrix}.$$

where s denotes the slip

$$S = \frac{\omega_1 - \omega_m}{\omega_1} \qquad \text{(II.13)}$$

and the modified machine parameters are given in appendix 1.

Letting p denote the time derivative operator, terms can be combined to obtain the complete voltage equations in terms of direct and quadrature axis components:

$$\begin{bmatrix} V_{d1} \\ V_{q1} \\ V_{d2} \\ V_{q2} \end{bmatrix} = \begin{bmatrix} r_1 + L_1 p & -\omega_1 L_1 & Mp & -\omega_1 M \\ \omega_1 L_1 & r_1 + L_1 p & \omega_1 M & Mp \\ Mp & -s\omega_1 M & r_2 + L_2 p & -s\omega_1 L_2 \\ s\omega_1 M & Mp & s\omega_1 L_2 & r_2 + L_2 p \end{bmatrix} \begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{d2} \\ i_{q2} \end{bmatrix} .$$

(II.14)

II.5 Generated Mechanical Power and Torque

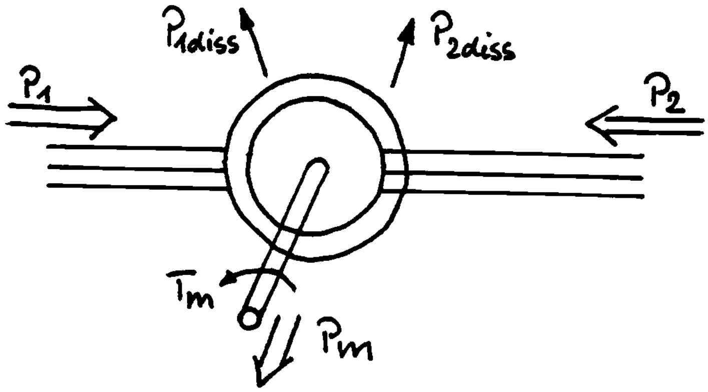

Fig. II.4 Power flow

From Fig. II.4, the equation for the external power balance of the doubly-fed machine can be written:

$$P_1 + P_2 = P_{1diss} + P_{2diss} + P_m \qquad \text{(II.15)}$$

in stable operation.
The constant factor $\sqrt{2/3}$ in Park's transformation matrix was chosen for power invariance (Ref. 1). Thus, electrical power input can be written in terms of d,q-voltages and -currents:

$$P_1 = v_{dq1}^T \; i_{dq1} , \qquad P_2 = v_{dq2}^T \; i_{dq2} . \qquad \text{(II.16)}$$

Stator and rotor voltages are substituted from equ. (II.14) to obtain:

$$P_1 = r_1(i_{d1}^2 + i_{q1}^2) + \frac{d}{dt} \underline{I}_{dq}^T \begin{bmatrix} L_1 & 0 \\ 0 & L_1 \\ M & 0 \\ 0 & M \end{bmatrix} \begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} +$$

$$+ \underline{I}_{dq}^T \begin{bmatrix} 0 & \omega_1 L_1 \\ -\omega_1 L_1 & 0 \\ 0 & \omega_1 M \\ -\omega_1 M & 0 \end{bmatrix} \begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} ;$$

$$P_2 = r_2(i_{d2}^2 + i_{q2}^2) +$$

$$+ \frac{d}{dt} \underline{I}_{dq}^T \begin{bmatrix} M & 0 \\ 0 & M \\ L_2 & 0 \\ 0 & L_2 \end{bmatrix} \begin{bmatrix} i_{d2} \\ i_{q2} \end{bmatrix} + \underline{I}_{dq}^T \begin{bmatrix} 0 & -s\omega_1 M \\ -s\omega_1 M & 0 \\ 0 & s\omega_1 L_2 \\ -s\omega_1 L_2 & 0 \end{bmatrix} \begin{bmatrix} i_{d2} \\ i_{q2} \end{bmatrix} . \tag{II.17}$$

The first terms in these two power equations represent the power dissipated in stator and rotor windings, respectively. The second terms correspond (Ref. 1) to the time rate of change of the magnetic field energy; for steady operating conditions with constant currents these terms vanish.

Comparing (II.17) with (II.15) one can conclude that the mechanical power generated must equal the sum of the last terms in equs. (II.17a,b).

$$P_m = (\omega_1 M - s\omega_1 M)(i_{q1} i_{d2} - i_{d1} i_{q2})$$

$$= \omega_m M \,(i_{q1} i_{d2} - i_{d1} i_{q2}), \tag{II.18}$$

and as

$$T_m = \frac{P_m}{\omega_M} = \frac{P \cdot P_m}{\omega_m}, \tag{II.19}$$

the produced torque expressed in terms of d,q-currents is

$$T_m = PM\,(i_{q1} i_{d2} - i_{d1} i_{q2}). \tag{II.20}$$

II.6 Thevenin Equivalent and Phasor Diagrams for Stator

For steady operating conditions, the time derivative terms in equ. (II.14) vanish.

Let complex phasors be defined as

$$\begin{aligned} \underline{V}_1 &= v_{d1} + j\, v_{q1} \\ \underline{I}_1 &= i_{d1} + j\, i_{q1} \\ \underline{I}_2 &= i_{d2} + j\, i_{q2} \end{aligned} \qquad \text{(II.21)}$$

and define the voltage induced on the stator as

$$\underline{E}_2 = j\omega_1 M\, \underline{I}_2 \,. \qquad \text{(II.22)}$$

Then, equ. (II.14) can be written as

$$\underline{V}_1 = \underline{E}_2 + (j\omega_1 L_1 + r_1)\, \underline{I}_1 \,. \qquad \text{(II.23)}$$

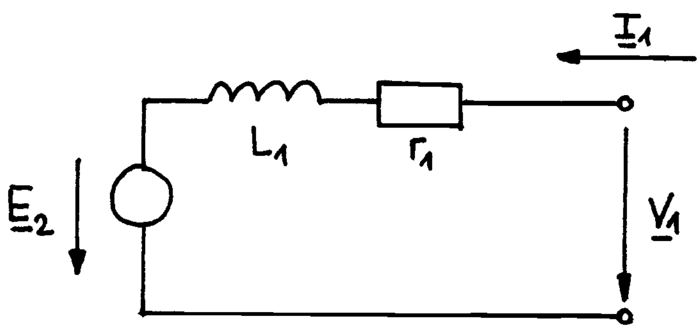

Fig. II.5 Thevenin equivalent circuit for the stator

From this equation, the single-phase equivalent circuit for the stator of the doubly-fed machine can be drawn. Shown in Fig. II.5, it consists of three elements:

- stator winding resistance r_1
- stator self inductance L_1
- a voltage source representing the voltage induced by the rotating rotor field.

From equ. (II.23), one can also draw phasor diagrams for the voltages and the currents. These phasor diagrams will be useful to have a better insight into the physically realizable steady state conditions. They also provide, to some extent, an understanding of the dynamical stability limits of the machine, as will be discussed in Part IV.

Fig. II.6 is a representation of equ. (II.23). From this equation, we also have

$$\underline{I}_1 = \frac{\underline{V}_1}{r_1 + j\omega_1 L_1} - \frac{j\omega_1 M}{r_1 + j\omega_1 L_1}\underline{I}_2 \qquad \text{(II.24)}$$

for which a phasor diagram is given in Fig. II.7. In both phasor diagrams, $\omega_1 L_1 \gg r_1$ is assumed, which holds for all practical machines. The load angle δ, defined as the angle by which the emf phasor induced by rotor current is lagging with respect to stator voltage $\underline{V}_1$, is shown in both diagrams.

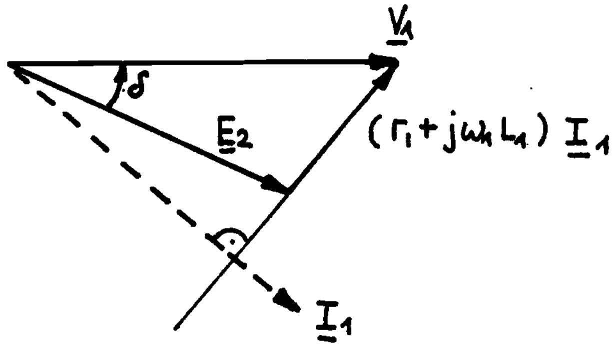

Fig. II.6 Phasor diagram for the voltages

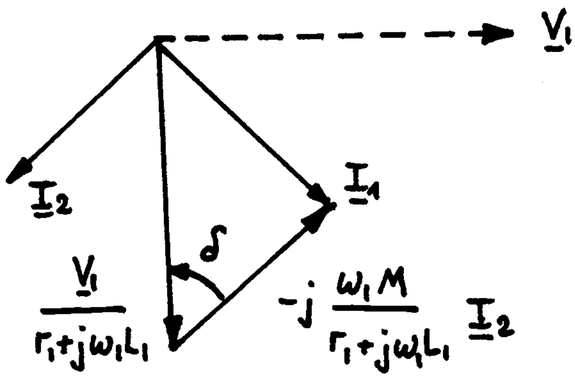

Fig. II.7 Phasor diagram for the currents

PART III : ROTOR EXCITATION BY VOLTAGE SOURCE

III.1 Voltage Equations

If a power electronic converter is used for rotor excitation of the doubly-fed machine, it will provide a set of symmetric three-phase voltages with variable frequency. An investigation of the properties of such a voltage source-excited machine has been conducted by Ohi and Kassakian of the Electric Power Systems Engineering Laboratory at the Massachusetts Institute of Technology (Ref. 6). Part III of this thesis relies heavily on their publication.

With stator and rotor quantities of the doubly-fed machine transformed to a reference frame rotating with synchronous speed ω_1, the equations for direct and quadrature axis quantities were found in chapter II.4 to be

$$\begin{bmatrix} v_{d1} \\ v_{q1} \\ v_{d2} \\ v_{q2} \end{bmatrix} = \begin{bmatrix} r_1+L_1p & -L_1\omega_1 & Mp & -M\omega_1 \\ L_1\omega_1 & r_1+L_1p & M\omega_1 & Mp \\ Mp & -Ms\omega_1 & r_2+L_2p & -L_2s\omega_1 \\ Ms\omega_1 & Mp & L_s s\omega_1 & r_2+L_2p \end{bmatrix} \begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{d2} \\ i_{q2} \end{bmatrix} \tag{III.1}$$

where p denotes the time derivative operator.
Note that in both the equations for the rotor voltages, the coefficients depend on the slip s, and thus on the actual rotational speed of the rotor.

Slip s was defined in part II as the relative deviation from synchronous speed:

$$s = \frac{\omega_1 - \omega_m}{\omega_1} . \tag{III.2}$$

The stator of the doubly-fed machine is connected directly to an infinite bus that provides symmetric three-phase voltages with constant amplitude $\sqrt{2}\, V_1$ and frequency ω_1 . This results in the following direct and quadrature axis stator voltages:

$$\begin{bmatrix} V_{d1} \\ V_{q1} \end{bmatrix} = \sqrt{3}\, V_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} . \tag{III.3}$$

Rotor excitation voltages as provided by a power converter are assumed to be

- symmetric
- of constant rms value V_2
- of constant frequency ω_2;
- the emf wave produced by the rotor lags the one produced by the stator by an electrical angle
- higher harmonics of the converter output voltages are neglected.

Under these assumptions, the following expressions for direct and quadrature axis rotor voltages are obtained:

$$\begin{bmatrix} V_{d2} \\ V_{q2} \end{bmatrix} = \sqrt{3}\, V_2 \begin{bmatrix} \cos\delta \\ -\sin\delta \end{bmatrix} . \tag{III.4}$$

III.2 Torque-Speed Characteristics

From chapter II.5 it is known that the electromechanical torque produced by the doubly-fed machine can be expressed in terms of the d- and q-currents as

$$T_e = PM\,(i_{q1}\, i_{d2} - i_{d1}\, i_{q2}). \qquad \text{(III.5)}$$

In References 6, 7 and 8 the steady-state torque-slip characteristic in terms of stator and rotor applied voltages is calculated by inverting equation (III.1). The result is given by Ref. 8 as

$$\begin{aligned} T_e = K(s)\Big[& s r_2 V_1^2 - r_1 V_2^2 \\ & + \frac{V_1 V_2}{\omega_1 M}\Big\{\big[r_1 r_2 + s\omega_1^2 (L_1 L_2 - M_2^2)\big]\sin\delta \\ & - (s\omega_1 r_1 L_2 - \omega_1 r_2 L_1)\cos\delta\Big\}\Big] \end{aligned} \qquad \text{(III.6)}$$

where

$$K(s) = \frac{3P\omega_1 M^2}{\big[r_1 r_2 - s\omega_1^2 (L_1 L_2 - M^2)\big]^2 + \omega_1^2 (r_2 L_1 + s r_1 L_2)^2}. \qquad \text{(III.7)}$$

The important result of these calculations, in which all three sources agree, is that the steady state torque consists of three major parts.

$$T_{I1} = K(s)\, s\, r_2\, V_1^2 \tag{III.8}$$

is called the primary induction torque and is caused by the relative motion between the rotor and the rotating electromagnetic field induced by the stator currents for nonsynchronous operation, that is, at slip different from zero.

$$T_{I2} = -K(s)\, r_1\, V_2^2 \tag{III.9}$$

is called the secondary induction torque. Its source is the difference in speed between the rotor body and that part of the rotating electromagnetic wave that is induced by the currents in the rotor windings.

$$T_s = \frac{K(s)}{\omega_1 M} V_1 V_2 \left\{ \left[r_1 r_2 + s\omega_1^2 (L_1 L_2 - M_2^2) \right] \cdot \sin\delta - (s\omega_1 r_1 L_2 - \omega_1 r_2 L_1) \cos\delta \right. \tag{III.10}$$

forms the synchronous part of the total torque generated. This part of the torque stems from the phase difference between the electromagnetic waves, both rotating at synchronous speed, created by stator and rotor currents, respectively. This kind of torque propels the synchronous machine, which has just a rotating permanent or electromagnet as the source of its rotor field. The synchronous torque component depends primarily on the load angle, that is, the phase difference between stator and rotor electromagnetic waves. In the case of the doubly-fed machine, however, as can be seen from equ. (III.10), the synchronous torque is also slip-dependent.

Of the three electromagnetic torque components, the primary induction torque T_{I1} turns out to be the decisive factor for dynamic stability. As is seen in Fig. III.1 (taken from Ref. 6), the torque-speed characteristic of this particular torque component looks very similar to that of an induction machine. For a small magnitude of the slip, the characteristic shows a steep descent. In this operating range, a speed decrease caused by an increase in load torque will be effectively counteracted by an increase in electromechanical torque. For larger slips, however, the induction torque-speed characteristic has a positive slope. A slowdown of the rotor in this operating range will decrease the electromechanical torque, which in turn will slow down the machine even further. Thus, the doubly-fed machine is inherently unstable for all but a small slip range. This fact has prevented its successful and economical application so far.

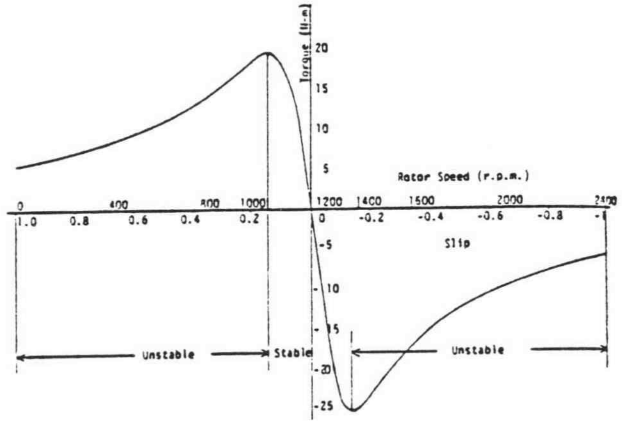

Fig. III.1 Induction torque-speed characteristic (Ref.6)

III.3 State Model

One can now proceed to develop a state model for the voltage source excited doubly-fed machine with the goal of finding an appropriate feedback that will stabilize the machine at all slips. It is the possibility of operating the doubly-fed machine at any speed between zero and twice synchronous speed that makes its application interesting at all.

For modeling purposes, the mechanical load torque is assumed to consist of a constant part and of a damping part proportional to rotational speed. Thus,

$$T_L = T_{L0} + K_L \omega_M . \tag{III.11}$$

According to Newton's law, the equation of motion for the rotating machine is

$$J \frac{d\omega_M}{dt} = T_e - T_L . \tag{III.12}$$

For a machine with more than one pole pair, actual angular speed and angular speed in electrical degrees are not equal. Rather,

$$\omega_m = P\,\omega_M \tag{III.13}$$

where ω_M is the actual rotor speed, ω_m is rotor speed in electrical degrees and P is the number of pole pairs in the machine.

Equs. (11) and (13) are substituted into equ. (12) to obtain

$$\frac{J}{p}\,\frac{d\omega_m}{dt} = T_e - \frac{K_L}{p}\,\omega_m - T_{LO}. \qquad \text{(III.14)}$$

The load angle δ was defined as the angle by which the rotor field <u>lags</u> the stator field. An increase in rotor speed above steady state speed will advance the rotor and its field, thus decreasing the load angle. The differential equation linking speed and load angle will then be given by

$$\frac{d}{dt}\,\delta = -\Delta\,\omega_m \qquad \text{(III.15)}$$

where $\Delta\omega_m$ denotes the deviation of rotor speed from steady state speed.

For a state model, the time derivatives of all the state variables have to be expressed as functions of the state variables themselves and of inputs to the system.

To this end, equ. (III.1) is solved for the time derivatives of all four currents involved. Together with equations III.5, 14 and 15, the complete, nonlinear state model for the doubly-fed machine can be written in matrix form.

$$\frac{d}{dt}\begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{d2} \\ i_{q2} \\ \delta \\ \Delta\omega_m \end{bmatrix} = \left[F \right] \begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{d2} \\ i_{q2} \\ \delta \\ \Delta\omega_m \end{bmatrix} + \left[G \right] \begin{bmatrix} V_{d1} \\ V_{q1} \\ V_{d2} \\ V_{q2} \\ T_{Lo} \end{bmatrix} \tag{III.16}$$

where

$$[F] = \begin{bmatrix} r_1 \frac{L_2}{Z} & \omega_1 + \omega_m \frac{M^2}{Z} & -r_2 \frac{M}{Z} & -\omega_m \frac{ML_2}{Z} & 0 & 0 \\ -\omega_1 + \omega_m \frac{M^2}{Z} & r_1 \frac{L_2}{Z} & \omega_m \frac{ML_2}{Z} & -r_2 \frac{M}{Z} & 0 & 0 \\ -r_1 \frac{M}{Z} & \omega_m \frac{ML_1}{Z} & r_2 \frac{L_1}{Z} & \omega_s + \omega_m \frac{L_1 L_2}{Z} & 0 & 0 \\ -\omega_m \frac{ML_1}{Z} & -r_1 \frac{M}{Z} & -\omega_s - \omega_m \frac{L_1 L_2}{Z} & r_2 \frac{L_1}{Z} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ -\frac{P^2}{J} M i_{q2} & \frac{P^2}{J} M i_{d2} & 0 & 0 & 0 & -\frac{K_L}{J} \end{bmatrix},$$

$$[G] = \begin{bmatrix} -\frac{L_2}{Z} & 0 & \frac{M}{Z} & 0 & 0 \\ 0 & -\frac{L_2}{Z} & 0 & \frac{M}{Z} & 0 \\ \frac{M}{Z} & 0 & -\frac{L_1}{Z} & 0 & 0 \\ 0 & \frac{M}{Z} & 0 & -\frac{L_1}{Z} & 0 \\ 0 & 0 & 0 & 0 & -\left(P + \frac{K_L \omega_{mo}}{T_{Lo}}\right)\frac{1}{J} \end{bmatrix},$$

with the abbreviation $Z = M^2 - L_1 L_2$,

and ω_{mo} denoting steady state mechanical speed:

$\omega_{mo} = \omega_1 - \omega_2$ for identical and

$\omega_{mo} = \omega_1 + \omega_2$ for opposite phase sequences .

III.4 Linear Model

The derived model now has to be linearized, so that the well-known methods of analysis in the Laplace domain and state variable feedback can be applied.

For this purpose, a setpoint must be defined at which the machine is assumed to be operating steadily when no disturbances are present. The setpoint values of all variables are denoted by the subscript "0". The incremental values of all those quantities, on the other hand, will be denoted with a prefix Δ.

The state equations are then expanded into Taylor series, and only the linear parts will be considered. Taking into account that all state equations equal zero on both sides for setpoint values, the linearized model for the incremental quantities is obtained. Note that this model is valid only in a small subspace of the entire state space in the proximity of one setpoint, and that its coefficients all depend on the chosen setpoint.

The linear state model consists of the following set of six equations, which are written in matrix form:

$$\frac{d}{dt}\begin{bmatrix} \Delta i_{d1} \\ \Delta i_{q1} \\ \Delta i_{d2} \\ \Delta i_{q2} \\ \Delta \delta \\ \Delta \omega_m \end{bmatrix} = \left[F' \right] \begin{bmatrix} \Delta i_{d1} \\ \Delta i_{q1} \\ \Delta i_{d2} \\ \Delta i_{q2} \\ \Delta \delta \\ \Delta \omega_m \end{bmatrix} + \left[G' \right] \begin{bmatrix} \Delta V_{d1} \\ \Delta V_{q1} \\ \Delta V_{d2} \\ \Delta V_{q2} \\ \Delta T_{LO} \end{bmatrix} \qquad \text{(III.17)}$$

with

$$\left[F'\right] = \begin{bmatrix} r_1 \frac{L_2}{Z} & \omega_1 - \frac{M^2}{Z}\omega_{mo} & -r_2 \frac{M}{Z} & -\frac{ML_2}{Z}\omega_{mo} & \sqrt{3}\frac{M}{Z}V_2 \sin\delta_0 & -\frac{M^2}{Z} i^{\circ}_{q1} - \frac{ML_2}{Z} i^{\circ}_{q2} \\ -\omega_1 + \frac{M^2}{Z}\omega_{mo} & r_1 \frac{L_2}{Z} & \frac{ML_2}{Z}\omega_{mo} & -r_2 \frac{M}{Z} & -\sqrt{3}\frac{M}{Z} V_2 \cos\delta_0 & \frac{M^2}{Z} i^{\circ}_{d1} + \frac{ML_2}{Z} i^{\circ}_{d2} \\ -r_1 \frac{M}{Z} & \frac{ML_1}{Z}\omega_{mo} & \frac{r_2 L_1}{Z} & \omega_1 + \frac{L_1 L_2}{Z}\omega_{mo} & -\sqrt{3}\frac{L_1}{Z} V_2 \sin\delta_0 & \frac{ML_1}{Z} i^{\circ}_{q1} + \frac{L_1 L_2}{Z} i^{\circ}_{q2} \\ -\frac{ML_1}{Z}\omega_{mo} & -r_1 \frac{M}{Z} & -\omega_1 - \frac{L_1 L_2}{Z}\omega_{mo} & r_1 \frac{L_1}{Z} & \sqrt{3}\frac{L_1}{Z} V_2 \cos\delta_0 & -\frac{ML_1}{Z} i^{\circ}_{d1} + \frac{L_1 L_2}{Z} i^{\circ}_{d2} \\ 0 & 0 & 0 & 0 & 0 & -1 \\ -\frac{P^2}{3} M i^{\circ}_{q2} & \frac{P^2}{3} M i^{\circ}_{d2} & \frac{P^2}{3} M i^{\circ}_{q1} & \frac{P^2}{3} M i^{\circ}_{d1} & 0 & -K_L/J \end{bmatrix},$$

$$\left[G'\right] = \begin{bmatrix} -\frac{L_2}{Z} & 0 & \frac{M}{Z} & 0 & 0 \\ 0 & -\frac{L_2}{Z} & 0 & \frac{M}{Z} & 0 \\ \frac{M}{Z} & 0 & -\frac{L_1}{Z} & 0 & 0 \\ 0 & \frac{M}{Z} & 0 & -\frac{L_1}{Z} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{P}{J} \end{bmatrix},$$

$$Z = M^2 - L_1 L_2 .$$

In this linearized model, Δv_{d2} and Δv_{q2} denote the deviations of the Park domain rotor voltages that are caused by changes in the parameters of the applied voltages, i.e. changes in amplitude and frequency only, but <u>not</u> those increments that are due a relative position change of the rotor with respect to the stator. These latter voltage increments have been worked into the system matrix F', since they depend on the state variable δ and are thus linearly dependent on the increment $\Delta\delta$.

III.5 Stability

To determine the stability properties, one now proceeds to numerical evaluation of the eigenvalues of this linearized model. Ref. 6 uses machine parameters of a 1.5 hp wound rotor induction machine which are listed in Appendix 2. It is found that at slips between -1 and 1, the dynamic behavior of the machine is always dominated by a pair of poles close to zero. Unfortunately, the authors do not say under which load conditions their results were obtained.

As is shown in Fig. III.2, taken from Ref. 6, the real parts of the dominant system poles are negative for slips close to zero. Over the remainder of the speed operating range, however, the dominant poles are found to lie in the right half of the complex frequency plane. Analysis of the linearized model thus leads to the result obtained earlier by examining the torque-speed characteristic of the doubly-fed machine with voltage-source rotor excitation: the machine without feedback is unstable at speeds substantially different from synchronous speed.

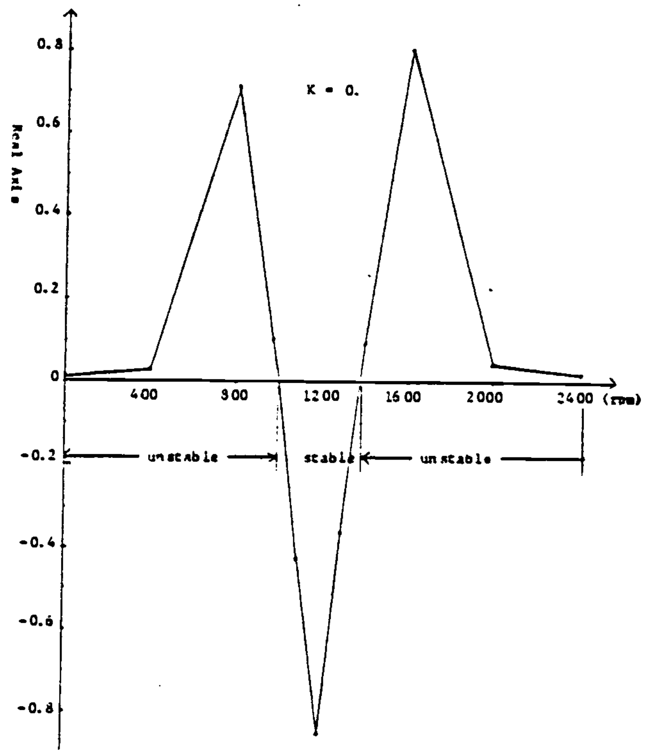

Fig. III.2 Real parts of dominant poles over the speed range (Ref. 6)

III.6 Stabilization by Speed Feedback

In order to stabilize the doubly-fed machine throughout its operating range, suitable feedback control must be applied. In Ref. 6, a "speed feedback" is used, meaning that the rotor exciter-converter is controlled to change its output frequency proportional to the deviation of the rotor speed from steady state. The feedback equation is given by

$$\Delta \omega_2 = - K \Delta \omega_m . \qquad \text{(III.18)}$$

Ref. 6 claims that this feedback will lead to a modification of just five elements in the last column of the state transition matrix F'. However, no explanation of the way towards obtaining this result is given. Several attempts to retrace the proceedings have led to different results, depending upon different ways of defining the state variable δ_0 in the model with feedback. The authors give no indication on whether in the modified model the phase difference between stator and rotor fields due to the variation in rotor frequency is included in the state variable δ_0 or not. In both cases, however, the result I obtained is different from theirs; therefore, an error in Ref.6 is assumed.

One then proceeds to find the root loci of the modified system at different slips. The feedback constant k is chosen so that the damping ratio of the resulting dominant poles, defined as

$$\zeta = \frac{\mathrm{Re}\, p}{|p|} , \qquad \text{(III.19)}$$

is between 0.4 and 0.8 , which will lead to a good damping of oscillations in the system response. Ref. 6 finds that the feedback gain has to be adjusted approximately proportional to the operating slip.

As a verification of this result, the slip-dependent speed feedback is implemented on the nonlinear machine model. This simulation gives the desired result: for slip -1, corresponding to twice synchronous speed, the system model shows a stable response to a step load increase.

III.7 Comments on Ohi and Kassakian

The paper presented by Ohi and Kassakian shows a method of stabilization for the voltage-source excited doubly-fed machine. In their mathematical presentation, the authors sometimes are not clear enough for the reader to retrace the path of their calculations. They reach, however, the desired result: stable operation of the doubly-fed machine can be secured by a speed feedback through control of the frequency of the converter-supplied rotor voltage. The drawback of this control strategy is the fact that the feedback gain must be adjusted to varying conditions of load and slip at which the machine is operating.

PART IV : ROTOR EXCITATION BY CURRENT SOURCE

IV.1 Voltage equations

Modern power electronic converters, such as the type developed by Schwarz (Ref. 5), allow excitation of the rotor with a controlled current rather than a voltage. This method proves to be beneficial for stability, as the dependence of rotor current on the mechanical speed of the rotor is eliminated. The Park domain rotor currents i_{d2}, i_{q2} are now controlled input variables instead of state variables. Consequently, only the voltage equations for the stator windings of the doubly-fed machine need to be considered:

$$\begin{bmatrix} V_{d1} \\ V_{q1} \end{bmatrix} = \begin{bmatrix} r_1 + L_1 p & -\omega_1 L_1 & Mp & -\omega_1 M \\ \omega_1 L_1 & r_1 + L_1 p & \omega_1 M & Mp \end{bmatrix} \begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{d2} \\ i_{q2} \end{bmatrix} . \qquad \text{(IV.1)}$$

Linearizing yields:

$$\begin{bmatrix} \Delta V_{d1} \\ \Delta V_{q1} \end{bmatrix} = \begin{bmatrix} \cdot & & \\ & \diagup & \\ & & \cdot \end{bmatrix} \begin{bmatrix} \Delta i_{d1} \\ \Delta i_{q1} \\ \Delta i_{d2} \\ \Delta i_{q2} \end{bmatrix} . \qquad \text{(IV.2)}$$

It is to be noted that these equations do not contain slip-dependent elements, as opposed to the case where the doubly-fed machine is excited by voltage sources.

A system using a DC-link converter with controlled DC current has been investigated in Ref. 7. The high smoothing inductance, however, leads to a slow response of this converter to any change in load conditions.
The recent Schwarz converter promises to provide fast change of the source currents as demanded by the control system.
In this analysis, exciting currents will be assumed to be controlled in amplitude and frequency. Changes in any of these parameters are assumed to be instantaneous, as

the speed of response of the Schwarz converter is high in comparison to events linked to the time constants of the machine.

Rotor winding currents i_{a2}, i_{b2} and i_{c2} are thus fixed in their amplitude $\sqrt{2}\, I_2$ and frequency ω_2. Deviations $\Delta \dot{i}_{d2}$, $\Delta \dot{i}_{q2}$ of the Park domain rotor currents i_{d2} and i_{q2} are caused by a deviation of the rotor angle from a certain setpoint value δ_0.

Converter output currents are given as

$$\begin{bmatrix} \dot{i}_{a2} \\ \dot{i}_{b2} \\ \dot{i}_{c2} \end{bmatrix} = \sqrt{2}\, I_2 \begin{bmatrix} \sin(\omega_2 t - \varphi_2) \\ \sin(\omega_2 t - \varphi_2 - \frac{2\pi}{3}) \\ \sin(\omega_2 t - \varphi_2 + \frac{2\pi}{3}) \end{bmatrix} . \qquad \text{(IV.3)}$$

Applying Park's transformation gives the rotor currents for a reference frame rotating with frequency ω_1:
(T_2 is taken from equ. II.4)

$$\begin{bmatrix} \dot{i}_{d2} \\ \dot{i}_{q2} \end{bmatrix} = \begin{bmatrix} & T_2 & \end{bmatrix} \begin{bmatrix} \dot{i}_{a2} \\ \dot{i}_{b2} \\ \dot{i}_{c2} \end{bmatrix} , \qquad \text{(IV.4)}$$

which is equivalent (Ref.3) to:

$$\begin{bmatrix} \dot{i}_{d2} \\ \dot{i}_{q2} \end{bmatrix} = \sqrt{3}\, I_2 \begin{bmatrix} \sin(\omega_2 t - \varphi_2 - \omega_1 t + \Theta_m) \\ -\cos(\omega_2 t - \varphi_2 - \omega_1 t + \Theta_m) \end{bmatrix} . \qquad \text{(IV.5)}$$

Considering steady-state conditions, $\frac{d\Theta_m}{dt} = \omega_m$ =constant which results in $\Theta_m = \omega_m t$ when $\Theta_m(0) = 0$ is assumed. Consequently,

$$\begin{bmatrix} i_{d2} \\ i_{q2} \end{bmatrix} = \sqrt{3}\, I_2 \begin{bmatrix} \sin(\omega_2 t - \varphi_2 - \omega_1 t + \omega_m t) \\ -\cos(\omega_2 t - \varphi_2 - \omega_1 t + \omega_m t) \end{bmatrix}. \quad \text{(IV.5a)}$$

The load angle δ is defined as the lag angle of rotor emf phasor with respect to stator voltage : $\delta = \omega_1 t - \omega_2 t - \Theta_m + \varphi_2$.
If phase sequence of stator and rotor windings are identical, then $\omega_1 = \omega_m + \omega_2$. For opposite phase sequences $\omega_1 = \omega_m - \omega_2$ applies. From this follows that $\delta = \varphi_2$.
Thus, for both directions of rotor field rotation the following result holds:

$$\begin{bmatrix} i_{d2} \\ i_{q2} \end{bmatrix} = -\sqrt{3}\, I_2 \begin{bmatrix} \sin\delta \\ \cos\delta \end{bmatrix}. \quad \text{(IV.6)}$$

If the rotor deviates from stable-state torque angle δ_0 by a small increment $\Delta\delta$, such that $\delta = \delta_0 + \Delta\delta$, then Taylor series expansion yields

$$\begin{bmatrix} i_{d2} \\ i_{q2} \end{bmatrix} \approx -\sqrt{3}\, I_2 \begin{bmatrix} \sin\delta_0 \\ \cos\delta_0 \end{bmatrix} - \sqrt{3}\, I_2 \begin{bmatrix} \cos\delta_0 \\ -\sin\delta_0 \end{bmatrix} \Delta\delta. \quad \text{(IV.7)}$$

Therefore,

$$\begin{bmatrix} \Delta i_{d2} \\ \Delta i_{q2} \end{bmatrix} = \sqrt{3}\, I_2 \begin{bmatrix} -\cos\delta_0 \\ \sin\delta_0 \end{bmatrix} \Delta\delta. \quad \text{(IV.8)}$$

This equation for the incremental quantities $\Delta i_{d2}, \Delta i_{q2}$ is substituted into the voltage equation (IV.2) to give

$$\begin{bmatrix} \Delta V_{d1} \\ \Delta V_{q1} \end{bmatrix} = \begin{bmatrix} r_1 + L_1 p & -\omega_1 L_1 \\ \omega_1 L_1 & r_1 + L_1 p \end{bmatrix} \begin{bmatrix} \Delta i_{d1} \\ \Delta i_{q1} \end{bmatrix} + \sqrt{3}\, I_2 \begin{bmatrix} Mp & -\omega_1 M \\ \omega_1 M & Mp \end{bmatrix} \begin{bmatrix} -\cos\delta_0 \\ \sin\delta_0 \end{bmatrix} \Delta\delta. \quad \text{(IV.9)}$$

Considering that with the given orientation of δ,

$$\frac{d\delta}{dt} = \frac{d}{dt}\Delta\delta = -\Delta\omega_m \tag{IV.10}$$

where $\Delta\omega_m$ is the deviation from steady-state speed:

$$\Delta\omega_m = \omega_m - \omega_{m0} = \omega_m - (\omega_1 \mp \omega_2), \tag{IV.11}$$

(IV.9) can be written as

$$\begin{bmatrix}\Delta v_{d1}\\ \Delta v_{q1}\end{bmatrix} = \begin{bmatrix} \cdot & / \\ / & \cdot \end{bmatrix}\begin{bmatrix}\Delta i_{d1}\\ \Delta i_{q1}\end{bmatrix} + \sqrt{3}I_2 M\begin{bmatrix}-\omega_1\sin\delta_0 & \cos\delta_0\\ -\omega_1\cos\delta_0 & -\sin\delta_0\end{bmatrix}\begin{bmatrix}\Delta\delta\\ \Delta\omega_m\end{bmatrix}. \tag{IV.12}$$

For the machine connected directly to an infinite bus with constant three-phase voltages, $\Delta v_{d1} = \Delta v_{q1} = 0$. Equ. (IV.12) can now be turned into state equation form:

$$\frac{d}{dt}\begin{bmatrix}\Delta i_{d1}\\ \Delta i_{q1}\end{bmatrix} = \begin{bmatrix}-\frac{r_1}{L_1} & \omega_1 & \sqrt{3}\frac{M}{L_1}I_2\omega_1\sin\delta_0 & -\sqrt{3}\frac{M}{L_1}I_2\cos\delta_0\\ -\omega_1 & -\frac{r_1}{L_1} & \sqrt{3}\frac{M}{L_1}I_2\omega_1\cos\delta_0 & \sqrt{3}\frac{M}{L_1}I_2\sin\delta_0\end{bmatrix}\begin{bmatrix}\Delta i_{d1}\\ \Delta i_{q1}\\ \Delta\delta\\ \Delta\omega_m\end{bmatrix}. \tag{IV.13}$$

IV.2 Torque Characteristic

For the complete state model, equations for the mechanical load angle, speed and torque must be established.
As in Part III, the equation of motion is

$$\frac{d}{dt}\Delta\omega_m \frac{J}{p} = T_e - T_m \qquad \text{(IV.14)}$$

where the mechanical load torque is assumed as

$$T_m = T_L + K_L \frac{\omega_m}{p} \qquad \text{(IV.15)}$$

and the electromagnetic torque as derived in Part II can be expressed as

$$T_e = pM\,(i_{q1} i_{d2} - i_{d1} i_{q2}). \qquad \text{(IV.16)}$$

From this equation, the steady-state torque as a function of input stator voltage and rotor current amplitude can be derived.
Defining complex phasors as explained in chapter II.6, stator current in steady state is written as

$$\underline{I}_1 = \frac{\underline{V}_1(r_1 - j\omega_1 L_1) - \omega_1 M \underline{I}_2 (j r_1 + \omega_1 L_1)}{\omega_1^2 L_1^2 + r_1^2}, \qquad \text{(IV.17a)}$$

which by substituting (IV.6) for the rotor current results in

$$\underline{I}_1 = \frac{\underline{V}_1(r_1 - j\omega_1 L_1) + \sqrt{3}\,\omega_1 M I_2 (\sin\delta_0 + j\cos\delta_0)(j r_1 + \omega_1 L_1)}{\omega_1^2 L_1^2 + r_1^2}. \qquad \text{(IV.17b)}$$

Separating $i_{d1} = \operatorname{Re} \underline{I}_1$ and $i_{q1} = \operatorname{Im} \underline{I}_1$ we have the Park domain currents

$$\dot{i}_{d1} = \frac{\sqrt{3}\,V_1 r_1 + \sqrt{3}\,\omega_1 M I_2 (\omega_1 L_1 \sin\delta_0 - r_1 \cos\delta_0)}{\omega_1^2 L_1^2 + r_1^2} , \tag{IV.18a}$$

$$\dot{i}_{q1} = \frac{-\sqrt{3}\,V_1 \omega_1 L_1 + \sqrt{3}\,\omega_1 M I_2 (r_1 \sin\delta_0 + \omega_1 L_1 \cos\delta_0)}{\omega_1^2 L_1^2 + r_1^2} . \tag{IV.18b}$$

Substitution into the torque equation (IV.16) yields

$$T_e = \frac{3PM}{\omega_1^2 L_1^2 + r_1^2} \left\{ V_1 I_2 (\omega_1 L_1 \sin\delta_0 - r_1 \cos\delta_0) - \omega_1 M r_1 I_2^2 \right\} . \tag{IV.19}$$

The torque for current source excitation can be considered to consist of two terms:

- the first, proportional to V_1 and I_2 , is called the synchronous torque and depends on the load angle.
- the second, proportional to I_2^2 , is called the induction torque and is relatively small.

Note that the produced torque is independent of the machine slip.

For the case where stator loss is negligible (r_1=0)

$$T_e \approx \frac{3PM}{\omega_1 L_1} V_1 I_2 \sin\delta_0 . \tag{IV.20}$$

This synchronous torque is, as in a synchronous machine, proportional to the sine of the load angle, and independent of speed.

In Fig. IV.1, the torque-speed characteristic for the current excited machine is shown, with load angle δ as a parameter.

Fig. IV.2 shows the dependence of the produced torque on the load angle. From this diagram it can be concluded that like a synchronous machine, the doubly-fed machine can be operated at a load angle between -90° and $+90^\circ$. Beyond these limits, torque decreases and the machine pulls out of synchronism. To secure stable operation under changing load conditions, one will usually observe a safety margin of about 30° from the limit of $\pm 90^\circ$.

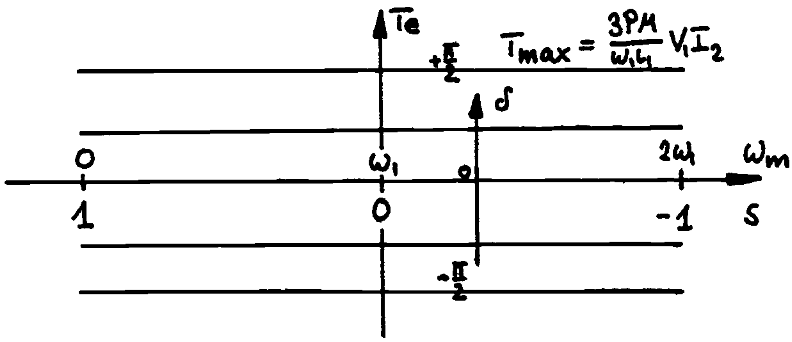

Fig. IV.1 Torque-speed characteristic

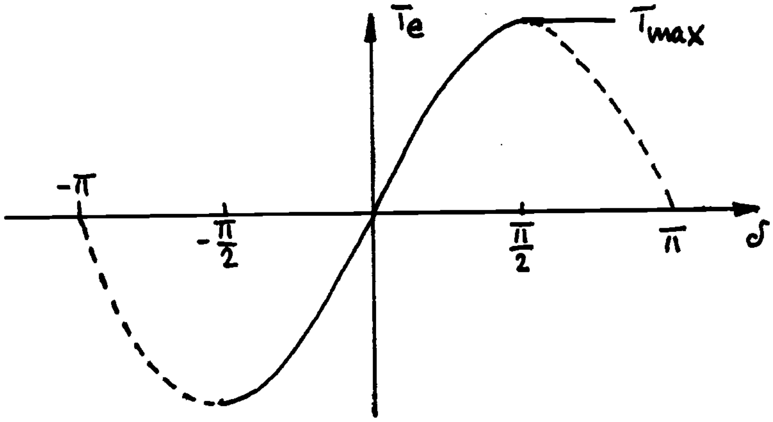

Fig. IV.2 Torque-load angle characteristic

IV.3 Linear model

For the linear state model, the electromechanical torque is linearized and incremental quantities are considered only.

$T_e^o = T_m^o$ (IV.21) holds in steady state. Then,

$$\Delta T_e = PM(\Delta i_{q1} i_{d2}^o + \Delta i_{d2} i_{q1}^o - \Delta i_{d1} i_{q2}^o - \Delta i_{q2} i_{d1}^o). \qquad \text{(IV.22)}$$

With

$$i_{d2}^o = -\sqrt{3} I_2 \sin\delta_o, \quad \Delta i_{d2} = -\sqrt{3} I_2 \cos\delta_o \Delta\delta,$$
$$i_{q2}^o = -\sqrt{3} I_2 \cos\delta_o, \quad \Delta i_{q2} = \sqrt{3} I_2 \sin\delta_o \Delta\delta,$$

$$\Delta T_e = \sqrt{3} PMI_2 (-\sin\delta_o \Delta i_{q1} + \cos\delta_o \Delta i_{d1} - i_{q1}^o \cos\delta_o \Delta\delta - i_{d1}^o \sin\delta_o \Delta\delta) \qquad \text{(IV.23)}$$

is obtained.

Let $A = -(i_{q1}^o \cos\delta_o + i_{d1}^o \sin\delta_o)$. With (IV.18),

$$A = \frac{1}{\omega_1^2 L^2 + r_1^2}\left[\sqrt{3} V_1(\omega_1 L_1 \cos\delta_o - r_1 \sin\delta_o) - \sqrt{3}\omega_1^2 M L_1 I_2\right] \qquad \text{(IV.24)}$$

and $\Delta T_e = \sqrt{3} PMI_2 \left[\Delta i_{d1} \cos\delta_o - \Delta i_{q1} \sin\delta_o + A\Delta\delta\right]$. (IV.25)

Writing (IV.14) as a state equation and substituting (IV.25) results in:

$$\frac{d}{dt}\Delta\omega_m = \frac{\sqrt{3} P^2 M I_2}{J}\left[\Delta i_{d1}\cos\delta_o - \Delta i_{q1}\sin\delta_o + A\Delta\delta\right] - \frac{P}{J}\Delta T_L - \frac{K_L}{J}\Delta\omega_m \qquad \text{(IV.26)}$$

where ΔT_L is an increment of the speed-independent load torque.

At this stage, the complete set of equations for the state model is established as

$$\frac{d}{dt}\begin{bmatrix}\Delta i_{d1}\\ \Delta i_{q1}\\ \Delta \delta\\ \Delta \omega_m\end{bmatrix}=\begin{bmatrix}-\frac{r}{L} & \omega & \sqrt{3}\frac{M}{L}I_2\omega\sin\delta_0 & -\sqrt{3}\frac{M}{L}I_{20}\cos\delta_0\\ -\omega & -\frac{r}{L} & \sqrt{3}\frac{M}{L}I_2\omega\cos\delta_0 & \sqrt{3}\frac{M}{L}I_{20}\sin\delta_0\\ 0 & 0 & 0 & -1\\ CI_2\cos\delta_0 & -CI_2\sin\delta_0 & CAI_{20} & -k_L/J\end{bmatrix}\begin{bmatrix}\Delta i_{d1}\\ \Delta i_{q1}\\ \Delta \delta\\ \Delta \omega_m\end{bmatrix}$$

$$+\begin{bmatrix}0 & 0 & 0 & -\frac{P}{J}\Delta T_L\end{bmatrix}^T$$

(IV.27)

where $$C=\frac{\sqrt{3}\,P^2 M}{J}\,. \qquad \text{(IV.28)}$$

Note that from now on, r, L and ω will stand for stator resistance, inductance and frequency.

IV.4 Characteristic Equation

The dynamic properties of the system are governed by the system matrix. To determine wether the system is stable or not, its characteristic equation is found from

$$\det (pI - F) = 0 .$$

Development of this determinant gives a polynomial of fourth order in p:

$$
\begin{aligned}
& p^4 \cdot J L^2 \\
+ \; & p^3 \cdot (K_L L^2 + 2 J r L) \\
+ \; & p^2 \cdot (J(\omega^2 L^2 + r^2) + 2 K_L r L + I_2 P^2 M(\sqrt{3} A L^2 + 3 I_2 M L) \\
+ \; & p \cdot (P^2 M r I_2 (2\sqrt{3} A L + 3 I_2 M) + K_L(\omega^2 L^2 + r^2) \\
+ \; & 1 \cdot (P^2 M I_2 (\sqrt{3} A(\omega^2 L^2 + r^2) + 3\omega^2 I_2 M L).
\end{aligned}
\qquad \text{(IV.29)}
$$

IV.5 The Routh-Hurwitz Criterion

The roots of the polynomial (IV.29) can be found numerically when the machine values and the setpoint of a certain system are given and can be substituted into the equation.

It is, however, much more convenient to apply the Routh-Hurwitz Criterion which provides the necessary and sufficient conditions for stability in terms of the parameters and setpoint quantities (Ref. 2).
The Hurwitz Criterion states that for a fourth order linear system to be stable, it is necessary that

- all five coefficients in the characteristic equation have the same sign and
- none of these coefficients vanishes.

In addition, the stronger, sufficient conditions for stability are that the Hurwitz determinants D_1 - D_4 be all positive.
The evaluation of the sufficient Hurwitz conditions can be carried out more conveniently by the method known as Routh Tabulation (Ref. 2).
For the fourth order polynomial,

$$a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4 = 0,$$

the sufficient conditions for stability are found to be

$$a_0 > 0 \qquad a_1 > 0 \qquad a_4 > 0$$

$$a_2 > \frac{a_0 a_3}{a_1} \qquad a_3 > \frac{a_4 a_1^2}{a_1 a_2 - a_0 a_3} .$$

(IV.30)

IV.6 Necessary Stability Conditions for the Uncontrolled Machine

The Hurwitz criterion states that in a stable system all the coefficients of the characteristic equation must be positive.

$a_0 = JL^2 > 0$ is satisfied, so is

$a_1 = k_L L^2 + 2JrL > 0.$

The third condition calls for

$$a_2 = J\,(\omega^2 L^2 + r^2) + 2k_L rL + P^2 MI_2\,(\sqrt{3}AL^2 + 3\,I_2 ML) > 0. \tag{IV.31}$$

A is substituted from (IV.24) to obtain the condition

$$a_2 = J(\omega^2 L^2 + r^2) + 2k_L rL + P^2 MI_2\left[\frac{3V\omega L^3(-\frac{r}{\omega L}\sin\delta_0 + \cos\delta_0) - 3\omega^2 L^3 MI_2}{r^2 + \omega^2 L^2} + 3I_2 ML\right] > 0. \tag{IV.32}$$

For all but very small machines it is reasonable to assume that $r \ll \omega L$. With this assumption the above condition can be simplified:

$$a_2 \approx J\omega^2 L^2 + 2k_L rL + 3VI_2 P^2 M \frac{1}{\omega} L\cos\delta_0 . \tag{IV.33}$$

This sum is positive for any δ_0 within the operating range $[-\frac{\pi}{2}, \frac{\pi}{2}]$ and therefore satisfies the Hurwitz condition.

The next coefficient to check is a_3:

$$a_3 = P^2 M r I_2 (2\sqrt{3} AL + 3 I_2 M) + K_L(\omega^2 L^2 + r^2)$$
$$= P^2 M r I_2 \left(2L \frac{3V(\omega L \cos\delta_0 - r\sin\delta_0) - 3\omega^2 M L I_2}{r^2 + \omega^2 L^2} + 3 I_2 M\right) + K_L(\omega^2 L^2 + r^2), \tag{IV.34}$$

which, under the same assumptions on negligible stator resistance, can be simplified to yield:

$$a_3 \approx 3 P^2 M r I_2 \frac{1}{\omega}(2V\cos\delta_0 - \omega M I_2) + \omega^2 L^2 K_L. \tag{IV.35}$$

For a_3 to be positive, the damping coefficient k_L must satisfy a certain minimum requirement:

$$K_L > \frac{3P^2 M r I_2}{\omega^3 L^2}(\omega M I_2 - 2V\cos\delta_0). \tag{IV.36}$$

This necessary condition for stability will be examined in detail later on, after checking the remaining coefficient

$$a_4 = P^2 M I_2 (\sqrt{3} A(\omega^2 L^2 + r^2) + 3\omega^2 I_2 M L)$$
$$= P^2 M I_2 \cdot 3V\omega L \left(\cos\delta_0 - \frac{r}{\omega L}\sin\delta_0\right). \tag{IV.37}$$

With rsmall, again, a_4 is positive throughout the operating range.

So, the only one of the necessary stability conditions that is not invariably met under any operating conditions is the one on a_3 or k_L, respectively:

$$K_L > \frac{3P^2 M r I_2}{\omega^3 L^2}(\omega M I_2 - 2V\cos\delta_0).$$

The right side of this inequality will, for $\delta_0 \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ never exceed an upper bound that is a function of I_2^2: so, if

$$K_L > \frac{3P^2 M^2 r I_2^2}{\omega^2 L^2}, \tag{IV.38}$$

the coefficient a_3 will meet the stability requirement for any load angle.

The criterion (IV.38) can serve as a primary check on stability. If k_L does not meet this requirement, the machine will become dynamically unstable at a certain load angle.

The decisive term in (IV.36) is $\omega M I_2 - 2V\cos\delta_0$.
For the case of very small damping, $k_L=0$, this inequality becomes

$$2V\cos\delta_0 > \omega M I_2 \tag{IV.39}$$

and will now be interpreted in terms of operating conditions by looking at some phasor diagrams.
For convenience, the factor $\sqrt{3}$ is left out of considerations in these phasor diagrams.
Fig. IV.3a shows the phasors for underexcited motor operation of the doubly-fed machine. The quantities of inequ. (IV.39) correspond to the length of the induction emf phasor $\underline{E}_2 = j\omega M I_2$ and to the projection of the voltage phasor $\underline{V}$ onto the direction of $\underline{E}_2$. Clearly, under the conditions shown in Fig. a, the condition $2V\cos\delta_0 > \omega M I_2$ is met.

As, under fixed load angle, excitation current amplitude is increased, a critical operating point will be reached when $\omega M I_2 = 2V\cos\delta_0$. Fig. IV.3b shows this operating point for overexcited motoring.
The same result holds for generating. As excitation is increased from underexcited (Fig. IV.3c) to overexcited (Fig. IV.3d), the critical operating point will be reached.

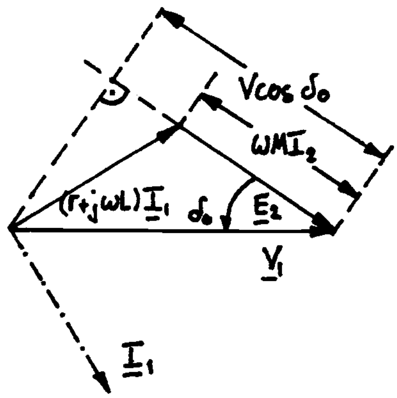

Fig. IV.3a Underexcited motoring

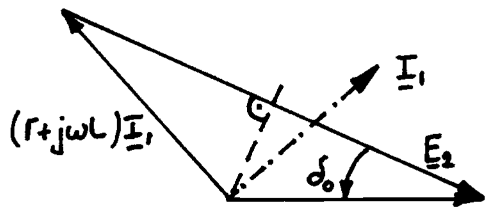

Fig. IV.3b Overexcited motoring

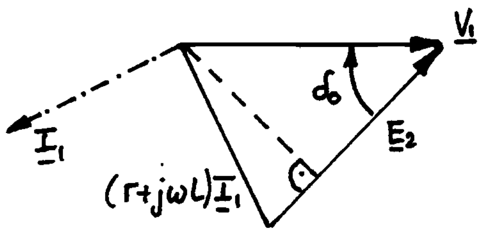

Fig. IV.3c Underexcited generating

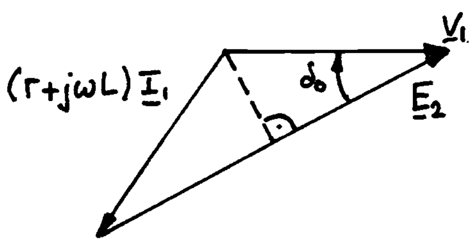

Fig. IV.3d Overexcited generating

Fig. IV.3 Stability in terms of phasors

It is now interesting to find the critical value I_{2max} for which, under a given load P_m for a motor or with a given power input P_m for a generator, the machine becomes unstable. Using the approximated formula for the torque,

$$T_e = \frac{3PM}{\omega L} V I_2 \sin\delta_0 ,$$

we have the electromechanical power

$$P_m = \omega_M T_e = \omega_M \frac{3PM}{\omega L} V I_2 \sin\delta_0 , \qquad \text{(IV.40)}$$

from which we conclude that

$$\sin\delta_0 = \frac{P_m \cdot \omega L}{3\omega_M PMVI_2} .$$

P_m is defined as the electro-mechanical power output of the doubly-fed motor and would be negative for generator operation.

The critical condition (IV.39) is

$$\omega M I_2 = 2V\cos\delta = 2V\sqrt{1-\sin^2\delta} \quad \text{for } \delta \in [-\tfrac{\pi}{2}, \tfrac{\pi}{2}] .$$

Squaring and solving for the positive solution for I_2 yields

$$I_2 = \frac{1}{\omega M}\left[2V^2 + \left(4V^4 - \left(\frac{2P_m\omega^2 L}{3\omega_M P}\right)^2\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} . \qquad \text{(IV.41)}$$

The inner square root will always be real, as it was derived from $(1 - \sin^2\delta)$.

One might also be interested in the critical value of the load angle, at which - for a fixed excitation current - stability is lost. From (IV.36),

$$\cos \delta_0 > \frac{\omega M I_2}{2V} - \frac{K_L \omega^3 L^2}{6P^2 M r I_2 V} = \cos \delta_{0max} . \tag{IV.41a}$$

So, for fixed excitation current, the machine becomes unstable when the load, and with it the load angle is increased beyond a certain limit δ_{0max} which can be positive or negative.

For large excitation current I_2, it can be concluded that if

$$\frac{\omega M I_2}{2V} - \frac{K_L \omega^3 L^2}{6P^2 M r I_2 V} > 1 , \tag{IV.41b}$$

the machine is dynamically unstable for any load angle and even for no-load conditions.

IV.7 Stabilizing by Feedback

Knowing that the doubly-fed machine will not work stably under some possible operating conditions, a means of control has to be found to stabilize it.

There are two parameters in the rotor excitation currents that can be used as control inputs: current amplitude and frequency. In the following, it will be examined how the machine can be stabilized by varying either of these parameters proportionally to the deviation of rotor speed from the steady-state speed.

IV.8 Effect of Rotor Frequency Control

If rotor frequency is increased by a small amount $\Delta\omega_2$, such that $\omega_2 = \omega_{20} + \Delta\omega_2$, then the load angle δ will change with $\Delta\omega_2$ as the time rate of change. At time t, the load angle will have deviated by

$$\Delta\varphi_2 = \int_0^t \Delta\omega_2(\tau)\, d\tau \qquad \text{(IV.42)}$$

so that $\delta(t) = \delta_0 + \Delta\varphi_2(t)$ (IV.43)

Applying Park's transformation to the rotor currents, the d,q-currents now are

$$\begin{bmatrix} i_{d2} \\ i_{q2} \end{bmatrix} = -\sqrt{3}\, I_2 \begin{bmatrix} \sin(\delta_0 + \Delta\delta - \Delta\varphi_2) \\ \cos(\delta_0 + \Delta\delta - \Delta\varphi_2) \end{bmatrix} \qquad \text{(IV.44)}$$

where $\Delta\delta$ is the change in load angle due to a deviation of rotor speed from synchronism, and $\Delta\varphi_2$ is the described deviation of the load angle due to a control variation of the rotor frequency.

Through Taylor expansion, the current increments are found to be

$$\begin{bmatrix} \Delta i_{d2} \\ \Delta i_{q2} \end{bmatrix} = \sqrt{3}\, I_2 \begin{bmatrix} -\cos\delta_0 \\ \sin\delta_0 \end{bmatrix} \Delta\delta' \qquad \text{(IV.45)}$$

$\Delta\delta'$ is defined as the combined load angle increment:

$$\Delta\delta' = \Delta\delta - \Delta\varphi_2 \qquad \text{(IV.46a)}$$

Its time derivative is

$$\frac{d}{dt}\Delta\delta' = -\Delta\omega_{m} - \Delta\omega_2 \qquad \text{(IV.46b)}$$

In the voltage equations of the state model (i.e., the top two equations) $\Delta\delta'$ and $\Delta\omega_m + \Delta\omega_2$ now replace $\Delta\delta$ and $\Delta\omega_m$. The third line is replaced by (IV.46b), which does not alter the third matrix row.

The linearized torque equation was given as

$$\frac{d}{dt}\Delta\omega_m = P^2\frac{M}{J}\left[\Delta i_{q1}\,\overset{\circ}{i}_{d2} + \Delta i_{d2}\,\overset{\circ}{i}_{q1} - \Delta i_{d1}\,\overset{\circ}{i}_{q2} - \Delta i_{q2}\,\overset{\circ}{i}_{d1}\right] - \frac{P}{J}\Delta T_L - \frac{K_L}{J}\Delta\omega_m$$

and $\Delta i_{d2}, \Delta i_{q2}$ are now given by (IV.45).

The only term in this expression in which the old variables cannot be replaced by the new ones is the damping term, $K_L/J \cdot \Delta\omega_m$. This is obvious, as a change in rotor frequency does not add mechanical friction. It will, however, affect all the electrical quantities and the torque angle.

The state model for a machine with controlled rotor frequency is thus given by

$$\frac{d}{dt}\begin{bmatrix}\Delta i_{d1}\\ \Delta i_{q1}\\ \Delta\delta'\\ \Delta\omega_m\end{bmatrix} = \begin{bmatrix} F' \end{bmatrix}\begin{bmatrix}\Delta i_{d1}\\ \Delta i_{q1}\\ \Delta\delta'\\ \Delta\omega_m + \Delta\omega_2\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\\ -\frac{P}{J}T_L\end{bmatrix} \tag{IV.47}$$

with

$$\begin{bmatrix} F' \end{bmatrix} = \begin{bmatrix} -\frac{r}{L} & \omega & \sqrt{3}\frac{M}{L}I_2\omega\sin\delta_0 & -\sqrt{3}\frac{M}{L}I_2\cos\delta_0 \\ -\omega & -\frac{r}{L} & \sqrt{3}\frac{M}{L}I_2\omega\cos\delta_0 & \sqrt{3}\frac{M}{L}I_2\sin\delta_0 \\ 0 & 0 & 0 & -1 \\ CI_2\cos\delta_0 & -CI_2\sin\delta_0 & CAI_2 & -\frac{K_L}{J}\frac{\Delta\omega_m}{\Delta\omega_m+\Delta\omega_2} \end{bmatrix}.$$

IV.9 Effect of Rotor Current Amplitude Control

Under this control strategy, the rms value of the rotor currents is increased: $I_2 = I_{20} + \Delta I_2$.

This results in the following Park domain currents:

$$\begin{bmatrix} i_{d2} \\ i_{q2} \end{bmatrix} = -\sqrt{3}\,(I_{20} + \Delta I_2)\begin{bmatrix} \sin\delta_0 \\ \cos\delta_0 \end{bmatrix},$$

so that the increments in these quantities due to the increment in current amplitude will be

$$\begin{bmatrix} \Delta i_{d2}^{a} \\ \Delta i_{q2}^{a} \end{bmatrix} = -\sqrt{3}\,\Delta I_2 \begin{bmatrix} \sin\delta_0 \\ \cos\delta_0 \end{bmatrix}. \qquad \text{(IV.48)}$$

Adding this term to equ. (IV.12) results in

$$\begin{bmatrix} \Delta V_{d1} \\ \Delta V_{q1} \end{bmatrix} = \begin{bmatrix} r+Lp & -\omega L \\ \omega L & r+Lp \end{bmatrix}\begin{bmatrix} \Delta i_{d1} \\ \Delta i_{q1} \end{bmatrix}$$
$$+\sqrt{3}\,I_{20}M\begin{bmatrix} -\omega_1\sin\delta_0 & \cos\delta_0 \\ -\omega\cos\delta_0 & -\sin\delta_0 \end{bmatrix}\begin{bmatrix} \Delta\delta \\ \Delta\omega_m \end{bmatrix}$$
$$+\begin{bmatrix} \sqrt{3}\,\omega M\cos\delta_0 & -\sqrt{3}\,M\sin\delta_0 \\ -\sqrt{3}\,\omega M\sin\delta_0 & -\sqrt{3}\,M\cos\delta_0 \end{bmatrix}\begin{bmatrix} \Delta I_2 \\ p\,\Delta I_2 \end{bmatrix}. \qquad \text{(IV.49)}$$

In the torque equation, too, the increments in Park domain rotor currents have to be substituted by

$$\begin{bmatrix} \Delta i_{d2} \\ \Delta i_{q2} \end{bmatrix} = \begin{bmatrix} -\sqrt{3}\,I_{20}\cos\delta_0\,\Delta\delta - \sqrt{3}\,\Delta I_2\sin\delta_0 \\ \sqrt{3}\,I_{20}\sin\delta_0\,\Delta\delta - \sqrt{3}\,\Delta I_2\cos\delta_0 \end{bmatrix} \qquad \text{(IV.8,48)}$$

and the torque equation (IV.14,22) becomes

$$\frac{d}{dt}\Delta\omega_m = \sqrt{3}\,p^2\frac{M}{3}\Big[-I_{20}\sin\delta_0\,\Delta i_{q1} + I_{20}\cos\delta_0\,\Delta i_{d1} - I_{20}\cos\delta_0\,\Delta\delta\, i_{q1}^{\circ} - I_{20}\sin\delta_0\,\Delta\delta\, i_{d1}^{\circ} - \Delta I_2\sin\delta_0\, i_{q1}^{\circ} + \Delta I_2\cos\delta_0\, i_{d1}^{\circ}\Big]. \quad \text{(IV.50)}$$

For convenience, the following abbreviations will be used:

$$A = -(i_{q1}^{\circ}\cos\delta_0 + i_{d1}^{\circ}\sin\delta_0),$$
$$B = -i_{q1}^{\circ}\sin\delta_0 + i_{d1}^{\circ}\cos\delta_0,$$
$$C = \sqrt{3}\,p^2\frac{M}{3}. \quad \text{(IV.51)}$$

Earlier, A was shown to be equal to

$$A = \frac{1}{\omega^2L^2+r}\Big[\sqrt{3}V(\omega L\cos\delta_0 - r\sin\delta_0) - \sqrt{3}\omega^2 MLI_{20}\Big]. \quad \text{(IV.24)}$$

In the same manner, by substituting (IV.18) into (IV.51),

$$B = \frac{1}{\omega^2L^2+r}\Big[\sqrt{3}V(\omega L\sin\delta_0 + r\cos\delta_0) - \sqrt{3}\omega M r I_{20}\Big]. \quad \text{(IV.52)}$$

The linear state model for the doubly-fed machine for the case of amplitude control of the rotor currents can now be formulated as

$$\frac{d}{dt}\begin{bmatrix}\Delta i_{d1}\\ \Delta i_{q1}\\ \Delta\delta\\ \Delta\omega_m\end{bmatrix} = \begin{bmatrix} & F' & \end{bmatrix}\begin{bmatrix}\Delta i_{d1}\\ \Delta i_{q1}\\ \Delta\delta\\ \Delta\omega_m\end{bmatrix}$$
$$+\begin{bmatrix}-\sqrt{3}\frac{M}{L}\omega\cos\delta_0 & \sqrt{3}\frac{M}{L}\sin\delta_0 & 0\\ \sqrt{3}\frac{M}{L}\omega\sin\delta_0 & \sqrt{3}\frac{M}{L}\cos\delta_0 & 0\\ 0 & 0 & 0\\ BC & 0 & -\frac{p}{J}\end{bmatrix}\begin{bmatrix}\Delta I_2\\ \frac{d}{dt}\Delta I_2\\ \Delta T_L\end{bmatrix}$$

with F' as given in equ. (IV.27). (IV.53)

IV.10 Rotor Current Frequency Control by Speed Feedback

A feedback is now built into the system that increases or decreases rotor frequency proportionally to the deviation of the rotor speed from steady state.
The feedback equation is given by

$$\Delta\omega_2 = -k\Delta\omega_m . \tag{IV.54}$$

Thus, in the state equations,

$$\Delta\omega_m + \Delta\omega_2 = (1-k)\Delta\omega_m \tag{IV.55}$$

is substituted into (IV.47) and the system matrix becomes

$$F' = \begin{bmatrix} -\frac{r}{L} & \omega & \sqrt{3}\frac{M}{L}I_2\omega\sin\delta_0 & -\sqrt{3}\frac{M}{L}I_2\cos\delta_0(1-k) \\ -\omega & -\frac{r}{L} & \sqrt{3}\frac{M}{L}I_2\omega\cos\delta_0 & \sqrt{3}\frac{M}{L}I_2\sin\delta_0(1-k) \\ 0 & 0 & 0 & -(1-k) \\ CI_2\cos\delta_0 & -CI_2\sin\delta_0 & CAI_2 & -\frac{k_L}{J} \end{bmatrix} . \tag{IV.56}$$

By the same procedure as in chapter IV.4, the determinant det (pI - F') is evaluated to find the characteristic equation

$$\begin{aligned} 0 = {} & p^4 \cdot JL^2 \\ & + p^3 (k_L L^2 + 2JrL) \\ & + p^2 [J(r^2+\omega^2L^2) + 2k_L rL + P^2 I_2 M(\sqrt{3}AL^2 + 3I_2 ML) \cdot \\ & \qquad \cdot (1-k)] \\ & + p\,[k_L(r^2+\omega^2L^2) + P^2 M I_2 r(2\sqrt{3}AL + 3I_2 M)(1-k)] \\ & + P^2 M I_2 [\sqrt{3}A(r^2+\omega^2L^2) + 3\omega^2 I_2 ML](1-k) \\ = {} & a_0 p^4 + a_1 p^3 + a_2 p^2 + a_3 p + a_4 . \end{aligned} \tag{IV.57}$$

For the system to be stable, k has to be chosen in a way so that the Hurwitz conditions are met.

Clearly, coefficients a_0 and a_1 in (IV.57) are positive.

The third coefficient,

$$a_2 = J(r^2+\omega^2L^2) + 2K_L rL + P^2 M I_2(\sqrt{3}AL^2 + 3I_2 ML)(1-k), \qquad \text{(IV.58)}$$

is approximated for small stator resistance by

$$a_2 \simeq J\omega^2L^2 + 2K_L rL + P^2 M I_2(1-k)(3\tfrac{1}{\omega}L\cos\delta_0). \qquad \text{(IV.59)}$$

This leads to the following condition on the feedback gain k :

$$k < 1 + \frac{J\omega^3 L + 2K_L \omega r}{3P^2 M V I_2 \cos\delta} . \qquad \text{(IV.60)}$$

This condition will be satisfied if k, as is discussed later on, is chosen to be smaller than 1.

The coefficient a_3, with r neglected and A substituted, yields

$$a_3 \simeq K_L\omega^2L^2 + 3\tfrac{1}{\omega}P^2 M I_2 r(1-k)(2V\cos\delta_0 - \omega M I_2) \qquad \text{(IV.62)}$$

or, equivalently,

$$K_L\frac{\omega^3L^2}{3P^2MI_2r} > (k-1)(2V\cos\delta_0 - \omega M I_2). \qquad \text{(IV.63)}$$

In section IV.6 it has been shown that the system could be stable for $2V\cos\delta_0 > \omega M I_2$. Choosing $k<1$ will preserve stability in this case.

If, however, the uncontrolled system is unstable because $2V\cos\delta_0 < \omega M I_2$, (IV.63) can be reformulated to give a condition on k:

$$k > 1 - \frac{K_L\omega^3L^2}{3P^2MI_2r(\omega M I_2 - 2V\cos\delta_0)} . \qquad \text{(IV.64)}$$

Stability for any δ_0 can be reached by adjusting k to the worst case when $\cos\delta_0 = 0$:

$$K > 1 - \frac{K_L \omega^2 L^2}{3P^2 M^2 I_2^2 r} \tag{IV.65}$$

The last coefficient yields, after neglecting r and substituting for A :

$$a_4 \approx P^2 M I_2 \cdot 3V(\omega L \cos\delta_0)(1-k) > 0 \tag{IV.66}$$

Within the operating range for δ_0, $\omega L \cos\delta_0 > 0$ and the Hurwitz condition is met by choosing $k < 1$.

So far, only the necessary conditions for stability have been met. Also, the sufficient conditions need to be investigated.

The first one, $a_2 a_1 > a_0 a_3$ is, with the usual approximation, evaluated as

$$\begin{aligned} &2K_L^2 r L^2 + 3K_L P^2 I_2 M L^2 (1-k) V \tfrac{1}{\omega} \cos\delta_0 + 2\xi^2 r \omega^2 L^2 + 4K_L \xi r^2 L \\ &> -3\xi L P^2 M^2 I_2^2 r (1-k) \end{aligned} \tag{IV.67}$$

and is met for $k < 1$.

The second sufficient condition,

$$a_3(a_1 a_2 - a_0 a_3) > a_4 a_1^2$$

yields a quadratic expression in (1-k):

$$[d + (1-k)e][c + (1-k)b] > a(1-k) \tag{IV.68}$$

with

$$a = 3P^2 M I_2 V \omega L \cos\delta_0 (K_L^2 L^2 + 4K_L \xi r L + 4\xi^2 r^2) > 0$$

$$b = 3K_L P^2 I_2 M L V \tfrac{1}{\omega} \cos\delta_0 + 3P^2 M^2 I_2^2 \xi r > 0$$

$$c = 2K_L^2 r L + 2\xi^2 r \omega^2 L + 4K_L \xi r^2 > 0$$

$$d = K_L \omega^2 L^2 > 0$$

$$e = 3\tfrac{1}{\omega} P^2 M I_2 \, r(2V\cos\delta_0 - \omega M I_{20})$$

Note that e is negative under unstable operating conditions.

Condition (IV.68) is equivalent to

$$(1-k)^2 be + (1-k)[bd + ec - a] + dc > 0,$$

and from this, the maximal value of (1-k) is found to be

$$(1-k)_{max} = -\frac{1}{2}\left(\frac{c}{b}+\frac{d}{e}-\frac{a}{be}\right)+\sqrt{\frac{1}{4}\left(\frac{c}{b}+\frac{d}{e}-\frac{a}{be}\right)^2-\frac{cd}{be}} \; . \qquad \text{(IV.69)}$$

Since with $e<0$ the parabola given in the expression (IV.68) is opened towards negative function values, the condition (IV.68) will only be satisfied for $(1-k)<(1-k)_{max}$.

Solving for k yields the condition

$$k > 1 + \frac{1}{2}\left(\frac{c}{b}+\frac{d}{e}-\frac{a}{be}\right) - \sqrt{\frac{1}{4}\left(\frac{c}{b}+\frac{d}{e}-\frac{a}{be}\right)^2-\frac{cd}{be}} \; . \qquad \text{(IV.70)}$$

Since $e<0$ for unstable conditions of the uncontrolled machine, this lower bound for k is less than one, which agrees with the conditions obtained earlier.

Summarizing, k for speed feedback frequency control must satisfy

$$k < 1,$$

$$k > 1 - \frac{K_L \omega^3 L^2}{3p^2 M I_2 r\,(\omega M I_2 - 2V\cos\delta_0)},$$

$$k > 1 + \frac{1}{2}\left(\frac{c}{b}+\frac{d}{e}-\frac{a}{be}\right)-\sqrt{\frac{1}{4}\left(\frac{c}{b}+\frac{d}{e}-\frac{a}{be}\right)^2-\frac{cd}{be}} \; .$$

IV.11 Rotor Current Amplitude Control by Speed Feedback

A different way to stabilize the doubly-fed machine is to vary the amplitude of rotor currents. Heuristically, the effectiveness of this control strategy can be explained as follows: if the rotor speed retards in motor operation, additional electromechanical torque to maintain synchronism is needed. The electromechanical torque is proportional to rotor current amplitude I_2. So, increasing I_2 would result in increased torque production and rotor speed acceleration can be expected. Likewise, in generator operation the electrical torque should be <u>de</u>creasing with decreasing speed as for this operation, directions of mechanical load torque and electrical torque are reversed in comparison to motoring.

Therefore, a feedback strategy according to

$$\Delta I_2 = K\Delta\delta \qquad \text{(IV.71)}$$

with positive k for motor and negative k for generator operation shall be investigated.

Equ. (IV.71) is substituted into (IV.53) to give:

$$\frac{d}{dt}\begin{bmatrix}\Delta i_{d1}\\ \Delta i_{q1}\\ \Delta\delta\\ \Delta\omega_m\end{bmatrix} = \begin{bmatrix} F' \end{bmatrix}\begin{bmatrix}\Delta i_{d1}\\ \Delta i_{q1}\\ \Delta\delta\\ \Delta\omega_m\end{bmatrix} + \begin{bmatrix} -\sqrt{3}\frac{M}{L}\omega\cos\delta_0 & \sqrt{3}\frac{M}{L}\sin\delta_0 \\ \sqrt{3}\frac{M}{L}\omega\sin\delta_0 & \sqrt{3}\frac{M}{L}\cos\delta_0 \\ 0 & 0 \\ BC & 0 \end{bmatrix}\begin{bmatrix} K\Delta\delta \\ -K\Delta\omega_m \end{bmatrix} + \begin{bmatrix} 0\\ 0\\ 0\\ -\frac{P}{J}\Delta T_L \end{bmatrix}$$

where F', again, is the unaltered system matrix from equ. (IV.27).

The feedback terms are now incorporated into the system matrix to yield:

$$\frac{d}{dt}\begin{bmatrix}\Delta i_{d1}\\ \Delta i_{q1}\\ \Delta \delta\\ \Delta \omega_m\end{bmatrix} = \begin{bmatrix}0\\0\\0\\-\frac{P}{3}\Delta T_L\end{bmatrix} +$$

$$\begin{bmatrix}-\frac{r}{L} & \omega & \sqrt{3}\frac{M}{L}\omega(I_{20}\sin\delta_0 - k\cos\delta_0) & -\sqrt{3}\frac{M}{L}(I_{20}\cos\delta_0 + k\sin\delta_0)\\ -\omega & -\frac{r}{L} & \sqrt{3}\frac{M}{L}\omega(I_{20}\cos\delta_0 + k\sin\delta_0) & \sqrt{3}\frac{M}{L}(I_{20}\sin\delta_0 - k\cos\delta_0)\\ 0 & 0 & 0 & -1\\ CI_{20}\cos\delta_0 & -CI_{20}\sin\delta_0 & C[AI_{20} + Bk] & -k_L/J\end{bmatrix}$$

$$\cdot\begin{bmatrix}\Delta i_{d1} & \Delta i_{q1} & \Delta\delta & \Delta\omega_m\end{bmatrix}^T. \qquad \text{(IV.72)}$$

(Abbreviations A, B and C were explained in (IV.51)).

The stability of this system, again, is evaluated by means of the determinant det (sI - F'). This gives the following characteristic equation for the system with rotor current amplitude control by speed feedback:

$$\begin{aligned}0 = {} & p^4 \cdot JL^2\\ & + p^3 \cdot (k_L L^2 + 2JrL)\\ & + p^2 \cdot [J(\omega^2L^2 + r^2) + 2k_L rL + \sqrt{3}P^2L^2M[AI_{20} + Bk]\\ & \qquad + 3P^2M^2LI_{20}^2]\\ & + p \cdot [k_L(\omega^2L^2 + r^2) + 2\sqrt{3}P^2MLr(AI_{20} + Bk) + 3P^2M^2I_{20}^2r]\\ & + 3P^2M^2I_{20}(\omega^2LI_{20} - k\omega r) + \sqrt{3}P^2M(AI_{20} + Bk)(\omega^2L^2 + r^2)\\ = {} & a_0p^4 + a_1p^3 + a_2p^2 + a_3p + a_4.\end{aligned} \qquad \text{(IV.73)}$$

Coefficients a_0 and a_1 clearly are positive.

$$a_2 = J(\omega^2 L^2 + r^2) + 2K_L r L + \sqrt{3}\, P^2 L^2 M (A I_{20} + Bk) + 3P^2 M^2 L I_{20}^2 \tag{IV.74}$$

and can be simplified by neglecting terms with stator resistance and by substituting for A and B to give:

$$a_2 \approx J\omega^2 L^2 + 2K_L r L + 3P^2 M L V I_{20} \frac{1}{\omega} \cos\delta_0 + 3P^2 M L V \frac{1}{\omega_1} \cdot k \sin\delta_0 \, . \tag{IV.75}$$

In accordance to the preliminary considerations positive k will be chosen for motor and negative k for generator operation. Thus, k sinδ_0 will always be positive; and this leads to positive a_2.

The coefficient a_3 turns out to be the crucial coefficient, as it did for the uncontrolled and the frequency-controlled machine.

$$a_3 = K_L(\omega^2 L^2 + r^2) + 2\sqrt{3}\, P^2 M L r [A I_{20} + Bk] + 3P^2 M^2 I_{20} r \tag{IV.76}$$

can be simplified to obtain

$$a_3 = K_L \omega^2 L^2 + \frac{3P^2 M r}{\omega} [2V I_{20} \cos\delta_0 - \omega M I_{20}^2 + 2Vk \sin\delta_0] \tag{IV.77}$$

This gives the following condition for the feedback gain k to ensure stability:

$$k \sin\delta_0 > \frac{I_{20}}{2V} [\omega M I_{20} - 2V \cos\delta_0] - \frac{K_L \omega^3 L^2}{6VP^2 M r} \, . \tag{IV.78}$$

This important condition will be discussed later.

The last coefficient in the characteristic equation is

$$a_4 = 3P^2 M^2 I_{20} (\omega^2 L I_{20} - k\omega r) + \sqrt{3}\, P^2 M (A I_{20} + Bk)(\omega^2 L^2 + r^2) \tag{IV.79}$$

and can be simplified in the same manner as before:

$$a_4 \approx 3MP^2 (V I_{20} \omega L \cos\delta_0 + k\omega L V \sin\delta_0) \, . \tag{IV.80}$$

With k sin δ_0 being positive for both motor and generator operation, a_4 is positive.

From the necessary stability conditions, it results that k must satisfy inequ. (IV.78):

$$|K| > \frac{1}{|\sin\delta_0|}\left(\frac{I_{20}}{2V}\left[\omega M I_{20} - 2V\cos\delta_0\right] - \frac{K_L \omega^3 L^2}{6 V P^2 M r}\right).$$

This condition shall now be discussed.
The most significant feature of this inequality is that k_{min} contains a factor $1/\sin\delta_0$. This makes sense, physically: from equ. (IV.20), the developed electromechanical torque is proportional to the product $I_2 \sin\delta_0$. To maintain stability, a certain additional accelerating torque must be produced to maintain synchronism of the rotor and the rotating field when the rotor speed retards. This is done, under this feed-back strategy, by increasing the rotor current rms-value by a certain amount I_2. For the accelerating torque to have the same value at different load angles, it must be that $\Delta I_2 = K\Delta\omega_m$ is proportional to $1/\sin\delta_0$. Then the accelerating torque will be proportional to the deviation of rotor speed and independent of the load angle.

One might think that due to the factor $1/\sin\delta_0$, the feedback gain will have to be very high for small load angles, and in fact, would approach infinity for operation at zero load angle. This, however, is not necessary under some minor constraints, as will now be shown.

From chapter IV.6 it is known that under given excitation current amplitude, the machine becomes unstable at a certain load angle, namely

$$\cos\delta_{0max} = \frac{\omega M I_{20}}{2V} - \frac{K_L \omega^3 L^2}{6 P^2 M r I_2 V} \qquad \text{(IV.41 a)}$$

and that the uncontrolled machine is never stable if

$$\frac{\omega M I_{20}}{2V} - \frac{K_L \omega^3 L^2}{6P^2 M r I_{20} V} > 1 . \qquad \text{(IV.41b)}$$

For unstable conditions, the feedback gain must satisfy

$$|K| > \frac{1}{|\sin\delta_0|}\left[\frac{\omega M I_{20}^2}{2V} - \frac{K_L \omega^3 L^2}{6VP^2 M r} - I_{20}\cos\delta_0\right] \qquad \text{(IV.78)}$$

which is simplified for convenience:

$$|K| > \frac{1}{|\sin\delta_0|}\,(a - b\cos\delta_0) \qquad \text{(IV.81)}$$

If $a > b > 0$, we have the case of (IV.41b). The machine is unstable without control at every load angle including zero. The machine cannot be stabilized by current amplitude control at zero load angle, since the accelerating torque is proportional to $\sin\delta_0$. It must therefore be secured that the uncontrolled machine is stable at $\delta_0 = 0$, which can be done by limiting I_{20}. From (IV.41b), the limit on the excitation current rms value is found to be

$$I_{20max} = \frac{V}{\omega M} + \sqrt{\left(\frac{V}{\omega M}\right)^2 + \frac{K_L \omega^2 L^2}{3P^2 M^2 r}} . \qquad \text{(IV.82)}$$

This upper limit is greater than twice no-load excitation current and is not a significant restriction, as the doubly-fed machine is usually operated at excitation current amplitudes not greater than 1.5 times no-load amplitude.

With $I_{20} < I_{20max}$ from (IV.82), $b > a > 0$ and the uncontrolled system is stable on the range $[\delta_{0max}, \delta_{0max}]$ as defined by (IV.41a). For this case, the limit for k is the product of the two functions $\frac{1}{\sin\delta_0}$ and $(a - b\cos\delta_0)$ which are sketched in Fig. IV.4.

The product function is shown in Fig. IV.5. The shaded areas represent the acceptable values of the feedback constant k which satisfy

$k > 0$ and $k > \frac{1}{\sin \delta_o} (a - b \cos \delta_o)$ for motor operation and

$k < 0$ and $k < \frac{1}{\sin \delta_o} (a - b \cos \delta_o)$ for generator operation.

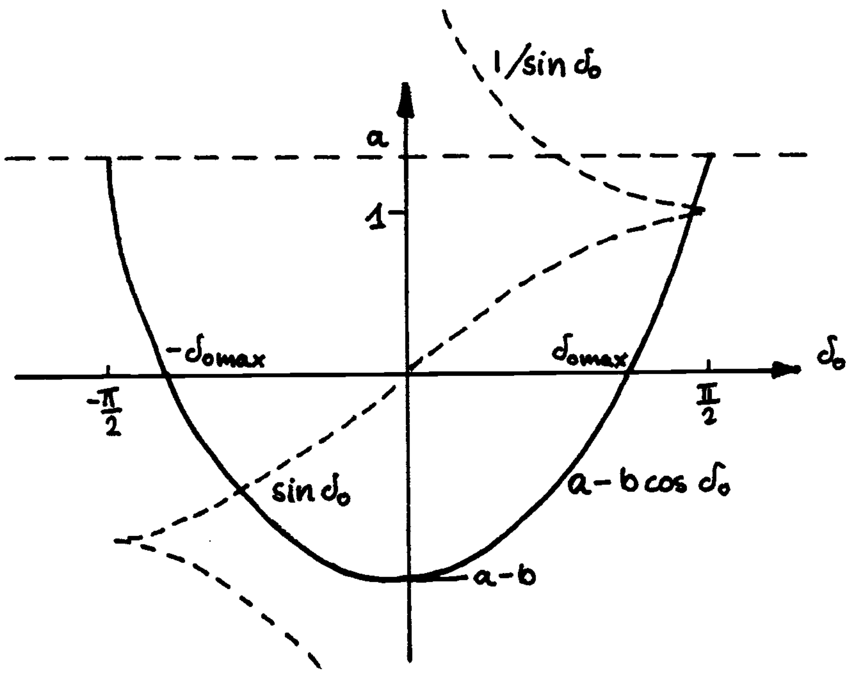

Fig. IV.4 Component functions for k_{min}

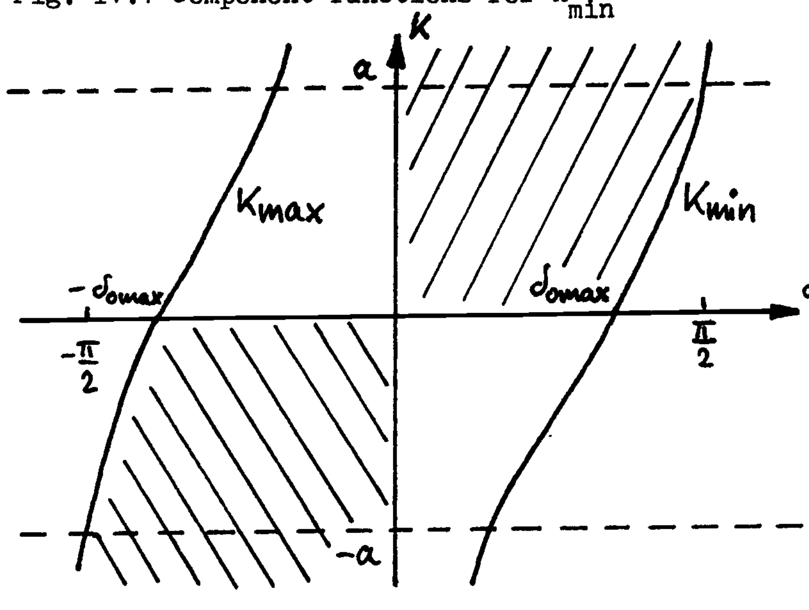

Fig. IV.5 Minimal feedback gain vs. load angle

From this qualitative analysis, it can be seen that a constant feedback gain k can be found to meet the stability condition at any load angle:
Taking, for motoring mode,

$$K > a = \frac{\omega M I_{20}^2}{2V} - \frac{K_L \omega^3 L^2}{6VP^2 M r}, \tag{IV.83}$$

or

$$K < -a = -\frac{\omega M I_{20}^2}{2V} + \frac{K_L \omega^3 L^2}{6VP^2 M r}$$

for generator operation, respectively, the stability condition (IV.78) is satisfied for all δ_0 .

The doubly-fed machine may be operated at various setpoint excitation current amplitudes, as this allows a varying absorption or generation of reactive power. In this case, k either has to be adjusted to the rotor current amplitude I_{20}, or can be chosen to satisfy the stability condition for the case of maximum excitation current, which is the worst case that requires the highest feedback gain.

The first sufficient condition for stability,

$a_2 a_1 > a_0 a_3$

is evaluated with the usual approximations as

$$2K_L^2 rL + 2J^2\omega^2 Lr + 3VMP^2 \frac{1}{\omega} K_L L (I_{20}\cos\delta_0 + K\sin\delta_0) > -3JP^2 M^2 r I_2^2$$

and is satisfied with the previous assumptions on the sign of k.

The second sufficient condition leads to a quadratic expression:

$$(c + dx)(a + bx) > ex \tag{IV.84}$$

where the parameters are defined as

$$a = 2K_L^2 r L + 2J^2\omega^2 L r + 3J p^2 M^2 r I_{20}^2 > 0$$

$$b = 3VMp^2 \frac{1}{\omega} K_L L > 0$$

$$c = K_L \omega^2 L^2 - 3p^2 M r \frac{1}{\omega} \cdot \omega M I_{20}^2 < 0$$

$$d = 6Vp^2 M r \frac{1}{\omega} > 0$$

$$e = (K_L L + 2Jr)^2 \cdot 3Mp^2 V\omega L > 0$$

$$x = (I_{20}\cos\delta_0 + K\sin\delta_0).$$

(IV.84) is equivalent to

$$K\sin\delta_0 > x_{min} - I_{20}\cos\delta_0$$

with $x_{min} = -\frac{1}{2}\left(\frac{a}{b} + \frac{c}{d} - \frac{e}{bd}\right) + \sqrt{\frac{1}{4}\left(\frac{a}{b} + \frac{c}{d} - \frac{e}{bd}\right)^2 - \frac{ac}{bd}} = f(I_{20}^2).$

This condition has to be numerically evaluated to find a possibly stricter lower bound on k than in the necessary conditions: no general conclusions can be made from this expression. For the case where $x_{min} > I_{20}$, similar to the considerations made for the necessary condition on k, excitation current I_{20} has to be lowered so that the machine is stable without control at least for $\delta_0 = 0$. (Note that x_{min} depends on I_{20}^2 from coefficients a and c.)
Then, $k > x_{min}$ will satisfy the sufficient stability condition for all load angles , as can be concluded from a graph analog to Fig. IV.4 and IV.5.

It has thus been shown that the doubly-fed machine can be stabilized by a proportional feedback of the speed that controls the rotor current amplitude, as long as the setpoint rotor current does not exceed a certain limit. This feedback strategy is significantly easier to implement on a power electronic converter than a frequency variation. It is thus a valuable method of stabilizing the doubly-fed machine with simple means.

PART V : CONCLUSIONS

When rotor excitation of the doubly-fed machine is provided by a power electronic converter as operated in voltage source mode, the machine has been shown to be unstable except for a very limited slip range. Speed feedback controling the frequency of rotor voltages has been investigated. This control strategy is able to stabilize the machine effectively. A major drawback to that strategy, however, is the fact that feedback gain needs to be adjusted with changing load conditions and operating speed.

New types of power electronic converters, such as the Schwarz converter, can be operated as controlled current sources rather than voltage sources. With rotor excitation provided by such a converter, it was found that the doubly-fed machine has significantly more favorable stability properties:

- The uncontrolled machine is stable as long as the excitation current and load angle are kept within certain limits.
- The stability properties are independent of slip.
- Stability does depend, however, on the excitation current, the load angle, and the mechanical damping coefficient of the machine-load or turbine-generator system.

In investigating the uncontrolled machine characteristics, a criterion was found to determine whether the damping coefficient is sufficient, at a given excitation current amplitude, for the machine to operate stably under any load condition.

For the case that the machine is operated beyond the described stability region for the uncontrolled case, two stabilizing control strategies have been investigated.

Under the first strategy, rotor current frequency is controlled by a speed feedback. The Hurwitz necessary and sufficient stability conditions demand that the feedback gain must not exceed unity and that it be chosen greater than a certain lower bound depending on machine parameters and operating conditions. A maximum value for this bound was found, such that if the feedback gain is greater than this greatest lower bound the machine will operate stably even under changing load conditions.

Under the second control strategy, speed feedback controls the rms value of the rotor currents. The Hurwitz conditions show that the feedback gain must be positive for motor operation and negative for generator operation of the machine. The Hurwitz conditions give a lower bound for the magnitude of the feedback gain that depends on excitation current and load angle. It has been shown that for this control strategy to be effective, the machine must, on a small interval in the proximity of zero load angle, be stable without control. This can easily be achieved by keeping the excitation current amplitude below a damping-dependent limit which is always greater than twice no-load current. A minimal feedback gain can then be found that will stabilize the machine under any load condition within its rated capacity.

Thus, it has been shown that the doubly-fed machine with current source rotor excitation can be stabilized by either of these control strategies. The second one promises to be easy to implement on a power electronic converter, since it only varies the amplitude of the rotor currents. The feedback gain in both cases is constant and thus independent of slip and load.

For all these qualitative investigations on stability and bonds on the feedback gain, the state model of the machine was linearized. This method showed to be very convenient for obtaining the desired results. Because of the linearization, however, these results contain inaccuracies. For a more exact study, Liapunov's direct method can be applied to the nonlinear machine model; linearization errors are thus avoided. Even these calculations, however, can only be as exact as the measurements on the machine parameters were.

Because of the neglect of stator resistance in most of the Hurwitz stability conditions, the results will not hold in the proximity of $\delta_o = \pm 90^\circ$. As was explained when examining the torque-slip characteristic, load angles near 90° need to be avoided for reasons of transient stability. The neglect of r_1 thus does not affect the validity of the derived conditions on the load angle range used in actual machine operation.

A limitation of the Hurwitz criterion is that no information is given about the quality of stability if the criterion is satisfied; i.e., nothing can be said about the real parts of the resulting system poles or about their damping ratios. These properties may be investigated by numerical evaluation of the characteristic equation. If the results are not satisfactory, the feedback gain has to be adjusted accordingly while still satisfying the Hurwitz conditions.

For the determination of the feedback gain in an actual machine system, the theory derived in this thesis gives an estimate of the minimal gain. Starting from this value, the optimal feedback gain for the system can be found experimentally in a very convenient way: feedback gain is increased until the systems dynamic properties are satisfactory.

In further studies, the theoretical results of this thesis may be verified by simulation or by actual experiments.
The effect of certain parameters representing a sample machine can be studied on a simulation program. Tests can then be made by substituting different values for the damping coefficient, excitation current, slip and load conditions.
Also, feedback of electrical quantities, especially the stator currents, may be investigated. Preliminary studies have shown that the analysis of such a feedback be most conveniently carried out in the Laplace domain. A successful method of stabilizing the machine by electrical feedback would eliminate such sensitive instrumentation devices as the tacho-generator from the system.

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APPENDIX

APPENDIX 1

The machine parameters used in the equations throughout this thesis are defined as

r_1 Stator winding resistance per phase

r_2 Rotor winding resistance per phase

l_1 Stator winding self-inductance

l_2 Rotor winding self-inductance

m_1 mutual inductance between two stator windings

m_2 mutual inductance between two rotor windings

m mutual inductance between a stator and a rotor winding when the winding axes are fully aligned

The machine parameters in the Park domain are obtained from the actual parameters by

$L_1 = l_1 + m_1$

$L_2 = l_2 + m_2$

$M = \sqrt{3/2}\ m$

APPENDIX 2

Machine specifications used for simulation in Ref. 6.

Rated Power: 1.5 hp

Pole pairs: 3

Rated voltage V_1: 220 v

Moment of inertia: 1.4 kg m^2

Damping constant: 0.06 Nmsec.

Turns ratio: 3.6

r_1 = 1.09 V/A

r_2 = 0.084 V/A

L_1 = 0.208 Vs/A

L_2 = 0.016 Vs/A

m = 0.037 Vs/A