The high resistance to corrosion and its moderate tensile strength gives zirconium potential possibilities in structural parts. One of the properties of structural parts is their resistance to repeated stress. The lack of available data on fatigue strength of zirconium, and the realization that expensive and long time tests of this nature are not suited for acceptance tests, has brought about this investigation.

Four different lots of arc melted zirconium sheets were received from the U. S. Bureau of Mines at Albany, Oregon. The specimens prepared from the first lot were annealed before testing. The remaining three lots were cold rolled 8.6%, 30%, and 42%, respectively, with a slight deviation from stated percentages in each lot. Within the knowledge of the author the correlation between endurance limit, cold rolling, and other physical properties of zirconium had not been studied prior to this test.

The fatigue specimens used in this test were specially designed for the Sonntag Flexure Fatigue Testing Machine SF-2. The factory designation is specimen No. 3, which was designed for sheet material 0.125 inches to 0.25 inches thick. It consisted essentially of a 4-inch strip, 1 1/2 inches wide, with a reduced section 3/4 inches wide at the midpoint.

The endurance ratio of the annealed zirconium averaged about 46% as compared to about 50% for rolled or forged steel. A number of the fatigue specimens developed fatigue cracks away from the reduced section of the specimen and failed because of fretting corrosion where they joined the grips of the machine. Cold rolling caused the zirconium to increase in tensile strength and also caused a reduction in the fatigue strength.

No correlation between endurance limit, hardness, and cold
working could be determined from these tests which could partially be accounted for by the condition of the sheets from which the specimens were prepared. The sheets of 30% and 42% cold worked had severe surface cracks in some sections and were severely bent in other sections. The 8.8% rolled sheet had ripples on the rolled surface caused by the breakdown of the flame-hardened surface of the rollers. The amount of nonmetallic inclusions varied with each lot, which may also partially explain the lack of correlation.
FATIGUE PROPERTIES OF ZIRCONIUM

by

JOSEPH HARTVIG SELLIKEN

A THESIS

submitted to

OREGON STATE COLLEGE

in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

June 1951
APPROVED:

Redacted for Privacy

Associate Professor of Mechanical Engineering
In Charge of Major

Redacted for Privacy

Chairman, Department of Mechanical Engineering

Redacted for Privacy

Chairman of School Graduate Committee

Redacted for Privacy

Dean of Graduate School

Date thesis is presented       May 11, 1951       

Typed by Zora Selliken
ACKNOWLEDGMENT

The author is sincerely grateful to S. H. Graf, Director of Oregon State College Engineering Experiment Station, for his helpful comments; to O. G. Paasche, Associate Professor of Mechanical Engineering, Oregon State College, for his help and suggestions; to Dr. E. T. Hayes and A. H. Roberson of the U. S. Bureau of Mines at Albany, Oregon, for their kind help and cooperation in securing the material and permitting the use of their shop and equipment.
## TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Introduction</td>
<td>1</td>
</tr>
<tr>
<td>II. Theory</td>
<td>3</td>
</tr>
<tr>
<td>III. Materials and Testing</td>
<td>10</td>
</tr>
<tr>
<td>IV. Discussion of Results</td>
<td>23</td>
</tr>
<tr>
<td>V. Conclusions</td>
<td>36</td>
</tr>
<tr>
<td>VI. Bibliography</td>
<td>39</td>
</tr>
<tr>
<td>VII. Appendix</td>
<td>40</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

Figure | Page
-------|------
1. Standard Rectangular Tension Test Specimen with 2-inch Gage Length | 11
2. Specimen and Mounting for Fatigue Testing Machine | 12
3. Sonntag Flexure Fatigue Testing Machine SF-2 (side view) | 14
4. Sonntag Flexure Fatigue Testing Machine SF-2 (front view) | 15
5. Forced Vibration | 16
6. Inertia Compensator Adjustment Chart | 19
7. Specimen Stress per Pound of Force | 20
8. Baldwin Tate-Emery Hydraulic Testing Machine | 22
9. Flexural Fatigue of Annealed Zirconium | 27
10. Flexural Fatigue of 8.6% Cold Rolled Zirconium | 28
11. Flexural Fatigue of 30.5% Cold Rolled Zirconium | 29
12. Flexural Fatigue of Cold Rolled Zirconium (40.8% to 43.2% cold rolled) | 30
13. Proportional Limit and Tensile Strength of Annealed Zirconium | 31
14. Proportional Limit and Tensile Strength of 8.6% Cold Rolled Zirconium | 32
15. Proportional Limit and Tensile Strength of Cold Rolled Zirconium (26% to 30.3% cold rolled) | 33
16. Proportional Limit and Tensile Strength of Cold Rolled Zirconium (40.8% to 43% cold rolled) | 34
<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Chemical Composition of Zirconium Sheet</td>
<td>13</td>
</tr>
<tr>
<td>2. Summary of Fatigue, Tensile, and Hardness Tests of Arc Melted Zirconium</td>
<td>35</td>
</tr>
<tr>
<td>3. Test Data of Flexure Fatigue Tests of Annealed Zirconium</td>
<td>40</td>
</tr>
<tr>
<td>4. Test Data of Flexure Fatigue Tests of 8.57% Cold Rolled Zirconium</td>
<td>41</td>
</tr>
<tr>
<td>5. Test Data of Flexure Fatigue Tests of 30.5% Cold Rolled Zirconium</td>
<td>42</td>
</tr>
<tr>
<td>6. Test Data of Flexure Fatigue Tests of 40.8% to 43.2% Cold Rolled Zirconium</td>
<td>43</td>
</tr>
<tr>
<td>7. Tensile Properties of Annealed Zirconium</td>
<td>44</td>
</tr>
<tr>
<td>8. Tensile Properties of 8.57% Cold Rolled Zirconium</td>
<td>45</td>
</tr>
<tr>
<td>9. Tensile Properties of 30.5% Cold Rolled Zirconium</td>
<td>46</td>
</tr>
<tr>
<td>10. Tensile Properties of 42% Cold Rolled Zirconium</td>
<td>47</td>
</tr>
<tr>
<td>11. Rockwell C Hardness of Annealed and Cold Rolled Zirconium</td>
<td>48</td>
</tr>
</tbody>
</table>
I. INTRODUCTION

Zirconium is a bright, silvery, tarnish-proof metal two and one-half times as abundant in the earth's crust as copper and thirteen times as plentiful as lead. Ductile zirconium possesses many useful and desirable characteristics but has remained a scarcity due to the difficulties encountered in its recovery process. The affinity of zirconium at elevated temperature for nitrogen, oxygen, and other gases has made its recovery process uneconomical. However, the recent demand for zirconium, at any price, by the Atomic Energy Commission has stepped up its production.

Much has been learned about the basic properties of zirconium such as working, strength, modulus of elasticity, melting point, density, crystallographic structures, etc. Zirconium possesses the unusual property to resist corrosion in all dilute acids and in hot concentrated solutions of nitric or hydrochloric acid. Unlike other corrosive-resistant metals, it will stand up to hot concentrated caustic solution and can be used where resistance to both acid and basic conditions is needed.

The moderate strength of zirconium together with its outstanding resistance to sea water offers numerous possibilities for its use, such as for pump rods or rotor shafts where stainless steel and monel metal
fail in fatigue through pitting. By the use of zirconium to increase
the life span of equipment now corroding, the demand for zirconium may
increase the production until it can compete economically with other
materials.

The lack of available data on the fatigue properties has
restricted the use of zirconium. Realizing the need for such data,
this investigation was made with the following purposes: (a) To
determine the endurance limit, which is a limiting stress below which
a load may be repeatedly applied an indefinitely large number of
times without causing failure, of annealed and cold rolled zirconium;
(b) To study the relationship of cold rolling, modulus of elasticity,
proportional limit, ultimate tensile strength, ductility, and hardness
to the endurance limit.
II. THEORY

Metal parts of machines which are subjected to repeated loads sometimes fracture suddenly even though the magnitude of the applied stress is considerably less than the static strength of the material. This type of failure is called fatigue failure, and because of the crystalline appearance of the break, early investigators were led to believe that vibration caused a change in the structure of the metals, making a fibrous material crystalline and brittle. This theory is erroneous for it is well known that all metals are crystalline and, by use of the microscope, no evidence of change of structure can be observed in a metal subjected to repeated stress (8, p.15).

A crystal in a metal is made up of a definite pattern or symmetrical arrangement of atoms that is repeated at regular intervals. The plasticity of a single crystal is attributed to movements of atoms along certain crystallographic planes. The plane in which slip is most apt to occur is the plane of greatest atomic density. In a single crystalline structure the atoms are symmetrically orientated, and the slip planes would, therefore, be parallel causing the properties to be isotropic. However, in commercial metals which are polycrystalline, the crystals are usually randomly orientated; therefore, the properties of the metal are usually nonisotropic.

Crystals that are unfavorably oriented with respect to the load will develop slip lines first. The distribution of stress from crystal
to crystal is probably nonuniform and when a piece is subjected to cyclic stress vibration, the particles tend to move slightly with respect to one another (6, p.184). This movement causes minute cracks to form which act as stress raisers, thus multiplying the stress on the particles immediately adjacent, and in turn increasing the size of the crack causing an accelerated crack propagation. This accounts for the sudden failure of the material.

When a beam is subjected to a bending load, the forces act in such a way as to induce compressive stresses over one portion of the beam and tensile stresses over the other portion with a neutral axis separating the two portions. The stress distribution theoretically varies as the moments of all the forces from the neutral axis. Actually this variation is not directly proportional to the distance from the neutral axis, but it does increase as this distance is increased. Machine parts subjected to reverse flexural stresses, therefore, would be most greatly stressed at the surface, causing the fatigue cracks to form at the surface of the material rather than in the core. Because of this stress distribution in metals, almost all the tests that are intended to measure the fatigue strength of metals only measure the strength of the surface fibers.

Abrupt changes in cross section, scratches, corroded areas, notches, and other surface discontinuities detract from the strength of fatigue specimens (1, p.118). These surface imperfections are points of high localized stress and are known as stress raisers.
Specimens that have been carefully polished so that all visible stress raisers are removed will give the true fatigue strength of the material. However, this does not duplicate service conditions in which commercial parts made of this material will show a lower fatigue strength due to poorer surface preparations. The results of fatigue tests on carefully prepared specimens give the ultimate which can be reached and will show the true comparison between different metals which can only be obtained by some form of standards in the test procedure. Although machined surfaces of a specimen may have a fatigue strength about 18 percent lower than a polished specimen, it is somewhat doubtful whether any appreciable effect on fatigue strength of steel is obtained by a finer grade of polishing than that obtained by No. 00 paper (8, pp.108-111).

Pits and flaws within the sub-surface are also stress raisers, but these are not so critical in fatigue as surface imperfections. These imperfections become more critical as they approach the surface and are only critical when the localized stress becomes greater than the surface stress.

It has been pointed out that fatigue strength of a metal is in reality a measure of its surface strength. If this is true, any increase in strength of the surface would cause a corresponding increase in the endurance limit. The increase in fatigue strength that follows shot peening appears to demonstrate this to be true. This increase in strength of the surface may be explained by two theories.
The first theory holds that: (a) surface fibers are exposed to greater stresses than the sub-surface material; (b) fatigue fracture can develop only from tensile stresses (1, p.119).

By this theory the strength is increased because shot peening induces residual compressive stresses in a thin surface layer of the material which must first be overcome by an externally applied tension load, thus increasing the surface strength by a corresponding increase in tensile strength. The material remains in a state of equilibrium, therefore the compressive force in the surface layer must be balanced by an equal tensile force in the core. The entire core being in tension, it can be assumed that this force is uniformly distributed resulting in little change in the strength of the core for thick material. However, the core of thin specimens would be weakened in tensile strength because of the greater intensity of induced tensile stress. Therefore, the induced tensile stress in the core is a function of the thickness of the material, varying inversely as the cross sectional area.

The tensile stresses in a material externally loaded in bending is assumed to increase directly, within its elastic limit, as the distance from the neutral axis. That portion of the core in tension immediately adjacent to the shot peened surface would have very nearly the same impressed tensile stress as the surface layer, but it also has an induced residual tensile stress which would result in this portion being under greater total stress than the surface. This would result
in failure just below the surface of a shot peened specimen. Thus light peening would be beneficial because it would increase the surface strength by the amount of residual compressive stress without any detrimental increases in residual tensile stress within the core. On the other hand, it would be possible to reduce the fatigue strength by severe peening because of the increase in tensile residual stresses within the core and by bruising the surface causing stress raisers.

The other theory holds that the increase in fatigue strength is due to "work hardening" of the surface fibers. The unpeened metal of the core is stronger in fatigue because the core stress is less than the surface stress in a beam loaded in bending.

Residual compressive stresses can be produced in metals by other means such as nitriding, and carburizing (these two apply only to ferrous alloys), filing, honing, tumbling, hammering, and cold rolling. Cold rolling has an advantage over shot peening because: (a) the compressed residual stress is uniform, (b) the depth of induced residual stress is controllable and reproducible, (c) the surface is free from stress raisers, (d) excessive local cold working can be avoided (1, p.130). This superiority has not been proven since a comparison has never been made. Cold rolling can reduce the fatigue strength by excessive rolling causing the surface to develop cracks formed by failure in compression of the surface fibers, or because the greater internal tensile residual stress offsets the gain in surface strength.

The preparation of fatigue testing specimens may change the
fatigue strength of the material. Annealing will remove any residual stresses. Machining and grinding may develop residual tensile stresses if localized heat is generated. Filing and honing develop residual compressive stress by cold working. The amount of residual stresses can be measured fairly accurately by the dissecting method which consists essentially of removing successive layers of material from the specimen and measuring the deformation of the remaining metal. The remaining metal must restore itself to a state of equilibrium and will deform an amount proportional to the strain removed. It is also possible to visualize the type of residual stress. For example, grinding a specimen on the surface will bend that surface into a convex shape if the residual stress is compressive.

Poor technique in preparation of specimens destroys the accuracy of the test for a specific material and only indicates the fatigue strength of the test specimen. In research and scientific tests, the variables must be controlled in order that the results may be interpreted so that fatigue properties of new materials may be discovered and their position in relation to other materials may become known. They also reveal the ultimate that may be reached.

In order to determine the endurance limit of a new material, tests must be conducted over a range of stresses until the limiting stress that will not cause failure has been found. Tests of this nature may require several months on a machine that operates at 1800 cycles per minute. Because of the time involved in determining the
fatigue strength of a metal, this form of testing could hardly be of any value as an acceptance test for the consumer, or where the purpose of the test is to control production. Therefore, any correlation between endurance limit and some other easily and readily determinable property would be desirable.

The ratio of endurance limit to ultimate tensile strength (the endurance ratio) of cast steel averages 0.42 (9, p.18), and the endurance limit under reversed flexure for rolled or forged steel and iron averages about 50 percent (6, p.193). The Brinell hardness number for steels seems to be correlated with tensile strength and is a fair indication for endurance limit. The tensile strength of steel in psi is about 500 times the Brinell hardness number, and the fatigue strength in psi is about 250 times the Brinell hardness number.
III. MATERIALS AND TESTING

Four different lots of arc melted zirconium were furnished by the U. S. Bureau of Mines at Albany, Oregon and are hereafter referred to by the laboratory identification letters A, B, C, and D. The chemical compositions are given in Table 1.

The specimens of zirconium A were cut from a 0.30-inch sheet forged and rolled at 1600 °F. Rectangular tension test specimens with 2-inch gage length (Figure 1) were prepared according to ASTM Standards, E8-46 (3, p. 994).

The fatigue specimen shown in Figure 2 is designated by the Sonntag Scientific Corporation as specimen No. 3 and is specially designed for sheet material, 0.125 inches to 0.25 inches thick, to be tested by model SF-2 Fatigue Machine. This specimen is to be used with the short arm (loading yoke AA) as shown in Figure 2.

The fatigue specimens A were polished with No. 00 metallographic polishing paper, removing all diagonal scratches. All the steps in machining and polishing were carefully controlled so as not to overheat the specimens by too rapid removal of the material. The tension and fatigue specimens were then annealed by the Bureau of Mines at 1370 °F for one hour in a helium atmosphere.

Specimens of zirconium B, C, and D were prepared from arc melted zirconium, forged and rolled at 1600 °F and cold rolled to about 3 1/3%, 30%, and 42%, respectively. The reduction by cold working varied from one end to the other of the sheet of zirconium D.
Figure 1 Standard Rectangular Tension Test Specimen with 2-Inch Gage Length.

- 2 3/8" min. in grips
- 3 1/4" min. between grips
- 2 3/8" min. in grips

Gage Length
2" ± 0.008"

Reduced Section
8" min.

W + 0.003" to 0.005" (see note)
W + 0.003" to 0.006"

2 1/4" min.

Radius 0.5" to 3"

t = thickness of material

Note: Gradual taper from ends of reduced section to middle.
FIGURE 2 SPECIMEN AND MOUNTING FOR FATIGUE TESTING MACHINE
TABLE 1

Chemical Composition of Zirconium Sheet
(Remainder Taken as Zirconium)

<table>
<thead>
<tr>
<th></th>
<th>Carbon percent</th>
<th>Iron percent</th>
<th>Nitrogen percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.05</td>
<td>0.12</td>
<td>0.015</td>
</tr>
<tr>
<td>B</td>
<td>0.01</td>
<td>0.07</td>
<td>0.016</td>
</tr>
<tr>
<td>C</td>
<td>0.04</td>
<td>0.08</td>
<td>0.05</td>
</tr>
<tr>
<td>D</td>
<td>0.02</td>
<td>0.06</td>
<td>0.016</td>
</tr>
</tbody>
</table>

The machined specimens of zirconium C and D were free from visible scratches as a result of rolling and no further treatment was necessary. The fatigue specimens of zirconium B were polished in successive stages with emery cloth from No. 1 to No. 000 with a final polish on crocus cloth using 600 mesh alundum as an abrasive. No additional treatment was necessary for the tension test specimens.

All specimens were cut parallel with the direction of rolling and stamped with lettered dies to correspond to the lot number from which it was prepared. The tension specimens were also numbered consecutively for each separate lot.

The Sonntag Flexure Fatigue Machine SF-3 was chosen for this test because of availability, simplicity of operation, and because sheet material was furnished. The side view of the machine is shown in Figure 3 and the front view is shown in Figure 4.
FIGURE 3  SONNTAG FLEXURE FATIGUE TESTING MACHINE SF-2 (side view)
FIGURE 4 SONNTAG FLEXURE TESTING MACHINE SF-2
This machine consists essentially of a synchronous motor to which is attached a rotating cantilever beam. The beam is loaded on the end with an eccentric mass A which generates the force applied to the specimen. The amount of force generated is controlled by adjusting the distance of its mass from the shaft. This force is transmitted by a linkage system through rod B to load yoke AA and the attached short arm and then to the specimen which is clamped rigidly to a pedestal as shown in Figure 2. Pivot rod C absorbs the side forces of the eccentric by limiting the vibrating motion of the shaft to a vertical direction. This results in a single degree of freedom system with forced vibration, the shaft and the specimen acting as springs parallel to each other.

\[ F_0 \sin wt = m \ddot{x} + k_s x \]

**Figure 5. FORCED VIBRATION**

In such a vibrating system the impressed force \( F_0 \sin wt \) is equal to the sum of the mass times the acceleration plus the spring constant \( k \) of the parallel system times the displacement.
The solution of this is shown below.

\[ m \frac{d^2x}{dt^2} + kx = F_0 \sin wt \]  

For a parallel system of springs,

\[ k = k_s + k_c \]  

\[ x = \text{displacement of mass } m \]

\[ t = \text{time, seconds} \]

\[ k = \text{spring constant, pounds per inch} \]

\[ w = \text{circular frequency or angular velocity} \]

The solution of this differential equation is,

\[ x = x_0 \sin wt \]  

\[ \ddot{x} = x_0w \cos wt \]  

\[ \ddot{x}_0 = -x_0w^2 \sin wt \]

By substituting equations (3), (4), and (5) in equation (1),

\[ -mx_0w^2 \sin wt + kx_0 \sin wt = F_0 \sin wt \]

\[ F_0 = x_0(k - mw^2) \]  

By controlling the value of \( m \) in such a way that \( \frac{k_c}{w^2} = m \), \( k_c \) and \( w \) being constant,

\[ F_0 = x_0(k - k_c) \]

\[ F_0 = x_0\left[(k_s + k_c) - k_c\right] \]

\[ F_0 = x_0k_s \]  

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]

\[ F_0 = x_0k_s \]
For a rigid specimen having a high $k_b$ value the deflection would be small, and for a soft specimen with a small $k_b$ the deflection would be large. The eccentric force $F_0$ is, therefore, independent of the rigidity of the specimen since the deflection $x_0$ will vary in such a way that the force transmitted to the specimen will remain constant even with a change in the rigidity of the specimen during the test.

To make $mw^2$ equal to $k_c$, poises are positioned with respect to the shaft. When using the loading yoke arm $AA$, the specimen weight is negligible, and it is necessary to correct only for the effective weight of the yoke arm. The zero line is marked on the graph for this particular machine by the manufacturer as shown in Figure 6. The weight of the yoke was 0.042 pounds which is equal to 21 divisions on the graph below the zero line, thus positioning the poises 4.23 inches from the pivot shaft.

This machine is controlled by a start and stop switch which automatically shuts off when the specimen breaks, and it is also equipped with a cycle counter.

Before the actual operation of the machine was begun, the thickness of the specimen was measured. The position of the eccentric mass was determined from the graph in Figure 7 which gives the specimen stress per pound of force for various thicknesses of specimen. Dividing the desired stress by this reading gives the force setting or position of the eccentric mass.

The first specimen was tested at a relatively high stress so that
FIGURE 6  INERTIA COMPENSATOR ADJUSTMENT CHART
FIGURE 7
SPECIMEN STRESS PER POUND OF FORCE
failure would occur at a small number cycles of applied stress. Each succeeding specimen was tested at a lower stress until it was determined from the S-N curve that the endurance limit was reached.

The tension tests were carried out on a 60,000 pound capacity Baldwin Tate-Emery hydraulic testing machine shown in Figure 8. For this test, flat two-inch gage length specimens (Figure 1), a Model T-1 extensometer, and a TA-1 Southwark stress-strain automatic recorder were used.

The aim of the tension test was to determine the modulus of elasticity, the proportional limit, ultimate strength, and the percent elongation in a two-inch gage length. The automatic recorder was used to record the load-strain curve. A short distance past the proportional limit, the extensometer was removed and only the load was recorded automatically. The ultimate load was read on the load indicator and compared with the recorded reading. The percent elongation in two-inch gage length was then determined from the broken specimen.

The Rockwell C hardness tests were made on the surfaces of the uninjured ends of the tension and fatigue test specimens.
FIGURE 8  BALDWIN TATE-EMERY HYDRAULIC TESTING MACHINE
IV. DISCUSSION OF RESULTS

The results of the fatigue tests are given in Tables 4, 5, 6, and 7 of the appendix, and the S-N diagrams obtained from them are shown in Figures 9, 10, 11, and 12 at the end of this section.

Tables 8, 9, 10, and 11 give the results of the Rockwell C hardness test and Table 2 gives a summary of results obtained from Fatigue Tensile and Hardness tests.

The S-N (stress-number of cycles) curves were plotted on semilogarithmic paper with the N, or number of cycles, plotted on the logarithmic coordinate. This enables N to be plotted with a fair degree of accuracy for both small and astronomical numbers.

For the annealed zirconium the endurance limit was 31,200 psi and the endurance ratio averaged 0.46. This ratio may be compared with 0.50 for rolled and forged steel. The S-N curve follows the usual pattern of a fatigue curve and makes a well-defined break indicating a decided change in the relationship between S and N. It is altogether possible that beyond the point $10^7$ cycles the curve may drop down again. However, previous work in fatigue testing has shown no such drop has occurred in a reasonable number of cycles. This, then, gives confidence that the horizontal line represents the endurance limit. All the specimens tested below the endurance limit broke in the grips, but they were subjected to so many cycles of stress that they had no bearing on the results.

The interpretation of the results of the 8.6% cold rolled zirconium
is almost impossible since all but two specimens broke in the grips. This was probably due to the condition of the sheet as received. This sheet had ripples on the surface caused by the rolling operation. The surface of the flame-hardened rolls broke down during the rolling operations on this sheet. These ripples were carefully removed by grinding and polishing, which may have removed that portion of the surface containing induced compressive stresses. A number of specimens broke in the grips at a high number of cycles, and there is some indication that the endurance limit may run as high as 31,000 psi, which would compare very closely to the annealed zirconium. Further indication of the indefinite results may be seen in the variations of proportional limits in Figure 14 and Table 8.

For the 30.5% cold rolled zirconium, the endurance limit dropped to 25,000 psi, and the endurance ratio dropped to 0.236. The fatigue strength was considerably higher at high stresses than the annealed or 8.6% cold worked specimen. There is some possibility that this sheet had been overworked and infinitely small fractures existed within the grains of the metal. The sheet as received was severely damaged in some areas and was bent into a sine-wave. Some parts of the sheet had to be discarded because of visible surface defects. One other factor that may have some effect on the results is the chemical composition. A. H. Roberson, metallurgist at the U. S. Bureau of Mines at Albany, Oregon, stated that an increase in nitrogen content will have a great influence on the tensile strength of zirconium. The nitrogen content, 0.05%, of this lot was three times as large as that found in the other
lots. This could account for the high tensile strength of 106,000 psi, and may also have some bearing on the low endurance limit. To the writer's knowledge no research has been made to study the effects of nitrogen content on the endurance limit of zirconium. A study of this nature should be important since the increase in tensile strength may be misinterpreted as also increasing the endurance limit.

The S-N curve for the 42% cold rolled zirconium is shown in Figure 12. The scattered results were at first disconcerting until it was found that the specimens were of varied thicknesses or different degrees of cold working. By connecting the points together of nearly equal thickness, it was found that two distinct S-N curves resulted. The sudden break in the shape of both curves gives confidence that the endurance limit had been reached. It is interesting to note the amount of reduction in endurance limit as the result of a small amount of over-working. The endurance limit of the 41½% cold worked specimens is 26,000 psi as compared to 24,000 psi for the 43% cold worked specimens.

Work done by A. H. Roberson at the Salt Lake City Station of the U. S. Bureau of Mines on the fatigue properties of titanium indicates that annealed and cold worked titanium have the same endurance limit. Sheet specimens and the same type of machine as used in this experiment were used in the titanium tests. Titanium is just above zirconium on the periodic chart, and their properties are similar. The summary of results in Table 2 indicates no increase in endurance limit due to cold working. However, these results can not be taken as conclusive evidence
since each sheet was made from a different ingot, and their chemical composition was not the same. Fatigue testing technique could be improved by making all sheets from the same ingot.
FIGURE 9 FLEXURAL FATIGUE OF ANNEALED ZIRCONIUM
FIGURE 10 FLEXURAL FATIGUE OF 8.6% COLD ROLLED ZIRCONIUM
FIGURE 11 FLEXURAL FATIGUE OF 30.5% COLD-ROLLED ZIRCONIUM
FIGURE 12 FLEXURAL FATIGUE OF COLD ROLLED ZIRCONIUM
FIGURE 13  PROPORTIONAL LIMIT AND TENSILE STRENGTH OF ANNEALED ZIRCONIUM
Strain gage removed beyond the proportional limit
FIGURE 15  PROPORTIONAL LIMIT AND TENSILE STRENGTH OF COLD-ROLLED ZIRCONIUM

(26% to 30.3% cold rolled)
FIGURE 16  PROPORTIONAL LIMIT AND TENSILE STRENGTH OF COLD-ROLLED ZIRCONIUM

(40.8% to 43% cold rolled)
### TABLE 2

**Summary of Fatigue, Tensile, and Hardness Tests of Arc Melted Zirconium**

<table>
<thead>
<tr>
<th></th>
<th>Annealed</th>
<th>8.6% Cold Rolled</th>
<th>30.5% Cold Rolled</th>
<th>42% Cold Rolled</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Endurance limit, psi</strong></td>
<td>31 200</td>
<td>31 000*</td>
<td>25 000</td>
<td>25 000</td>
</tr>
<tr>
<td><strong>Tensile strength, psi</strong></td>
<td>68 200</td>
<td>75 100</td>
<td>106 000</td>
<td>84 000</td>
</tr>
<tr>
<td><strong>Endurance ratio</strong></td>
<td>0.46</td>
<td>0.41*</td>
<td>0.236</td>
<td>0.30</td>
</tr>
<tr>
<td><strong>Proportional limit, psi</strong></td>
<td>23 700</td>
<td>9 500 to 20 200</td>
<td>34 300</td>
<td>25 700</td>
</tr>
<tr>
<td><strong>Modulus of elasticity, (10^6) psi</strong></td>
<td>13.74</td>
<td>13 to 25</td>
<td>14.1</td>
<td>14</td>
</tr>
<tr>
<td><strong>Elongation for 2-inch gage length, percent</strong></td>
<td>22.5</td>
<td>16</td>
<td>11.5</td>
<td>11</td>
</tr>
<tr>
<td><strong>Hardness, Rockwell C</strong></td>
<td>13</td>
<td>14</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

*approximation
V. CONCLUSIONS

The results of the fatigue test as found in this experiment should be interpreted to indicate the fatigue strength of the metal itself rather than of zirconium as it is used in machine parts.

From an examination of the graphs and the discussion of results in the previous section, the following summary of conclusions is given:

1. The diagrams plotted for the annealed zirconium indicate a well-defined endurance limit. The small amount of scatter is not alarming in view of the fact that the plate did have some visible surface defects, indicating poor fabrication technique.

2. The modulus of elasticity remained nearly constant for all of the specimens tested except for the 8.6% rolled zirconium and gives no indication of the endurance limit.

3. An increase in cold working did not increase the endurance limit as it would for ferrous metals. This is not surprising since similar characteristics were found for titanium by A. H. Roberson using the same type fatigue machine.

4. An increase in the nitrogen content in zirconium showed
a decided increase in the tensile strength and brittleness. There is a possibility that the nitrogen content had a detrimental effect on the endurance limit since the endurance limit was decidedly reduced in the sheet containing a high nitrogen content.

5. The effect of cold working in reducing the endurance limit of zirconium may be explained in these ways: (a) the presence of surface defects in the material due to poor rolling technique; and (b) the material had been worked to its limit, causing the induced residual stresses within the core to more than offset the increase in surface fiber tensile strength.

6. The endurance limit increases with the ductility for the specimens having very nearly the same chemical composition. Additional tests should be conducted at various degrees of ductility in order to establish a definite relationship.

7. An increase in cold rolling decreased the endurance limit and the endurance ratio.

8. The results of this experiment are not conclusive since each lot tested was forged and rolled from different ingots having different chemical composition.
The technique of the rolling procedure could decidedly be improved upon. For other tests of this nature, it is also recommended that the sheets tested shall be made from the same ingot in order that the results may be consistent and interpreted correctly.

9. No conclusion could be made for the 8.57% cold rolled zirconium because of the excessive number of specimens that broke in the grips. The endurance limit could possibly be higher than that indicated on the S-N diagram.

10. Five possible explanations are offered for the lack of definite correlation between endurance limit and other physical properties: (a) difference in chemical composition; (b) internal breakdown of structure due to rolling technique; (c) the material had been cold worked beyond its limit; (d) the lack of homogeneity; and (e) the breaking of specimens in the grips at the testing machine.
VI. BIBLIOGRAPHY


VII. APPENDIX
## TABLE 3

Test Data of Flexure Fatigue Tests of Annealed Zirconium

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Stress in lb. per sq. in.</th>
<th>Cycles for Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-1</td>
<td>40 800</td>
<td>43 000</td>
</tr>
<tr>
<td>A-2</td>
<td>38 400</td>
<td>67 000</td>
</tr>
<tr>
<td>A-3</td>
<td>36 000</td>
<td>78 000</td>
</tr>
<tr>
<td>A-4</td>
<td>33 600</td>
<td>120 000</td>
</tr>
<tr>
<td>A-5</td>
<td>31 200</td>
<td>12 432 000*</td>
</tr>
<tr>
<td>A-6</td>
<td>31 200</td>
<td>11 843 000*</td>
</tr>
<tr>
<td>A-7</td>
<td>30 240</td>
<td>720 000**</td>
</tr>
<tr>
<td>A-8</td>
<td>30 240</td>
<td>7 243 000*</td>
</tr>
<tr>
<td>A-9</td>
<td>30 240</td>
<td>5 355 000*</td>
</tr>
<tr>
<td>A-10</td>
<td>30 240</td>
<td>5 156 000*</td>
</tr>
<tr>
<td>A-11</td>
<td>30 240</td>
<td>12 704 000*</td>
</tr>
<tr>
<td>A-12</td>
<td>30 240</td>
<td>8 192 000*</td>
</tr>
<tr>
<td>A-13</td>
<td>30 240</td>
<td>5 885 000</td>
</tr>
<tr>
<td>A-14</td>
<td>36 000</td>
<td>22 000</td>
</tr>
<tr>
<td>A-15</td>
<td>33 600</td>
<td>39 000</td>
</tr>
<tr>
<td>A-16</td>
<td>31 200</td>
<td>13 744*</td>
</tr>
<tr>
<td>A-17</td>
<td>32 400</td>
<td>152 000</td>
</tr>
</tbody>
</table>

* Broke in grip
** Inclusion in break
TABLE 4

Test Data of Flexure Fatigue Tests of 8.57% Cold Rolled Zirconium

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Stress in lb. per sq. in.</th>
<th>Cycles for Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-1</td>
<td>40 000</td>
<td>78 000*</td>
</tr>
<tr>
<td>B-2</td>
<td>36 000</td>
<td>131 000</td>
</tr>
<tr>
<td>B-3</td>
<td>32 000</td>
<td>160 000*</td>
</tr>
<tr>
<td>B-4</td>
<td>32 000</td>
<td>849 000*</td>
</tr>
<tr>
<td>B-5</td>
<td>31 000</td>
<td>977 000*</td>
</tr>
<tr>
<td>B-6</td>
<td>30 000</td>
<td>673 000*</td>
</tr>
<tr>
<td>B-7</td>
<td>29 000</td>
<td>1 807 000*</td>
</tr>
<tr>
<td>B-8</td>
<td>29 000</td>
<td>2 072 000*</td>
</tr>
<tr>
<td>B-9</td>
<td>29 000</td>
<td>2 819 000*</td>
</tr>
<tr>
<td>B-10</td>
<td>28 000</td>
<td>273 000</td>
</tr>
<tr>
<td>B-11</td>
<td>28 000</td>
<td>442 000*</td>
</tr>
</tbody>
</table>

* Broke in grip
TABLE 5

Test Data of Flexure Fatigue Tests of 30.5% Cold Rolled Zirconium

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Stress in lb. per sq. in.</th>
<th>Cycles for Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-1</td>
<td>40 000</td>
<td>30 000</td>
</tr>
<tr>
<td>C-2</td>
<td>40 000</td>
<td>61 000</td>
</tr>
<tr>
<td>C-3</td>
<td>32 000</td>
<td>86 000</td>
</tr>
<tr>
<td>C-4</td>
<td>32 000</td>
<td>164 000</td>
</tr>
<tr>
<td>C-5</td>
<td>30 000</td>
<td>377 000</td>
</tr>
<tr>
<td>C-6</td>
<td>30 000</td>
<td>109 000</td>
</tr>
<tr>
<td>C-7</td>
<td>28 500</td>
<td>301 000</td>
</tr>
<tr>
<td>C-8</td>
<td>27 500</td>
<td>415 000</td>
</tr>
<tr>
<td>C-9</td>
<td>26 000</td>
<td>275 000</td>
</tr>
<tr>
<td>C-10</td>
<td>25 000</td>
<td>5 000 000*</td>
</tr>
</tbody>
</table>

* Broke in grip
TABLE 6

Test Data of Flexure Fatigue Tests of 40.8% to 43.2% Cold Rolled Zirconium

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Percent Cold Rolled</th>
<th>Stress in lb. per sq. in.</th>
<th>Cycles for Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>D-1</td>
<td>42.5</td>
<td>40 000</td>
<td>36 000</td>
</tr>
<tr>
<td>D-2</td>
<td>42.5</td>
<td>34 000</td>
<td>81 000</td>
</tr>
<tr>
<td>D-3</td>
<td>42.5</td>
<td>30 000</td>
<td>140 000</td>
</tr>
<tr>
<td>D-4</td>
<td>43.2</td>
<td>28 000</td>
<td>156 000</td>
</tr>
<tr>
<td>D-5</td>
<td>43.2</td>
<td>28 000</td>
<td>162 000</td>
</tr>
<tr>
<td>D-6</td>
<td>43</td>
<td>25 000</td>
<td>447 000</td>
</tr>
<tr>
<td>D-7</td>
<td>43</td>
<td>24 000</td>
<td>10 000 000*</td>
</tr>
<tr>
<td>D-8</td>
<td>40.8</td>
<td>40 000</td>
<td>72 000</td>
</tr>
<tr>
<td>D-9</td>
<td>40.8</td>
<td>34 000</td>
<td>149 000</td>
</tr>
<tr>
<td>D-10</td>
<td>40.8</td>
<td>28 000</td>
<td>437 000</td>
</tr>
<tr>
<td>D-11</td>
<td>40.8</td>
<td>28 000</td>
<td>325 000</td>
</tr>
<tr>
<td>D-12</td>
<td>41.4</td>
<td>27 000</td>
<td>241 000</td>
</tr>
<tr>
<td>D-13</td>
<td>41.4</td>
<td>27 000</td>
<td>608 000</td>
</tr>
<tr>
<td>D-14</td>
<td>40.8</td>
<td>26 000</td>
<td>2 050 000*</td>
</tr>
</tbody>
</table>

* Broke in grip
### Tensile Properties of Annealed Zirconium

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Tensile Strength</th>
<th>Proportional Limit</th>
<th>Modulus of Elasticity</th>
<th>Elongation in 2 inches</th>
<th>Strain at Proportional Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-1</td>
<td>66 100</td>
<td>23 500</td>
<td>13.5 (10)$^6$</td>
<td>22.5</td>
<td>0.00174</td>
</tr>
<tr>
<td>A-2</td>
<td>70 000</td>
<td>22 300</td>
<td>14.4 (10)$^6$</td>
<td>21.5</td>
<td>0.00155</td>
</tr>
<tr>
<td>A-3</td>
<td>66 700</td>
<td>23 800</td>
<td>13.67 (10)$^6$</td>
<td>22.7</td>
<td>0.00174</td>
</tr>
<tr>
<td>A-4</td>
<td>70 000</td>
<td>25 100</td>
<td>13.36 (10)$^6$</td>
<td>23.5</td>
<td>0.00188</td>
</tr>
</tbody>
</table>
### TABLE 8

Tensile Properties of 8.57% Cold Rolled Zirconium

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Tensile Strength</th>
<th>Proportional Limit</th>
<th>Modulus of Elasticity</th>
<th>Elongation in 2 inches</th>
<th>Strain at Proportional Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>pounds per square inch</td>
<td>percent</td>
<td>inches per inch</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B-1</td>
<td>75 500</td>
<td>9 500</td>
<td>$25 ,(10)^6$</td>
<td>15.5</td>
<td>0.00038</td>
</tr>
<tr>
<td>B-2</td>
<td>79 100</td>
<td>15 300</td>
<td>$17 ,(10)^6$</td>
<td>16</td>
<td>0.0009</td>
</tr>
<tr>
<td>B-3</td>
<td>69 900</td>
<td>20 200</td>
<td>$13 ,(10)^6$</td>
<td>16</td>
<td>0.00155</td>
</tr>
</tbody>
</table>
### Table 9

Tensile Properties of 30.5% Cold Rolled Zirconium

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Tensile Strength</th>
<th>Proportional Limit</th>
<th>Modulus of Elasticity</th>
<th>Elongation in 2 inches</th>
<th>Strain at Proportional Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-1</td>
<td>105 000</td>
<td>38 700</td>
<td>13 (10)</td>
<td>11.5</td>
<td>0.00296</td>
</tr>
<tr>
<td>C-2</td>
<td>108 000</td>
<td>33 800</td>
<td>14.4 (10)</td>
<td>11.5</td>
<td>0.00235</td>
</tr>
<tr>
<td>C-3</td>
<td>83 000</td>
<td>21 100</td>
<td>14.4 (10)</td>
<td>13.5</td>
<td>0.00144</td>
</tr>
<tr>
<td>C-4</td>
<td>106 000</td>
<td>30 400</td>
<td>15 (10)</td>
<td>11.5</td>
<td>0.00202</td>
</tr>
</tbody>
</table>
TABLE 10

Tensile Properties of 42% Cold Rolled Zirconium

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Tensile Strength</th>
<th>Proportional Limit</th>
<th>Modulus of Elasticity</th>
<th>Elongation in 2 inches</th>
<th>Strain at Proportional Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>D-1</td>
<td>85 500</td>
<td>29 300</td>
<td>14 (\times 10^6)</td>
<td>11</td>
<td>0.0021</td>
</tr>
<tr>
<td>D-2</td>
<td>85 500</td>
<td>24 500</td>
<td>14.1 (\times 10^6)</td>
<td>11</td>
<td>0.00174</td>
</tr>
<tr>
<td>D-3</td>
<td>83 500</td>
<td>24 600</td>
<td>13.3 (\times 10^6)</td>
<td>11</td>
<td>0.00185</td>
</tr>
<tr>
<td>D-4</td>
<td>100 100</td>
<td>23 900</td>
<td>13.5 (\times 10^6)</td>
<td>11.5</td>
<td>0.00177</td>
</tr>
</tbody>
</table>
### TABLE I1

**Rockwell C Hardness of Annealed and Cold Rolled Zirconium**

<table>
<thead>
<tr>
<th>Annealed</th>
<th>Cold Rolled 8.57%</th>
<th>Cold Rolled 30.5%</th>
<th>Cold Rolled 42%</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>14</td>
<td>19</td>
<td>18</td>
</tr>
<tr>
<td>14</td>
<td>13</td>
<td>19</td>
<td>17</td>
</tr>
<tr>
<td>13</td>
<td>14</td>
<td>23</td>
<td>19</td>
</tr>
<tr>
<td>13</td>
<td>13</td>
<td>21</td>
<td>20</td>
</tr>
<tr>
<td>14</td>
<td>14</td>
<td>20</td>
<td>19</td>
</tr>
<tr>
<td>15</td>
<td>14</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>16</td>
<td>15</td>
<td>21</td>
<td>19</td>
</tr>
<tr>
<td>12</td>
<td>16</td>
<td>18</td>
<td>20</td>
</tr>
<tr>
<td>12</td>
<td>13</td>
<td>21</td>
<td>24</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
<td>19</td>
<td>23</td>
</tr>
<tr>
<td>14</td>
<td>15</td>
<td>20</td>
<td>19</td>
</tr>
<tr>
<td>13</td>
<td>14</td>
<td>22</td>
<td>18</td>
</tr>
<tr>
<td>10</td>
<td>13</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>13</td>
<td>13</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>13</td>
<td>16</td>
<td>20</td>
<td>21</td>
</tr>
</tbody>
</table>