AN ABSTRACT OF THE THESIS OF

George Modesto Adotevi-Akue for the M.S. in Meteorology
(Name) (Degree) (Major)

Date thesis is presented 13 May 1965

Title EVALUATION OF ALTERNATIVE MARKOVIAN MODELS FOR PRECIPITATION OCCURRENCE IN OREGON

Abstract approved [redacted]
(Major professor)

Precipitation occurrence in three winter periods and three summer periods is examined for two Oregon stations: Seaside and Squaw Butte. The winter periods are January, February, and the interval January 15 to February 20. The summer periods are July, August, and the interval July 10 to August 15. The two intervals were selected as representing stable periods in the Natural Calendar of the Atmosphere proposed in the literature. Seaside is a coastal station with a marine climate, while Squaw Butte is in the high desert of eastern Oregon. At both stations winter is the rainy season and summer is dry.

By first defining a wet day as any on which .01 inch or more of precipitation is measured, and then proceeding from the assumption that daily precipitation occurrence is a two-state Markov Chain of order 4, the hypotheses are tested that the order of dependence is 0, 1, 2, and 3 within the fourth order. At Seaside for the winter

periods another threshold was employed, with wet days being those having .20 inch or more of precipitation.

In no case did a Chi-square test for goodness of fit show a model of zero order to be suitable. For winter periods at Squaw Butte, a model of order as high as 3 was not clearly suitable. For all other tests, however, a Markovian model of an order within the range tested was judged suitable. The variation of threshold tested at Seaside in winter has no major significance for the periods examined.

The tests as conducted neither confirm nor deny the validity of the concept of the Natural Calendar. As a result, however, recommendations for further tests of this validity are made.

EVALUATION OF ALTERNATIVE MARKOVIAN MODELS FOR PRECIPITATION OCCURRENCE IN OREGON

by

GEORGE MODESTO ADOTEVI-AKUE

A THESIS

submitted to

OREGON STATE UNIVERSITY

in partial fulfillment of
the requirements for the
degree of

MASTER OF SCIENCE

June 1965

APPROVED:

[redacted]

Assistant Professor of Biometeorology

In Charge of Major

[redacted]

Head of Department of Physics

[redacted]

Dean of Graduate School

Date thesis is presented May 13, 1965

Typed by Eula Weathers

ACKNOWLEDGMENT

I wish to express my sincere gratitude to: Dr. W. P. Lowry for bringing up the problems examined in this study and for ready help given throughout the entire period of the investigation; to Dr. D. Guthrie for advice given on the theoretical aspects of the investigation and for unstinted help with computer programming; to the team of computer technicians, led by Mr. T. Yates, for help with computer runs; and finally to the many friends, too many to mention individually, who contributed to making my two-year stay in the United States a most pleasant experience.

Perhaps I should seize this opportunity to place on record my gratitude to the U.S. State Department for awarding the two-year scholarship that made it possible for me to undertake graduate studies in the United States.

TABLE OF CONTENTS

	Page
INTRODUCTION	1
Markovian Models	1
Singularities	2
Definition	2
Objections Against Singularities	3
Implications of Recent Research	4
Primary Singularities and the Natural Calendar	5
STATEMENT OF THE PROBLEM	7
APPLICABLE THEORY OF MARKOV CHAINS	9
Definition	9
Multiple Markov Chains: Order of Markov Chains	10
METHODS	14
Source of Data	14
Analysis of Data	14
RESULTS	17
DISCUSSION	19
CONCLUSIONS	21
BIBLIOGRAPHY	22
APPENDIX 1	24
APPENDIX 2	27

LIST OF TABLES

Table | Page

1 Summary of Chi-square analyses 16

EVALUATION OF ALTERNATIVE MARKOVIAN MODELS FOR PRECIPITATION OCCURRENCE IN OREGON

INTRODUCTION

Markovian Models

The fact that present weather is not independent of past weather has been known for some time. In 1924, Louis Besson (1) studied the probability of occurrence of rainy spells of various lengths at Montsouris, France, arriving at the conclusion that past weather exerts an influence on future weather. Gabriel and Neumann (7) investigated the distribution of dry and wet spells and their combinations at Tel Aviv, Israel. They derived distribution functions from a Markov model and found them to fit observed data of daily rainfall occurrence. Caskey (6) derived theoretical probabilities of precipitation occurrence in intervals of various lengths from a Markov model, and found them to agree with empirical values for Denver, Colorado. In Canada, Hopkins and Robillard (9) applied a two-state Markov Chain to the durations of spells recorded in the months of April through September over a period of 45 years in the Great Plains provinces. They concluded the model provided workable approximations to observed values. Finally, Weiss (14) made a notable contribution not only by publishing frequencies of wet and dry spells calculated from a Markov model for comparison with a wide variety of sources, but also by showing that such a model is a useful tool when applied to occurrences of meteorological variables other than daily precipitation. More specifically, he investigated stormy periods and the interstorm periods for four 10 degree squares of latitude and longitude near England, Newfoundland, the Great Lakes, and the Aleutians.

While by no means explaining temporal patterns of the occurrence of weather events, the relative successes of simple Markovian models in fitting observations tempt one to study other properties of the data in light of these models.

Singularities

Definition

The term "singularity" has been used to refer to a recognizable weather pattern or atmospheric pattern change which recurs near certain average dates in either most years or in certain types of years (3, p. 34). It has been suggested, by analogy with the electron energy-level concept, that with given boundary conditions the atmosphere has preferred patterns or energy levels on a hemispheric scale (5, p. 2). At any given time, the atmosphere can either exist in one quasi-stable pattern or be in a state of transition from one pattern to another. On this basis one may argue that the preferred patterns or the transitions are singularities.

Objections Against Singularities

The concept of singularities in the annual course of weather elements dates back to the days before scientific meteorology. It originated with people, like farmers, who are closely connected with the weather in their daily occupations. With the beginning instrumental observations in meteorology on a more intensive scale, and with the rise of statistical methods in the last century, investigators applied these methods to the more abundant data to evaluate the concept of singularities. As a result two types of objections to the concept emerged. The first dealt with the failure of certain anomalous events, the singularities being studied at the time, to repeat on the same date each year. The occurrences were said to be either absent in a given year or to have occurred on other than the expected date.

The second type of objection concerned the variation in intensity of the singularities, that is to say the amount by which the anomalous observation departed from the value which would have been expected on a seasonal basis. The belief was held among early meteorologists who had investigated singularities, therefore, that they were the chance results of random variations in short records. These variations, and therefore the singularities, would disappear when longer records became available.

Implications of Recent Research

By now facts have emerged from recent research which indicate that singularities do indeed exist. Wahl (12) investigated the "January Thaw" in New England and concluded that the phenomenon was linked to weather patterns of a singular nature at the same time in Europe. He also studied (13) the behavior of some other marked singularities in the New England area in relation to the general circulation of the atmosphere. Using as index the strength of the surface zonal westerlies, as defined by the monthly mean pressure difference at sea level between latitudes 35N and 55N, he concluded that the occurrence of singularities was related to the value of this index.

The results of Wahl's work have a twofold significance. First, in addition to supporting the validity of the concept of singularities, they strongly suggest individual singularities have more than local importance. Second, if the state of the general circulation influences the occurrence or non-occurrence of singularities, not all singularities need appear in a given year. Further, any long-term changes in the character of the general circulation would be reflected in the disappearance of some singularities and the appearance of others in the course of events. This line of thinking tends to refute the two types of general objections cited above.

Yet another recent study by Bryson and Lowry (4, 5) dealt with the onset of summertime precipitation in Arizona. It not only supported the notions of Wahl, but also introduced the idea that each broad geographical region might have a suite of recognizable singularities different from that of another region.

Primary Singularities and the Natural Calendar

Bryson and Lahey (3, p. 34) have termed "primary singularities" those periods of transition during which the general circulation changes from a regime typical of one season to a regime typical of the next season. After consideration of a wide variety of variables and indices, they conclude these primary singularities exist, represent relatively abrupt changes of regime, and may be considered as reference times in a Natural Calendar of the Atmosphere. On the strength of their results, Bryson and Lahey have proposed such a calendar, with seasons as follows:

Winter:	November 1 - March 21
Spring:	March 21 - June 25
Summer:	June 25 - August 21
Autumn:	August 21 - November 1

Each season is proposed as having three or four sub-periods consisting of periods exhibiting a stable regime which becomes inappropriate to the boundary conditions about the time the next sub-period begins. The work leading to the description of this Natural Calendar is sufficiently convincing as to prompt further investigation of the concepts involved.

More of a feeling for the details of the proposed Natural Calendar may be gained from the following. First, the approximate dates (3, Fig. 27) for the sub-periods of winter are: Winter I (November 1 - December 6), Winter II (December 6 - January 12), Winter III (January 12 - February 13), and Winter IV (February 13 - March 21). Finally, the description of winter given is (3, p. 36):

> Winter -- November 1 to March 21. A period of high storminess with the major polar frontal system far to the south (in the Northern Hemisphere, of course).
>
> Subtropical anticyclones generally weak
> Continental highs well developed
> Circulation indices highly variable
> A general inflow of air from Canada
> Usually only one Aleutian and one Icelandic low

STATEMENT OF THE PROBLEM

Two problems are dealt with in this thesis. First is the degree to which sequences of wet and dry days may be represented by Markovian models for Oregon stations. The second is the possibility of applying methods of Markov Chain analysis to assess the validity of the proposed Natural Calendar of the Atmosphere.

Two Oregon stations, representing two different types of climate and for which long records of daily precipitation occurrence were readily available on punched cards, were chosen for analysis. Winter periods and summer periods were investigated for Seaside, a coastal station with winter rainy season, and for Squaw Butte, a station in the high desert east of the Cascade Mountains.

Winter periods selected for study were the months of January and February and the interval from January 15 to February 20. Summer periods were the months of July and August and the interval from July 10 to August 15. The two intervals of 35 days each were chosen to represent sub-periods of the proposed Natural Calendar, as discussed below.

To obtain information on the success of Markovian models in describing sequences of daily precipitation occurrence at the stations, empirical transition probabilities were obtained for a two-state process from the historical records in such a way as to inquire of the results whether the order of dependence of the process were 0, 1, 2, or 3. In the works cited above on Markovian models in climatology, orders of dependence greater than 1 were not considered.

On the assumption that the notion of a "stable regime" in a sub-period of the Natural Calendar would be equivalent to a stationary transition probability structure in the Markovian sense, it was postulated that any indication of a better agreement between theory and observation during the two 35-day intervals (mid-winter and mid-summer) than during the standard months associated with them would be strong indication that the proposed Natural Calendar is valid during those periods of the year. While recognized as not being a definitive test of that validity, the comparison seemed such a manageable extension of the methodology being employed as to be worthwhile making.

APPLICABLE THEORY OF MARKOV CHAINS

Definition

Assume we have a sequence of experiments with the following properties. The outcome of each experiment is one of a finite number of possible outcomes $a_1, a_2, \ldots, a_s$. It is assumed that the probability of the outcome a_j of any experiment depends on the outcome of the previous experiment. Assume further that there are given numbers p_{ij} which represent the probability of outcome a_j given that the outcome a_i occurred on the preceding experiment. The outcomes $a_1, a_2, \ldots, a_s$ are called states and the numbers p_{ij} are called transition probabilities. If there are s possible outcomes to each experiment, we speak of an s-state Markov Chain (10, p. 171).

The dependence referred to in the definition may go backward 1 or more steps. If only 1 step, the process is a simple Markov Chain or a process of order 1. If the dependence goes backward r steps, $r > 1$, then we speak of an r^{th} order process or a multiple Markov Chain of order r.

In a simple Markov Chain with a chosen value of daily precipitation serving as threshold to define wet days and dry days, the transition probability of a wet day following another wet day, for example, may be written p_{ww}, while that for a wet day following a dry day may be p_{dw}.

<u>Multiple Markov Chains: Order of Markov Chains</u>

The following treatment of multiple Markov Chains has been given by Billingsley (2, p. 28-29).

"Let $\{x_n\}$ be a t^{th} order Markov Chain with transition probabilities

$$p_{a_1 \ldots a_t : a_{t+1}} = P\{x_n = a_{t+1} \mid x_{n-t} = a_1, \ldots, x_{n-1} = a_t\}$$

assumed for simplicity to be positive. If $t > 1$, $\{x_n\}$ is called a multiple Markov Chain. Problems involving multiple Markov Chains are easily reduced to problems about simple ones by the following device. Consider the process $\{y_m;\ m=1, 2, \ldots\}$, where

$$y_m = (x_m, x_{m+1}, \ldots, x_{m+t-1}).$$

Then y_m is a simple Markov Chain the state space of which consists of the s^t different t-tuples, the transition probabilities being

$$p_{(a_1 \ldots a_t)(b_1 \ldots b_t)} = \begin{cases} p_{a_i \ldots a_t : b_t} & \text{if } b_i = a_{i+1},\ i=1, \ldots, t-1 \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$

A knowledge of the first $n+t$ steps of the original process $\{x_n\}$ is obviously equivalent to a knowledge of the first $n+1$ steps of the new process $\{y_m\}$. For example, let $f_{a_1 \ldots a_\nu}$ be the number of m, with $1 \leq m \leq n$, such that $(x_m, \ldots, x_{m+\nu-1}) = a_1, \ldots, a_\nu$.

The quantity $f_{a_1 \ldots a_t}$ plays such a role that the statistic

$$\sum_{a_1 \ldots a_{t+1}} \frac{\left(f_{a_1 \ldots a_{t+1}} - f_{a_1 \ldots a_t} p_{a_1 \ldots a_t \nu : a_{t+1}}\right)^2}{f_{a_1 \ldots} p_{a_1 \ldots a_t : a_{t+1}}} \qquad (2)$$

is asymptotically Chi-square with $s^{t+1} - s^t$ degrees of freedom. It can be shown that this statistic is asymptotivally equivalent to the appropriate Neyman-Pearson criterion.

"There is a possibility of estimating the parameters upon which the $p_{a_1 \ldots a_t : a_{t+1}}$ may depend. For example, if $r < t$, then the parameters may be so defined as to correspond to the hypothesis that $\{x_m\}$ is a Markov Chain of order r. In this case the $p_{a_1 \ldots a_t : a_{t+1}}$ in equation (2) are to be replaced by

$$\hat{p}_{a_1 \ldots a_t : a_{t+1}} = \frac{f_{a_{t-r+1} \ldots a_{t+1}}}{f_{a_{t-r+1} \ldots a_t}} \qquad (3)$$

If this is done, the resulting statistic, appropriate for testing the null hypothesis that $\{x_m\}$ is an r^{th} order Markov Chain within the hypothesis that it is of the t^{th} order, is asymptotically Chi-square with $(s^{t+1} - s^t) - (s^{r+1} - s^r)$ degrees of freedom, provided the null hypothesis is true."

For the problem at hand in this thesis, the appropriate forms of Billingsley's formulae for the orders 1, 2, and 3 are set out below with $s = 2$ and $t = 4$.

For order 1,

$$\chi^2 = \sum_{a_1 \ldots a_5} \frac{\left(f_{a_1 \ldots a_5} - \frac{f_{a_1 \ldots a_4} f_{a_4 \ldots a_5}}{f_{a_4}} \right)^2}{\frac{f_{a_1 \ldots a_4} f_{a_4 \ldots a_5}}{f_{a_4}}} \tag{4}$$

with 14 degrees of freedom.

For order 2,

$$\chi^2 = \sum_{a_1 \ldots a_5} \frac{\left(f_{a_1 \ldots a_5} - \frac{f_{a_1 \ldots a_4} f_{a_3 \ldots a_5}}{f_{a_3 \ldots a_4}} \right)^2}{\frac{f_{a_1 \ldots a_4} f_{a_3 \ldots a_5}}{f_{a_3 \ldots a_4}}} \tag{5}$$

with 12 degrees of freedom.

For order 3,

$$\chi^2 = \sum_{a_1 \ldots a_5} \frac{\left(f_{a_1 \ldots a_5} - \frac{f_{a_1 \ldots a_4} f_{a_2 \ldots a_5}}{f_{a_2 \ldots a_4}} \right)^2}{\frac{f_{a_1 \ldots a_4} f_{a_2 \ldots a_5}}{f_{a_2 \ldots a_4}}} \tag{6}$$

with 8 degrees of freedom.

A straightforward extension of the above, the following is the appropriate form for the test of order zero:

$$\chi^2 = \sum_{a_1 \ldots a_5} \frac{\left(f_{a_1 \ldots a_5} - \dfrac{f_{a_1 \ldots a_4} f_{a_4}}{N} \right)^2}{\dfrac{f_{a_1 \ldots a_4} f_{a_4}}{N}} \qquad (7)$$

with 15 degrees of freedom (8), with N the total number of transitions.

METHODS

Source of Data

The observations used in this investigation were daily precipitation data for Seaside and for Squaw Butte, extracted from published records of the U.S. Weather Bureau (11). Seaside data were for the 33 years 1931-1963; while the Squaw Butte data were for the 28 years 1937-1964.

Analysis of Data

To define the two states of the process examined, a threshold was chosen so that a day on which .01 inch or more of precipitation was measured was considered a wet day. Using this definition with an appropriate computer program, data were extracted from the punched cards in such a way as to categorize each run of five days into one of the 32 possible permutations of sequences. Frequencies of each of these formed the basis by which the computer produced values of Chi-square according to equations (4) through (7). Sample calculations are shown in Appendix 1.

In an extension of the same computer program, estimates of the transition probabilities were prepared according to equation (3), and are exhibited in Appendix 2.

The same program just mentioned, together with a second threshold value of .20 defining a wet day, was employed in an additional analysis of the data for the three winter periods at Seaside.

In summary of the various values of Chi-square calculated by these methods, Table 1 presents the results for the two stations, four orders of dependence, six periods of the year, and three additional periods in connection with the second threshold at Seaside. In the customary manner, a value of Chi-square larger than a certain critical value was interpreted as meaning the model being tested produced calculated results at odds with observations, the model thereupon being judged inappropriate. The five percent level of significance was selected as the basis for judgment.

Table 1. Summary of Chi-square analyses

	WINTER					SUMMER			
Order (i.e., days of dependence)	0	1	2	3		0	1	2	3
Degrees of Freedom	15	14	12	8		15	14	12	8
SEASIDE ("Wet" ⩾ .01")									
January	175.10**	8.54	8.19	6.26	July	80.70**	15.17	9.54	4.11
February	172.45**	26.96*	22.35*	6.65	August	65.95**	20.21	14.52	8.88
"Winter" (Jan 15 to Febr 20)	219.33**	19.78	16.55	7.64	"Summer" (Jul 10 to Aug 15)	69.13**	25.46*	16.53	13.53
SEASIDE ("Wet" ⩾ .20")									
January	150.10**	27.21*	9.38	1.94					
February	112.20**	25.86*	22.78*	8.13					
"Winter"	181.87**	29.98**	18.05	6.87					
SQUAW BUTTE ("Wet" ⩾ .01")									
January	64.16**	33.53**	23.80*	18.34*	July	70.21**	27.58*	12.71	7.50
February	168.42**	31.89**	30.33**	18.98*	August	67.52**	15.75	9.51	4.80
"Winter"	67.78**	29.93**	21.12	11.60	"Summer"	58.34**	26.40*	15.13	9.10

* Indicates statistical significance at the 5 percent level, ** at the 1 percent level.

RESULTS

With the results of Table 1, the hypotheses tested are that within the assumption the process is Markovian of order 4, the order of dependence of precipitation occurrence is 0, 1, 2, or 3. The lowest order at which the value of Chi-square in each case becomes small enough to be significant only at a level greater than five percent is judged to be the order of dependence of the process, since here calculated and observed frequencies become sufficiently congruent.

Table 1 reveals at once the absence of any basis for believing that any process of order zero is represented. Thus, all processes examined exhibited significant temporal dependence of some sort. In addition, all processes except those for Squaw Butte in winter (i.e., January and February) appear to be Markovian of an order 1, 2, or 3. While some further consideration might have been given the practical basis for selecting the five percent level of significance rather than, say, 25 percent, in most cases here we see the matter is sufficiently clear as to make such consideration immaterial. This contention arises from the fact that in most instances Chi-square falls in a one-day change of order from a value clearly significant to a value rather near the mean for the distribution.

For the most part, these results give no clear evidence for or against the validity of the portions of the proposed Natural Calendar dealt with. Only in the case of winter at Squaw Butte was the fit for the "natural" period consistently better for all orders than either of the standard months. On the other hand, only during summer at Seaside was the fit for the "natural" period consistently poorer for all orders. On balance, the results for the suggested sub-periods of the Natural Calendar gave results very much like those for the standard periods.

In consideration of the effects of the change of threshold for wintertime data at Seaside, there appears to be no evidence in Table 1 for believing the change has much significance. The higher threshold value was employed because, at a station where large daily amounts of precipitation are a regular occurrence during the rainy season, a threshold as low as .01 inch yields very few dry days. It was felt, therefore, that the higher threshold would give a more meaningful test of whether or not a Markovian model was appropriate. In both cases, as it turns out, January and "winter" are more similar to one another than either is to February, the order being 1 or 2 for the two similar periods and 3 for February. Clearly, a higher and higher threshold might at some point produce a real change in results as fewer and fewer days were classified wet. Within the range of any reasonable definition of a wet day, however, the threshold does not appear critical.

DISCUSSION

Following upon the remarks above concerning the lack of clear evidence supporting the validity of the proposed natural sub-periods, it would appear worthwhile considering why this might be so. One explanation would be, of course, that indeed the natural calendar is not a valid concept and that no stable regimes (i.e., stationary transition structure) with distinct and abrupt changes between them are to be found. There are other possibilities, however; and judgment of the concept must be postponed until these are investigated.

One alternative explanation would be that, as observations from the months of January and February, for example, consist of information on two distinct sub-periods of the Natural Calendar, so the observations from the "natural" winter consist of information from more than one basic type of year. Thus, in both instances the estimated matrices really are weighted means of the true values from more than one process.

A second alternative explanation might be, in the case of winter at Seaside, for example, that the processes during January and "Winter" are really approximately the same with order 1, while during the last half of February a distinctly different process is in evidence. That is, in the terms suggested by Bryson and Lahey, at Seaside Winter II and Winter III are quite similar as compared with

Winter IV.

In future investigations, an additional stratification of data by type of year, in the sense suggested by Wahl in using the Zonal Index, should reveal which of the above three alternatives is most likely. The most stable regime, and thus the most stationary transition matrix, should be found not only within a proposed natural sub-period, but also within a distinct type of general circulation pattern.

As with questions of the validity of the Natural Calendar, the results of Table 1 likewise raise questions about whether or not spells of wet and dry days form a Markov Chain during winter at Squaw Butte. While it is possible that the process is Markovian of order greater than 4, it is also possible the process is simply not Markovian. In the latter case there might be an alternation between dominance by marine and by continental air masses or regimes, with the alternation following, for example, a regular period of a certain number of days. In view of the fact that increasing the order of the Markovian model tends to result in ever better fit of theory and observation, selection of an explanation must await further investigation.

CONCLUSIONS

While refinements of test procedures beyond those employed in this thesis appear to be required before more firm conclusions may be drawn as to the suitability of Markov models for describing sequences of wet and dry days in Oregon and as to the validity of the concept of the Natural Calendar, several tentative conclusions may be drawn from the results presented.

1) Markov probability models of order 1, 2, or 3 provide a great improvement in characterizing sequences of wet and dry days in the two major climate types of Oregon as compared with a model assuming random occurrences (i. e., Markov of order zero).

2) Within the range of thresholds representing any reasonable definition of a "wet" day at a given station, the value chosen for the threshold is probably not critical to an analysis of the suitability of a Markov model or to a decision on the order of the process.

3) Since consideration of hypotheses allowing for processes of order greater than 1 has produced results strongly suggesting that multiple Markov Chains are as prevalent as simple chains, investigators should resist the temptation to stop once they have demonstrated the improvement a simple Markov Chain provides over a random (zero order) model.

BIBLIOGRAPHY

1. Besson, Louis. On the probability of rain. Monthly Weather Review 52:308. 1924.

2. Billingsley, P. Statistical methods in Markov chains. Annals of Mathematical Statistics 32:12-71. 1961.

3. Bryson, R. A. and J. F. Lahey. The march of the seasons. Madison, 1958. 41 p. (University of Wisconsin. Department of Meteorology. U.S. Air Force Contract AF 19(604)-992, final report)

4. Bryson, R. A. and W. P. Lowry. Synoptic climatology of the Arizona summer precipitation singularity. Bulletin of the American Meteorological Society 36:329-339. 1955.

5. ______________. Synoptic climatology of the Arizona summer monsoon. Madison, Wisconsin, 1955. 25 p. (University of Wisconsin. Department of Meteorology. Scientific Report No. 1. U.S. Air Force Contract AF 19(604)-992)

6. Caskey, J. E., Jr. A Markov chain model for the probability of precipitation occurrence in intervals of various lengths. Monthly Weather Review 91:298-301. 1963.

7. Gabriel, K. R. and J. Neumann. A Markov chain model for daily rainfall occurrence at Tel Aviv. Quarterly Journal of the Royal Meteorological Society 88:90-95. 1962.

8. Guthrie, D., Assoc. Prof. of Statistics, Oregon State University. Personal communication. April 16, 1965.

9. Hopkins, J. W. and P. Robillard. Some statistics of daily rainfall occurrence for the Canadian prairie provinces. Journal of Applied Meteorology 3:600-602. 1964.

10. Kemeny, J. G., Laurie Snell, J. and G. L. Thompson. Introduction to finite mathematics. Englewood Cliffs, Prentice-Hall, 1961. 372 p.

11. U.S. Weather Bureau. Climatological data. Oregon Section. vols. 37-70. 1931-1964.

12. Wahl, E. W. The January thaw in New England. Bulletin of the American Meteorological Society 33:380-386. 1952.

13. ________________. Contributions to the study of planetary atmospheric circulations. Journal of Meteorology 10:42-45. 1953.

14. Weiss, L. L. Sequences of wet or dry days described by a Markov chain probability model. Monthly Weather Review 92:169-176. 1964.

APPENDIX

APPENDIX 1

Sample Calculations of Chi-square

The explanation of the method for computing the Chi-square statistic is based on the frequencies tabulated below.

Sequence Preceding Today	Today Wet	Today Dry	Sequence Preceding Today	Today Wet	Today Dry
WWWW	10	13	DWWW	12	17
WWWD	7	23	DWWD	5	15
WWDW	5	8	DWDW	8	8
WWDD	10	27	DWDD	10	50
WDWW	6	7	DDWW	23	13
WDWD	2	13	DDWD	14	47
WDDW	7	14	DDDW	27	45
WDDD	13	64	DDDD	56	313

The formula used for computing the Chi-square needed to test the hypothesis that $r = 2$ is:

$$\chi^2 = \sum_{a_1, \ldots a_5} \frac{\left(f_{a_1 \ldots a_5} - \dfrac{f_{a_1 \ldots a_4} \, f_{a_3 \ldots a_5}}{f_{a_3 \ldots a_4}} \right)^2}{\dfrac{f_{a_1 \ldots a_4} \, f_{a_3 \ldots a_5}}{f_{a_3 \ldots a_4}}}$$

Appendix 1 (continued)

While the summation is over the 32 five-day sequence in the table, only three sequences (WWWWW, WDWDW, and DDWWD) are used here as samples.

For the sequence WWWWW:

$$a_1 \ldots a_5 = \text{WWWWW}, \quad f_{a_1 \ldots a_5} = 10$$

$$a_1 \ldots a_4 = \text{WWWW}, \quad f_{a_1 \ldots a_4} = 23$$

$$a_3 \ldots a_5 = \text{WWW}, \quad f_{a_3 \ldots a_5} = 51$$

$$a_3 \ldots a_4 = \text{WW}, \quad f_{a_3 \ldots a_4} = 101$$

$$\Delta = \frac{\left(10 - \frac{(23)(51)}{101}\right)^2}{\frac{(23)(51)}{101}} = 0.2232$$

For the sequence WDWDW:

$$a_1 \ldots a_5 = \text{WDWDW}, \quad f_{a_1 \ldots a_5} = 2$$

$$a_1 \ldots a_4 = \text{WDWD}, \quad f_{a_1 \ldots a_4} = 15$$

$$a_3 \ldots a_5 = \text{WDW}, \quad f_{a_3 \ldots a_5} = 28$$

$$a_3 \ldots a_4 = \text{WD}, \quad f_{a_3 \ldots a_4} = 126$$

$$\Delta = \frac{\left(2 - \frac{(15)(28)}{126}\right)^2}{\frac{(15)(28)}{126}} = 0.5376$$

Appendix 1 (continued)

For the sequence DDWWD:

$$a_1 \dots a_5 = \text{DDWWD}, \quad f_{a_1 \dots a_5} = 13$$

$$a_1 \dots a_4 = \text{DDWW}, \quad f_{a_1 \dots a_4} = 36$$

$$a_3 \dots a_5 = \text{WWD}, \quad f_{a_3 \dots a_5} = 50$$

$$a_3 \dots a_4 = \text{WW}, \quad f_{a_3 \dots a_4} = 101$$

$$\Delta = \frac{\left(13 - \frac{(36)(50)}{101}\right)^2}{\frac{(36)(50)}{101}} = 0.1295$$

By computing the 32 Δ's and finding their sum, the appropriate Chi-square is obtained. The frequencies of the sequences $a_1 \dots a_5$ are read directly from the table, while those for such sequences as $a_1 \dots a_4$ and $a_3 \dots a_4$ are obtained by adding frequencies from several parts of the table.

APPENDIX 2

Summary of Transition Probabilities

SEASIDE ("Wet" ≥ . 01")

Sequence Preceding Today	January Today Wet	January Today Dry	February Today Wet	February Today Dry	"Winter" Today Wet	"Winter" Today Dry
Order 1						
W	.825	.175	.828	.172	.825	.175
D	.391	.609	.397	.603	.395	.605
Order 2						
WW	.821	.179	.838	.162	.835	.165
WD	.385	.615	.458	.542	.394	.606
DW	.845	.155	.775	.225	.775	.225
DD	.394	.606	.355	.645	.396	.604
Order 3						
WWW	.827	.123	.853	.147	.843	.157
WWD	.380	.620	.453	.547	.385	.615
WDW	.842	.158	.825	.175	.765	.235
WDD	.433	.567	.529	.471	.513	.487
DWW	.791	.209	.757	.243	.798	.202
DWD	.412	.588	.476	.524	.429	.571
DDW	.846	.154	.735	.265	.782	.218
DDD	.369	.631	.253	.747	.320	.680

	July Today Wet	July Today Dry	August Today Wet	August Today Dry	"Summer" Today Wet	"Summer" Today Dry
Order 1						
W	.439	.561	.453	.547	.393	.607
D	.175	.825	.223	.777	.196	.804
Order 2						
WW	.505	.495	.500	.500	.491	.509
WD	.222	.778	.279	.721	.235	.765
DW	.385	.615	.414	.586	.329	.671
DD	.164	.836	.207	.793	.186	.814
Order 3						
WWW	.423	.577	.579	.421	.556	.444
WWD	.240	.760	.345	.655	.218	.782
WDW	.448	.552	.375	.625	.368	.632
WDD	.206	.794	.168	.832	.202	.798
DWW	.592	.408	.421	.579	.423	.577
DWD	.211	.789	.232	.768	.243	.757
DDW	.366	.634	.430	.570	.317	.683
DDD	.155	.845	.217	.783	.183	.817

Appendix 2 (continued)

SEASIDE ("Wet" ≥ .20")

Sequence Preceding Today	January Today Wet	January Today Dry	February Today Wet	February Today Dry	"Winter" Today Wet	"Winter" Today Dry
Order 1						
W	.658	.342	.607	.393	.651	.349
D	.280	.720	.273	.727	.272	.728
Order 2						
WW	.643	.357	.620	.380	.644	.356
WD	.410	.590	.333	.667	.375	.625
DW	.686	.314	.586	.414	.667	.333
DD	.225	.775	.248	.712	.233	.767
Order 3						
WWW	.630	.370	.635	.365	.651	.349
WWD	.449	.551	.321	.679	.365	.635
WDW	.609	.391	.644	.356	.635	.365
WDD	.280	.720	.391	.609	.346	.654
DWW	.667	.333	.592	.408	.630	.370
DWD	.326	.674	.353	.647	.396	.604
DDW	.750	.250	.554	.446	.687	.313
DDD	.208	.792	.197	.803	.196	.804

Appendix 2 (continued)

SQUAW BUTTE ("Wet" ⩾ .01")

Sequence Preceding Today	January Today Wet	January Today Dry	February Today Wet	February Today Dry	"Winter" Today Wet	"Winter" Today Dry
Order 1						
W	.448	.552	.419	.581	.464	.536
D	.244	.756	.229	.771	.259	.741
Order 2						
WW	.438	.562	.427	.573	.476	.524
WD	.349	.651	.269	.731	.346	.654
DW	.457	.543	.412	.588	.453	.547
DD	.211	.789	.216	.784	.229	.771
Order 3						
WWW	.500	.500	.500	.500	.521	.479
WWD	.322	.678	.259	.741	.311	.689
WDW	.381	.619	.333	.667	.364	.636
WDD	.262	.738	.333	.667	.308	.692
DWW	.386	.614	.367	.633	.432	.568
DWD	.373	.627	.277	.723	.375	.625
DDW	.494	.506	.444	.556	.500	.500
DDD	.197	.803	.178	.822	.204	.796

	July Today Wet	July Today Dry	August Today Wet	August Today Dry	"Summer" Today Wet	"Summer" Today Dry
Order 1						
W	.279	.721	.273	.727	.210	.790
D	.059	.941	.054	.946	.054	.946
Order 2						
WW	.294	.706	.200	.800	.286	.714
WD	.159	.841	.135	.865	.122	.878
DW	.273	.727	.300	.700	.188	.812
DD	.052	.948	.050	.950	.050	.950
Order 3						
WWW	.000	1.000	.333	.667	.000	1.000
WWD	.273	.737	.100	.900	.200	.800
WDW	.286	.714	.600	.400	.286	.714
WDD	.051	.949	.065	.935	.023	.977
DWW	.417	.583	.167	.833	.444	.556
DWD	.121	.879	.148	.852	.103	.897
DDW	.270	.730	.257	.743	.171	.829
DDD	.052	.948	.049	.951	.051	.949