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Title: A Discrete Multi-Facility Location Analysis of a Public Sector Office Siting Problem

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A discrete multi-facility plant location algorithm developed by Basheer Khumawala is modified and applied to the location analysis of offices of the State of Oregon Health Facilities Licensing and Certification Section (HFLC). Choice of the algorithm is based upon a literature survey of available computational procedures which compares their relative efficiencies and appropriateness to the problem at hand.

The problem studied includes 211 health care facilities in seventy-seven cities throughout the state of Oregon. These facilities are regulated through inspections by the HFLC staff which is presently located in a single Portland office. The hypothesis is tested that a feasible HFLC multi-office plan can be formulated which reduces the agency's costs of operation. Khumawala's algorithm is used to compute the total costs of the present single-office, sixteen-inspector operation and to optimize the cost of a multi-office plan. Costs under centralized and decentralized office plans are evaluated; the results indicate that a fifteen per cent reduction in the annual cost of agency operations could be realized under a multi-office plan.

A sensitivity analysis is performed using a multi-factor design to study the effect of changes in parameters of cost, demand, and efficiency of personnel on the total cost and facility location results. The multiple office model is shown to remain optimal under a range of future conditons foreseen for the next four years.

A Discrete Multi-Facility Location Analysis of a Public Sector Office Siting Problem

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I. INTRODUCTION

The Problem

Two hundred and eleven (211) nursing care facilities are located in Oregon, with about 60% in the Willamette Valley and 40% at population centers across the state. These are classified as Skilled Nursing Facilities (SNF), Intermediate Care Facilities (ICF), and facilities for the mentally retarded (SNF/MR and ICF/MR). The nursing facilities are regulated through inspection by the State Health Division, Health Facilities Licensing and Certification Section. The inspection group's name describes its functions. About 80% of the Section's workload entails Federal Medicare and Medicaid certification inspections, entailing about 2000 reports per year. Annual inspections for state licensing, plus investigations of complaints from nursing care clients and their families constitute the remaining 20% of the Section's workload.

At present, all nursing homes, including hospital nursing units, are served by one staff of inspectors centrally located in Portland. Following a State Executive Department Administrative Analysis [14] of the Health Facilities Section, a question arose: can the total cost of inspections be reduced by locating inspectors at decentralized offices across the state? This thesis tackles that question.

Model Formulation

The Oregon State Health Facilities Licensing and Certification Section (HFLC) is charged with responsibility for inspecting nursing homes across the state to insure that homes comply with state licensing and Federal certification requirements. Nursing facilities, including facilities for the mentally retarded and hospital nursing care units, are presently located at 77 Oregon locations as shown in Figure 1. The distribution of homes changes little from year to year. Usually, three to five homes are added or dropped from the total each year. Each home requires several visits by an inspector each year, as outlined in Figure 2. Homes must be inspected for annual state licensing and qualification for Federal Medicare and Medicaid payments. The Medicare certification information satisfies the requirements of Medicaid, so it is reasonable that the Federal inspections in each home should be made at the same time. Provision is made in current Federal regulations to adjust the two certification periods so that the two inspections are coterminous, that is, the inspections fall due on the same date. Further, the state and Federal inspections are complementary, utilizing the same survey data, but requiring different write-ups. Hence, one survey visit is utilized for both, and again, the Federal certification period may be adjusted so that the State and Federal inspections are coterminous.

In essentially all cases, the inspector finds deficiences that must be corrected. Previously, the inspector had to make a second visit to cite and, if necessary, fine a home for a deficiency. A third visit was made to check the deficiency correction. The 1977 Oregon State Legislature changed the law so that a home may be cited and fined on the first visit; the second visit is the follow-up visit. The state requires the follow-up visit within 30 days of the original survey; the Federal Government requires a follow-up within 60 days of the inspection.

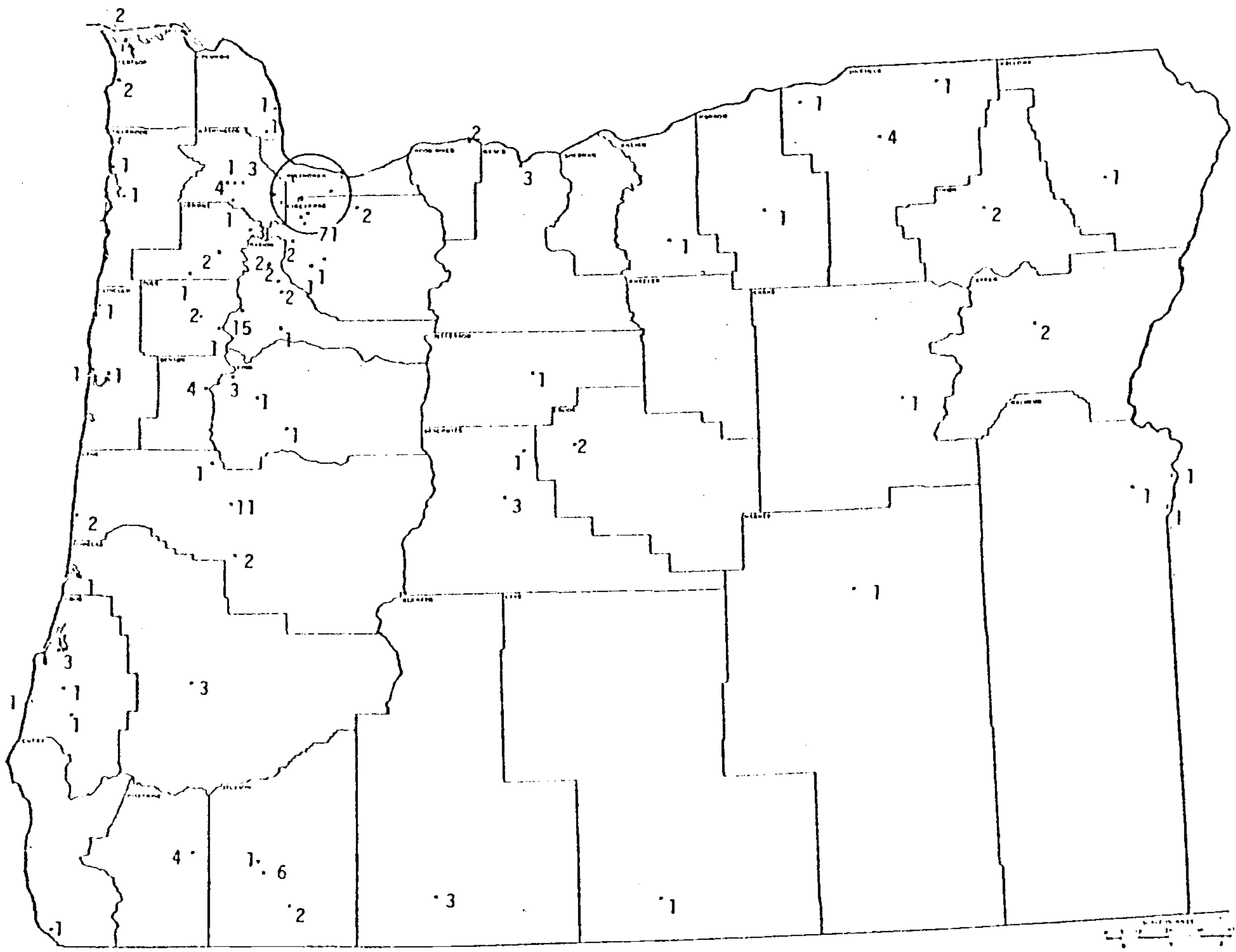

Figure 1. Distribution of nursing facilities in Oregon

Demand Per Facility	Number of Visits	Duration Each Visit (Days)	Description
1	1	3	Original survey for State licensing and Federal Medicare and Medicaid Certification; a sample of clients is interviewed, homes are inspected, and facilities may be cited and fined for deficiencies.
1	1	1	State Licensing follow-up to check corrections to deficiencies under State regulations; must be within 30 days of original survey.
1	1	1	Federal certification follow-up to check Medicare and Medicaid deficiency corrections; must be within 60 days of original survey.
1	1	1	MIPRT follow-up - review findings of Medical Independent Professional Review Team and cite deficiencies which have not been corrected.
0 - 4	2	1	Each complaint requires one investigation visit and one follow-up visit. Facilities usually receive between zero and four complaints per year; "bad" homes may be higher.
0 - 1			Changes of Ownership require repetition of the first four visits:
	1	3	Licensing and certification survey
	1	1	State licensing follow-up
	1	1	Federal certification follow-up
	1	1	MIPRT follow-up
			CHOW have varied between 19 and 30 total for the state in recent years.

Figure 2. Required Annual Visits to Each Nursing Facility

The inspector schedules the first licensing and certification visit at his discretion; the visit is unannounced, in order to maintain an element of surprise. Generally, the second and third visits then fall near the end of the 30 and 60 periods during which homes correct reported deficiencies. All visits must be completed before the end of the one-year licensing for continued state licensing and 45 days before the end of the period for renewed Federal Medicare and Medicaid certification. Approximately twenty of the 211 Oregon nursing care facilities are state-licensed but choose to remain uncertified for Medicare and Medicaid payments. In this analysis, it is assumed that all homes are both licensed and certified.

At present, a follow-up visit is also required to review deficiencies found by a Medical Independent Professional Review Team - MIPRT - from the State Division of Adult and Family Services (formerly Public Welfare Division). A MIPR team visits each home once per year to interview each client in the home and review that person's medical care. Deficiencies which MIPRT notes can only be acted upon by the Health Facilities Licensing and Certification Section, so HFLC conducts the follow-up. This interaction between state agencies causes delay, according to the State Executive Department report [14]. HFLC may assume the total MIPRT function in the future; here it is assumed, as recommended by the Executive Department [24], that HFLC conducts only the follow-up.

Beyond these required annual inspection visits, inspectors' time is required to service demands which occur more randomly. Complaints occur that require investigation, usually followed by a citation for a deficiency and a follow-up visit to check on the correction. Changes of ownership (CHOW) require that a home be relicensed and recertified under the operation of the facility's new management.

Infrequently, a home is decertified because of deficiencies. This process requires several visits by the inspector, often accompanied by the inspector's supervisor. The inspector must file reports and lengthy correspondence, and perhaps meet with the HFLC legal counsel, as facilities often initiate lawsuits to maintain their certification. Decertification is very rare, and is excluded in our study.

Inspectors are occasionally called upon for consultation visits by homes requiring information. These short visits are usually scheduled along with other visits in the area, and are here viewed to be included in the demand data for licensing and certification.

The inspector's time is taken up with travelling, on-site visits, meetings, and also with report writing. Fourteen hours are budgeted to write a Medicare/Medicaid report to the Federal Government following each certification visit; state licensing requires no report beyond the survey form completed during the visit. Follow-up and complaint visits generate additional paperwork.

Costs of inspection activities fit well into fixed, variable, and step cost categories. The inspector's office location generates a fixed cost. The inspector's salary is a fixed cost on an annual basis, or variable on a per-hour basis. Transportation cost is variable, based on mileage and cost of an inspector's time. Step costs consist of the per diem paid for meals and lodging when an inspector or the supervisor is away from home base. If reasonable cost figures can be developed, then a mathematical model can be used to express total cost of operating from some number of optimally located HFLC offices. The optimal plan can be compared with the present single-office plan to identify any potential cost savings to the State government and, ultimately, to Oregon taxpayers.

Several factors must be considered in the location analysis. Demand must be satisfied to insure that nursing home clients receive proper care. Travel expenses and the costs of operating an additional HFLC office must be determined. The required number of inspectors must be estimated. The cost implications of these factors are shown in Figure 3. As the number of offices increases, the travel cost decreases and the fixed office cost increases. Total cost is the sum of the fixed and variable cost functions, each of which includes step cost elements. It can be seen that in order to minimize cost, we must analyze the trade off between fixed office costs and variable transportation costs.

Search for a Solution Algorithm

The decentralized inspection office location problem fits well into the category of problems known as discrete multi-facility plant (warehouse) location. Some characteristics of the problem, however, are more appropriately classified by the so-called multi-terminal problem found in vehicle routing problem (VRP) literature. Both groups of literature are related to the problem at hand. The discrete multi-facility plant location model is discussed first.

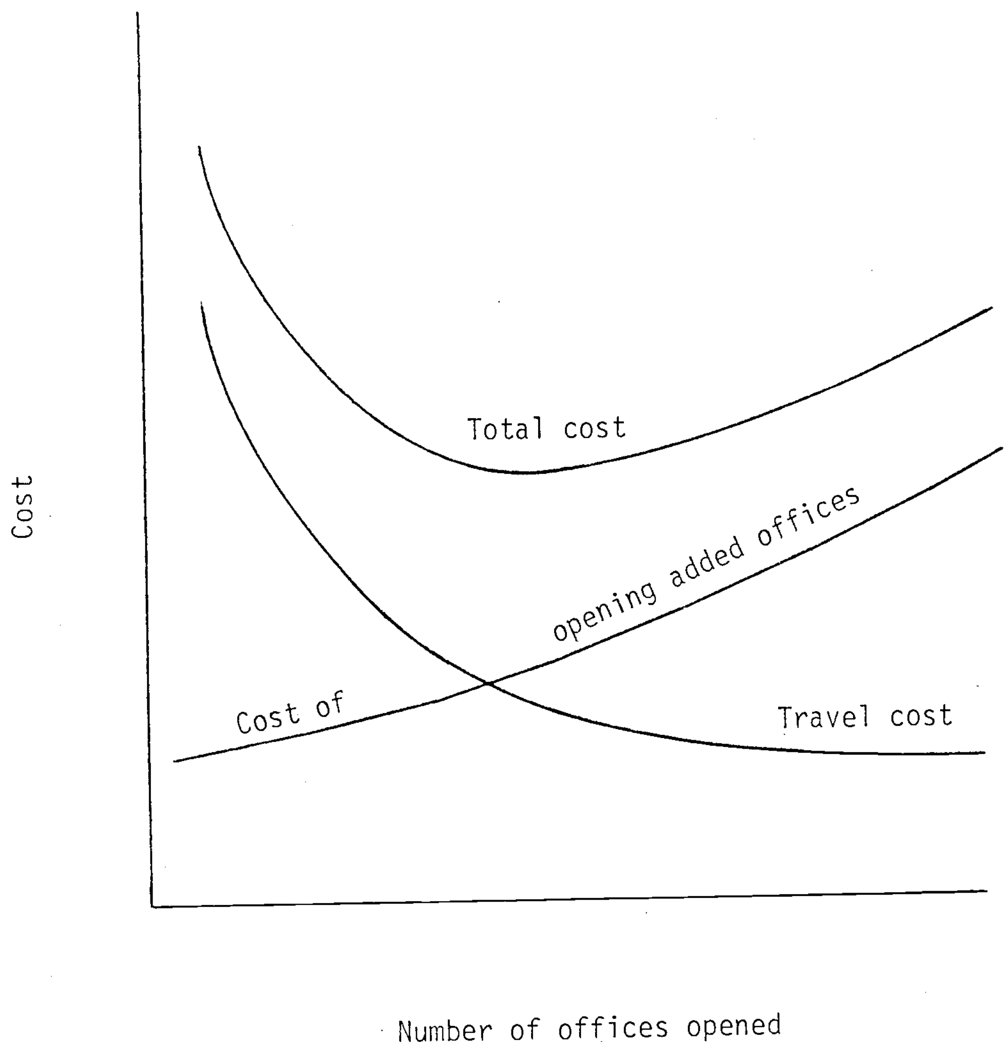

Figure 3. Relationship of Multi-Office Location Costs

II. DISCRETE MULTI-FACILITY PLANT LOCATION LITERATURE SURVEY

Problems which can be modelled in discrete multi-facility plant location algorithms are characterized by a known set of customer demands which must be satisfied by a least-cost geographic arrangement of service facilities. Common problems include the location of warehouses in a distribution system and the location of plants in a multi-plant production system. Discreteness of the problem resides in the requirement that service facilities must be located at points taken from a list of candidate sites. Given the customer demands and candidate server sites, we wish to find both the number and location of service facilities, and the size or capacity of each that minimizes the total cost.

Small multi-facility problems can be solved by inspection. With increasing size, the computational burden of the problem - a combinational one - increases swiftly. Two principal techniques are available to solve moderate to large-sized problems: heuristics and integer programming (IP).

Heuristics

Heuristic solutions employ rules or guidelines to find a good, but not necessarily optimal, solution. Kuehn and Hamburger [33] developed a heuristic solution to the location problem about fifteen years ago. Their program starts with no warehouses and locates warehouses one by one until any additional warehouse increases total cost. It then enters a "bump and shift" routine that computes the savings that would result from dropping or relocating individual warehouses. The authors ran a 50 customer, 24 warehouse problem in two minutes, 30 seconds on a rather slow IBM 650 computer.[1] Lovro [38] notes that their run times appear to increase with the number of warehouses times the number of customers.

Feldman, Lehrer and Ray [15] in 1966 extended the Kuehn and Hamburger algorithm to handle a concave cost function, F_i, the fixed annual cost of operating a warehouse. Feldman, et. al., modelled F_i as being proportional to the size of the warehouse, while Kuehn and Hamburger employed a constant fixed cost of warehouse operation. The Feldman heuristic starts with a full list of warehouses, and drops warehouses from the list to produce cost savings. The authors found that their solutions were as good as Kuehn and Hamburger's, with run times on an IBM 7094 averaging less than one minute for the 50 customer, 24 warehouse problem.[2]

Ross and Soland [48] have recently developed a heuristic based on the generalized assignment problem. They found in dealing with the uncapacitated plant location problem that their procedure was less efficient than other existing procedures because it failed to capitalize on the problem's special structure. The authors suggest that further specialization of their heuristic is necessary in order to efficiently solve the problem.

Heuristics avoid two problems inherent in integer programming solutions: large computer memory requirements and long computer processing

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[1] Kuehn and Hamburger do not identify the computer language used.

[2] Computer run times should not be taken as a strict measure of program efficiency; different machines vary widely in computing speed.

TABLE I. SOLUTION TECHNIQUES INVESTIGATED

Discrete Multi-Facility Plant Location Technique

	Exact Solutions			
Heuristics	Cutting Planes	Bender's Decomposition	Group Theoretic	Enumeration
Kuehn and Hamburger (p. 9) Feldman, Lehrer and Ray (p. 10) Ross and Soland (p. 10)	Gomory (p.19) Bowman and Nemhauser (p. 21)	Spielburg (p. 24) Ellwein and Gray (p. 24)	Shapiro (p. 22) Wolsey (p. 22)	Balas (p. 23) Land and Doig (p. 23) Dakin (p. 23) Tomlin (p. 23) Geoffrion (p. 23) Effroymson and Ray (pp. 23,26) Khumawala (pp. 23, 39, 41)
			Gorry and Shapiro (p.24) Rardin and Unger (p. 24)	
				Spielburg (pp. 24, 40) Ellwein and Gray (p. 24) Akinc (p.39)

Vehicle Routing Problem

Heuristics	Exact Solutions
Wren and Holliday (p. 34)	Svestka and Huckfeldt (p. 33)
Gillett and Johnson (p. 35)	Golden, Magnanti and Nguyen (p. 34)
Russel (p. 35)	Chistofides and Eilon (p. 33)
Tillman (p. 35)	
Golden, Magnanti and Nguyen (p. 35)	
Lin (p. 33)	
Lin and Kernigham (p. 33)	
Clarke and Wright (p. 35)	
Dantzig and Ramser (p. 34)	
Little, Murty, Sweeney, and Karel (p. 33)	

times. Traditionally, heuristics were the only practical tool for solving large problems. The first generation of computers ran ten to twenty times slower than the present generation machines, and limited memory was often a restriction on problem size. Larger, faster computers and the development of efficient integer programming formulations of the multi-facility location problem have made IP an attractive solution technique for many problems of a useful size. As McGinnis notes in a recent survey:

> "With advances in computer technology and the state of the art in integer programming, many previously intractable problems are now being solved." (McGinnis, [41], p. 11)

IP Principles

While heuristic solutions are approximate, IP yields exact solutions that optimize total cost. An IP can be described as a linear program (LP) in which all coefficients X_{ij} in the objective function are constrained to take on integer values. A mixed integer program (MIP) requires that some subset of the X_{ij} be integers. The facilities location problem is appropriately modelled by an MIP in which the integer subset of variables is constrained to values 0 and 1 to denote, respectively, a warehouse closed or opened at a prospective site. We shall denote the integer variables Y_j as a Y vector, corresponding to j possible office sites Y_j. The problem is stated most simply as Problem "P_o."

Problem P_o: minimize $$\sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}X_{ij} + \sum_{j=1}^{n} f_jY_j \qquad (1)$$

$$\text{s.t.} \quad \sum_{j=1}^{n} X_{ij} = D_i \qquad i = 1, 2, 3, \ldots, m \tag{2}$$

$$\sum_{i=1}^{m} X_{ij} \leq Q_j Y_j \qquad j = 1, 2, 3, \ldots, n \tag{3}$$

$$X_{ij} \geq 0 \tag{4}$$

$$Y_j = 0, 1 \tag{5}$$

where: m = number of customers

n = number of possible plants (candidate office sites)

X_{ij} = fraction of demand of customer i which is satisfied by a plant located at site j

$Y_j = \begin{cases} 1 \text{ if a plant is located at site j} \\ 0 \text{ otherwise} \end{cases}$

C_{ij} = cost of supplying the entire demand of customer i from a plant located a site j

f_j = fixed cost resulting from locating a plant at site j

Q_j = capacity of candidate site j

D_j = customer i demand

The first set of constraints (Equation 2) requires that each demand be satisfied while the second set of constraints (Equation 3) puts capacity constaints on the candidate facility sites. The capacity constraints, Q_i, can be dropped if there are no capacity restrictions or made arbitrarily large and then tightened to anlyze the effect of imposing such restrictions. The first term of the objective function summarizes all variable costs (VC); the second term collects all fixed costs (FC) associated with opening a plant site. The model can be viewed as a trade off between FC and VC.

Existing IP techniques fall into four categories: enumeration, Bender's Decomposition, cutting planes, and group theory. All techniques for solving the IP formulation use three basic strategies: separation, relaxation, and fathoming. These procedures provide a framework for study of the various solution algorithms.

Separation

In most algorithms, the first step is to make a reasonable attempt to solve Problem P_0. If, in the result, all integer variables, Y_j, are not integer-valued, P_0 is separated into several subproblems, called descendents, each of which constrains one Y_j to each of the integer values it may assume. In Problem P_0, two subproblems are formed, with a particular Y_j set equal to 0 and 1. This initializes a list of subproblems or candidate problems (CP). One CP is selected from the list and its solution is attempted. If it can be solved, a new problem is selected from the candidate list and its solution is attempted; otherwise, its descendents are separated and added to the candidate list. This separation procedure continues until the candidate list is exhausted. If no CP is feasible, then P_0 is infeasible. The best solution at any point is the lowest cost solution that possesses an all-integer Y vector. The final such minimum cost solution must be the optimum solution of P_0, provided that degeneracy in any separation of a CP is limited by only one descendent of the separation being feasible.

Relaxation

Any IP problem can be relaxed by loosening or dropping any of its constraints. The only restriction on the relaxed problem is that its set of feasible solutions falls within the feasible solution space of the original Problem P_0. Dropping the integer constraints on the Y vector is a relaxation of P_0. Call the relaxed problem P_r. According to the solution space restriction, if P_r has no feasible solutions, then neither has P_0. The minimum P_0 objective function must be greater than or equal to the P_r minimum, and if the P_r result is feasible in P_0, then it must be an optimum solution to P_0. Regrettably, the objectives of finding a P_r that is easy to solve, and one whose solution satisfies P_0, conflict. As we make P_r easier to solve, in general, the gap between P_r and P_0 grows larger.

Fathoming

The separation procedure yields a combinatorial number of candidate problems which must be evaluated or fathomed. It is desired to find whether or not the feasible solution space of each CP may possibly contain an optimal solution of P_0. If not, the CP can be eliminated from further study; if so, one goes on to try and find the CP optimum. If it can be determined that the CP cannot yield a feasible solution better than that found so far, then again the CP can be dropped from further consideration. The best solution found at any point that satisfies the original Problem P_0 is called the incumbent.

Suppose a particular CP has been relaxed to CP_r. If CP_r has no feasible solution, then the same is true of CP. It is said that CP has been fathomed; it can be eliminated from further investigation. If CP_r has some feasible solution, CP_r^*, and that solution is greater than the current incumbent, then CP is again fathomed, and can be eliminated. Otherwise, the relaxed problem optimum CP_r^* must be further evaluated. If CP_r^* is feasible in P_0, it becomes the new incumbent. Alternately, if CP_r^* is not feasible in P_0, it must be separated and its descendents added to the CP list, or one must persist in trying to fathom CP by choosing a new relaxation, CP_r'.

Geoffrion and Marsten [19] summarize this procedure for fathoming, or completely evaluating, a CP in terms of three fathoming criteria (FC). A CP is fathomed if any one of the criteria is satisfied. F(P) denotes the solution set of problem P. Z* is the current incumbent. The fathoming criteria are:

(FC1) An analysis of CP_r reveals that CP has no feasible solution; e.g., $F(CP_r) = 0$;

(FC2) An analysis of CP_r reveals that CP has no feasible solution better than the incumbent; e.g., $CP_r^* \geqq Z^*$;

(FC3) An analysis of CP_r reveals an optimal solution of CP; e.g., an optimal solution of CP_r is found which happens to be feasible in CP.

Separation, relaxation, and fathoming form a circuitous procedure, as shown in Figure 4. Decision rules within the procedure have a marked effect on the speed with which the optimal result is found. It is desirable to quickly obtain a good solution to P_0. The first incumbent will then lead to elimination of, it is hoped, many CP's by FC2. The candidate selection rule, also, is important in efficiently eliminating CP by FC2, and also in reaching the optimal result for P_0 by FC3. Two candidate selection rules are used: Last-In-First-Out (LIFO) and

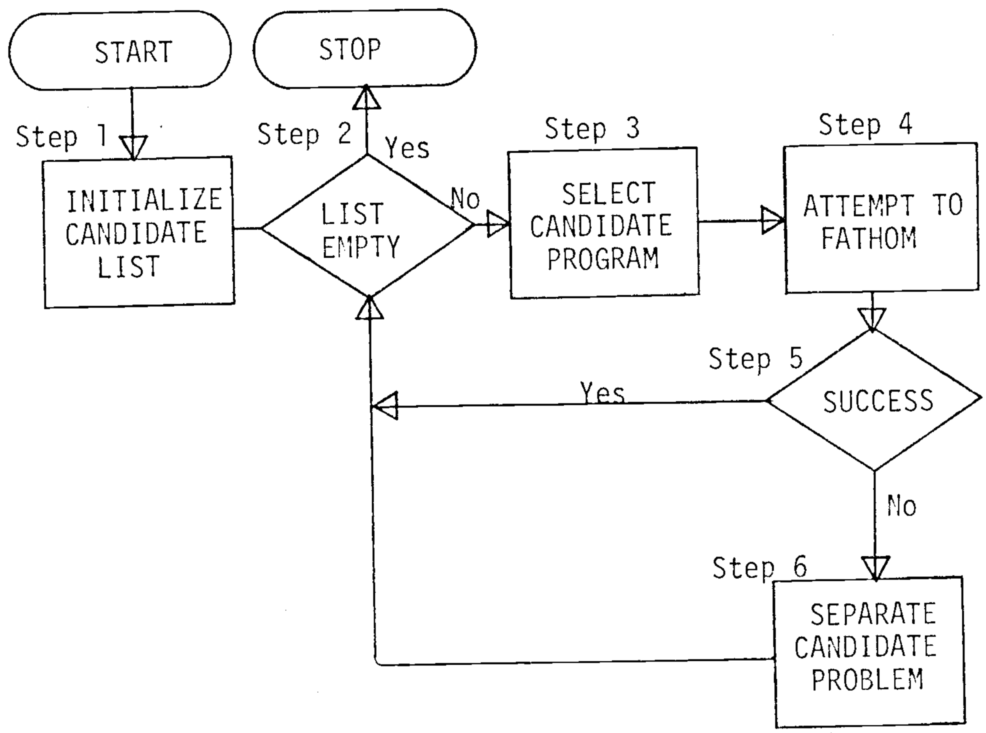

Figure 4. General form of enumerative algorithms.
(from [41], p. 13)

Priority. Under the LIFO rule, the last problem to be added to the candidate list is selected. Under the Priority rule, an index is assigned to each CP that determines the order of CP selection. An example of a priority index is the lower bound (LB) on the optimal solution of a CP. This is the value CP_r^*, not feasible in P_0, which is set aside in the fathoming procedure. The need to store the unfathomed (CP_r^*) values in memory increases the information storage requirements of Priority over LIFO.

IP Algorithms

The three strategies of separation, relaxation, and fathoming are, in general, standard procedures in all IP algorithms. Current IP algorithms can be segregated into four categories that differ principally in the approach used to fathom the candidate problems.

Cutting-Plane Algorithms

Historically, cutting-planes was the first approach used in solving the IP formulation. First used by Gomory [22], the approach relaxes all integrality constraints and solves the associated LP to obtain an initial feasible solution, CP_r^*. FC1 and FC3 are then applied until termination. The separation routine resulting from FC2 is never used. Rather, each time CP_r passes FC1 and FC3 without being fathomed, the problem is tightened by adding a linear constraint, or "cutting plane" (see Figure 5). The linear program to be solved at the n^{th} execution of CP_r, consists of the original LP with all integrality constraints dropped and n-1 linear constraints added.

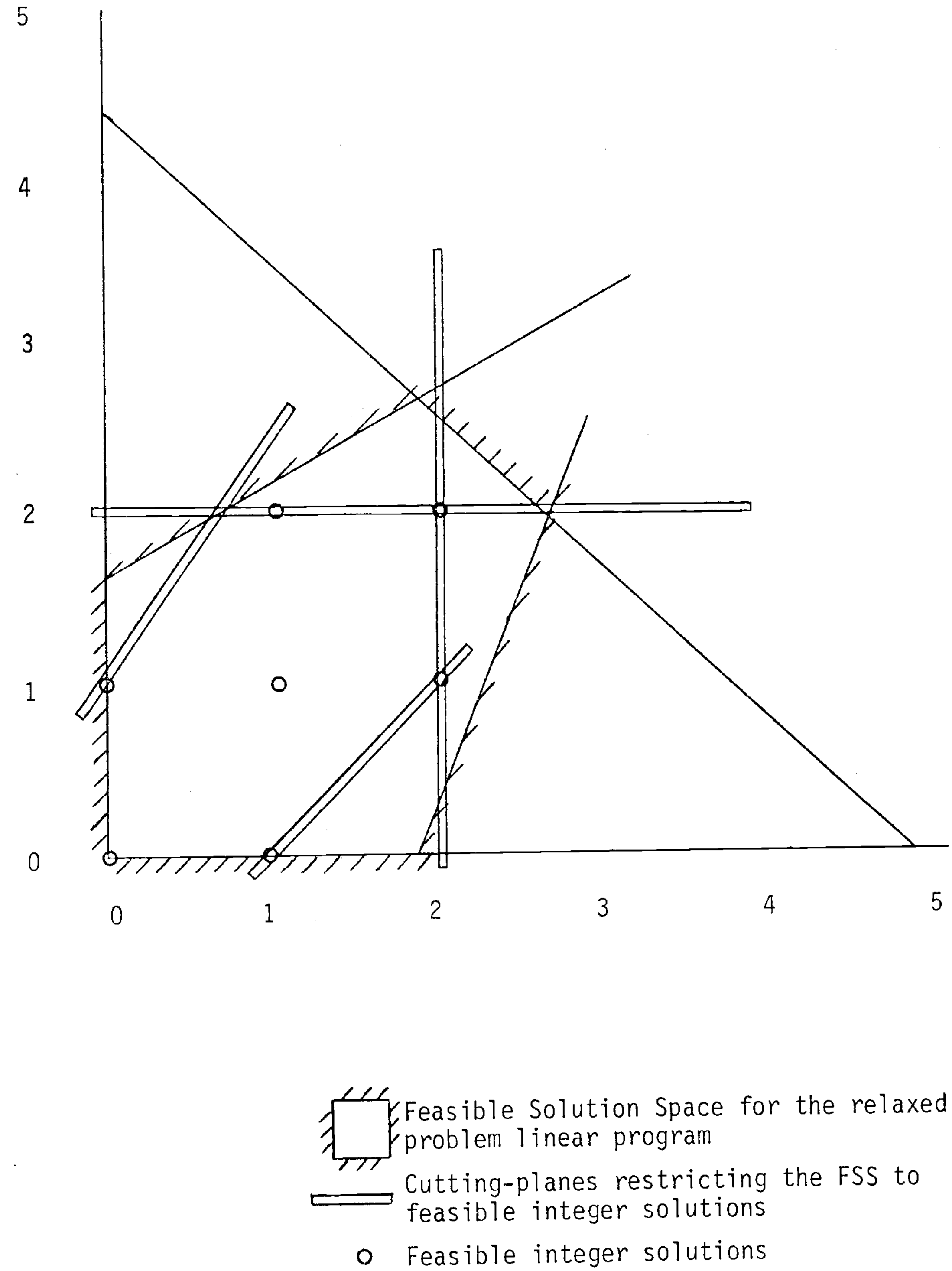

Figure 5. Illustration of Gomory Cutting-Planes

Each new cut must correctly tighten the previous relaxation, yet still yield a valid relaxation of the CP. That is, a portion of the feasible solution space of a current LP must be eliminated without lopping off any feasible integer solutions of the original LP. Several researchers besides Gomory have developed methods to do this. One example is Bowman and Nemhauser [5].

Bender's Decomposition

In certain integer programs, and MIP problems including the warehouse location problem P_0, fixing the values of the integer-valued Y vector results in a special problem structure for the remaining continuous variables X_{ij}. For any feasible solution containing an all integer-valued Y vector, the remaining optimization problem is an LP network optimization as Wagner [63] points out.

Bender's Decomposition assigns feasible values to the discrete variables Y_j, $j = 1, 2, 3, \ldots, n$; in the warehouse location case, these are binary assignments of 0 or 1. The remaining LP is then solved, and the entire discrete/continuous solution (the first incumbent) is recorded. The solution is modified by a system of linear restrictions on its integer-valued variables. If untested feasible integer Y vector values are found to still exist in the feasible solution space, one (or more) Y_j assumes one (or more) untested integer value(s) to form a new feasible Y vector solution. The remaining continous X_{ij} variables are again included in the solution of the linear program, and the complete solution is compared with the incumbent and replaces it if the objective function value is lower. This process continues until all the Y vector alternatives have been explored. A branch and bound process is often used to keep track of the search process.

Group Theoretic Approach

This technique solves integer programs by exploiting the so-called group theoretic properties of the problem. The group theoretic approach was initiated by Gomory and extended by Shapiro [51], [52]. The procedure has been applied almost exclusively to the pure integer programming formulation, that is, the MIP without any continuous variables X_{ij}. Wolsey [65] applied the Group Theory technique to MIP in 1971 to solve a small problem.

In general terms, the group theoretic approach forms the dual of the linear program relaxation of Problem P_0. This dual relaxation is then tightened by adding constraints on the integer variables. The feasible solution space is narrowed to include only feasible integer solutions from which the optimum is then selected.

Enumerative Algorithms

Enumerative algorithms encompass those procedures which use implicit enumeration or branch and bound (B & B) to methodically search the set of all possible integer solutions. B & B algorithms use various strategies to "prune" from the B & B "tree" all CP which cannot lead to an optimal result. In B & B, when a problem is separated into descendent subproblems, each new CP becomes a branch off of the problem node. A tree is formed, as shown in Figure 8 (p. 43), since each Y vector value is represented by a path from the original problem node - the root on the left - to the final branch node at which the Y vector value, or CP, is fathomed. A branch is bounded when, at some node in the branch, the objective function value exceeds the present least upper bound obtained from some other branch. The branch is then pruned or eliminated from further investigation under FC2.

Enumerative algorithms fall into two general categories resulting from early work in two separate areas. The first, originally developed in this country by Balas [2], applies only to all-integer problems. Such algorithms fathom candidate problems using logical implications found within the problem constraints. The application of these procedures has been limited, according to Geoffrion and Marsten [19] by computing times that grow exponentially with the number of variables.

A second category, pioneered by Land and Doig [34], bases the fathoming test on the linear programming relaxation of P_0. This procedure has lead to several efficient computer programs for mixed as well as pure integer programs, including Dakin [10], Tomlin [61], Geoffrion [18], and the B & B procedure of Efroymson and Ray [12] subsequently modified by Khumawala [30].

Overview of IP Model Formulations

Of the four categories of integer programming techniques delineated here, the bulk of development work has been in enumerative algorithms. This was pointed out by Geoffrion and Marsten in their 1972 survey [19], and again in 1977 by McGinnis [41]. Computer program efficiency is very important in solving problems of a useful size, as the model is combinatorial. The early cutting-plane algorithms showed highly irregular computational performance; McGinnis contends that this discouraged their development. Geoffrion and Marsten suggest a method for using the cutting-plane approach to fathom candidate problems in an enumerative algorithm. This has not been applied in the literature. Gorry and Shapiro [23] outlined a similar strategy of combining enumeration with a group theory approach.

Rardin and Unger [46] used a group theoretic approach to develop tight bounds on an optimal solution in the branch and bound procedure. After solving CP_r to obtain CP_r^* as the lower bound on the optimal result, they attempted to improve the LB using group theory.

Bender's inequalities have found application as a fathoming procedure in enumerative algorithms by Spielburg [54] and Ellwein and Gray [13], though neither appears to be as efficient as the simplified procedure used by Khumawala [30] (see Table II).

As noted previously, the bulk of development of exact IP solution techniques to solve the discrete multi-facility location problem has centered around enumeration technqiues. The special structure of P_0 lends itself well to implicit enumeration or B & B strategies. McGinnis states:

> " ... problem (P_0) has been a popular subject for study, primarily because it has a structure to which general purpose algorithms may readily be adapted." (McGinnis,[41], p. 12)

This special structure can be exploited by rearranging the objective function:

$$\text{Problem } P_1: \quad \text{minimize } \{ \sum_{j=1}^{n} f_j Y_j + \text{minimum} \sum_{j=1}^{n} \sum_{i=1}^{m} C_{ij} X_{ij} \} \qquad (6)$$

TABLE II. COMPUTATIONAL RESULTS FOR STANDARD PROBLEMS (CPU TIMES IN SECONDS).

Capacities Q_k	5000				15000				Uncapacitated			
Fixed Costs f_k	7.5	12.5	17.5	25	7.5	12.5	17.5	25	7.5	12.5	17.5	25
*(Open Init./Open Opt.)	(11/12)	(11/12)	(11/12)	(11/12)	(9/11)	(7/9)	(3/7)	(3/5)	(9/11)	(7/9)	(3/5)	(3/4)
Sa [50]	90.6	108.0	96.0	87.6	94.2	457.2	900+	-	-	-	-	-
Soland [53]	38.7	5.8	4.2	-	-	271.3	-	-	-	-	-	-
Akinc [1]	10.21	9.15	9.26	9.58	0.23	0.43	38.65	34.39	47.5	0.44	0.30	0.15
Ellwein & Gray [13]	-	88.8	92.4	-	-	127.8	-	-	-	15.0	63.0	-
Buffin [6]	-	27.0	25.0	-	-	87.0	-	-	-	43.8	28.2	-
McGinnis [40]	2.1	2.5	2.8	3.5	-	-	-	-	-	-	-	-
					16x50 problems							
(Open Init./Open Opt.)	(11/17)	-	-	-	(10/15)	(6/11)	-	-	(10/15)	(6/11)	(4/8)	(2/4)
Khumawala [30]	-	-	-	-	-	-	-	-	1.50	1.36	1.66	0.85
Akinc [1]	0.23	120+	120+	120+	0.75	7.75	120+	120+	0.68	1.65	1.34	0.46
Ellwein & Gray [13]	-	-	-	-	-	-	-	-	196.2	-	-	-
Buffin [6]	-	-	-	-	-	-	-	-	123.0	-	-	-
					25x50 problems							

+ : time limit exceeded

* : refers to the number of sites fixed open initially by the Δ- and Ω-simplifications discussed in Chapter V, Section 3

Table is from McGinnis [41], p. 16.

s.t. (5) s.t. (2), (3), (4)

The inner minimization can be solved as a transportation problem if the capacity constraints (3) hold, or, if not, by assigning each demand to its least cost source and solving n one-row optimizations. This partitioning is then used to solve P_0 by implicity enumeration, as in Balas [3]. Frequently, a B & B procedure is used to keep track of subproblems which have been solved, until all Y vector values have been treated.

Efroymson and Ray [12] exploited the structure of the problem to obtain an efficient B & B optimization procedure. The authors relaxed the integer constraints on the Y_j (Equation 5) to

$$0 \leq Y_j \leq 1. \tag{7}$$

Their 1966 paper noted that, with this change, the constraints (Equation 3) would hold as equalities in the optimum solution. Therefore, the integer Y_j were eliminated from the objective function to form the relaxed problem:

Problem P_2: minimize $$\sum_{i=1}^{m} \sum_{j=1}^{n} [C_{ij} + \frac{f_j}{Q_j}] X_{ij} \tag{8}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} X_{ij} \leq Q_j \tag{9}$$

$$\sum_{j=1}^{n} X_{ij} = D_i \quad i = 1, 2, 3, \ldots, m \tag{2}$$

$$X_{ij} \geq 0 \tag{4}$$

$$0 \leq Y_j \leq 1 \tag{7}$$

Both the implicit enumeration (Equation 6) and relaxed (Equation 8) problems have a transporation problem structure. Thus, procedures more efficient than a general IP may be used to enumerate and solve candidate problems.

Limitation of Discrete Multi-Facility Algorithms for the HFLC Analysis

The most suitable technique for modelling the HFLC facility location problem is the discrete multi-facility formulation. Various problem characteristics support this. First, the relevant costs of the HFLC problem fit well into categories of variable cost based on mileage and fixed cost attributable to opening an inspection office. Second, the objective function of minimized cost of HFLC operations matches the model objective function, with its trade off between variable cost of transportation and added fixed cost of facility location. Third, the number of inspectors located at each office location should be unconstrained, and the present office in Portland should have no particular advantage or weight over the other possible locations. The model encompasses these assumptions.

HFLC office site selection requires that certain cities be denoted as possible office locations; correspondingly, the model locates warehouses at given (discrete) locations. Finally, efficient optimizing procedures exist for solving the discrete location model.

These characteristics all support selection of a discrete multi-facility location algorithm to model the HFLC problem. The fit is not perfect, however. All such multi-facility algorithms are modelled after a problem in which the distance function to be minimized is the total of straight line distances from each warehouse to each customer. In the HFLC case, this distance would be the Euclidean path from an inspection office to each of the cities served by the office (see Figure 6a). This travel pattern assumes that an inspector makes a single visit to a home and then returns to the office. This is a valid assumption for optimal location of facilities, as the random nature of demands widely spaced in time will result in most trips following the office-to-facility-and-back round trip. Figure 6b illustrates this for a small randomly generated problem in which inspectors visit more than one facility on a single trip. Interestingly, in the absence of data on the interaction between facilities, an exact multi-facility algorithm now yields an approximate best-guess result if it is possible to visit more than one demand location on a single trip.

The total cost, however, for centralized and decentralized plans reflect a bias in the following manner. With the inspection office located in Portland, inspectors frequently travel long distances to nursing facilities, and are lodged for one or more nights at the distant location. Inspectors fill out travel proposals for overnight lodging. In reviewing their agendas for a three month period in 1976, it was noted that inspectors usually stayed overnight if one way distance to the facility was greater than 60 miles. Forty-seven cities, or 62% of Oregon cities served are more than 60 miles from Portland. A lodging per diem is paid if the inspector is more than 50 miles from home. With travel over long distances required to many Oregon cities, inspectors will often schedule several days worth of work in the general area visited. In essence, they form tours. Often the tour is

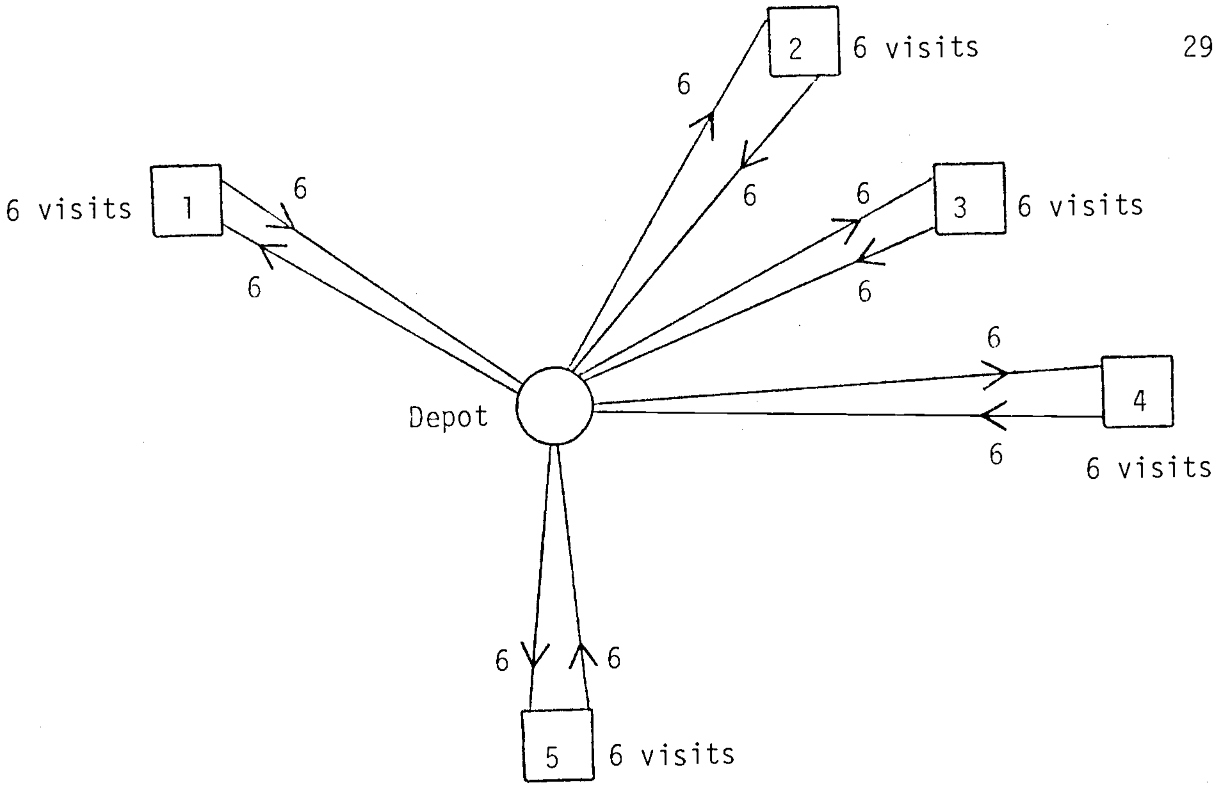

Figure 6a. Travel pattern under discrete multi-facility location with six visits required to each demand site.

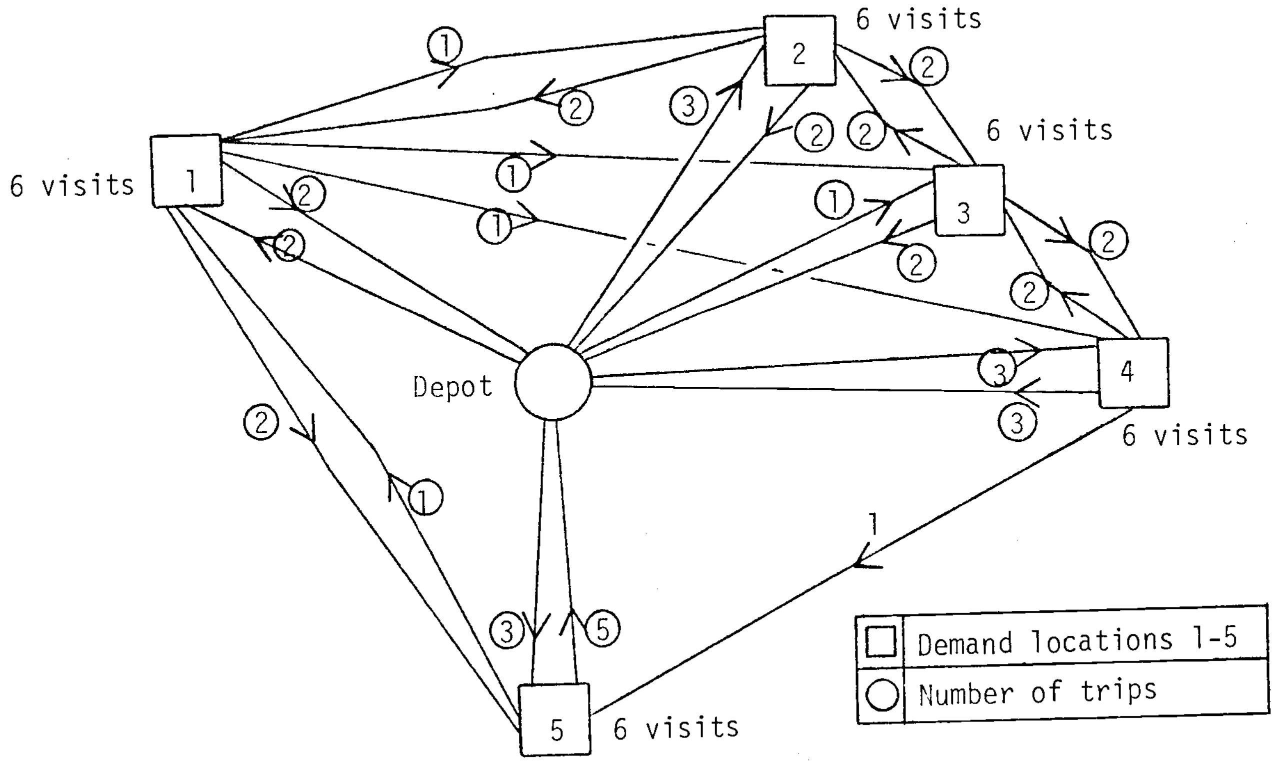

Figure 6b. Travel patterns with tours, for a set of randomly generated tours with six visits required to demand sites.

formed by travelling to a particular facility, and scheduling visits to other nursing homes in the area on the return trip if time permits. It would be expected that inspectors return home on weekends, so tours would be a maximum of one week in length. This occasionally is not true, with inspectors remaining lodged at a distant city over a weekend. Tour building is not a general rule, either. An inspector will often travel more than 60 miles, conduct a short visit, and return. Even when visiting a facility at a long distance, inspectors will return after a visit because of meetings or a report deadline.

III. VEHICLE ROUTING PROBLEM LITERATURE SURVEY

A problem of n demand centers and m possible supply points can be modelled as a multiple terminal delivery problem. Formulation for this problem follows the physical model of a vehicle routing problem (VRP) in which several trucks are dispatched from m depots to supply single demands of n customers. The routing models with fixed depots are constrained by fleet, delivery point, and route structure restrictions.

Travelling Salesman Problem

The vehicle routing problem is one of several variants and extensions of the ubiquitous Travelling Salesman problem. The problem has received much attention, as chronicalled by Bellmore and Nemhauser [4]. The Travelling Salesman problem can be formulated as an integer program as follows, with integer variables X_{ij} valued:

$$X_{ij} = \begin{cases} 1 \text{ if the salesman goes from city i to city j} \\ 0 \text{ if otherwise} \end{cases}$$

Then if c_{ij} denotes the cost of travel or distance between city i and city j, the objective function is:

$$\text{minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} X_{ij} \tag{10}$$

The following 2n constraints insure that each city will be included in the tour once and only once:

$$\sum_{i=1}^{n} X_{ij} = 1 \qquad j = 1, 2, 3, \ldots, n \tag{11}$$

$$\sum_{j=1}^{n} X_{ij} = 1 \qquad j = 1, 2, 3, \ldots, n \tag{12}$$

In the travelling salesman problem, the salesman must return to the depot from which the route originated. These constraints do not eliminate the possibility that a route will end somewhere other than the central depot. Such a tour through a subset of the n cities is termed a subtour. Constraints must be added to eliminate all subtours of length (n-1) or less. The following $(n-1)^2 - (n-1)$ constraints are added:

$$Y_i - Y_j + nX_{ij} \leq n-1 \qquad \begin{array}{l} i = 2, 3, \ldots, n \\ j = 2, 3, \ldots, n \\ i \neq j \end{array} \tag{13}$$

where (n-1) additional variables (Y_j, j = 2, 3, ..., n) are added. These constraints exclude subtours by the following argument. Any solution containing a subtour must have at least two subtours, since each city must be visited once, by the constraints (11) and (12). Therefore, a subtour always exists which does not contain some city s. Suppose that such a subtour of length K exists, K ≤ (n-1), with K variables X_{ij} equal to one. Adding up the K constraints from (13) for these K variables yields the inequality

$$Kn \leq K(n-1)$$

which is false for all K > 0.

The constraints in (13) do not eliminate any feasible tours, as shown by Cooper and Steinburg [9]. Suppose a feasible tour exists in which city i is the m_i-th visited, i = 1, 2, 3, ..., n, and $Y_i = m_i$, i = 1, 2, 3, ..., n. If $X_{ij} = 1$ in this tour, city j is visited immediately after city i, so that $Y_j = m_i + 1$ and constraint (13) is satisfied for X_{ij} since

$$Y_i - Y_j + nK_{ij} = m_i - (m_i + 1) + n\,(1) = n - 1.$$

If $X_{ij} = 0$ in this tour, (13) is also satisfied for X_{ij}, since $(Y_i - Y_j)$ is at most equal to (n-1). The largest Y_i is n; the smallest is one.

The IP Travelling Salesman formulation has n^2 variables X_{ij}, (n-1) variables Y_i, and a total of $n^2 - n + 2$ constraints. Other approaches, both heuristic and exact, include Lin [35], Lin and Kernighan [36], Christofides and Eilon [7], and Little [37].

The Travelling Salesman problem is a vehicle routing problem with one depot, and one vehicle or server whose capacity meets or exceeds total demand. The model can be extended to several vehicles, several depots, different vehicle capacities, and restrictions on route length.

Multiple Travelling Salesman Problem

The Multiple Travelling Salesman Problem (MTSP) accounts for more than one vehicle (salesman). All salesmen report to one central depot. The objective is to visit each demand point exactly once by one of m salesmen, so that total demand is satisfied and total distance travelled by all m salesmen is minimized. There are no capacity or route length restrictions; that is, each salesman has sufficient capacity to satisfy total demand. Svestka and Huckfeldt [55] solved the problem exactly, using a B & B procedure, with up to 60 demand locations.

Vehicle Dispatching Problem

The direct extension of the MTSP again deals with a set of m delivery routes terminating at one central depot. Each demand point has a known demand requirement that is satisfied by one visit of a salesman.

The problem adds capacity and maximum route time constraints to the tours. The problem was first considered by Dantzig and Ramser [11].

Multi-Depot Vehicle Routing Problem

The vehicle dispatch problem can be altered slightly to allow multiple depots. Again, a single visit satisfies all demand of each location; demand at any location does not exceed the capacity of any truck (salesman). Integer programming and heuristics are possible solution techniques.

IP Formulation

An IP for the multi-depot VRP was formulated by Golden, et.al. [21]. The formulation is of interest for very small problems only, with the number of variables equal to $(n^2)*(NV)$, where NV is the total number of possible salesmen, and n is the number of demand points. For comparison, the MPOS (IP) system on Oregon State University's CDC Cyber 73 will accept up to 100 variables; the HFLC problem encompasses about $(n^2)*(NV) = (77^2)*(20) = 118{,}580$ variables. With the HFLC problem beyond the scope of the optimizing procedure, heuristic solutions were next explored.

Heuristic Solution Techniques

While the VRP has been widely studied, the multi-depot problem is represented in the literature by only a few papers.

Wren and Holliday [66] generate one solution arbitrarily, and then improve the solution by exchanging nodes one at a time between routes until no further improvement can be made. The authors report results with problems of up to four depots and 320 demand points.

Gillett and Johnson [20] solve the multi-depot problem in two stages. First, locations are assigned to depots by partitioning the problem into subproblems. Then, several single depot VRP's are solved independently.

Russel [49] developed a multi-depot heuristic which showed good results using the Lin and Kernighan heuristic [36] for the single-tour travelling salesman problem. The heuristic is attractive, since it is the only multi-depot heuristic which incorporates sequencing and due date restrictions, and time constraints on tours. Unfortunately, it cannot solve moderate-sized or large problems due to high computer storage space requirements:

$$[2 * (N + M)^2 + 52* (N + M) + 9000] = [2 * (77 + (20*50))^2 +$$
$$52 (77 + (20*50)) + 9000] = 2{,}843{,}862 \text{ words.}$$

The HFLC problem thus requires approximately 2.8 million words of core storage, far beyond current computer capacity.

Tillman [57] developed a heuristic based on the single-terminal heuristic of Clarke and Wright [8]. Tillman's heuristic forms an initial solution by assigning each demand point to the nearest depot, with one salesman serving each demand point. The solution is by joining points on a route to minimize the distance travelled. The route formed is assigned to the terminal associated with the improvement, and the number of salesmen necessary to meet demand on the route are assigned. See Figure 7.

Golden, et.al. [21] also proposed an algorithm based on Clarke

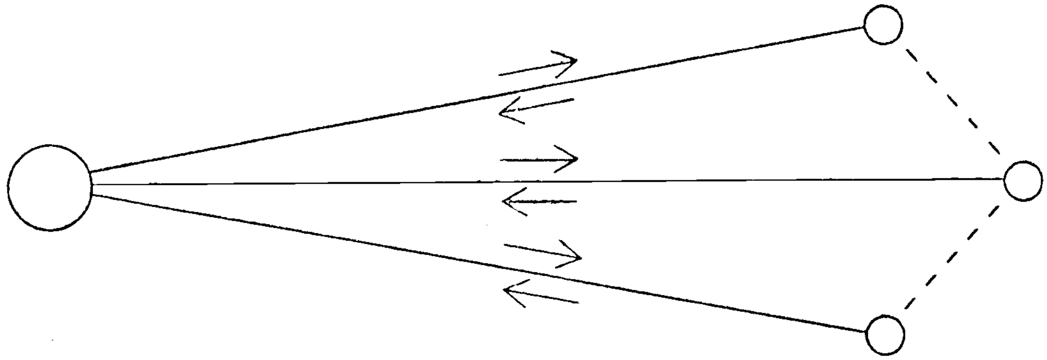

Figure 7a. Link demand sites F_1, F_2, F_3 to Depot.

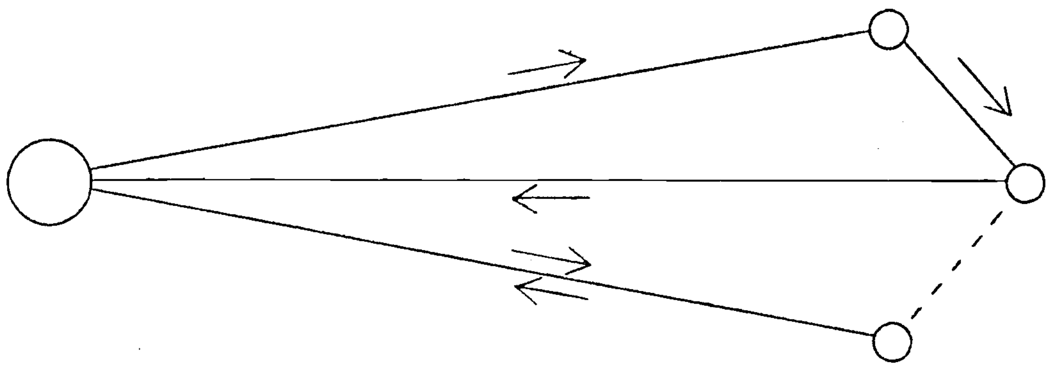

Figure 7b. Link demand sites F_1 and F_2.

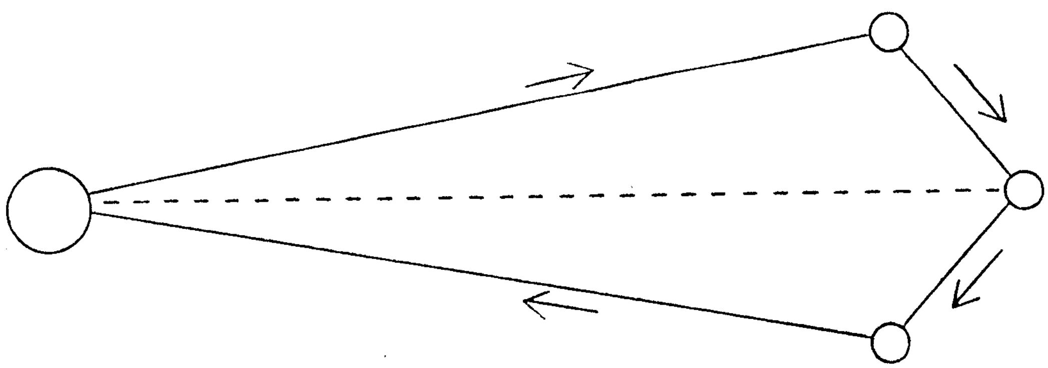

Figure 7c. Link demand sites F_1 and F_2 and demand sites F_2 and F_3.

and Wright's single-terminal heuristic. The authors obtained solutions to five problems from Gillett and Johnson [20] in 4-10 times less CPU time, but having 2-8% higher solution values. The largest problem attempted by Golden, et. al. was the largest problem solved by any heuristic. A problem with two depots and 600 demand points was solved in less than 55 seconds on an IBM 370/168.

IV. SELECTION OF A SOLUTION PROCEDURE

Multi-Depot VRP Heuristics Versus Discrete Multi-Facility Algorithms

The multi-depot VRP has been investigated with an eye toward modelling the HFLC problem. In order to model the problem, inspectors would make weekly tours for a total of about (50 working weeks/inspector) * (1 tour/week) * (20 inspectors total) = 1000 tours. Multiple visits to each demand location would occur, and time constraints on follow-up visits would be satisfied by the tours formed. Total cost of transportation and facility location would be minimized. None of the VRP heuristics can do this. All reflect the structure of the truck routing problem, with a single demand at each location satisfied by the single visit of a server. None of the programs developed so far can model weekly tours, yet also satisfy total demand over a year. The one heuristic [49] which does model sequencing and time constraints cannot model a problem of this size. All the multi-depot VRP heuristics are involved, lengthy and inefficient as compared to the more efficient discrete multi-facility plant location algorithms. A highly specialized and inefficient algorithm would be of little future use to the State Executive Department.

Adaptation of one of the multi-depot VRP heuristics to solve a problem with the HFLC problem characteristics was judged to be prohibitive, if not impossible.

With multi-depot VRP heuristics ruled out as infeasible at their current state of development, the problem solving focus returned to the discrete plant location algorithms.

If additional data were known, tours would be simulated in a discrete multi-facility model. If the transition probabilities - the probabilities of travel from one location i to another demand or office location j - were known, tours could be simulated. Sequencing and time constraints on tours could be satisfied; tour building would continue until a counter in the program determined that demand at all locations had been satisfied. Several sets of simulated data could be entered into the discrete multi-facility algorithm, with each tour's mileage and demand treated as one demand location. Such data is not known, even approximately, in this problem. Without data to support a simulation, this analysis reserves that approach for the area of further study.

Algorithm Selection

With multi-depot VRP formulations proving to be infeasible, and simulation impossible, it was decided that an efficient multi-facility plant location algorithm, with its close representation of the problem, should be pursued for office location with an error term attached to resulting cost estimates.

The discussion of discrete multi-facility plant location algorithms noted that heuristics have been superseded by MIP solutions for problems of moderate or large size. Only really large problems of several hundred demand locations must rely on heuristics. Of the MIP techniques, enumerative algorithms have been most well-developed. Of these, the branch and bound algorithms of Akinc [1] and Khumawala [30] offer the greatest efficiency, according to McGinnis' March 1977 survey [41].

Table II, p. 25, shows computational run times for several MIP formulations. In addition, Lovro [38] in 1975 compared the B & B algorithms of Spielburg [54] and Khumawala. Spielburg's algorithm offers the capability to make use of a previous solution or a good solution that is not optimal. This gives Spielburg's algorithm the capability of solving large problems. Khumawala's algorithm, on the other hand, appears to be more efficient, although a direct comparison on similar computers has not been published. Akinc's and Khumawala's algorithms offer similar efficiency. Due to its high efficiency, and the availability of the computer code, Khumawala's algorithm was selected to model the problem.

V. DESCRIPTION OF THE ALGORITHM

The Objective Function

Khumawala's algorithm uses an improved version of a branch and bound formulation originally formulated by Efroymson and Ray [3]. Those authors developed a simpler formulation based on P_0. Relaxing the capacity constraints Q_j and simplifying, Efroymson and Ray produced the following relaxed problem:

$$P_1: X_{ij} = 1 \text{ if } (c_{ij} + \frac{g_i}{n_j}) = \min_{k\varepsilon k_1 UK_2} [C_{ki} + \frac{g_k}{n_k}] \tag{14}$$

$$= 0 \text{ otherwise}$$

$$Y_j = 0, \qquad j\varepsilon k_0 \tag{15}$$

$$= \sum_{i=1}^{n_j} X_{ij}/n_j, \quad j\varepsilon k_2$$

$$= 1, \qquad j\varepsilon k_1$$

$$g_k = f_k,\ k\varepsilon\ k_2 \tag{16}$$

$$= 0,\ k\ k_1$$

where k_0 = the set of closed offices, not available for use. Y_j's are set equal to zero.

k_1 = the set of open offices, available for use. Y_j's are set equal to one.

k_2 = the set of offices which are not assigned open or closed. Y_j's are fractional, the office is "free".

c_{ij} = cost of transportation from city i to city j

f_j = fixed cost of locating office j

P_j = set of those customers which can be supplied by office j

n_j = the number of elements in P_j

The problem now has the structure of a transportation problem with the set of supply constraints removed. Problem P_1 is used to solve the linear program at nodes in the B & B tree.

Branch and Bound Procedure

The procedure first drops all integrality constraints on the Y_j and solves the initial LP. Its solution, Z_0, becomes a lower bound (LB) for subsequent, more highly constrained subproblems. If all Y_j in the LP result are integer-valued, then the problem is solved. Otherwise, the Y_j must be integerized. Any feasible all-integer solution is a "terminal" solution. All other feasible solutions are "nonterminal." A B & B procedure is entered into in which each service facility j is assigned to be used ($Y_j = 1$) or assigned not to be used ($Y_j = 0$). Bookkeeping is accomplished by assigning the Y_j to sets K_0, K_1, or K_2.

Each assignment of a Y_j to values 0 and 1 produces a new pair of candidate problems to be fathomed. Figure 8 illustrates the procedure, with Y_k set equal to

0 to obtain X_{k0},

1 to obtain Z_{k1}.

At each stage, the algorithm branches from successive non-integer (nonterminal) Y_k nodes until all Y_k are integer valued, a terminal solution. The objective function becomes an upper bound (UB) for future solutions. Infeasible solutions are eliminated according to FC1.

All remaining nonterminal nodes must be investigated, so the algo-

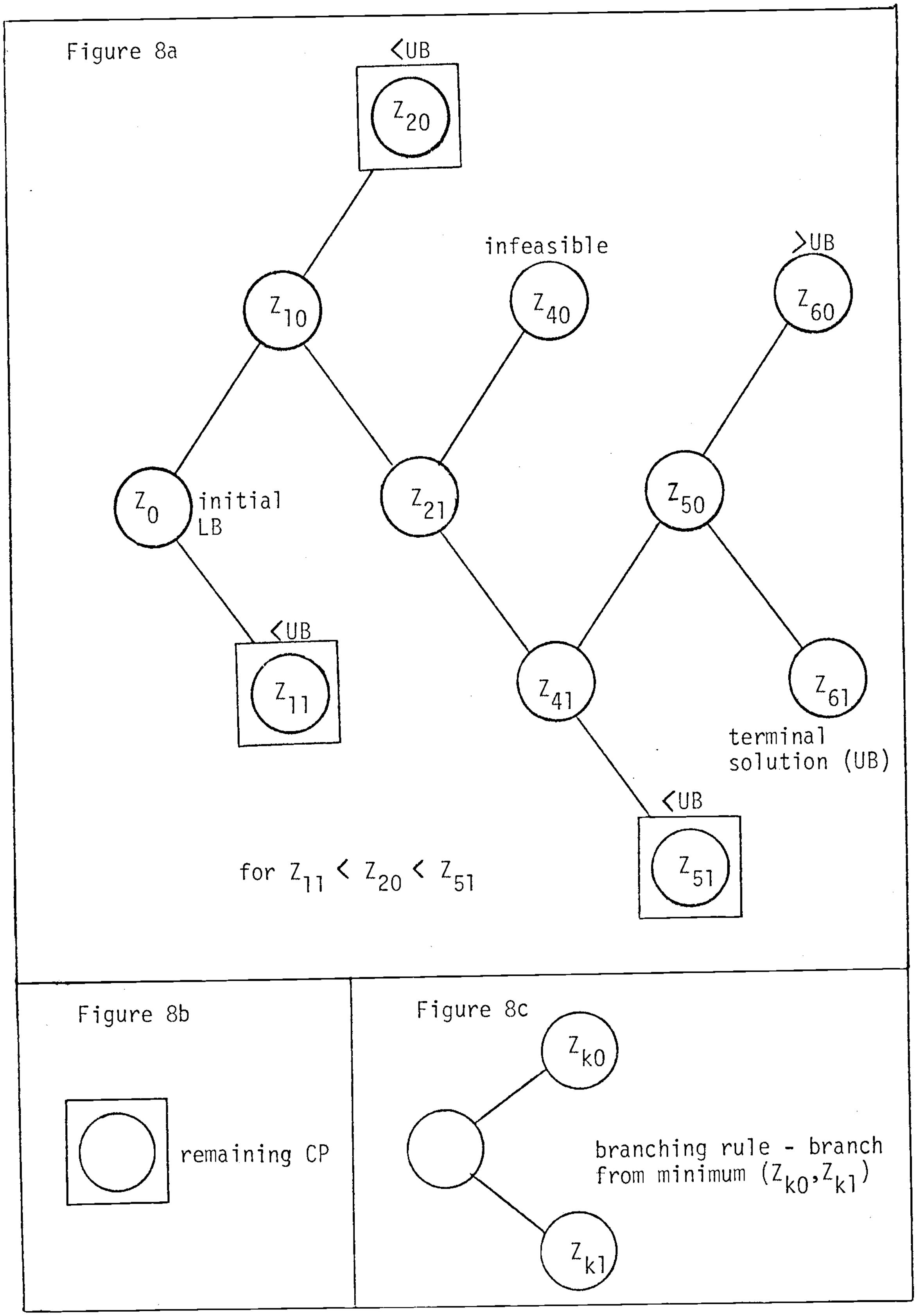

Figure 8. Branch and bound procedure after finding one terminal solution

rithm returns to nodes (CP) that have objective function values less than the current UB, and begins branching. The branching rule followed is least lower bound (LLB), that is, the nonterminal node with the smallest objective function value LB is selected (see Figure 8c). (LIFO is used in some algorithms. In Figure 8a, for example, LLB would select node X_{11} as the CP. LIFO would select node Z_{51}. Geoffrion and Marsten [19] note, in general, better results with LLB). The fractional Y_j at that node is constrained to 0 and 1, and LP's are solved at the two additional nodes. The solution value at each node (if feasible) becomes a new LB for all branches emanating from that node. Infeasible nodes are "pruned." If a nonterminal solution is greater than the current UB, then the branch is pruned. Figure 8a illustrates these rules after one terminal solution has been found.

Once a node (CP) has been shown to be infeasible, greater than the current UB, or terminal, the CP is fathomed. The initial IP has been <u>relaxed</u> to an LP, <u>separated</u> into CP, and each CP <u>fathomed</u>. The optimal solution is, clearly, the minimum terminal node value - the least UB.

COMPUTATIONAL EFFICIENCY

Khumawala improved the computational performance of the Efroymson and Ray procedure by adding three types of efficiencies:

1) At each step of the B & B algorithm, an LP relaxed problem is to be solved. Khumawala used information already available at that stage to solve the LP very rapidly.
2) At each stage, the B & B algorithm selects a free office from set K_2 and constrains it open and closed. Khumawala developed

and tested formal rules for selecting the free office, and found one very efficient rule. These rules are referred to as branching decision rules.

3) Several improvements were made to the computer program, so that storage space was used more efficiently.

Efficiencies (1) and (2) follow in more detail.

LP Simplifications

1. A minimum possible savings value is determined for opening a field office. If it is positive, then the office is fixed open, i.e., Y_j is assigned to K_1. Mathematically, the computation is:

$$\nabla_{ij} = \text{Min}_{K \varepsilon N_i \cap (K_1 \cup K_2);\ k \neq j} [\text{MAX}\ (C_{kj} - C_{ij},\ 0)]$$

$$\Delta_j = (\Sigma_{i \varepsilon P_j} \nabla_{ij}) - f_j$$

where N_i = the set of warehouses j which can supply customer i. (If prohibitive routes exist, not all warehouses will be able to supply all customers.)

Khumawala notes that for $\Delta_j > 0$, $Y_j = 1$ for all branches emanating from the node under consideration. Delta (∇_{ij}) is the minimum savings that results if office j is opened to service city i. If the sum of all deltas for office j is greater than f_j, the cost of opening office j, then it pays to open the office.

2. The second simplification is an updating procedure. It reduces n_j, the number of cities which office j can serve.

"If for $j \varepsilon k_2$, $i \varepsilon P_j$

$$\text{Min}_{k \varepsilon k_1 \cap N_i} (C_{ki} - C_{ij}) < 0$$

then n_j is reduced by one." (Khumawala, [30], p. B-721; some changes in notation)

If a fixed-open warehouse can supply demand center i cheaper (at lower variable cost) than any of the "free" offices at the node, then demand center i should not be considered as a possible customer of the free field offices.

3. The third simplification contrasts with the first. While the first simplification determines if the minimum possible cost savings warrants the opening of a field office, the third determines whether the cost reduction resulting from an office already open is still warranted. Hence, it determines whether the open office can be closed, and, also, whether a free office can be closed. Khumawala states:

"For $j \varepsilon k_2$, $i \varepsilon P_j$

$$W_{ij} = \text{Min}_{k \varepsilon N_i \cap k_1} [\text{MAX } (C_{ki} - C_{ij}, 0)]$$

$$\Omega_j = (\Sigma_{i \varepsilon P_j} W_{ij}) - f_j$$

If $j < 0$, the $Y_j = 0$ for all branches emanating from the node." (Khumawala [80], p. B-721; some changes in notation)

W_{ij} is the minimum savings resulting from city i being served by office j. If the sum of all such savings for office j is less than the cost of opening office j, then the office is closed.

These simplifications are cycled through at each node, as shown in Figure 9. When no further simplifications can be made, the LP is solved. The entire solution procedure is shown in Figure 10.

Khumawala ([30], p. B-721) reduces the size of the LP at each node by opening only the offices that will minimize the objective function

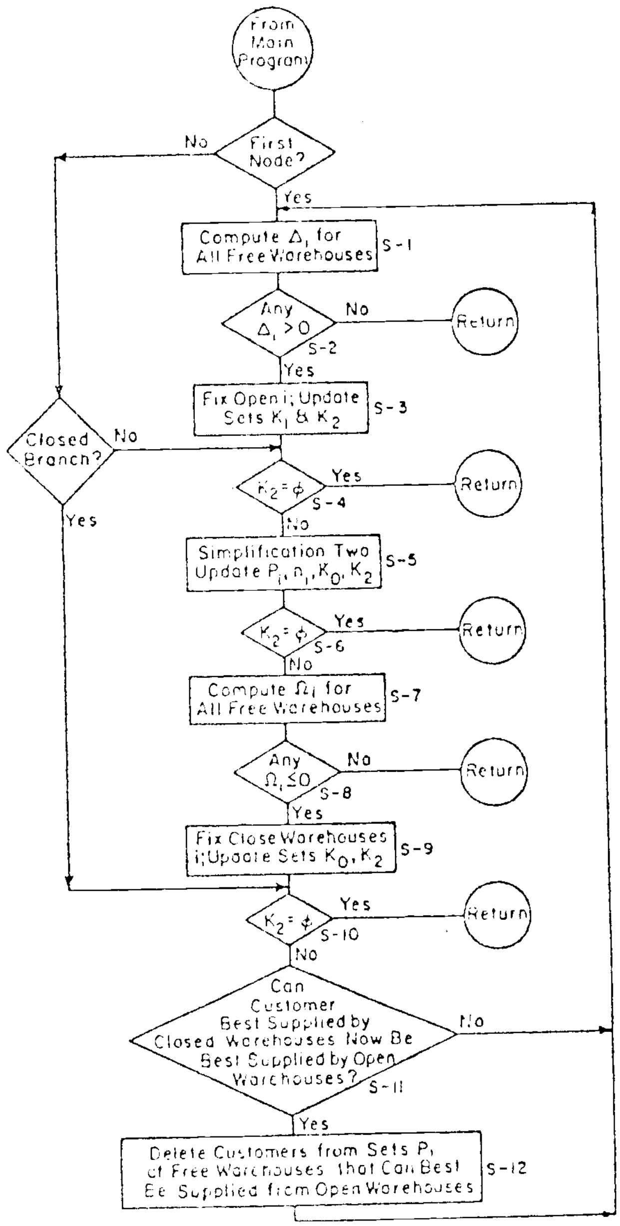

Figure 9. Simplification flow chart
(from [30] p. B-724)

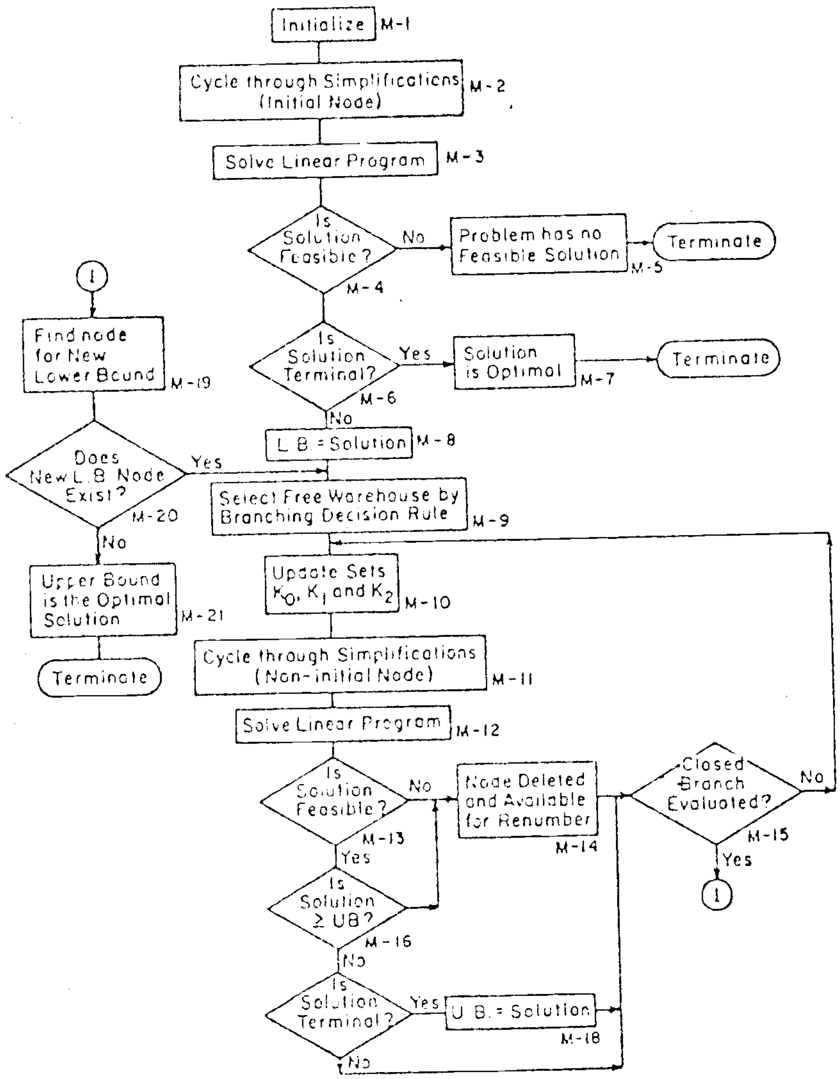

Figure 10. Branch and bound procedure flow chart
(from [30] p. B-725)

at the node, based on the already-computed deltas. If (j_i) is the value of j that minimizes C_{ij} over all j in $N_i \cap (K_1 \cup K_2)$, then customer i should be supplied from j_i if:

$$\nabla_i\ (j_i) \geq f(j_i)/n(j_i) \qquad \text{if } (j_i) \in K_2,$$

$$\nabla_i\ (j_i) > 0 \qquad \text{if } (j_i) \in K_1,$$

where the ∇_{ij}'s, the minimum savings resulting from opening office i, were computed in simplification 1 (steps S-1, Figure 9). Proof that the result is optimal is given in Khumawala's dissertation [31].

Branching Decision Rules

In order for branching to continue after the LP at a node has been solved, an office must be selected for the set of free offices, K_2, at the node selected under the LLB branching criteria. This office selection is performed by the branching decision rule. Khumawala [30] tested eight such rules and found, in most cases, selection of the office with the largest positive omega gave the best performance. Efficiency is again gained by using the Ω_j computed in simplification 3 (step S-7, Figure 9).

To summarize Khumawala's algorithm, an initial LP is simplified and solved. An office is then selected by the branching decision rule and constrained open and closed. In both cases, the resulting pair of problems are simplified and solved. The first terminal solution obtained becomes the UB. All nonterminal solutions are retained as CP. A new CP is selected by the branching decision rule, and the new pair of LP's are simplified and solved. Each resulting solution, if terminal, is compared with the current UB and, if less, becomes the new UB. If the solution is nonterminal, it is compared with the current LLB, and the minimum value denotes the next node selection. When no nonterminal nodes with solutions less than the current UB can be found, the procedure ends; the current UB is optimal.

VI. HFLC PROBLEM ANALYSIS

Demand Data

The number and locations of nursing facilities in Oregon is quite stable. Demand at each location is comprised of predictable inspection visits, plus a less certain number of complaint and change of ownership visits. Licensing and certification visits and follow-up visits, including MIPRT follow-ups, can be modelled well as deterministic, if it is assumed that the inspector staggers inspections throughout the year, so that all demand is satisfied. Sequencing requirements could be violated if an inspector is too overloaded at a particular time of year to service all demands.

Complaint and change of ownership visits, however, were seen to occur randomly during the year. Available records of complaints and CHOW consisted of four years of data listing the number of complaints by county per year, and the total number of CHOW for the state each year.[3] It was seen that the average number of complaints per home varied by county, and by year within counties. Figures 11 a-d show complaint totals per county for each year; the number of homes in each county was assumed to be constant. It appeared that the number of complaints in each county hovered around some high or low level. The fluctuation from year to year had no apparent pattern. Frequency histograms for the number of complaints per home each year (Figure 12 a-d) and for the four year data (Figure 13) showed no obvious underlying distribution.

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[3] Data is listed in Appendix 1

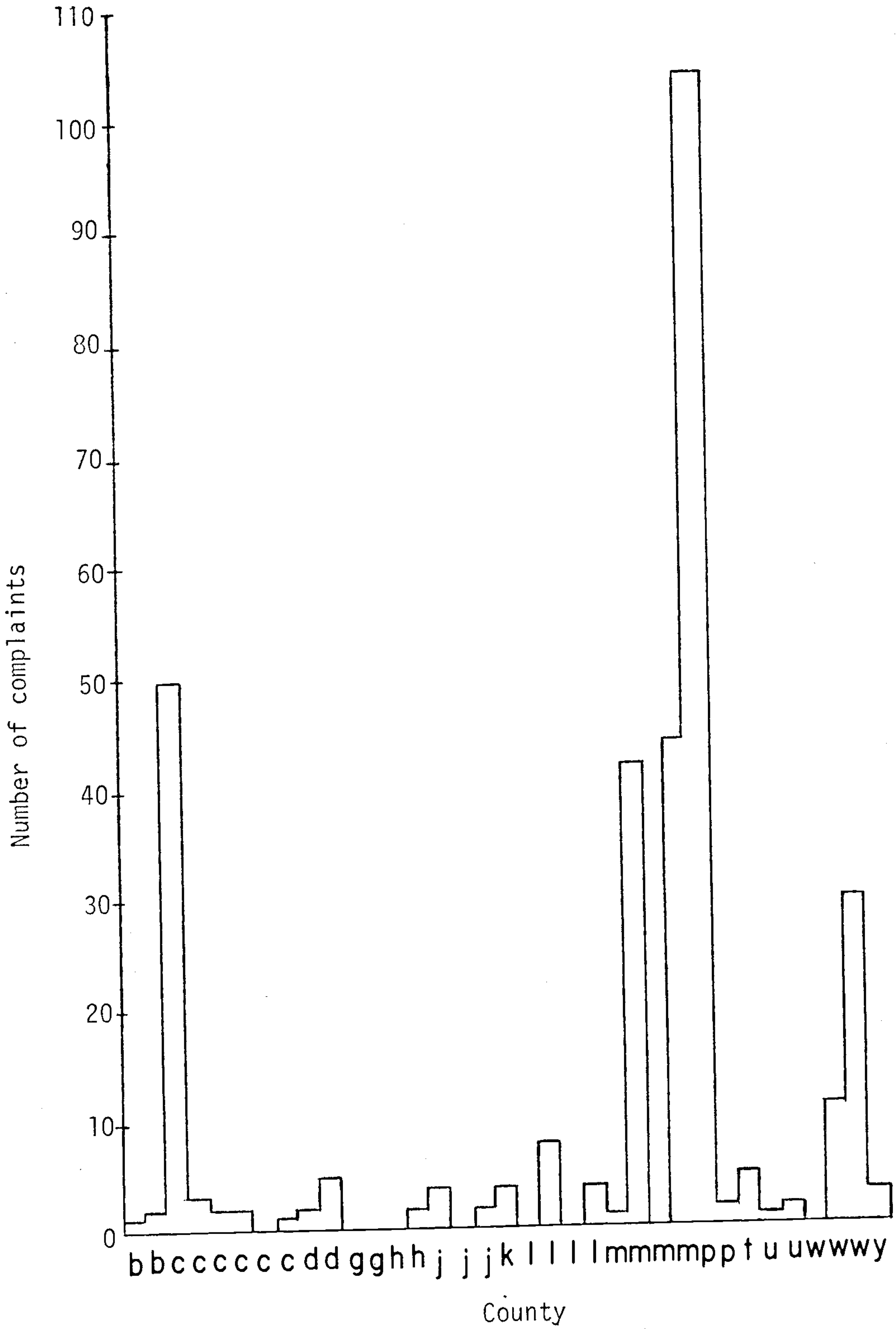

Figure 11a. Number of complaints by county, 1977

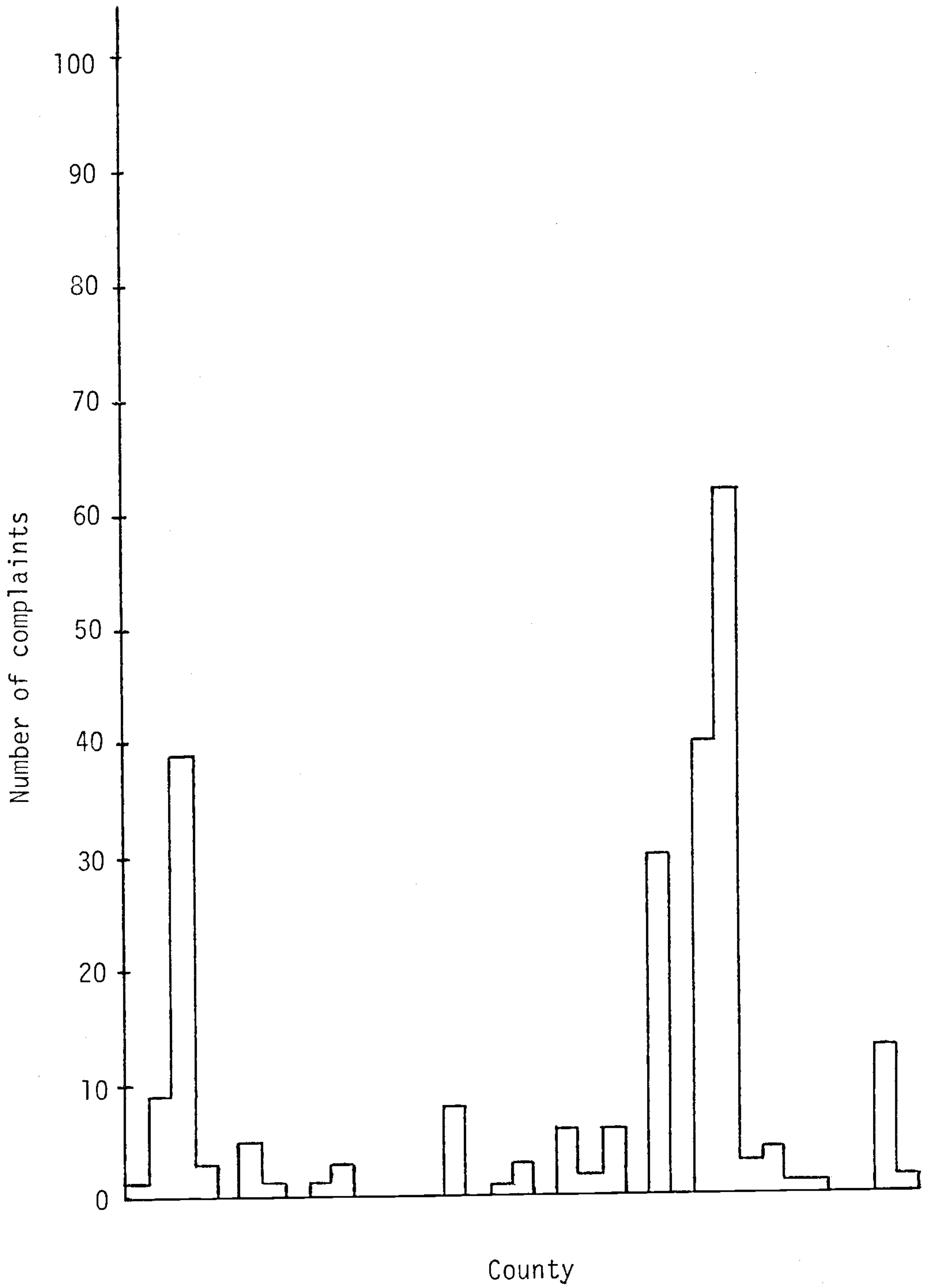

Figure 11b. Number of complaints by county, 1976

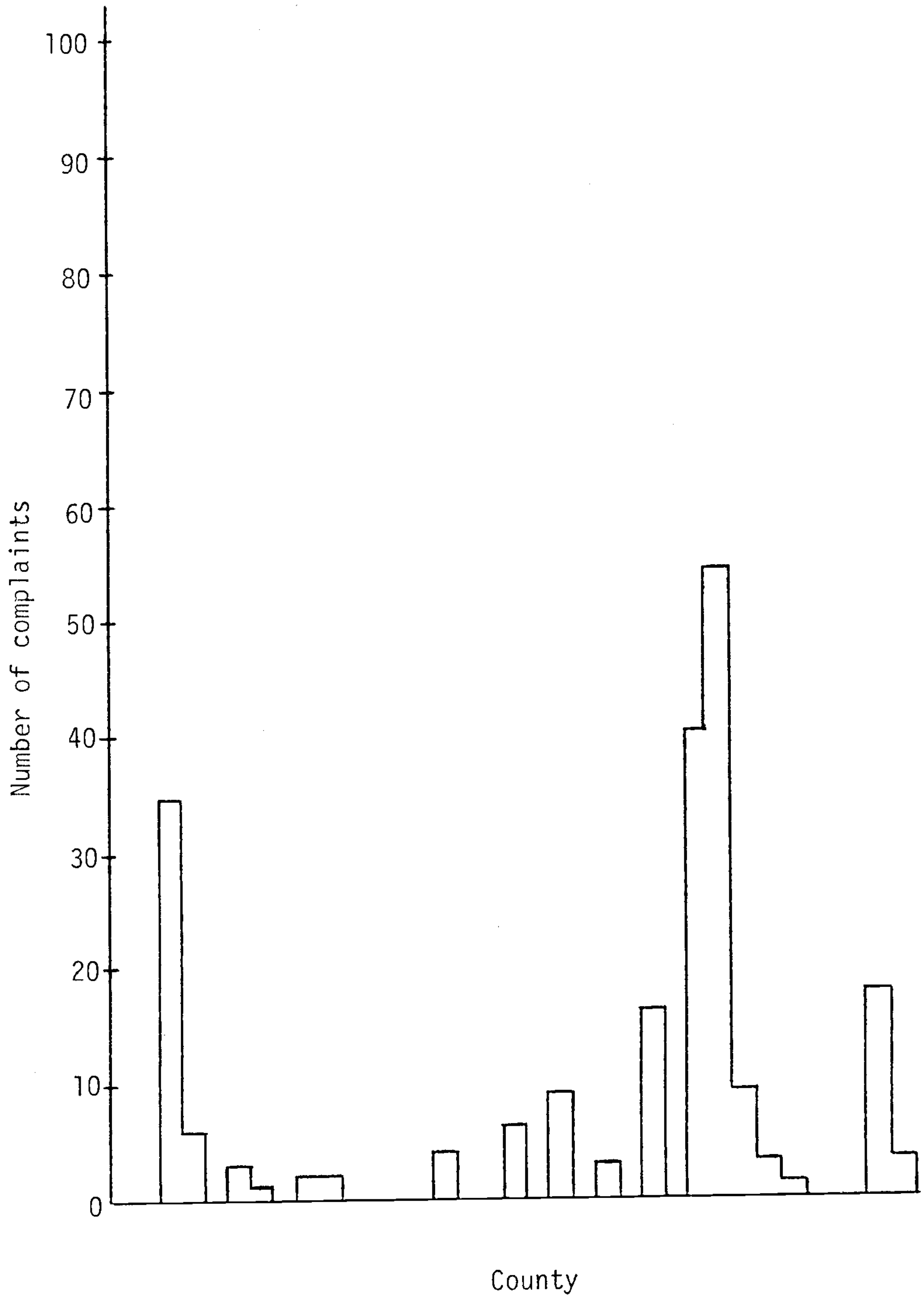

Figure 11c. Number of complaints by county, 1975

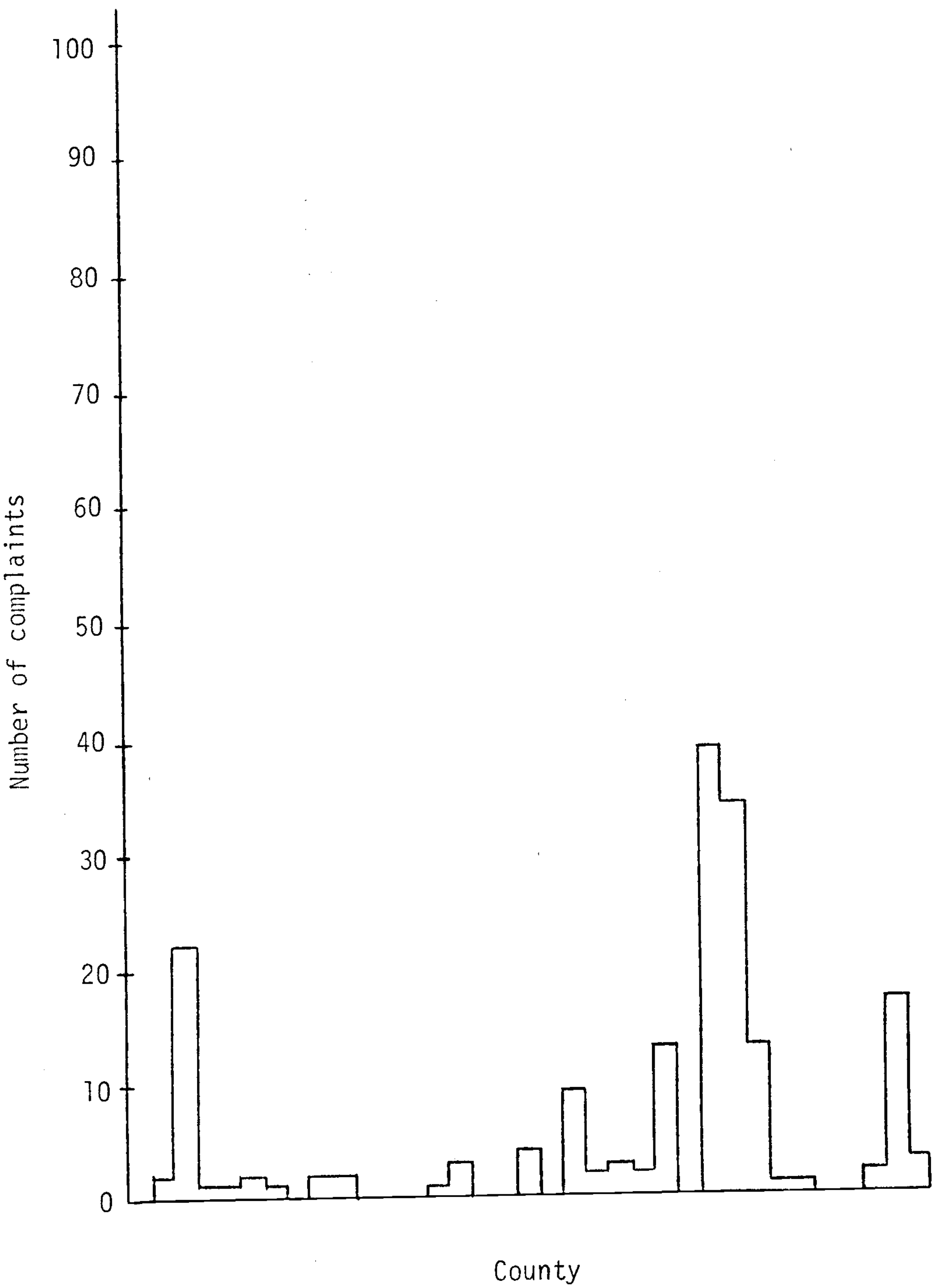

Figure 11d. Number of complaints by county, 1974

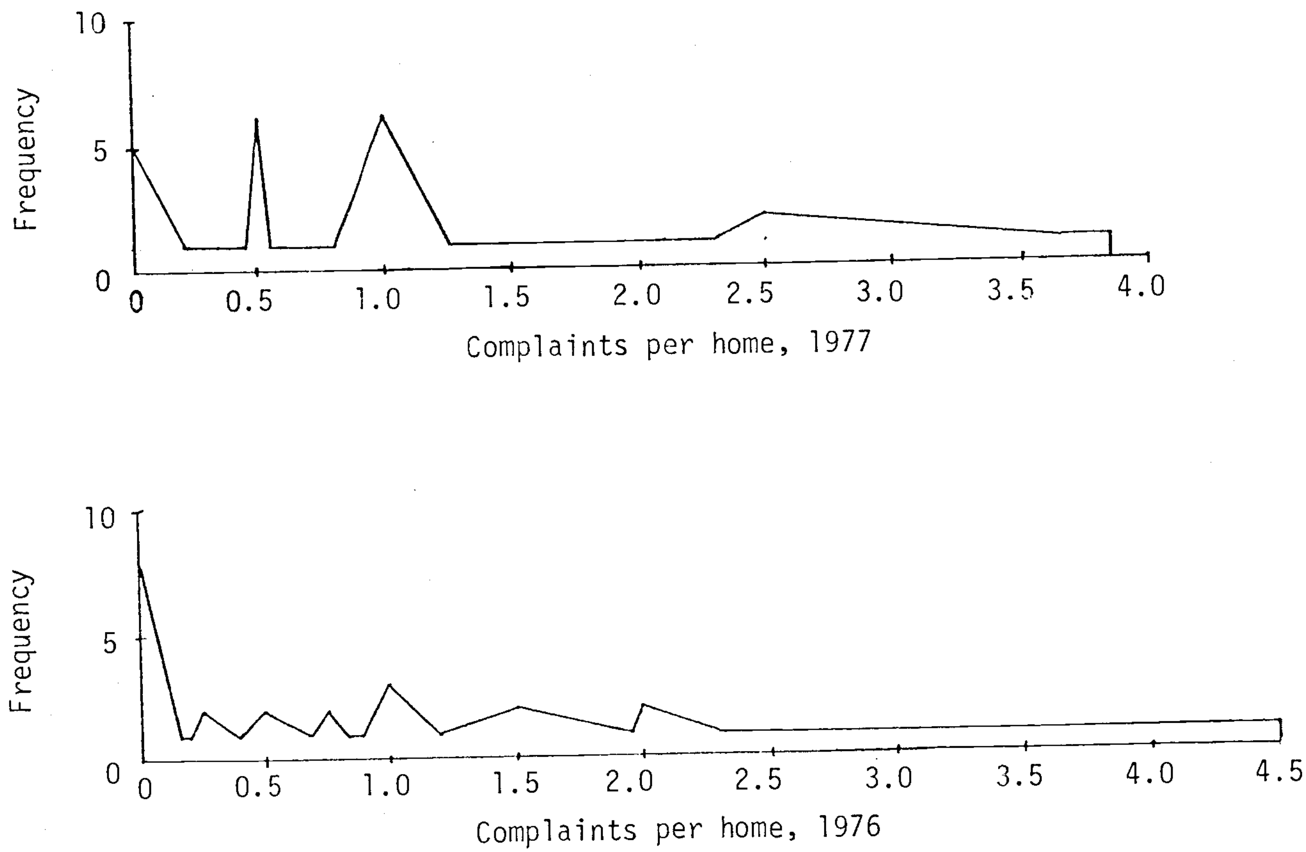

Figure 12. Frequency versus complaints per home

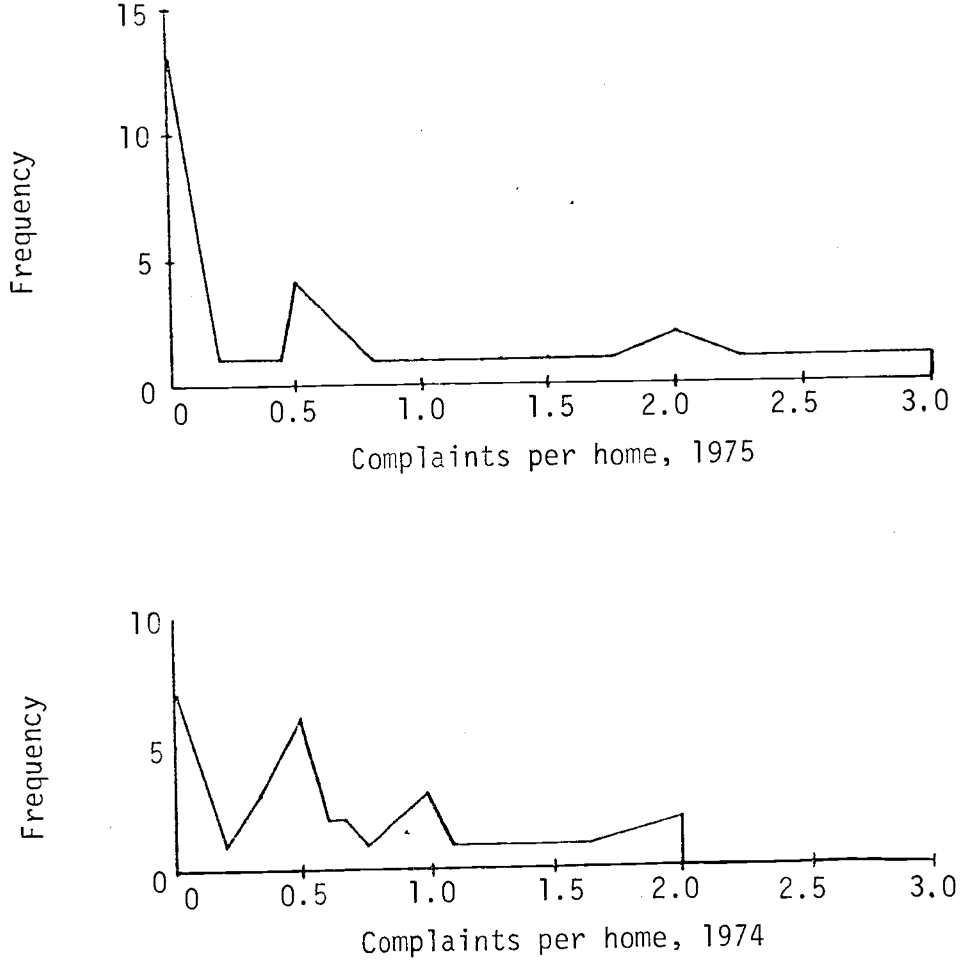

Figure 12 (continued). Frequency versus complaints per home

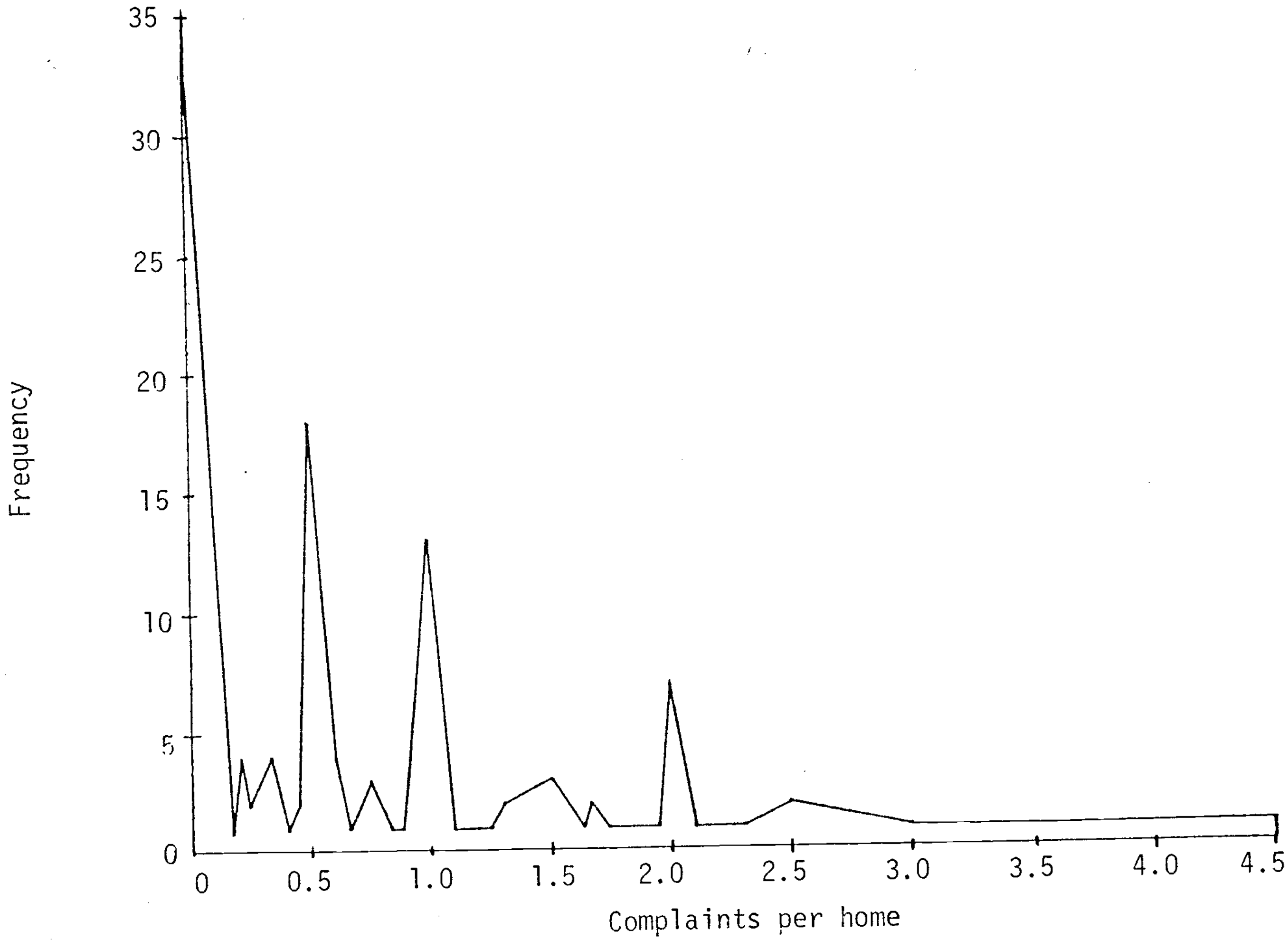

Figure 13. Frequency versus complaints per home, 1974-1977

Further, the HFLC inspection group supervisor [64] suggested that complaints have tended to be higher in metropolitan areas such as Portland and Salem, and lower in remote cities, largely, perhaps, as a result of media exposure and proximity to the State government. While complaint data showed different levels per home by county, and each single county level fluctuated without trend over time, the state totals for four years suggested an increase over time. While the data for only four years is a poor basis for long-range forecasting, it does indicate an upward trend in recent years.

The simplest method for handling complaint data would be to determine the average or mean number of complaints per home and assign that number of complaints to all homes. The wide variability in both the level of complaints by county and the number of complaints per home by county leads one to suspect that this would be an erroneous assumption.

Comparing each year of data in Figure 11 suggests that the levels of complaints by county are fairly stable from year to year, with some counties high and others low in complaints per home. The total number of homes in the state remained nearly constant during the four year period, with a change of only one or two each year. Unfortunately, no record was kept for the number of homes in each county per year. If we assume that the number of homes per county has been constant at the present level over the past four years, then the level of complaints per home fluctuates in the same manner as total complaints in Figure 11.

It would be of interest to determine whether complaint data can be modelled by the density function of a known distribution such as, say the standard normal.

Four questions, thus, arise in analyzing complaint data:

1) Is an average number accurate?

2) If not, does the level of complaints per home in a county remain about the same?

3) Does the data indicate that an underlying distribution is present? and,

4) Can we be confident that the statewide data showed an increase?

Several statistical tests were performed to answer these questions. The question of whether one mean is valid for all four years was treated by a comparison of means and variances for the four years. The means were compared in an F-test; it was concluded, for $\alpha = .05$, that the hypothesis of equal means for all four years could not be rejected. An F-test comparison of variance inferred that all years do not have equal variances.[4] This would suggest that the use of a four-year mean to assign an average number of complaints to each home is poorly supported due to the presence of changes or shifts in the number of complaints in the counties from year to year.

The level of complaints per home in each county appears to be stable, within some range for each county, over the four years. This second area of inquiry was treated by a contingency table test for homogeneity of the data. This contingency table tests the hypothesis that the proportion of total complaints which falls in a county varies from year to year.[5] The F-test results showed, for $\alpha = .01$, .05 and .10, that the data does not present evidence sufficient to conclude

[4] F-Tests are listed in Appendix 5.

[5] Contingency Table tests are found in Appendix 6.

that the proportion of complaints in a county varies from year to year. Therefore, a pattern of levels of complaints per home by county - suggested by Figure 11 - cannot be refuted statistically. In modelling complaint demand, this pattern should be preserved.

The third question concerning an underlying density function was investigated by an Approximate Chi-Square Test for Poisson and Normal models.[6] The results showed that the Normal distribution is not appropriate for the data. For the Poisson distribution, however, the null hypothesis,

H_0: the distribution is Poisson

was accepted. We concluded that the Poisson is a reasonable model for the data.

The last question asks whether trends can be detected in complaint and CHOW levels. Admittedly, a total of four data points is too small a snapshot for long run forecasting. It does tell us, however, whether the complaint and CHOW means appear to be stationary or changing in recent years.

Interestingly, a least squares analysis of complaints showed that 93% of the change in yearly complaint totals is explained by the fit of the data to a straight line.[7] The prediction intervals (α = .05) for future years were quite small, considering the small sample size of four.

The least squares fit of CHOW data was very poor, with only 8% of the variation explained by the regression. This could have been expected; the CHOW figures appeared to fluctuate or cycle.

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[6] Found in Appendix 7.

[7] Calculation is found in Appendix 8.

To summarize, the statistical tests pointed toward an increasing number of complaints based on the past four years' experience. The statewide distribution of complaints per home changed from year to year, but the number of complaints in each county varied in some high or low range, and the number of complaints/home were not the same, county by county.

In addition, the nature of complaint data should be considered. Complaint records for individual facilities or cities, if they existed, might indicate that particular homes were high or low in complaints, or followed a trend in time. Use of that information would have been a poor data base for long run location of inspectors. A home may change management or physical facilities, and may improve or deteriorate, changing the level of complaints. Media, also, seems to have an effect. A series of articles in the March 6-10, 1977 issues of the Portland-based newspaper "The Oregonian" prominantly displayed nursing home problems and HFLC activities in Oregon. Publicity such as this may contribute to an increasing level of complaints.

It could be possible that a would-be complaintant outside of the Portland area might not expect to receive action on a complaint from a faraway state agency, and could decline to complain. The same person might be induced to complain to an inspector at a nearby Adult and Family Service Office. Complaints might increase under a decentralized office plan.

For these reasons, it was decided that several levels of complaints per county should be analyzed in order to determine the sensitivity of office location with varying demand. Also, a method was sought to utilize the county complaint data, since the consistent high or low levels of complaints per home in different counties precluded the use of a statewide average. Complaint demand by county appears to occur randomly, influenced by many factors, including visibility of HFLC operations, and the level of care that clients and their families perceive in each home.

The Demand Model

Monte Carlo simulation was chosen to assign complaints to homes in a county. The number of complaints per county was an inputted variable, so that the structure of the complaint/home data by county was maintained; it was varied in the sensitivity analysis.

The fallacy of maintaining that one home in a county generated a fixed average number of complaints over time was eliminated. It was assumed that, within a county, each facility was equally likely to generate a complaint. Complaints were assigned to homes using a uniform distribution and a Monte Carlo simulator; several simulation runs analyzed the sensitivity of results with various numbers of complaints assigned to each county. Change in the number of complaints per county analyzed the problem sensitivity to changes in the number of complaints statewide.

Change of ownership of a nursing facility requires that the facility be relicensed and recertified, with initial and follow-up visits; the four-year data varied between 19 and 30 CHOW per year. With no additional data to support a pattern of CHOW across the state, it was assumed that each home in the state had the same probability of changing ownership. Monte Carlo simulation was again used, given a certain number of CHOW, to randomly assign CHOW to homes.

Each CHOW was assigned four visits, consisting of the licensing and certification visit, two follow-ups, and one MIPRT follow-up. Each complaint was modelled as requiring two visits - an initial investigation and one follow-up visit. The number of visits per complaint or CHOW was a usual or average figure obtained from HFLC, with no data available to support a range of values.

Each facility also requires the annual licensing and certification visit, two follow-ups, and MIPRT follow-up. The first licensing visit requires three full days of on-site inspection. The other visits require one day on-site. This time duration for visits is, again, a common or average figure that reflects the usual time required for each type of visit. Inspections require sampling of patient records, but, overall, the time required for inspection is independent of the size of the home; the standard routine is not highly variable. Follow-up visits are somewhat more variable, and depending on the number and gravity of deficiencies, a follow-up may take two hours to two days. The majority require one day. Complaint visits are also variable in duration, but, again, the one day visit is a reasonable assumption and the correct value in nearly all cases.

Model of Present Operations

At present, the staff of sixteen health facility inspectors and one supervisor are located at a central office in downtown Portland. A large nonproductive time and travel expense is incurred by inspecting all facilities within the state from the Portland office. The most distant facility is 386 miles away. 27% of the Oregon cities with nursing facilities are more than 200 miles from Portland. 48% are more than 100 miles away. To reach the more distant locations, inspectors spend up to one full day travelling in each direction. The inspector may spend one to three nights lodged in the distant city.

A per diem is paid for meals by the State if the inspector is 25 or more miles from the office, and for lodging if he is 50 or more miles away.

State cars are used for transportation, charged to HFLC at $0.11/ mile. Air travel has recently been approved, on occasion, for travel to Pendleton and Klamath Falls. The arrangement is not typical or presently feasible at other locations. Air travel was excluded from this analysis.

Decentralized Plan Model

Locating inspectors at decentralized offices has the benefit of reduced travel time and cost. Nineteen cities were chosen as candidates for inspection offices after consultation [24] with State Management Analyst Mike Greany. It was felt that the cities should be natural centers in the Oregon roadway system, and should offer services sufficient to attract potential inspectors and their families. Adult and Family Services has offices in the candidate cities (and in nearly every Oregon town of any size). A major assumption of the plan required that HFLC inspectors be located in existing State Adult and Family Service Offices. Secretarial service that is presently obtained at the Portland Office would be obtained at AFS offices. This includes services such as photocopying, telephone answering, and typing. Telephone service cost would remain the same, since one extension would be removed in Portland and added elsewhere. The state tie (SPAN) line has the same charge regardless of where the telephone is located. It is assumed that office furniture would be available at AFS locations; otherwise, an additional purchase and/or moving cost would be incurred initially.

Lease on office space would increase $495 per office per year.[8] This was based on an increased cost allocated to the location of the first inspector in each office as follows. Beyond the first desk in any office, the rate per additional desk drops. An inspector in Portland is charged for office space at a lower additional-office rate. If that inspector position is relocated to another city, the primary position is charged the higher rate. Additional personnel are charged at the same rate at the outlying office as in Portland. Hence, the added cost for the inspection office space, including maintenance, at an outstate location is the differential added cost of locating the first inspector.

It was anticipated that postage cost would increase with frequent report mailings to Portland and memorandum mailings from Portland supervision to inspectors. Thirty dollars per month was allocated for postage per office as a rough but adequate estimate.

Presently, inspectors meet with their supervisor every Monday morning to discuss Federal and State regulation changes, problems, procedures, and for training. These weekly meetings have been described both as vital and unnecessary. At any rate, travel to Portland for a weekly

[8] Fixed costs are formulated in Appendix 4.

four-hour meeting by all inspectors is infeasible under a decentralized plan. An alternate plan, endorsed by Management Analyst Mike Greany, was modelled. It was assumed that once a month the Portland-based supervisor would travel to each inspection office to brief the inspectors during a half-day meeting. This should allow ample time to discuss developments which could not be resolved by mail or telephone. In addition, the supervisor would take on the role of quality controller, able to sample both the inspector's work and the level of care in homes that were visited.

A qualitative difficulty under decentralization is the possibility that inspectors serving the same group of nursing facilities will become so empathetic with home operators that they will lose their objectivity, and bend rules to the advantage of the facilities. The periodic appearance of the supervisor on-site could help the inspector maintain a stance of objectivity. The supervisor could also act as a source of information and advice to facility administrators and staff.

Management Analyst Mike Greany also noted that the element of surprise would probably be increased under a decentralized plan. Currently, an inspector, having travelled a long distance from Portland, sometimes visits several facilities on a single trip. Home operators anticipate this, and forewarned by a nursing facility grapevine, can prepare for the visit. With mulitple offices, the inspector is close to a greater number of facilities. In many cases, the inspector's next move would not be so obvious.

Summary of Costs

Fixed Costs

Fixed costs included the incremental or added FC which would be incurred by opening an added inspection office outside Portland, plus the added cost of supervisor's travel to monthly meetings.

The cost of opening each inspection office included the cost of office space (including maintenance) and postage. The cost per year of each added decentralized office included:[9]

Office lease	\$495
Postage	360
Subtotal	\$855/year

In addition, the supervisor's travel cost for monthly meetings was a cost incurred in decentralized location. Since the cost was based on the travel from Portland to each office location, the travel cost had a fixed value associated with each possible office. The cost was:[10]

transportation cost + supervisor wage cost per mile + per diems for out-of-town travel = {[(\$0.11/mile + \$0.14/mile)*2*(mileage)] +(per diems based on mileage)}*(12 meetings) (17)

The first term in the expression gives the state car and supervisor time costs of round trip mileage. The second bracketed term, the per diem, is a step cost estimate of the meals and lodging that are required by the supervisor, based on mileage. Per diems ranging from \$2.75 to \$59.25 were assigned;[11] gives ranges of mileage were used, with a per diem as-

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[9] Developed in Appendix 4.

[10] Developed in Appendix 3.

[11] Developed in Appendix 2.

signed to each. These step costs were included in field office costs for the location analysis.

Variable Costs

Variable costs that changed with office locations, again based on the common denominator of mileage were:

cost of state car per mile,
inspector's wage and travel cost per mile, and,
per diem cost based on mileage.

For each nursing facility, these costs were incurred for every visit. Each facility received four visits per year - one three-day licensing and certification visit, two one-day follow-ups, and one MIPRT follow-up. In addition, the inputted number of complaints and changes of ownership, entailing two and four visits respectively, were assigned to demand locations by Monte Carlo assignment. The annual number of visits per facility was:

Visits = 4 + (Assigned complaint and CHOW visits). (18)

The per diem and mileage costs depended on distance and whether the visit lasted one day or three days. Figure 14 illustrates this. On a one day visit, if one-way mileage was greater than 60 miles, the inspector stayed overnight; otherwise, the inspector returned to home base. It was assumed that travel time was scheduled so that the inspector had ample time to conclude the visit in one day. If the one day visit required travel to a facility more than 60 miles from the office, the inspector received a per diem that included lodging. If the facility was less than 60 miles away, the per diem included meals, but not

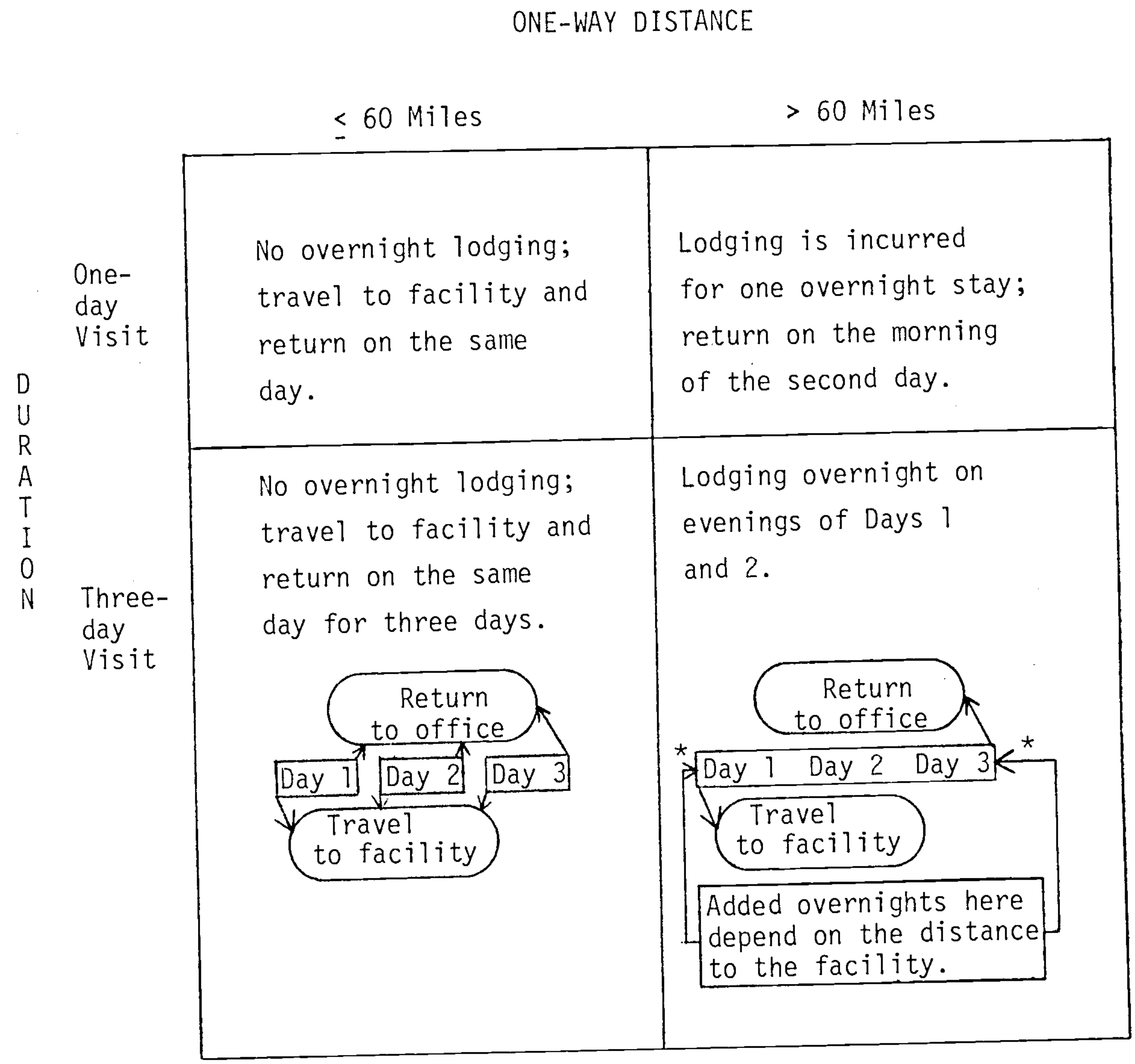

Figure 14. Overnight lodging per diems.

lodging.[12]

Three-day per diems were assumed to also show two patterns, based on the 60 mile cut-off point. For facilities less than 60 miles away, it was assumed that the inspector left the office early - before 8:00 a.m. if necessary, and returned each day. If one-way distance was greater than 60 miles, the inspector was lodged for four nights at the facility location, as shown in Figure 14. Meal per diems were paid during the three days of inspection, and for travel prior to and after the three days.[13] These inspector per diem step costs were included in the variable costs, c_{ij}.

As Figure 14 illustrates, for a three-day licensing and certification visit to a facility 60 miles away or less, two added round trips were necessary in the midst of the inspection. Therefore, in this one case, the number of visits per home was the number of annual visits, VISITS, in expression (18), plus two:

(VISITS + 2). (19)

For each nursing facility, the total variable cost of meeting its annual inspection requirements was formulated as follows.

If the facility was 60 miles or less from a HFLC office, the VC expression for supplying the demand of facility i from office location j was:

VC (i,j) = {[(1 3-day visit)*(round trip mileage)*(3 round trips)*
(inspector wage and travel cost per mile)] + (3-day per diem cost) }
+ {(Visits - 1 1-day visits)*[((round trip mileage)*(1 round trip)*

---

12 Developed in detail in Appendix 2.

13 Developed in Appendix 2.

(inspector wage and travel cost per mile)) + (1-day per diem cost)]}

(20)

where the first bracketed term expressed the cost of the three-day licensing and certification visit, and the second bracketed term yielded the cost of the remaining (visits-1) one-day trips. Note that the total number of visits were:

[(1 3-day visit)*(3 round trips)] + [(visits-1 1-day visits)*
(1 round trip)] = (visits + 3-1) round trip visits
= (VISITS + 2) round trip visits

as in (19).

Similarly, if a facility i was more than 60 miles from a HFLC office site j, the corresponding variable cost expression was:

VC (i,j) = {[(1 3-day visit)*(round trip mileage)*(1 round trip)*
(inspector wage and travel cost per mile)] + (3-day per diem cost) }
+ {(visits-1 1-day visits)*[(round trip mileage)*(1 round trip)*
(inspector wage and travel cost per mile) + (1-day per diem cost)]}

(21)

where the number of visits in this case was:

[(1 3-day visit)*(1 round trip)] + [(visits-1 1-day visit)*(1 round
trip)] = (Visits) round trips

as in (18).

The variable costs were computed for all demand locations and office site locations (i,j), and each was multipled by the number of nursing facilities at the demand locations.

Office Personnel Requirements

Once nursing facilities had been assigned to offices, the corresponding demand at each office was expressed as the number of inspectors required at each office. The optimal results of the algorithm were not necessarily integer inspector requirements. State budgeting procedures are geared to expressing demand in non-integer terms; each inspector, or full-time equivalent (FTE) position, is expressed fractionally as twelve man-months. For this reason, it was desired to find the optimal, even if non-integer, result - an absolute best solution.

To assign inspectors to offices required integer demand. To obtain integer office demands, the optimal result was perturbed until an integer or near-integer demand resulted.

The number of inspectors required at each office $XINSP_j$, was computed as follows:

$$XINSP_j = \{[(Homes_j)*(62 \text{ hours/home})] + [(\text{Complaints and CHOW's})*(8 \text{ hours})] + (\text{Total miles travelled/average MPH})\} \div [(2080 \text{ work hours/year})*(\text{Efficiency Factor})] - 48 \text{ meeting hours}$$

where:

$Homes_j$ = number of nursing facilities i serviced by office j

Complaints and CHOW's = total number of complaints and ownership change visits at facilities served by office j

Total miles Travelled = total round trip mileage required to meet all demands of facilities served by office j

Average MPH = the average speed, throughout the state, of inspector travel, in miles per hour

Efficiency Factor = a rating factor that determined the percentage of work hours that an inspector was available for normal inspection duties; the excluded time included vacations, personal and fatigue time, training, meetings and conferences, and exceptional demand time requirements such as decertification hearings, special reports to the Federal government; the factor yielded standard time.

62 hours/home were allotted to each home for inspections, with the following breakdown:

1 3-day certification and licensing visit	24
2 follow-up visits, 1-day duration each	16
1 MIPRT follow-up, 1-day duration	8
Time required to complete Federal certification report (state budget figure)	14
	62 total hours

8 hours or one full day were required for each complaint or CHOW visit. 48 meeting hours equal to 12 four-hour meetings per year with the supervisor were required of each inspector.

2080 work hours/year was the amount of time available annually per inspector based on 52 weeks and 40 hours per week.

The numerator of the demand expression computed the number of man-hours needed to meet the regular expected inspection requirements of all homes served by office j. The denominator was the number of hours available per inspector to serve the inspection demand.

An efficiency factor of 70% was chosen. This factor is used as a rule of thumb by the Budget and Management Division, Executive Department for estimating the rating factor in State agencies.

VII. THE COMPUTER ANALYSIS

The computer code for Khumawala's branch and bound algorithm was modified and extended to tackle the HFLC model. Provision was made to assign complaints and changes of ownership to nursing facilities using a Monte Carlo Simulator. A routine was developed to calculate step costs as well as fixed and variable costs, and changes were incorporated so that the algorithm could make use of actual distance data, rather than relying on less accurate Euclidian distances.[14] A routine was added to compute the total cost of annual operations, including the total cost of employee salaries. The solution technique is compared with other major solution procedures in Table III.

Model Result Analysis

Analysis of the data required multiple computer runs. Results were obtained for the centralized and decentralized costs of operation under these model conditions

Model Conditions

present costs, demand and efficiency	model results
variation of parameters of cost, demand and efficiency	model sensitivity
worst possible case	
best possible case	

[14] Inter-city distances were compiled for the study by the State of Oregon Mileage Control Unit.

TABLE III. COMPARISON OF SOLUTION TECHNIQUES

Author	Distance Norm	Maximum n	Maximum m	Step Costs?	Form Tours?	Demand Satisfied by Multiple Visits?	Exact Solution Procedure?	Allow Maximum Load?	Allow Maximum Distance?	Allow Office Capacity Constraints?
Multi-Depot Vehicle Routing Procedures										
Gillett and Johnson [20]	Euclidean	5	250	No	Yes	No	No	Yes	Yes	No
Wren and Holliday [66]	Euclidean	4	320	No	Yes	No	No	Yes	Yes	No
Golden, et.al. [21]	Euclidean	2	600	No	Yes	No	No	Yes	No	No
Tillman and Cain [58]	Euclidean	5	50	No	Yes	No	Yes	No	No	No
Tillman [57]	Euclidean	5	50	No	Yes	No	No	No	No	No
Discrete Multi-Facility Plant Location Algorithms										
Khumawala [30]	Euclidean	25	50	No	No	No	Yes	No	Yes	No
Sa [50]	Euclidean	25	50	No	No	No	No	No	No	No
Soland [53]	Euclidean	25	50	No	No	No	Yes	No	No	Yes
Ellwein and Gray [13]	Euclidean	25	50	No	No	No	No	No	No	Yes
Ryan's Application of Khumawala's Branch and Bound Algorithm										
Ryan (1977)	Actual or Euclidean	30	100	Yes	No	Yes	Yes	No	Yes	No

For both centralized and decentralized cases, twenty computer runs were performed in which, by changing the sequence of random numbers in the Monte Carlo Simulator, complaints and CHOW's were assigned to different locations.

In assigning complaints, our technique preserved the approximate Poisson distribution of complaints per county in the data. It also modelled the assumption that complaints occur randomly. The following example illustrates the Monte Carlo technique (see Figure 15).

In 1977, three complaints occurred in Yamhill County. Six nursing facilities were located in three cities in Yamhill County: two homes in McMinnville, three in Newburg, and one in Sheridan. Our example divides the interval zero to one into a number of increments equal to the number of homes in the county. Each home has an equal share of the interval; here it equals one-sixth. We then generate three random numbers between zero and one, corresponding to the three complaints. Each random number is assigned to the interval which encloses its value. If the first random number generated is 0.250, then a complaint would be assigned to home 1. The second and third random numbers would assign the second and third complaints as shown in Figure 15. For our example, home 1 receives two complaints, home 5 has one complaint, and the other homes would have no complaints. In the location analysis, demand in Newberg and McMinnville would have increased by one and two complaints, respectively. Changes of ownership were assigned in a similar manner across the state.

A sequence of random numbers, then, generated the assignment of complaints and CHOW. The computer's random number generator required an initial value, or seed, to generate a sequence of numbers which, because the sequence would have started over after several thousand numbers, were pseudo-random numbers. A particular seed always generates the same sequence of pseudo-random numbers. Hence, to analyze a change in assignment of complaints and CHOW, twenty program runs were performed with twenty seed values.

YAMHILL COUNTY		
City	Number of Nursing Facilities	Number of Complaints
McMinnville	2	N/A
Newberg	3	
Sheridan	1	
Total	6	3

Facilities	Home 1	Home 2	Home 3	Home 4	Home 5	Home 6	
Cumulative Probability	0	1/6	1/3	1/2	2/3	5/6	1
City	McMinnville		Newburg			Sheridan	
Random Number:							
1/7	*						
3/4					*		
1/60	*						
Number of Complaints per home	2	0	0	0	5	0	
Number of Complaints per city	2		1			0	

* indicates assignment of one complaint

Figure 15. Example of Assignment of Complaints in a County.

The results, listed in Table IV, were deterministic because each solution was uniquely determined by the model parameters, including the Monte Carlo sequence of random numbers. No guarantee of normality in the distribution could be made. Rather than express the results in terms of a mean and confidence interval, which assumes normally distributed random error terms were present, we used values from the twenty runs which evaluated a worst case savings figure. The highest cost figure for decentralized location, and the lowest cost figure for centralized location were used in the determination of an expected minimum savings figure. The optimal decentralized cost figure resulted from non-integer assignments of inspectors to offices. This figure is noted for the State budgeting requirement which is based on man-months. The results showed that six offices should always be opened: Astoria, Bend, Eugene, Medford, Portland, and Salem. The seventh office opened, either Pendleton or LaGrand, was sensitive to the pattern of complaint and CHOW demand in northeastern Oregon. The choice of either city as the seventh office opened could rest on qualitative considerations. Pendleton was here selected because it appeared in the majority of cases - eighteen out of twenty present cost runs. Likewise a selection had to be made between Coos Bay and Reedsport in the southern coast area. Reedsport appeared only once in twenty runs; Coos Bay was therefore selected. Additional support for the selection of these eight sites was given by the sensitivity analysis, since the eight appeared in nearly all computer runs and were included in the best and worst cases. The minimum cost savings with non-integer assignment of inspectors was $57,461.

In order to apply the decentralized result, integer numbers of in-

TABLE IV. SOLUTION RESULTS

Decentralized Location

RND Seed	Run	Facility Location Cost	Total Cost	Number of Inspectors
2323	1	36885	260853	14.691
9999	2	37030	260755	14.677
5656	3	37397	261442	14.707
8134	4	36417	260322	14.678
7345	5	38419	262394	14.729
6321	6	36374	260260	14.674
3789	7	36989	261009	14.702
1313	8	37170	261160	14.696
7058	9	37533	261569	14.705
8176	10	37763	261866	14.719
1111	11	39177	263214	14.742
9876	12	37642	260240	14.626
3333	13	37936	262069	14.725
9898	14	36508	260406	14.677
5555	15	37572	261620	14.707
2345	16	36896	260853	14.689
6754	17	37602	261697	14.717
1921	18	37426	261481	14.709
8633	19	37844	260635	14.646
4444	20	38387	262598 [1]	14.741
		Centalized Portland Office		
2323	1	85245	322973	16.739
9999	2	84782	322449	16.727
5656	3	86505	324448	16.784
8134	4	85735	323607	16.769
7345	5	89166	327521	16.869
6321	6	83396	320796	16.672
3789	7	84373	321959	16.711
1313	8	82746	320059 [2]	16.654
7058	9	87399	325471	16.811
8176	10	85227	322927	16.734
1111	11	88170	326422	16.848
9876	12	89933	328493	16.911
3333	13	86505	324433	16.781
9898	14	84441	321983	16.701
5555	15	85601	322785	16.739
2345	16	86901	324903	16.796
6754	17	84599	322192	16.712
1921	18	85192	322929	16.742
8633	19	89535	328024	16.897
4444	20	85768	323582	16.758

[1] Highest result with Pendleton included in the solution

[2] Lowest cost result

Decentralized Location, Integer Staff Assignments

RND Seed	Run	Facility Location Cost	Total Cost	Number of Inspectors
2323	1	41250	268024	14.936
9999	2	41817	268682	14.954
5656	3	42849	269871	14.987
8134	4	40682	267376	14.919
7345	5	44024	271221	15.023
6321	6	40857	267541	14.917
3289	7	42651	269635	14.979
1313	8	41710	268544	14.948
7058	9	43766	270894	15.009
8176	10	42752	269758	14.984
1111	11	44501	271760 [3]	15.036
9876	12	42693	269690	14.982
3333	13	43082	270125	14.991
9898	14	41670	268493	14.946
5555	15	42192	269084	14.960
2345	16	42797	269781	14.979
6754	17	42487	269451	14.975
1921	18	42184	269095	14.964
8633	19	43918	271109	15.022
4444	20	43482	270578	15.002

[3] Highest result with five facilities

spectors had to be assigned to the offices. The optimal non-integer result was perturbed slightly to find a near-optimal plan that resulted in integer assignments. It was noticed that demand in the area clustered around a possible office site might be sufficient to merit the opening of the office, but was never large enough to require one full inspector full time equivalent (FTE) even in the highest demand, worst case solution. This was true of Astoria and Bend. Since there were no other office sites that could possibly serve these areas plus adjoining demands to produce integer inspector requirements at lower cost, these two cities were dropped from the list of candidate sites (See Figure 16). The opening of an office to serve Northeastern Oregon demand resulted in all three cases. Location at Pendleton, however, produced lowest cost. In twenty present case complete runs, 192 sensitivity analysis runs, and this integer capacity analysis, Pendleton was nearly always selected. The other offices opened in the non-integer solution exhibited integer assignments. Hence, with integer assignment of inspectors, five offices were opened: Eugene, Medford, Pendleton, Portland, and Salem. The cost savings of this plan, again based on the highest cost assignment of complaints and CHOW from twenty computer runs, was $54,020; the number of inspectors at each site was 2, 1, 1, 8, and 3 respectively, as outlined in Figure 17.

It has been noted that the visiting of more than one facility on a round trip by inspectors - the formation of tours - could reduce the travel time and distance in the preceeding results. This gain from the formation of tours was difficult to estimate, since the sequencing of demands that leads to tour formation was not known. Alternately

- offices opened with integer-valued staffs
- additional offices opened in the non-integer-staff optimum

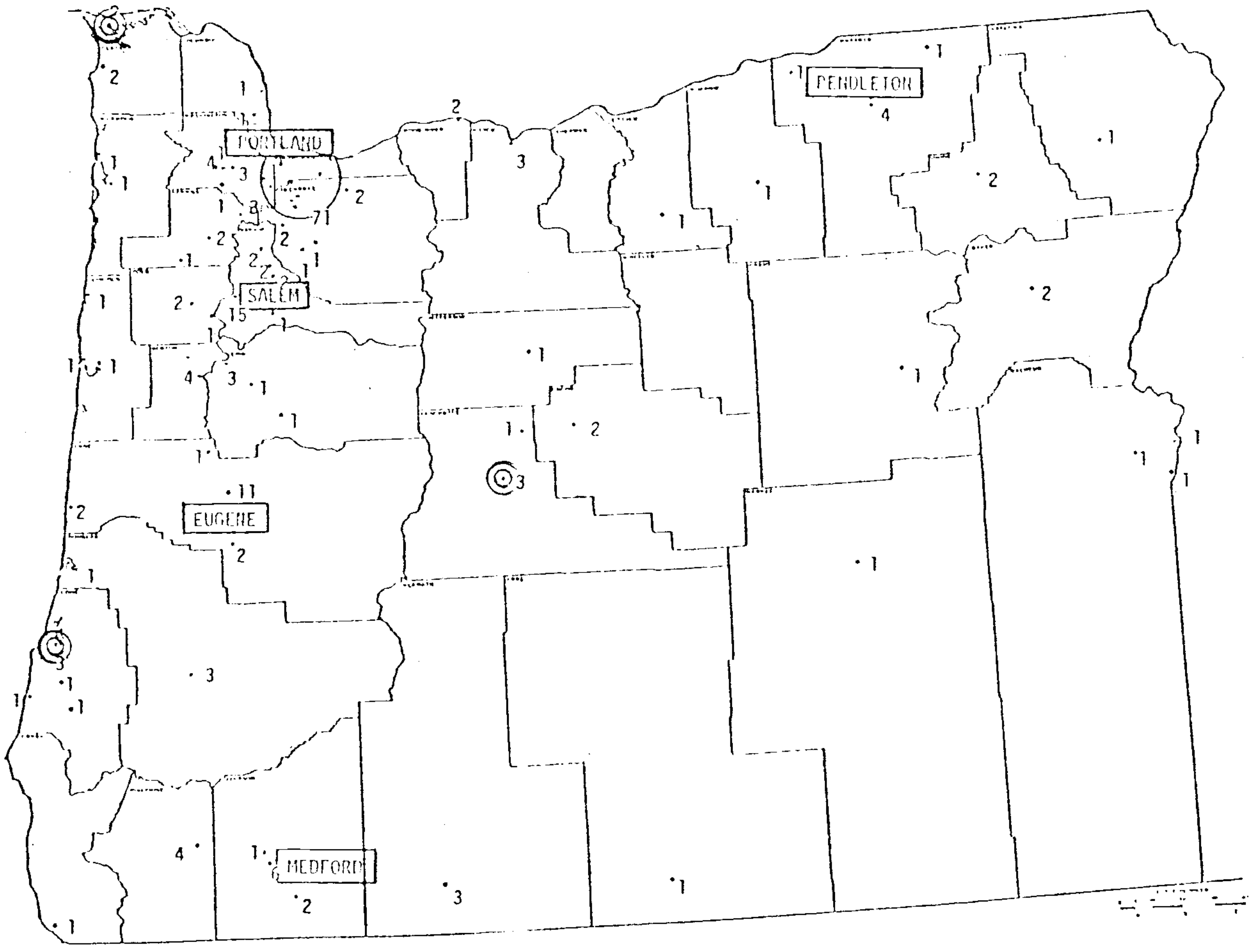

Figure 16. Multi-office locations with integer-valued staff assignments

I. Solution values for:	Total Cost	Number of Inspectors
1. Centralized Portland office location - least cost from 20 computer runs	$320,059	16.654
2. Multi-facility location with integer assignments of staff - highest cost result from 20 computer runs	$271,760	15.036

II. Modification of computer result to reflect integer assignments.

Cost Term	Value	
	Portland	Multi-office
Fixed cost - office opening costs and supervisor's travel cost*	0	5,201
Variable cost - state car cost and inspectors per diem	53,803	24,570
Inspectors' wages, Portland - (17.0 inspectors * $14,994)	254,898	
Inspectors' wages, Multi-office - (15.0 inspectors * $14,994)		224,910
Supervisor's wages	16,544	16,544
Total	$325,245	$271,225

III. Minimum cost savings

$$\begin{array}{r} = \ \ 325,245 \\ - \ \underline{271,225} \\ \$ \ 54,020 \end{array}$$

IV. Detail of multi-office inspector assignments:

Location	Computer Value	Integer Value
Eugene	2.073	2
Medford	1.160	1
Pendleton	1.109	1
Portland	7.669	8
Salem	3.025	3
TOTAL		15

* Values from computer results

Figure 17. Minimum multi-office cost savings with integer assignments.

stated, the transition matrix probabilities of moving from location to location cannot presently be determined.

In order to analyze the effect of reduced travel time and cost due to formation of tours, a simple assumption was made. If we assume the total distance travelled should be reduced by some percentage, and the per diem costs should remain approximately the same, then total cost varies as shown in Figure 18. Figure 18c shows that, under these assumptions, the integer staff decentralized location plan should become more attractive for reduction of ten and twenty per cent in total mileage travelled in both centralized and decentralized plans. At 40% reduction in distance for both plans, these savings in favor of decentralization drop about $4,000, from $54,020 to $50,002. The long distances from Portland to many nursing facilities versus the shorter distances under the multi-facility plan should yield a greater reduction of distance for the single-office case if tours are formed. If we model this more stringent assumption that the centralized Portland location of inspectors should have gained an estimated 40% efficiency while the decentralized plan gained only say, 10%, then the savings for decentralized offices should have been $45,254 (Figure 18d). Without further information on the formation of tours by inspectors, this 14.34% savings figure is a rough but reasonable lower bound on the savings that would have been realized with tour formation under the multiple office plan.

Sensitivity Analysis

It was desired to know how the multi-facility location reacted to changes in cost, demand, and efficiency. In order to do this in an

Reduction in total mileage of integer central-office result	Integer number of inspectors required	① Total Cost	Reduction in total cost from integer central office solution of $325,245
10%	17	$322,816	$ 2,429
20%	16	320,387	4,858
30%	16	317,958	7,287
③ 40%	16	315,529	9,716

a. Reduction in travel for Portland central office plan (integer staff).

Reduction in total mileage of integer multi-office result	Integer number of inspectors required	② Total Cost	Reduction in total cost from integer multi-office office solution of $271,225
④ 10%	15	$270,275	$ 950
20%	15	269,326	1,899
30%	15	268,376	2,849
40%	15	265,527	5,698

b. Reduction in travel for the multi-office plan (integer staff).

Reduction in mileage	Savings
0%	$ 54,020
10%	52,541
20%	51,061
30%	49,582
40%	50,002

c. Savings figures for reduction in mileage (①-②) applied to both Portland and multi-office cases.

10% reduction in distance for multi-office plan,

40% reduction in distance for central-office plan;

$\boxed{③ - ④ = \$45,254 = 14.34\%}$

d. Savings figure.

Figure 18. Estimate of reduction in travel distance with tour formation.

exact, organized manner, a multi-factor experiment was designed. The best and worst values for the parameters in the model were estimated and the number of complaints was set at three levels. These three levels were worst, present, and best case values. The worst case value assumed that the present trend in a rising number of complaints will continue and used the approximate number of complaints four years from now - 544[15]. The recent marked rise in complaints should level off at some point; it was estimated that this value lies in the area of the level-off point. The present case used the number (377) and distribution of complaints that occurred in Oregon in 1977. The best case modelled a return to the level (and distribution) of complaints in 1976 - 243 complaints.[16]

The other parameters were varied between high and low values. These were:

	best	worst	for comparison: modelled value
Changes of Ownership	20	40	30
Efficiency	75%	60%	70%
Speed, MPH	60	45	55
Number of Visits	3	6	4
Salary: Inspector	$14,994	$17,993	$14,994
Supervisor	16,544	19,853	16,544
Office Opening Cost	600	1,200	855

The best case salary levels reflected current costs; the worst case values were present cost plus 10%. Other values were estimates of reasonable

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[15] Value is based on least squares calculation found in Appendix 8.

[16] Complaint data are listed in Appendix 9.

endpoint values for the parameters.

Multi-factor design of the experiment required $3*2^6 = 192$ model results. This large multi-factor design allowed us to explicity analyze the many cases or scenarios modelled by variation of the parameters. Neter and Wasserman [44, p. 551] point out the efficiency of the procedure when used with an F-test analysis of variance. In our case, the analysis of variance would not be strictly correct. As in the Model Result analysis of the previous section, these deterministic results lack the unknown random error terms which the F-test analysis of variance analyzes. Degrees of freedon, here, equalled zero. We made use of the multi-factor format to present the mean, high, and low solution results for each parameter value when all other parameters were varied - a total of thirty-two cases for each value of each parameter. The mean averaged the effects of all other parameters to estimate the change in results due to the parameter in question. The high and low values established the expected range for all possible cases at the given parameter value. These values are displayed in Figures 19.1 - 19.3, and mean figures plus the range of solution values at the high and low parameter values are listed in Table V.

Total cost appeared to be most sensitive to the efficiency rating factor for inspector's time and to salary levels for inspectors and the supervisor, exhibiting about twenty per cent change between high and low parameter values. The number of complaints had a noticeable but less marked effect on total cost. The number of CHOW, the travel speed, the number of visits per facility, and office opening cost had small observed effects on total cost (see Figure 19.1 - 19.3).

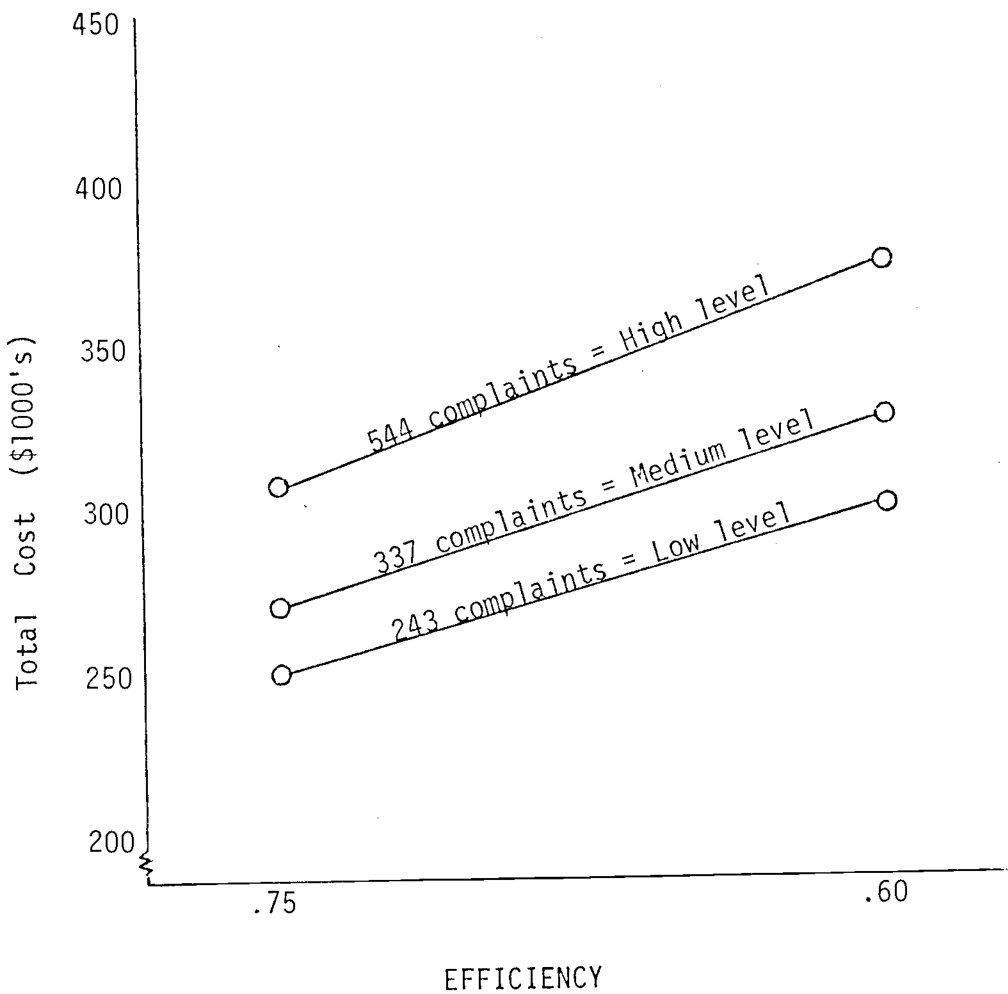

Figure 19.1a. Total cost versus efficiency at three levels of complaint demand.

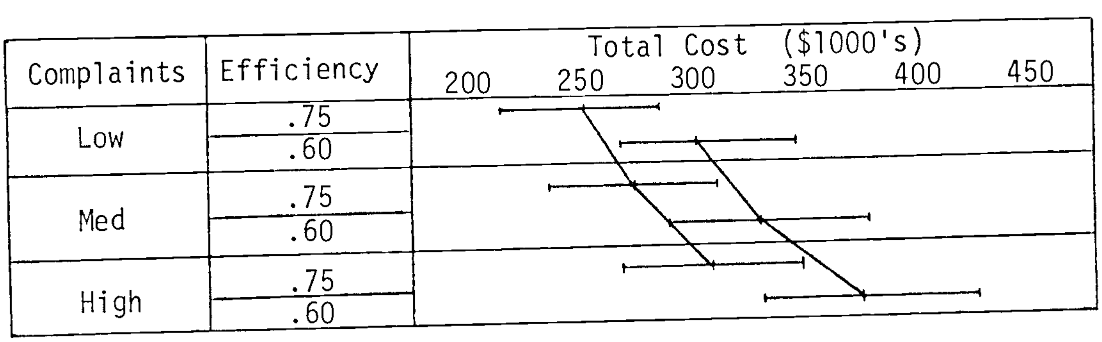

Figure 19.1b. Range of total cost for efficiency equal to 75% and 60% at three levels of complaint demand. Means are shown connected between complaint levels.

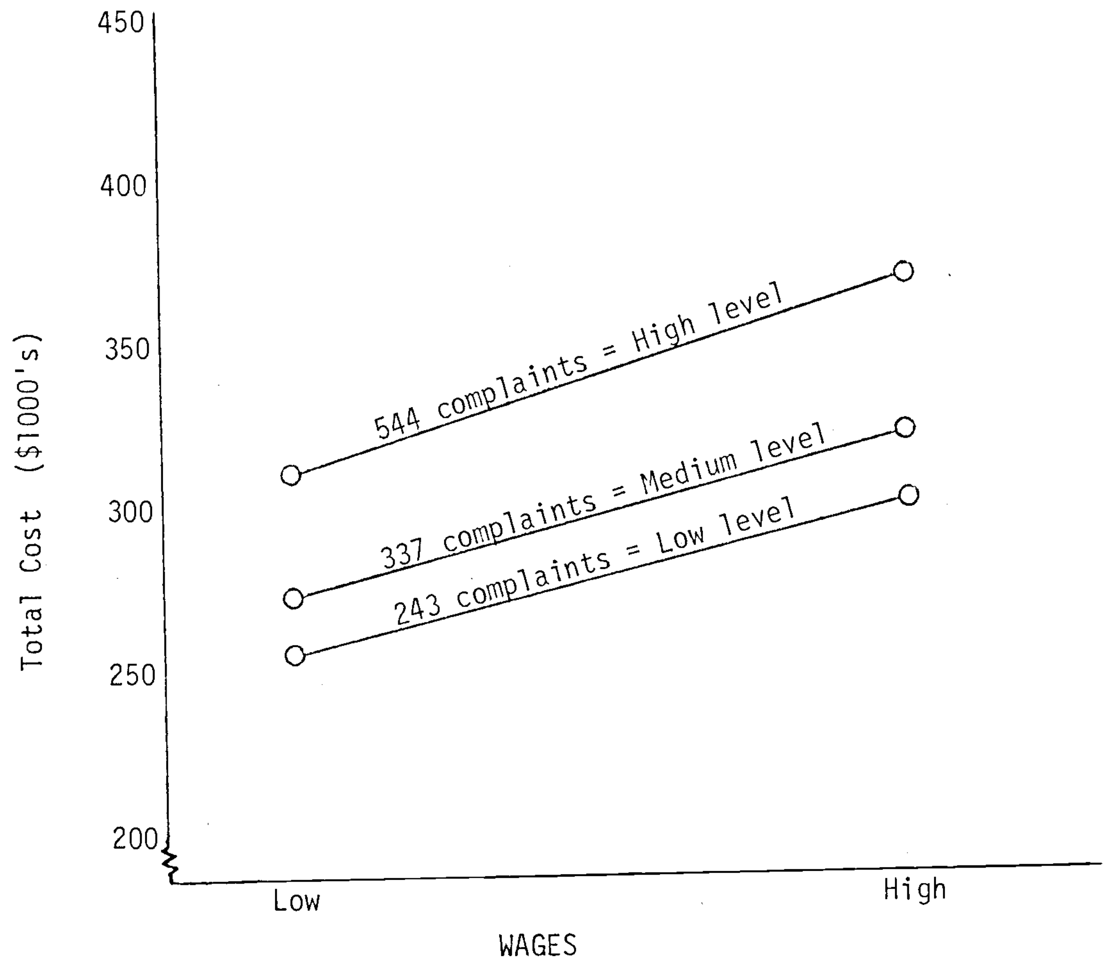

Figure 19.2a. Total cost versus low and high wage levels at three levels of complaint demand.

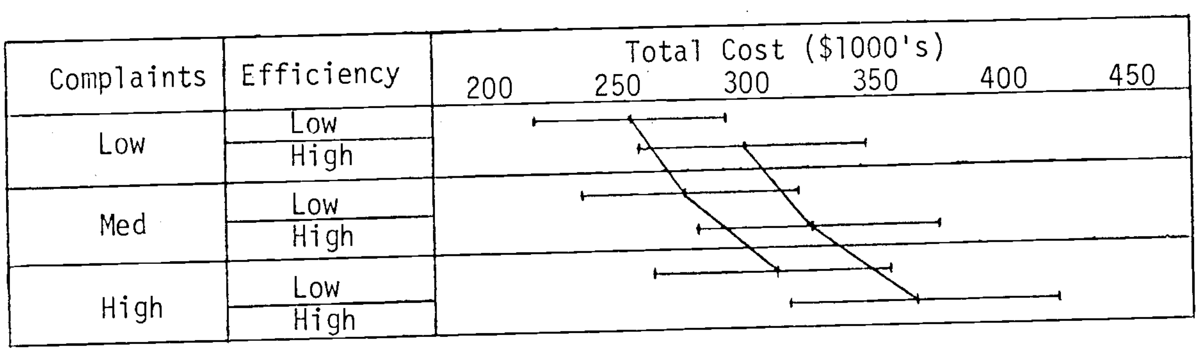

Figure 19.2b. Range of total cost for low and high wage levels at three levels of complaint demand. Means shown connected between complaint levels.

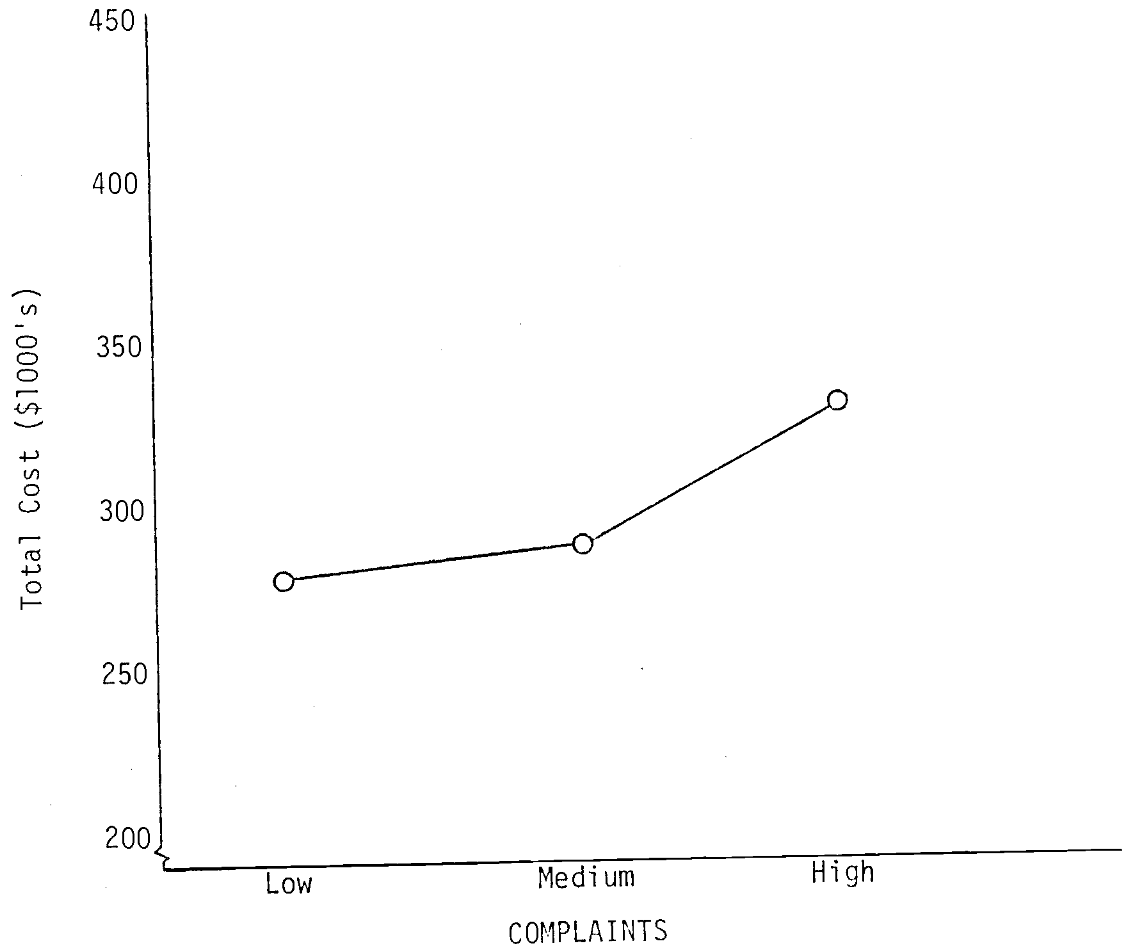

Figure 19.3a. Total cost versus complaint level.

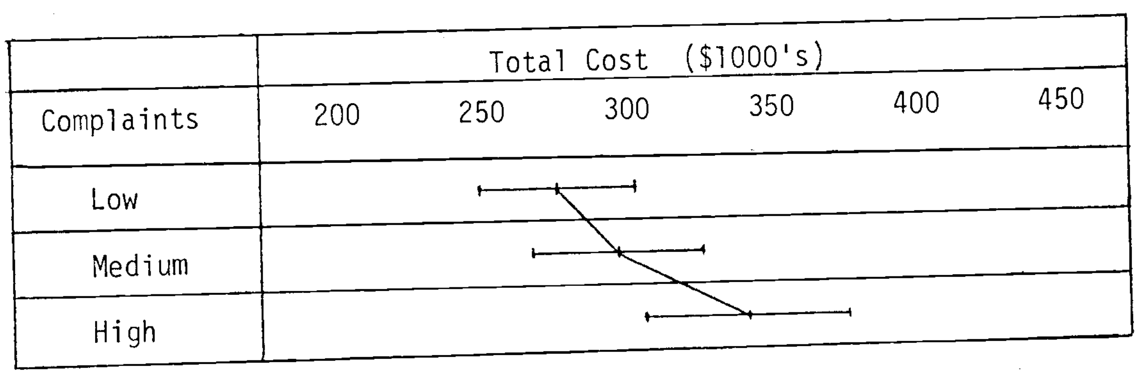

Figure 19.3b. Range of total cost at three levels of complaint demand. Means are shown connected between complaint levels.

TABLE V. SENSITIVITY ANALYSIS

Number of Complaints = 243

Parameter	* Solution	Lowest Solution Result	Average at Low Parameter Value	Highest Solution Result	Lowest Solution Result	Average at High Parameter Value	Highest Solution Result	Change in Average Value Δ	%
CHOW	1	27826	37526	49013	28466	38229	49665	703	1.87
	2	217752	278320	338317	224639	278320	346182	5061	1.85
	3	12.200	14.539	16.124	12.639	14.539	16.547	.476	3.38
EFF	1	27826	37877	49665	27826	37877	49665	0	-
	2	217752	250216	284744	265312	301363	346182	51147	20.44
	3	12.200	12.656	13.133	15.372	15.946	16.547	3.290	26.00
MPH	1	27826	35792	44397	31007	39963	49665	4171	11.65
	2	217752	275229	340159	220516	276350	341382	1121	.41
	3	12.200	14.173	16.213	12.384	14.430	16.547	.257	1.81
VISITS	1	27826	32978	38590	36355	41090	49665	8112	24.60
	2	217752	272086	333472	225693	282617	346182	10531	3.87
	3	12.200	14.151	16.163	12.381	14.327	16.547	.176	1.24
WAGES	1	27826	36550	46153	29735	39205	49665	2655	7.26
	2	217752	254253	293242	257648	300451	346182	46198	18.17
	3	12.200	14.274	16.547	12.200	14.328	16.547	.054	.38
OFFICE COST	1	27826	35621	44865	31930	40134	49665	4513	12.67
	2	217752	274677	335359	222296	280027	346182	5350	1.95
	3	12.200	14.274	16.547	12.234	14.328	16.547	.054	.38

*1 = Facility Location Cost Result 2 = Total Cost 3 = Number of Inspectors

Number of Complaints = 337

Parameter	* Solution	Lowest Solution Result	Average at Low Parameter Value	Highest Solution Result	Lowest Solution Result	Average at High Parameter Value	Highest Solution Result	Change in Average Value	
CHOW	1 2 3	30374 235532 13.272	40292 294677 15.259	51974 362090 17.352	30797 244160 13.701	40742 302842 15.743	52399 367531 17.914	450 8165 .484	1.12 2.77 3.17
EFF	1 2 3	30374 235532 13.272	40517 269347 13.718	52399 305819 14.218	30374 287273 16.723	40517 328172 17.284	52399 372331 17.914	0 58825 3.566	- 21.84 26.00
MPH	1 2 3	30374 235532 13.272	38285 296422 15.360	46837 365798 17.551	33837 238648 13.482	42749 301096 15.642	52399 372331 17.914	4464 4674 .282	11.66 1.58 1.84
VISITS	1 2 3	30374 235532 13.272	35652 293865 15.371	41362 359658 17.530	38863 243426 13.452	45382 303653 15.631	52399 372331 17.914	9730 9788 .260	27.29 3.33 1.69
WAGES	1 2 3	30374 235532 13.272	39097 273824 15.449	48691 315298 17.914	32452 278644 13.272	41937 323644 15.503	52399 372331 17.914	2840 49870 .004	7.26 18.21 .03
OFFICE COST	1 2 3	30374 235532 13.212	38256 296232 15.484	47599 367531 17.914	34516 240122 13.308	42778 301286 15.518	52399 372331 17.914	4522 5054 .034	11.82 1.71 .22

*1 = Facility Location Cost Result   2 = Total Cost   3 = Number of Inspectors

Number of Complaints = 544

Parameter	* Solution	Lowest Solution Result	Average at Low Parameter Value	Highest Solution Result	Lowest Solution Result	Average at High Parameter Value	Highest Solution Result	Change in Average Value	
CHOW	1	31621	41599	53334	32945	42847	54721	1248	3.00
	2	268728	335830	411369	275872	344949	422917	9119	2.72
	3	15.461	17.730	20.070	15.884	18.243	20.677	.513	2.89
EFF	1	31621	42223	54721	31621	42223	54721	0	-
	2	268728	306263	346146	329001	374516	422917	68253	22.29
	3	15.461	15.917	16.410	19.481	20.056	20.677	4.139	26.00
MPH	1	31621	39838	48795	35373	43983	54721	4145	10.405
	2	268728	338030	416369	271870	342748	422917	4718	1.40
	3	15.461	17.844	20.313	15.670	18.129	20.677	.285	1.60
VISITS	1	31621	37875	44688	39273	46571	54721	8696	22.96
	2	268728	336371	413921	275475	344408	422917	8037	2.39
	3	15.461	17.887	20.548	15.614	18.086	20.677	.199	1.11
WAGES	1	31621	40705	50770	33872	43741	54721	3036	7.46
	2	268728	311724	357597	318404	369055	422917	57331	18.39
	3	15.461	17.985	20.677	15.461	17.989	20.677	.004	.02
OFFICE COST	1	31621	39638	48729	36269	44808	54721	5170	13.04
	2	268728	337091	415713	274552	343688	422917	6597	1.96
	3	15.461	17.944	20.594	15.531	18.029	20.677	.085	.47

*1 = Facility Location Result 2 = Total Cost 3 = Number of Inspectors

Between-Complaint-Level Solutions

	Solution	Complaint Level 243	337	544
Lowest Solution Result	1 2 3	32978 250216 12.656	35652 269347	37875 306262
Highest Solution Result	1 2 3	41090 301363 15.946	45382 328172 17.284	46571 374516 20.056
Average Solution Result	1 2 3	37737 276571 14.291	40517 298759 15.501	42119 340389 17.987
Change (%)	1 2 3	2780 (7.37) 22180 (8.02) 1.210 (8.47)		1602 (3.95) 41630 (13.93) 2.486 (16.04)

*1 = Facility Location Result   2 = Total Cost   3 = Number of Inspectors

The facility location analysis incremental costs were a part of total costs. It is of interest to separate these costs to observe their contribution to total cost. The location analysis cost was affected most by the change in the number of visits, travel speed, and office operating cost (see Figure 19.4). The change in number of complaints appeared to be less important, while the level of CHOW had little effect.

The number of inspectors required to meet demand was apparently influenced predominately by the efficiency rating factor, and was also affected somewhat by the number of complaints. Other factors did not appear to significantly affect the number of inspectors required.

Best and worst case values are listed in Table VI. Worst case costs and inspector requirements resulted from running the model with the most adverse values assigned to all parameters; best case results came from the assignment of the most favorable parameter values. The expected total savings incurred by a multi-office plan was $44,869 in the best case. Under the increased demand and higher costs of the worst case, the savings magnitude was much greater, equal to $141,412.

In addition to these analyses, a study was performed to determine the effect of increased office opening cost on the five-office integer-assignment solution. All other cost, efficiency and demand parameters were fixed at present case levels, and office opening costs, for office locations other than Portland, were incremented by $500 steps. The results, listed in Table VII showed that the five offices - Eugene, Medford, Pendleton, Portland and Salem - should be opened at fixed added-office cost levels up to $6,000. For the range of added-office costs from $6,000 to $9,500, Medford, Pendleton, Portland, and Salem

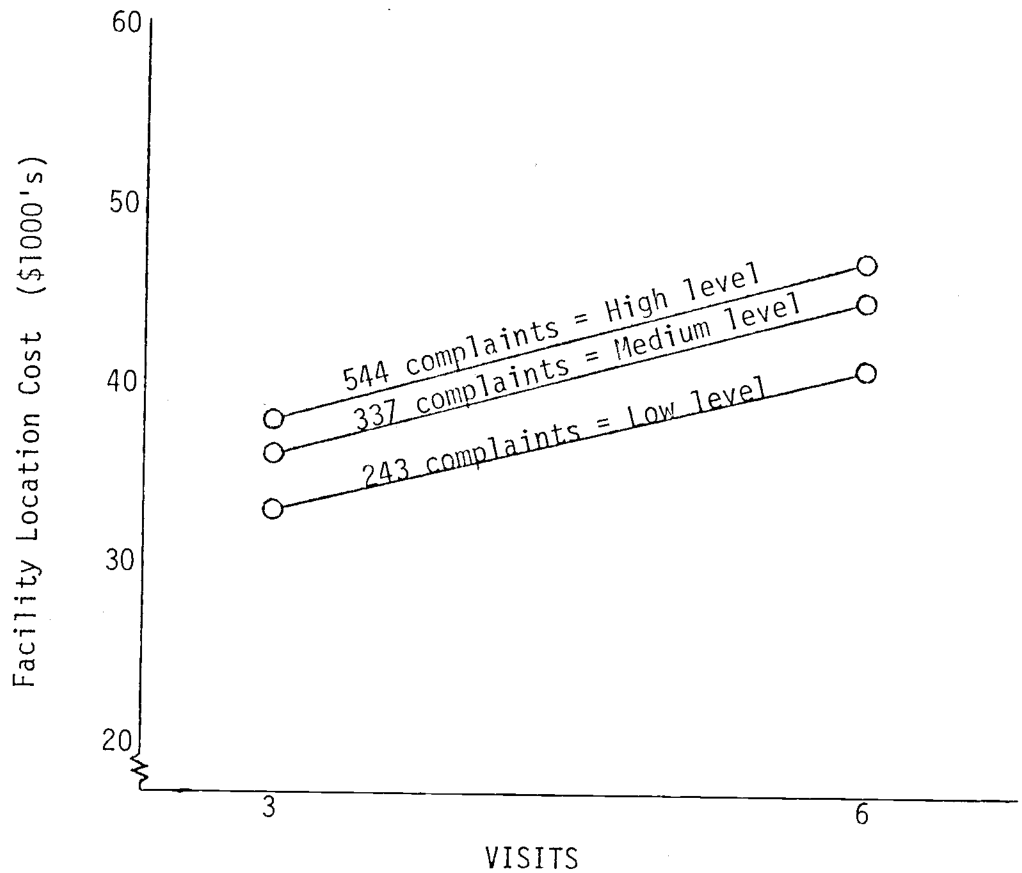

Figure 19.4a. Total cost versus low and high numbers of visits at three levels of complaint demand.

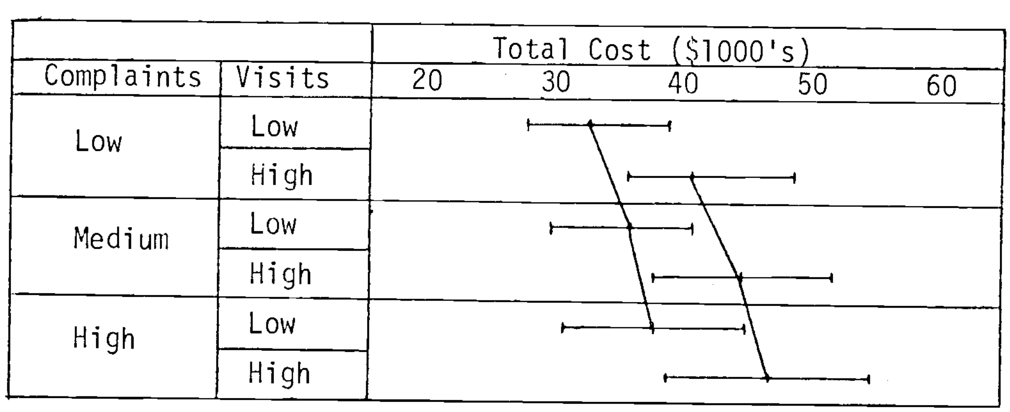

Figure 19.4b. Range of total cost for a low and high number of visits at three levels of complaint demand. Means are shown connected between complaint levels.

TABLE VI. BEST AND WORST CASE SOLUTIONS

Best Case			Worst Case	
Facility Location Cost		$27,826	Facility Location Cost	$54,721
Total Cost		217,752	Total Cost	422,917
Number of Inspectors		12.200	Number of Inspectors	20.677
Facilities Opened and (Number of Inspectors)			Facilities Opened and (Number of Inspectors)	
Astoria	(.346)		Astoria	(.755)
Bend	(.373)		Baker	(.777)
Coos Bay	(.493)		Bend	(.620)
Eugene	(.902)		Coos Bay	(.761)
Medford	(.879)		Eugene	(1.389)
Pendleton	(.861)		Klamath Falls	(.428)
Portland	(5.871)		Medford	(.902)
Salem	(2.474)		Pendleton	(.502)
			Portland	(10.678)
			Salem	(3.866)

TABLE VII. SOLUTION RESULTS WITH VARIOUS FACILITY OPENING COSTS

OPENING COST	VC	FC	TOTAL COST	NUMBER OF INSPECTORS	OFFICES *
855	24570	5201	271760	15.036	E, M, Pe, Pt, S
1000	24570	5781	272340	15.036	E, M, Pe, Pt, S
5000	24570	21781	288340	15.036	E, M, Pe, Pt, S
5500	24570	23781	290340	15.036	E, M, Pe, Pt, S
6000	29215	19465	293568	15.229	M, Pe, Pt, S
6500	29215	20965	295068	15.229	M, Pe, Pt, S
7000	29215	22465	296568	15.229	M, Pe, Pt, S
7500	29215	23965	298068	15.229	M, Pe, Pt, S
8000	29215	25465	299568	15.229	M, Pe, Pt, S
8500	29215	26965	301068	15.229	M, Pe, Pt, S
9000	29215	28465	302568	15.229	M, Pe, Pt, S
9500	29215	29965	304068	15.229	M, Pe, Pt, S
10000	36089	20882	308118	15.646	M, Pt, S
10500	42992	10815	311694	16.096	E, Pt
11000	42992	11315	312194	16.096	E, Pt
11500	42992	11815	312694	16.096	E, Pt
12000	42992	12315	313194	16.096	E, Pt
12500	42992	12815	313694	16.096	E, Pt
13000	42992	13315	314194	16.096	E, Pt
13500	42992	13815	314694	16.096	E, Pt
14000	42992	14315	315194	16.096	E, Pt
14500	42992	14815	315694	16.096	E, Pt
15000	42992	15315	316194	16.096	E, Pt
20000	42992	20315	321194	16.096	E, Pt
20500	42992	20815	321694	16.096	E, Pt
21000	42992	21315	322194	16.096	E, Pt
21500	57260	0.00	326422	16.848	Pt
22000	57260	0.00	326422	16.848	Pt
22500	57260	0.00	326422	16.848	Pt
23000	57260	0.00	326422	16.848	Pt
23500	57260	0.00	326422	16.848	Pt
24000	57260	0.00	326422	16.848	Pt
24500	57260	0.00	326422	16.848	Pt
25000	57260	0.00	326422	16.848	Pt
30000	57260	0.00	326422	16.848	Pt

* E Eugene
M Medford
Pe Pendleton
Pt Portland
S Salem

should be opened; at a $10,000 opening cost, Medford, Portland, and Salem should be opened. For fixed added-office costs from $10,500 to $21,000, Eugene and Portland should be HFLC office sites; for office opening costs of $21,500 or more, Portland only should be the site of an HFLC office. It should be noted that this result demonstrated that, at a $21,500 cost of opening each office outside of Portland, the additional offices should not be opened. The result did not indicate that Portland is the best office site in the state under a single-office plan.

VIII. CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDY

Conclusions

The first objective of this analysis was to find a method for solving a multi-office location problem. After a literature survey that investigated and compared the available solution techniques for discrete multi-facility location and multi-depot vehicle routing problems, a discrete multi-facility location algorithm by Khumawala was selected. This branch and bound procedure was chosen because it modelled the important characteristics of this multi-facility problem, yielded minimum cost solutions, and the computer code was available. The procedure represents the current state-of-the-art in exact multi-facility location algorithms, and was found to run efficiently. On Oregon State University's CDC Cyber 73, the Fortran IV program required 127,500 octal words of memory space; average run times ranged from .75 seconds for a one office, seventy-seven demand location problem to 3.85 seconds for a nineteen office, seventy-seven demand site problem.

The second objective was to obtain a result to support or reject the hypothesis of reduced HFLC inspection cost under a multiple office plan. Demand and cost data were collected and analyzed graphically and statistically (Chapter VI). In order to model the agency's activities, the computer program was modified to allow random elements in demand, include step costs, permit the use of actual or straight line distances, compute the number of staff required at each office, and evaluate the total cost of annual operations. The program determined the optimal office locations and staff requirements based on the trade-off between added fixed costs of locating offices and variable costs of transportation to demand sites that minimized cost.

In our study, the model indicated a fifteen percent improvement in cost with decentralization, based on integer-valued office staffs. Multiple computer runs established the minimum expected cost reduction.

The final objective of the study was to determine the practicality of the optimal multiple office plan. The plan has the following merits:

1) The sensitivity analysis showed that multiple offices opened under present conditions should remain open under a variety of foreseeable future circumstances, including the most adverse conditions of demand and cost projected over a four-year period.

2) Assumptions of increased future cost and demand improve the cost savings attributable to decentralization; decreased cost and demand reduce the savings by a smaller amount (Sensitivity Analysis, Chapter VII).

3) Staffing requirements are integer-valued.

4) The analysis places a bound on the reduction in the modelled costs due to the gain in efficiency from the formation of tours.

5) Multiple visits to each facility have been considered.

The decentralized plan has other attributes. The supervisor's role as a health care expert, inspection observer, and quality controller is enhanced. The element of surprise in inspection visits is increased. The cost results and qualitative items must be weighed against the loss of centralized control under decentralization. This task lies with the appropriate State decision maker. The study results have been presented to the Budget and Management Division of the State Executive Department, and the computer program has been supplied to the Division for use in future multi-office location problems.

Recommendations for Further Study

The present model can be improved in the following ways:

1) Compile data for the transition matrix probabilities of travel between locations. Then simulate weekly tours - that is, a path of visits to one or more facilities that returns to the office at week's end - using an approach such as the Markov Chain in Inoue [28] until all demands are satisfied. Modify the computer program to assign each tour as one round trip, with distance and demand equal to the total tour values. The distance function for a possible office site is then the sum of the tour length plus the minimum distance from the facility to the tour path. The branch and bound algorithm can then be applied. In the result, blocks of fifty tours can be assigned to one inspector to compute office staff sizes. Tours can thus be modelled external to the present multi-facility algorithm.

2) Forecast the long-term changes in demand and costs, and rerun the model.

3) Compile better data for the frequency versus length of visits needed to service various types of demand. More intensively standardized inspection procedures may be needed.

A need for development of a new algorithm to optimize facility location when office staff, salesmen, or vehicles can visit more than one demand location on a round trip is made obvious by this study. While much research has been directed toward both discrete multi-facility plant location and vehicle routing problems (VRP), we found it difficult to marry the two bodies of techniques.

Present multi-depot VRP's rely on inefficient and inexact heuristics that do not minimize the fixed cost/variable cost trade-offs. The heuristics allow only single visits to each demand site. The factorial number of possible tours in a network makes the problem very difficult to solve. Multi-facility algorithms, on the other hand, optimize cost, can be programmed to allow multiple visits, and balance fixed and variable costs. The existence of efficient discrete multi-facility optimizing procedures encourages their extension to implicitly model tours.

This study has utilized the discrete multi-facility location model to solve a practical multi-office problem. The need has been demonstrated for a new class of solution techniques for large problems that blends elements of discrete multi-facility location models and multi-depot vehicle routing models. Hopefully, we have marked the proper direction for the future development of such a technique.

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APPENDICES

APPENDIX 1

County	Nursing Home Complaints By County				Complaints per Home By County*				
	5/76-4/77	5/75-4/76	5/74-4/75	5/73-4/74	5/76-4/77	5/75-4/76	5/74-4/75	5/73-4/74	Number of homes,4/76
1. Baker	1	1	0	0	0.50	0.50	0.00	0.00	2
2. Benton	2	9	0	2	1.00	4.50	0.00	1.00	2
3. Clackamas	50	39	35	22	2.50	1.95	1.75	1.10	20
4. Clatsop	3	3	6	1	1.00	1.00	2.00	0.33	3
5. Columbia	2	0	0	1	1.00	0.00	0.00	0.50	2
6. Coos	2	5	3	2	0.33	0.83	0.50	0.33	6
7. Crook	0	1	1	1	0.00	1.00	1.00	1.00	1
8. Curry	1	0	0	0	1.00	0.00	0.00	0.00	1
9. Deschutes	2	1	2	2	0.50	0.25	0.50	0.50	4
10. Douglas	5	3	2	2	1.25	0.75	0.50	0.50	4
11. Gilliam	0	0	0	0	0.00	0.00	0.00	0.00	1
12. Grant	0	0	0	0	0.00	0.00	0.00	0.00	1
13. Harney	0	0	0	0	0.00	0.00	0.00	0.00	1
14. Hood River	2	0	0	1	1.00	0.00	0.00	0.50	2
15. Jackson	4	8	4	3	0.44	0.89	0.44	0.33	9
16. Jefferson	0	0	0	0	0.00	0.00	0.00	0.00	1
17. Josephine	2	1	0	0	0.50	0.25	0.00	0.00	4
18. Klamath	4	3	6	4	2.00	1.50	3.00	2.00	2
19. Lake	0	0	0	0	0.00	0.00	0.00	0.00	1
20. Lane	8	6	9	9	0.53	0.40	0.60	0.60	15
21. Lincoln	0	2	0	2	0.00	2.00	0.00	2.00	1
22. Linn	4	6	3	3	0.80	1.20	0.60	0.60	5
23. Malheur	1	0	0	2	0.50	0.00	0.00	1.00	2
24. Marion	42	30	16	13	2.10	1.50	0.80	0.65	20
25. Morrow	0	0	0	0	0.00	0.00	0.00	0.00	1
26. Polk	2	3	9	3	0.50	0.75	2.25	0.75	4
27. Tillamook	5	4	3	1	2.50	2.00	1.50	0.50	2
28. Umatilla	1	1	1	1	0.20	0.20	0.20	0.20	5
29. Union	2	1	0	0	1.00	0.50	0.00	0.00	2
30. Wallowa	0	0	0	0	0.00	0.00	0.00	0.00	1
31. Wasco	11	0	0	2	3.67	0.00	0.00	0.66	3
32. Washington	30	13	17	17	2.31	1.00	1.31	1.31	13
33. Yamhill	3	1	3	3	0.50	0.17	0.50	0.50	6
34. Multnomah	44	40	40	39	1.83	1.67	1.67	1.63	24
35. Portland	104	62	54	34	3.85	2.30	2.00	1.26	27
TOTAL	337	243	214	170					

Changes of Ownership			
5-76 to 4-77	5-75 to 4-76	5-74 to 4-75	7-73 to 4-74
30	24	19	28

*Based on the number of homes per county in April, 1976.

Nursing Facility Locations

City	Number of Facilities	City	Number of Facilities
Albany	3	Madras	1
Ashland	2	McMinnville	2
Astoria	2	Medford	6
Baker	2	Milton-Freewater	1
Bandon	1	Milwaukie	2
Beaverton-Marylhurst	3	Mollala	1
Bend	3	Mount Angel	2
Brookings	1	Myrtle Point	1
Burns	1	Newberg	3
Canby	2	Newport	1
Central Point	1	Nyssa	1
Colton	1	Ontario	1
Condon	1	Oregon City	4
Coos Bay-North Bend	3	Pendleton	4
Coquille	1	Portland	46
Cornelius	1	Prairie City	1
Corvallis	4	Prineville	2
Cottage Grove	2	Redmond	1
Dallas	2	Reedsport	1
Enterprise	1	Rockaway	1
Eugene-Springfield	11	Roseburg	3
Florence	2	Salem	15
Forest Grove	4	Sandy	2
Gaston	1	Scappoose	1
Gladstone	3	Seaside	2
Grants Pass-Merlin	4	Sheridan	1
Gresham	6	Silverton	2
Heppner	1	Saint Helens	1
Hermiston	1	Sublimity	1
Hillsboro	3	Sweet Home	1
Hood River	2	The Dalles	3
Independence	1	Tigard	3
Junction City	1	Tillamook	1
Klamath Falls	3	Toledo	1
La Grande	2	Troutdale	2
Lake Oswego	1	Vale	1
Lakeview	1	West Linn	1
Lebanon	1	Woodburn	2
Lincoln City	1		

APPENDIX 2

STEP COSTS: PER DIEM COST FORMULATIONS

State Employee Per Diems

Breakfast	$ 2.75
Lunch	2.75
Dinner	6.50
Lodging	13.00
Total	$25.00

Key

B
L
D
Lodg

Meal per diems are paid if an inspector is more than 25 miles from the office. Lodging is paid if an inspector is more than 50 miles from the office. It is assumed that per diem cost behaves as a step cost based on mileage. Travel speed is estimated at 50-60 mph.

Supervisor Per Diem Costs

Distance to Outlying Office (Miles)	Per Diem Cost	Per Diems Paid (Round Trip)		
		Day 1	Day 2	Day 3
0 - 24	$ 0.00	-		
25 - 59	2.75	L		
60 - 99	9.25	L,D		
100 - 199	25.00	L,D	Lodg, B	
200 - 299	34.25	L,D	Lodg, B,L,D	
300 +	59.25	L,D	Lodg, B,L,D	Lodg, B,L,D

We assume that the supervisor travels to half-day meetings at outlying HFLC offices. Per diems are based on distance and time. For example, if mileage to an office is 60-99 miles, the time required for travel in each direction is one to one and one half hours. The inspector, though eligible for a lodging per diem, can travel the round trip distance and conduct the meeting in one day. Suppose distance is 300 miles or greater. The inspector must spend at least six hours travelling in each direction. We tabulate the maximum number of per diems that could be expected to occur, assuming that the inspector travels the first day, travels and/or meets with inspectors during the second day, and returns on the third day.

Inspector Per Diem Costs for One-Day Visits

Distance to Nursing Facility (Miles)	Per Diem Cost	Per Diems Paid (Round Trip)		
		Day 1	Day 2	Day 3
0 - 24	$ 0.00	-		
25 - 59	2.75	L		
60 - 99	25.00	L,D	Lodg, B	
100 - 174	47.25	D	Lodg, B,L,D	B
175 - 299	52.75	L,D	Lodg, B,L,D	Lodg, B,L
300 +	62.00	B,L,D	Lodg, B,L,D	Lodg, B,L,D

Inspectors visits are modelled as requiring one full day on-site. We assume, for distances up to 60 miles, that the inspector returns to the office on the day of the visit. For a distance between 60 and 99 miles, we modelled a case frequently found in the inspector's travel agendas. The inspector travels to the facility on the afternoon of day one and begins the inspection. On day two the inspector finishes the visit and returns. For distances greater than 99 miles, we visualize the inspector travelling on the first day, inspecting on the second, and returning on the third. Added per diems for days 1 and 3 reflect added time "on the road" which includes more meal per diems as distance increases.

Inspector Per Diems for Three-Day Visits

Distance to Nursing Facility (Miles)	Per Diem Cost	Per Diems Paid (Three Round Trips)		
		Day 1	Day 2	Day 3
0 - 24	$ 0.00	-	-	-
25 - 59	$ 8.25	L	L	L

Distance to Nursing Facility (Miles)	Per Diem Cost	Per Diems Paid (One Round Trip)				
		Day 1	Day 2	Day 3	Day 4	Day 5
60 - 99	97.25	D	Lodg,B,L,D	Lodg,B,L,D	Lodg,B,L,D	B
100 - 174	100.00	L,D	Lodg,B,L,D	Lodg,B,L,D	Lodg,B,L,D	B
175 - 299	102.75	L,D	Lodg,B,L,D	Lodg,B,L,D	Lodg,B,L,D	B,L
300 +	112.00	B,L,D	Lodg,B,L,D	Lodg,B,L,D	Lodg,B,L,D	B,L,D

The licensing and certification visit requires three days on-site. We assume that the inspector returns to the office each day if one-way distance is less than 60 miles. If distance is 60 miles or more, the inspector is lodged at the nursing facility location. Per diems on days 1 and 5 reflect added travel time.

APPENDIX 3

VARIABLE COST FORMULATION

Three types of costs are incurred by the HFLC Section in travelling from office j to nursing facility k:

1. $(\text{cost of state car})_{kj}$
2. $(\text{per diem cost})_{kj}$
3. $(\text{inspector time cost})_{kj}$

The variable cost of satisfying all demand of facility k from office j is:

$$VC_{kj} = (\text{number of visits/year})*\{[(\text{distance}_{kj}\cdot \$2)* (\text{state car cost/mile} + \text{inspector time cost/mile})] + \text{per diem cost/visit}\}$$

1. State car cost is $.11/mile.
2. Per diems vary with steps in distance $_{kj}$; the approximation is outlined in Appendix 2.
3. An inspector's time is used nonproductively for travel. We base the worth of an inspector on his salary. The budgeting figure used by the State for inspectors is $14,994.

$$\text{Cost per mile of inspector's time} = \text{WAGEMI} = \frac{(\text{salary/year})}{(\text{hours/year})*(\text{miles/hour})}$$

At an average of 55 mph, this cost is $\frac{\$14,994}{(2080)*(55)} = \$.1311/\text{mile}$.

We have assumed that homes are visited four times per year, plus twice for each complaint and four times for each change of ownership. For a facility with one complaint and no changes of ownership, the variable cost would be:

$$VC_{kj} = [4 + 1*(2) + 0*(4)] \text{ visits} * \{[(\text{distance}_{kj}*2) * (\$.11 + \$.13)] + (\text{one three-day per diem} + \text{five one-day per diems})\}.$$

Our assumptions require that if the distance is less than sixty miles, the inspector makes three trips to license and certify; if the distance is more than sixty miles, the inspector makes one round trip and stays overnight at the facility location. The VC expressions for distances greater than sixty miles and distances less than or equal to sixty miles differ in the number of licensing visits modelled.

If (one-way mileage $_{kj}$ ≤ 60):

VC_{kj} =

{[(one 3-day visit)*(2*distance $_{kj}$)*(3 round trips)*(car cost.mile + wagemi)] + (3-day per diem)}

+

{(visits-1 1-day visits)*[((Distance $_{kj}$*2)*(1 round trip)*(car cost/mile + wagemi)) + (1-day per diem)]}

The first bracketed term is the cost of the three-day licensing and certification visit; the second term is the cost of the remaining one-day visits.

Likewise, if (one-way mileage $_{kj}$ > 60):

VC_{kj} =

{[(one 3-day visit)*(2*distance $_{kj}$) + (1 round trip)*(car cost/mile + wagemi)] + (3-day per diem)}

+

{(visits-1 1-day visits)*[((distance$_{kj}$*2)*(1 round trip)*(car cost/mile + wagemi)) + (1-day per diem)]}.

To determine the total variable cost of serving the demand of some city i, we sum up the variable costs of serving all facilities k in that city.

$$VC_{ij} = \sum_{k=1}^{homes} VC_{kj}$$

where homes = the number of facilities in city i.

APPENDIX 4

FIXED COSTS

Fixed costs are associated with the cost of operating an additional office j. These are office space and maintenance cost, office support costs, and supervisor's travel cost.

1. OFFICE SPACE

State budgeting model: one-man office requires 200 $ft.^2$ at $\$.55/ft.^2/month.$, (1)

additional office staff requires 125 $ft.^2$ at $\$.55/ft.^2/month.$ (2)

This cost includes maintenance. The resulting incremental cost to establish an outlying office is the higher rate (1) paid for the first inspector, less the lower rate paid in Portland for the inspector (2); this applies only to the first inspector. Additional staff at each outlying office are charged under (2) as they were charged in Portland.

$$\text{Office Space Cost} = (12\ \text{mo./year})*(200\text{-}125\ \text{ft.}^2)*(\$.55/\text{ft.}^2/\text{mo.})$$

$$= \$495/\text{year}, \quad j=1,2,3,\ldots,n$$

2. OFFICE SUPPORT COSTS

Telephone and SPAN-line cost is the same at centralized and decentralized offices, as is secretarial service. Extra postage is required at outlying offices. If reports average about five ounces, a montly postage allowance of $30.00 would allow about 45 first-class report mailings.

$$\text{Support Cost} = (\$30/\text{mo.})*(12\ \text{mo.}) = \$360/\text{year}, \quad j=1,2,3,\ldots,n$$

3. SUPERVISOR'S TRAVEL COST

The supervisor must travel to outlying offices once a month for meetings with inspectors. This cost is:

$$SUPR_j = (\text{transportation cost } j) + (\text{wage cost per mile}_j) + (\text{per diems }_j).$$

Salary is used as the measure of the worth of the supervisor's time and a speed of 55 mph is assumed.

$$\text{WAGE SP} = \frac{\$16{,}544/\text{year}}{(2080 \text{ hours/year})*(55 \text{ miles/hour})} = \$.1446/\text{mile}$$

State car cost = $.11/mile

$$SUPR_j = \{[(\$.11/\text{mile} + \$.1446/\text{mile})*(\text{Distance}_j*2)] + (\text{per diem based on distance})\}* 12 \text{ meetings}.$$

4. FIXED COST_j

$$FC_j = \text{Office Space Cost} + \text{Office Support Cost} + SUPR_j$$
$$= \$495 + \$360 + SUPR_j$$

APPENDIX 5

COMPARISON OF MEANS*

Complaints/Home

	1977	1976	1975	1974	
mean	.952	.775	.603	.564	$\overline{X}$ = .724
$T_i = \Sigma x$	33.310	27.110	21.120	19.750	
Σx^2	68.212	52.531	35.474	22.178	

$n_i = 35$ $\quad$ $p = 4$

$n = 140$

$$\text{Total SS} = (68.212 + 52.531 + 35.474 + 22.178) - 140(.724)^2 = 105.010$$

$$\text{SST} = \frac{33.310^2 + 27.110^2 + 21.120^2 + 19.750^2}{35} - 140(.724)^2 = 3.205$$

$$\text{SSE} = \text{Total SS} - \text{SST} = 101.806$$

$$\text{MSE} = \frac{\text{SSE}}{n_1 + n_2 + n_3 + n_4 - p} = \frac{\text{SSE}}{136} = .749$$

$$\text{MST} = \frac{\text{SST}}{p - 1} = \frac{\text{SST}}{3} = 1.068$$

HYPOTHESIS

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 \qquad \nu_1 = p - 1 = 3$

$H_1: \mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4 \qquad \nu_2 = n - p = 136$

TEST

Reject H_0 if $F = \frac{\text{MST}}{\text{MSE}} > F_{\alpha,\nu_1,\nu_2}$

$F_{.10,\ 3,\ 136} = 2.08$

$F_{.05,\ 3,\ 136} = 2.60$

$F_{.01,\ 3,\ 136} = 3.78$

* Taken from Mendenhall [43] p. 330.

$$F = \frac{MST}{MSE} = 1.427$$

CONCLUSION

Cannot reject (H_0: equal means) at α = .10, .05, or .10

COMPLAINTS

	1977	1976	1975	1974	
mean	9.63	6.94	6.11	4.86	$\overline{X} = 6.89$
$T_i = \Sigma x$	337	243	214	170	
Σx^2	18253	8339	6582	3786	

$n_i = 35$

$n = 140$

$p = 4$

$$\text{Total SS} = (18253 + 8339 + 6582 + 3786) - 140 \left[\frac{337+243+214+170}{140}\right]^2 = 30322$$

$$SST = \frac{337^2 + 243^2 + 214^2 + 170^2}{35} - 140 \left[\frac{337+243+214+170}{140}\right]^2 = 428$$

$$SSE = \text{Total SS} - SST = 29894$$

$$MSE = \frac{SSE}{n_1 + n_2 + n_3 + n_4 - p} = \frac{29894}{136} = 220$$

$$MST = \frac{SST}{3} = 143$$

HYPOTHESIS

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$ $\qquad \nu_1 = p - 1 = 3$

$H_1: \mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4$ $\qquad {}_2 = n - p = 136$

TEST

Reject H_0 if $F = \frac{MST}{MSE} > F_{2,\nu_1,\nu_2}$

$F_{.10,\ 3,\ 136} = 2.08$

$F_{.05,\ 3,\ 136} = 2.60$

$F_{.01,\ 3,\ 136} = 3.78$

$$F = \frac{MST}{MSE} = 0.650$$

CONCLUSION

Cannot reject (H_0: equal means) at $\alpha = .10, .05,$ or $.01$

COMPARING VARIANCES*

Complaints/Home

	1 1977	2 1976	3 1975	4 1974
mean	.952	.775	.603	.564
s	1.036	.963	.818	.570
s^2	1.074	.927	.669	.325

Year	F Value
(1,2)	F = 1.074/.927 = 1.159
(1,3)	F = 1.074/.669 = 1.605
(1,4)	F = 1.074/.325 = 3.305
(2,3)	F = .927/.669 = 1.386
(2,4)	F = .927/.325 = 2.852
(3,4)	F = .669/.325 = 2.058

$n_1 = n_2 = n_3 = n_4 = 35$

HYPOTHESIS $H_0: \sigma_i^2 = \sigma_j^2$, $i = 1,2,3,4$ $j = 2,3,4,$ $i < j$

$H_1: \sigma_i^2 \neq \sigma_j^2$, $i = 1,2,3,4$ $j = 2,3,4$ $i < j$

TEST STATISTIC $F = \dfrac{S_i^2}{S_j^2}$, $S_i^2 > S_j^2$

REJECTION REGION $F > F_{\alpha/2,\ 34,\ 34}$ $F_{.05,\ 30,\ 30} = 1.84$

CONCLUSION

Cannot say with 90% confidence that these years have unequal variance:

(1977. 1974)
(1976, 1974)
(1975, 1974)

* From Mendenhall [43], p. 238.

Can reject (H_0: equal variances) for these years:

(1977, 1976)
(1977, 1975)
(1976, 1975)

COMPARING VARIANCES

COMPLAINTS

	1977	1976	1975	1974
mean	9.629	6.943	6.029	4.857
s	21.010	13.987	12.394	9.331
s^2	441.420	195.636	153.611	87.068

Year	F Value			
(1,2)	$F = 21.010^2/13.987^2$	$= 441.420/195.636$	$= 2.256$	$n_1=n_2=n_3=n_4=35$
(1,3)	$F = 21.010^2/12.394^2$	$= 441.420/153.611$	$= 2.874$	
(1,4)	$F = 21.010^2/\ 9.331^2$	$= 441.420/\ 87.068$	$= 5.070$	
(2,3)	$F = 13.987^2/12.394^2$	$= 195.636/153.611$	$= 1.274$	
(2,4)	$F = 13.987^2/\ 9.331^2$	$= 195.636/\ 87.068$	$= 2.247$	
(3,4)	$F = 12.394^2/\ 9.331^2$	$= 153.611/\ 87.068$	$= 1.764$	

HYPOTHESIS $\quad H_0: \sigma_i^2 = \sigma_j^2, \quad i = 1,2,3,4 \quad j = 2,3,4 \quad i < j$

$\quad H_1: \sigma_i^2 \neq \sigma_j^2, \quad i = 1,2,3,4 \quad j = 2,3,4 \quad i < j$

TEST STATISTICS $\quad F = \dfrac{S_i^2}{S_j^2}, \quad S_i^2 > S_j^2$

REJECTION REGION $\quad F > F_{\alpha/2,\ 34,\ 34}$

$$F_{.05,\ 30,\ 30} = 1.84$$

CONCLUSION

Can say with 90% confidence that these years have unequal variances:

(1977, 1976)
(1977, 1975)
(1977, 1974)
(1976, 1974)

Cannot reject (H_0: equal variances) for these years:

(1976, 1975)
(1975, 1974)

APPENDIX 6

CONTINGENCY TABLE - COMPLAINTS*

n_{ij}:

Year_i	COUNTY_j																																			Total	
	B	B	C	C	C	C	C	C	D	D	G	G	H	H	J	J	J	K	L	L	L	L	M	M	M	P	T	U	U	W	W	W	Y	M	P	Total	r_i
1977	1	2	50	3	2	2	0	1	2	5	0	0	0	2	4	0	2	4	0	8	0	4	1	42	0	2	5	1	2	0	11	30	3	44	104	337	
1976	1	9	39	3	0	5	1	0	1	3	0	0	0	0	8	0	1	3	0	6	2	6	0	30	0	3	4	1	1	0	0	13	1	40	62	243	
1975	0	0	35	6	0	3	1	0	2	2	0	0	0	0	4	0	0	6	0	9	0	3	0	16	0	9	3	1	0	0	0	17	3	40	54	214	
1974	0	2	22	1	1	2	1	0	2	2	0	0	0	1	3	0	0	4	0	9	2	3	2	13	0	3	1	1	0	0	2	17	3	39	34	170	
Total c_j	2	13	146	13	3	12	3	1	7	12	0	0	0	3	19	0	3	17	0	32	4	16	3	101	0	17	13	4	3	0	13	77	10	163	254	964	n

* From Mendenhall [43], p. 288.

$$\hat{E}\,(n_{ij}) = \frac{r_i c_j}{n} \quad :$$

$Year_i$	COUNTY$_j$																		
	B	B	C	C	C	C	C	C	D	D	G	G	H	H	J	J	J	K	L
1977	.70	4.54	51.04	4.54	1.05	4.20	1.05	.35	2.45	4.20	0	0	0	1.05	6.64	0	1.05	5.94	0
1976	.50	3.28	36.8	3.28	.76	3.02	.76	.25	1.76	3.02	0	0	0	.76	4.79	0	.76	4.29	0
1975	.44	2.89	32.41	2.89	.67	2.66	.67	.22	1.55	2.66	0	0	0	.67	4.22	0	.67	3.77	0
1974	.35	2.29	25.75	2.29	.53	2.12	.53	.18	1.23	2.12	0	0	0	.53	3.35	0	.53	3.00	0
Total c_j	2	13	146	13	3	12	3	1	7	12	0	0	0	3	19	0	3	17	0

$Year_i$	COUNTY$_j$																	
	L	L	L	M	M	M	P	T	U	U	W	W	W	Y	M	P	Total	r_i
1977	11.19	1.40	5.59	1.05	35.31	0	5.94	4.54	1.40	1.05	0	4.54	26.92	3.50	56.98	88.79	337	
1976	8.07	1.01	4.03	.76	25.46	0	4.29	3.28	1.01	.76	0	3.28	19.41	2.52	41.09	64.03	243	
1975	7.10	.89	3.55	.67	22.42	0	3.77	2.89	.89	.67	0	2.89	17.09	2.22	36.18	56.39	214	
1974	5.64	.71	2.82	.53	17.81	0	3.00	2.29	.71	.53	0	2.29	13.58	1.76	28.74	44.79	170	
Total c_j	32	4	16	3	101	0	17	13	4	3	0	13	77	10	163	254	964	n

$$X^2 = \sum_{i=1}^{4} \sum_{j=1}^{35} \frac{[n_{ij} - \hat{E}(n_{ij})]^2}{\hat{E}(n_{ij})} = \begin{array}{r} 32.84309078 \\ 30.80024717 \\ 26.54904905 \\ +\ 23.62751049 \\ \hline \end{array}$$

$$= 113.8198975 \qquad \text{d.f.} = (r-1)\ (c-1) = (4-1)(35-1) = 102$$

Use $\alpha = .05$, reject null hypothesis, H_0: cell probability $(\hat{E}(n_{ij})) = n_{ij})$, all (i,j) ; that is, the two classifications are independent if the computed statistic $X^2 > X^2_{102,\ .95}$

$\approx X^2_{100,\ .95} = 124.342$

$X^2_{100,\ .90} = 118.498$

Data does not present sufficient evidence to indicate that the proportion of complaints in a county varies from year to year. Pattern, by counties, of a level of complaints, cannot be refuted statistically.

CONTINGENCY TABLE - COMPLAINTS/HOME

n_{ij}:

Year_i	COUNTY_j																				
	B	B	C	C	C	C	C	C	D	D	G	G	H	H	J	J	J	K	L	L	L
1977	.50	1.0	2.5	1.0	1.0	.33	0	1.0	.5	1.25	0	0	0	1	.44	0	.5	2	0	.53	0
1976	.5	4.5	1.95	1	0	.83	1	0	.25	.75	0	0	0	0	.89	0	.25	1.5	0	.4	2
1975	0	0	1.75	2	0	.5	1	0	.5	.5	0	0	0	0	.44	0	0	3	0	.6	0
1974	0	1	1.1	.33	.5	.33	1	0	.5	.5	0	0	0	.5	.33	0	0	2	0	.6	2
Total	1	6.5	7.3	4.33	1.5	1.99	3	1	1.75	3	0	0	0	1.5	2.10	0	.75	8.5	0	2.13	4

Year_i	COUNTY_j															
	L	M	M	M	P	T	U	U	W	W	W	Y	M	P	Total	r_i
1977	.8	.5	2.1	0	.5	2.5	.2	1.0	0	3.67	2.31	.5	1.83	3.85	33.310	
1976	1.2	0	1.5	0	.75	2	.2	.5	0	0	1	.17	1.67	2.3	27.110	
1975	.6	0	.8	0	2.25	1.5	.2	0	0	0	1.31	.5	1.67	2	21.120	
1974	.6	1	.65	0	.75	.5	.2	0	0	.66	1.31	.5	1.63	1.26	19.750	
Total c_j	3.2	1.5	5.05	0	4.25	6.5	.8	1.5	0	4.33	5.93	1.67	6.80	9.41	101.290	n

$$\hat{E}\,(n_{ij}) = \frac{r_i c_j}{n} :$$

$Year_i$	$COUNTY_j$																	
	B	B	C	C	C	C	C	C	D	D	G	G	H	H	J	J	J	K
1977	.329	2.138	2.401	1.424	.493	.654	.987	.329	.576	.987	0	0	0	.493	.691	0	.247	2.795
1976	.268	1.740	1.954	1.159	.401	.533	.803	.268	.468	.803	0	0	0	.401	.562	0	.201	2.275
1975	.209	1.355	1.522	.903	.313	.415	.626	.209	.365	.626	0	0	0	.313	.438	0	.156	1.772
1974	. 95	1.267	1.423	.844	.292	.388	.585	.195	.341	.585	0	0	0	.292	.409	0	.146	1.657
Total	1	6.5	7.3	4.33	1.5	1.99	3	1	1.75	3	0	0	0	1.5	2.10	0	.75	8.5

$Year_i$	$COUNTY_j$																	
	L	L	L	L	M	M	M	P	T	U	U	W	W	W	Y	M	P	Total
1977	0	.700	1.315	1.052	.493	1.661	0	1.398	2.138	.263	.493	0	1.424	1.95	.549	2.236	3.095	33.310
1976	0	.57	1.071	.856	.401	1.352	0	1.138	1.74	.214	.401	0	1.159	1.587	.447	1.82	2.519	27.110
1975	0	.444	.834	.667	.313	1.053	0	.886	1.355	.167	.313	0	.903	1.236	.348	1.418	1.962	21.120
1974	0	.415	.78	.624	.292	.985	0	.829	1.267	.156	.292	0	.844	1.156	.326	1.326	1.835	19.750
Total c_j	0	2.13	4	3.2	1.5	5.05	0	4.25	6.5	.8	1.5	0	4.33	5.93	1.67	6.80	9.41	101.290

$$\chi^2 = \sum_{i=1}^{4} \sum_{j=1}^{35} \frac{[n_{ij}-\hat{E}(N_{ij})]^2}{\hat{E}(n_{ij})} = 11.61892442$$

9.64448972

$$
\begin{array}{r}
9.81258588 \\
+\ 6.75257751 \\
\hline
= \quad 37.82857753
\end{array}
$$

d.f. = (r - 1) (c - 1) = (4 - 1) (35 - 1) = 102

Cannot reject H_0: Two classifications are independent for

$$X^2_{100,\ .95} = 124.342$$

$$X^2_{100,\ .90} = 118.498\ .$$

APPENDIX 7

APPROXIMATE CHI-SQUARE TEST FOR POISSON AND NORMAL MEANS*

NORMAL DISTRIBUTION:

HYPOTHESIS H_0: the distribution is normal

H_1: the distribution is not normal

K = Four categories

$$Y'_{k-3} = \sum_{i=1}^{k} \frac{(x_i - n\hat{p}_i)^2}{n\hat{p}_i} \qquad \text{d.f.} = k-3 = 1,\ \alpha = .05$$

REJECTION REGION

Reject H_0 if $Y'_{k-3} > \chi^2_{k-3,\ 1-\alpha} = \chi^2_{1;.95} = 3.84$

Category	Number of observations
$-\infty$ - 0	33
0 - 1	59
1 - 2	22
2 - 5	10

$n = 140 \qquad \bar{X} = .721$

$n\hat{p}_i = 35 \qquad \Sigma X = 101.290$

$\hat{p}_i = .25 \qquad \Sigma X^2 = 178.395$

$$S = \left[\frac{178.395 - \frac{(101.290)^2}{140}}{140 - 1}\right]^{1/2} = .870$$

From standard normal table, the four intervals on Z meeting the .25 probability requirement are

$(-\infty, -.675)$, $(-.675, 0)$ (), $.675)$, $(.675, +\infty)$

* From Guenther [26], p. 316.

$$Z = \frac{X-\mu}{\sigma} \; ; \; X = \sigma Z + \mu \approx SZ + \overline{X}$$

Intervals on X are:

$(-\infty, .134)$, $(.134, .721)$, $(.721, 1.308)$, $(1.308, +\infty)$

number of observations in interval	49	39	24	28

TEST STATISTIC

$$Y_1' = \frac{(49-35)^2}{35} + \frac{(39-35)^2}{35} + \frac{(24-35)^2}{35} + \frac{(28-35)^2}{35} = 10.91$$

CONCLUSION

Reject H_0: distribution is normal at $\alpha = .05$

(can reject at $\alpha = .005$, $\chi^2_{1;\,.995} = 7.88$)

POISSON DISTRIBUTION:

HYPOTHESIS

H_0: $p_1 = p(0;\mu)$, $p_2 = p(1;\mu)$, ..., $p_{k-1} = p(k-2;\mu)$, $p_k = 1 - \sum_{i=1}^{k-1} p_i$

H_1: p's are not given by Poisson

REJECTION REGION

Reject H_0 if $Y'_{n-2} > \chi^2_{138;.95} = 124.3$ $\alpha = .05$

$n = 140$ observations, d.f. $= n-2 = 140-2 = 138$

Category	Number of observations
0 - 1	49
1 - 2	59
2 - 3	22
3 - 4	7
4 - 5	2
5 - $+\infty$	1

$\overline{X} = .721$

For $\hat{p}_i$ from cumulative Poisson distribution:

Category	X_i	$\hat{p}_i$	$n\hat{p}_i$	$\frac{(X_i - n\hat{p}_i)}{n\hat{p}_i}$
1	49	.49658 - 0 = .49658	69.521	6.057
2	59	.84419 - .49658 = .34761	48.665	2.195
3	22	.96586 - .84419 = .12167	17.034	1.448
4	7	.99425 - .96586 = .02839	3.975	2.302
5	2	.99921 - .99425 = .00496	.694	2.458
6	1	1.0000 - .99921 = .00079	.111	7.120
Total	140	1.0000	140.00	21.580

TEST STATISTIC

$Y'_{138} = 21.580$

CONCLUSION

Do not reject H_0 and conclude that Poisson is a reasonable model.

APPENDIX 8

LEAST SQUARES REGRESSION*

$$\hat{Y} = (-104460.50) + 53.0\ X_p$$

Complaints

Year	i	Y_i	X_i	X_i^2	X_iY_i	Y_i^2
1974	1	170	1974		335580	
1975	2	214	1975		422650	
1976	3	243	1976		480168	
1977	4	337	1977		666249	
	$\sum_{i=1}^{4}$	964	7902	15610406	1904667	247314

$\overline{X}$ = 1975.50

$\overline{Y}$ = 241.00

n = 4

$$\hat{Y} = (-2147.80) + 1.10\ X_p$$

CHOW

Year	i	Y_i	X_i	X_i^2	X_iY_i	Y_i^2
1974	1	28	1974		55272	
1975	2	19	1975		37525	
1976	3	24	1976		47424	
1977	4	30	1977		59310	
	$\sum_{i=1}^{4}$	101	7902	15610406	199531	2621

$\overline{X}$ = 1975.50

$\overline{Y}$ = 25.250

n = 4

* From Mendenhall [43], p. 254.

Prediction Interval $\hat{Y} \pm t_{\alpha/2}\, S \sqrt{1 + \frac{1}{n} + \frac{(X_p - \overline{X})^2}{SSX}}$; $t_{.025,2} = 4.303$

$t_{.05,2} = 2.92$

d.f. $= n - 2$

Coefficient of Determination $r^2 = 1 - \frac{SSE}{SS_Y}$

$$SS_X = \sum_{i=1}^{n} X_i^2 - \frac{(\sum_{i=1}^{n} X_i)^2}{n}$$

$$SS_{XY} = \sum_{i=1}^{n} X_i Y_i - \frac{(\sum_{i=1}^{n} X_i)(\sum_{i=1}^{n} Y_i)}{n}$$

$$SS_Y = \sum_{i=1}^{n} Y_i^2 - \frac{(\sum_{i=1}^{n} Y_i)^2}{n}$$

$n = 4$

$$SSE = SS_Y - \hat{\beta}_1\, SS_{XY}$$

$$S^2 = \frac{SSE}{n - 2}$$

	COMPLAINTS	CHOW
SS_X	5.000	5.00
SS_Y	14990.00	70.750
SS_{XY}	265.00	5.50
SSE	945.00	64.70
S^2	472.50	32.350
S	21.73	5.687

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_p$$

	COMPLAINTS	CHOW
r^2	93.70%	8.55%
Pred. Int.	95%	95%
1978	373.50 + 147.89	28.00 + 38.70
1979	426.50 + 179.92	29.10 + 47.08
1980	479.50 + 215.33	30.20 + 56.34
1981	532.50 + 252.72	31.30 + 66.13
1982	585.50 + 291.31	32.40 + 76.22
1983	638.50 + 330.69	33.50 + 86.53
1984	691.50 + 370.61	34.60 + 96.97

APPENDIX 9

THREE COMPLAINT LEVELS MODELLED

	May '76 - Apr '77 Present-Σ= 337	May '75 - Apr '76 Low-Σ= 243	(May '74 - Apr '75)*(2.5) High-Σ= 544
Baker	1	1	1
Benton	2	9	1
Clackamas	50	39	87
Clatsop	3	3	15
Columbia	2	0	0
Coos	2	5	7
Crook	0	1	3
Curry	1	0	0
Deschutes	2	1	5
Douglas	5	3	5
Gilliam	0	0	0
Grant	0	0	3
Harney	0	0	1
Hood River	2	0	0
Jackson	4	8	10
Jefferson	0	0	0
Josephine	2	1	0
Klamath	4	3	15
Lake	0	0	0
Lane	8	6	23
Lincoln	0	2	1
Linn	4	6	8
Malheur	1	0	0
Marion	42	30	40
Morrow	0	0	0
Polk	2	3	22
Tillamook	5	4	8
Umatilla	1	1	2
Union	2	1	1
Wallowa	0	0	0
Wasco	11	0	1
Washington	30	13	43
Yamhill	3	1	7
Multnomah	44	40	100
Portland	104	62	135

APPENDIX 10

Low Complaints

$\Sigma = 243$

Run No.	Facility Location Solution	Run No.	Facility Location Solution	Run No.	Total Cost of Operation	Run No.	Total Cost of Operation	Run No.	Number of Inspectors	Run No.	Number of Inspectors
1	31007	33	31757	1	268795	33	277489	1	15.604	33	16.163
2	35134	34	35957	2	273684	34	281689	2	15.661	34	16.163
3	33553	35	34390	3	318901	35	329272	3	15.604	35	16.163
4	37696	36	38590	4	323961	36	333472	4	15.661	36	16.163
5	40638	37	41353	5	278585	37	288448	5	15.909	37	16.547
6	45569	38	46153	6	286653	38	293248	6	16.124	38	16.547
7	44065	39	44865	7	329604	39	341382	7	15.909	39	16.547
8	49013	40	49665	8	338317	40	346182	8	16.124	40	16.547
9	27826	41	28466	9	265312	41	273909	9	15.372	41	15.925
10	31930	42	32666	10	269989	42	278109	10	15.414	42	15.925
11	29735	43	30441	11	314721	43	324976	11	15.372	43	15.925
12	33852	44	34641	12	319526	44	329176	12	15.414	44	15.925
13	36355	45	36963	13	273960	45	283428	13	15.600	45	16.213
14	41264	46	41763	14	281222	46	288228	14	15.762	46	16.213
15	38925	47	39597	15	324054	47	335359	15	15.600	47	16.213
16	43847	48	44397	16	331800	48	340159	16	15.762	48	16.213
17	31007	49	31757	17	220516	49	227480	17	12.384	49	12.828
18	35134	50	35957	18	225230	50	231680	18	12.429	50	12.828
19	33553	51	34390	19	260965	51	269260	19	12.384	51	12.828
20	37696	52	38580	20	265814	52	273460	20	12.429	52	12.828
21	40638	53	41353	21	229363	53	237250	21	12.626	53	13.133
22	45569	54	46153	22	236766	54	242050	22	12.797	54	13.133
23	44065	55	44865	23	270537	55	279944	23	12.626	55	13.133
24	49013	56	49665	24	278452	56	284744	24	12.797	56	13.133
25	27826	57	28466	25	217752	57	224639	25	12.200	57	12.639
26	31930	58	32666	26	222296	58	228839	26	12.234	58	12.639
27	29735	59	30441	27	257648	59	265851	27	12.200	59	12.639
28	33852	60	34641	28	262294	60	270051	28	12.234	60	12.639
29	36355	61	36963	29	225693	61	233266	29	12.381	61	12.867
30	41264	62	41763	30	232456	62	238066	30	12.509	62	12.867
31	38925	63	39597	31	266133	63	275164	31	12.381	63	12.867
32	43847	64	44397	32	273280	64	279964	32	12.509	64	12.867

Medium Complaints

$\Sigma = 377$

Run No.	Facility Location Analysis				Total Cost of Operation				Number of Inspectors		
1	33837	33	34333	1	291244	33	299577	1	16.988	33	17.530
2	38008	34	38533	2	296197	34	303777	2	17.048	34	17.530
3	36608	35	37162	3	345500	35	355458	3	16.988	35	17.530
4	40801	36	41362	4	350632	36	359658	4	17.048	36	17.530
5	43422	37	43870	5	300971	37	309292	5	17.290	37	17.846
6	48315	38	48691	6	306742	38	315298	6	17.352	38	17.914
7	47070	39	47599	7	356132	39	367531	7	17.290	39	17.914
8	51974	40	52399	8	362090	40	372331	8	17.352	40	17.914
9	30374	41	30797	9	287273	41	295572	9	16.723	41	17.263
10	34516	42	34997	10	292001	42	299772	10	16.768	42	17.263
11	32452	43	32919	11	340734	43	350652	11	16.723	43	17.263
12	36611	44	37119	12	345597	44	354852	12	16.768	44	17.263
13	38863	45	39194	13	295868	45	304104	13	16.949	45	17.500
14	43742	46	44055	14	301406	46	309853	14	16.996	46	17.551
15	41599	47	42000	15	350008	47	359894	15	16.949	47	17.500
16	46486	48	46837	16	355686	48	365798	16	16.996	48	17.551
17	33837	49	34333	17	238684	49	245339	17	13.482	49	13.913
18	38008	50	38533	18	243452	50	249539	18	13.530	50	13.913
19	36608	51	37162	19	282427	51	290372	19	13.482	51	13.913
20	40801	52	41362	20	287337	52	294572	20	13.530	52	13.913
21	43422	53	43870	21	247477	53	254077	21	13.722	53	14.163
22	48315	54	48691	22	253055	54	259871	22	13.771	54	14.218
23	47070	55	47599	23	291938	55	301019	23	13.722	55	14.218
24	51974	56	52399	24	297665	56	305819	24	13.771	56	14.218
25	30374	57	30797	25	235532	57	242160	25	13.272	57	13.701
26	34516	58	34997	26	240122	58	246360	26	13.308	58	13.701
27	32452	59	32919	27	278644	59	286558	27	13.272	59	13.701
28	36611	60	37119	28	283341	60	290758	28	13.308	60	13.701
29	38863	61	39194	29	243426	61	249960	29	13.452	61	13.889
30	43742	62	44055	30	248820	62	255550	30	13.489	62	13.929
31	41599	63	42000	31	287078	63	294921	31	13.452	63	13.889
32	46486	64	46837	32	292582	64	300634	32	13.489	64	13.929

High Complaints

$\Sigma = 544$

Run No.	Facility Location Analysis	Run No.	Facility Location Analysis	Run No.	Total Cost of Operations	Run No.	Total Cost of Operations	Run No.	Number of Inspectors	Run No.	Number of Inspectors
1	35373	33	36871	1	332961	33	341753	1	19.745	33	20.278
2	39981	34	41568	2	339501	34	349259	2	19.863	34	20.458
3	38375	35	40013	3	395484	35	405874	3	19.745	35	20.278
4	42951	36	44688	4	402379	36	413921	4	19.863	36	20.458
5	44090	37	44825	5	341265	37	350462	5	20.001	37	20.594
6	49511	38	50770	6	347871	38	357597	6	20.070	38	20.677
7	47934	39	48729	7	405969	39	415713	7	20.070	39	20.594
8	53334	40	54721	8	411369	40	422917	8	20.070	40	20.677
9	31621	41	32945	9	329001	41	337795	9	19.481	41	20.014
10	36269	42	37669	10	335099	42	344626	10	19.569	42	20.149
11	33873	43	35301	11	390732	43	401124	11	19.481	43	20.014
12	38497	44	40009	12	397096	44	408361	12	19.569	44	20.149
13	39273	45	39945	13	336343	45	345496	13	19.673	45	20.251
14	44734	46	45832	14	342693	46	352141	14	19.724	46	20.313
15	42163	47	42873	15	398652	47	409538	15	19.673	47	20.251
16	47601	48	48795	16	405155	48	416369	16	19.724	48	20.313
17	35373	49	36871	17	271870	49	279014	17	15.670	49	16.093
18	39981	50	41568	18	278046	50	285963	18	15.764	50	16.236
19	38375	51	40013	19	322175	51	330587	19	15.670	51	16.093
20	42951	52	44688	20	328632	52	337965	20	15.764	52	16.236
21	44090	53	44825	21	279381	53	286923	21	15.874	53	16.345
22	49511	54	50770	22	285775	54	293622	22	15.928	54	16.410
23	47934	55	48729	23	331453	55	339250	23	15.928	55	16.345
24	53334	56	54721	24	336853	56	346146	24	15.928	56	16.410
25	31621	57	32945	25	268728	57	275872	25	15.461	57	15.884
26	36269	58	37669	26	274552	58	282286	26	15.531	58	15.991
27	33873	59	35301	27	318404	59	326817	27	15.461	59	15.884
28	38497	60	40009	28	324438	60	333552	28	15.531	60	15.991
29	39273	61	39945	29	275475	61	282839	29	15.614	61	15.072
30	44734	62	45832	30	281665	62	289291	30	15.654	62	16.122
31	42163	63	42873	31	325609	63	334349	31	15.614	63	16.072
32	47601	64	48795	32	331922	64	340949	32	15.654	64	16.122

Factor levels for runs 1-64 at each level of complaints:

Run	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
CHOW	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
EFF	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2
MPH	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1
VISITS	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1
WAGES	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
OPCOST	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1

Run	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
CHOW	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
EFF	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
MPH	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
VISITS	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
WAGES	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
OPCOST	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2

Run	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
CHOW	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
EFF	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
MPH	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
VISITS	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
WAGES	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
OPCOST	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2

Run	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64
CHOW	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
EFF	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
MPH	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
VISITS	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
WAGES	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
OPCOST	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2

	1 = Low Value	2 = High Value
CHOW = Changes of ownership	20	40
EFF = Efficiency rating factor	.75	.60
MPH = Travel Speed	60	45
VISITS = Number of visits excluding complaint and CHOW visits	3	6
WAGES = Salary levels for inspectors (I) and the supervisor (S)	$14,994 $16,594	$16,493 $18,198
OPCOST = Fixed cost	$ 600	$1,200

BEST CASE - Run 25		WORST CASE - Run 40	
Facility Location Cost	$ 27,826	Facility Location Cost	$ 54,721
Total Cost	$217,752	Total Cost	$422,917
No. of Inspectors	12.200	No. of Inspectors	20.677
Facilities Opened and (No. of Inspectors)		Facilities Opened and (No. of Inspectors)	
Astoria	(.346)	Astoria	(.755)
Bend	(.373)	Baker	(.777)
Coos Bay	(.493)	Bend	(.620)
Eugene	(.902)	Coos Bay	(.761)
Medford	(.879)	Eugene	(1.389)
Pendleton	(.861)	Klamath Falls	(.428)
Portland	(5.871)	Medford	(.902)
Salem	(2.474)	Pendleton	(.502)
		Portland	(10.678)
		Salem	(3.866)

Best scan available. Original copy faded.

APPENDIX 11

The Computer Program

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PROGRAM HEALTH1     73/73    OPT=1                      FTN 4.6+446          77/09/09. 01.5

      PROGRAM HEALTH1 (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      REAL IFC,IVC,MDEL,MDELS,MINC,MEGAS,JD,LLN,MINC1,MINC2,IJD
      DIMENSION IFC(30),IVC(30,100),MDEL(60,100),MDELS(60,30),
     1MEGAS(60,30),JD(60,30),MINC(100),Z(60),Y(60,30),IJD(30),
     2ISCL(60,100),D(100),OFF(30),OFFF(30),AMILES(30,100),
     3KZ(60,30),K1(60,30),K2(60,30),LN(60,30),IDEL(60,100),ILN(30),
     4SUPR(2,10),PDM(2,10),CALPDM(2,10),NUMCPL(50),NUMCTY(50),TOTHM(40),
     5PTIDX(50),ICHNG(50),HOMES(50,15),NCITY(50,15),DEM(100),DEMCPL(100)
     6,DEMCHOW(100),CUS(100),CUST(100)
C
C
C  PROGRAM VARIABLE LIST
C
C           K2 - THE SET OF OFFICES NOT OPENED OR CLOSED (FREE)
C           K1 - THE SET OF OFFICES THAT HAVE BEEN OPENED
C           KZ - THE SET OF OFFICES THAT HAVE BEEN CLOSED
C           LN - THE SET OF CUSTOMERS WHICH CAN BE SUPPLIED BY OFFICE IK
C           IDEL - THE SET OF OFFICES THAT HAVE BEEN OPENED AS A RESULT OF
C             THE DELTA CALCULATIONS AND THEIR RESPECTIVE CUSTOMERS
C           IFC - FIXED OFFICE COST
C           D - DEMAND FOR SERVICE, NUMBER OF HOMES
C           IVC - THE VARIABLE COSTS RESULTING FROM TRAVEL COSTS AND
C             OPERATING COSTS
C           MDEL - DELTA
C           MDELS - SUM OF THE DELTAS FOR A SPECIFIC OFFICE AND NODE
C           MEGAS - OMEGAS
C           JD - THE SET OF OFFICES WHICH CAN SUPPLY CUSTOMER ID
C           Z - TOTAL COST
C           Y - EQUALS 0 OF THE OFFICE IS CLOSED AND 1 IF THE OFFICE IS
C             OPEN
C           SCL - THE SET OF OPEN OFFICES IN THE TERMINAL SOLUTIONS
C           NK - THE NUMBER OF POSSIBLE OFFICE LOCATIONS
C           NC - THE NUMBER OF CUSTOMERS
C           UBD - UPPER BOUND
C           LBD - LOWER BOUND
C           NUBDN - NEW UPPER BOUND NODE
C           NLBDN - NEW LOWER BOUND NODE
C           NODE - NUMBER OF DISTINCT NODES INVESTIGATED
C           ITER - INTERATIONS
C           SDIEM - SUPERVISORS PER DIEM FOR TRAVEL TO INSPECTOR STATION
C             FROM PORTLAND
C           PDIEM - INSPECTORS PER DIEM FOR ONE-DAY VISIT
C           PDIEML - INSPECTORS PER DIEM FOR THREE-DAY VISIT
C           SUPR(1,I) - MILEAGE, AND SUPR(2,I) - ASSOCIATED SUPERVISOR
C             PER DIEM COST FOR TRAVEL FROM PORTLAND
C           PDM(1,I) - MILEAGE, AND PDM(2,I) - ASSOCIATED INSPECTOR
C             ONE-DAY PER DIEM
C           CALPDM(1,I) - MILEAGE, AND CALPDM(2,I) - ASSOCIATED INSPECTOR
C             THREE-DAY PER DIEM
C           PTIDX - ENDPOINT OF A CITYS MONTE CARLO INTERVAL
C           ICHNG(I) - NUMBER OF CHANGES OF OWNERSHIP IN COUNTY I
C           TTHM - TOTAL NUMBER OF HOMES IN THE STATE
C           NCOMP - TOTAL NUMBER OF COMPLAINTS IN THE STATE
C           WAGEMI - THE COST PER MILE OF AN INSPECTORS TIME
C           WAGESP - THE COST PER MILE OF THE SUPERVISORS TIME
C
C  VARIABLES WHICH MAY BE CHANGED
```

```
C          VISITS - THE AVERAGE NUMBER OF TRIPS TO A HOME PER YEAR,
C                   EXCLUDING COMPLAINT VISITS AND CHANGES OF OWNERSHIP
C          CAR    - THE COST PER MILE OF OPERATING A STATE CAR
C          ASALI  - INSPECTORS ANNUAL SALARY
C          ASALS  - SUPERVISORS ANNUAL SALARY
C          OFCOST - THE ADDED FIXED COST OF OPENING A BRANCH OFFICE
C          AMPH   - THE AVERAGE TRAVEL SPEED
C          EFF    - THE EFFICIENCY FACTOR FOR INSPECTORS TIME
C
C   SOME COMMENTS ABOUT THE INPUT DATA
C          1. THE FOLLOWING MUST BE SUPPLIED -
C             NUMBER OF NURSING FACILITY LOCATIONS (CITIES), NC
C             NUMBER OF POSSIBLE INSPECTOR LOCATIONS, NW
C             NUMBER OF PER DIEM STEP COST POINTS - NI,NS,NL
C             NUMBER OF COUNTIES WITH HOMES, NUMCO
C             TOTAL NUMBER OF CHANGES OF OWNERSHIP, NCHNG
C             MONTE CARLO RANDOM NUMBER SEED, ISEED
C             WAREHOUSE NUMBER (NW) FOR PORTLAND, NPTLD
C             CUSTOMER NUMBER (NC) FOR PORTLAND DATA, IPTLD
C          2. ACTUAL MILEAGE BETWEEN INSPECTOR LOCATION I AND CUSTOMER
C             CITY J, AND NUMBER OF HOMES IN J MUST BE SUPPLIED
C          3. STEP COST MILEAGE AND COST FIGURES ARE ENTERED NEXT -
C             PDM, SUPR, CALPDM
C          4. NUMBER OF COMPLAINTS (NUMCPL), NUMBER OF CITIES (NUMCTY),
C             AND NUMBER OF HOMES PER COUNTY (TOTHM) MUST BE GIVEN
C          5. ALPHABETIC NAMES FOR INSPECTOR LOCATIONS AND HOME LOCATIONS
C             ARE READ
C          6. BY COUNTY, A CITY IDENTIFYING NUMBER - NCITY -SAME AS INDEX
C             I=1,NC, AND NUMBER OF HOMES PER CITY IN THE COUNTY - HOMES -
C             MUST BE GIVEN.
C
C
C   THIS PROGRAM ASSIGNS TO HOMES A USER-SUPPLIED NUMBER OF COMPLAINTS AND
C    CHANGES OF OWNERSHIP BY MONTE CARLO ASSIGNMENT.  ACTUAL DISTANCE
C    IS USED.
C
C
C   AN EXPLANATION OF THE ALGORITHM USED TO SOLVE THIS PROBLEM CAN BE
C    FOUND IN AN ARTICLE BY KHUMAWALA, B. M., ≠AN EFFICIENT BRANCH AND
C    BOUND ALGORITHM FOR THE WAREHOUSE LOCATION PROBLEM,≠ ≠≠MANAGEMENT
C    SCIENCE≠≠, VOL. 18, NO. 12, (AUGUST 1972).
C
C
C   THE ORIGINAL COMPUTER PROGRAM CAN BE FOUND IN AN UNPUBLISHED PH.D.
C    DISSERTATION.  KHUMAWALA, B. M., ≠AN EFFICIENT BRANCH AND BOUND
C    ALGORITHM FOR WAREHOUSE LOCATION≠, KRANNERT GRADUATE SCHOOL OF
C    INDUSTRIAL ADMINISTRATION, PURDUE UNIVERSITY, JUNE 1970.
C
C
  303 FORMAT(≠1≠,40X,≠HELD OFFICE LOCATIONS≠//≠0≠,29X,≠AVERAGE VISITS PER
     1 FACILITY = ≠,F4.2,≠ PER YEAR≠/30X,≠AUTOMOBILE COST IS $≠,F3.2,≠ P
     1ER MILE≠/30X,≠COST OF INSPECTORS TIME IS $≠,F4.2,≠ PER MILE≠/30X,≠
     1COST OF SUPERVISORS TIME IS $≠,F4.2,≠ PER MILE≠/30X,≠COST OF OPERA
     1TING AN INSPECTION OFFICE IS $≠,F8.2,≠ PER YEAR≠/30X,≠AVERAGE TRAV
     1EL SPEED IS ≠,F3.0,≠ MPH≠/30X,≠EFFICIENCY FACTOR IS ≠,F3.2/30X,≠RA
     1NDOM NUMBER GENERATOR SEED IS ≠,I5/30X,≠TOTAL NUMBER OF COMPLAINTS
     1 = ≠,I4/30X,≠TOTAL OWNERSHIP CHANGES = ≠,I3)
```

```
 5833 FORMAT(//41X,≠FACILITY LOCATION ANALYSIS≠/≠-≠,4X,≠FIRST TERMINAL S
     1OLUTION FOUND WAS $≠,F15.2,≠.  IT WAS FOUND AT ITERATION NUMBER ≠,
     1I10)
10001 FORMAT(10I5)
10003 FORMAT(///5X,≠SOLUTION INFEASIBLE≠)
10004 FORMAT(//5X,≠THE OPTIMAL SOLUTION FOUND AFTER ≠,I7,≠ ITERATIONS≠,4
     1X,≠=≠,2X,F15.2/5X,≠TOTAL FIXED COST≠,39X,≠=≠,2X,F15.2/5X,≠TOTAL VA
     1RIABLE COST≠,36X,≠=≠,2X,F15.2/5X,≠THE SOLUTION WAS FOUND AT ITERAT
     1ION NUMBER≠,13X,≠=≠,7X,I7/5X,≠THE MAXIMUM NUMBER OF NODES USED WAS
     1≠,19X,≠=≠,7X,I7)
10005 FORMAT(//3X,A4,A4,5X,≠SUPPLIES THE FOLLOWING FACILITIES≠/≠-≠,28X,≠
     1VARIABLE COST≠,6X,≠COMPLAINTS≠,6X,≠OWNERSHIP CHANGES≠,4X,≠STAFF RE
     1QUIRED≠,2X,≠DISTANCE TO ≠,A4,A4,3X,≠NO. OF HOMES≠/1X)
10006 FORMAT(/16X,A4,A4,5X,≠$≠,F10.2,10X,F4.0,17X,F4.0,13X,F7.3,10X,F5.0
     1,≠ MILES≠,12X,F4.0)
10007 FORMAT(//41X,≠TOTAL COST ANALYSIS≠/≠-≠,4X,≠FC - DECENTRALIZED OFFI
     1CE COST + SUPERVISOR TRAVEL(CAR COST AND PER DIEMS)≠,6X,≠=≠,F15.2/
     15X,≠VC - INSPECTOR TRAVEL(CAR COST AND PER DIEMS)≠,35X,≠=≠,F15.2/5
     1X,≠INSPECTORS WAGES, BASED ON SALARY OF $≠,F7.0,≠ AND ≠,F7.3,≠ INS
     1PECTORS REQUIRED≠,7X,≠=≠,F15.2/5X,≠SUPERVISORS ANNUAL SALARY≠,55X,
     1≠=≠,6X,F9.2//5X,≠TOTAL COST OF OPERATIONS≠,56X,≠=≠,F15.2)
10008 FORMAT(/9X,A4,A4,2X,≠REQUIRES≠,F7.3,2X,≠STAFF.≠/9X,≠TOTAL ROUND TR
     1IP MILEAGE IS ≠,F10.2,≠ MILES.≠/9X,≠COST OF OPENING OFFICE IS $≠,F
     110.2,≠.≠/9X,≠NUMBER OF COMPLAINT AND CHANGE OF OWNERSHIP VISITS IS
     1 ≠,F4.0)
10009 FORMAT(5X,A4,A4)
10011 FORMAT(//5X,≠TOTAL INSPECTOR MILES TRAVELLED≠,6X,≠=≠,F11.2/5X,≠SUP
     1ERVISOR MILES TRAVELLED≠,11X,≠=≠,F11.2/5X,≠TOTAL NUMBER OF INSPECT
     1ORS≠,11X,≠=≠,F11.3)
19999 FORMAT(20(I3,1X))
20000 FORMAT(16(I2,F3.0))
20002 FORMAT(20(F4.0))
20003 FORMAT(5(2F7.2))
20004 FORMAT(11(I3,F4.0))
97791 FORMAT(//5X,≠COMPUTATIONS DISCONTINUED FOR MORE STORAGE. SOLUTION
     1GIVEN BELOW MAY NOT NECESSARILY BE OPTIMAL≠)
      READ(5,10001)NW,NC,NI,NS,NL,NUMCO,NCHNG,ISEED,NPTLD,IPTLD
      DO 158 I=1,NW
      READ(5,20002) (AMILES(I,J),J=1,NC)
  158 CONTINUE
      READ(5,20002) (D(I),I=1,NC)
      READ(5,20003) (PDM(1,I),PDM(2,I),I=1,NI)
      READ(5,20003) (SUPR(1,I),SUPR(2,I),I=1,NS)
      READ(5,20003) (CALPDM(1,I),CALPDM(2,I),I=1,NL)
      READ(5,19999) (NUMCPL(I),I=1,NUMCO)
      READ(5,20000) (NUMCTY(I),TOTHM(I),I=1,NUMCO)
      READ(5,10009) (OFF(J),OFFF(J),J=1,NW)
      READ(5,10009) (CUS(J),CUST(J), J=1,NC)
      DO 159 I=1,NUMCO
      N1=NUMCTY(I)
      READ(5,20004) (NCITY(I,J),HOMES(I,J),J=1,N1)
  159 CONTINUE
      AMPH=55.
      EFF=.70
      VISITS=4.0
      CAR=.11
      ASALI=14994.
```

```
            WAGEMI=ASALI/(2080.*AMPH)
            ASALS=16544.
            WAGESP=ASALS/(2080.*AMPH)
            OPCOST=855.60
      C
      C     SET UP INTERVALS FOR MONTE CARLO ASSIGNMENT BY COUNTY OF CHANGES
      C     OF OWNERSHIP
      C
            CALL RANSET(ISEED)
            TTHM=0.
            DO 160 I=1,NUMCO
            TTHM=TTHM+TOTHM(I)
            ICHNG(I)=0
        160 CONTINUE
            DO 163 I=1,NC
            DEM(I)=DEMCPL(I)=DEMCHOW(I)=0.
        163 CONTINUE
            SINCR=1.0/TTHM
            DO 162 J=1,NUMCO
            IF(J.GT.1) GO TO 164
            PTIDX(J)=TOTHM(J)*SINCR
            GO TO 162
        164 PTIDX(J)=PTIDX(J-1)+TOTHM(J)*SINCR
        162 CONTINUE
      C
      C     ASSIGN CHANGES OF OWNERSHIP BY COUNTY ASSUMING UNIFORM
      C     DISTRIBUTION FOR CHANGES PER HOME
      C
            DO 165 I=1,NCHNG
            RNO=RANF(1.0.)
            DO 166 J=1,NUMCO
            IF(RNO.LE.PTIDX(J)) GO TO 167
        166 CONTINUE
        167 ICHNG(J)=ICHNG(J)+1
        165 CONTINUE
      C
      C     SET UP INTERVAL SCALE FOR MONTE CARLO ASSIGNMENT OF COMPLAINTS
      C     BY COUNTY
      C
            NCOMP=0.
            DO 170 I=1,NUMCO
            N1=NUMCTY(I)
            N2=NUMCPL(I)
            N3=ICHNG(I)
            NCOMP=NCOMP+N2
            RINCR=1.0/TOTHM(I)
            DO 172 J=1,N1
            IF(J.GT.1) GO TO 174
            PTIDX(J)=HOMES(I,J)*RINCR
            GO TO 172
        174 PTIDX(J)=PTIDX(J-1)+HOMES(I,J)*RINCR
        172 CONTINUE
      C
      C     ASSIGN COMPLAINTS TO HOMES IN A COUNTY USING UNIFORM DISTRIBUTION
      C
            IF(N2.EQ.0) GO TO 177
            DO 175 IRND=1,N2
```

```
      RND=RANF(100.)
      DO 178 JCTY=1,N1
      IF(RND.LE.FTIDX(JCTY)) GO TO 180
  178 CONTINUE
  180 DEM(NCITY(I,JCTY))=DEM(NCITY(I,JCTY))+2.
      DEMCPL(NCITY(I,JCTY))=DEMCPL(NCITY(I,JCTY))+1.
  176 CONTINUE
  177 IF(N3.EQ.0) GO TO 170
C
C     ASSIGN CHANGES OF OWNERSHIP TO HOMES IN A COUNTY USING UNIFORM
C     DISTRIBUTION
C
      DO 182 IRND=1,N3
      RND=RANF(100.)
      DO 184 JCTY=1,N1
      IF(RND.LE.FTIDX(JCTY)) GO TO 186
  184 CONTINUE
  186 DEM(NCITY(I,JCTY))=DEM(NCITY(I,JCTY))+4.
      DEMCHOW(NCITY(I,JCTY))=DEMCHOW(NCITY(I,JCTY))+1.
  182 CONTINUE
  170 CONTINUE
      DO 2 IW=1,NW
C
C     ASSIGN SUPERVISOR PER DIEM
C
      IF(IW.NE.NFTLD) GO TO 8
      IFC(IW)=0.
      GO TO 35
    8 DO 27 IS=1,NS
      IF(AMILES(IW,IPTLD).LT.SUPR(1,IS)) GO TO 28
   27 CONTINUE
      SDIEM=SUPR(2,NS)
      GO TO 29
   28 SDIEM=SUPR(2,IS-1)
   29 SPCOST=AMILES(IW,IPTLD)*24.*(CAR+WAGESP)+SDIEM
C
C     FIXED COST
C
      IFC(IW)=SPCOST+OPCOST
   35 DO 2 IC=1,NC
C
C     ASSIGN INSPECTOR ONE-DAY PER DIEM
C
      DO 4 IS=1,NI
      IF(AMILES(IW,IC).LT.PDM(1,IS)) GO TO 5
    4 CONTINUE
      PDIEM=PDM(2,NI)
      GO TO 6
    5 PDIEM=PDM(2,IS-1)
C
C     ASSIGN INSPECTOR THREE-DAY PER DIEM
C
    6 DO 31 ILC=1,NL
      IF(AMILES(IW,IC).LT.CALPDM(1,ILC)) GO TO 32
   31 CONTINUE
      PDIEML=CALPDM(2,NL)
      GO TO 33
```

```
      32    PDIEML=CALFDM(2,ILC-1)
      C
      C     VARIABLE COST
      C
      33    IF(AMILES(IW,IC).GT.60.) GO TO 34
            IVC(IW,IC)=AMILES(IW,IC)*2.*(CAR+WAGEMI)*((VISITS+2.)*D(IC)+DEM(IC
           1))+PDIEM*((VISITS-1.)*D(IC)+DEM(IC))+PDIEML*D(IC)
            GO TO 2
      34    IVC(IW,IC)=AMILES(IW,IC)*2.*(CAR+WAGEMI)*(VISITS*D(IC )+DEM(IC))+P
           1DIEM*((VISITS-1.)*D(IC)+DEM(IC))+PDIEML*D(IC)
      2     CONTINUE
            WRITE(6,303) VISITS,CAR,WAGEMI,WAGESP,OPCOST,AMPH,EFF,ISEED,NCOMP,
           1NCHNG
      C
      C     INITIALIZATION
      C
            NFIRST=0
            NKTR=0
            NKTR1=0
            LLN=9.999E38
            XLBC=0.0
            UBC=LLN
            MODE=1
            NODE=1
            NUBCN=NODE
            ITER=1
            KODE=0
            DO 1000 IW=1,NW
            JD(NODE,IW)=0
            KZ(NODE,IW)=0
            K1(NODE,IW)=0
            K2(NODE,IW)=1
            LN(NODE,IW)=NC
            DO 1001 IC=1,NC
            JD(NODE,IW)=JD(NODE,IW)+D(IC)
            IF(IW.GE.2)GO TO 1001
            IDEL(NODE,IC)=0
      1001  CONTINUE
            ILN(IW)=LN(NODE,IW)
            IJD(IW)=JD(NODE,IW)
      1000  CONTINUE
            GO TO 786
      C
      C     SETS ARE UPDATED
      C
          1 CONTINUE
            ITER=ITER+1
            IF(NLBDN.EQ.1)GO TO 4193
            IF(NKTR.EQ.1.OR.NKTR1.EQ.1) GO TO 4192
            IF(KODE.NE.0)GO TO 4195
      4193  NODE=MODE+1
            MODE=NODE
      C
      C     STORAGE ALLOTMENT CHECK
      C
            IF(MODE.GT. 60) GO TO 9779
            GO TO 4196
```

```
      4195 NODE=KODE
           KODE=0
      4196 DO 5167 IC=1,NC
           IDEL(NODE,IC)=IDEL(NLBON,IC)
           MDEL(NODE,IC)=MDEL(NLBON,IC)
      5167 CONTINUE
           DO 92 IW=1,NW
           JD(NODE,IW)=JD(NLBON,IW)
           KZ(NODE,IW)=KZ(NLBON,IW)
           K1(NODE,IW)=K1(NLBON,IW)
           K2(NODE,IW)=K2(NLBON,IW)
           LN(NODE,IW)=LN(NLBON,IW)
           MDELS(NODE,IW)=MDELS(NLBON,IW)
           MEGAS(NODE,IW)=MEGAS(NLBON,IW)
      92   CONTINUE
           GO TO 4194
      4192 NODE=NLBON
      4194 IF(NKTR.EQ.0)GO TO 3786
           GO TO (3912,3911),NKTR
      3786 GO TO (911,912),NKTR1
      3911 NKTR=NKTR-1
           GO TO 3913
      911  NKTR1=NKTR1-1
      3913 KZ(NODE,KKW)=1
           K2(NODE,KKW)=0
           GO TO 786
      3912 NKTR=NKTR-1
           GO TO 3914
      912  NKTR1=NKTR1-1
      3914 K1(NODE,KKW)=1
           K2(NODE,KKW)=0
           GO TO 787
      C
      C    SIMPLIFICATION CYCLE
      C
      786  CONTINUE
           DO 20 IC=1,NC
           KKK=0
           KTR=0
           DO 10 IW=1,NW
           IF(KZ(NODE,IW).EQ.1)GO TO 10
           IF(K1(NODE,IW).EQ.1.AND.IDEL(NODE,IC).EQ.IW)GO TO 20
           KTR=KTR+1
           IF(KTR.EQ.1) GO TO 11
           IF(KTR.EQ.2) GO TO 12
           IF(IVC(IW,IC).GE.MINC2) GO TO 10
           GO TO 12
      11   MINC1=IVC(IW,IC)
           MW=IW
           GO TO 10
      12   CONTINUE
           MINC1=AMIN1(MINC1,IVC(IW,IC))
           IF(MINC1.EQ.IVC(IW,IC)) GO TO 13
           MINC2=IVC(IW,IC)
           GO TO 10
      13   MINC2=IVC(MW,IC)
           MW=IW
```

```
      10    CONTINUE
            IF(KTR.EQ.0) GO TO 19
            IF(KTR.EQ.1) GO TO 14
            IDEL(NODE,IC)=MW
            MDEL(NODE,IC)=MINC2-MINC1
            GO TO 20
      14    K1(NODE,MW)=1
            K2(NODE,MW)=0
            KKK=KKK+1
            GO TO 20
      C
      C     FEASIBILITY CHECK
      C
      19    IF(NODE.NE.1) GO TO 74
            WRITE(6,10003)
            STOP
      20    CONTINUE
            KTR=KKK
            DO 25 IW=1,NW
            IF(K2(NODE,IW).EQ.0) GO TO 25
            MDELS(NODE,IW)=-IFC(IW)
            DO 30 IC=1,NC
            IF(IDEL(NODE,IC).NE.IW) GO TO 30
            MDELS(NODE,IW)=MDELS(NODE,IW) + MDEL(NODE,IC)
      30    CONTINUE
            IF(MDELS(NODE,IW)) 25,26,26
      26    KTR=KTR+1
            K1(NODE,IW)=1
            K2(NODE,IW)=0
      25    CONTINUE
            DO 4386 IW=1,NW
            IF(K2(NODE,IW).EQ.0) GO TO 4386
            GO TO 43861
      4386  CONTINUE
            GO TO 789
      43861 IF(KTR.EQ.0) GO TO 789
      787   CONTINUE
            DO 42 IW=1,NW
            IF(K2(NODE,IW).EQ.0) GO TO 42
            LN(NODE,IW)=ILN(IW)
            JD(NODE,IW)=IJD(IW)
            DO 41 IC=1,NC
            MW=IDEL(NODE,IC)
            IF(K1(NODE,MW).EQ.0) GO TO 41
            LN(NODE,IW)=LN(NODE,IW)-1
            JD(NODE,IW)=JD(NODE,IW)-D(IC)
      41    CONTINUE
      42    CONTINUE
            JW=1
      43    IF(K1(NODE,JW).EQ.1) GO TO 44
            JW=JW+1
            GO TO 43
      44    DO 45 IC=1,NC
      45    MINC(IC)=INC(JW,IC)
            JW=JW+1
            IF(JW.GT.NW) GO TO 47
            DO 46 IW=JW,NW
```

```
      IF(K1(NODE,IW).EQ.0) GO TO 46
      DO 48 IC=1,NC
   48 MINC(IC)=AMIN1(MINC(IC),IVC(IW,IC))
   46 CONTINUE
   47 KTR=0
      DO 49 IW=1,NW
      IF(K2(NODE,IW).EQ.0) GO TO 49
      MEGAS(NODE,IW)=-IFC(IW)
      DO 50 IC=1,NC
      MEGAS(NODE,IW)=MEGAS(NODE,IW)+AMAX1(0.,MINC(IC)-IVC(IW,IC))
   50 CONTINUE
      IF(MEGAS(NODE,IW).GT.0.) GO TO 49
      KZ(NODE,IW)=1
      K2(NODE,IW)=0
      KTR=KTR+1
   49 CONTINUE
      DO 4329 IW=1,NW
      IF(K2(NODE,IW).EQ.0) GO TO 4329
      GO TO 43291
 4329 CONTINUE
      GO TO 789
43291 IF(KTR) 789,789,786
  789 Z(NODE)=0.
      DO 60 IW=1,NW
      IF(K1(NODE,IW).EQ.1) GO TO 52
      Y(NODE,IW)=0.
      GO TO 60
   52 Y(NODE,IW)=1.
C
C     LINEAR PROGRAM
C
   60 CONTINUE
      DO 53 IC=1,NC
      KW=IDEL(NODE,IC)
      IF(KZ(NODE,KW).EQ.1) GO TO 538
      IF(K1(NODE,KW).EQ.1)GO TO 54
      IF(LN(NODE,KW).EQ.0) XX=9.999999E 50
      IF(LN(NODE,KW).EQ.0) GO TO 151
      XJN=FLOAT(LN(NODE,KW))
      XX=IFC(KW)/XJN
  151 IF(MDEL(NODE,IC).GT.XX)  GO TO 54
  538 JW=1
  540 IF(KZ(NODE,JW).EQ.0) GO TO 539
      JW=JW+1
      GO TO 540
  539 AA=IVC(JW,IC)
      IF(LN(NODE,JW).EQ.0) XX=9.999999E 50
      IF(LN(NODE,JW).EQ.0) GO TO 152
      XJN=FLOAT(LN(NODE,JW))
      XX=IFC(JW)/XJN
  152 IF(K2(NODE,JW).EQ.1) AA=AA + XX
      KW=JW
      JW=JW+1
      IF(JW.GT.NW) GO TO 54
      DO 55 IW=JW,NW
      IF(K2(NODE,IW).EQ.1) GO TO 55
      BB=IVC(IW,IC)
```

```
      IF(LN(NODE,IW).EQ.0) XX=9.999999E 50
      IF(LN(NODE,IW).EQ.0) GO TO 153
      XJN=FLOAT(LN(NODE,IW))
      XX=IFC(IW)/XJN
153   IF(K2(NODE,IW).EQ.1)BB=BB + XX
      IF(AA.NE.BB) GO TO 56
      IF(K1(NODE,IW).EQ.1) GO TO 57
      GO TO 55
56    AA=AMIN1(AA,BB)
      IF(AA.NE.BB) GO TO 55
57    KW=IW
55    CONTINUE
54    XJN=FLOAT(LN(NODE,KW))
      IF(LN(NODE,KW).EQ.0) XX=9.999999E 50
      IF(LN(NODE,KW).EQ.0) GO TO 154
      XX=1./XJN
154   IF(K1(NODE,KW).EQ.1) GO TO 58
      Y(NODE,KW)=XX + Y(NODE,KW)
58    Z(NODE)=Z(NODE)+IVC(KW,IC)
53    ISOL(NODE,IC)=KW
      KTR=0
      DO 4173 IW=1,NW
      IF(Y(NODE,IW).EQ.0.) GO TO 4174
      Z(NODE)=Z(NODE)+IFC(IW)*Y(NODE,IW)
4174  IF(Y(NODE,IW).EQ.0..OR.Y(NODE,IW).EQ.1.) GO TO 4173
      KTR=KTR+1
4173  CONTINUE
      IF(KTR) 71,71,72
71    CONTINUE
C
C     IS THE SOLUTION TERMINAL
C
      IF(NFIRST.EQ.1) GO TO 711
      WRITE(6,5833) Z(NODE),ITER
      NFIRST=1
C
C     IS THE SOLUTION OPTIMAL
C
711   IF(NODE.EQ.1) GO TO 6789
      IF(UBD.GT.Z(NODE)) GO TO 790
      Z(NODE)=LLN
      KODE=NODE
      IF(NKTR.NE.0.OR.NKTR1.NE.0) GO TO 1
      GO TO 1236
790   UBD=Z(NODE)
      IF(NUBDN.NE.1) KODE=NUBDN
      NUBDN=NODE
      ITROPT=ITER
      Z(NODE)=LLN
      IF(NKTR.NE.0.OR.NKTR1.NE.0) GO TO 1
1236  CONTINUE
      JW=1
      XLN=LLN
7911  JW=JW+1
      IF(Z(JW).LT.UBD) GO TO 7910
      IF(Z(JW).LT.LLN) KODE=JW
      Z(JW)=LLN
```

```
      C
      C     IS THE SOLUTION OPTIMAL
      C
            IF(JW-MODE) 7911,7789,7789
      7910  XLBC=Z(JW)
            NLBCN=JW
            IF(JW.EQ.MODE) GO TO 7914
            JW=JW+1
            DO 7913 I=JW,MODE
            IF(Z(I).LT.UBD) GO TO 77913
            IF(Z(I).LT.LLN) KODE=I
            Z(I)=LLN
            GO TO 7913
      77913 IF(XLBC.LE.Z(I)) GO TO 7913
            XLBC=Z(I)
            NLBCN=I
      7913  CONTINUE
      7914  CONTINUE
      C
      C     IS THE SOLUTION OPTIMAL
      C
            IF(LBC.LE.XLBC) GO TO 7789
            Z(NLBCN)=LLN
            GO TO 2163
      72    IF(NODE.NE.1) GO TO 791
            XLBC=Z(NODE)
            NLBCN=NODE
            Z(NODE)=LLN
            GO TO 2163
      74    Z(NODE)=LLN
            KODE=NODE
            GO TO 7791
      791   IF(Z(NODE).LT.UBD) GO TO 7791
            Z(NODE)=LLN
            KODE=NODE
      7791  IF(NKTR.NE.0.OR.NKTR1.NE.0) GO TO 1
            GO TO 1236
      2163  JW=1
            NODE=NLBCN
      5781  IF(K2(NODE,JW).EQ.1) GO TO 5782
            JW=JW+1
            GO TO 5781
      5782  CONTINUE
      C
      C     A FREE OFFICE IS SELECTED BY A BRANCHING DECISION RULE
      C
            KKW=JW
            JW=JW+1
            DO 6721 I=JW,NW
            IF(K2(NODE,I).EQ.0) GO TO 6721
            IF(MEGAS(NODE,KKW).GE.MEGAS(NODE,I)) GO TO 6721
            KKW=I
      6721  CONTINUE
            NKTR1=2
            GO TO 1
      6789  UBD=Z(NODE)
            ITROPT=ITER
```

```
      GO TO 7789
 9779 WRITE(6,97791)
      GO TO 97792
 7789 CONTINUE
97792 CONTINUE
C
C     FACILITY LOCATION ANALYSIS
C
      FUBD=0.
      DO 83 I=1,NK
      IF(Y(NUBDN,I).EQ.0.) GO TO 83
      FUBD=FUBD+IFC(I)
   83 CONTINUE
      VUBD=UBD-FUBD
      WRITE(6,10004) ITER,UBD,FUBD,VUBD,ITROPT,MODE
C
C     TOTAL COST ANALYSIS
C
      TSMI=TIMI=XDEM=XD=0.
      DO 93 I=1,NK
      IF(Y(NUBDN,I).EQ.0.) GO TO 93
      TSMI=TSMI+AMILES(I,IPTLD)*24.
      DO 94 J=1,NC
      IF(ISOL(NUBDN,J).NE.I) GO TO 94
      XDEM=XDEM+DEM(J)
      XD=XD+D(J)
      IF(AMILES(I,J).GT.60) GO TO 95
      TIMI=TIMI+AMILES(I,J)*2.*(DEM(J)+(VISITS+2.)*D(J))
      GO TO 94
   95 TIMI=TIMI+AMILES(I,J)*2.*(DEM(J)+VISITS*D(J))
   94 CONTINUE
   93 CONTINUE
      TINSP=(XD*62.+XDEM*8.+TIMI/AMPH)/(2080.*EFF-48.)
      SALI=TINSP*ASALI
      FUBD=FUBD-TSMI*WAGESP
      VUBD=VUBD-TIMI*WAGEMI
      TOTAL=FUBD+VUBD+SALI+ASALS
      WRITE(6,10007)FUBD,VUBD,ASALI,TINSP,SALI,ASALS,TOTAL
      WRITE(6,10011)TIMI,TSMI,TINSP
C
C     OUTPUT FOR EACH FACILITY
C
      DO 82 I=1,NK
      IF(Y(NUBDN,I).EQ.0.) GO TO 82
      WRITE(6,10005) OFF(I),OFFF(I),OFF(I),OFFF(I)
      TINSP=TDEM=TOTMI=0.
      DO 81 J=1,NC
      IF(ISOL(NUBDN,J).NE.I) GO TO 81
      TDEM=TDEM+DEM(J)
      IF(AMILES(I,J).GT.60.) GO TO 85
      HOMEMI=AMILES(I,J)*2.*(DEM(J)+(VISITS+2.)*D(J))
      GO TO 84
   85 HOMEMI=AMILES(I,J)*2.*(DEM(J)+VISITS*D(J))
   84 XINSP=(D(J)*62.+DEM(J)*8.+HOMEMI/AMPH)/(2080.*EFF-48.)
      TINSP=TINSP+XINSP
      TOTMI=TOTMI+HOMEMI
      WRITE(6,10006) CUS(J),CUST(J),IVC(I,J),DEMOPL(J),DEMCHOW(J),XINSP,
```

```
     1AMILES(I,J),D(J)
  81  CONTINUE
      WRITE(6,10008) OFF(I),OFFF(I),TINSP,TOTMI,IFC(I),TDEM
  82  CONTINUE
      STOP
      END
```