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COMPRESSIVE BUCKLING CURVES FOR FLAT SANDWICH PANELS WITH DISSIMILAR FACINGS

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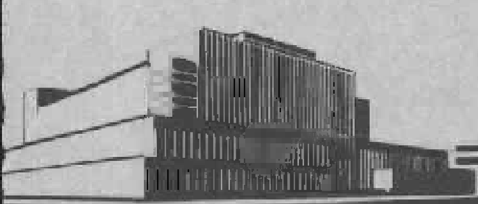

FOREST PRODUCTS LABORATORY
MADISON 5, WISCONSIN

UNITED STATES DEPARTMENT OF AGRICULTURE
FOREST SERVICE
In Cooperation with the University of Wisconsin

COMPRESSIVE BUCKLING CURVES FOR FLAT SANDWICH PANELS WITH DISSIMILAR FACINGS [1]

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Abstract

In this report are presented curves and formulas for use in calculating the buckling of flat, simply supported panels of sandwich construction under edgewise compressive loads. The curves apply particularly to sandwich panels having one facing of glass-fabric laminate, the other facing of an isotropic material, and a honeycomb or isotropic core.

Introduction

The derivation of formulas for the buckling loads of rectangular sandwich panels subjected to edgewise compression is given in Forest Products Laboratory Report No. 1583-B.[3] These formulas apply to

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[1]This progress report is one of a series (ANC-23, Item 58-2) prepared and distributed by the Forest Products Laboratory under U. S. Navy Bureau of Aeronautics No. NAer 01974 and U. S. Air Force No. DO 33(616)58-1, Amendment 4(60-755). Results here reported are preliminary and may be revised as additional data become available.

[2]Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

[3]Erickson, W. S., and March, H. W. "Effects of Shear Deformation In the Core of a Flat Rectangular Sandwich Panel." "Compressive Buckling of Sandwich Panels Having Dissimilar Facings of Unequal Thickness" Forest Products Laboratory Report No. 1583-B. Revised November 1958.

panels having dissimilar orthotropic facings and orthotropic cores. The cores are of such a nature that the stresses in them associated with strains in the plane of the panel may be neglected in comparison with the similar stresses in the facings.

These formulas, reduced to apply to sandwich panels with isotropic facings and cores, are given with design curves in section 3.2.1.1 of ANC Bulletin 23.[4]

For honeycomb cores it was found that the modulus of rigidity associated with the directions perpendicular to the ribbons of which the honeycomb is made and the length of the cells is roughly 40 percent of the modulus associated with the directions parallel to those ribbons and the length of the cells. Making use of this fact, design curves for sandwich panels having isotropic facings and honeycomb cores were calculated and published in Forest Products Laboratory Report No. 1854.[5]

For the glass-fabric laminates currently used for facings it was found that the numerical values of the combinations of the elastic properties that enter the formula for the buckling coefficient could be divided into three groups so that in each group the values do not vary greatly from one laminate to another. Using this fact, design curves for sandwich panels having similar glass-fabric-laminate facings were calculated and published in Forest Products Laboratory Report No. 1867.[6]

The values of the combinations of the elastic properties of most of the glass-fabric laminates currently used fall into one of the three groups. Compressive buckling curves were calculated for sandwich panels with honeycomb and isotropic cores and with one facing consisting of a glass-fabric laminate of this group and the other facing of an isotropic material, and are presented in the present report.

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[4] U. S. Forest Products Laboratory. Sandwich Construction for Aircraft. Air Force-Navy-Civil Aeronautics Bulletin 23, Part II, Second Edition; 1955.

[5] Norris, Charles B. Compressive Buckling Curves for Sandwich Panels with Isotropic Facings and Isotropic or Orthotropic Cores. Forest Products Laboratory Report No. 1854. Revised January 1958.

[6] Norris, Charles B. Compressive Buckling Curves for Simply Supported Sandwich Panels with Glass-Fabric-Laminate Facings and Honeycomb Cores. Forest Products Laboratory Report No. 1867, December 1958.

Elastic Properties

One of the facings of the sandwich panel is taken to be orthotropic and oriented so that its natural axes are parallel to the edges of the panel. Its elastic properties are taken to be such that

$$\alpha_2 = \sqrt{\frac{E_{Fa2}}{E_{Fb2}}} = 1.0$$

$$\beta_2 = \alpha_2 \mu_{Fba2} + 2\gamma_2 = 0.6 \qquad (1)$$

$$\gamma_2 = \frac{G_{Fab2} \lambda_{F2}}{\sqrt{E_{Fa2} E_{Fb2}}} = 0.2$$

where E_{Fa2} and E_{Fb2} are the moduli of elasticity and G_{Fab2} the modulus of rigidity associated with these axes, μ_{Fba2} is Poisson's ratio of the contraction in the a direction to the extention in the b direction due to a tensile stress in the b direction and λ_{F2} is one minus the products of the two Poisson's ratios. The 2 in each subscript refers to this facing. The particular values used apply approximately to most of the glass-fabric laminates listed in table 1 of the Report No. 1867.[6] Their use allows the following simplification of nomenclature:

$$E_{Fa2} = E_{Fb2} = E_{F2}$$

The other facing of the sandwich is taken to be isotropic with a Poisson's ratio of 1/4. Equations (1) with the well-known relation between the modulus of elasticity, modulus of rigidity, and Poisson's ratio for an isotropic material give

$$\alpha_1 = \beta_1 = 1 \text{ and } \gamma_1 = 3/8 \qquad (2)$$

and the simplification in nomenclature:

$$E_{Fa1} = E_{Fb1} = E_{F1}$$

The numeral 1 used in each subscript applies to this facing.

The core of the sandwich panel is taken to be orthotropic and oriented so that its natural axes are parallel to the edges of the panel. It is assumed that its elastic properties associated with strains in the plane of the panel are so small in comparison with those of the facings that the related stresses, may be neglected, and that its modulus of elasticity in a direction perpendicular to the facings is so great that the related strain may be neglected. These assumptions apply particularly well to honeycomb cores.

Formulas

Load is applied to two opposite edges of the panel, as shown in figure 1. The length of these edges is a, and the length of the other two edges is b. The load is applied at the neutral axis of the panel so that the panel does not bend until the critical load is reached. It follows that the strains in the two facings are equal and the critical stress in each facing is given by:

$$f_{F1} = \frac{E_{F1}\lambda_{F2}}{t_{F1}E_{F1}\lambda_{F2} + t_{F2}E_{F2}\lambda_{F1}} P \quad (3)$$

and

$$f_{F2} = \frac{E_{F2}\lambda_{F1}}{t_{F1}E_{F1}\lambda_{F2} + t_{F2}E_{F2}\lambda_{F1}} P \quad (4)$$

where P is the critical load of the sandwich panel in pounds per inch of edge, and t_{F1} and t_{F2} are, respectively, the thickness of the isotropic and the orthotropic facing. The other symbols have been defined.

The formula for P, taken from Report No. 1583-B[3] reduced to the specific case under considation, is:

$$P = \frac{\pi^2}{4a^2}\frac{t_{F1}E_{F1}t_{F2}E_{F2}}{t_{F1}E_{F1}\lambda_{F2} + t_{F2}E_{F2}\lambda_{F1}}(t + t_C)^2 K_M + \frac{\pi^2}{12a^2}\frac{t^3_{F1}E_{F1}}{\lambda_{F1}}K_1 + \frac{\pi^2}{12a^2}\frac{t^3_{F2}E_{F2}}{\lambda_{F2}}K_2 \quad (5)$$

where $\underline{t}$ and $\underline{t_C}$ are the thickness of the sandwich panel and the thickness of the core, $\underline{a}$ is the length of the loaded edge of the panel, and $\underline{K_M}$, $\underline{K_1}$, and $\underline{K_2}$ may be read from the curves presented or calculated from the following formulas:

$$K_M = \frac{TK_2F_1 + (1-T)K_1F_2 + [r(\frac{a}{b})^2 + 1]\,V\,F_1F_2}{T^2F_1 + 2T(1-T)F_{12} + (1-T)^2F_2 + TL_2F_1 + (1-T)L_1F_2 + r(\frac{a}{b})^2V^2F_1F_2} \qquad (6)$$

$$K_1 = (\frac{b}{a})^2 + 2 + (\frac{a}{b})^2 \qquad (7)$$

$$K_2 = (\frac{b}{a})^2 + 1.2 + (\frac{a}{b})^2 \qquad (8)$$

$$F_1 = 0.375\,K_1 \qquad (9)$$

$$F_2 = 0.64 + 0.2\,K_2 \qquad (10)$$

$$F_{12} = 0.4 + 0.1875\,K_2 + 0.10\,K_1 \qquad (11)$$

$$L_1 = (\frac{a}{b})^2\,(0.375\,r + 1)\,V + (r + 0.375)\,V \qquad (12)$$

$$L_2 = (\frac{a}{b})^2\,(0.20\,r + 1)\,V + (r + 0.20)\,V \qquad (13)$$

The parameters of these formulas are given by the following expressions.

$$T = \frac{t_{F1}E_{F1}\lambda_{F2}}{t_{F1}E_{F1}\lambda_{F2} + t_{F2}E_{F2}\lambda_{F1}} \qquad (14)$$

$$V = \frac{t_{F1}E_{F1}t_{F2}E_{F2}}{t_{F1}E_{F1}\lambda_{F2} + t_{F2}E_{F2}\lambda_{F1}}\;\frac{\pi^2 t_C}{a^2\,G_{Cbz}} \qquad (15)$$

$$r = \frac{G_{Cbz}}{G_{Caz}} \qquad (16)$$

where $\underline{G_{Caz}}$ and $\underline{G_{Cbz}}$ are the moduli of transverse rigidity of the core associated with the directions of the loaded and unloaded edges of the panel as shown in figure 1.

These formulas apply to simply supported panels that buckle into a single half wave. They do not give the cusps that are due to buckles forming in greater numbers of half waves (shown in Report No. 1854[5]); thus they are accurate for values of $\frac{b}{a}$ only between zero and their minimum points. Report No. 1854[5] shows that the values do not greatly exceed these minimums for greater values of $\frac{b}{a}$ when the formation of a greater number of half waves is taken into account.

Discussion of Design Curves

The curves given in figures 2, 3, and 4 are plotted from formula (6) and show values of $\underline{K_M}$ in terms of the aspect ratio $\frac{b}{a}$ of the panel and the parameters $\underline{r}$, $\underline{V}$, and $\underline{T}$ defined by formulas (16), (15), and (14). Each figure shows curves for a different value of $\underline{r}$. The values 0.4, 1.0, and 2.5 of $\underline{r}$ were chosen so that the curves would apply to hexagonal honeycomb and isotropic cores.

Each figure shows a family of curves, each curve for a different value of $\underline{V}$. Four of these curves are shown. Each one of these curves is shown as a subfamily of five curves for different values of $\underline{T}$.

Figure 5 is a plot, on logarithmic paper, of $\underline{K_1}$ and $\underline{K_2}$ against the aspect ratio $\frac{b}{a}$. The dashed line is the asymptote of these curves and is useful in determining values of $\underline{K_1}$ and $\underline{K_2}$ for values of $\frac{b}{a}$ less than 0.1.

All of the curves apply to panels having simply supported edges that buckle into a single half wave. The curves associated with a greater number of half waves are not shown. Their minimum points are equal to the minimum points of the curves given. Such curves are shown in figures 2, 3, and 4 of Report No. 1854[5] for panels with isotropic facings. These added curves, with their wave-like cusps, add little to the design information.

These curves apply accurately only if the isotropic facing has a Poisson' s ratio near 1/4 and the glass-fabric-laminate facing has values of α, β, and γ (as given by formulas 1) near 1, 0.6, and 0.2.

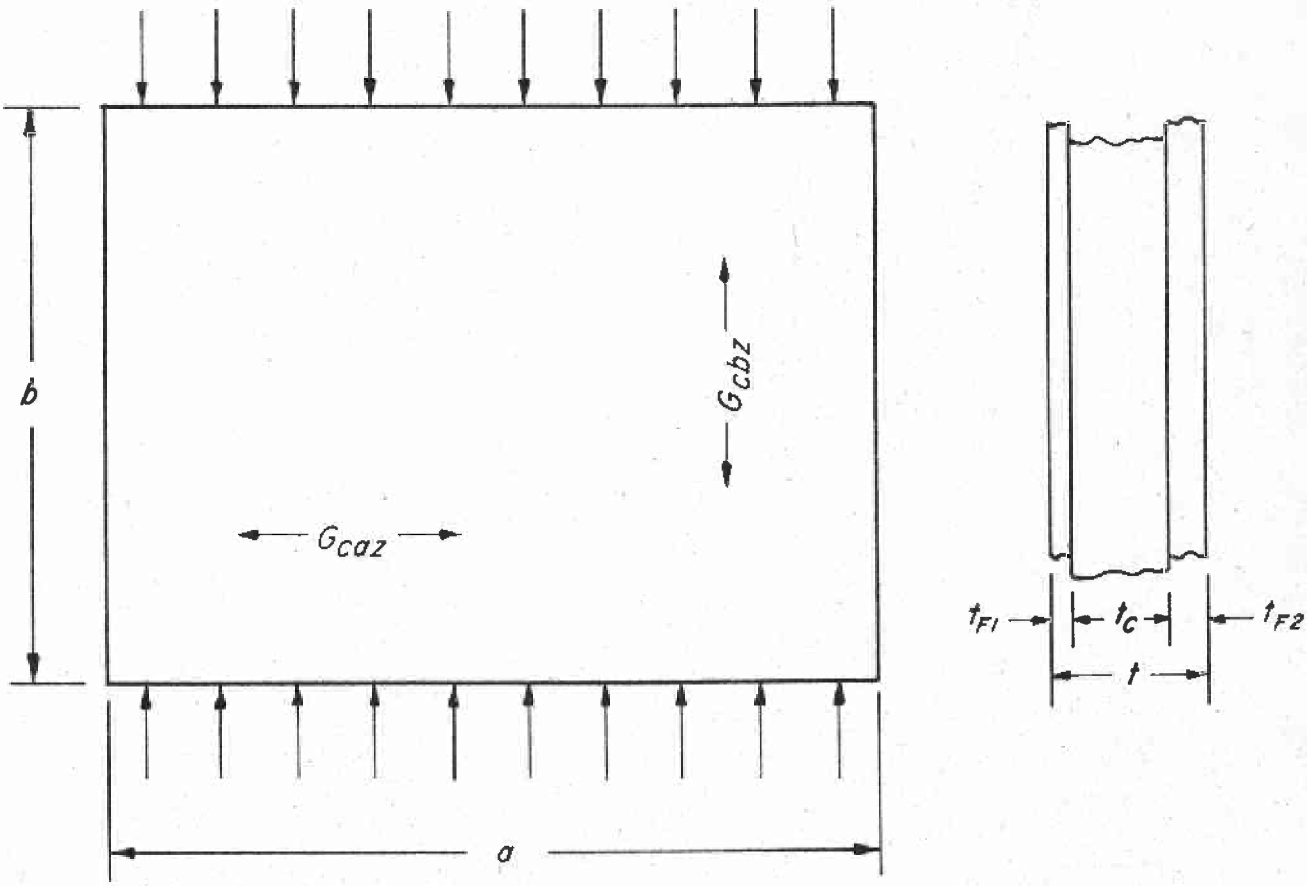

Figure 1. --Sketch showing notation used in this report.

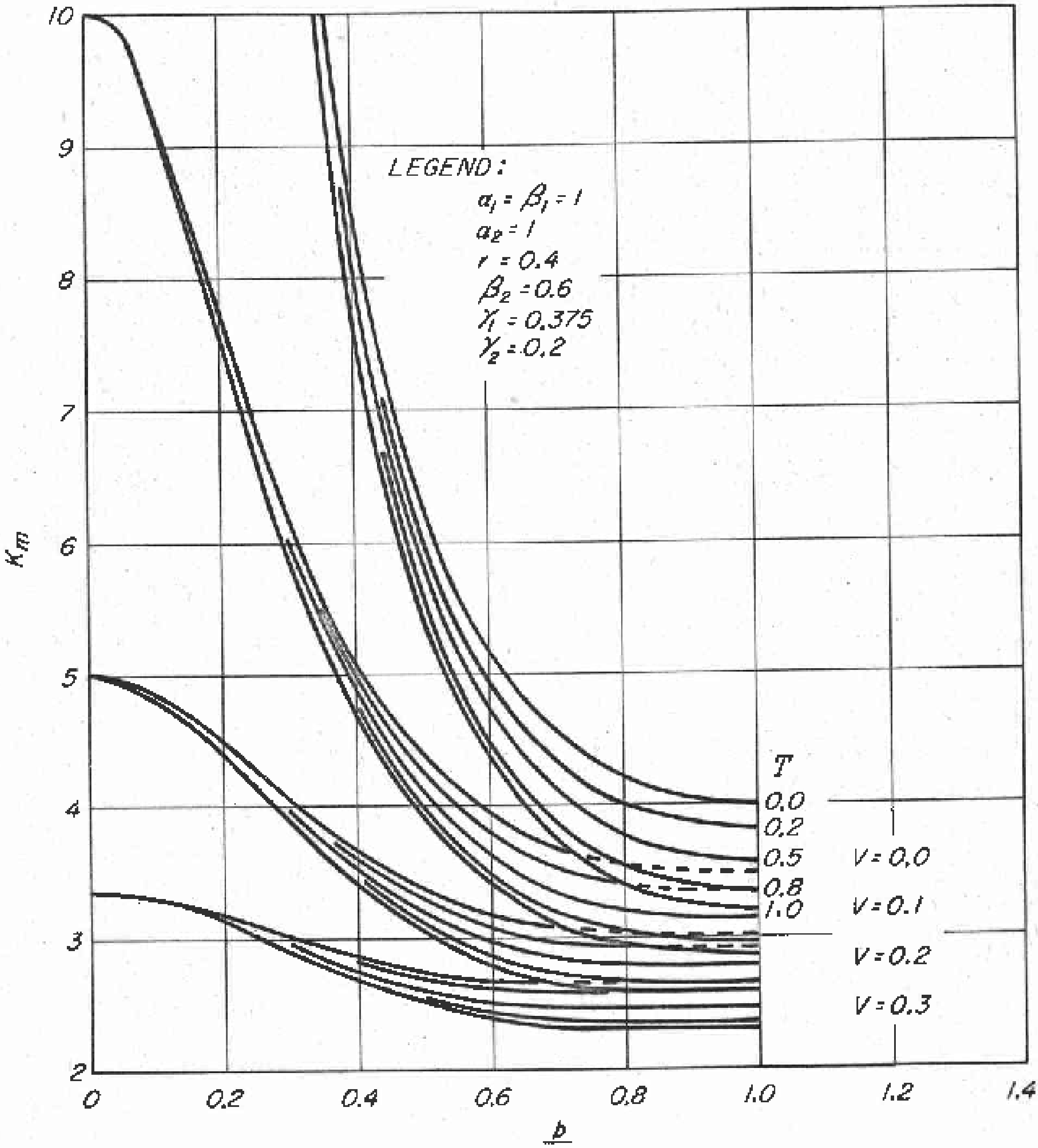

Figure 2. --Values of K_M for r = 0.4 and various values of V and T.

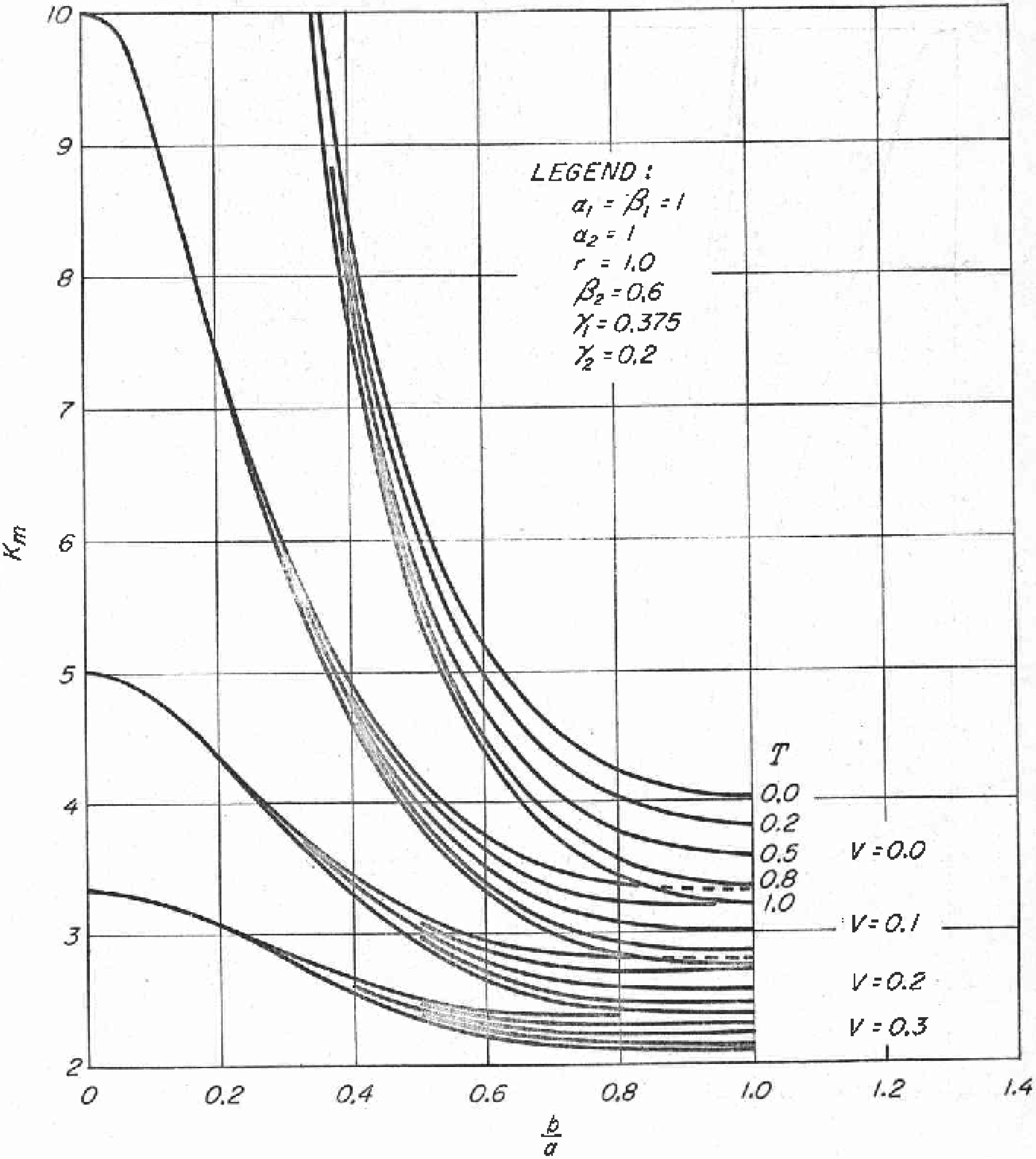

Figure 3. --Values of K_M for r = 1.0 and various values of V and T.

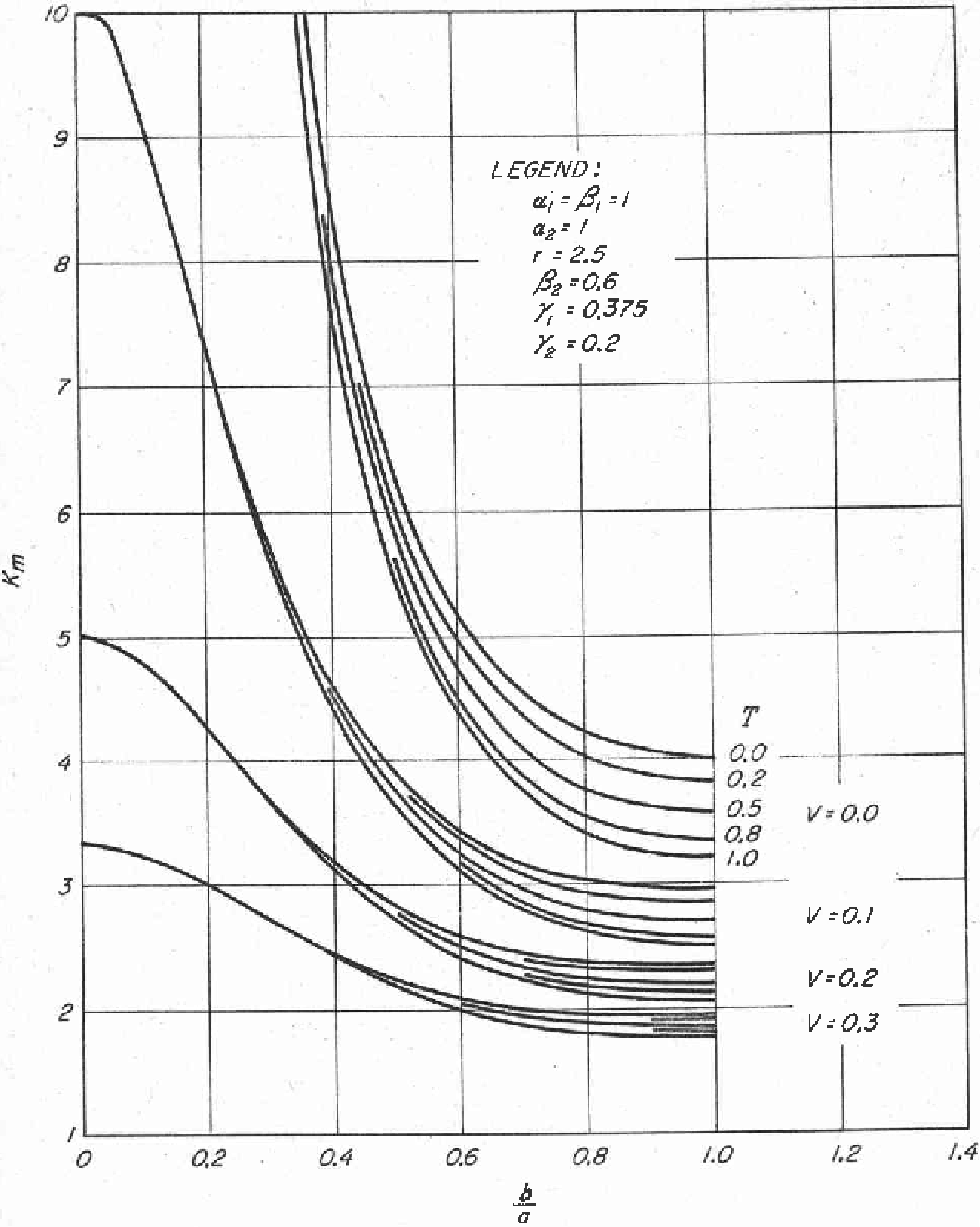

Figure 4.--Values of K_M for r = 2.5 and various values of V and T.

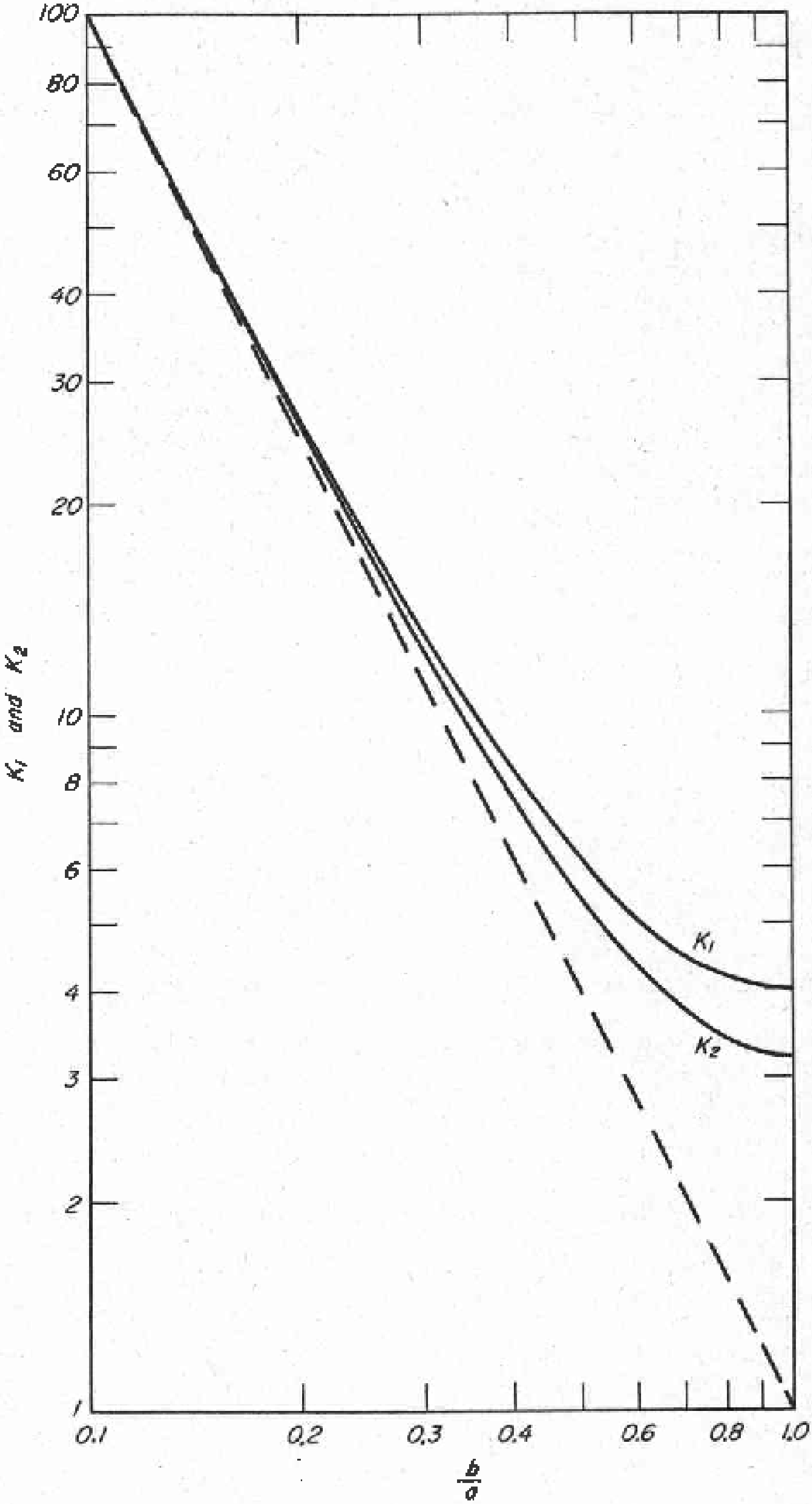

Figure 5. --Values of $\underline{K_1}$ and $\underline{K_2}$.

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