OBJECTIVES OF A TWELVE-YEAR MATHEMATICS PROGRAM FOR
ELEMENTARY AND SECONDARY SCHOOLS

by

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CHAPTER I

INTRODUCTION

Accelerating developments of our dynamic society cause stresses and strains on its parts, among them public education. Inevitable social change creates cultural lags in various phases of social organization including the realm of public education (1). In view of these sociological 'laws,' it is advisable occasionally to re-examine and to reappraise procedures, objectives, and assumptions of public education.

There has been, and is, criticism of public mathematics education (78, pp. 64-65) (59, pp. 88-89). There is not unanimity of opinion among experts and laymen concerning what mathematics should be taught to whom, and when (11, p. 164). Hence, it seems appropriate that a study be undertaken to find valid grounds for determining objectives of public mathematics education at this particular stage in the development of American culture, and then to formulate tenable objectives for public mathematics education.

Statement of the Problem

The problem of this study is to establish a currently, reasonably tenable basis for determining objectives of mathematics education in public elementary and secondary schools, and then to propose some objectives of mathematics education in accordance therewith.
The use of the terms, 'currently,' 'dynamic society,' 'social change,' and 'stage in development,' in the preceding paragraphs connotes the concept that there are no final or eternal answers to the problem. The problem might be stated, "What should the public schools be attempting to do now with respect to mathematics education?"

It should be noted that there are repeated references to public education and to public schools. The study is concerned with objectives of mathematics education in public elementary and secondary schools as opposed to private schools, tutoring, or mathematical learning in the abstract. What limitation this puts on the problem remains to be seen.

Educational programs and investigations relating to them have geographic and social settings. Although this study is general in nature and application insofar as the United States is concerned, it has its setting in Oregon. Oregon has a relatively homogeneous population, high literacy, and well-rated schools (44). Data and findings may not apply equally to cultures and schools which are significantly different from Oregon's.

**Values of the Study**

Values are subjective and depend upon the evaluator. Possible values of this study are:

The study may provide a frame of reference for programs of mathematics education in public schools.
The study may stimulate further consideration of the problem and serve as an example of a procedure to be used in its solution.

The study may clarify some issues regarding mathematics education for educators and laymen.

The report may serve as a source of information and data bearing on objectives of mathematics education.

**Assumptions**

There are some assumptions that form a background for the study. The assumptions have varying degrees of significance for the problem and are not rigorously validated. Highly significant assumptions are examined later on, and reasons are given for their postulation.

Background assumptions are as follows:

It is assumed that some mathematical knowledge is an essential for living in today's world.

It is assumed that the public schools are obligated to provide instruction in mathematics.

It is assumed that the pattern of public education is, and will be for the next few years, a twelve-year program for practically all the children of practically all the people (49, pp. 5-6).

It is assumed that the 'no-failure' policy is, and will increasingly become, the vogue; and that children will tend to pass through the public schools with their age groups with little regard to their academic achievement (62).
Procedure

The study followed the form of a classical logical argument not unlike mathematical reasoning itself. The first step consisted of investigating a mass of data and opinion which might contain matter useful in the solution of the problem. These sources were examined to discover if they tended to lead inductively toward reasonable generalizations, tenable assumptions, or basic policies, suitable for the solution of the problem.

The second step was to formulate a set of usable postulates which were compatible with the sources of data and opinion, and which were not mutually contradictory.

The third step was to reason deductively from the postulates or basic assumptions toward logical outcomes related to the problem.

Lastly, conclusions resulting from the process were stated.

Sources of Data, Opinions, and Ideas

The sources investigated in the first step for assembling data, ideas, and opinions were:

A longitudinal study of mathematics education was made from histories; and for the last century, by analysis of the yearbooks of educational associations, and by a study of the promulgations of various committees and commissions appointed by recognized educational agencies. Concomitant with this historical analysis, a
cursory study of changes in psychology and theories of learning as they related to mathematics education was made.

Recent writings of experts and authorities in the field were checked for their views on principles and objectives of mathematics education, and their underlying assumptions.

Official courses of study relating to mathematics education from nearly every state in the Union were secured and analyzed for their objectives of mathematics education.

Current mathematics texts and sets of texts with accompanying teachers' manuals were investigated and compared, to glean from them their stated and implicit premises and aims of mathematics education.

Standardized mathematics and general educational achievement tests in use in public schools were surveyed for their implicit objectives of mathematics education.

Eleven Oregon school districts were visited; informal interviews with administrators, teachers, and students were held; a few class visitations were made; pertinent practices and printed policies were perused in an effort to discover what were the presumptive principles and objectives of mathematics education.

A fruitful source of data, information, and opinion was that of scanning the rich and varied literature and research relating directly and indirectly to mathematics education. This included psychological, educational, and sociological research and professional opinion as well as lay views.
A few simple surveys concerning mathematical practices and opinion were made by telephone, questionnaire, letters, and interviews with mathematicians, educators, business folk, students, and parents.

Definitions

The meanings of three terms, 'objectives,' 'mathematics,' and 'education,' used in this paper will bear consideration. Concerning the first and third, 'objectives' and 'education,' there is little disagreement, but 'mathematics' has a multiplicity of meanings.

Objectives. According to Bossing (5, p. 289), the term, objective, is synonymous with purpose or aim. He finds virtually no attempt in contemporary educational literature to distinguish meanings between these terms. The Dictionary of Education (23, p. 278) defines objective as:

(1) a standard or goal to be achieved by the pupil when the work in the school activity or school division is completed; (2) the end toward which a school-sponsored activity is directed; (3) a desired change in the behavior of a pupil as a result of experience directed by the school.

An objective connotes a purpose. Objectives of mathematics education will vary with individuals involved. Students, parents, business men, school administrators, and teachers may have different goals in mind with reference to mathematics education.
The term, objectives, in this paper usually refers to aims of classroom teachers with regard to:

a. standards or goals to be achieved by pupils when the work in a school activity or school division is completed; and

b. changes in behavior of pupils desired by classroom teachers as a result of experiences directed by the school.

When objectives of others than classroom teachers are meant, it will be indicated.

Education.

The aggregate of all the processes employed under the auspices of the schools by means of which a person develops abilities, attitudes, and other forms of behavior of positive value in the society in which he lives (23, p. 145).

The term, education, refers in general to the American system of public instruction including elementary and secondary schools, and in particular to facets of the curricula and programs sponsored by the schools.

Mathematics education is the aggregate of efforts by public school instructors to promote and enhance mathematical knowledge and skills of pupils.

Mathematics. Webster's (36, p. 1514) unabridged dictionary defines mathematics as that:

... science or class of science which treats of the exact relations existing between quantities or magnitudes and operations, and of the methods by which, in accordance with these relations quantities sought are deducible from others known or supposed; the science of serial, spatial, quantitative,
and magnitudinal relations; the science of order. Mathematics is usually classified as follows:
(1) pure mathematics...; (2) abstract mathematics...also called pure mathematics; (3) applied mathematics.

Funk and Wagnall's (20, p. 1527) unabridged dictionary has a little different emphasis in its definition of mathematics:

The science that treats of quantity or magnitude, and of their measurements, especially by the use of symbols, and that investigated deductively the spatial, serial and numerical relations existing between objects of perception; in a wider sense, the group of allied sciences concerned with the concrete application of such abstract data. Mathematics embraces pure, or abstract, mathematics of (1) arithmetic...; (2) algebra...; (3) theory of numbers; (4) theory of probabilities; (5) the analysis of real quantities...; (6) the analysis of complex quantities...; (7) pure geometry...; (8) algebra and analysis as applied to geometry...; (9) differential geometry...; and applied, or mixed, mathematics treating of (1) mechanics...; (2) physics...; (3) geodesy and geophysics...; (4) astronomy. In both pure and applied mathematics the treatment may be (1) by synthesis or (2) by analysis.

The 1955 World Book Encyclopedia (91, p. 4373) says:

Mathematics is usually thought of as a branch of human activity which is concerned with the logical arrangements of such things as number, quantity, and form. Modern mathematics is concerned with even broader ideas than these. Many of these ideas seem to have no relation to objects in the real world, although some of them were suggested to mathematicians by ordinary things... There are really two kinds of mathematics...applied...(and)...pure.

The Dictionary of Education does not define mathematics but does define several different kinds of mathematics. It says that mathematical education is the body of mathematical knowledge,
together with the techniques for enlarging and applying the knowledge, that can be used to advantage in the social and intellectual enlightenment of the individual or the group.

Other definitions or descriptions of mathematics are:

Freeman (19, p. 251)

Mathematics may properly be thought of as a language—that is, as a particular set or particular sets of symbols which represent special aspects of reality.

Rapoport (57, p. 122)

Mathematics is another system of logic. It is a system of rules applied to assertions in the form of equations, inequalities, and other relations, so that given certain assertions, we may derive others from them. For example, if it is asserted that the difference of two numbers is equal to the difference of their squares, then we can assert that either the two numbers are equal, or their sum is 1.

Kinney (29, pp. 497-498), writing his chapter on "Mathematics in the Curriculum" in THE HIGH SCHOOL CURRICULUM discusses the nature of mathematics under three headings:

1. Mathematics as a Way of Thinking. Mathematical symbols, formulas, units of measurement, and statistical expressions are essentially devices to facilitate mathematical thinking. The increasing refinement of measurement in industry and elsewhere as well as the extended use of new types of maps, and definition of time zones, are typical of the increased precision in dealing with social problems.

2. Mathematics as a Means of Communication. A quantitative idea can pass from one individual to another without loss of meaning only in case both have the ability to use or interpret quantitative modes of expression. These include such devices as numbers and other symbols, formulas, charts, tables, and the terminology of counting, measuring, and defining
shape, size, and position. It is interesting to note, in the literature of recent years, the growing extent to which competence on the part of the general public is assumed in this area.

3. Mathematics in Reflective Thinking. Problem solving is not the special province of any one field of subject matter. It is rather the concern of all fields of science. In a problem dealing with quantitative data, however, the processes of problem solving are thrown into clear relief. For this reason, the study of problem solving as such has become a major responsibility in mathematics teaching.

A public relations brochure put out by the General Electric Company (21, p. 5) has this to say about mathematics:

When you come right down to it, all—or nearly all—of mathematics, no matter how advanced, no matter how strange it may seem, is just four simple parts of arithmetic: addition, subtraction, multiplication, and division. The more advanced branches of mathematics teach you how to use these four parts of arithmetic to solve harder problems, and they teach you how to do those four things fast. . . Geometry, however, isn't really mathematics at all.

A mathematician, Bertrand Russell, says (3, p. 17):

Mathematics may be defined as the subject in which we never know what we are talking about nor whether what we are saying is true.

Another mathematician, Albert Einstein, has this to say about mathematics (17, p. 244):

Insofar as the propositions of mathematics refer to reality, they are uncertain, and insofar as they are certain, they do not refer to reality.

Countless other definitions or descriptions of mathematics could be listed; some ponderous, some ecstatic; some concise, some
voluminous; some profound, some trivial; but seldom would two definitions be identical. Most definitions tend to reflect that mathematics has two phases but even though it is agreed that mathematics has two aspects, it may not be agreed as to just what mathematics includes. In the final analysis, mathematics is a word symbolizing a generalization which incorporates many ideas which, to large numbers of people, seem to be related. Mathematics has no existence in the real world like a house, or power, or heat. It cannot be counted, or measured, or clearly defined. It exists only in the minds of people as a distillation of learning and experience. Probably no two people would put the same boundaries around the field of mathematics.

For the purpose of this paper, mathematics will be an undefined term. However, it is recognized that mathematics has two phases: pure and applied. Pure mathematics refers to the structure of logic involving number systems, axioms, lawful operations, definitions, and symbols, which man has invented. It extends from the simple sequence of the natural numbers to the complete abstractions of multi-dimensional fields. Applied mathematics refers to man's use of this structure of pure mathematics to help him think, get answers, and communicate, regarding how many and how much.

Mathematics education then becomes the business of teaching public school children concepts, understandings, and skills of pure and applied mathematics.
The objectives of mathematics education refer to the mathematical concepts, understandings, and skills which teachers (and others) want children to learn.
CHAPTER II

INVESTIGATION

Historical Summary of Objectives of Mathematics Education

For centuries pure mathematics had been the province of philosophers and learned men. They developed and tested theories, devised kinds of numbers, invented symbols and terminology and algorithms, and gradually developed a huge rambling structure of the science of mathematics. The construction is continuing at an accelerating pace.

While these philosophers were engaged in their contemplations, people in ordinary walks of life were making use of elementary numbers to count their possessions and to make their calculations; men in the commercial world were figuring and recording their transactions.

These two phases of mathematics, the theoretical and the practical, have been in evidence since the dawn of history. On the one hand, number has been clothed in mysticism, and on the other, it has been used to drive hard bargains.

The earliest mathematics books printed in England in the early 16th century were almost entirely commercial or practical in nature. They were little more than mathematical handbooks for the merchant or bookkeeper, stating definitions and giving rules
for making computations involved in business transactions.

Mathematics was so commercially-centered in England of the seventeenth century that it was largely neglected except by those whose way of life demanded it. People of culture considered mathematics as beneath them and suitable only for such underlings as shopkeepers and tradesmen. Young gentlemen were not supposed to evince any interest in mathematics and young ladies should avoid arithmetic.

In the eighteenth century some scholars and men of culture were interested in pure mathematics, but mundane computation and clerical work was the chore of menials. It was beneath the dignity of a person in the upper classes to stoop to doing arithmetic.


"Though there be many truths discovered in the theory of arithmetic of which there has been no use or application yet found, there is no reason why these things should be neglected or kept out of the system; for they are still a part of the Science which we ought to enlarge more and more as far as we can: one age may find the use of the Theory which a former has invented. . .

"The mind of man is made for knowledge and contemplation, and the pleasure arising from the perception of Beauty and order in other things is allowed to be worthy of rational natures: the contemplation of the surprising connections, the beautiful order and harmony of relations and dependencies found among numbers, is not less reasonable. . .
"Others ask no more than plain rules for the practice so far as they have use for it."

At the end of the eighteenth century we find arithmetic being incorporated as a common school subject. The objective was utilitarian and the organization was characterized by its logical arrangement. Lists of rules were presented to be memorized and to be mechanically applied to dictated problems (32).

In the nineteenth century much emphasis was being put on the cultural and mental training values of mathematics in academies and colleges. However, it seems that the primary purpose of mathematics education in the common schools was utilitarian. The emphasis was on rote computation.

Led by Pestalozzi in Europe and Warren Colburn in this country in the early part of the nineteenth century, we find an emphasis on meaning and understanding being important objectives of mathematics education.

By 1890 psychology was becoming generally recognized in theory as an independent science although there were many vestiges of the metaphysical in psychological writings. It was about this time that American educators began to write about systematic theories of teaching, scientific pedagogy, and what later became known as educational psychology. Theory began to influence practice.

An attempt will be made to trace in a very streamlined fashion shifts in teaching-learning theory and concomitant changes in
objectives of mathematics education during the past three score years (33). This is a simplified version of a complex story and is one man's abstraction from reading the literature. Some suggested functional relationships may not actually have existed.

The third "R," 'rithmetic, had been in the school curriculum during most of the nineteenth century. There was no question that one objective of mathematics education was to teach children the rudiments of computation necessary in the practical world. There were questions of the what, when, and how of this strictly utilitarian value but these were minor. All children should learn to read, to write, and to compute, as much as they could and would. Since their time in school was apt to be limited, each child should be pushed to his capacity by any available means; the rod, the dunce cap, school marks, honors, praise, competition. If he didn't respond, let him quit school.

Many elements of this situation have prevailed since 1890 and prevail today. But few people were, or are, satisfied to limit the objectives of mathematics education to the learning of ordinary practical usages. A theme runs through much literature of mathematics that it has inherent esoteric values.

Educational practice tends to lag ten to twenty years behind educational theory (90, p. 321). While "modern" educational psychologists are hammering out their theories on the anvil of professional debate and research the public schools tend to pursue
"traditional" practices with occasional innovations. There are many social forces operating to keep public schools conservative and "traditional."

Prior to 1890, learning theory was tied up with faculty psychology and mental discipline. It used to be assumed that human beings possessed certain mental faculties which could and should be trained. The mind in general could be developed and toughened by hard work. The harder the mental task, the better. The will was trained by vigorous exercise. Teachers should be authoritarians and drive students to do difficult mental tasks. A subject ideally suited to training the faculties and providing mental exercise was mathematics. A prime objective of mathematics education apart from any mundane utilitarian values was to train the faculties and provide mental discipline. Long after faculty psychology and mental discipline had been discounted by leading educators, these objectives of mathematics education were prevalent in schools and generally accepted by people.

By 1890 educational psychologists were tending to reject the concept that the mind was a collection of faculties and were thinking in terms of the mind being an integrated whole. They spoke of powers of the mind but did not conceive of these powers as being separate entities or faculties. Older doctrines of mental discipline were giving away. Teachers should be more selective in giving
assignments. Mental powers should be developed with certain ends in view. What better vehicle could be used to develop general mental potentialities than mathematics! An objective of mathematics education of the 1890's was to develop mental power.

In the early decades of the twentieth century, psychologists were trying to become more scientific. Psychological measurement, brain structure, laws of learning became important. In 1905, E. L. Thorndike was developing the S-R bond theory. In 1914, John B. Watson was actively promoting behaviorism. About 1915, there was a rather abrupt and comprehensive change in psychological theory from that of the mind being an integrated entity to that of human behavior being constituted of countless S-R bonds subject to formation and conditioning. The function of education was to teach subject matter and to transmit racial experiences. This could be done most effectively by atomizing subject matter into S-R units and making the correct S-R connections. Mathematics fitted into this picture very well. The fundamental processes of arithmetic could be broken down into so many neat combinations for learning purposes. (A moot point was whether $5 \times 3 = 8$, and $3 \times 5 = 8$, was one or two items.) Mathematics was a collection of laws with definite rules for application. Rules, formulas, procedures, etc. should be learned, the more, the better. So the objectives of mathematics education in this era was to teach by establishing S-R bonds as many items of mathematics as could be taught and for which
a person might at some time have some use.

Vestiges of this type of thinking are still manifest in present day mathematics education.

A gradual revolt against extremes of behaviorism took place in the nineteen-thirties, having been preceded by shifts in the thinking of the theorists. Instead of so much emphasis on discrete units of subject matter, the center of gravity gradually shifted to learning activities for building up broader skills, concepts, meanings, problem solving abilities. The teacher was to direct learning activities in such a way as to have these desirable outcomes result. It was difficult to be precise about the nature of these general skills and concepts which were to be the outcome of the teaching-learning process. Frequently, skills or concepts were logically analyzed into constituent parts in the manner of the behaviorists and were sometimes lost, or disappeared, in the process. During this period of educational theory, which is still somewhat current, the stated objectives of mathematics education tended to be generalized statements of desirable skills, concepts, quantitative and critical thinking, meanings, etc. However, in actual practice, an implicit objective of mathematics education continued to be the accumulation of items of knowledge and specific skills, with elements of mental discipline and mind-power development thrown in for good measure.

The next significant shift in teaching-learning theory was toward being more child centered. It is not yet clear just what this
will result in. Instead of the teacher directing the learning activities, the function of the teacher is to guide pupil experiencing. Note the word change from "directing" to "guide" and from "activities" to "experiencing." There is increasing emphasis on the needs of each child as an unique individual, and there is much more concern about the total impact on the child of his experiences under school auspices. What are the needs of this particular child? How can we guide him to have experiences that will satisfy his needs? What all is happening within this child as he undergoes an "experience"? Within this sort of educational framework, subject matter as subject matter takes secondary place and the welfare and wholesome development of the child are paramount. It is not easy for a subject matter specialist to adjust to this point of view. Even if the tenets were apparently philosophically and psychologically sound, it is unlikely that the public and educational authorities would or could move very far very fast toward incorporating them in the schools. Nevertheless, teaching-learning theory is steadily moving toward an individual child-centered frame of reference for education

**Findings from Committee Reports and Yearbooks**

Excerpts from committee reports and association yearbooks of the last 60 years given in chronological order will tend to
reflect trends in guiding principles and objectives of mathematics education.

The Committee of Ten on Secondary School Studies of the National Educational Association made its report in 1894 (h8). The committee had set up several conferences to consider various phases of the secondary school curriculum. Among these conferences was one on mathematics which made statements as follows:

"The conference on mathematics wish to have given in elementary schools not only a general survey of arithmetic, but also elements of algebra, and concrete geometry in connection with drawing..." (h8, p. 14)

"As things now are, the high school teacher finds in pupils fresh from the grammar schools no foundation of elementary mathematics conceptions outside of arithmetic; no acquaintance with algebraic language; and no accurate knowledge of geometrical forms." (h8, p. 15)

"The course in arithmetic (should) be at once abridged and enriched; abridged by omitting entirely those subjects which perplex and exhaust the pupil without affording any really valuable mental discipline, and enriched by a greater number of exercises in simple calculation, and in the solution of concrete problems." (h8, p. 23)

"The method of teaching (mathematics in high school) should be throughout objective, and such as to call into exercise the pupils' mental activity." (h8, p. 105)

The NEA Committee of Fifteen on Geometry published their report about 1911. Quotations from it are as follows:

"In the high school geometry has long been taught because of its mind-training value only. This exclusive attention to the disciplinary side may be fascinating to mature minds, but in the case of young pupils, it may lead to a dull formalism which is unfortunate."
"Among the claims in behalf of geometry, the committee would emphasize the following: Geometry is taught because of the pleasure it gives when properly presented to the average mind. Geometry is taught because of the profit it gives when properly presented, e.g. (1) It is an exercise in logic. . . Closely connected with the logic element is the training in accurate and precise thought and expression and the mental experience and contact with exact truth." (47, p. 32)

The National Committee on Mathematical Requirements worked on the Reorganization of Mathematics in Secondary Education for several years during and after World War I making their final report in 1923. This was a significant report in mathematics education. Some excerpts are:

"It has been customary to distinguish three classes of aims (of mathematics education: (1) practical or utilitarian, (2) disciplinary, (3) cultural; and such a classification is indeed a convenient one. It should be kept clearly in mind, however, that the three classes mentioned are not mutually exclusive and that convenience of discussion rather than logical necessity often assigns a given aim to one or the other of these classes. Indeed, any truly disciplinary aim is practical and, in a broad sense, the same is true of cultural aims.

"PRACTICAL AIDS - By a practical or utilitarian aim, in the narrower sense, we mean the immediate or direct usefulness in life of a fact, method, or process in mathematics.

"1. The immediate and undisputed utility of the fundamental processes of arithmetic in the life of every individual demands our first attention. The first instruction in these processes, it is true, falls outside the period of instruction which we are considering. By the end of the sixth grade the child should be able to carry out the four fundamental operations with integers and with common and decimal fractions accurately and with a fair degree of speed. This goal can be reached in all schools—as it is being reached in many—if the work is done under properly qualified teachers and if drill is confined to the simpler cases which alone are of importance.
in the practical life of the great majority. Accuracy and facility in numerical computation are of such vital importance, however, to every individual that effective drill in this subject should be continued throughout the secondary school period, not in general as a separate topic, but in connection with numerical problems arising in other work. In this numerical work, besides accuracy and speed, the following aims are of greatest importance.

(a) A progressive increase in the pupil's understanding of the nature of the fundamental operations and power to apply them in new situations. The fundamental laws of algebra are a potent influence in this direction.

(b) Exercise of common sense and judgment in computing from approximate data, familiarity with the effect of small errors in measurements, the determination of the number of figures to be used in computing and to be retained in the result, and the like.

(c) The development of self-reliance in the handling of numerical problems through consistent use of checks on all numerical work.

"2. Of almost equal importance to every educated person is an understanding of the language of algebra and the ability to use the language intelligently and readily in the expression of such simple quantitative relations as occur in every-day life and in the normal reading of the educated person.

"Appreciation of the significance of formulas and ability to work out simple problems by setting up and solving the necessary equations must nowadays be included among the minimum requirements of any program of universal education.

"3. The development of the ability to understand and to use such elementary algebraic methods involves a study of the fundamental laws of algebra and at least a certain minimum of drill in algebraic technique, which when properly taught, will furnish the foundation for an understanding of the significance of the processes of arithmetic already referred to. The essence of algebra as distinguished from arithmetic lies in the fact that
algebra concerns itself with the operations upon numbers in general, while arithmetic confines itself to operations on particular numbers.

"4. The ability to understand and interpret correctly graphic representations of various kinds, such as nowadays abound in popular discussions of current scientific, social, industrial, and political problems, will also be recognized as one of the necessary aims in the education of every individual. This applies to the representation of statistical data which are becoming increasingly important in the consideration of our daily problems, as well as to the representation and understanding of various sorts of dependence of one variable quantity upon another.

"5. Finally, among the practical aims to be served by the study of mathematics should be listed familiarity with the geometric forms common in nature, industry, and life; the elementary properties and relations of these forms, including their mensuration; the development of space-perception; and the exercise of spatial imagination. This involves acquaintance with such fundamental ideas as congruence and similarity and with such fundamental facts as those concerning the sum of the angles of a triangle, the pythagorean proposition, and the areas and volumes of the common geometric forms.

"Among directly practical aims should also be included the acquisition of the ideas and concepts in terms of which the quantitative thinking of the world is done, and of ability to think clearly in terms of those concepts..."

"DISCIPLINARY AIDS - We include here those aims which relate to mental training, as distinguished from the acquisition of certain specific skills discussed in the preceding section. Such training involves the development of certain more or less general characteristics and the formation of certain mental habits which, besides being directly applicable in the setting in which they are developed or formed, are expected to operate also in more or less closely related fields—that is, to "transfer" to other situations.

"The subject of the transfer of training has for a number of years been a very controversial one. Only recently has there been any evidence of agreement among the body of educational psychologists. We need not at this
point go into detail as to the present status of disciplinary values. . . It is sufficient for our purpose to call attention to the fact that most psychologists have abandoned two extreme positions as to transfer of training. The first asserted that a pupil trained to reason well in geometry would thereby be trained to reason equally well in any other subject; the second denied the possibility of any transfer and, hence, the possibility of any general mental training. That the effects of training do transfer from one field of learning to another is now, however, recognized. The amount of transfer in any given case depends upon a number of conditions. If these conditions are favorable, there may be considerable transfer, but in any case the amount of transfer is difficult to measure. Training in connection with certain attitudes, ideals, and ideas is now almost universally admitted by psychologists to have general value. It may, therefore, be said that with proper restrictions, general mental discipline is a valid aim in education.

"The aims which we are discussing are so important in the restricted domain of quantitative and spatial (i.e., mathematical or partly mathematical) thinking which every educated individual is called upon to perform that we do not need for the sake of our argument to raise the question as to the extent of transfer to less mathematical situations.

"In formulating the disciplinary aims of the study of mathematics the following should be mentioned: (1) the acquisition, in precise form, of those ideas or concepts in terms of which the quantitative thinking of the world is done. Among these ideas and concepts may be mentioned ratio and measurement (lengths, areas, volumes, weights, velocities, and rates in general, etc.), proportionality and similarity, positive and negative numbers, and the dependence of one quantity upon another. (2) The development of ability to think clearly in terms of such ideas and concepts. This ability involves training in: (a) Analysis of a complex situation into simpler parts, This includes the recognition of essential factors and the rejection of the irrelevant. (b) The recognition of logical relations between interdependent factors and the understanding and, if possible, the expression of such relations in precise form. (c) Generalization; that is, the discovery and formulation of a general law and an understanding of its properties and applications.
"3. The acquisition of mental habits and attitudes which will make the above training effective in the life of the individual. Among such habitual reactions are the following: a seeking for relations and their precise expressions; an attitude of enquiry; a desire to understand, to get to the bottom of a situation; concentration and persistence; a love for precision, accuracy, thoroughness, and clearness, and a distaste for vagueness and incompleteness; a desire for orderly and logical organization as an aid to understanding and memory.

"4. Many of these disciplinary aims are included in the broad sense of the idea of relationship or dependence—in what the mathematician in his technical vocabulary refers to as a "function" of one or more variables. Training in "functional thinking," that is thinking in terms of an about relationship, is one of the most fundamental disciplinary aims of the teaching of mathematics.

"CULTURAL AIDS — By cultural aims we mean those somewhat less tangible but none the less real and important intellectual, ethical, esthetic or spiritual aims that are involved in the development of appreciation and insight and the formation of ideals of perfection. As will be at once apparent, the realization of some of these aims must await later stages of instruction, but some of them may and should operate at the very beginning.

"More specifically, we may mention the development or acquisition of:

"1. Appreciation of beauty in the geometrical forms of nature, art, and industry.

"2. Ideals of perfection as to logical structure, precision of statement and of thought, logical reasoning (as exemplified in the geometric demonstration), discrimination between the true and the false, etc.

"3. Appreciation of the power of mathematics—of what Byron expressively called 'the power of thought, the magic of the mind'—and the role that mathematics and abstract thinking, in general, have played in the development of civilization; in particular, in science, in industry, and in philosophy. In this connection, mention should be made of the religious effect, in the broad sense, which the study of the infinite and of the permanence of laws in mathematics tends to establish...
"The primary purposes of the teaching of mathematics should be to develop those powers of understanding and of analyzing relations of quantity and of space which are necessary to an insight into and control over our environment and to an appreciation of the processes of civilization in its various aspects, and to develop those habits of thought and of action which will make these powers effective in the life of the individual (36, pp. 6-10)."

The voluminous 1923 report of the National Committee on Mathematical Requirements has been quoted at length, because it is a landmark in the development of mathematics education. It represents a clear exposition of enlightened thinking a third of a century ago.

The 1926 yearbook of the NAA Department of Superintendence stressed utilitarian aspects of mathematics based on research.

"Arithmetic is not an end in itself. It is a tool. To meet the demands of social utility, three phases of arithmetic need attention: (1) The basic experience which is necessary in order to make manipulative work meaningful, (2) the mastery for automatic reproduction of the useful number facts, (3) training in application of life and business situations (46, p. 35)."

In 1930, the National Society for the Study of Education published a yearbook concerned with arithmetic from which the following excerpts were taken:

"THE PSYCHOLOGY OF LEARNING ASSUMED IN THE YEARBOOK. Theoretically, the main psychological basis is a behavioristic one, viewing skills and habits as fabrics of connections. This is in contrast, on the one hand, to the older structural psychology which has still to make direct contributions to classroom procedure, and on
the other hand, to the more recent Gestalt psychology, which, though promising, is not yet ready to function as a basis of elementary education...

"The psychological point of view pervading this Yearbook emphasizes the fact that teaching based upon felt needs and interest only is inadequate. Not that felt needs and interest are lacking in vitality and importance, but that neglect of other matters (even if the neglect is only by way of inference) weakens effective teaching and learning to an intolerable degree. Use of all the dynamics of learning, rather than a use of some and a neglect of others, is the position taken. In the older school there was an overconfidence in drill—too often so stupidly administered that it could not possibly affect learning—and a corresponding neglect of interest and of the significance of the work of the worker... There is almost an emotional antipathy to anything that in any way reminds us of the kind of schools we attended as children.

"The committee takes the point of view that the purpose of teaching mathematics is to develop the social values, social utility." (50, p. 5)

"...it is held that whatever is of greatest good to the individual is likewise of greatest good to the aggregate of individuals. Whatever makes the life of the individual fuller and richer does, in fact, upon any valid idea of fullness and richness, make him a better member of society. Moreover, whatever accomplishes or contributes to the accomplishment of this result in any widely diffused way is of service to the community of individuals which we call society." (50, p. 37)

The yearbook expresses concern with the great elimination of items from the curriculum and deplores this chipping away.

"The doctrine of social utility is often interpreted to mean that that only is useful which is shown to be actually in use. In reality, utility should be broadly enough envisaged to include the value of a subject to mankind in all its phases. Arithmetic, for example, has information value. It functions in the concept that a citizen has of his world. It forms a basis for the orientation of a person to many fundamental conceptions..."
of modern society. It is necessary to examine carefully, therefore, the topics advocated for elimination in the light of all the values which arithmetic can offer."

(50, p. 80)

The tenth yearbook of the National Council of Teachers of Mathematics published in 1935 (l3) isolates for analysis three theories of learning arithmetic: (1) the drill theory, (2) the incidental learning theory, and (3) the meaning theory. The yearbook strongly endorsed the "meaning" theory and launched this theory onto a career which has become increasingly important in educational circles.

"The record of arithmetic in the school is an unenviable one. The position taken in this chapter is that the fault lies in the type of instruction given... too much toward drill... The basic tenet in the proposed instructional reorganization is to make arithmetic less a challenge to the pupil's memory and more a challenge to his intelligence." (l3, p. 30)

"Arithmetic should be judged primarily in terms of its social values. These values are not to be determined alone by a survey of computational practices. The fallacy of the previous application of the theory of social utility lies primarily in the narrow and restricted definition given to social utility... Arithmetic consists of two major types of material, first, a number system which must be learned as a system with all of its common interrelations as expressed in the operation of the four fundamental processes. Computational ability is essential and necessary for this type of mastery. Second, arithmetic consists in the socialization of this type of number experience until it permeates the common thinking practices of individuals. Overemphasis on computation has produced a lopsided arithmetic. The recent movement to balance the teaching of arithmetic by giving emphasis to its social and informational values is a movement so significant that it may well become the outstanding reform which this generation will contribute to the subject." (l3, p. 84)
Chapter 12, The New Psychology of Learning, written by Raymond Holder Wheeler anticipates much of present day learning theory.

"The main principles of the New Psychology. . . (1) Learning is a function of maturation and insight. It is a growth process that follows laws of dynamics, that is, laws of structured, unitary, energy systems or fields. (2) First impressions are of total situations, but are undifferentiated. . . (3) Learning is not exclusively an inductive process. . . (4) Learning does not proceed by trial and error. . . (5) More important, by far, than formal prescribed methods of instruction are the personality of the learner and of the teacher, and the relationship between these personalities. . . (6) Learning depends upon the will to learn, which cannot be forced by requirements or authority, but must be challenged by dynamic teachers and dynamic teaching. (7) Learning depends on clearness of goals, and the fitness with which tasks are adjusted to the pupil's level of maturation and insight. Progress is made by pacing. (8) Goals are their own rewards, under natural law. Grades, grade points, many forms of motivation by social competition, and other hypocrisies are detrimental to learning. . . (9) A large part of the most efficient learning is incidental, that is, learning a special subject with reference to some broader interest or aim without realizing it. . . (10) Learning depends on transposition, that is, discovery of form, system, order, pattern, logical relations, analogies, the repeated use of a hidden logical principle, and the making of relational judgments. It is not a matter of combining skills. (11) Subjects are inadequately learned when in isolation. (12) No transfer will occur unless the material is learned in connection with the field to which transfer is desired." (43, p. 237)

Under hints to teachers of mathematics:

"16. Eliminate from your mind that mathematics is, first, the science of number, quantity, and measurement. It is not. Primarily, it is a rigid logic, a science of precise order, pattern, transposition, invariants, matching. Study such phrases as the following, all of which are taken from the history of mathematics: the part is equal to the whole; the part has the power of the whole; you can prove the special case only when the general case
is subject to proof; divide means contain; infinity is not a noun, it is an adjective; there are no infinitesimals; a 'point' is in reality a 'system'; a given number is a class. The logic of these assertions is the same as the logic of Gestalt psychology." (43, p. 215)

In 1938, the National Council of Teachers of Mathematics' committee on arithmetic published a report in the journal of the organization in which the following statement appears:

"The functions of instruction in arithmetic, then, are to teach the nature and uses of the number system in the affairs of daily and to help the learner to utilize quantitative procedures effectively in the achievement of his purposes and those of the social order of which he is a part. This conception recognizes two major mutually related and interdependent phases, namely, the mathematical and the social. . .

"The purposes of stressing the mathematical phase are (1) to develop the ability to recognize among all the attractive and objective elements of appropriate situations the number element that does not meet the eye but nevertheless, gives them order and exactness; (2) to introduce pupils to the systematic methods of attack upon the number elements of situations that have finally been brought together into the unified number system as we know and use it today; (3) to develop in pupils both confidence in the reliance one can place upon such methods of attack and facility in their use by guiding practice in number thinking to higher and higher levels of effectiveness; and (4) to train pupils to carry to later and more advanced studies of important personal, business, social, and civic situations these methods of attack with the deliberate purpose of analyzing out their number elements and determining their relations to the other elements which with the number elements comprise the whole.

"The purposes of stressing the social phase are: (1) to insure understanding of the contributions number has made to the development of the social institutions that have made possible the progress of the human race; (2) to insure that the work of pupils in arithmetical processes will have meaning and significance; (3) to
develop in pupils the disposition and ability to apply increasingly mature procedures in the quantitative situations of life." (34, pp. 267-268)

The sixteenth yearbook (1941) of the National Council of Teachers of Mathematics contains the final report of the committee on arithmetic (37). The report develops the point of view that arithmetic makes a contribution to the development of personality. It rejects drill and incidental learning theories. It expresses concern over mathematical fears engendered in children and considers postponing arithmetic instruction. It stresses the importance of learning the rationale of the number system.

"Arithmetic is a system of ideas. Arithmetic exists and grows for the learner only in the mind of the learner." (37, p. 109)

"Those who are conducting research in arithmetic should realize how vigorously many psychologists are disputing the validity of the principal connectionist maxims. The issues are clear-cut. The newer point of view emphasizes relatedness rather than itemization. It stresses generalization instead of extreme specificity. It conceives of learning as a meaningful, not a mechanical process. It considers understanding more important than mere repetition or drill. It looks upon learning as a developmental process, not one of fixation of stereotyped reactions. It encourages discovery and problem-solving rather than rote learning and parrot-like repetition. It is with the matrix of these issues that the new research in arithmetic should be conducted." (37, p. 288)

During World War II, military leaders and others became concerned about the lack of arithmetic competence and understanding among draftees and young employees. The schools were blamed for this situation. Some investigations were made and reports printed. Among these were:
"Pre-Induction Courses in Mathematics" issued jointly by the U. S. Office of Education and the NCTM (77).


1944 and 1945 First report and second report of the Commission on Post-War Plans appointed by the National Council of Teachers of Mathematics (38) (42).


These reports emphasize computation and "essential" mathematics. The final report of the Commission on Post-War Plans was published in 1947 and revised in 1953. The principal contribution of this report is the checklist of functional competences in mathematics which the commission believed should form the basis for objectives of high school mathematics education. The report also lists mathematical needs for many vocations (39, pp. 14-23). The 29 functional competences in mathematics proposed by the commission in the final report are expressed as follows:

1. Computation. Can you add, subtract, multiply, and divide effectively with whole numbers, common fractions, and decimals?

2. Percents. Can you use percents understandingly and accurately?
3. Ratio. Do you have a clear understanding of ratio?

4. Estimating. Before you perform a computation, do you estimate the result for the purpose of checking your answer?

5. Rounding numbers. Do you know the meaning of significant figures? Can you round numbers properly?

6. Tables. Can you find correct values in tables; e.g., interest and income tax?

7. Graphs. Can you read ordinary graphs: bar, line, and circle graphs? the graph of a formula?

8. Statistics. Do you know the main guides that one should follow in collecting and interpreting data; can you use averages (mean, median, mode); can you draw and interpret a graph?

9. The nature of a measurement. Do you know the meaning of a measurement, of a standard unit, or the largest permissible error, of tolerance, and of the statement that 'a measurement is an approximation'?

10. Use of measuring devices. Can you use certain measuring devices, such as an ordinary ruler, other rulers (graduated to thirty-seconds, to tenths of an inch, and to millimeters), protractor, graph paper, tape, caliper, micrometer, and thermometer?

11. Square root. Can you find the square root of a number by table, or by division?

12. Angles. Can you estimate, read, and construct an angle?

13. Geometric concepts. Do you have an understanding of point, line, angle, parallel lines, perpendicular lines, triangle (right, scalene, isosceles, and equilateral), parallelogram (including square and rectangle), trapezoid, circle, regular polygon, prism, cylinder, cone, and sphere?

14. The 3-4-5 relation. Can you use the Pythagorean relationship in a right triangle?
"15. Constructions. Can you with ruler and compases construct a circle, a square, and a rectangle, transfer a line segment and an angle, bisect a line segment and an angle, copy a triangle, divide a line segment into more than two equal parts, draw a tangent to a circle, and draw a geometric figure to scale?

"16. Drawings. Can you read and interpret reasonably well, maps, floor plans, mechanical drawings, and blueprints? Can you find the distance between two points on a map?

"17. Vectors. Do you understand the meaning of vector, and can you find the resultant of two forces?

"18. Metric system. Do you know how to use the most important metric units (meter, centimeter, millimeter, kilometer, gram, kilogram)?

"19. Conversion. In measuring length, area, volume, weight, time, temperature, angle, and speed, can you shift from one commonly used standard unit to another widely used standard unit; e.g., do you know the relation between yard and foot, inch and centimeter, etc.?

"20. Algebraic symbolism. Can you use letters to represent numbers; i.e., do you understand the symbolism of algebra—do you know the meaning of exponent and coefficient?

"21. Formulas. Do you know the meaning of a formula—can you, for example, write an arithmetic rule as a formula, and can you substitute given values in order to find the value for a required unknown?

"22. Signed numbers. Do you understand signed numbers and can you use them?

"23. Using the axioms. Do you understand what you are doing when you use the axioms to change the form of a formula or when you find the value of an unknown in a simple equation?

"24. Practical formulas. Do you know from memory certain widely used formulas relating to area, volumes, and interest, and to distance, rate, and time?
"25. Similar triangles and proportion. Do you understand the meaning of similar triangles, and do you know how to use the fact that in similar triangles the ratios of corresponding sides are equal? Can you manage a proportion?

"26. Trigonometry. Do you know the meaning of tangent, sine, cosine? Can you develop their meanings by means of scale drawings?

"27. First steps in business arithmetic. Are you mathematically conditioned for satisfactory adjustment to a first job in business; e.g., have you a start in understanding the keeping of a simple account, making change, and the arithmetic that illustrates the most common problems of communications and every day affairs?

"28. Stretching the dollar. Do you have a basis for dealing intelligently with the main problem of the consumer; e.g., the cost of borrowing money, insurance to secure adequate protection against the numerous hazards of life, the wise management of money, and buying with a given income so as to get good values as regards both quantity and quality?

"29. Proceeding from hypothesis to conclusion. Can you analyze a statement in a newspaper and determine what is assumed, and whether the suggested conclusions really follow from the given facts or assumptions?"


The Davis (David J.) Test of Functional Competence in Mathematics, World Book Company, 1950, is a high school mathematics achievement test based on the checklist.
Part II of the fiftieth yearbook (1951) of the National Society for the Study of Education is devoted to the teaching of arithmetic. It lists as desirable outcomes of elementary school arithmetic instruction:

"(1) Computational skill: facility and accuracy in operations with whole numbers, common fractions, decimals, and per cents. (This group of outcomes is here separated from the second and third groups which follow because it can be isolated for measurement. In the separation, much is lost, for computation without understanding when as well as how to compute is a rather empty skill. Actually, computation is important only as it contributes to social ends.)

"(2) Mathematical understandings: a. meaningful conceptions of quantity, of the number system, of whole numbers, of common fractions, of decimals, of per cents, of measures, etc. b. A meaningful vocabulary of the useful technical terms of arithmetic which designate quantitative ideas and the relationships between them. c. Grasp of important arithmetical generalizations. d. Understanding of the meanings and mathematical functions of the fundamental operations. e. Understanding of the meanings of measures and of measurement as a process. f. Understanding of important arithmetical relationships, such as those which function in reasonably sound estimations and approximations, in accurate checking, and in ingenious and resourceful solutions. g. Some understanding of the rational principles which govern number relations and computational procedures.

"(3) Sensitiveness to number in social situations and the habit of using number effectively in such situations: a. Vocabulary of selected quantitative terms of common usage (such as kilowatt hour, miles per hour, decrease and increase, and terms important in insurance, investments, business practices, etc. b. Knowledge of selected business practices and other economic applications of number. c. Ability to use and interpret graphs, simple statistics, and tabular presentations of quantitative data (as in study in school and in practical activities outside of school). d. Awareness of the usefulness of quantity and number in dealing with many aspects of
life. Here belongs some understanding of social institutions in which the quantitative aspect is prominent, as well as some understanding of the important contribution of number in their evolution. e. Tendency to sense the quantitative as part of normal experience, including vicarious experience, as in reading, in observation, and in projected activity and imaginative thinking. f. Ability to make (and the habit of making) sound judgments with respect to practical, quantitative problems. g. Disposition to extend one's sensitiveness to the quantitative as this occurs socially and to improve and extend one's ability to deal effectively with the quantitative when so encountered or discovered."
(51, p. 6)

Recent yearbooks have been much less subject centered and much more child centered in their approach to mathematics education. Meeting the needs of youth is becoming the objective of education. Recognition of individual differences is a watchword, and guidance is a key part of the program. A striking contrast may be noted between the third (1926) and the thirty-first (1953) yearbooks of the American Association of School Administrators (46). Both yearbooks deal with curriculum. The former is entirely subject centered and the latter is individual centered. General and specific objectives of education are written in entirely different terms. Quotations from the 1953 yearbook may illustrate this:

"Belief in the essential worth of each individual demands educational opportunities for all. Schools... influence... better life for all... Home is the foundation of citizenship and our social structure... We must make clear the ethical and moral ideals which sustain our pattern of government... Our society is an evolving one based on the freedom of the individual. The school curriculum and its processes of development
should stimulate faith in our form of government." (1, p. 22)

"What then are the essentials that our citizens should possess in order to fulfill the requirements. Civic competence... Educational flexibility... Social understanding... Occupational efficiency... Home loyalty... Religious consciousness... Leisure time opportunities... Social outcomes determine content... Something is learned... Learning is an active process... Learning is affected by the total situation... Learning should be meaningful... Learning must be adjusted to individual differences... Learning should result in versatile, adaptable behavior... Emotional and social learning are important." (1, p. 22)

On page 159, the yearbook endorses the idea of functional mathematics. (1, p. 159)

The twenty-first yearbook (1953) of the National Council of Teachers of Mathematics is devoted to learning theory and practice. It is individual centered and emphasizes that useful motives in mathematics education are internal (subjective) in the nature of purposes, interests, attitudes, needs. (40, p. 44)

"...incentives in the form of marks or prizes, and the type of competition which tend to be associated with them, are less favorably regarded today than they were formerly. This is true because educators have become aware of the unfavorable effects of these incentives upon many students." (40, p. 62)

There is a section on individual differences (Chapter 9) which notes differences in ability as well as attitudes and interests. The implication of the yearbook for objectives of mathematics education is that although adults may have certain aims in mind, the effective goals are those of the student, and every student is different.
Opinions of Authorities Concerning Objectives of Mathematics Education

What constitutes an authority or expert in the field of mathematics education? He might be one with whom you agree; he might be one who speaks or writes prolifically; he might be one with the most academic degrees, or the widest or longest experience. For the purpose of this paper, an authority is a person who has had a book on mathematics education printed by a reputable publishing house since World War II. This is a narrow definition, but it is probably sounder than a less discriminating one.

It should be recognized that vested interests enter into the picture. Most authorities are also authors of mathematics texts or text series for use in public schools. Their publication on mathematics education are usually intended for sale to teachers or teacher trainees. Whether these commercial facets of the situation tend to influence an author or publishing house to be more conservative is a question for the reader to decide.

A second kind of authority is also considered in this section of our study. He is the so-called curriculum expert. For our purposes, a curriculum expert is one who has had a book on public school curricula printed by a reputable publishing house since World War II. The opinions of experts on the mathematics education phases of curricula were sought.
It is a common practice among mathematics education authorities to discuss two aspects of mathematics education: the mathematical phase, and the social phase. The former is sometimes called 'pure' mathematics and the latter 'applied' mathematics. Writers also usually think in terms of elementary school mathematics and secondary school mathematics.

Most of the authorities who write concerning elementary school mathematics education assume that a basic objective is to develop accuracy and speed in the four fundamental processes of addition, subtraction, multiplication, and division with whole numbers, fractions, and decimals. When such skills have been developed, they are to be maintained by drill under various euphonious terms. Almost invariably, elementary school mathematics education is referred to as arithmetic, which has come to be synonymous with computation with real positive numbers.

There is great stress on the importance of teaching for understanding of the structure of our current decimal notation system in elementary schools.

All authorities agree that children must understand the meaning of the terms, numbers, and processes used in their mathematics. Thus, 'meaning' in mathematics is now a universal object of instruction. The word 'meaning' is not defined and sometimes it is used in the plural. It seems to be the consensus that maturation, extensive significant experiences, and time are essential to the development of meaning.
All authorities reviewed recommend that individual differences be recognized and provided for in the elementary mathematics education program, but none challenges the current conventional grade-placement of the various phases of arithmetic. The outcome of elementary school mathematics education programs are sometimes deplored, but the bad results are attributed to poor teaching rather than to faulty objectives, improper grade-placement, or course content. An implication is that mathematics authorities believe there are certain effective teaching methods; and if these are properly used, the bad outcomes of elementary school mathematics education will be minimized.

Not all writers on elementary school mathematics education specifically state their views on objectives, but their opinions may usually be discerned from a careful reading of their publications.

Bruechner and Grossnickle (6) have listed in some detail what they construe to be objectives of a modern arithmetic program. These are recorded here as being representative of most authorities with reference to elementary school mathematics education.

"The two primary objectives of the modern arithmetic program are: (1) to develop in the learner the ability to perform the various number operations skillfully and with understanding, and (2) to provide a rich variety of experiences which will assure the ability of the pupil to apply quantitative procedures effectively in social situations in life outside the school."
1. Outcomes related to the mathematical phase of arithmetic.

   a. An understanding of the structure of the decimal number system and an appreciation of its simplicity and efficiency.

   b. The ability to perform computations connected with social situations with reasonable speed and accuracy, both mentally and with mechanical computing devices.

   c. The ability to make dependable estimates and close approximations.

   d. Resourcefulness and ingenuity in perceiving and dealing with quantitative aspects of situations.

   e. Understanding of the technical vocabulary used to express quantitative ideas and relations.

   f. Ability to use and to devise formulas, rules of procedure, and methods of bringing out relations.

   g. Ability to represent designs and spatial relations by drawings.

   h. The ability to arrange numerical data systematically and to interpret information which is presented in graphic or tabular form.

2. Outcomes related to the social phase of arithmetic.

   a. Understanding of the process of measurement and skill in the use of instruments of precision.

   b. Knowledge about the development and social significance of such institutions as money, taxation, banking, standard time, and measurement.

   c. Knowledge of the kinds and sources of information essential for intelligent buying and selling and for general economic competence.

   d. Understanding of the quantitative vocabulary encountered in reading, in business affairs, and in social relations.
e. Appreciation of the contributions number has made to the development of social cooperation and to science.

f. Ability and disposition to secure and utilize reliable information in dealing with emerging personal and community problems.

g. Ability to rationalize and analyze experience by utilization of quantitative procedures." (6, pp. 2-3)

"The specific abilities that are involved in the study and solution of significant social problems which are particularly important as far as arithmetic is concerned may be listed as follows:

"1. The ability to sense problems and to formulate them clearly and specifically.

"2. The ability to formulate methods of arriving at the solutions of these problems.

"3. The ability to sense and to identify by suitable means the kinds of quantitative information inherent in a situation which can then be extracted so as to make it more meaningful and comprehensible.

"4. The ability to locate, gather, organize, and present essential pertinent information, both social and quantitative in nature.

"5. The ability to perceive relations between quantitative elements in situations and to present or express them by an appropriate terminology.

"6. The ability to arrive at correct conclusions and also to demonstrate their correctness.

"7. The ability and disposition to work cooperatively with others in group activities of various kinds.

"To make certain that these broad social objectives are achieved, the teacher should provide a wide variety of systematic life-like experiences in which number functions directly. The more closely the learning of arithmetic is integrated with its uses in the affairs of daily life, the more productive the experience will be. To be most beneficial, these activities should be conducted in
such a way that the children participate in genuinely democratic enterprises." (6, p. 5)

Experts on secondary school mathematics seem to agree that high schools should operate on at least a double track plan:

(a) The traditional sequence of pure mathematics; algebra, geometry, trigonometry, etc., and

(b) Non-traditional mathematics variously called applied mathematics, consumer mathematics, etc.

Sometimes, it is suggested that a third track of vocational mathematics be provided; bookkeeping, accounting, etc. What track a particular student should follow should be determined through the guidance program of the school. The general recommendation is that those students who have the ability should be guided into taking the traditional high school mathematics sequence.

Secondary education mathematics authorities assume that high school students have achieved reasonable proficiency in computation in their elementary school training and that the high school should build on this. However, it is generally conceded that there is need for arithmetic refresher and maintenance programs in conjunction with secondary mathematics.

Authorities subscribe to a wide range of stated and implied objectives of secondary mathematics education including cultural, psychological, and utilitarian values. Davis (13) lists the objectives of teaching (secondary) mathematics as follows:
"I. Abilities

1. To express thoughts clearly and accurately.

2. To systematically organize and interpret data.

3. To reach correct conclusions by accurate and logical reasoning.

4. To analyze a problem discovering fundamental relationships.

5. To perform original thinking and investigation.

6. To exercise intuitive powers and common sense.

7. To accurately generalize special concepts.

"II. Appreciations

1. Of the contributions of mathematics to physical and natural sciences, engineering, philosophy, and other fields.

2. Of the influence of mathematics upon human progress and our modern civilization.

3. Of the vocational value of mathematics in modern business and industrial activities.

4. Of mathematics as a mode of thought which serves as a model for scientific thinking in other fields.

5. Of the rigor and power of mathematical processes and of the accuracy and precision of the results obtained.

6. Of the cultural values of mathematics.

7. Of mathematics for leisure time activity.

"III. Attitudes

1. To form the habit of systematically and logically pursuing a task to completion.
2. To cultivate proper habits of study and power of concentration.

3. To train the mind in scientific thinking and in reasoning logically toward conclusions.

4. To attain the power of clear and accurate expression.

5. To seek the ability to do independent and original thinking.

6. To seek knowledge with an open mind for the sake of its possible usefulness.

7. To build self-confidence and reserve powers which constitute a strong personality." (13, p. 3)

Kinney and Purdy (30) state briefly their objectives of secondary mathematics education as to train leaders and to develop citizenship competence.

Dr. Reeve who was for many years the editor of "The Mathematics Teacher," the journal of the National Council of Teachers of Mathematics, in a summary of his views on secondary school mathematics states that junior and senior high school mathematics education must be completely reorganized along the lines of a general mathematics program and recommends that instruction be made vital and significant to youth (53).

Curriculum experts seem to believe that public education efforts should be individualized. Programs should be based on the unique needs, purposes, goals, and growth patterns of each individual child. This involves comprehensive guidance set-ups within the schools.
In the modern curriculum, the aim is for the children to have purposeful experiences and to work on felt problems arising from their environment and within their ken. The sequence of learning experiences cannot be fixed ahead of time. The formal study of logically organized subject matter is postponed until late in the high school years.

Students have an important part in planning activities and delineating problems to be solved. A democratic climate must prevail in classrooms. The teacher is a guide rather than a boss. Texts and books are source materials rather than syllabi for assignments.

Curriculum experts do not list objectives of mathematics education as such. Arithmetic skills are learned as the need and desire arises in the individual's life. Facilities for learning arithmetic skills are available when the need or desire is felt. Present rather than future mathematical needs are concentrated on especially in the younger years on the assumption that mature learnings will take place with maturity.

The curriculum experts seem to premise that children will seek to know their environment and will enjoy quantitative as well as qualitative explorations within the limits of their understanding. They recommend that mathematics be tied in closely with youthful scientific investigations. They recommend that the quantitative aspects of community, social and economic life be stressed.
Where mention is made of arithmetic as a subject, the recommendation is that it be upgraded.

Curriculum experts invariably stress some form of field learning theory and are usually concerned about the impact of educational methods on the emotional development of children.

Representative quotations which tend to support the above statements are as follows:

Jersild:

"Much of the failure of courses to take hold and add something significant to the intellectual life of the student stems from failure to make a developmental approach to the learner. A logical presentation of subject matter that is the fruit of generations of scholarship will often be quite out of step with the psychological processes of learning." (27, p. 230)

". . . happenings which produce fear, lack of confidence in self, or distrust in others may bar the child from plunging into activities which he could handle quite ably if he were not emotionally blocked." (27, p. 76)

"Many findings based on studies of children in formal school situations as well as in 'progressive' schools indicate that schools have either (a) failed to discover and apply effectively teaching techniques or (b) tended to push children too soon into subject matter and ideas relating to adult political, economic, and social affairs." (27, p. 106)

". . . assuming that it is important for children sooner or later to master arithmetic. . . , to what extent is the superiority of the conventionally taught child likely to be temporary in nature? Given added maturity and experience, will the child who has come through the newer program eventually catch up. The evidence on this point is not complete or conclusive, but such indications as we have point to an affirmative answer." (27, p. 160)
Stratemeyer, Forkner, and McKim:

"We are committed to a belief in the worth and dignity of the individual." (74, p. 444)

"Each learner is unique; the learner reacts as a whole; the normal child has both capacity and appetite for learning; the learner learns those things which have meaning for him." (74, p. 56)

"Meeting the criterion that subject study must be meaningful will mean that for most students systematic study of subjects will come much later in the school curriculum than is now generally the practice." (74, p. 362)

Kinney:

"Those planning the mathematics program must be prepared to deal with the individual pupil, must understand his capacities and deficiencies, and be equipped to use the procedures necessary for discovering capacities and deficiencies in the pupil." (29, p. 497)

", . . . there is . . . a reaction against crowding the entire curriculum in arithmetic into the grades." (29, p. 500)

Objectives of Mathematics Education in State Courses of Study.

Bulletins referring to mathematics education issued by State departments of education are variously called courses of study, guides, handbooks, curriculum committee reports, standards, etc. Some of them are well printed on fine paper with many illustrations; others are simple mimeographed outlines. Some are well edited; others show many typographical errors. Some states publish syllabi or guides covering all phases of the curriculum. Others publish bulletins devoted to specific areas such as social science,
mathematics, etc. A few states, notably in the south, issue printed material developing the curriculum on a twelve-year basis. Most states differentiate between elementary and high school. Some have developed courses of study on a 6-3-3 basis. Several guides are for a six-year secondary school from grades VII to XII, inclusive. There is no consistent pattern, and there is evidence of change within states over short periods of time. The trend seems to be toward a twelve-year program on a 6-6 plan. Another trend is toward issuing guide types of publications.

Official publications concerning mathematics education in the post-war period were secured from 34 states and from the District of Columbia. Those states from which no publication concerning mathematics was procured were: Connecticut, Georgia (making transition from 11 years to 12 years of public education; no State bulletin available.), Idaho, Kentucky, Maryland, Michigan, Montana, New Hampshire, Rhode Island, South Carolina (No course of study issued; local schools prepare their own guides.), Tennessee (No material available at the time.), Vermont, Washington (No guides printed.), and West Virginia (No longer have courses of study.).

Courses of study relating to mathematics education from many cities, counties, and Territories were perused, but the findings are not included in this digest. In general, they are quite similar to State bulletins except that they give more attention to classroom procedures.
Bulletins relating to mathematics education issued by states may roughly be classified into three types depending on the extent of detailed directions concerning the scope and sequence of the curriculum. The terms used here for classifying them are: (1) course of study, (2) study guide, (3) handbook. The course of study is a detailed syllabus; the study guide is a general outline of training; and the teachers' handbook gives practical suggestions on class organization, teaching methods, evaluation of instruction, etc. These terms are used in the bulletin titles and in most instances, the title and the classification as defined here agree. For convenience, the term course of study is used in a generic sense, here-in, to imply all types of publications.

Because of the great diversity in the nature of the publications they did not lend themselves to any sort of statistical treatment without perverting basic principles of mathematical statistics. Instead, a summary was made of the pertinent matter in each publication. These summaries were carefully analyzed for points of agreement concerning objectives of mathematics education; points which were stated in some, implied in others, and where it was not unreasonable to infer that there would be agreement if the issue could be raised; minority or unique points were also noted. From these data a digest of State courses of study as they relate to objectives of mathematics education was drafted.
Those State courses of study which considered general objectives of education almost invariably referred either to "The Purposes of Education in American Democracy" (15) or to the seven cardinal principles (45) or both as acceptable statements of purpose. In general, the point of view taken was that the schools exist for the benefit of both the individual and society; what is good for one is good for the other in a democratic social order.

It seemed to be the consensus of the courses of study that the primary objective of mathematics education is to teach 'functional' mathematics to the children in the public schools. By functional mathematics in this instance is meant those skills, concepts, knowledges, and facts which an individual must possess or know in order to get along in our complex society. The check list of functional competences in mathematics drafted by the National Council of Teachers of Mathematics in 1947 (39, pp. 4-5) was frequently referred to as indicating the content of functional mathematics. In those instances where a state drew up its own list of functional competences, the content did not differ materially from the NCTM check list.

It was a common practice for State courses of study to point out two aspects of mathematics: social mathematics and pure mathematics. Other names were used but the dichotomy was the same. Social mathematics implied functional or practical mathematics. Pure mathematics referred to mathematics as a logical system, or
classic mathematics, or the science of mathematics. It was generally agreed by the courses of study that along with training in functional (social) mathematics, children should also receive instruction in traditional (pure) mathematics of arithmetic, algebra, geometry, etc.

Whether it should classify under social mathematics or pure mathematics, the courses of study were almost unanimous in recommending that children should develop skill in the fundamental processes: addition, subtraction, multiplication, and division of integers, common fractions, and decimal fractions.

Although varying degrees of importance were ascribed to the recommendation, there was general agreement to the effect that understanding or meaning of a quantitative idea or process is important and that meaning or understanding should precede drill or extensive use by children. Children should not be taught to solve problems by rule or rote without understanding.

Nearly every course of study stressed that consideration should be given to individual differences. In some instances, this referred to differences in mathematical ability while in others, it referred to a gamut of differences in interests, background, needs, etc., as well as ability.

Thus, there were four areas of practically universal agreement among courses of study:
1. Training in functional mathematics is a primary objective of mathematics education.

2. Children should have some training in pure mathematics.

3. Meaning is essential and should precede drill.

4. Consideration should be given to individual differences.

There were some matters concerning which it is reasonable to assume that there was agreement among the courses of study although clearcut implications or specific statements were sometimes lacking.

One of these areas of probable concurrence is that an objective of mathematics education should be to teach mathematics in such a way that children will gain insight into their environment.

In this connection, reference was sometimes made to a statement in the Report of the National Committee on Mathematical Requirements:

"The primary purposes of the teaching of mathematics should be (1) to develop those powers of understanding and of analyzing relations of quantity and space which are necessary to an insight into and control over our environment, and to an appreciation of the progress of civilization in its various aspects, and (2) to develop those habits of thought and of action which will make these powers effective in the life of the individual." (36, p. 10)

Another probably generally accepted objective of mathematics education is that children should learn to enjoy mathematics.

Some courses of study say this in so many words, and others imply that it is important that children get satisfaction and pleasure from their number activities.
It also seemed to be generally agreed that children should have first-hand meaningful experiences involving a phase of mathematics before being given formal instruction in that phase. It was believed that instruction without significant experience by children was somewhat futile.

It was a common statement or implication in the courses of study that teachers would do well to adhere to the content and method (and objectives) of the mathematics textbooks and accompanying teachers' manuals rather than go on their own or deviate radically from the texts.

Thus, there were four matters concerning which there was probable concurrence in the courses of study:

1. Mathematics should help children become acquainted with their environment.

2. Children should enjoy mathematics.

3. Vital experiences should precede instruction.

4. Teachers should follow (not slavishly) adopted texts and manuals.

Two ideas concerning objectives of mathematics education were expressed or inherent in several courses of study and which, if taken seriously, would tend to revolutionize mathematics instruction. Both ideas imply a profound regard for individual differences and respect for the individual.
1. Mathematics education should be adapted so that it meets the needs of each child as an individual.

2. In the mathematics education program, attention should constantly be given by the teacher to the individual's feelings and attitudes toward the self, mathematics, the school, and society generated by the instructional situation. Many learnings besides mathematics are developing in a mathematics class. In trying to teach each whole child mathematics, a teacher should try to assure more good than harm.

Most courses of study indulged in splendid sounding objectives of mathematics education without defining the terms. This writer likes the statements but hesitates to try to interpret the psychology or mathematics involved. Following are some stated objectives of mathematics education which fit in this category:

To develop critical thinking.
To develop quantitative thinking.
To develop habits of order, neatness, precision, logical analysis.
To develop practical ability to solve life's problems.
To develop character.

There were some other objectives of mathematics education listed by a minority of the courses of study:

To develop an appreciation of number as a part of the social heritage.
To train specialists.
To prepare students for college.
To give vocational training.

The only place noted where there seemed to be a possible disagreement among the courses of study was in connection with testing. Some states favored extensive standardized testing plans while others mildly deplored the use of standardized tests and competitive grading, suggesting that many forms of evaluation were possible, just as valid, and fraught with less chance of psychological harm.

The variety of ideas concerning the nature of the human individual seemed to be inherent in the courses of study. Some of the proposals for mathematics education as stated in the courses of study seemed to be based on faculty psychology, that there are certain attributes of the human mind and personality which can be trained in and through mathematics.

Some proposals reflected behaviorism as the psychological base for education particularly in regard to automatic responses in the fundamental processes.

Other proposals seemed to be based on field theories of human behavior. In nearly every course of study could be found indications of all three types of psychology.

The rather common practice of printing grade placement charts in courses of study possibly indicates the view that mathematics
education can be standardized although many of the states said
that the grade placement charts were not to be followed too
literally.

There seemed to be agreement that the fundamental processes
should be mastered by the end of grade VI and that thereafter only
maintenance and remedial instruction should be necessary.

The frequent mention of fundamental processes implies that
there is one standard way to perform each process. It was indi-
cated that accuracy and speed in performing computations in accord-
ance with these standardized procedures were desirable.

Investigation of Mathematics Texts

The earliest arithmetic text in the English language was
The Groundes of Artes written by Robert Records, in 1540, over 400
years ago. The early texts on this side of the Atlantic were al-
most entirely of English origin. Arithmetic texts in use in
America 150 years ago were written for adults rather than for chil-
dren and were in the nature of handbooks for business men and ap-
prentices. They told how to solve problems of business.

Arithmetic became a part of the public school program in
America in the early part of the nineteenth century. The first
arithmetic text written for children was published in 1821. It
was Warren Colburg's Intellectual Arithmetic and set the vogue for
public school arithmetic down to the present day.
The earlier arithmetics were one volume handbook book affairs, but in the later part of the nineteenth century most American arithmetic texts came in two and three volume series. For the past twenty years, nearly all arithmetic texts for use in the public schools have been published in a series with one volume for each grade beginning with grade three. Most of these series have considerable supplementary material such as teachers' manuals or guides, children's workbooks, visual aids, and manipulative materials.

There have been changes in the format of arithmetic texts during the past 150 years (65, 66, 67, 68, 69), but there have been very few important changes in the processes and principles included in the instructional program developed in the texts. The main shift has been from a textbook being primarily a book of directions on how to solve problems to being a source book of practice problems to be solved. The content has usually been determined by what was considered practical. The underlying objective of arithmetic texts has been (and is) to teach children how to apply standard computation procedures to solve quantitative problems that arise in the business and social worlds.

Seven current arithmetic text series and their supplementary materials were investigated in an effort to discover stated and implied objectives of mathematics education.

Advertising matter and statements in teachers' manuals or
guides for arithmetic text series usually indicate that the aim of teaching arithmetic is to help children learn to do quantitative thinking, to understand the meaning of numbers and of number relationships, and to teach those mathematical skills, knowledges, and concepts necessary for children and adults in the modern world.

An analysis of the contents of the text series indicates that the principal objective of arithmetic instruction is to teach children computational skills. The grade placement of subject matter is fairly standardized, including:

- Grades 1 and 2: learning how to count, write numbers, and developing a quantitative vocabulary.
- Grades 3 and 4: addition and subtraction of whole numbers.
- Grades 4 and 5: multiplication and division of whole numbers.
- Grades 5 and 6: the four fundamental processes with common fractions.
- Grades 6 and 7: percentage, ratio, and proportion.
- Grades 7 and 8: business practices, including bank accounts, installment buying, investments, insurance, profit and loss, taxes, etc.
- Grades 1 to 8: measurements.

Most elementary school arithmetic texts are organized on a 'spiral' system and include plans for skill maintenance. By grade eight, it is expected that the four fundamental processes of addition, subtraction, multiplication, and division of whole numbers,
common and decimal fractions will be mastered by all students. A tacit assumption seems to be that skill in computation develops quantitative thinking and number meanings.

A smattering of signed numbers, roots and powers, algebra and geometry is sometimes included in elementary school mathematics texts syllabi.

Provision for individual differences is supposed to be taken care of by varying the assignments. Incorporated in most text series are achievement tests to discover how well the children are mastering the textbook content.

The almost universal practice of using arithmetic textbooks as sources of problems seems to connote the acceptance on the part of educators of a theory of mathematics education that not nearly enough effective mathematics learning situations come out of children's experiences, and that mathematics is best learned by children by the textbook method.

Arithmetic text writing and publishing is a very big business. A shift back to a one or two volume mathematics handbook idea, or toward an experience curriculum would tend to curtail the business. After examining many arithmetic texts and text series, the writer came to feel that these comprise a limited source of data on valid objectives of mathematics education, because it is more profitable to text writers and publishers to continue doing what they are doing.
Similar remarks may be made about high school mathematics texts. However, high school mathematics courses are usually in some degree elective, and the texts are written in specific areas such as algebra, or consumer mathematics. Such texts indicate what the author believes should be included in a specific course.

The vast expansion in the adoption and use of consumer mathematics texts for high schools in the past two decades is indicative that such practical mathematics is considered to be an appropriate objectives of secondary school mathematics education.

Standardized Tests and Objectives of Mathematics Education

Cursory analysis of several standardized tests, literature concerning them, and considerable thinking about them revealed that they are a poor source of data and information concerning valid objectives or guiding principles of mathematics education.

Typical tests are achievement tests which purport to measure the relative standing of a testee in his ability to solve impersonal mathematical problems similar to those listed in mathematics textbooks compared with an extensive population. The tests may also be used as diagnostic devices to discover areas of weakness or strength in conventional mathematics.

The implication for objectives of mathematics education is that one should aim at achieving well in the kind of mathematics measured by the tests. This can most profitably be done by
strengthening the weak areas. A premium is put on conformity, and speed and accuracy of computation.

The standardized administration procedure, proctoring, and statistical treatment of the tests emphasize an element of adult enforced competition in mathematics education with an implicit objective of beating the other fellow.

Standardized tests have value and will continue to be used, but they have limitations in their value as a source of ideas for objectives of mathematics education.

Two quotations will indicate the limited value of standardized tests as a source of ideas for developing objectives of mathematics education.

Smith:

"Probably the most disappointing of all the efforts of the psychologists is seen in their unintentional efforts to standardize parts of arithmetic that are tending to disappear, thus leading the schools to retain them. The psychologist seems to devote his efforts to showing only the best methods of teaching what is, not in finding out what ought to be . . ."

"There is no agency that so tends to the maintenance of the traditional, and the most useless part of the traditional, as some of the modern psychological tests."

(64, p. 31)

Monroe:

"The outcome of learning activity are now generally conceived of as controls of conduct, i.e., as dispositions, tendencies, and abilities to do, not more acquisitions of subject matter. . . Some authors, however, by their emphasis upon the use of objective tests and in
other ways suggest the subject-matter concept of the goal of instruction.

**Objectives of Mathematics Education in the Classroom**

It was originally planned to make a somewhat objective survey of several schools to discover their aims of mathematics education in actual practice. Tentative questionnaires were devised for use with administrators, teachers, and high school students. A little experimentation with them resulted in discovering that administrators usually referred to printed courses of study or syllabi; and teachers and students verbalized various orthodoxies. Replies indicated theory rather than practice. It was thought that such a survey would make little or no contribution to the investigation and would fail to bring out the aims of mathematics education manifested on the firing line. So another approach was made.

Eleven school districts in western Oregon were visited and re-visited. Formal and informal, individual and group, interviews with administrators and teachers were held. Classes were observed. The matter of mathematics education was discussed in the interviews, but the questions, "What are your objectives of mathematics education?" or, "What are you trying to accomplish?" were not asked directly. Instead, a discussion was provoked on what was the mathematics program in the school and in the classroom. An effort was then made by the interviewer to guide the discussion toward the
problems, successes, and failures encountered in the program. It was thought that the nature of what was considered by teachers to be their problems, successes, and failures in mathematics education would reveal what were the felt objectives of mathematics education in the school or classroom.

Subjective analysis, interpretation and evaluation of the interviews and observations give the following findings:

In the schools and classrooms of this limited survey, the scope and sequence of the elementary school mathematics program was practically entirely determined by the adopted text. A principal objective of teachers (and students) was to 'cover' the prescribed text.

Wilson made a similar discovery about textbooks determining the curriculum for mathematics in his investigation of trends in mathematics education. He says:

"It is evident that many (school) systems were relying on textbooks to set the pace. In most cases, the outlines of material (for mathematics in courses of study) were so sketchy that they presented nothing more than an over-all picture of the kinds of topics to be taught. Additional evidence of the importance of textbooks are certain facts already noted, namely that nearly forty state departments had no curriculum materials in mathematics published in the last ten years, that nearly one-half of the cities contacted for courses did not reply, that of those replying, many regretted that they had no available material." (90, p. 389)

A major concern of intermediate and upper grade teachers was how to teach so that their pupils would make a good showing on
standardized tests. Hence, another principal objective of classroom teachers was to have students make good scores in achievement testing programs. Some teachers 'taught for the tests.' Many teachers were sure that 'other schools' did, also.

A concern of teachers was what the 'next teacher' would think of them when incompetent pupils were passed on under the no-failure policy. Many teachers tended to consider it a reflection on them when they 'promoted' a mathematically illiterate student. Thus, an objective of mathematics education in the classroom seems to be producing students in whom one may take pride and minimizing poor products.

It amazed the writer to discover the extent to which elementary school teachers' feelings of self respect were tied up with their mathematics education efforts, and the extent to which they felt obligated to push their students through the texts and tests with little regard for other considerations.

Very few elementary teachers had much mathematical comprehension of the mathematics they were trying to teach. (This was indicated by the type of questions they asked the interviewer and is confirmed by two studies to be mentioned later (22) (54).) Teachers devoted most of their efforts to getting across rote computational skills. When help on problems was offered, a common query was, "How do you teach fractions?" or, "How do you teach long division?" The plea was for sure-fire methods of instruction which would guarantee
that children could solve the problems set before them by text or test with a minimum of errors.

No elementary teacher voluntarily made reference to having as objectives of mathematics education the development of quantitative thinking, or reasoning, or personal problem solving, or the meeting of the needs of the individual children. This does not mean that such were not objectives.

A harsh, unfair, but not completely wrong statement to make about objectives of mathematics education in the limited survey of schools investigated is that teachers were trying to drive home rote computational skills in order to save face of the teachers and the school.

Administrators and teachers seemed to be somewhat on the defensive with regard to mathematics education. In two of the eleven districts visited, the schools were under minor attack by community factions partly because they weren't teaching the fundamentals, and because high school graduates 'couldn't even add and subtract.' In some instances, high school teachers complained about the low aptitudes of children from the elementary schools.

Competence in rote computation is perhaps the easiest academic skill to measure. The measurements are calibrated in terms of grade standards or norms. It can readily be determined that many eighth graders and high school students have lower than fifth grade competence. People can and do use such data to attack teachers and
schools. Such attacks are not easy to combat and put educators on the defensive. Subtle pressures develop. That these pressures bear on administrators, and through them on teachers (and students), was sensed in the school visitations. Reactions to the pressures vary from individual to individual.

Among high school teachers generally there was less concern about mathematics. However, teachers who were directly involved in mathematics education had their trials, tribulations, and successes.

In Oregon, the State standards require that all high school students earn one credit in mathematics to graduate. The prevailing practice is to require this one year in the ninth grade. Among the school districts visited, the policy was for all ninth graders to take either general mathematics or algebra. Tests were given to eighth graders or entering ninth graders to prognosticate their algebra aptitude. From a fourth to a half of the students entered in algebra classes on the basis of the tests, and the rest were steered into general mathematics. The majority of high school students take no more mathematics.

Interest in high school mathematics is quite low. Most students take it because it is required. Some take more mathematics because it fits a schedule neatly. A few study mathematics because they really like it, or want to earn pre-requisites for college. Class enthusiasm, especially in general mathematics and refresher mathematics is not high.
The objective of mathematics education for the majority of those taking high school courses is to get through the courses and receive the credits.

Teachers' objectives of high school mathematics education seem to be to get students through the course as prescribed in the official texts. High school mathematics seemed to be textbook centered.

Some of the schools visited had a plan of giving high school juniors or seniors an arithmetic test involving computation with integers, fractions, and decimals. Those students who failed to make a certain cutting score on the test were required, or strongly urged, to take a refresher course, usually without credit, before graduating. The refresher courses ranged in time from six weeks to a semester. The obvious objective of this plan was to try to assure that no student was graduated from high school who was incompetent in computation.

In conclusion, it may be said that aims of mathematics education which emerged from a narrow investigation of schools are:

1. To 'cover' the prescribed texts.
2. To 'get by' the tests.

There are other objectives and motivations operating among people engaged in mathematics education. Statements of these may be found in courses of study. However, it would seem to be less than honest to disregard the fact that texts and tests provide a
framework for objectives of mathematics education in the classroom.

**Related Literature and Research**

During the course of the investigation, many by-paths in literature, research, and experience were followed to glean data, information, and opinion on matters more or less related to the problem of determining objectives of mathematics education. Brief summaries of some of these findings follow. They do not lend themselves to integrated consideration and are listed in no particular sequential order.

**Committee of Seven**

Findings of research on grade-placement in arithmetic of the Committee of Seven of the Northern Illinois Conference on Supervision has significance for considering objectives for mathematics education (95). The Committee of Seven began their work in 1926 and continued until the war conducting controlled, cooperative experiments in 255 cities and towns in 16 states, involving 1190 teachers and 30,724 children. Phases of the research have been criticized but not negated. The committee sought to find the minimum mental ages at which certain arithmetic concepts and skills may be taught, and the optimum ages for instruction. Typical findings are: Twelve years, seven months is the minimum mental age at which
a person may comprehend the process of long division. Manipulation of denominate numbers involving the more complicated forms of applying the four fundamental processes to them, and also involving changing of denominators cannot be effectively taught and retained before a mental age of 1½ or 1½.

**Manchester Experiment**

In the early 1930's, Superintendent Benezet of the Manchester, N. H., (a city of 75,000) schools directed that systematic instruction in arithmetic be deferred until the sixth grade for selected groups of children. At the beginning, middle, and end of the sixth grade, computation tests were given to children of the experimental and regular groups. The September scores made by experimental children were very low compared to regular children, especially in division. By January, the experimental group had made great advances. The May tests showed the experimental group just about on a par with the regular children. Wilson, who was a close observer of the entire experiment, makes the following comment concerning the Manchester Study:

"These data indicate that the pupils who began on systematic master of processes of arithmetic at the beginning of the sixth grade are practically equal, in the fundamentals, to pupils who began their work in the second grade." (89, p. 144)
The literature is replete with evidence of the inadequate mastery of mathematics by pupils whether it is considered from the 'meaning' point of view, or the 'skill' point of view. Vast numbers of children who go through our public schools do not understand arithmetic. A comprehensive study which may be cited is that of Glennon (22) who concluded that, "...the seventh grade pupils tested have mastered an average of 12.5% of the understandings basic to computational processes taught in grades one through six." (Ninth grade pupils, 18%)

Research also shows that arithmetic teachers understand only about half of what they are supposed to teach. To quote Glennon (22) again (conclusion #15), "The teachers-in-service tested have mastered an average of 55% of the understandings basic to the computational processes taught in grades one through six."

Orleans (51) study bears out the conclusion that the mathematical understandings of arithmetic teachers are inadequate.

In his study concerning the mathematical background and competence of elementary teacher trainees, Philips (55) lists some findings as follows: "The elementary and high school mathematics completed gives little indication of the achievement in meaning, understanding, and mechanical mastery. Lack of achievement in mechanical mastery starts with the topic of fractions and continues with decimal fractions and percent. (It is interesting to note that
there is a greater negative reaction to arithmetic starting in the intermediate grades.) Problem solving achievement involving measurement, fractions, and percent is very low. Achievement in meaning is extremely low."

**Learning Theory**

Psychological research and literature relating to learning theory and personality development seem to be more strongly stressing subjective (as opposed to objective) aspects of human motivation, and organismic (as opposed to connectionist) theories of response. This is not to say that objective analysis and connectionism have been repudiated, but rather that the consideration of the feelings and attitudes and desires of the individual as an organized personality is a profitable pedagogical path to pursue.

More attention is being given to a developmental approach to learning; that readiness and ability to learn and learning itself are functions of individual growth and maturity which in turn are partial functions of time.

There is a decided trend toward recognizing that serving the individual's subjectively felt needs is a legitimate aim of education (33). There also seems to be a trend toward assuming that the basic motivation of human behavior is ego-satisfaction (60) or the maintenance and enhancement of the phenomenological self (71).

There seems to be considerable consensus among educators and psychologists that there are at least three factors in mathematical
learning: 1. meaningful experiences; 2. ability; 3. desire or interest. To the degree that an individual lacks in any of the three, mathematical learning does not take place. Related to the above is the view almost unanimously held by educators that mathematics must have meaning to the learner. Rote learning without understanding is usually considered ineffective if not harmful.

Some experimentation has been done conducting learning experiences under democratic vs. autocratic (authoritarian) classroom climates (31) (12). The results tend to be favorable to a democratic atmosphere (63).

In the field of guidance, there is a tendency toward more non-directive guidance recognizing that in the final analysis the individual must face and solve his own problems and make his own decisions. No one else can do it for him. This policy is in accord with a concept of democracy which recognizes the rights, responsibilities, liberty, and worth of the individual.

School Organizational Trends

The word, 'grade,' as applied in public schools is tending to become a misnomer. In its earlier meaning, it conveyed the impression of a level of ability or achievement. A child passed from one grade to the next as he achieved the standards set for the grades. If he passed prescribed examinations, he graduated. If a child could not make a grade, he was failed and repeated until he could,
or quit school. This operated to keep the academic achievement spread of the children in a given grade or class group to a minimum. It also served to concentrate low IQ children in the elementary schools and high IQ children in the high schools. However, the chronological age range in any one grade could be very large. This situation prevailed at the turn of the century and for some years later.

The picture is different now. A 'no-failure' policy is in operation and the tendency is for children to remain with their age group as they pass through school (25) (62). A 'grade', now, is essentially an age division. In the Oregon courses of study, the term 'grade' is not used. Instead, they talk about six-year-olds, ten-year-olds, etc., instead of first grade or fifth grade (52). In any one grade (classroom group) now, the age range seldom exceeds two years, but the ability and achievement spreads are frequently five years.

Coupled with the tendency to have children pass along with their age group regardless of their ability or achievement is the fact that children are remaining in school longer. Two forces operate to accomplish this: (1) Compulsory education laws and policies require children to stay in school in some states until they are 18 years old. (2) Child labor laws and mores forbid children to work and provide little place in the social order for out-of-school children during the school year. Children are placed into
the schools and required to stay there. Children and parents do not have the freedom to choose whether to go to school or not. Regardless of desires, interests, or ambitions, the child must go to school for many years. He cannot be repeatedly failed or expelled unless he becomes a juvenile delinquent, in which case he is placed in another institution.

A present tendency also is to take boys into the military for two years shortly after they are released from high school.

There seems to be no alternative to this pattern of regimentation of children at present.

Data to support the statement that children are tending to remain longer in schools may be secured from many sources. One such source is in Edwards' and Richey's book where a graph shows the percentage of teen-age children remaining in U. S. schools for different years as follows: (16, p. 634)

<table>
<thead>
<tr>
<th>Year</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1890</td>
<td>6%</td>
</tr>
<tr>
<td>1900</td>
<td>11%</td>
</tr>
<tr>
<td>1910</td>
<td>16%</td>
</tr>
<tr>
<td>1920</td>
<td>32%</td>
</tr>
<tr>
<td>1930</td>
<td>51%</td>
</tr>
<tr>
<td>1940</td>
<td>69%</td>
</tr>
</tbody>
</table>

Arithmetic in Modern Life

Computational practices in the business world have been shifting from the human to the mechanical. At present, the following
conditions prevail:

Most retail stores use cash registers and adding machines for over-the-counter exchanges. They have weighing scales that automatically compute prices of commodities purchased by weight. They have tables and computers for figuring other costs and mark-ups and discounts. Little longhand computation is done.

Gasoline is vended from pumps which automatically register gallons and dollars.

Bank bookkeeping is done with machines. Interest and lapsed time are figured from tables. Automatic change makers are not uncommon.

Lumber measures and costs are figured from tables.

Personal income tax returns involve simple addition and subtraction, and tables, or the most elementary forms of multiplication. Joint returns necessitate dividing by two. Internal revenue personnel will figure the tax for the individual if desired.

Statistical computations are done on machines as are all forms of scientific and engineering calculations. Estimates and approximations are worked on slide rules.

Aviators and meteorologists use computers and nomograms.

Such a list could go on for a long time. The number system lends itself so ideally to mechanical computation that it is not easy to find a business which does not use machines. A similar number system with the base two is suited to electronic computing
machines which have been invented to solve the most involved problems.

It seems that as soon as there develops a situation necessitating fast and accurate computation, someone devises a machine, gadget, table, or nomogram to do the job. There is little demand by business for a human rapid calculator, and there will probably be less.

In the 1930's, when the criterion that social usage should determine arithmetic instruction content was more widely accepted than it is now, there was considerable research to determine the nature and extent of arithmetic actually used in the adult world. The amount was surprisingly low to many educators. Wilson's book (88) contains summaries of many of the investigations made then and since. The details of the findings are not important, but a general conclusion is inescapable that few people engage in extensive computation or involved mathematics.

Most authorities seem to believe that the nearest thing to a universal need as far as mathematics is concerned is for an understanding of our decimal notation system. This is a prerequisite for all intelligent computation, estimating, etc. Understanding the number system makes possible computation by various means (shortcuts!) in addition to the standard algorithms.

The standard algorithms we use for pencil and paper computations were conventionalized about a hundred years ago and were compromises
between shorter and longer procedures. They were presumed to be the most efficient methods to use where much computation had to be done long-handed by folks of moderately superior attainments.

Schools taught these algorithms. Students who could and did learn them were passed. Students who could not, or would not, learn them, failed and dropped out of school.
CHAPTER III

POSTULATION

Preview

This chapter attempts to establish reasonably tenable premises for the solution of the problem of determining objectives of mathematics instruction in public schools. The sources of data and opinion outlined in Chapter II are drawn upon for ideas concerning appropriate educational and philosophical doctrines. History and psychology are scanned for acceptable generalizations pertinent to the problem.

Two assumptions are formulated, examined and postulated to serve as guiding principles for the solution of the problem. One of the guiding principles concerns accepting a social setting for mathematics education; the other provides for adopting a theory of learning.

By definition, a postulate is accepted without proof. However, the postulates forming a set should be reasonable, serviceable, and not incompatible with each other. The line of reasoning leading to the postulation of each guiding principle is indicated.

Point of View

Man seems to be a purposing creature. His schemes for educating his young do not exist in an intellectual vacuum.
Implicit in his programs for training children are his ideas about
the nature of society and the nature of people.

Different cultures have differing goals in their educational
efforts. A little reflection will suggest varying aims and methods
of organized education among a theocracy, an aristocracy, a totali-
tarian state, a welfare state, a caste system, and an atheistic
society. One may contemplate how differing concepts of human nature
would influence the purposes of organized instruction. Suppose the
people who are setting up an educational program believe that
children are born evil, and that education is needed to eradicate
sins or demons. Or suppose that humans were considered to be
collections of mental faculties which are capable of being trained
or sharpened. Or suppose educators assumed that children were
infinitely modifiable and that under proper education anyone could
become a genius, or an artist. Suppose it was believed that indi-
viduals varied in value and talent according to their race, color,
caste, or nationality. What would happen in education if the
prevailing view was that the universe and the people in it were
completely mechanistic; that all events were fore-ordained, and
that human efforts to change man or his world were futile and
absurd. It may be realized how objectives and patterns of education
may vary in accordance with concepts of human nature, epistemology
and philosophy.

It is right and proper that the social and psychological
theories on which an educational plan is founded be clearly
An attempt will be made to suggest the sociological and psychological premises used for solving the problem of this study. The premises will be postulated as guiding principles.

**Guiding Principle Number One**  
**Social and Political Theory**

The story of the development of public education in America is written in histories of education. Briefly, education of children in America was once considered to be the enterprise of their parents. Where feasible, parents occasionally pooled their efforts to train their young. Sometimes civic-minded people set up community schools, charity or tuition. More than a hundred years ago in Horace Mann's time, a movement for free public elementary education was accelerated. Later, the idea of free public schools expanded to included secondary education. Since the turn of the twentieth century, public high schools have, to a considerable extent, replaced private academies.

During the past century and a half, the relationship between government and education in America has gone through a process of growth, development, and change. Government in the United States is divided into various levels; federal, state, county, township, municipality, district, etc. The use of the word, "level," does not imply a "chain of command" or authority. It may be thought of as indicating a degree of remoteness from the people. Constitutions, charters, and laws define the functions, powers, and
limitations of the various levels, or agencies, of government, with federal and state constitutions embodying the supreme laws. Early constitutions made little, or no, reference to education. The tenth amendment to the United States constitution states:

The powers not delegated to the United States by the constitution, nor prohibited by it to the states, are reserved to the states respectively, or to the people. (53, p. 356)

From this it may be inferred that education was construed not to be a function of the federal government but rather that if anything were to be done about education it was up to the people, or to the state governments, to do it.

The early schools were set up under the auspices of the "people". In time, permissive legislation was passed by state legislatures empowering local communities to tax for school support, and later for school building construction or purchase. Gradually, state governments gave more attention to public education.

As the western states came into the union, it was a practice to set aside certain lands, the proceeds from which were to be used for educational purposes.

The Oregon Constitution, drawn up in 1857, said:

Article VIII, Section 3. The legislative assembly shall provide by law for the establishment of a uniform and general system of common schools. (53, p. 340)

Oregon's constitution also made provision for a Common "irreducible" School Fund to be derived from the sale or disposition of school
lands, and other sources. (53, p. 310)

Until comparatively recently, the control and direction of the public schools was almost entirely in the hands of the people of the local community or school district with little or no state or federal domination.

Of late years, the control and direction of the public schools has been shifting to higher, more remote, levels of government. State governments have been providing increasing proportions of school funds. State legislatures have passed increasing numbers of laws affecting the curriculum (concerning health, physical education, alcohol, etc.). State-wide compulsory school attendance laws are including wider age ranges. State departments of education have been expanded and given more and more power to set up and enforce standards regarding school programs, buildings, teachers and teaching. Textbooks to be used in public schools are sometimes controlled at the state level. Some states provide that public school teachers are eligible for state retirement assistance along with other state employees (civil service). Public school teachers usually must receive state certification to teach; must sometimes take special loyalty oaths required by the state; and are sometimes subject to special teacher minimum salary laws. In many states, education is responsible for impressive shares of the state budget. There is some public pressure to move responsibility for public education to the highest, most remote, level by having the federal government step in with financing.
Although local school districts continue to have local fights, many school issues now have larger arenas. National magazines, as well as local newspapers, publish attacks on, and defenses of, the public schools, almost as if the schools were a national enterprise. There are lobbyists for and against the public schools in legislatures and congress. Publishers wage great sales campaigns to win state-wide textbook contracts. Adopted texts are analyzed for possible subversive material. Teachers are banded together in politically-powerful state and national organizations and wage state and national campaigns on behalf of "education".

Through legislation and changing public opinion the control and direction of the public school seems to be shifting from the people to representatives of the people (government officials); or shifting from close-to-home control to more remote control. The change is tending to be from a "pure" democratic to a republican form of school administration.

It is not implied that the shifting control of public education is good or bad, nor that it is completed, nor irreversible, nor inevitable. It is recognized that there is such a shift and it is believed that it should be reckoned with in determining objectives of education.

The problem of this paper is being considered as if the public school were no longer under the primary control of the people of the district, but has, instead, become an agency or function of
state government. It is as if public education, as a residual power referred to in the tenth amendment to the United States Constitution, has shifted from being the people's project to being vested in the state.

The assumption that public education is basically a state function, is only partially valid because various levels of government from federal to sub-district, not to mention the people and the PTA's, have parts to play. However, in order to provide a focus for the problem, it is deemed necessary to designate somewhat arbitrarily who, or what political or social agency, is considered as having the primary responsibility for prescribing content and objectives of public mathematics education. For working toward a solution of the problem, the state, as a political division is so designated. Hence, professional educators who participate in public mathematics education planning or instruction are state functionaries as well as teachers, and are involved with political as well as educational considerations.

An alternative assumption to the concept that the public school is a state agency is that the public school is, or properly should be, controlled by local laymen, and that the state does not or should not, have the authority and should not set up standards of education and prescribe curricula, or objectives. Under this assumption the problem of this thesis would tend to disappear inasmuch as each local group would be empowered to handle the matter as they wished.
Another alternative assumption could be that the public school is, and properly should be, controlled and directed by designated professional educators who are given power and authority to do as they see fit without reference to political considerations. This would still necessitate designating the geographic or political area where they have jurisdiction, but it would relieve the professional educators from having to conform to the bill of rights and other legal or philosophical mandates.

For the purpose of this study, it is postulated that public education, in general, and public mathematics education, in particular, is a state function, and, therefore, should be conducted in accord with American political philosophy and constitutional government.

Although many other phases of American life (milk, labor, sanitation, roads, etc.) have left the home and local community control and started climbing the ladder of levels of government, public education being considered basically a state function as opposed to being a primarily local responsibility is particularly fraught with grave implications. People who scan the schools for subversion may be aware that dictators and demagogues may mold minds and dominate populations through children in public schools which are state-controlled. They may realize that educators and psychologists who are only indirectly answerable to the people may be tempted to use potent means of thought-control to further their own
ends or those of their superiors. An integrated system of education controlled and directed at the state level establishes machinery for effecting the lives and fortunes of larger masses of people for good or ill by competence and incompetence, wisdom and folly, ethics and unscrupulousness.

It seems to be especially incumbent upon public education officials that they take every reasonable precaution to safeguard people and future generations against possibilities of evil which may be caused by themselves or their successors, and which are inherent in a state-controlled and directed public educational program.

Insofar as the public school is a governmental (state) agency, it seems almost axiomatic that it should be conducted in accordance with constitutional guarantees and American political philosophy. When objectives of mathematics education in the public schools are being formulated, serious consideration should be given to the foundations in law and political theory. Proposals and plans should be in harmony with written and unwritten law.

It may have been noted that many of the sources of data and opinion investigated (Chapter II) indicated that the public schools should teach for citizenship, democracy, respect for law. These seem to be ways of saying that our schools should have due regard for American tradition and law. No source indicated that public schools and public mathematics education should be conducted contrary to, or without respect for, or above and beyond American
social and political philosophy.

It is therefore postulated that:

GUIDING PRINCIPLE #1: PUBLIC EDUCATION IN GENERAL AND PUBLIC MATHEMATICS EDUCATION IN PARTICULAR MUST BE IN ACCORD WITH AMERICAN POLITICAL PHILOSOPHY AND CONSTITUTIONAL GUARANTEES.

Having postulated this guiding principle, it immediately becomes necessary to explain what is meant by American political philosophy and constitutional guarantees.

According to American political philosophy, what is government? What is government for?

The Declaration of Independence states that governments are social compacts instituted among men to secure their inalienable rights of life, liberty, and the pursuit of happiness (53, p. 350).

The Constitution of Oregon says:

We declare that all men, when they form a social compact, are equal in right; that all power is inherent in the people, and all free governments are founded on their authority, and instituted for their peace, safety, and happiness; and they have at all times a right to alter, reform, or abolish the government in such manner as they may think proper (53, p. 328).

Lincoln, in his Gettysburg address, spoke of our government of the people, by the people, and for the people. It was conceived in liberty and dedicated to the proposition that all men are created equal.
American literature and songs contain references to freedom and liberty and the fear of tyrants. The framers of the federal constitution were united in the view that powers of government and government officials should be circumscribed and checked in such a way that ambitious men in any area of government could not capture control and infringe upon the inherent liberties of the people (26, pp. 335-341).

The fundamental doctrine of American political philosophy is that governments are social compacts, and that governmental agencies exist to maintain the liberties of the people and to protect them individually and collectively from undue exploitation or domination. Governments are supposed to defend the rights and liberties of individuals and minorities in the face of majorities and power groups according to bills of rights written into American constitutions.

In further support of this doctrine of government are two statements of American philosophy which have legal standing. They are the Pledge to the Flag (79, p. 1077) and the American’s Creed (53, p. 361). The first of these stresses "... liberty and justice for all." The second states:

"I believe in the United States of America as a government of the people, by the people, and for the people; whose just powers are derived from the consent of the governed; a democracy in a republic; ... established upon those principles of freedom, equality, justice, and humanity..."

1 See: the National Anthem and America.
What do constitutions say about the functions of government and governmental agencies?

The preamble to the Constitution of the United States lists the purposes (objectives) of the federal government. They are:

a. To form a more perfect union.
b. To establish justice.
c. To insure domestic tranquility.
d. To provide for common defense.
e. To promote general welfare.
f. To secure the blessings of liberty to ourselves and our posterity (53, p. 352).

The preamble to the constitution of Oregon lists only three broad purposes of the state government:

a. To establish justice.
b. To maintain order.
c. To perpetuate liberty. (53, p. 328)

Although the preamble to Oregon's constitution follows the pattern of that of the federal constitution, it may be significant that the framers of Oregon's constitution did not include anything about promoting or providing general welfare as an objective of Oregon's government.

The purposes, powers, and limitations of American governments are set forth in federal and state constitutions. The constitutions are the supreme laws.
Although the constitution of a state is the supreme law for that state regarding many matters, nevertheless, a state's authority over people is not unlimited. Legislative enactments or governmental practices which run counter to guarantees in the federal or state constitution may be voided by state or federal courts. Customs and practices long accepted by a state as being legal and proper may, when challenged in the light of constitutional guarantees, be modified by courts. An example is the United States Supreme Court's action of May 17, 1954, on segregated schools.

In a concurring opinion on the famous McCollum case (83), Mr. Justice Jackson, while deploiring the tendency for the United States Supreme Court to become the national school board, declared that it is proper for the court to listen to appeals in instances in which constitutional guarantees may be being jeopardized in public education.

There are many instances of the federal supreme court declaring a state statute, or school district regulation, concerning public school matters to be unconstitutional because it violated one or more of the Bill of Rights. A few United States Supreme Court opinions may serve to clarify the powers and limitations of the state for education under American constitutional government.

In 1923, the court held that a state statute which prohibited the teaching of foreign languages in private schools clashed with the fourteenth amendment, and was also an arbitrary interference
with the liberty of parents to control and educate their children (80, p. 403).

In the famous Oregon case of 1925, the court held that children of compulsory school age could not be compelled to go to public schools. A part of the court opinion in this case reads:

"... The fundamental theory of liberty under which all governments in this Union repose excludes any general power of the State to standardize its children by forcing them to accept instruction from public school teachers only. The child is not the mere creature of the State; those who nurture him and direct his destiny have the right, coupled with the high duty, to recognize and prepare him for additional obligations." (81, p. 535)

In 1943, the Supreme Court reversed a ruling made two years earlier and found that children may not be expelled from school for refusing to salute the national flag. The tribunal said:

"The Fourteenth Amendment, as now applied to the States, protects the citizen against the State itself and all of its creatures—Boards of Education are not excepted. They have, of course, important, delicate, and highly discretionary functions, but none that they may not perform within the limits of the Bill of Rights. That they are educating the young for citizenship is reason for scrupulous protection of Constitutional freedoms of the individual, if we are not to strangle the free mind as its source and teach youth to discount important principles of our Government as mere platitudes.

"... we apply the limitations of the Constitution with no fear that freedom to be intellectually and spiritually diverse or even contrary will disintegrate the social organization... Freedom to differ is not limited to things that do not matter much. That would be a mere shadow of freedom. The test of its substance is the right to differ as to things that touch the heart of the existing order." (32, pp. 637-642)
A recent Supreme Court ruling concerning racial segregation in public schools reiterates the theme of liberty under law. The unanimous opinion says:

"Although the Court has not assumed to define 'liberty' with any great precision, that term is not confined to mere freedom from bodily restraint. Liberty under law extends to the full range of conduct which the individual is free to pursue, and it cannot be restricted except for a proper governmental objective. Segregation in public education is not reasonably related to any proper governmental objective, and thus it imposes on Negro children of the District of Columbia a burden that constitutes an arbitrary deprivation of their liberty in violation of the Due Process Clause." (54, p. 499)

There is written into the basic law of our land ideas of freedom of speech, freedom of press, religious liberty, etc., all of which may be construed as guaranteeing the people against organized efforts on the part of government officials or government agencies to standardize the thinking and behavior of the people. The laws not only guarantee the right to be different, but almost seem to be aimed at cultivating differences in thinking and behavior among people. These constitutional guarantees are the basis for academic freedom which educators cherish.

The doctrine of the States' police power is well established in American government. This holds that one of the powers 'reserved to the States' under an interpretation of the Tenth Amendment to the United States Constitution is police power under which a State may pass any law not prohibited by the Constitution and clearly designed to promote the public health, morals, safety, and welfare (75, p. 95).
Summary Concerning Guiding Principle No. 1

The public school is an agency of state government. Governments exist to maintain the liberties of people as guaranteed in federal and state constitutions. Restrictions on individual freedom in the name of public education must be clearly tied to some proper governmental objective such as establishing justice, maintaining order, perpetuating liberty, or promoting public health, morals, safety, and welfare. Since public school officials are under oath to support constitutional government, they are bound to keep these things in mind when they are developing objectives of public mathematics education.

It is the parents who have the primary responsibility of the guidance and nurture of their children.

Guiding Principle No. 2. Learning Theory

It is proper that premises concerning the nature of human individuals and theories concerning human learning be postulated to serve as guides in developing educational objectives.

Organized efforts to instruct children presuppose that instructors have some sort of ideas relative to the essential nature of the instructees, how and why they learn, or may learn, under instruction. By their actions, if not by their words, teachers disclose their conscious and unconscious views and attitudes concerning human
nature. When one is trying to formulate objectives of education, it seems sensible to state what is an acceptable underlying psychological philosophy. If there were universally accepted, positive knowledge about what human beings are and how they learn, it would be unnecessary to discuss the point, but since there is not complete agreement, it is advisable to postulate an educational psychology to serve as a basis for developing educational objectives.

Such an educational psychology may not be just drawn by chance from a hatful of psychologies, so to speak. There ought to be reasoned criteria for accepting this point and rejecting the other one. Three such criteria for adopting tenets about human behavior and learning for use in public education and devising objectives of mathematics education suggest themselves.

Criterion #1. Educational psychology must be compatible with current scientific knowledge.

Psychologists have learned much about human beings, individually and collectively. They have studied persons, groups, and masses of people longitudinally and 'latitudinally.' They have compiled many statistics and made countless case studies; they have set up hypotheses and checked them for validity. In the process, they have discovered and verified that a few things which were once thought so probably are not; and also that some things once thought not so, could be. The investigations of psychologists are published in various professional and lay journals and books.
It would be absurd to postulate a learning theory which runs counter to the findings of scientists, and so we have at least a reasonably valid negative criterion for establishing an educational psychology: that it must not be incompatible with current scientific knowledge.

Current scientific knowledge is fragmentary, perhaps, because psychology is a young science. Scientifically speaking, analysis precedes synthesis. Of necessity psychologists have been more concerned during the early days of their investigations with analyzing the human individual into his component parts; giving the parts suitable labels like neurons, motor areas, drives, habits, abilities, complexes, mechanisms, etc., than they have been with trying to synthesize individual behavior into universal laws or law. When psychologists have defined and labeled phenomena, the scientific procedure is to try to measure them. This scientific approach to exploring the nature of human beings is extremely valuable, but the mass of data produced tends to remain undigested. In the meantime, public education must go on. An educational psychology has to be built on scientific data and reasonable inferences therefrom. Two extremes are to be avoided. The inferences drawn from psychological research should not be too sweeping. On the other hand, educators should not be too timid about taking a stand. This brings us to a second criterion for postulating an educational psychology as a foundation for establishing educational objectives.
Criterion #2. The adopted educational psychology should be reasonably comprehensive and yet specific enough to be serviceable for establishing objectives of mathematics education, but it need not be broad enough to explain any and all phases of human behavior.

It is the contention here that moderately broad generalizations, laws, or rules about human behavior have to be hypothesized to provide a modus operandi for public mathematics education; and that these laws or rules will have to go beyond our present demonstrated scientific knowledge to do us any good. The postulated psychological generalizations may eventually be proved wrong or inadequate, but if they are temporarily serviceable, it is better than to have had no guides or hypotheses at all.

Criterion #3. The educational psychology postulated as a basis for American public mathematics education should be compatible with American political philosophy and law.

This may seem like a strange position to take, because it smacks of science bowing to politics, or of making education follow the party line. However, let the matter be examined further.

Science has not produced a universal 'law' concerning human behavior and learning. In the meantime, real, or would-be, philosophers; earnest, able, and crack-pot, religious enthusiasts; wise and smart educators; occasionally come up with their positive answers concerning the nature of man, and why, what, and how children should (or do) learn. Curriculum makers are not entirely unsusceptible to
believing they have been anointed and have found the truth. It is possible that some religious zealot or psychologist has discovered the universal law of human learning, but if it runs counter to American political philosophy and law, it may not be used as a basis for American public education without going through a legal process.

Catholic authors have written texts on educational psychology intended primarily for use in Catholic colleges, universities, and teacher-education institutions. Such texts incorporate Church doctrine and basic and leading principles of Scholastic Psychology. One such text (28) has a chapter titled, "The Soul," and discusses its spirituality, immortality, and relationship with God. The psychology in this text does not conflict with the findings of modern science, but it has a strong religious slant.

It is possible that this orthodox Catholic psychology is correct, or at least is on the right track, but regardless of its correctness, such metaphysical doctrine may not form a basis for public school educational psychology, because our constitutional law maintains a 'wall of separation' between church and state. Religious dogmata (2, p. 164), including special brands of educational psychology, are outside the province of the public schools.

Separation of church and state is written into the law, but tradition tends also to preclude the promotion, directly or indirectly, of any social, economic, scientific, psychological, or political isms through the public schools.
The conclusion is reached that the educational psychology used in the public schools must be in harmony with American political philosophy.

To summarize: To guide our thinking in developing objectives of public mathematics education, it is desirable to postulate an educational psychology which is usably comprehensive in scope, is in accord with scientific findings, and which is not antagonistic to American political philosophy.

Three phases of educational psychology pertinent to mathematics education will be considered in the light of the criteria for adopting an educational psychology and theory of learning.

a. How may an educator regard man?
b. What motivates man?
c. How does man learn?

Working hypotheses relating to each of these phases will be predicated.

What is man? Is he a composite, collection, or conglomeration of discrete elements such as arms, legs, will-power, wishes, attitudes, faculties, abilities, S-R bonds, mind, body, spirit, etc., or is the human individual an organism which reacts as a whole and whose separate parts are convenient fictions which have no existence except in relation to the complete organism? Of course, this may not be an either-or question, but for the sake of brevity, it will be considered such.
The dichotomy in educational psychology of man being composed of discrete elements versus man being an organic whole may be examined in the light of the criteria to evaluate theories of behavior and learning for the public schools.

a. Both ideas have some scientific support.

b. Either could provide a comprehensive enough foundation for mathematics education, but the discrete-element philosophy could involve almost an infinite number of elements.

c. The concept of man being composed of discrete elements is hardly in keeping with American political philosophy, which seems to assume that a man is a man with rights, liberties, duties, and responsibilities rather than being composed of ego, id, complexes, soul, etc.

The decision is made to reject the 'discrete-element' point of view and to postulate that the human individual is an organism; and that any stimulus, educational or otherwise, impinges on the total being. In other words, a part of a child may not be trained without affecting the rest of the whole.

What constitutes human motivation, particularly motivation to learn? Is it rewards and penalties? Is it coercion? Is it duty, or conscience? Is it love, or patriotism? Is it fear? Is it a finite or infinite list of motivations? Does motivation abide in the individual, waiting to be tapped by himself or by his teacher; can it be produced within the individual by some process? Does
motivation come from within, or from without the individual, or both, or is it an interaction? In other words, what makes a human being tick? The motivation phase of educational psychology may be considered in the light of the criteria for selecting an educational psychology.

Scientific investigations find all kinds of things and circumstances which seem to motivate some individuals to learn, but investigations constantly run into intangibles expressed in reports as attitudes, interests, feelings, meanings, rapport, etc., which foul up clear-cut unqualified conclusions.

Attributing all human motivation to any specific source which scientific investigation has proven fruitful, such as rewards and penalties, immediately appears to be incomplete. Adding all the fruitful sources together produces a conglomeration. Repeating the definition that motivation is that which motivates is not useful.

In postulating an educational psychology, we must make some sort of generalisation concerning motivation which has predictive value for why children behave and learn. For drawing the inference, the idea of man being an organism may be used as a guide.

At birth, the neonate man (an organism) seems to be endowed with a few difficult-to-define biological urges or drives. As he grows and develops, man's primitive urges are modified. There seems to be no standard pattern of motivating forces which all men acquire
or develop in the process of growth and maturation.

It is possible to predict with fair accuracy what an infra-human of a certain species and age will do in a given situation. It is much more difficult to predict what a member of the human species will do under prescribed conditions. Infra-human patterns of behavior seem to be more stereotypical. Man's patterns of behavior take all kinds of forms. The manifold yems that men manifest may be cataloged in countless categories; death-wishes, sex-drives, desire to excel, desire for power, wish for security. But such terms represent generalizations about behavior rather than specific forces or entities.

Although the individual's primitive urges seem to undergo change as he develops, it does not mean that tendencies and tensions to act cease to exist. As long as man lives and breathes, he seems constantly to be seeking or purposing. He is not satisfied, or sated, for any length of time. The objectives of his seeking change from time to time, and from moment to moment? Now it might be food, now rest, now adventure, now a thrill, now a mate to love, now a body to seduce.

Is there any generalization that can be made which will tend to synthesize the conglomeration of scientific observations on human behavior and human motivation?

Snygg and Combs attempt to unify human behavior in their book, INDIVIDUAL BEHAVIOR. Three short quotations from it will indicate the train of their thought:
"To maintain their organizations is the dominant characteristic of all living things; indeed, it appears true of non-living things and of the universe itself." (71, p. 54)

"What the (human) individual is seeking to preserve is not his physical self but the self of which he is aware, his phenomenal self." (71, p. 56)

"The basic human need... (is) the preservation and enhancement of the phenomenal self." (71, p. 58)

A generalization about human motivation may be made by the statement that "the individual is trying to preserve and/or enhance his self-concept." Does this conclusion meet the criteria for inclusion into an adopted educational psychology?

(a) Is it unscientific? No. It goes beyond the present findings of science, but it is not contrary to scientific knowledge.

(b) Does it have practical value for a working hypothesis for public mathematics education? It could have!

(c) Would the adoption of it as a phase of educational psychology for public mathematics education run counter to American political philosophy? No. On the contrary, American political philosophy seems to guarantee the individual the right and liberty to preserve and enhance his self-concept, subject to others having the same right and liberty.

The criteria for adopting an educational psychology will permit the assumption that man (an organism) is motivated to preserve and/or enhance his self-concept. This assumption is made and comprises an element in the contemplated educational psychology.
If we assume that the motivation of individuals lies in their trying to maintain and enhance their phenomenal selves, or self concepts, the question arises, "What is a self-concept?"

The self-concept is what a person conceives himself to be: butcher, baker, thief; martyr; man-about-town, mendicant; leader, follower, reject; brilliant, dumb, bum. The self-concept includes that with which the individual identified himself. He will work, fight, bleed, and die for his self, which may include his alma mater, his country, his fuhrer; his family, his ancestors, his descendants; his party, his religious sect, his craft; his gang, his neighborhood, his code. Self-concept-preservation is the first law of motivation!

Two properties of the self-concept may be stated as follows:

(1) The self-concept is developed as a result of interaction of the human organism and its environment.

(2) The self-concept is not static, but is dynamic.

Lengthy discussions of the development and nature of the self-concept, or phenomenal self, or self-system, may be found in various psychology books (11) (71).

It was mentioned above that the individual's primitive urges undergo changes as he grows and develops in a dynamic relationship with his environment. It was also noted that the 'self-concept' is developed and changes through an interaction process. The picture is of man being a developing and changing creature. Although
his physical growth is somewhat standardized, his behavior is far from stereotyped. To indicate and emphasize this basic concept of the contemplated educational psychology, the term 'developmental psychology' will be used.

GUIDING PRINCIPLE #2. PUBLIC EDUCATION IN GENERAL AND PUBLIC MATHEMATICS EDUCATION IN PARTICULAR SHOULD BE CONSONANT WITH DEVELOPMENTAL PSYCHOLOGY.

Facts of individual differences, factors in learning-readiness, and facets of human behavior, led inductively toward the hypothesis that the individual personality is a developing system. Certain corollaries regarding individual differences, readiness, and learning, follow from the basic assumption if developmental psychology is postulated as a working hypothesis for devising objectives of public mathematics education.

If it is postulated that each human personality develops through a process of interaction of the organism with its environment, and that motivation of individuals lies in their trying to maintain and/or enhance their self-concepts or self-systems, then it logically follows that each developing personality will be different from every other, because no two individuals can have exactly the same combinations of the infinite numbers of hereditary and environmental factors. No two human beings can have occupied the same life space for the same life time nor can they have had identical heritages. Facts of individual differences have been well-established through psychological research.
Corollary. ALL HUMAN BEINGS DIFFER FROM EACH OTHER.

Instead of his behavior being predetermined (as in infra-human creatures) by well-defined instincts and skills (nest-making and hunting methods), and instead of his motivation being predetermined (as in infra-humans) by specific biological drives (seeking certain foods and mates at certain times), man develops his individual conceptual patterns and skills and develops his individual motivational patterns. He develops ideas and attitudes. Thus, it may be said that postulating developmental psychology implies at least a bilaterality in the self-concept; a cognitive aspect and a motivating aspect.

Corollary. EVERY HUMAN BEING MAY BE CONSIDERED AS POSSESSING BOTH COGNITIVE AND MOTIVATING SYSTEMS.

Implicit in the concept of personality growth and development (developmental psychology) is the idea of the individual person being ready or unready to grow, or expand, or progress in some particular direction. Developmental psychology implies learning readiness (or unreadiness) at stages or times in development. A child may not learn or perform before he is physiologically and psychologically ready.

Corollary. A HUMAN BEING LEARNS WHEN READINESS AND OPPORTUNITY COINCIDE. READINESS IS, IN PART, A FUNCTION OF PHYSICAL AND MENTAL GROWTH AND DEVELOPMENT.

Developmental psychology with its corollaries outlined above constitutes an underlying assumption for the solution of the
problem of this paper. Extended discussion of the corollaries follows, with special reference to a learning theory inherent in the assumption.

Corollary #1. All human beings differ from each other.

Educational literature is replete with research on the extent to which individuals differ from one another in manifold ways. Common observation also tells us that humans differ in abilities, talents, interests, needs, temperament, background, opportunities, and social status, as well as in physical characteristics.

Most of the sources consulted for this study agreed that recognition should be given to individual differences in formulating any phase of mathematics education. It is essential that some consideration more than lip service be given to the obvious fact of extensive individual differences among pupils.

Some of the areas relating to mathematics education in which children differ may be noted as follows:

1. Children differ in mathematical aptitudes or potentialities to learn mathematics. Usually mathematical ability is manifested early, but this is not always true.

2. Children differ in experiential background. Mathematical learning is based on experience.

3. Children differ in the time it takes to learn mathematical concepts.

4. Children differ in the time they need or want to learn
mathematical skills or concepts.

(5) Children differ in the type of mathematics they need or that will be of value to them.

(6) Children differ in their attitudes and feelings toward mathematics, the school, teachers, home, and themselves.

(7) Training increases differences between individuals.

The observation in the biblical parable seems to be born out, "For unto everyone that hath shall be given and he shall have abundance; but from him that hath not shall be taken away even that which he hath."2

(8) Not only do children differ among themselves, but no child is exactly the same from one day to the next.

The twin trends of increasing numbers of students staying in school longer coupled with the no-failure policy in vogue in public schools thrusts new importance onto the need for recognizing individual differences. The net result tends to be wider ranges of differences in each classroom.

A generation or two ago it was accepted policy to set standards for continuance in school. Individual differences existed, but if a child could not, or did not, reach the prescribed standards, he was usually retained. Conversely, an exceptionally able child could sometimes jump academic hurdles faster than the prescribed schedule. The tendency was to have the child conform to school

standards. Comparatively little consideration was given to individual differences.

It is the opinion here that this policy is no longer tenable. If the schools are to undertake the responsibility of training all the children of all the people for as long as they want to go to school, it will be necessary for the schools to adjust to the fact of individual differences. Blanket standards of achievement, arbitrary curricula, uniformity, and regimentation go out the window when serious consideration of individual differences comes in.

A tenet for mathematics education, and devising the objectives thereof, is the fact of individual differences must be given right-of-way.

Corollary #2. Every human being may be considered as possessing both cognitive and motivating systems.

Ordinary observation tells us that human individuals develop concepts and skills, memorize facts. Psychological research tells us that individuals have differing abilities or capacities for learning; that they have varying intelligence quotients; and that humans tend to generalize and systematize their knowledge.

Ordinary observation tells us that individuals have feelings and desires, develop attitudes and prejudices, generate likes and hates. Psychologists and psychoanalysts attempt to measure attitudes and to discover the value systems of individuals.

Thus, it appears that the self-system, or self-concept, may be
thought of as having two phases, cognitive and preciative. The cognitive phase has to do with the processes involved in knowing, or that pertain to the intellect. The preciative phase has to do with processes involved in subjective valuing, or the assigning of valences (positive and negative) to elements in the field of the individual.

It is contended that the individual is continuously developing, expanding, and changing his cognitive maps of his world; and that he is just as continuously (and simultaneously) assaying the elements of his cognitive maps on the scales of their contribution to the maintenance and/or enhancement of his self-concept. It is as if he puts colors on all the features of his cognitive maps. Areas with positive valences have bright colors. Other areas have somber or ugly hues. Sometimes the colors run into adjacent areas with various consequences. The brightly colored areas which promise to pay dividends are the areas explored. The ugly or threateningly colored areas are to be avoided.

Thus the individual in interaction with his environment acquires knowledge, systematizes knowledge, and incorporates knowledge into

3 "preciative" is coined from a root of the words 'appreciate' and 'depreciate'; from Latin, pretitare—to prize; from pretium—price. According to unabridged dictionaries, "preciation" is an old term (rare) meaning presumably in the mercantile world, the assigning of prices or values. If merchants do not need or use the word anymore, it was deemed ethical to resurrect it from the scrap heap and embellish its meaning to serve new purposes. The word can have three convenient forms: noun—preciation; verb—preciate; and adjective—preciative; all having to do with the subjective process of assigning of positive and negative values or valences.
his self-system. At the same time, he is developing attitudes and values. The knowledge is good, especially if it is reasonably accurate, but it is the "personal and social values" he has developed with their positive and negative valences which energize and direct his behavior."

From the moment of birth, if not before, the human individual is engaged in cognitive and preciative activities. By the time he enters school, he has developed many cognitive and preciative patterns. In school, he continues to expand his cognitive maps and to preciate their parts.

It is contended, for the purposes of this study, that learning is two-dimensional. One dimension is cognitive; the other is preciative. One dimension does not exist without the other. It is contended that teachers should be sensitive to both dimensions in their instructional efforts. It is contended that objectives of mathematics education which are developed in terms of only one dimension are inadequate.

How does the two-dimensional learning theory fare when considered in the light of criteria for an acceptable educational psychology? No scientific studies refute it. It could be a usable hypothesis for teachers. It does not run counter to American political philosophy and law.

Corollary #3. A human being learns when readiness and opportunity coincide. Readiness is, in part, a function of physical and
mental growth and development.

A corollary of developmental psychology was stated as follows: A human being learns when readiness and opportunity coincide. Readiness is defined in the Dictionary of Education as: "willingness, desire, and ability to engage in a given activity, depending on the learner's level of maturity, previous experience, and mental and emotional set." (23, p. 329)

It is obvious that there are stages in an individual's development when he is not yet ready to learn a certain skill or concept. An example is that a six-months-old child is ordinarily not yet ready to learn to walk. Because some, or most, children become ready to learn a skill or concept within a certain age range is not conclusive evidence that readiness is an automatic function of growth of all children. Some may never be ready, because they never develop all the essential ingredients of readiness.

Developmental psychology implies that readiness to learn develops in a process of growth and in the interaction of the organism with his environment. The definition for readiness given above mentions that willingness, desire, ability, maturity, previous experience, mental and emotional sets are all involved in readiness. Cole notes that:

"...we can see at least three factors that are crucial for the perceptive type of learning:

(1) The background of experience.

(2) The motivation, interest, purposes of the learner, sharpened until the approach to the learning situation becomes a directed search."
(3) The layout of the learning situation, the perceptual field with its groupings, emphasis." (11, p. 331)

Other authors discuss readiness, usually including maturity, ability, experience, and a motivating factor such as interests, desire, etc.

A definition of readiness within the framework of the educational psychology being expounded for this paper would include elements of growth and maturity, cognition, and preciation. Cognition and preciation imply prior experience. Briefly stated, readiness means possessing the requisite maturity, background of experience, and motivation to learn a concept or skill.

The implication of the corollary (#3) concerning readiness is that a person learns that which he is ready to learn, if the readiness and opportunity coincide; and that he does not learn that which he is not ready to learn regardless of the opportunity. In a given instructional situation, he will learn something (two-dimensional learning), but it will not be what the instructor intends if readiness is lacking.

Of course, the ideal situation is one in which the instructor is correctly sensitive to the readiness of the pupil, and helps make available proper learning conditions at the appropriate time.

Summary of the educational psychology postulated for the purposes of this paper.

It is assumed that:

(1) Man is an organism endowed with relatively few instincts
or inherited patterns of behavior.

(2) Man's primitive urges early sublimate into a constant motivation to preserve and maintain his self-concept.

(3) Man's self-concept is developed and expanded in interaction with his environment.

(k) Man's self-concept has two phases, a cognitive system and a value system.

(5) The individual's self-concept including his cognitive maps and his value systems changes from moment to moment, but tends to have stability and continuity.

For convenience, this assumed "psychology" is called developmental psychology.

It is further assumed that:

(1) All human beings differ from each other physically and mentally.

(2) The human being learns when readiness and opportunity coincide.

(3) Human learning is inescapably two-dimensional; cognitive and preciative.

(4) It is the individual's value systems (precitations) which energize and direct his behavior.

The developmental psychology and related learning theory outlined above are construed as being reasonably tenable and as providing an adequate educational psychology for devising objectives of mathematics education.
It is stipulated that mathematics education and objectives thereof should be consonant with developmental psychology.

Summary of Postulation

Guiding Principle No. 1. Public education, in general, and public mathematics education, in particular, must be in accord with American political philosophy and constitutional guarantees.

The public school is a state function. As such, it should encourage a maximum of individual liberty and academic freedom within a framework of justice, law, and order.

Parents have the primary responsibility for the guidance and nurture of their children.

Guiding Principle No. 2. Public education, in general, and public mathematics education, in particular, should be consonant with developmental psychology.

Each person in interaction with his environment develops and expands a concept of himself. He seeks to maintain and enhance his self-concept.

All human beings differ from each other.

Every human being possesses personally developed cognitive and motivating systems.

A human being learns when readiness and opportunity coincide.

These two guiding principles are postulated to serve as a frame of reference for developing objectives of mathematics education for public elementary and secondary schools.
CHAPTER IV

DEDUCTION

Preview

This chapter attempts to trace some of the implications of adopting the guiding principles postulated in Chapter III. The public school is presumed to be an agency of state government rather than being people-controlled, and therefore, its program should be conducted in accord with American political philosophy and constitutional guarantees. Developmental psychology is accepted as the correct guide to human behavior and learning. What do these assumptions mean for mathematics education and especially what are implications for developing objectives of public mathematics education?

Public school mathematics education is divided into two phases for consideration: (1) compulsory (mandatory) mathematics; and (2) optional mathematics. Criteria are devised for validating objectives of public mandatory mathematics, and the criteria are applied to some objectives suggested by authorities in Chapter II. The procedure results in some suggested elements being included in mandatory mathematics, and other proposed elements being rejected.

Objectives for optional mathematics must perforce be more flexible. Criteria for assisting students and teachers select tailor-made objectives of optional mathematics are proposed.

This chapter is concerned with deductive reasoning from the
postulated guiding principles with special reference to their logical significance for objectives of public mathematics education.

**Distinction Between Mandatory and Optional Mathematics**

American public schools tend to have required phases to their mathematics curricula, and also to have optional or elective phases. The question of whether this is good or bad is not raised. However, it is a purpose of this study to consider objectives of both required and optional phases of mathematics education in the public schools. The objectives accepted must be valid in the light of the guiding principles postulated in Chapter III.

A dividing line between mandatory mathematics and optional mathematics is not as easy to draw as it appears at first glance. For instance, a child may be required to 'take' mathematics and to get a mark in it before being promoted but this does not necessarily mean that he must attain prescribed standards, nor that he has no freedom of choice within the prescribed course. On the other hand a mathematics course may be listed as being elective but the circumstances may be such that the student has no choice but to take a designated course and meet required standards of achievement. In other words, the labels of 'required' or 'optional' with reference to certain phases of mathematics education are not always true indicators of the actual situation or practice.

Mandatory mathematics in a school set-up is construed as being comprised of those areas of mathematics education for which specific
objectives and standards of achievement by pupils are prescribed by school officials presumably acting in their official capacities. Students are led to believe, or allowed to believe, that they must meet the required standards. Motivation for the learning of mandatory mathematics is largely extrinsic for recalcitrant students, and rests on systems of rewards and penalties including rewards of high marks and threats of low marks or failure. Although it may be glossed over, there is inherent in the situation an element of official compulsion to meet prescribed standards.

Optional mathematics is construed as being comprised of areas of mathematics which the student believes he has the controlling decision concerning what and whether he may learn. In optional mathematics the pupil and his parents have a definite responsibility for setting or choosing objectives and standards of achievement for the individual. Motivation to learn optional mathematics is more intrinsic and rests on the individual's desire or will to learn.

Teachers and other government officials may have in mind objectives or aspirations concerning mathematics education for children, but these hoped-for goals do not have teeth, or authority back of them, either in spirit or in fact. Teachers may urge and encourage students to reach standards of achievement in optional mathematics but in the final analysis the student knows he may accept or reject for himself the mathematical goals without being penalized by the school.

The key to the difference between mandatory and optional
mathematics lies in the student's differentiation between what is demanded of him by the school in the way of mathematical achievement, and what is not. If a child is permitted to believe that the authority of government is backing a mathematical requirement and that his refusal, unwillingness, or inability to conform is tantamount to revolting against the government and constituted authority, then that mathematical requirement is mandatory. On the other hand, if a child believes that an opportunity to achieve in an area of mathematics is made available to him by the public schools and he believes he has freedom of choice in the matter without revolting against legal authority or being punished by it any way, then that mathematics is optional.

An analogy to show what could tend to become mandatory mathematics may be illustrated with a traffic cop, a driver and traffic laws. The driver assumes that there are traffic laws and that the traffic cop is correctly interpreting and enforcing the spirit of the laws. However, the traffic cop may exceed his authority and prescribe behavior for the driver which has little or no basis in law. Nevertheless, if the driver is unversed in law, obedience to the decree of the traffic cop becomes mandatory under penalty, as far as the driver is concerned.

In a similar way, public school officials may exceed their prerogatives and prescribe educational objectives beyond the spirit and letter of the law. As far as the student who is unversed in
educational law is concerned, such prescriptions are mandatory under penalty.

A difference between mandatory and optional mathematics education may be expressed in this way. The objectives of mandatory mathematics may be sought by authoritarian instruction, if necessary; whereas, in the final analysis, a permissive atmosphere prevails in optional mathematics instruction. Teachers, parents, and students would presumably be aware of this situation.

This distinction between mandatory and optional mathematics is consonant with developmental psychology. The student forms his cognitive map in accordance with the way he perceives the situation. If he thinks official authority, with teeth, is prescribing standards of mathematical achievement (mandatory mathematics), that is the way it is for him. But his learning is two-dimensional. As he is developing his cognitive maps to include an element of mandatory mathematics, he is also assigning positive and negative valences to all perceived aspects of the situation. The perceived aspects may include the student himself, the teacher, the state, the school, mathematics, and so forth. The learning of attitudes and appreciations and depreciations along with knowledge, understandings and skills cannot be prevented.

On the other hand, if a student is challenged to strive for a mathematical achievement and perceives the challenge to be presented on a friendly take-it-or-leave-it basis, he forms his cognitive maps and preceptive colorings accordingly. The
developmental (educational) philosophy postulated for this paper indicates that it is the value systems the student forms in such situations that energize and direct his present and future behavior with reference to the perceived aspects, mathematical or otherwise, of his field.

Since every student differs from every other there is not assurance that only wholesome mathematical and self-concept, cognitive and preceptive changes will occur in every individual under either mandatory or optional mathematics instruction. No two individuals will react or learn the same in any learning situation. Some students may resent an element of compulsion in mandatory mathematics; others may find relief and satisfaction in the prescription. Some students may loathe themselves for submitting to the compulsion; others may find self-enhancement in fulfilling every jot and tittle of the requirements.

In optional mathematics, too, either or both positive and negative valuations of the self-concept may accrue. Some students may form a low opinion of themselves because of their laziness in the absence of compulsion; others may take intense pride in their self-direction without teacher guidance. Mathematical cognitive learnings may be righ and/or poor, and preceptive valences negative and/or positive under either and both systems of instruction, mandatory or optional.

There seems to be no conclusive evidence to prove whether
authoritarian or non-authoritarian (permissive) instruction is consistently more successful although perhaps the odds favor a so-called democratic instructional climate in American schools. If every individual differs from every other, it would be predicted that there would be no uniformity of behavior under any particular method of instruction.

From a developmental psychology standpoint, a logically efficient way to assure that most children will acquire the general and specific objectives in both dimensions of a given mandatory mathematics curriculum is for the instructors to control as many of the conditioning factors which affect the developing cognitive and value systems of the individual child as possible. If public school officials presume to set extensive standards (objectives) of mandatory mathematics education, then they should also assume the means of accomplishing their purposes; i.e. greater control of the environment. But here we run up against the first guiding principle concerning American political philosophy and constitutional guarantees.

**Deductions Concerning Mandatory Mathematics**

If it is assumed that the public school is a state function; that public school teachers are state officials; and that therefore mathematics education in the public school must be in accord with American political philosophy and constitutional guarantees; then any deprivation of individual liberty, or infringement on
intellectual freedom, as implied by the term 'mandatory', must be justified by demonstrating that it clearly serves some proper function of government such as perpetuating freedom, maintaining order, or promoting public safety or public welfare.

A tenet of American political philosophy is that laws and ordinances must clearly state what is required of citizens. Interpretations, changes and substitutions in laws and regulations should not be left to the whim of individual officials. Objectives of mandatory mathematics education must not be stated in such generalities that a student may not know what is required of him; or that permits a teacher to read into mandatory mathematics what he wishes. The mathematical requirements which a state expects all children to achieve should be clearly stated.

American political philosophy holds that in the eyes of the law, all people are to be treated equally. Logically, therefore, the specific requirements of mandatory mathematics established for public schools should apply to all students equally.

We come to the position that the mathematical skills and concepts which the state may require children to learn willy-nilly are those reasonably specific skills and concepts which are essential to the preservation of a free society.

How may it be determined what mathematical skills and concepts are essential to the preservation of a free society?

Two criteria are suggested:
a. those mathematical skills and concepts which are frequently used by all normal people in American society are construed as being essential to its preservation.

b. those mathematical skills and concepts the lack of which in a citizen would tend clearly to jeopardize public safety, law and order, are construed as being essential to the preservation of free society.

Normal people may be defined as those who are not abnormal, or who tend to be within two standard deviations of the mean in mental, moral and physical measures of large groups of citizens. Sharp boundary lines between normal and abnormal people cannot be drawn. For the purposes of setting up specific objectives of compulsory mathematics education for normal people, the 70 per cent to 80 per cent of people grouped around measures of central tendency in areas pertaining to education are considered normal. Psychologists usually consider the middle two-thirds of a mass of people as normal; one-sixth subnormal; and one-sixth above normal. If there is a doubt whether an individual fits into the category of normal with reference to compulsory learning it would be in keeping with American philosophy of liberty to exempt him from the learning requirements demanded of normal folks.

The term 'frequently', as it applies to the matter under discussion, should be defined. The question is, "How often should a particular mathematical skill or concept be used by all normal people to warrant including it as an item in the compulsory education of normal children, when the basic theory under which we are operating is that there should be a maximum of academic freedom
and personal liberty and ample free educational opportunity for all?" If a skill has too infrequent use, it gets rusty. It a skill has too infrequent use it may not be worth the time and effort of either the learner or the teacher to develop it especially if the learner is not interested in the learning. It would seem that if 90 per cent of all normal adults use a mathematical skill or concept once a month, and the other 10 per cent use it at least once a year, then there might be justification within the framework of the guiding principles to include it as a part of mandatory mathematics. If there is a doubt concerning whether to classify a given mathematical skill or concept in the mandatory category, then it should be left out. Such a policy is in keeping with American tradition that the least government is the best government.

It is possible that there are some mathematical skills or concepts which are not frequently used, or which many citizens lack, but which are demonstrably essential to the maintenance of public safety, welfare, law and order. If such there be, it would be legitimate to require children to acquire these mathematical skills and concepts.

During the 1930's and since, there have been many studies to find out the extent and use of mathematical skills and concepts by various segments of society. These studies tended to show that scientists, engineers, statisticians, economists, business men are more and more using the fruits of mathematics, especially in their
specialties. The studies also indicate that the rank and file of folks, generally, perform few mathematical operations, and those operations that they do perform are quite elementary.

Quantitative thinking of common people seems largely to be limited to ordinary measures, round numbers, and the simplest of proportions. Not many citizens think of, or apply, mathematics as a classic system of deductive logic. Increasingly, routine computations in our culture are performed with machines operated by technicians. Estimates, appraisals, accounting, actuarializing are done by specialists.

Almost every conceivable quantitative calculation is already done for the modern consumer. There is very little bartering or haggling over prices in shops. Everything from houses to cakes are pre-figured, pre-cut, and pre-mixed; installment payments, loan interest, insurance premiums, risks and life-expectancies are calculated by experts.

Two criteria for determining elements of mandatory mathematics which are deduced from the guiding principles postulated in Chapter III are:

Criterion #1. All normal children may be required to learn those mathematical skills and concepts which are frequently used by all normal adult citizens in American society.

Criterion #2. All normal children may be required to acquire those mathematical skills and concepts, the lack of which in citizens would tend clearly to jeopardize public safety, welfare,
law and order.

These criteria are to be applied to various proposals for objectives of mathematics education in order to develop valid objectives of public mandatory mathematics education. Two policies should prevail in the procedure:

(1) Objectives expressed in broad generalizations and hence are subject to a multiplicity of interpretations for classroom practice, should be avoided. The reason being that compulsions on American citizens should be clearly defined.

(2) Since every person differs from every other; since compulsion on a child to learn may be perceived as a threat to his self-concept with consequent negative attitudes; since parents have the prime responsibility for the guidance and nurture of their children; since the state should not try to standardize children; and since an important function of public agencies is to perpetuate freedom including academic liberty, then a policy in developing objectives of mandatory mathematics education would be to keep the mandatory program to a minimum and include nothing that is not clearly justifiable in terms of the criteria above.

Objectives of mathematics education suggested by various authorities and recorded in Chapter II may be examined in the light of the criteria for inclusion in a program of mandatory mathematics education for all normal children. There is little to be gained by considering proposed objectives of mathematics education which are
expressed in broad generalities because the objectives of mandatory mathematics education expected to result from the process are not to be expressed in generalities.

The 29 functional competences in mathematics proposed by the Commission on Post-War Plans (39, pp. 4-5) for a basis for objectives of high school mathematics education will be considered first. The proposed functional competences which have universal and frequent adult use, or are demonstrably necessary to the preservation of a free society will be included in public mandatory mathematics objectives.

"1. Computation." Research shows that comparatively few people compute with fraction or decimal symbols except adding dollars and cents. Multiplication with more than two-place numbers is very seldom performed by many people. Probably a majority of people have frequent occasion to add and subtract whole numbers. It would seem that skill and understanding of addition and subtraction of integers; multiplication of two-place integers by two-place integers, and the corresponding division might be justified in mandatory mathematics because of their universal and frequent use. Skill in computation with fractions or decimals, or involved multiplication and division of integers is hardly justifiable for inclusion in compulsory mathematics either from a use or a public safety standpoint.

4 A brief explanation of each of the 29 functional competences is given on page 33 above.
"2. Percents." The majority of normal American adults hear or read per cent concepts expressed in publications and over the ether waves. Not enough people compute per cents to justify making it mandatory learning. Acquiring an understanding of the symbolism and meaning of simple per cent situations may be permissible objectives for mandatory mathematics.

"3. Ratio." Too few people understand or use ratio (an abstract quotient) to make the learning of it mandatory.

"4. Estimating." Estimating is included in the 29 functional competences as a policy rather than a skill. It is too general a statement to be included in the mandatory mathematics objectives although it is an excellent practice to keep before students.

"5. Rounding numbers." The concept and skill of rounding off abstract numbers is not widespread. They should not be included in mandatory mathematics.

"6. Tables." Practically everybody in our culture frequently meets information and data in tabular form: e.g., time-tables, height and weight tables, calendars, and so forth. Also, it may be essential to public safety, welfare and order that people be able to glean correct information from simple tables. It would be permissible to require children to learn to read a few most commonly used tables like calendars.

"7. Graphs." Practically everybody in our culture meets graphs involving statistical data although they seldom use them.
It would be permissible to require children to have some experience with interpreting simple graphs.


"9. The nature of measurement." Technical knowledge of measurement theory is not widespread. It should not be required.

"10. Use of measuring devices." All people have occasion to use foot-rules, yardsticks, weighing scales, pint, quart, and gallon measures. Mandatory mathematics may include these.


"12. Angles." Estimating, reading, and constructing angles is not a frequent activity of many people. This competence is not justifiable for inclusion in mandatory mathematics.

"13. Geometric concepts." Many geometric forms, such as circles, squares, triangles, etc., will and should be learned by students, but technicalities of geometry are not justifiable for mandatory mathematics.

"14. The 3-4-5 relation." Cannot be justified for inclusion in mandatory mathematics on either a use or safety basis.

"15. Constructions." This specialized geometric procedure cannot be justified for inclusion in mandatory mathematics.

"16. Drawings." Most people in our culture have occasion to use or refer to diagrams or maps. It may be essential to public
welfare, safety, and order that all people be able to read simple diagrams and maps. It is permissible to include acquiring modest skill and understanding in reading diagrams and maps in mandatory mathematics.

"17. Vectors." Not justifiable for mandatory mathematics.


"19. Conversion." Conversions involving inches to feet and yards, ounces to pounds, pints to quarts and gallons may be permissible in mandatory mathematics.

"20. Algebraic symbolism." Not justifiable in mandatory mathematics.


"22. Signed numbers." Not justifiable for mandatory mathematics.

"23. Using the axioms." Not justifiable for mandatory mathematics.

"24. Practical formulas." Formulas, as such, are little used except by specialists. The memorizing of them is not justifiable in mandatory mathematics.

"25. Similar triangles and proportions." Not justifiable for mandatory mathematics.


"27. First steps in business arithmetic." Making out lists of purchases, adding of costs, making change, are fairly universal practices. Children may be required to learn some of these.
"28. Stretching the dollar." This is a broad generalization and not specific enough to be included in mandatory mathematics.

"29. Proceeding from hypothesis to conclusion." This functional competence is not specifically enough stated to be included in mandatory mathematics. Most thinking is done from hypothesis to conclusion. The problem lies in the nature of the hypotheses.

It is conceded that the NCTM committee on post-war plans drafted an excellent list of functional competences in mathematics. It is conceded that it would be fine if all people were proficient in every one of the competences. But if objectives of mandatory public mathematics education are to be devised in accordance with the postulated guiding principles, comparatively few of the functional competences will qualify for inclusion in compulsory mathematics under the criteria of frequent widespread use, or of essentiality to the preservation of a free society. However, it is also conceded that the 29 competences provide an excellent check list for optional mathematics.

The check list of functional competences was compiled for high school students and advisers. Typical objectives of elementary school mathematics education should also be considered for validity.

Brueckner's and Grossnickle's representative list of objectives (6, pp. 3-5) of a modern arithmetic program may be examined in the light of the criteria for mandatory mathematics. Those proposed specific objectives which are universally used, or are indispensable
for public welfare, will be included in the list of objectives for public mandatory mathematics education.

"1-a. An understanding of the structure of the decimal number system and an appreciation of its simplicity and efficiency."

The decimal number system is universally used by all normal adults. It may be argued that understanding of the number system and the ability to count, and read and write numbers, are essential to public safety and welfare.

"1-b. The ability to perform computations connected with social situations with reasonable speed and accuracy, both mentally and with mechanical computing devices."

Every child may be required to learn to perform computations connected with social situations in which all normal adults are frequently involved. They should learn to do such computation in a serviceable way, or ways, whether on paper, mentally, or with mechanical devices.

"1-c. The ability to make dependable estimates and close approximations."

Since most applications of mathematics involve approximations, most usable computations actually involve approximations or estimates, with approximations or estimates resulting. Children may be required to learn to make approximations or estimates to the extent that all normal people do frequently.

"1-d. Resourcefulness and ingenuity in perceiving and dealing with quantitative aspects of situations."

This proposed objective is too imponderable for use as an objective of mandatory mathematics education.
"1-e. Understanding of the technical vocabulary used to express quantitative ideas and relations."

Understanding commonly used quantitative terms may be included in mandatory mathematics on the basis of universal frequent use and public welfare.

"1-f. Ability to use and to devise formulas, rules of procedure, and methods of bringing out relations."

Not specific enough. Many people need to use some mathematical procedures, but few people have occasion to devise formulas, etc.

"1-g. Ability to represent designs and spatial relations by drawings."

Does not qualify for inclusion in mandatory mathematics.

"1-h. The ability to arrange numerical data systematically and to interpret information which is presented in graphic or tabular form."

Arranging numerical data systematically is not a common practice except perhaps in a most elementary sense. Interpreting information which is presented in graphic or tabular form is more common. It is permissible to require children to have experiences with simple diagrams and tables.

"2-a. Understanding of the process of measurement and skill in the use of instruments of precision."

See the discussion under functional competences Nos. 9 and 10 above.

"2-b. Knowledge about the development and social significance of such institutions as money, taxation, banking, standard time, and measurement."

Such knowledge is not universally used nor does it seem to be essential for public welfare. Social institutions and their
histories take a high degree of maturity to comprehend. The objective does not qualify for inclusion in mandatory mathematics in public schools.

"2-c. Knowledge of the kinds and sources of information essential for intelligent buying and selling and for general economic competence."

Such knowledge is not universally used, nor universally agreed upon, nor is it essential for public welfare.

"2-d. Understanding of the quantitative vocabulary encountered in reading, in business affairs, and in social relations."

A minimum vocabulary based on universal use may be required.

"2-e. Appreciation of the contributions number has made to the development of social cooperation and to science."

Such appreciations are not universally used, nor are they essential for public welfare.

"2-f. Ability and disposition to secure and utilize reliable information in dealing with emerging personal and community problems."

The state has no authority to try to standardize thinking or action by making this a part of mandatory mathematics education.

"2-g. Ability to rationalize and analyze experiences by utilization of quantitative procedures."

Does not qualify for inclusion in mandatory mathematics.

It may be noted that the application of the criteria tends to limit the content of the mathematics which all normal children may be required by the school to learn. It now becomes necessary to consider when a child may be required to reach the objectives of mandatory mathematics.
Since mandatory mathematics is largely comprised of the mathematical skills and concepts which a youngster may observe in frequent use, it is likely that most children will attain the required objectives in the course of in-school and out-of-school activities, without the necessity of teachers using harsh pressure. However, there should be some sort of a deadline for children generally to achieve the prescribed goals of mandatory mathematics. Such a deadline should not be set without regard to the guiding principles postulated in Chapter III.

A corollary of developmental psychology concerns readiness: that a person does not learn until he is mentally and physiologically ready. Another corollary of developmental psychology holds that every individual differs from every other. Hence, the age of readiness to learn a specified concept will differ from individual to individual. However, a time is reached when children who have not yet achieved the objectives of mandatory mathematics by some age must be dealt with by the schools in terms of, "Ready or not, here I come!" The age of checking for achievement and instituting unrelenting pressure to reach objectives of mandatory mathematics should come as late in the school course as feasible in order to permit maximum time for readiness and consequent learning to occur prior to resorting to compulsion, with possible consequent negative reactions, and still early enough to allow time for the child to complete the requirements before he leaves the public education program.
Statistics indicate that practically all normal children remain in school until the age of sixteen (L9, p. 6)

Assuming that it may take as long as two years for an adolescent to achieve the objectives of mandatory mathematics, if he has missed mathematics previously, and recognizing that there is little increase in mental maturity after puberty (2h, p. 160), lead to a conclusion that age fourteen is about as late as feasible for the schools to use strong measures, if necessary, to assure that a child will attain the objectives of mandatory mathematics before he leaves school.

To summarize concerning objectives of mandatory mathematics education: it is deduced that the compulsory mathematics program should be limited in scope, and that coercive measures should not be invoked prior to age fourteen. These conclusions will be elaborated on in the next chapter.

**Deductions Concerning Optional Mathematics**

American political philosophy and constitutional government permit a wide range of services which the various levels of government may provide for citizens. Although the public school is a state agency and may be limited in the extent and nature of the mathematical skills and concepts which it may require children to learn (mandatory mathematics), it is not so limited in the opportunities and facilities it may provide for optional mathematics learning.
If a rich range of mathematics educational opportunity is going to be arranged under the auspices of the public school, the lion's share of it must be in the optional category. The term optional implies that the educatee holds the right to choose freely whether to learn all, some, or none of a mathematical unit. It also implies that the educatee may apply his own criteria in making a decision. Hence, absolute criteria for evaluating objectives of optional mathematics for an individual cannot exist apart from the one affected. It is incongruous for the state or the teacher to set blanket objectives for optional mathematics education.

The objectives of optional mathematics education are, in the final analysis, determined by the individual learner for himself. The school may help him learn those things that he wants to know, or that he believes he needs to know. Teachers and counselors and parents may help the individual discover his talents, abilities, and deficiencies. They can show him advantages of mathematical training.

Selecting objectives of optional mathematics education becomes essentially a matter of non-coercive guidance. Optional mathematics education objectives and programs should be cooperatively tailor-made to suit each individual. He has the right and privilege to expand or curtail the scope of his mathematics program in the public school. The teacher may serve as an adviser and be available to help the student achieve his aims. The teacher should try to avoid making a child who does not conform to normal patterns of
It would seem to be permissible for teachers to have in mind some objectives for their optional mathematics instruction of students as long as they (the teachers) do not exercise pressure to the point that the mathematics becomes mandatory. Allowing for academic freedom and individual differences, and being sensitive to readiness implied by adopting the guiding principles, would tend to make teachers' objectives of optional mathematics for children somewhat general.

Parents and teachers are sometimes prone to make plans for their children without giving due consideration to the child's point of view. When a teacher finds himself formulating objectives of the mathematics education for a certain child, it behooves him to keep in mind acceptable criteria for evaluating the merit of his proposals for the child. The criteria should be consonant with the guiding principles. Criteria for evaluating teachers' specific objectives (designated mathematical concepts and skills) for specific students may be suggested as follows:

Teacher's criterion #1. Readiness. Is this child ready to learn this skill? Is this the best time for him to do it?

Teacher's criterion #2. Will the total effect, cognitive and preciative, of the instruction and learning be beneficial to the individual? How?

Teacher's criterion #3. Does the individual feel he has freedom of choice in the matter as far as the school authority is
concerned?

Teacher's criterion #1. Does the individual want to work toward this specific objective under consideration?

If the answer to all of these questions (criteria) is, "Yes," then the teacher has a go-ahead signal.

Parents, or economic, social, military, and religious groups, may want certain specific objectives of mathematics promoted through the public schools. The demand of a pressure group is not a wholly valid criterion for a teacher to use in judging the merit of specific objectives of mathematics education for individuals. Such pressure groups may put pressure by legitimate means such as advertising and exhortation on the student to learn.

It may be assumed that many children in our society will want to learn more mathematics than that encompassed by mandatory mathematics. Most of these students would probably welcome guidance to help them formulate, anticipate, and meet their desires and needs in mathematics. It is a legitimate function of public school teachers to make their services available to children for guidance and instruction in optional mathematics. Teachers should have at hand some instruments for guidance and instruction of optional mathematics. Among these instruments are some suggested reasonable criteria for evaluation of proposed objectives of mathematics education for application by individual students with or without the cooperation of teachers and counselors.
Student's criterion #1. Am I ready to profit from instruction in the mathematical area under consideration. In other words, do I have the requisite maturity, interest, and background of experience?

Student's criterion #2. Will the proposed mathematics learning make a contribution to my long-range goals? How?

Student's criterion #3. Does the proposed mathematics learning meet my immediate needs? How?

Student's criterion #4. Is this the best time for me to work on the proposed mathematics, considering present and prospective personal and school situations, and alternative activities?

Student's criterion #5. How do my parents, teachers, and respected friends counsel me regarding this mathematics?

Student's criterion #6. Am I ready to cooperate in group instruction?

Components of the broad fields of mathematics may be indicated to students by teachers and counselors. Various proposals for objectives of mathematics education assembled in Chapter II may be used by educators as sources of suggestions for students. A student may apply the criteria listed above to aims of mathematics for him as he sees fit.

Even if a fine set of objectives and an excellent program of mathematics education is developed through guidance for a certain student and he accepts the plan whole-heartedly, it is logical to deduce from the guiding principle of developmental psychology that
the whole scheme is subject to change and rejection without notice. It is also logical to assume from the first guiding principle that a teacher may exceed his authority as a public official if he tries unduly to pressure the student into holding to the objectives and plans of his mathematics training set-up when the student wants to change.

**Summary of Chapter IV**

There are usually two phases of mathematics education in the public schools: that which is required, and that which is elective. For this study, mathematics education is dichotomized into mandatory and optional mathematics. The distinction between mandatory and optional mathematics lies in the student's interpretation or understanding of which is which.

The state may require all children to learn the mathematical skills and concepts which all normal adults use frequently, or which are clearly essential to public safety, order, justice, and the preservation of a free society. These criteria limit the scope of mandatory mathematics.

The public schools may provide extensive opportunities for optional mathematics education of children. Aims and programs for optional mathematics education should be tailor-made for each student, probably through a guidance process. If the mathematics is truly optional, any plan made for or by a student is subject to change.
Criteria for teachers to use in judging the merit of objectives of mathematics education for a particular student should be conducive to permitting the student to choose freely to learn what he is ready and needs to learn.

A student's criteria for evaluating proposed objectives of optional mathematics education should be conducive to his discovering his needs, desires, and readiness.
CHAPTER V

CONCLUSIONS

It was premised in Chapter III that educational policies, including objectives of mathematics education, of American public schools should be in accord with developmental psychology and American political philosophy.

Developmental psychology is the theory that the human individual, in interaction with his environment, gradually develops and continues to develop and expand a concept of himself—who and what he is, and with whom and with what he identifies himself—and that he seeks constantly to maintain and enhance his self-concept. Included in developmental psychology is the theory that the individual develops, in relation to his self-concept, a system of values (positive and negative) and a structure of knowledge. This could be stated another way to emphasize personality integration—that the individual develops, in relation to his self-concept, a knowledge system which has two aspects, preciative and cognitive. Developmental psychology holds that it is the individual's value system, developed in consideration of the maintenance and enhancement of his self-concept, that energizes and directs his behavior. Developmental psychology contends that the self-concept, including the preciative and cognitive systems, develop in accordance with the way the individual perceives (or believes) things to be.
American political philosophy is the theory that governments and governmental agencies, including the public schools, exist to perpetuate liberty, maintain law and order, and to provide for public welfare; that the normal individual is responsible for his own destiny and decisions; that, in the case of a normal child, the parents are responsible for his nurture and guidance; and that no infringement on individual freedom or rights may be instituted by governmental agencies except for proper government function.

Developmental psychology leads to the conclusion that if a child develops a concept of himself as being rejected by the school, or as being a rebel against the school or society, then he will seek to maintain that self-concept and behave like an outcast or a rebel. "As a man thinketh in his heart, so is he."5

Developmental psychology leads to the conclusion that if a person feels his self-concept to be threatened in an instructional situation, he will take measures to meet the threat—he will cheat, withdraw, become hostile, submit, etc.

Developmental psychology leads to the conclusion that instruction to unready children is futile and possibly detrimental.

Developmental psychology leads to the conclusion that if a person develops the concept of himself as being inept or incompetent in mathematics, then he will seek to maintain that self-concept and be inept or incompetent in mathematics.

5 Proverbs 23:7
Developmental psychology leads to the conclusion that if a youngster under public school tutelage develops the self-concept that he is mathematically incompetent, then further cognitive training in mathematics is of limited value.

American political philosophy is opposed to regimentation, or standardization, or control of thought or activity of individuals by officials except in the clear interest of public welfare or for perpetuation of freedom. The U. S. Supreme Court has said that it is not a proper function of the public schools to try to standardize children.

The above conclusions lead to a further conclusion that coercion in public mathematics education should be used sparingly.

Mandatory mathematics is the mathematics which all normal children may be required to learn under coercion, if necessary. Mathematical concepts and skills which are universally used, or are essential to the preservation of a free society, are considered to be justifiable objectives of mandatory mathematics.

**Recommended Objectives of Mandatory Mathematics**

1. All normal children should be required to learn to count and to read and write numbers in the Hindu-Arabic notation system.

2. All normal children should be required to develop an understanding of the decimal number system, i.e., that place determines the value of a digit; that numbers are additive between places; and
that the value relationship of adjacent places is 10 to 1.

3. All children should be required to have experience with the most commonly used measuring devices.

4. All children should be required to have experience in interpreting ordinary graphs, charts, maps, and tables.

5. All children should be required to develop modest skill in counting by small, equal and unequal, groups.

6. All children should be required to develop modest skill in commonly used addition, subtraction, and multiplication.

7. All children should be required to have experience in interpreting the most common used of percent.

8. All children should be required to learn how to recognize money in its various commonly used forms (coin, currency, checks, money-orders, postage stamps). They should learn to count money and to make change.

Placement of Elements of Mandatory Mathematics

Since mathematics instruction by teachers prior to readiness of the child is ineffective and potentially detrimental, the time that a topic should be presented to a child for learning will depend upon his readiness. Teachers should constantly be alert to the readiness of the child to learn mathematics. Teachers should try to develop the child's readiness for mathematics learning by involving him in perceived, meaningful, mathematical situations. If, in the course
of ordinary non-coercive instruction and quantitative experience, a child has not achieved the objectives of mandatory mathematics by age fourteen, then co-ercive measures may be taken to assure completion of the requirements by age sixteen.

Objectives for Optional Mathematics Education in the Public Schools

Beyond the minimum requirements of mandatory mathematics which the state may demand of all normal children, the objectives of mathematics education are an individual matter. As far as mandatory mathematics is concerned, public school teachers are masters of the children. But, for other phases of mathematics, public school teachers are servants, teachers, and counselors.

Programs and objectives of optional mathematics education for individuals should be devised and revised through guidance. Teachers should help the student discover his readiness and potential need for various phases of mathematics in accordance with his own aspirations. The teacher should be especially sensitive to the individuals' self-concepts and value systems developing in the interaction of the individuals with the mathematics program. If negative attitudes and valences are tending to develop, the teacher should try to facilitate a change even to the point of dropping mathematics education for the individuals for awhile. Since it is the self-concept and precarious colorings which energize and direct present and future behavior, these matters may be more important than cognitive learnings.
The guiding principles leave no choice but to make the selection of objectives of mathematics education of an individual, for an individual, and by an individual, above and beyond mandatory mathematics, a matter for friendly guidance by school officials. The opportunities for individual and group instruction in any and all phases of mathematics which a public school may provide for students are unlimited, but compulsion to take advanced mathematical training is psychologically unsound and politically "unconstitutional."

Appropriate criteria for teachers and students to make use of in the guidance of an individual toward determining his objectives and program for his mathematics education are suggested in Chapter IV.

**Suggestions for Further Study**

1. Spell out for use in a public school a mathematics education program based on developmental psychology and American political philosophy and incorporating suggestions for aims embodied in this paper.

2. How can an effective guidance program for optional mathematics education of children be implemented?

3. What are effects of mathematical testing programs on self-concepts of individual students, parents, and teachers? What changes in children's cognitive and perceptive systems take place under the impact of standardized testing?
4. What are values of public school marking systems in mathematics in the light of developmental psychology? What are effects of school marks in mathematics on the self-concepts and cognitive and value systems of students, parents, and teachers?

5. Is there any relationship between public mathematics education as now conducted and juvenile delinquency?6

6 This topic is suggested because the principal of the education department of a state girls' reform school once stated to the writer that every girl in the institution seemed to have an antagonism, aversion, or block toward mathematics.
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Appendix A

State Courses of Study Analyzed


Delaware. Department of Public Instruction. Goals and objectives through curriculum experiences in the elementary school by subjects and year levels, I-VIII. Dover, State of Delaware, n.d. 154p. (Bulletin no. 51-49.)


Florida. Department of Education. A guide to a functional program in the secondary school. Tallahassee. The Department, 1940. 491p. (Bulletin no. 10)

Indiana. Department of Public Instruction. Elementary school guide. Indianapolis, Superintendent of Public Instruction. 1946. 51p. (Bulletin no. 150)


New Jersey. Department of Education. Arithmetic in child development. Trenton, The Department, 1950. 89p. (Elementary school bulletin no. 16)


Texas. Department of Education. Basic learning areas in the elementary school. Austin, The Department, 1946. 311p. (Bulletin no. 471)


Virginia. Board of Education. Course of study for Virginia elementary schools, grades I - VIII. Richmond, Division of Purchase and Printing, 1943. 553p.


Appendix B

Elementary School Arithmetic Text Series Examined


Osborn, Jesse, and Adeline Riefling, Adventures with Numbers (Grades 3 - 8) St. Louis, Webster Publishing Co., 1953.


Appendix C

Oregon School Districts Visited

Benton County School District No. 90J, Corvallis
Linn County School District No. 5, Albany
Linn County School District No. 16 and UH1, Lebanon
Linn County School District UH no. 2, Sweet Home
Marion County School District No. 2hCJ, Salem
Marion County School District No. 88, Keizer
Multnomah County School District No. 1, Portland
Polk County School District No. 2, Dallas
Polk County School District No. 13C, Monmouth-Independence
Polk County School District No. 33, Buena Vista
Yamhill County School District No. 29 and UH6J, Newberg