

Errors in Determining Cubic Foot Volumes
of Western Hemlock logs by Various Rules

by

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TABLE OF CONTENTS

	Page
Introduction	1
Proposed Cubic Foot Rules	6
The Huber Rule	7
The Smalian Rule	7
The One-in-Eight Rule	8
The One-in-Ten Rule	8
The Newton Rule	9
Appraisal of The Cubic Foot Log Rules with Respect to Practical Application . .	9
Errors Involved in Cubic Foot Scaling by Proposed Rules	11
The Huber Rule	12
The Smalian Rule	12
The One-in-Eight Rule	15
The One-in-Ten Rule	15
Summary and Conclusions	16

ERRORS IN DETERMINING CUBIC FOOT VOLUMES OF WESTERN HEMLOCK LOGS BY VARIOUS RULES

Introduction

Log volumes may be ascertained by one of three methods. The board foot method, which is a unit of measurement peculiar to the North American continent alone, has been used in the United States for more than a century. Various board foot rules were advanced; and apparently there were many different opinions pertaining to their accuracy, for new ones were always being devised and old ones being reconstructed. The second method is the cubic foot, which has been more universally used throughout other timber producing regions of the world. Although this method seems more appropriate than the board foot, it has been used less extensively. The third method, the cord measurement, has been used widely in the measurement of pulpwood and fuelwood.

The controversy over which to use as a standard unit for measuring the volumes of logs, the board foot or the cubic foot, has been the object of frequent discussions. Belyea and Sheldon (1) have made the following statement regarding the problem.

"A unit of measure for timber, like the unit of measure for any other commodity, should be uniform, exact, and standardized, and should mean the same thing at all times and in all places and under all conditions. The thousand board feet log scale, as related to a thousand board feet of lumber, is not such a unit. What is really needed is a unit which is an absolute measure, is simple in its application, is not presumptuous or preemptory regarding the finished

product, and obviates by its precision the necessity of the acceptance of the perennial dispute of overrun and underrun."

The board foot as a basis of measurement of wood supplies has been used both in the east and in the west. However, the cubic foot and cord standards have been used to a considerable extent for the measurement of pulpwood and fuelwood products, especially in the east. When all logs are sent to the mill to be produced into lumber the board foot serves as a fairly satisfactory unit of measure. Even then there is a difference between log scale and lumber tally, the amount depending on the log rule used, the size of the logs sawed, who saws the logs, and other factors.

Most board foot log rules underscale small logs to a greater extent than they do large. Thus in sawing second growth timber, where the logs are relatively small, there is a tendency for a large overrun. This brings up the problem of revising existing log rules or making new ones, since most of the timber cut in the future will be second growth.

Many foresters have already taken the lead in advocating the measurement of wood products on a cubic foot basis. T. T. Munger (2) has listed 16 advantages of the cubic foot. They bear repeating here since they include most of the important advantages made by other authors. They are listed as follows:

"1. The cubic foot is an absolute measure of volume, which the board foot as applied to logs or trees is not.

"2. The use of the cubic foot necessitates no assumption as to the products to be made from a log or as to the

intensity of manufacture and therefore is particularly suitable for measuring standing trees, logs in transit on common carriers, logs in the open market, or those in any stage before reaching the manufacturing plant.

"3. Cubic foot measure could be applied universally to trees and logs, whether they were to be cut into lumber, ties, shingles, fuel, pulpwood, veneer.

"4. Adoption of cubic foot measure of unmanufactured forest products and trees would do away with the confusion and disparity that now exists between 'log scale in board feet' and 'lumber tally in board feet' in handling any business negotiations, statistics, bookkeeping, etc.

"5. Cubic foot would furnish a unit of measure more easily understood by laymen, investors, and the trades using wood who are now confused by the complications of the sundry board foot rules, and in that way help the forest industries.

"6. Cubic foot measure does away with that perennial basis for dispute -- 'overrun' and 'underrun'.

"7. The conversion from cubic foot to the unit of measure appropriate to each manufacturing plant could be done very simply and would involve no greater uncertainty or difficulties than now in converting log scale to lumber tally.

"8. Cubic foot is a much better unit of measure for taking account of the taper and exact geometrical contents of the tree than in the board foot unit, and lends itself to formulae or rule-of thumb methods of computing exact tree or log contents; this is important when total solid wood contents are desired, as with pulp, excelsior, or extractive products.

"9. The disputes over log freight charges between shipper and common carrier could be much more easily ironed out if there were a better unit for measuring loads, such as the cubic foot, and there would be a better chance for establishing rates that were equitable to both parties.

"10. The cubic foot can be applied either with or without allowance for defect, by merely using the terms gross and net.

"11. The mechanics of obtaining the volume in cubic feet of a log or tree is just about as easy as obtaining the volume in board feet.

"12. Log scaling and cruising using the cubic foot unit of measure is easier if anything than when using any

board foot scale rule, and there are no difficulties in adapting scale sticks and cruiser' volume tables to the new unit which any scaler or cruiser might readily learn.

"13. The use of cubic foot measure would remove to a considerable degree the personal element of scaler or cruiser who, under the present system of scaling or cruising, is more or less influenced by his judgment as to how the logs are to be cut up.

"14. Cruising according to cubic foot, either gross or net, would be on a more accurate and scientific basis than the present attempt to express the stand in terms of inch boards it might cut in a hypothetical mill.

"15. Already there is much precedent for using the cubic measure of forest products in the rough. It is common in foreign countries.

"16. On the pacific coast the Forest Service is already using the cubic foot as the unit of cruising and selling pulpwood stumpage in Alaska; also in Maine and New Hampshire."

Practically the only reason for continuing the use of the board foot measurement seems to be one of custom. In the past most raw material has been used for lumber, and the board foot unit was probably a satisfactory measure. However, a greater proportion of wood products now goes into industrial processes in the bulk form. The board foot unit is a poor index of bulk volume. Therefore, the need arises for some unit which is more indicative of the volume of the raw material for bulk use. The answer seems to be the adoption of the cubic foot as a standard unit for measuring all wood supplies.

Since the board foot use has grown with the lumber industry of the United States to such an extent, it will be rather difficult to change to the cubic foot. Perhaps this conversion will not come in the very near future, but when it does come it should meet the approval of men who are

concerned with log buying and selling. It is the writer's belief that every effort should be made by foresters to urge such a conversion.

Since cubic foot measurement is a fairer and more appropriate method of ascertaining log volumes, especially when bulk use is intended, now is the time for research to be made in connection with the development of cubic foot scaling techniques. The problem confronting mensurationists is to develop a practical as well as an accurate method in which cubic foot log volumes can be determined. Very little study has been made in computing the errors involved in applying the various cubic foot rules that have been advanced. The objective of this paper is to compare the various cubic foot rules by applying them to actual log measurements of Western hemlock.

The data, used in determining the errors involved in scaling, was taken from taper measurements collected under the direction of Mr. Hanzlick in 1913 on 965 Western hemlock trees. Diameter measurements were taken at 16 foot intervals of length to a top of approximately 8 inches. By taking a log of 32 feet as a standard, the diameter was therefore available at the top, base, and midpoint of each log. Since time was very limited 28 trees were picked at random from the total of 965, which yielded 100 logs. The diameter inside the bark of each log at the top, base, and midpoint were listed. This was sufficient data to make volume computations for the more important cubic foot rules that have been proposed for use in the Pacific Northwest region.

Proposed Cubic Foot Rules

A number of cubic foot rules have been proposed for obtaining log volumes. Even though some of these have a high degree of accuracy, they are not practical when applied to actual scaling conditions. Of the rules advanced the Huber formula, which is used by the Forest Service as described in the National Forest Scaling Handbook, offers some promise. The Smalian formula has been in use for a number of years, but it requires the taking of two diameter measurements and on a theoretical basis is not as accurate as the Huber formula. Rapraeger (3) has suggested a modification of both of these rules wherein the small end diameter only is measured, and the middle diameter or large end diameter is estimated by allowing a taper of 1 inch per 8 feet of length to the point in question. This modification will be called hereafter the one-in-eight rule. Another practical rule has been described by Sorensen (4), which is based on the formula for determining the volume of a frustum of a cone. In its unmodified form this formula requires the two end diameters, but Sorensen would eliminate measurement of the large end by allowing a taper of 1 inch in 10 feet from the diameter of the top end. Newton's formula has been adapted to measurement of log volumes from an engineering formula, which is used in calculating cubic volumes for earthwork. This rule is the most accurate of all the above rules and was used as a basis in determining the errors. A discussion of the above rules are given below in greater detail.

In calculating the cubic volumes by the Huber, Smalian, one-in-eight, and Newton rules, the basic formula for the area of a circle (πR^2) was used in obtaining the cross-sectional area for each diameter measurement required. In terms of the diameter squared the following basic formula was devised:

$$A = \frac{\pi \times D^2}{4 \times 144}$$

where A = The cross-sectional area in square feet.

D^2 = Diameter squared inside bark in square inches.

The Huber Rule

Using the basic formula as shown above the following formula was derived for log volume determinations by the Huber rule, length being constant at 32 feet.

$$V = \frac{\pi \times L \times D^2}{4 \times 144}$$

$$V = \frac{3.1416 \times 32}{4 \times 144} \times D_m^2$$

$$V = 0.174528 \times D_m^2$$

where D_m^2 = Diameter squared at the log's midpoint in square inches.

144 = Used in order to change square inches into square feet.

L = Log's length in feet.

The Smalian Rule

The Smalian formula requires the inside bark measurement of both the top and butt diameters. In terms of diameter squared and log length the following formula was derived:

$$V = \frac{\pi \times L}{4 \times 144} \times \frac{D_b^2 + D_t^2}{2}$$

$$V = \frac{3.1416 \times 32}{4 \times 2 \times 144} \times (D_b^2 + D_t^2)$$

$$V = .037264 \times (D_b^2 + D_t^2)$$

where V = Volume in Cubic feet.

D_b^2 = Diameter at the butt squared in square inches.

D_t^2 = Diameter squared at the top in square inches.

The One-in-Eight Rule

The one-in-eight rule allows for the taper of 1 inch in 8 feet from the top of the log for estimating the middle diameter inside the bark. From this estimated diameter the Huber rule is applied as described previously. For the standard log of 32 feet the rule becomes $V = 0.174528 \times (D_t + 2)^2$.

The One-in-Ten Rule

The one-in-ten rule allows for the taper of 1 inch in 10 feet from the top of the log to the other end. It treats the log as a frustum of a cone, and in terms of diameters squared and log length is as follows:

$$V = \frac{3.1416 \times 32}{3 \times 4 \times 144} \times (D_b^2 + D_t^2) + (D_b \times D_t)$$

$$V = .05817 \times (D_b^2 + D_t^2) + (D_b \times D_t)$$

where V = Volume in cubic feet.

D_b^2 = Diameter squared at the butt in square inches.

D_t^2 = Diameter squared at the top in square inches.

D_b = Diameter at the butt in inches.

D_t = Diameter at the top in inches.

Newton's Rule

Newton's rule considers the log as a frustum of a solid having a smooth curvilinear form. Three diameter measurements are necessary: the top diameter, the middle diameter, and the bottom diameter. The middle diameter is weighted 4 times as heavily as either the top or bottom diameter. It has been adapted for squared diameters in the following manner:

$$V = \frac{3.1416 \times 32}{4 \times 144} \times \frac{(D_b^2 + 4D_m^2 + D_t^2)}{6}$$

$$V = \frac{3.1416 \times 32}{4 \times 6 \times 144} \times (D_b^2 + 4D_m^2 + D_t^2)$$

$$V = 0.02909 \times (D_b^2 + 4D_m^2 + D_t^2)$$

where V = Volume in cubic feet.

D_b^2 = Diameter at the butt squared in square inches.

D_t^2 = Diameter at the top squared in square inches.

D_m^2 = Diameter at the middle squared in square inches.

Appraisal of the Cubic Foot Log Rules

with Respect to Practical Application

Each of the above rules have their advantages and disadvantages. The Huber rule is widely applied under some circumstances, since it requires only the middle diameter measurement as compared to two diameter measurements for the Smalian rule. However, logs are quite often decked together, which makes it impossible to obtain the middle diameters by direct measurement. In scaling decked logs of irregular lengths, both the two end diameters as well as the middle diameters are difficult to determine. When logs

are scaled in a log pond there is also an inconvenience of chopping bark to measure the diameter inside the bark. The two end diameters of the Smalian rule would generally be more accessible than the middle diameters, when logs are decked or boomed in a log pond. Since it is generally possible to scale at some point in the operation other than where logs are decked or located in a pond, the Huber formula would seem the most satisfactory of the two.

The one-in-eight rule and the one-in-ten rule would do away with the necessity of obtaining the middle diameter or the large end diameter by direct measurement. Each top diameter would be measured inside the bark and the arbitrary amount of taper would be applied for each rule. Therefore, even though logs were scaled in a mill pond, at the point of operation, or when decked, there would be no difficulty in estimating the middle diameters or the large end diameters. However, the accuracy of setting arbitrary taper correction values is questionable, since taper varies with different species, different side conditions, and the log's position in the individual tree.

Scientifically speaking Newton's rule will give volumes that are nearer the absolute volumes of logs than any of the other rules. However, for actual application in scaling, this rule would be impractical because it requires three different measurements. In the calculation of the data for this paper, the assumption was made that the volumes obtained by Newton's rule were exact. All log volumes determined by other rules were compared with Newton's rule to ascertain the deviation from the true volumes.

Errors Involved in Cubic Foot Scaling
by Proposed Rules

Discussion of Errors

Table I shows the volumes of an individual butt, top and intermediate log, picked at random from the entire group of 100 logs by each of the five rules. On the original record sheet presented herewith, there were 500 log volumes recorded; 100 recorded for each rule.

An examination of the entire data shows that errors in individual logs amount to as much as -17.0% by Huber's rule, +34% by Smalian's rule, -58.5% by the one-in-eight rule, and -61.4% by the one-in-ten rule. On 6 occasions the volume for each individual log was exactly the same by Huber's rule as it was by Newton's rule. The one-in-eight and one-in-ten rules showed no instance of having the same volume as the Newton rule.

Table II shows the aggregate volumes in cubic feet of butt, intermediate, and top logs by each rule. From this, Table III has been derived which shows the differences in aggregate volumes of the Huber, Smalian, one-in-eight, and one-in-ten rules from the Newton volumes. The Newton volumes were used as checks or absolute volumes. Table IV reveals the aggregate errors expressed as percentages of the aggregate Newton volumes. A study of this table is indicative of the value of each rule for cubic foot scaling.

The Huber Rule

This rule scaled the 26 butt logs 5.6 per cent low. Since Huber's rule considers the middle diameter only, it does not account for any additional volume due to butt swell. The flare generally found in butt logs would result in a larger actual volume than indicated by the rule.

In the application of this rule to 51 intermediate logs the scale was 0.5% high. In this case the logs were nearer the form of a paraboloid for which the Huber rule was intended. However, the average cross-sectional areas of the middle diameters were slightly in excess of the average diameter corresponding to that of the volume by Newton's formula.

The top logs scaled 2.1% low, which would indicate that the form of these logs tended towards that of a cone or neiloid rather than that of a paraboloid.

In aggregate the Huber rule was 2.2% low over all groups of logs. Because the proportion of intermediate logs to butt logs was 51 to 26, the aggregate error was between the butt log value and the intermediate log value. Of all the formulas tested the Huber rule was the most consistent over all the log groups.

The Smalian Rule

Smalian errors were twice as great as those for the Huber rule and in the opposite direction, +11.2% for 26 butt logs, -1.0% for 51 intermediate logs, and +4.4% for 23 top logs.

Since the Smalian formula is based on the average of the two end cross-sectional areas, the values derived from its use in butt logs would be in excess of the true volume. This is accounted for by reasoning that butt logs are more or less neloidal in shape and that the average of the two measurements over-estimates the diameter corresponding to the Newton volume.

In the application of this rule to intermediate logs and top logs, the average of the two end cross-sectional areas under-estimated the diameter corresponding to the Newton volume for intermediate logs and over-estimated the diameter for top logs.

It may be shown algebraically that the errors of the Smalian formula are twice as great and of the opposite sign to those of the Huber formula. The proof is developed below.

In terms of cross-sectional area and length the Newton, Huber and Smalian formulas may be expressed as follows:

$$V_n = \frac{1}{6} (A_b + 4A_m + A_t) L$$

$$V_h = A_m \times L$$

$$V_s = \frac{1}{2} (A_b + A_t) L$$

where V_n = Volume by Newton formula.

V_h = Volume by Huber formula.

V_s = Volume by Smalian formula.

L = Length of Log.

A_b , A_m , and A_t = Cross sectional area at base, midpoint, and top of log respectively.

It follows that

$$\begin{aligned} 6V_n &= (A_b + 4A_m + A_t) L \\ &= (A_b + A_t) L + 4A_m \times L \end{aligned}$$

The first term of the latter expression is equal to twice the Smalian volume, and the second term is equal to four times the Huber volume.

Therefore:

$$\begin{aligned} 6V_n &= 2V_s + 4V_h \\ 6V_n - 6V_h &= 2V_s - 2V_h \\ \text{and} \\ 6V_n - 6V_s &= -4V_s + 4V_h \end{aligned}$$

By division,

$$\begin{aligned} \frac{6V_n - 6V_h}{6V_n - 6V_s} &= \frac{2V_s - 2V_h}{-4V_s + 4V_h} \\ \frac{6(V_n - V_h)}{6(V_n - V_s)} &= \frac{2(V_s - V_h)}{-4(V_s - V_h)} \\ \frac{V_n - V_h}{V_n - V_s} &= \frac{-1}{2} \end{aligned}$$

The left hand portion of the latter equation represents the Huber error over the Smalian error, the ratio of which is $-1/2$ which was to have been shown.

The One-In-Eight Rule

This rule gave errors of +0.5% for butt logs, +0.3% for intermediate logs, and -23.0% for top logs. Small errors in the butt and intermediate logs indicate that a 1 inch in 8 feet taper is a satisfactory allowance for taper for estimating middle diameters and applying the Huber rule. However, the large error of -23.0% for top logs clearly reveals that these logs have a taper greater than 1 inch in 8 feet.

The aggregate error for all groups of logs by this rule was -1.4%, which is the lowest aggregate value of all the rules applied. Such a small error was made possible by the proportionately larger number of butt logs and intermediate logs than top logs. Also, there was a greater volume contained in the butt and intermediate logs than in the top logs.

The One-In-Ten Rule

In all groups of logs calculated by the one-in-ten rule, the values received were lower than the corresponding Newton values by 2.7% for butt logs, 3.7% for intermediate logs, and 28.6% for top logs. Consequently, this rule proves to be too conservative in estimating the large end diameters by allowing a taper of 1 inch in 10 feet. The largest error occurred in the top logs, where the taper was decidedly greater than 1 inch in 10 feet. As a result of the negative values over all groups of logs, the aggregate error (-5.2%) was greater than the aggregate values of all other formulas used.

Summary and Conclusions

Previous experience has proven that board foot rules are inadequate for scaling log volumes. Some standard unit is needed which will measure absolute log volumes regardless of the end products derived from the logs. This is especially true for logs used in the manufacture of bulk products. Although the cubic foot unit is considered to measure absolute volumes, it does not necessarily do so, since logs quite often deviate in form from the shape for which a cubic foot formula is intended. Nevertheless, the cubic foot unit is superior to the board foot unit and its adoption should be urged.

Various cubic foot rules have been proposed. The most promising of these from a practical standpoint are the Huber, Smalian, one-in-eight, and one-in-ten rules. In an investigation of the errors involved in applying the above rules, the Huber rule proved to be the most accurate considering all log groups in general. The one-in-eight rule was very accurate except in the top log group. Smalian rule gave errors twice as great as Huber values and of the opposite sign. Because the one-in-ten gave values lower than the Newton rule in all log groups, this rule can be discounted as an accurate method to use.

In logging operations where top logs are to be utilized for pulpwood or some other bulk product, the Huber rule should be used in preference to all other rules.

Further study should be carried out in determining errors involved in cubic foot scaling. Although the Newton formula is considered an accurate basis of determining deviations of other cubic foot rules, it does not measure the absolute volume of a log. It is true that this formula will measure the exact cubic content of any regular curved solid having a smoother surface, whether the solid be a neiloid, paraboloid, or cone. However, the butt logs give the greatest error due to the butt swell forming an irregular curved solid. It is recommended, for future study, that diameters be measured inside the bark at 4 foot intervals along each log. By averaging the cross-sectional areas of each diameter and computing the volume, a value can be obtained which will be absolute enough for scientific purposes.

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APPENDIX

TABLE I

Cubic Volumes of Individual Logs

	<u>Top Dia.</u>	<u>Middle Dia.</u>	<u>Butt Dia.</u>	<u>Newton</u>	<u>Huber</u>	<u>Smalian</u>	<u>1-in-8</u>	<u>1-in-10</u>
Butt Log	26.8	27.6	42.0	160.8	132.9	216.6	144.3	141.0
Intermediate Log	19.3	21.0	23.8	78.6	77.0	81.9	79.2	76.4
Top Log	7.5	14.4	16.8	34.0	36.1	29.5	15.8	14.6

TABLE II

Aggregate Cubic Foot Volumes by Log Groups

<u>Groups</u>	<u>No. Logs</u>	<u>Newton</u>	<u>Huber</u>	<u>Smalian</u>	<u>1-in-8</u>	<u>1-in-10</u>
Butt Logs	26	2255.5	2129.3	2507.3	2268.8	2195.9
Intermediate	51	2876.0	2890.5	2846.2	2884.9	2771.2
Top Logs	23	453.4	443.2	473.6	349.4	324.0
All Logs	100	5584.9	5463.0	5827.1	5503.1	5291.1

TABLE III

Errors in Cubic Foot Volume by Log Groups

<u>Groups</u>	<u>No. Logs</u>	<u>Huber</u>	<u>Smalian</u>	<u>1-in-8</u>	<u>1-in-10</u>
Butt Logs	26	-126.2	+251.8	+13.3	-59.6
Intermediate	51	+14.5	-29.8	+8.9	-104.8
Top Logs	23	-10.2	+20.2	-104.0	-129.4
All Logs	100	-121.9	+242.2	-91.4	-293.8

TABLE IV

Aggregate Cubic Foot Volume Errors in Percentages

<u>Groups</u>	<u>No. Logs</u>	<u>Huber</u>	<u>Smalian</u>	<u>1-in-8</u>	<u>1-in-10</u>
Butt Logs	26	-5.6%	+11.2%	+0.5%	-2.7%
Intermediate	51	+0.5%	-1.0%	+0.3%	-3.7%
Top Logs	23	-2.1%	+4.4%	-23.0%	-28.6%
All Logs	100	-2.2%	+4.4%	-1.4%	-5.2%

Newton Volume	Huber Volume Error	Smalian Volume Error	1" in 8' Rule Volume Error	1" in 10' Rule Volume Error				
Butt Logs								
94.1	86.8	-7.3	108.8	+14.7	88.9	-5.2	87.0	-7.1
66.8	67.0	+0.2	66.4	-0.4	65.7	-1.1	63.2	-3.6
112.3	109.9	-2.4	117.0	+4.7	114.4	+2.1	111.0	-1.3
145.4	136.8	-8.6	162.5	+17.1	134.9	-10.5	131.2	-14.2
93.6	93.1	-0.5	94.6	+1.0	101.4	+7.8	98.2	+4.6
84.2	78.4	-5.8	95.8	+11.6	78.4	-5.8	75.7	-8.5
86.3	76.2	-10.1	106.5	+20.2	91.5	+5.2	88.5	+2.2
77.5	71.2	-6.3	90.1	+12.6	86.0	+8.5	83.1	+5.6
160.8	132.9	-27.9	216.6	+55.8	144.3	-16.5	141.0	-19.8
113.8	111.7	-2.1	118.1	+4.3	118.5	+4.7	115.5	+1.7
49.4	47.5	-1.9	53.0	+3.6	54.5	+5.1	52.4	+3.0
32.0	31.3	-0.7	33.4	+1.4	35.2	+3.2	33.4	+1.4
95.5	95.5	0.0	95.3	-0.2	97.2	+1.7	94.1	-1.4
94.4	90.0	-4.4	103.4	+9.0	97.2	+2.8	94.1	-0.3
96.6	97.2	+0.6	95.3	-1.3	97.2	+0.6	94.1	-2.5
91.7	89.9	-1.8	95.3	+3.6	97.2	+5.5	94.1	+2.4
80.5	74.8	-5.7	91.8	+11.3	79.9	-0.6	77.1	-3.4
49.3	45.8	-3.5	56.4	+7.1	54.7	+5.4	52.4	+3.1
78.9	76.2	-2.7	84.1	+5.2	86.8	+7.9	83.9	+5.0
71.1	68.4	-2.7	76.4	+5.3	70.5	-0.6	67.9	-3.2
90.3	84.5	-5.8	101.9	+11.6	81.4	-8.9	78.6	-11.7
117.8	107.3	-10.5	138.8	+21.0	116.2	-1.6	112.8	-5.0
59.2	57.8	-1.4	62.0	+2.8	59.1	-0.1	56.7	-2.5
97.3	86.8	-10.5	118.4	+21.1	90.7	-6.6	87.8	-9.5
65.9	63.0	-2.9	71.6	+5.7	70.5	+4.6	67.9	+2.0
50.8	49.3	-1.5	53.8	+3.0	56.5	+5.7	54.2	+3.4
2255.5	2129.3	-126.2	2507.3	+251.8	2268.8	+13.3	2195.9	-59.6

Intermediate Logs

60.5	60.3	-0.2	60.8	+0.3	59.1	-1.4	56.7	-3.8
36.0	35.2	-0.8	37.5	+1.5	37.7	+1.7	35.8	-0.2
50.4	50.4	0.0	50.2	-0.2	59.7	+9.3	57.3	+6.9
85.5	86.8	+1.3	82.8	-2.7	82.9	-2.6	80.1	-5.4
50.7	50.4	-0.3	51.1	+0.4	43.6	-7.1	42.1	-8.6
27.0	27.3	+0.3	26.5	-0.5	29.5	+2.5	27.9	+0.9
99.7	100.5	+0.8	98.0	-1.7	95.5	-4.2	92.5	-7.2
76.3	78.4	+2.1	72.1	-4.2	78.4	+2.1	75.7	-0.6
57.1	59.7	+2.6	52.0	-5.1	50.4	-6.7	48.3	-8.8
73.5	74.1	+0.6	72.5	-1.0	73.3	-0.2	70.7	-2.8

Newton Volume	Huber Volume Error	Smalian Volume Error Intermediate	1" in 8' Rule Volume Error Logs	1" in 10' Rule Volume Error
45.6	46.4 +0.8	44.2 -1.4	38.2 -7.4	36.4 -9.2
61.0	62.3 +1.3	58.3 -2.7	65.0 +4.0	62.5 +1.5
39.4	40.3 +0.9	37.7 -1.7	31.8 -7.6	29.9 -9.5
70.9	71.2 +0.3	70.3 -0.6	78.4 +7.5	75.7 +4.8
54.1	55.3 +1.2	51.8 -2.3	50.4 -3.7	48.3 -5.8
59.3	57.8 -1.5	62.3 +3.0	66.4 +7.1	63.8 +4.5
46.4	46.4 0.0	46.4 0.0	50.4 +4.0	48.3 +1.9
113.0	112.6 -0.4	113.8 +0.8	119.4 +6.4	116.4 +3.4
77.0	75.5 -1.5	80.0 +3.0	71.0 -6.0	68.6 -8.4
89.0	89.1 +0.1	88.8 -0.2	91.2 +2.2	88.5 -0.5
60.2	63.0 +2.8	54.5 -5.7	42.9 -17.3	41.0 -19.2
37.1	37.7 +0.6	36.0 -1.1	38.6 +1.5	36.9 -0.2
23.7	24.3 +0.6	22.4 -1.3	26.8 +3.1	25.3 +1.6
77.8	79.2 +1.4	74.9 -2.9	83.0 +5.2	80.1 +2.3
60.3	61.0 +0.7	58.8 -1.5	61.7 +1.4	59.3 -1.0
74.6	75.5 +0.9	72.9 -1.7	78.4 +3.8	75.7 +1.1
52.1	53.4 +1.3	50.0 -2.1	45.2 -6.9	43.2 -8.9
77.8	79.2 +1.4	74.9 -2.9	82.9 +5.1	80.1 +2.3
60.3	61.1 +0.8	58.8 -1.5	61.7 +1.4	59.3 -1.0
72.6	72.6 0.0	72.5 -0.1	77.7 +5.1	75.0 +2.4
52.0	53.4 +1.4	49.2 -2.8	45.2 -6.8	43.2 -8.8
53.5	52.8 -0.7	54.9 +1.4	55.9 +2.4	53.6 +0.1
35.0	35.2 +0.2	34.6 -0.4	34.2 -0.8	32.4 -2.6
38.5	38.2 -0.3	39.1 +0.6	45.8 +7.3	43.7 +5.2
28.0	28.1 +0.1	27.8 -0.2	28.6 +0.6	27.0 -1.0
57.2	55.9 -1.3	59.7 +2.5	59.7 +2.5	57.3 +0.1
38.3	39.3 +1.0	36.3 -2.0	34.2 -4.1	32.4 -5.9
52.0	52.2 +0.2	51.5 -0.5	57.8 +5.8	55.5 +3.5
33.8	34.2 +0.4	32.9 -0.9	28.1 -5.7	26.6 -7.2
53.8	51.6 -2.2	58.2 +4.4	61.7 +7.9	59.3 +5.5
38.2	38.2 0.0	38.3 +0.1	36.7 -1.5	34.9 -3.3
78.6	77.0 -1.6	81.9 +3.3	79.2 +0.6	76.4 -2.2
53.9	55.3 +1.4	51.1 -2.8	48.1 -5.8	46.0 -7.9
39.1	39.3 +0.2	38.7 -0.4	40.3 +1.2	38.4 -0.7
63.7	64.3 +0.6	62.4 -1.3	61.7 -2.0	59.3 -4.4
37.8	37.2 -0.6	39.0 +1.2	38.2 +0.4	36.4 -1.4
51.1	49.8 -1.3	53.5 +2.4	62.3 +11.2	59.9 +8.8
39.1	38.7 -0.4	39.7 +0.6	39.3 +0.2	37.3 -1.8
30.9	29.5 -1.4	33.7 +2.8	31.3 +0.4	29.7 -1.2
82.2	82.9 +0.7	80.6 -1.6	80.7 -1.5	77.9 -4.3
50.4	50.4 0.0	50.3 -0.1	44.7 -5.7	42.6 -7.8
2876.0	2890.5 +14.5	2846.2 -29.8	2884.9 +8.9	2771.2 -104.8

Newton Volume	Huber Volume Error	Smalian Volume Error	1" in 8' Rule Volume Error	1" in 10' Rule Volume Error
Top Logs				
24.6	22.3 -2.3	29.3 +4.7	17.4 -7.2	15.9 -8.7
18.7	17.5 -1.2	21.1 +2.4	14.1 -4.6	13.1 -5.6
11.7	11.5 -0.2	12.2 +0.5	7.1 -4.6	6.4 -5.3
25.7	25.1 -0.6	26.7 +1.0	21.1 -4.6	19.8 -5.9
25.3	24.3 -1.0	27.2 +1.9	22.2 -3.1	20.9 -4.4
30.6	28.6 -2.0	34.5 +3.9	17.4 -13.2	16.2 -14.4
20.5	19.6 -0.9	22.4 +1.9	23.1 +2.6	21.7 +1.2
15.2	15.4 +0.2	14.7 -0.5	16.7 +1.5	15.6 +0.4
33.8	36.2 +2.4	29.0 -4.8	14.1 -19.7	13.1 -20.7
20.0	18.9 -1.1	22.3 +2.3	15.8 -4.2	14.6 -5.4
34.0	36.1 +2.1	29.5 -4.5	15.8 -18.2	14.6 -19.4
20.0	18.8 -1.2	22.3 +2.3	15.7 -4.3	14.6 -5.4
16.0	16.4 +0.4	15.1 -0.9	15.8 -0.2	14.6 -1.4
6.4	6.3 -0.1	6.7 +0.3	7.4 +1.0	6.6 +0.2
15.5	15.8 +0.3	14.9 -0.6	9.0 -6.5	8.2 -7.3
16.8	16.4 -0.4	17.7 +0.9	13.5 -3.3	12.5 -4.3
22.3	20.7 -1.6	25.4 +3.1	20.4 -1.9	19.0 -3.3
22.9	23.9 +1.0	20.8 -2.1	17.4 -5.5	16.2 -6.7
12.0	11.2 -0.8	13.7 +1.7	7.4 -4.6	6.6 -5.4
15.5	14.1 -1.4	18.3 +2.8	13.5 -2.0	12.5 -3.0
20.1	20.0 -0.1	20.5 +0.4	17.8 -2.3	16.6 -3.5
12.2	11.5 -0.7	13.7 +1.5	12.6 +0.4	11.6 -0.6
13.6	12.6 -1.0	15.6 +2.0	14.1 +0.5	13.1 -0.5
453.4	443.2 -10.2	473.6 +20.2	349.4 -104.0	324.0 -129.4