AN ABSTRACT OF THE DISSERTATION OF

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This dissertation addresses issues of strategic behavior of firms in oligopoly markets. In the first study we analyze how generic advertising affects brand advertising and firm profits in differentiated oligopoly markets. We develop two models, one with vertical differentiation and another with horizontal differentiation. In the case of vertical differentiation, we amend Crespi's (2007) model to show that only the high quality firm will use brand advertising. We also show that when differentiation is horizontal, the equilibrium is likely to be more symmetric in terms of each firm's profits, spending on brand advertising, and response to generic advertising. We also demonstrate that generic advertising will increase expenditures on brand advertising when firms play a supermodular game.

In the second study, we analyze the interaction between generic advertising, brand advertising, and firm profits when products are differentiated either vertically or horizontally and brand advertising is purely informative. That is, brand advertising lowers consumer search costs of identifying brand characteristics. The model demonstrates that firms can benefit from investing in brand advertising that lowers consumer search costs as well as from brand advertising that is purely persuasive. In addition, the results demonstrate that whether brand advertising is persuasive or informative, the outcome is more likely to be symmetric with horizontal differentiation than with vertical differentiation. This study shows that brand advertising is a strategic complement when persuasive and a strategic substitute when informative.

In the third study, we allow the choice of strategic variable, output and price, to be endogenous to the firm. We consider the case where one firm chooses output and the other firm chooses price, which we call a Cournot-Bertrand model. We provide a real world example of this "Cournot-Bertrand" behavior and show that the outcome can be a Nash equilibrium. Allowing the timing of play (early or late) as well as the strategic variable (output or price) to be endogenous, we demonstrate an outcome where one firm competes in output and the other firm competes in price can be a subgame perfect Nash equilibrium.

Essays on Strategic Behavior in Oligopoly Markets: Advertising, Output, and Price Competition

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I understand that my dissertation will become part of the permanent collection of Oregon State University library. My signature below authorizes release of my dissertation to any reader upon request.

Kosin Isariyawongse, Author

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CONTRIBUTION OF AUTHORS

Dr. Victor Tremblay provided ideas and assistance in all aspects of this dissertation. Dr. Carol Tremblay contributed ideas and helped with the implementation of the third manuscript. Yasushi Kudo helped with modeling in the first manuscript.

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Chapter 1

General Introduction

This dissertation addresses issues of strategic behavior of firms in imperfectly competitive markets. We investigate the interaction of generic advertising and brand advertising when brand advertising is purely persuasive and when it is purely informative. Motivated by a real world example, we also develop a hybrid "Cournot-Bertrand" model where one firm competes in output and the other competes in price in both static and dynamic settings.

In Chapter 2, we analyze how generic advertising affects brand advertising and firm profits in differentiated oligopoly markets where brand advertising is persuasive. There is a major issue pertaining to markets with generic advertising where some producers argue that generic advertising is harmful to them because it will reduce perceived product differentiation and thus will make differentiated products look similar to consumers. Using duopoly models with vertical and horizontal product differentiation, we argue that this is not necessarily correct. We also establish conditions under which firms play a supermodular game (Milgrom and Roberts, 1990). In this setting, comparative static results emerge from a relatively general model so we can analyze the relationship between generic and brand advertising in markets with n firms regardless of the type of product differentiation.

In Chapter 3, we analyze the interaction between generic advertising, brand advertising, and firm profits where brand advertising is purely informative. The literature on the economics of advertising distinguishes between informative advertising, which informs consumers of a product's objective characteristics, and persuasive advertising, which is designed to change consumer tastes in favor of the advertised brand. To date, none of the research has considered a market with a generic advertising program where brand advertising is purely informative. Ward (2006) argues that generic advertising is primarily informative, and the same can be true for brand advertising. We develop duopoly models with vertical and horizontal differentiation when brand advertising lowers consumer search costs of identifying brand characteristics. The key distinction between models is that brand advertising is a strategic complement when persuasive and a strategic substitute when informative.

Chapter 4 studies the strategic interaction among firms focusing on the choice of strategic variable and the timing of play. Cournot and Bertrand are classic models of Oligopoly. Cournot derived the Nash equilibrium to a static duopoly game where firms compete in quantity. Bertrand analyzed the same model except that the choice variable is price instead of output. Both authors treat the strategic variable as exogenous and assume that the strategic variable (price or quantity) is the same for both firms. We develop a model where the choice of strategic variable is endogenous to the firm, allowing each firm to compete in either output or price. We also provide a real world example of this "Cournot-Bertrand" behavior and show that the outcome can be a Nash equilibrium in the static setting. In our model where the timing of play (early or late) as well as the strategic variable (output or price) are endogenous to the firm, we demonstrate that an outcome where one firm competes in output and the other firm competes in price can be a subgame perfect Nash equilibrium.

This dissertation analyzes the strategic actions of firms. This work is important because it provides a better understanding of generic and brand advertising and the feasibility and desirability of such programs in the future. In addition, this study develops a model that provides reasons why firms choose different strategic variables.

Chapter 2

Generic and Brand Advertising in Markets with Product Differentiation [1]

2.1 Introduction

Most of research on economics of advertising focuses on brand advertising in imperfectly competitive markets. Previous work has considered markets with differentiated products, whether real or subjective, and has clearly developed models to explain the mechanism by which advertising affects consumer choice. That is, advertising may change tastes through persuasive means or provide consumers with useful information that reduces the search cost of finding a brand with desirable characteristics. It may also serve as a complement to output by creating a desirable image or by raising the social status of the product. This body of work explains and predicts how brand advertising might affect firm behavior and the welfare of society.[2]

Research on the economics of generic commodity advertising and its relationship to brand advertising has just begun. Generic advertising is common in markets for agricultural commodities or processed foods, where producers frequently cooperate to supply a joint advertising campaign. Such campaigns are

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[1] Part of this chapter is published in *Journal of Agricultural & Food Industrial Organization*: 5 (6), 2007, 1-15.

[2] See Bagwell (2005) for an excellent review of the literature on brand advertising, and see Stivers and Tremblay (2005) for a review of the welfare effect of brand advertising.

commonly financed through an institutional structure known as a commodity checkoff program that impose a mandatory assessment on producers in the form of a sales or per-unit tax.[3] Marketing boards within the program develop and promote advertising campaigns designed to emphasize the universal characteristics of the product and increase market demand. When products are perfectly homogeneous, such mandatory programs avoid the free-rider problem and distribute program benefits equitably among producers.[4]

In markets with commodity checkoff programs, it is becoming more and more common for major producers to use brand advertising to differentiate their products. This raises questions about the relationship between generic and brand advertising. It also provides one reason for lawsuits by almond, peach, mushroom, plum, beef, and pork producers over mandatory generic advertising programs (Chakravarti and Janiszewski, 2004). In these markets, leading producers that have invested heavily in brand advertising oppose mandatory programs because they fear that generic advertising provides a disproportionate benefit to non-branded producers. This can occur, for example, if generic advertising causes consumers to believe that branded and non-branded goods are of like quality. If true, such inequalities are a concern to marketing boards, as one

[3] Most assessments are based on a per-unit basis and constitute less than 1 percent of the dollar value of the good (Ward, 2006).
[4] For a more complete description of commodity checkoff programs and generic advertising, see Chakravarti and Janiszewski (2004), Chung et al. (2006), Crespi and McEowen (2006), Ward (2006), and Williams and Capps (2006).

of their goals is to assure that generic advertising produces an equitable distribution of benefits among producers (Ward, 2006).

These issues have motivated a series of recent theoretical papers on the economics of generic and brand advertising. Notable examples include the research by Crespi and Marette (2002), Hunnicutt and Israelsen (2003), Bass et al. (2005), and Crespi (2007).

Although the theoretical models developed in these papers make important contributions to our understanding of generic and brand advertising, they either analyze limiting cases or make substantive errors. Hunnicutt and Israelsen (2003) develop a useful model of generic and brand advertising in a monopolistically competitive industry. Their model clearly shows how the free-rider problem associated with firm advertising diminishes with product differentiation and demonstrates that the industry's optimal level of generic advertising diminishes when products become more differentiated.

The main limitation of the Hunnicutt and Israelsen model is that the type of product differentiation characterized by monopolistic competition is not always consistent with that found in agricultural and other food markets. It assumes that consumer preferences are symmetric and that one brand is an equally good substitute for any other brand.[5] Archibald and Rosenbluth (1975) argue that this type of differentiation is most likely to occur in markets where the characteristic space is very large. In agricultural and food markets, however, brands compete on a limited number of characteristics. These might include quality (e.g., premium versus generic brands of bananas, almonds, soft drinks, etc.) or a simple taste characteristic (e.g., sweet versus tart apples). In addition, because product demand does not derive directly from consumer utility functions, the model does not explain why consumers respond to advertising.[6]

The paper by Bass et al. (2005) uses optimal control methods to analyze the effects of generic and brand advertising in a duopoly market. Each firm sets its price, generic advertising level, and brand advertising level. Commodity checkoff programs are assumed not to exist. The main conclusions are that generic advertising suffers from the free-rider problem and that a firm's market share is determined primarily by brand advertising. Like Hunnicutt and Israelsen, the Bass et al. model does not explain why consumers respond to advertising. It is also of limited use when analyzing issues important to agricultural markets because the model assumes that commodity checkoff programs do not exist and that the effects of generic and brand advertising are separable.

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[5] For a discussion of the form of product differentiation in monopolistic competition, see Beath and Katsoulacos (1991).
[6] As Bagwell (2005, p. 3) indicates, "An economic theory of advertising can proceed only after this question is confronted." For example, does advertising lower consumer search costs or change consumer tastes.

The papers by Crespi and Marette (2002) and by Crespi (2007) are related, so we discuss them together. Both start with models of consumer preferences, which explicitly show how generic and brand advertising affect utility (by changing tastes through persuasion) and formally characterize product differentiation as being vertical (i.e., there are real and subjective quality differences between brands). Firms play a three-stage game: (I) a marketing board sets the assessment rate,[7] (II) firm(s) choose brand advertising levels, and (III) firms choose prices. Backwards induction is used to identify the sub-game perfect Nash equilibrium. In the Crespi and Marette model, the goal of the marketing board is to choose an assessment rate (or the level of generic advertising) to maximize industry profits. The model assumes that a single high quality firm uses brand advertising; all other firms produce homogeneous goods of low quality and cannot use brand advertising.

In the more recent model by Crespi, there are two firms, one with a high and the other with a low quality brand, and both can use brand advertising. To facilitate comparative static analysis in the recent Crespi model, firms are assumed to have no control over the assessment rate (g). The models in both papers demonstrate that generic advertising may influence subjective product differentiation and benefit the low quality firm more than the high quality firm(s).

[7] Because the level of generic advertising (G) is defined as the assessment rate (g) times total industry output, determining the optimal g also determines the optimal G at the Nash equilibrium level of output.

This is an important result that is consistent with concerns raised by many brand name producers of agricultural products about the adverse effects of generic advertising.

In spite of their contributions, however, the Crespi and Marette and the Crespi models suffer from several weaknesses. They both ignore the fact that generic advertising may be informative rather than persuasive. According to at least one expert (Ward, 2006, p. 55), "Generic advertising is all about information – information about a specific commodity and its underlying characteristics."[8] The Crespi and Marette model assumes that low quality producers cannot use brand advertising, a constraint that does not generally exist in real world markets and an assumption that may or may not be consistent with optimal behavior.

Although this constraint is relaxed in the more recent Crespi paper, the new model is limited in other ways. First, it is built from two assumptions that are inconsistent: that generic advertising attracts new customers to the market and that the number of consumers is fixed (i.e., the market is covered).[9] Second, his conclusion that the low quality firm will choose a positive level of brand advertising is incorrect. The firm's first-order condition with respect to brand

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[8] Even the popular "Got Milk" ads, which are designed primarily to capture attention, provide some information. For example, in magazine ads Batman states that "milk's 9 essential nutrients" give him strength; Superman says that calcium in milk makes strong bones; the recording artist, Alondra, says that "Milk provides potassium, minerals, and vitamins needed for growth."

[9] We show in the next section of the paper, that this inconsistency can be rectified by assuming that generic advertising has an informative component.

advertising is always negative (equation 6), implying that the low quality firm will never use brand advertising. This is a standard result in models of brand advertising and vertical product differentiation (e.g., Tremblay and Martins-Filho, 2001; Tremblay and Polasky, 2002).

In the sections that follow, we avoid some of the weaknesses found in previous studies and derive new results concerning the relationship between generic and brand advertising. As in Crespi (2007), our purpose is to show how generic advertising affects the brand advertising behavior and profitability of firms in differentiated oligopoly markets. Unlike previous studies, we consider models with horizontal as well as vertical product differentiation. We also show how the notion of supermodularity aids in our understanding of the relationship between generic and brand advertising.

2.2 A Duopoly Model with Vertical Differentiation

We begin by developing a duopoly model with vertical product differentiation, as in Crespi (2007).[10] Brands produced by firms *1* and *2* differ in quality, indexed by k, and firm *1* is defined to be the high quality firm (i.e., $k_1 > k_2 > 0$). Real differences in quality are assumed to be exogenously determined, which can occur if firm *1* has more favorable weather conditions in agricultural

[10] To aid comparison, we use the same notation except for the consumer taste parameter. In the Crespi model, θ is the vertical taste parameter. We choose to use ϕ as the vertical taste parameter and θ as the horizontal taste parameter.

production or some other idiosyncratic advantage that cannot be replicated by its competitor.

We use an indirect utility function, developed by Mussa and Rosen (1978), to describe consumer preferences. When prices are the same, all consumers prefer the high quality brand, but consumer strength of preference or willingness to pay for quality varies by person. This strength of preference is captured by the taste parameter ϕ. Tastes are distributed over the interval [0, 1], with *N* consumers dispersed uniformly over the taste interval. Thus, the indirect utility function for brand *i* (*1* or *2*) is $V_i = y + \phi k_i - P_i$, where *y* is consumer income and P_i is the price of brand *i*. Given prices and quality levels, preferences are illustrated in Figure 2.1. Notice that consumers with relatively high values of quality (i.e., a high ϕ relative to that of the marginal consumer, ϕ_M) prefer brand *1* and consumers with relatively low values prefer brand *2*.

Demand depends on consumer preferences, income, product quality, and market prices. To simplify the derivation of demand functions, we assume that consumers have unit demands and that the market is covered (i.e., each consumer buys one unit of either brand *1* or brand *2*).[11] As is evident from Figure 2.1, demand for brand *1* is $D_1 = N(1 - \phi_M)$, and demand for brand *2* is $D_2 = N\,\phi_M$.

[11] As Crespi (2007, footnotes 5 and 8) indicates, this makes the model more tractable and does not appreciably alter the main results. See Wauthy (1996) and Tremblay and Martins-Filho (2001) for discussion of models with uncovered markets, vertical differentiation, and brand advertising.

Evaluating ϕ when $V_1 = V_2$ identifies ϕ_M, which equals $(P_1 - P_2)/(k_1 - k_2)$. Thus demand functions for brands *1* and *2* are

$$D_1(P_1, P_2, k_1, k_2) = N\left[1 - \left(P_1 - P_2\right)/k\right], \tag{2.1}$$

$$D_2(P_1, P_2, k_1, k_2) = N\left(P_1 - P_2\right)/k\ , \tag{2.2}$$

where $k \equiv k_1 - k_2$ or the degree of vertical product differentiation.

Firms compete in prices and can use advertising to persuade consumers that the advertised brand is of higher quality. This can be accomplished by changing consumer tastes or by creating a premium image that becomes tied to the product. This form of advertising creates subjective product differentiation, as it only affects consumer perceptions of product quality or desirability. Pure image creating advertising can be seen in the market for premium cola, where Coke's marketing themes emphasize family values, while Pepsi's are designed to appeal to a younger, more rebellious generation. Similarly, in the early 1990s Anheuser-Busch created a blue-collar image for its Budweiser brand of beer, while Coors created a white-collar image for its flagship brand (Tremblay and Tremblay, 2005). Examples more relevant to subjective vertical differentiation include the Chiquita bananas and Bayer aspirin, brands that are heavily advertised to create a premium or high quality image.[12]

To distinguish this type of advertising from generic commodity advertising, it is called branded or brand advertising. In this model, a firm can use brand advertising (B_i) to increase consumer utility by enhancing the perceived quality of its brand. That is, $k_i = k_i(\kappa_{0i}, B_i)$, where κ_{0i} is the level of brand i's objective quality, $\kappa_{01} > \kappa_{02} > 0$. Brand advertising increases perceived quality, such that $\partial k_i/\partial B_i > 0$, and $\partial^2 k_i/\partial B_i^2 < 0$.

In the early stages of market evolution there is no real or subjective difference between brands (i.e., k is close to 0), making generic advertising a worthwhile way of avoiding the free-rider problem associated with product advertising. Through a commodity checkoff program, firms are forced to fund generic advertising, financed by a per-unit assessment rate, g, imposed on each firm by a marketing board. Institutional inertia keeps the program in place, even as brand advertising begins to create subjective differentiation.[13]

In order to compare our results with those of Crespi (2007), we start by assuming that the market is covered and that generic advertising can increase the

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[12] For greater discussion of these and other examples where advertising creates subjective product differentiation, see Tremblay and Polasky (2002).

[13] As brand advertising becomes more prominent and enhances perceived differentiation, Hunnicutt and Israelsen (2003) show that generic advertising will diminish when voluntary. Once in place, however, the evidence shows that the legal cost of rescinding a mandatory commodity checkoff program is high (Chung et al., 2006; Crespi and McEowen, 2006; Crespi 2007).

number of consumers. Then, we overcome this inconsistency by assuming that generic advertising has an informative as well as a persuasive component. Regarding information, assume that the market consists of two sets of people: (1) those who know of a product's existence and (2) those who do not know of a product's existence (e.g., an unusual fruit such as lychee). If consumers are defined as informed people, the market could be covered in that all consumers purchase one or another brand of lychee. The informative component of generic advertising then attracts new people to the market, increasing N; the persuasive component enhances subjective differentiation, increasing k_i. Thus, $N = N(g)$ and $k_i = k_i(\kappa_{0i}, B_i, g)$, such that $\partial N/\partial g > 0$, $\partial^2 N/\partial^2 g < 0$, $\partial k_i/\partial g > 0$, and $\partial^2 k_i/\partial g^2 < 0$.

At issue is the effect of generic advertising on each firm's brand advertising and profit levels under two scenarios. The first scenario has a symmetric effect on perceived quality, and the second raises the perceived quality of brand *2* relative to brand *1*.[14]

Scenario 1: Generic advertising has a symmetric effect on brand quality. This implies that g attracts new consumers but has no effect on the quality gap or the degree of vertical differentiation (i.e., $\partial k_1/\partial g = \partial k_2/\partial g > 0$ or $\partial k/\partial g = 0$).

[14] A third scenario is also possible, one where g increases vertical differentiation (i.e., $\partial k/\partial g > 0$). As this is a non-issue with generic advertising, we ignore this case. If g were to increase product differentiation, whether differentiation is vertical or horizontal, both firms would benefit from and support commodity checkoff programs.

Scenario 2: Generic advertising enhances the quality of brand *2* relative to the quality of brand *1*. In this case, generic advertising attracts new customers and lowers vertical differentiation (i.e., $\partial k_2/\partial g > \partial k_1/\partial g > 0$ or $\partial k/\partial g < 0$).

In order to focus on strategic issues, firm cost functions are very simple. Unit production costs are assumed to be the same for both firms and are normalized to 0.[15] Costs include only marketing expenditures, resulting in the following profit equation for firm $i = 1, 2$:

$$\pi_i = (P_i - g)Q_i - B_i, \qquad (2.3)$$

where $Q_i \equiv D_i$. Firms are assumed to play a three-stage game. In the first stage, the marketing board sets g.[16] In the second stage, firms compete in brand advertising. In the final stage, they compete in price. Firms are assumed to have perfect and complete information. That is, each firm knows the profits of each player and structure of the game (Gibbons, 1992).

We use backwards induction to obtain the sub-game perfect Nash equilibrium to the game, which produces a Nash equilibrium in each sub- or stage-game. At each stage, we assume that a unique equilibrium exists. Working backwards, the Nash equilibrium prices and profits in the final stage are

[15] Thus, price in this model can be though of as the markup of price over the marginal cost of production.
[16] Because we are only interested in comparative static analysis and not in obtaining a closed form solution, the objective of the marketing board is ignored. This is consistent with Crespi (2007).

$$P_1^* = \frac{2}{3}(k_1 - k_2) + g\,, \tag{2.4}$$

$$P_2^* = \frac{1}{3}(k_1 - k_2) + g\,, \tag{2.5}$$

$$\pi_1^* = \frac{4}{9}(k_1 - k_2)N - B_1\,, \tag{2.6}$$

$$\pi_2^* = \frac{1}{9}(k_1 - k_2)N - B_2\,. \tag{2.7}$$

With perfect and complete information, firms are able to look forward and reason back to forecast Nash prices and profits in the final stage of the game. Given this information, the first-order conditions in the second stage are

$$\frac{\partial \pi_1^*}{\partial B_1} = \frac{4}{9}\frac{\partial k_1\left(B_1^*, g\right)}{\partial B_1}N - 1 = 0\,, \tag{2.8}$$

$$\frac{\partial \pi_2^*}{\partial B_2} = -\frac{1}{9}\frac{\partial k_2\left(B_2^*, g\right)}{\partial B_2}N - 1 < 0\,. \tag{2.9}$$

Notice that firm *1* will use brand advertising as long as the marginal benefits are sufficiently high. Equation (2.9) will always be negative, however, implying that the optimal value of firm *2*'s brand advertising is 0. Thus, in Nash equilibrium firm *1* will choose a positive level of brand advertising and firm *2* will not advertise at all ($B_1^* > 0$ and $B_2^* = 0$). This result is consistent with the assumption made in the Crespi and Marette (2002) model, but firm *2*'s first-order condition is misinterpreted in Crespi (2007).

With vertical differentiation, it makes intuitive sense that only the high quality firm will use brand advertising, because advertising that increases product differentiation will dampen competition and raise prices [see equations (2.4) and (2.5)]. Because k is defined as $k_1 - k_2$, firm *1*'s advertising increases k by raising k_1, and firm *2*'s advertising lowers k by raising k_2. Because firm *2*'s brand advertising is costly and lowers vertical differentiation, it is optimal for firm *2* not to advertise.[17]

Our first issue of interest is the effect of generic advertising on Nash equilibrium levels of brand advertising. Because B_2^* equals zero, g has no effect on firm *2*'s brand advertising, and firm *1*'s first-order condition is not a function of B_2.[18] Applying the implicit-function theorem to equation (2.8), which is identically equal to zero at B_1^*, produces

$$\frac{\partial B_1^*}{\partial g} = -\frac{\dfrac{\partial^2 k_1}{\partial B_1 \partial g}}{\dfrac{\partial^2 k_1}{\partial B_1^2}} - \frac{\dfrac{\partial k_1}{\partial B_1}\dfrac{\partial N}{\partial g}}{\dfrac{\partial^2 k_1}{\partial B_1^2}N}. \tag{2.10}$$

[17] This result is driven by the assumptions of vertical product differentiation and a uniform distribution of consumers. If, for example, the majority of consumers are clustered near k_1, then it may be worthwhile for firm *2* to use brand advertising to position its brand closer to k_1.

[18] Crespi performed comparative static analysis assuming that $B_2^* > 0$. If this were true, his analysis would still be in error, because it ignores the fact that the optimal values of brand advertising are embedded in the system of first-order conditions. The proper procedure is to implicitly differentiate both first-order conditions with respect to g and then use Cramer's rule to obtain comparative static results, which will depend upon the second-order conditions of profit maximization and the condition required for the Nash equilibrium to be stable (Bulow et al. 1985 and Baldani et al., 2005, Chapter 6). We use this technique in the next section of the paper.

Crespi correctly points out that the sign of $\partial B_1^*/\partial g$ is indeterminate, does not depend upon the scenario, and does depend critically upon the sign of $\partial^2 k_1/(\partial B_1 \partial g)$. If the only effect of generic advertising is to increase the size of the market, then $\partial^2 k_1/(\partial B_1 \partial g) = 0$ and generic advertising causes firm *1* to increases its expenditures on brand advertising. The result that an increase in the size of a market leads to an increase in endogenous sunk costs such as brand advertising is standard in the literature (Sutton, 1991). It also verifies Crespi's (p. 8) point that just because generic advertising leads to an increase in brand advertising does not necessarily imply that generic advertising lowers product differentiation. In addition, if generic advertising increases the marginal returns associated with brand advertising [i.e., $\partial^2 k_1/(\partial B_1 \partial g) > 0$], then an increase in generic advertising will also lead to an increase in firm *1*'s brand advertising. In this case, g and B_1 are said to be strategic complements (Bulow et al., 1985).

Next, we explore the effect of generic advertising on each firm's second-stage optimal profit functions (π_i^{**}). Differentiating equations (2.6) and (2.7) when brand advertising is set to its optimal level produces

$$\frac{\partial \pi_1^{**}\left(B_1^*, B_2^*, g\right)}{\partial g} = \frac{4}{9}\left[\left(k_1^* - k_2^*\right)\frac{\partial N}{\partial g} + \left(\frac{\partial k_1^*}{\partial g} - \frac{\partial k_2^*}{\partial g}\right)N\right], \tag{2.11}$$

$$\frac{\partial \pi_2^{**}\left(B_1^*, B_2^*, g\right)}{\partial g} = \frac{1}{9}\left[\left(k_1^* - k_2^*\right)\frac{\partial N}{\partial g} + \left(\frac{\partial k_1^*}{\partial g} - \frac{\partial k_2^*}{\partial g} + \frac{\partial k_1^*}{\partial B_1}\frac{\partial B_1^*}{\partial g}\right)N\right], \tag{2.12}$$

where $k^* = (k_1^* - k_2^*)$. This differs from the Crespi result, because $\partial B_2^*/\partial g$ is correctly set to zero as discussed above. The implications of these results can be seen more clearly by considering three cases. First, consider the case where generic advertising attracts new customers but has no effect on brand advertising ($\partial B_1^*/\partial g = 0$). This implies that

$$\frac{\partial \pi_1^{**}\left(B_1^*, B_2^*, g\right)}{\partial g} = \frac{4}{9}\left[\left(k_1^* - k_2^*\right)\frac{\partial N}{\partial g} + \left(\frac{\partial k_1^*}{\partial g} - \frac{\partial k_2^*}{\partial g}\right)N\right], \qquad (2.13)$$

$$\frac{\partial \pi_2^{**}\left(B_1^*, B_2^*, g\right)}{\partial g} = \frac{1}{9}\left[\left(k_1^* - k_2^*\right)\frac{\partial N}{\partial g} + \left(\frac{\partial k_1^*}{\partial g} - \frac{\partial k_2^*}{\partial g}\right)N\right]. \qquad (2.14)$$

Under scenario 1 where $\partial k^*/\partial g = 0$, generic advertising increases the profits of both firms by increasing market demand (i.e., $\partial N/\partial g > 0$). Under scenario 2 where $\partial k^*/\partial g < 0$, there is a tradeoff between the market demand effect (i.e., $\partial N/\partial g > 0$), which increases the profits of both firms, and the product differentiation effect (i.e., $\partial k^*/\partial g < 0$), which lowers the profits of both firms. The dominant effect will determine the influence of generic advertising on firm profits.

In the second case, consider the comparative static results in equations (2.11) and (2.12) when $\partial B_1^*/\partial g > 0$. In this case, our predictions are different from those of Crespi. Under scenario 1, where $\partial k^*/\partial g = 0$, generic advertising benefits both firms. Under scenario 2 where $\partial k^*/\partial g < 0$, the results are indeterminate for both firms. This setting is most likely to produce an outcome where firm *1*'s profits fall and firm *2*'s profits rise. This could occur if generic advertising sufficiently lowers product differentiation (lowering profits of both firms) and sufficiently induces firm *1* to increase spending on brand advertising (raising profits of firm 2 relative those of firm *1*).

In the third case where $\partial B_1^*/\partial g < 0$, our comparative static results are the same as those found in the Crespi model. Generic advertising benefits firm *1* under scenario 1. Otherwise the effect on firm profits is indeterminate.

Our amended version of the vertically differentiated model produces several important results. First, only the high quality firm uses brand advertising, a common feature in such markets as bananas, almonds, and aspirin where branded goods are heavily advertised and generic products are not advertised at all. Second, generic advertising is more likely to be beneficial to both firms when it attracts new customers, does not lower subjective product differentiation, and causes the high quality firm to use more brand advertising. Third, the low quality firm is likely to benefit and the high quality firm to be harmed by generic advertising when generic advertising sufficiently lowers subjective product differentiation and causes firm *1* to sufficiently increase spending on brand advertising.

2.3 A Duopoly Model with Horizontal Differentiation

Next, we develop a duopoly model that differs from the model above only in that differentiation is horizontal rather than vertical. To do this, we use a simple linear-city or address model (Hotelling, 1929, d'Aspremont et al., 1979). Brands *1* and *2* differ in a single horizontal characteristic, $\theta \in [\theta_1, \theta_2]$ and $0 \leq \theta_1 < \theta_2$. There are *N* consumers with preferences over θ who are uniformly distributed over the interval θ_1-θ_2. A consumer's ideal level of θ identifies the consumer's type or location. Unlike the case with vertical differentiation, consumers disagree over which value of θ is ideal or most preferred.

The market for breakfast cereal provides an example where there are real horizontal differences among brands. To illustrate, consider a market with just two brands, unsweetened corn flakes (brand *1*, located at $\theta = 0$) and sweetened corn flakes (brand *2*, located at $\theta = 1$). If $P_1 = P_2$, then consumers who prefer a sweeter cereal (with preference locations $½ < \theta \leq 1$) will prefer brand *2* and consumers who prefer a cereal that is less sweet (with preference locations $0 \leq \theta < ½$) will prefer brand *1*.

The premium cola market provides an example of a market where horizontal differentiation is subjective or perceived. Following Tremblay and Polasky, 2002, assume two brands, Coke (brand *1*) and Pepsi (brand *2*). Without advertising $\theta_1 = \theta_2 = ½$. Brand advertising can create subjective differentiation by producing distance between θ_1 and θ_2, at least in the eyes of the consumer. As discussed above, θ might index the degree of youth appeal. Aware of this characteristic, Coke has responded by using brand advertising to lower θ_1, and Pepsi has responded by using brand advertising to raise θ_2. Although advertising is expensive, benefits accrue to both firms because increased product differentiation dampens price competition.

This model may also apply to agricultural products where one brand is organic and the other is not. Although many consumers may prefer organic, others may prefer non-organic foods. The latter group may not believe that organic foods are superior and may be concerned that organic brands are linked to a liberal, environmental image.[19] Thus, the presence of organic and non-organic brands creates horizontal differentiation over an environmental characteristic. In such a market, generic advertising may exist to boost market demand, while individual firms use brand advertising to create a pro- or anti-organic/environmental image.

To parallel the vertical differentiation case, we use an indirect utility function to characterize consumer preferences when brands are horizontally differentiated. In the linear city model, the indirect utility function for a particular consumer considering brand i is $V_i = y - P_i - t\, d_i$, where $t > 0$ is the disutility

[19] For a discussion of this issue, see the web page of The Food Standards Agency (www.food.gov.uk).

associated with purchasing a brand that is not ideal and d_i is the distance from the consumer's ideal brand (i.e., the consumer's location or type) to the θ associated with brand i (θ_i). Figure 2.2 illustrates this case assuming brands *1* and *2* are located at θ_1 and θ_2. Notice, for example, that a consumer located at θ_M is a distance of d_1 from θ_1 and a distance of d_2 from θ_2.

As before, the market is covered and consumers have unit demands. The market demand for each brand depends on the location of the marginal consumer (θ_M), located where $V_1(\theta_M) = V_2(\theta_M)$. Assuming that a firm's horizontal location is arbitrary and that $0 \leq \theta_1 \leq ½ \leq \theta_2 \leq 1$, the marginal consumer is defined as

$$\theta_M = \left[t\left(\theta_1 + \theta_2\right) - P_1 + P_2 \right] / 2t. \tag{2.15}$$

With N consumers located within the preference interval, the demand functions are

$$D_1\left(P_1, P_2, \theta_1, \theta_2\right) = Nd_1 = N\left(\theta_M - \theta_1\right) = N\left[\frac{t\left(\theta_2 - \theta_1\right) - P_1 + P_2}{2t}\right], \tag{2.16}$$

$$D_2\left(P_1, P_2, \theta_1, \theta_2\right) = Nd_2 = N\left(\theta_2 - \theta_M\right) = N\left[\frac{t\left(\theta_2 - \theta_1\right) + P_1 - P_2}{2t}\right]. \tag{2.17}$$

In this model, θ_2 and θ_1 represent each brand's perceived or subjective locations. Without brand advertising $\theta_2 = \theta_1 = ½$ (i.e., there is no product differentiation). We assume that brand advertising can increase subjective horizontal differentiation, $\partial\theta_1/\partial B_1 < 0$ and $\partial\theta_2/\partial B_2 > 0$.[20] Generic advertising increases N and may decrease or have no effect on subjective horizontal differentiation. Under scenario 1, generic advertising has no effect on horizontal differentiation ($\partial\theta_2/\partial g$ - $\partial\theta_1/\partial g = 0$); under scenario 2, generic advertising reduces horizontal differentiation ($\partial\theta_2/\partial g$ - $\partial\theta_1/\partial g < 0$). The remaining structure of the model is the same as with vertical differentiation. Given the degree of symmetry in the model, we can write the profit equation as

$$\begin{aligned}\pi_i &= (P_i - g)Q_i - B_i \\ &= (P_i - g)N\left[t(\theta_2 - \theta_1) - P_i + P_j\right]/2t - B_i, \quad \forall\, i, j = 1, 2, i \neq j,\end{aligned} \tag{2.18}$$

where $Q_i \equiv D_i$.

Recall that in the final stage of the game, firms compete in price. The Nash equilibrium for this sub-game is described below:

$$P_1^* = t(\theta_2 - \theta_1) + g\,, \tag{2.19}$$

$$P_2^* = t(\theta_2 - \theta_1) + g\,, \tag{2.20}$$

$$\pi_1^* = \frac{1}{2}Nt(\theta_2 - \theta_1)^2 - B_1\,, \tag{2.21}$$

$$\pi_2^* = \frac{1}{2}Nt(\theta_2 - \theta_1)^2 - B_2\,. \tag{2.22}$$

[20] To ensure that second-order conditions hold, we assume that $\partial^2\theta_1/\partial B_1^2 > 0$ and $\partial^2\theta_2/\partial B_2^2 < 0$. This implies diminishing returns to brand advertising.

Notice that in the limit as θ_1 approaches θ_2, the degree of product differentiation diminishes and the Nash equilibrium approaches simple Bertrand, where price equals marginal cost and profits are zero.

In the second stage, firms compete in brand advertising. The first-order conditions for this stage game are

$$\frac{\partial \pi_1^*}{\partial B_1} = Nt\left(\theta_1 - \theta_2\right)\frac{\partial \theta_1}{\partial B_1} - 1 = 0, \tag{2.23}$$

$$\frac{\partial \pi_2^*}{\partial B_2} = Nt\left(\theta_2 - \theta_1\right)\frac{\partial \theta_2}{\partial B_2} - 1 = 0. \tag{2.24}$$

Unlike the case with vertical differentiation, both firms will use brand advertising in a horizontally differentiated market as long as the marginal benefits from advertising are sufficiently high.[21] Furthermore, if each firm has equally effective brand advertising (i.e., $\partial\theta_2/\partial B_2$ = -$\partial\theta_1/\partial B_1$), then the level of brand advertising will be the same for both firms. This is consistent with the outcome in the market for premium cola, where the amount of advertising spending by Coke and Pepsi is nearly the same (Tremblay and Polasky, 2002).

Analyzing the effect of generic advertising on the optimal level of brand advertising is more complex in this model. Given the nature of the game and the

[21] Notice that the marginal benefits are positive for both firms. For firm *1*, *N* and *t* are positive, while (θ_1 - θ_2) and $\partial\theta_1/\partial B_1$ are negative. For firm *2*, *N*, *t*, (θ_2 - θ_1), and $\partial\theta_2/\partial B_2$ are positive.

fact that both firms advertise when differentiation is horizontal, the first-order conditions are interdependent. In this case, comparative static results are obtained by implicitly differentiating both first-order conditions with respect to *g* and then using Cramer's rule. This produces the following comparative static results:

$$\frac{\partial B_1^*}{\partial g} = \frac{\begin{vmatrix} -\pi_{1g} & \pi_{12} \\ -\pi_{2g} & \pi_{22} \end{vmatrix}}{|\Pi|} = \frac{-\pi_{1g}\pi_{22} + \pi_{2g}\pi_{12}}{|\Pi|}, \tag{2.25}$$

$$\frac{\partial B_2^*}{\partial g} = \frac{\begin{vmatrix} \pi_{11} & -\pi_{1g} \\ \pi_{21} & -\pi_{2g} \end{vmatrix}}{|\Pi|} = \frac{-\pi_{11}\pi_{2g} + \pi_{1g}\pi_{21}}{|\Pi|}, \tag{2.26}$$

$$\text{where } \Pi = \begin{pmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{pmatrix}.$$

For notational convenience, we define π_{ij} to equal the second derivative of firm *i*'s profit function with respect to B_i and variable *j*.[22] For the Nash equilibrium to be stable, the determinant of matrix Π must be positive. Thus, the

$$\operatorname{sign} \partial B_1^* / \partial g = \operatorname{sign}\left(-\pi_{1g}\pi_{22} + \pi_{2g}\pi_{21}\right), \tag{2.27}$$

$$\operatorname{sign} \partial B_2^* / \partial g = \operatorname{sign}\left(-\pi_{11}\pi_{2g} + \pi_{21}\pi_{1g}\right). \tag{2.28}$$

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[22] That is,

$$\pi_{11} = \partial^2 \pi_1 / \partial B_1^2 = Nt\left[\left(\partial\theta_1/\partial B_1\right)^2 + \left(\theta_1 - \theta_2\right)\partial^2\theta_1/\partial B_1^2\right],$$

$$\pi_{12} \equiv \partial^2 \pi_1 / (\partial B_1 \partial B_2) = \pi_{21} \equiv \partial^2 \pi_2 / (\partial B_2 \partial B_1) = -Nt(\partial\theta_1/\partial B_1)(\partial\theta_2/\partial B_2),$$

$$\pi_{22} \equiv \partial^2 \pi_2 / \partial B_2^2 = Nt\left[\left(\partial\theta_2/\partial B_2\right)^2 + \left(\theta_2 - \theta_1\right)\partial^2\theta_2/\partial B_2^2\right].$$

For the second-order conditions of profit maximization to hold, π_{11} and π_{22} must be negative. Because $\partial\theta_1/\partial B_1 < 0$ and $\partial\theta_2/\partial B_2 > 0$ in this model, π_{12} and π_{21} are positive. This implies that B_1 and B_2 are strategic complements, such that an increase in B_i increases the marginal returns to B_j and causes B_j^* to increase.

Given these conditions, the signs $\partial B_1^*/\partial g$ and $\partial B_2^*/\partial g$ depend only on the sign of π_{ig}, which depends upon how generic advertising affects demand. Under scenario 1, generic advertising attracts new consumers (i.e., $\partial N/\partial g > 0$) but has no effect on horizontal differentiation (i.e., $\partial\theta_2/\partial g - \partial\theta_1/\partial g = 0$). In this case,

$$\pi_{ig} \equiv \frac{\partial^2 \pi_i}{\partial g \partial B_i} = t\left(\theta_i - \theta_j\right)\frac{\partial \theta_i}{\partial B_i}\frac{\partial N}{\partial g} > 0. \qquad (2.29)$$

Thus, under scenario 1 generic advertising increases the brand advertising of both firms.[23] For both vertical and horizontal differentiation, this demonstrates that generic advertising can increase brand advertising without reducing product differentiation.

Comparative static analysis is more complex under scenario 2, where generic advertising reduces horizontal differentiation (i.e., $\partial\theta_2/\partial g - \partial\theta_1/\partial g < 0$). In this case,

[23] Notice that when $i = 1$, $\partial\theta_1/\partial B_1 < 0$ and $(\theta_1 - \theta_2) < 0$, so $\pi_{1g} > 0$; when $i = 2$, $\partial\theta_2/\partial B_2 > 0$ and $(\theta_2 - \theta_1) > 0$, so $\pi_{2g} > 0$.

$$\pi_{1g} \equiv \frac{\partial^2 \pi_1}{\partial g \partial B_1} = \frac{\partial N}{\partial g} t\left(\theta_1 - \theta_2\right) \frac{\partial \theta_1}{\partial B_1} + Nt\left(\frac{\partial \theta_1}{\partial g} - \frac{\partial \theta_2}{\partial g}\right) \frac{\partial \theta_1}{\partial B_1} + Nt\left(\theta_1 - \theta_2\right) \frac{\partial^2 \theta_1}{\partial g \partial B_1}, \tag{2.30}$$

$$\pi_{2g} \equiv \frac{\partial^2 \pi_2}{\partial g \partial B_2} = \frac{\partial N}{\partial g} t\left(\theta_2 - \theta_1\right) \frac{\partial \theta_2}{\partial B_2} + Nt\left(\frac{\partial \theta_2}{\partial g} - \frac{\partial \theta_1}{\partial g}\right) \frac{\partial \theta_2}{\partial B_2} + Nt\left(\theta_2 - \theta_1\right) \frac{\partial^2 \theta_2}{\partial g \partial B_2}. \tag{2.31}$$

As before, the first terms on the right had side of equations (2.30) and (2.31) are positive. The second terms are negative, however, and the signs of the third terms are unknown. Thus, generic advertising may raise or lower brand advertising in this case. If generic advertising and brand advertising are strategic complements, however, the third terms will be positive and sufficiently large so that both π_{ig} and $\partial B_i^*/\partial g$ are positive. This means that under scenario 2, generic advertising will lead to an increase in brand advertising when it sufficiently raises the marginal effectiveness of brand advertising.

Next, we analyze the effect of generic advertising on firm profits. Given that second-stage profits are similar for both firms, we can write the comparative static effect generally as

$$\frac{\partial \pi_i^{**}}{\partial g} = \frac{1}{2} t\left(\theta_2 - \theta_1\right)^2 \frac{\partial N}{\partial g} + Nt\left(\theta_2 - \theta_1\right)\left[\frac{\partial \theta_2}{\partial B_2}\frac{\partial B_2^*}{\partial g} - \frac{\partial \theta_1}{\partial B_1}\frac{\partial B_1^*}{\partial g} + \frac{\partial \theta_2}{\partial g} - \frac{\partial \theta_1}{\partial g}\right] - \frac{\partial B_i^*}{\partial g}. \tag{2.32}$$

Assuming the first-order conditions hold, this simplifies to

$$\frac{\partial \pi_i^{**}}{\partial g} = \frac{1}{2}t\left(\theta_j - \theta_i\right)^2 \frac{\partial N}{\partial g} + Nt\left(\theta_j - \theta_i\right)\left[\frac{\partial \theta_j}{\partial B_j}\frac{\partial B_j^*}{\partial g} + \frac{\partial \theta_j}{\partial g} - \frac{\partial \theta_i}{\partial g}\right]. \quad (2.33)$$

Under scenario 1, the profits of both firms will increase with generic advertising. With complete symmetry, where the brand advertising of each firm is equally effective at creating subjective differentiation and the effect of generic advertising on the amount of brand advertising is the same for both firms, the effect of *g* on profits will be the same for both firms. Under scenario 2, the effect is indeterminate, because $\partial B_j^*/\partial g$ may be positive or negative and because $(\theta_j - \theta_i)\cdot(\partial\theta_j/\partial g - \partial\theta_i/\partial g) < 0$.

Given the degree of symmetry inherent in this model, generic advertising is more likely to have a symmetric effect on the brand advertising and profits of each firm when differentiation is horizontal rather than vertical. An asymmetric result can occur with horizontal differentiation, however, if generic advertising induces one firm to use more brand advertising than the other firm. In this case, the heavy advertiser will have relatively lower profits, because both firms benefit equally from advertising that increases horizontal differentiation but only one firm pays for it. It could also occur if generic advertising attracts relatively more consumers who favor brand *2* (i.e., it skews the distribution toward θ_2), generating greater gains for firm *2* relative to firm *1*. This outcome would be of obvious concern to firm *1* and may motivate legal actions to eliminate mandatory checkoff programs.

2.4 Generic and Brand Advertising in a Supermodular Setting

An alternative way to analyze the relationship between generic and brand advertising in an oligopoly setting is to assume that firms play a supermodular game. As the analysis above indicates, the effect that generic advertising has on brand advertising depends critically on whether or not one agent's advertising raises the marginal returns of another agent's advertising. When the effect is positive, this causes the best reply of each firm to increase in generic advertising and in each of its rival's brand advertising, a defining characteristic of a supermodular game. In a supermodular setting, comparative static results emerge from a relatively general model, even when the assumptions of the implicit function theorem do not hold (Milgrom and Roberts, 1990); Milgrom and Shannon, 1994; Shannon 1995; and Vives, 1999).

To illustrate, consider the case of a smooth supermodular game where best reply functions are differentiable.[24] Firms compete in a two-stage game. In the first stage, a marketing board sets generic advertising (g). In the second stage, two or more firms in a market compete by simultaneously choosing price (P) and

[24] Alternatively, one could assume that best-replies are complete lattices instead of smooth functions without affecting the main conclusions, as discussed in Milgrom and Roberts (1990), Milgrom and Shannon (1994), and Vives (1999).

brand advertising (B). For the game to be supermodular, the following assumptions must hold for each firm $i = 1, 2, 3, \ldots$ and each of its rivals, indexed by j (Milgrom and Roberts, 1990, p. 1264).

(A1) *Bounded Strategies*: P_i and B_i each lie within a closed interval where $\{P_i \mid 0 < P_{iL} \leq P_i \leq P_{iH} < \infty\}$ and $\{B_i \mid 0 < B_{iL} \leq B_i \leq B_{iH} < \infty\}$.

(A2) *Differentiability of the Profit Function*: Firm i's profit (π_i) equation is twice continuously differentiable with respect to P_i and B_i.

(A3) *Complementary Strategies*: $\partial^2\pi_i / \partial P_i \partial B_i \geq 0$.

(A4) *Strategic Complementarity Strategies*: $\partial^2\pi_i / \partial P_i \partial P_j \geq 0$, $\partial^2\pi_i / \partial P_i \partial B_j \geq 0$, $\partial^2\pi_i / \partial B_i \partial P_j \geq 0$, and $\partial^2\pi_i / \partial B_i \partial B_j \geq 0$.

(A5) *Complementary Exogenous Variable*: $\partial^2\pi_i / \partial P_i \partial g \geq 0$ and $\partial^2\pi_i / \partial B_i \partial g \geq 0$.

The key assumptions are A3-A5. When strictly positive, A3 implies that P_i and B_i are complements in the demand function, which assures that there are increasing differences or increasing marginal returns between the pair of firm i's strategies (P_i and B_i). This means that an increase in P_i (B_i) causes B_i (P_i) to increase. When the restrictions in A4 are strictly positive, the best reply functions have a positive slope with respect to a firm's own and its rival's strategies. In other words, the pairs of strategies P_i-P_j, P_i-B_j, B_i-P_j, and B_i-B_j are strategic complements. When the restrictions in A5 are strictly positive, there are increasing marginal returns between the exogenous variable g and each strategic variable of firm i, P_i and B_i.

When these assumptions hold, Milgrom and Roberts prove that the game will have at least one Nash equilibrium. Assuming a unique solution and that strict inequalities hold for A3-A5, they also prove that an increase in the exogenous variable g will cause Nash equilibrium prices (P^*) and brand advertising (B^*) to increase for each firm (Milgrom and Roberts, 1990, Theorem 6). This result holds for all markets with more than one firm and for a discrete as well as a continuous change in g.

The Milgrom-Roberts theorem is driven by the fact that the market exhibits super-complementarity. That is, assumptions A3-A5 imply that the exogenous variable and all strategic variables in the model are complements. Because of increasing marginal returns, an increase in generic advertising causes P_i^* (and B_i^*) to increase. The increase in P_i^* (B_i^*) in turn causes B_i^* (P_i^*) to rise because the firm's own choice variables are complements (A3). It also causes P_j^* and B_j^* to increase for all j because rival choice variables are strategic complements (A4). Finally, this causes a chain of feedback effects: the resulting increases in P_j^* and B_j^* cause further increases in P_i^* and B_i^*, etc. Because all of these direct and indirect effects work in the same direction, an increase in g will cause the Nash level of brand advertising to unambiguously increase for each firm.

The recent claim that generic advertising has forced some producers to respond by increasing their brand advertising raises questions concerning the motivation for this response. According to Supreme Court testimony in a case involving tree fruit, one high quality producer claims to have increased brand advertising in order to undo the negative impact of generic advertising on product differentiation (Glickman v. Wileman Frothers & Elliot, 1997; Crespi, 2007). As Crespi (2007, p. 8) points out, however, this need not be the only reason why brand advertising increases in response to generic spending. Another possibility is that brand advertisers spend more to take advantage of gains in the marginal effectiveness of brand advertising produced by generic advertising.[25]

2.5 Conclusion

This paper extends previous work and produces several new insights concerning the relationships between generic advertising and a firm's brand advertising and profitability. In a duopoly model with vertical product differentiation, we revise previous work to show that only the high quality firm will use brand advertising. In this case, generic advertising is likely to benefit the low quality firm more than the high quality firm when generic advertising lowers

[25] Unfortunately, the Milgrom and Roberts result is not powerful enough to rule out the possibility that generic advertising increases brand advertising when the game in not supermodular. Thus, other explanations are still possible.

product differentiation and induces the high quality firm to spend more on brand advertising.

In a duopoly model with horizontal differentiation, we show that both firms advertise to promote their brands and that a symmetric outcome is more likely. When this occurs, profits and expenditures on brand advertising will be the same, and each firm will respond in the same way to an increase in generic advertising. This suggests that producers will be more likely to be either uniformly in favor or uniformly opposed to commodity checkoff programs when differentiation is horizontal. Asymmetries can arise in the horizontally differentiated model, however, if generic advertising induces one firm to spend more on brand advertising than the other firm. In this case, the heavy advertiser will have lower profits. Differences in profits can also occur if generic advertising increases the demand for one brand relative to that of the other brand.

Finally, we show that the relationship between generic advertising and brand advertising is clear when the structure of the model is supermodular. That is, generic advertising will induce firms to spend more on brand advertising when firms play a supermodular game. This requires that generic and brand advertising are strategic complements, which occurs when generic advertising increases the marginal returns of brand advertising. Regardless of the type of differentiation, the results confirm Crespi's conjecture that generic advertising may induce firms to spend more on brand advertising even when generic advertising does not reduce perceived product differentiation.

Future research might move in two directions. First, our theoretical analysis identifies conditions under which generic advertising will have symmetric and asymmetric effects on brand advertising and firm profits. Future research might focus on empirically estimating these relationships for different horizontally and vertically differentiated industries to determine if model predictions are consistent with the data, as in Crespi and Marette (2002). One could also test whether firms behave as if generic and brand advertising are strategic complements or substitutes, as in Seldon et al. 1993. Second, to date brand advertising has been assumed to be purely persuasive. In future research we plan to analyze the relationship between generic and brand advertising when brand advertising is purely informative, as in the brand advertising model developed by Stivers and Tremblay (2005).

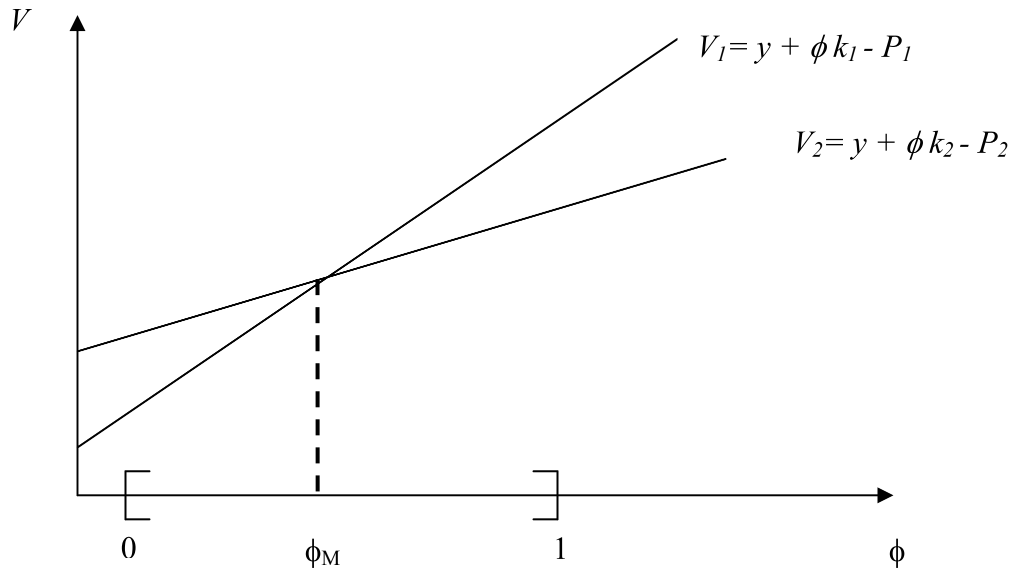

Figure 2.1: Indirect Utility Functions for Vertically Differentiated Brands

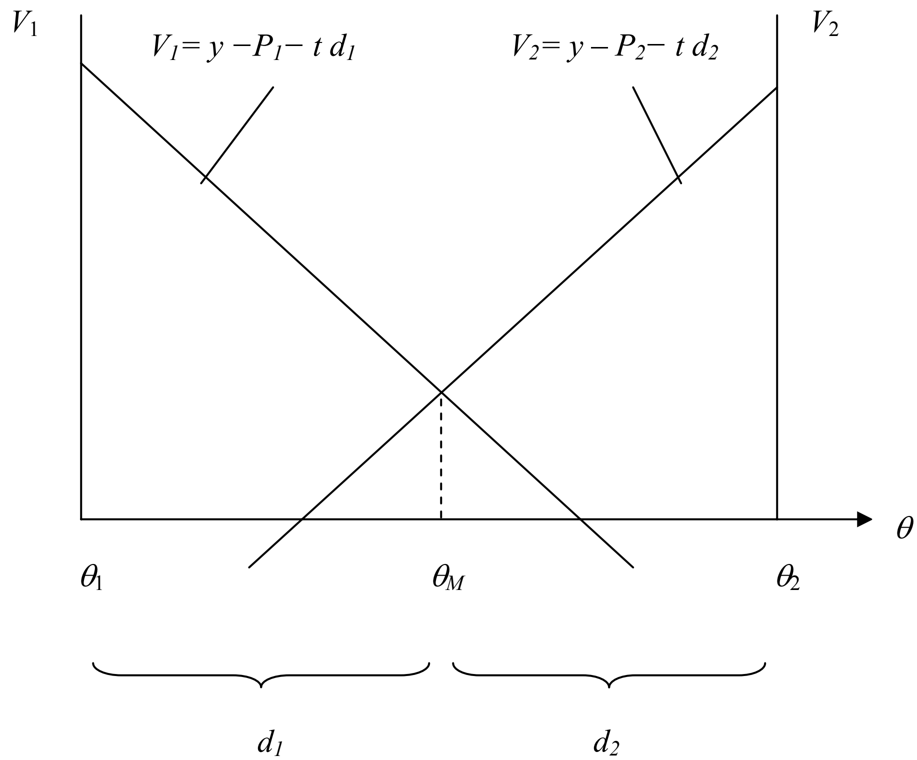

Figure 2.2: Indirect Utility Functions for Horizontally Differentiated Brands

Chapter 3

Generic Advertising in Markets with Informative Brand Advertising [26]

3.1 Introduction

Research on the economics of advertising began since Marshall (1890) and gained momentum with Chamberlin's (1933) work on monopolistic competition. Since then, there have been hundreds of studies analyzing the nature of advertising and its effect on price, profit, and social welfare (see Bagwell, 2005, for an excellent survey). In general, these studies analyze markets with differentiated products in the context of monopoly or imperfectly competitive market, where a firm uses advertising to increase demand for its product or brand. This can be accomplished by attracting new customers to the market; Marshall calls this constructive advertising. It can also occur if advertising causes customers to switch brands, a form of advertising that Marshall calls combative. The literature also distinguishes between informative advertising, which informs consumers of a product's objective characteristics, and persuasive advertising, which is designed to change consumer tastes in favor of the advertised brand.

Because of the free rider problem, it can be uneconomic for an individual firm to advertise when there is adequate competition and little or no product

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[26] Part of this chapter is published in *Journal of Agricultural & Food Industrial Organization*: 7 (1), 2009, 1-20.

differentiation. This setting is common to many agricultural commodity markets, where products are homogeneous or nearly so. To address the problem, in many industries producers coordinate on advertising. That is, firms produce a joint advertising campaign that is designed to increase market demand by emphasizing the universal characteristics of the product. This form of advertising, called generic advertising, is financed through a commodity checkoff program that imposes a mandatory assessment rate (i.e., a sales or per-unit tax) on every producer in the industry. Mandatory participation eliminates the free-rider problem. It also provides an equitable split of benefits among producers in markets with perfectly homogeneous goods. A prominent example is the “Got Milk” commercial, where a famous celebrity promotes the benefits of drinking milk. The beef, pork, peach, mushroom, and almond industries also produce generic advertising.[27]

It is becoming increasingly common for some producers in markets such as these to use brand advertising to augment generic advertising campaigns. With the success of brand advertising, however, these producers have begun to express an interest in pulling out of commodity checkoff programs. They claim that generic advertising dilutes the effectiveness of brand advertising, forcing them to spend more on brand advertising which lowers profits. This raises legal concerns,

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[27] Much of the work on generic advertising has been empirical. Examples include Reberte et al. (1996), Kaiser (1997), Pritchett et al. (1998), Schmit et al. (2002), and Crespi and Marette (2002).

because such programs are not only mandatory but are required to provide proportional benefits to all producers in the industry (Chakravarti and Janiszewski, 2004; Ward, 2006). At issue is whether generic advertising causes brand advertisers to spend more on brand advertising and receive disproportionately less benefit from generic advertising.[28]

Given concerns raised by brand advertisers, recent research has focused on developing models to analyze the interaction between brand and generic advertising.[29] These studies generally assume that brand advertising is persuasive and demonstrate that the effect of generic advertising depends upon the type of subjective differentiation created by brand advertising (vertical vs. horizontal) and the effect of generic advertising on demand. Two scenarios are considered: (1) generic advertising attracts new customers to the market but has no effect on the degree of subjective differentiation between brands and (2) generic advertising attracts new customers and lowers the degree of subjective differentiation between brands. The results show that only the high quality firm will advertise when differentiation is vertical. When differentiation is horizontal, a symmetric outcome is more likely, one where both firms choose the same level of advertising

[28] According to Chakravarti and Janiszewski (2004), this has motivated lawsuits by brand advertisers of almonds, peaches, mushrooms, plums, beef, and pork over mandatory generic adverting programs. For further discussion of the legal and historical background of these concerns with generic advertising, see Ward (2006), Crespi and McEowen (2006), Crespi (2007), and Isariyawongse et al. (2007).

[29] The primary studies are Crespi and Marette (2002), Hunnicutt and Israelsen (2003), Bass et al. (2005), Crespi (2007), and Isariyawongse et al. (2007).

and earn the same profit. The evidence also shows that the effects of generic advertising on brand advertising and firm profits depends on the scenario.

To date, no one has considered a market with generic advertising programs and brand advertising that is purely informative. Ward (2006) argues that generic advertising is primarily informative, and the same can be true for brand advertising. The beef industry provides a good example where differentiation is vertical. The Snake River Farm of Boise, Idaho produces American style Kobe beef that meets the USDA grade of prime quality and also meets the more stringent marbling requirement of Japan, information that is communicated to consumers in its marketing (Parsons, 2005). Cheese presents an example of the horizontal differentiation case, with generic ads promoting cheese generally ("Ahh, the Power of Cheese") and local distributors providing their own informative ads, such as blue cheese makes an excellent topping on salads and mozzarella cheese is low in saturated fat.

The main purpose of this study is to investigate the effect of generic advertising on firm profits and on brand advertising that is purely informative. To compare our analysis with that of previous studies, we develop duopoly models with vertical and horizontal differentiation that are similar to those of Crespi (2007) and Isariyawongse et al. (2007). In our model, brand advertising lowers the consumer search costs of identifying brand characteristics. The key distinction between models is that brand advertising is a strategic complement when persuasive and a strategic substitute when informative. This small change leads to several new results.

3.2 A Duopoly Model with Vertical Differentiation

We consider a market with two profit-maximizing firms (1 and 2) that produce differentiated products. Product characteristics, such as the grade of beef or the fat content of cheese, are predetermined. Firms play a three-stage game with perfect and complete information. In Stage I, a marketing board sets an assessment rate of g dollars per unit.[30] In stage II, firms choose simultaneously their levels of brand advertising, B. In stage III, firms choose their prices, P, simultaneously.

Consumers face a search cost associated with obtaining information about product characteristics. Following Stivers and Tremblay (2005), we assume that consumers pay the full price for *i's* brand (P_i^f) which equals the market price (P_i) plus a per-unit search cost ($s_i \geq 0$). That is, $P_i^f = P_i + s_i$. Firm i can then invest in informative brand advertising (B_i) to lower search costs, such that $\partial s_i/\partial B_i < 0$ and $\partial^2 s_i/\partial B_i^2 > 0$. With this notation, subscript i represents firm 1 or 2 and j identifies the other firm.

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[30] Total spending on generic advertising is then g times the aggregate production of both firms. The political process that determines g is assumed to be predetermined.

First, we analyze the case of vertical product differentiation (VPD). The specification derives from Mussa and Rosen (1978) and is similar to the model found in Crespi (2007) and Isariyawongse et al. (2007). In our model, the vertical characteristic is defined as product quality, with ϕ_i indexing the quality of firm i's brand, where $\phi_1 > \phi_2 > 0$. The degree of VPD is $\phi \equiv \phi_1 - \phi_2$. With N consumers in the market, this model generates the following demand functions for firms 1 and 2 (D_1 and D_2).

$$D_1(P_1, P_2, B_1, B_2) = N\left[\frac{1+\left(-P_1^f + P_2^f\right)}{\phi}\right] \tag{3.1}$$

$$= N\left[\frac{1+\left\{-P_1 - s_1\left(B_1\right) + P_2 + s_2\left(B_2\right)\right\}}{\phi}\right]$$

$$D_2(P_1, P_2, B_1, B_2) = N\left[\frac{\left(P_1^f - P_2^f\right)}{\phi}\right] \tag{3.2}$$

$$= N\left[\frac{\left\{P_1 + s_1\left(B_1\right) - P_2 - s_2\left(B_2\right)\right\}}{\phi}\right]$$

Firm demand falls with the firm's own price and in rival advertising and increases with the number of consumers, its rival's price, the firm's own brand advertising, and the degree of product differentiation. Firm i's profit function is $\pi_i = (P_i - g)\, Q_i - B_i$, where $Q_i \equiv D_i$ is firm output. Production costs are normalized to zero for convenience.

The effect of generic advertising differs by scenario. In scenario 1, generic advertising attracts new customers to the market (i.e., $\partial N/\partial g > 0$). In scenario 2, generic advertising attracts new customers and lowers VPD (i.e., $\partial N/\partial g > 0$ and $\partial \phi /\partial g < 0$).

We use backwards induction to obtain the sub-game perfect Nash equilibrium (SPNE), which requires a Nash equilibrium in every sub-game. The Nash equilibrium in the final pricing stage of the game is:[31]

$$P_1^* = \frac{1}{3}\left(2\phi - s_1 + s_2\right) + g, \qquad P_2^* = \frac{1}{3}\left(\phi + s_1 - s_2\right) + g, \qquad (3.3)$$

$$Q_1^* = N\left(\frac{2\phi - s_1 + s_2}{3\phi}\right), \qquad Q_2^* = N\left(\frac{\phi + s_1 - s_2}{3\phi}\right), \qquad (3.4)$$

$$\pi_1^* = \frac{N}{9\phi}\left(2\phi - s_1 + s_2\right)^2 - B_1, \quad \pi_2^* = \frac{N}{9\phi}\left(\phi + s_1 - s_2\right)^2 - B_2. \qquad (3.5)$$

To ensure firm participation and that second order conditions of profit maximization are met, ϕ must be sufficiently large.[32] We will later show that s_2 will be greater than s_1 in equilibrium, which implies that the higher quality brand

[31] In this model, second-order conditions of profit maximization hold for each firm and the Nash equilibrium is stable and unique. For firm i, $\partial^2\pi_i/\partial P_i^2 = -(2N)/\phi$, which is negative given that $N, \phi > 0$. Note that $\partial^2\pi_i/\partial P_i\partial P_j = N/\phi$. A stable Nash equilibrium exists because the $|\partial^2\pi_i/\partial P_i^2|$ is greater than $|\partial^2\pi_i/\partial P_i\partial P_j|$ (Dixit, 1986).

[32] First, because s_1 will be less than s_2 in equilibrium, ϕ must be sufficiently larger to assure firm 2's participation [i.e., $\phi > (s_2 - s_1) > 0$]. Second, to assure that the second-order conditions hold, ϕ must be greater than $(s_i')^2/s_i'' + (s_2 - s_1)$, where $s_i' \equiv (\partial s_i/\partial B_i)$ and $s_i'' \equiv (\partial s_i^2/\partial^2 B_i)$.

(brand 1) will sell for a higher price and have a larger market share. It will also earn a higher profit as long as its advertising expenditures are not excessive compared with that of brand 2. Notice that the simple Bertrand paradox, which implies that Nash prices approach marginal cost as the degree of product differentiation diminishes, does not hold in this model when $s_1 \neq s_2$. Instead, P_1^* approaches $g + \alpha$ and P_2^* approaches $g - \alpha$, where g is the effective marginal cost and $\alpha \equiv (s_2 - s_1)/3 > 0$. This suggests that firms benefit from product differentiation and that the high quality firm gains a strategic advantage from having relatively low search costs.[33]

In the second stage, firms compete in informative brand advertising given that they can look forward and predict Nash prices in the final stage of the game. This produces the following first-order conditions:

$$\frac{\partial \pi_1^*}{\partial B_1} = \left[\frac{2N}{9\phi} \left(2\phi + s_2 - s_1 \right) \left(-\frac{\partial s_1}{\partial B_1} \right) \right] - 1 = 0, \tag{3.6}$$

$$\frac{\partial \pi_2^*}{\partial B_2} = \left[\frac{2N}{9\phi} \left(\phi + s_1 - s_2 \right) \left(-\frac{\partial s_2}{\partial B_2} \right) \right] - 1 = 0. \tag{3.7}$$

Each bracketed term is the marginal benefit of brand advertising, and the marginal cost of brand advertising is -1. Assuming that a stable and unique Nash equilibrium exists, Nash levels of brand advertising for each firm (B_i^*) are

[33] This need not lower social welfare, as Stivers and Tremblay (2005) show that firm investment in informative brand advertising can benefit consumers by lowering the full (consumer) price and benefit producers by raising the market (producer) price.

embedded in equations (3.6) and (3.7). In order to avoid any undue asymmetry in the model, we assume throughout the paper that the marginal effectiveness of informative brand advertising is the same for each firm (i.e., $\partial s_1/\partial B_1 = \partial s_2/\partial B_2, \forall B_1 = B_2$).

Given the assumptions above, equations (3.6) and (3.7) imply the following proposition.

***Proposition 3.1*:** Consider the duopoly model with VPD ($\phi > 0$) described above. With N sufficiently large (i.e., the marginal benefit of brand advertising is sufficiently high), $B_1^* > B_2^* > 0$.

***Proof*:** Proofs of all propositions can be found in the appendix A.3.

In this model, both firms advertise, but the high quality firm advertises more than the low quality firm.[34] This is in contrast to the case of purely persuasive brand advertising, where only the high quality firm advertises. With informative brand advertising, however, each firm benefits from brand advertising that lowers the search costs for its own product.

Next, working backwards to the first stage of the game, the marketing board sets generic advertising by determining g. Assuming optimal play in the

[34] Note that if firm 2 has a substantial marketing advantage over firm 1, firm 2 would invest in a greater amount of informative brand advertising than firm 1.

later stages of the game, we can now determine how a change in g will affect informative brand advertising.

***Proposition 3.2*:** Consider the duopoly model with VPD as described above. Under both scenarios 1 and 2, $\partial B_1^*/\partial g$ and $\partial B_2^*/\partial g$ are indeterminate.

The intuition is as follows. In scenario 1, an increase in g has a direct effect and a strategic or indirect effect. The direct effect is positive. That is, an increase in g increases the size of the market (N), which increases B_i, ceteris paribus. The strategic effect is negative, however, because B_1 and B_2 are strategic substitutes.[35] That is, when an increase in g causes B_j to rise, this causes B_i to fall, ceteris paribus. Because the direct and strategic effects work in opposite directions, the signs of $\partial B_1^*/\partial g$ and $\partial B_2^*/\partial g$ are indeterminate. The results in scenario 2 are complicated by an additional third effect, where an increase in g lowers ϕ, making the net effects of g on brand advertising indeterminate. These results are different from the persuasive case, where B_2 is constant and equal to 0 when brand advertising is purely persuasive.

Next, we investigate the effect of g on firm profits.

[35] Following Bulow et al. (1985), B_1 and B_2 are strategic substitutes when $\partial^2\pi_i/\partial B_i\partial B_j < 0$ and strategic complements when $\partial^2\pi_i/\partial B_i\partial B_j > 0$.

***Proposition 3.3*:** Consider this duopoly model with VPD. Under both scenarios, $\partial\pi_1^*/\partial g$ and $\partial\pi_2^*/\partial g$ are indeterminate.

The reasoning is that the direct and the strategic effects work in opposite directions, and there are no other restrictions that allow us to sign $\partial\pi_i^*/\partial g$. The only difference from the case where brand advertising is persuasive is that when persuasive, an increase in *g* increases firm 1's profits under scenario 1 because B_2 is zero and is unaffected by *g*. That is, only the direct effect comes into play. In all other cases, the effect of *g* on firm profit is the same under persuasive and informative brand advertising.

The model can also be used to identify conditions under which generic advertising benefits the low quality producer more than the high quality producer. If we assume that the concern of high quality producers is true that $\partial B_1^*/\partial g > \partial B_2^*/\partial g > 0$, then the following proposition holds.[36]

[36] This concern of leading brand advertisers is discussed in Crespi (2007). Crespi (2003) points out that not just leading brand advertisers have opposed generic advertising. Marketing to different buyers and biasing the marketing board can lead to disproportionate benefits of generic ads. For example, in Cal-Almond, Inc. v. U.S. Department of Agriculture (1993), the major producer, Blue Diamond, who sold directly to consumers favored generic advertising, while small generic producers who sold to food processors opposed generic advertising. The reason for this is that generic ads marketed almonds to consumers, not food processors. It is also clear from the Glickman v. Wileman (1997) case that a producer with greater political power within the marketing board for nectarines was able to receive disproportionately greater benefits by biasing generic ads to favor that producer's own variety of nectarines.

***Proposition 3.4*:** Consider this duopoly model with VPD. If $|\partial s_2/\partial B_2|$ is sufficiently large relative to $|\partial s_1/\partial B_1|$ and $\partial B_i^*/\partial g > 0$, then $\partial \pi_2^*/\partial g > \partial \pi_1^*/\partial g$ under both scenarios.

The assumptions of the proposition are certainly plausible. From *Proposition 1*, B_1 will exceed B_2 in equilibrium. Given diminishing returns to brand advertising, $s_2 > s_1$ and $|\partial s_1/\partial B_1| < |\partial s_2/\partial B_2|$. As the proof demonstrates, the concern of the high quality firm that invests more in brand advertising is supported by the model of VPD and informative brand advertising as long as $|\partial s_2/\partial B_2|$ is sufficiently large relative to $|\partial s_1/\partial B_1|$. This is similar to the outcome when brand advertising is persuasive.

Table 3.1 provides a summary of these results and the results found in Crespi (2007) and Isariyawongse et al. (2007) when brand advertising is persuasive.

3.3 A Duopoly Model with Horizontal Differentiation

Next, we analyze this market when there is horizontal product differentiation (HPD). Following Isariyawongse et al. (2007), we characterize horizontal differentiation using Hotelling's (1929) linear city model. In the discussion here, the horizontal characteristic could be the fat content of different brands of cheese, which is indexed by θ_i for brand i, with $\theta_1 \geq \theta_2 > 0$. The degree of HPD is $\theta \equiv \theta_1 - \theta_2$, the difference in fat content between brands. The Hotelling model includes an additional parameter, *t*, which is defined as the unit cost a consumer faces when purchasing a brand with a characteristic that is less than ideal.[37]

Firm demand derives directly from the Hotelling model. As in the case with VPD, consumers pay the full price for a particular brand. Thus, firm demand functions are:

$$D_1\left(P_1, P_2, B_1, B_2\right) = N\left[\frac{t\theta - P_1^f + P_2^f}{2t}\right] \tag{3.8}$$

$$= N\left[\frac{t\theta - P_1 - s_1(B_1) + P_2 + s_2(B_2)}{2t}\right]$$

$$D_2\left(P_1, P_2, B_1, B_2\right) = N\left[\frac{t\theta + P_1^f - P_2^f}{2t}\right] \tag{3.9}$$

$$= N\left[\frac{t\theta + P_1 + s_1\left(B_1\right) - P_2 - s_2\left(B_2\right)}{2t}\right]$$

Firm demand falls with its own price and rival advertising and increases with the number of consumers, the price of the rival brand, the firm's own advertising, and the degree of product differentiation. Again, production costs are normalized to zero. Firm *i*'s profit is $\pi_i = (P_i - g)\, Q_i - B_i$.

[37] In Hotelling's interpretation, firms have different locations along a main street and are differentiated because they have different locations and distances from particular consumers. In this case, *t* represents the transportation cost per unit of distance (e.g., per mile) along main street.

In the last stage of the game, firms compete by simultaneously choosing price. The Nash equilibrium for firm i in this sub-game is described below.[38]

$$P_i^* = t\theta + g + \frac{\left(-s_i + s_j\right)}{3}, \tag{3.10}$$

$$Q_i^* = N\left(\frac{-s_i + s_j + 3t\theta}{6t}\right), \tag{3.11}$$

$$\pi_i^* = \frac{1}{18t} N\left(-s_i + s_j + 3t\theta\right)^2 - B_i. \tag{3.12}$$

Because s_i may be greater than, less than, or equal to s_j, θ must be positive and greater than $(s_i - s_j)/(3t)$ to assure firm participation. If s_i equals s_j, the Nash equilibrium in this stage will be symmetric. In this case, the Bertrand paradox holds: the Nash price approaches marginal cost (g) as θ goes to zero.

In the second stage, firms compete in brand advertising. Using backwards induction, each firm correctly anticipates the Nash equilibrium in the final stage, and the first-order condition for firm i in this stage is

$$\frac{\partial \pi_i^*}{\partial B_i} = \left[-\frac{N}{9t}\left(-s_i + s_j + 3t\theta\right)\frac{\partial s_i}{\partial B_i}\right] - 1 = 0. \tag{3.13}$$

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[38] In this model, second-order conditions of profit maximization hold for each firm and the Nash equilibrium is stable and unique. For firm i, $\partial^2\pi_i/\partial P_i^2 = -N/t$, which is negative given that N, $t > 0$. Note that $\partial^2\pi_i/\partial P_i\partial P_j = N/(2t)$. A stable Nash equilibrium exists because the $|\partial^2\pi_i/\partial P_i^2|$ is greater than $|\partial^2\pi_i/\partial P_i\partial P_j|$ (Dixit, 1986).

As in the vertical case, we continue to assume a unique and stable equilibrium and that the marginal effectiveness of advertising is the same for both firms (i.e., $\partial s_1/\partial B_1 = \partial s_2/\partial B_2, \forall\ B_1 = B_2$).

The first-order condition in equation (3.13) implies the following proposition.

Proposition 3.5**:** In this model with HPD ($\theta > 0$), if N is sufficiently large, $B_1^* = B_2^* > 0$.

A symmetric outcome is not surprising, given the natural symmetry of the model and the assumption that neither firm has a marketing advantage over the other firm.[39] This produces the same outcome as with purely persuasive brand advertising.

Assuming optimal play in the later stages of the game, we now determine the effect of g on informative brand advertising.

Proposition 3.6**:** Consider this duopoly model with HPD, where θ is sufficiently large. Under these conditions, $\partial B_1^*/\partial g > 0$, $\partial B_2^*/\partial g > 0$ under scenario 1. Under scenario 2, the signs of $\partial B_1^*/\partial g$ and $\partial B_2^*/\partial g$ are indeterminate.

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[39] If firm i has a marketing advantage over firm j, however, $B_i^* > B_j^*$ and $s_i < s_j$. Thus, $P_i^* > P_j^*$ and $Q_i^* > Q_j^*$.

This is the same result as found with persuasive advertising. Under scenario 1, the direct effect of an increase in g is to increase informative brand advertising. In spite of the fact that the strategic effect works in the opposite direction, the direct effect dominates the strategic effect in this model. Thus, $\partial B_i^*/\partial g > 0$. This demonstrates that when firms use informative brand advertising, generic advertising can increase brand advertising even though it has no effect on HPD. Under scenario 2, an increase in g also leads to a reduction in θ, which puts downward pressure on informative brand advertising, resulting in an indeterminate net effect.

The effect of g on firm profits is as follows.

***Proposition 3.7*:** Given this duopoly model with HPD, the signs of $\partial\pi_1^*/\partial g$ and $\partial\pi_2^*/\partial g$ are indeterminate under both scenarios.

This is different from the case where brand advertising is persuasive under scenario 1. For both persuasive and informative brand advertising, the direct effect of g on profits is positive. When brand advertising is persuasive, the strategic effect reinforces the direct effect, because B_1 and B_2 are strategic complements ($\partial^2\pi_i/\partial B_i\partial B_j > 0$). The opposite is true when brand advertising is informative, however, because B_1 and B_2 are strategic substitutes ($\partial^2\pi_i/\partial B_i\partial B_j < 0$). Thus, under scenario 1 brand advertising increases firm profits when persuasive but has an indeterminate effect when informative. Under scenario 2, the effect of g on firm profits is indeterminate whether brand advertising is persuasive or informative.

An asymmetry can occur if, for example, firm 1 has a marketing advantage over firm 2, resulting in B_1^* being greater than B_2^*. In this case, the condition needed to support the claim that generic advertising causes firm 1 to increase its brand advertising and causes firm 1's profits to fall relative to firm 2's profits is revealed in the following proposition.

***Proposition 3.8*:** In this duopoly model with HPD, assume further that firm 1 has a marketing advantage over firm 2. If $|\partial s_2/\partial B_2|$ is sufficiently large relative to $|\partial s_1/\partial B_1|$ in equilibrium and $\partial B_i^*/\partial g > 0$, then $\partial \pi_2^*/\partial g > \partial \pi_1^*/\partial g$ under scenarios 1 and 2.

A similar result emerges when brand advertising is persuasive. In both the persuasive and informative cases, however, this result requires that firm 1 has a marketing advantage over firm 2. When true, generic advertising can produce a relative decline in the profits for the heavy advertiser. Because the outcome that one firm has a marketing advantage over another is more natural when differentiation is vertical, the concern that some firms benefit more from generic advertising is more likely in the vertical differentiation case. Again, a comparison of results for the persuasive and informative cases can be found in Table 3.1.

3.4 Conclusion

In this paper, we analyze the effect of generic advertising on firm behavior and profits when brand advertising is purely informative. We also compare our results with those of Crespi (2007) and Isariyawongse et al. (2007), who analyze the case where brand advertising is purely persuasive. Our work shows that firms can benefit from investing in brand advertising that lowers consumer search costs as well as from brand advertising that is purely persuasive. The results also show that whether brand advertising is persuasive or informative, the outcome is more likely to be symmetric with horizontal differentiation than with vertical differentiation. In addition, the effect of generic advertising on brand advertising and firm profits is generally indeterminate under scenario 2, regardless the type of advertising and the type of product differentiation.

Under scenario 1, however, important differences emerge. When brand advertising is purely persuasive, generic advertising always raises firm profits and causes firms that choose to advertise to increase spending on persuasive brand advertising, whether differentiation is vertical or horizontal. When brand advertising is purely informative, however, generic advertising generally has an indeterminate effect on firm profits and brand advertising. This difference is due to the fact that the brand advertising between firms is a strategic complement when persuasive and a strategic substitute when informative. Thus, the direct and strategic effects of an increase in generic advertising move in the same direction when brand advertising is persuasive but in opposite directions when brand advertising is informative.

The one exception is that generic advertising increases informative brand advertising when differentiation is horizontal. The reason for this is that the direct effect dominates the strategic effect due to the symmetry of the model. This is not true with regards to firm profits, however. Generic advertising increases firm profit when brand advertising is persuasive but has an indeterminate effect when it is informative.

Finally, some firms that invest heavily in brand advertising have expressed a concern that generic advertising is of less value to them. Whether brand advertising is persuasive or informative, we show that this is likely to occur when the heavy advertiser has a marketing advantage over its competitor. This marketing advantage occurs more naturally when differentiation is vertical than horizontal, so this expressed concern is more likely when brand advertising is used to increase subjective quality or to inform consumers of a real quality advantage. In any case, these and the results of Crespi (2007) and Isariyawongse et al. (2007) demonstrate that there is considerable theoretical ambiguity regarding the effect of generic advertising on brand advertising and firm profit. Thus, many of the issues raised in these papers remain empirical questions.

Table 3.1: The Effects of Generic Advertising under Persuasive and Informative Brand Advertising

	VPD		HPD	
	Persuasive Advertising	Informative Advertising	Persuasive Advertising	Informative Advertising
Brand Advertising	$B_1^* > 0$ $B_2^* = 0$	$B_1^* > B_2^* > 0$	$B_1^* = B_2^* > 0$	$B_1^* = B_2^* > 0$
$\partial B_i^*/\partial g$				
Scenario 1	$\frac{\partial B_1^*}{\partial g} > 0, \frac{\partial B_2^*}{\partial g} = 0$	$\frac{\partial B_1^*}{\partial g} \gtreqless 0, \frac{\partial B_2^*}{\partial g} \gtreqless 0$	$\frac{\partial B_1^*}{\partial g} > 0, \frac{\partial B_2^*}{\partial g} > 0$	$\frac{\partial B_1^*}{\partial g} > 0, \frac{\partial B_2^*}{\partial g} = 0$
Scenario 2	$\frac{\partial B_1^*}{\partial g} \gtreqless 0, \frac{\partial B_2^*}{\partial g} = 0$	$\frac{\partial B_1^*}{\partial g} \gtreqless 0, \frac{\partial B_2^*}{\partial g} \gtreqless 0$	$\frac{\partial B_1^*}{\partial g} \gtreqless 0, \frac{\partial B_2^*}{\partial g} \gtreqless 0$	$\frac{\partial B_1^*}{\partial g} \gtreqless 0, \frac{\partial B_2^*}{\partial g} \gtreqless 0$
$\partial \pi_i^{**}/\partial g$				
Scenario 1	$\frac{\partial \pi_1^*}{\partial g} > 0, \frac{\partial \pi_2^*}{\partial g} > 0$	$\frac{\partial \pi_1^*}{\partial g} \gtreqless 0, \frac{\partial \pi_2^*}{\partial g} \gtreqless 0$	$\frac{\partial \pi_1^*}{\partial g} > 0, \frac{\partial \pi_2^*}{\partial g} > 0$	$\frac{\partial \pi_1^*}{\partial g} \gtreqless 0, \frac{\partial \pi_2^*}{\partial g} \gtreqless 0$
Scenario 2	$\frac{\partial \pi_1^*}{\partial g} \gtreqless 0, \frac{\partial \pi_2^*}{\partial g} \gtreqless 0$	$\frac{\partial \pi_1^*}{\partial g} \gtreqless 0, \frac{\partial \pi_2^*}{\partial g} \gtreqless 0$	$\frac{\partial \pi_1^*}{\partial g} \gtreqless 0, \frac{\partial \pi_2^*}{\partial g} \gtreqless 0$	$\frac{\partial \pi_1^*}{\partial g} \gtreqless 0, \frac{\partial \pi_2^*}{\partial g} \gtreqless 0$
$\frac{\partial \pi_1^{**}/\partial g}{\partial \pi_2^{**}/\partial g}$	$\frac{\partial \pi_1^{**}/\partial g}{\partial \pi_2^{**}/\partial g} < 0$ if	$\frac{\partial \pi_1^{**}/\partial g}{\partial \pi_2^{**}/\partial g} < 0$ if	$\frac{\partial \pi_1^{**}/\partial g}{\partial \pi_2^{**}/\partial g} < 0$ if	$\frac{\partial \pi_1^{**}/\partial g}{\partial \pi_2^{**}/\partial g} < 0$ if
Scenario 1	$\frac{\partial k_1^*}{\partial B_1}, \frac{\partial B_1^*}{\partial g}, N$ are positive and sufficiently large	$\lvert\partial s_2/\partial B_2\rvert$ is sufficiently large relative to $\lvert\partial s_1/\partial B_1\rvert$	$\lvert\partial \theta_1/\partial B_1\rvert$ is sufficiently large relative to $\lvert\partial \theta_2/\partial B_2\rvert$	$\lvert\partial s_2/\partial B_2\rvert$ is sufficiently large relative to $\lvert\partial s_1/\partial B_1\rvert$
Scenario 2	Same as in scenario 1	Same as in scenario 1	Same as in scenario 1	Same as in scenario 1

Sources: The evidence for persuasive advertising comes from Crespi (2007) and Isariyawongse et al. (2007), and the evidence for informative advertising comes from this paper.

Appendix A.3

Proof of Proposition 3.1:

The bracketed terms on the right hand side of equations (3.6) and (3.7) are the marginal benefits of brand advertising, which are both positive and have a negative slope given that $N > 0$ and sufficiently large, $\phi > (s_2 - s_1) > 0$ in equilibrium, and $\partial s_i/\partial B_i < 0$. The marginal cost of brand advertising is -1, the last term in equations (3.6) and (3.7). With N sufficiently large (i.e., the marginal benefit of brand advertising is sufficiently high), each firm will invest in brand advertising. Because the marginal benefit of brand advertising for firm 1 is greater than for firm 2, $B_1^* > B_2^* > 0$. ■

Proof of Proposition 3.2:

To analyze the effect of generic advertising on the optimal level of brand advertising requires implicitly differentiating both first-order conditions in equations (3.6) and (3.7) with respect to g and using Cramer's rule to obtain the desired comparative static results. This produces the following:

$$\frac{\partial B_1^*}{\partial g} = \frac{\begin{vmatrix} -\pi_{1g} & \pi_{12} \\ -\pi_{2g} & \pi_{22} \end{vmatrix}}{|\Pi|} = \frac{-\pi_{1g}\pi_{22} + \pi_{2g}\pi_{12}}{|\Pi|}, \qquad (3.14)$$

$$\frac{\partial B_2^*}{\partial g} = \frac{\begin{vmatrix} \pi_{11} & -\pi_{1g} \\ \pi_{21} & -\pi_{2g} \end{vmatrix}}{|\Pi|} = \frac{-\pi_{11}\pi_{2g} + \pi_{1g}\pi_{21}}{|\Pi|}, \tag{3.15}$$

$$\text{where } \Pi = \begin{pmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{pmatrix}.$$

Notationally, under scenario 1:

$$\pi_{11} \equiv \frac{\partial^2 \pi_1}{\partial B_1^2} = \frac{2N\left(\frac{\partial s_1}{\partial B_1}\right)^2}{9\phi} - \frac{2N}{9\phi}\left(2\phi - s_1 + s_2\right)\left(\frac{\partial s_1^2}{\partial^2 B_1}\right) < 0\,, \tag{3.16}$$

$$\pi_{22} \equiv \frac{\partial^2 \pi_2}{\partial B_2^2} = \frac{2N\left(\frac{\partial s_2}{\partial B_2}\right)^2}{9\phi} - \frac{2N}{9\phi}\left(\phi + s_1 - s_2\right)\left(\frac{\partial s_2^2}{\partial^2 B_2}\right) < 0, \tag{3.17}$$

$$\pi_{12} \equiv \frac{\partial^2 \pi_1}{\partial B_1 \partial B_2} = \pi_{21} \equiv \frac{\partial^2 \pi_2}{\partial B_2 \partial B_1} = -\frac{2N}{9\phi}\frac{\partial s_1}{\partial B_1}\frac{\partial s_2}{\partial B_2} < 0, \tag{3.18}$$

$$\pi_{1g} \equiv \frac{\partial^2 \pi_1}{\partial B_1 \partial g} = -\frac{2}{9\phi}\left(2\phi - s_1 + s_2\right)\frac{\partial N}{\partial g}\frac{\partial s_1}{\partial B_1} > 0, \tag{3.19}$$

$$\pi_{2g} \equiv \frac{\partial^2 \pi_2}{\partial B_2 \partial g} = -\frac{2}{9\phi}\left(\phi + s_1 - s_2\right)\frac{\partial N}{\partial g}\frac{\partial s_2}{\partial B_2} > 0. \tag{3.20}$$

Assuming that the Nash equilibrium is stable, the determinant of matrix Π must be positive. This implies that:

sign $\partial B_1^*/\partial g$ = sign $(-\pi_{1g}\,\pi_{22} + \pi_{2g}\,\pi_{12})$,

sign $\partial B_2^*/\partial g$ = sign $(-\pi_{11}\,\pi_{2g} + \pi_{21}\,\pi_{1g})$

Assuming further that the Nash equilibrium is also unique, the absolute value of the slope of the best reply function must be less than 1 or that $|\pi_{ii}| > |\pi_{ij}|$ (Dixit, 1986).[40] However, because $s_2 > s_1$ in equilibrium, π_{1g} may be greater than, equal to, or less than π_{2g}. Thus, $\partial B_1^*/\partial g$ and $\partial B_2^*/\partial g$ are indeterminate under scenario 1.

Under scenario 2, the only changes will be to π_{1g} and π_{2g}, which are:

$$\pi_{1g} \equiv \frac{\partial^2 \pi_1}{\partial B_1 \partial g} = x_1 + \left[\frac{4N}{9\phi}\frac{\partial \phi}{\partial g} - \frac{2N}{9\phi^2}\left(2\phi - s_1 + s_2\right)\frac{\partial \phi}{\partial g}\right]\left(-\frac{\partial s_1}{\partial B_1}\right), \tag{3.21}$$

$$\pi_{2g} \equiv \frac{\partial^2 \pi_2}{\partial B_2 \partial g} = x_2 + \left[\frac{2N}{9\phi}\frac{\partial \phi}{\partial g} - \frac{2N}{9\phi^2}\left(\phi + s_1 - s_2\right)\frac{\partial \phi}{\partial g}\right]\left(-\frac{\partial s_2}{\partial B_2}\right), \tag{3.22}$$

where x_1 equals the terms on the right-hand side of equation (3.19) and x_2 equals the terms on the right-hand side of equation (3.20). Note that $x_1 > 0$, $x_2 > 0$, and -$\partial s_i/\partial B_i > 0$. In scenario 2, $\partial \phi/\partial g < 0$. Given that ϕ is sufficiently large, the signs of the terms in square brackets in equations (3.21) and (3.22) are indeterminate. Thus, the effect g on brand advertising is indeterminate in scenario 2. ■

Proof of Proposition 3.3:

Following the envelope theorem in Caputo (1996), the effects of g on Nash profits in the final stage of the game (π_i^{**}) under scenario 1 are:

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[40] For firm i, for example, its first-order condition implicitly defines B_i^* as a function of B_j. this is called firm i's best-reply function (r_i). From the implicit-function theorem, the slope of r_i = $|-\pi_{ij}/\pi_{ii}|$.

$$\frac{\partial \pi_1^*}{\partial g} = \left[\frac{\left(2\phi - s_1 + s_2\right)^2}{9\phi}\frac{\partial N}{\partial g}\right] + \left[\left(2\phi - s_1 + s_2\right)\frac{2N}{9\phi}\frac{\partial s_2}{\partial B_2}\frac{\partial B_2}{\partial g}\right], \tag{3.23}$$

$$\frac{\partial \pi_2^*}{\partial g} = \left[\frac{\left(\phi + s_1 - s_2\right)^2}{9\phi}\frac{\partial N}{\partial g}\right] + \left[\left(\phi + s_1 - s_2\right)\frac{2N}{9\phi}\frac{\partial s_1}{\partial B_1}\frac{\partial B_1}{\partial g}\right]. \tag{3.24}$$

Notice that the terms in the first bracket of each equation are positive. This is the direct effect. The sign of the second set of bracketed terms in each equation is the strategic effect. Its sign is indeterminate in equation (3.23) and (3.24) because $\partial B_1/\partial g$ and $\partial B_2/\partial g$ are indeterminate from *Proposition 2*. The signs of the second set of bracketed terms in equations (3.23) and (3.24) are indeterminate because $2N/9\phi > 0$, $\partial s_i/\partial B_i < 0$, and $\partial B_i/\partial g$ is indeterminate. Therefore, $\partial \pi_i^*/\partial g$ is indeterminate.

The effects of g on Nash profits in the final stage of the game under scenario 2 are:

$$\frac{\partial \pi_1^*}{\partial g} = x_3 + \left[\left(2\phi - s_1 + s_2\right)\frac{N}{9\phi}\frac{\partial \phi}{\partial g}\left(2 + \frac{\left(s_1 - s_2\right)}{\phi}\right)\right], \tag{3.25}$$

$$\frac{\partial \pi_2^*}{\partial g} = x_4 + \left[\left(\phi + s_1 - s_2\right)\frac{N}{9\phi}\frac{\partial \phi}{\partial g}\left(1 + \frac{\left(-s_1 + s_2\right)}{\phi}\right)\right]. \tag{3.26}$$

where x_3 equals the terms on the right-hand side of equation (3.23) and x_4 equals the terms on the right-hand side of equation (3.24). In scenario 2, $\partial\phi/\partial g < 0$, and the signs of the bracketed terms are negative because $\left(2\phi - s_1 + s_2\right) >$

0, $\left(\phi + s_1 - s_2\right) > 0$, $2N/9\ \phi > 0$, and $\left(2 + \frac{\left(s_1 - s_2\right)}{\phi}\right) > 0$, $\left(1 + \frac{\left(-s_1 + s_2\right)}{\phi}\right) > 0$.

However, because x_3 and x_4 are indeterminate and we cannot determine whether x_3 and x_4 are larger than the bracketed terms in (3.25) and (3.26), the effects of g on Nash profits of both firms are also indeterminate in scenario 2.

Proof of Proposition 3.4:

Under scenario 1, inspection of equations (3.23) and (3.24) shows that the first set of bracketed terms in each equation is positive, and the first set of bracketed terms in equation (3.23) is greater than the first set of bracketed terms in equation (3.24). The second set of bracketed terms is negative in both equations under the assumptions of the proposition. Because $B_1 > B_2$ in equilibrium and given diminishing returns to advertising, $|\partial s_1/\partial B_1| < |\partial s_2/\partial B_2|$ in equilibrium. Furthermore, the set of second bracketed terms in (3.23) is larger in magnitude than the second set of bracketed terms in (3.24) when $|\partial s_2/\partial B_2|$ is sufficiently large relative to $|\partial s_1/\partial B_1|$. Therefore, when $|\partial s_2/\partial B_2|$ is sufficiently large, $\partial\pi_2^*/\partial g > \partial\pi_1^*/\partial g$. This also holds under scenario 2. From equations (3.25) and (3.26), $x_4 > x_3$. As long as $|\partial s_2/\partial B_2|$ is sufficiently large, the relative effect of x_4 continues to play a dominant role in equations (3.25) and (3.26). Therefore, $\partial\pi_2^*/\partial g > \partial\pi_1^*/\partial g$ under scenario 2. ■

Proof of Proposition 3.5:

The bracketed term on the right hand side of equation (3.13) equals the marginal benefit of brand advertising, which is positive and declines in own brand advertising. The marginal cost of brand advertising equals -1. Because $\partial s_i/\partial B_i < 0$ and θ is positive and greater than $(s_i - s_j)/(3t)$, each firm will advertise as long as N is sufficiently large. Given the symmetry of brand advertising effectiveness (i.e., $\partial s_1/\partial B_1 = \partial s_2/\partial B_2, \forall\ B_1 = B_2$) search costs will be the same and each firm will invest equally in informative brand advertising in equilibrium because the marginal benefit of advertising is the same for each firm. ■

Proof of Proposition 3.6:

We use the implicit-function theorem and Cramer's rule in equation (3.13) to determine the effect of generic advertising on each firm's optimal choice of brand advertising. This produces the following results.

$$\frac{\partial B_1^*}{\partial g} = \frac{\begin{vmatrix} -\pi_{1g} & \pi_{12} \\ -\pi_{2g} & \pi_{22} \end{vmatrix}}{|\Pi|} = \frac{-\pi_{1g}\pi_{22} + \pi_{2g}\pi_{12}}{|\Pi|}, \tag{3.27}$$

$$\frac{\partial B_2^*}{\partial g} = \frac{\begin{vmatrix} \pi_{11} & -\pi_{1g} \\ \pi_{21} & -\pi_{2g} \end{vmatrix}}{|\Pi|} = \frac{-\pi_{11}\pi_{2g} + \pi_{1g}\pi_{21}}{|\Pi|}, \tag{3.28}$$

where $\Pi = \begin{pmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{pmatrix}$.

Under scenario 1 and with this notation,

$$\pi_{11} = \frac{N}{9t}\left[\left(\frac{\partial s_1}{\partial B_1}\right)^2 - \left(-s_1 + s_2 + 3t\theta\right)\left(\frac{\partial^2 s_1}{\partial B_1^2}\right)\right] < 0, \tag{3.29}$$

$$\pi_{22} = \frac{N}{9t}\left[\left(\frac{\partial s_2}{\partial B_2}\right)^2 - \left(s_1 - s_2 + 3t\theta\right)\left(\frac{\partial^2 s_2}{\partial B_2^2}\right)\right] < 0, \tag{3.30}$$

$$\pi_{12} = \pi_{21} = -\frac{N}{9t}\frac{\partial s_1}{\partial B_1}\frac{\partial s_2}{\partial B_2} < 0\,, \tag{3.31}$$

$$\pi_{1g} = \left[-\frac{1}{9t}\left(-s_1 + s_2 + 3t\theta\right)\frac{\partial s_1}{\partial B_1}\frac{\partial N}{\partial g}\right] > 0\,, \tag{3.32}$$

$$\pi_{2g} = \left[-\frac{1}{9t}\left(s_1 - s_2 + 3t\theta\right)\frac{\partial s_2}{\partial B_2}\frac{\partial N}{\partial g}\right] > 0. \tag{3.33}$$

Notice that that $\pi_{1g} = \pi_{2g}$ in equilibrium because $s_1 = s_2$. For the Nash equilibrium to be stable, the determinant of matrix Π must be positive. This implies that:

sign $\partial B_1^*/\partial g$ = sign $(-\pi_{1g}\,\pi_{22} + \pi_{2g}\,\pi_{12})$,

sign $\partial B_2^*/\partial g$ = sign $(-\pi_{11}\,\pi_{2g} + \pi_{21}\,\pi_{1g})$

If the equilibrium is also unique, $|\pi_{ii}| > |\pi_{ij}|$ (Dixit, 1986). Thus, $\partial B_1^*/\partial g$ and $\partial B_2^*/\partial g$ are positive under scenario 1.

Under scenario 2, the only changes will be to π_{1g} and π_{2g}, which are:

$$\pi_{1g} = x_5 + \left[\frac{N}{3}\frac{\partial\theta}{\partial g}\left(-\frac{\partial s_1}{\partial B_1}\right)\right], \tag{3.34}$$

$$\pi_{2g} = x_6 + \left[\frac{N}{3} \frac{\partial \theta}{\partial g} \left(-\frac{\partial s_2}{\partial B_2} \right) \right]. \tag{3.35}$$

where x_5 equals the terms on the right-hand side of equation (3.32) and x_6 equals the terms on the right-hand side of equation (3.33), with $x_5 > 0$ and $x_6 > 0$. Note further that $-\partial s_i/\partial B_i > 0$ and that $\partial\theta/\partial g < 0$ under scenario 2. Thus, the signs of the bracketed terms in equations (3.34) and (3.35) are negative, and the effect of g on brand advertising is indeterminate in scenario 2. ■

Proof of Proposition 3.7:

Following the envelope theorem in Caputo (1996), the effects of g on Nash profits in the final stage of the game (π_i^{**}) under scenario 1 are:

$$\frac{\partial \pi_1^{**}}{\partial g} = \left[\frac{(-s_1 + s_2 + 3t\theta)^2}{18t} \frac{\partial N}{\partial g} \right] + \left[-\frac{N}{9t}(-s_1 + s_2 + 3t\theta)\left(-\frac{\partial s_2}{\partial B_2}\frac{\partial B_2}{\partial g} \right) \right], \tag{3.36}$$

$$\frac{\partial \pi_2^{**}}{\partial g} = \left[\frac{(s_1 - s_2 + 3t\theta)^2}{18t} \frac{\partial N}{\partial g} \right] + \left[-\frac{N}{9t}(s_1 - s_2 + 3t\theta)\left(-\frac{\partial s_1}{\partial B_1}\frac{\partial B_1}{\partial g} \right) \right], \tag{3.37}$$

The first set of bracketed terms in each equation represents the direct effect of g on profit and is positive. The signs of the terms in the second bracket represent the strategic effect of g on profit, which is negative because the first set of terms in parentheses are positive and, the second set of terms in parentheses are positive, and N, θ, $t > 0$. Because the magnitudes of $\partial N/\partial g$, $\partial s_i/\partial B_i$, and $\partial B_i/\partial g$ are unknown, $\partial \pi_i^*/\partial g$ is indeterminate.

Under scenario 2, the added effect of g on θ (i.e., $\partial\theta/\partial g < 0$), produces the following:

$$\frac{\partial \pi_1^{**}}{\partial g} = x_7 + \left[\frac{N}{3}\left(-s_1 + s_2 + 3t\theta\right)\frac{\partial \theta}{\partial g} \right], \tag{3.38}$$

$$\frac{\partial \pi_2^{**}}{\partial g} = x_8 + \left[\frac{N}{3}\left(s_1 - s_2 + 3t\theta\right)\frac{\partial \theta}{\partial g} \right], \tag{3.39}$$

where x_7 equals the terms on the right-hand side of equation (3.36) and x_8 equals the terms on the right-hand side of equation (3.37). Because the signs of x_7 and x_8 are indeterminate and the relative effect of g on B_i versus θ is unknown, the effect of g on Nash profits is also indeterminate in scenario 2. ■

Proof of Proposition 3.8:

Under scenario 1, inspection of equations (3.36) and (3.37) show that the first set of bracketed terms in each equation is positive and the first bracketed term in (3.36) is greater than the first bracketed term in (3.37) because $s_2 > s_1$ in equilibrium. The second set of bracketed terms is negative in both equations under the assumptions of the proposition. Furthermore, the second bracketed term in (3.36) is larger in magnitude than the second bracketed term in (3.37) when

$|\partial s_2/\partial B_2|$ is sufficiently large relative to $|\partial s_1/\partial B_1|$. In this case, $\partial\pi_2^*/\partial g > \partial\pi_1^*/\partial g$. This holds under scenario 2 as well. Notice that the set of bracketed terms in (3.38) is larger than (3.39) because $s_2 > s_1$. Therefore, when $|\partial s_2/\partial B_2|$ is sufficiently large relative to $|\partial s_1/\partial B_1|$, $\partial\pi_2^*/\partial g > \partial\pi_1^*/\partial g$. ■

Chapter 4

Endogenous Timing and Strategic Choices: The Cournot-Bertrand Model

4.1. Introduction

The study of oligopoly theory began since the classic models of Augustin Cournot (1838) and Joseph Bertrand (1883). Cournot derived what came to be known as the Nash equilibrium to a static duopoly game where firms produce homogeneous goods and simultaneously choose output knowing market demand and cost conditions. In equilibrium, price is below the monopoly price but above marginal cost. Bertrand analyzed the same model except that the choice variable is price instead of output. Interestingly, Bertrand found a different Nash equilibrium, one where prices equal marginal cost and firms earn zero profit. Unlike the case of monopoly, the choice of output or price as the strategic variable can have a dramatic effect on the Nash outcome in an oligopoly market.

This has caused extensive debate concerning which choice variable better reflects firm behavior. The genesis of Bertrand's model came in response to his "Cournot criticism." Bertrand argued that there is no price-setting mechanism in the Cournot model and that real firms set prices, not quantities. The main criticism of the Bertrand model is the implication that no more than two firms are needed to generate perfect competition (the Bertrand Paradox).

An important concern with both models is that the strategic variable (output versus price) is exogenously determined. If firms had the choice, they would clearly prefer output competition in this simple static setting with homogeneous goods. With vertical product differentiation, however, Häckner (2000) shows that price competition can be more profitable than output competition when the quality difference between brands is sufficiently large.

A model that combines Cournot and Bertrand, allowing one firm to chose output and the other to choose price, has received little attention in the literature. To our knowledge, Singh and Vives (1984) are the only authors to consider this possibility.[41] In their duopoly model, they find that the dominant strategy is for both firms to compete in output when products are substitutes and for both firms to compete in price when products are complements. Thus, they do not allow different firms to pursue differing strategies. Nevertheless, such behavior does exist. In the retail market for small cars, for example, Honda dealers set quantities (i.e., monthly inventories) and adjust prices to clear the market. Scion (and Saturn) dealers, however, set prices and then fill all orders at the given price. Scion dealers behave as Bertrand-type firms, while Honda dealers behave as

[41] Their example is also mentioned in Hamilton and Slutsky (1990), Amir (1995), Amir and Grillo (1995) ,Vives (1999), and Häckner (2000).

Cournot-type firms, while. This example suggests that further study is needed.[42] Additionally, there may be conditions unexplored by Singh and Vives that support the Cournot-Bertrand outcome.

Another important criticism of the simple Cournot and Bertrand models is that they are static. Irving Fisher (1898) is the first to raise this concern, observing that real firms engage in various forms of dynamic behavior, what we would call predatory pricing and trigger strategies today. Stackelberg (1934) addresses this issue by identifying what is now called the subgame perfect Nash equilibrium (SPNE) to a dynamic Cournot-type model, where one firm produces output in the first period and the other firm produces output in the second period. The weakness with the Stackelberg model, however, is that the timing of play is exogenously determined. This assumption has an important effect on the outcome of the game, as the firm that moves first (the leader) earns greater profit than the firm the moves second (the follower).

Recently, Hamilton and Slutsky (1990), Amir (1995), and Amir and Grilo (1999) have made important contributions by developing models that allow the timing of play to be endogenous. In a model with two firms and two periods (early and late), two types of outcomes are possible: (1) Static, where both firms act in the early period or both firms act in later period, and (2) Dynamic, where

[42] This follows from Kreps and Scheinkman's (1983) argument that it is "witless" to choose one static oligopoly model (e.g., Cournot) over another (Bertrand), as it is an empirical question whether or not firms compete in output or price.

one firm acts in the early period and the other in the later period. This new body of research shows that the SPNE to an output game is for both firms to act simultaneously. In a pricing game, there are multiple equilibria with each player preferring a dynamic setting.

In this paper, we derive a duopoly model where both the timing of play and the choice of strategic variable (quantity or price) are endogenous, a model not previously developed in the literature. We first consider the case where only the choice of strategic variable is endogenous and show that an outcome where one firm competes in output and the other in price can be a Nash equilibrium; we call this a static Cournot-Bertrand model.[43] We also investigate the general properties of the model and provide a simple example. Finally, we develop a model where firms can choose to act in an early or late period and can choose to compete in quantity or price. We show that an asymmetric choice of strategic variables, where one firm competes in output and the other competes in price, can be a SPNE outcome.

[43] When at least one firm has a different objective function from other firms in an oligopoly market, De Fraja and Delbono (1990) define this as a mixed oligopoly. Similarly, we could define our model as a mixed oligopoly in output and price. Because this may lead to confusion with oligopoly models with pure vs. mixed strategies, we prefer the title "Cournot-Bertrand" model. In our analysis, we consider only pure strategy Nash equilibria.

4.2 A Static Model with an Endogenous Quantity-Price Choice

In this section, we review the Singh and Vives (1984) result and show how their model can be modified to support a Cournot-Bertrand outcome. Their model assumes that two firms (1 and 2) produce products that are imperfect substitutes and that each firm faces a negatively sloped demand function.[44] Marginal cost is normalized to zero, and information is complete. The endogenous choice of the strategic variable is modeled by assuming that firms have the option of choosing to compete in output or price in a preplay stage of the game. Once the choice is made, they then simultaneously choose the optimal level of their choice variable.[45] This leads to four possible outcomes:

1. Cournot (CC): both firms compete in output.
2. Bertrand (BB): both firms compete in price.
3. Cournot-Bertrand (CB): firm 1 competes in output and firm 2 competes in price.
4. Bertrand-Cournot (BC): firm 1 competes in price and firm 2 competes in output.

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[44] The best-reply functions normally have a negative slope when both firms compete in output (Cournot) and have a positive slope when both compete in price. Although there are cases where these sign conditions need not hold, Amir and Grilo (1999) call this the "typical geometry" for the Cournot and Bertrand models. We assume this typical geometry for the remainder of the paper.

[45] If the preplay stage does not take place in real time, as in Hamilton and Slutsky (1990), this is a single stage game. Singh Vives (1984) assumed real time, making this a two stage game. Our description follows Hamilton and Slutsky.

Nash equilibrium profits for each outcome are defined as π_i^k, where subscript i refers to firm 1 or 2 and superscript k refers to the particular outcome, CC, BB, CB, or BC. In the game such as this, Singh and Vives prove that the dominant strategy for each firm is to compete in output (Proposition 2).[46] That is, $\pi_1^{CC} > \pi_1^{BC}$ and $\pi_1^{CB} > \pi_1^{BB}$ for firm 1, and $\pi_2^{CC} > \pi_2^{CB}$ and $\pi_2^{BC} > \pi_2^{BB}$ for firm 2. Thus, Cournot is the dominant-strategy equilibrium. The problem with this result is that even though the model rules out the possibility of a Cournot-Bertand outcome, it is observed in the real world. One way to salvage the Cournot-Bertrand model is to assume that the choice of strategic variables is exogenously given, as is done in the traditional Cournot and Bertrand models. For example, institutional constraints require that firm 1 compete in output and firm 2 compete in price.

Perhaps a better solution is to assume that firms face different fixed costs. Let firm 2 face relatively high fixed costs of holding inventory (F) when competing in output, such that $F_2 > F_1$ (with F_1 normalized to zero for convenience). If F_2 is sufficiently high, the Cournot-Bertrand outcome becomes the Nash equilibrium. That is, firm 1's dominant strategy remains to compete in output, but firm 2 prefers to compete in price, because $\pi_2^{CB} > \pi_2^{CC} - F_2$ and $\pi_2^{BB} > \pi_2^{BC} - F_2$ where π_2^{CC} and π_2^{BC} are variable profits. Thus, sunk costs can affect a firm's choice of strategic variable.

[46] They also consider the case where firms produce complements instead of substitutes. When firms produce complements, the dominant strategy for each firm is to compete in price.

This scenario appears to describe our automobile example. Fixed costs would be higher for output competition than price competition, because output competition requires a dealer to hold a large inventory relative to a firm that competes in price and ships only after an order is placed. Scion (firm 2) is a relatively new brand that is distributed by Toyota dealers. When Scion was introduced, existing Toyota dealers faced a capacity constraint, with sales lots full of new Toyotas and used cars. By choosing to compete in price and shipping to order, little inventory is required, giving Scion relatively low fixed costs. If Scion had chosen to compete in output, its fixed costs would have been high, because Toyota dealers would have had to expand their capacity, which would be costly in densely populated areas. Thus, Scion prefers price competition. In contrast, Honda (firm 1) has established dealers that have competed in output for decades. Switching to price competition would incur a switching cost, as Honda has existing storage capacity which would be costly to liquidate. With sufficiently high switching costs or sufficiently low fixed costs associated with holding inventory, Honda sticks with output competition.

4.3 Properties of the Static Cournot-Bertrand Model

4.3.1 General Properties

Given that there are several ways to justify the Cournot-Bertrand model, we now describe its characteristics. We consider a differentiated duopoly game where firms have complete information. Firm 1 competes in output, $q_1 \in [0, \infty)$, and firm 2 competes in price, $p_2 \in [0, \infty)$. Firm *i*'s profit function depends on both strategic variables, $\pi_i(q_1, p_2)$, and is concave and twice continuously differentiable. Best-reply functions (correspondences) are strictly monotone, and an equilibrium exists on the interior of the action space. The model has a natural strategic asymmetry, with firm 1's profit function exhibiting increasing marginal returns to p_2 (that is, q_1 and p_2 are strategic complements) and firm 2's profit function exhibiting decreasing marginal returns to q_1 (that is, q_1 and p_2 are strategic substitutes). In other words, $\pi_{12} > 0$ and $\pi_{21} < 0$, where π_{ij} is defined as the second derivative of firm *i*'s profit function with respect to firm *i*'s choice variable and its rival's choice variable.

The effect of this asymmetry can be seen in each firm's incentive to cheat on a cartel agreement. If firms 1 and 2 can write an enforceable agreement that allows them to maximize joint profits, $\Pi = \pi_1 + \pi_2$, the first-order conditions are

$$\frac{\partial \Pi}{\partial q_1} = \frac{\partial \pi_1}{\partial q_1} + \frac{\partial \pi_2}{\partial q_1} = 0, \tag{4.1}$$

$$\frac{\partial \Pi}{\partial p_2} = \frac{\partial \pi_1}{\partial p_2} + \frac{\partial \pi_2}{\partial p_2} = 0. \tag{4.2}$$

Because $\partial\pi_2/\partial q_1 < 0$, $\partial\pi_1/\partial q_1$ must be positive, and firm 1 has an incentive to cheat on the cartel agreement by increasing its strategic variable, q_1. Similarly, $\partial\pi_2/\partial p_2$ must be negative because $\partial\pi_1/\partial p_2 > 0$. Thus, firm 2's incentive is to cheat by lowering its strategic variable, p_2.

In a non-cooperative setting, the structure of the model guarantees a unique Nash equilibrium. If we define firm 1's best-reply function as $r_1(p_2)$ = argmax $\pi_1(q_1, p_2)$ and firm 2's best-reply function as $r_2(q_1)$ = argmax $\pi_2(p_2, q_1)$, then the Nash equilibrium occurs where the best-reply functions intersect. From the first-order conditions and the implicit-function theorem, the slope of r_1 equals $-\pi_{12}/\pi_{11}$ which is positive because $\pi_{12} > 0$ and $\pi_{11} < 0$ (from concavity). The slope of r_2 equals $-\pi_{21}/\pi_{22}$, which is negative because $\pi_{21} < 0$ and $\pi_{22} < 0$ (from concavity). Along with continuity of the best-reply functions, this guarantees uniqueness.[47]

4.3.2 An Example

We illustrate these results with a linear model, similar to that found in Singh and Vives (1984). We begin with the static Cournot-Bertrand model where

[47] Stability requires that when out of equilibrium, the dynamic (myopic) adjustment process converges to the Nash equilibrium. According to Dixit (1986), stability requires that $|\partial^2\pi_1/\partial q_1^2| > |\partial^2\pi_1/\partial q_1\,\partial p_2|$ and $|\partial^2\pi_2/\partial p_2^2| > |\partial^2\pi_2/\partial p_2\partial q_1|$.

firm 1 competes in output and firm 2 competes in price. Variable costs are the same for both firms and are normalized to zero. With no fixed costs, the profit functions are $\pi_1(q_1, p_2) = p_1\, q_1$ and $\pi_2(q_1, p_2) = p_2\, q_2$. This is accurate when firm 2 competes in price, because $F_2 = 0$, but when firm 2 competes in output, its full profit would be $\pi_2 - F_2$, where $F_2 > 0$. We assume simple linear demand functions in choice variables[48]

$$p_1(q_1, p_2) = a - q_1 + b\, p_2, \tag{4.3}$$

$$q_2(q_1, p_2) = a - p_2 - d\, q_1, \tag{4.4}$$

where $a \in (0, \infty), b \in \left(0, \frac{2}{3}\right),$ and $d \in (0,2)$.[49]

Assuming profit maximization and ignoring fixed costs, the Nash equilibrium values are

$$p_1^* = \frac{a(2+b)}{4+bd} > p_2^* = \frac{a(2-d)}{4+bd}, \tag{4.5}$$

$$q_1^* = \frac{a(2+b)}{4+bd} > q_2^* = \frac{a(2-d)}{4+bd}, \tag{4.6}$$

48 Assume that $a_1 = (a + a\, b)$, $a_2 = (a - a\, d)$, $a_3 = a_1/d_1$, $a_4 = a_2/d_1$, $b_3 = b/d_1$, $d_1 = (1 + b\, d)$, $d_3 = 1/d_1$, and $d_4 = d/d_1$. This structure produces the following inverse demand functions in the CC model: $p_1 = a_1 - d_1\, q_1 - b\, q_2$ and $p_2 = a - q_2 - d\, q_1$. In the BB model, the demand functions are $q_1 = a - p_1 - b\, p_2$ and $q_2 = a_2 - b_2\, p_2 + d\, p_1$. In the BC model, the demand functions are $q_1 = a_3 - d_3\, p_1 - b_3\, q_2$ and $p_2 = a_4 - d_3\, q_2 + d_4\, p_1$.

49 These parameter restrictions ensure firm participation in Nash equilibria for all possible strategic choice combinations and when the model is static and dynamic.

$$\pi_1^* = \frac{a^2(2+b)^2}{(4+bd)^2} > \pi_2^* = \frac{a^2(2-d)^2}{(4+bd)^2}. \tag{4.7}$$

The model has a natural asymmetry, with the strategic advantage going to firm 1.[50]

This equilibrium is illustrated in Figure 4.1, which graphs the best-reply and isoprofit functions assuming that a = 19, b = 0.5, and d = 1.5. Because firm 1's best reply has a positive slope and firm 2's has a negative slope, the Nash equilibrium is unique at $p_1^* = q_1^* = 10$ and $p_2^* = q_2^* = 2$, $\pi_1^* = 100$, and $\pi_2^* = 4$.

The other three individual Nash equilibria (CC, BB, and BC) are determined similarly. When fixed costs are zero, Singh and Vives (1984) prove that when the choice of strategic variable (quantity or price) is endogenous, each firm's dominant strategy is to compete in output. Thus, when given the option, firms will always compete in output in a static setting. Table 4.1 illustrates this for the example above. If $10.4 \leq F_2 < 11.2$, however, the Cournot-Bertrand outcome is the Nash equilibrium. Firm 1's dominant strategy remains the same, but firm 2's dominant strategy is now to compete in price. This illustrates that the Cournot-Bertrand outcome is possible when fixed costs are included in the model.

[50] In contrast, when variable costs are greater than zero, demand is log-convex, and the model is dynamic, Amir and Grilo (1999) find that firm 1 (the leader) is the weaker firm.

4.4 A Model with Endogenous Timing and Quantity-Price Choice

In this final section, we investigate a dynamic duopoly model where both the timing and the choice of strategic variable are endogenous. That is, each firm has the choice of competing in output or price and has the option of acting in an early (E) or late (L) stage of the game. Information is perfect and complete.
For example, if firm 1 chooses to act early and firm 2 chooses to act late, firm 2 knows the history of play (i.e., firm 2 observes firm 1's action). Other characteristics of the game remain the same as in section 4.3.1.

Of the 16 possible outcomes in the extensive form of the game, 8 are static and 8 are dynamic. In the static cases where both firms act in the same period and choose the same strategic variable, this leads to one early and one late Cournot outcome (E-CC and L-CC) and one early and one late Bertrand outcome (E-BB and L-BB). Similarly, if different strategic variables are chosen, this leads to the mixed Cournot-Bertrand cases: E-CB, L-CB, E-BC, and L-BC. Assuming no discounting, only 4 of the 8 static cases are distinct:

1. Both firms choose output, early and late (E-CC and L-CC).
2. Both firms choose price, early and late (E-BB and L-BB)
3. Firm 1 chooses output and firm 2 chooses price, early and late (E-CB and L-CB).

4. Firm 1 chooses price and firm 2 chooses output, early and late (E-BC and L-BC).

The 8 dynamic outcomes also have interesting features. For example, if firm *i* competes in output and firm *j* (defined as not firm *i*) competes in price, both firms are better off if firm *i* chooses to act early and firm *j* chooses to act late (Hamilton and Slutsky, 1990; Amir, 1995). We call this a dynamic Cournot-Bertrand model This can be seen in Figure 4.1. If firm $i = 1$ moves first, it will maximize its profits given the best reply of firm 2, which will occur in the Pareto superior region (i.e., the shaded region) of the diagram. Both firms are better off when firm 1 moves first because both move to a higher isoprofit function. In our numerical example above, by moving dynamically rather than statically firm 1's profits increase from 100 to 102.6, and firm 2's profits increase from 4 to 9.1. Thus, when the timing of play is endogenous in a Cournot-Bertrand setting, firms will prefer to play a dynamic rather than a static game.

A model where both the timing and the choice of strategic variable are endogenous has not been addressed in previous research. An interesting feature of the model is that once firm 1 has made its choice, firm 2's outcome is the same whether firm 2 decides to maximize profit with respect to output or price.

Proposition 4.1: Firm i chooses its action, q_i or p_i, in the first period, which is observed by firm j. In the second period, firm j maximizes its profit with respect to its action, q_j or p_j. Each firm's demand is continuous and has a negative slope. Demand is positive when the price is below some positive but finite number x, and demand zero for $p_i \geq x$. Each firm's cost function is twice continuously differentiable and strictly convex. Fixed costs are zero. A unique equilibrium exists on the interior of the action space. Under these conditions, firm j's profits will be the same, whether firm j competes in output or price.

Proof: Consider the case where firm i moves first and competes in output. In the second period, firm j chooses its optimal level of output or price. If it competes in output, its problem is to

$$\max_{q_j} \pi_j = p_j(q_i, q_j)\, q_j - C(q_j), \tag{4.8}$$

where $p_j(q_i, q_j)$ is firm j's inverse demand function and $C(q_j)$ is its cost function. Because q_i is predetermined and equal to a constant in the second period, firm j's (residual) demand function is only a function of q_j. The firm's first-order condition is

$$\frac{\partial \pi_j}{\partial q_j} = p_j + \frac{\partial p_j}{\partial q_j} q_j - \frac{\partial C}{\partial q_j} = 0 \tag{4.9}$$

If instead firm j competes in price, its problem is to

$$\max_{p_j} \pi_j = p_j\, q_j(q_i, p_j) - C\left[q_j(q_i, p_j)\right], \tag{4.10}$$

where $q_j(q_i, p_j)$ is firm *j*'s demand function. Again, because q_i is predetermined and equal to a constant, firm *j*'s (residual) demand function is only a function of p_j. The firm's first-order condition is

$$\frac{\partial \pi_j}{\partial p_j} = q_j + \frac{\partial q_j}{\partial p_j} p_j - \frac{\partial C}{\partial q_j}\frac{\partial q_j}{\partial p_j} = 0 \qquad (4.11)$$

Multiplying both sides of equation (4.11) by $\partial p_j/\partial q_j$ produces equation (4.9). Thus, the first-order conditions and the optimal values of p_j and q_j will be the same whether firm *j* competes in price or quantity. A similar argument applies when firm *i* competes in price in period 1. ■

The intuition behind Proposition 1 is that firm *i* sets q_i (or p_i) equal to a constant in the first period. In the second period, firm *j* faces a residual demand function that depends only on its own action. Therefore, the structure of firm *j*'s problem is just like that of a monopolist, where the monopoly outcome is the same whether the firm maximizes profit with respect to output or price.

Thus, for a given choice by the early firm, the equilibrium outcome is invariant to the strategic choice variable of the later firm. This result is important, because it implies that when firms are forced to compete in a dynamic setting, as in Stackelberg, blended q_i-p_j competition is just as likely as traditional q_i-q_j or p_i-p_j competition.

Proposition 1 also reduces the set of possibilities. Of the 8 dynamic cases, therefore, only 4 are distinct. These are:

1. Firm 1 chooses output early and firm 2 chooses output late (q_{1E}, q_{2L}); firm 1 chooses output early and firm 2 chooses price late (q_{1E}, p_{2L}).
2. Firm 1 chooses price early and firm 2 chooses output late (p_{1E}, q_{2L}); firm 1 chooses price early and firm 2 chooses price late (p_{1E}, p_{2L}).
3. Firm 2 chooses output early and firm 1 chooses output late (q_{2E}, q_{1L}); firm 2 chooses output early and firm 1 chooses price late (q_{2E}, p_{1L}).
4. Firm 2 chooses price early and firm 1 chooses output late (p_{2E}, q_{1L}); firm 2 chooses price early and firm 1 chooses price late (p_{2E}, p_{1L}).

We now consider the equilibrium outcome for the linear model described in section 4.3.2. Proposition 2 indicates our finding.

Proposition 4.2*:* Firms compete in the general linear game describe in section 4.3.2, except that they have no fixed costs. Firms have the choice of strategic variable (output or price) and the timing of play (early or late). Under these conditions, the SPNE is the early Cournot outcome, were both firms compete in output in the early period.[51]

[51] This is an iterated-dominant outcome, as only firm 1 has a dominant strategy for all permissible parameter values: $a > 0$, $0 < b < 2/3$, and $0 < d < 2$.

Proof. See the Appendix A.4.

One can think of this as a dynamic version of Proposition 2 in Singh and Vives (1984), who considered only a static setting, or an endogenous strategic version of Theorem V in Hamilton and Slutsky (1990), who assume that strategic choices are exogenous.

As before, however, this conclusion can be overturned with the addition of appropriately defined fixed costs. Our findings are:

***Proposition 4.3**:* Firms compete in same game as described in Proposition 2, except that firm 2 faces a positive fixed cost when competing in output (F_2). When F_2 is sufficiently large, the SPNE is the dynamic Cournot-Bertrand outcome, where firm 1 competes in output early and firm 2 competes in price late.

Proof. See the Appendix A.4.

In this setting, firm 1's optimal choice remains q_{1E}, while firm 2's optimal choice switches from q_{2E} to p_{2L}. Thus, when both the strategic variable and the timing of play are endogenous, a dynamic Cournot-Bertrand outcome can be a SPNE. This also appears to be consistent with the market for small cars, where the established firm, Honda, competes in output and the new entrant, Scion, competes in price.

4.5 Conclusion

Although the timing of play and the choice of strategic variable are assumed to be exogenously determined in the classic models of oligopoly, these assumptions have been relaxed in more recent studies. Several authors endogenize the timing of play, allowing each firm to decide when to enter and compete in the game (early or late), but the choice of strategic variable remains exogenous. Others endogenize the choice of strategic variable (output or price), but only consider a static setting. We develop a general model where both the timing of play and the choice of strategic variable are endogenously determined.

The strategic choices of Honda and Scion in the market for small cares, in part, motivates our work. We build a “Cournot-Bertrand” model where one firm competes in output and the other competes is price in both a static and a dynamic settings. Three important results emerge from our analysis. First, when two firms must compete in a two-stage game of perfect information (with firm 1 moving first) but have a choice of strategic variable (output or price), the outcome of the game is invariant to whether firm 2 decides to compete in output or price. Thus, an outcome where one firm competes in output and the other in price is just as likely as one where both firms compete in the same choice variable. Second, when both the timing of play and the choice of strategic variable are endogenous and there are no fixed costs, the SPNE is for both firms to compete in output in the first stage of the game, that is, an early Cournot outcome. Finally, we show that with appropriately defined fixed costs, a Cournot-Bertrand model where one firm competes in output and the other competes in price can be a Nash equilibrium when firms are forced to play a static game and can be a SPNE when both the timing of play and the choice of strategic variable are endogenous.

Appendix A.4

Proof of Proposition 4.2:

The proof of Proposition 2 is as follows. Note that most steps in the proof are based on our Proposition 1, Theorem V in Hamilton and Slutsky (1990) and Amir and Grilo (1999), and Proposition 2 in Singh and Vives (1984).

1. Firm 1 has a dominant strategy, to play output early (q_{1E}). The argument is as follows.

   (a) When firm 2 chooses output early (q_{2E}), firm 1's choice of q_{1E} dominates:

      i. Output late (q_{1L}) (Hamilton and Slutsky; Amir and Grilo).

      ii. Price early (p_{1E}) (Singh and Vives).

      iii. Price late (p_{1L}), because q_{1E} dominates q_{1L}, and firm 1's profit is the same whether it competes in q_{1L} or p_{1L} (Proposition 1). Therefore, q_{1E} dominates p_{1L}.

   (b) When firm 2 chooses output late (q_{2L}), q_{1E} dominates:

      i. q_{1L} (Hamilton and Slutsky; Amir and Grilo 1999).

      ii. p_{1E}. For these alternatives, firm 1's profits are:

      $\pi_1(q_{1E}, q_{2L}) = [a^2(2 + b)^2]/[8(2 + b\, d)]$, and

$\pi_1(p_{1E}, q_{2L}) = [a^2(2 + b + b\,d)^2]/[8(2 + 3\,b\,d + b^2\,d^2)]$.

$\pi_1(q_{1E}, q_{2L}) - \pi_1(p_{1E}, q_{2L}) =$

$[a^2\,b^2\,d\,(2 + b - d)]/[8(2 + 3\,b\,d + b^2\,d^2)]$, which is positive because d must be less than 2.

iii. p_{1L}. From above, q_{1E} dominates q_{1L}, and q_{1L} dominates p_{1L} (Singh and Vives). Therefore, q_{1E} dominates p_{1L}.

(c) When firm 2 chooses price early (p_{2E}), q_{1E} dominates:

i. q_{1L}. For these alternatives, firm 1's profits are:

$\pi_1(q_{1E}, p_{2E}) = [a^2(2 + b)^2\,]/(4 + b\,d)^2$, and

$\pi_1(q_{1L}, p_{2E}) = [a^2(4 + b(2 + d))^2]/[16(2 + b\,d)^2]$.

$\pi_1(q_{1E},p_{2E}) - \pi_1(q_{1L}, p_{2E}) =$

$\{a^2b^2d(2\text{-}d)[32+16b(1+d)+b^2d(6+d)]\}/[16(8+6bd+b^2d^2)^2]$,

which is positive because d must be less than 2. Therefore, q_{1E} dominates q_{1L}.

ii. p_{1E} (Singh and Vives).

iii. p_{1L}, because q_{1E} dominates q_{1L} and firm 1's profit is the same whether it competes in q_{1L} or p_{1L} (Proposition 1). Therefore, q_{1E} dominates p_{1L}.

(d) When firm 2 chooses price late (p_{2L}), q_{1E} dominates:

i. q_{1L} (Hamilton and Slutsky; Amir and Grilo).

ii. p_{1E}. $\pi_1(q_{1E}, q_{2L}) > \pi_1(p_{1E}, q_{2L})$ from b.ii above.

$\pi_1(q_{1E}, q_{2L}) = \pi_1(q_{1E}, p_{2L})$ and $\pi_1(p_{1E}, q_{2L}) = \pi_1(p_{1E}, p_{2L})$ from our Proposition 1. Therefore q_{1E} dominates p_{1E}.

iii. p_{1L}, because q_{1E} dominates q_{1L} (Hamilton and Slutsky; Amir and Grilo) and q_{1L} dominates p_{1L} (Singh and Vives).

2. Given that firm 1's dominant strategy is to play q_{1E}, firm 2's best reply is to play q_{2E}. That is, when firm 1 chooses q_{1E}, competing in q_{2E} dominates:

(a) q_{2L} (Hamilton and Slutsky; Amir and Grilo).

(b) p_{2E} (Singh and Vives).

(c) p_{2L}, because q_{2E} dominates q_{2L} and firm 2's profit is the same whether it competes in q_{2L} or p_{2L} (Proposition 1). ■

Proof of Proposition 4.3:

The proof of Proposition 3 is as follows. Because fixed costs are only imposed on firm 2 (F_2), firm 1's dominant strategy remains q_{1E} (Proposition 2). Ignoring fixed costs for the moment, the following profit conditions hold for firm 2: $\pi_2(q_{1E}, q_{2E}) > \pi_2(q_{1E}, q_{2L}) = \pi_2(q_{1E}, p_{2L}) > \pi_2(q_{1E}, p_{2E})$. This is because $\pi_2(q_{1E}, q_{2E}) > \pi_2(q_{1E}, q_{2L})$ by Proposition 2, $\pi_2(q_{1E}, q_{2L}) = \pi_2(q_{1E}, p_{2L})$ by Proposition 1, and $\pi_2(q_{1E}, p_{2L}) > \pi_2(q_{1E}, p_{2E})$ by Hamilton and Slutsky (1990) and by Amir and Grilo (1999). With positive fixed costs when firm 2 competes in output, this condition becomes

$\pi_2(q_{1E}, q_{2E}) - F_2 > \pi_2(q_{1E}, q_{2L}) - F_2 < \pi_2(q_{1E}, p_{2L}) > \pi_2(q_{1E}, p_{2E})$.

When F_2 is sufficiently high, $\pi_2(q_{1E}, p_{2L}) > \pi_2(q_{1E}, q_{2E}) - F_2$. Thus, Firm 2's best reply to firm 1's decision to compete in q_{1E} is to compete in p_{2L}. ■

Table 4.1

Nash Equilibrium Profits for Cournot (CC), Bertrand (BB), and Cournot-Bertrand (CB) Outcomes

Firm 2

Firm 1		q_2	p_2
	q_1	$\pi_1^{CC} = 101.1$, $\pi_2^{CC} - F_2 = 14.4 - F_2$	$\pi_1^{CB} = 100$, $\pi_2^{CB} = 4$
	p_1	$\pi_1^{BC} = 88$, $\pi_2^{BC} - F_2 = 11.2 - F_2$	$\pi_1^{BB} = 97.6$, $\pi_2^{BB} = 4.04$

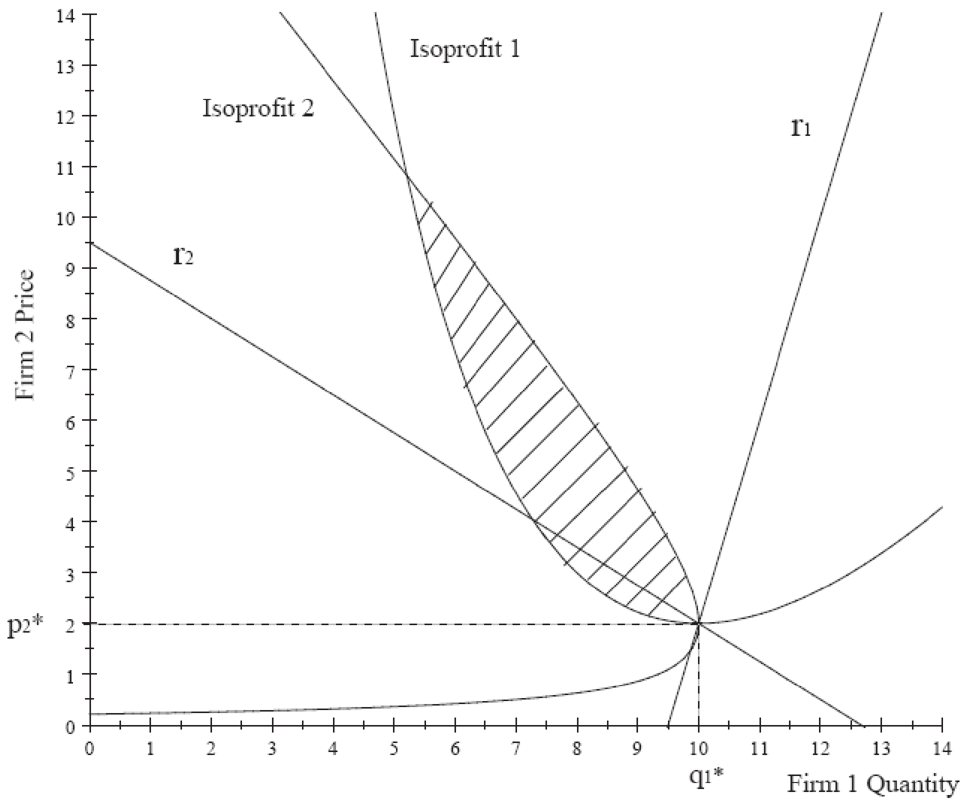

Figure 4.1: Best Reply and Isoprofit Functions in a Cournot-Bertrand Duopoly.

Chapter 5

General Conclusion

In this dissertation, we address strategic issues of imperfectly competitive firms that compete in output or price and in informative or persuasive brand advertising.

The results of Chapter 2 show that in a duopoly model with vertical product differentiation and persuasive brand advertising, only the high quality firm will use brand advertising. In this case, generic advertising is likely to benefit the low quality firm more than the high quality firm when generic advertising lowers product differentiation and induces the high quality firm to spend more on brand advertising. In a duopoly model with horizontal differentiation, we show that both firms advertise to promote their brands and that a symmetric outcome is more likely. When this occurs, profits and expenditures on brand advertising will be the same, and each firm will respond in the same way to an increase in generic advertising. This suggests that producers will be more likely to be either uniformly in favor or uniformly opposed to commodity mandatory generic advertising programs when differentiation is horizontal. We also show that the relationship between generic advertising and brand advertising is clear when the structure of the model is supermodular. That is, generic advertising will induce firms to increase their expenditures on brand advertising when firms play a supermodular game regardless of the type of product differentiation.

In Chapter 3, we analyze the interaction of generic and brand advertising when brand advertising is purely informative. We show that firms can benefit from investing in brand advertising that lowers consumer search costs as well as from brand advertising that is purely persuasive. The results also show that whether brand advertising is persuasive or informative, the outcome is more likely to be symmetric with horizontal differentiation than with vertical differentiation. In addition, the effect of generic advertising on brand advertising and firm profits is generally indeterminate when advertising increases the size of the market and reduces product differentiation, regardless the type of advertising and the type of product differentiation. When advertising only increases the size of the market, however, important differences emerge. When brand advertising is purely persuasive, generic advertising always raises firm profits and causes firms that choose to advertise to increase spending on persuasive brand advertising, whether differentiation is vertical or horizontal. When brand advertising is purely informative, however, generic advertising generally has an indeterminate effect on firm profits and brand advertising. This difference is due to the fact that the brand advertising between firms is a strategic complement when persuasive and a strategic substitute when informative.

This study helps us better understand why some producers oppose generic advertising programs. The model also provides alternative explanations to explain why firms increase their expenditures on brand advertising. For example, firms may oppose generic advertising because of the free rider problem rather than because generic advertising reduces product differentiation.

In Chapter 4, we develop a general model where both the timing of play and the choice of strategic variable are endogenously determined. The strategic choices of Honda and Scion in the market for small cares, in part, motivates our work. We develop a hybrid "Cournot-Bertrand" model where one firm competes in output and the other competes is price in both a static and a dynamic setting. Three important results emerge from our analysis. First, when two firms must compete in a two-stage game of perfect information (with firm 1 moving first) but have a choice of strategic variable (output or price), the outcome of the game is invariant to whether firm 2 decides to compete in output or price. Second, when both the timing of play and the choice of strategic variable are endogenous and there are no fixed costs, the Subgame perfect Nash equilibrium is for both firms to compete in output in the first stage of the game, that is, an early Cournot outcome. Finally, we show that with appropriately defined fixed costs, a Cournot-Bertrand model where one firm competes in output and the other competes in price can be a Nash equilibrium when firms are forced to play a static game and can be a Subgame perfect Nash equilibrium when both the timing of play and the choice of strategic variable are endogenous. This Cournot-Bertrand model is important because it helps us understand the real world observation that firms choose different strategic variables and provides a theoretical justification for these differences.

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