

SD144
07
A45
no.51
cop.2

Equations for Predicting Basal Area Increment in Douglas-fir and Grand Fir

Martin W. Ritchie
David W. Hann



FOREST RESEARCH LAB



The Forest Research Laboratory of Oregon State University was established by the Oregon Legislature to conduct research leading to expanded forest yields, increased use of forest products, and accelerated economic development of the State. Its scientists conduct this research in laboratories and forests administered by the University and cooperating agencies and industries throughout Oregon. Research results are made available to potential users through the University's educational programs and through Laboratory publications such as this, which are directed as appropriate to forest landowners and managers, manufacturers and users of forest products, leaders of government and industry, the scientific community, and the general public.

As a research bulletin, this publication is one of a series that comprehensively and in detail discusses a long, complex study or summarizes available information on a topic.

The Authors

Martin W. Ritchie is a research assistant and David W. Hann is an associate professor, Department of Forest Management, Oregon State University, Corvallis.

Notice

This paper represents one chapter from the senior author's thesis, "Development of tree height and diameter growth equations for mid-Willamette Valley Douglas-fir," which was submitted in partial fulfillment of the Master of Science Degree, Department of Forest Management, Oregon State University.

To Order Copies

Copies of this and other Forest Research Laboratory publications are available from:

Forestry Business Office
College of Forestry
Oregon State University
Corvallis, Oregon 97331

Please include author(s), title, and publication number if known.

4-10-11
C
H
W
H
10/11

Equations for Predicting Basal Area Increment in Douglas-fir and Grand Fir

Martin W. Ritchie
David W. Hann

*Oregon State University, Forest Sciences Laboratory
Research Institute*

10/11

Contents

- 1 Abstract**
- 1 Introduction**
- 1 Data Collection**
- 3 Model Development**
 - 3 Model Selection**
 - 3 Variable Selection**
- 5 Final Models of Basal Area Increment**
- 6 Predicting Future Diameters**
- 6 Conclusion**
- 7 Literature Cited**
- 8 Appendix A: Maximum Crown Width**
- 8 Appendix B: Foliage Weight**
- 9 Appendix C: Height Growth**

Abstract

Equations are presented for predicting basal area increment for individual Douglas-fir and grand fir trees in the east-central Coast Range of Oregon. Final parameter estimates were obtained using weighted nonlinear regression analysis of a simple exponential model. Two equations are presented for each species: one has site index, and the other has predicted height growth as independent var-

iables in the model. The other variables used are diameter, crown ratio, crown competition factor in larger trees on the sample point, and stand basal area. Techniques for predicting future diameters from these equations are also presented. A number of methods of expressing stand density or structure are compared for the log-linear model of basal area growth.

Introduction

Simulators of growth and yield can be valuable tools in the management of forest stands. For example, when considering the application of various treatments to a given stand, growth responses can be predicted. Also, when conducting long-term planning, yield over the length of a rotation can be estimated. However, the results of simulators can differ depending on the variables incorporated into the design of each simulator.

Simulators can be classified on the basis of the primary modeling unit used in projecting growth (Munro 1974). In simulators for which the individual tree is the primary modeling unit, projections of stand growth and yield depend on estimates of the components of individual tree development, which are then aggregated to produce stand level estimates. The individual tree components may include diameter growth, height growth, crown change, and mortality models. Growth models for individual trees may be further classified by the presence or

absence of intertree distances in measures of competitive stress. Models which incorporate some measure of distance between the subject tree and its competitors are referred to as distance-dependent. Conversely, if competitive stress is quantified by some measure of overall density in a stand or plot, the model is referred to as distance-independent.

This study was conducted as one part of a project to develop an individual-tree/distance-independent growth simulator for Oregon State University's Research Forest Properties in the east-central Coast Range of Oregon. The objective was to develop distance-independent type equations for projecting individual tree diameter growth for Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco) and grand fir (*Abies grandis* (Dougl.) Lindl.). To meet this objective, it was necessary to evaluate various model forms and to compare a number of expressions of density or structure relative to the basal area increment of individual trees.

Data Collection

The data base for model development was established simultaneously with the installation of inventory plots on the Research Forest Properties. A total of 136 stands were selected, each of which had to have a significant amount of stand basal area in Douglas-fir and grand fir. Also, the stands had to be free from silvicultural treatment during the 5 years prior to measurement. This latter requirement was to insure that factors affecting growth were not disrupted during the growth period.

Plots were established on a systematic grid within each stand. Most stands contained one plot every 2 acres. However, some of the younger stands were sampled with one plot per acre to insure an

adequate representation on those age classes, and some of the older stands were sampled at one plot every 4 acres.

Among the stands measured, basal area ranged from 10 square feet to over 270 square feet per acre. Site index (King 1966) ranged from 90 to 142 feet at a base age of 50 years. Stand ages varied between 20 and 120 years, with a high concentration of stands between 40 and 60 years.

Each plot consisted of a variable radius point and two nested sub-plots with fixed radii. The 20 basal area factor (BAF) variable radius point was established for measuring all trees greater than 8.0 inches in diameter at breast height (4.5 feet). The larger of the two fixed-area sub-plots had a

radius of 15.56 feet for measuring all trees 4.1 to 8.0 inches in diameter. Finally, a plot with a radius of 7.78 feet was established at the point for measuring all trees 4.0 inches or smaller.

All trees, including hardwoods, were measured on each plot for total height, height to crown base, and diameter at breast height. Tree height and height to crown base were measured to the nearest 0.1 foot using the pole tangent method described by Curtis and Bruce (1968). Diameters were measured to the nearest 0.1 inch using a diameter tape. The previous five-year radial growth was measured on increment cores taken from all live conifers greater than 3.0 inches in diameter. Height growth was measured directly on 866 Douglas-firs. Radial growth was measured to the nearest 1/40 inch. All mortality trees estimated to have died in the last 5 years were included on the plot and noted as such with a mortality code.

A total of 9526 Douglas-fir trees were measured for radial growth on the growth plots. An additional 595 grand fir and 152 other conifers were also measured. The other conifers are not considered in this analysis because they represented a wide variety of species, many of which are not native to the area. A number of different tree species have been introduced on the forest over the years, either individually or in small groups, but none of the species in this group were represented well enough to form a data base of any use in modeling.

A computer program was written to backdate stands. Backdating was necessary for the data developed from temporary plots because predictor variables should represent measurements taken at the beginning of the growth period of interest. Bruce's (1981) height equation was used to backdate all trees for which height growth was not measured directly. Crown ratio was assumed constant over the 5-year growth period and diameters were backdated using the radial growth measurements.

Variables expressing stand and point density were then calculated. These variables included crown competition factor (CCF) (Krajicek et al. 1961), basal area per acre (BA), and foliage weight per acre (FW).

CCF was calculated by summing individual tree (CCF_i) values over all trees in the plot or stand, respectively:

$$PCCF = \sum_{i=1}^{np} (CCF_i \cdot EXPN_i) \quad [1]$$

$$SCCF = \sum_{i=1}^{ns} (CCF_i \cdot EXPN_i/M) \quad [2]$$

where:

PCCF = CCF at the plot level,
 SCCF = CCF at the stand level,
 CCF_i = 0.001803 · MCW_i²,
 np = number of trees on a point,
 ns = number of trees in a stand,
 M = number of points in a given stand,
 EXPN_i = expansion factor of each tree,
 and
 MCW_i = maximum crown width of each tree, estimated from crown width equations (Appendix A).

Since the foliage weight on a plot or stand should be indicative of demand for nutrients and water in the soil, as well as competition for light, it was felt that FW estimators may provide an improved means of estimating competitive stress.

Foliage weight was calculated for each tree using the species-specific equations developed from Brown's (1978) data and the local maximum crown width equations (Appendix A):

$$FW = k_1 (CL \cdot MCW \cdot CR^{k_2})^{k_3} \quad [3]$$

where:

FW = foliage weight of a given tree,
 CL = crown length,
 CR = crown ratio, and
 k₁, k₂, k₃ = species-specific parameter estimates (Appendix B).

Foliage weight per acre was then calculated by summation, in the same manner as CCF.

All measures of density were computed at both the point level and stand level. Competitive stress experienced by a given tree was characterized by subdivisions of these three density variables based on diameter intervals.

Two methods were used to define the diameter intervals. One method characterized density by three diameter intervals (Hann 1980). These intervals were defined such that the middle interval was composed of three, 1-inch diameter classes with the subject tree's diameter falling in

the central class. The remaining two intervals were defined as comprising all diameters above and below this middle interval. For each tree, the sum of the three variables equals total density per acre on the stand or point.

The second approach at indexing competition used a two-diameter-interval subdivision of density. Wykoff et al. (1982) defined a variable expressing basal area (BA) of trees larger in diameter than the subject tree. Basal area in smaller trees can be calculated in a similar fashion. This breakdown into two intervals was also applied to the CCF and FW density measures.

Because we also wished to examine species effects, levels of both CCF and BA were further subdivided by three species groups: Douglas-fir, other conifers, and hardwoods.

Model Development

Model Selection

The first step in the modeling effort was to define those factors which would characterize the best model. For this study, the best model was defined as that which minimized the residual mean squared error, came closest to meeting the assumptions of regression analysis, and characterized the relationship between basal area growth and the independent variables in a biologically meaningful fashion.

Diameter growth can be estimated by the use of either basal area growth equations or diameter growth equations. Both methods have been used in previous studies. Cole and Stage (1972) compared basal area and diameter increment, as well as logarithmic transformations of each. They concluded that the log of basal area growth best met the regression assumptions of normally distributed residuals and constant variance. However, in a study comparing basal area increment and diameter increment, West (1980) concluded that there was no a priori justification for choosing one over the other in projecting tree diameters.

In our analysis, basal area increment was used for parameter estimation because it is more easily extrapolated to alternative growth-period lengths (Cole and Stage 1972). The natural logarithm of basal area increment was used for variable screenings. The logarithmic transformation makes it possible to linearize some nonlinear

Because of the large size of the Douglas-fir data set and the cost involved with repeated regression analyses on a data file of such magnitude, a random subsample of approximately 20 percent (1910 trees) was selected for the variable screening phase of the analysis. The remaining 80 percent (7616 trees) was reserved for final parameter estimation. The grand fir data set was left intact because there was not a large enough number of observations for a subsample.

The primary advantage of subdividing a large data set is that parameter estimation can be done independently of variable selection. This allows for more reliable significance testing because there is no loss of degrees of freedom caused by variable selection. This reduction in sample size for variable screening also resulted in a significant savings in computing costs.

models. Such models are called intrinsically linear and make it possible to apply variable selection procedures with linear least-squares to nonlinear models.

Variable Selection

The Douglas-fir data were used for all variable screening and selection and for model comparisons. A combinatorial variable screening routine was used to select independent variables. The index used for comparison in this routine was the adjusted multiple correlation coefficient, R^2 (Draper and Smith 1981). This index can be thought of as a relative mean-squared error, such that a value of 1 indicates a perfect fit to the data, and a value of 0 indicates that the regression is no better than a simple mean.

The basic model used in variable selection is expressed as:

$$\ln(\text{BAG}) = b_0 + b_1 \cdot X_1 + b_2 \cdot X_2 + \dots + b_p \cdot X_p \quad [4]$$

where:

BAG = 5-year basal area growth inside bark, in square inches
 X_1, X_2, \dots, X_p = independent variables, and
 $b_0, b_1, b_2, \dots, b_p$ = parameter estimates.

The variables influencing basal area growth can be divided into three groups: site productivity, competition, and tree size or vigor. Site productivity was characterized by site index (King 1966), transformations of slope and aspect (Stage 1976), and potential height growth and predicted height growth. For a more thorough description of the height growth prediction, refer to Appendix C.

One of the most influential factors on basal area growth of individual trees is the level of competition for light, water, and nutrients. Density variables can be employed in models of basal area growth as indicators of the competitive stress being experienced by a given tree. In such models, high levels of density will generally indicate increased competition and, thus, reduced growth rates.

By subdividing density variables with respect to the size of the subject tree, it may be possible to more accurately model the response of basal area growth to various levels of competition. For example, stand basal area in larger trees and basal area in smaller trees, rather than total basal area, may be a better means of describing the stress on a given tree. If a stand is quite dense, but the subject tree is larger than all of its competitors, less growth reduction could be expected than if the same level of density was in trees larger than the subject tree. It is possible that density in smaller trees will show no significant effect on growth whatsoever. A similar analogy can be made with the three-diameter-class subdivisions of density described earlier.

Another possibility is that the growth response to competition is, to some degree, dependent on the species of the competition. If a given tree is competing with hardwoods, that tree's growth response may be different than if the competition is primarily other conifer species. However, because species composition is not independent of site index and other productivity measures, it may be difficult to assess these types of relationships.

Finally, it might be expected that density on a given plot is a more precise expression of the competitive stress experienced by a given tree than the stand level estimate of the same density variable. This effect should be more pronounced where density varies greatly within a given stand. If, however, a stand is perfectly homogeneous (tree diameters and spacing constant, as in some very young plantations), any difference between point and stand estimates of density are due solely to sampling error and are not indicative of any real differences in competitive stress.

Tree variables include diameter outside bark at breast height (DOB) squared and the natural logarithm of DOB. In addition, crown ratio and foliage weight were considered as indicators of tree vigor.

The results of the screening portion of the analysis can best be summarized as follows:

- (1) The combination of DOB squared and logarithm of DOB form a peaking function in basal area growth over DOB. These two variables alone account for over 38 percent of the variability in the logarithm of basal area growth.
- (2) The addition of crown ratio as an independent variable resulted in a significant improvement in fit ($R^2 = 0.5974$). Crown ratio provided a slightly higher R^2 (improvements less than 1 percent) than did various transformations of the subject tree's foliage weight or crown length.
- (3) When DOB squared, logarithm of DOB, and CR were forced into the model, the addition of site productivity variables failed to improve R^2 by more than 2 percent. Transformations of site index generally performed better than predicted height growth. Nevertheless, both were significant at the 99 percent confidence level. Stage's (1976) transformations of slope and aspect were not significant either with or without site index in the model.
- (4) The measures of density compiled by point-level summations were generally superior to their corresponding stand-level variables.
- (5) The various transformations of CCF generally resulted in better fits than basal area per acre. BA and CCF both performed better than foliage weight in this model.
- (6) The three-interval subdivision of density did not improve fits over use of the two-interval subdivision. In either case, only the larger trees had a significant effect on basal area growth. Density in smaller trees or in the middle-diameter interval of the three-diameter-class subdivision was insignificant ($P > .10$). As expected, increasing values of basal area and CCF in larger trees (CCFL) indicated a reduction in growth rates.

- (7) The density variables summed over all diameter classes provided poorer fits than did the same variables expressed in both larger and smaller trees.
- (8) The subdivisions of density variables by species did not provide any improvement over diameter-interval subdivisions.
- (9) The best combination of density variables was CCFL on the plot and stand BA.

From these analyses, it was determined that either predicted height growth or site index could be used as an indicator of site productivity. Furthermore, transformations of DOB and crown ratio constituted the main variables in the model.

The density/position variable that best expressed the effects of competing vegetation was CCFL. However, BA was also significant in this analysis, indicating that the effect of larger trees on basal area growth is not independent of stand density.

Final Models of Basal Area Increment

The logarithmic transformation, as used in the variable selection phase of the analysis, will result in some degree of bias being introduced in predictions of the untransformed dependent variable (Flewelling and Pienaar 1981). Although adjustments can be made for the bias, they are generally dependent on meeting the assumption of normality of the residuals with respect to the logarithm of basal area growth. The logarithmic transformation was rejected for final parameter estimation because of the severe non-normality of the residuals about the log-linear model.

An alternate approach to parameter estimation is to apply nonlinear regression to fit the exponential of the log-linear model, thus eliminating the need for the logarithmic transformation of the dependent variable. In this regression, untransformed basal area growth is the dependent variable. An additive error is assumed with nonlinear regression. Furthermore, the resulting model is asymptotically unbiased, regardless of the distribution of the residuals.

The log-linear and weighted nonlinear regressions were compared on the same model using Furnival's (1961) index of fit, which indicated that the log-linear fit was only slightly better than nonlinear with a weight of $1.0/DOB^2$. Test contours for skewness and kurtosis (Bowman and Shenton 1975) indicated that both models had non-normally distributed residuals. From this it was concluded that weighted nonlinear regression was preferable to the log-linear estimation procedure. A weight of $1.0/DOB^2$ was used for final parameter estimation.

The equations for the two models chosen to fit the remaining 80 percent (7616 observations) of the Douglas-fir data and the total grand fir data set are, respectively:

$$BAG = EXP [c_0 + c_1 \cdot \ln(DOB) + c_2 (DOB^2) + c_3 \cdot CR + c_4 \cdot \Delta H + c_5 \cdot PCCFL + c_6 \cdot SBA] \quad [5]$$

and

$$BAG = EXP [c_0 + c_1 \cdot \ln(DOB) + c_2 (DOB^2) + c_3 \cdot CR + c_4 \cdot \ln(S) + c_5 \cdot PCCFL + c_6 \cdot SBA] \quad [6]$$

where:

CR = crown ratio,
 S = King's (1966) site index in feet,
 PCCFL = point CCF in trees larger than the subject tree, and
 SBA = stand basal area in square feet.

The parameter estimates, \bar{R}^2 , and mean square errors (MSE) for both Douglas-fir and grand fir are presented in Table 1.

The appropriate model for predicting growth depends on, among other things, how the different components of growth are incorporated in the framework of a stand simulator. Basal area and height increment models may be linked in a two-stage fashion similar to that used by Stage (1973). If such an approach is used, then predicted height growth would be used in the basal area increment model as a means of compensating for correlated errors between the two models. A random variable may then be introduced to the height growth predictions and carried through to the diameter growth predictions.

An alternative approach is to assume that the height and basal area growth equations are seemingly unrelated. The use of site index alone as the productivity variable in both height and diameter growth equations is an application of this approach.

TABLE 1.

PARAMETER ESTIMATES (c), STANDARD ERRORS OF THE ESTIMATES (IN PARENTHESES), MEAN SQUARED ERRORS (MSE), AND ADJUSTED MULTIPLE CORRELATION COEFFICIENT (R^2) FOR BASAL AREA INCREMENT MODELS OF DOUGLAS-FIR AND GRAND FIR.

Item	Douglas-fir model		Grand fir model	
	Eq. [5]	Eq. [6]	Eq. [5]	Eq. [6]
c ₀	0.275735 (.0526)	-4.51337 (.328)	-0.313843 (.202)	-2.23906 (1.25)
c ₁	1.0060822 (.0140)	1.0190520 (.0138)	1.318159 (.0675)	1.370403 (.0669)
c ₂	-0.0002754 (.0000166)	-0.0003326 (.0000160)	-0.0008066 (.000162)	-0.0009438 (.000158)
c ₃	1.13501 (.0396)	1.33031 (.0357)	1.09101 (.142)	1.21110 (.137)
c ₄	0.0310146 (.00231)	1.07106 (.0673)	0.0241041 (.00793)	0.448166 (.255)
c ₅	-0.0027791 (.000103)	-0.0028096 (.000102)	-0.0017633 (.000384)	-0.0018079 (.000387)
c ₆	-0.0013096 (.000166)	-0.0019739 (.000143)	-0.0022058 (.000639)	-0.0030200 (.000546)
MSE	0.586661	0.580577	0.821505	0.829858
\bar{R}^2	0.5571	0.5616	0.4713	0.4659

In seemingly unrelated regressions, some correlation is assumed to exist between the errors of the two models (Kmenta 1971). If this correlation exists, and these two regressions are developed separately, the parameter estimates are unbiased and consistent. However, in order to obtain efficient estimates, the error correlation can be accounted for through generalized least-squares. Although generalized least-squares can be applied in nonlinear regression, the procedure is quite complex and was rejected for our analysis.

Predicting Future Diameters

The primary utility of Equations [5] and [6] is in estimating future DOB. The method by which this can be accomplished was described by Ritchie and Hann (1984). The first step is to calculate projected tree basal area as a function of current diameter squared and estimated basal area growth. This is done by assuming the following relationship between squared diameters inside and outside the bark:

$$DIB^2 = a_1(DOB^2)^{a_2} \quad [7]$$

where:

DIB = diameter inside bark at breast height,
and

a_1, a_2 = species-specific coefficients.

Then:

$$BA_2 = (\pi/4) [(4 \cdot BAG)/(\pi \cdot a_1) + DOB^{2 \cdot a_2}]^{1/a_2} \quad [8]$$

where:

BA_2 = projected basal area outside bark in square inches,

BAG = predicted basal area growth inside bark in square inches (from Equations [5] or [6]).

The equations for projected diameter outside bark (DOB_2) for Douglas-fir and grand fir, respectively, can be derived then from the square root of BA_2 :

$$DOB_2 = [1.35171 \cdot BAG + DOB^{1.93369}]^{0.517145} \quad [9]$$

$$DOB_2 = [1.34747 \cdot BAG + DOB^{1.95478}]^{0.511565} \quad [10]$$

Conclusion

Our calculations indicate that basal area growth can be adequately modeled by a simple exponential function. A procedure using nonlinear least-squares was applied in parameter estimation because of problems with non-normality in the residuals of the log-linear fit to the same model.

Competitive stress is expressed in the model by crown competition factor in larger trees on the plot and total stand basal area. More complex

expressions of stand or plot density, including species subdivisions, did not improve the explanatory power of the model.

Our model should function well over a wide range of site index, stand density, and tree size.

Literature Cited

- ARNEY, J.D. 1973. Tables for quantifying competitive stress on individual trees. Canadian Forestry Service, Victoria, British Columbia. Information Report BC-X-78. 45 p.
- BOWMAN, K.O., and L.R. SHENTON. 1975. Omnibus test contours for departures from normality based on β_1 and β_2 . *Biometrika* 62:243-249.
- BROWN, J.K. 1978. Weight and density of crowns of Rocky Mountain conifers. USDA Forest Service, Intermountain Forest and Range Experiment Station, Ogden, Utah. Research Paper INT-197. 56 p.
- BRUCE, D. 1981. Consistent height-growth and growth rate estimates for remeasured plots. *Forest Science* 27:711-725.
- COLE, D.M., and A.R. STAGE. 1972. Estimating future diameters of lodgepole pine trees. USDA Forest Service, Intermountain Forest and Range Experiment Station, Ogden, Utah. Research Paper INT-131. 20 p.
- CURTIS, R.O., and D. BRUCE. 1968. Tree heights without a tape. *Journal of Forestry* 66:60-61.
- DRAPER, N.R., and H.R. SMITH. 1981. Applied regression analysis. 2nd edition. John Wiley and Sons, New York. 709 p.
- FLEWELLING, J.W., and L.V. PIENAAR. 1981. Multiplicative regression with lognormal errors. *Forest Science* 27:281-289.
- FURNIVAL, G.M. 1961. An index for comparing equations used in constructing volume equations. *Forest Science* 7:337-341.
- HANN, D.W. 1980. Development and evaluation of an even- and uneven-aged ponderosa pine/Arizona fescue stand simulator. USDA Forest Service, Intermountain Forest and Range Experiment Station, Ogden, Utah. Research Paper INT-267. 106 p.
- KING, J.E. 1966. Site index curves for Douglas-fir in the Pacific Northwest. Weyerhaeuser Co., Centralia, Washington. Weyerhaeuser Forestry Paper No. 8. 49 p.
- KMENTA, J. 1971. Elements of econometrics. Macmillan Pub., New York, New York. 655 p.
- KRAJICEK, J.E., K.A. BRINKMAN, and S.F. GINGRICH. 1961. Crown competition—a measure of density. *Forest Science* 7:35-42.
- MUNRO, D.D. 1974. Forest growth models—a prognosis. P. 7-72 in *Growth Models for Tree and Stand Simulation*. (J. Fries, ed.). Royal College of Forestry, Stockholm. Research Note 30. 379 p.
- PAINE, D.P., and D.W. HANN. 1982. Maximum crown-width equations for southwestern Oregon tree species. Forest Research Laboratory, Oregon State University, Corvallis. Research Paper 46. 20 p.
- RITCHIE, M.W., and D.W. HANN. 1984. Non-linear equations for predicting diameter and squared diameter inside bark at breast height for Douglas-fir. Forest Research Laboratory, Oregon State University, Corvallis. Research Paper 47. 12 p.
- SMITH, J.H.G. 1966. Studies of crown development are improving Canadian forest management. Proceedings, Sixth World Forestry Congress (Madrid), Vol. II:2309-2315.
- STAGE, A.R. 1973. Prognosis model for stand development. USDA Forest Service, Intermountain Forest and Range Experiment Station, Ogden, Utah. Research Paper INT-137. 32 p.
- STAGE, A.R. 1976. An expression for the effect of slope, aspect and habitat type on tree growth. *Forest Science* 22:457-460.
- WEST, P.W. 1980. Use of diameter increment and basal area increment in tree growth studies. *Canadian Journal of Forest Research* 10:71-77.
- WYKOFF, W.R., N.L. CROOKSTON, and A.R. STAGE. 1982. User's guide to the Stand Prognosis Model. USDA Forest Service, Intermountain Forest and Range Experiment Station, Ogden, Utah. General Technical Report INT-133. 112 p.

Appendix A

Maximum Crown Width

Equations for maximum crown width are necessary for calculating the crown competition factor (CCF) and for estimating foliage weight (FW). CCF is defined as the sum of the maximum crown areas for all trees in a stand or at a given point (Krajicek et al. 1961). The species-specific maximum crown width (MCW) equations used to calculate CCF and FW in this study are all quadratic functions of diameter:

$$MCW = d_0 + d_1 \cdot DOB + d_2 \cdot DOB^2 \quad [11]$$

However, for most of these equations, the value of d_2 is zero. In these cases, the equation reduces to a linear function over diameter. The parameter estimates of d_1 , d_2 , and d_3 for MCW of each species, as well as their source in the literature, are presented in Table 2.

TABLE 2.

COEFFICIENTS OF EQUATIONS FOR MAXIMUM CROWN WIDTH BY SPECIES AND THE LITERATURE SOURCE FOR EACH.

Species	d_0	d_1	d_2	Source
Douglas-fir	4.7071	2.0168	-0.0186	Arney (1973)
Yew/hemlock	4.20	1.42	0.00	Smith (1966)
Western redcedar	4.0	1.6	0.00	Smith (1966)
Grand fir	5.0	1.5	0.00	Smith (1966)
Sitka spruce	6.5	1.8	0.00	Smith (1966)
Oregon white oak	3.0785	1.9242	0.00	Paine and Hann (1982)
Madrone	3.4299	1.3532	0.00	Paine and Hann (1982)
Alder	8.0	1.53	0.00	Smith (1966)
Ash/cottonwood	0.5	1.62	0.00	Smith (1966)

Appendix B

Foliage Weight

Foliage weight estimates were developed in a two-stage process using the data from Brown (1978). The first stage involved the development of crown width equations. The crown width model was assumed to be:

$$CW/MCW = (CL/H)^{k_2} \quad [12]$$

where:

CW = crown width, and
 k_2 = species-specific parameter estimate.

Then, predicted crown width ($\hat{C}W$) is simply:

$$\hat{C}W = CR^{k_2} \cdot MCW \quad [13]$$

This predicted crown width was then used as an independent variable in fitting a model of the form:

$$FW = k_1 (\hat{C}W \cdot CL)^{k_3} \quad [14]$$

where:

FW = foliage weight of an individual tree,
 CL = crown length, and
 k_1, k_3 = species-specific parameter estimates.

The complete model for predicting tree foliage weight is then:

$$FW = k_1 (CR^{k_2} \cdot MCW \cdot CL)^{k_3} \quad [15]$$

The parameter estimates of k_1 , k_2 , and k_3 can be found in Table 3.

TABLE 3.
SPECIES-SPECIFIC PARAMETER ESTIMATES FOR THE MODEL OF TREE FOLIAGE WEIGHT.

Species	k_1	k_2	k_3
Douglas-fir	0.0261580	0.7652813	1.1889078
Ponderosa pine	0.0200009	0.6974895	1.2528906
Western redcedar	0.0267653	0.3550908	1.1418090
Spruce	0.0170113	1.1700041	1.4016785
Grand fir	0.0033270	0.6843938	1.5192013

Appendix C

Height Growth

Height growth was predicted for each tree in order to be used as an independent variable in the basal area increment models. The equation was developed for Douglas-fir trees as a separate phase of the overall modeling project¹. The height growth estimator is a product of a potential height growth function (PHG) and a modifier (MHG) which adjusts the potential according to tree position and vigor. Potential height growth is expressed as a function of site index and tree height. The modifier of height growth is a function of crown ratio and tree position. Predicted height growth (ΔH) is then estimated with the equation:

$$\Delta H = (PHG) \cdot (MHG) \quad [16]$$

where:

$$\begin{aligned} PHG &= 1.14906 (PH - H), \\ PH &= S \cdot \text{EXP} [t_1 ((A + S + 13.25 - S/20)^{t_2} \\ &\quad - (63.25 - S/20)^{t_2})], \\ S &= \text{King's (1966) site index in feet,} \end{aligned}$$

H = tree height in feet,

$$\begin{aligned} t_2 &= -0.447762 - 0.894427(S/100) \\ &\quad + 0.793548(S/100)^2 - 0.17166(S/100)^3, \end{aligned}$$

$$t_1 = \ln(4.5/S) / [(13.25 - (S/20))^{t_2} - (63.25 - (S/20))^{t_2}].$$

$$A = [\ln(H/S)/t_1 + (63.25 - S/20)^{t_2}]^{1/t_2} - 13.25 + S/20.$$

OFFICIAL
CORRECTION

The modifier is defined as:

$$\begin{aligned} MHG &= q_1 (1 - \text{EXP}(q_2 \cdot CR)) \\ &\quad \cdot (\text{EXP}(q_3 ((H/SH)^{q_4} - 1))) \end{aligned} \quad [17]$$

where:

$$\begin{aligned} CR &= \text{crown ratio,} \\ H/SH &= \text{tree height divided by dominant} \\ &\quad \text{stand height,} \\ q_1 &= 1.117148, \\ q_2 &= -4.26558, \\ q_3 &= 2.54119, \text{ and} \\ q_4 &= 0.250537. \end{aligned}$$

¹ Ritchie, M.W., and D.W. Hann. Development of a tree height growth model for Douglas-fir. Manuscript in preparation.

Ritchie, M.W., and D.W. Hann. 1985. EQUATIONS FOR PREDICTING BASAL AREA INCREMENT IN DOUGLAS-FIR AND GRAND FIR. Forest Research Laboratory, Oregon State University, Corvallis. Research Bulletin 51. 9 p.

Equations are presented for predicting basal area increment for individual Douglas-fir and grand fir trees in the east-central Coast Range of Oregon. Final parameter estimates were obtained using weighted nonlinear regression analysis of a simple exponential model. Two equations are presented for each species: one has site index, and the other has predicted height growth as independent variables in the model. The other variables used are diameter, crown ratio, crown competition factor in larger trees on the sample point, and stand basal area. Techniques for predicting future diameters from these equations are also presented. A number of methods of expressing stand density or structure are compared for the log-linear model of basal area growth.

Ritchie, M.W., and D.W. Hann. 1985. EQUATIONS FOR PREDICTING BASAL AREA INCREMENT IN DOUGLAS-FIR AND GRAND FIR. Forest Research Laboratory, Oregon State University, Corvallis. Research Bulletin 51. 9 p.

Equations are presented for predicting basal area increment for individual Douglas-fir and grand fir trees in the east-central Coast Range of Oregon. Final parameter estimates were obtained using weighted nonlinear regression analysis of a simple exponential model. Two equations are presented for each species: one has site index, and the other has predicted height growth as independent variables in the model. The other variables used are diameter, crown ratio, crown competition factor in larger trees on the sample point, and stand basal area. Techniques for predicting future diameters from these equations are also presented. A number of methods of expressing stand density or structure are compared for the log-linear model of basal area growth.