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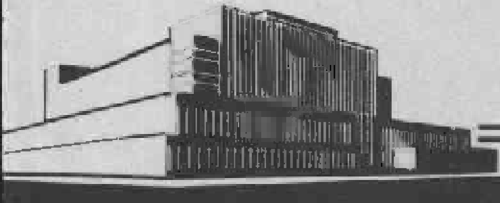

FOREST PRODUCTS LABORATORY
MADISON 5, WISCONSIN

UNITED STATES DEPARTMENT OF AGRICULTURE
FOREST SERVICE
In Cooperation with the University of Wisconsin

PLASTIC FLOW THROUGHOUT VOLUME OF THIN ADHESIVE BONDS[1]

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Introduction

When two adherends are bonded together by an adhesive, the strength of the bond often greatly exceeds the strength of the adhesive material.[3] This fact has been explained in many ways -- among them one that relates the restraint of the strains in the adhesive to the presence of the adherends.

If a butt joint is tested in tension by applying a force normal to the plane of the bond, stress concentrations occur at the boundary edges of the joint because of the different elastic properties of the adhesive and the adherends. The cross-sectional area of the adherends is reduced, because of the longitudinal strain, by the effect of Poisson's ratio. If

[1]Work here reported was done in cooperation with the Office of the Chief of Ordnance, Department of the Army, under contract ORDOR-AC-P-1473/A-10625, dated 5/11/55.

[2]Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

[3]DeBruyne and Houwink, Adhesion and Adhesives, Elsevier Publishing Company. Chap. 3, part 3, Chap. 4, part 10. 1951.

the adhesive has different elastic properties than do the adherends, its cross-sectional area tends to be reduced a different amount than that of the adherends. Because the adhesive adheres to the more rigid adherend, the contraction of the adhesive film is constrained (in the extreme) to that of the adherends. This gives rise to stress concentrations.

As the force is increased, the concentrated stresses reach the yield stress of the adhesive. Further increase of the force increases these stresses at a reduced rate (a strain-hardening material is assumed) and causes further stress concentrations slightly removed from the bounding edges of the joint. The line at which the yield stress of the adhesive is reached thus travels inward from the bounding edges of the joint as the force is increased. The details of this process for a plastic adhesive and infinitely rigid adherends have been worked out.[4]

In the examination of broken specimens from such tests, it was noted that the fracture was smooth in a very narrow area adjacent to the bounding edges of the joint but rough and jagged over the rest of the area. This suggested that the fracture consisted of two types of failures: The first was caused by plastic flow and started at the bounding edges as described above; the second was caused by instability throughout the bond. The relative amount of area associated with plastic flow was so small that it seemed reasonable to assume that failure took place

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[4] Shields, R. T. The Application of Limit analysis to the Determination of the Strength of Butt Joints, Brown University Technical Report No. 10.

when a condition of instability existed throughout the entire bond. This condition is developed when the stress at every point in the bond satisfies the von Mises yield condition.[5]

The following mathematical analysis assumes that the adhesive in the bond is isotropic and that the strains in the adhesive, parallel to the plane of the bond, are equal to those in the adherends. Equations are derived for the start of plastic flow at the bounding edges of the joint and for instability throughout the bond.

In the sections that follow this analysis, a method is developed for the determination of the elastic properties of adhesives as they exist in bonds. These properties are substituted in the formula for the determination of the stress at which instability becomes general throughout the bond, and this stress is compared with the results of tests.

Mathematical Analysis

An isotropic adhesive bond between two adherends is located parallel to the _y-z_ plane of a rectangular coordinate system. A uniformly distributed tensile stress is applied to the adherends in the direction of the x-axis -- that is, normal to the plane of the bond. The strains in the _y_ and _z_ directions in the adherends are:[6]

$$e_y = e_z = - \frac{vs_x}{E} \qquad (1)$$

[5]Nadai, A. Theory of Flow and Fracture of Solids, McGraw-Hill. Vol. 1, 2nd Ed. p. 210 and 212, 1950.

[6]Timoshenko and Goodier. Theory of Elasticity, McGraw-Hill. 2nd Ed. p. 7, equation (3), 1951.

where s_x is the applied stress, E is the modulus of elasticity of the adherends, and v is the Poisson's ratio.

The strains in the adhesive in the y and z directions are given by:[6]

$$\epsilon_y = \frac{1}{\Sigma} [\sigma_y - \nu (\sigma_z + \sigma_x)] \quad (2)$$

$$\epsilon_z = \frac{1}{\Sigma} [\sigma_z - \nu (\sigma_x + \sigma_y)]$$

where σ_x, σ_y, and σ_z are the stresses in the adhesive in the directions of the x-, y-, and z-axes; and Σ and ν are the modulus of elasticity and Poisson's ratio of the adhesive, respectively.

Assume that the bond is so thin with respect to its extent that the strains in the y and z directions in the adhesive are identical, at points remote from the edges of the bond, to the strains in the adherends. Thus, the right hand member of equation (1) may be equated to the right hand members of equations (2); the two resulting equations are solved simultaneously for σ_y and σ_z resulting in:

$$\sigma_y = \sigma_z = \frac{\nu - \frac{\Sigma v}{E}}{1 - \nu} s_x \quad (3)$$

According to the von Mises yield condition[4] the adhesive will yield throughout its volume when:

$$(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 = 2\sigma_o^2 \quad (4)$$

where σ_o is the yield stress in unrestrained tension. Putting $\sigma_z = \sigma_y$ as indicated by equation (3), equation (4) becomes:

$$\sigma_x - \sigma_y = \sigma_o \quad (5)$$

Substituting equation (3) in equation (5) and noting that the stress s_x on the adherends is equal to the stress σ_x in the adhesive, the value of s_x that will cause yielding throughout the volume of the adhesive is obtained as:

$$s_x = \frac{1 - \nu}{1 - 2\nu + \frac{\Sigma}{E} v} \sigma_o \tag{6}$$

Now consider a point at the edge of the bond midway between the two adherends. Choose the coordinates so that the bond again lies parallel to the y-z plane and so that, at the point considered, the z direction is normal to the edge of the bond and the y direction tangential. Assume that the bond is so thin with respect to the length of its edge that the strain in the y direction is identical to the strain in the adherends. The strain in the z direction will not be limited in this way but will assume a value that will cause the stress in this direction to be zero. Equating the right hand member of equation (1) to the right hand member of the first of equations (2), and placing $\sigma_z = 0$, yields:

$$\sigma_y = (\nu - \frac{\Sigma v}{E}) \sigma_x \tag{7}$$

Substituting this value of σ_y and zero for σ_z in equation (4) gives the stress at which yielding at the edge of the bond will occur as:

$$s_x = \frac{\sigma_o}{\sqrt{1 - (\nu - \frac{\Sigma v}{E}) + (\nu - \frac{\Sigma v}{E})^2}} \tag{8}$$

Determination of Elastic Properties of Adhesives in Bonds[7]

The specimens for the determination of the elastic properties of adhesives in bonds consisted of pairs of metallic tubes. Between the tube ends were interposed a number of metallic rings, with cross sections equal to the cross section of the tubes. Thus, a series of similar bonds were tested at one time.

These tubes were tested in torsion and then in tension as illustrated in figures 1 and 2. Displacements were measured by Tuckerman gages as shown. The modulus of rigidity was computed from the torsion test by means of the formula:

$$\frac{1}{\Omega} = \frac{dJ}{aT\lambda} - \frac{\ell}{G\lambda} \qquad (9)$$

where Ω and G are the moduli of rigidity of the adhesive and the tube, λ is the sum of the thicknesses of the adhesive bonds, ℓ is the length of the metal tube and rings in the gage length ($\ell + \lambda$ is the gage length), J is the polar moment of inertia of the tube cross section, T is the applied torque, a is the outside radius of the tube, and d is the displacement measured by the gage.

The apparent modulus of elasticity of the bond was computed from the tensile test by means of the formula:

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[7]Kuenzi, E. W. Determination of Mechanical Properties of Adhesives for Use In the Design of Bonded Joints. Forest Products Laboratory Report No. 1851, 1956.

$$\frac{1}{\Sigma'} = \frac{Ad}{P\lambda} - \frac{\ell}{E\lambda} \qquad (10)$$

where Σ' is the apparent modulus of elasticity of the bond, E is the modulus of elasticity of the tube, A is the cross-sectional area of the tube, and P is the applied force. The values of ℓ and d need not have the same values in this test that they had in the torsion test.

The value of the apparent modulus of elasticity of the adhesive (Σ') is different from the true modulus of elasticity unless the tendency of the adhesive to shrink transversely because of the effect of Poisson's ratio is the same as that of the tube. Taking the co-ordinate system previously used, the strain normal to the plane of the adhesive joint is:

$$\epsilon_x = \frac{1}{\Sigma} [\sigma_x - \nu (\sigma_y + \sigma_z)] \qquad (11)$$

It is assumed, again, that the transverse strains in the adhesive are equal to those in the tube. Using equation (3) and remembering that:

$$s_x = \sigma_x \text{ and } \frac{\sigma_x}{\epsilon_x} = \Sigma'$$

equation (11) becomes:

$$(1 - \nu) \frac{\Sigma}{\Sigma'} = (1 + \nu) (1 - 2\nu) + 2 \frac{\Sigma}{E} v \nu \qquad (12)$$

Substituting the relation:

$$\Sigma = 2\ (1 + \nu)\ \Omega \tag{13}$$

in equation (12) yields:

$$\nu = \frac{\frac{1}{2}\Sigma' - \Omega}{\Sigma' - \Omega\left(1 + 2\nu\frac{\Sigma'}{E}\right)} \tag{14}$$

The modulus of elasticity of the adhesive is obtained by substituting this value of Poisson's ratio in equation (13).

Description of Specimens

Two different kinds of specimens were made. One was the tube specimen previously mentioned. The other was a boilerplate-type specimen made from a sheet of the cured adhesive. Two different unsupported film adhesives were used. One was a phenol polyvinyl formal type cured at a pressure of 100 pounds per square inch and the other was a phenol buna N rubber type cured at a pressure of 150 pounds per square inch. The polyvinyl formal adhesive made a hard, brittle sheet; the buna N rubber adhesive made a soft, rubbery sheet that exhibited the non-elastic properties of creep and recovery.

Description of Tube Specimens

Aluminum tubing, 2 inches outside diameter and 1-1/2 inches inside diameter, was used to make these specimens. The tubing was tested in torsion and in axial tension. Its moduli of rigidity and elasticity were found to be 3.7 and 10.5 million pounds per square inch respectively. Poisson's ratio was calculated to be 0.42 by use of equation (13). Pieces of this tubing were cut 1-3/4 inches long and internally threaded at one end to fit studs that were clamped in the testing machines. Rings about 1/8 inch long were also cut from this tubing.

The desired number of rings were assembled endwise between two pieces of tubing and all pieces were clamped together in the jig shown in figures 3 and 4. The rings are not shown in the figures. The distance between the shoulders of the jig was measured at several points to an accuracy of a ten thousandth of an inch. The assembly was then disassembled, films of adhesive were placed between the rings, and between the rings and the tubes, and the entire arrangement assembled again.

The assembly was then placed in an oven and heated to cure the glue. The temperature was measured periodically by means of a thermocouple fastened to one of the pieces of tubing at a bond. The temperature-time curves obtained as a result of these recordings, are plotted in figures 5 and 6.

The assembly was removed from the oven and allowed to cool. When it reached the temperature at which it was previously measured, the distances between shoulders of the jig were measured again. The difference between the average values of the two sets of measurements was taken to be the thickness of the adhesive bond.

The jig was then removed and the bonded tubes and rings were tested as described later.

Description of Sheet Specimens

Sheets of cured adhesive were made by assembling stacks of adhesive films in a hot plate press. The stacks were separated from steel cauls 0.064 inch thick by sheets of teflon 0.005 inch thick. Stacks of cardboard 0.1 inch thick were placed on each side of the steel cauls to reduce the rate of temperature increase to roughly match that obtained for the tube specimens. Heat was applied and the temperatures were measured by thermocouples placed in the sheets of adhesive. Comparisons of the time-temperature curves with those for the tube assembly are shown in figures 5 and 6.

The sheet of the cured phenol polyvinyl formal adhesive was 0.065 inch thick and the sheet of the cured phenol buna N rubber adhesive was 0.041 inch thick. Each of these sheets was cut into four tensile specimens 3/4 inch wide and 10 inches long. These specimens were necked down at their centers to a 1/4-inch width for a length of 2 inches as shown in figure 7. These specimens were tested in tension as subsequently described.

Description of Tests

Four kinds of tests were made, three on the bonded tubes and rings and one on the sheet adhesive. Torsion and tension tests were made on each tube assembly to determine the modulus of elasticity and Poisson's ratio of the adhesive. An additional tension test was made on the tube assemblies to determine the proportional limit of the bond. Tension tests were made on the specimens cut from the sheets of adhesive to determine modulus of elasticity and proportional limit of the unrestrained material.

Torsion Tests

The bonded tubes and rings were fixed in a torsion machine and a Tuckerman extensometer mounted on them as shown in figure 1. The relative displacement in a 1-1/2 inch length of the tube was measured by the extensometer. This length contained all of the adhesive bonds as shown. The shear stress at the outside surface of the tube was applied at a rate of about 25 pounds per square inch per second to a value of 1,700 pounds per square inch. The testing machine was then reversed and the stress removed at the same rate. Readings of load and time were taken at displacement increments of 0.0001 inch. This load was applied four times to remove slack from the apparatus. The average of the loads determined as the displacements were increasing and decreasing exhibited a linear relationship to the displacements, and were used to determine the modulus of rigidity of the adhesive according to equation (9).

Tension Test of Tube Specimens

The specimens tested in torsion were also tested in tension. They were clamped in a testing machine and the adhesive bonds bridged by Tuckerman extensometers as shown in figure 2. Load was applied at a rate of about 8 pounds per second up to a maximum of 2,000 pounds for the phenol polyvinyl formal specimens and of 1,000 pounds for the phenol buna N rubber specimens. The load was then reduced at the same rate. This was repeated three times on each specimen. The average displacement at a given load was calculated. These displacements plotted against the load gave a curve from whose slope the apparent modulus of elasticity was calculated with the help of equation (10).

The load was then increased again at the same rate and the strains read until failure occurred.

Tension Tests of Sheet Material

The specimens were clamped in an Instron testing machine in the usual way and the crosshead run at a speed of 0.01 inch per minute for the phenol polyvinyl formal specimens and for one of the phenol buna N rubber specimens. The speed was 0.05 inch per minute for the remainder of the phenol buna N rubber specimens. Tuckerman extensometers were used on the phenol polyvinyl formal specimens but the phenol buna N rubber specimens were too flexible for these gages and a special gage designed for paper was used. This gage is illustrated in figure 8 and described in.[8]

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[8] Setterholm, V. C., and Kuenzi, E. W. Method of Determining Tensile Properties of Paper. Forest Products Laboratory Report No. 2066, Dec. 1956.

Presentation of Data

The results of the tests and the values calculated from these results are given in table 1 and in figures 9, 10, and 11.

The moduli of rigidity of the adhesives as determined from the torsion tests of the tubes and equation (9) are given in the fourth row of table 1. The fifth row gives the apparent modulus of elasticity determined by the tensile tests of the tubes and equation (10). The sixth row contains proportional limit values from the tensile tests of the tubes. The load-elongation curves for these tests on the phenol buna N rubber adhesive are shown in figure 9. The proportional limits of the phenol polyvinyl formal adhesive coincided with the load at failure so that similar curves were merely straight lines that terminated at the ultimate loads. These proportional limits are not the normal values, but are the values modified by the presence of lateral restraint.

The Poisson's ratios determined by means of equation (14) are listed in the seventh row. The average values shown are computed from the average values listed in rows four and five. The eighth row lists the moduli of elasticity of the adhesives obtained by means of equation (13). The ratios of the proportional limit of an adhesive bond to that of the adhesive, computed by use of equation (6), are given in the ninth row. The tenth row contains the predicted values of the proportional limits of the adhesive. These values are obtained by dividing the values in the sixth row by those in the ninth row.

The stress-strain curves obtained from the tensile tests of the sheets of cured adhesive are given in figures 10 and 11. In figure 10 the horizontal lines at the upper right indicate the stresses at failure. In figure 11 the second curve from the left is obtained from a test run at one-fifth the speed of the others, as previously described. The phenol buna N rubber specimens were not broken. Their elongations became excessive.

Discussion of Results

The mathematical development is made on the assumption that the adhesive in the bonds and in the sheets have the same properties and are isotropic and elastic up to a yield stress given by the von Mises criterion. These assumptions were more nearly true for the phenol polyvinyl formal adhesive than for the phenol buna N rubber adhesive. The stress-strain curve of the latter adhesive is not linear and its slope is affected by the speed of testing as shown in figure 11. Also, it seems unlikely that an adhesive bond formed under unilateral pressure is strictly isotropic. The direction of the tensile stress in the tube specimens is in the direction of this pressure and the direction of the tensile stress in the sheet specimens is at right angles to it.

Even though these assumptions do not agree with fact, a rough agreement between experiment and theory was obtained. The two moduli of elasticity of the phenol polyvinyl formal adhesive, obtained from tests of the bond, were 404,000 and 334,000 pounds per square inch (row 8, table 1); the average value from the sheet specimens (obtained from figure 10) is 558,000 pounds per square inch. The pairs of straight lines on figure 11 for the phenol buna N rubber adhesive are drawn in accordance with the two values from the tests of bonds given in row 8 of table 1. They roughly agree with the stress-strain curves for the sheet specimens. Comparison of the computed proportional limits given in the tenth row of table 1 with those of figure 10 show reasonable agreement. Proportional limits cannot be located in figure 11 for the phenol buna N rubber adhesive because the material is not sufficiently elastic; however, the experimental curve intersects the straight lines, which indicate modulus of elasticity, at about the average computed proportional limit values given in table 1.

It is evident that the values of the apparent modulus of elasticity (row 5, table 1) are not obtained with sufficient accuracy, especially for the phenol buna N rubber adhesive. The values obtained from duplicate specimens of both adhesives have ratios of about 2 to 1 and 10 to 1. Better methods of obtaining this measurement are required. This variation does not greatly affect the values of modulus of elasticity obtained (row 8), but does greatly affect the values of Poisson's ratio (row 7). The average values of the apparent modulus of elasticity probably yield the best values of Poisson's ratio and modulus of elasticity.

Table 1.--Experimental and computed values from the torsion and tension tests of the tube specimens

Row No.	Elastic properties of adhesives	Phenol polyvinyl formal adhesive			Phenol buna N rubber adhesive		
1	Specimen No.	1	2	Av.	3	4	Av.
2	Thickness of bonds, in.	0.0143	0.0089	....	0.0024	0.0022	....
3	Number of bonds	5	5	....	2	2	....
4	Modulus of rigidity, p.s.i.	155,000	161,000	155,500	1,720	1,625	1,672
5	Apparent modulus of elasticity, p.s.i.	632,000	333,000	482,500	55,200	5,040	20,120
6	Proportional limit, p.s.i.	6,330	4,490	5,410	1,020	946	983
7	Poisson's ratio	0.345	0.033	0.267	0.484	0.262	0.47
8	Modulus of elasticity, p.s.i.	404,000	334,000	394,000	5,100	4,110	4,920
9	Proportional limit ratio	2.01	1.03	1.52	16	1.55	8.84
10	Computed proportional limit, p.s.i.	3,150	4,360	3,560	64	610	111

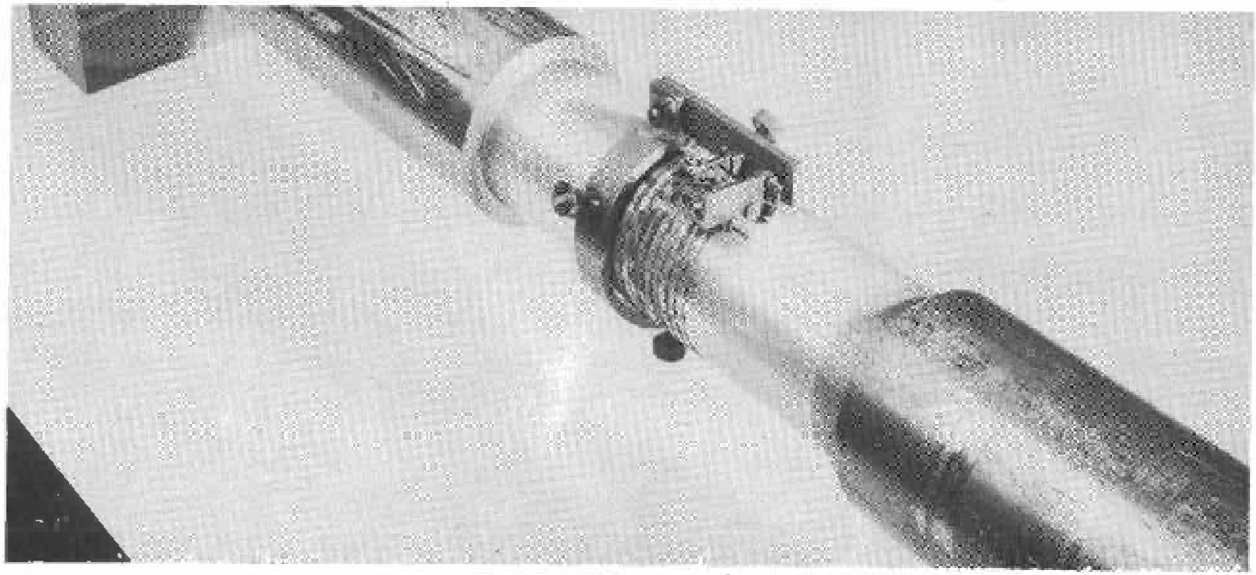

Figure 1.--Torsion test tubular specimen with Tuckerman extensometer and holder for measuring total displacement across bonds.

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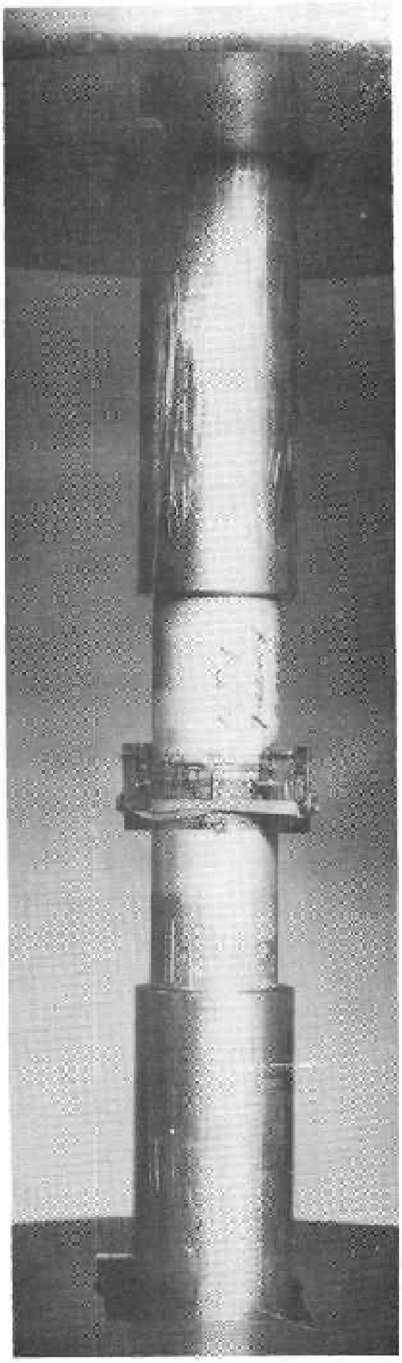

Figure 2. --Tension test tubular specimen with Tuckerman extensometers for measuring total displacement across bonds.

Z M 96918 F

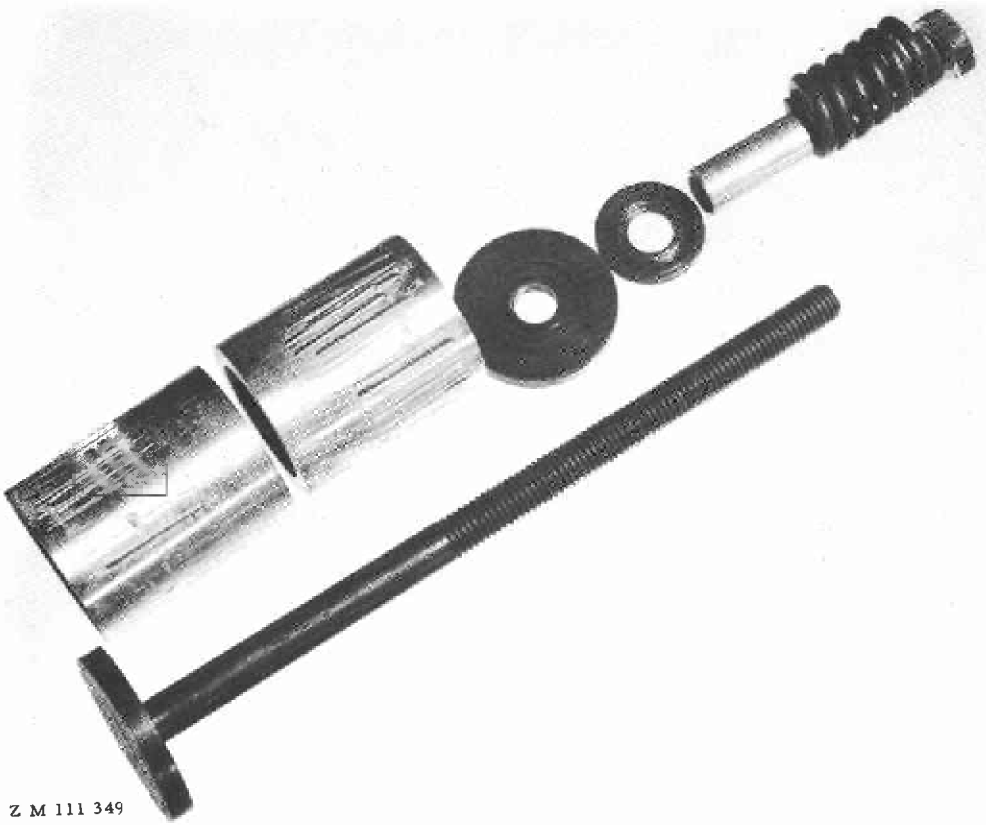

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Figure 3.--Jig for bonding tubular specimens--disassembled.

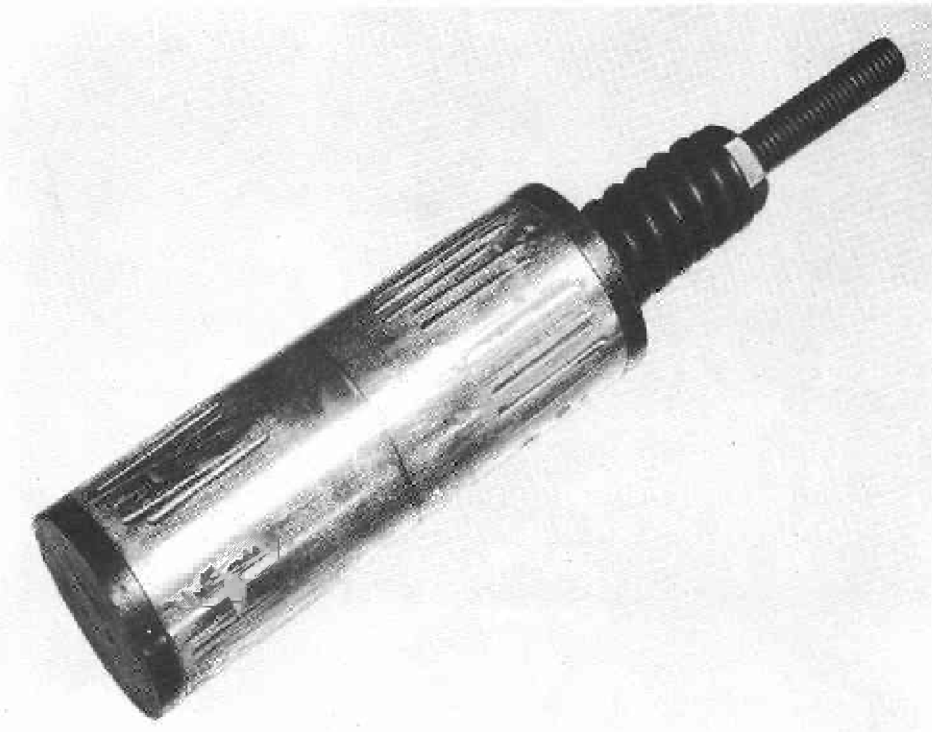

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Figure 4.--Jig for bonding tubular specimens--assembled.

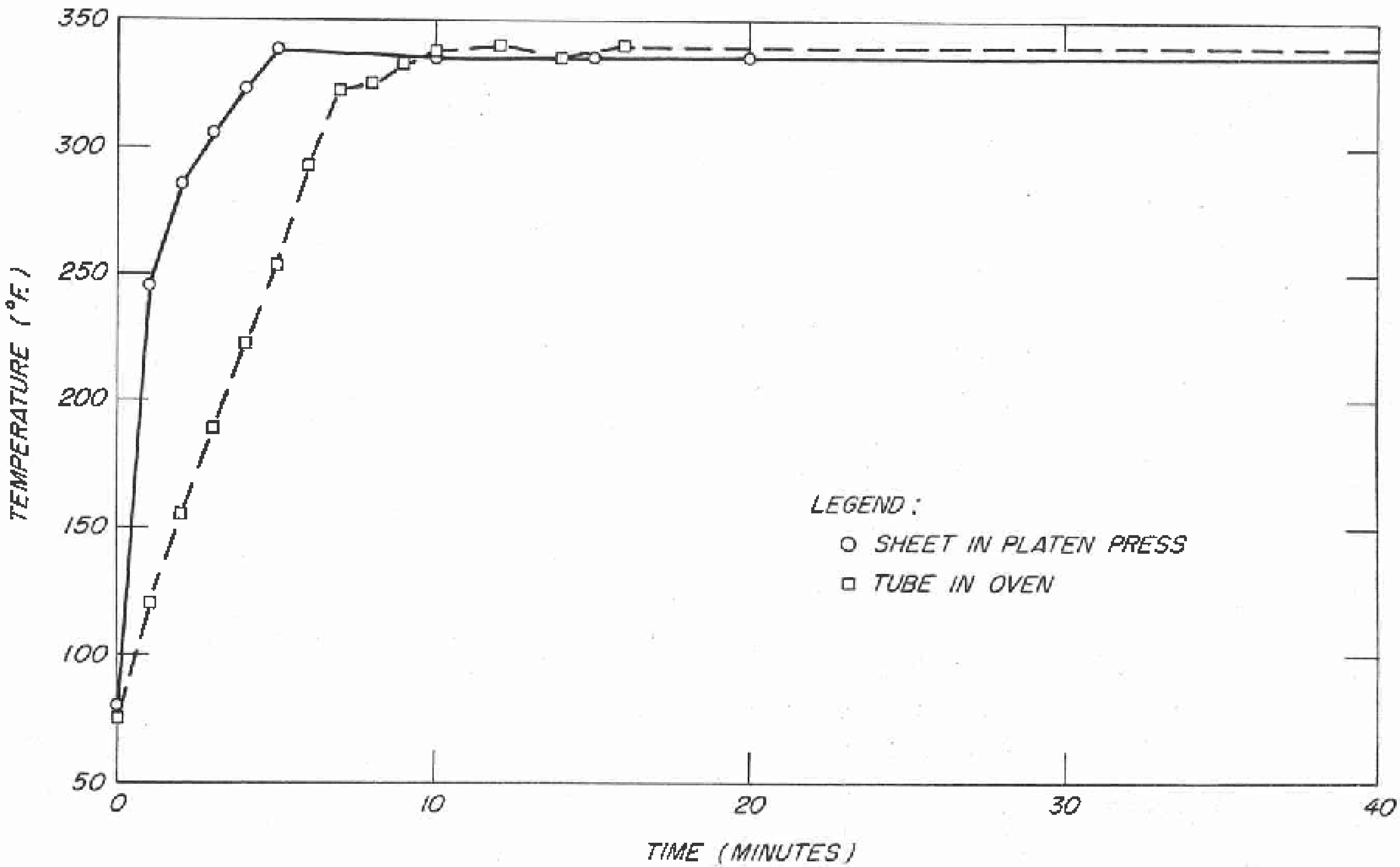

Figure 5.--Time-temperature curves for cure of phenol buna N rubber specimens.

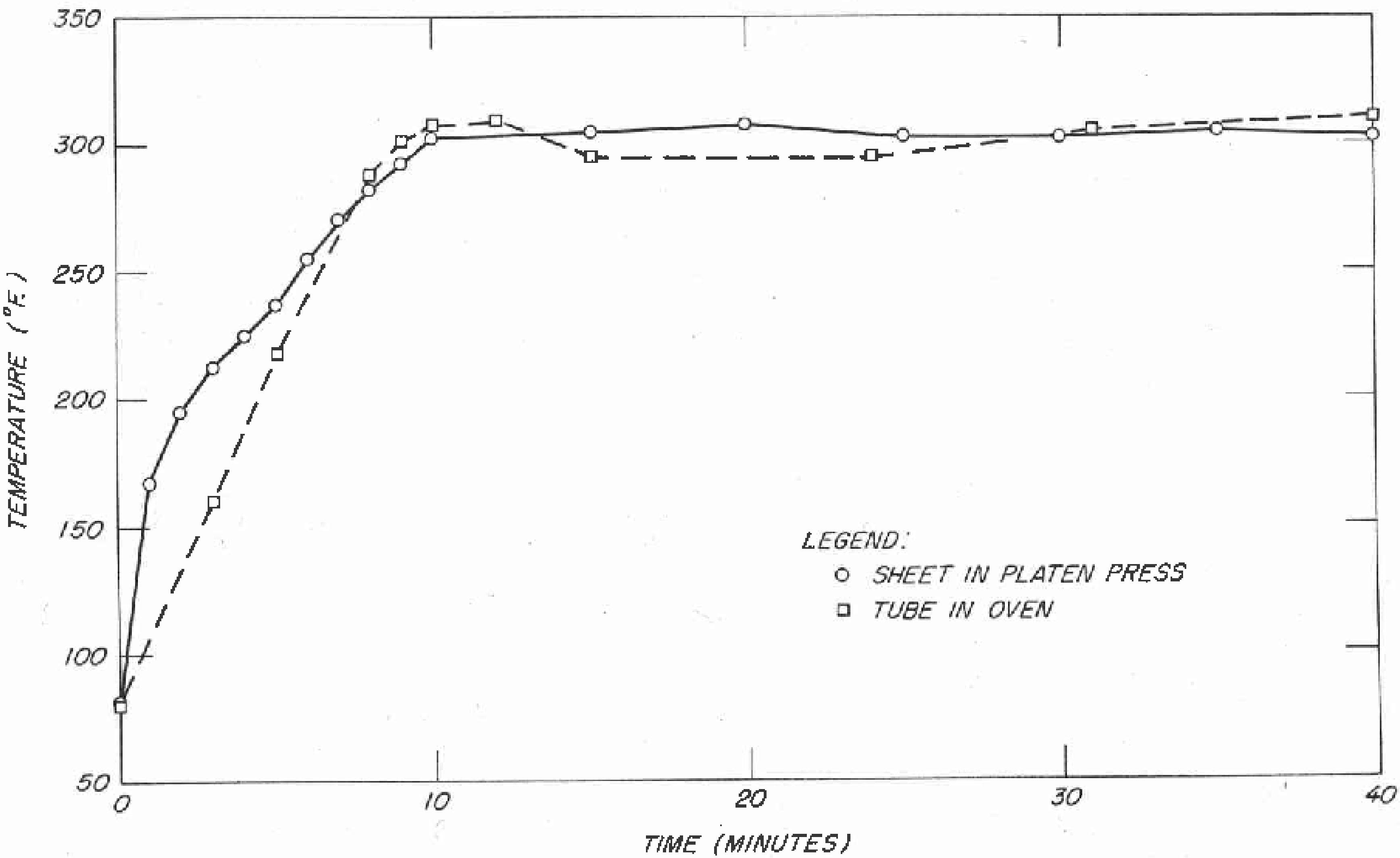

Figure 6.--Time-temperature curves for phenol polyvinyl formal specimens.

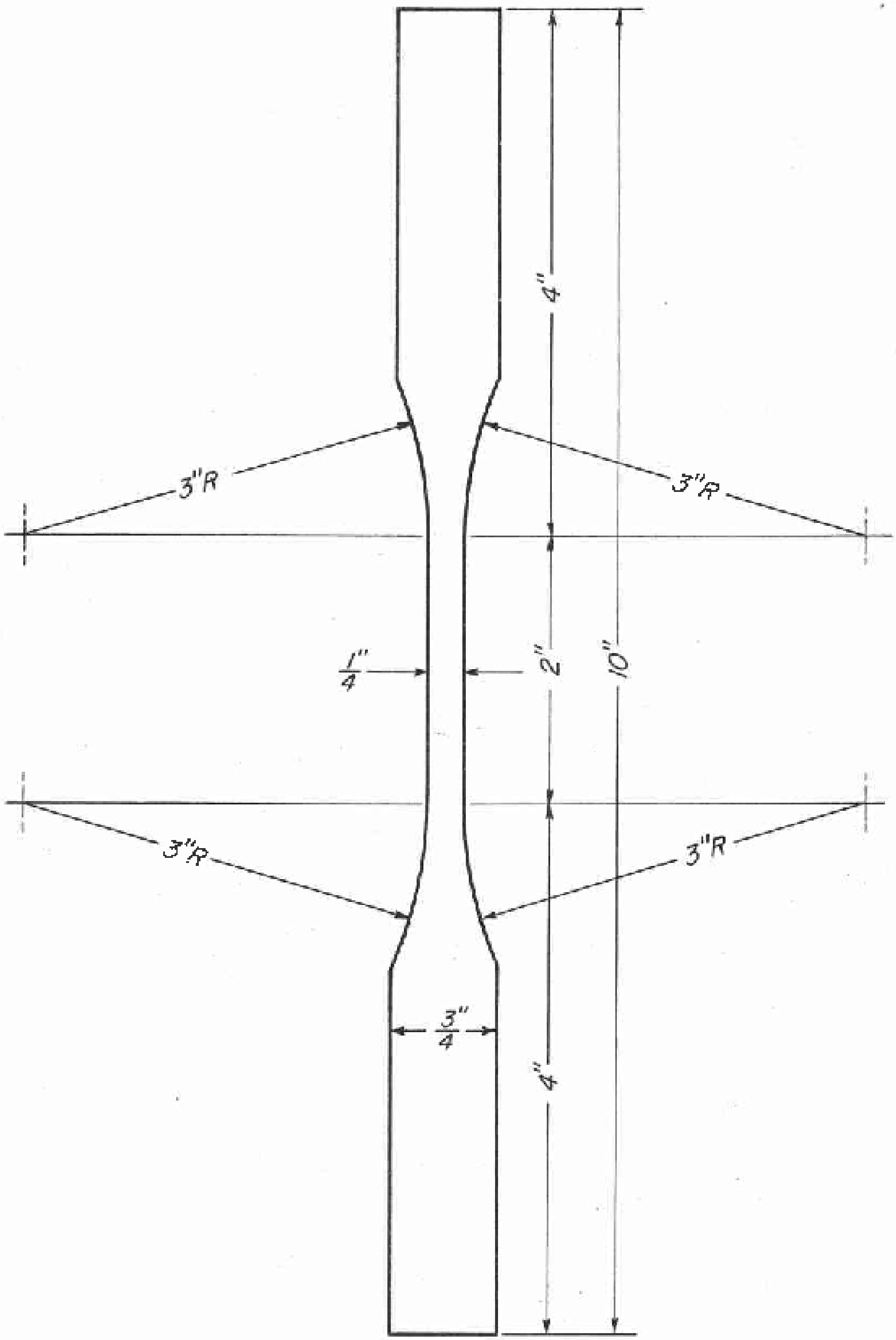

Figure 7.--Dimensions of specimens cut from sheets of cured adhesives.

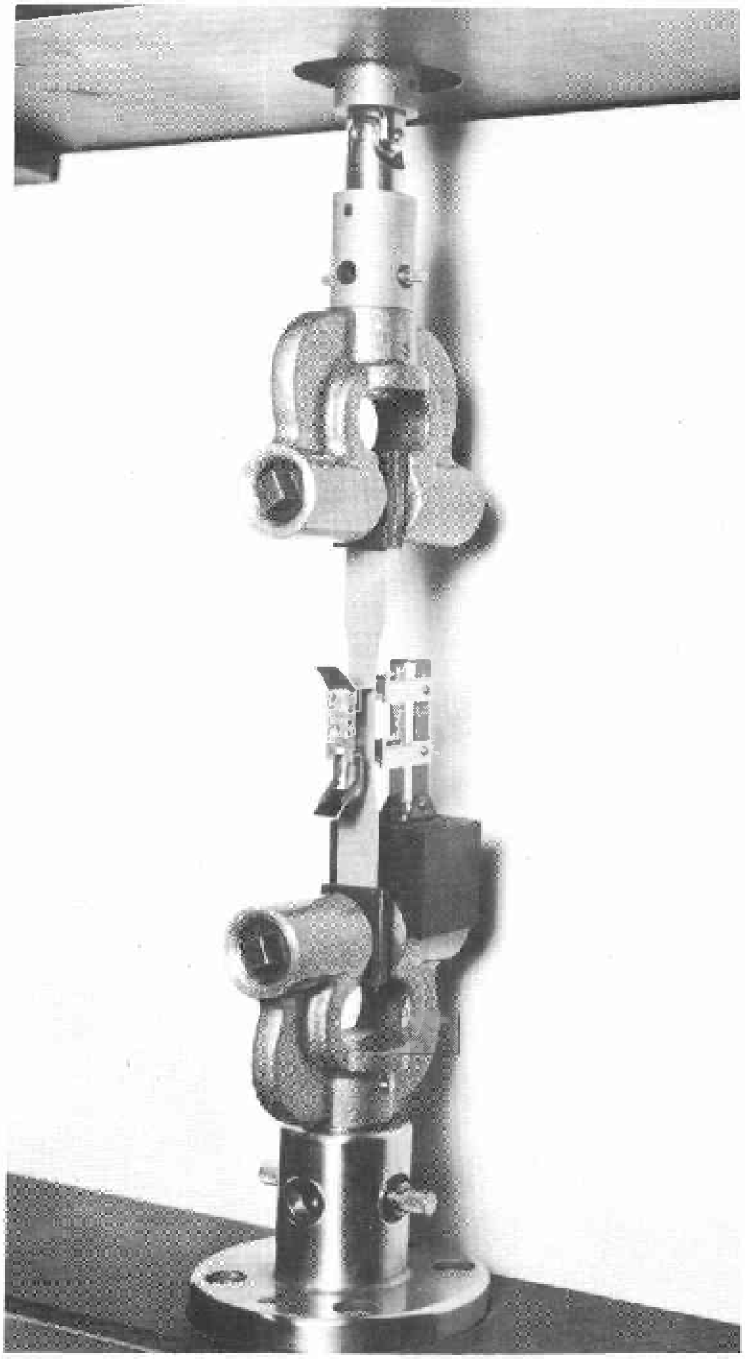

Figure 8.--Extensometer used in tension tests of phenol buna N rubber sheet specimens.

Z M 108 272

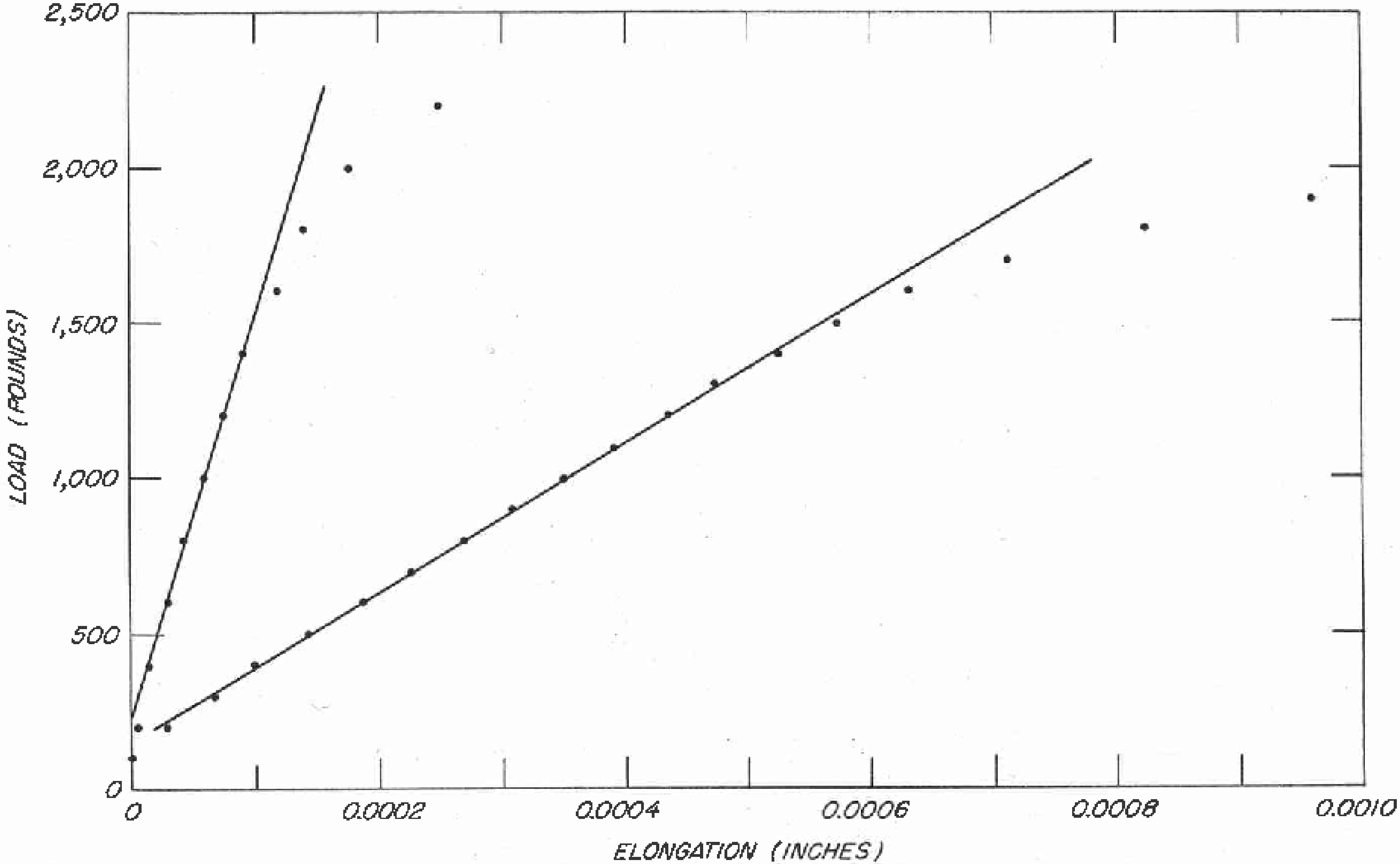

Figure 9.--Load-elongation curves from tensile tests of tubular specimens bonded with phenol buna N rubber adhesive.

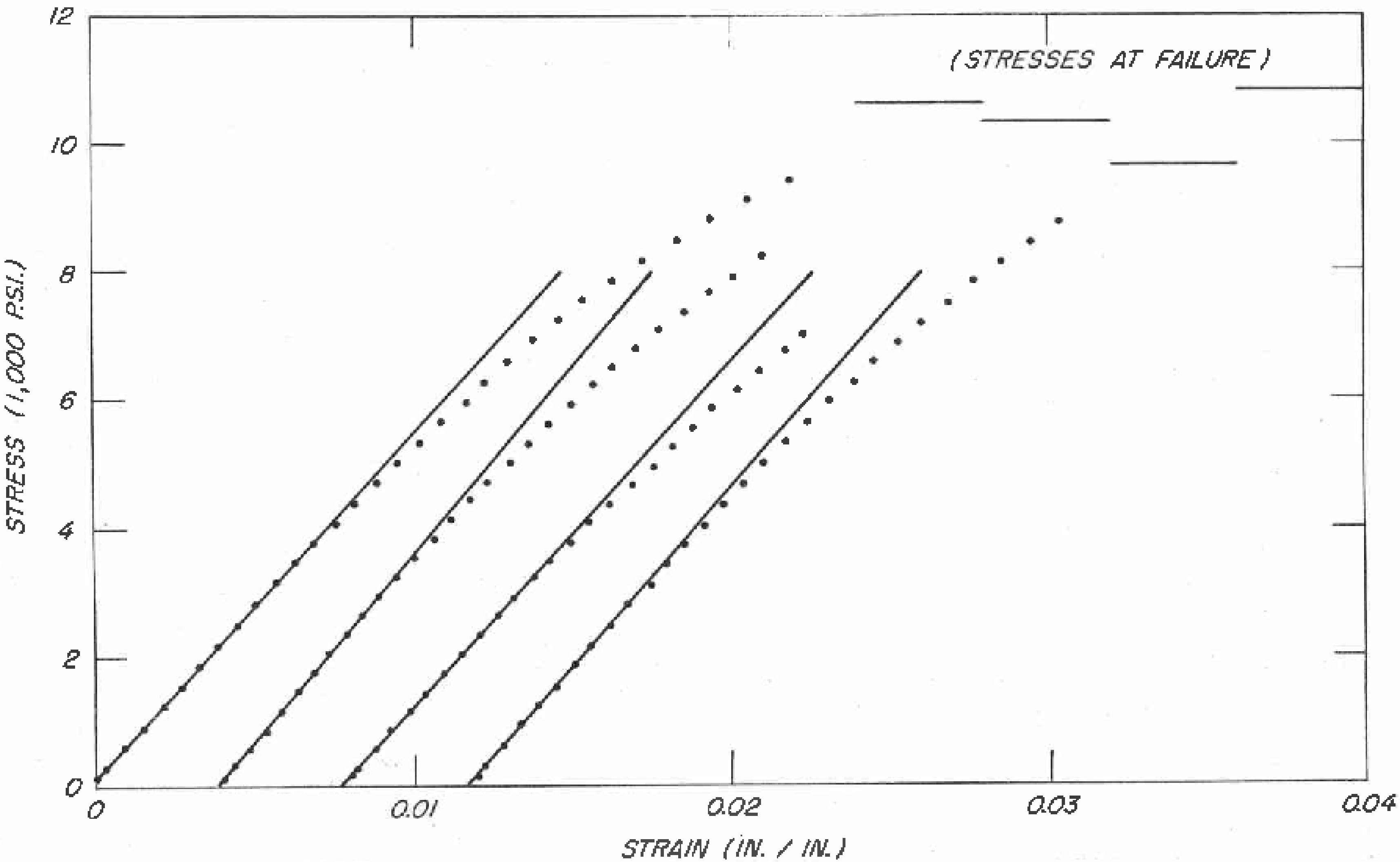

Figure 10.--Stress-strain curves from tensile tests of specimens cut from sheets of cured phenol polyvinyl formal adhesive.

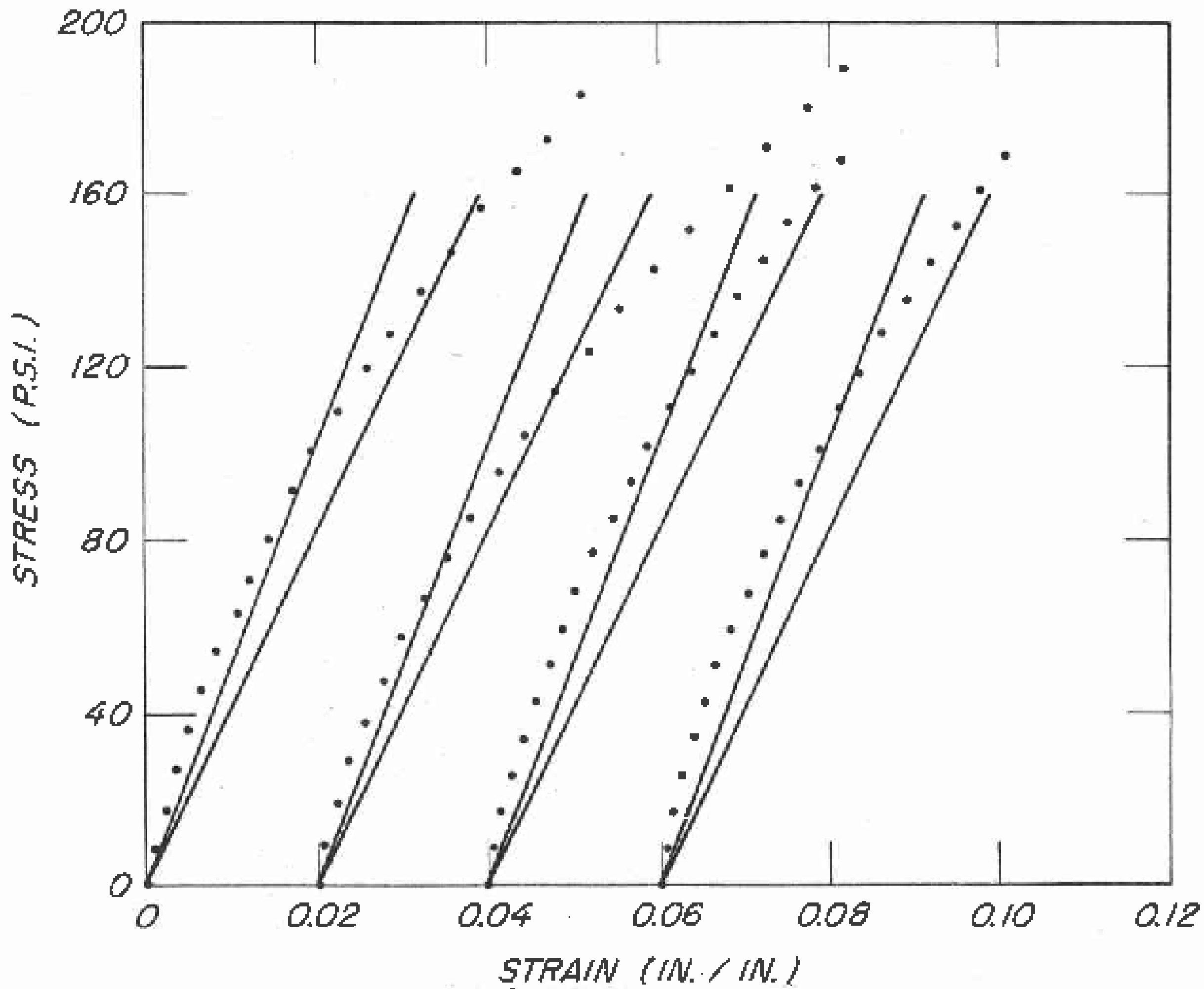

Figure 11.--Stress-strain curves from tensile tests of specimens cut from sheets of cured phenol buna N rubber adhesive.

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