GEOMETRIC LAWS FOR MOIRE PATTERNS PRODUCED BY CARTOGRAPHIC TINT SCREENS

by

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GEOMETRIC LAWS FOR MOIRE PATTERNS PRODUCED BY CARTOGRAPHIC TINT SCREENING

ABSTRACT. Moire patterns have been identified by printers as undesirable screen patterns caused by incorrect angling of overprinted halftones. These patterns are being examined by cartographers as a possible alternative to traditional methods of producing patterned areas on maps. Map design guidelines are needed before the use of moires can become prevalent, and to develop the necessary guidelines the characteristics of moires must be thoroughly defined. Four important aspects for studying moires have been identified as arrangement, orientation, gray tone, and moire pattern size. Kimerling examined these attributes for superimposed square dot arrangement screens, and formulated predictive equations. This study analyzes the four attributes for both triangular and parallelogram dot arrangement screens to determine if equations which are universally applicable to all types of dot arrangement screens exist.

INTRODUCTION

The interaction of the often imperceptible dot arrangement patterns of two tint screens through superimposing can produce a much larger and perceptible third pattern--called a moire. In the color printing process the moire has been defined by printers as, "The undesirable screen pattern caused by incorrect screen angles of overprinting halftones."[1] Printers and cartographers have basically viewed the moire as a disrupting intrusion and sought to minimize its occurrence. Moire patterns have been found to be dependent upon the angular separation between two or more screens (referred to as the "separation angle"), and with proper angling the patterns which result can be reduced. In color printing a separation angle of 30° minimizes the moire and up to three screens can be employed before repetition of the separation angle occurs when square dot arrangement screens (most commonly used in the United States) are superimposed. A fourth screen is accommodated by printing at a 15° separation angle to the other screens. The separation angles commonly used with the screens required in four color printing are: black, 45°; magenta, 75°; yellow, 90°; and cyan, 105°. An error as small as 0.1° between screen angles can cause noticeable moire patterns to form.[2]

Cartographers are beginning to review the possible usefulness of the moire. It is being recognized moire patterns may be an alternative to the patterns produced by traditional techniques, and could be particularly applicable in work with black and white (or gray tone) maps. If acceptable patterns were created by superimposing two screens, the need for additional overprinting of a coarse line or dot pattern (the current method of producing patterned areas on maps) could be eliminated. Reducing the number of required overlays would reduce both the time and cost of the printing process. Fewer separate negatives would be requisite, and less chance of negativemisregistry would exist. As a wide range of moires can be produced by altering the separation angle between two screens (Figures 1, 2, and 3), only two screens could conceivably be used to produce a complex gray tone map requiring several patterned areas (Figure 4). The success of this relies on several factors--including the size of the patterned areas, the interactions with and effects of neighboring areas, special pattern requirements due to specific representational prerequisites, and other considerations.

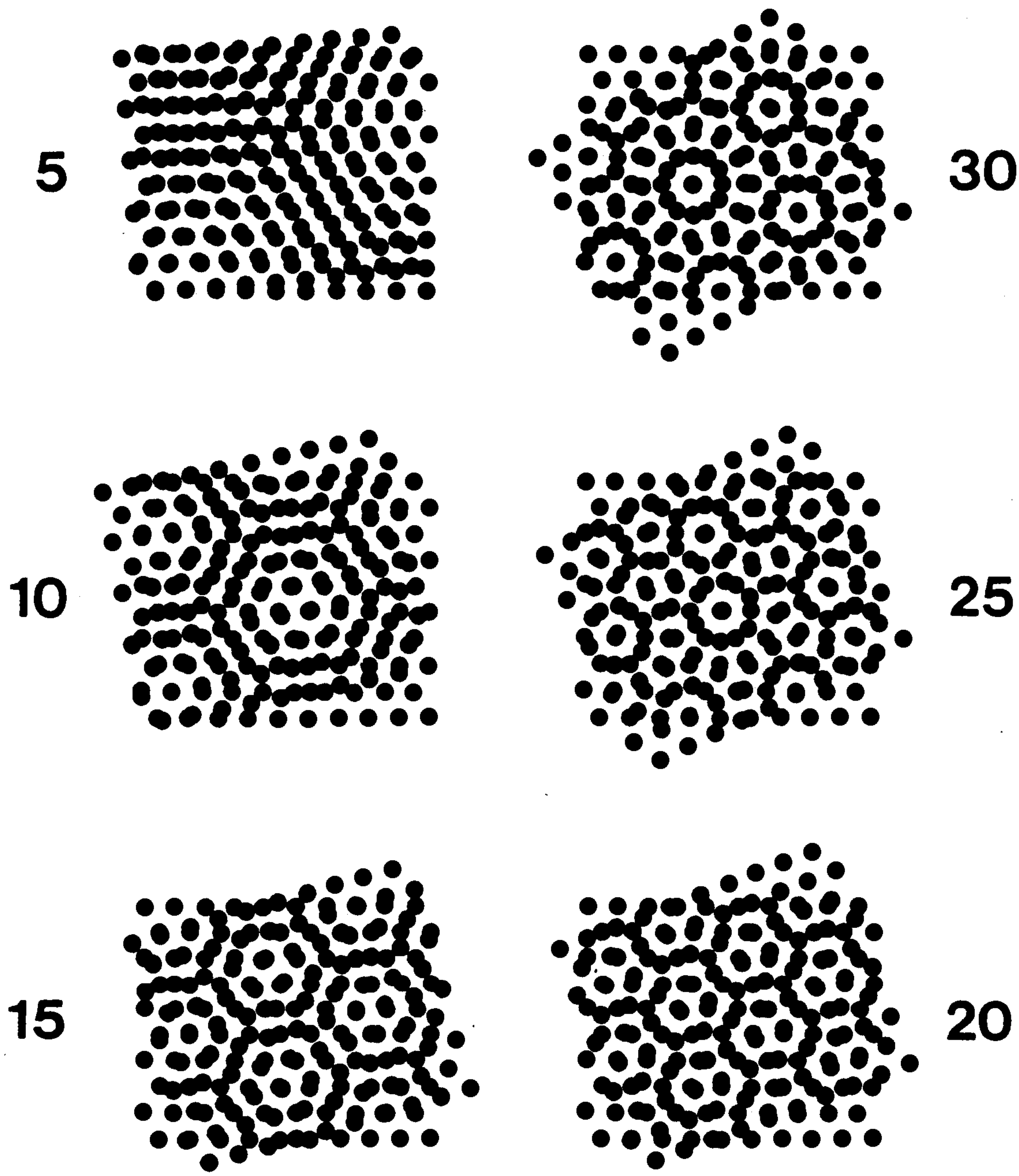

Figure 1. Moire pattern arrangements for superimposing two triangular dot arrangement screens at 5° separation angle intervals.

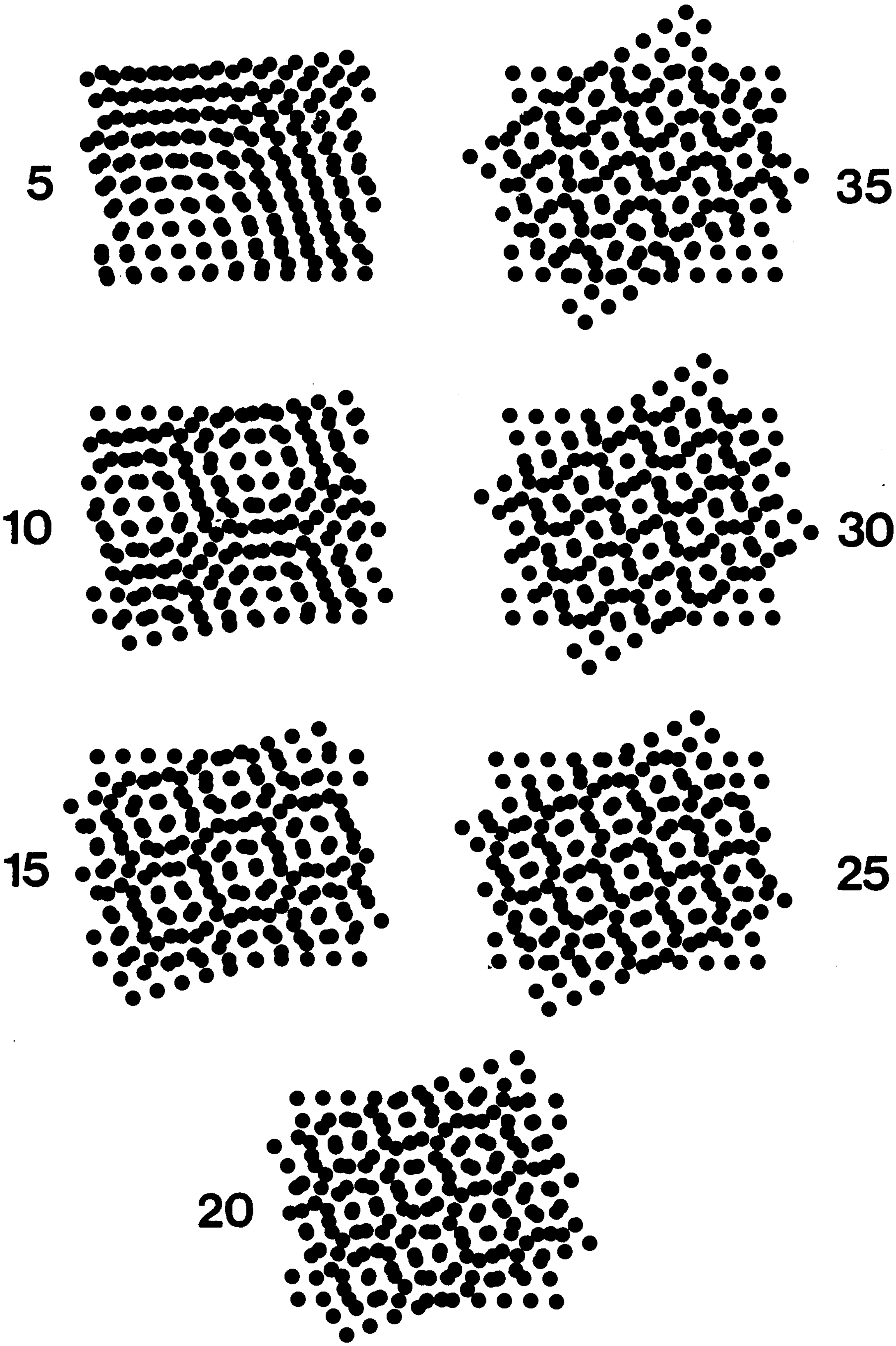

Figure 2. Moire pattern arrangements for superimposing two parallelogram dot arrangement screens at 5° separation angle intervals.

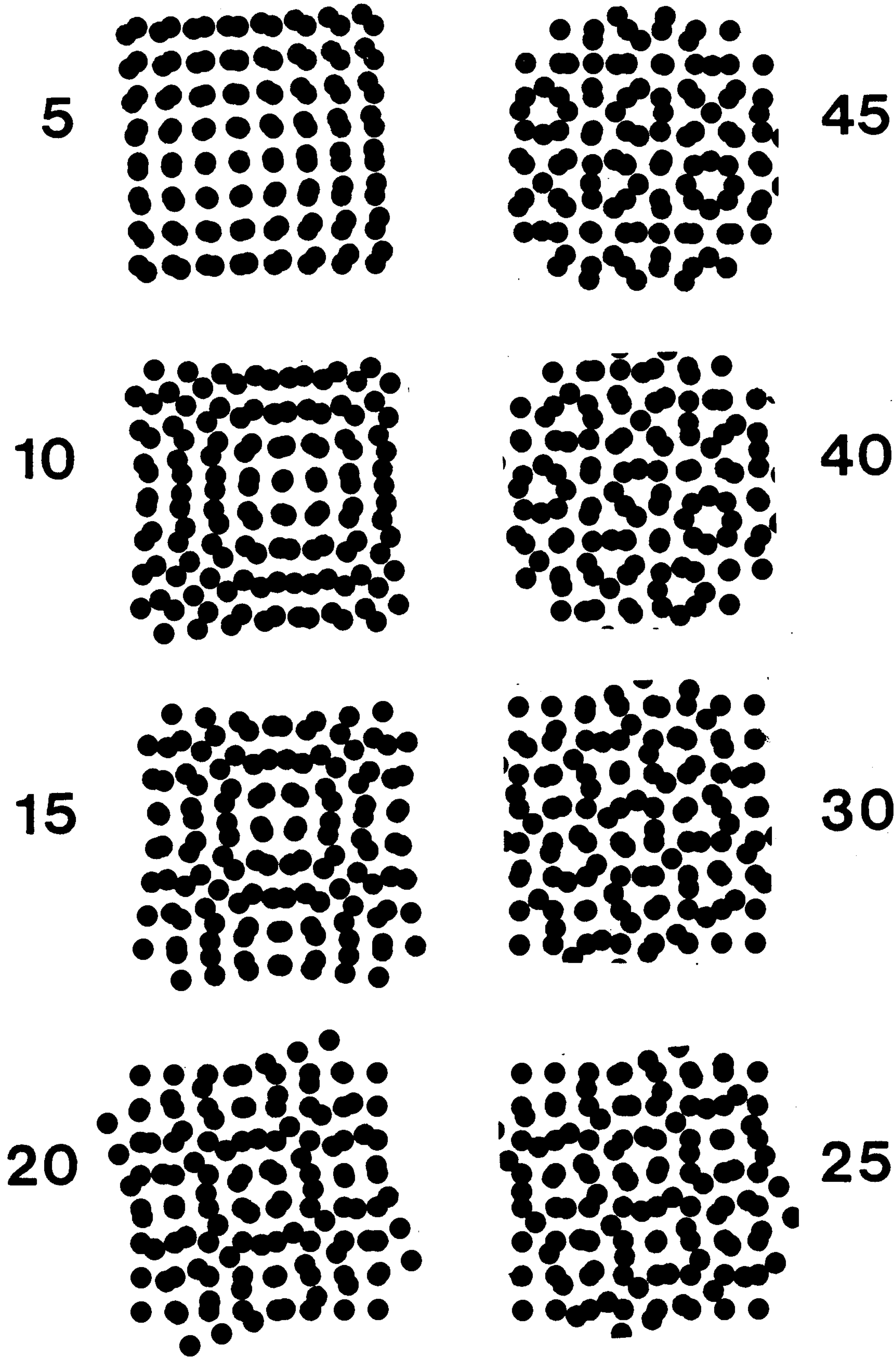

Figure 3. Moire pattern arrangements for superimposing two square dot arrangement screens at 5° separation angle intervals.

Source: A.J. Kimerling
Cartographic Guidelines For the Use of Moire Patterns Produced by Dot Tint Screens

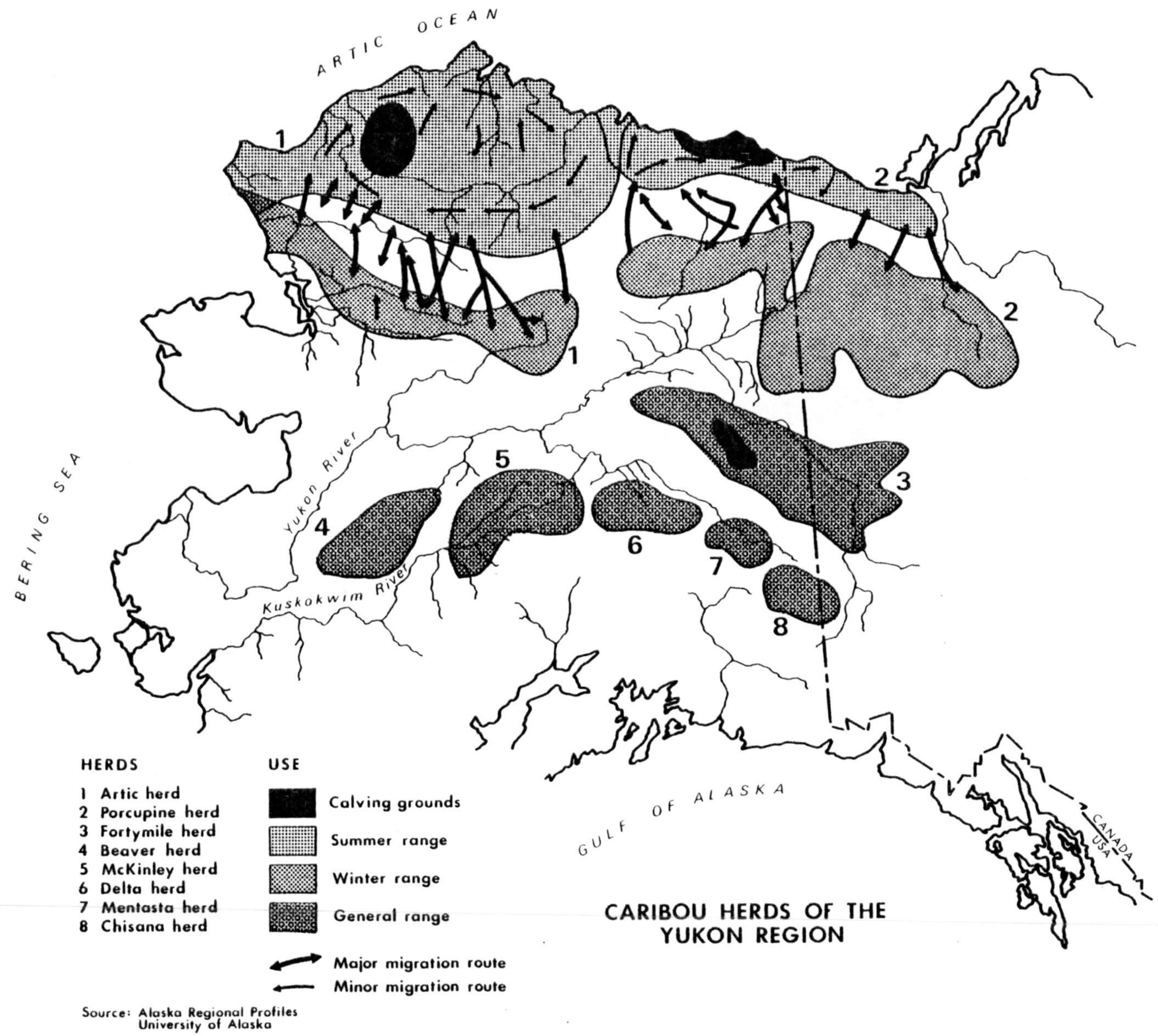

Figure 4. The use of moire patterns in map design.

Another interesting fact favoring use of moires produced from dot arrangement screens is the preference of map users for dot patterns. It has been found that "Dot patterns are considered to be more pleasant and more accurate for area shadings than other styles."[3]

The lack of availability of map design guidelines has contributed to the somewhat limited use of moire patterns. Kimerling identified four characteristics or physical attributes necessary for understanding moires, and analyzed the moire patterns resulting from the superimposing of two square dot arrangement screens in terms of these factors.[4] Predictive mathematical equations were established for square dot arrangement screens. This study analyzes additional dot arrangements to determine if equations are universally applicable to all types of dot arrangements. If universal equations are identified, it can be concluded that general laws for the geometric aspects of moire patterns exist. This fact would greatly aid in the task of developing map design guidelines.

In order to determine if universally applicable equations can be formulated, additional dot arrangement screens must be looked at. Screens with triangular and parallelogram dot arrangements were selected for study. These choices can be explained in terms of the triangle. The square dot arrangement screens initially reviewed by Kimerling can be considered a special triangular case--as a square is two right triangles mirrored about a common hypotenuse. A triangular dot arrangement screen can also be considered a special triangular case, being comprised of equilateral triangles. A parallelogram dot arrangement screen represents the case of the general triangle, as a parallelogram is actually two triangles mirrored about a common hypotenuse. Due to the twofold symmetry of these types of dot arrangements, the three cases of square, triangular, and parallelogram dot arrangement screens are representative of all possible angular configurations which can be superimposed at varying separation angles (Figure 5). A parallelogram dot arrangement screen having a base angle of 75° was arbitrarily chosen for this study, as 75° falls halfway between the angles of 90° for the square and 60° for the equilateral triangle. Generalized conclusions can be drawn from this sampling.

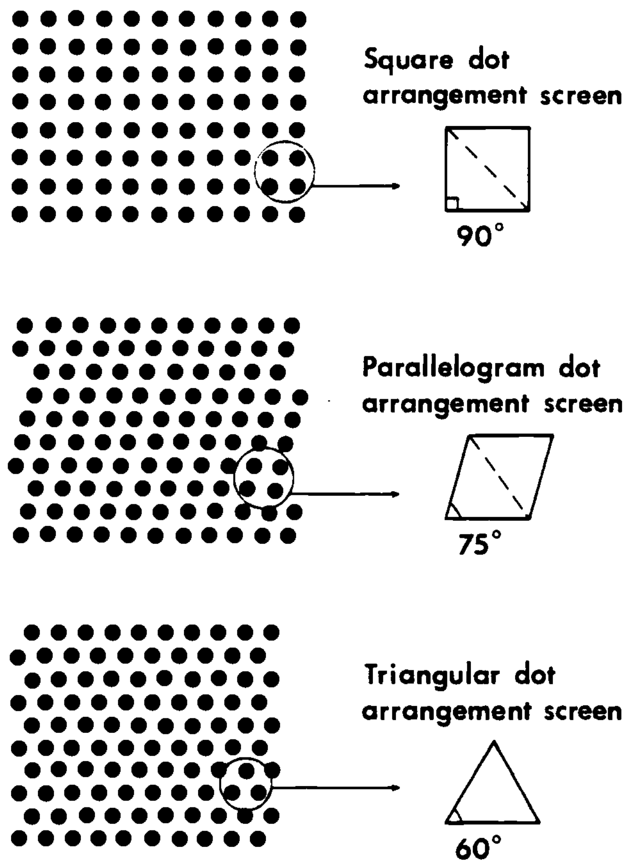

Figure 5. The three dot arrangement screens

The four physical attributes to be examined are: arrangement, orientation, gray tone, and moire pattern size. All are considered within the context of the separation angle, which has been identified as playing an important role in determining the appearance of the resulting moire patterns.

ARRANGEMENT

Arrangement, the relation of the positioning of dots, is a somewhat unique attribute. The moire patterns obtained from superimposing square dot arrangement screens have been found to form a continuing transition of arrangements which varied with separation angle. The moires could be grouped according to separation angle, with descriptive categories rather than a single mathematical equation most adequately summarizing the arrangements formed. Instead of a universal equation, a generalized rule for arrangement must be examined. It is hypothesized the arrangement of moires will reflect the geometry of their composite screens. This was found in square dot arrangement screens--square moire arrangements reflected the initial square dot arrangement screens.

Figure 1 illustrates the arrangement of moire patterns resulting from superimposing two triangular dot arrangement screens at 5° separation angle intervals.[5] Hexagonal arrangements are formed at 5°, 10°, and 15° while circular arrangements are formed at 25° and 30°. The hexagons decrease in size with increasing separation angle, and at a separation angle of around 20°, the transition from hexagons to circles occurs. Because of the continuous nature of the arrangements--which change with separation angle--the exact angle at which the transition occurs cannot be pinpointed. Due to the transitional aspect of the arrangements, and also keeping in mind the manner in which moires will be used by the cartographer, the arrangements can best be described by placing them into categories. The cateogries were somewhat arbitrarily selected as:

0°- 10° = large hexagons
10°- 20° = small hexagons
20°- 30° = circles

It should be kept in mind "large" and "small" are relative, and the transition is indeed a continuum.

These categories describe the first-order moire patterns produced. Yule[6] points out the existence of additional second-order moire patterns in his work with parallel line screens. Second-order patterns are visible at the larger separation angles for triangular dot arrangement screens. At 25° to 30°, the circles appear to interact with one another, forming larger, wagon-wheeled rosettes.

The moire pattern arrangements can be seen to be based on, and a logical outcome of, the arrangements of the composite screens. This is best shown at smaller separation angles--a hexagon can be broken down into the components of six interacting triangles, which clearly reflects the make-up of the composite triangular screens.

Figure 2 illustrates the arrangement of moire patterns resulting from superimposing two parallelogram dot arrangement screens at 5° separation angle intervals.[7] Elongated hexagonal arrangements are formed at 5°, 10°, and 15° while rectangular arrangements are formed at 25°, 30°, and 35°. The elongated hexagons decrease in size with increasing separation angle, and at a separation angle of around 20° the transition from elongated hexagons to rectangles occurs. Again, there is a continuum of moire patterns, and they can arbitrarily be placed into the following categories:

0° - 10° = large elongated hexagons
10° - 20° = small elongated hexagons
20° - 37.5° = rectangles

The existence of second-order moire patterns is visible at the larger separation angles. At 30° to 35°, the rectangles appear to interact with one another, forming wave-like patterns.

The moire pattern arrangements can also be seen as a logical outcome based on the arrangements of the composite screens. Again, the elongated hexagon formed at the smaller separation angles most clearly illustrates this.

The above two examples, considered in conjunction with the case of the square dot arrangement screen in which square to rosette patterns were formed reflecting the arrangement of the composite square dot arrangement screens (Figure 3), indicate a general rule for the arrangement of moire patterns does exist. It can be simply stated that the arrangement of moire patterns reflects the arrangements of their composite screens.

ORIENTATION

Orientation is the dominant direction of the pattern arrangement, generally defined in terms of the map border. When working with moires, it can be observed that the orientation changes continuously with separation angle. As with the attribute of arrangement, the continual change in orientation can be said to be reliant on separation angle, and is best defined in terms of it. A specific orientation angle law has been identified for the moires of square dot arrangement screens.

Figures 6 and 7 illustrate the dominant direction of the moires at 5° separation angle intervals for triangular and parallelogram dot arrangement screens. Orientation was measured with respect to the separation angle, and in all cases found to be equal to one half of the separation angle. Kimerling[8] observed this same relationship between orientation and separation angle with moires generated from superimposing square dot arrangement screens (Figure 8), naming the line through the center of the pattern at the orientation angle the <u>pattern axis</u>.

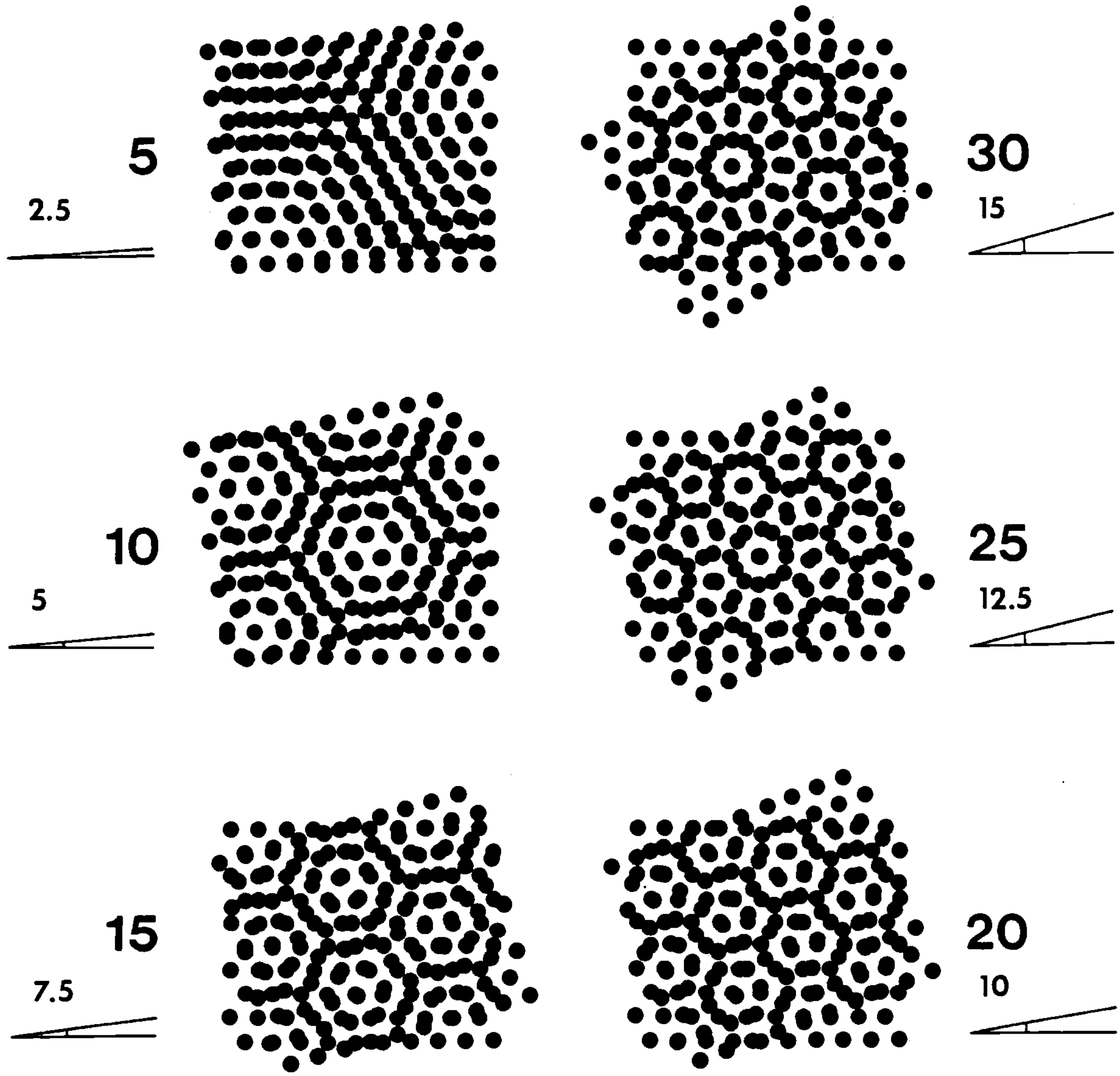

Figure 6. Moire pattern orientation for superimposing two triangular dot arrangement screens at 5° separation angle intervals.

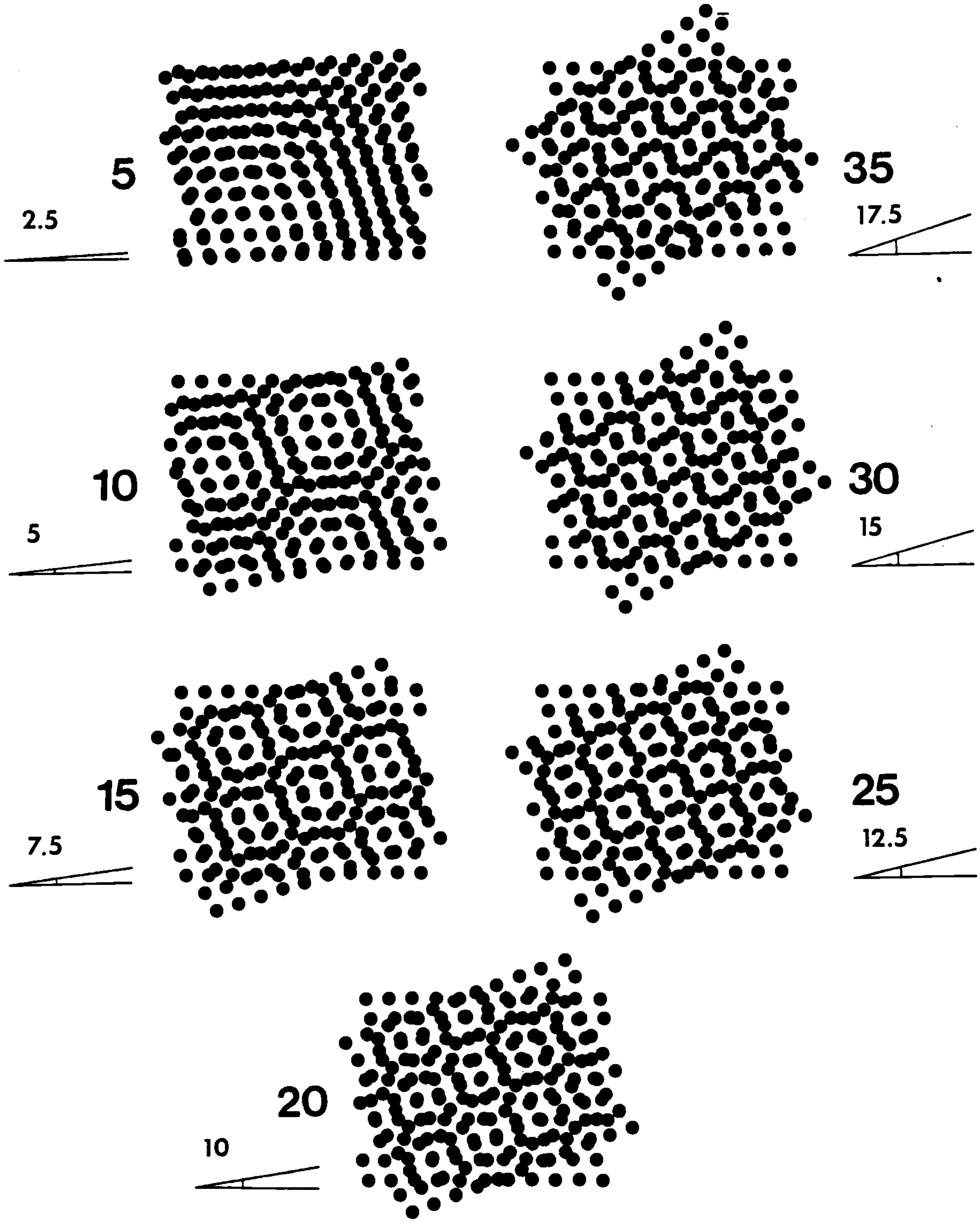

Figure 7. Moire pattern orientation for superimposing two parallelogram dot arrangement screens at 5° separation angle intervals.

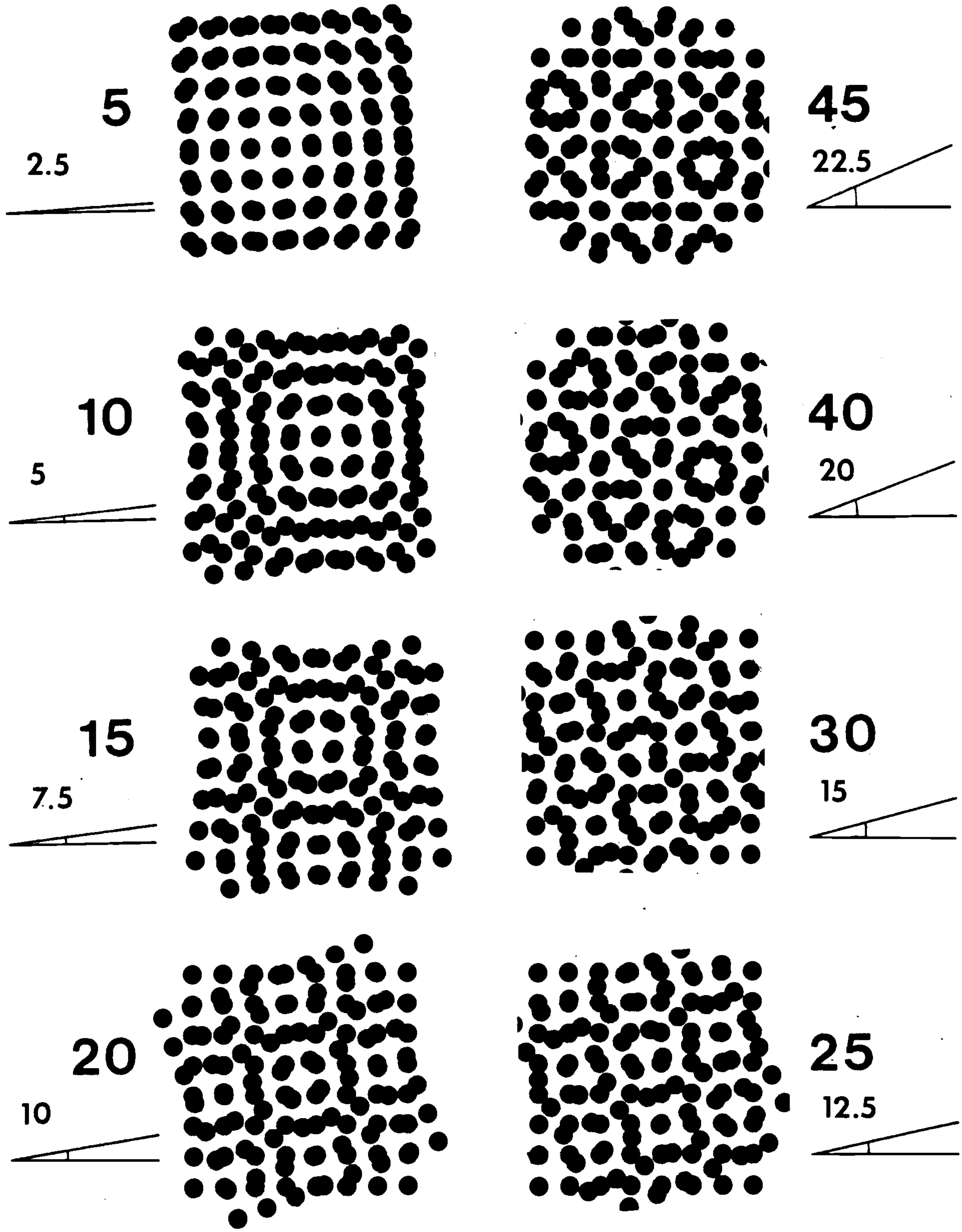

Figure 8. Moire pattern orientation for superimposing two square dot arrangement screens at 5° separation angle intervals.
Source: A.J. Kimerling
Cartographic Guidelines For the Use of Moire Patterns Produced by Dot Tint Screens

As one equation holds true for the three cases of triangular, parallelogram, and square dot arrangement screens, it can be concluded the orientation of moires resulting from superimposing any type of dot arrangement screens can be determined from the formula:

Orientation angle = $\theta/2$, where θ is the separation angle.

GRAY TONE

Gray tone is the perceived lightness or darkness of the arrangement as rated on a gray scale. An equation for the gray tone of moires resulting from superimposing square dot arrangement screens is dependent on measuring the per cent area inked of the initial screens (percent area inked is one physical measurement of gray tone).

In order for moire patterns to be of applicable use to the cartographer, a means of predetermining gray tone specification is required. The gray tone specification of moire patterns generated from varying the separation angle between two screens of known gray tone values must be consistently predictable--if the gray tone of the resultant moire patterns varied inconsistently with separation angle (deviated from an expected value), an important parameter would persist as an unknown and applicability of such patterns would be limited.

The determination of percent area inked is the key to analyzing gray tone specification as it is the physical measure of gray tone and can be readily calculated. Probability Theory is utilized to determine percent area inked of moire patterns. Kimerling illustrates the applicability of Probability Theory with an easily visualized example.[9] First, imagine a screen (Screen A) produces a dot pattern on a wall. The probability of hitting a black dot is analogous to determining the percent area inked of the screen, and is notated P(A). The relation of probability to percent area inked can be written P(A) = PIA(A)/100%. Now imagine two superimposed non-overlapping screens (Screens A and B). The percent area inked of the first screen (Screen A) is not affected by the percent area inked of the second screen (Screen B), and Probability Theory assures the screens pattern on the wall can be viewed as independent probability events with the probability of hitting a dot produced by Screen A still being P(A), and the probability of hitting a dot produced by Screen B being P(B). The probability of hitting any dot would be the sum of the two independent provabilities:

$$P(A + B) = P(A) + P(B)$$

Now imagine one screen is rotated with respect to the second screen so that a pattern containing dot overlap is produced on a wall. The probability of hitting any dot is now the sum of the two independent probabilities minus dot overlap. This can be expressed as:

$$P(A + B) = P(A) + P(B) - P(A \times B)$$

In the analysis, P(A) is also represented as being equal to PAI(A)/100%. By substituting into the above equation, the formula for determining the percent area inked of a cross screen product is:

$$\frac{PAI(A + B)}{100} = \frac{PAI(A)}{100} + \frac{PAI(B)}{100} - \frac{PAI(A)}{100} \times \frac{PAI(B)}{100}$$

This can be rewritten as:

$$PAI(A + B) = PAI(A) + PAI(B) - \frac{PAI(A) \times PAI(B)}{100}$$

The formula indicates the percent area inked of any moire pattern generated from superimposing two screens is entirely dependent upon the known per cent area inked values of the two initial screens. As the separation angle does not appear to contribute to the percent area inked of the resultant combination, it should be evident all non-zero separation angle moire patterns which result from the superimposing of two screens will have an identical, predictable percent area inked values.

Previous study has shown the formula was applicable to moires produced from superimposing two square dot arrangement screens. In order to generalize upon this, the percent area inked of moire patterns produced from superimposing two triangular dot arrangement screens was tested. A screen can be viewed as a coordinate system, and in the case of the triangular dot arrangement screen the coordinate system is unique in that every other row of X coordinates is offset from the origin by one half of the triangular base. The change of X (ΔX) can be chosen to represent the triangular base (commonly notated a). Given the definition of an equilateral triangle, the change of Y (ΔY) is determined from a, and is equal to a(cos 30) (commonly notated h). Each dot of the first screen (Screen A) has a fixed location of (X,Y) (Figure 9). A defined matrix of X columns and Y rows can be chosen for Screen A. The second screen (Screen B) which is to be superimposed on Screen A can also be conceived of as a coordinate system, with each dot having a fixed location (X',Y'). When the two screens are aligned, the separation angle equals 0° or 60° and $(X',Y') = (X,Y)$. The coordinate systems have the same orientation or X and Y axes. If, however, Screen B is rotated and the separation angle falls between but is not equal to 0° or 60°, then $(X',Y') \neq (X,Y)$. The coordinate systems have different orientations or a different set of X and Y axes. In order to compare

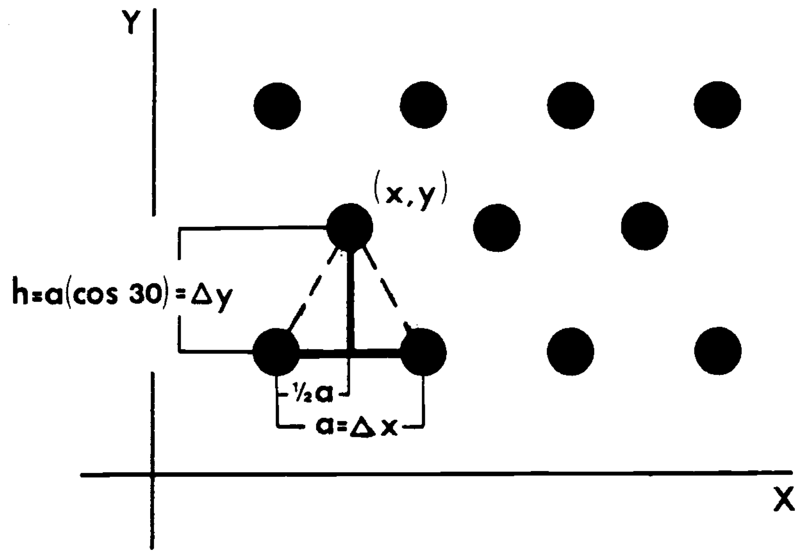

Figure 9. The triangular dot arrangement coordinate system

the dots of the two coordinate systems and determine the percent area inked for the chosen matrix, the second coordinate system of Screen B must be rotated or transformed to the first coordinate system of Screen A. This is done with the use of the transformation equations:

$$X' = X \cos \theta + Y \sin \theta$$

$$Y' = Y \cos \theta + X \sin \theta \quad , \text{ where } \theta = \text{separation angle}/180$$

Once the transformation has been completed, the two coordinate systems can be compared and the percent area inked within the defined matrix of Screen A determined (Figure 10). Percent area inked will be equal to the sum of the area of all dots within the matrix minus the total area of dot overlap occurring. The area of all dots was determined by simply counting the dots and multiplying this total by πr^2. The area of dot overlap was determined with the aid of Oregon State University's CDC CYBER Computer. A program was written that compared all dots of the matrix of Screen A with all dots of Screen B falling within the matrix. If the distance between

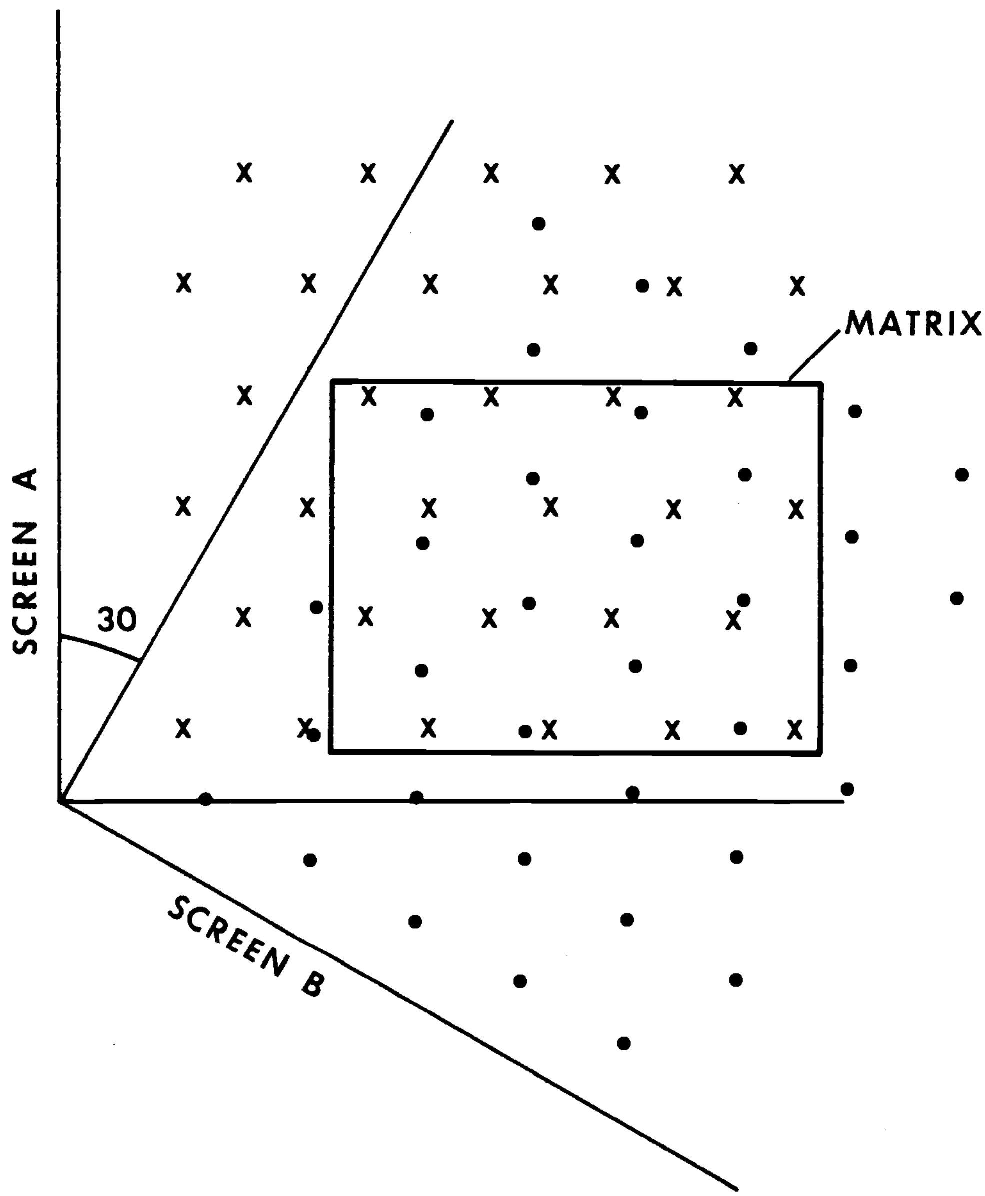

Figure 10. A sample matrix used in percent area inked measurement.

the centers of two dots was found to be greater than the sum of their respective radii, the program concluded no overlap existed and proceeded to the next comparison. If the distance between the centers of the two dots was found to be less than the sum of their respective radii, overlap was calculated using trigonometric and circle mensuration formulii. Dot overlap is the sum of A and B as shown in Figure 11. It is reliant upon:

1) the respective radii of the two dots, r_1 and r_2,
2) the distance between the centers of the two dots, d, and
3) the angles a and b.

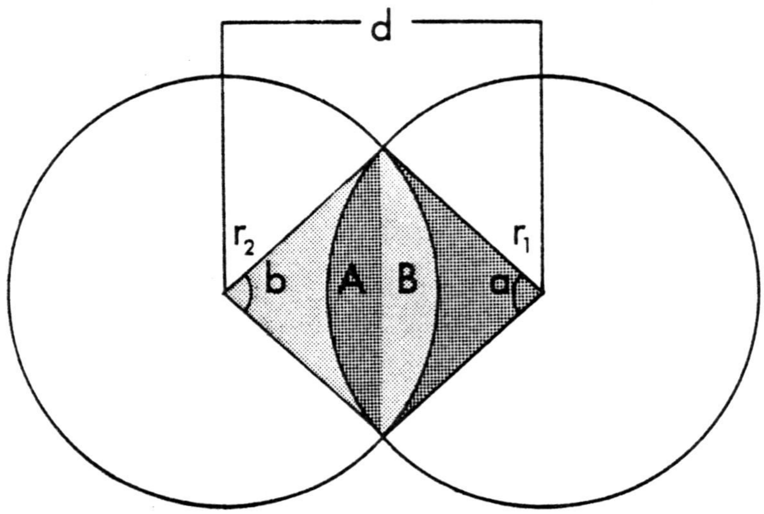

Figure 11. Geometric basis of dot overlap calculation. Circle segments A and B are summed. The distance between dots (d), single dot radii (r_1 and r_2), and angles a and b must be determined.

Angles a and b are defined as:

$$a = (2 \cos^{-1} \times (d^2 + r_1^2 - r_2^2))/(2dr_1)$$

$$b = (2 \cos^{-1} \times (d^2 + r_2^2 - r_1^2))/(2dr_2)$$

If the product $(d^2 + r_1^2 - r_2^2)$ is found to be greater than one, complete dot overlap exists, and total area of overlap is simply assigned the area of a single dot, r_1^2.

The formulii for areas A and B can be caluclated as:

$$A = 0.5\ r_1^2(a - \sin a)$$

$$B = 0.5\ r_2^2(b - \sin b)$$

Total area of overlap for two dots is simply A + B.

In order to ensure the above was indeed applicable to all possible moire patterns generated by two triangular dot arrangement screens being superimposed, examples of the two possible screen combinations were looked at: superimposing two screens having the same gray tone specification, and superimposing two screens having differing gray tone specifications. Five degree separation angle intervals ranging from 5° through 30° were tested. As Kimerling had previously demonstrated, a pattern segment at least as large as the width of an arrangement was required for generation of moire patterns. Separation angles less than 5° were not considered, due to the large amount of computation that would be required. As the triangular dot arrangement screen has a twofold symmetry, the full range of moire patterns occurs between 0° and 30° (the bisecting angle)--because of this, angles greater than 30° were not considered. Varied matrix or cell pattern sizes were also examined. The results (Tables 1 and 2) clearly show a close agreement between the expected and observed percent area inked values--the majority of observed values fall within ±1.0% of the expected values.

The third case of parallelogram dot arrangement screens was also tested. The coordinate system for this type screen is slightly different.

Table 1. PERCENT AREA INKED VALUES FOR TWO SUPERIMPOSED TRIANGULAR DOT ARRANGEMENT SCREENS OF IDENTICAL VALUE AT VARIOUS SCREEN SEPARATION ANGLES AND PATTERN CELL SIZES

PAI Values of initial screens: 30 PAI, 30 PAI

Expected cross screen PAI Value: $30 + 30 - \frac{30 \times 30}{100} = 51$

Screen Separation	Cell Size	Observed PAI	Deviation
5	20 x 10	49.2	-1.8
10	20 x 10	50.9	-0.1
	10 x 10	51.7	0.7
	10 x 10	50.2	-0.8
	10 x 10	52.3	1.3
15	20 x 10	51.1	0.1
	8 x 8	51.0	0
	8 x 8	50.3	-0.7
	8 x 8	51.2	0.2
20	20 x 10	51.0	0
	6 x 6	51.3	0.3
	6 x 6	51.2	0.2
	6 x 6	51.5	0.5
	8 x 8	51.3	0.3
	8 x 8	51.0	0
	8 x 8	50.6	-0.4
25	20 x 10	51.1	0.1
	8 x 8	50.5	-0.5
	8 x 8	50.9	-0.1
	8 x 8	51.0	0
30	20 x 10	50.9	-0.1
	10 x 10	50.7	-0.3
	10 x 10	50.4	-0.6
	10 x 10	51.3	0.3

Table 2. PERCENT AREA INKED VALUES FOR TWO SUPERIMPOSED TRIANGULAR DOT ARRANGEMENT SCREENS OF DIFFERING VALUE AT VARIOUS SCREEN SEPARATION ANGLES AND PATTERN CELL SIZES

PAI Values of initial screens: 10 PAI, 40 PAI

Expected cross screen PAI Value: $10 + 40 - \frac{10 \times 40}{100} = 46$

Screen Separation	Pattern Cell Size	Observed PAI	Deviation
5	20 x 10	44.8	-1.2
10	20 x 10	45.5	-0.5
	10 x 10	45.9	-0.1
	10 x 10	45.0	-1.0
	10 x 10	46.2	0.2
15	20 x 10	45.5	-0.5
	8 x 8	45.3	-0.7
	8 x 8	45.3	-0.7
	8 x 8	45.5	-0.5
20	20 x 10	45.6	-0.4
	6 x 6	45.9	-0.1
	6 x 6	45.7	-0.3
	6 x 6	46.0	0
	8 x 8	45.6	-0.4
	8 x 8	45.7	-0.3
	8 x 8	45.7	-0.3
25	20 x 10	45.5	-0.5
	8 x 8	45.6	-0.4
	8 x 8	45.2	-0.8
	8 x 8	45.6	-0.4
30	20 x 10	45.5	-0.5
	10 x 10	45.5	-0.5
	10 x 10	45.7	-0.3
	10 x 10	45.6	-0.4

The change of X (ΔX) was chosen to represent the base of the parallelogram (commonly notated b). Given the definition of a parallelogram, the change of Y (ΔY) is determined from b, and is equal to 1/b (commonly notated h) (Figure 12).

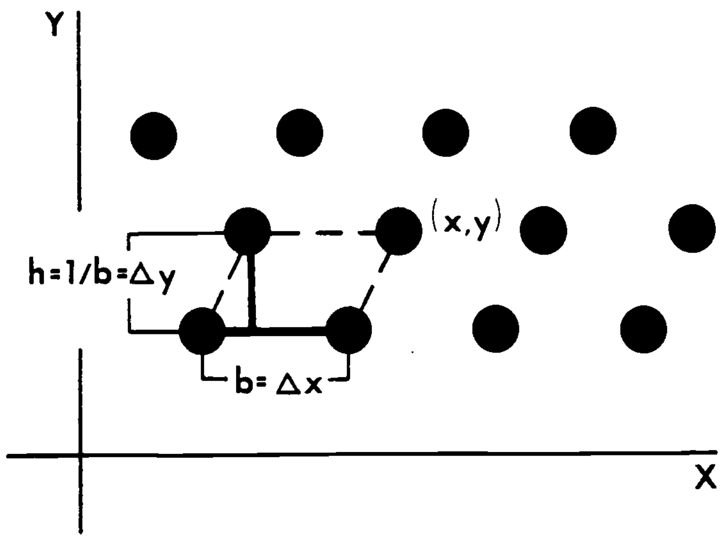

Figure 12. The parallelogram dot arrangement coordinate system

The process covered for the triangular dot arrangement screens was repeated. The results (Tables 3 and 4) clearly show a close agreement between the expected and observed percent area inked values--a majority of the observed values fall within ±1.0% of the expected values.

The formula has now been successfully applied to square, triangular, and parallelogram dot arrangement patterns. These are all examples of highly structured screens. Another type of screen is one which is completely random. To test the case of random dot arrangement screens, two 25 x 25 segments of entirely random dot arrangement patterns were generated by computer. Each had a 30 per cent area inked surface. The random dot arrangement patterns have a "1" corresponding to an inked square, and a "0"

Table 3. PERCENT AREA INKED VALUES FOR TWO SUPERIMPOSED PARALLELOGRAM DOT ARRANGEMENT SCREENS OF IDENTICAL VALUE AT VARIOUS SCREEN SEPARATION ANGLES AND PATTERN CELL SIZES

PAI Values of initial screens: 30 PAI, 30 PAI

Expected cross screen PAI Value: $30 + 30 - \frac{30 \times 30}{100} = 51$

Screen Separation	Cell Size	Observed PAI	Deviation
5	10 x 10	53.4	2.4
10	12 x 12	51.0	0
	10 x 10	50.8	-0.2
	10 x 10	49.4	-1.6
	10 x 10	50.1	-0.9
15	12 x 12	50.7	-0.3
	10 x 10	50.7	-0.3
	10 x 10	52.4	1.4
	10 x 10	51.2	0.2
20	10 x 10	51.8	0.8
	8 x 8	51.7	0.7
	8 x 8	52.0	1.0
	8 x 8	49.9	-1.1
25	8 x 8	51.2	0.2
	6 x 6	51.6	0.6
	6 x 6	51.9	0.9
	6 x 6	51.9	0.9
30	6 x 6	51.0	0
	4 x 4	49.4	-1.6
	4 x 4	51.7	0.7
	4 x 4	50.6	-0.4
35	4 x 4	53.3	2.3

Table 4. PERCENT AREA INKED VALUES FOR TWO SUPERIMPOSED PARALLELOGRAM DOT ARRANGEMENT SCREENS OF DIFFERING VALUE AT VARIOUS SCREEN SEPARATION ANGLES AND PATTERN CELL SIZES

PAI Values of initial screens: 10 PAI, 40 PAI

Expected cross screen PAI Value: $10 + 40 - \frac{10 \times 40}{100} = 46$

Screen Separation	Pattern Cell Size	Observed PAI	Deviation
5	10 x 10	46.1	0.1
10	12 x 12	45.4	-0.6
	10 x 10	45.3	-0.7
	10 x 10	44.8	-1.2
	10 x 10	45.4	-0.6
15	12 x 12	45.3	-0.7
	10 x 10	45.3	-0.7
	10 x 10	45.6	-0.4
	10 x 10	45.2	-0.8
20	10 x 10	46.0	0
	8 x 8	45.7	-0.3
	8 x 8	45.5	-0.5
	8 x 8	45.8	-0.2
25	8 x 8	45.6	-0.4
	6 x 6	45.8	-0.2
	6 x 6	46.6	0.6
	6 x 6	45.5	-0.5
30	6 x 6	45.5	-0.5
	4 x 4	45.0	-1.0
	4 x 4	45.7	-0.3
	4 x 4	45.6	-0.4
35	4 x 4	46.5	0.5

SCREEN 1

```
0100010011011010000000100
0000010010000010000100000
0011000101110000000010000
0001010001001001000110111
0011001000000100010001111
0110000100110010000101000
1010001000000000111100010
0111001000000000000100001
1000010010010010010100000
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1010000000101110000101011
1000001100000100000000100
0100000001100010011101010
0100000100100010000000000
```

```
NO SCREEN: 304 = 48.64%
SCREEN 1: 134 = 21.44%
SCREEN 2: 134 = 21.44%
SCREEN 1+2: 53 = 8.48%

GOOD BYE
```

SCREEN 2

```
1000000100010011110010101
0010100001110111101111000
0100000000010000101000110
1000110010100000001001010
1010000000000000010101011
1100000110010001001011101
0001100001100000100000001
0100100010000000100000000
0000000000000000101110000
1000000100000100100000001
0010100000100101111010000
0110100000000101000000000
0000000010000011001010010
1011010000001011010001000
0101000000100000001000000
0010001101001000010000010
0010100101000010010101001
1100000100101101010000111
0000100100010000011000101
0101010000000100000110000
0011000111000000000101110
0000110000000100100110000
1000000001000001100101101
0000110001110010001101000
0011010001001100000001000
```

$$PAI(A + B) = PAI(A) + PAI(B) - \frac{PAI(A) \times PAI(B)}{100}$$

$$= (\text{Screen 1}) + (\text{Screen 2}) + (\text{Screen 1 + 2})$$

$$= 21.44 + 21.44 + 8.48$$

$$= 51.36\ PAI$$

Figure 13. Computer generated random dot arrangement segments. (Program written by A. Jon Kimerling)

indicating a non-inked square (Figure 13). The two "screens" were superimposed with the separation angle kept constant (there was no screen rotation). A cell by cell search was made in order to tabulate which of four possible outcomes resulted from the superimposition:

1) No ink on the resultant cell - a "0" had been superimposed over a "0",

2) Inked cell due to overlap of the screens - a "1" had been superimposed over a "1",

3) and 4) Inked cell due to the initial inked configuration of

Screen 1 or Screen 2 - a "1" had been superimposed over a "0", or a "0" had been superimposed over a "1".

The formula predicts the resultant pattern will have a gray tone specification of an expected 51 percent area inked:

$$PAI(A + B) = PAI(A) + PAI(B) - \frac{PAI(A) \times PAI(B)}{100}$$

$$\underline{51} = 30 + 30 - \frac{30 \times 30}{100}$$

The observed value of 51.36 percent area inked was in close accordance with the expected value.

This example, taken together with the initial study on square dot arrangement screens and the analysis of triangular and parallelogram dot arrangement screens substantiates that a universal formula that can be successfully applied to all types of dot arrangement screens exists. This formula is:

$$PAI(A + B) = PAI(A) + PAI(B) - \frac{PAI(A) \times PAI(B)}{100}$$

SIZE

Moire pattern size is simply the largeness or smallness of the identified arrangements. The size of moire pattern arrangements has been examined in two ways. Yule[10] considered size in terms of separation angle and the screen period of the initial screens. The resultant value was a measure of the physical width of the moire pattern arrangement. Kimerling[11] considered size in terms of separation angle and columns or rows of dots. The resultant value was a relative width, expressed in number of dots. Number of dots is a more general value than physical width, being independent of screen ruling. As such, it can be applied to the myraid of screen line rulings or periods that are commonly used.

Size in terms of numbers of dots was measured for both triangular and parallelogram dot arrangement screens at 5° separation angle intervals. As described earlier, triangular screens produce a continuum of moires which shift from hexagonal to circular arrangements. The largest distance across a hexagon and the diameter of a circle seemed the logical distance to measure. The largest distance across a hexagon was found to correspond to what has been identified as the pattern axis. To ensure compatability of measurement throughout the transition of arrangements, all measurements were taken along the pattern axis. Parallelogram dot arrangement screens, which produce a continuum of moires shifting from elongated hexagons to rectangles, were also measured at their largest distance which again corresponded to the pattern axis. The measurements of both types of screens were found to form nearly identical curves. The trigonometric solution to these curves was identified as $0.5/(\sin(\theta/2)) + 1$. The curves (expected and measured values) are shown in Figure 14. Measurement values are given in Table 5.

Table 5. MOIRE PATTERN SIZE MEASUREMENT (in number of dots)

screen separation	Expected $0.5/(\sin(\theta/2)) + 1$	Observed TRIANGLE	Observed PARALLELOGRAM
5	13.735	13	12
10	7.372	7	6
15	5.250	5	4.5
20	4.196	4	4
25	3.562	3.2	3.2
30	3.140	3	3
35	2.844	-	3

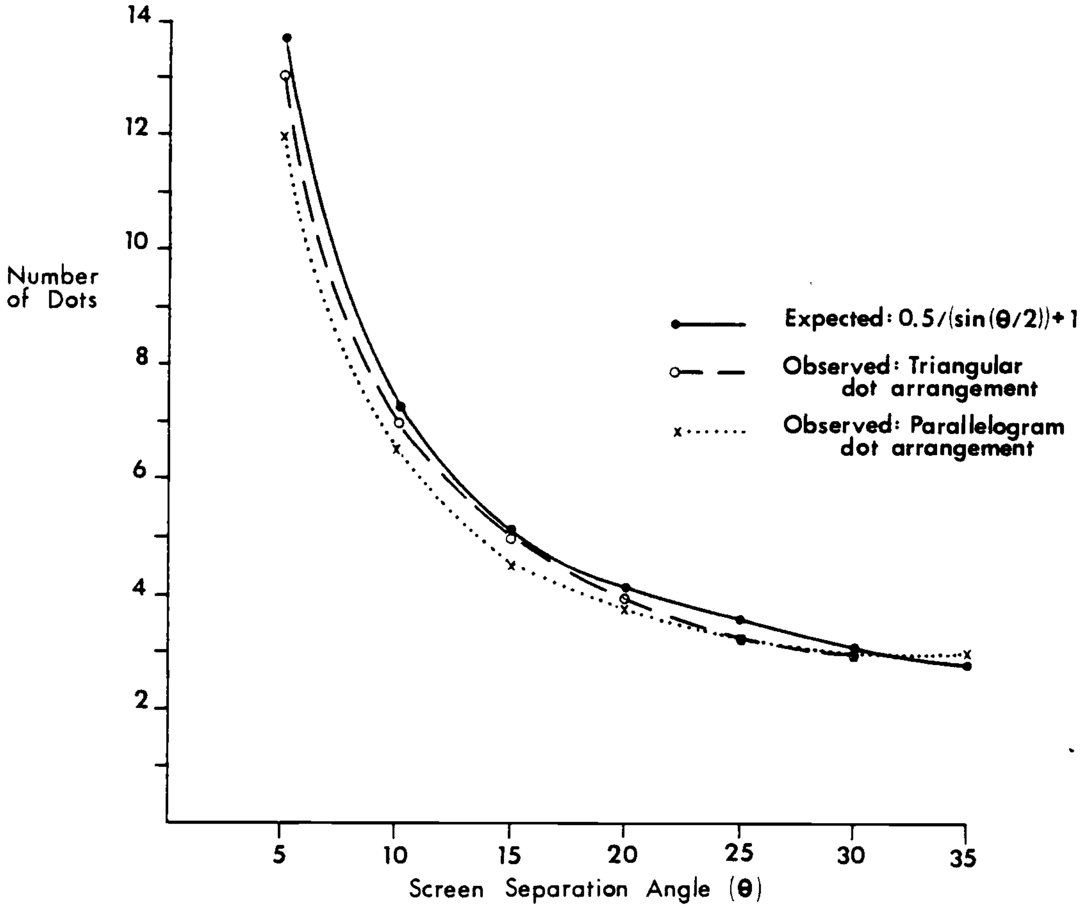

Figure 14. Graph of the moire pattern size equation, with measured size of triangular and parallelogram dot arrangement moires shown

The formula can be most clearly explained in terms of the separation angle. The number of dots measured at the separation angle would have been equal to the amount of dots of the first screen needed to traverse one row of dots of the second screen (Figure 15). The formula for measurement at the separation angle is $1.0/(\sin\theta) + 1$. As only one half of the separation angle is being considered when measurement is made along the pattern axis, the amount of dots of the first screen needed to traverse one half of a row of dots of the second screen is being considered. The formula $0.5/(\sin(\theta/2)) + 1$ is, as expected, one half of the formula $1.0/(\sin\theta) + 1$.

Kimerling's predictive formula for the size of moires of square dot

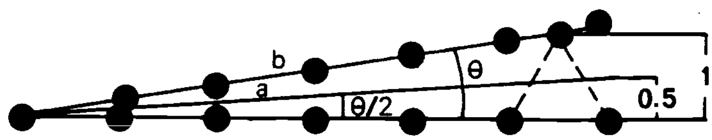

Figure 15. Trigonometrical basis of pattern size calculation. Side b is the pattern width along a row of dots on one screen; side a is the width along the pattern axis.

arrangements was $1.5/(\sin(\theta/2)) + 1$. This is identical to the formula for triangular and parallelogram dot arrangement screens, with the exception of the numerator. The difference in the numerator of 1.5 as compared to 0.5 can be explained by looking at the continuum of arrangements which are formed by different dot arrangement screens. In order to obtain compatable measurements throughout the transition of square to rosette patterns produced by square dot arrangement screens, Kimerling measured <u>three</u> rows of dots at the separation angle, or 1.5 rows of dots at the pattern axis. With both triangular and parallelogram dot arrangement screens, the transition from hexagons to circles and elongated hexagons to rectangles could be adequately accommodated by considering just <u>one</u> row of dots at the separation angle, or 0.5 rows of dots at the pattern axis. The formulii reflect the difference of rows of dots which must be measured along the separation angle.

In order to arrive at a measurement which best describes the size of the continuum of moire patterns produced from two initial dot arrangement screens, a decision must be made as to what measurement will most adequately reflect all categories of the continuum. Once this has been ascertained, the following general formula can be applied to all types of dot arrangement screens:

$$\text{Size of moire pattern} = \frac{\text{number of rows of dots}}{2\sin(\theta/2)} + 1$$

CONCLUSION

The existence of universally applicable equations and/or rules for determining the geometric aspects of moire patterns was verified. The four physical attributes and their predictive equations and/or rules can be summarized as:

Arrangement: The arrangement of moire patterns reflects the arrangements of the composite screens.

Orientation: Orientation angle and pattern axis are dependent upon the separation angle, and can be determined from the relationship:

Orientation angle = $\theta/2$, where θ = separation angle

Gray Tone : There is a single gray tone for moires produced at all separation angles from two composite screens, reliant upon the gray tones of the composite screens. Gray tone is measured in percent area inked, and can be determined from the relationship:

$$PAI(A + B) = PAI(A) + PAI(B) - \frac{PAI(A) \times PAI(B)}{100}$$

Size : Moire pattern size measurement must be chosen so as to completely represent the entire spectrum of moire pattern arrangements produced from superimposing two screens. Once this is ascertained, size can be determined from the relationship:

$$\text{Size of moire pattern} = \frac{\text{number of rows of dots}}{2 \sin(\theta/2)} + 1$$

, where θ = separation angle

The equations and/or rules provide the cartographer with basic information for determining suitable moire patterns in map design. The cartographer must, however, remember both practical and perceptual aspects of map design cannot be solved by simply applying the equations and/or rules. Pertaining to the practical aspects, the cartographer must convert the size equation from number of dots to a physical size measurement using the rulings of the particular screens available to him/her. Pertaining to the perceptual aspects, although the size of the moire is known for the various arrangements, the minimum moire size which is distinguishable as well as the maximum moire size which is acceptable for use is still to be determined. Further research in this area is necessary. In the area of gray tone, the equation tells the cartographer the specific gray tone of the moires, but this gray tone must be considered with respect to its position on the equal value gray scale (which must be determined) and the manner in which the gray tone is perceived in relation to surrounding areas must be taken into consideration. This is another area in which further study is required.

FOOTNOTES

1. International Paper Company, Pocket Pal, A Graphic Arts Production Handbook, twelfth edition (New York: International Paper Company, 1979), p. 82.

2. International Paper Company, op. cit., footnote 1, p. 83.

3. G. Jenks, "Selection of Area Shading Patterns on Maps," Annals, Association of American Geographers, Vol. 49 (1959), p. 190.

4. A.J. Kimerling, "Cartographic Guidelines For the Use of Moire Patterns Produced by Dot Tint Screens," The Canadian Cartographer, Vol. 16, No. 2 (1969), p. 160.

5. The six examples are an adequate sampling of the moires that are produced at all separation angles. Complete realignment of two triangular dot arrangement screens occurs at 60°, and every 60° thereafter. Rotation of separation angle from 60° to 120° would simply repeat the rotation from 0° to 60°, producing identical moires. Due to the twofold symmetry of the equilateral triangle about its bisecting angle of 30°, the moire patterns produced from 30° to 60° mirror those produced from 0° to 30°. Therefore, the entire spectrum of moires can be found between 0° and 30°.

6. J.A.C. Yule, Principles of Color Reproduction, (New York: John Wiley and Sons, 1967), p. 336.

7. For the parallelogram dot arrangement screen used in this study, the bisecting angle is equal to 37.5°. Therefore, given the twofold symmetry of the parallelogram, the entire spectrum of moires produced can be found between 0° and 37.5°. The seven examples are an adequate sampling.

8. A.J. Kimerling, op. cit., footnote 4, pp. 164 - 65.

9. A.J. Kimerling, "Visual Value as a Function of Percent Area Inked for the Cross-Screening Technique," The American Cartographer, Vol. 6, No. 2 (1979)

10. J.A.C. Yule, op. cit., footnote 6, p. 335.

11. A.J. Kimerling, op. cit., footnote 4, pp. 165 - 66.

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Yule, J.A.C., Principles of Color Reproduction, New York: John Wiley and Sons, 1967. Chapt. 13, pps. 328 - 345.