AN ABSTRACT OF THE THESIS OF


Abstract Approved:______________________________________________

Dianne K. Erickson

This investigation of secondary geometry teachers’ decision making in a mathematics curricular reform context examined the following questions: (a) What planning and interactive decisions were secondary geometry teachers making during this time of reform, and (b) what factors influenced the decisions that these teachers made? In addition, comparisons were generated between influential factors identified during a mathematics reform context and the stable context of previous decision making studies.

A multi-case study approach involving detailed examination of five geometry teachers’ decision making was used. The data collected and analyzed included a questionnaire, interviews, observational field notes, audiotapes and videotapes of classroom instruction, and written instructional documents. Teachers’ profiles were created describing geometry and teaching biographies, views toward curricular change, the classroom, planning decisions and influential factors, and interactive decisions and influential factors. Findings were developed by searching
for similarities and differences across the sample.

Teachers' decisions generated descriptions of their geometry courses. One teacher promoted geometry as a mathematical system using predominantly a lecture approach. The other four teachers advocated a multifaceted view of geometry recognizing geometry as a mathematical system and as a setting for developing communication and problem solving skills. In addition, the four teachers' courses included references to connections between geometry and the real world. These four teachers used a variety of instructional approaches that encouraged students' active involvement in their geometry learning with an emphasis on developing student understanding.

Factors influencing teachers' decisions included: (a) past geometry experiences, (b) professional development experiences, (c) articulated course goals, (d) advanced planning decisions, (e) teachers' beliefs, (f) the geometry textbook and other materials, (g) teachers' school settings, and (h) students' needs and actions. Some findings highlighted differences between this study and previous decision making studies. All teachers in this study appeared to be influenced by their beliefs about the nature of geometry as a discipline. Teachers were also influenced by whether they viewed the process of becoming an effective teacher as a life-long process. For four of the teachers, reform agendas were influential as another source of curriculum ideas.
Decision Making in a Mathematics Reform Context: Factors Influencing Geometry Teachers' Planning and Interactive Decisions

by

Kay A. Wohlhuter

A THESIS submitted to Oregon State University in partial fulfillment of the requirements for the degree of Doctor of Philosophy

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Kay A. Wohlhuter, Author
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CHAPTER I
THE PROBLEM

Introduction

Mathematics teachers have been challenged by state legislatures and by national organizations such as the National Council of Teachers of Mathematics (NCTM) and the National Research Council (NRC) to assist in the preparation of students who will live the majority of their lives in the twenty-first century. To be productive citizens in the next century, students must be able to live in a society that is changing from an industrial age to an information age. As society changes, so do the needs of students. Essentials for students in the information age include: (a) the ability to reason, to design models, and to solve problems since brain power is now the driving force of the economy; and (b) the acquisition of knowledge and skills which will enable high school graduates to pursue a series of careers (Sowder, 1989). As schools implement reforms to meet the changing needs of students, mathematics - when viewed as problem solving, communication, reasoning, and connections - is an important component of the reform.

The process of reform, by its very nature, is a process of change. As indicated by previous reform movements, teachers must be recognized as having an
important role in the process of change (Ball, 1992; Chambers, 1990; Cooney, 1988; NRC, 1989; Romberg, 1988).

For example, the new mathematics movement of the 1960s that was based on mandates from college subject matter specialists focused on the structure of mathematics rather than the issue of how teachers could best help students learn mathematics. Learning from the past, Cooney (1988) recommended viewing reform as an undertaking that "emphasizes the teacher as a decision maker who determines what mathematics students are capable of learning and what strategies are appropriate given the mathematical maturity of the students" (p. 355). This emphasis, along with reform history, identifies the need to examine and to understand teachers' decision making during the present reform movement.

As a context for the examination of teachers' decision making, reform in mathematics education presents itself as a strong candidate for influencing teachers' decisions about course goals, content, and instruction. However, due to the lack of observational studies during previous reform movements the exact nature of the reform that occurred in mathematics classrooms is not known (Good & Biddle, 1988). It is clear from previous reforms that calls for change alone were not enough to persuade teachers to make decisions to implement change (Lovitt, Stephens, Clarke, &
Romberg, 1990), but it is not clear how a time of reform affected teachers' decision making.

Support for the importance of examining teacher decision making can be found in the research that explored the role of the teacher in the teaching and learning process. Much of this research has addressed the following two domains: (a) teachers' actions and their observable effects, and (b) teachers' thought processes (Clark & Peterson, 1986). The process-product approach was used to investigate teachers' actions and their observable effects. The consensus of this vast body of research was that teachers' actions affected students' achievement (Shulman, 1986). The research approach to teaching, however, expanded to include the second research paradigm when teachers became regarded as professionals whose thinking guided their behavior. An important component of teacher thought processes is the decision making process. These decision making studies found that decisions teachers made before, during, and after instruction (preactively, interactively, and postactively) were a major influence on what students learned (Clark & Peterson, 1986; Fennema & Franke, 1992; Shulman, 1986).

In teachers' decision making literature, researchers have retained the distinction between preactive and interactive decisions and the distinction between interactive and postactive decisions. Collectively
preactive and postactive decisions have been labeled as planning decisions. Planning decisions include decisions such as what content to teach, what instructional approach to use, what questions to ask, and how much time to spend on an activity. Decision making research has indicated that various factors influenced teachers' planning decisions. For example, teachers' planning decisions were influenced by factors such as teachers' goals and objectives for students (Putnam, 1984; Ropo, 1987; Westerman, 1991; Zahorik, 1975), teachers' reflection on student behavior (Borko & Livingston, 1989; Clark & Elmore, 1981; Putnam, 1984), curriculum (Borko & Livingston, 1989; Clark & Elmore, 1981; Morine-Dershimer, 1978-1979; Nesselrodt, 1990; Peterson, Marx, & Clark, 1978; Putnam, 1984; Westerman, 1991; Yinger, 1980; Zahorik, 1975), and teachers' knowledge of content, students, and teaching (Borko & Livingston, 1989; Clark & Elmore, 1981; Putnam, 1984; Westerman, 1991; Yinger, 1980).

Interactive decisions include decisions made during instruction about such details as whether to implement the lesson as planned, how to respond to students' questions, whether to provide an alternative explanation, and when to pursue a student-generated discussion. Early research on interactive decision making revolved around the following: (a) the Peterson and Clark model, and (b) the Shavelson and Stern model (Clark & Peterson, 1986). In both models,
teachers' decision making was influenced by whether observed student behavior was within tolerance. Within tolerance was defined as the teacher's judgement of whether the students were understanding the lesson and were participating appropriately. In addition, decision making in the Shavelson and Stern model was described as implementing well-established routines.

Further research on interactive decision making indicated, however, that observed student behavior that is not within tolerance was not the only influence on teachers' decision making. For example, teachers' interactive decisions were also influenced by factors such as teachers' preactive goals (Borko & Livingston, 1989; Morine-Dershimer, 1978-1979; Nesselrodt, 1990; Westerman, 1991), teachers' perceptions of students' abilities (Borko & Livingston, 1989; Fogarty, Wang, & Creek, 1983; MacKay & Marland, 1978; McNair & Joyce, 1979; Nesselrodt, 1990; Putnam, 1984), teachers' ability to integrate knowledge (Westerman, 1991), and students' questions (MacKay & Marland, 1978).

The settings for previous decision making studies can be characterized as classrooms working within a stable curriculum (goals, content, instructional methods) environment. In contrast, this study begins to examine teachers' decision making in an environment of curriculum reform.
To focus the examination of teachers' decision making during the reform process, this study addresses mathematics reform in the context of geometry classrooms. Traditionally, secondary geometry has been approached from the viewpoint that geometry is an example of a mathematical system. Using deductive instructional strategies, teachers have focused on the development of students' proof skills. Students' success in a proof-oriented geometry course, however, has been described as poor (Clements & Battista, 1992; Usiskin, 1987).

The NCTM has outlined a reform agenda in two companion documents. The first document is titled *Curriculum and Evaluation Standards for Teaching Mathematics* (1989), hereafter referred to as the *Curriculum Standards*. The second document is titled the *Professional Teaching Standards for Mathematics* (1991), hereafter referred to as the *Professional Standards*. For the reform agenda advocated by NCTM, geometry teachers are asked to recognize that geometry is more than deductive reasoning and proof. "Equally important is the continued development of students' skills in visualization, pictorial representation, and the application of geometric ideas to describe and answer questions about natural, physical, and social phenomena" (NCTM, 1989, p. 160). With respect to instruction, teachers are encouraged to foster the interplay of deductive and inductive reasoning by using a
variety of instructional strategies. For example, students can make conjectures based on informal explorations and then verify their conjectures using deductive methods.

The reform agenda stated by NCTM promotes geometry's presence in the secondary curriculum as valuable. Geometry's role is due to its rich environment for student development of their abilities to explore, conjecture, reason logically, and solve non-routine problems. Furthermore, much of the content of geometry (e.g., transformations, spatial visualization, measurement) is not developed in other parts of the mathematics or general school curriculum.

Support for promoting a multifaceted geometry in the secondary curriculum has been provided by research focused on determining why students have difficulty learning deductive geometry and what materials and strategies are available to foster students' understanding of geometry. Studies involving the van Hiele levels of geometric thought (van Hiele, 1986; or see Appendix A for details) and studies involving computer instruction have addressed these two issues.

The traditional goal of writing proofs requires deductive (level 4) thinking. The difficulty occurs with the realization that many students entering geometry have only developed their visual (level 1) geometric thinking skills (van Hiele, 1986; Wirszup, 1976). Implications of a
mismatch between levels include the following: (a) students and teachers reasoning at different levels do not understand each other (Burger & Shaughnessy, 1986; Mayberry, 1983), (b) students learn geometry by memorization (Fuys, 1985; Lowry, 1986; Mayberry, 1983), and (c) students lack opportunities to develop visual, analytic, and abstract thinking skills (Fuys, 1985; Senk, 1989; Shaughnessy & Burger, 1985; van Hiele, 1986).

Research has also generated suggestions for facilitating students' understanding of geometry. The van Hiele research has indicated the use of phase-based instruction for developing students' geometric thinking (Bobango, 1987; Fuys, 1985; Pastor, 1993; Lowry, 1986; E. O. Thompson, 1992). Technology studies have suggested the use of the Geometric Supposer program (Schwartz & Yerushalmy, 1985) with a guided inquiry approach (Yerushalmy, Chazan, & Gordon, 1987; McCoy, 1991) and the use of the Logo programming language in a guided-discovery environment (Clements & Battista, 1989, 1990) for promoting students' problem solving and reasoning skills. Using Driscoll's (1982) words:

The strides made in delineating the several levels of geometric understanding, combined with the tremendous potential for using microcomputers to aid in geometry instruction, make it appear that geometry, somewhat changed in content and presentation, will gain new life in the high school curriculum. (p. 138)
Statement of the Problem

The mathematics education community has called for a reform that requires change in the expected outcomes for student learning of mathematics. In particular, students are expected to learn conceptual mathematics that is viewed as problem solving, communication, reasoning, and connections. This call necessitates a change in the mathematical curriculum as well as a change in the mathematical instruction. As demonstrated in the teachers' thought processes literature, classroom teachers are the primary agents of this change. Described by Clark and Peterson (1986), classroom teachers' thinking, planning, and decision making establish the psychological context of teaching:

The thinking, planning, and decision making of teachers constitute a large part of the psychological context of teaching. It is within this context that curriculum is interpreted and acted upon; where teachers teach and students learn. (p. 255)

Even though teachers and their decisions are key to this reform, the teaching literature has not specifically addressed the decision making process during a general school reform movement.

Due to the vast difference between the expected geometry curriculum involving goals, content, and instructional methods and the traditional geometry curriculum, secondary geometry classrooms are the setting for this study on teacher decision making. The goal of
this study is to investigate teachers' decision making in a mathematics reform context. A comprehensive look at geometry teachers' planning and interactive decisions as related to curriculum issues provide the basis for this investigation. Specifically, the study addresses the following research questions: (a) What planning and interactive decisions are secondary geometry teachers making during this time of reform, and (b) what factors influence the decisions that these teachers make?

Significance of Study

"The role of research in the reform movement is to provide reliable knowledge about important aspects of the reform" (Sowder, 1989, p. 10). The important aspects of the reform addressed by this study are geometry teachers' planning and interactive decisions and factors influencing these decisions.

The planning and interactive decisions made by teachers are major determinants of what goes on in the classroom. By generating descriptions of geometry goals, content, and instruction planned for and presented in the classroom, this study provides rich pictures of geometry classrooms. Descriptions of geometry classrooms based on teachers' decisions supply information about the geometry classroom from a perspective that was not utilized in previous geometry research.
Since teachers' decisions are great determinants of reform implementation, the geometry classroom descriptions provide an indication of how well actual classroom occurrences match with the vision of reform. Besides adding to the geometry literature base, these pictures of geometry classrooms provide a foundation for future research designed to address the curriculum suggestions presented in the geometry reform agenda.

The examination of teachers' decision making in a mathematics reform context considers whether a time of reform alters how teachers make decisions. Previous research has not addressed this issue. Furthermore, this study's examination of teachers' decision making in a mathematics reform context identifies factors that influence the decisions teachers make. The nature of these factors has implications for the reform process and for teacher education.

With respect to the reform process, these results provide valuable information for reform implementation. Mathematics teachers' planning and interactive decisions described in this study indicate whether teachers viewed reform agendas as top-down mandates, a resource for content and instructional ideas, or a source of curriculum ideas that are difficult to implement. Mathematics educators who are encouraging teachers to implement the reform need to be
aware of reform-related factors influencing teachers' decision making.

With respect to teacher education, the identification of factors that influence teachers' work in the classroom provides information for teacher educators facilitating their work with preservice and inservice teachers. As an example, the possibility exists that teachers' unfamiliarity with the proposed geometry content and instruction influences teachers' decision making. Research supports the idea that teachers teach what they know (A. G. Thompson, 1984) and that generally teachers teach how they are taught (Ball, 1990; A. G. Thompson, 1984). Thus, teacher educators or other staff developers must provide opportunities for teachers to learn the geometry content via the suggested instructional practices. Factors identified in this study provide similar insight for teacher educators.

Since decision making has been identified as an important component of teaching (Clark & Peterson, 1986; Shavelson, 1982), teacher educators are also interested in the general process of decision making. The view of the teacher as a decision maker emphasizes the role of the teacher in the complex process of teaching. The complex nature of teaching acknowledges the existence of more than one way to teach. Teacher educators' awareness of the factors influencing teachers' decision making provides
guidance for their work in helping both preservice and inservice teachers' develop their own teaching styles.

This study provides information on geometry teachers' decision making in a mathematics reform context. The determination of the planning and interactive decisions made by the teachers and identification of factors that influence these decisions provide a basis for future work regarding the geometry classroom, teacher decision making, the reform process, and teacher education.
CHAPTER II
REVIEW OF THE LITERATURE

Introduction

This study investigated geometry teachers' decision making in a mathematics reform context. For each teacher, a profile identifying their planning and interactive decisions as well as factors that influenced these decisions was generated.

Previous research has not addressed teachers' decision making in the context of a reform agenda in the geometry classroom. However, research in three main areas related to the investigation has been completed and are reviewed in this chapter. The first area to be considered involves teachers' decision making. Studies discussed in this section address teachers' decisions made before, during, and after classroom instruction and identify factors that influenced those decisions. A second area related to the investigation is teachers' actions during mathematical reforms. Literature reflecting on past reform movements provides a basis for examining teachers' actions during the present time of reform. Concluding the review of literature is an examination of studies involving the geometry classroom. Studies in this section support as well as encourage the continued exploration of geometry classrooms.
Teachers' Decision Making

Teachers' decisions made before, during, and after classroom instruction (also described as preactive, interactive, and postactive decisions) determine what happens in the classroom. Extensive research has been conducted in these three areas of teacher decision making. In this research, preactive and postactive decisions are collectively known as planning decisions. Thus, this literature review focused on studies that examined teachers' planning and interactive decisions.

Studies selected for this review included teacher decision making studies addressed in Clark and Peterson's (1986) literature review as well as decision making studies that have been completed since the review. The studies in Clark and Peterson's review, subsequently referred to as the early decision making studies, established a basis for understanding the factors influencing teachers' planning and interactive decisions. Studies completed since the review, subsequently referred to as recent decision making studies, used these early studies as a foundation. The results of the recent decision making studies provided additional information about factors influencing teachers' decision making by using methods that were more exploratory than methods used in early decision making studies. In addition, many of the researchers conducting the recent decision making studies recognized the importance of
examining the complete picture of decision making and thus focused on planning (preactive and postactive) and interactive decisions.

**Early Decision Making Studies**

The first three studies in this section, Zahorik (1975), Clark and Elmore (1981), and Yinger (1980), examined teachers' planning. Zahorik examined planning models of kindergarten through 12th-grade teachers. Clark and Elmore focused on the yearly planning of one elementary teacher. Yinger investigated the planning decisions of one elementary teacher over a five-month period. The fourth study described in this section was a laboratory research project involving 12 elementary teachers. As part of this project, Peterson et al. (1978) investigated teachers' planning and Peterson and Clark (1978) examined teachers' interactive decisions. Studies by MacKay and Marland (1978) and by Fogarty et al. (1983) examined elementary teachers' interactive decisions. Concluding this section is a discussion of the South Bay Study (McNair & Joyce, 1979; Morine-Dershimer, 1978-1979). As part of this project, Morine-Dershimer investigated the relationship between elementary teachers' plans and classroom reality, whereas McNair and Joyce examined the interactive decisions of these teachers.

The purpose of Zahorik's (1975) study was to investigate planning models used by teachers. One
component of the study was the determination of what planning decisions were made by teachers and in which order the teachers made their planning decisions. The analysis of the data provided information about factors affecting teachers' planning decisions.

The sample for the study consisted of 194 teachers who volunteered to participate. The teachers taught in schools located in a large, metropolitan city and in many of its suburbs. All grade levels were represented in the sample. Specifically, 63 teachers taught in grades kindergarten through six, 63 teachers in grades seven through nine, 42 teachers in grades 10 through 12, 17 teachers in adult education, and nine teachers in schools that span grades kindergarten through 12. The teaching experience of the teachers ranged from 1 to 20 years with 122 of the 194 teachers having five years or less experience.

The planning decision data for the study were collected using a questionnaire. Teachers were asked to list in order the decisions they made before teaching a class. The researcher used the following eight categories to analyze the lists: objectives, content, activities, materials, diagnosis, evaluation, instruction, and organization. The researcher also determined the frequencies and percentages of decisions made and of decisions made first.
The results of the researcher's analyses showed that activities (81%) and content (70%) were the two most frequently reported types of decisions made by the teachers. Other types of decisions and their percentages included objectives (56%), materials (56%), evaluation (35%), diagnosis (25%), organization (21%), and instruction (16%). The results also indicated that content (51%) and objectives (28%) were the two most frequently reported initial decisions. Based on frequency, the teachers' planning decisions were mainly influenced by activities, content, objectives, and materials.

Differing from Zahorik's (1975) study in sample size and method, the study by Clark and Elmore (1981) examined the yearly planning completed by one elementary teacher in think aloud sessions. One component of Clark and Elmore's data analysis identified factors influencing the teacher's planning decisions.

The subject for the study was a second-grade teacher with eight years of teaching experience. The teacher had taught in a self-contained classroom during her entire teaching career. The teacher volunteered to participate in the study, but also was paid an hourly wage for the time spent in planning sessions.

Prior to the beginning of the school year, data were collected during three two-hour planning sessions. Once a week for three weeks, the teacher met with the two
researchers to discuss her yearly planning regarding mathematics, science, and writing. Each session focused on one curriculum area. During these audiotaped sessions, the teacher planned aloud as the researchers took notes. Occasionally, the researchers asked clarifying questions.

The analysis of the data showed that during the yearly planning sessions the teacher focused on setting a projected schedule of instruction for each subject, analyzing and rearranging curriculum materials, and establishing an overall framework within which other levels of planning and action occurred. Factors influencing her yearly planning included curriculum materials (especially the teacher’s guide), her memory of classroom interaction during the previous year, and a calendar for the upcoming school year. In addition, the teacher made decisions based on the typical abilities of second graders, her knowledge of incoming students’ prior experiences, and her beliefs and values about the relative importance of subject matter.

In contrast to the previous two studies, Yinger (1980) collected planning as well as interactive data while examining one elementary teacher’s planning decisions over a five-month period. The focus of the detailed study was the description of the mental processes in which the teacher engaged while making planning decisions. As part of the description, factors affecting the teacher’s planning decisions were identified. The subject for the
study taught a combined first- and second-grade class in a Michigan school district. The year of the study marked the beginning of her sixth year of teaching, three years of which had been in a special education classroom and three in a combination first- and second-grade classroom.

The data for the study were collected in two phases. During the first 12 weeks, the researcher spent approximately 40 full school days observing and recording the teacher’s activities in the preactive and interactive phases of teaching. When students were not in the classroom, preactive information was gathered informally as the researcher shadowed the teacher, recording her behavior and her statements as she "thought aloud." Preactive information was also collected during deliberate instructional planning sessions that were audiotaped. When students were in the classroom, interactive information was gathered as the researcher recorded classroom activities involving the teacher.

Based on the information collected during the first 12 weeks of the study, the researcher wrote a detailed written description of the teacher’s behavior as she made planning decisions. During the remainder of the study (approximately eight weeks), the researcher further examined the teacher’s planning by conducting additional observations and interviews and by having the teacher participate in three judgement tasks designed to show the
teacher's perceptions of her students and her instructional activities.

The data analyses identified factors influencing the teacher's planning decisions. The identification of two factors affecting planning decisions was based on the central role they played in the teacher's planning. The first factor, instructional activities, was the teacher's most important and most frequent planning concern. As she made decisions about her instructional activities, she made decisions regarding seven features of the activity: location, structure and sequence, duration, participants, acceptable student behavior, teacher's instructional moves, and content and materials.

The second factor identified by its role in teacher's planning was the use of routines. The teacher's planning was described as decision-making about the selection, organization, and sequencing of routines. The four categories of routines used by the teacher were activity, instructional, management, and executive planning.

The results of the study also showed that the teacher's planning was influenced by her repertoire of knowledge. Most notable components of her repertoire of knowledge was her experience with routines and her awareness of pupils' backgrounds.

Like Yinger's study (1980), planning and interactive data were also collected in studies by Peterson et al.
(1978) and Peterson and Clark (1978). The two types of data, however, were used differently than Yinger used his data. As part of the same laboratory research study, Peterson et al. addressed teachers' planning decisions and Peterson and Clark addressed teachers' interactive decisions. The planning component of the research examined teachers' planning decisions whereas the interactive part of the research investigated teachers' cognitive processes during teaching.

The sample for this study consisted of 12 experienced elementary teachers who were recruited and paid to take part in the study. The teachers' average number of years of teaching experience was 8.3. For this laboratory study 288 junior high students were recruited and paid to participate in the study. Thirty-six groups of eight students were randomly formed. Each group was randomly assigned to participate in one teacher's class on one of the three days of the study.

Three days before data collection began, teachers read the social studies content materials and examined their assigned classrooms. On each day of the study, every teacher taught a three-hour lesson to a different group of students. Before each day's instruction, teachers were provided with content materials and objectives and were given 90 minutes to plan the day's lesson. During the
planning time, teachers were asked to "think aloud." The planning sessions were audiotaped.

All lessons of each teacher were videotaped. At the end of the school day, teachers were shown segments of their videotapes. The segments consisted of the first two to three minutes of the teaching, the last two to three minutes of the teaching, and two two-to-three minute segments randomly selected from the middle portion of the presentations. After teachers watched each selected segment of the videotape, they participated in an audiotaped interview with one of the researchers. Sample questions included: (a) What were you doing in this segment and why, (b) what were you noticing about the students, (c) were you thinking of any alternative actions or strategies at that time, and (d) did any students' reactions cause you to act differently than you had planned?

Factors that influenced the teachers' planning decisions were implied by the lesson components on which teachers spent the most time. The analysis of the planning data showed that the teachers spent the largest part of their planning time on content to be taught. After content to be taught, teachers' planning statements were most concerned with instructional process, defined to include intended student activities and planned teacher strategies.

The audiotaped interviews of teachers discussing their interactive decisions were coded as belonging to one of the
four paths of a decision making model patterned after Snow's (1972) model. Path one represented the situation in which the teacher judged students' behavior to be within tolerance and instruction continued as usual. Within tolerance was defined as the teacher's judgement of whether the students were understanding the lesson and were participating appropriately. In Path two, the teacher determined students' behavior was not within tolerance, but the teacher had no alternative strategies to implement. Path three depicted the situation in which the teacher decided students' behavior was not within tolerance, and the teacher had alternative strategies, but chose not to implement them. In Path four, the teacher judged students' behavior not to be within tolerance, had alternative strategies, and chose to implement one of them in order to change students' demeanor.

The analysis of the interactive data showed that the greatest majority of teachers' cognitive processes were categorized as Path one. Across the three days of teaching, the average percent of Path one ranged from 71% to 61%. The researchers described the choice of Path one as teachers conducting "business as usual." In the remaining cases where students' behavior was not within tolerance, Path four was selected the most often. The selection of Path four indicated that teachers had alternative strategies and chose to implement one of them.
The model for interactive decision making described by Peterson and Clark (1978) was based on whether students' behavior was within tolerance. One of the first studies to show that interactive decisions were based on more than students' behavior was a study conducted by MacKay and Marland (1978). Their study examined teachers' interactive thoughts and cognitive processes. Six elementary teachers representing two schools in a Canadian urban school district volunteered to participate in this study. One teacher from each of the first-, third-, and sixth-grade levels in both schools was selected.

Data collection revolved around each subject teaching two one-hour lessons, one in language arts and one in mathematics. A pre-lesson audiotaped interview was done with each teacher. The purpose of the interview was the determination of teachers' goals for the lesson as well as their intended procedures for helping students achieve the goals.

The teachers' lessons were videotaped. At the end of the school day, the interviewer and the observed teacher participated in an audiotaped stimulated recall interview. Before the interview, the interviewer selected 20 to 30 minutes of lesson segments to use for the stimulated recall session. The teacher was given some time to review the tape before the interviewer began the stimulated recall session.
During the stimulated recall interview, teachers usually made the decision regarding when to stop the tape. The interviewer made the decision when teachers did not stop the tape at interviewer-identified spots rich with interactive data. When the tape was stopped, teachers reflected on their thought processes and were encouraged to provide a detailed account of the following ideas: (a) thoughts, feelings, and moment-to-moment reactions, and (b) conscious choices, alternatives considered before making a choice, and the reasons for making a choice.

Results reported by MacKay and Marland (1978) addressed the content of the interactive thoughts. The analysis of the stimulated recall data found that teachers' thoughts were primarily concerned with lesson strategies to be used next, events from past lessons, teachers' expectations for students, and predictions about future lesson developments. The researchers also found that teachers had few thoughts about their previous behavior in the lesson, their behavior's impact on the lesson, and lesson objectives.

Additional analysis of the stimulated recall data indicated there were usually at least 30 or more potential decision points in a lesson. A decision point occurred when teachers made a decision (a conscious choice between alternatives) or a deliberate act (planning the next strategy without considering an alternative). Factors
influencing teachers' actions at a decision point included student inattentiveness or disruption, incorrect or incomplete student response, students' apparent lack of understanding, student questions, and the need to select a student respondent, a specific teaching technique, or appropriate examples.

Like the study conducted by MacKay and Marland (1978), Fogarty et al.'s (1983) study examined teachers' interactive decisions. One part of the study conducted by Fogarty et al. was the examination of experienced and novice teachers' performance and cognition during interactive instruction. The study focused on student performance cues that led to teachers' implementation of instructional actions, instructional actions used, and information teachers considered during the instructional process.

The setting for this study was a university laboratory school that consisted of three multi-age groupings. This school used two main instructional approaches: (a) individualized instruction with independent learning, and (b) small-group (five to eight students) instruction. The subjects of this study consisted of three experienced teachers asked to participate in the study based on administration recommendations. The average years of teaching experience for these three teachers was 10.1. The five novice teachers were described as student teachers at
the school. Two of the experienced teachers and one of the
novice teachers taught in classrooms with first- and
second- grade students. The rest of the teachers taught in
a classroom with integrated third-, fourth-, and fifth-
grade students.

Two sources of data were used to collect information
on teachers' interactive decisions as reflected in their
instructional thoughts and actions. One source of data was
videotapes of regularly scheduled small group lessons.
Teachers selected the lessons to be videotaped. The
videotaping occurred a total of three times during a 1.5
month period. For each lesson, a 15-minute segment was
videotaped after the first five minutes of the lesson had
passed. In order to acclimatize students and teachers to
the equipment and the experimenter pre-study taping was
done.

A second source of data was the stimulated recall
interview conducted with each teacher for each of their
videotaped lesson segments. Each interview occurred soon
after the lesson segment had been taped. Before the
researcher met with the teacher, the researcher viewed the
tape to identify situations on the tape where it appeared
the teacher made an interactive decision. During the
stimulated recall interview teachers were instructed to
stop the tape at the place where the teacher recalled any
thoughts or decisions. If the teacher did not stop at the
researcher's identified decision spots, the researcher stopped the tape and asked the teacher if a decision had been made at this point. If teachers made a negative response, the reviewing of the tape was continued. If teachers made a positive response, the researcher asked probing questions to encourage the teacher to talk about the instructional goals and knowledge associated with the decision point. Sample questions used by the researcher were: (a) What were you aiming at there, (b) at that point, what were your thoughts, and (c) what was the reason for that decision? The same probing questions were asked of the teacher for the teacher-identified decision spots.

The analysis of the stimulated recall data provided the results that pertained to factors influencing teachers' interactive decisions. The stimulated recall information described the instructional goals pursued and prior knowledge utilized by the teachers during the instructional process. The instructional goals identified by the teachers as reasons for their decisions included student understanding, student motivation and involvement, group management, curriculum integration, social development, and subject matter content. When the teachers described knowledge used in making interactive decisions, they referred to pedagogical principles, student preferences, students' academic skills and abilities, important content, students' social behavior, and students' knowledge. Even
though the study focused on student performance cues, the results indicated that non-student factors (e.g., curriculum integration, pedagogical principles) also influenced teachers' interactive decision making.

Comparison of the performance and thinking processes of the experienced and novice teachers when making interactive decisions indicated that novice teachers did not consider as large a variety of instructional goals as did the experienced teachers. Influences on interactive decisions made by the novice teachers were less likely to include their prior knowledge about subject matter content, student history, and pedagogical principles.

The South Bay Study (McNair & Joyce, 1979; Morine-Dershimer, 1978-1979) investigated teachers' teaching styles, decision-making behavior, and concepts of students. As one part of the study, Morine-Dershimer investigated the relationship between teachers' plans and classroom reality. McNair and Joyce's component of the South Bay Study examined interactive teacher decision making.

The subjects for the South Bay Study consisted of 10 teachers who taught at a single elementary school in a large metropolitan area. Included in the sample were three first-grade teachers, two third-grade teachers, one fourth-grade teacher, two fifth-grade teachers, and two special education teachers. All teachers had at least three years of teaching experience.
Each teacher was observed and videotaped during six reading lessons. For each teacher, a data collection day occurred in fall, winter, and spring. On a data collection day, teachers were observed for two reading lessons, one for more advanced students and one for less advanced students. On observation days, teachers participated in pre-lesson interviews before the school day began. The interview began with teachers explaining their plans for the reading lessons that were to be observed later in the day. The interview continued with teachers responding to a series of questions designed to generate more information about their planning. Examples of questions included: (a) Is there anything about the pupils that you want to comment on in relation to your planning, (b) is there anything about the materials that you have selected that you want to comment on in relation to your planning, (c) did your planning result in a specific dominant objective for this particular lesson, and (d) could you comment on your teaching strategy or the instructional process you are planning to use?

At the end of the observation day, a stimulated recall interview was conducted. During the audiotaped interview, teachers were instructed to stop the tape whenever they remembered hesitating, assessing the situation, or making a decision. The interviewer stopped the tape at two predetermined places: (a) the first time a pupil gave an
incorrect answer to a teacher question, and (b) the second or third time the teacher made a transition from one activity to another. In addition, the researcher stopped the tape at two randomly selected places. When the tape was stopped, the interviewer asked questions such as the following: (a) What were you thinking at that point, (b) what did you notice that made you sort of stop and think, (c) what did you decide to do, and (d) was there anything else you thought of doing at that point, but decided against?

Factors influencing planning decisions were implied by the substance of the teachers' stated plans. In response to a general request to state their plans for the day, the teachers consistently mentioned content to be addressed and activities in which students were to be engaged. In addition, materials to be used were frequently mentioned. The teachers rarely mentioned pupil ability, objectives, or teaching strategies when responding to the general planning question. However, when the teachers were asked probing questions concerning these ideas, they had ready responses. The results suggested the teachers' mental plans were more detailed and spanned more aspects of the lesson than did the stated plans.

The analysis of the stimulated recall data generated categories describing teachers' concerns during instruction. As teachers made decisions, 40% of their
reported concerns were with pupil learning, attitudes, and behavior and 30% were with learning tasks, facts, and ideas. The remaining 30% of the concerns were spread over categories involving procedures, materials, and time. The analysis also showed that teachers made "fine tuning" adjustments during the lessons, but they did not make any major alterations.

Additional analyses were performed with the planning and stimulated recall data. The results showed that the amount of discrepancy between teacher plan and classroom reality influenced teachers' decision making. When little or no discrepancy occurred, teachers proceeded with established routines. In the situation with a minor discrepancy, teachers made "inflight" decisions. With a major discrepancy between plan and reality, decisions were postponed.

Recent Decision Making Studies

The first four studies (Borko & Livingston, 1989; Nesselrodt, 1990; Putnam, 1984; Westerman, 1991) in this section examined the complete process of teacher decision making. Studies in the complete process of decision category were studies that addressed preactive, interactive, and postactive decisions through the use of observations and interviews where the interviews occurred pre- and post-observation. Borko and Livingston's study and Westerman's study both examined expert and novice
complete process of decision making. The Borko and Livingston sample consisted of elementary and secondary mathematics teachers, whereas Westerman's sample consisted of elementary teachers teaching a variety of subjects. The complete decision making process of one teacher was investigated in Putnam's study and in Nesselrodt's study. Putnam focused on the decision making of an elementary Title I teacher, and Nesselrodt examined the decision making of a secondary English teacher. Like the first four studies, Ropo's study (1987) examined planning and interactive decisions using interviews and observations. In contrast to these studies, Ropo did not interview the teachers after the observations. Additional information about interactive decisions was provided by the Parker and Gehrke (1986) study.

Planning decisions made preactively, interactive teaching, and postactive reflection on interactive decisions were three components of a study conducted by Borko and Livingston (1989) that examined the thoughts and actions of expert as well as novice teachers with respect to teachers' knowledge structures. The analysis of the planning decisions, interactive teaching, and postactive reflection included the determination of factors that influenced teachers' planning and interactive decisions.

The sample of interest consisted of one elementary teacher and two secondary mathematics teachers who were
cooperating teachers for student teachers enrolled in a large eastern university's preservice teacher preparation program. The teachers were selected to participate in the study based on their willingness to take part in the study and based on recommendations by their principal and by the county teacher center coordinator.

The organization of the data collection process revolved around observations of each teacher teaching mathematics in their own classes on successive days for one week of instruction. For each lesson, the entire mathematics instructional period of the elementary teacher, approximately one hour in length, was observed. The secondary teachers were observed teaching two sections of the same course. Before each lesson teachers participated in an audiotaped semi-structured interview. During this interview the teachers were asked to discuss the nature of the upcoming lesson, explain how they planned for the lesson, describe what they thought about as they planned, and identify factors that influenced their plans. Photocopies of texts, written plans, and content notes supplemented the planning information collected during the interviews.

Information about the nature of instructional activities, classroom routines, and teachers' instructional and management strategies guided the researcher's observations of the teachers. The researcher collected
field notes during the observation, a sample of which was audiotaped. Immediately after each lesson, field notes were expanded to generate as complete and accurate a picture as possible of the teacher's actions in the classroom. When available, audiotapes helped with this process.

After each observation, teachers participated in an audiotaped semistructured interview designed to examine teachers' reflections about their teaching. During the interview, teachers were asked to discuss the prominent features of the lesson, talk about unexpected occurrences, explain changes from plans, and give reasons for those changes.

Analysis of the pre-observation and post-observation transcripts identified factors that influenced teachers' planning decision making. The yearly planning of all three teachers was influenced by the need to establish the general content and curriculum sequence for the class and to determine time allocation for coverage of the content. The secondary teachers' chapter planning and the elementary teacher's unit planning were influenced by the portion of the text to be covered and previous year's plans. Teacher-developed course notebooks also played a role in the secondary teachers' planning. The complexity of the topics, the teaching styles of the teacher, the ability of the teacher to identify potential content difficulties, and
student performance in class as related to instructional goals affected the detailed planning needed for the teaching of the content.

For all three teachers, the planning details materialized as mental plans for their lessons. These mental plans guided their interactive teaching. In addition, teachers' interactive decisions were influenced by their desire to keep the lesson on track, meet their objectives for the class, and respond to students' questions and comments.

The decision making results of the novice teachers in this study provided additional information for understanding the decision making process of the expert teachers. The novices were student teachers of the expert teachers. Factors influencing novice teachers' planning decisions revolved around their inexperience as a teacher. In contrast to the expert teachers, the novice teachers' decisions were influenced by their difficulty making priority decisions about content coverage, their lack of professional knowledge, and the context of the student teaching experience. Like the expert teachers, the novice teachers had developed mental plans for their lessons. However, the implementation of these plans was influenced by the novice teachers' difficulty addressing students' questions and their inability to provide explanations that connected topics.
Whereas Borko and Livingston (1989) observed each teacher for one continuous week of instruction, Westerman (1991) observed each teacher for two lessons. The focus of Westerman’s study was the examination of expert and novice teachers’ thinking during three stages of decision making defined as preactive (planning), interactive (teaching), and postactive (evaluating and reflecting). This study was part of a larger project on expert and novice teachers’ reflective thinking.

The sample consisted of five teachers and their five student teachers from a public elementary school in a middle-class suburb of Washington, D.C. The teachers had volunteered to be a part of the reflective teaching project. Teachers were selected to participate in this study based on their ability to implement an integrated curriculum, promote reflection in student teachers, and willingness to incorporate a problem solving orientation toward teaching. Subjects of lessons observed included language arts, mathematics, social studies, and spelling. Each of the teachers had been teaching at the elementary level for at least five years.

The teachers each taught two lessons during the study. Data for each lesson were collected in four phases. During the first phase each teacher participated in a structured interview before the lesson was taught. Interview questions developed by a university team and based on
teacher thinking and decision making research were designed to collect information on teachers’ planning decision making. Sample questions included: (a) Is this lesson related to anything else you are doing, and (b) where do you start when you plan a lesson?

The second phase consisted of the interactive stage of teaching. The lesson was videotaped as the teacher taught. Soon after the lesson was completed the researcher conducted a stimulated recall interview. As the teacher and researcher watched the entire videotape the teacher was instructed to stop the tape each time the teacher remembered making a decision. The teacher was then asked to explain the thinking that went into the decision.

The third phase described as postactive evaluation and reflection occurred immediately after the stimulated recall interview. During this phase, the researcher interviewed the teacher. Questions such as the following guided the interview: (a) Would you rate this lesson as successful why, and (b) did you gain information during the teaching of this lesson that will be useful in planning future lessons?

Several months later, the fourth phase involved the teacher watching the videotape of the lesson without the sound. Instead of stopping the tape, the teacher was asked to talk continuously while the tape was played. The goal
of this phase was to capture any decision making that had not been identified during the stimulated recall interview.

The interviews and videotapes were transcribed. School system curriculum guidelines, classroom handouts, students' work, and field notes taken during the classroom observations completed the set of data. The data were analyzed inductively using a constant comparative method developed by Glaser and Strauss (1967).

The examination of expert teachers' planning decision making found that the teachers began the planning process by viewing the curriculum guidelines. Adaptations of the suggested curriculum were done based on teachers' knowledge of the following: (a) overall curriculum, (b) subject matter, (c) students' abilities, (d) students' learning styles, and (e) students' interests. Postactive reflection indicated that the planning process was also influenced by the teachers' perception of how well the classroom instructional process facilitated students' achievement of lesson goals.

The goals formulated during the preactive decision making process was one of the driving forces for teachers' interactive actions. When students' behaviors indicated a need for a change in approach, teachers recognized the need to modify their preactive decisions during classroom instruction. Teachers' interactive decision making was also influenced by the teachers' desire to repeat an
important idea, to relate new information to previous knowledge, and to assess students’ understanding.

Additional information for understanding the decision making process of the expert teachers is found in comparisons with novice teachers’ decision making results. The novice teachers were student teachers of the expert teachers in this study. The novice teachers also began their planning process by viewing the curriculum guidelines. In contrast to the expert teachers, the novice teachers did not have the overall curriculum knowledge or student characteristic knowledge needed to adjust the curriculum. Instead, the novice teachers’ planning focused on the curriculum objectives. Interactive decisions were influenced by the novice teachers’ need to follow their lesson plans, their inability to incorporate ideas generated by students, and their classroom management style.

In contrast to the studies by Borko and Livingston (1989) and Westerman (1991), Putnam’s (1984) sample consisted of one teacher. The goal of Putnam’s study was to investigate one teacher’s decisions in an attempt to determine whether this teacher used a systematic decision-making model. The determination of what governed this teacher’s decisions was one focus of the investigation.

The subject for this study was a Title I teacher for a rural, midwestern consolidated school district. The
teacher met with the students (25 first- and second-graders) four half days a week, beginning on September 24. These meetings continued until March when the teacher took a parenting leave of absence.

Methods used to collect data included participant observation, debriefing and verification sessions, interviews, and document collection. The first observation began on the first day of school and was four hours in length. The observation process was quite intensive at the beginning of the school year with observations occurring on 12 of the first 17 school days and on three of the next 10 school days. Between November and March the researcher conducted nine more observations for a total of 24 four-hour observations. Field notes, photographs, and audiotapes were used to record the observations.

For each classroom observation, the researcher held either a debriefing discussion or a verification discussion with the teacher. During the debriefing discussion the teacher made comments about the forthcoming lesson and responded to questions the researcher had generated from the field notes. The verification discussion involved the teacher commenting on the completed lesson and answering the researcher’s questions.

The interview portion of the data collection consisted of three types of interviews. The first type of interview had two purposes: (a) to collect demographic data, and (b)
to clarify and explain observations and preliminary data analyses. These interviews were scheduled for days when no observations were to be done and on the average were two hours in length. It is not clear how many of these interviews occurred.

The second type of interview was a simulation. The simulation involved the sample teacher responding to a colleague's spoken ambition to learn how to teach like the sample teacher. The sample teacher explained her process of teaching by responding to questions like the following: (a) What guidelines would you give your colleague to help her/him select a topic of instruction, (b) what questions would you ask your colleague, (c) what information do you need from your colleague, and (d) what do you tell your colleague?

The final type of interview occurred as the researcher and the sample teacher collaborated on an article for publication. During this process discussions between the researcher and the teacher revealed the teacher's perception of the important elements of the decision making model and the accuracy of the decision making model generated by the researcher's analysis of the data.

The collection of documents such as students' work, maps of spatial and temporary relationships in the classroom, and teacher's planning products completed the set of data.
With the intention of creating a decision making model, the researcher analyzed, categorized, ordered, and verified the collected data. In its simplest form, the teacher's decision making model was described as a process of data collection, data synthesis, and data-based decision making. Guiding forces in the teacher's decision making process were her long-term outcomes (e.g., goals and objectives for pupils to master by the end of the year, how pupils were to behave in the classroom) and her belief that the main purpose of schooling was teaching personal and social responsibility.

The teacher's preactive decisions made at the start of the school year were influenced by the teacher's synthesis of the following information: (a) teacher's knowledge about her interests, strengths, philosophy, and weaknesses, (b) teacher's assumptions about teaching and learning based on knowledge and experience, (c) students' previous experience and records, (d) classroom environment, (e) curriculum, and (f) community. Factors influencing the teacher's preactive decisions during the school year included teacher's content knowledge, students' content knowledge, students' interests, reflection on students' performances in the classroom, and the goal of using an interactive instructional style.

Interactive decisions were influenced by students' knowledge, skills, and interests, the teacher's need to
assess students during instruction, the teacher’s goal of maintaining activity flow, and the teacher’s desire to bring organized closure to a study unit.

In contrast to Putnam’s (1984) sample of one elementary teacher, Nesselrodt’s (1990) sample consisted of one secondary teacher. The purpose of Nesselrodt’s study was to investigate whether a secondary English teacher’s planning and interactive decisions for a class with perceived low-ability students were different from those planning and interactive decisions made for a class with perceived high-ability students.

The subject for this study was a secondary female English teacher with 10 years of teaching experience. Two ninth-grade English classes from a school with approximately 1,750 students in grades 9 through 12 were used to study the teacher’s planning and interactive decisions. The class perceived by the teacher to be high-ability contained 21 female and four male students; the class perceived to be low-ability contained 12 male and 12 female students.

During approximately a six-week period in the spring of the school year the researcher collected data using multiple sources. In the first phase of data collection, the researcher interviewed the teacher using a structured interview designed to establish the context of teacher’s decision making. Sample questions included: (a) Upon what
kinds of information do you base your perception of the ability level of these classes, (b) what are your sources of information, and (c) do you use this information related to each class in making instructional decisions and if so, how?

The second phase of the data collection focused on the teacher’s unit, weekly, and daily planning. The teacher audio-recorded her plans. As part of the think aloud process, the teacher was encouraged to talk about the planning elements of objectives, activities, content, materials, and evaluation. For the first unit, the teacher audio-recorded her unit plans, plans for week one and week three, and plans for one lesson from each of these weeks. For the second unit, the teacher audio-recorded her unit plans, plans for the first week, and one lesson for this week.

For the next phase of data collection, the researcher observed the teacher teaching the lesson plans that had been audio-recorded during the previous phases. During the videotaped observations the researcher collected field notes. As soon as possible after each lesson, the teacher participated in an audiotaped stimulated recall interview. As they watched the tape, the researcher stopped the tape when the teacher identified a decision point and at two researcher predetermined points (the first change in activity and the second incorrect student response).
Whenever the tape was stopped, the researcher asked probing questions such as the following: (a) What were you thinking at that point, (b) what did you notice that made you stop and think, and (c) what did you decide to do? After the complete lesson had been viewed, the teacher was asked to identify two or three decisions key to the success of the lesson. To complete the data collection process, the researcher asked the teacher to explain how her postactive reflections on the lesson influenced the next lesson.

Inductive analyses of the transcribed audiotapes occurred during each phase of the data collection process and generated data-based categories related to the decision making process. The results indicated that the teacher’s planning was influenced by the content of the ninth grade English class which in turn was driven by the school’s English curriculum. Permeating her planning process was the teacher’s perceptions of her students’ abilities. The teacher made content adjustments based on these perceptions, when necessary. For example, when the two classes were studying *Romeo and Juliet*, the perceived high-ability class read the whole play while the perceived low-ability class read selected portions of the play.

The analysis of the interactive decision making data indicated that the teacher’s decisions were influenced by her teaching style and by the students. The interactive decisions were a fine tuning of her preactive decisions.
During instruction the decisions appeared to be influenced by her need to complete planned tasks. Student factors that influenced the teacher's interactive decisions included the teacher's perception of students' abilities, her academic expectations of the students, and her behavioral expectations of the students.

Like the previous studies, Ropo (1987) examined teachers' planning and interactive decisions. The purpose of Ropo's study was to learn more about the development of expertise in teaching while comparing expert and novice teachers' thought processes, knowledge, and teaching behavior in a pilot study. Eight mathematics teachers volunteered to participate in the study. The students of these teachers ranged from seventh to twelfth graders. Grade level details for individual teachers were not given. The teachers with at least five years of teaching experience were classified as experts. The novice teachers for this study included four individuals completing their teacher training and one individual teacher completing a second year of teaching.

Clinical interviews and classroom observations were the two sources of data for this study. Each teacher participated in an audiotaped clinical interview that was approximately one hour in length. During the interview the teachers were asked to talk freely about the following: (a) their goals and objectives for education in general and in
their own subject matter, (b) their conceptions about interaction in the class, (c) their conceptions about their own behavior in interactional situations, and (d) their planning processes before and during the teaching. Talking freely was defined as not being told specifically how to talk about each topic. When necessary, the researcher asked the teachers to clarify their responses. A research assistant observed the teachers during the instruction of three or four lessons. The audio-recorded observations occurred during the last two months of the school year.

Analysis of the interview data showed that teachers’ planning decisions were influenced by their need to write plans that were flexible and for a specific class. Teachers’ plans were also influenced by their need to emphasize the process of learning rather than the product of learning and by their desire to take students into account when planning the lesson.

The observation data were analyzed with respect to how the teachers reacted to a student’s incorrect answer or to the failure to get any answer. Analysis of these data confirmed teachers’ interview findings that the teachers analyzed students’ answers and ways of thinking when determining how to proceed with instruction.

The decision making results of the novice teachers in this study provide additional information for understanding the decision making process of the expert teachers. The
focus of the novice teachers' planning was on the outcomes of learning rather than the process of learning. Interactive decisions were influenced by the novice teachers' perception of lack of time for interaction during lessons, their focus on their own teaching behavior, and their focus on classroom management.

In contrast to the previous studies examining planning and interactive decisions, the research by Parker and Gehrke (1986) addressed only teachers' interactive decisions. The goal of Parker and Gehrke's study was to use an exploratory approach for examining elementary teachers' interactive decision making. The intention of this exploration was to generate categories and to produce hypotheses grounded in the interactive decision making data.

The sample for this study consisted of 11 female elementary teachers and one male elementary teacher who were randomly selected from a predominantly white, middle class, suburban western school district. Random selection of the teachers occurred because the researchers had access to this sample that was connected with another project. The teachers taught in regular, self-contained classrooms; teachers ranged in experience from one year to retirement year.

The data collection process began with teachers teaching a lesson of choice. An audio recording of the
chosen lesson was made by each teacher. Within 48 hours of
the lesson, each teacher participated in an audiotaped
stimulated recall interview. The teachers were in charge
of controlling the playing of the audiotape and were
directed to stop the tape each time they remembered a
decision they had made during the lesson. When teachers
stopped the tape identifying a decision spot, the
researcher asked them to explain the decision as completely
and accurately as possible. To promote clarity and
descriptive detail in the teachers’ verbal report the
researcher would ask questions such as "What do you mean by
... " and "Do you remember anything else about that
decision?"

The researchers used an inductive process on the data
from the transcribed interviews to generate hypotheses
based in the data on teachers’ interactive decision making.
Three hypotheses were presented as the results for this
study: (a) teachers’ interactive decision making is
embedded in classroom learning activities, (b) a primary
teacher goal during interactive teaching is moving learning
activities toward completion, and (c) teachers’ intention
of moving learning activities forward to completion is
supported by their decision rules and routines.

Discussion of Teachers’ Decision Making

Results of studies that addressed teachers’ planning
decisions revealed factors related to the course taught,

Results of the studies addressing teachers' interactive decision making identified a variety of
influential factors that revolved around the students and the teachers. Specifically, these studies found that interactive decisions were driven by preactive goals (Borko & Livingston, 1989; Morine-Dershimer, 1978-1979; Nesselrodt, 1990; Westerman, 1991), elaborate mental plans (Borko & Livingston, 1989; Morine-Dershimer, 1978-1979; Westerman, 1991), students' behavior which indicated a need to use another instructional approach (Fogarty et al., 1983; Peterson & Clark, 1978; Westerman, 1991), teachers' perceptions of students' abilities (Borko & Livingston, 1989; Fogarty et al., 1983; MacKay & Marland, 1978; McNair & Joyce, 1979; Nesselrodt, 1990; Putnam, 1984), and students' questions (MacKay & Marland, 1978). Interactive decisions were also found to be influenced by teachers' need to complete the lesson (Borko & Livingston, 1989; MacKay & Marland, 1978; Nesselrodt, 1990; Parker & Gehrke, 1986; Putnam, 1984), teachers' desire to assess students (Putnam, 1984; Westerman, 1991), teachers' ability to integrate knowledge (Borko & Livingston, 1989; Westerman, 1991), teachers' goal of students' social development (Fogarty et al., 1983; Putnam, 1984), and the discrepancy between teachers' plans and classroom reality (Morine-Dershimer, 1978-1979).

The results of the early teacher decision making studies (Clark & Elmore, 1981; Fogarty et al., 1983; MacKay & Marland, 1978; McNair & Joyce, 1979; Morine-Dershimer,
1978-1979; Peterson & Clark, 1978; Peterson et al., 1978; Yinger, 1980; Zahorik, 1975) must be considered in terms of their limitations. For example, some of the studies used data collection methods that did not connect the planning to classroom occurrences (think aloud sessions, Clark & Elmore, 1981; questionnaires, Zahorik, 1975) and settings not representing actual classrooms (laboratory classrooms, Peterson & Clark, 1978; Peterson et al., 1978). Another limitation was seen in those studies with a limited time in the field (three 15-minutes lessons, Fogarty et al., 1983; two one-hour sessions, MacKay & Marland, 1978; three one-hour lessons, McNair & Joyce, 1979; Morine-Dershimer, 1978-1979). In addition, this collection of early teacher decision making studies was limited since all studies except one used elementary teachers.

In spite of their limitations, this group of early decision making studies recognized the idea of the teacher as a decision maker, established a foundation for subsequent decision making studies, and provided suggestions for ensuing studies involving planning and interactive decisions. The most prominent suggestion was the necessity of studying planning and interactive decisions together while exploring relationship between the two types of decisions (Morine-Dershimer, 1978-1979; Yinger, 1980).
More recent decision making studies that addressed the complete process of decision making (Borko & Livingston, 1989; Nesselrodt, 1990; Putnam, 1984; Westerman, 1991) were the core of the literature base. The described methods of each of these studies indicated a use of multiple sources of data, discussions with the sample teachers, and a detailed data collection and analysis process. Even though a sample of one (Nesselrodt, 1990; Putnam, 1984) or limited time in the field (one week of lessons per teacher, Borko & Livingston, 1989; two lessons per teacher, Westerman, 1991) may be considered limitations of the studies, the intensity with which the studies were completed appeared to indicate results were supported by their respective data.

The length of time in the field may also be a limitation of the results of Parker and Gehrke's (1986) study which addressed only interactive decision making. The description of the data collection and analysis process, however, suggested the result of this study was of value. In addition, this study exemplified researchers' movement away from the use of the process-product paradigm for examining teachers' interactive decisions. In Ropo's (1987) pilot study, the details of the data collection and analysis procedures were not clear. The results of the study, however, supported conclusions made by other studies in this review.
Teachers' Actions During Mathematical Reforms

The reform movement advocated by NCTM in the *Curriculum Standards* (1989) was not the mathematics education community's first effort to reform mathematics education. Examples of other reforms were the new mathematics movement in the 1960s, the back-to-basics mathematics movement in the 1970s, and the problem solving movement in the early 1980s. Expert reflections on the new mathematics reform and the problem solving reform indicated disappointment that ideas from these reforms were not implemented in mathematics classrooms (Cooney, 1988; Crosswhite, 1987; NRC, 1989). Additional reflections suggested that during these two reforms teachers were not able to change their instructional methods to meet the needs of the new curriculum (Cooney, 1988; Crosswhite, 1987; Hill, 1976). However, due to the lack of observations studies during these reforms, the exact nature of teachers' actions in the mathematics classroom was not known (Good & Biddle, 1988). The notion that was clear from reflections on past reforms was that teachers must be an integral part of the development and implementation of reform (Ball, 1992; Chambers, 1990; Cooney, 1988; NRC, 1989; Romberg, 1988).

Additional support for the integral role of the teacher during the process of reform is provided by the educational change literature. The role of the teacher is
evident in Fullan's (1982) reflection on the reform process. Fullan stated that some reforms do not work due to the following reasons: (a) reform was based on sound theory and principles, but was not translatable to practice due to limited teachers' resources, (b) reform was based on conditions different from those faced by teachers, (c) reform did not include clear procedures for implementation, and (d) reform did not acknowledge the time and effort it would take teachers to implement.

Experts seem to agree that the teacher has a prominent role in the implementation of a reform. Even though factors such as state legislation, school administration, district curriculum, available resources, school traditions, and school community shape curriculum (goals, content, and instructional methods), the factor on which educational change depends is the teacher (Fullan, 1982). Teachers, through their thoughts and actions, determine, interpret, and implement their classroom curriculum.

The Geometry Classroom

In the present reform agenda, geometry is recognized as a rich environment for development of students' abilities to explore, conjecture, reason logically, and connect mathematics with the real world (NCTM, 1989). This view of geometry is vastly different from the traditional view of geometry - an environment for developing logical skills. Observational studies of geometry classrooms
during a time of reform have not previously been
undertaken. However, a basis for understanding the
geometry classroom is provided by studies related to
geometry classroom components and studies related to
researcher-implemented geometry instruction.

Geometry Classroom Components

Research addressing geometry classroom components were
minimal. The few studies concerning classroom content or
instructional methods used by the teacher are discussed in
this section. Additional information about components of
the geometry classrooms are provided by researchers'
comments pertaining to students' performance in a proof-
oriented geometry classroom.

The goals and content of secondary geometry classrooms
were part of the survey used by the NCTM (1981) in the
Priorities in School Mathematics Project (PRISM).
Approximately 5,000 people comprised the sample consisting
of elementary and secondary teachers, junior college and
college mathematics teachers, supervisors of mathematics,
mathematics teacher educators, principals, presidents of
school boards, and presidents of parent-teacher
organizations. It was not clear how many of the subjects
were geometry teachers. Results of the surveys indicated
that this group of individuals felt that geometry should be
taught to develop logical thinking abilities, to develop
spatial intuitions about the real world, to acquire the
knowledge needed for study of more mathematics, and to
learn to read and interpret mathematical arguments.
According to the group surveyed, the content used to meet
these goals should be the same that was presently in the
curriculum. Specifically, this content focused on
Euclidean geometry and included topics such as properties
of triangles and rectangles, similar figures, geometry of
distance and direction, three-dimensional geometry, and
logical reasoning principles including axioms and proofs.
The findings of this study provided support for geometry's
role in the secondary curriculum.

Generating information about geometry content and
instruction, Brown's (1974) study explored geometry
teachers' use of textbooks. Data consisted of interviews
with 42 teachers and observations of their geometry
classes. Results of the study indicated that teachers
relied heavily on the textbook for content selection and
sequencing. The teachers usually presented textbook
content section by section and used methods similar to
those in the text. Teachers' instruction was described as
a sequence of homework, discussion, and new homework.

Additional information about the geometry classroom
was generated by researchers' reflections on students'
performance in a proof-oriented geometry classroom. In
general, the performance was described as poor (Clements &
Battista, 1992; Usiskin, 1987). As the mathematics
education community tried to understand why students have trouble with geometry, the van Hiele theory (van Hiele, 1986) has been one school of thought which has examined and attempted to inform the mathematics community about teaching geometry (Burger & Culpepper, 1993; Clements & Battista, 1992; Crowley, 1987; Dessart & Suydam, 1983).

According to van Hiele (1986), the traditional goal of writing proofs required deductive (Level 4) thinking (see Appendix A). The difficulty occurred with the realization that many students entering geometry had only developed their visual (Level 1) geometric thinking skills (van Hiele, 1986; Wirszup, 1976). Results of Senk's (1989) study indicated that a student with entry Level 1 had a one out of three chance of demonstrating proficiency at proof-writing by the end of the geometry year, a student with an entry Level 2 had at least a 50% chance, and a student with entry Level 3 had a slightly higher chance. Other researchers identified students' difficulty with geometry resulting from classroom occurrences like the following: (a) teachers and students reasoned at different levels and were not able to understand each other (Burger & Shaughnessy, 1986; Mayberry, 1983), (b) students learned geometry by memorization (Fuys, 1985; Hoffer, 1983; Lowry, 1986; Mayberry, 1983), and (c) students lacked opportunities to develop visual, analytic, and abstract
thinking skills (Fuys, 1985; Senk, 1989; Shaughnessy & Burger, 1985; van Hiele, 1986).

Researcher-implemented Geometry Instruction

Studies in this section investigated researcher-implemented geometry instruction. Results of these studies indicated the type of geometry outcomes that could possibly be reached with certain teaching approaches. The studies in this section were divided into two categories: (a) studies involving the van Hiele phases of instruction, and (b) studies involving computer instruction.

Studies involving the van Hiele phases of instruction. Research studies involving the van Hiele phases of instruction (see Appendix A) have examined the possibility of developing instructional ideas that support the van Hiele theory (Fuys, 1985; Pastor, 1993) and investigated possible student geometry outcomes when van Hiele phases of instruction were implemented (Bobango, 1987; Fuys, 1985; Lowry, 1986; E. O. Thompson, 1992). The results of the two studies involving the development of instructional ideas showed the feasibility of designing van Hiele phase-based instruction. In particular, as one part of the Brooklyn College van Hiele Project, Fuys developed instructional modules involving the classification of two-dimensional shapes, angle measurement and relationships, and area and perimeter. The result of Pastor's research with elementary
children was the development of phase-based instructional units for all levels of thinking. Isometries in the plane were the content focus of these instructional units.

The studies that examined student outcomes resulting from the implementation of van Hiele phase-based instruction generated information about possible student benefits from van Hiele phase-based instruction (Bobango, 1987; Fuys, 1985; Lowry, 1986; E. O. Thompson, 1992). An example from Fuys is provided to explain instruction that incorporates the van Hiele phases of instruction. In the first phase, the teacher gave the students an opportunity to get acquainted with the topic of rectangles by looking at examples and non-examples. For the second phase, the teacher provided students with tasks such as folding rectangular figures, measuring angles or sides of rectangles, or looking for symmetry in rectangular figures in order for the students to identify properties of the rectangle. This guided orientation process was followed by a third phase in which the teacher asked students to express their ideas about the properties of a rectangle and to learn the language associated with properties of a rectangle. In the fourth phase, the teacher asked students to use their knowledge about rectangles to investigate a new shape, a square for example. Students conducted their own investigation and in the final phase summarized what they learned about rectangles and squares. Included in the
students' summaries were their reflection on the learning process.

Two of the researchers that examined student geometry outcomes resulting from the implementation of van Hiele phases of instruction used the phases of instruction as the instructional component of clinical interviews (Fuys, 1985; Lowry, 1986). The third researcher in this section (Bobango, 1987) was one of the first researchers to examine the implementation of van Hiele phase-based instruction in the geometry classroom. The last researcher (E. O. Thompson, 1992) looked at the implementation of van Hiele phase-based instruction in the context of small cooperative learning groups in geometry classrooms. Bobango and E. O. Thompson both used computers as an instructional tool in their studies. Since the focus of their respective studies was on the influence of the van Hiele phases of instruction rather than the influence of the computer instruction, Bobango's research and E. O. Thompson's research were included in this section.

As one part of the Brooklyn College van Hiele Project, Fuys (1985) interviewed sixth-grade students with clinical interviews that had an instructional component. The purpose of the study was to determine to what extent the van Hiele model was useful for characterizing student thinking in geometry. Specifically, the project researchers were interested in determining the van Hiele
level of each student, students' progress within a level or to a higher level as a result of instruction, and students' difficulties encountered during the process.

The sample consisted of 16 sixth-graders from two inner-city New York City public schools. Subjects were chosen from three achievement levels as determined by grade equivalency scores on mathematics and reading tests. The group consisted of three below average subjects (one to two years below grade level), five average, and eight above average (at least one year above grade level).

All except two of the subjects were interviewed for eight 45-minute sessions by trained interviewers (members of the project staff). During the videotaped interviews, students worked on three van Hiele phase-based geometric modules. The topics of Modules one, two, and three were classification of two-dimensional shapes, angle measurement and relationships, and area, respectively. All students started with Module one. An individual's progress in the interview sessions determined when work on Module two began. Two of the students were interviewed for four one-hour sessions, worked only on Module one, and were given extra instruction and a review on basic geometric concepts. All 16 students completed Module one, eight students finished Module two, and three students completed Module three.
The results of the study showed that all students' van Hiele levels could be determined and that many of the students showed progress within a level or to a higher level. Three of the 16 students began at Level 1 and even after instruction remained at Level 1. Five of the 16 students made progress from Level 1 to Level 2. Eight of 16 had an entry level that was described as 1-2 (meaning the student's thinking was in transition from Level 1 to Level 2). They made progress within Level 2 and showed a transition toward Level 3 thinking. The researchers concluded that van Hiele phased-based instruction facilitated students movement through the levels.

Using an adaptation of Module three from the study conducted by Fuys (1985), Lowry (1986) investigated whether the van Hiele model was useful with nine-year-old children when assessing their thought processes about area and perimeter. The instructional aspect of the investigation involved the following questions: (a) Can students move to the next level through teaching protocols, and (b) is there any difference in readiness for movement between children who have had formal instruction on area and perimeter and those who have not?

The sample for this study consisted of third- and fourth-grade students from a Washington, D. C. private school. One of the third-grade teachers and one of the fourth-grade teachers at the school volunteered to
participate in the study. One month prior to the study, the fourth-grade students completed the textual unit on area and perimeter. The third-grade students worked on linear measure and multiplication prior to the study and did not receive instruction in area and perimeter until after the study.

An age-appropriate adaptation of the Brooklyn College Project interview protocol for the area module was developed for this study. The first two activities were assessment activities while the remaining five activities were a blend of assessment and instruction. Each of these activities began with Level 1 thinking but included instruction and exploration of materials potentially leading to Level 2 or Level 3 thinking.

The researcher interviewed each child individually in sessions that were 40 minutes in length. During the first session, each child worked through activities one and two. Either one or two activities were presented in each session depending on the child’s progress with the activities. The total interview time for each child ranged from three to five hours during a two week time period.

Data collected included videotaped interview sessions and written descriptions of the children’s nonverbal behaviors and procedures used with materials. Descriptors of the van Hiele levels were used to evaluate children’s responses to each activity.
Based on the analyses of students’ responses, Lowry (1986) concluded that the van Hiele-based protocol activities were successful in moving children to the next level. Lowry also concluded that the formal, rule-based instruction received by the fourth-grade students in other classroom settings fostered reduction of level thinking by the students. Reduction of level thinking is demonstrated, for example, when students use Level 1 thinking to understand a Level 2 idea. In addition, the rule-based instruction hindered the fourth-grade students’ movement through the levels. The third-grade students who had not received any instruction on area and perimeter did not demonstrate reduction of level thinking and showed more progress through the levels than the fourth-grade students.

In contrast to using van Hiele phase-based instruction during clinical interviews, Bobango (1987) used van Hiele phase-based instruction in the geometry classroom and examined the effect of this instruction on raising students’ van Hiele levels of geometric thought.

Students from one teacher’s four intact geometry classes in a rural senior high school were used as subjects for this study. Two geometry sections were honors classes with 16 and 18 students respectively, and two geometry sections were regular classes with 21 students each. One honors class and one regular class were used as the experimental classes.
Pretreatment assessments included the determination of students’ van Hiele level of geometric thought and of students’ standard content knowledge by an Educational Testing Service geometry test. Under the researcher’s guidance, the treatment occurred for 20 school days and consisted of students working van Hiele-based exercises designed by the researcher. To do these exercises students worked in groups of two, three, or four, and used two computer programs, Geometric Supposer: Triangle (Schwartz & Yerushalmy, 1985) and Geometric Supposer: Quadrilateral (Schwartz & Yerushalmy, 1985).

For ten days, the comparison classes worked on computers using nine segments of a non-Supposer triangle program. Even though students were free to discuss the exercises within their groups, they did not meet as a whole class to discuss their findings. When the comparison students finished with the computer programs they spent class time doing algebraic proofs.

At the completion of the treatment, each student’s van Hiele level was assessed. The computers were also removed from the classroom at this time. Following six weeks of the teacher’s constructed lessons and of work from the geometry textbook, all students took a standard content test and two proof tests.

Results of the analysis involving the comparison of students’ before- and after-treatment van Hiele levels
indicated a significant difference existed between the number of students in the experimental groups and the number of students in the comparison groups whose van Hiele levels had increased. The analysis of variance regarding treatment and achievement did not show any significant interactions. Thus, the researcher concluded that the van Hiele phased-based instruction was successful with increasing students' van Hiele level of thinking but not with increasing geometry achievement.

Like Bobango (1987), E. O. Thompson (1992) studied secondary geometry classroom instruction. Specifically, E. O. Thompson examined the effect that three methods of geometry instruction had on achievement and retention of certain geometrical concepts. One method of instruction involved small cooperative learning groups doing paper and pencil van Hiele-based activities. A second method of instruction had small cooperative learning groups using the computer and the Geometric PreSupposer (Chazan, 1989) and Geometric Supposer: Triangles (Schwartz & Yerushalmy, 1985) software together with van Hiele-based activities. The third type of instruction was identified as whole class instruction based on traditional textbook procedures.

The sample consisted of 14 intact geometry classes selected from five Montana high schools. Each class used only one of the methods of instruction. One of the schools had two participating classes; the other four schools each
had three participating classes. At four of the schools, the same teacher taught each class. One of the schools with three participating classes had one teacher teaching two classes and another teacher teaching the third class. Each of the teachers involved in the study had experience with cooperative learning techniques and had recently participated in an Integrating Mathematics Programs and Computer Technology (IMPACT) project or workshop in which the teachers had been trained to use the computer in the classroom.

The instruction of the congruent triangles unit occurred over the same four week period for all teachers. Before the instruction began the students took a test designed to determine a student’s van Hiele level, completed a form designed to measure students’ attitudes toward mathematics and toward geometry, and a test (form A) designed by the researcher to measure students’ geometry achievement. At the conclusion of the treatment, students took a geometry achievement test (form B). Four weeks later the students took the achievement test (form A) as a retention test.

Analysis of covariance was used to analyze the data collected during the study. The independent variables were gender, socio-economic status, school, student age, van Hiele level, attitude toward mathematics, attitude toward geometry, and the pretest geometry achievement score.
Dependent variables were posttest and retention test geometry achievement scores. Results of the study indicated that the classes using either of the van Hiele based treatments scored significantly higher than the control group on the low level (knowledge and comprehension) items on the posttest. The analysis with the retention test scores indicated that the groups using the van Hiele based paper-and-pencil activities scored significantly higher than the control group on the high level (application, analysis, synthesis, and evaluation) items. The researcher concluded that instructional methods using small cooperative learning groups and van Hiele phase-based materials were workable ideas for the geometry classroom.

Studies involving computer instruction. Each of the studies (Clements & Battista, 1989, 1990; McCoy, 1991; Yerushalmy et al., 1987) in this section examined the implementation of computer instruction in the mathematics classroom as students learned geometry concepts. The first two studies in the section (Clements & Battista, 1989, 1990) involved elementary students using Logo, a programming language developed to serve as a conceptual framework for learning mathematics. These studies suggested that Logo could be an environment for geometric investigations by secondary students (Burger & Culpepper, 1993). The last two studies in this section (McCoy, 1991;
Yerushalmy et al., 1987) involved the use of the Geometric Supposer (Schwartz & Yerushalmy, 1985), a construction program designed to facilitate the process of making and testing conjectures.

Clements and Battista (1989) directed a study that looked at how third-grade children’s geometric conceptualizations were affected by a Logo environment emphasizing projects and problems solving and including intensive teacher guidance and peer interaction.

The sample of the study consisted of 48 third-grade students from a middle class midwestern school system. Twenty boys and 28 girls were randomly selected from a pool of children who returned a parental permission form. The children were randomly assigned to either a Logo computer programming group or a control group. The pretreatment level of mathematics achievement of the two groups was not significantly different.

Both the Logo group and the control group were involved with computer activities for 26 weeks. The Logo group met for three 45- to 55-minute sessions per week. Sessions involved a review of previous work, presentation of material by the teacher to the students in a large group setting, and students independently working problems that involved such ideas as constructing and manipulating squares, rectangles, and equilateral triangles. The control group met once a week for 26 weeks and worked on
lessons consisting of computer activities designed to
develop creativity and literacy.

The first author interviewed the children to determine
their conceptualizations of angle and shape. The
interviews occurred one week after the sessions were
completed and lasted from 45 to 70 minutes. All children
studied the geometry chapter of their mathematics textbook
within two months before the interview. Analysis of the
interviews indicated that the Logo environment used in this
study appeared to foster children's geometric explorations
and discussions such that the students' geometric
conceptualizations were positively affected.

In another study by Clements and Battista (1990), the
researchers interviewed 12 fourth graders to determine the
effects of the study of Logo on the students' conceptualizations of geometric topics. The focus of this
study was to determine whether Logo programming experience assisted children's development of geometric concepts such as angle, angle size, and related arithmetic ideas.

The sample consisted of four boys and eight girls from
a single fourth-grade classroom in a middle-class school
system. From a pool of all those who returned a parental
permission form, pairs of children were formed based on
gender and pretreatment mathematics achievement. The Logo
programming group and the comparison (composition/word
processing) group were created by having the children in each pair randomly assigned to one of the groups.

The two computer treatments were implemented in two 40-minute sessions per week for 40 sessions. Under the guidance of an experienced teacher, the children worked in pairs on the activities. At least one of the authors observed the computer lessons for about half of the sessions. The focus of the Logo group's activities was the use of programming in turtle graphics as an environment for problem solving. For example, students were asked to determine what input with the LEFT command made the turtle turn half-way around. The focus of the comparison group was the use of an integrated package of writing/word processing programs to emphasize the process approach to composition.

During the course of the study the researchers interviewed all the children three times. The three interviews, ranging in length from 40 to 60 minutes, occurred at the beginning of the study, two-thirds of the way through the study, and at the end of the 40 computer sessions. Between the second and the third interviews all children participated in the normal classroom instruction on geometry.

Analyses of the interviews indicated that Logo children appeared to develop more mathematically accurate conceptualizations of angle, angle size, and rotation than
the comparison children did. In addition, more Logo children demonstrated they thought about geometric shapes in terms of properties rather than in terms of the visual picture the shape presented. The researchers concluded that the Logo programming environment appeared to facilitate children's development of geometric concepts.

In contrast to examining geometry at the elementary level, McCoy's (1991) study examined geometry at the high school level. Specifically, McCoy's study compared the geometry achievement of a high school class that used the Geometric Supposer (Schwartz & Yerushalmy, 1985) regularly throughout the school year with the geometry achievement of a similar class that did not use the software.

The sample consisted of two intact high school geometry classes at two parochial high schools located in the same metropolitan area. Twenty-nine 10th-grade students were in each class, and both classes were identified as "honors" geometry. Both teachers were described as having had several years teaching experience in high school mathematics. Informal observations by the researcher were used to verify that, except for the Geometric Supposer (Schwartz & Yerushalmy, 1985) activities, the two teachers used similar teaching methods. In addition, the same textbook was used by both teachers.

The experimental class used a Geometric Supposer (Schwartz & Yerushalmy, 1985) activity approximately once
every two weeks throughout the school year. Following the suggestions in the Geometric Supposer manual, students worked in groups of two or three on adapted problems from the text. As students worked on the activities in their groups they were required to make conjectures, to find examples or counterexamples, to write a report of their investigation, and to discuss their reports in class. When the students were not doing Geometric Supposer activities they were involved with a traditional instructional process of lecture, demonstration/discussion, and homework. The control class did not do any Geometric Supposer activities and followed the traditional instructional process throughout the school year.

Students' geometry achievement at the end of the school year was measured by a multiple choice final examination provided by the publisher of the textbook used by both classes. To control for initial differences between the two classes, the total mathematics percentile scores from the SRA Achievement Series (Level H) were used as a covariate in the analysis of the data. Results of the analysis indicated that the treatment group scored significantly higher on the final examination than the control group. Further analysis showed that the difference in achievement between the two groups was due to the treatment group's performance on questions identified by Bloom's Taxonomy (Bloom, 1956) as application, analysis,
synthesis, and evaluation. The researcher concluded that the Geometric Supposer (Schwartz & Yerushalmy, 1985) when used as part of geometry instruction was a tool that appeared to foster the development of students' higher level thinking skills.

Like McCoy's study (1991), Yerushalmy et al.'s (1987) study involved the Geometric Supposer (Schwartz & Yerushalmy, 1985). The goal of Yerushalmy et al.'s year-long research project was to examine the implementation of a guided inquiry approach using the Geometric Supposer to teach high school geometry. One component of the research project focused on student learning. The research team was interested in students' knowledge of standard geometry content and concerned with students' ability to generalize from collected data and students' ability to move from generalizations to proofs.

The setting for this study consisted of three high schools in three different Boston area suburbs. The selection of the schools was based on the schools' interested teachers, supportive mathematics department chairpersons and principals, and available computer hardware. An experimental geometry class and a comparison geometry class were selected at each school. The experimental classes used the Geometric Supposer (Schwartz & Yerushalmy, 1985) with a guided inquiry approach that asked students to integrate inductive reasoning with
deductive reasoning and to combine empirical work with conceptual work when solving problems and planning proofs. In the comparison classes, the content came mainly from the textbook and was presented using an approach that focused on deductive reasoning and two-column proofs. Based on the feeling that the teaching style of the teachers using the Geometric Supposer would change over the year, the comparison class teacher was not the same as the experimental class teacher.

A variety of sources were used for collecting data on students and teachers. Experimental classrooms were observed roughly once every three weeks from October through mid-June. The observer took field notes during the observation and then wrote a report on the class after each visit. The researchers collected all of the students' written work on Geometric Supposer (Schwartz & Yerushalmy, 1985) problems. All students took a pretest and a posttest designed to assess students' ability to make conjectures or general statements. At the end of the year all students except those in one comparison class took a test designed to assess students' ability to write proofs. Supplemental student data were also collected from teacher interviews, teachers' regularly scheduled meetings with the research team, teacher reflections, and selected experimental students' year-end interviews.
The research team made a commitment to the schools to address standard geometry content in their experimental course. Based on teachers' reports of students' midyear and final exams, the researchers found that the students working with the Geometric Supposer (Schwartz & Yerushalmy, 1985) learned as least as much geometry as their non-Geometric Supposer counterparts. The experimental students performed significantly better on the generalization test than did the comparison students. With respect to devising informal arguments and writing traditional proofs, the experimental students were equal to or a little better than the comparison students. In summary, the analysis of all data sources indicated that using a guided inquiry approach with the Geometric Supposer was feasible for helping students create mathematics.

Discussion of the Geometry Classroom

The discussion of the literature related to the geometry classroom provides a basis for understanding the context of the conducted study. The PRISM (NCTM, 1981) report advocated the presence of geometry in the secondary classroom and provided an indication of how the mathematics education community viewed geometry. The content and goals stated 10 to 15 years ago implied that the focus of secondary geometry should be on the logical development of geometry - a traditional view of the purpose for teaching geometry.
Brown's (1974) finding of geometry teachers' heavy reliance on textbooks as an authority for content and for topic sequence was also a normal occurrence in a typical mathematics classroom (Robitaille & Travers, 1992; Romberg & Carpenter, 1986). Similarly, Brown's finding of the instructional routine of homework, discussion, and new homework was a normal occurrence in a typical mathematics classroom (NCTM, 1989; NRC, 1989; Owens, 1989; Romberg & Carpenter, 1986; Ropo, 1987). In general, the geometry classroom portrayed by these results did not appear to match the NCTM's reform agenda that promoted student learning of a multifaceted geometry course through students' active participation in the learning process.

Research that attempted to implement reform ideas have displayed promise for developing a geometry classroom that was not a typical mathematics classroom. The van Hiele research indicated the use of phase-based instruction for developing students' geometric thinking (Bobango, 1987; Fuys, 1985; Lowry, 1986; Thompson, 1992). Technology studies suggested the use of the Geometric Supposer (Schwartz & Yerushalmy, 1985) with a guided inquiry approach (Yerushalmy et al., 1987; McCoy, 1991) and the use of the Logo programming language in a guided-discovery environment (Clements & Battista, 1989, 1990) promoted students' problem solving and reasoning skills.
One important aspect of the results of all the geometry studies was simply the fact that geometry was being studied. The implication was that the mathematical topics studied were relevant for mathematics classrooms. The results of the studies addressing geometry classroom components were limited by the use of a survey (PRISM, 1981) and by an emphasis of the use of the textbook in the classroom (Brown, 1974). These studies suggest that a more open-ended, detailed investigation of the geometry classroom was needed.

The results of the researcher-implemented studies involving the van Hiele theory were limited by the following: (a) the researcher's role in the clinical interviews (Fuys, 1985; Lowry, 1986), (b) the lack of researcher observation of the instruction (Lowry, 1986; E. O. Thompson, 1992), and (c) the study's length and the sample teacher's knowledge of the van Hiele theory (Bobango, 1987). The results of the researcher-implemented studies involving technology (Clements & Battista, 1989, 1990; McCoy, 1991; Yerushalmi et al., 1987) were limited by the realization that classroom success with the technology depended on the method of instruction and the content of the lesson. The strength of the results of all the researcher-implemented studies, however, was that collectively the results demonstrated that the teaching of geometry could be perceived from a non-traditional
perspective. In addition, these studies provided possible suggestions for implementing this non-traditional geometry.

Conclusion

The decision making studies in this review indicated the importance of examining teachers' planning and interactive decisions together. This literature also showed that teachers' planning and interactive decisions were influenced by a variety of factors. The identified factors were based on studies whose settings were characterized as classrooms working within a stable curriculum (goals, content, and instructional methods) environment. This study examined teachers' decision making in a mathematics reform context.

During previous mathematics reforms, minimal observational studies in the classroom were conducted. As a result of this deficiency, the exact nature of mathematics teachers' actions during reform was unclear. Since teachers' decisions determine what occurs in classrooms, it is important to examine their decisions during a time of reform. This study attempts to understand how individual teachers are making decisions in a mathematics reform context.

Exploring teachers' planning and interactive decisions in the context of the geometry classroom also fills a void in the geometry literature. Previous research has examined researcher-implemented instruction quite extensively, but
has minimally addressed exploratory investigations of the secondary geometry classroom. This study provides a beginning look at the type of secondary geometry courses that are occurring in a major mathematics reform context.

Based on the discussions of the literature related to teachers' decision making, teachers' actions during mathematical reforms, and the geometry classroom, a study bringing these three areas together generates valuable information about the secondary geometry teaching process.
CHAPTER III  
DESIGN AND METHOD

Introduction

The purpose of this study was to investigate secondary geometry teachers' decision making in a mathematics reform context. To focus the investigation, two questions were examined: (a) What planning and interactive decisions were secondary geometry teachers making during this time of reform, and (b) what factors influenced the decisions that these teachers made? Planning decisions are made before a lesson is taught (preactively) and with reflection after the lesson (postactively) in anticipation of the next lesson. Interactive decisions include decisions reflecting an implementation of plans and decisions made during day-to-day lessons.

Given the exploratory nature of the research questions, a descriptive design using qualitative methods was employed. Since the intent of the study was to investigate geometry teachers' decisions and the context in which the decisions were made, a case study approach involving a detailed investigation of a small number of secondary geometry teachers was considered most suitable. The design and method discussion includes detailed descriptions of the setting, the sample, the research method, and the data collection and analysis procedures.
Setting

Preparing students to live in the twenty-first century is the challenge facing the mathematics education community. To assist teachers with this challenge, organizations at the national and state level have provided curriculum (including goals, content, and instructional methods) guidelines. At the time of the study at the national level, the National Council of Teachers of Mathematics (NCTM) provided two companion documents for teachers, namely the Curriculum Standards (1989) and the Professional Standards (1991). This set of standards advocated the study of mathematics as problem solving, reasoning, and communication in an environment where students are active participants. In addition, an expectation existed that the study of mathematics focused on promoting mathematical connections to other mathematics, other disciplines, and the real world.

At the state level, a state-legislated reform that focused on the preparation of students for the work place was being implemented. A main goal of the reform was students’ attainment of the Certificate of Initial Mastery (CIM) by the completion of their tenth grade year. For the mathematics component of CIM, the Curriculum Standards (1989) had been adopted.
The sample for this study consisted of five secondary mathematics teachers whose geometry classes consisted of students who had been successful in first year algebra. The extensive data collection procedures used for this study indicated the need for easy accessibility to the teachers. Thus, this sample of convenience consisted of teachers located within 60 miles of the researcher.

Selection of the teachers was based on two criteria. First, all of the teachers participating in the study had to have at least three years of geometry teaching experience. With this criterion, the study focused on the planning and interactive decisions of geometry teachers with developed ideas about geometry course goals, content, and instruction.

Secondly, teachers who viewed geometry content and instruction from different perspectives were selected. Textbooks used by geometry teachers that emphasized different aspects of the nature of geometry guided the selection of the subjects. Teachers' initial comments about their general instructional approaches provided additional guidance for selection. The geometry textbooks used by schools in the vicinity of the researcher were divided into three categories - traditional, inductive, and multiple perspective - with the majority of the schools using a traditional textbook. Traditional geometry
textbooks used by schools emphasized a logical development of geometry content and included an introduction to two-column proofs in the first chapter. An inductive textbook was defined as a textbook that used an intuitive and inductive approach for the development of geometry concepts. A multiple perspective textbook developed geometry concepts in the context of plane, coordinate, and transformational geometries.

Pseudonyms were used for each of the teachers in order to provide anonymity. Three teachers who used a traditional textbook were asked to participate in the study. Ardella's textbook was *Geometry* (Clemens, 1990). Emily used *Geometry* (Jurgensen, Brown, & Jurgensen, 1988) as her textbook. Jon's textbook was *Geometry* (Kalin & Corbitt, 1992). Based on these three teachers' comments about their instruction, one indicated using small group instruction and hands-on activities (Ardella), one used the lecture/discussion format while incorporating visualization activities (Emily), and one used the lecture mode of instruction (Jon). To complete the sample, one teacher who used the inductive approach textbook (Jordan) and one teacher who used the multiple perspective textbook (Lynne) were asked to participate in the study. Jordan used *Discovering Geometry: An Inductive Approach* (Serra, 1989), a prominent textbook at the time of the study that used an inductive approach for developing geometry concepts.
Lynne's multiple perspective textbook was the *University of Chicago School Mathematics Project Geometry* textbook (Coxford, Usiskin, & Hirschorn, 1991), the most prevalent textbook addressing geometry from multiple perspectives at the time of the study. Teachers' participation in the study was an indication of their willingness to be part of the sample.

Method

The methodology followed in this study was a comparative case study. With this method, each teacher was treated as a comprehensive case study. Comparisons were then made across the individual cases in order to determine whether differences and similarities existed between the cases. The data were analyzed using an inductive process as described by Bogdan and Biklen (1992). In brief, this process involved a systematic searching of the data in order to identify categories and themes prevalent in the data and the use of this information to guide the data collection and analysis process.

The collected data for this study consisted of a questionnaire, audiotaped interviews, videotaped observations, observation field notes, transcripts from audiotaped observations, informal interviews, written documents, and a researcher journal. The use of multiple data sources allowed for the triangulation of the data, enhancing the researcher's ability to understand teachers'
decision making and to be more certain that categories and themes arising from the data matched the five geometry teachers' planning and interactive decision making.

Data Collection

"To understand teaching, one must understand the decisions teachers make during the planning and interactive stages, as well as the factors influencing those decisions" (Brown & Borko, 1992, p. 215). The recognition of the importance of planning and interactive decisions to the process of teaching was supported by a broad research base (Clark & Peterson, 1986) and provided the framework for the data collection process which took place from August to December, 1994. The decision to conduct the investigation at the beginning of the school year was based on the premise that decisions made at the start of the school year were a major influence on classroom occurrences for the remainder of the school year (Shavelson, 1982). The curriculum decisions involving goals, content, and instructional methods made at the beginning of the year were anticipated to communicate the view of geometry promoted throughout the year.

A variety of sources were used to collect data from each geometry teacher. The data collected prior to the start of the school year included a questionnaire and an interview. Beginning with the start of the school year, weekly classroom observations and teacher interviews were
conducted. In addition, written document data were collected. At the conclusion of the observation process, a final interview was conducted with each teacher.

Questionnaire

Prior to the start of the school year, a written questionnaire was sent to each of the five geometry teachers (Appendix B). The purpose of the questionnaire was to gather information on teachers' geometry and teaching background. Objectives were written for the questionnaire from which questions were developed. Content validity of the questionnaire was determined independently by three science and mathematics education professors. Provided with a list of questionnaire objectives and a list of questions, the three professors individually completed a table of specifications. When the professors were in complete agreement about the table of specifications, the questionnaire was considered valid.

First Interview

Before the start of the observation phase of the study, each teacher participated in an audiotaped semi-structured interview. The purpose of this first interview was to gather planning information and to establish a baseline for teachers' views toward reform curricular ideas including goals, content, and instructional methods. The interview consisted of core questions (Appendix C) asked of
each teacher, follow-up questions as needed to clarify teachers' responses during the interview, and follow-up questions to teachers' questionnaire responses. The content validity of the core questions was determined in the same way as the content validity of the written questionnaire.

Weekly Observations and Interviews

A weekly observation of each teacher beginning with the first week of school and continuing until the completion of instruction of the third unit or third chapter of material was intended. Due to teachers' personal and school schedules, weekly observations were not always possible. Beginning with the second week of school, Ardella (traditional textbook) was observed nine times over a nine-week period. Emily's (traditional textbook) observations began with the first week of school and occurred nine times over a 13-week period. Beginning with the first week of school, Jon (traditional textbook) was observed 10 times over an 11-week period. Jordan's (inductive approach textbook) weekly observations began with the first week of school and continued for 11 weeks. Beginning with the third week of school, Lynne (multiple perspective textbook) was observed eight times over a 10-week period. All observations were of complete class periods.
Comprehensive field notes on classroom occurrences relating to curriculum issues including goals, content, and instructional methods were recorded by the researcher during each observation. The researcher’s reflective comments about the lesson were also recorded in the field notes during the observation process. The observations made by the researcher were videotaped and audiotaped. The videotapes were used to complete the researcher’s field notes concerning classroom occurrences. Audiotape transcriptions provided the researcher with a word-for-word account of the classroom dialogue.

Each observation was preceded and followed by an informal interview. The purpose of the preceding interview was to determine planning decisions the teachers had made for given lessons and factors that influenced those decisions. Sample questions for the preceding interview included the following: (a) Describe today’s lesson with respect to its goals, content, and instruction, (b) where and when did you start when planning for this lesson, (c) how did you decide what to include in this lesson, and (d) how is this lesson related to the previous lesson and to the course content?

The purpose of the post-lesson interview was to collect information on the teacher’s view of the lesson, interactive decisions that were made, and decisions made based on reflections about the lesson. Sample questions
for the post-lesson interview included the following: (a) Describe how you feel about today’s lesson, (b) describe decisions you made during this lesson, (c) explain why these decisions were made, (d) when a teacher did not mention any decisions, the researcher asked about possible teacher decisions identified in the researcher’s field notes, (e) how is today’s lesson related to the succeeding lesson, (f) how will you plan for the next lesson, and (g) explain why these will be your plans. During the pre-and post-lesson interviews, teachers’ responses were recorded by the researcher.

Written Documents

Throughout the study written documents involved in the planning and implementation of geometry instruction were collected. These documents included weekly lesson plans, lesson plans for observed lessons, textbooks, classroom handouts, and assessment instruments. The information acquired from these documents were used to support or negate the categories or themes generated from the analysis of the interviews and observations.

Final Interview

A final interview was conducted with each teacher within one week after the final observation of a geometry lesson. The purpose of these final interviews was to gather information concerning the teachers’ thoughts as
they looked back upon the progression of their geometry course, as they clarified issues that emerged during the analysis of data, and as they looked ahead to their subsequent teaching of geometry. The interview consisted of core questions asked of each teacher as well as teacher-specific questions concerning their goals, content, instruction, reform, and factors (Appendix D).

Data Analysis

The analysis of the data occurred in four phases. During the first phase of analysis prior to any observations, questionnaire responses and interview transcripts were analyzed. The results of this analysis guided the data collection and analysis occurring in the second phase.

The second phase of the analysis began with the first observation of each teacher and continued through each teacher’s final interview. After each teacher’s observed lesson, the videotape of the lesson was watched and notes were added to the researcher’s field notes for that lesson. Following the observed lesson, the preceding and follow-up interview notes, the researcher’s field notes that contained information from the videotapes, and written documents were analyzed using an ongoing, inductive process. Results of this analysis was used to guide subsequent data collection. In preparation for the final
interview with each teacher, the data for all observations of that individual teacher were analyzed.

The third phase of analysis began with the process of completing transcripts for each teacher’s lesson. The audiotape transcripts were merged with the teacher’s field notes of each lesson in order to make the transcript complete. Each transcript was then analyzed using an ongoing, inductive process. A preliminary profile consisting of the teacher’s planning decisions and influential factors and of the teacher’s interactive decisions and influential factors was prepared.

The ongoing, inductive process used to analyze the data was a process described by Bogdan and Biklen (1992). This general inductive process involved reading the data, organizing it, breaking the data into manageable units such as codes or categories, searching the data for patterns, refining categories as needed, and identifying themes that were prevalent in the decision making data. From the repeated use of the inductive process, prevalent decision making categories and themes emerged from the data.

Once individual profiles for each teacher had been written, the researcher re-examined each teacher’s data. During this part of the analysis the researcher examined the data for situations that did not support existing categories. When this circumstance occurred, the researcher refined the category. The re-examination
process continued until a proposed refinement had no additional implications.

In the analysis of the variety of sources of collected data, the videotaped and audiotaped classroom observations were the primary sources of data because the observations represented what the teachers actually said and did in the classroom. The questionnaire and the first interview were critical for collecting background information about the teachers and overall course information. The interviews preceding and following the lessons were also important for understanding teachers' planning and their decision making as their geometry course progressed. Written documents such as textbooks, daily lesson plans, activities, and assessment instruments were used to support or negate the categories or themes generated from the analysis of the primary sources of data. The informal interviews and the final interview enabled the researcher to collect teacher feedback on themes, categories, and conjectures that emerged during the analysis of data.

The results of the analysis of each individual's data generated a description of each geometry teacher's decision making. Specifically, this description consisted of a narrative of teacher's planning decisions and factors influencing those decisions and a narrative of teacher's interactive decisions and factors influencing those decisions.
After each of the individual decision making descriptions were developed, the descriptions were examined collectively in search of similarities and differences across the sample. The resulting comprehensive description produced an initial description of these five secondary geometry teachers' planning and interactive decisions in a mathematics reform context and of factors that influenced both types of decisions.

The Researcher

In this study using qualitative methods, the primary instrument for collecting and analyzing data was the researcher. Thus, the researcher's background pertained to this study. The researcher taught four years of high school geometry and at the time of the study was teaching mathematics education courses at the college level. During the two years prior to the study, the researcher taught a visualization geometry course for inservice teachers, taught a pedagogy course on teaching geometry for preservice teachers, and supervised student teachers teaching in secondary geometry classrooms. Based on geometry teaching experience and extensive reading on the van Hiele levels of geometric thought, the researcher supported the views advocated by the reform. In particular, the researcher supported students' actively learning geometry concepts through a variety of instructional methods. Furthermore, the researcher
believed geometry should be viewed as a rich environment for students' development of their ability to explore, conjecture, reason logically, and solve non-routine problems in mathematics.

The researcher's background has potential for biasing the data collection and analysis process. In an attempt to recognize and transcend personal bias, the researcher kept a journal recording thoughts, feelings, and judgements made during the data collection and analysis process. During the analysis process, the researcher read the journal and used the recorded information while attempting to transcend personal bias by using precautions when analyzing data, examining conflicting evidence and alternative hypotheses, and exploring the influence of the researcher's presence in these geometry teachers' classrooms.
CHAPTER IV
RESULTS

Introduction

This study examined secondary geometry teachers’ decision making in a mathematical reform context. Five secondary geometry teachers agreed to participate in this study. The selection of the teachers was based upon their years of teaching experience, variations in their geometry curricula, their willingness to participate, and their proximity (within 60 miles) to the researcher’s location.

Using an ongoing, inductive process recommended by Bogdan and Biklen (1992), individual teacher profiles were developed. The profile of each teacher was prepared in six sections: (a) geometry and teaching biography, (b) teacher’s view toward curricular change, (c) an introduction to the teacher’s classroom, (d) planning decisions and influential factors, (e) interactive decisions and influential factors, and (f) summary.

Each profile begins with a geometry and teaching biography. This section provides background information concerning the teacher’s geometry experiences as a student and as a teacher, professional development experiences, and reflections on these experiences.

The next section of the profile describes the teacher’s view toward curricular change. This section includes the teacher’s reflections on the Curriculum
Standards (1989) and the state-legislated reform movement as well as an interpretation of the reform setting in which the teacher worked.

The third section provides an introduction to the teacher's classroom. The portrait of the teacher's classroom includes descriptions of the school setting, the course and section selected for the study, the physical nature of the classroom, a characterization of the teacher, the general instructional routine, and the context in which the teaching and learning occurred.

The fourth section addresses the teacher's planning decisions and factors affecting those decisions. The section begins with a description of the teacher's general planning process and continues with a detailed look at the teacher's decisions concerning the goals, content, and instruction of the individual's course. The discussions involving goal, content, and instructional decisions consist of a description of the substance of their decisions and the reasons for their decisions.

The fifth section discusses the teacher's interactive decisions and factors influencing those decisions. Interactive decisions include decisions reflecting an implementation of plans and decisions made during day-to-day lessons. The discussions include information on the comparison of planning and interactive decisions involving goals, content, and instruction, the teacher's reflections
concerning decisions made during instruction, and descriptions of decision-related issues that contribute to the portrait of the teacher’s geometry course. The factors influencing the teacher’s interactive decisions are incorporated throughout the discussion. Finally, each teacher’s profile concludes with a summary highlighting the main findings concerning planning and interactive decisions and factors influencing those decisions.

After detailed individual teacher profiles were developed, the profiles were examined collectively in search of similarities and differences across the sample. Results of this examination are described in a cross-case profile. These descriptions, which correspond to the two research questions proposed for this study, are presented in two sections: (a) planning and interactive decisions, and (b) factors influencing planning and interactive decisions.

Individual Teacher Profiles

The individual teacher profiles are presented in the following order - Ardella, Emily, Jon, Jordan, and Lynne. The order of the profiles is related to the textbook used by each teacher’s school. The schools of Ardella, Emily, and Jon each used a traditional geometry textbook. Jordan’s school used a textbook that examined geometry content via an inductive approach. Lynne’s school used a textbook that examined geometry from multiple perspectives.
Distinctions between teachers' courses were suggested by the teachers' use of their textbook and their general approach to instruction. Ardella used her textbook as a guide for her course. Her instruction revolved around the use of student groups and hands-on activities. Emily also used her textbook as a guide for her course, but her instruction focused on the incorporation of writing, reading, thinking, visualizing, and measuring skills. Jon followed his textbook quite closely and his predominant mode of instruction was lecture. Jordan also followed his textbook quite closely. However, in contrast to Jon, Jordan used a variety of instructional approaches. Lynne used her textbook as a guide for her course, and her instruction centered on the use of student groups.

Ardella

Geometry and teaching biography. Ardella had studied geometry in high school and in college before teaching the subject. Ardella's description of her high school geometry course indicated proofs were an important component of the course. "I took geometry as a sophomore in high school by a very traditional (goofy) man. He did proof after proof after proof on the overhead and tested us on our ability to do proofs." Ardella said that she "didn't like it [geometry] in high school." At the time of the study, Ardella felt she would not "learn well any more" with the
"listen and do the drill" method she learned by in high school. Approximately four years after her high school geometry experience, Ardella took a transformational geometry course in college. She stated that "for the first time [I] saw the connections between algebra and geometry and I loved it."

Ardella's first teaching experience was at a seventh through twelfth grade school. Three years later she started her present position at a ninth through twelfth grade high school located in a small university city. Ardella was beginning her twentieth year of teaching mathematics and her fifteenth year of teaching geometry at the time of this study. During the fall of the research project, the school enrollment was approximately 1,050 students.

During her fifth year of teaching, Ardella began her Master of Science degree at a local university. As part of her degree she took a geometry course from a mathematics education professor who was studying kindergarten through twelfth-grade student learning of geometry. The following year the professor interviewed students from Ardella's geometry class. In addition, Ardella made two presentations with him, one at the local university and one at a conference, using her favorite geometry activities. The following year, Ardella and the professor received permission from the local school board to pilot a new
geometry course at her high school. The new course was a hands-on approach to geometry. The textbook for the course was *Geometry* (Hoffer, 1979). The distinguishing characteristic of this textbook was that it delayed the discussion of formal proofs until the second half of the textbook. Ardella described how she and the professor shared the teaching responsibilities:

We received permission from the school board to pilot a new geometry course in which he volunteered three times a week. He was the guest lecturer on a continuing basis and I did the other day's lessons and wrote tests, and contacted parents and awarded the grades.

Ardella said that the pilot course "helped to cement ideas I had about sequencing the course and manipulatives and gave me permission to let proofs have their place, but not dominate all of geometry." Even though Ardella was not able to convince any of her colleagues that it was worth the work required to implement the new curriculum, she continued teaching geometry within the pilot course framework for another year. She then realized that she could "still be within the same frame [use the same textbook] as the rest of the teachers" by varying her instruction because "the content, the end content was still the same."

Ardella completed the description of her geometry background with a reference to a geometry workshop in which she had participated in 1993. The 40-hour one week workshop was taught by high school teachers for other
teachers on ways to teach geometry in non-traditional ways. During this experience, Ardella realized that she already used many of the ideas presented in the workshop:

> It was excellent. But to my amazement, I found that I had already changed my style to incorporate much of what they were advocating. I came away with more ideas and a desire to have computers more available for my students. There is a notebook of ideas from this class.

Through experiences such as taking a college transformational course, teaching a pilot geometry course, and taking a workshop of teaching geometry, Ardella grew to like geometry. In addition, she used these experiences as opportunities to learn more about teaching geometry.

**View toward curricular change.** Ardella’s view toward curricular change including goals, content, and instructional methods was suggested in her geometry and teaching biography. Ardella’s biography suggested that she has sought out opportunities to improve her geometry instruction through college classes, a geometry pilot study, and a geometry workshop. Based on interviews with Ardella and observations of her teaching, she appeared to be a teacher who continually strived to improve her teaching. When asked to reflect on this supposition about her attitude toward change, Ardella responded "Yeah, I agree. I don’t sit still."

During the first interview, the initial reference to reform occurred in Ardella’s response to the question "How
did you decide what content was to be included in your course?" Ardella replied "Well, in working in this district I've been on committees to decide with other teachers and looking at the national [Curriculum Standards recently]." In response to later questions in the interview about the Curriculum Standards (1989), Ardella felt that she was incorporating the standards in her lesson planning and that they provided a validation of what she was already doing with her geometry course:

Well I think I've read them [the Curriculum Standards] enough times to know that I'm incorporating them in my big picture, lesson planning. I guess if I saw the standards written down in front of me I'd probably say, "Yes I'm doing this here and this here," but I feel confident that I'm using the spirit of the standards in my teaching. When I read them I said, "Yes, great job." It's not like it's new to me. It was nice to get permission to not teach some stuff that I'd often asked myself, "Why are we doing this?"

Ardella's reflection on the state-legislated reform movement indicated her willingness to consider some of the reform ideas and showed her concern with what would be done with the content:

I would like all the math programs to adapt and change to the kids' needs. And if this [state reform effort] bumps us towards that a little quicker, that's fine with me. . . . I don't object to the thought behind it, to get kids ready for the work force, to help them be accountable to do the integration portion. . . . I think we need to make sure we keep the content intact at some point in their training. I don't want to just throw it out and not replace it.
As indicated by her descriptions, Ardella did not want to change her practice simply because reform was popular. Ardella felt the challenge was to determine how reform ideas were to be incorporated and which ones were going to be excluded:

But what allows us to get that stuff in [reform ideas] and what do we throw out? You know it is just some really tough decisions and I don’t think you can just be a non-teacher or even a beginning teacher and know how to answer these questions yet. It takes so much time.

As Ardella made decisions regarding her geometry course, a reform agenda from any source was another resource to be explored.

An introduction to Ardella’s classroom. Ardella’s school was in its second year of a block schedule implementation. Her geometry classes were taught every day for a 90-minute block, which meant that a year-long course was completed in one semester. During the course of this study, Ardella taught two sections of formal geometry and team taught a newly-developed second year algebra combined with physics course. Both formal and informal geometry courses were offered at Ardella’s high school. Both courses were designed so that students could prepare for the Scholastic Aptitude Test (SAT) and for college courses. Students who demonstrated previous success in first year algebra took the formal geometry course. Ardella also supervised a Master of Arts in Teaching (MAT) preservice
teacher intern who worked with the geometry section of his choice. The section selected for this study was the remaining first-period geometry section taught exclusively by Ardella. This geometry section consisted of 13 male and 12 female students. All the students in this section were sophomores.

Ardella's classroom was a spacious room with 35 desks arranged in five rows, an overhead in the front left part of the room, and the teacher's desk in the front right. During a partner activity, desks were moved so that partners were "elbow to elbow." As the school year progressed, the room decor was enhanced by displays of students' projects such as enlargements of Far Side cartoons, origami bow ties, and heart posters.

Throughout the observation period, Ardella's dynamic presence in the classroom and positive interactions with her students were evident. Each 90-minute lesson followed a similar format. When the bell rang, students were seated in their designated seats near an assigned partner. Students checked their homework from a list of even answers on the overhead. After a greeting, Ardella told the students to get out their homework as announcements were broadcast over the public address system. With her clipboard in hand, Ardella walked up and down the rows looking at students' homework, recording the results on her clipboard. Ardella explained that each assignment was
worth 10 points and that she graded homework on "work and completeness, not so much correctness." This five-minute task completed, Ardella typically asked students for homework questions. For the next 15 minutes or so, Ardella worked student-requested problems on the overhead before starting the day's activities. These activities assumed varying forms such as large group discussions, large group investigations, example explanations, partner discussions, and hands-on activities.

The context in which the teaching and learning occurred in Ardella's classroom was characterized by her desire that all students learn mathematics. Ardella described this desire:

Just a sense of doing right by kids and making it worthwhile for long time learners and trying to reach every kid. I want to reach the kid who graduates from high school and stops there and is just part of the community to the kid who's going to be a rocket scientist. I want to meet those different levels of learning with everybody feeling they have some success if they work for it.

Ardella explained that "reaching kids" in geometry meant that students "were interested in geometry, thought about geometry even when they were not in the classroom, and had an active role in their own learning." In trying to reach every kid, Ardella felt that "enthusiasm on her part is crucial." Ardella also characterized the teaching and learning context in her classroom with respect to her expectations:
I think that’s what learning is about - is helping them be excited about it. All of this also includes taking them beyond what they know. And so, I don’t ever let my expectations of them to stay at where they’re at. I’m known to be a teacher that expects them to risk new ideas, to check it out. . . . I think that’s what teaching’s about - is letting them feel success way beyond what they think they’re able to do. I like to push kids and see their excitement for that. Most kids meet that challenge.

The characterization of the context indicated that Ardella had expectations of students succeeding beyond their perception.

Planning decisions and influential factors. Ardella’s planning for her geometry course began with intense planning one year before the study was conducted. At that time Ardella returned to the teaching of geometry after a three year hiatus, after training in cooperative group learning, and at the start of the implementation of block scheduling at her school. As she made plans for her geometry course, Ardella stated that she found herself "pouring through books to see what I could bring in to help students invent their understanding of geometry."

Ardella’s training in group learning, consultant work with colleagues, and research reading also were sources of ideas for her course planning. From this intense planning, Ardella made decisions regarding the scope and sequence of the course and developed written lesson plans. While reflecting on the overall plan for her geometry course
based on the previous year's planning, Ardella felt that she had "the course pretty well developed."

For the year of the study Ardella stated that she "used the general ideas" from her overall curriculum plan, but emphasized that she "never does the course again the same way." Ardella stated that "It is kind of hard for me to do the same thing in the two periods I have because I am open to what the kids want and where we go with stuff [discussion of concepts]." Ardella explained that on a day-to-day basis she made decisions about whether she had the best sequence, whether a hands-on activity existed for the lesson, and whether to skip a concept because students already understood it.

Ardella's course goals consisted of ideas related to the significance of geometry in the real world, the relevance of geometry as an example of a mathematical system, and the suitability of geometry as a setting for developing communication as well as problem solving skills. Ardella started the discussion about her course goals for students by stating that she wanted her students to "appreciate their world, to see that mathematics is really everywhere around them." Supporting her goal that mathematics was everywhere around students, Ardella felt that geometry was a class that had "applications to the real world." Believing that "kids have intuitions about geometry already," Ardella wanted to develop these natural
abilities as students pursued the goal of "developing an appreciation of the relationships of shapes."

As part of her course goals, Ardella wanted her students "to sense the mathematicians' deriving of the course." Meeting this goal meant students experienced how mathematics was put together through the interplay of inductive and deductive reasoning. Another course goal Ardella had for her students was "to be good problem solvers." She felt that part of being a problem solver was having the "tool of perceiving shapes." Believing that "when kids articulate what they are doing in a math classroom that the mathematics stays with them," Ardella also wanted her students "to communicate mathematics." For Ardella, communicating mathematics meant reading, writing, and talking mathematics.

Ardella's reflection on these goals for her students' geometry learning indicated that her planning decisions were influenced by her research reading, her intuition on what students need, her teaching experience, her work with colleagues and different textbooks, and her feelings:

I've read some research as I've taught these last 19 years, but I also feel that teachers work on intuition and if we really get into what the kids' need, then we sense that; and over time it's just through my experience, through collaborating with other math teachers, taking geometry workshops, teaching out of eight different textbooks by now, doing things differently every year. These all add to my feelings about what's most important here.
Ardella described her anticipated content sequence of the geometry course as undefined terms, geometry vocabulary, constructions (incorporated throughout the course), triangles, quadrilaterals, deductive thinking with triangles and quadrilaterals, similarity, special right triangles, circles, three-dimensional objects, perimeter, area, volume, and projects.

Ardella identified the textbook, Geometry (Clemens, 1990), as a guide for the content sequence. She stated "I agree with it well enough." Ardella had two planned deviations from the textbook's sequence: (a) delay the work with logical reasoning and proof, and (b) use constructions (chapter nine) throughout the course. An explanation for these deviations was based on Ardella's beliefs about constructions:

Constructions are much more fundamental in my book [not a reference to the textbook] than jumping into proofs because reaching the kids through kinesthetic, tactile kind of learning is essential. I think we grab more kids and allow kids to feel success.

Constructions were another example of Ardella’s intentions for students "to learn problem solving via playing with things."

In addition to the textbook, Ardella identified other factors affecting her advanced content planning. Factors identified by Ardella included district committees, the Curriculum Standards (1989), and her collaborative work with other teachers:
Well, in working in this district I’ve been on committees to decide with other teachers and looking at the national [Curriculum] Standards recently. But even before then, collaborative working with other teachers as to what people thought should be in a geometry course and generally going with the traditional view of geometry, but making the technique of teaching that traditional content fairly non-traditional.

When asked to explain what she meant by a non-traditional teaching approach Ardella responded by first describing a traditional approach as the way she was taught with "lecture followed by an assignment" and on the next day "correcting the assignment followed by more lecture." Ardella included in her description of traditional teaching the idea of "following the book, getting through what someone before has deemed important." She continued by stating that she "still does some traditional teaching when it’s appropriate." As an example, Ardella explained that "I use drill and practice to back up what students know intuitively and what they have learned concretely." For example, students explored angles of a polygon using pattern blocks and then worked on a book assignment to apply the information discovered during their exploration.

Ardella’s instructional plans supported her intention to use techniques of teaching that involved more than lecturing by the teacher. As previously indicated, Ardella planned to use constructions throughout the course. Ardella’s plan indicated that she used a construction related to a given concept when the concept was being
developed rather than as part of a construction chapter. For example, Ardella planned to do the "copy an angle" construction when students were learning about the angle concept in chapter two.

The use of constructions, as well as other hands-on activities, supported Ardella’s desire to use kinesthetic learning as often as possible. Examples of other planned hands-on activities included the use of miras to explore parallel and perpendicular lines, the use of paper folding to investigate angle bisectors of a triangle, and the use of origami to practice geometry vocabulary. Ardella labeled the use of hands-on materials as the "toy aspect" of geometry. Ardella stated that "I didn’t used to think that [playing with toys] was ok for high school students." She found that "kids who’ve had that kind of learning already find it enjoyable and don’t object and become great helpers for other kids too." As stated previously, through Ardella’s experience with kinesthetic learning she felt that she had engaged more students in their learning and had allowed students to feel success.

Ardella’s plans for instruction also included a commitment to using cooperative groups as a predominant method of instruction. For Ardella, cooperative group activities ranged from partners working a worksheet to a small group completing an activity in which each person had an assigned duty. Ardella used cooperative group
activities for varying purposes such as exploring, practicing, and applying a concept. Previously, Ardella felt that "high school students came with the expectation of a social environment" and noticed that "when I called for it [learning in a social environment] I got a high response."

With the desire for more cooperative learning techniques, Ardella chose to take a workshop three years ago on cooperative learning, meeting for a three-hour block once a week for six weeks. While Ardella’s training in cooperative learning techniques encouraged her to try to use these techniques in the classroom, her students’ success with cooperative learning supported her continual use. Describing her experience with using a cooperative learning teaching strategy Ardella stated, "When I cautiously tried it [cooperative learning] three years ago, kids responded well, their learning was stronger. When I gave a test they seemed to do better on it. . . . They’re getting their social needs met and still learning more geometry."

Based on her belief that students need a variety of methods of instruction, Ardella also planned to use lectures, drills, games, and student written and verbal explanations of their mathematics. Ardella provided additional reasons for her choice of instructional methods by explaining that "it has to be fun teaching or why are we
doing it" and by referring to the "enthusiasm in the kids" when she varied her approach.

Incorporated into Ardella's overall planning was a designated day of the week on which quizzes and tests occurred. Each Wednesday students took either a 30-minute quiz or 60-minute chapter test. Ardella explained the reason for a set quiz and test day, "And so this quiz and test day just helps them to know that every weekend they should be thinking about it [geometry] and that even on weekends when I am not in their face with geometry, that they should still study geometry." With this designated written evaluation day, students often started new material before being tested on the old material. Ardella stated that students "complain bitterly" about having new material, but she found that this schedule allowed her to help students "keep the material alive longer," which in turn supported her goal for students to "internalize the material" rather than "to know it for the test." In addition, Ardella found that parents supported having a designated written evaluation day.

Interactive decisions and influential factors. An examination of Ardella's instruction provided information about her interactive decisions and factors influencing those decisions. Interactive decisions include decisions reflecting an implementation of plans and decisions made during day-to-day lessons. The decisions informing this
study were those regarding the geometry course involving goals, content, and instruction.

Students were provided with opportunities to make progress toward the course goals based on observations of Ardella and conversations with her. Ardella's course goals consisted of ideas that implied the significance of geometry in the real world, the relevance of geometry as an example of a mathematical system, and the suitability of geometry as a setting for developing communication as well as problem solving skills. The foundation for all of these ideas was students' learning and appreciation of shapes.

Opportunities for students to learn about shapes were evident in the content of the lessons. As expected, shapes in the form of triangles, quadrilaterals, and polygons were addressed during many of the lessons. The content of the lessons also included the use of shapes to explore other concepts. An illustration of using shapes to explore other content included the use of origami bow ties and origami birds as students learned about geometry vocabulary such as line segments, angles, and triangles. Another example of using shapes to explore other content included the enlargement of a pictorial poster as students studied similarity and the use of tangrams as students studied polygons.

In the process of learning about shapes, students had opportunities to make progress toward the goals of sensing
mathematicians' derivation of geometry, communicating mathematics, and problem solving. The goal that students be able to understand mathematicians' derivation of geometry was demonstrated in observations of Ardella's teaching. She attempted to include both inductive and deductive reasoning in her lessons. Examples of Ardella's dialogue during instruction showed how she explained the two types of reasoning:

Again, this is inductive reasoning not deductive and I want to show you both. . . . Inductive reasoning is where we reason by many examples. By many examples. Okay. And here we are, 27, or there abouts, different examples, and all getting very close to this. [The measure of the exterior angle of a triangle equals the sum of the measures of the two remote interior angles.]

[10 minutes later in lesson.] That's a deduction because I could state this, and use an old theorem and an old definition to tell you the reason why this is true and come up with the exact same thing that you knew 10 minutes ago. The sum of the two remote interior angles of a triangle equals the exterior angle.

Ardella's description showed that deductive reasoning was used to verify the conjectures made with inductive reasoning. A second type of verification of conjectures occurred in Ardella's classroom when students used a second method to explore their conjectures. For example, students generated conjectures about the triangle congruence postulates based on their work with angles and straws. Using constructions, Ardella led a discussion on the verification of the conjectures. Ardella explained that it was her philosophy to "give students ownership [through
creating their own mathematics] and then to let them see what a mathematician needs to go through to make that verification."

Each observed lesson also contained opportunities for students to communicate mathematics. For Ardella, part of the communication emphasis was on students learning the language of geometry. When a student asked during class "Why does it matter that it's called that [segment addition theorem]," Ardella replied "Because if I say those words, I want you to do 'click I know what she is talking about'. We have to talk the same language here." The communication emphasis was also on students' development of their abilities for describing mathematical concepts, processes, and procedures. For example, when the class was learning about conditionals, students wrote down the converse of a given statement, discussed their work with their partner, and then shared their results with the class.

With respect to problem solving, Ardella wanted her students to be visual learners. For Ardella, this meant students would be able to use the tool of shape perception when solving problems. Observations of Ardella indicated that she regularly provided students with opportunities to become aware of shapes through visual and tactile experiences. Students then used this shape awareness to discuss a concept or to solve a problem. For example, during an investigation involving congruence postulates
students created triangles with straws and paper angles. Students compared their work with each other and then as a class made a conjecture based on their investigation. Other visual and tactile experiences that promoted visual problem solving included representing intersecting planes with pieces of paper in order to look at line and plane relationships, using miras to explore parallel and perpendicular lines, and utilizing patty paper foldings to explore lines of concurrency.

The classroom observations did not provide the researcher with enough information about students' opportunities to meet the goals identified by Ardella as "to appreciate their world" and "to see that mathematics is really everywhere around them." However, Ardella explained how these opportunities were given at times other than the observations. For both of these goals, Ardella used writing assignments to foster students' appreciation of the world around them and students' abilities to see mathematics everywhere around them. The most recent writing assignment involved students taking a picture from their family photo album or a magazine and describing all the geometry in it. Ardella explained that the culminating assessment for both of these goals was a final project (end of the year project) where students were asked to select an idea that would communicate to their audience (classmates and Ardella) that geometry existed outside the textbook.
After students presented and explained their projects to the class, Ardella asked them to explain what they learned from their project. Ardella explained the success she had with this project last year. Her description indicated that her students had made progress toward appreciating their world and seeing that mathematics was everywhere around them:

And that is definitely where, when I asked them [as part of the project] to self-assess in the end. And what did they learn? Almost every student last year when I did this showed their appreciation for geometry in the world. It was just all around them, and they were even pretty excited about it and they weren’t shy to tell me that in their write-up.

Support for Ardella’s attempt to implement her envisioned geometry course was provided by the notion that topics presented by Ardella during instruction matched the topics planned before the start of the course. Observed lessons included topics such as conditionals, perpendicular lines, angles, triangles, and quadrilaterals. The sequence of topics, however, did not occur as planned. Departures from Ardella’s planned sequence occurred in two areas: (a) the placement of proofs, and (b) the placement of the similarity chapter. In planning discussions Ardella had indicated that she planned to address proofs after students learned the content related to triangles and quadrilaterals. In preparation for proofs, Ardella explained that "converse me" arguments were to be a regular part of discussion of course material. For example, as the
class worked with supplementary angles, Ardella asked the students to convince her that a supplement to an acute angle was obtuse.

While Ardella was working with her students on the congruent triangle postulates, she changed her mind about waiting to do proofs until the quadrilateral unit was completed. Ardella explained "I changed my mind about waiting longer on proofs. These kids were ready. They were giving me clues. It didn't make sense to do more convince me stuff." Ardella then proceeded with the congruence postulates and two-column proofs.

The second change in sequence involved the placement of the chapter containing quadrilaterals. Originally, Ardella planned to follow the general sequence of topics presented in the textbook. According to the textbook's table of contents, chapter four addressed congruent triangles, chapter five addressed congruent triangles and parallel lines (quadrilaterals were included in this chapter), chapter six addressed similarity, and chapter seven addressed right triangles. As part of the work with her MAT intern, Ardella predicted that the geometry classes would be ready for the similarity chapter at the time that he was scheduled to teach. As the intern's teaching time drew near, Ardella realized that the classes had not had time to do the quadrilateral chapter before starting the similarity chapter. Thus, Ardella made the decision to
complete the congruent triangle chapter in both geometry sections and then to start the similarity chapter in both sections. Ardella continued to teach the first block geometry section while her intern taught the third block geometry section.

When the similarity chapter was completed, Ardella returned to teaching both sections. At this time Ardella chose to work with the right triangle chapter before going back to address the chapter containing quadrilaterals. Ardella's reason for this decision was based on her use of similar triangles (including ratios and proportions) when addressing right triangle proportions and special right triangles. Ardella's reflection on the placement of the quadrilateral chapter indicated that the changed arrangement of geometry chapters was a new decision for her and was successful:

Just the surprise that the chapter on quadrilaterals moved around, put it later, two chapters later, has continued to be real successful. . . . I have toyed with changing and rearranging things before in my [teaching] life. I do that most of the time I have taught geometry, but this one [placement of quadrilateral chapter] I haven't tried before with geometry.

Additional comments by Ardella about this rearrangement of chapters suggested that the focus on triangles for three consecutive chapters was helpful to the students. Ardella also found that the rearrangement provided her with the opportunity to have students address proofs again.
Ardella’s focus on proofs was restricted to her work with congruent triangles and quadrilaterals. With this rearrangement of chapters, she was able "to hit proofs with triangles, back off, and then hit proofs again with quadrilaterals." This "recycling approach" to proofs enabled Ardella "to see who had kept the basic proof ideas." For Ardella, keeping the basic proof ideas meant students were able to use deductive reasoning to verify conjectures.

Ardella’s teaching showed that her overall plans for instruction were implemented. Ardella’s planning decisions predicted the use of cooperative groups as a major instructional method utilized during instruction. During cooperative group activities Ardella expected students to learn and apply mathematics while interacting with others. Interactions provided students with opportunities to communicate mathematics while they strove to come to an agreement upon the concept being addressed in the activity.

As suggested by the use of partners in all nine observations and the frequent reference to partner activities in Ardella’s lessons plans, partners were the predominant form of cooperative group work. Examples of partner use during instruction were provided by samples of Ardella’s dialogue during instruction. In the first example involving an investigation of the relationship between an exterior angle and its remote interior angles,
Ardella’s statements showed that in partnerships students verbalized ideas with each other:

How could I write this down? You formulate it in your mind and see if you can use what I named as these angles. What was one called and what were three and four called and how could we write this as a theorem? Would you turn to your partner and whisper this to them?

The second example involved students working as partners on an angle relationship activity. Ardella’s statements showed that in partnerships students worked together on an angle relationship activity before comparing results with another partnership:

And we are going to play that little game. Okay, will you get with your partner. . . . Okay, then what you are going to do is you are going to spread these pieces out on your desk and you are going to decide which ones of these [problems involving angle relationships] are true and which ones are false. . . . Then, I want you not to say "I’ll do this half and you do this half." I want you to do each one at a time. . . . What I want you to do is to verify that you did it right by comparing it with another group only when you are done and another group is done.

The partner format was also used for worksheet activities, for each person checking on the partner’s progress during large group instruction, and, occasionally, for taking quizzes. The importance of the partner format to Ardella’s classroom was also supported by the fourth pairing of partners that occurred during the tenth week of school.

Even though the predominant form of cooperative groups was partnerships, students also did work with more people than their partners. One example occurred when two
partnerships formed a group in order to compare their partner work as described in the preceding quote. Another situation in which students worked with more than their partners transpired during lessons designed to generate conjectures. In this setting, Ardella asked her students to "compare your work with your partner and at least two other people."

In addition to the use of cooperative groups, Ardella's instruction confirmed her intention to use hands-on activities as a major instructional method. Constructions were one type of hands-on activity that Ardella used in her classroom. The first introduction of constructions to the students occurred on the sixth day of class. For the introductory lesson, Ardella demonstrated the basic constructions on the overhead while her students practiced them. Examples of the basic constructions included copying a segment or constructing an angle bisector of a given angle. Basic constructions were incorporated into the lessons at this time to foster the discussion of geometry vocabulary. The significant role constructions played in the course were reflected in Ardella's comments to her students on the first day constructions were used. In her comments, Ardella indicated to her students that constructions would be used throughout the course to learn geometry relationships:

For your sake I would like your constructions to be in a space so that three-and-one-half months
later when I say, "Remember the constructions on [pause] and you can say 'Oh, yeah'." You can look back, if you don't. . . . These constructions are in your book, but they are not in your book until chapter nine, of all the most ridiculous places. The authors decided to wait until you know a whole bunch of geometry instead of saying this is the way to learn more relationships. So I am doing my own stuff here, but if you really need to read the book, chapter nine has these in there.

Examples of how constructions were later incorporated into Ardella's lessons included the use of constructions to further explore manipulative-based conjectures about triangle congruence, the use of constructions to examine the geometric mean with a second method, and the use of constructions during an impromptu exploration of finding the center of a circle. Ardella's commitment to use of constructions throughout the course was one example of Ardella defining her geometry course rather than the textbook defining it.

Besides constructions, Ardella used activities involving hands-on materials such as miras, pattern blocks, origami, patty paper, and straws. Ardella used these hands-on materials for exploration of concepts and relationships between concepts. During the final interview Ardella was asked for her response to the comment "You can find a way to do most things hands-on." Ardella agreed with the comment. Ardella's reasons for agreeing with the comment indicated that, when she used hands-on activities,
students retained the mathematics longer and were excited about learning:

And when I can't I feel real bad. So, yeah, it has been a real major goal of mine. Just because when I have included it in the past it sticks more and the kids are excited about learning. When I have to just go on about the stuff in front of them I almost feel like "read it yourself." I don't want to be that kind of teacher where all I am doing is blabbing for months in the book. So yeah, the hands-on is definitely a way that I try to teach consistently.

Other instructional methods used by Ardella, together with her use of cooperative groups and hands-on activities, illustrated an implementation of her plan to use "a huge variety of experiences" in order to "meet the kids on the different levels." Additional instructional techniques used included Ardella using lecture/recitations when addressing theorems, Ardella working examples on the overhead (often involving diagrams), students making conjectures based on work done by the whole class, and students practicing problems on white boards and showing their work to Ardella as students completed problems.

Variety also meant looking at the same topic in more than one way. Ardella's response to a student's question about learning the geometric mean by constructions after having learned it algebraically characterized her feelings about the importance of using variety and making mathematical connections in her classroom. Ardella's explanation indicated that she recognized the variety of
learners in her classroom and that she wanted to help her students see the bigger picture:

See we have all kinds of learners in here. Some people might say, "Oh I can do that but I want to know why it works, and I want to know why with shapes." And so here is another way. I like you to see as many different perspectives as we have so that things start making sense and that you see the bigger picture. Part of my job is to show you the connections to this. And if I can show you the geometric mean definition that works with right triangles in general and works with a construction, then you get more confident.

The variety of instructional methods used by Ardella had one element in common - student involvement. Students were engaged in Ardella’s lessons while taking notes, volunteering to explain an idea, answering questions as a non-volunteer, working with manipulatives, making conjectures, and working with a partner. Observations verified that students did not just sit in the classroom without participating in the lesson. Ardella’s commitment to having her students involved in the lesson supported a statement she made during the first interview: "I like to let the kids feel responsible for their learning and to give them opportunity to show me."

In addition to decisions made regarding Ardella’s overall content and instruction, decisions were made during daily lessons with respect to the implementation and the context of the lesson. When Ardella reflected upon decisions made during the lesson, her comments revolved around decisions made about instruction and student
behavior. Ardella’s reflections on decisions made about the instruction during a given lesson indicated she made decisions about the use of partners, about how the class worked with the content, and about how she used class time. For example, a partner-work-decision occurred when Ardella and her students practiced writing conditionals. At this time, she decided to have the students share their example with their partner before volunteering to share their example with the class. Ardella explained that "I know that sharing with a partner is effective and it gets more kids on track." In general, decisions involving partner work were made based on Ardella’s belief in the benefits of partner work, her feelings about what students needed, and her wish to maximize participation.

An example of a decision relating to how Ardella wanted the students to work with the content occurred when Ardella was discussing theorems from chapter one. As Ardella was presenting the first theorem, "all right angles are congruent," she decided to have students write the theorem in if/then form. Ardella explained that "I wanted to incorporate the stuff in the book, the given and the [to] prove. I wanted to give them more practice with conditionals." Related to decisions about how students needed to work with the content were Ardella’s decisions about how she used class time. Examples involving content and time included varying situations such as how many
examples can be completed while still giving students enough time to take the quiz, how quickly minor theorems can be addressed, and how long the class worked on review problems. Decisions involving how students needed to work with the content and how Ardella used class time reflected Ardella's continual attempts to meet the needs of her students.

Occasionally, during Ardella's post-lesson reflections on decisions made during the lesson, she stated "I had to decide how comfortable I was with students' behavior." Ardella had to make decisions about individual students whose behavior either appeared to disrupt the lesson or dominate the discussion. Ardella's response to disruptive students included a statement of the student's name in order to get the individual back on task. If students continued to disrupt, Ardella said "Linda, this is my time now" or "Chris, move to the last desk in the first row."

Ardella's response to the researcher's comment, "It appeared that two students dominated the discussion today," indicated that she considered the welfare of the whole class when deciding what to do. Ardella stated, "I knew they were dominating, but I also knew that 90% could do it. I don't know if you noticed that I called on everyone today. They were getting it. I could give Oliver and Jane time."
On occasion the class was vocal when they did not want to do another activity. Ardella explained how she dealt with their whining. Sometimes Ardella "needed to be sure that I wasn't assigning busy-work with partners." In other situations, she "had to get on their case" because the students did not know the material and were using their whining in hopes of not having to do any more thinking. In general when making decisions about students' behaviors, Ardella seemed to balance her agenda with what was best for her students.

Ardella's actions during instruction provided the researcher with decision-related issues to pursue and routines to describe. The importance of homework discussion was apparent in every lesson. Ardella set time aside for the discussion and gladly answered students' questions. When asked "how much homework discussion do you want during your class and why," Ardella explained "I don't want the students to get bored or to do the homework for them. During the checklist I get a sense of what they know and what they need. If they check out during the discussion, I will stop." As Ardella worked and discussed the homework problems, she tried to involve the student who asked the question in the discussion. Ardella's attempts to involve the student usually included having the student read the problem and answer Ardella's question regarding format of the problem. If the individual was not able to
answer Ardella’s questions, she either gave some hints or called on another student to explain the problem.

During lessons, Ardella and her students had conversations on the lesson’s topic. Based on the willingness of students to participate and the variety of responses that occurred, Ardella had decisions to make regarding the progression of the discourse. When asked about this, Ardella replied "I just do it. I don’t think about it." Students’ participation in discussion seemed to be fostered by Ardella’s encouragement and her validation of their responses. An example of dialogue during a lesson on biconditionals illustrated how Ardella promoted students’ involvement in the discussion:

Ardella: So let’s write this sentence and its converse as a biconditional. In fact, let’s practice it out loud before we write it down. Who wants to do this? Write it as an if and only if sentence. If and only if. . . . Ray would you try [this] for us?

Ray: A number is prime if and only if it has exactly two divisors.

Ardella: Did you hear him? He started out saying "A number is prime if and only if it has exactly two divisors." Isn’t that just nice and clean?

[Comment by Cindy that is not audible.]

Ardella: I know, I heard you start to say that Cindy, that’s great.

Examples of other comments by Ardella that validated students’ responses and work were "I’m glad you asked that question," "All right, very good," and "You read the book;
good job." Along with the enthusiastic manner in which Ardella spoke, her eye contact with the students and her movement around the classroom as she spoke also fostered students' involvement in the discourse.

The progression of discourse also appeared to be influenced by the information gathered by Ardella's use of ongoing assessment methods during the lesson. One method used by Ardella was polling her students during the lesson. Ardella polled students after making a statement like "Raise your hand when you have an answer" and then watching for students to respond. When asked how she used this information Ardella replied "I know if they know." Ardella continued by stating "Hands go up to say they are listening. Eyes are the key." Observations suggested that Ardella used the information from students' eyes to guide the discourse and the instruction of the lessons. For example, when students were trying to create triangles with two sides (straws) and one non-included angle, only two students indicated with their hands they were able to make a triangle. Ardella continued with the lesson by asking questions about students' triangles as if all the students had created triangles. Ardella explained later, "Many of the students had created triangles, but lost confidence when their triangle did not match their neighbors' [triangle]." Additional comments by Ardella indicated that she saw the lack of confidence in students' eyes and guided
the discussion to help the students understand why their triangles were different.

Other ongoing assessment methods used by Ardella included a class discussion of practice problems and her circulation around the classroom as students worked individually or in groups. In general, Ardella appeared to use the gathered information to make instructional decisions that enabled her to foster students' understanding of the concepts while keeping her goals for the day in mind.

Summary of Ardella. Ardella was a secondary teacher beginning her twentieth year of teaching mathematics and her fifteenth year of teaching geometry at the time of this study. Ardella’s school was in its second year of a block schedule implementation. In the block schedule, Ardella taught formal geometry for 90 minutes every day. Both formal and informal geometry courses were offered at Ardella’s school and were designed so that students could prepare for college courses. The formal geometry course, however, was intended for students who demonstrated previous success in first year algebra.

Ardella’s planning decisions defined a multifaceted geometry course. The basis for the definition of Ardella’s geometry course were her course goals. Ardella’s course goals consisted of students appreciating the relationships of shapes, seeing a mathematician’s derivation of geometry,
being able to communicate mathematics, being a problem
solver, appreciating their world, and seeing that
mathematics is everywhere around them.

Ardella’s teaching reflected an implementation of her
planning decisions. In an energized, cooperative
classroom, students had opportunities to learn about
different aspects of geometry while striving to meet the
course goals. Aspects of geometry emphasized by Ardella
were geometry as a content knowledge base, an example of a
mathematical system, and a setting for developing
communication and problem solving skills. In addition,
Ardella’s instruction hinted at the connection between
geometry and the real world.

While working toward the course goals, students
studied geometry content as Ardella used instructional
methods such as cooperative groups, hands-on activities,
drill and practice, guided discussions, and student written
and verbal explanations of their mathematics.

The foundation for the factors influencing Ardella’s
decision making were her commitment to being a life-long
professional learner and her desire to help all students
learn geometry. As a life-long professional learner,
Ardella continually strove to improve her geometry
teaching. Components of Ardella’s process of improving her
teaching consisted of reading research, participating in
workshops, teaching a pilot course, collaborating with
colleagues, examining textbooks, and utilizing the *Curriculum Standards* (1989). The process of improving her teaching also enabled Ardella to define her geometry course. While striving to help all students learn geometry, Ardella determined what students needed to meet her course goals and considered how to involve students in their own learning. Thus, Ardella made decisions about her geometry course based on how she could best balance her geometry agenda with the needs of her students.

Ardella used her textbook as a guide as she made decisions about the content addressed in her course. The content represented typical topics found in a secondary geometry class. Students first learned about undefined terms and basic geometry vocabulary such as line segments, angles, and parallel lines. As the course progressed, students worked with triangles, proofs, similarity, special right triangles, and quadrilaterals.

The textbook was used as a guide, but did not define the geometry course for Ardella. Examples that demonstrated how Ardella defined the course rather than her textbook included her decisions about the use of constructions and the placement of students' work with logical reasoning and proof. Ardella decided to use constructions throughout the course rather than waiting until chapter nine to discuss them. Ardella used basic constructions such as copying a line segment or finding the
perpendicular bisector of a line segment to discuss geometry vocabulary. Constructions were also used to create figures and to verify conjectures made from students' work with manipulatives. Ardella felt that the use of constructions provided students with kinesthetic learning which in turn enabled students to be successful with learning geometry.

Ardella's textbook addressed logical reasoning and proof in the first chapter. Ardella's choice was to delay the discussion on proof until after students had learned about triangles and quadrilaterals. This delay supported Ardella's decision that proofs be part of her geometry course without dominating the course. Ardella's decision about when to do proofs was also influenced by students' needs. During this study, Ardella chose to do proofs after students learned about triangles rather than after they learned about quadrilaterals because she felt the students were ready to do proofs.

Ardella's willingness to differ from the textbook was also shown when she decided to alter the planned content sequence for the course. Instead of following the textbook's outline of triangles, quadrilaterals, similarity, and special right triangles, Ardella delayed the study of quadrilaterals until after special right triangles. Even though the decision to make this change was instigated as a result of her MAT intern's schedule,
Ardella’s confidence in herself enabled her to work with a new sequence in a manner that was beneficial to her students.

For her instruction Ardella used the routine of homework discussion followed by presentation and then practice and application of new content. The details of the routine’s components characterized Ardella’s instruction. Part of the homework discussion involved a visual check of homework by Ardella and a student answer key check with solutions on the overhead. Ardella’s focus during this part of the discussion was students’ accountability for their own learning.

Students’ accountability for being involved in the lesson was a theme throughout Ardella’s instruction. Students were expected to ask homework questions, to take notes during discussions, to participate in class discussion, to discuss ideas with their partners, to make conjectures based on work with manipulatives, to do practice problems on white boards, and to work with their partners on activities. These various ways in which students were expected to be involved in Ardella’s lessons demonstrated the variety of instructional approaches used in her classroom. Ardella varied her instructional approaches because she wanted to accommodate her students’ different styles of learning.
In addition, Ardella’s decisions about instructional approaches were influenced by her wish to meet students’ social needs. As part of her instruction, Ardella chose to incorporate cooperative groups. Cooperative groups in the form of partnerships were an integral part of Ardella’s classroom. Every observation included students working with their partners. With their partners, students compared work, discussed definitions, compared investigation results, and completed practice worksheets. Ardella’s experience with the use of cooperative groups in the form of partnerships demonstrated that students learned more geometry as well as met their social needs.

Ardella’s awareness of her students’ needs was apparent as she utilized ongoing assessment techniques during instruction. To gather information about her students’ understanding of the concepts, Ardella called on volunteers and non-volunteers during classroom discussions, had students show their work on white boards, and watched students’ eyes during classroom discussions. In addition, Ardella systematically walked around the room looking at students’ work and consulting with individuals and groups about their work during all parts of the lesson. Summative assessments such as written quizzes and tests, cartoon enlargement project, and an upcoming geometry and the real world project also provided Ardella with information about
her students' understanding of geometry concepts and relationships.

Ardella’s awareness of her students’ needs was also evident in her reflections upon decisions made during instruction. Ardella’s reflections on her decisions indicated that she tried to meet the needs of her students while focusing on the goals for the day. In most situations, Ardella determined how class time should be used, how students should work with the content, and whether students’ behaviors were interrupting other students’ learning.

Ardella’s planning and interactive decisions demonstrated that she had defined and implemented her desired geometry course. The predominant factors influencing Ardella’s decisions were her definition of her geometry course, her willingness to work to improve her geometry teaching, and her desire to meet the needs of her students.

Emily

Geometry and teaching biography. Emily had studied geometry in junior high mathematics classes, in a high school geometry class, and in college classes prior to teaching the subject. Emily described her earliest recollections of learning geometry in her seventh and eighth grade mathematics classes:
My earliest recollections of learning geometry are my seventh and eighth grade math classes. I was fortunate to have a couple of sisters (nuns) for teachers who truly enjoyed mathematics. This was the mid-sixties and women were just beginning to be encouraged to be "math-nerds." Both of these women had us making conjectures, measuring objects, developing our reasoning skills, doing paper and pencil calculations that I wish my students would (could???) do, and I loved every second of it.

Emily’s positive experience with learning geometry continued in the "formal" geometry class she took two years later. Reflecting on her high school geometry experience Emily remembered thinking that "these proofs were no big deal." She also remembered that "my friends would ask me 'how'd ya do that'"? Emily’s geometry experiences continued in college but not in the form of specific geometry classes. Instead, her experiences included the study of geometry as part of other mathematics classes. For example, she studied some non-Euclidean geometry in a topology course. Emily’s general feelings about her advanced courses were that "they were less challenging and not as much fun" as her early years of learning geometry.

Emily began her teaching career at a seventh through twelfth grade high school. She then taught seventh through ninth grade junior high for five years before starting her present position at a ninth through twelfth grade high school located in a small industrial town. At the time of this study Emily was beginning her twenty-first year of teaching mathematics and her sixteenth year of teaching
geometry. During the fall of the research project, the school enrollment was 1,150 students.

With respect to the teaching of geometry, Emily identified two important geometric experiences. The first was a 1985 through 1988 National Science Foundation Alternative Curriculum Seminar led by two mathematics educators from a local university. The seminar met for three 40-hour weeks during the first summer and one 40-hour week for each of the following three years. The focus of the geometry seminar was on the exploration of instructional methods other than lecture. Emily stated that this seminar "helped me bridge the gap between early seventies and early eighties teaching styles." Emily explained, "My first contact with using manipulatives in the classroom occurred during this seminar. I was not in college when manipulatives became popular education." In addition to learning about manipulatives, this seminar enabled Emily to be revitalized as a teacher, to gain a network of colleagues (high school teachers and college professors), and to later collaborate with one of the college professors. Specifically, Emily and the college professor wrote graphing calculator materials and made presentations at conferences. Emily characterized her feelings about this seminar by stating "this seminar changed my life forever."
The second important geometry experience was a 40-hour one week 1993 Woodrow Wilson TORCH (Teacher Outreach) Seminar on Geometry led by four dynamic secondary mathematics teachers. During this seminar the four leaders as well as the participants shared ideas about the teaching of geometry. For Emily the benefits from this seminar were her continued revitalization with respect to teaching and an acquisition of additional ideas for teaching geometry.

Emily completed the description of her geometry background by referring to the opportunities she had to attend conferences sponsored by the NCTM and the state organization of the Council of Teachers of Mathematics. Emily stated that these conferences "have given me tidbits of improvement for my geometry teaching." Emily’s geometry background indicated that she liked geometry and that she had continued to work to improve her teaching of geometry.

View toward curricular change. Emily’s view toward curricular change in goals, content, and instructional methods was suggested in her geometry biography. The biography suggested that Emily had sought out opportunities to improve her geometry teaching through participation in seminars and conferences. Emily had also been involved in recent curriculum work at her school. This work indicated that Emily continued to make a commitment to the improvement of her geometry course. For the 1992 to 1993 school year Emily and three of her colleagues "decided to
reinvent" what they wanted in geometry. Each of the teachers wrote a nine-week unit. Emily said, "I didn't like it; it was too fragmented."

In the summer of 1993, Emily met with four of her colleagues "to take the year before and pin it down." Emily explained that decisions were made after the group of teachers examined the school's mathematics curriculum and NCTM's *Curriculum Standards* (1989):

Last summer, the summer of 93, I got together with four of my colleagues and we examined our own curriculum and the NCTM *Curriculum Standards* and came up with what we felt would be appropriate goals for our geometry classes as we start into the next century.

In response to later questions in the first interview about the *Curriculum Standards* Emily explained that she felt the standards were directing them to help students use geometry in the real world. Emily stated, "What I saw them directing us to do was to get kids to be able to represent real world situations in geometric models, and then, from those models, be able to pose questions on their own and answer questions about those situations." Emily concluded her interview discussion on the *Curriculum Standards* by depicting them as an "underlying guideline" for what she did in her classroom.

For the 1994 to 1995 year Emily was "trying to build on last year's work" and at the same time "deal with 95-minute periods." Emily's school set up their block schedule in order for the teachers to have a common
preparation time. Emily explained that "their school was committed to the state-legislated restructuring movement" and that "the common preparation period was to help with the restructuring." Additional thoughts by Emily on the state-legislated reform indicated that she felt the intent of the bill was the same as her reason for being a teacher: "My own reason for being here is the kids. I want what’s best for kids. And I think that is the intent of House Bill 3565." For Emily, doing what’s best for kids meant "preparing students for life-long learning, for the world of work, and for higher education." Determining what was best for kids in the context of the *Curriculum Standards* (1989) and 95-minute periods was Emily’s challenge as she progressed through the 1994 to 1995 school year.

**An introduction to Emily’s classroom.** Emily’s school was in its first year of a block schedule implementation. In this schedule Emily taught all of her classes for 50 minutes on Mondays. On Tuesdays and Thursdays, Emily taught precalculus and two sections of geometry in 95-minute blocks. On Wednesdays and Fridays, Emily taught two sections of geometry and one section of first year algebra. The majority of students at Emily’s school studied geometry as tenth-grade students. A small group of accelerated students studied geometry in ninth grade as part of a grade-level block system within the school’s block schedule.
The geometry section selected for this study met during the last block of the day on Tuesdays and Thursdays. The class enrollment consisted of 11 male and 17 female students. The selection of this section provided the best opportunity for conducting informal interviews with Emily immediately before and after the lesson. This geometry class was the fourth time Emily taught a particular geometry lesson on Mondays when all classes met. The class was the second presentation of a given lesson on Tuesdays and Thursdays when only half of the classes met.

Emily's square-shaped classroom was almost filled to capacity with 28 desks arranged in five rows. In the left front of the room was the teacher's desk. In the front center of the room was a table and an overhead on a cart. Next to the overhead was a captain's chair with extended legs. The theme of the room's decor was color. At the top of each wall was a row of posters with bright colored paper behind them. Most of these posters consisted of sayings and pictures illustrating the sayings. For example, one poster stated "When life gives you lemons, make lemonade." Posters also were displayed around the bulletin boards. These posters had a geometric theme, for example a poster of regular polyhedra. Emily's room also contained an Escher bulletin board and an assignment calendar bulletin board. Toward the end of the study, students' line design visualization projects were displayed on a bulletin board.
Emily's classroom was characterized by her jovial presence and by her good rapport with students. Each observed lesson followed a similar format. Students walking into the classroom usually found a smiling Emily sitting in her captain's chair next to the overhead. The overhead had a message about the beginning of class such as "pick up a protractor" and "warm-up B /30 pts." With a statement of "ok, folks let's get started" or "ok, ladies and gentlemen, boys and girls" Emily started to explain what the students needed to do for the warm-up. Emily's reasons for using warm-ups included "to have something ready to go so class starts at the time it should, to review information that they [students] will use in today's lesson or the next lesson, and to be part of the notebook grade."

As students worked on the warm-up, Emily took roll and then walked around the room. As Emily circulated around the room she watched students work and answered students' questions. Warm-up activities ranged from 5 to 20 minutes in length and concluded with a class discussion guided by Emily. Emily called on volunteers and non-volunteers to state and explain their results. If needed, Emily provided additional explanation on the concepts presented in the warm-ups. At the conclusion of the warm-up discussion, Emily explained the point values for the problems and directed the students to record their scores on the
assignment sheet. This assignment sheet was a form prepared by Emily on which the students kept an inventory of warm-up, homework, quiz, and test scores. Students kept the assignment sheets in their notebooks until Emily collected the assignment sheets as well as the notebooks once every six weeks.

After the warm-up, Emily led a 10- to 20-minute homework discussion. Solutions for all assigned problems were addressed. Emily either read the solutions, had the problems and solutions written on the overhead, or called on students for solutions. Occasionally, solution discussions occurred because students had questions or because Emily chose to discuss a problem. Emily’s selected problems usually were problems about which students from earlier geometry sections had asked and whose solutions were already written on an overhead acetate roll. At the conclusion of the discussion, Emily asked the students to record their scores on their assignment sheets and then put their homework in the designated box on her desk. After the homework discussion, Emily proceeded with the day’s activities. These activities assumed varying forms such as lecture, large group discussion, small group activity, hands-on activity, reading, and individual work time.

The context in which the teaching and learning occurred in Emily’s classroom was characterized by her desire "to make geometry less threatening to students."
Emily explained that this ambition was based on her feelings about what students heard from their parents about the parents' geometry experience in high school:

I think part of my reason for choosing what I've chosen is that geometry can be very threatening to kids. One of the things they hear their parents say at home is that you know it was the most awful class I ever had to take, and I'm sorry you have to go through it, but that's life. You'd better get in there and do it.

Student comfort level was established through Emily's encouragement of student involvement in the lessons, visible concern for her students, and sense of humor. Students' actions during observed lessons also suggested an established comfort level. For example, students actively engaged in hands-on activities and readily participated in discussion as volunteers and as non-volunteers. Emily's approach to teaching geometry appeared to convince students that learning geometry was a real possibility. Her approach included the use of visualization activities, a focus on vocabulary, and a departure from the textbook's early focus on two-column proofs.

**Planning decisions and influential factors.** Emily's planning decisions for her geometry course were closely related to the work she had done as part of her department's ongoing plan to revise their geometry course. As stated earlier, Emily's decisions for this course revolved around building on department curriculum work from the previous year and managing the 95-minute periods.
During the summer prior to the start of this study's geometry course, Emily made calendar planning decisions for the course and pre-planned in detail about three weeks of material. Planning information was recorded in lesson plan format. Two-and-one half weeks into the school year, Emily continued with the lesson plan writing process. Emily made plans as she referred to the *Curriculum Standards* (1989) and to the scope (see Figure 1) created by Emily and her colleagues during the summer one year before the start of this study. The scope was based on the school's present curriculum and the *Curriculum Standards*.

Emily stated that the overall course goal for her students was to "develop their knowledge of geometry." When asked about her process of planning to help students meet this goal, Emily explained that she planned differently this year from previous years as she tried to use the blocks of time wisely. "This is generally what I do, no actually I'm planning differently then I have before. I make a map of time and a list of concepts. I try to use blocks of time wisely and try to stick to my five things." The five things to which Emily referred were five types of skills that were part of her course goals: (a) writing, (b) reading, (c) thinking (thinking in general, but also inductive and deductive reasoning), (d) visualizing, and (e) measuring. Emily explained that these
fives skills "could be thought of as overriding goals for the course and also as daily objectives."

GEOMETRY

GOAL:
The student will be able to represent real-world phenomena using geometric models; and from the models pose and/or answer questions about the phenomena.

REQUIRED SKILLS:
- use deductive and inductive reasoning techniques to identify and/or establish properties of or relationships among geometric figures
- classify figures (including parallel, perpendicular, congruent, similar, 2-D, 3-D)
- draw, interpret, and mensurate 2-D and 3-D figures
- use coordinate geometry to verify/establish properties of figures
- use transformations and coordinates to deduce the properties of figures (vectors)
- use transformations to deduce congruence or similarity
- use the properties of right triangles (Pythagoras, trig ratios)

Figure 1. Scope Prepared by Emily’s Department

Emily’s reflections on factors that influenced her course goals identified the scope created by her and her colleagues. Emily stated, "From those [scope ideas] I came up with my own personal slant on them." Emily’s personal slant meant that she approached this course by using lessons that enabled the students to build a vocabulary
base, involved the use of her five skills (writing, reading, thinking, visualizing, measuring), and were fun.

Even though Emily's stated course goals did not include the application goal cited in the scope, her first interview comments about what she found valuable about teaching geometry in secondary schools included references to application ideas. Emily associated the value of students learning geometry with students learning spatial visualization skills, developing a variety of reasoning skills, becoming acquainted with formulas, and reflecting on real-world mathematics:

Probably for me the greatest value is the need for students to learn spatial visualization skills and secondly the development of a variety of methods of reasoning and then to acquaint them with the kinds of formulas . . . rather than memorizing them like we had to . . . . I think it accesses a part of the brain that the algebra track does not. It stimulates the kid who maybe didn't enjoy the number crunching of algebra, allows them some freedom to reflect on mathematics and art, mathematics and architecture.

With the goal of developing students' knowledge of geometry, Emily identified the content for which the students would be responsible. Emily described the sequence at the beginning of the course as undefined terms, writing an effective definition (a technique to be revisited throughout the course), and geometry vocabulary. As Emily started to describe the content that followed geometry vocabulary she said "I can't really give these in any kind of sequence because I don't know what it's going
to be yet." The content that would be addressed included plane figures (polygons), triangles, properties of triangles, right triangles, and right triangle trigonometry. Emily felt that the content described would take them through at least first semester.

When asked "How did you decide what content was to be included in your course," Emily's reply indicated her planning decisions for content were influenced by her goals (influenced by her department's scope), her attempt to find activities students enjoyed and she liked, and the NCTM Curriculum Standards (1989). Emily described her reasons for her choice of content:

Well, looking at the goals. Hopefully your goals determine your content and at the same time trying to find activities that the kids will enjoy, that they'll feel good about taking part in. The Woodrow Wilson class that I mentioned awhile ago had some really neat nineties kind of materials. . . . If I find something neat it becomes part of my content because I really like it. I also think that articles I read in the Math Teacher and TOMT [the state mathematics journal] magazines tend to push you in a direction that you might not have gone on your own. And of course, the NCTM [Curriculum] Standards pushed us or should I say are pushing us.

Additional comments by Emily revealed that her textbook, Geometry (Jurgensen et al., 1988), as well as five other textbooks influenced her content selection. Emily explained that as she looked through the textbooks, she made decisions about content on "clarity, humor, the degree of difficulty to a certain extent."
Emily’s description of her reasons for content choices suggested that she had been influenced to make changes in her content. Emily stated "The most significant change for myself personally and probably for most geometry teachers is going away from formal, two-column proof." Another specific content change to which Emily referred was "the addition of things like transformational geometry to the curriculum."

Emily also felt that she had made changes in her instruction since she first started teaching. In reference to the first time she taught geometry Emily stated, "I probably lectured for 160 out of 180 days." Last year, Emily felt, "I had about a 50-50 split." For this year, she hoped that she would be using lecture for less than half of the instructional time. Emily’s change in instruction over the years was fostered by the research involving the use of hands-on manipulatives:

I think the evidence in research pointing to the need for hands-on kinds of manipulatives to stimulate the mind really helped me change to a different style of teaching. I don’t know why anybody ever thought that talking to somebody could help them visualize, but I guess I did.

Emily explained that her instructional plans for this study’s course included some lecture because "some of the time I think in order to accomplish my goals I need to do teacher-directed-lecture-type lessons." Emily also used some lecture because in her continual search for different ways to address geometry concepts "there’s some stuff I
haven't figured out any other way [than lecture] to do it."
Examples of concepts on which Emily lectured included basic
definitions such as rays, angles, and line segments,
definitions of convex and concave polygons, and the
exterior angle sum of a polygon.

At the same time, Emily's instructional plan indicated
that she planned to use instructional methods other than
lecture. Emily's choice to use more than lecture was based
on her belief that variety was needed "to try to do what's
best for kids" and "to keep students active and
interested." Part of Emily's plan to encourage students to
be active and to be interested in geometry lessons was to
have students involved in "explorations while working with
their peers, particularly in pairs or small groups."
Another part of Emily's plan was to use hands-on activities
involving tools such as miras, geoboards, compasses,
protractors, and wooden cubes.

Emily's instructional plans also consisted of students
reading and writing mathematics. Based on her experience
that "most high school kids have a hard time reading
anything technical," Emily planned to incorporate directed
reading into her lessons. As described by Emily, directed
reading comprised of students taking notes on their
reading. Her explanations indicated that the directed
reading could occur during class or as an assignment:

So we're going to experiment with reading it
[technical mathematics] in small groups, helping
each other take notes. Part of their assignment each day or often will be to read a section of their textbook and take notes and then the next day the partner will be responsible for reading their notes and checking for the highlights to be there and giving them a grade for that.

The writing process started on the first day of class when Emily had the students write her a letter of introduction. As the school year progressed, Emily planned to incorporate writing into her classroom in the forms of reflections, journal style writing, and paragraph narratives.

Emily identified two other factors influencing her selection of instructional methods. The first factor was her personal preference. Emily stated, "I also think that there's a lot to be said for knowing what you like to do. What's fun for me lots of time is fun for the kids." The second factor was influence of the people with which she worked, such as her colleagues and her student interns. From these people Emily acquired ideas for instruction. Emily expressed that she's "lucky to work with extremely clever and creative people." With this instructional plan Emily hoped to help her students develop their geometric knowledge as well as their reading, writing, visualizing, measuring, and thinking skills.

Interactive decisions and influential factors. An examination of Emily's instruction provided information about her interactive decisions and factors influencing those decisions. Interactive decisions included decisions
made reflecting an implementation of plans and decisions made during day-to-day lessons. The decisions informing this study were those regarding the geometry course involving goals, content, and instruction.

Students were provided with numerous opportunities to make progress toward the course goals based on observations of Emily and conversations with her. Emily’s overall goal for her students was for them to develop their knowledge of geometry. Students’ opportunities to develop their knowledge of geometry were evident in the content of the lessons, by students’ involvement in classroom discussion and activities, and by students’ work on warm-ups and assignments. The content of the observed lessons included undefined terms, angles, parallel lines, polygons, triangles, and quadrilaterals. Students’ contributions to discussions about this content included call-out answers to Emily’s questions that required one or two word responses, definitions for the lesson’s concepts, and generation of conjectures from student investigations. During homework and warm-up discussions students provided answers and explained their results. Many of the students actively volunteered to participate in discussions. By calling on non-volunteers, Emily was able to get most of the students involved in the discussions.

Emily had also identified five skills (writing, reading, thinking, visualizing, measuring) as overriding
goals for her course. When Emily described these five skills as goals, she also labeled them as daily objectives. Emily’s intention was to provide students with the opportunities to use these skills in each of her lessons. During the nine observed lessons all five skills were incorporated in two lessons, four of the five skills were incorporated in five lessons, and three of the five skills were incorporated in two lessons.

One example of incorporating all five skills in one class period occurred when the students first studied polygons. For the introduction to polygons students did a connect-the-dot activity that required students to use their visualization skills to interpret the picture made with polygons. Students read material that addressed polygons and that presented examples of polygons and non-polygons. From the reading material students independently attempted to define polygon, writing their definitions in their notebooks. Using their thinking skills students discussed their definitions and decided on a class definition for polygon. As part of their assignment, students used protractors to measure angles of polygons.

In instances when Emily did not provide students with opportunities to apply all five skills in one lesson to learn about a concept, she often used a combination of skills while focusing on one part of the lesson. For example during a warm-up, students had to use their
visualization skills to determine whether a given figure was a triangle, their thinking skills to determine a response and to explain their response, and their writing skills to record the response and the explanation for their answer.

Three skills out of the five specifically related to geometric knowledge. These were visualizing, measuring, and mathematical thinking. On the third day of class Emily started to establish the importance visualization had in the geometry class. Emily explained that visualization involved students being able to see, copy, draw, and compare items:

The past week we have spent a little bit of time talking about visualization, the idea of trying to get a picture in your mind. . . . During this class there will be a lot of times when I ask you to try being able to see things, copy things, compare things. These are all part of the visualization skills that we are going to use.

Opportunities for students to develop their visualization skills occurred in every observed lesson. Examples of how students worked on visualization skills included students creating and using a three-dimensional cube to discuss undefined terms, devising and drawing line designs, and counting triangles by types in a triangle collage.

In most of the lessons, students had opportunities to develop their measurement skills. These skills were practiced during activities involving the use of a protractor either to draw angles with a specified measure
or to determine the measure of the angles of a triangle. Measurement skills were also practiced when students were asked to estimate distances or angle measure.

Opportunities for students to develop their general thinking skills were apparent in every lesson as students were expected to be involved in the lesson. Students were asked to participate in warm-up activities such as finding missing angle measures in diagrams involving parallel lines and transversals and classifying triangles in a triangle collage. Examples of students' participation in classroom discussions included student questions when they did not understand a concept, student explanations of their solutions, student examples of points, lines, and planes, and student-generated lists about important unit concepts during a review.

Emily also identified inductive reasoning and deductive reasoning as part of the thinking skill focus. Observations provided information about student opportunities for developing their inductive and deductive reasoning skills. Students' inductive reasoning skills were used when students were asked to identify segment measurement patterns on geoboards and to generate a formula for angles of polygons based on an investigation. Inductive and deductive reasoning were used when students wrote definitions of special polygons based on pictures of examples and non-examples. Students used their deductive
reasoning skills when they applied their definitions to problems written in drill and practice form.

Emily's reflections during the final interview on inductive reasoning and deductive reasoning provided additional information about students' opportunities to develop their thinking skills. Emily's comments about inductive reasoning indicated that she felt her students were capable of thinking inductively:

Inductively, I think they [the students] are all pretty much at a point now where they are able to examine information, understand how the parts of, I am not saying this very well, . . . given a bunch of stuff to look at they are able to draw a conclusion.

With respect to deductive reasoning, Emily explained that "I don't really do a whole lot of that [deductive reasoning] formally." Formal deductive reasoning was interpreted to mean formal logic and proofs. Additional comments by Emily during the final interview indicated she had decided to include more formal deductive reasoning this year than she had in the past two or three years. Emily stated, "I am going to do a short unit after spring break on formal logic, starting up with different kinds of statements to an abbreviated version of truth tables, and then I thought I would do a quick unit of two-column proof." The basis for Emily's decisions was her guilty reaction to an article that she had read about students going to college and needing some knowledge of formal logic in high school.
Reading and writing mathematics were the remaining two skills that Emily had described as overriding goals for her geometry course. Emily's reading focus for her students included reading mathematical materials and then taking notes on the reading. The observed lessons and plans for other lessons indicated that students were asked to read and take notes approximately once every five or six lessons. Other general reading opportunities occurred when Emily asked the students to read part of the chapter for their assignment and to read supplemental material during class in preparation for working problems presented in the reading.

Writing took on varying forms in Emily's geometry classroom. The first writing assignment required that students write Emily a letter introducing themselves. Even though the assignment was not directly related to writing mathematics, the assignment suggested that writing was going to be part of this course's activities. For many observed lessons students had opportunities to practice their writing skills. Examples of this student practice included writing notes as guided by Emily, writing student-generated definitions of special polygons based on examples and non-examples, writing responses to explain why they got the answers they did, and in groups writing a chapter outline in the form of a poster.
Comments by Emily indicated she was aware of her progress toward the goal of including all five skill-goals in her lessons "I got all five skills in today" and "the use of my five things is dwindling a little bit, usually three out of five." During the final interview, Emily commented that she was doing better with incorporating all five skill-goals, "I feel like I, if anything, have pulled it [using the five skills] back up some from the lowest points." Emily stated, "The hardest one for me to get into is having them write in the geometry classroom." Emily was referring to writing that included more than students taking notes in class. In addition to having difficulty with including writing as part of the lesson, the observed lessons suggested that Emily did not often choose to include directed reading as part of the lesson.

A goal of applying geometry ideas in the real world was suggested by Emily’s description of what was valuable in the teaching of geometry and by her reference to the school’s geometry scope. Since the focus of observed lessons was on the establishment of a geometric knowledge base and development of skills, the application of real-world ideas did not appear to be a goal for the first third of the course. In addition, incidental references to the real world in the textbook such as careers that used geometry and homework problems based in the real world (e.g., intersection of two city streets and perpendicular
lines) supported the notion that real-world applications were not a focus of the early part of this course.

The content addressed during observed lessons matched the topics stated by Emily before the start of the course. A comparison between content addressed in observations and content stated in the department's scope indicated that there was agreement between the two with respect to expectations for the first third of the course.

Before the start of the observation process Emily was not sure of the exact sequence of the course. At the completion of the observation process, Emily's contemplation on the content sequence showed she made a small adjustment from what she thought she would be doing. Emily explained that she had planned to address area and perimeter of plane figures after students completed learning about plane figures. Her plans were changed due to the school schedule for winter vacation:

I had planned that as soon as I finished working with the line designs, which was sort of like the end of learning about plane figures. I had planned to go directly into this last couple of weeks doing area and perimeter for plane figures and, when I realized I only had three class periods of time left, I decided that it really wasn't enough time to start that and do it very well.

Instead Emily chose to complete the students' learning of plane figures with a trihexaflexagon activity and to help students tie together some geometric ideas with a cumulative review. In addition, Emily chose to have fun
with the students by watching *Donald in Mathmagic Land* (Luske & Colman, 1959) as well as *Stand and Deliver* (Musca & Menendez, 1988). Emily's comments regarding class discussions about the content of the movies suggested that her use of movies was one way to foster students' interest in mathematics.

An indication of the influence of advanced planning on Emily's instruction was the instructional approaches she used in her classroom. Specifically, Emily's plans indicated lecture, student participation in hands-on activities, and student exploration of mathematics while working in small groups.

Observations indicated that during some portion of each lesson Emily presented material. The presentation was not a "formal" lecture in which Emily did all the talking. Even though a topical outline for the lesson often occurred on the overhead and students took notes during the presentation of material, the presentation was best characterized as lecture/recitation. As Emily proceeded with the lecture/recitation format, students asked, as well as answered questions, explained their answers, provided suggestions for working practice problems, and made conjectures based on patterns. From the observations, Emily appeared to involve volunteers and non-volunteers in the discussion and seemed to involve students from all parts of the room.
Each observation also showed that students spent part of their class time working with hands-on materials. As part of a class discussion students worked with a model of a cube while discussing points, lines, and planes, used geoboards to practice segment and angle vocabulary, and used packaging tape to create a trihexaflexagon with which students practiced polygon vocabulary and visualization skills. In addition, students used protractors to measure and draw angles while completing a class time worksheet and used rulers while creating line designs.

Emily incorporated varying forms of group work in her classroom. During work time students were always encouraged to work with their peers comparing answers and answering each other's questions. One of the worksheet activities was designed for students to work with the partner of their choice. As part of the design, each student was responsible for completing part of the worksheet before comparing results with each other. In another lesson, a structured group format was used for the chapter one review activity. For this activity, Emily arranged students in groups of three or four and instructed them to create a poster reflecting the main concepts of chapter one. During the next class period, each group presented its poster to the class. Group members were then randomly asked questions about their poster. Having each group member responsible for understanding what was on
their poster encouraged students to work cooperatively on the activity. Additional comments by Emily during the final interview indicated that she wanted to include more formal cooperative group activities: "I want to do more formal groups, formal cooperative things than I have done. I think I really have only done about four lessons that were specifically designed and turned out to be cooperative lessons."

In addition to decisions made regarding Emily's overall content and instruction, decisions were made during daily lessons with respect to the implementation and the context of the lesson. Emily's reflections on how her lessons had proceeded usually began with phrases like "I think the lesson went really well" or "I think the lesson went pretty much okay." When Emily reflected upon decisions made during the lesson, her comments referred to decisions made about instruction and about student behavior. Emily's reflections on decisions made about the instruction during a given lesson indicated she needed to determine the amount of time spent on lesson segments. For example, as Emily led the discussion on an activity in which students were demonstrating line segments and angles on a geoboard, she made the decision to shorten the activity. Emily explained, "The students were getting it and today was the first time they had more than five minutes to do homework." Emily felt "with 95-minute
periods students should have some time to do homework." In
general, decisions involving the amount of time spent on
lesson segments reflected Emily's awareness of what her
students needed.

Emily's reflections about decisions involving student
behavior occurred three times during the post-lesson
interviews and were all in reference to one student, Ross.
In reflection Emily said, "I had to decide to move Ross
today" or "I decided not to 'strangle' Ross." Additional
comments by Emily indicated Ross was a talented and gifted
student who was able to goof-off in class and still learn
the geometry that was expected of him. Observations
supported these comments since Ross often made positive
contributions to classroom discussions. In general,
Emily's decisions regarding Ross' behavior seemed to be
based on whether his conduct caused difficulty for other
students.

Emily's actions during instruction generated decision-
making issues to pursue as well as routines to describe.
The overhead as an important instructional tool was
apparent in every observation. Emily used the overhead to
display the warm-up, to outline the point values for the
assignments, and to show the outline for notes. Emily also
used the overhead to answer homework questions by showing a
solution that had previously been written, to display
definitions and diagrams, and to show directions for an
activity. The information indicated in these descriptions was usually pre-written on a roll of acetate attached to the overhead. Emily's transitions to different parts of the lesson often included her turning the acetate roll to a different spot. When asked about her usage of the overhead, Emily's reply stated that her reasons included her ability to maintain eye contact with students, the change in the chalkboard due to her sweaty hands, and her need for organization:

**Practicality.** It's easier to use the overhead for eye contact, and I have sweaty hands that turn chalkboard to limestone. I found with block, actually I knew this before, that it's nice to have some predictable types of things. I'm less likely to forget something.

The observations also showed that assigned homework and homework discussions were important components of Emily's classroom routine. Emily set aside class time for discussion of homework, involved her students in the discussion, and answered students' questions. As the class discussed the homework, students graded their own papers. At the conclusion of the homework discussion, students recorded their scores on their assignment sheet and then turned in their papers.

When asked about her general homework policy, Emily stated, "I think of homework as a learning tool first and foremost. . . . I'm very much concerned with homework being turned in [within late policy limits] and complete than on time and incomplete." Emily wanted her students to focus
on understanding the concepts. Emily's homework plans had been affected slightly by the 95-minute periods. Emily found that she did not "have to send them home with so much homework."

Part of homework discussions included Emily calling on volunteers and non-volunteers for answers and for explanations of their results. On occasion when a student did not have an answer, Emily said to the student "Okay, let's look at it [the problem] now." To help the student, Emily rephrased the question or provided a hint about the problem. If it was apparent that the student still did not understand the problem, Emily called on another student. At the completion of the discussion Emily checked back with the first student to see if the individual understood. Emily briefly reexplained the problem if the student still did not understand.

Students were also involved in discussions of warm-ups and activities. Examples of dialogue during instruction illustrated how Emily encouraged students to provide the main ideas for the discussion. In the first example, the class had generated a set of information (see Figure 2) regarding the total interior angle measure of a polygon based on the number of triangles created when all the diagonals from one vertex of the polygon had been drawn.
<table>
<thead>
<tr>
<th>Polygons</th>
<th># of sides</th>
<th>Total interior measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>3</td>
<td>1(180) = 180</td>
</tr>
<tr>
<td>Quadrilateral</td>
<td>4</td>
<td>2(180) = 360</td>
</tr>
<tr>
<td>Pentagon</td>
<td>5</td>
<td>3(180) = 540</td>
</tr>
<tr>
<td>Hexagon</td>
<td>6</td>
<td>4(180) = 720</td>
</tr>
</tbody>
</table>

Figure 2. Angle Sums Generated by Emily’s Students

Emily now wanted the students to find the pattern that occurred in the set of information. The example of dialogue showed how Emily asked Christie to clarify her comment:

Emily: 180. One triangle equals 180. Quadrilaterals are two triangles and so on down to a hexagon. So what is the pattern that is going on there? What are we going to be able to predict will happen in any old polygon if we know the number of sides? Does anybody see? Christie, what do you see?

Christie: There will be two less.

Emily: Two less than?

Christie: Two less than the number of sides.

Emily: The number of sides, that’s right. The number of triangles that are formed each time is two smaller than the number of sides.

The discussion continued with the class looking at more examples and then with Emily’s guidance generating the formula $180(n-2)$, where $n$ was the number of sides.

An example of dialogue during a review of the three characteristics of a polygon demonstrated how students
provided the review information and how Emily validated the students’ responses:

Emily: Okay, three characteristics that we have repeated over and over again. Paul, what’s one of them?

Paul: In the plane.

Emily: Okay, lies in a plane - it’s flat. What’s another characteristic of them, Julie?

Julie: All parts are attached to each other - like a closed figure.

Emily: Okay, it’s a closed figure. It lies in a plane. What else? What’s the third thing?

Jody: Endpoint.

Emily: Endpoint to endpoint. Good. So those are the three things that we’ve said a bunch of times - a closed plane figure formed by connecting segments endpoint to endpoint.

Students’ participation in discussion seemed to be fostered by Emily’s expectation that students participate, her validation of their responses, and her eye contact with the students.

Observed lessons indicated that Emily used the routine of repetition of ideas over the course of a week or two. Emily stated, "It [use of repetition] is just part of my repertoire, my own teaching repertoire." Additional comments by Emily supported the idea that learning through repetition was one way to learn. When asked, "When is it enough and is there some type of pattern to the repetition," Emily stated "I really don’t have any set
pattern and as far as when it is enough, I usually judge more by the kids than I do anything else."

Another routine identified by observations was Emily's circulation around the room as students worked on warm-ups, class time worksheets, group activities, and homework work time. When asked about her circulation around the room, Emily's comments indicated that circulating around the room was an ongoing assessment technique that enabled her to see what the students were doing and allowed her to help them as needed without embarrassing them. Emily explained her reasons for circulating around the room:

I feel that you put up a barrier when you flop down at your desk. There's enough students so that I can't sit at my table, I have to be out there to see what the students are doing. . . . If I'm not walking around, I can't see who needs help or who needs to be on task without embarrassing them.

Summary of Emily. Emily was a secondary teacher beginning her twenty-first year of teaching mathematics and her sixteenth year of teaching geometry at the time of this study. Emily's school was in its first year of a block schedule implementation. In this schedule Emily taught geometry once a week for 50 minutes and twice a week for 95 minutes. Although a small group of accelerated students took geometry in ninth grade as part of a grade-level block system within the school block schedule, the majority of students at Emily's school studied geometry as tenth graders.
Emily’s planning decisions described a geometry course that acknowledged a variety of geometric skills. The basis for the description of Emily’s geometry course were her five course goals that involved skills. Emily wanted her students to learn geometry concepts through their skills of writing, reading, thinking, visualizing, and measuring.

Emily’s instruction reflected an implementation of her planning decisions. Students had opportunities to learn about different facets of geometry in a classroom where the teacher strove to establish students’ positive comfort level with geometry. Facets of geometry emphasized by Emily were geometry as a content knowledge base and as a setting for developing communication and problem solving skills. A smaller facet of Emily’s course was geometry as an example of a mathematical system as seen through her inclusion of inductive and deductive reasoning skills. In addition, Emily’s course included some references to the connection between geometry and the real world. While learning about the different facets of geometry, students studied geometry content as Emily used instructional methods such as guided discussions, hands-on activities, group activities, student reading, and student writing.

The foundation for factors influencing Emily’s decision making was her wish to make geometry less threatening to students and her personal commitment to being a life-long professional learner. While striving to
make geometry less threatening to students, Emily made decisions about her geometry course and determined how students can be active and interested in their learning of geometry. As a life-long professional learner, Emily continually worked to improve her geometry teaching. Emily’s process of improving her teaching consisted of her collaboration with colleagues, her participation in workshops and conferences, her reading of the *Mathematics Teacher* and the state mathematics teacher journal, her examination of textbooks, and her use of the *Curriculum Standards* (1989) as a guideline for her geometry course.

As part of the process of improving her geometry teaching, Emily was involved in the process of defining her geometry course. The basis for Emily’s geometry course was her personal slant of the scope created during her collaboration with departmental colleagues a year before the start of the study. The restructuring of Emily’s geometry course continued during the year of the study as she made a commitment to student learning of geometry through writing, reading, thinking, visualizing, and measuring skills. In addition, Emily made decisions about her course as she used her textbook and supplemental material from five other textbooks.

This year Emily decided that two-column proofs were to be re-included in the curriculum. This decision suggested she was still in the process of defining her geometry
course. During the first interview, Emily stated that one of the changes in her course over the years had been the movement away from two-column proofs. At that time her content description did not include references to any type of proofs. During the final interview, Emily indicated that she had changed her mind about two-column proofs and that she would be addressing two-column proofs as well as some formal logic during the second semester. Emily explained that a professional journal article had persuaded her to provide college-bound students with an opportunity to learn about logic and two-column proofs. Emily's continual process of defining her course appeared to be supported by her wish to prepare students for future work in mathematics.

The school's use of a block schedule challenged Emily as she worked with 95-minute blocks for the first time during the year of this study. With the 95-minute blocks Emily tried to use portions of time wisely by matching activities with time segments and by providing students with opportunities to practice concepts as needed.

Emily's instruction during a 95-minute block followed the general routine of warm-up activity, homework discussion, and presentation and practice of new content. The routine appeared to provide structure to the instruction but did not completely describe Emily's instruction. Descriptions of other factors that influenced
Emily's teaching generated a more complete picture of her instruction.

One of the main factors affecting Emily's decision making was her commitment to student use of writing, reading, thinking, visualizing, and measuring skills to learn geometry. As Emily planned her geometry course she made decisions based on how best she could incorporate these skills into her classroom. Throughout the observation period, Emily was aware of her progress toward meeting the goal of using all five skills during instruction each day. Through her awareness, Emily made adjustments in her plans in order to use all five skills. Emily's commitment to providing students with opportunities to use the five skills supported her acknowledgement of different facets of geometry. In addition, Emily's use of these different skills supported her wish to have students active in their learning of geometry.

The implementation of Emily's plan for students' active learning was also evident during observations in which students participated in hands-on activities, group activities, and teacher-guided discussions. Student generation of mathematical concepts and generalizations appeared to be the focus of these activities. Emily also felt that the use of activities that encouraged student involvement met students' needs for learning geometry.
Emily's awareness of her students' understanding of geometry concepts and relationships was apparent as she used ongoing assessment techniques during instruction. Emily learned about her students' understanding of content by calling on volunteers and non-volunteers during classroom discussions, systematically walking around the room while students were participating in an activity or working on an assignment, and by having students show her their geoboard-displayed solutions. Summative assessments such as notebooks, written tests, and a line design visualization project also provided Emily with information about her students' understanding of geometry concepts and relationships.

Emily's concern for meeting students' needs was evident in her reflections about decisions made during instruction. Emily's reflections demonstrated her consideration of students' needs while determining how best to proceed with her plans for the lesson. In most cases, Emily decided how class time could best be used and determined whether an individual's behavior was interrupting other students' learning.

Emily's planning and interactive decisions indicated that she was in the process of defining her geometry course and was able to implement a course that matched her plans for the first third of the course. The predominant factors influencing Emily's decisions were her willingness to work
to improve her geometry teaching, her commitment to the incorporation of five skills into her course, and her wish to meet the needs of her students.

Jon

Geometry and teaching biography. Prior to teaching geometry, Jon had studied the subject during high school and college. Reflecting on his personal tenth-grade geometry experience, Jon described a class that "was the most fun and was the most enjoyable because of the logic involved." At the college level, Jon studied a variety of geometries in three or four classes including projective geometry, modern geometry, finite geometry, and topology. In reference to his high school and college experiences, Jon stated that he "always liked math" and that "geometry probably became the most important math class" he had ever taken. Today, Jon's feelings about the importance of geometry were reflected by what he planned to tell his students, "I tell the geometry kids this is probably the most important math class you'll ever take. I really believe that. Not that they're going to remember the geometry but they're going to learn to think logically - hopefully."

Jon's first year of teaching was at his present school - a ninth through twelfth grade high school located in a small industrial city. His teaching career was then
interrupted for 2.5 years while he served in the army. When Jon returned to teaching, he taught mathematics at a junior high school for six years before starting his present position. Jon was beginning his twenty-seventh year of teaching mathematics and his twenty-fourth year of teaching geometry at the time of this study. During the fall of the research project, the ninth through twelfth grade enrollment was 1,830 students.

**View toward curricular change.** Reflections by Jon on the state-legislated reform movement and the NCTM *Curriculum Standards* (1989) provided some insight on Jon's view toward curricular change including goals, content, and instructional methods. With respect to the state-legislated reform movement, Jon's reflection suggested that a school curriculum and instructional change was not needed:

> I'm not sure it's going to raise the levels in those kids that have those degrees anymore than it is right now without the Certificate of Initial Mastery (CIM) and the Certificate of Advanced Mastery (CAM). I think the kids are going to be just as intellectually ready for college as they are now. I think they'll be just as intellectually ready for work whatever it may be as they are now.

Jon indicated that he's "probably read the [Curriculum Standards] at one time or another" but that he "wasn't familiar with them." Upon further consideration, Jon stated that his department talked about the *Curriculum Standards*. Jon's recollections of the *Curriculum Standards*
indicated that he was familiar with part of the technology component and with part of the geometry component of the Curriculum Standards:

If I remember right they [Curriculum Standards] talked about things they felt should be changed. One is use of more technology, one was less proof in geometry. That's the one I disagree with. The technology - that's good; it's coming about and kids ought to be able to use the technology.

Jon used graphing calculators in his algebra classes, but did not use any technology in his geometry classes. Jon explained his reason for not using technology in geometry, "I know there is some software out there. There is just too much to teach to take off a day or two for that." Additional comments regarding Jon's interpretation of the Curriculum Standards' (1989) recommendation for formal proofs demonstrated a difference in Jon's expectations for honors and regular geometry students:

But I'm also really a firm believer in logic and proof in which they said should be cut back. Now maybe it should for regular geometry, but I don't think for honors geometry. I think rigorous proof is important, especially if those kids are going to become mathematicians, and some of them will.

This distinction between beliefs about emphasis on proofs for honors students and beliefs about proofs for regular students suggested that a class consisting of both types of students provided a challenge for Jon for the 1994 to 1995 school year.
An introduction to Jon's classroom. The daily class schedule at Jon's school consisted of six 54-minute periods. During the course of this study, Jon taught two sections of formal geometry, two sections of advanced algebra, and one section of pre-algebra. Both formal and informal geometry courses were offered at Jon's high school. Jon was one of three teachers who usually taught the formal geometry course. Students who previously demonstrated A and B work in mathematics usually enrolled in a formal geometry class, whereas students that demonstrated C and D work usually enrolled in an informal geometry course. The majority of students enrolled in a formal geometry class were ninth graders, although the classes also contained students from eighth, tenth, eleventh, and twelfth grades. Up until the start of this school year, the formal geometry classes were either labeled as "honors" or "regular." Due to scheduling difficulties, no honors geometry classes were offered this year.

The formal geometry course selected for this study was Jon's first period class. Enrollment in this class consisted of 13 male and 12 female students. The class of 25 students contained two eighth-graders, fifteen ninth-graders, six tenth-graders, one eleventh-grader, and one twelfth-grader. The selection of the first-period class for observations provided the best opportunity for informal
interviews with Jon immediately before and after each lesson.

Jon's classroom was longer than it was wide and spacious enough for 34 desks arranged in five rows. In the front center of the room was Jon's desk. Adjacent to his desk were two two-drawer filing cabinets which usually held Jon's open textbook and lesson plan book. In this room with a plain decor, a wall containing windows provided a contrast to the two-and-one-half walls of blackboard and the one-half wall of bookshelves.

Throughout the observation period Jon's business-like presence and respectful interactions with his students were evident. Each lesson followed a similar format. Before the bell rang, students were seated in their assigned seats, quiet and ready to begin class. After the bell rang, Jon closed the door and started to take roll as announcements were broadcast over the public address system. Once announcements were completed Jon asked the students to get out their homework assignments. Homework discussion began with Jon reading the answers to the assigned problems, occasionally providing oral explanations for the more challenging problems. The discussion continued with Jon working student-requested problems at the board. The length of homework discussion ranged from 10 to 28 minutes, depending on the difficulty of the material. If the homework assignment included proofs in
open-ended form (not a fill-in proof), then Jon asked for student volunteers to put their proofs on the board and to explain the proofs to their classmates for extra credit. After homework discussion, new material was presented during 20- to 30-minute lectures. As Jon lectured, he wrote main ideas on the board, asked students a few questions, and answered questions raised by students. Following the presentation of content, students had time to individually work on the assignment as Jon walked around the room answering questions.

The context in which the teaching and learning occurred in Jon's classroom is described as both formal and comfortable. The formal atmosphere of the classroom was implied by the quiet classroom, Jon's requirement of homework done in pencil, and Jon's request that students not move desks together to work on homework. Students' comfort level in the classroom was evident in the observations. All students appeared to be able to work within Jon's classroom system. The students were ready to begin lessons when Jon was ready to begin, they took notes, they worked on their homework during work time, and they asked Jon for help during work time. Jon did not require students to be part of the classroom discourse. However, Jon's eye contact and his validations of students' comments did encourage those students who wanted to participate.
Planning decisions and influential factors. Jon described the planning process in which he made decisions for his geometry course as a process in which he "doesn't do a lot of planning." Jon's explanation for his approach to planning showed that his planning process was influenced by his teaching experience and his textbook:

I've taught this class so often that I know what I'm going to do. I glance over the chapter to find the words I want to emphasize, to think about examples I want to present, and to determine whether I need back-up examples. . . . I've taught geometry for so long I just jot down and use examples from my head.

Further discussion with Jon about his planning revealed that he wrote brief lessons in a weekly lesson plan book using the previous year's plan book as a guide. When describing his plans Jon stated, "This year's plans would be pretty close to last year's."

Jon's discussion of course goals for his geometry students focused on his feeling that "the most important goal was that they learn to think logically." According to Jon, "Thinking logically means if you have an argument in which you make a statement, you gotta be able to back it up." Jon stated that in geometry, "The teaching of proof is the best way to teach students to think logically."

Jon's reflection on how he decided on the goal of thinking logically indicated his decisions were influenced by colleagues when he first began teaching and by his teaching experience over the years:
Good question. How did I decide that? To be honest, I was probably told that when I first started teaching, either verbally or told that this was how we want you to teach it. But in doing that over so many years I really have come to the conclusion that it's probably correct.

In addition to thinking logically, Jon wanted his students to "remember enough geometry to do well on the Scholastic Aptitude Test (SAT)." He explained that doing well on the SAT was important to many of his students who went to college.

With respect to the planned content of the geometry course, Jon stated that "it really breaks down into two distinct semesters. First semester is logic and proof. Second semester is geometric relationships such as areas, perimeters, volumes, and power theorems for circles." Jon explained that proofs still occurred second semester, but the proofs were not the focus of the second half of the year:

It traditionally has pretty much broken right at about the end of the semester, just because in almost all of the books that we have had they started looking at geometric relationships more so than proof. And so second semester even though you still do proof, you still have theorems, and you still prove those. There is just not as many.

Jon described the beginning sequence of his geometry course as undefined terms, beginning postulates and theorems, applications of theorems, and proofs. When asked "How did you decide what content was to be included in your geometry course," Jon replied "pretty much by the textbook that was
selected by the district." The book selection committee consisted of eight teachers representing the five schools in the district. Jon had been on the committee for the last three book selections. He explained that the selection process involved the committee members compromising in order to make a decision:

I have been on the selection committee the last three times, even the time they threw out the book I liked. . . . I thought the one that this one guy wanted was way too easy. He thought the one I wanted was way too difficult. So there was a compromise. We both compromised.

Jon stated his feelings about the present textbook and explained how he supplemented the textbook: "The book that was selected - I can live with it. I am not as happy as I would have been, but I am still going to do geometry out of it. . . . I just supplement certain things, and I will pull problems from other books I have used in the past."

Expanding on his reference to supplementing the textbook, Jon explained that he felt the present textbook was weak on fill-in proofs (two-column proofs in which some of the statements and reasons were given). In his explanation Jon stated that he supplemented the present textbook by using handouts from the previous textbook:

Like one area that our book I think is weak in is it doesn't have enough fill-in proof. . . . And so I do an awful lot of supplementing from another book we have used, the last textbook selection we used it for eight years and it was quite good. . . . There were a lot of handouts that it had and we used those handouts with that last book and that seemed to work
really well. So I have used a lot of those handouts.

Jon's discussion of supplemental material also included references to a set of formal logic materials and to a set of construction materials. Previously, this extra material had been part of the content for his honors geometry class. Jon explained that this year students who wanted honors credit were to do this content on their own:

Now see, all the ninth graders last year would get honors credit, and they all did the constructions, they all did the logic section, they just did as a class. And this year, they took that class away. . . . So now you're teaching half a class of regulars and half a class of honors, you can't go as fast, except your honors kids are still going to be able to go as fast. So, if they want honors credit it's going to be outside the actual classroom instruction.

Jon still presented the logic and construction information in the textbook, but during instruction he did not go as in depth with this class as he usually did with an honors class. Thus, the content of his course was affected by having honors and regular students in the same class.

Jon's sole reference to the textbook as the reason for the content of his course suggested that the sequence followed the textbook's organization. Based on the table of contents of Jon's textbook, *Geometry* (Kalin & Corbitt, 1992), the content for first semester was the language of geometry, the logic of geometry, parallelism, congruent triangles, inequalities in triangles, and quadrilaterals.
Additional discussions with Jon about his planning indicated that his content decisions were also influenced by district content guidelines and the amount of class time available. Jon explained that decisions about what to do from the textbook were "kind of dictated by the district." The district provided the geometry teachers with a list identifying the chapters that the teachers were to address and chapters that were optional if the teachers had time. As an example of an optional section Jon stated "I think in our book there is a section on reflections, translations, and so on, I just don't get to it. I mean I would love to, I just don't have the time."

The manner in which the content was addressed in Jon's classroom was described by Jon as lecture/recitation:

It's probably lecture/recitation type for maybe, I probably lecture for about 20 to 30 minutes. And then make an assignment and the kids will work on that. I'll just walk around and help individual kids if I notice they need help or they'll come up and I'll go over a problem with them.

Further comments by Jon about his instructional approaches indicated that he tried to do everything on the board. As Jon described his reasons for using the board he explained the importance of students having the opportunity to copy work, especially proofs, off the board:

I really believe, especially with geometry, that if you don't put it up there and let the kids see, especially a proof, and for some of them they even have to copy them down and look at it a bit later, they're not going to get it. . . . I found over the years that a lot of times we'll
put up the proofs on the board, might have six or seven, and maybe three or four of them they didn’t get . . . they just got totally lost.

Proof work on the board was done by both Jon and his students. Jon stated that he liked students to put proofs on the board because "it gives them a chance to do some extra credit" and "it gives them kind of a feeling of accomplishment when they put one up there."

When talking about planned instructional approaches, Jon also commented that "occasionally we’ll do cooperative learning, depends on the class and how well they can work together." For Jon, cooperative learning meant students working in groups to complete problems. Jon explained "I’m not comfortable with it [cooperative learning] and so I don’t think I do a good job with cooperative learning."

Interactive decisions and influential factors. An examination of Jon’s instruction provided information about his interactive decisions and factors influencing those decisions. Interactive decisions included decisions reflecting an implementation of plans and decisions made during day-to-day lessons. The decisions informing this study were those regarding the geometry course involving goals, content, and instruction.

Students were provided with many opportunities to make progress toward the course goals based on observations of Jon and conversations with him. Jon’s main goal was for his students to learn to organize a mathematical argument
through the process of making statements and providing reasons to back up these statements (i.e., to think logically). Jon’s commitment to helping his students meet this goal was evident throughout the study. As Jon discussed theorems from the first chapter, he made references to the idea that the class was to prove theorems later. When Jon started the second chapter, "The Logic of Geometry" (Kalin & Corbitt, 1992), he set the context for the course by stating his goal of learning to think logically and by relating logical thinking to conditionals:

One of the most important things in geometry is learning to think logically, and we spend almost the whole first semester using that idea of thinking logically and putting statements and reasons together. And those statements and reasons that we put together are either going to be theorems or problems. And every theorem of every problem is in the form of a conditional.

Continuing with chapter two, Jon introduced his students to two-column proofs while discussing algebraic properties and equivalence relations. By the end of chapter two, Jon’s students were working with two-column fill-in proofs and two-column proofs that students set up themselves. The content contained in these proofs involved angles, segments, and perpendicular lines. As the course progressed, Jon and his students worked with proofs involving congruent triangles. During these sections, Jon demonstrated proofs on the board and students continued to work proofs as part of their assignments.
In Jon's classroom, geometry content was the context for students to learn to think logically. In addition, Jon wanted his students to know enough geometry to do well on the SAT. The content of the lessons and students' work on assignments as indicated by homework discussions demonstrated that students had opportunities to learn geometry concepts. Observed lessons included topics such as undefined terms, perpendicular lines, parallel lines, conditionals, algebraic properties, polygons, and triangle congruences. These topics presented by Jon matched the topics planned before the start of the course.

As planned, the overall sequence of the content followed the textbook's outline. Jon usually presented one textbook section of material a day. The lesson outline for perpendicular lines was typical for textbook sections addressed in one day. Jon began the class by asking the students to open their books to page 28, the start of the perpendicular lines section. He then identified the content that was to be discussed by defining perpendicular lines and by demonstrating with two pencils what perpendicular lines looked like. Once the definition of perpendicular lines and the symbol for perpendicular had been explained, Jon proceeded to state and to explain the textbook's eight theorems involving perpendicular lines. Sometimes the explanation consisted of Jon orally leading the students through an informal proof of the theorem (a
formal proof occurred at a later date). Other times the explanation consisted of Jon drawing and using diagrams to explain why the theorem made sense. At the conclusion of the presentation on perpendicular lines, Jon gave the students an assignment from the book.

Exceptions to the practice of presenting one textbook section of material a day occurred during two situations. The first situation occurred when Jon made the decision to spend two or three days on one section. Jon made this pacing decision when he taught with this textbook for the first time. During this study, Jon was working with this textbook for the fourth time. He explained "I know which sections will give students difficulty and I’ve made them two-day sections." Additionally, based on Jon’s lesson plans, some of the sections were three-day sections. Examples of sections described by Jon as difficult ones for students and addressed for more than one day were algebra field properties and equivalence relations, triangle congruence postulates, and proofs with overlapping triangles. The first day of a two- or three-day section usually included a presentation of the concepts as described for the one-day sections. During the second and third days on a section Jon demonstrated problems on the board and gave students time to work on their assignments.

Jon also decided to supplement the textbook’s approach to the logic chapter. Jon chose to supplement the textbook
with two-column fill-in proof material. For three days, students worked individually on a set of handouts that contained two-column fill-in proofs. The fill-in proof materials were inserted between sections titled "Strategy: Using Logical Reasoning" and "Strategy: Prove Theorems" (Kalin & Corbitt, 1992). Jon explained that his concentration on two-column fill-in proofs exposed students to the proof format and pushed the students to learn the ideas that were used for reasons (the second column):

It [two-column fill-in proof] kind of does two things. Number one, it gives the kids a chance to look at the format and to see this format. Just the more they see it, the more it is going to start to sink into their heads that things kind of go-together. Plus it gives the kids a chance or maybe it pressures them into really learning theorems, postulates, and definitions because you are just filling in reasons which are those.

Additionally, Jon emphasized "The idea of the fill-ins is not for the kids to write the proofs. The idea for the fill-ins is for the kids to see the connection between a statement and a corresponding reason that was used with that." Jon’s inclusion of this supplement to the textbook’s approach to the logic chapter indicates his commitment to helping his students learn to think logically.

Jon’s overall plans for instruction were implemented. Jon’s planning decisions predicted the use of lecture and the use of the board as the main instructional approaches utilized during instruction of new content. Confirmation
of Jon's intentions occurred with his use of lecture and the board in every observed lesson. Jon's lectures on new content usually were teacher-led presentations during which students took notes and occasionally provided input. Student input consisted of students responding to six or seven questions Jon asked during his lectures. Examples of Jon's questions included the following: "What happens when you have two angles that have the same measures, they are what," "how many diagonals from this vertex," and "what do you think my next statement might want to be?" Students responded in a call-out manner when questions required one or two word answers. When questions required a longer response, Jon called on volunteers.

As Jon lectured, he wrote definitions and theorems on the board. When needed, Jon drew diagrams on the board to supplement his comments. Other visual models of his words sometimes occurred. Visual model examples included his use of pencils to symbolize lines, his use of paper to represent a plane, and his use of a kleenex box to exemplify a right rectangular prism.

Observations of Jon's teaching confirmed his planned use of the board as an opportunity for students to write and explain their proofs. Once the students began to work with open-ended two-column proofs, Jon invited students to volunteer to put their proofs on the board. When Jon asked for volunteers, he gave preference to students who had not
previously volunteered. While volunteer students wrote their proofs on the board, the rest of the class either sat quietly or read their textbooks. Once all proofs were on the board, volunteers individually explained the proofs to the class and then asked for questions on the proof. Jon became involved in the discussion of the proof if the presenter could not clearly answer a student's question.

During the first formal interview Jon had said that he occasionally used cooperative learning as a teaching strategy. No use of cooperative learning or small groups occurred during the 10 classroom observations. When Jon was asked about this, he explained: "Mostly that is done at the end of the chapter when we are doing a review."

Additional comments by Jon illustrated that cooperative learning in his classroom occurred in the form of students working together on a chapter review assignment:

Like I will give them an assignment and say "Do these problems on both pages. And if you get done you might look at these other examples. Now you can move around and work in groups of like two or three or four as long as you're relatively quiet." Then I just kind of float around the room and answer questions or whatever.

Jon admitted, "It [cooperative learning] is not as in control for me." At the same time, Jon felt "It allows the kids to question each other and maybe challenge their friend with their knowledge of geometry or quiz each other. It helps them learn."
In addition to decisions made regarding Jon's overall content and instruction, some decisions were made during daily lessons with respect to the implementation and the context of the lesson. Jon's reflection on how his lessons had proceeded usually began with phrases like "the lesson went pretty good" or "the lesson went pretty much as planned." Jon's comments about decisions made during the lesson usually implied that he did not have to make any adjustments during the lesson. An occasional comment by Jon about decisions made during the lesson showed the decisions involved content and instruction. For example, Jon chose to explain a diagram in the text because the diagram was not drawn well. In another situation during work time, Jon called the class back together to talk about corresponding angles. Jon had not addressed corresponding angles during lecture, but the topic was included in the homework problems.

Jon's actions during instruction generated decision-making issues to pursue as well as routines to describe. The discussion of the previous lesson's homework appeared to be an important component of Jon's lessons. With the exception of a day after a test, Jon's class began with homework discussion. During the discussion he read the answers to the homework problems and then worked student-requested problems. When asked "How long do you like to go over homework," Jon replied "I want to answer as many
questions as possible, but I want to have enough time for the new material." Additional comments by Jon indicated that his previous experience with the problems enabled him to know on which homework assignments he would need to spend more time.

Even though most of the homework discussion was done by Jon, students were occasionally involved in the discussion. Once in a while Jon asked students for the equation needed to solve a given problem or to state needed information by completing statements such as "The measure of the exterior angle equals what?" Some individuals took an active role in the homework discussion when they volunteered to put proofs on the board and to explain the proofs to their classmates.

Another component of homework was Jon’s assigning of honors problems. Most assignments included one to five additional problems for the students taking Jon’s course for honors credit. Jon described the honors’ problems as "the most challenging ones in the textbook assignment." These problems were not discussed in class but were turned in to Jon as part of students’ notebooks. Additional comments by Jon indicated that having to decide on an assignment for regular students and on an assignment for honors students was a major change for him this year.

As stated previously, homework discussion was followed by Jon’s presentation of material. In general, the
observations suggested that the presentations of concepts were teacher-centered. An exception to a teacher-centered discussion was the discussion on parallel and skew lines. Jon began the lesson by asking students for the definition of parallel lines. When a student responded "lines that never intersect," Jon showed the students two pencils that were skew and asked the students if the two pencils were parallel. Jon continued to work with the students until the students stated that "parallel lines were lines in the same plane that do not intersect." Jon described this process of working off of students' definitions as "discovery." When asked "Were there other lessons in which this [discovery] happened," Jon replied "Yeah, I think there is. Some of the areas where that happens is when you start doing some spatial relationships." Further comments by Jon indicated the spatial relationships to which he was referring involved relationships between lines and planes.

Observations indicated that sometimes Jon asked students questions that suggested he was assessing their progress. Examples of questions included "How many think that theorem is obvious," "how many have worked with the closure property," and "does that make sense?" These type of questions, however, were not a predominant part of Jon's lessons. The majority of Jon's ongoing assessment of students' progress was done during work time as Jon walked around the room and answered students' questions. When
asked "What type of information do you gather as you walk around the room," Jon's response indicated that he wanted to make sure the students were on task and he wanted to see how they were doing with the problems. Jon stated:

I am looking at how far along they are for one [thing], keeping them on task. . . . And then, of course, I am looking at how well they are doing the problems, the thinking that goes on in writing the problem. It is interesting to see kids work a proof differently from what you did.

Jon's post-lessons reflections usually indicated that while circulating around the room Jon had determined that students were "getting the material."

Summary of Jon. Jon was a secondary teacher beginning his twenty-seventh year of teaching mathematics and his twenty-fourth year of teaching geometry. Jon's school had a daily class schedule that consisted of six 54-minute periods. Both formal and informal geometry classes were offered at Jon's school. Jon was one of three teachers who usually taught the formal geometry classes. The majority of his geometry students were ninth graders, although some of the students were eighth, tenth, eleventh, and twelfth graders.

Jon's planning decisions defined a geometry course that focused on geometry as an example of a mathematical system. The foundation for the definition of Jon's course was his major course goal of learning to think logically
and his secondary goal of learning enough geometry to do well on the SAT.

As described by his interactive decisions, Jon's teaching reflected an implementation of his planning decisions. In a formal, quiet classroom students had opportunities to learn about the mathematical system aspect of geometry while striving to meet the primary and secondary course goals. Students worked toward the course goals and studied geometry content while listening to Jon's lectures, taking notes during lectures, looking at Jon's work and classmates' proofs on the board, presenting proofs on the board, and working on assignments during work time.

The foundation for the factors influencing Jon's decision making was his belief that geometry was probably the most important mathematics class taken by students due to their opportunity to learn to think logically. Jon's experience as a geometry student and as a geometry teacher fostered his belief about the importance of learning to think logically. As a high school geometry student, Jon felt that geometry was the most enjoyable mathematics class because of the logic involved. When he first started teaching geometry, his colleagues told him that thinking logically was an important goal for a high school geometry class. Based on his teaching experience over the years, Jon concluded that his colleagues were right.
Jon relied heavily on a textbook that he helped select as he made decisions about the content studied in his geometry course. The content represented typical topics found in a secondary geometry course. Students first studied undefined terms and basic geometry vocabulary such as rays, angles, and perpendicular lines. As the course progressed, students worked with conditionals, proofs, parallel lines, and triangles.

The pacing decision regarding Jon's progression through the textbook was made when Jon first taught with the textbook three years before the start of this study. Jon's combined class of honors and regular students did not appear to affect the pacing of the course, but the combination class affected what problems Jon assigned for homework. As additional work for honors credit, Jon assigned honors students the more challenging problems at the end of the section and required them to work through logic and construction packets on their own.

Even though Jon relied heavily on his textbook, he was willing to supplement the textbook material in order to accommodate perceived weaknesses of the textbook. With the present textbook, Jon used supplemental material from other textbooks to provide students with extra practice on two-column fill-in proofs as students were first learning about proofs. The extra fill-in-proof work provided students with an opportunity to become familiar with the two-column
proof format. Jon was also willing to supplement the textbook with material from other textbooks in order to provide honors students with logic and construction packets to do on their own. Both packets went more in-depth with the topics than what Jon could do in his class.

Jon's instruction followed the general routine of homework discussion, presentation of new content through lectures with brief recitation, and student work time on the assignment. Structure for Jon's instruction was provided by the routine as well as Jon's role and the students' roles during the parts of the routine. Jon's role during the homework discussion was to read answers and work student-requested problems. The usual role for students during homework discussion was asking questions. The homework discussion was more student-centered when students started to do open-ended two-column proofs and individuals volunteered to write proofs on the board as well as to explain their work to their classmates.

Jon presented new content through lectures because he was the most comfortable with that instructional method. As Jon presented the material, the students took notes and answered Jon's occasional questions. Jon's reflections on his instruction indicated that he did not make many adjustments during lecture. If Jon did make an adjustment, the adjustment was made in response to student questions. Adjustments included getting a kleenex box out of his desk
drawer to explain the relationship between lines and planes and calling the students back together to explain a definition that had not been addressed during lecture.

Jon also appeared to be the most comfortable with a quiet work time. During this time students individually worked on their assignment and asked Jon for help as needed. In addition to helping students during work time, Jon informally assessed students' progress on the assignment while walking around the room. More formal assessment of students' progress in the course occurred through collected notebooks and written exams.

Jon's planning and interactive decisions demonstrated that Jon defined geometry as an example of a mathematical system and was able to implement his desired course. The main factors influencing Jon's decisions were his goal of students learning to think logically, his reliance on the textbook which he had chosen, and his comfort with using lecture as a predominant instructional method.

Jordan

Geometry and teaching biography. Jordan had studied geometry in high school and college prior to teaching the subject. Jordan's description of his high school geometry experience indicated he had not understood geometry at that time:

About all that I remember of that [high school geometry] is that we started into proofs very
early and not only did I not understand geometry, but I did not understand how or why we were doing proofs and I never did really gain the proper understanding of them.

Additional comments by Jordan indicated that he did proofs successfully without understanding them or their purpose. With his success with writing proofs, Jordan did quite well grade-wise in his high school geometry course even though he did not like the way geometry had been presented.

Jordan's next focus on geometry was during his mathematics education preparation courses at a local university. At this time Jordan started to develop an appreciation for geometry. He described this experience:

There [in mathematics education preparation courses] I learned an appreciation and an understanding of geometry and began building and refining my philosophy of how geometry should be taught - emphasizing hands-on, intuitive, and inductive processes before deductive reasoning is emphasized.

Jordan's first year of mathematics teaching was at a seventh and eighth grade middle school. The following year he started his present position at a ninth through twelfth grade high school located in a small lumber town. Jordan was beginning his tenth year of teaching mathematics and his ninth year of teaching geometry at the time of this study. During the fall of the research project, school enrollment was approximately 560.

As part of his inservice work, Jordan participated in workshops. Jordan took workshops on cooperative learning and on using graphing calculators and computers in
geometry. Jordan explained that these workshops fostered his development of learning "how to teach an understanding of geometry to students before emphasizing the process of deduction and proof."

This focus on teaching toward student understanding of geometric concepts before emphasizing the process of deduction characterized Jordan’s philosophy for teaching geometry and represented the manner in which his mathematics department wanted to teach geometry. With this teaching goal in mind, Jordan and his colleagues looked for a textbook that did not follow the traditional deductive approach to geometry. For Jordan, a textbook with "the traditional curriculum of hitting you with proofs the first chapter and going from there" followed the traditional approach to geometry. The textbook adoption process at Jordan’s school occurred six years ago and resulted in the selection of *Discovering Geometry: An Inductive Approach* (Serra, 1989). Jordan described this text as one that "did de-emphasize the deduction and emphasized the intuition and inductive reasoning." He said "It still contains all the traditional topics of geometry, it’s just organized in a different way." Based on his experiences with this textbook and this approach to teaching geometry, Jordan said "I like it a lot better than the traditional way of teaching it." Jordan’s present interest in geometry was
indicated by his comment "There's a lot to geometry. I find that it's my favorite course to teach."

View toward curricular change. Jordan's view toward curricular change in goals, content, and instructional methods was suggested in his geometry and teaching biography. The biography indicated that Jordan built and refined a philosophy about the teaching of geometry, one that emphasized student learning of geometric concepts through inductive processes and downplayed the traditional role of deductive reasoning. Jordan's experiences with high school geometry, college geometry courses, workshops, and a textbook adoption process fostered the development of his philosophy.

Jordan's reflections on the state-legislated reform movement and the NCTM Curriculum Standards (1989) reform agenda gave his view of the two reforms and how they affected his classroom. Jordan's reflection on the state-legislated reform indicated that he felt the goal of the reform movement was a more accurate way of determining whether students were learning what they should be learning:

I just see it as a way to more accurately, hopefully, more accurately determine that students are learning what we want them to learn effectively and be able to demonstrate it in a more, hopefully, realistic setting. Something to show that they've learned this and can use it.
Further remarks by Jordan implied that "what we want them to learn" meant "a mastery of more material and subject matter." Jordan also felt that the state-legislated reform had not really affected him at this time, "It hasn't changed a lot yet; it will somewhat, but even so I think that some of what I do [already] relates directly to what the intent of the bill and the whole act." Jordan anticipated that using rubrics and working with portfolios were challenges that he needed to address in the future.

For the year of the study, Jordan stated that his school’s focus was on the planning for the Certificate of Initial Mastery (CIM). Jordan expressed his opinion that the geometry course was not part of this planning: "Right now most of our efforts in reform have been going to planning for the CIM and restriction for CIM, which most students in geometry should already have passed all the math they would even need to achieve the CIM level." Even though Jordan felt that geometry was not part of the CIM planning, he believed his geometry planning was affected by the shorter school year due to two weeks of professional days set aside for state reform planning.

Jordan’s opinion on the *Curriculum Standards* (1989) expressed during the first interview indicated that he felt that much of what he did in the classroom was related directly to them:

I think a lot of what I do relates directly to the *Curriculum Standards*. I try to emphasize
writing and reading more in the classroom, writing across the curriculum. I try to emphasize problem solving. Right off the top of my head I don’t remember what all makes the list of standards, but there’s a lot.

Subsequent interview comments by Jordan seemed to suggest that he was comfortable with the Curriculum Standards:

The [Curriculum] Standards, when they came out, they made perfect sense to me. . . . I’ve been through the research and the different stuff that’s been coming out over the years, and I felt it was just something that was sort of stating the obvious and stating what was necessary to change, you know, and be emphasized in mathematics education.

Jordan’s approach to curricular change was most affected by his personal philosophy of teaching geometry. His comments about the state legislated reform and the Curriculum Standards (1989) reform agenda suggested that the reforms validated what he was already doing in the classroom.

An introduction to Jordan’s classroom. Jordan’s school was in its third year of a block schedule implementation. His geometry classes were taught every other day for a 90-minute block. During the course of this study, Jordan taught three sections of geometry, two sections of college algebra, and one section of informal geometry. The decision regarding whether a student enrolled in the geometry class or the informal geometry class was made by the first year algebra teacher and the individual student. Students who demonstrated previous
success in mathematics as well as readiness for a college-preparatory course took the geometry class.

Jordan also supervised a Master of Arts in Teaching (MAT) preservice teacher intern who worked with the two morning sections of geometry. The geometry section used for this study occurred during the last block of the day on the first day of the two-day teaching cycle. This geometry section contained 13 male and 14 female students.

Jordan's classroom was longer than it was wide and spacious enough for 30 individual tables and chairs arranged in six rows. When students worked in groups, four or five tables were pushed together. An overhead on a cart was in the front center of the room. Adjacent to the cart was a music stand which held Jordan's textbook and other materials needed for the lesson. The decor of the room was enhanced with an Escher bulletin board created by Jordan's MAT intern, a bulletin board for notes, a display of Far Side cartoons on the back blackboard, and mathematics posters.

Throughout the observation period, Jordan's encouraging and composed presence in the classroom was apparent. Jordan's established rapport with his students was also evident during the observation period. Jordan's 90-minute lessons usually followed a similar format. When the bell rang students were seated in their assigned seats. Designated groups of four or five students sitting in the
same area of the room were also part of the seating chart. As Jordan took roll, he reminded students to get out their homework. Jordan then addressed students' questions on the homework by working problems on the overhead for approximately 15 minutes. After the discussion of homework, Jordan began the activities addressing the new material. These activities assumed varying forms such as large group discussions, large group investigations, small group investigations described in the text, hands-on activities, group work time, and individual work time.

The context in which the teaching and learning occurred in Jordan's classroom was characterized by his desire to foster students' interest in geometry. Jordan expressed his concern about the traditional approach to geometry turning students off: "Traditionally, I've felt that the traditional geometry curriculum starts students too heavily in deduction too early before they understand much geometry so they get doubly confused and turns a lot of them off." For Jordan, the traditional approach to geometry was defined as one in which "the teacher talked, students listened and took notes, and students then worked on the assignment." As Jordan worked "to break the mold of the traditional approach" he encouraged students' involvement in their learning of geometry through the use of guided discussions, group investigations, and hands-on activities.
Planning decisions and influential factors. Jordan's decisions for his overall course planning for geometry were closely tied to his textbook Discovering Geometry: An Inductive Approach (Serra, 1989) and accompanying materials. At the time of this study, Jordan and his departmental colleagues began their sixth year with this textbook. Jordan and his colleagues had chosen a textbook that emphasized intuitive and inductive processes and de-emphasized deductive processes. As indicated by the first four chapters of the text "Geometric Art," "Inductive Reasoning," "Introducing Geometry," and "Geometric Construction" (Serra, 1989), the organization of the textbook supported a switch from an early deductive emphasis to primarily an intuitive and inductive emphasis.

Jordan stated with respect to his general planning, "The book is written so well I can follow it for my formal class." At the same time Jordan indicated that each year he still had some planning decisions to make: "At the beginning of the year (or late summer) I review the scope and sequence for the overall year and think about any modifications that might need to be made." Planning comments made early in the study indicated that Jordan had decisions to make about his course due to two weeks less class time because of school-wide work on the state restructuring process. However, specific decisions about what content was to be left out were not made at this time.
Jordan’s course goals supported his philosophy of learning geometry content through instruction which emphasized intuitive and inductive processes before the emphasis on deductive processes. As his overall course goal, Jordan wanted "to teach students the processes of mathematics through using geometry." Jordan’s explanation indicated that the processes of mathematics included the use of inductive and deductive reasoning: "The way math has been developed is you first come up with an idea (the same in science) what you think is going to happen, you test it out and then you try to prove it." In the context of geometry, Jordan stated that he also wanted "to develop students’ broad base of geometric vocabulary" and "to develop spatial visualization skills."

Jordan also wanted his students "to develop an appreciation for different areas of mathematics, specifically geometry." Part of this appreciation was showing how geometry related to other areas of mathematics. Jordan explained why he felt it was important to show the connection between geometry and other areas of mathematics:

I think there are too many students that come in and say, "Oh it’s geometry, it’s not math." So to show how geometry relates to algebra, how it relates to arithmetic, to probability, to, you know, . . . a number of different areas of mathematics that it’s integrally tied with.

Jordan also stated that appreciation for geometry included students seeing "how geometry has direct application to their lives in some way."
Jordan’s statement of factors affecting his selected goals first included a reference to his philosophy of teaching geometry, that is, using intuitive and inductive processes to learn geometry concepts and relationships before using deductive reasoning to prove theorems. The development of Jordan’s philosophy was fostered by his confusing high school experience and his insightful college experience. In addition, Jordan’s experience with his students supported his use of an approach described by his philosophy of teaching geometry:

And just my experience with students’ reactions to different ways of teaching geometry that I’ve tried over the years. I found they tend to retain more, enjoy it more, and get more out of it when I’ve taught it this way [using intuitive and inductive processes before using deductive reasoning to prove theorems] as opposed to more traditional ways of approaching it.

When Jordan described the content of the geometry course he emphasized that the "major content of geometry is still the same." He discussed this idea further:

I mean you still learn about triangles and quadrilaterals and all the different polygons. And you learn about angle bisectors, and parallel lines, and perpendicular lines and all the properties of all, you know, it’s still all there it’s just the way you approach it is different.

Jordan described his first semester’s anticipated sequence as inductive reasoning (definition and examples), undefined terms, geometry vocabulary, constructions, use of constructions for discovering properties of triangles and quadrilaterals, a little deductive reasoning, and
properties of circles. Problem solving and application problems were to be included throughout the year. After the first semester Jordan was not sure about the sequence of the topics but stated that topics like transformational geometry, area, Pythagorean theorem and its applications, similarity, and different types of proofs were probably going to be included.

Jordan's main influence on content selection was the textbook. Jordan's explanation included a reminder that the textbook was chosen for its content and its approach to the teaching of geometry:

The content was mostly determined by the text, the text that was adopted for the course. But that was determined when we adopted the text we picked a text that had the content we wanted. . . . We had the idea of what we wanted and how we wanted to teach geometry. We went out and looked for a text and a program that had as much of that as possible.

Jordan felt that "in a district with limited resources" that the content was "pretty much limited in a lot of ways to what their text was." Jordan had one planned deviation from the textbook. Chapter zero, "Geometric Art" (Serra, 1989), was used throughout the year rather than at the beginning of the course.

Jordan's description about the content addressed in his classroom included references to instruction involving that content. Reiterating an important characterization of his geometry course Jordan stated, "The core content hasn't changed really, but the way in which that core content is
dealt with has changed a lot." Jordan explained that presently in his classroom students were doing more discovering, problem solving, reading, writing, and working with hands-on materials than his students had done in the past:

There's more discovery, there's more problem solving I feel. Students are required to write more, are required to read more. There is more reading and writing involved. There is more hands-on and manipulative work than what I had done in the past.

For Jordan, "in the past" referred to the time when he first started teaching and was using a more traditional approach where the teacher talked and the students took notes.

Jordan's description of his instructional changes in the classroom implied that he incorporated a variety of instructional approaches that encouraged students to be active participants in their learning of mathematics. Additional comments by Jordan indicated that hands-on work involved the use of constructions, paper folding, and wooden cubes.

The use of a variety of instructional approaches was also advocated by Jordan's description of his instructional plans for his overall course. One area in which variety was planned was in the role Jordan played in the classroom. Jordan stated, "I like to mix up some lecture, some guided discovery, and a little bit of true discovery where I just let them loose and say figure it out on your own." Variety
also referred to whether students worked alone or with classmates. Jordan explained, "Within those [lecture, guided discovery, true discovery] there's always a variety of whether they are working alone, in pairs, in larger groups, cooperatively or not."

Students' learning style was one reason identified by Jordan's reflection on why he tried to use a variety of instructional approaches within a lesson as well as throughout the school year. Jordan stated, "I think having a variety of different types of teaching activities and things to accommodate different learning styles is important so that they're not always just sitting in their desks while I'm lecturing to them." In addition, Jordan felt "To use them [textbook and materials] effectively you have to hit it in a variety of ways."

Additional reasons for his instructional choices were related to his philosophy of teaching geometry. Jordan felt that "with the traditional format, not only is it not as effective, I don't think for a lot of students to learn, but you also tend to bore them, lose them, and they become disinterested and disenchanted." Jordan explained that he made instructional choices in order to "keep students interested and motivated."

As part of the instructional process, Jordan had his students keep notebooks. Jordan's description of how the class worked with some of the content identified notebook
requirements such as a glossary of geometry vocabulary and conjectures about geometric relationships as well as indicated students' involvement in their learning. Jordan described the class work:

We start discovering all those properties [triangle and quadrilateral] and building up our bases of knowledge, which they keep in notebooks, and they build up their own glossaries. They build up lists of these conjectures that they've developed and they learn about and discover properties of geometric figures.

In addition to glossaries and conjectures, the notebooks contained constructions and homework assignments. The notebooks were used as a reference by students as they studied succeeding geometry topics and as they participated in written assessments.

Jordan's instructional plans also included a commitment to frequent evaluation of students' progress. Jordan explained that he liked to evaluate students' progress with more than unit tests:

I feel it's also important to have frequent evaluation of their [students'] progress. I don't believe in giving one unit test and the next time they get evaluated give them another unit test three weeks down the line. I give lots of quizzes; I give lots of very quick little evaluative checks along with their major unit test.

Jordan explained that he typically used the four or five quizzes per chapter included in the textbook's supplemental materials.

**Interactive decisions and influential factors.** An examination of Jordan's instruction provided information
about his interactive decisions and factors influencing those decisions. Interactive decisions included decisions made reflecting an implementation of plans and decisions made during day-to-day lessons. The decisions informing this study were those regarding the geometry course involving goals, content, and instruction.

Students were provided with many opportunities to make progress toward his geometry course goals based on observations of Jordan and conversations with him. Jordan's overall course goal was for his students to learn the processes of mathematics through the use of geometry. For Jordan, the processes of mathematics meant the development of mathematical ideas through "the pattern of first using inductive reasoning and intuition and experimentation and discovery and then lead in to deductive reasoning later."

The first part of the process that focused on inductive reasoning was evident throughout the observation period. Jordan's comments during the third day of class characterized the role of inductive reasoning in his geometry course:

This first chapter [inductive reasoning] may not seem a lot like geometry to you so far because we have been working with inductive reasoning and number patterns. It is very closely related to a lot of the work we do in geometry because of the stuff that we do with inductive reasoning, looking for patterns, looking for relationships, trying to discover things.
As stated by Jordan, students first learned about inductive reasoning while looking at number patterns during the first chapter. Student work in chapter one also included pattern work with pictures, geometric shapes, and Pascal's triangle. Student use of inductive reasoning as the course progressed occurred in activities such as looking at examples and non-examples of angles and triangles to generate definitions, using paper folding to investigate concurrent lines of triangles, and examining numerical patterns to determine the interior angle sum of a polygon.

As suggested by Jordan's plans and confirmed by observations, the deductive part of the mathematical process did not occur until students met Jordan's goals of developing a foundation of geometric knowledge, a foundation that included a vocabulary base and spatial visualization skills. Student opportunities for developing their basic geometry knowledge were evident by the content of the lessons, involvement in homework and classroom discussions, as well as their involvement in classroom activities. The content of the observed lessons included topics such as undefined terms, line segments, angles, altitudes, medians, angle bisectors, triangles, quadrilaterals, polygons, and three-dimensional geometry. Students learned about this content by asking questions, providing answers to Jordan's questions, making conjectures based on investigations, working assignments, and comparing
assignment and investigation results with their group members. Students’ opportunities to develop their visualization skills occurred throughout the observation process. Examples of how students worked on visualization skills included students drawing three-dimensional figures, making conjectures based on picture patterns, and developing construction routines based on paper folding explorations.

When Jordan began chapter four material on discovering angle relationships and properties of polygons, the inductive process had been defined and utilized to foster the development of students’ geometric knowledge involving vocabulary, properties, and spatial visualization skills. Following the textbook’s guidelines and his feelings that his students were ready, Jordan started to incorporate the deductive part of the process of mathematics through the use of algebraic proofs in paragraph form. An example of Jordan’s incorporation of the complete process of mathematics (using inductive processes to learn the geometry concepts and then using deductive reasoning to prove theorems) was given by Jordan’s instruction leading to the application and proof of the conjecture - the sum of the angles of a quadrilateral equals 360 degrees. During the lesson, each student drew a general quadrilateral and measured the angles of the figure. Jordan asked the students to find the sum of the angles, to compare their
work with other group members' work, and to make a conjecture regarding the sum of the angles of a quadrilateral. As a class, students discussed the conjectures made by the groups and then agreed upon a class conjecture. To verify the conjecture, Jordan led the students through an algebraic proof in paragraph form by setting up the first statement of the proof and proceeding with the proof as directed by students' suggestions.

Jordan's goals also included student development of an appreciation of geometry. For Jordan, student appreciation meant seeing how geometry related to other areas of mathematics and applied to their lives. Even though observations showed that Jordan used algebraic processes when solving problems and writing proofs and used real-world representations for points, lines, planes, and concurrent lines, the observations also indicated that neither relating geometry to other mathematics nor applying geometry to the real world was a major focus during the first third of the course. Instead, during this part of the course inductive reasoning, geometry vocabulary, and geometric properties were emphasized. Jordan's final reflections on the course supported this suggestion. Jordan explained that not many opportunities for relating geometry to other areas of mathematics and for applying geometry existed during this early part of the course:

I have tried to show, the few times so far that I have had, to show how algebra fits in, how it
[geometry] relates to other areas, and there haven’t been a whole lot of opportunities for that, yet. . . . I have tried to throw in places where it [geometry] can be used. Practical applications, but then a lot of this early stuff because of the vocabulary and the constructions . . . at this point in the year they haven’t seen a lot of the practical uses yet because we haven’t gotten to a lot of the stuff that can be really used practically.

Additional comments by Jordan indicated that opportunities for relating and applying geometry occurred later in the course. Jordan stated, "A lot of the applications come later in the year. Especially when we get into work with formulas and things where we can relate even more algebra and more physical world examples."

Topics presented by Jordan during the observations matched the topics planned before the start of the course. Observations verified that Jordan’s content sequence progressed as planned. Jordan’s reflection on the content sequence revealed that at this stage in the course he had emphasized deductive reasoning a little more than planned. Jordan explained how he had incorporated deductive reasoning in the course:

I am emphasizing it more. Whenever it shows up in the book, as well as sometimes when it doesn’t. I have brought it out where I can show some basic algebraic proofs and basic paragraph proofs, and just logical thinking skills showing how this results from this, and this results from this, trying to show them a little bit of linear deductive logic like that. And I am hitting it more than I have in previous years.

Jordan’s decision to include more deductive reasoning at this time was because his students were "more open and seem
to be understanding, or following, the deductive process a little more quickly than I would have expected." Students' beginning work with formal deductive reasoning was observed during the last observation. During this lesson, students worked on polygon angle sum proofs as Jordan guided them.

Jordan's comments indicated that his overall approach for students' use of deductive reasoning while writing proofs was to gradually expose students to different methods of proof. At the time of the final interview, Jordan indicated that he did not expect to wait as long as the textbook did to do two-column proofs, but he was not sure of the details for incorporating different methods of proof as he progressed through the course. Jordan stated his thoughts about incorporating different proof methods:

The book presents what are called flowchart proofs in chapter five, and I have shown them algebraic proofs and how those relate and can be mixed with or combined with paragraph proofs... I will also show them two column proofs. And I am not sure if we will get that in chapter five or not. I may do that in addition to, because the book does not show two-column proofs until chapter 14.

Jordan's use of varied approaches to proofs once students developed their geometric knowledge supported his belief that students needed to be involved with deductive reasoning as part of a geometry course. Jordan explained his beliefs: "I do think that now, once they get to the point where they have learned all this geometry and they know geometry pretty well, now do some deductive logic and
proof with it [geometry knowledge]." Additional comments by Jordan indicated that he wanted students be able to use deductive logic when verifying conjectures.

Review of Jordan's instruction determined that his overall plans for instructional approaches were implemented. Jordan's planning decisions predicted that Jordan had different roles in the classroom as he lectured, guided students' discovery of new relationships, or let students work in a true discovery mode. Observations showed that in the 10 lessons addressing new material Jordan presented some of the content through lectures. The lectures, however, were not presentations in which Jordan did all the talking. Instead, the presentations were guided discussions. For example, as Jordan demonstrated how to find the nth term of a sequence using a factoring method, he asked students for the factors and for the pattern displayed by the factors. During guided discussions Jordan called on volunteers and non-volunteers and involved students from all parts of the room.

Guided discovery activities were designed so that with Jordan's guidance students explored and explained the main concepts of the lesson. Jordan's use of guided discovery activities occurred after the first lesson on basic geometry vocabulary. As a facilitator, Jordan helped his students generate definitions of geometry vocabulary and identify geometric properties. As an example of a guided
discovery activity, Jordan had the students make measurements as they were exploring the concept of perpendicularity. Prior to the measurement activity, students had drawn a line on a piece of patty paper and had put a dot above the line. The students then folded the patty paper such that the fold represented a line through the point that was perpendicular to the given line. For the measurement part of the activity Jordan directed the students to put four points on the original line (see Figure 3) and to measure the distance from each of the points to the original point.

![Figure 3. A Graphical Representation of Results Generated by Paper Folding](image)

After students had a chance to measure the segments, they told Jordan "The ones [segments OA and OD] on the outside are longer." The following dialogue showed how Jordan used the students' idea while guiding one of the students to state the perpendicular property being examined:
Jordan: [Pointing at figure 3 on the overhead] This one is longer then that one, and that one is longer than that one, that one is longer than that. . . . Okay, so the farther you get away from, what? I mean if you are saying they get longer as you get farther away, what happens to this point over here? [Jordan pointed to another point on the line.] Longer than what? For example, right here I would argue that C is longer maybe than B, so what determines the fact [of] how long they are?

Tina: From the center.

Jordan: From the center, you mean right here? So what is true about that dotted segment then?

Tina: The perpendicular is the shortest distance.

Jordan: The perpendicular is the shortest. Does everybody agree with that?

Students: Yes.

As demonstrated in this dialogue, a student stated the main concept being studied. Even though Jordan's goal during guided discovery lessons was to have students state the main ideas, occasionally the statement of the conjecture became more teacher-centered when Jordan referred to the textbook's statement of the conjecture.

Jordan did not have students participate in true discovery activities during observations. These activities were defined as activities where students worked with a topic to see what they were able to create on their own. Occasionally, students worked on a section of material without comments from Jordan, but the material involved applications of understood concepts rather than concepts to
be learned. Jordan’s reflection during the final interview indicated that due to time constraints he had not been able to let students participate in true discovery activities:

I feel that I haven’t been able to do quite as much letting them on their own in discovery type of thing. It has been more guided discovery where I guide it a little more strictly or more closely, rather than just sort of saying "Here’s a topic, play with it, run with it, see what you can come up with." I feel just because of time restraints that I am not able to do that much.

The use of hands-on activities during instruction was another implementation of Jordan’s plan for students to be engaged in their learning of geometry. One type of hands-on activities used in Jordan’s classroom was constructions. Jordan’s focus on constructions began when he started the material in chapter three, "Geometric Construction" (Serra, 1989). The chapter began with the basic constructions such as copy an angle and bisect a segment. Students did these constructions as Jordan demonstrated them on the overhead. As the class progressed through the chapter, students applied the basic constructions to solve problems and to make conjectures involving geometric properties. Examples of how students applied constructions during homework and during classroom discussion included the duplication of a given quadrilateral and the exploration of the three angle bisectors and concurrence in a triangle.

Jordan’s explanation of the textbook organization as well as the course organization suggested that students’ work with constructions was part of their foundation of
geometric knowledge. Jordan's comments during the second observed lesson on constructions reiterated that constructions were an important component throughout this course. As students were taking notes on basic constructions, Jordan explained his expectations for their future work with application of constructions:

> Your notes should be clear enough that when you come back and look at these six months later you will understand what you did. . . . You will be expected to be able to produce constructions six months from now. And when we get into the spring we will be doing investigations that involve constructing different things.

In addition to constructions, Jordan included activities that used hands-on materials such as patty paper for folding activities, protractors for angle measurement, and wooden cubes for visualizing numerical patterns.

Jordan's instruction confirmed his intention of using cooperative groups as a major instructional method. Jordan's general description of cooperative learning was "working in groups." When Jordan elaborated on his description of cooperative learning, his comments suggested two interpretations of cooperative learning. The first interpretation of cooperative learning was students answering each other’s questions in a group setting. Jordan’s comments reflecting this view of cooperative learning included the following:

> It is a lot more helpful to have somebody right there to compare with and to maybe give advice or to ask questions or to give help. Just the idea of having somebody else to work with to help you
get a more complete understanding of what it is you are studying.

Evidence of this type of cooperative learning occurred when students worked in their groups on their assignments during class work time.

The second interpretation of cooperative learning was students working together to meet a common goal. Jordan's remarks reflected this view of cooperative learning:

The purpose is to use the group's abilities and each other to come up with ideas and just compare and bounce ideas off of each other when they are doing the same investigations, but they are doing it with different figures. . . . I see it more of just a way to get people to learn how to work with each other towards a common goal, whether that be a fairly simple little goal or when we get later in the year when we do more project oriented things.

Evidence of this type of cooperative learning occurred when students worked together to generate definitions based on examples and non-examples of angles and triangles, to make a conjecture about the polygon angle sum based on numerical patterns, and to complete an algebra-based problem solving activity. During the definition writing, group members worked together to come up with one definition. During the other two activities, each group member had an individual task to do. Results from the individual tasks were used to complete the task. For example, during the polygon angle sum activity, each member drew a different polygon (e.g., quadrilateral, pentagon, hexagon) and measured the angle
sum of the polygon. The group members then examined the numerical values for a pattern.

In addition to decisions made regarding Jordan's overall content and instruction, decisions were made during daily lessons with respect to the implementation and the context of the lesson. When asked to describe how he felt about a lesson, Jordan usually responded with "it went about as expected" or "it went very well, but it took longer than expected." When Jordan reflected upon decisions made during the lesson, his comments indicated that decisions were made about instruction and student behavior. Jordan's self-reported instructional decisions made during the lesson involved how to use class time and how to have students work with the content. An example of a decision relating to how Jordan used class time occurred during a lesson when he had to decide when to give students a quiz. Jordan decided to break up the block of time and give the quiz between the two sections of material. One example of deciding how the class needed to work with the content occurred when Jordan determined students do an exterior angle investigation with him instead of on their own. Jordan made this decision due to limited class time. Another example included Jordan asking the students to individually work on the assignment in class rather than in groups because he wanted to see what the students were able to do on their own.
A few of Jordan’s reflections indicated that he made decisions based on students’ behavior. An example of a decision relating to how Jordan was affected by students’ behavior occurred when he was leading a discussion on points, lines, and planes. As Jordan led an oral discussion on undefined terms (descriptions and student examples of each) he made the decision to write the ideas on the board. Jordan explained, "Not everyone was following along so I decided to write the ideas on the board." In general, Jordan’s decisions made during instruction reflected an awareness of his students and his desire to foster students’ learning of geometry.

Jordan’s actions during instruction generated decision-making issues to pursue as well as routines to describe. Observations indicated that assigned homework and homework discussions were important components of Jordan’s classroom. Homework was assigned daily and collected every two or three days as part of students’ notebooks. Jordan spot-checked the notebooks at this time and then more thoroughly graded the notebooks when students handed them in at the end of a chapter.

As part of the homework discussion routine, Jordan expected his students to assume responsibility during deliberations on assignments. On the second day of class Jordan explained that students were expected to ask questions during homework discussions:
The way I will typically run the beginning of the period when we’re going to go over an assignment is that I’ll ask "Are there any questions?" If there’s not, I’ll go on. So it’s very important that if you have a question that you ask it because I am not going to read the answers or give you every answer.

When students asked questions, Jordan usually sought suggestions from students on how to work the problem. Using short call out responses from the class or longer responses from volunteers, Jordan guided the discussion while working the problem on the overhead. Jordan’s willingness to answer questions on homework problems was aligned with his description about the purpose of homework: "Homework is where they [students] should learn and if they make mistakes, fine, correct them and figure out what you did wrong." Observations showed that during homework discussions Jordan was able to address problems that caused students difficulty without working all of the assigned problems for the students.

During the majority of Jordan’s observations, two sections of material were addressed each day. Part of the routine on a day when two sections of material were addressed included students’ working on the assignment over the first section of material before Jordan addressed the second section of material. Jordan’s comments suggested that he used work time between the two sections to break up the 90-minute block of time and to provide students with
opportunities to work with some of the concepts that were supportive information for the next section.

On days when one section of material was addressed, work time, either individual or group, was still part of the lesson. The basis for Jordan for using both types of work time was his belief that cooperative group work and individual work were both appropriate. Jordan stated, "I’m a proponent of some form of cooperative learning. I think that there’s definitely a time and place for individual work and learning, but there’s also a place for working cooperatively."

Work time provided students with opportunities to study new concepts discussed during class before they left the classroom for the day. In addition, work time provided Jordan with the opportunity to assess students’ progress. As students worked, Jordan walked around glancing at students’ work and answering students’ questions. When students worked in groups, he tried to limit his assistance to answering group questions. Jordan also acted on the information that he gathered as he walked around the room. For example, Jordan’s actions included shortening work time because students were done with the assignment and were ready to move on to the next section. Another example of Jordan acting on gathered information while walking around the room included Jordan reminding the students that they
needed to construct the right triangle before they did the problem.

Assessment of students' progress also occurred in written form. During the 11 observations, students took seven quizzes and one test. Frequent occurrences of written assessments was a component of Jordan's plan for his classroom. Part of Jordan's routine for administering written evaluations was allowing students to use their notebooks as they took the quiz or test. When asked about this policy, Jordan's reply indicated that he allowed students to use their notebooks because students did not use the notebook much and the availability of the notebook alleviated stress for his students. Jordan explained his reasons for allowing students to use their notebooks:

If you look carefully on tests and quizzes, most students really aren't looking up a whole lot of stuff. In a lot of cases because of time. And they have learned it well enough by working with it that they really don't have to look up a whole lot of it. . . . It [the notebook] is almost like a security blanket and they [students] know they have it but they don't really have to use it. I find that it alleviates a lot of stress.

In addition, Jordan found that use of notebooks on quizzes and tests encouraged the students to keep an organized notebook. Jordan explained, "If they are going to be able to use it on a test it has got to be something they can access quickly because they have a limited amount of time on a test."
Summary of Jordan. Jordan was a secondary teacher beginning his tenth year of teaching mathematics and his ninth year of teaching geometry. Jordan’s school was in its third year of a block schedule implementation. In the block schedule, Jordan taught geometry every other day for 90 minutes. Both geometry and informal geometry classes were offered at Jordan’s school. The geometry class was designed for those students who demonstrated previous success in mathematics as well as readiness for a college-preparatory course.

Jordan’s planning decisions defined a geometry course in which various facets of geometry were addressed. The basis for these different facets were his course goals. Jordan’s overall course goal was for students to learn the processes of mathematics through using geometry. The processes of mathematics were defined as the use of intuitive and inductive reasoning to generate ideas followed by the use of deductive reasoning to prove the idea. Other course goals included student development of a broad base of geometry vocabulary, spatial visualization skills, an appreciation of geometry, and a sense of how geometry has application to their lives.

Jordan’s teaching reflected an implementation of his planning decisions. In a classroom where students were encouraged to be active participants in their learning, students had opportunities to learn about geometry as a
content knowledge base, an example of a mathematical system and as a setting for developing communication and problem solving skills. In addition, the observed lessons hinted at student examination of geometry connections with the real world.

To accommodate different learning styles of students, Jordan used various forms of instruction such as guided discussions, group work, guided discovery activities, and hands-on activities. These learning opportunities helped students establish a foundation of geometric knowledge consisting of the process of inductive reasoning, basic geometry vocabulary, and basic constructions. The foundational knowledge was the basis for students' subsequent geometry work.

The basis of the factors influencing Jordan's decision making was his philosophy for teaching geometry. Jordan's philosophy consisted of student use of intuitive and inductive processes to learn geometry concepts before using deductive reasoning to prove theorems. Related to Jordan's philosophy was his wish to keep students interested and motivated in their learning of geometry. Jordan felt that geometry curriculum in which students started too heavily with deductive reasoning before they understood much geometry "turned students off."

Influencing Jordan's development of his philosophy were his experiences as a geometry learner. In high school
Jordan successfully worked proofs, but he never understood why he was doing proofs. Jordan felt that his misunderstanding was due to the course's early focus on proofs. During his mathematics education preparation courses Jordan was introduced to learning geometry by emphasizing inductive processes before emphasizing deductive reasoning. In addition, Jordan started to appreciate and to understand geometry.

Further support for Jordan's philosophy came from his departmental colleagues and his teaching experiences while implementing his philosophy. Jordan's colleagues shared his philosophy and together they selected a textbook that emphasized intuitive and inductive processes and de-emphasized deductive processes. During his five years of teaching with his philosophy and textbook, Jordan found that students retained more geometry and enjoyed more the learning of geometry than they did with a more deductive, traditional approach.

As implied by the selection of a textbook based on his philosophy, Jordan relied heavily on the textbook as he made decisions about the content addressed in his course. After students learned about the process of inductive reasoning, the content addressed was typical for a high school geometry class. Students' first geometry content included undefined terms, line segments, angles, altitudes, medians, and angle bisectors. As the course progressed
students worked with constructions, triangles, quadrilaterals, polygons, and space geometry.

Even though Jordan relied heavily on his textbook, he was also willing to depart from the textbook. Jordan felt that, once students had established a foundation of geometric knowledge, they were ready to do deductive logic and proofs. The textbook introduced algebraic proofs in chapter four and flowchart proofs in chapter five, but the textbook’s concentration on deduction did not occur until chapter 14, when two-column proofs were introduced. Jordan began students’ deductive reasoning work with chapter four material - angle relationships and polygon properties. Feeling that students were understanding the deductive process, Jordan emphasized deductive reasoning more than the textbook did and planned to address two-column proofs either with material in chapter five (congruence) or chapter six (circles). Jordan’s decision about when and how to emphasize deductive reasoning reflected his wish for students to explore the deductive part of the processes of mathematics without losing interest in the study of geometry.

Jordan’s instruction followed the general routine of homework discussion, presentation of new content, and student work time on the assignment. The routine, however, did not completely characterize Jordan’s instruction due to his commitment to using a variety of instructional
approaches. Jordan assumed different roles during instruction. As a recitation leader, Jordan presented geometric concepts and demonstrated how to work problems. As a facilitator during guided discovery activities, Jordan encouraged students to explore and to explain the main concepts. As a resource person, Jordan walked around the room and answered questions while students worked in their groups. Jordan’s different roles were one way he tried to accommodate students’ different learning styles.

Students’ different learning styles were also accommodated with Jordan’s use of a variety of activities. Jordan expected students to participate in activities such as group generation of definitions, individually performing basic constructions as Jordan demonstrated them, individually applying constructions to create a square, individually investigating points of concurrency with paper folding and comparing results with group members, and group exploration of a numerical pattern involving polygon angle sums. Jordan’s use of individual work and group work reflected his feelings that both types of work were appropriate in the classroom.

Jordan’s awareness of his students’ learning needs as well as their progress toward understanding geometry concepts and relationships was evident due to his use of ongoing assessment techniques during instruction. During classroom discussions, Jordan called on volunteers and non-
volunteers to provide responses and reasons for their responses. In addition during all parts of the lesson, Jordan walked around the room looking at students' work and consulting with individuals and groups about their work. Jordan also used summative assessments such as written tests and notebooks to determine students' understanding of geometry concepts and relationships.

Jordan appeared to use the ongoing assessment information about students as he made decisions during instruction. Jordan’s reflections on decisions made during instruction supported this idea. In particular, Jordan’s reflections showed that when making decisions about how to use class time and how students needed to work with the content, he determined how to best foster students’ learning of geometry.

Jordan’s planning and interactive decisions showed that he defined and implemented a geometry course that matched his philosophy for the teaching of geometry. Additional predominant factors affecting Jordan’s decisions were his chosen textbook and his wish to keep students interested in their learning of geometry.

Lynne

Geometry and teaching biography. Prior to teaching geometry, Lynne had studied the subject in high school and in college. Lynne’s reflection on her geometry learning
experiences indicated that it was her exposure to a variety of geometries in college rather than her high school formal Euclidean geometry course that interested her in the study of geometry:

I took my first geometry class in high school when I was in the tenth grade. It was a formal Euclidean geometry course based on building a geometry through proofs. Although the teacher was great, I was one of those students who preferred algebra. It was only when I got to college and was exposed to a variety of geometries that I really started to appreciate and enjoy the beauty of geometry.

Lynne explained that her college courses helped her "understand the wide variety of applications of geometry as well as the natural occurrences of geometric patterns in the world in which we live."

Lynne had taught all grade levels seventh through twelfth with classes ranging from basic mathematics to advanced placement calculus. In 1983 she started her present position at a ninth through twelfth grade high school located in a university city. Lynne was starting her twenty-sixth year of teaching mathematics and her sixteenth year of teaching geometry at the time of this study. During the fall of this research project, the school enrollment was approximately 960 students. For a year and a half prior to the start of this study Lynne was not in the mathematics classroom. During this time she substituted for the assistant principal who was on leave.
For teaching geometry, Lynne felt that her college courses, as well as her experiences at regional and national mathematics conferences, prepared her for "teaching geometry in ways that are quite different from my own high school experience." Lynne's high school experience was with a "straight Euclidean geometry" class in which she learned by listening to lectures and working on homework. Lynne explained that in her high school geometry course "very little collaboration was done, no projects were assigned, no oral presentations were given, and no real problem solving was done - only simple applications of proofs."

Lynne completed her description of geometry and teaching background by indicating the Curriculum Standards (1989) played a role in her teaching: "The NCTM [Curriculum] Standards are also quite helpful in establishing guidelines for the teaching of high school mathematics." Lynne's teaching and geometry background suggested that she liked geometry and that she had prepared herself to teach a geometry that was different from the one she learned in high school.

View toward curricular change. Lynne's view toward curricular change in goals, content, and instructional methods was suggested in her geometry biography. The biography indicated that Lynne used course work and conferences to prepare for teaching a geometry course that
was different from the formal Euclidean geometry course she experienced in high school. The staff-generated restructuring process at Lynne's school indicated that Lynne and her colleagues were committed to a curricular change involving goals, content, and instructional methods that they felt best prepared their students for the twenty-first century. Lynne had a leadership role in the development of her school's restructuring plan.

The restructuring process at Lynne's school began four years ago, before the present state-legislated reform was mandated. With the aid of a state grant, Lynne's high school spent one year looking at research to determine the skills students would need in the twenty-first century. Following the focus of research, the staff worked for a year on writing school standards emphasizing career and life options, technology, community involvement, content, and process. During this process the staff realized that 50-minute periods did not allow students enough time to work in groups. Plans were made to pilot 90-minute courses. Based on the success of the pilot block classes, the staff agreed to try a block schedule (90-minute classes taught every day) for one year for all classes. At the end of the year, staff, students, and parents evaluated the block schedule and made the decision to continue with this schedule. A reevaluation of the block schedule was to be
done at the end of the second year which was the same year in which the study was conducted.

Lynne's description of her school's system of standards indicated that students had three levels within which to create portfolios and to meet benchmarks related to staff-identified school standards:

We have a system here where the kids come in at entry level and they put portfolios together and meet certain benchmarks. Then they move to core level and continue to gather materials in their portfolio until they reach that benchmark. Then they go on to application level which is kind of their exit out. . . . We have as a staff identified about 12 or 13 standards that we want our kids to meet, and so we as a staff have made a commitment to helping kids learn the skills they need to reach those standards.

The system at Lynne's school consisted of 13 standards which students had to meet. These standards were divided into three areas: (a) universal, (b) content, and (c) process. The universal standards were described as career and life options, systems awareness, technology, and community involvement. The content standards were identified as global awareness, personal awareness and relationships, humanities, science, and mathematics. The process standards included responsibility, learning and thinking skills, communication, and teamwork. Lynne indicated that part of the restructuring process included the process of reexamining and adjusting their standards, "Every year we keep looking at them and fine-tuning. What
that does is just reasserts that you feel they are valuable."

When asked about the state-legislated reform, Lynne’s reply reiterated that her school had their own restructuring process in progress, but also indicated that her school’s system was related to the reform advocated by the state:

Like I said before, we’re kind of on our own here and have moved ahead and have our own portfolios and our own standards and they’re very closely related to the CAMs [Certificate of Advanced Mastery] so I feel pretty comfortable with that [state reform] in theory. . . . I believe down the road if and when the CAMs become a reality that it will be a rather smooth transition for us to take our 12 or 13 standards and mesh them with the CAMs at the state level.

Lynne’s opinion about the Curriculum Standards (1989) indicated that she liked the basic ideas presented in them: "I think the basic ideas in there are tremendously important. I think that they not only have content, but they also deal with process." Lynne explained that this focus on process was "one of the ways that math has changed." Lynne stated that she and her departmental colleagues used the Curriculum Standards when designing ideas to use in their classrooms: "As we design lessons, projects, and our approaches we take a look at what kinds of skills and knowledge our kids should have from the [Curriculum] Standards, and we try to reflect that in our design."
An introduction to Lynne's classroom. Lynne's school was in its second year of a block schedule implementation. In this schedule classes were taught every day for a 90-minute block, which meant that a year-long course was completed in one semester. For the duration of this study, Lynne was a half-time mathematics teacher and half-time administrator. As a half-time mathematics teacher Lynne taught one section of Integrated II Mathematics (the school's only geometry course) first semester and two sections of Integrated III Mathematics second semester. This schedule was set right before school started and was a change in Lynne's first semester schedule.

The Integrated II Mathematics course emphasized a variety of geometries such as plane, coordinate, transformational, and solid. Algebra, probability, and statistics were also incorporated into the course. Lynne's administrative duties included serving on a district assessment committee and other tasks as assigned by the principal. Lynne's section of Integrated II Mathematics consisted of 16 male and 10 female students. The majority of these students were either ninth- or tenth-grade students.

Lynne's classroom was one of six mathematics classrooms located around a mathematics/computer center. Lynne's room was spacious enough to accommodate 11 groupings of desks with three desks in each group. Lynne
indicated that she started the beginning of the year with the desks in rows. Two weeks into the school year, after Lynne got to know her students, she arranged the students and desks in groupings. Groups were changed after the group members had worked together for two chapters. In each of the front two corners of the room was a teacher's desk. Lynne shared this room with another teacher. In the front center was a table and a podium which often held Lynne's textbook and materials for a lesson. Displayed above the front blackboard was a computer-generated banner which asked "Have you helped someone be successful today?" As the school year progressed, the room decor was enhanced by career and quadrilateral exploration projects done by students of both teachers using the room.

Throughout the observation period, Lynne presented a serious and enthusiastic approach to the teaching and learning of geometry. Lynne's positive interactions with her students were also evident during the observation period. Each observed lesson followed a similar format. Students entering Lynne's classroom usually found Lynne organizing information at her desk or writing the warm-up on the board. When the bell rang, students were seated in their desks, waiting for announcements to be broadcast over the public address system. As announcements were made, Lynne took roll and then walked around the room glancing at students' work as they began the warm-up. Warm-up
activities ranged from 7 to 17 minutes in length and concluded with a class discussion guided by Lynne. Prompted by Lynne, students stated and explained their warm-up results. If needed, Lynne provided additional concept explanations. Lynne’s reasons for using warm-ups included "to get class started and to get kids focused on mathematics as quickly as possible, . . . to review some concepts and to check their [students’] understanding, . . . and to pose interesting questions."

After the warm-up, Lynne spent the next 10 to 15 minutes working student-requested problems at the board. Occasionally, Lynne selected a problem for the class to consider during homework deliberation. Homework discussion continued until Lynne decided that her intended ideas had been addressed. At the completion of the homework discussion, one member from each group collected the homework and turned it in to a basket sitting on a counter near Lynne’s desk. Lynne then proceeded with the day’s activities. These activities assumed varying forms such as large group discussion, small group work, small group presentations, hands-on activities, reading, and work time. Based on the work students did during the day’s lesson, Lynne had students hand in the in-class work before the end of the period. The classwork assignments enabled Lynne "to check to see if they [the students] have the understanding to do the homework." With respect to the in-class work
Lynne stated, "They may work in their groups and ask any questions, and we try to clear those up."

The context in which the teaching and learning occurred in Lynne's classroom was characterized by her desire to move away from a teacher-centered classroom and toward a student-centered classroom. Lynne explained that, with a movement toward a student-centered classroom, she wanted her students to be active learners and to take charge of their learning:

I guess basically it gets down to moving away from a teacher-centered classroom and moving towards a student-centered class. I really want the students to be active learners. I want them to be taking charge of their learning, and so I try to move away from being the center of the room and let them be the center and let them interact and take over.

The banner across the front of the room which asked "Have you helped someone be successful today" and the arrangement of desks in groups of three suggested that Lynne encouraged her students to work together. Lynne's description of her changed classroom implied that working together meant more than completing an assignment: "The classroom has changed as well. There's a lot more interactions, a lot more teamwork (group work) going on, and a lot more investigations."

Planning decisions and influential factors. Lynne's planning decisions for her geometry course were closely tied to her work with colleagues who also taught Integrated
II Mathematics. Lynne explained that she and her colleagues determined the textbook content coverage and decided what supplementary material made the course better:

At the beginning we sit down and look at what content in this text do we want to make sure we cover. And when do we want to supplement; what kind of things are not here that we want to improve the course. And usually all of the teachers of geometry [the Integrated II Mathematics course] do that.

The planning-generated information was organized by Lynne according to the chapter outline in their textbook, The University of Chicago School Mathematics Project Geometry (Coxford et al., 1991). Additional planning comments by Lynne indicated that the planning process continued as she made decisions about the course while taking into consideration the amount of time available, the concepts that needed to be addressed, and how she was going to use the textbook. Lynne explained the planning decisions that had to be made:

Then it depends on about how long we are going to take on this [the content] this year and the given amount of time and what are the key concepts I want to cover. Which ones can be combined? . . . Which ones do I want to enrich? And how am I going to enrich them?

Additional comments by Lynne indicated that as she made planning decisions she wrote notes regarding the ideas she wanted to include in lessons, the good examples to use, and the instructional approach for addressing a topic.

As could be expected with an Integrated II Mathematics course that emphasized geometry, Lynne’s course goals for
her students included references to both integrated mathematics ideas and geometric ideas. Lynne explained that her goals for Integrated II Mathematics evolved as she and her colleagues went through the textbook adoption process:

Well I think they [course goals] kind of evolved as basically when we started looking at the last textbook adoption. We started looking philosophically what direction do we need to go with the kids, and we decided that we really needed to look toward an integrated approach. We looked at what was on the market and really got involved with the Chicago materials and philosophy behind that approach to mathematics.

Lynne's interpretation of the philosophy behind the Chicago materials indicated that the materials presented a mathematics course that integrated mathematics including algebra, geometry, as well as some probability and statistics rather than isolating each of the subjects. Additionally, these materials displayed the usefulness of mathematics, focused on real-world applications, and fostered the movement away from drill and practice.

Lynne stated "The general goal is to help kids develop an appreciation for geometry as well as the other areas of math and how they interrelate." Expanding on this general goal, Lynne wanted her students "to become more sophisticated in their approach to mathematics, to see some of the applications, and to understand that geometries are systems and ways of interpreting the world around us."

Lynne’s discussion of each of these goals illuminated
their meanings. For Lynne, students becoming more sophisticated in their approach to mathematics meant students were "more independent problem solvers having more than math skills." Specifically, Lynne wanted students to develop their abilities to "ask good questions, be logical thinkers, to express situations mathematically, to analyze data, to draw conclusions, and to make valid arguments." These abilities implied that a sophisticated approach to mathematics was one in which students used multiple processes while learning and applying the mathematics.

In regard to applications, Lynne wanted students to see real-world problems in her classroom:

I like to use a lot of examples . . . looking out there at the world, and seeing all the geometric shapes and taking situations such as earthquakes and stress on bridges and those kinds of things and bring them back to the classroom and talking about that.

Lynne explained that understanding geometries as systems meant understanding the process of coming up with different systems: "We start out with certain premises and if we change those, if we change the rules, we come up with different systems."

Lynne's description of her anticipated content sequence showed that the Integrated II Mathematics course began with the undefined terms, points and lines, in the context of the different geometries. Lynne explained the starting point of her course:
Well, basically points and lines and working with the different definitions of the point. You start initially talking about different geometries, depends on how you define point as to what kind of geometry system you’re in. Points as dots, and we’ll get into pixels on screens in the discrete area of mathematics and geometry.

Lynne identified the different geometries that are considered part of this initial exploration of geometric systems: "It starts out initially talking, like I said, about a point and compare this system. So you talk about transformational geometry, coordinate geometry, plane geometry, all those different kinds of geometries."

Once the different geometric systems were initially examined, Lynne stated that the content sequence continued with topics such as proofs, polygons, triangles, quadrilaterals, circles, a little trigonometry, surface area, volume, similarity, and three-dimensional figures. Lynne’s content comments also indicated that probability, statistics, and algebra were integrated into the program.

When asked "How did you decide what content was to be included in your Integrated II Mathematics course," Lynne’s reply suggested that the textbook adoption process and the selection of the Chicago materials were two main factors. Lynne explained the reasons for her content choice:

Again I think, you know, initially it started by looking at the sequence and the availability of textbooks and so when we bought into the Chicago materials, we bought into basically that concept as well as that content. We supplement that [the Chicago materials], but that kind of guides our program.
Lynne referred to the Chicago materials "as basically one of the stepping stones to a totally integrated program."

Her reference provided explanation for why Lynne and her departmental colleagues supplemented the Chicago materials.

Expanding on the process of supplementing the textbook, Lynne's comments explained that she acquired supplementation ideas from her departmental colleagues and from materials located in their office. Lynne described the sources for her ideas:

Everybody is quite creative and does a lot of sharing and we have little notebooks sitting there [on an extra desk in the office] saying this is what I did and if you want to use it go ahead. . . . We have a whole storeroom full of sources and supplementary materials and stuff like that we use quite a bit.

Lynne's plans for instruction indicated her commitment to moving toward a student-centered classroom. Lynne's description of her plan for helping students meet course goals showed that she wanted her students to be actively involved in their learning by working on investigations and by communicating mathematics with their classmates. Lynne explained how she helped her students meet the course goals:

I think the goals are best met if you give the students a lot of opportunities to do some investigations so that they begin to see the patterns; they begin to see the relationships. I feel strongly about students working together and discussing their findings and having some practice and feeling confident in their ability to do so.
For Lynne, working together meant that students worked as a team toward a common goal with each member having responsibilities to fulfill. Lynne’s plans indicated that students were to work together on investigations, proofs, and problems designed for applications of concepts.

Communicating mathematics with their classmates was an important component of students working together. This communication involved comparing and contrasting ideas, writing mathematical ideas, and making oral presentations. Students also used their communication skills while reading the textbook and by recording class notes in an organized notebook.

Lynne’s planning comments also indicated that technology was to be used during instruction. With the computer center next to the classroom, the computers were the technological tool utilized. Lynne’s comments implied that students were to use the computers to do investigations. For example, using GeoExplorer (Senk, Hirschorn, & Usiskin, 1992) students created a quadrilateral hierarchy based on relationships between diagonals.

In addition to utilizing instructional methods that fostered students’ active participation in their learning, Lynne used a lecture/recitation format in which she explained ideas and led discussions. Lynne emphasized,
however, that her talking was not to be the focus of instruction:

I think it's much more effective to be a facilitator-type person in the classroom. . . . Although there are still times that I'll be up in front of the room, talking and explaining, the more that they [students] can interact and the more they can take charge, that's the direction I'm going.

Lynne's plans for content choices and instructional approaches indicated her commitment to the restructuring process at her school. Two standards identified as part of the restructuring process were teamwork and communication. Previous instructional comments addressed how Lynne provided opportunities for students to meet these standards. Lynne identified information from engineers as a reason for focusing on teamwork and communication skills:

I guess one of the motivations for moving them [students] this direction is the information we got from engineers. They said basically that kids were coming out of schools knowing their math quite well, but really not being very effective in being able to communicate about it nor were they very effective in working in teams which they [engineers] really had to do out in their work force.

Lynne's instructional plan provided other examples of how she helped her students meet school standards. Lynne stated "I will incorporate some projects so that they [students] can look at completing things for their portfolio." As examples, Lynne's students worked on the communication standard where the form of communication was writing, on the teamwork standard by helping students
develop their small group process skills, and worked on the mathematics standard by developing an exemplary piece of mathematics. An exemplary piece of mathematics was a project in which students effectively analyzed or solved a clearly defined problem, applied and connected at least two mathematics themes (geometry, numeration, mathematical language and procedures, measurement, statistics and probability, algebra, functions and relations), and clearly communicated results.

Interactive decisions and influential factors. An examination of Lynne’s instruction provided information about interactive decisions and factors influencing those decisions. Interactive decisions included decisions reflecting an implementation of plans and decisions made during day-to-day lessons. The decisions informing this study were those regarding the geometry course involving goals, content, and instruction.

Based on observations of Lynne and conversations with her, students were provided with a variety of opportunities to make progress toward the course goals. Lynne’s general goal for her Integrated II Mathematics course was for students to appreciate geometry as well as other areas of mathematics and to see how these areas interrelate. Student opportunities to meet this goal were articulated by Lynne in terms of becoming more sophisticated in their approach to mathematics, seeing the applications of the
Integrated II Mathematics course, and understanding that geometries were systems and ways of interpreting the world.

According to Lynne, students who had become more sophisticated in their approach to mathematics meant that they could demonstrate they were independent problem solvers while exhibiting skills that involved more than computation of numbers or statement of knowledge. One part of students becoming more sophisticated in their approach to mathematics included being able to write proofs using three different forms: (a) flowchart, (b) paragraph, and (c) two-column.

Becoming more sophisticated in their approach to mathematics also included student learning and application of mathematics while using different mathematical methods. For example, students were asked to compare and to contrast their work with group members, to read the text and to answer "applying the reading" questions, and to prepare for and to make oral presentations. Comments by Lynne also indicated that students completed small group investigations on the computer. During these investigations students were asked to generate data and to draw conclusions about the data. Depending on the nature of the investigations, results were either recorded on a worksheet or displayed as a poster.

Classroom observations did not provide enough information to discuss in detail student opportunities to
meet the application goal. Lynne’s explanations of how these opportunities were given indicated that applications meant more than application to the real world. Lynne stated that part of the application goal was students working in groups to discuss and to work mathematics problems:

We talk about how important that [discuss and problem solve mathematics in groups] is in terms of careers and applications, that not only do you understand the mathematics but that you can work together with problem solving and communicate about it. So that’s part of the application.

Additional comments suggested that real-world connections were also addressed as students worked with tessellations and art, the study of triangles and architecture, and problems involving area, perimeters, and volumes.

The observation schedule did not provide the opportunity to collect complete information about students’ study of different geometry systems. Students’ introduction to different geometry systems occurred when the class studied the material in chapter one. The first observation of Lynne occurred when students were studying material in chapter two. The observations did show, however, that students had opportunities to work with different systems of geometry throughout the course. For example, students looked at parallel lines from a coordinate geometry perspective and then examined two parallel lines cut by a transversal in a plane geometry setting. Another example of integrating the varied systems
of geometry occurred when transformations were used to discuss corresponding parts of congruent triangles.

As students were provided with many opportunities to meet Lynne's course goals they encountered a variety of geometry topics. Observed lessons included topics such as writing definitions, angles, parallel and perpendicular lines, applications of constructions, proofs, isosceles triangles, transformations, and congruent triangles. With the exception of constructions, topics presented by Lynne in the course matched the planned topics indicated during the first interview with her. When the textbook first addressed constructions in chapter three, the textbook only included constructions involving perpendiculars. Lynne supplemented the textbook's information by assembling a packet of materials that consisted of other basic constructions (e.g., copy an angle, construct a line parallel to a given line through a given point) and utilization of constructions to explore a geometric concept or to solve a problem (e.g., investigating concurrent lines of triangles, finding a buried treasure on a map). The concentration on constructions during the first part of the course provided students with a reinforcement of the vocabulary currently being studied and furnished students with problem solving skills that were used when students later worked with transformations.
Observations verified that Lynne's sequence for content went as planned. Lynne's reflection on the content sequence indicated that the sequence had gone as expected, but she was concerned about the content she was able to address before the course concluded at the end of the semester. Lynne explained "I am seeing that I am running out of time and now I am having to really be selective in what I do for the remainder of the course." Remarks by Lynne indicated that she made decisions based on concepts needed "to compete on some of those standardized national tests" and "to progress to the next level of mathematics." Lynne's tentative plans focused on the content that was to be included. Lynne's comments implied that she planned to address ratios, proportions, similarity, some trigonometry, as well as some work with circles, tangents, segments, and chords.

Lynne's teaching demonstrated that her overall plans for instruction were implemented. Lynne's plans predicted the use of instructional methods that signified a movement toward a student-centered classroom. Confirmation of these plans occurred with Lynne's use of student groups as a major instructional method utilized during her lessons. The permanent arrangement of desks in groupings of three suggested the importance of the group structure to Lynne's classroom. Lynne used the group format when collecting papers or handing out materials for an activity. In these
situations, one member from each group was responsible for turning in papers or picking up materials.

As part of the class structure, students were encouraged to help each other while working on in-class assignments and homework assignments. Lynne's comments during instruction indicated that she expected students to explain their work while helping each other and that these explanations were helpful to each student. For example, as students worked on a proof as part of their in-class assignment Lynne encouraged students to share their logic with a partner:

Now for some of us proofs are, for some reason or another, are a little bit easier than for others so this is really a way you can help each other out here. Explain the logic you use in a given proof to one of your partners. What that does is not only it helps them, but it helps you really solidify your understanding.

Observations showed that working in groups also meant students working as a team to complete a task. Examples of group tasks included reading the textbook and writing definitions based on the reading, working problems that required the utilization of constructions, creating flowchart proofs, and presenting a group-worked problem to the class. Based on Lynne's comments and on project results displayed on posters, students also participated in mathematical explorations using the computer. Examples of students using the GeoExplorer (Senk et al., 1992) for explorations included learning about transformations and
included creating a quadrilateral hierarchy based on diagonals.

Lynne's commitment to providing an instructional environment where students were taking charge of their learning was supported by her expectations of students while working in groups. As indicated by samples of Lynne's comments during instruction, she expected students to communicate with each other and to work as a team. In the first example where groups worked on flowchart proofs, Lynne's statements showed that students were expected to discuss their work while striving to come to an agreement about the proof:

When you're finished your group should have the same flowchart. So you need to talk it through and agree. Just don't simply sit there and do a flowchart. You want everyone to have the same flowchart because when you go to the next group, whoever goes is representing your group work, not individual work. You can go ahead and start on your own, but then start comparing and talking.

The context for the second dialogue was a situation in which each group worked on a different problem involving transformations and prepared to present their assigned problem to the class. Lynne's comments demonstrated that group members were to agree on the solution and to share the responsibilities of the presentation. Lynne stated, "Once you get your problem finished and you agree on it in your group then you might decide how you are going to present it, who's going to do the writing, who's going to do the explaining, who will take care of questions, that
sort of thing." As suggested by the type of tasks completed by groups, students were provided with opportunities to be active participants in their own and others' learning.

The restructuring process at Lynne's school advocated a movement toward a student-centered classroom. Lynne's plans and her instruction displayed a commitment to her school's restructuring process. Observations of Lynne's lessons confirmed that she wanted to help each student make contributions to their school portfolio - a compilation of a student's progress toward meeting the school standards. During the eight observations, Lynne made references to students' school portfolios during five of the lessons. Lynne's references explained how students' projects, notebooks, and peer- and self-evaluations could be part of their portfolios.

Samples of Lynne's statements during instruction provided more insight into the contents of the portfolio. The first example of Lynne's classroom dialogue on this topic was from a lesson in which she explained the foundation for students' work with logical thinking. Lynne's statements showed that students could include in their portfolios a project involving logical thinking, which was part of the school process standards:

This section is setting you up to develop that skill [logical reasoning]. I would like to see as a paper in your portfolio in this class a real nice justification to show or demonstrate that
you can think logically, that you can use a lot of information, understand terminology and get a nice piece together.

A second example of Lynne’s classroom dialogue contained a description of how she and an individual student prepared a portfolio note-taking entry, which was one of the school process standards. Lynne’s description showed that she wrote an introduction to explain the purpose of the note-taking activity and the student wrote a reflection on the project:

I will write-up a brief summary of what this activity was like so that you have it in your portfolio and someone [who] looks at it just doesn’t say "This is just class notes." It [Lynne’s write-up] will explain what you are doing, and then I would like you to just put a little paragraph of a reflection. You should describe what you did and how it was helpful.

Students were ready to make this note-taking entry once they had met Lynne’s note-taking goal for class notebooks three times.

Lynne’s instructional plans indicated that she also explained concepts. Lynne’s explanations, in the form of guided discussions, occurred during some part of each observed lesson. During discussions about warm-up, homework, and in-class problems, Lynne asked students for solutions to the problems and for suggestions on how to proceed with the problem. In addition, Lynne summarized class deliberations about the problem being discussed. Lynne’s use of guided discussions were also evident in those observed lessons in which new material was presented.
For example, as Lynne demonstrated how to work miniature golf reflection problems, she asked students to tell her what step occurred next and to explain why that was the next step. During all discussions Lynne called on volunteers and non-volunteers and appeared to involve students from all parts of the room.

In addition to decisions made regarding Lynne’s overall content and instruction, decisions were made during daily lessons with respect to the implementation and the context of the lesson. Lynne’s reflection on how her lessons had proceeded included comments such as "I thought the lesson went well," "I felt good about the quality of work," and "I didn’t get as far as I wanted because I saw problems." When Lynne reflected on decisions made during her lessons, her remarks consisted of decisions concerning instruction and student understanding. Lynne’s decisions regarding instruction referred to her determination of how to deal with content when class time was limited. For example, this type of decision occurred when students were working on congruent triangle proofs based on congruent theorems for any type of triangle. Part way through the lesson Lynne chose to have students use class time to work more proofs rather than to participate in a planned class discussion on the next section (congruence theorems for right triangles). Lynne felt that students needed a chance to get questions answered while working on proofs and that
they were able to start their study of the next section by reading the textbook.

Lynne’s remarks about decisions concerning student understanding indicated her awareness of students’ progress toward understanding a concept or relationship. Her remarks also showed that if students were not understanding a concept or relationship, she had to determine how to help her students. An example of this type of decision occurred when students were examining parts of a triangle during a classwork assignment. As Lynne walked around to see how students were progressing on the in-class assignment she discovered they were having difficulty identifying the side opposite a given angle (and vice versa) in a triangle. Lynne called the students together, went to the board, and led students through examples in which they practiced naming parts of a triangle. Lynne explained "I didn’t go to the board because of the one student I was helping; it’s because I could see that many students were having difficulties." Lynne’s decisions about how to use class time and about what to do when students did not fully understand a concept suggested that she tried to meet the needs of her students while implementing her plans.

Lynne’s actions during instruction provided decision-related issues to pursue and routines to describe. The importance of homework was apparent throughout the observation period. Lynne allocated class time for
discussion of homework and readily addressed students’ questions. When students asked questions, Lynne read the problem and then asked students for ideas on how to do the problem. Using students’ ideas, Lynne led the discussion of the problem. At the completion of the discussion, students handed in their assignments. When grading the homework, Lynne focused on four or five significant problems. When papers were returned to students, Lynne briefly discussed problems that gave students trouble as well as complimented students for problems that were done well.

To foster students’ success with homework, Lynne used in-class assignments. During most of the observations, students were asked to work a few problems on the lesson’s topic and to hand in the in-class assignment before the end of the period. As students worked on these problems they were able to get help from their group members and Lynne. Collected in-class assignments were graded by Lynne and then returned during the following class period.

During all classroom discussions student participation in discussion appeared to be fostered by Lynne’s work with them. Examples of dialogue during instruction demonstrated how Lynne encouraged students to participate in discussion as well as nurtured their understanding of the concept under discussion. In the first example, students looked at the graphs and equations of two perpendicular lines \( y = 2x \)
In order to determine the pattern of the slopes of these lines. In this dialogue Lynne worked with students' ideas while encouraging students to state the main idea of the discussion:

Lynne: Does anybody see a pattern between these two slopes?

Art: They are kind of backwards and one is negative.

Lynne: Okay. The negative, we might say opposite, the negatives are opposites. Backwards - I know what you mean but can you think of another way of saying that?

Art: Flip-flopped.

Lynne: Flip-flopped. What is that word we call when we . . .

Rose: Opposite

Lynne: No, opposite is . . .

Rose: Inverse. Reciprocal.

Lynne: Reciprocal. Okay. So they are negative reciprocals.

In a second example, Lynne led the discussion on a homework problem which asked the students to draw a triangle with sides 3 cm, 11 cm, and 6 cm. This dialogue demonstrated how Lynne tried to involve the student who asked the question (Knute) as well as other students from the class:

Lynne: Okay, take a look at those numbers. If I gave you a ruler, do you think you could draw that triangle?

Knute: Yeah.
Lynne: Does everyone agree that Knute could draw that triangle?

[Many students shake their heads no.]

Meta: No.

Lynne: Meta says she couldn't. Do you know why she couldn't?

Knute: No.

Lynne: Why does Meta say Knute won't be able to draw that? Wade?

Wade: Because if you add up the sides the second side, the second one is 11 cm; the others are 3 and 6. If you add 3 and 6 together they would not be bigger than 11. So you can't draw it.

Lynne: Okay, and what do we know has to be true about the sides of any triangle? Knute?

Knute: The triangle inequality theorem.

Lynne: Yeah. What's the triangle inequality theorem? Wade explained why it won't work and so now I am just asking for a summary of what the theorem says. Knute, do you know what that is?

Knute: No.

The discussion continued as Wade reexplained the theorem, as Lynne paraphrased Wade's words, and Lynne worked with Knute until he understood the summary of the theorem. Student participation in discussion also appeared to be promoted by Lynne's expectation that students participate, her validation of their responses, and her eye contact with the students.

Observations indicated that Lynne's movement around the classroom was a general routine for her. Lynne walked
around the classroom while leading discussions and making announcements and as students worked on in-class exercises and assignments. Lynne’s circulation around the room as students worked on in-class exercises and assignments functioned as an ongoing assessment technique for her. Throughout her movement, Lynne gathered information regarding students’ progress on their work and acted on the information she gathered as needed. Sometimes the action involved Lynne stopping and talking to a group and answering their questions. At times when many students were having difficulty with a given concept, Lynne called the students together and addressed a topic at the board. An example of this occurred when students were working construction application problems. Students were having difficulty understanding inscribed circles and circumscribed circles. Lynne called the students together and explained how the terms were used in their problems. Lynne’s actions during work time also included comments that validated students’ work. Example comments included "I hear some good discussions going on" and "Everybody is just about finished."

Summary of Lynne. Lynne was a secondary teacher beginning her twenty-sixth year of teaching mathematics and her sixteenth year of teaching geometry. Lynne’s school was in its second year of a block schedule implementation. In this schedule Lynne taught Integrated II Mathematics
every day for 90 minutes. This course was the school’s only geometry course. The majority of Lynne’s students were either ninth- or tenth-grade students.

Lynne’s planning decisions for her Integrated II Mathematics class described a course that addressed geometry concepts from multiple perspectives. The basis for the description of Lynne’s course were her goals. Lynne’s general goal for her students was their development of an appreciation for geometry as well as the other areas of mathematics and to see how these areas interrelate. One component of this general goal was Lynne’s focus on students becoming more sophisticated in their approach to mathematics. The process of students becoming more sophisticated in their approach to mathematics included their abilities to ask good questions, be logical thinkers, express situations mathematically, analyze data, draw conclusions, and make valid arguments. Other components of Lynne’s general goal for her students included doing applications and understanding that geometries were systems and ways of interpreting the world.

Lynne’s teaching indicated an implementation of her planning decisions. In a group-structured classroom, students had many opportunities to learn about different facets of geometry. Facets of geometry emphasized in Lynne’s course were geometry as a content knowledge base, an example of a mathematical system, and a setting for
developing communication and problem solving skills. Connections within mathematics were seen in Lynne’s course since the basis for the course was multiple perspectives of geometry (plane, transformational, coordinate, solid). In addition, Lynne’s instruction hinted at the connections between geometry and the real world.

While working toward course goals, students studied geometry content relating different geometries and including connections to algebraic ideas. During instruction, Lynne used instructional methods such as group activities and investigations, group presentations, group work time, reading, and guided discussions.

The foundation for the factors influencing Lynne’s decision making was her commitment to moving toward a student-centered classroom and her dedication to the school’s staff-generated restructuring process. Lynne felt that her course goals were best met in a classroom where students were active learners and were group workers. Lynne strove to be a facilitator of students’ mathematics learning and to be a guide for students as they worked toward the school standards. In her geometry-focused course, Lynne provided students with many opportunities to work on process standards such as thinking skills, communication, and teamwork.

In addition to self-motivation, Lynne’s involvement with the school’s restructuring process was motivated by
her teaching colleagues. Lynne and the rest of the staff developed the school standards and made a commitment to help their students meet these standards. Lynne and her mathematics colleagues worked together to design courses that supported the school standards.

The geometry-focused class that Lynne taught differed from the high school geometry course she learned in high school. One of the factors influencing Lynne's decision making was her willingness to teach a course that included different geometries as well as other areas of mathematics in a manner that suggested a movement away from a teacher-centered classroom. Lynne's preparation for teaching her present geometry-focused course was a result of her experiences in college courses and at mathematics conferences, her work with her teaching colleagues, and her use of the *Curriculum Standards* (1989) as a guideline for her teaching.

The textbook selection was another factor that influenced Lynne's decision making. During the selection process, Lynne and her departmental colleagues made the decision to move toward an integrated approach for their geometry class and chose a textbook that supported this move. In addition, the teachers were influenced by the philosophy behind the adopted materials. The philosophy advocated integrating geometry with other mathematics,
incorporating the usefulness of mathematics and real-world applications, and moving away from drill and practice.

Lynne used the textbook, chosen for its philosophy, as a guide when making decisions about content. Lynne's comments indicated that she started the course with undefined terms from the perspective of different geometries. Observed lessons showed that as the course progressed students studied topics such as writing definitions, angles, parallel and perpendicular lines, applications of constructions, proofs, isosceles triangles, transformations, and congruent triangles. Even though the textbook's philosophy coincided with the direction Lynne and her departmental colleagues wanted to take their geometry course, the textbook did not define the course. To implement their envisioned Integrated II Mathematics course in the context of their school standards, Lynne and her departmental colleagues supplemented their textbook with computer activities, self-designed projects, and printed resources.

Lynne's instruction followed the routine of warm-up, homework discussion, and presentation and practice of new content. Lynne's decisions regarding instructional methods used during the daily routine and factors influencing her decisions characterized Lynne's instruction. A predominant characteristic of Lynne's instruction was her use of groups. The importance of the group format was suggested
by the permanent arrangement of desks in groupings of three, the use of groups when collecting papers and organizing activities, and the frequency with which students worked with each other. With the group format, students helped each other on assignments, completed group tasks and computer investigations, and made group presentations. The majority of group work emphasized students’ ability to work as a team.

Students’ ability to communicate with each other was an important component of students’ group work as well as a predominant characteristic of Lynne’s instruction. In addition to communicating during group work, students were asked to participate in class discussions, write self- and peer-evaluations, keep a notebook, and communicate mathematics through project work. An emphasis on teamwork skills and communication skills was supported by Lynne’s goal to move toward a student-centered classroom, her school standards, and her acquired information from engineers working in industry.

Lynne’s utilization of ongoing assessment techniques was also a predominant component of her instruction. By calling on volunteers and non-volunteers for a response and reasons for their response and by having students present problems at the blackboard, Lynne became aware of her students’ understanding of geometric concepts and relationships being studied. Lynne also gathered
information on her students' understanding by systematically circulating around the room looking at students' work and consulting with individuals and groups about their work. Additional information about students' understanding was gathered through summative assessments including written tests, notebooks, self- and peer-evaluations, and projects based on computer investigations.

Lynne's awareness of her students was evident in her reflections about factors influencing her decisions during instruction. Specifically, Lynne's reflections indicated that when making decisions during instruction she sought to balance what her students needed in order to understand a concept or relationship with what she had planned for the lesson. In most cases, Lynne made decisions about how to address the content when class time was limited and about how to help students understand a concept or relationship better.

Lynne's planning and interactive decisions showed that she had defined her geometry-focused course and had implemented the course depicted in her plans. Predominant factors affecting Lynne's decisions were her commitment to moving toward a student-centered classroom, her dedication to her school's restructuring process, and her willingness to teach a geometry-focused course that was different than the geometry course she learned in high school.
Cross-case Profile

The geometry teaching experience of the teachers in this study ranged from 8 years to 23 years ($X = 15.2$), suggesting these teachers had developed ideas about geometry course goals, content, and instruction. Both formal and informal geometry courses were offered at the high schools of Ardella, Jon, and Jordan. Students who had previously demonstrated success in first year algebra took the formal geometry course. The majority of students at Emily’s school took geometry as tenth graders; a few accelerated students took the same course as part of a special grade block within the school’s block system. At Lynne’s school, only one geometry course was offered and was titled Integrated II Mathematics. During the course of this study, four of the teachers (Ardella, Emily, Jordan, Lynne) worked within a block schedule and the fifth teacher (Jon) worked within a six-period schedule.

Further comparisons between individual teacher profiles generated additional similarities and differences across the sample. The resulting description highlighting the similarities and differences is presented in two sections: (a) planning and interactive decisions, and (b) factors influencing planning and interactive decisions.

Planning and Interactive Decisions

Planning decisions made about goals, content, and instructional methods generated descriptions of each
teacher's intended geometry course. Their interactive
decisions demonstrated the extent to which teachers
implemented their stated plans. For all teachers in this
study, observed classroom teaching reflected the geometry
course generated by their earlier planning decisions with
the possible exception of the application of geometry to
the real world. Geometry course descriptions include
different facets of secondary geometry, selection and
interpretation of geometry content, and instructional
components.

Facets of secondary geometry. Different facets of
graphology were highlighted by teachers' stated goals and
their instruction that provided students with opportunities
for meeting these goals. Facets of geometry featured by
the teachers included geometry as a content knowledge base,
an example of a mathematical system, and a setting for
developing communication and problem solving skills. In
addition, teachers' instruction hinted at connections
between geometry and the real world and between geometry
and other areas of mathematics.

The facet of geometry described as a content knowledge
base was founded on teachers' course goals that stated the
"basic geometry stuff" they wanted their students to know.
All the teachers wanted their students to know foundational
geometry concepts such as points, lines, planes, rays,
angles, polygons in general, and specific polygons -
triangles and quadrilaterals. Relationships involving these concepts were also part of this knowledge base. As two examples: (a) vertical angles are congruent, and (b) the sum of the angles in a triangle equals 180 degrees in Euclidean geometry. The establishment of content knowledge base provided a foundation for other facets of geometry addressed in these teachers' classrooms.

A second facet of high school geometry addressed by all five teachers was that geometry is an example of a mathematical system. This aspect of geometry included both the logical development and the process of how mathematicians derive mathematics. The interpretation and the amount of emphasis given to this facet of the course varied considerably. In his course, Jon emphasized almost exclusively the deductive development of geometry content and the cultivation of logical thinking ability through the writing of two-column proofs. Both Jordan and Ardella wanted their students to get a sense of how mathematicians derive mathematics, using intuitive and inductive reasoning to make conjectures followed by deductive reasoning to verify the conjectures. For Jordan, students' ability to use inductive reasoning and deductive reasoning in this manner was his overriding course goal. For Ardella, this goal was one strand of her course. Lynne incorporated geometry as an example of a mathematical system through the use of multiple formats for deductive proofs including
flowchart, paragraph, and two-column proofs as well as through the use of projects requiring student investigative work and application of concepts. Emily gave the least emphasis to the facet that geometry is an example of a mathematical system. Inductive reasoning and deductive reasoning were part of the thinking skills she tried to instill in her students, but the interaction between the two types of reasoning was not addressed during the observed lessons. Her emphasis was on the intuitive development of geometric ideas. At the end of the study, however, Emily indicated that she planned to include a unit on deductive reasoning and logic during second semester. This decision was a change in her thinking influenced by her professional journal reading that recommended this unit for college bound students.

A third facet of geometry addressed by each teacher in some manner was geometry as a setting for developing communication and problem solving skills. In Jon’s classroom, student communication skills were practiced as students volunteered to write proofs on the board and to explain their proofs to the class. Jon’s students engaged in problem solving skills as they used diagrams and drew figures while learning vocabulary and completing proofs. Since the development of these communication and problem solving skills were not explicitly stated by Jon as a course goal nor were they prevalent during instruction, the
use of these skills by Jon's students appeared incidental.

In the classrooms of the other four teachers, student development of mathematical communication skills was explicitly stated as a course goal. In the classrooms of Ardella, Jordan, and Lynne, communication of geometry through talking, writing, and reading was an important component of student development of geometric concepts. For all three teachers, much of the talking and writing occurred in group activities. In addition in Lynne's classroom, student communication occurred through group presentations. The communication skills emphasized in Emily's classroom were reading and writing mathematics, but student talking about mathematics was also seen during classroom discussions.

As part of their stated course goals, Ardella, Emily, Jordan, and Lynne wanted students to develop problem solving skills in the context of student growth of visualization skills. Examples of how these four teachers helped students develop their visualization skills included using constructions to explore and apply concepts (Ardella, Lynne, Jordan), using paper folding to investigate a topic (Ardella, Emily, Jordan), writing definitions based on the comparison of figures (Emily, Jordan), and using two- and three-dimensional drawings to apply concepts (Ardella, Emily, Jordan, Lynne).
Based on observed lessons of Ardella, Emily, Jordan, and Lynne, student development of problem solving skills encompassed more than the development of visualization skills. In the classrooms of these four teachers, students were involved in developing a problem solving approach to learning geometry. A problem solving approach involved an active participation by students in their learning of geometry through explorations, discoveries, and construction of geometric concepts and relationships. For Ardella, Jordan, and Lynne, a problem solving approach was part of the group structure in their classrooms as well as part of their large group instruction. For Emily, the problem solving approach was embedded in her focus on student use of reading, writing, visualizing, thinking, and measuring skills to explore and apply geometry concepts and relationships. Included in the problem solving approach to learning geometry was student development of problem solving strategies. In particular, students were encouraged to look for patterns (Ardella, Emily, Jordan, Lynne), make systematic lists (Ardella, Lynne), and draw diagrams or use figures (Ardella, Emily, Jordan, Lynne).

Another facet of geometry addressed in this study was the connection of geometry with the real world. Included in this aspect was students finding geometry in the real world and students applying geometry to the real world. As a course goal, geometry's connection with the real world
was directly discussed by Ardella, Lynne, and Jordan. All three of these teachers wanted their students to appreciate geometry in the world around them. For example, the teachers stated they wanted students to see the symmetry present in a sunflower plant and to understand the role of triangles in bridge construction. Even though Emily and Jon did not directly talk about the connection between geometry and the real world as a course goal, their plans to address perimeter and area subsequently in the course suggested that real-world applications were to be incorporated in their respective geometry courses at a later time.

Teachers' view of the connection between geometry and the real world appeared to be unfocused since observed lessons included informal references to the real world, but did not predominantly address this connection. For example in observed lessons, Emily and Jordan related points, lines, and planes to physical representations such as tip of a pencil, a road, and a chalkboard. Jon used pencils and a kleenex box to help explain geometric concepts when students were confused. Ardella used cartoons to explore geometric concepts and right triangle word problems to apply geometric concepts. Lynne used cartoons as well as a miniature golf setting to explore and apply geometric concepts. In addition, teachers' textbooks and homework assignments included some references to geometry in the
real world. For example, a point represented by a pixel on a computer screen, perpendicular lines represented by an intersection of two city streets, and discussions of careers (e.g., artist, geologist) that use geometry. However, these geometry teachers did not emphasize the connection between geometry and the real world in their daily plans.

Connections between geometry and the real world as well as between geometry and other areas of mathematics were implied by some teachers' inclusion of transformational geometry and coordinate geometry. The incorporation of these two areas of geometry varied among these teachers - from Jon's indication that these were two chapters he rarely reached during the school year to Lynne's use of both geometries as part of the foundation for her course. The amount of inclusion of transformational geometry and coordinate geometry in the classrooms of Ardella, Emily, and Jordan was between that of Jon and Lynne. Ardella, Emily, and Jordan indicated that transformational geometry ideas were informally addressed through the study of tessellations. Ardella planned to briefly address coordinate geometry ideas such as midpoint and slope as well as coordinate proofs such as showing a particular quadrilateral to be a parallelogram or a rhombus. Jordan planned to address three lessons from his textbook: (a) coordinate geometry, (b) the slope of a
line, and (c) slopes of parallel and perpendicular lines. Jordan also indicated that he felt the textbook's work with coordinate geometry was too brief, but he was unsure whether to do the course differently. In the final interview, Emily stated that she planned to finish with coordinate geometry which she felt was a good lead-in for her students who continued with second year algebra in the fall.

Similarities in the five geometry courses were suggested by the facets of geometry - a content knowledge base, an example of a mathematical system, a setting for developing communication and problem solving skills, and a connection with the real world. However, the emphasis given to each facet varied among the teachers suggesting distinctions between the courses. Whether teachers' geometry course promoted one facet of geometry or multiple facets of geometry was one characteristic that suggested course distinctions.

**Selection and interpretation of geometry content.** Similarities in the teachers' selection of geometry content was seen in the topics addressed at the beginning of the course as well as topics discussed throughout the observation process. All teachers except Jordan began their geometry course with undefined terms (points, lines, planes) and basic geometry vocabulary (e.g., line segment, rays, angles). Jordan first did a chapter on inductive
reasoning before addressing the same topics. As the classes continued, students also studied topics such as perpendicular and parallel lines, triangles, quadrilaterals, other polygons, three-dimensional figures, and relationships within and between each of these topics.

Although similar topics were addressed in each of the geometry classes, differences in content focus also existed in these five geometry teachers' courses. As previously discussed in the preceding section, the amount and type of proofs included in the courses varied widely among the five teachers. Similarly, as previously discussed, the inclusion of transformational geometry and coordinate geometry varied widely among the five teachers. Additional content focus differences between the teachers were seen in their use of constructions. As part of the foundation of their respective courses, Ardella and Jordan used constructions to explore other geometry topics. Lynne also used constructions to explore other topics but she did not use them as frequently as did Ardella and Jordan. During the first interview, Jon made comments indicating that he planned to address constructions in his course, but it was not clear when. If Jon continued to follow the textbook guideline as he intended, constructions were to be addressed in chapter nine, "Constructions and Loci" (Kalin & Corbitt, 1992). During the final interview, Emily indicated that her students did mira as well as compass and
straightedge constructions during the second half of the course.

Content differences were implied by the interpretation of content suggested by each teacher’s general instructional theme and by the teachers’ general comments about their course. Jon used a lecture format for his general approach to instruction. In a business-like atmosphere Jon presented theorems, definitions, and examples. Jon’s comments about his course indicated that the content and the approach had been consistent during his teaching career.

The other four teachers’ interpretation of content was suggested by their use of instructional methods that promoted a learning environment in which students were actively involved in geometry learning. Ardella’s interpretation of content was based on her attempt to address content through the incorporation of constructions, hands-on activities, and partner work. The core of Jordan’s general instructional approach was student exploration of geometric content via an intuitive and inductive approach followed by the use of a deductive approach. Comments by Ardella and Jordan indicated that over the years the basic geometry content had stayed the same, but the manner in which students studied the content had changed. Emily and Lynne both indicated that the process by which students learned geometry had changed over
the years, but they interpreted the change in process to be a change in content. In addition, Emily's geometry content was interpreted through a general instructional approach consisting of the integration of five skills: reading, writing, thinking, measuring, and visualizing. Lynne's interpretation of content was based on the incorporation of student group activities, computer-based investigations, and presentations.

**Instructional components.** The teachers' instruction was characterized by their use of routines, varied teaching methods, and ongoing assessment techniques. All teachers' instruction followed a general routine of homework discussion, presentation of new content, and practice and application of new content. The discussion of homework in each of the classrooms also followed a routine. During homework discussions Jon and Emily usually addressed each assigned problem, whereas Ardella, Jordan, and Lynne focused on student-requested problems. A common component of the homework discussions was each teacher's willingness to answer students' questions. When the homework assignments involved proofs, Jon encouraged students to participate in the discussion. Student participation in homework discussion was customary for Ardella, Emily, Jordan, and Lynne.

The instructional methods used for presentation and practice of new content assumed varying forms such as
teacher explanation and questioning in large group settings, student group activities and investigations, individual student investigations, and student involvement with hands-on activities. All teachers in the study presented new content via the method of teachers talking about the new content. Jon’s teacher-centered presentations were best described as lectures, whereas presentations by the other four teachers were characterized as guided discussions. While guiding discussions, these four teachers sought input from students in situations such as recording of definitions and conjectures based on student investigations, solving problems, and stating theorems, some of which were to be proved.

Student groups were part of each teacher’s instruction. Working in groups occurred in two modes: (a) students working together on homework or in-class assignments by answering each other’s questions, and (b) students working cooperatively to complete a task in which each student had an obligation to fulfill. Jon only used the first group format and chose to do so on days when students did the chapter review as an in-class assignment. Emily’s occasional use of both modes of groups supported her intention to incorporate more group activities in her instruction. Substantial use of groups in both formats indicated that groups were part of the classroom structure and routines for Ardella, Jordan, and Lynne. Support for
groups as part of the regular classroom structure was also demonstrated by these three teachers' inclusion of assigned group memberships on their seating charts. In addition, these three teachers encouraged group members to be a source of information to the rest of the group (e.g., the directions for an activity, the identification and understanding of assigned problems).

The instructional methods of Ardella, Emily, Jordan, and Lynne also included the use of investigative activities. These activities were used to foster student exploration of geometric topics. For example, Ardella's students measured angles and examined their measurements looking for a relationship between the measure of a remote exterior angle and the sum of the measures of two interior angles. As another example, Emily's students used geoboards to explore relationships involving geometry vocabulary (e.g., the shortest line segment, two angles whose sum is 90 degrees).

When implementing investigative activities, varying formats were used by the geometry teachers. Variation occurred within individual teacher's practice and between teachers' practices. For example, Jordan's student investigations sometimes occurred in a large group setting and other times occurred while students worked in groups. Ardella guided large-group investigations, whereas Emily and Lynne set up investigations to be conducted in small
groups. Additional information about the instructional methods used by these teachers included whether hands-on activities were used for student investigations. Examples of the utilization of hands-on activities during student investigations were provided by Ardella and Jordan as they had their students use paper folding and constructions to analyze numerical and visual patterns. In contrast, investigations by Emily’s students entailed the utilization of numerical and visual patterns based on diagrams and investigations by Lynne’s students included a search for quadrilateral diagonal patterns generated by their work on the computer. The one common strand in these four teachers’ use of investigations was each teacher’s objective to have students actively involved in their learning of geometry. The manner in which these teachers involved their students suggested a problem solving approach to instruction.

The use of ongoing assessment techniques was another component of these geometry teachers’ instruction. All teachers used the assessment technique of systematically walking around the room looking at students’ work and consulting with individuals and groups about their work during all parts of the lesson. Ardella, Emily, Jordan, and Lynne circulated around the room while students were participating in an activity or working on an assignment. In Jon’s classroom, this looking and consulting with
students only occurred during assignment work time. All of the teachers appeared to do some type of assessment through verbal feedback to the students. Often the statement was in the form of "Do you have any questions?" Ardella, Emily, Jordan, and Lynne also appraised students' understanding through the process of calling on volunteers and non-volunteers during classroom discussions. When students were called upon, they were asked to provide a response and were usually encouraged to give support for their response. Some of the teachers assessed students' work by having students show them their work by means of white boards (Ardella), geoboards (Emily), or the blackboard (Jon, Lynne). Even though all teachers appeared to have good eye contact with their students, only Ardella specifically stated that she used students' eyes for determining their understanding of the concepts.

The teachers' use of routines, teaching methods, and ongoing assessment techniques characterized the way in which students encountered the selected geometry content. Jon's students studied selected geometry content via instruction that followed the routine of homework discussion, presentation of content through lectures, and practice and application of new content through assignment work time in class. Throughout the observation period, this routine appeared to be a predictable description of Jon's instruction.
The general routine of homework discussion, new content presentation, and new content practice and application was also the framework for the instruction of the other four teachers. The instruction of these four teachers, however, was not predictable, because of the variety of methods each teacher used as their students encountered new geometry content. In the classrooms of these four teachers, new content was introduced through teacher-led discussions (Ardella, Emily, Jordan, Lynne), student reading (Emily, Jordan, Lynne), student investigations (Ardella, Emily, Jordan, Lynne), and hands-on activities (Ardella, Emily, Jordan, Lynne). For these four teachers, instructional methods for student practice and application of new geometry material included small group work, individual homework assignments, and hands-on activities. In addition, Ardella's students worked problems on white boards and Lynne's students worked problems at the board, made group presentations based on their in-class assignment work, and used the computer to explore geometry content.

Factors Influencing Planning and Interactive Decisions

The identification of factors that influenced the planning and interactive decisions made by the teachers in this study provides a means for understanding geometry teachers' decision making. Identified factors include teachers': (a) past geometry experiences, (b) professional
development experiences, (c) articulated geometry course goals, (d) advanced planning decisions, (e) beliefs, (f) geometry textbooks and other materials, (g) school settings, (h) reform agendas, and (i) students' needs and actions.

Past geometry experiences. Teachers' planning and interactive decisions were influenced by their respective past experiences with geometry - as students, as teachers, or both. Jon and Jordan both made references to their experiences learning and teaching geometry when explaining reasons for their course decisions. For example, Jon's successful high school experience with logical thinking, as well as his teaching experience over the years, established the predominant role logical thinking had in his course. In Jordan's case, his frustrating experiences with learning high school geometry propelled him to develop a philosophy for teaching geometry that was different from the approach used in the traditional geometry class he took in high school. Jordan's success with teaching methods based on his philosophy fostered his continual use of methods that emphasized the use of intuitive and inductive reasoning for learning geometry concepts before using deductive reasoning to prove theorems.

Ardella, Emily, and Lynne indicated that their decision making was influenced by their experiences teaching geometry. Ardella found that student
understanding of geometry was enhanced when she used a variety of instructional approaches, encouraged student active involvement in lessons through the use of partners, constructions, and investigations, and did not address proofs in the first chapter. Enhanced student understanding was also the basis for Emily’s continual use of manipulatives, incorporation of writing, reading, thinking, visualizing, and measuring skills, and de-emphasis on two-column proofs. Lynne’s positive teaching experiences with group activities and investigations, group presentations, and an integrated approach to geometry fostered her continued use of these teaching methods.

Although Ardella, Emily, and Lynne did not identify their high school geometry experience as a reason for their course decisions, each teacher indicated that the geometry course they taught was different from the one they learned, suggesting their high school experience had influenced their decision making. For all three teachers, the course differences were seen as the course they taught moved away from a course that emphasized lecture and two-column proofs and toward a multifaceted geometry course that fostered active student involvement in their learning.

Professional development experiences. Teachers’ planning and interactive decisions appeared to be influenced by their professional development experiences. The amount of influence of teachers’ professional
development experiences was implied by references made and by details provided by the teachers. Ardella’s comments indicated that her experience with teaching a pilot geometry course cemented her ideas about sequencing the course, including the placement of constructions and proofs, in order to have a class that was not dominated by the proof process. In addition, Ardella’s experience with a cooperative learning workshop and a geometry teaching workshop encouraged her to incorporate the use of cooperative learning groups and manipulatives to foster students’ conceptual understanding. Emily’s experience in two inservice geometry seminars provided her with new ways to approach the teaching of geometry. In addition, the seminars helped her realize that students needed to work with manipulatives while learning geometry in order to visualize concepts and to identify relationships between concepts. In Lynne’s case, the influence of professional development experiences was suggested by her references to her participation in mathematics conferences that helped prepare her for teaching a multifaceted geometry course in a student-centered classroom. A fourth teacher, Jordan, indicated that a cooperative learning workshop and a geometry and technology workshop influenced his philosophy for teaching geometry from an intuitive, inductive perspective. However, due to lack of access to computers, technology was not incorporated into his geometry course.
The remaining teacher, Jon, did not make any references to professional development experiences.

**Articulated geometry course goals.** Teachers' articulated geometry course goals were factors influencing teachers' planning and interactive decisions. Even though only Emily directly stated that her course goals influenced her planning and interactive decisions on a daily basis, the influence of course goals was seen in every teacher's case. Emily's decisions were influenced by her wish to develop students' reading, writing, visualizing, measuring, and thinking skills while learning geometry concepts and relationships between concepts. Decisions made by Jon were influenced by his main goal of developing students' logical thinking skills. Many of Jordan's decisions were based on his aspiration for students to first learn geometry concepts intuitively and inductively and then verify their work with deductive thinking. Ardella's decisions were influenced by her goals for students to visualize geometric concepts, investigate concepts, communicate geometry, and verify their mathematical thinking. The influence of Lynne's goals was apparent as geometry discussions were based on different geometry perspectives (e.g., discrete, plane, transformational, coordinate) and as students worked in groups exploring and communicating mathematics using a variety of tools.
As a course goal, Jordan, Ardella, and Lynne also wanted their students to appreciate geometry in the real world. The influence of this goal was not obvious during observed lessons. Incidental references to the real world made during observed lessons included the use of cartoons and the use of physical representations of points, lines, and planes. In addition, textbooks and homework assignments involved references about geometry and the real world. Comments by Jordan, Ardella, and Lynne indicated that the connection between geometry and the real world were emphasized to a greater extent as the course progressed.

**Advanced planning decisions.** The influence of teachers' advanced course development planning decisions on their decision making was seen throughout the study. Prior to the beginning of the observation process, all five teachers had made planning decisions related to goals of their course, content to be addressed, and instructional methods to be used. These decisions provided a framework in which the teachers made planning and interactive decisions as the course progressed.

Planning decisions made as the course progressed included intermediate decisions as well as daily decisions. All of the teachers appeared to divide their course material into chapters and then daily lessons. Jon's decision making process seemed static as his chapter and
daily planning decisions closely followed those outlined in previous lesson plan books. Also, Jon indicated that he did not do a lot of planning. Even though the other four teachers also used previous planning as a guideline for their chapter and daily decisions, their decision making process appeared to be more dynamic than Jon's. In particular, these four teachers' comments conveyed the notion that they were continually thinking about how best to organize the content of their course and which instructional methods to use. For all five teachers, the influence of intermediate and daily planning decisions was seen as each teacher implemented their envisioned geometry course.

Consistency between planning and implementation was also seen on a daily basis when teachers' actions during the observed lessons matched the general planning information discussed during the pre-lesson interview. On occasion, teachers' post-lesson reflections indicated slight changes in plans due to students' needs for clarification, practice of new concepts or skills, or because of time limitations. These adjustments, however, were not structural changes in the lessons.

**Teachers' beliefs.** The planning and interactive decisions made by the teachers appeared to be influenced by their beliefs about the nature of geometry as a discipline and about how they viewed their own teaching. Teachers'
beliefs about the nature of geometry as a discipline appeared to be related to whether they focused on one facet of geometry or on multiple facets of geometry and how they implemented these various facets of geometry. Although Jon addressed the content knowledge aspect of geometry in his course, he emphasized the system aspect of geometry. Jon's emphasis on geometry as a mathematical system via a lecture approach suggested that his beliefs about the nature of geometry characterized geometry as a static and structured body of knowledge and procedures that students must master. The other four teachers emphasized geometry as a knowledge base, an example of a mathematical system, and as a setting for developing communication and problem solving skills while students actively participated in their learning of geometry. In addition, these four teachers hinted at the exploration of the connection between geometry and the real world. Thus, these four teachers appeared to believe that geometry was a multifaceted body of knowledge that needed to be examined, explored, and constructed by students.

Teachers' planning and interactive decisions also appeared to be influenced by their beliefs about how they viewed their own teaching. One part of this belief was whether teachers viewed the process of becoming an effective teacher as a life-long process. In their self-descriptions of their teaching and geometry history as well as of their present geometry teaching, Ardella, Emily, and
Lynne implied that their teaching had changed since they first started teaching. In addition, their comments indicated they were continuing to explore ways to provide their students with better opportunities for learning geometry. These three teachers appeared to view the process of becoming an effective geometry teacher as a life-long process. Jordan’s self-description of his teaching suggested that his teaching had also changed since his first teaching position. In addition, his comments indicated that he was open to seeking out new ways to teach geometry, but at the same time he was satisfied with the structure of his course. His openness to future change indicated he probably viewed the process of becoming an effective geometry teacher as a life-long process. On the other hand, Jon’s self-description of his teaching implied that he had found an instructional approach early in his career with which he was comfortable, and he continued to use that approach. Thus, it appeared that Jon might not view the process of becoming an effective geometry teacher a life-long process.

A second part of teachers’ beliefs about how they viewed their own teaching concerned whether teachers appeared to think of themselves as curriculum builders. As curriculum builders, teachers defined their own geometry courses rather than implemented courses defined by others. In this study, teachers’ beliefs about viewing themselves
as curriculum builders were implied by the role the textbook played in determining their curriculum.

**Geometry textbooks and other materials.** Teachers' planning and interactive decisions were influenced by geometry textbooks and other geometry materials. Both Jon and Jordan relied heavily on textbooks that matched their philosophies for teaching geometry. Jon and Jordan each helped select their respective textbooks. A heavy reliance on a self-selected textbook suggested that Jon and Jordan were implementing a course defined by others with which they were in agreement. Presenting a traditional deductive approach to geometry, Jon's textbook matched his belief that the logical reasoning focus of geometry was the most important component of geometry learning. For Jon, this logical reasoning focus made geometry the most important mathematics class taken by students. In Jordan's case, the textbook's inductive approach agreed with his philosophy of using intuitive and inductive processes to learn geometry concepts and relationships before using deductive reasoning to prove theorems.

Even though the majority of the course taught respectively by Jon and by Jordan was defined by their textbooks, both of them were willing to depart from their textbooks in small ways. To strengthen his textbook's approach to logical thinking and writing proofs, Jon supplemented his course with material from other textbooks
he had previously used in teaching geometry. Jordan's departure from his textbook was seen in the emphasis given to deductive reasoning. While following the textbook's suggestion of guiding students through informal proofs to verify their conjectures, Jordan chose to emphasize the deductive reasoning component more than the textbook did. In addition, Jordan planned to look at more formal proofs in the form of two-columns with either the congruence chapter or the circle chapter, whereas the textbook did not address two-column proofs until chapter 14, "Geometric Proof" (Serra, 1989), which was the second-to-last chapter of the textbook.

Lynne, Emily, and Ardella used the textbook as a guideline for making decisions about their courses, implying that the textbook suggested but did not define the course for these teachers. Instead, each teacher used the textbook as one resource to implement their envisioned geometry class. Lynne followed the general sequence of the textbook that she helped select, but supplemented the textbook with activities and projects that supported an integrated mathematics class, her school's restructuring plan, and the Curriculum Standards (1989). Emily used the textbook she helped select, five other textbooks - a potpourri of previous classroom textbooks and textbooks used as a general resource, the department's scope, and the Curriculum Standards to define her course. She emphasized
learning geometry through reading, writing, thinking, measuring, and visualizing. Ardella used ideas from the district textbook that she helped select but followed her own sequence of topics. Ardella's selection and organization of topics were influenced by the Curriculum Standards and her wish to use hands-on activities – in particular, constructions – and group exercises. Thus, other resources also influenced these teachers' decisions about their geometry courses.

Teachers' school setting. Teachers' interpretation of their school setting, as well as how they worked within their particular school system, appeared to influence the decision making of the teachers in this study. The amount and type of influence differed among the teachers. A little influence of the school setting was indicated in Jon's situation. The school setting influence was seen in Jon's involvement with the district selection of the textbook. In addition, he made a few references to district guidelines for content to be addressed. However, he never mentioned discussing his course with colleagues or working with them to improve his course. On the other end of the spectrum, the school setting heavily influenced Lynne's decisions. Her school made a commitment to a restructuring process in which Lynne and her colleagues actively assisted with the development of school-wide universal, content, and process standards. To foster the
implementation of these standards, Lynne's school set up a block schedule. In addition, during the course of the study, Lynne and her departmental colleagues made decisions together regarding the outline of the course and shared teaching ideas with each other. The amount of influence of the school setting on the other teachers was somewhere between its influence on Jon and Lynne. Ardella, Jordan, and Emily were influenced by the block schedule followed by each of their schools and by the freedom each one felt to develop their course within departmental guidelines. All three of these teachers indicated that their teaching was influenced by their departmental colleagues. Ardella's comments suggested that she obtained teaching ideas from colleagues, whereas Jordan's comments implied he was influenced by his scope and sequence work with departmental colleagues when they adopted their present textbook. For Emily, the influence of her colleagues was seen through the acquisition of activities from them as well as her geometry course scope work with them prior to the study.

Reform agendas. Ardella, Emily, Lynne, and Jordan explicitly identified reform movements as a factor that influenced their planning and interactive decisions. The amount and type of references made about reform by these four teachers varied among the individuals. Ardella made references to the *Curriculum Standards* (1989) when she initially talked about her overall approach to teaching
geometry and when she talked about her lesson planning. Her comments indicated that the Curriculum Standards validated her thoughts about what was important in geometry and was a resource for activities. Emily referred to the Curriculum Standards during her initial discussion about her overall approach to teaching geometry and during post-lesson interviews. Emily indicated that she used the standards as a guide for planning her lessons.

Lynne's general comments indicated that the Curriculum Standards (1989) guided her teaching, but most of her comments about reform ideas referred to her school's restructuring process that was based on research. To help her students develop their school portfolio, Lynne provided her students with opportunities to do investigations with computers, to complete self- and peer-evaluations on group work, and to communicate mathematics through written reports and oral presentations. During the first interview, Jordan's reference to the Curriculum Standards indicated that he agreed with the ideas presented in the standards, but he did not specifically refer to them as a reason for decisions made about goals, content, and instruction. He did, however, frequently indicate that he made decisions about his course to accommodate the shorter school year due to inservice days for the state educational restructuring process.
Ideas that agreed with suggestions presented in reform agendas were also identified by Ardella, Emily, Jordan, and Lynne as factors influencing their decision making. These four teachers made planning and interactive decisions based on their wishes for students to be active participants in their learning as students communicated mathematics, explored concepts and relationships, used spatial visualization skills, made conjectures, and developed their thinking and problem solving skills. Students' active approach to learning is part of the reform agenda presented by the *Curriculum Standards* (1989) and the *Professional Standards* (1991). A student active approach for student learning is also part of the foundational skills component of the state-legislated reform.

**Students' needs and actions.** As teachers made planning and interactive decisions, they were also influenced by their students. In general, this influence was seen in the form of the genuine respect the teachers showed their students. During interviews and during observed lessons, these teachers conveyed the message that they made decisions based on what they felt was best for their students. The particulars of how the teachers were influenced by their students were seen in the following discussion about how teachers' viewed their students' learning needs and their students' actions during instruction and work time.
Ardella, Emily, Jordan, and Lynne believed students needed to be actively involved in learning activities. These four teachers felt that students' active participation in instruction kept students interested in geometry and fostered students' geometry learning. For these four teachers, students' active involvement in instruction occurred during instructional activities such as guided discussions, group activities, investigations, and hands-on activities. Many of Jordan's decisions about when to use different types of activities were influenced by his textbook, which was based on an investigative approach to geometry. For these four teachers, the decisions made involved when to use a specific type of instructional activity. These decisions appeared to be influenced by their files of collected activities, their previous teaching experiences with these activities, their wishes to use a variety of instructional methods, and their block schedules.

Ardella, Emily, and Jordan specifically identified the need for variety when choosing instructional methods for fostering students' involvement in the lesson. These three teachers also indicated that using a variety of instructional methods fostered student understanding of foundational concepts and made learning fun for their students. Based on her selection of instructional
activities, Lynne also supported the notion of using a variety of teaching methods.

All five teachers in the study felt that students needed time to practice solving problems while learning new concepts and applying new theorems. Thus, these teachers made decisions to provide students with practice time during class. The manner in which teachers incorporated practice time for their students varied. Jon wanted his students to have at least 10 to 15 minutes at the end of the 54-minute period to work on homework individually as he walked around the room answering students' questions. For the other four teachers who had 90- or 95-minute periods, practice occurred through group activities (Ardella, Emily, Jordan, Lynne), presentations of group problems (Lynne), and individual work time before addressing additional concepts and theorems (Ardella, Emily, Jordan, Lynne). Often this practice was used to "break up" the 90- or 95-minute block.

As suggested by teachers' comments and classroom observations, student actions during instruction and work time also were factors influencing teachers' decision making. When reflecting on decisions made during instruction, Ardella, Emily, Jordan, and Lynne talked about decisions made based on balancing the needs of students with their plans for the day. For example, students may have needed to practice applying concepts or theorems
before exploring subsequent concepts or theorems. Then, the teachers gave students an example to work that was to be discussed in a large group setting or provided students with a small group activity that addressed the particular concept or theorem. A second example, Ardella had her students solve on white boards right triangle problems using the trigonometric functions before proceeding with her angle of elevation and angle of depression triangle problems.

For Ardella and Emily, the influential factor of students’ actions during instruction also included whether an individual’s behavior was disruptive to the rest of the class. When reflecting on decisions made during instruction, both Ardella and Emily indicated that they needed to balance the needs of the potentially disruptive student with the needs of the other members of the class.

Teachers’ interactive decisions were also influenced by the envisioned role for students in classroom discourse. In the classrooms of Ardella, Emily, Jordan, and Lynne, students were encouraged to contribute to a discussion as well as to provide explanations. These expectations invited an active dialogue between students and teachers. In Jon’s classroom during discussions, students were expected to ask questions and to answer his questions. These expectations occasionally promoted a dialogue between Jon and his students.
As teachers and students participated in a dialogue supporting the role teachers wanted students to assume, teachers had decisions to make. These decisions appeared to be influenced by the manner in which students participated in the discussion as well as by the content of their responses. For example, if more than one student tried to talk at a time, the teacher needed to decide who would talk or whose idea was to be discussed. These five teachers' decisions about who would talk appeared to be based on a concern for giving all students an opportunity to participate and at the same time a concern for paying special attention to volunteers who rarely participated in discussion. In addition, if students' responses were not complete, the teacher needed to decide how to get a complete answer. Jon usually provided the answer himself, whereas, the other four teachers either asked the students probing questions based on their response or asked other students to build on an individual's initial idea.
CHAPTER V
DISCUSSION AND CONCLUSIONS

Introduction

The goal of this study was to investigate secondary geometry teachers' decision making in a mathematics reform context. To focus the investigation, two questions were asked: (a) What planning and interactive decisions were secondary geometry teachers making during this time of reform, and (b) what factors influenced the decisions that these teachers made? The answers to the research questions were generated by examining five geometry teachers' planning and teaching profiles and looking for similarities and differences across the sample.

This chapter begins with a summary and discussion of the main findings of the study and concludes with the limitations of the study and comments concerning implications for education and recommendations for future research.

Summary and Discussion of the Main Findings

Planning and Interactive Decisions

As implied by their planning comments and their written lesson plans, all five teachers in this study made curriculum decisions involving goals, content, and instructional methods prior to the start of their geometry course, at the beginning of each chapter, as well as prior
to a particular lesson. The teachers' interactive decisions were made about implementing a lesson as planned, responding to student questions, providing alternative explanations, and pursuing student-generated discussions. The examination of geometry teachers' planning and interactive decisions as related to curriculum issues generated a description of secondary geometry classrooms. Included in this description are facets of secondary geometry, selection and interpretation of geometry content, and instructional components.

Facets of secondary geometry. The findings from this study suggested that different facets of geometry were promoted in the five classrooms. Facets of geometry featured by the teachers included geometry as a content knowledge base, an example of a mathematical system, and a setting for developing communication and problem solving skills. In addition, teachers hinted at a facet described as connections between geometry and the real world and between geometry and other areas of mathematics.

All teachers in the study valued student development of a knowledge base of geometry concepts and relationships between these concepts. The knowledge base consisted of basic geometry ideas such as points, lines, planes, rays, angles, polygons in general, and specific polygons - triangles and quadrilaterals, and three-dimensional figures. Relationships involving these concepts were also
part of this knowledge base. These relationships included theorems such as: (a) vertical angles are congruent, and (b) the sum of the angles of a triangle equals 180 degrees in Euclidean geometry. This facet of geometry provided a foundation for other facets addressed by the teachers.

The findings from this study showed that teachers continue to consider geometry as an example of a mathematical system appropriate for study in the secondary mathematics curriculum. This facet of geometry included both the logical development of geometry and the process of how mathematicians derive mathematics. One teacher promoted the logical development of a proof-oriented geometry course through the use of lectures. In contrast, the other four teachers' interpretation of the mathematical system view indicated that the development of deductive reasoning skills would be one expected outcome of geometry, not the primary focus of their course. For three of the four teachers, proofs via deductive reasoning were one component of the process of developing mathematical concepts and relationships through exploring, conjecturing, and verifying geometric ideas. The system aspect of geometry promoted by these three teachers was in agreement with the *Curriculum Standards* (1989) idea that the deductive perspective of geometry needed to receive less emphasis than in the past and the interplay of inductive and deductive reasoning needed to be fostered more in
present and future geometry classes. The remaining teacher appeared to be moving toward this same interpretation of the system aspect of geometry. Throughout the study, she encouraged student development of inductive and informal deductive reasoning skills. In addition, during the process of the study, this teacher had decided to include a second semester unit on logical reasoning and two-column proofs. The teacher's decision was based on a professional journal article that promoted the study of logical reasoning and proofs for college-bound students.

Another facet of geometry addressed by the teachers during this study included geometry as a setting for developing communication and problem solving skills. For the proof-oriented teacher, students had some opportunity to communicate geometric ideas as they volunteered to write their proofs on the board and explain their work to their classmates. Students also used diagrams while answering homework knowledge questions and writing two-column proofs. However, the development of communication and problem solving skills was not prevalent during observations nor was articulated in his course goals.

For the other four teachers, geometry as a setting for developing communication skills was evident as their students were encouraged to read, write, and talk about geometry concepts and relationships. In agreement with the *Curriculum Standards*' (1989) description of communication,
the focus of these communication processes was on student understanding of the geometry ideas. As implied in the following paragraph, student development of communication skills was related to their development of problem solving skills.

The geometry courses of these four teachers were also seen as a setting for developing problem solving skills such as looking for patterns, making systematic lists, drawing and interpreting diagrams, measuring with protractors, performing and applying constructions, and exploring concepts and relationships through investigations. These mathematical skills which advocated students' active involvement in "doing mathematics" support teachers' efforts toward integrating a problem solving approach for learning mathematics as described by the Curriculum Standards (1989).

For all teachers in the study, the connection between geometry and the real world was incidently seen during observed lessons when they used cartoons to explore a topic or when they used real-world representations for vocabulary. Three of the teachers who specifically included geometry and the real world as a course goal indicated that the connection between geometry and the real world was to be emphasized in their classrooms in the latter part of the course. The other two teachers (one of whom was the proof-oriented teacher) indicated that area
and perimeter was to be addressed during the latter part of their course. In addition, for all of the teachers except the proof-oriented teacher, the incorporation of transformational and coordinate geometry involved real-world ideas as well as connections with other areas of mathematics. Thus, teachers in this study were beginning to promote the Curriculum Standards' (1989) idea that students be encouraged to see the connections between geometry and their daily lives and between geometry and other mathematical areas.

All the teachers presented a geometry course characterized by different emphases on the facets of geometry. Whereas one teacher emphasized one facet of geometry - namely, geometry as a logical mathematical system - the other four teachers advocated multiple facets of geometry. The depiction of geometry by these four teachers was similar to geometry researchers' previous descriptions of geometry. Burger and Culpepper (1993) described geometry as an abstraction of visual and spatial experiences, as a provision of approaches for problem solving, and as an environment for studying mathematical structure. Usiskin (1987) characterized geometry as the visualization, construction, and measurement of figures, as the study of the real, physical world, as a vehicle for representing other mathematical concepts, and as an example of a mathematical system. Even though researchers from
past studies and teachers in this study did not express the facets in exactly the same manner, all individuals supported the ideas that geometry involves connections with the real world, the use of problem solving skills, and the development of a mathematical system through logical reasoning and proof.

The findings from this study also provided school-based classroom support to research completed as part of the National Council of Teachers of Mathematics' (NCTM's) Priorities in School Mathematics Project (PRISM) in 1981. For the PRISM project, a group of people consisting of teachers, supervisors of mathematics, mathematics teacher educators, principals, school board presidents, and parent-teacher organization presidents were surveyed. The results of the PRISM survey indicated that this group of people felt that geometry needed to be studied to develop logical thinking abilities, to acquire the knowledge needed for study of more mathematics, to learn to read and interpret mathematical arguments, and to develop spatial intuitions about the real world. This study's findings showed that these geometry teachers agreed with the PRISM results pertaining to logical thinking abilities and partially agreed with PRISM results related to the spatial intuitions about the real world. In particular, the teachers in this study emphasized spatial visualization, but only hinted at the real world applications of geometry. The teachers in
this study also appeared to support the reading and interpreting mathematical arguments result as long as students were involved in the development of the arguments. Teachers in this study did not directly state that students needed to study geometry for the knowledge needed for subsequent mathematics courses, but their comments and actions implied that the majority of their students planned to continue to study mathematics. Due to the use of surveys for the PRISM project, it was not clear whether the results represented actual occurrences in secondary geometry classrooms or were a broad sampling of individual opinions. In contrast, this study’s findings were based on classroom observations of five experienced secondary geometry teachers.

Selection and interpretation of geometry content. The findings concerning content suggested that the teachers were not in complete agreement as to what geometry content needed to be studied in the secondary geometry classroom. On one hand, the results showed that teachers addressed similar topics in their classrooms such as undefined terms, line segments, rays, angles, congruence, perpendicular and parallel lines, triangles, quadrilaterals, other polygons, and three-dimensional figures. The similarities in content suggested that all students were learning basic geometry concepts and relationships. On the other hand, results exhibited broad content variations among topics such as
proofs, constructions, transformational geometry, and coordinate geometry. The differences in content were seen through teachers' decisions about which content to emphasize and possibly related to course time restrictions. For example, one teacher presented two-column proofs as a main component of his course, whereas, the other teachers showed that proofs have more than one format and were only one part of their respective courses. Some teachers used constructions as tools for exploring concepts and relationships such as congruent triangles and lines of concurrency in a triangle. Other teachers utilized constructions as a way to "draw" with a compass and straightedge to copy a line segment or to draw a square. One teacher presented coordinate and transformational geometry concepts as a major part of her geometry course. One teacher did not plan to address these concepts at all. The amount of class time planned by the other three teachers to discuss coordinate and transformational geometry was between the amounts used by the first two teachers.

The teachers' content selection was in partial agreement with Curriculum Standards' (1989) recommendations. All five teachers and the Curriculum Standards agreed that students needed to develop an understanding of two-dimensional shapes and their properties. The Curriculum Standards also advocated the
exploration of three-dimensional shapes. Three-dimensional
shape exploration was sometimes seen in the classrooms of
those teachers focusing on visualization. The findings of
this study also provided teachers' partial support for
Curriculum Standards' recommendations that two-column
proofs be de-emphasized, deductive arguments be expressed
orally and in paragraph form, and coordinate and
transformation approaches receive increased attention.
Minimal support was shown for the Curriculum Standards'
recommendations that advocated the use of real-world
applications and modeling and computer explorations of two-
and three-dimensional shapes. As stated in the facets of
secondary geometry discussion, teachers indicated that
real-world applications were emphasized later in the
course. Only one teacher incorporated computer-based
investigations in her course. The other teachers did not
use technological applications due to limited room in the
curriculum or no access to machines.

Differences in emphasis in geometry course content was
not an unexpected result. Historically, the mathematics
education community has had difficulty reaching a consensus
for the content of the high school geometry course (Dessart
& Suydam, 1983; Usiskin, 1987). Content differences in
this study were partially explained by the previously
discussed facets of secondary geometry promoted by the
individual teachers. One of the teachers focused primarily
on the system view of geometry. Besides promoting geometry as an example of a mathematical system, the other four teachers advocated geometry as a setting for developing communication and problem solving skills. In addition, the four teachers hinted at the connection between geometry and the real world and the connection between geometry and other areas of mathematics.

Additional explanation for content differences was provided by the interpretations of content suggested by each teacher's general instructional approach. For example, geometry content presented in a business-like manner through lectures had a different flavor from content examined with constructions in large group discussions or hands-on activities in small cooperative groups. Additionally, geometry content studied through the use of reading, writing, thinking, measuring, and visualizing skills had a different essence than content explored via an intuitive and inductive approach or by means of instruction that recognized multiple perspectives of geometry (e.g., discrete, plane, transformational, solid, coordinate). For example, three of the teachers addressed the triangle congruence postulates (or theorems, depending on the textbook) before the completion of the study. The proof-oriented teacher stated the theorems, followed by proofs on the board. One teacher who advocated hands-on activities engaged students in an investigation in which the students
determined which three pieces of information enabled them to create unique triangles using straws and angle wedges. In the classroom of the teacher who promoted multiple perspectives of geometry, the congruence postulates were examined from a transformational approach.

As shown in the previous example, students learned the same basic geometry concept - the triangle congruence statements - in three different ways. This intertwined relationship between content and instruction supported one of the ideas advocated by NCTM in the *Professional Standards* (1991). Specifically, the *Professional Standards* stated that mathematics teaching be based on the idea that "WHAT students learn is fundamentally connected with HOW they learn it" (p.21). The teachers in this study, as well as the *Professional Standards*, reinforced the notion that the process of mathematics is as important as the content of mathematics.

**Instructional components.** Teachers' instruction was characterized by their general instructional routines, teaching methods, and ongoing assessment techniques. Instruction results implied that all teachers used a daily routine of homework discussion, presentation of new content, and practice and application of new content. These results agreed with Brown's (1974) findings that geometry teachers' instruction followed this routine. In general, this routine of homework discussion, presentation

Brown's (1974) findings also suggested that geometry teachers' instruction was completely described by this routine of homework discussion, presentation of new content, and practice and application of new content. For one of the teachers in the study it was true that his instruction was completely described by the general routine. For the other four teachers, their instruction followed the general routine but was not completely described by it. Instead, the routine provided a framework for a variety of instructional methods used by the teachers to promote a learning environment that fostered student active involvement in their learning of geometry. Students interacted with their teachers during guided discussions and with their peers during group activities. In all four classrooms, students examined geometry content via investigations. Depending on the teacher, students folded patty paper, used geoboards, explored with miras, analyzed numerical and visual patterns, used constructions, or worked on the computer during their investigations. The teachers' use of a variety of methods precluded their instructional sequence being predictable. One possible reason for the difference between the first teacher's use of the routine and the other four teachers' use was that
the first teacher worked with 54-minute periods and the four teachers worked with 90- or 95-minute blocks.

All teachers in this study used some form of ongoing assessment during their instruction. Assessment techniques included walking around the room looking at students’ work and consulting with them, asking students questions, asking students to explain their responses, and having students demonstrate their work on white boards, geoboards, or blackboards. The teachers’ use of these assessment techniques supported the purpose of assessment described by Webb (1992) as "provide evidence and feedback on what students know and are able to do" (p. 663).

How frequently teachers used these ongoing assessment techniques during instruction and in what manner teachers used the gathered information determined whether assessment was described as an integral component of their instruction. Teachers who felt it was important for students to be actively involved in their learning appeared to use ongoing assessment techniques more often and made decisions based on the gathered information.

As an integral component of instruction, ongoing assessment techniques also supported the purpose of assessment described by Webb (1992) as "express what is valued regarding what students are to know, do, or believe" (p. 663). For example, teachers’ ongoing assessment techniques indicated students were expected to explain
their responses, to demonstrate their work on the board, and to realize that patterns were found during their investigations.

Additional information concerning what teachers valued was suggested by summative assessment data predominantly collected via anecdotes during informal teacher interviews. All five teachers gave written tests indicating their expectation that students know and apply geometric concepts and relationships. Four of the teachers collected student notebooks. Student notebooks for the teacher who had a more teacher-centered classroom consisted mainly of homework assignments. The student notebooks of the other three teachers, who fostered a more student-centered classroom, consisted of homework assignments and notes from teacher-led discussions as well as student investigations.

As part of their summative assessment three of the teachers used projects that required students to create a product that met the project’s guidelines. For one of the teachers, the project was an upcoming geometry in the real world project. In the case of a second teacher, an observed project was a line design visualization project. For the third teacher, the project involved a partner-computer investigation of quadrilaterals. The third teacher’s summative assessment plan also included student self- and peer-evaluation of group work. The evaluations
were possible entries for her geometry students' school portfolio.

Factors Influencing Planning and Interactive Decisions

The identification of factors influencing teachers' planning and interactive decisions provided more information concerning the workings of the geometry classroom and insight about teachers' decision making. Main identified factors included the following: (a) past geometry experiences, (b) professional development experiences, (c) articulated geometry course goals, (d) advanced planning decisions, (e) teachers' beliefs, (f) the geometry textbook and other materials, (g) teachers' school setting, (h) reform agendas, and (i) students' needs and actions. In addition, the design of the study provided an opportunity to examine whether main factors affecting geometry teachers' decision making identified during a context of national and state reform differed from main factors identified during the stable context predominant in previous decision making studies.

Past geometry experiences. As part of their geometry background, the teachers in this study talked about their geometry experiences as high school students, as college students, and as teachers. Teachers' general comments about whether they enjoyed geometry and the instructional methods used by their own teachers suggested their
experiences as students influenced their decision making as teachers of geometry. Two of the teachers explicitly identified their high school geometry experience as a factor influencing their geometry content and instruction. In high school, both of these teachers learned a logic-focused geometry via lectures. The teacher who learned to enjoy logical thinking in high school mathematics taught a logic-focused geometry course using lectures. Teaching how one was taught supported results from research that examined teachers' conceptions of mathematics teaching (Ball, 1990; Brown & Borko, 1992; A. G. Thompson, 1984).

In the case of the second teacher who made explicit references to his high school geometry experience, the logic-focused, lecture-oriented geometry course did not foster his conceptual understanding of geometry although he earned high marks in the subject. In reaction to his high school geometry experience, this teacher taught geometry using an intuitive and inductive approach with a decreased emphasis on deductive proof. Although the remaining three teachers did not identify their high school geometry experience as a reason for decisions made in their geometry courses, they did make references to the idea that the geometry course they taught was different from the one they learned. Thus, these five teachers provided evidence that teachers' learning experiences - successful or unsuccessful - informed and influenced their beliefs about teaching.
All teachers in the study selected and implemented content and instructional methods based on teaching experiences with students in the classroom. For example, some of the teachers used group activities because student understanding was fostered by the opportunity to work together to generate results during an investigation. Student communication was an important component of the group process. As another example, one teacher allotted assignment work time in each class period because this work time enabled students to practice concepts and apply relationships as well as get their questions answered. The identification of classroom teaching experiences as an influential factor supported other decision making studies that found teachers made decisions based on classroom experiences (Nesselrodt, 1990; Putnam, 1983; Westerman, 1991).

A. G. Thompson (1984) observed that teachers' experiences in the classroom shaped their beliefs. The results from this study agreed with A. G. Thompson's findings. For the teachers in this study, their beliefs about what instructional components, particularly use of manipulatives and group activities, fostered student understanding of geometry were shaped by their students' success with these components.

**Professional development experiences.** Four teachers' professional development experiences appeared to influence
their decisions made about their geometry courses. Their professional development experiences were related to learning about teaching geometry while participating in conferences, seminars, and workshops or team-teaching a pilot course with a university professor. Based on her experiences, one of the teachers decided that manipulatives and cooperative learning needed a predominant role in her instruction. A second teacher made this same decision about manipulative use as well as agreed with the third teacher who decided students needed to take an active role via group activities in their learning of geometry. A fourth teacher's experience with a cooperative learning workshop supported his goal of having students actively involved in learning geometry through an intuitive, inductive approach. The four teachers incorporated these ideas about manipulatives, cooperative learning, and active student participation and continued to use these techniques based on their beliefs and teaching experiences with students that these approaches fostered student understanding of geometry.

The identification of teachers' professional development experiences as a factor influencing teachers' planning and interactive decisions was not seen in previous decision making research and thus, extended previous decision making research. In addition, the identification of teachers' experiences outside of the classroom (e.g.,
workshops, mathematics education conferences) extended
A. G. Thompson's (1984) observation that teachers' experiences in the classroom shaped their beliefs about teaching geometry. In particular, opportunities to develop curriculum ideas, to learn about cooperative learning and manipulatives, and to see other mathematics teachers' instructional ideas were experiences that also altered these teachers' approach to teaching geometry.

Articulated geometry course goals. The influence of teachers' overall course goals on their planning and interactive decisions was seen throughout the study. Observed lessons showed that the content addressed and the instructional methods used supported most of the teachers' course goals. One possible exception was the course goal pertaining to geometry and the real world. Incidental references to the real world were made as teachers used cartoons and real-world representations to explore concepts and employed measurement and construction skills to apply concepts. Teachers' comments did suggest, however, that this goal was emphasized during the remainder of their respective courses. Even though the real-world connection was not an integral component of the observed lessons, the consistency with which these teachers made decisions to incorporate other course goals suggested that they provided opportunities for students to meet the course goal of connecting geometry and the real world after the
observation period. The identification of course goals as a factor influencing teachers' decision making was in agreement with results from previous decision making studies (Nesselrodt, 1990; Putnam, 1984; Ropo, 1987; Westerman, 1991; Zahorik, 1975).

The influence of teachers' geometry course goals was also implied when some of the teachers talked about decisions made during instruction. Specifically, these teachers made references to being influenced by their goals as they attempted to balance their goals with students' needs. The results of this study were in agreement with previous decision making studies that found teachers' interactive decisions were influenced by their preactive goals as well as their ability to adapt their instruction to students' needs (Borko & Livingston, 1989; Nesselrodt, 1990; Westerman, 1991). Together, Borko and Livingston's study and this study began to examine teachers' decision making in a mathematical context. In addition, this study examined teachers' decision making in a mathematical reform context following the publication of NCTM's *Curriculum Standards* (1989) and *Professional Standards* (1991).

**Advanced planning decisions.** Teachers' advanced planning decisions, both individual and departmental, were factors that influenced teachers' daily decision making. This influence was evident because a course plan was created within a framework based on teachers' geometry
curriculum decisions made before the observation process started. As the study progressed, teachers made intermediate and daily decisions based on their course framework. The influence of this planning-based framework was seen as all five teachers implemented their envisioned geometry course. The influence of early-in-the-school-year decisions on the progress of a course was in agreement with previous decision making research that examined the effect of teachers' planning decisions (Shavelson, 1982).

The influence of teachers' daily plans on their lessons in action was seen when teachers only made minor adjustments from their stated and written plans during instruction. It was possible that these teachers were not willing to make major adjustments in their daily plans. A more plausible explanation was that these successful experienced teachers had pedagogical content knowledge about what worked in their classroom to foster students' learning of geometry. Teachers appeared to have mental plans that were more elaborate than what they described in writing or during pre-lesson interviews. The teachers appeared to use these mental plans to guide their instruction. This finding was in agreement with results of decision making studies that found experienced teachers had elaborate mental plans that directed their teaching (Borko & Livingston, 1989; Morine-Dershimer, 1978-1979; Westerman, 1991). Of particular interest was the agreement between
the results of this study and the results of Borko and Livingston's study that also examined mathematics teachers' decision making, but not in geometry classrooms.

**Teachers' beliefs.** Teachers' beliefs have been recognized as an important component for understanding mathematics teaching from teachers' perspectives (A. G. Thompson, 1992). Consistent with this finding was this study's identification of teachers' beliefs as a factor influencing teachers' planning and interactive decisions. Specifically, this study suggested that teachers' beliefs about geometry as a discipline and about how they viewed their own teaching affected their planning and interactive decisions.

Findings from this study implied that teachers' beliefs about geometry as a discipline influenced their decisions regarding the nature of their geometry course. Teachers' beliefs about the nature of geometry appeared to be related to which facets of geometry they emphasized in their course. One teacher emphasized geometry as an example of a mathematical system as well as addressed geometry as a base of specific content knowledge. This teacher appeared to believe that geometry was a static and structured body of knowledge. The other four teachers emphasized geometry as a knowledge base, an example of a mathematical system, and as a setting for developing communication and problem solving skills. In addition,
these teachers incidently addressed the connection between geometry and the real world and talked about emphasizing this connection in the later part of the course. For these four teachers, their views suggested that they believed that geometry was a multifaceted body of knowledge that needed to be constructed by students.

The study's findings about these four teachers' views regarding facets of a secondary geometry course as well as their implied beliefs about the nature of geometry were in agreement with three of the four major themes of the Curriculum Standards (1989): (a) mathematics as reasoning, (b) mathematics as communication, and (c) mathematics as problem solving. In addition, these four teachers hinted at the fourth theme - mathematical connections.

Fennema and Franke (1992) stated that the themes of the nature of mathematics included "the ever-growing content of mathematics, the interrelationships of its major structural elements, its ability to represent the world, its use in communication, and its creative use in solving problems of many kinds" (p. 152). With an emphasis on reasoning, communication, and problem solving, the teachers in this study appeared to be in partial agreement with Fennema and Franke's (1992) statement.

Much of the influence of teachers' beliefs was seen in the consistency between the planned geometry course and the implemented one. For example, teachers who felt that
students needed to construct their own geometry knowledge through active involvement in purposeful activities made decisions to engage their students in investigations, discussions, and group work. The influence of teachers' beliefs was also evident as teachers gave reasons for their decisions. For example, one of the teachers focused on proofs during instruction because he believed that learning to think logically was the most important student outcome from geometry. A fairly strong relationship between teachers' beliefs about geometry as a discipline and their instructional practices partially supported previous research finding strong as well as weak relationships between teachers' beliefs about the nature of mathematics and their instructional practices (Kesler, 1985; A. G. Thompson, 1984). One possible weak component was the relationship between teachers' beliefs about the connection between geometry and the real world and their classroom instruction. Four of the teachers talked about the importance of the connection between geometry and the real world, but observed lessons of the teachers included only incidental real world references. These teachers indicated, however, that real world applications were to be incorporated more as their courses progressed. It is possible that the timing of the study highlighted the foundational component of the course rather than the application component of geometry.
Teachers' decisions also appeared to be influenced by their beliefs about how they viewed their own teaching. One aspect of this belief was whether teachers viewed the process of becoming an effective mathematics teacher as an ongoing and life-long process. Based on self-reported descriptions of their geometry background, three teachers clearly indicated they viewed preparation to teach as a dynamic, ongoing process. The fourth teacher probably agreed with the view of the first three teachers, and the fifth teacher probably did not. The life-long professional learner factor appeared to influence teachers' overall curriculum decisions including goals, content, and instructional methods. In previous decision making studies, the dynamic nature of the teaching process was implied by the identification of factors described as teachers' ability to integrate and apply knowledge during instruction (Borko & Livingston, 1989; Clark & Elmore, 1981; Putnam, 1984; Westerman, 1991; Yinger, 1980). Whereas, previous decision making studies generated information supporting the dynamic process of teaching, this study extended the findings of previous decision making research by addressing implied teachers' beliefs about the process of being and becoming an effective mathematics teacher.

A second aspect of teachers' beliefs about how these teachers viewed their own teaching involved whether they
appeared to think of themselves as curriculum builders. As
curriculum builders, teachers defined their geometry course
rather than implemented a course defined by others.
Teachers in studies conducted by Putnam (1983) and
Westerman (1991) took an active role in making decisions
that transformed their curriculum into their desired
course. For the teachers in this study, the role assumed
by them in the process of defining their geometry course
was influenced by whether they chose their textbook and
whether their textbook matched their beliefs about the
teaching and learning of geometry.

The geometry textbook and other materials. All
teachers in this study relied on their textbooks for making
decisions about their geometry course. The role the
textbooks played in determining the scope and sequence of
the course varied. Two of the teachers echoed Brown’s
(1974) finding of geometry teachers relying heavily on the
textbook for content selection and sequencing. For both of
these teachers, the reliance on their textbook was
attributed to the fact that they helped select a textbook
that matched their philosophies for teaching geometry.
Even though a third teacher also used a textbook that she
helped select and that supported her approach for teaching
geometry, she used her textbook as a guide. The textbook
did not completely define the teacher’s desired geometry
course. This geometry teacher supplemented her course with
activities and projects that supported an integrated mathematics class and her school's restructuring plan. The remaining two teachers used the textbook as a guide and made decisions about their course based on their wish to present a view of geometry that was broader than the proof-oriented scope and sequence emphasized in their textbooks. Both teachers participated in the book selection process at their respective schools. Their agreement to select a textbook that did not match their philosophies for teaching geometry appeared to be related to their willingness to work with their colleagues and their beliefs that their geometry course was not defined by the textbook.

For teachers who relied heavily on the textbook, the textbook appeared to be their written curriculum including goals and content. For the teachers who used the textbook as a guide, the textbook appeared to be part of their written curriculum. Other components of their curriculum included the Curriculum Standards (1989), departmental guidelines, other textbooks, conference, seminar, and workshop materials, colleague-created activities, and self-created ideas. In either case, the results from this study agreed with previous decision making studies that identified curriculum as a factor that influenced teachers' planning decisions (Borko & Livingston, 1989; Clark & Elmore, 1981; Morine-Dershimer, 1978-1979; Nesselrodt, 1990; Peterson et al., 1978; Putnam, 1984; Westerman, 1991;
Yinger, 1980; Zahorik, 1975). Part of curriculum’s influence on teachers’ planning decisions was seen through the influence of teachers’ school setting on their decision making.

**Teachers’ school setting.** The results from this study suggested that teachers’ interpretation of their school setting, as well as how they worked within their particular school system, influenced teachers’ decision making. In general, all teachers indicated their contentment with working within their respective school systems. Specific components of the systems that influenced teachers’ planning and interactive decisions included colleague collaboration, block schedules, selection of textbook, and a school restructuring process. The four teachers whose decisions were influenced by their colleagues and their school’s block schedules appeared to work within their particular school system as well as to participate in the process of creating their system. The creative process was particularly noticeable for the teacher whose school had developed its own school restructuring system. This teacher assumed a leadership role in the development process that included reading research, writing and revising standards, and piloting block classes. For the fifth teacher, the influence of his school’s setting was mainly seen through his participation in the textbook
selection process. This situation was interpreted as working within the system as an individual.

The identification of the school setting as a factor that influenced teachers' planning and interactive decisions has not directly been seen in previous decision making studies, but was possibly implied by the identification of curriculum as a factor influencing teachers' planning decisions (Borko & Livingston, 1989; Clark & Elmore, 1981; Morine-Dershimer, 1978-1979; Nesselrodt, 1990; Peterson et al., 1978; Putnam, 1984; Westerman, 1991; Yinger, 1980; Zahorik, 1975). However, the influence of the school setting on teachers' instruction has been directly seen in research involving teachers' conceptions of teaching and learning mathematics. Specifically, the research has identified the social context in which mathematics teaching takes place as one factor influencing a complex relationship between teachers' conceptions and their instructional practices (A. G. Thompson, 1992). Thus, the identification of teachers' school setting as a factor extended the decision making literature and supported teacher conceptions literature.

Reform agendas. The findings from this study suggested that for all but one of the teachers, national and local reform agendas and their ideas influenced teachers' planning decisions as well as those decisions reflecting an implementation of their plans. The influence
of reform agendas was seen as teachers stated the use of the *Curriculum Standards* (1989) as a source for ideas for geometry goals, content, and instructional methods, as one teacher referred to her school portfolio requirements as she included student peer-evaluation activities in her lessons, and as teachers had their students actively exploring concurrent lines of a triangle with constructions. Since previous decision making studies were not conducted in a specific discipline reform context, the identification of national and local reform agendas as a factor influencing teachers’ decision making extended the previous research.

Brown and Baird (1993) argued that in order for teachers to decide to teach according to a vision presented in a reform agenda, teachers must believe that the mathematics and teaching described in the agenda are valuable. The results of this study appeared to support Brown and Baird’s argument. Based on these geometry teachers’ comments and actions, the reform agendas (*Curriculum Standards* (1989) and local school restructuring process) that had the most influence on teachers’ decision making were those reform agendas that were in agreement with the teachers’ personal beliefs and successful teaching experiences. With respect to the state-legislated reform, the teachers and their colleagues were just beginning to interpret the top-down reform. At the time of the study,
teachers' comments implied that the value of the agenda had not been determined. Instead, the teachers were trying to obtain an understanding of the agenda's details in order to make decisions about possible implementation of the agenda.

Throughout the study, teachers' comments about the standards implied they were referring to the *Curriculum Standards* (1989). Although teachers' planning and instruction appeared to contain ideas related to the *Professional Standards* (1991), teachers did not make references to the *Professional Standards* or use vocabulary such as "worthwhile mathematical tasks," "the teacher's role in discourse," or "learning environment." It is possible that the teachers were not familiar with the *Professional Standards* or that they group the NCTM standards' documents together when thinking about them. Teachers may not have referred to the *Professional Standards* because they got their ideas from other resources such as a cooperative learning workshops, geometry presentations at conferences, or activities in *The Mathematics Teacher*. The possibility also exists that the sole reference to *Curriculum Standards* in the first interview may have biased the discussion.

**Students' needs and actions.** This study found that when geometry teachers talked about their decision making they indicated that students' needs influenced their planning and interactive decisions. In general, this
influence was seen in the form of genuine respect these teachers showed their students as well as represented by teachers' desire to do what they felt was best for their students.

A more specific influence on teachers' decision making was seen as the teachers in this study made planning decisions based on the perceived needs of their students. For example, some teachers' plans included the use of student groups because they felt the small group structure provided students with the needed opportunity to talk about geometry while meeting students' social needs. As another example, some teachers supplemented their textbook material because students needed additional practice with the geometry concepts and relationships or needed to see the concepts from another perspective. Teachers also made planning decisions to incorporate a variety of instructional methods because students had various learning styles and because variety encouraged a pacing in block schedules that fostered student interest in the course.

The identification of student needs as a factor appeared to support the results of decision making research that found teachers' planning decisions were influenced by the teachers' goal of students' social development (Fogarty et al., 1983; Putnam, 1984).

Teachers also made decisions based on students' needs as indicated by students' actions during instruction and
work time. On occasion, students' actions indicated they needed assistance with understanding the content. For example, if students had questions about the assignment or about the concept under discussion, teachers made decisions concerning whether to answer the question, ask probing questions, or have another student assist the individual by providing an alternate explanation. On other occasions, students' actions were seen in the form of their active participation in investigations and discussions. Teachers then needed to determine how to guide the discussion. The results of this study were consistent with previous decision making research that found teachers adapted their instruction to students' needs and actions (Borko & Livingston, 1989; Fogarty et al., 1983; Nesselrodt, 1990; Peterson & Clark, 1978; Westerman, 1991).

Even though observed lessons indicated geometry teachers' instruction was influenced by students' needs and actions, the lessons did not appear to be drastically altered by these needs and actions. It appeared that teachers' awareness of students' needs as well as the teachers' teaching experience and comfort level with the content enabled them to anticipate many students' needs during planning. This ability of teachers to use their knowledge of students, content, and instruction to make decisions was consistent with other decision making studies concerning experienced teachers (Clark & Elmore, 1981;
Putnam, 1984; Westerman, 1991; Yinger, 1980) and was supportive of research that has examined mathematics teachers subject matter and pedagogical conceptions (Borko & Livingston, 1989; Leinhardt & Smith, 1985).

The context of reform. As previously stated, the results of this study have contributed to the decision making literature by identifying reform agendas as factors influencing geometry teachers' planning decisions and their interactive decisions that showed their plans were implemented. The design of this study also made it possible to determine whether predominant factors identified during a mathematics reform context differed from predominant factors identified during the stable context of previous decision making studies. Comparisons between previous decision making research and this study showed that similarities as well as differences emerged. With the emergence of similarities and differences described below, the results of this study also contributed to the decision making literature.

Similar influential factors clearly identified in previous studies and this study included teachers' teaching experiences, course goals, advanced planning decisions, curriculum, and students' actions. Similarity in factors was also implied in the area of geometry teachers' consideration of their students' needs as they made decisions. In previous decision making research, student
influence was described in terms of their social needs and abilities. In this study, student influence was characterized in terms of their needs for understanding geometry concepts and relationships better, as well as their social needs.

Partial differences between previous decision making results and this study's findings were seen with the identification of teachers' beliefs as a factor. Agreement occurred with the factor described as teachers' beliefs about their role as curriculum builders. Results from both prior studies and this study showed that teachers believed they were empowered to transform their courses into their own.

Some differences in factors influencing teachers' planning and interactive decisions were implied by this study's identification of reform agendas, teachers' geometry experiences as students, teachers' professional development experiences, teachers' beliefs about the nature of geometry as a discipline, teachers' beliefs about the process of becoming an effective teacher, and teachers' school settings as influential factors. The identification of a general school reform agenda and NCTM's reform agenda as a factor supported the idea that the context of reform influenced teachers' decision making. In addition, this study's identification of teacher-related factors supported the notion that teachers are key components of the reform
process. Naturally, when teachers were making decisions during a context of reform, they made decisions based on self-related factors such as their experiences as geometry students, professional development experiences, beliefs, and school settings.

At the same time, it is possible that the data collection process, not the context of reform, was the reason for the emergence of some of the identified factors. In an attempt to understand the background of each teacher and the context in which each teacher worked, data were collected on teachers' geometry experiences as a student and as a teacher and on their school settings. Another plausible explanation is that, without teachers being asked about this information, the factors described as teachers' geometry experiences as a student, teachers' professional development experiences, teachers' beliefs about the process of becoming an effective teacher, and teachers' school setting would not have been identified.

Limitations of the Study

The findings of the study are limited in generalizability by various aspects of the study: the representativeness of the sample, the data collection process, and potential researcher bias as data collector.

One limitation of the study was the representativeness of the sample due to the small number of subjects. The nature of the study and the amount of data to be collected
and analyzed indicated the necessity of a small sample size. It is acknowledged that conclusions acquired from a few teacher profiles are not generalizable to a larger population of teachers. It is hoped, however, that geometry-related conclusions made across the teacher profiles generates conjectures for further exploration with a larger sample of secondary geometry teachers. Similarly, it is hoped that decision-making-related conclusions made across teacher profiles generates conjectures for further examination with a larger sample of teachers.

Even though the sample was selected to represent different views of geometry curriculum including goals, content, and instructional methods, the fact that all participants in the study were experienced secondary geometry teachers (having more than five years of teaching experience) may not be representative of geometry teachers in general. A larger representative sample may now be used to further examine various conjectures generated by this study.

The findings from the study were limited by the data collection process. Beginning with the first week of school, the teachers were interviewed and observed weekly for approximately one-third of their school year. Whereas the information gathered during this data collection period generated pictures of the five teachers' geometry courses, consecutive observations and a longer data collection
period, perhaps a full school year, may have provided a more complete picture of the teachers' sequencing of content and instruction. At the same time, the reader is reminded that from a decision making perspective, the data collection process was more intensive than previous decision making studies involving multiple teachers.

At times during the data collection, teachers' schedules did not allow for complete pre-lesson or complete post-lesson informal interviews. When schedule conflicts occurred, attempts were made during subsequent informal interviews and the final interview to address the decision information associated with a given lesson. However, some of the results of the study may have been slightly altered if the informal interviews had been more complete.

It is also recognized that the process itself of having teachers reflect on their planning and their interactive decisions may bias the data collection and serve as a treatment. For example, during the reflection process teachers may have been asked to think about decisions or factors of which they were not aware or to which they had given minimum importance. Even though the teachers in this study may not naturally have been reflective individuals, the information generated from their reflections provided fundamental data about the teachers' decision making.
Although a variety of data collection techniques were used in the study, the primary instrument for collecting and analyzing data was the researcher. While the researcher’s experiences with the teaching and learning of geometry helped with learning about teachers’ planning and interactive decisions, the experiences also had the potential for biasing the data collection and analysis process. During the data collection and analysis process, the researcher kept a journal recording thoughts, feelings, and judgements made throughout the course of the study. Reading the journal during the analysis process fostered the researcher’s attempt to recognize and transcend personal bias.

Implications and Recommendations for Future Research

The findings from this study suggest several implications for practice and recommendations for future research in the areas of the high school geometry course, decision making, the reform process, development of written materials, and teacher education.

Geometry course descriptions generated by this research were based on teachers’ planning and interactive decisions, supplying information about the geometry classroom from a perspective that was not utilized in previous geometry research. Teachers’ words and actions implied they felt geometry was an important component of their secondary curriculum. The essence of what these
teachers valued about geometry is implied by characterizations of their geometry course.

The geometry courses in this study were characterized by teachers' beliefs about the nature of geometry as a discipline. Based on the nature of geometry as a discipline and related instructional practices, the geometry classrooms in the study ranged from a classroom where students listened to lectures while learning a static and structured body of knowledge to a classroom where students constructed their own knowledge of geometry through active participation in purposeful activities.

The geometry courses were also characterized by various facets of geometry promoted by teachers as implied by their goals, content, and instruction. One teacher emphasized the mathematical system view of geometry and addressed geometry as a base of content knowledge. In contrast, the other four teachers promoted multiple facets of geometry including geometry as a base of content knowledge, an example of a mathematical system, and a setting for developing communication and problem solving skills. In addition, these four teachers talked about promoting connections between geometry and the real world. However, the connections between geometry and the real world were mostly seen in terms of what was to be addressed later in the course.
The characterizations of the geometry courses generated by this research have possible implications for classroom practice. Nickson (1992) stated that the beliefs held by teachers and students in terms of the nature of mathematics were important components of the culture of the classroom since these views were connected to the way mathematics was taught and received. Teachers' beliefs about the nature of geometry were part of the geometry classroom descriptions, but their students' beliefs concerning the nature of geometry were not known. Do students see geometry as a static and structured set of knowledge? Do they perceive geometry as a set of knowledge which they need to construct for themselves and to connect with other knowledge? How do teachers use the information about students' views to plan their instruction? How is students' understanding of geometry affected when their views of the nature of geometry are different than their teacher's views? To answer these questions future research needs to simultaneously examine teachers' and students' views about the nature of geometry.

As indicated by the characterizations of the geometry courses, differences in the courses included whether teachers emphasized one view of geometry or multiple views of geometry and whether the instruction fostered passive learning or active learning by their students. Differences also occurred in approaches used by the four teachers whose
instruction fostered students' active involvement in their geometry learning. One teacher's focus was the use of constructions, hands-on activities, and student-work with partners; another teacher's focus was the use of five skills. A third teacher approached the course through the use of intuitive and inductive reasoning, and the fourth teacher incorporated group activities and technology as well as addressed geometry from a more integrated perspective. With these different approaches to geometry the question again becomes, "What effect do these different approaches have on students' understanding of geometry?" Future research needs to address this issue.

The findings from this study provide a beginning look at geometry teachers' planning and interactive decision making in the context of both general school and mathematics education reform. This beginning look was based on weekly interviews and observations. As additional research involving decision making during reform is conducted, the inclusion of consecutive observations provides another perspective from which to examine teachers' decision making.

Brown and Baird (1993) hypothesized that teachers' knowledge of factors affecting their decision making is an important component of teachers' teaching and ability to manage classroom dilemmas. The influential factors identified in this study were based on teachers' comments
and implied from their actions in the geometry classroom. Teachers' awareness of factors that influenced their planning and interactive decisions seemed apparent. Attempts to document teachers' awareness of influential factors occurred during follow-up interviews. However, the notion of whether teachers' awareness of factors influenced their decision making was not documented. To further describe the process of teachers' decision making, additional decision making research needs to document the effect teachers' awareness of influential factors has on their own decision making.

Implications concerning the process of reform follow from the research-generated descriptions of geometry classrooms. As presented by four of the teachers in the study, the occurrence of a multifaceted geometry class in which students were active participants in their learning was possible. Geometry as reasoning, geometry as problem solving, and geometry as communication were clearly part of the multifaceted nature of these teachers' geometry courses. Thus, much of the general curriculum guidelines including goals, content, and instructional methods for geometry advocated by NCTM in the Curriculum Standards (1989) was occurring in the classrooms of these four teachers. However, the study’s findings also showed that geometry’s connections with the real world or with other subjects, the use of technology, and real-world
applications and modeling were not prevalent components of
the teachers' geometry courses. Additional research is
needed to determine the prevalence of this study's
occurrences in the population of secondary geometry
teachers. Furthermore, future research needs to determine
what additional factors influence different degrees of
implementation of the Curriculum Standards, the
Professional Standards (1991), as well as the post-study
released Assessment Standards for School Mathematics

Reform agendas were identified as an influential
factor for four of the teachers. The influence was seen
from the perspective that a reform agenda was one possible
source of ideas for teaching and learning geometry. The
explanations for the reasons these teachers implemented
some reform ideas and not others provide implications for
the reform process. These four teachers implemented reform
ideas that matched their beliefs about teaching and
learning geometry. Thus, it is important for reform
leaders to communicate and interact with teachers in order
to be aware of teachers' beliefs.

Teachers in the study also implemented reform ideas
because they had previous experience with reform-related
recommendations made at workshops and conferences and in
professional literature. The apparent influence of
teachers' professional development experiences suggests
that opportunities for inservice teachers to examine and learn about reform agendas continues to be an important component of the reform process. The importance of teacher development to the reform process suggests the need to require teachers to participate in inservice work during a time of reform. However, the findings of this study suggest that the success of required ongoing teacher development efforts depends on whether the teachers have assumed the role of a professional life-long learner.

Four teachers' indication that reform agendas were one source of ideas involving goals, content, and instructional methods for their geometry class and their teaching histories implied that these teachers took on the role of a life-long professional learner. The reform agendas related to this study challenge teachers to assist their students with developing as life-long learners. Research needs to address the issues of how teachers as life-long professional learners view students as learners and whether these teachers foster students' development as life-long learners.

The reform agendas with which the teachers in the study were associated also advocated the use of technology in the classroom. Since only one of the five teachers used technology, its use was not widespread. The teacher who used the technology had easy access to a computer lab and had a school restructuring agenda that contained a
technology component. Some of the teachers provided reasons for not using technology. The teacher who emphasized the logical component of geometry felt there was no room in his curriculum. Two teachers stated they would use computers for student investigations during their instruction if they had access to the technology. Interestingly, one of these teachers indicated that much of what she would do with technology she does in her classroom now with groups and hands-on activities. Even though previous research studies involving geometry and technology (McCoy, 1991; Yerushalmy et al., 1987) have supported the use of technology in the geometry classroom, it appears that additional research is needed to determine what role technology plays in successful teaching of geometry as well as in the reform process.

The results from this study also have implications for developers of written classroom materials. Specifically, textbook developers need to be aware of how teachers use their textbooks. Some teachers in this study used the textbook as a guide as they developed their geometry course. In this situation, teachers appeared to be curriculum builders. Other teachers in the study relied heavily on their textbook as they made decisions about their course. These teachers appeared to interpret a course defined by others. In either case, no teacher chose
to disregard the textbook and adapt a project or problem-centered curriculum.

The degree to which the teachers used their textbooks depended on whether the textbooks were closely aligned with the teachers' beliefs about the learning and teaching of geometry. When the textbook was closely aligned with their beliefs, the teachers relied heavily on the textbook. When the textbook was not closely aligned with their beliefs, the teachers used the textbook as a guide and supplemented with other materials. Thus, it seems that textbook developers need to be aware of teachers' beliefs about the learning and teaching of geometry in order to develop textbooks or other materials that match teachers' beliefs. As part of this awareness, textbook developers must also realize that teachers' beliefs change over time due to classroom experiences (A. G. Thompson, 1992). Thus, an active dialogue between teachers and textbook developers is needed. In addition, the variety of teachers' beliefs concerning approaches to organizing geometry suggests the need for the development of textbooks on CD or laser disk. Textbooks in this form provide teachers with the opportunity to print out materials they wanted and in their preferred order.

Even though it is not clear whether teachers' planning and interactive decisions were a direct result of their teacher education experiences, the findings from this study
do identify issues that have implications for teacher education. Based on their beliefs about the nature of geometry as a discipline, facets of geometry, student learning, and preferences for instructional approaches, the five teachers in the study made decisions that enabled them to implement their envisioned geometry course. Due to the potential influence on teachers' decision making, teacher education needs to provide preservice teachers with opportunities to examine their beliefs about content and instruction. Because of the possibility of teachers' beliefs changing (A. G. Thompson, 1992), inservice teachers also need to be provided with staff development that gives teachers opportunities to examine their own beliefs about geometry curriculum including goals, content, and instructional methods. As part of this examination process, inservice teachers must be exposed to materials and methods that assist them in implementing a geometry course that support their beliefs.

For four of the teachers, the geometry course they implemented in their classrooms was different from the one they experienced as high school students. These teachers felt their present geometry classes put less emphasis on two-column proofs than did their high school geometry course and included and emphasized the use of manipulatives, group activities, communication and problem solving skills, and multiple facets of secondary geometry,
none of which were addressed in their high school geometry courses. Research has suggested that teachers teach what they know (A. G. Thompson, 1984). If teachers are being asked to teach a geometry course that is qualitatively different from the one they experienced as a student, it seems imperative that preservice teachers are provided with occasions to experience geometry in the same manner in which they are expected to teach. Specifically, preservice teachers need to experience geometry as reasoning, problem solving, communication, and connections through instructional approaches requiring their active involvement.

Anecdotal references made by the geometry teachers indicated that they had preservice and inservice opportunities that enabled them to develop their desired geometry course. In particular, they learned about different facets of geometry and instructional approaches such as cooperative learning and hands-on activities. Since the knowledge base of mathematics is growing (e.g., fractals and their applications), preservice and inservice opportunities involving new curriculum ideas including goals, content, and instructional methods must continue to be available to teachers. In the case of preservice teachers, these opportunities need to be part of their required course work. The decision of whether to require inservice teachers to participate in these new curriculum
opportunities is not clear. Teachers' exposure to new ideas involving goals, content, and instructional methods has the potential to influence their beliefs about the teaching of geometry. However, it is also possible that requiring teachers to participate in staff development activities that do not match their beliefs about the teaching of geometry is not beneficial to them or their students.

For the geometry teachers in this study, participation in inservice opportunities appeared to be related to their beliefs about how they viewed their own teaching. Teachers who apparently took on the role of curriculum builder and life-long professional learner talked more about their inservice participation than those who did not take on that role. The curriculum-builder and life-long professional learner teachers appeared to be empowered with teaching tools that enabled them to meet any mathematics education challenge. It is not known whether teachers' preservice experiences were the catalysts for this empowerment, but the possibility exists that a preservice education that encourages teachers to be curriculum builders and life-long professional learners promotes this empowerment. Research specific to the influence of teacher education experiences on teachers' beliefs concerning their own teaching needs to address this issue.
REFERENCES


Van Hiele, P. M. (1980, April). Levels of thinking, how to meet them, how to avoid them. Paper presented at the meeting of the National Council of Teachers of Mathematics, Seattle.


APPENDICES
Appendix A

Van Hiele Theory

The van Hiele theory revolves around two ideas: (a) the van Hiele levels of geometric thought, and (b) the phases of instruction. The description of the levels which follows is the one used by Fuys, Geddes, and Tischler (1988). The numbering for the levels has varied in the literature. This paper consistently numbered the van Hiele levels one through five. The name of each level represents a consensus of labels used in the literature.

Level 1: Visual. The student identifies, names, compares and operates on geometric figures according to their appearance.

Level 2: Descriptive/analytic. The student analyzes figures in terms of their components and relationships among components and discovers properties/rules of a class of shapes empirically (e.g., by folding, measuring, using a grid or diagram).

Level 3: Abstract/relational. The student logically interrelates previously discovered properties/rules by giving or following informal arguments.

Level 4: Deductive. The student proves theorems deductively and establishes interrelationships among networks of theorems.

Level 5: Rigor. The student establishes theorems in different postulational systems and analyzes/compares these systems.

P. M. van Hiele's (1980, 1986) recent works included a reference to a three level model and a reference to a model without level 5. The original five level model, however, was used as a reference in this paper. The basis for this
decision is best described by Clements and Battista (1992): "empirical evidence and the need for precision in psychology oriented models of learning argue maintaining finer delineations" (p. 427).

Fuys et al. (1988) identified six characteristics of the levels as proposed by the van Hieles: (a) the levels represent jumps in the learning curve, and thus are discrete; (b) the levels are sequential (much of the literature used the word hierarchical); (c) each level has its own language, set of symbols, and network of relations; (d) what is implicit at one level becomes explicit at the next level; (e) material taught to students above their level is subject to reduction of level; and (f) progress from one level to the next is more dependent on instructional experience than on age or maturation.

The instructional component of the van Hiele theory is defined as the phases of instruction. The goal of the implementation of the phases of instruction is to facilitate students' movement through the van Hieles' levels of geometric thought. The description of the theoretical phases that follows was used by Fuys et al. (1988).

Phase 1: Information. Students get acquainted with the working domain (e.g., examines examples and non-examples).

Phase 2: Guided orientation. Students do tasks involving different relations of the network that is to be formed (e.g., folding, measuring, looking for symmetry).
Phase 3: Explicitation. Students become conscious of the relations, tries to express them in words, and learn technical language which accompanies the subject matter (e.g., expresses ideas about properties of figures).

Phase 4: Free orientation. Students learn, by doing more complex tasks, to find their own way in the network of relations (e.g., knowing properties of one kind of shape, investigates these properties for a new shape).

Phase 5: Integration. Students summarize all that they have learned about the subject, then reflect on their actions and obtain an overview of the newly formed network of relations now available (e.g., properties of a figure are summarized)
Appendix B

Geometry Questionnaire

Please give a written response to the following questions. The responses to these questions will provide demographic information and will provide guidelines for the first interview.

In order to protect your identity the last four digits of your social security number will be used as your identification number. Last four digits: ________.

1. How many years and at what levels have you been teaching mathematics?

2. How many years have you been teaching mathematics at your present school?

3. How many years have you been teaching geometry?

4. What mathematics classes are you teaching this year? If possible, provide me with your teaching schedule.

5. Write a brief geometry biography. Include in this biography a description of your experiences learning geometry from your earliest memories (what classes have you taken, what do you remember about the instruction of these classes, what do you remember about how you were evaluated in these classes, what did you like about these classes, what did you dislike, what workshops involving geometry ideas have you taken, what experience has been most beneficial for your understanding of geometry, . . . ) If needed, use the back side of this paper.
Appendix C
First Interview

Core questions for the interview are the following:

1. What do you find valuable about teaching geometry in secondary schools?

2. What are the goals of your geometry class?

3. How do you feel these goals are best met?

4. How did you decide what these goals would be?

5. What content will be covered in your geometry class this year? (Probe for sequence)

6. How did you decide what content was to be included in the geometry course?

7. Has this content changed over the years? Explain.

8. What instructional approaches will be used in your geometry class this year?

9. Explain why you selected these instructional approaches.

10. Have these approaches changed over the years? Explain.

11. Presently in the state, the state’s Educational Act for the 21st Century and CIM/CAM are topics of conversation. What is your interpretation of these topics?


13. Are you familiar with the National Council of Teachers of Mathematics’ Curriculum Standards? If no, no further questions. If yes--What is your interpretation of the agenda described in the Standards? Has the Standards’ affected what you do in your classroom? Explain.
Appendix D

Final Interview

1. Describe how feel about the progress of your geometry class. (Have students met your goals?)

2. Explain why you feel the class has progressed as it has. (Were there any surprises?)

3. Describe the view of geometry that has been portrayed in your classroom. (What is geometry?)

4. Goals
   (a) Have students met your goals?
   (b) Have your goals changed? Explain.
   (c) If goals have been changed, what influenced your changes?
   (d) In the first interview, you indicated that one of your course goals was ___________ (researcher selects a goal that has not been clearly addressed in the course), please explain how students have had opportunities to meet this goal. (repeat with a different goal if necessary)
   (e) Based on my observations and analysis work with the data, it appears that ___________ (researcher shares a decision or factor theory that has emerged from the data), please respond to my comments. (repeat as needed)

5. Content
   (a) Has the sequence gone as expected? Explain.
   (b) If changes have been made, what influenced your changes?
   (c) Based on my observations and analysis work with the data, it appears that ___________ (researcher shares a decision or factor theory that has emerged from the data), please respond to my comments. (repeat as needed)

6. Instruction
   (a) Have you been able to use the instructional approaches that you had hoped to use? Explain.
   (b) If changes have been made, what influenced your changes?
   (c) Based on my observations and analysis work with the data, it appears that ___________ (researcher shares a decision or factor theory that has emerged from the data), please respond to my comments. (repeat as needed)
7. What are your plans for this class for the remainder of the year? (goals, content, instruction)

8. Explain why these are your plans.

9. When you teach this class again, how will you approach this class? Explain.