

AN ABSTRACT OF THE THESIS OF

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Title Diffusion in a Variable Density Liquid Under

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The diffusion process in a variable density, turbulent flow is a factor which greatly influences the waste disposal characteristics of tidal estuaries. Waste disposal from cities and industries, is posing an increasing threat to the estuarine environment. Accurate prediction of the effects of proposed waste outfalls in well-mixed estuaries (which predominate during the most critical waste disposal season) is very desirable. The most used analytical tool for the prediction of waste concentrations in well-mixed estuaries, is the one-dimensional conservation of mass equation. A lack of knowledge concerning the diffusion process and a resulting inability to predict the coefficient of turbulent diffusion, is a major limitation to the use of this tool. Closed form solutions to the equation of mass conservation are not sufficiently accurate for most real estuaries because of the required

simplifying assumptions. Field determination of the average diffusion coefficients has allowed numerical solutions for average steady-state concentrations.

For the purpose of providing a tool for predicting the cyclical variation of waste concentrations and improving our understanding of the diffusion process, a dimensional analysis for the instantaneous diffusion coefficient (considering only bottom shear and longitudinal density gradients) was made. Experimental work was designed to determine the relationships between dimensionless parameters obtained by the systematic method of Buckingham. The narrow range of flows and depths for which consistent results could be obtained did not allow a complete determination of the relationships. However, the nature of the relationships obtained, indicated that they could serve as the correspondence between model and prototype for determining the longitudinal density gradient effects on the diffusion coefficient in well-mixed estuaries. Further studies are required to verify these relationships over a wider range of flows and depths.

DIFFUSION IN A VARIABLE DENSITY
LIQUID UNDER CONDITIONS OF
TURBULENT, STEADY FLOW

by

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LIST OF SYMBOLS

c	concentration of waste, pollutant, etc.
\mathbf{W}	vector velocity
H	depth
ρ	mass density
x	distance measured in the longitudinal direction
y	distance measured in the vertical direction
k	absolute roughness
D_t	coefficient of turbulent diffusion without density gradients
D_L	longitudinal coefficient of diffusion due to shear and eddy diffusion associated with the Taylor formula
D'	apparent diffusion coefficient
ppt	parts per thousand
B.O.D.	biochemical oxygen demand
D.O.	dissolved oxygen
A	cross-sectional area
n	designates cross section of estuary or channel
∇	del operator
$\nabla \cdot$	divergence of
τ	shear stress
R	hydraulic radius
G	average rate of energy dissipation
J	average rate of gain of potential energy
g	acceleration of gravity
S	slope of energy grade line

U velocity in the longitudinal (x) direction
 f river water concentration as a decimal fraction
 ϵ eddy viscosity
 ϕ denotes unknown function
 C_c Chezy coefficient
 Δ small increment in a variable quantity

DIFFUSION IN A VARIABLE DENSITY LIQUID UNDER CONDITIONS OF TURBULENT, STEADY FLOW

INTRODUCTION

Statement of Problem

The diffusion process in variable density, turbulent flow is of great importance in tidal estuaries. The ability of an estuary to assimilate waste is largely dependent upon this process. Since man is continually modifying tidal estuaries for purposes of waste disposal, flood control, improved navigation, and land reclamation, the engineer is increasingly being called upon to predict, beforehand, the probable effects of these changes.

This study is primarily concerned with the diffusion process as related to waste disposal in estuaries.

Tidal estuaries have long served as receiving bodies for the disposal of domestic and industrial wastes. The advantages of estuaries, particularly in the transportation of products and resources, make them desirable locations for cities and industries. While estuaries usually reduce the cost of waste disposal, the problems created are many. The damages to marine resources are probably the most serious in terms of economic value. Health hazards and aesthetic nuisances may also be considerable.

Due to a lack of knowledge of the factors involved, prediction of the effects of waste disposal in estuaries

has been haphazard and many times prejudiced by narrow interests. It is only in recent years that man's knowledge of estuaries has increased to the point where a scientific approach is at all possible. Often, the first step in the scientific approach to estuary pollution problems is to predict waste concentrations which will result from a given waste disposal at a specific location. The predicted waste concentrations may then be considered for their probable effects on the estuarine environment.

Purpose of the Study

In order to analytically predict waste concentration, a knowledge of the turbulent diffusion process in estuaries is necessary. An improvement in the knowledge of this diffusion process is the purpose of this study.

DESCRIPTION OF ESTUARIES

Pritchard (23, p. 245) has defined an estuary as "a semi-enclosed coastal body of water having a free connection with the open sea and containing a measurable quantity of sea salt." Under this broad definition, estuaries may be classified, for our purposes, according to structural types and circulation patterns.

The three main structural types are the fiord, bar-built, and coastal plain estuaries. Fiords are deep, narrow basins associated with regions of glaciation such

as the Norwegian coast and the Pacific coast of Canada. Bar-built estuaries are usually shallow, often more saline than the adjacent ocean, and are begun by the development of an off-shore bar. The most common type of estuary and the one which is of concern here, is the coastal plain estuary. This type varies greatly in size and shape, and is the result of the flooding of a river valley when either the valley has subsided or the sea level has risen.

Pritchard (22) has classified the "positive" (salinity increasing towards the ocean) coastal plain estuary according to four main circulatory patterns. The classifications vary from Type A (poorly mixed) to Type D (well-mixed). (See Figure 1.) The Type A estuary is a two-layered system with the fresh river water overlying a saline wedge. Some salt is advected into the upper fresh water but little turbulent exchange exists between the layers. Large river flow, large depths, and slight tidal action lead to this type system. Conversely, in the Type D estuary these factors are reversed. The mixing is sufficient to eliminate the vertical salinity gradients and the estuary is narrow enough so that lateral stratification due to the coriolis effect is negligible.

According to Pritchard's system of classification, an estuary will probably shift with the season from one

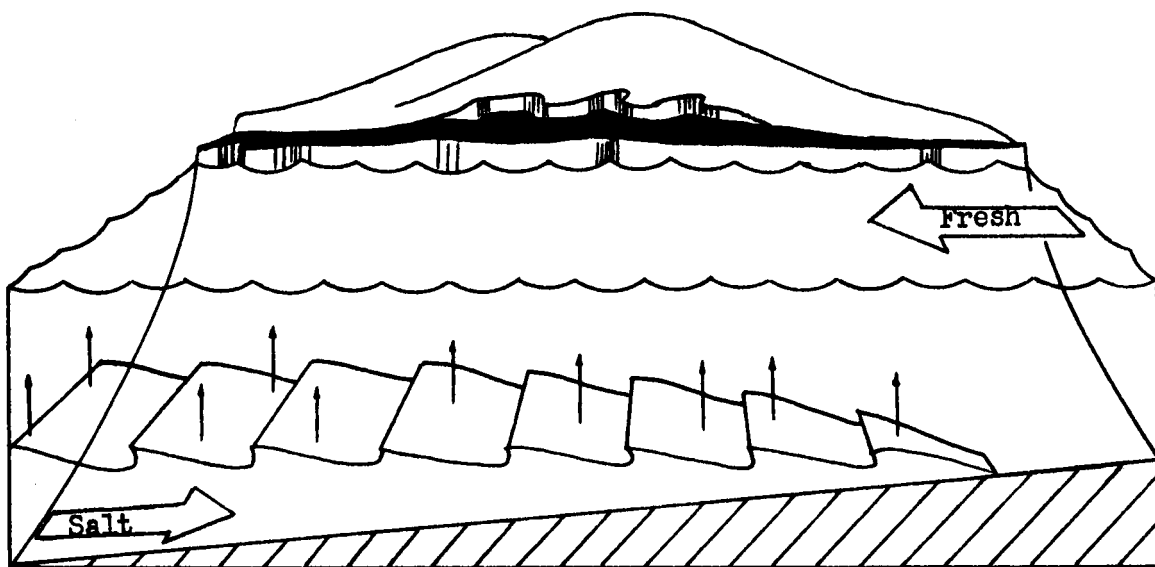


Figure 1(a) Type A Estuary (22, p. 10)

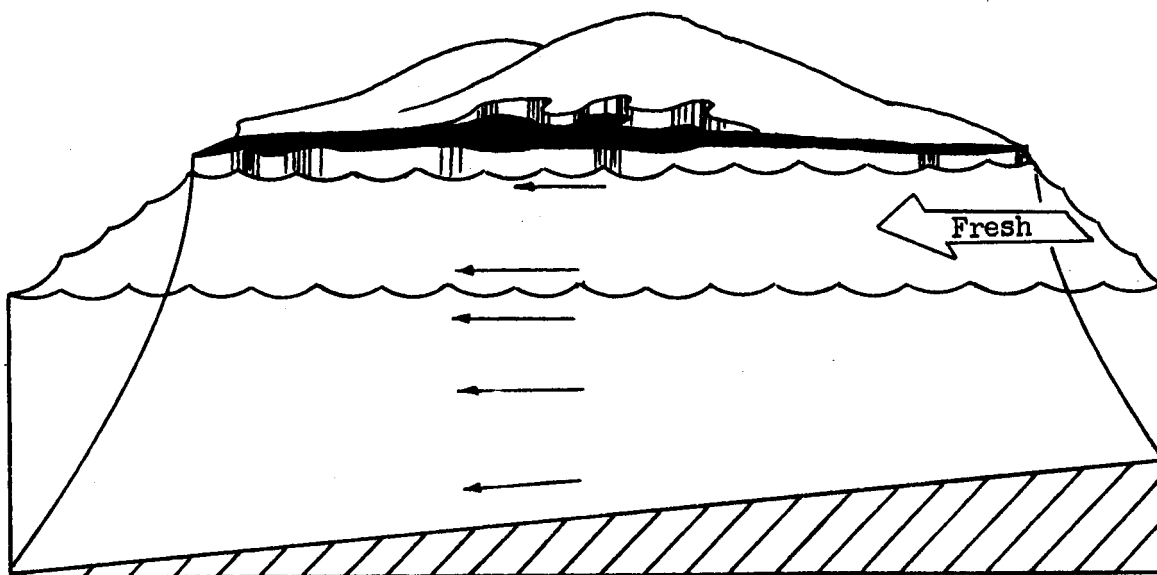


Figure 1(b) Type D Estuary (22, p. 11)

FIGURE 1.

type to another depending primarily on the variation in river flow.

Oregon Estuaries

A parameter which has been used by Burt and McAllister (3) to distinguish between the different types of estuaries is the salinity difference from top to bottom at the station where the average salinity is 17 parts per thousand (ppt). On this basis, Oregon estuaries were classified by them according to the following table:

Type	Vertical salinity variation from top to bottom (high tide)
A	20 ppt or more
B	4 to 17 ppt
D	less than 3 ppt

Actual measurements (3) indicated that nearly all of the Oregon estuaries were Type D (well-mixed) during their low river flow period.

From the viewpoint of waste disposal the low river flow period is usually the critical season because dilution is less and the time required for the waste to move out of the estuary is greater. Further, in all but the Columbia River, the low flow period of Oregon estuaries coincides with the late summer months when higher temperatures increase the rate of Biochemical Oxygen Demand (B.O.D.) and reduce the dissolved oxygen available to marine life.

Waste disposal problems certainly occur at times other than this low flow, well-mixed period, but it is usually the logical period for determining worst possible conditions. This work is limited to the well-mixed condition where significant salinity gradients exist only in the longitudinal direction.

PREDICTING WASTE CONCENTRATION IN WELL-MIXED ESTUARIES

Past Work and Present Methods

Presently there are two basic analytical tools for the prediction of waste concentrations in estuaries. These are the tidal prism concepts and the one-dimensional conservation of mass equation.

Model studies are sometimes indicated in larger systems where the initial cost of the model is usually justified for the study of problems related to navigation. However, until the "art" of modeling proceeds to the point where field data are no longer necessary for calibrating and checking the validity of these models, analytical methods will probably continue to be the most used methods for predicting waste concentration in estuaries.

Tidal Prism Concepts

The tidal prism has been defined by Ketchum as

"the difference between the volumes of water in the estuary at high and low tide" (14, p. 198). In classical tidal prism concepts, the ocean water on the incoming tide was assumed to mix completely with the polluted estuary water and, on the out-going tide, to remove a proportion of the estuary water equal to the tidal prism divided by the high water volume. The concept usually involved two incorrect assumptions: 1) that the entering ocean water mixed completely with the low water, and 2) that the water moving seaward on the ebb-tide escaped and was not returned. Ketchum (14) modified the concept by dividing the estuary into segments defined by the average excursion (longitudinal distance a particle moves) on the flood tide. This is assumed to be the length of the estuary over which complete mixing takes place. These modifications allow the calculation of the total accumulation of a pollutant, and the average length of time the pollutant will be in any section of an estuary. Ketchum used the method to correctly predict the salinity distribution of the Raritan River Estuary. However, the method has not worked well on shorter estuaries which appear to fit the requirements of the method. The assumption regarding the mixing process is open to question; therefore, the principal analytical tool now used for predicting waste concentration in estuaries is the one-dimensional conservation of mass equation.

Solutions to the Conservation of Mass Equation (Continuity)

The one-dimensional equation of continuity for a conservative (non-decaying) pollutant is

$$U \frac{\partial c}{\partial x} + c \frac{\partial U}{\partial x} - \frac{\partial}{\partial x} \left(D' \frac{\partial c}{\partial x} \right) + \frac{\partial c}{\partial t} = 0$$

where:

c = concentration of pollutant

x = distance measured in the longitudinal direction

U = velocity in the x direction

D' = diffusion coefficient

t = time

Closed Form Solutions

There have been several solutions to the above equation in closed form, subject to varying initial and boundary conditions and assumptions regarding the independent variables.

Kent (11, p. 27-29) presented several solutions but also indicated that these were only available when the variables D' , $\left(-U + \frac{\partial D'}{\partial x}\right)$, and $\frac{D'}{A} \frac{\partial A}{\partial x}$ are constants. (Here A refers to the cross-sectional area.) Since in any real estuary the above variables are usually non-linear, closed form solutions were not pursued further.

O'Conner (18) presented a solution to the continuity equation for dissolved oxygen (D.O.) in which a reaeration source and biochemical oxygen demand (B.O.D.) sink were

included. The increments of time for the differential equation were taken as a tidal cycle so that the term $\frac{\partial c}{\partial t}$ could be eliminated. The B.O.D. sink was first expressed as a function of x by solving a continuity equation for this quantity. In order to solve the equation in closed form, the following were assumed constant: U , A , D' , reparation rate and B.O.D. reaction rate. Initial values for the ultimate B.O.D. and D.O. deficit were also assumed. The resulting solution agreed reasonably well with observed values in the Delaware and James Rivers.

An attempt to relate the diffusion coefficient to measurable quantities was presented by Arons and Stommel (1). For a rectangular channel the following equation for the coefficient of turbulent diffusion was presented:

$$D' = B E Z$$

where

E = tidal excursion,

Z = tidal amplitude,

B = a dimensionless coefficient.

This is somewhat equivalent to Ketchum's assumption regarding the dimensions of the mixing volume. In the authors' words: " . . . the simplicity of these formulations results from a certain vagueness about the physical process involved."

After introducing a dimensionless parameter, F ,

called the flushing number, which included the expression for the coefficient of diffusion, the continuity equation was solved. This resulted in an expression for the salinity as a function of x . The observed values for Alberni Inlet on Vancouver Island and the Raritan River in New Jersey, followed the shape of the theoretical curves for constant values of the flushing number. However, the proportionality factor B was an order of magnitude different for the two inlets. Therefore, an a-priori calculation of the flushing number was not considered feasible.

Since Ketchum's and Aron and Stommel's a-priori assumptions about the diffusion process were apparently unjustified when applied to various well-mixed estuaries, Stommel (27) suggested using the equation of continuity and the observed salinity data to compute the diffusion coefficient. This method will be presented in a later section after discussion of the diffusion coefficients.

Numerical Solutions to the Equation of Continuity

In well-mixed real estuaries, where the expense of model studies may not be justified for other purposes, numerical solutions to the equation of continuity appear to be the preferred means of solving for waste concentrations. Stommel (27) presented a finite difference method in which the continuity equation, with time periods equal to a tidal cycle, was differentiated with respect to x .

This yielded an expression for the rate of change of pollutant flux with distance which was set equal to zero at all but the outfall section. At the outfall section the assumed rate of change of pollutant flux was $P/2a$ where P is the total amount of pollutant introduced per tidal cycle and a is the distance increment of the finite difference solution. The resulting equations, one for each segment of the estuary, were solved simultaneously by a relaxation technique. The assumption that the pollutant flux changes by an amount P over a distance $2a$, is open to question. Since, during an increment Δt (a tidal cycle), the pollutant is dispersed in a volume approximately equal to the tidal excursion at the outfall times the area (A) at the outfall, it might be expected that the solution would be most accurate if the tidal excursion at the outfall were approximately equal to $2a$.

Kent (11) presented a numerical solution to the equation of continuity in which he also used the observed salinity distribution to determine coefficients of turbulent diffusion. However, the coefficients used were the products of the coefficients calculated for salt and the ratio of the extent within the estuary of the pollutant to that of the salt. The transient dispersion of a pollutant introduced instantaneously into the Delaware Model was studied. The differential equation, in finite difference form, was arranged so that $c(n, t+\Delta t)$, the concentration at

cross section n , for a time $t + \Delta t$, is a function of c_n , c_{n+1} and c_{n-1} at a previous time t , and the known values of U , A , and D' at sections n , $n+1$, and $n-1$. (See below.)

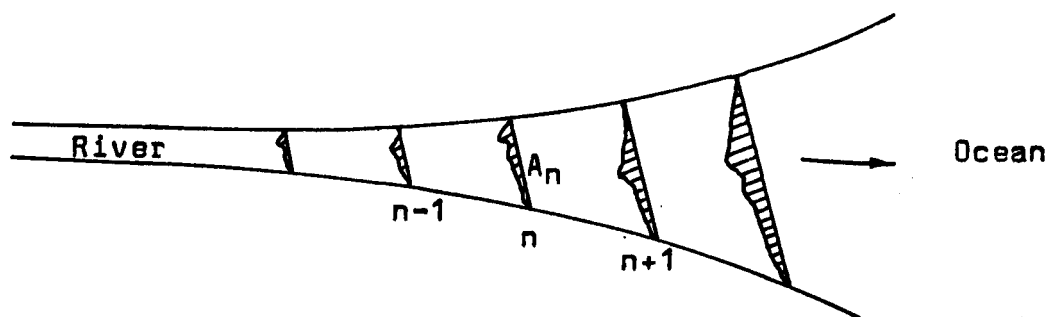


FIGURE 2 Segmentation of Estuary

Values of $c_{n,t}$ were observed after three tidal cycles and the computations continued from that point. A reasonably good correlation was obtained between observed and predicted values.

The author has used a similar solution for calculating steady state concentrations of D.O. and ultimate B.O.D. (8) The equations used are essentially those obtained by Kent with a pollution addition term at the outfall section. Decay, sinks and sources are also considered where appropriate. Solutions for these two pollution parameters were computerized and applied to the Yaquina Estuary, Oregon, using distance intervals of one nautical mile, time intervals of one day, and diffusion data from Burt and

Marriage (2). Ultimate B.O.D. values obtained by this method are approximately twice those obtained by Burt and Marriage using Stommel's method (27). Lacking a method of checking the results, further refinements or corrections are not possible at present.

Need for Predicting Cyclical Variations of Waste Concentrations

It should be pointed out that all of the methods now available give average concentrations over a tidal cycle. In many estuaries where the tidal prism is small compared to the low water volume, the steady-state, or average pollution distribution, could be assumed, without serious error, to move back and forth with the average tidal excursion. In other estuaries there may be large variations in tidal excursion, and the tidal prism may be of the same order of magnitude as the low water volume. Critical conditions may occur at low tide, because of the smaller dilution volumes, or at high tide if a critical area is upstream from the waste outfall. A means of determining concentrations at any time during the tidal cycle is therefore desirable.

The motion and tidal amplitudes at any point in an estuary may be approximated by solution to the differential equation of motion either in closed form or in finite difference form. With the determination of velocities and

cross-sectional areas as a function of x and t , all that is lacking for a finite difference solution to the equation of continuity, for small time intervals, is the instantaneous diffusion coefficient (as a function of x and t). This diffusion coefficient is the only unknown variable which is responsible for our present inability to trace, analytically, the movement of a conservative pollutant in a well-mixed estuary. This diffusion coefficient and the continuity equation are examined in the following section.

EQUATION OF CONTINUITY AND TURBULENT DIFFUSION

The vector form of the equation of continuity for a conservative pollutant of concentration c is

$$\nabla \cdot c\rho \mathbf{W} + \frac{\partial c\rho}{\partial t} = 0$$

where \mathbf{W} is the vector velocity and ρ the mass density. Since for two vectors \bar{A} and \bar{B} ,

$$\nabla \cdot \bar{A}\bar{B} = \nabla \bar{A} \cdot \bar{B} + \bar{B} \cdot \nabla \bar{A}$$

the vector equation can be written

$$\nabla c\rho \cdot \mathbf{W} + c\rho \nabla \cdot \mathbf{W} + \frac{\partial c\rho}{\partial t} = 0$$

For an incompressible fluid,

$$\nabla \cdot \mathbf{W} = 0 ,$$

and the equation becomes,

$$\nabla c \cdot W + \frac{\partial c \rho}{\partial t} = 0$$

Expanding,

$$\rho \nabla c \cdot W + c \nabla \rho \cdot W + c \frac{\partial \rho}{\partial t} + \rho \frac{\partial c}{\partial t} = 0$$

the two middle terms, being very small compared to the end terms, may be neglected, leaving

$$\rho \nabla c \cdot W + \rho \frac{\partial c}{\partial t} = 0$$

Now substituting for W and c , the mean time average denoted by $(\bar{\quad})$ and a turbulent deviation from this average denoted by a prime ($'$), $W = \bar{W} + W'$ and $c = \bar{c} + c'$, the equation becomes

$$\rho \nabla (\bar{c} + c') \cdot (\bar{W} + W') + \rho \frac{\partial (\bar{c} + c')}{\partial t} = 0$$

expanding, taking mean time averages and noting that \bar{c}' and \bar{W}' equal 0

$$\nabla (\bar{c} \cdot \bar{W}) + \nabla (\overline{c' \cdot W'}) + \frac{\partial \bar{c}}{\partial t} = 0$$

The term $(\bar{c} \cdot \bar{W})$ is the pollutant transfer due to the mean advective flow. The term $(\overline{c' \cdot W'})$ is the transfer due to the turbulent fluctuations. Due to the lack of adequate turbulence theory the transfer is usually assumed

analogous to mass transfer by molecular diffusion. According to Fick's second law of mass transfer, molecular diffusion may be expressed in terms of a coefficient of diffusion and a mean concentration gradient. Then, since this transfer is in the direction of the decreasing gradient,

$$\overline{c'v'} = -D' \nabla c$$

where D' is the coefficient of turbulent diffusion; so that

$$\nabla(\bar{c} \cdot \bar{v}) + \nabla(-D' \nabla \bar{c}) + \frac{\partial \bar{c}}{\partial t} = 0$$

For the well-mixed estuary the only gradient and velocities assumed to exist are in the x direction. The one-dimensional equation reduces to

$$u \frac{\partial \bar{c}}{\partial x} + \bar{c} \frac{\partial u}{\partial x} - \frac{\partial}{\partial x} \left(D' \frac{\partial \bar{c}}{\partial x} \right) + \frac{\partial \bar{c}}{\partial t} = 0$$

Although decay, sinks, and sources may be easily included in the expression, in application it is often very difficult to assign numerical values.

Diffusion Coefficients

As with molecular diffusion the term D' is used to describe an effective mass transfer from regions of higher concentration to those of lower concentration. The analogy here is not very strong because, while the molecular diffusivity is known to be a well defined liquid property, the

eddy viscosity, or eddy diffusivity, is a function of the velocity gradient and a mean eddy size both of which vary with flow conditions and position within the flow.

Eddy Viscosity

According to Reynold's analogy, in turbulent flow generated by shear, the transfer of mass is analogous to the transfer of momentum. The turbulent diffusion coefficient therefore equals the eddy viscosity as can be seen from the following equations.

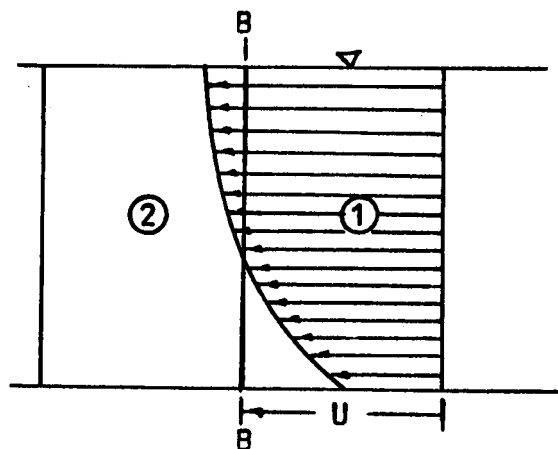
$$\epsilon = \frac{\tau}{\rho \frac{\partial U}{\partial y}} = \frac{m}{\frac{\partial c}{\partial y}}$$

Here, ϵ is the eddy viscosity, τ is the shear stress or momentum transfer rate per unit area in the y direction, and m is the mass transfer rate (of a solute of concentration c) in the y direction.

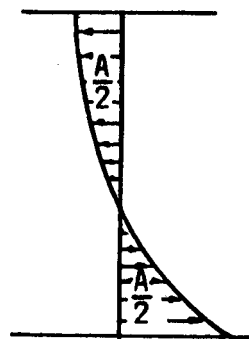
Longitudinal Diffusion Coefficient--Dispersion

In the direction of flow, it has been observed that the diffusion coefficient is several orders of magnitude greater than the eddy viscosity. This is due to the shear generated, vertical, velocity distribution. Here there is a distinction between transfer due to velocity fluctuations which are random, and that due to velocity deviations which are related to a vertical velocity gradient. The latter may be termed dispersion. Dispersion may be visualized by

referring to Figure 3.



Velocity Distribution.
Figure 3(a).



Velocity Deviations.
Figure 3(b).

FIGURE 3 Dispersion.

Section B-B of a wide rectangular channel moves downstream with a mean velocity U . The volume exchange per unit time, per unit width, between segments 1 and 2 is equal to the area A under the velocity deviation curve (Figure 3(b)). Letting the pollutant concentration of segment 1 be c_1 and that of segment 2 be c_2 , the net rate of pollutant transfer to segment 2 will be

$$\frac{A}{2} (c_1 - c_2)$$

It is seen that the pollutant transfer rate across section B-B due to dispersion, is directly proportional to the area under the velocity deviation curve and the extent of the difference in concentration between segments 1 and 2.

The longitudinal diffusion coefficient due to shear

generated dispersion (and, to a very small degree, to the eddy diffusion) was termed D_L by Taylor (28). On the assumption of the universal velocity distribution law for turbulent flow in pipes Taylor derived a theoretical expression for D_L . The expression, when applied to rivers and channels becomes:

$$D_L = \frac{14.28 (2g)^{\frac{1}{2}} UR}{C_c}$$

where:

C_c = Chezy coefficient

R = hydraulic radius

In real estuaries, the value of the turbulent diffusion coefficient is usually much larger than that calculated by Taylor's formula using estimated values of C_c . It is usually assumed that this difference is a result of bends, changes in cross-sectional area, and density gradients. These factors would definitely increase the vertical and lateral velocity gradients and the circulation.

The longitudinal diffusion coefficient which is the result of all factors which might be present in an estuary is designated D' and called the apparent diffusion coefficient. At present there is no known functional relationship between this coefficient and other factors such as river flow, tidal amplitude, bends, etc.

Field Determination of Average Diffusion Coefficients

Internal Tracers

The method proposed by Stommel (27) involved calculating the average diffusion coefficient from the observed distribution of a conservative property of the estuary (fresh water). This method is presented here because, at present, it appears to be the most used method for determining average diffusion coefficients in well-mixed estuaries. For the assumed critical period or period of interest in a given estuary, a time of observation is chosen when the tidal range and river flow are relatively constant. The estuary may then be considered to be in a semi-steady state, the average salinity at a point being constant from one tidal cycle to the next. The net flux of fresh water at any section is equal to the river flow Q . Since the net fresh water flux is equal to the sum of the advective and turbulent diffusion fluxes,

$$Q = Qf_n + D'_n A_n \frac{\partial f_n}{\partial x},$$

where f_n equals the concentration of river water as a decimal fraction at section n (refer to Figure 2).

Solving for D'_n in finite difference form,

$$D'_n = \frac{Q(1-f_n) \Delta x}{A_n(f_{n-1} - f_{n+1})} \quad \text{Equation 1}$$

The fresh water concentration f is determined from salinity observations, Q from river flow measurements, and A from actual field determination or from navigational charts.

The implicit assumption of no ground water inflow might lead to serious errors especially during the low flow period when the surface flow is smallest in terms of its percentage of total fresh water inflow.

External Tracers

Field tracer studies using external tracers may be necessary to determine diffusion coefficients in areas where either the salinity gradients are insufficient for application of Equation 1, or the mean advective flow is largely affected by ground water inflow (21). Generally, due to the expense of introducing a tracer, a non-steady state tracer distribution is studied and sampling is required for several tidal cycles, over the tracer field, in order to solve for the diffusion coefficients.

Recent Research--Average Diffusion Coefficients

In recent studies at Massachusetts Institute of Technology (5,9) an attempt has been made to relate the average apparent diffusion coefficient to the average rate of energy dissipation G , and the rate of gain of potential energy J . In the laboratory, an ideal estuary was constructed. Isotropic turbulence, generated by oscillating screens,

was substituted for tidally created turbulence and a uniform flow, representing the river flow, was superimposed on the turbulence. A salt solution was introduced at the lower end to create salinity gradients. (See Figure 4.) From the observed steady-state salinity distribution and the known advective velocity, values of the diffusion coefficient were calculated from the equation of continuity. These were related to G (power input of oscillating screens) and the rate of potential energy gain J (due to increasing density of the water as it moves downstream). (See Figure 5.)

In a rectangular channel with turbulence created by tidal motion, similar results were obtained. It was found, for a constant depth, that the diffusion coefficient, without density gradients is proportional to $G^{1/3}$. This was a verification of Kolmogoroff's similarity principal (19) which states that the diffusion coefficient, for isotropic turbulence, is proportional to $G^{1/3} L^{4/3}$ where L is a mean eddy size assumed here to be proportional to the depth. Then, including density gradients, the relationship shown in Figure 6 was obtained. The lower portion of the curve (G/J large) is the well-mixed condition and is rather poorly defined. The upper part of the curve is completely defined by the following expression (page 25).

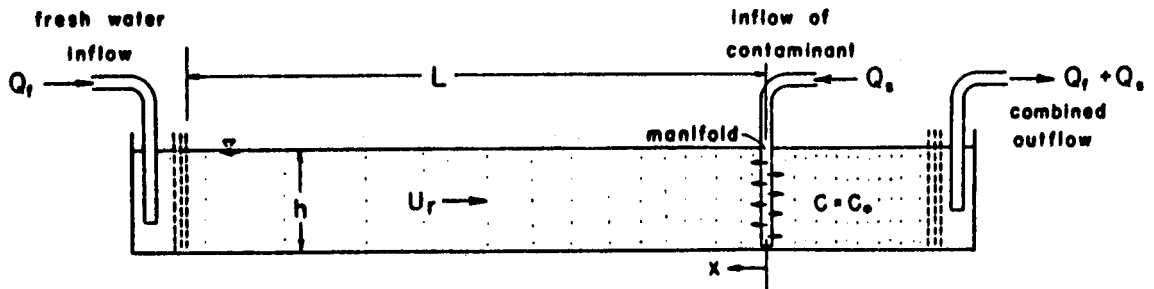


FIGURE 4 (5, p.L-f-5) Schematic Diagram of Idealized Estuary

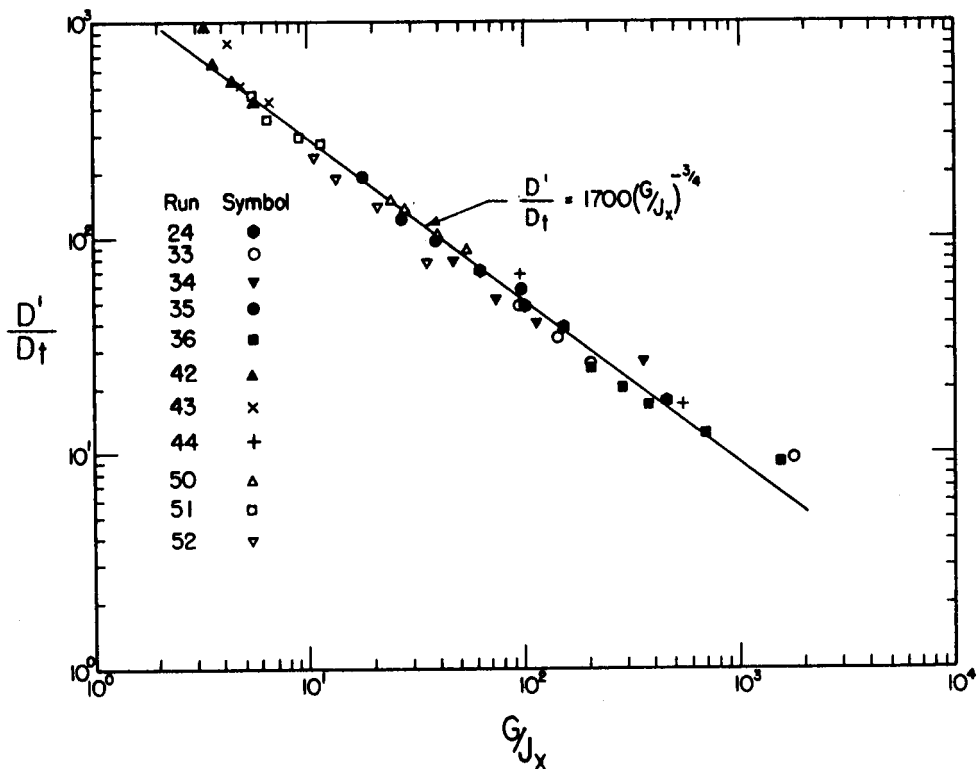


FIGURE 5 (5, p. L-f-7) Correlation of Ratio of Local Apparent Diffusion Coefficient D' (density gradients) and Turbulent Diffusion Coefficient D_t (no density gradients) with Local Stratification Parameter G/J_x .

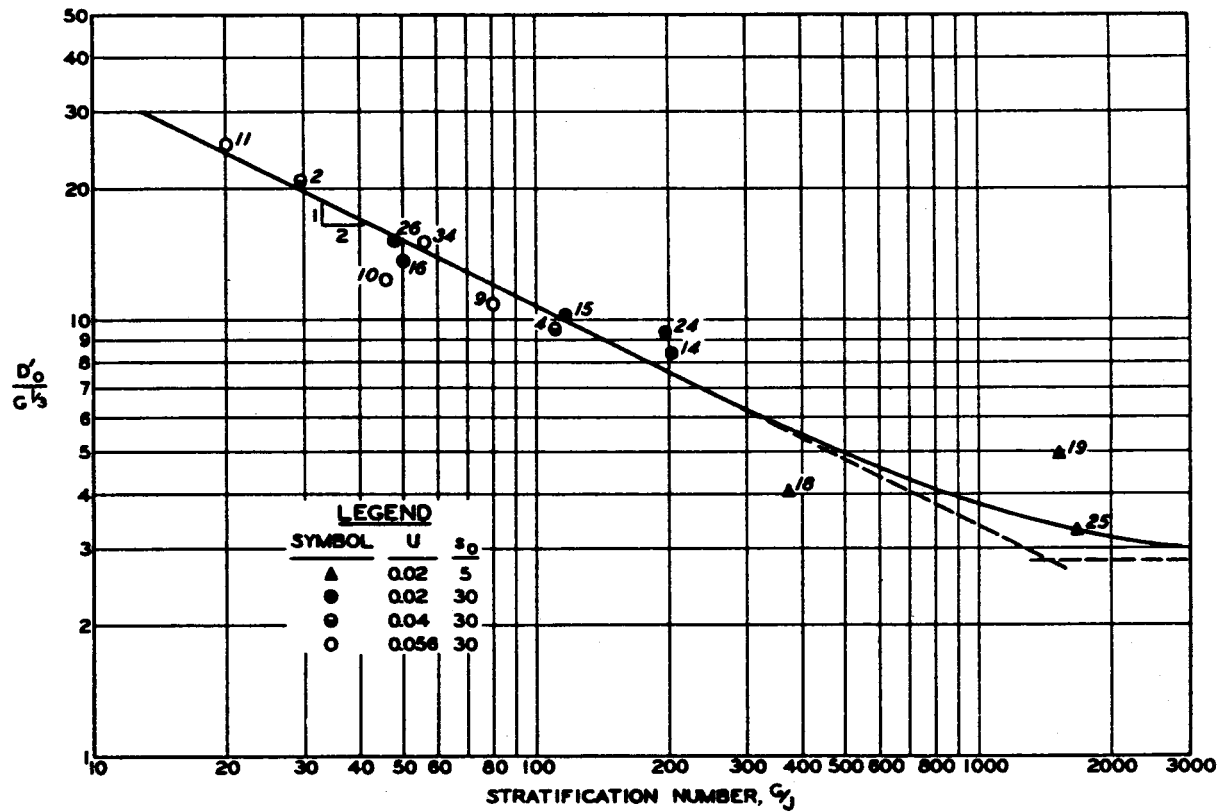


FIGURE 6 (9, p. M-f-5)
 Correlation of Apparent Diffusion Coefficient
 D'_0 with Stratification Number G/J . (Note: $D'_0 =$
 D' , S_0 = salinity (ppt) at channel mouth.)

$$\frac{D'}{G^{1/3} H^{4/3}} = 275 \left(\frac{J}{G} \right)^{+1/2}$$

The validity of the above expression is, unfortunately, restricted to the partially stratified range, and to the channel and depth for which it was obtained.

Since instantaneous values for rate of potential energy gain become negative on the flood tide, it is doubtful that a similar approach could be used for instantaneous diffusion coefficients. As noted previously, instantaneous diffusion coefficients are necessary for predicting the cyclical variation of waste concentrations. Determination of the instantaneous diffusion coefficient should also lead to a greater understanding of the important variables affecting the average diffusion coefficient. A dimensional approach for obtaining instantaneous diffusion coefficients (well-mixed conditions) is presented in the following section.

DIMENSIONAL ANALYSIS FOR INSTANTANEOUS DIFFUSION COEFFICIENTS

The variables considered in the dimensional analysis are: $U, \rho, \mu, g, D', H, k$ and x ,

where:

H = depth

μ = dynamic viscosity

g = acceleration of gravity

k = absolute roughness.

For this study the channel was sufficiently wide and the walls sufficiently smooth to eliminate the significance of the width. The hydraulic radius (R) and depth (H) were, therefore, approximately equivalent. Since the longitudinal dimension x for a straight rectangular channel has significance only with respect to the longitudinal density gradient, (assumed an important factor) it is replaced by the independent variable $\frac{\Delta \gamma}{\Delta x}$, where $\gamma = \rho g$.

The Pi terms obtained from the Buckingham Pi theorem, with U , D' , and H as the repeating variables are:

$$\begin{aligned}\pi_1 &= \frac{U^2}{Hg} = (N_F)^2 & \pi_2 &= \frac{U^2}{H^2 \frac{\Delta \gamma}{\Delta x}} \\ \pi_3 &= \frac{UH}{D'} & \pi_4 &= \frac{H}{k} \\ \pi_5 &= \frac{UH\rho}{\mu} = N_R\end{aligned}$$

where

N_F = Froude Number

N_R = Reynolds Number

In turbulent flow the effects of the Reynold's Number (π_5) are negligible. For relatively dense roughness patterns (grain-type, screens, etc.) the chezy coefficient C_c is a function solely of the relative roughness H/k (25, p. 148). Replacing π_2 by $\frac{\pi_2}{\pi_1}$ the following functional relationship is obtained

$$\frac{D'}{UH} = \phi \left[\frac{U^2}{Hg}, \frac{\Delta \gamma}{\Delta x} \frac{H}{\gamma}, C_c \right]$$

The purpose of the laboratory research was to determine the nature of the function ϕ .

LABORATORY EQUIPMENT AND PROCEDURE

Laboratory Equipment

The laboratory channel, constructed of plexiglass, was artificially roughened, by screens, on the bottom only. The channel was three feet wide, one foot deep, and 24 feet long. The samplers were vertical plastic tubes (1/8" I.D., 1/4" O.D.) with 1/16" holes drilled at 1" intervals. Eight pairs of these samplers were placed at two-foot intervals and connected to 125 ml. flasks for holding the samples. The flasks were in turn connected to a manifold leading to a vacuum reservoir and suction pump (Figure 7).

Laboratory Procedure

The laboratory technique was to obtain a steady turbulent flow, introduce a salt (NaCl) solution, well mixed, at the upper end of the channel and take samples on the trailing side of the salinity distribution curve. Here, a somewhat evenly sloping salinity distribution curve (Figure 8) allowed calculation of the diffusion coefficients. One set of eight samples (approximately 50 ml.) were withdrawn by operating a valve connected to the vacuum reservoir. Several seconds later, a second set of samples at the same

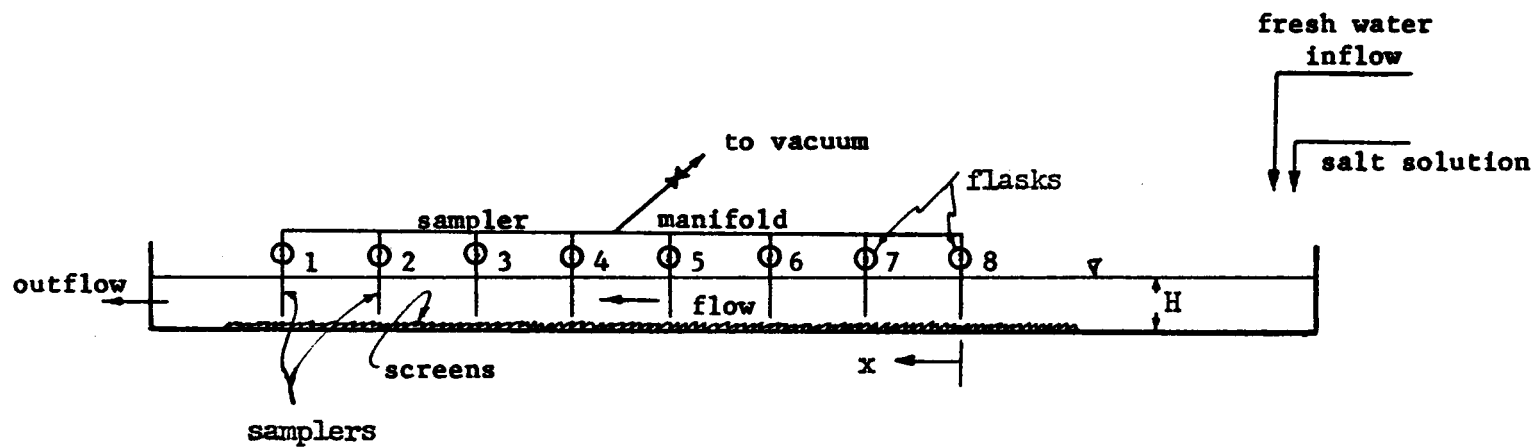


FIGURE 7
Schematic Diagram of Laboratory Channel and Apparatus.

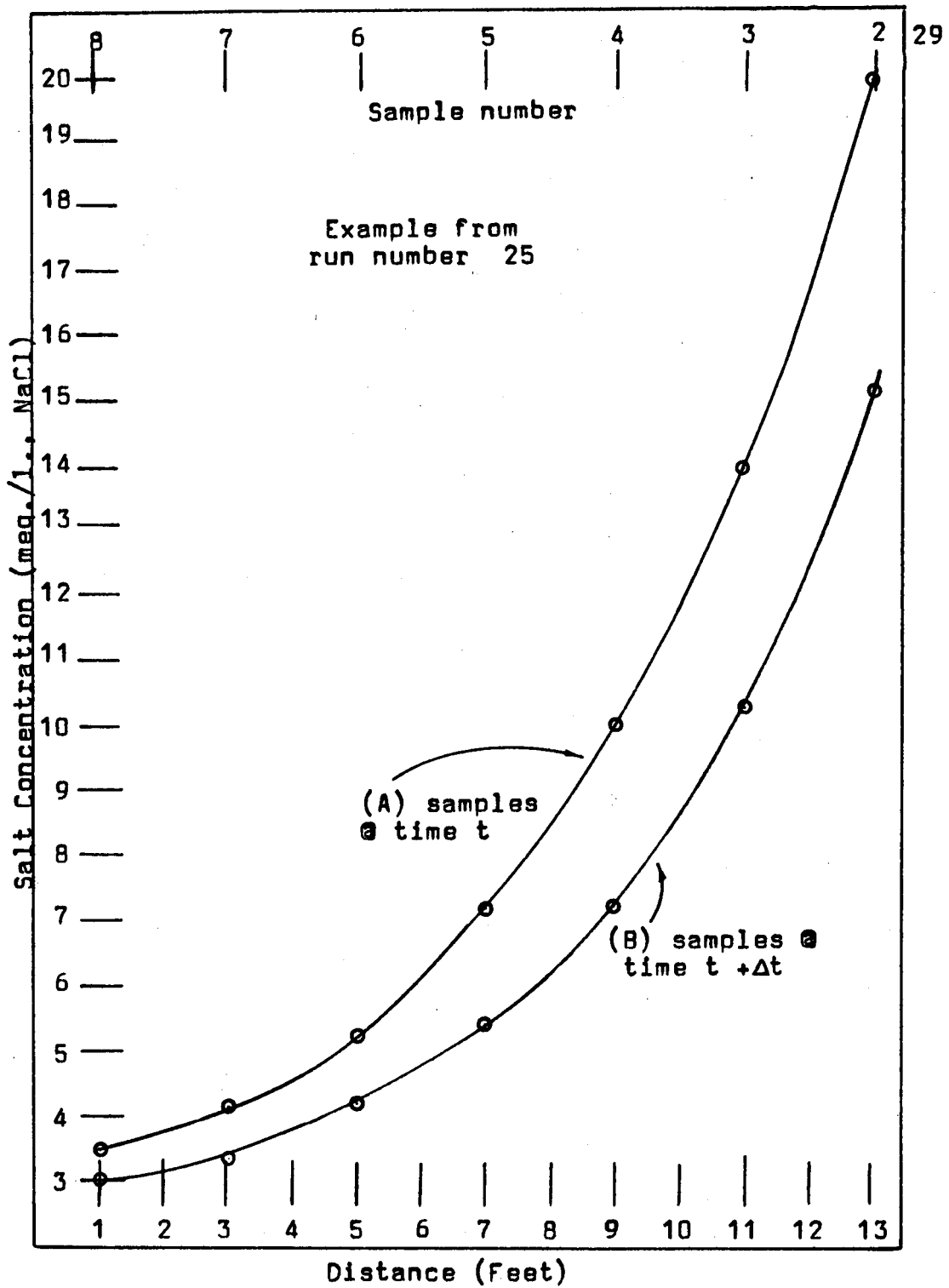
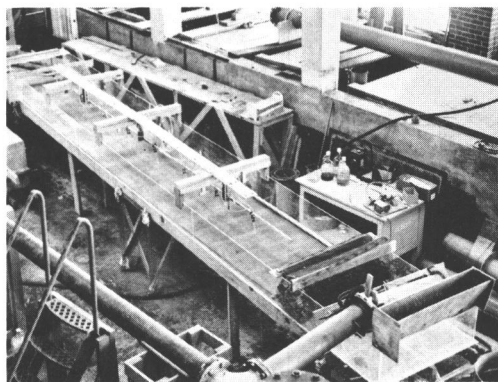
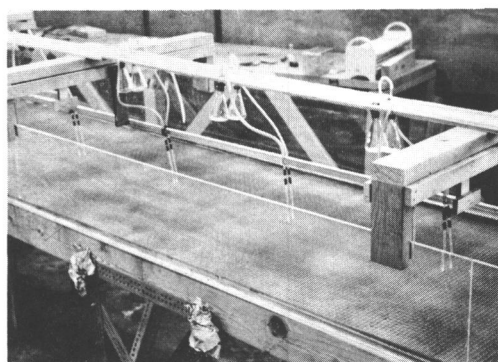


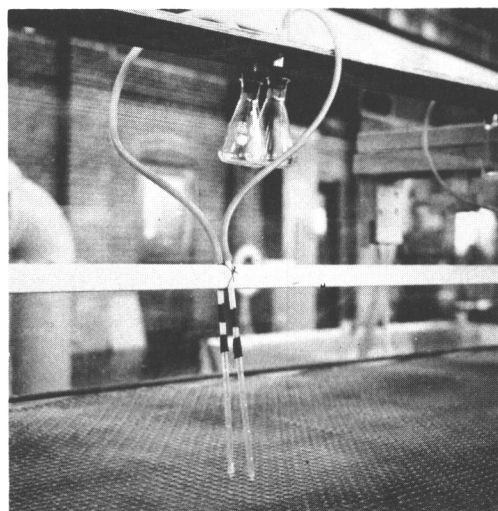
FIGURE 8 Concentration Vs. Distance



Over-all View



Close-up



Sampling Device

FIGURE A Photographs of Laboratory Channel and Apparatus.

locations were similarly withdrawn. Samples were analyzed for salt concentration by means of an electrical conductivity cell and Wheatstone bridge. Relationships for specific conductivity versus concentration (32) and concentration versus density (4) were obtained from the International Critical Tables. Smooth curves were drawn through the observed data and the appropriate conversion factors applied to obtain concentration versus distance curves (Figure 8). Diffusion coefficients were obtained from the one-dimensional equation of continuity for uniform, steady flow which is

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} = \frac{\partial}{\partial x} \left(D' \frac{\partial c}{\partial x} \right).$$

Substituting the following finite difference approximations:

$$\frac{\partial c}{\partial t} = \frac{\Delta c}{\Delta t}$$

and

$$\frac{\partial}{\partial x} \left(D' \frac{\partial c}{\partial x} \right) = \frac{1}{2\Delta x} \left[D'_{(n+1)} \left(\frac{\partial c}{\partial x} \right)_{(n+1)} - D'_{(n-1)} \left(\frac{\partial c}{\partial x} \right)_{(n-1)} \right],$$

then, solving for $D'_{(n+1)}$,

$$D'_{(n+1)} = \frac{2\Delta x \left[-\left(\frac{\Delta c}{\Delta t} \right)_{(n)} + u \left(\frac{\partial c}{\partial x} \right)_{(n)} \right] + D'_{(n-1)} \left(\frac{\partial c}{\partial x} \right)_{(n-1)}}{\left(\frac{\partial c}{\partial x} \right)_{(n+1)}}.$$

The slope of the curve (Figure 8, average of runs A and B)

at section n is substituted for $\frac{\partial c}{\partial x}(n)$. The calculations were begun at the trailing edge of the salinity distribution curve (near sample 8, Figure 8) where density gradients were small and the first $D'(n-1)$ could be calculated using Taylor's formula. Succeeding intervals were chosen so that $D'(n-1)$ could be taken as the previously calculated $D'(n+1)$.

Values of C_c were obtained from the Chezy formula

$$U = C_c (RS)^{\frac{1}{2}}$$

or

$$C_c = \frac{U}{(RS)^{\frac{1}{2}}}$$

where S = slope of energy grade line.

RESULTS AND CONCLUSIONS

Laboratory experiments to define the function ϕ revealed that only within a narrow range of depths and flows could consistent results be obtained. This was at least partially due to equipment deficiencies which did not allow proper mixing at high flows or adequate sampling at low flows. It was, therefore, not possible to determine the function ϕ . However, within the range of sampling, some interesting conclusions were indicated. Unfortunately, because of the difficulty in obtaining a smooth salinity distribution curve, the flood tide condition (density decreasing in the direction of flow) was not studied. For the ebb-condition Figure 9 defines the diffusion coefficient as a function

of U , H , and $\frac{\Delta \gamma}{\Delta x}$ with a constant value of N_F and C_c . (C_c was not quite constant but varied only about five percent.) For a wide rectangular channel (i.e. depth \cong hydraulic radius), N_F and C_c are conditions for dynamic and geometric similarity. There were two depths for which somewhat consistent results were obtained. One can be considered a model study of the other with a distorted vertical scale relative to the longitudinal horizontal scale (defined by longitudinal density gradients). The relationship between the dimensionless parameters for one depth was very close to that obtained for the second (Figure 9). It may be concluded that the diffusion coefficient (considering only bottom shear and longitudinal density gradients) can be determined by distorted model studies. The type of relationship shown in Figure 9 provides the correspondence between model and prototype. If the relationship of Figure 9 holds for greater depths, the magnitude of the density effects on the apparent diffusion coefficient in real estuaries can be determined.

Example:

Considering a wide rectangular channel (prototype estuary) under the following conditions:

well-mixed
 ebb-flow (density increasing in the direction
 of flow)
 Depth = 10 feet
 $C_c = 80 \text{ (ft.)}^{\frac{1}{2}}/\text{sec.}$
 Velocity = 1.4 feet per second
 Salinity gradient = one percent in five miles

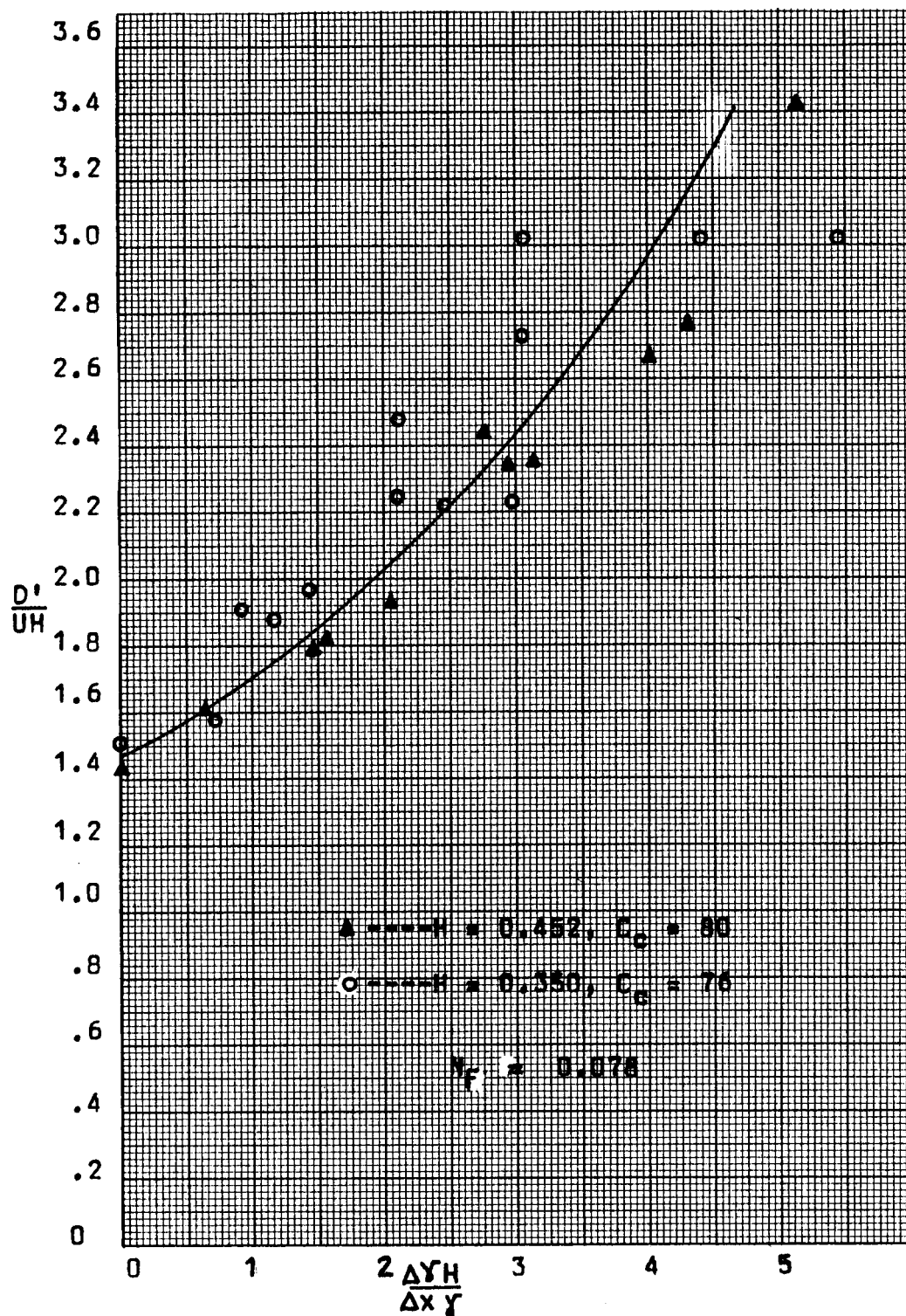


FIGURE 9
Relationship between Dimensionless Parameters.

The results of the model study (Figure 9) predict that the effects of this longitudinal density gradient would increase the diffusion coefficient from 20 ft.²/sec. predicted by the Taylor formula to 22 ft.²/sec.

While this is a significant increase over the coefficient obtained by the Taylor formula it is very small when compared to the overall or apparent diffusion coefficient which would be much larger in a real estuary under similar conditions (2). It may be concluded, for a relatively well-mixed condition with $N_f = 0.078$, that the longitudinal density effects on the instantaneous diffusion coefficient are relatively small. This is indicated for average diffusion coefficients by Ippen (9) for well-mixed conditions (i.e. large values of G/J). (See Figure 6.) The longitudinal density effects may well have significant effects on the instantaneous diffusion coefficients when the flow rate is less (near slack water).

It is assumed that vertical salinity stratification is negligible in a well-mixed estuary. However, there is always some stratification in any real estuary and it cannot be avoided in the laboratory. Since this is an important factor in the stratified range, it may be one cause for the data spread (Figure 9).

In this study the rate of salt transfer due to turbulent diffusion (the result primarily of dispersion and

longitudinal density gradients) was, on the average, approximately an order of magnitude less than that due to advection. Since the time rate of change in concentration at any section was a very significant term the non steady-state solution to the continuity equation was necessary. Obtaining data for a non steady-state solution presented numerous unexpected difficulties which are discussed below with some suggestions for further work:

1. Since vertical mixing was insufficient to eliminate vertical density gradients caused by shear, an improvement in the equipment was required to allow sampling over the entire depth. Additional improvement would allow distinct sampling with depth to allow determination of the vertical salinity gradient.
2. A satisfactory solution to the mixing problem was not obtained. A gravel barrier to still turbulence created at the entrance and even out salinity fluctuations was adequate only for a small range of depths and flows. A larger mixing chamber is required to allow a larger range of depths and flows and still maintain adequate mixing.
3. Samples should be large enough to eliminate the significance of concentration fluctuations, but not so large as to invalidate the finite difference approximation Δt . With adequate mixing, samples could be smaller.
4. Sampling at shorter distance intervals (about one foot) would allow a finite difference computer solution to the

continuity equation for the diffusion coefficients (assuming a smooth salinity distribution curve is obtained). This would increase the laboratory work but considerably decrease the data reduction and the necessary human judgments (i.e. drawing curves and determining slopes).

FUTURE STUDIES

Before generalizing about the density effects on the diffusion coefficient in a well-mixed real estuary, the type of result shown in Figure 9 should be verified over a larger range of depths and flows. Flood conditions (longitudinal density decreasing in the direction of flow) should also be studied. Further studies in other areas are also needed. The magnitude of the diffusion coefficient in well-mixed real estuaries indicates that factors other than the longitudinal density gradients have very significant effects on the apparent diffusion coefficients. Bends and changes in cross-sectional area are known to have important effects, but these require quantitative study. Further study, directed towards either of these factors, should provide a better understanding of the diffusion process and another step towards a rational formulation for the turbulent diffusion coefficient.

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APPENDIX

DATA: Runs Number 1 - 6

Flow Rate 39.0 seconds per 1000 lb.

Depth 0.452 ft.

Electrical Resistance (Ohms) At 18 Degrees Centigrade

Cell Constant = 1.11

Run #	Time Interval Between A and B.	Sample # -	1	2	3	4	5	6	7	8
1	7.3	A	730	1190	2000	2980	5000	6570		
		B	1040	1680	2580	4760	6560	7880		
2	8.5	A	435	590	780	1320	2120	3440	4910	
		B	635	835	1310	2000	3400	5040	6260	
3	8.1	A	415	520	725	1120	1790	3000	4030	
		B	553	730	1120	1640	2710	3970	5120	
4	9.3	A	750	773	1430	2000	3000	4250	5100	5560
		B	1200	1000	1960	2900	3780	5070	5460	5940
5	8.7	A	921	920	1270	1730	2550	3450	4020	4270
		B	1120	1010	1700	2420	3390	3910	3960	4270
6	7.7	A	817	807	1270	1840	2760	3250	3670	3880
		B	958	922	1670	2300	3300	3670	3680	4030

DATA: Runs Number 21 - 25

Flow Rate 58.5 seconds per 1000 lb.

Depth 0.350 ft.

Electrical Resistance (Ohms) At 18 Degrees Centigrade

Cell Constant = 1.11

Run #	Time Interval Between A and B	Sample # -	1	2	3	4	5	6	7	8
21	9.8	A	290	290	388	518	1120	1600	2330	3470
		B	358	460	590	1010	1580	2310	3020	4620
22	11.7	A	333	348	429	683	1410	1810	2780	3450
		B	443	521	793	1110	2120	2360	3130	3780
23	9.3	A	267	326	420	621	1060	1400	1830	2770
		B	400	349	560	800	1490	1890	2840	3480
24	13.0	A	442	470	704	1000	1720	2070	2750	3180
		B	560	658	1150	1580	2110	2550	2930	3260
25	10.5	A	404	496	720	1010	1430	1820	2410	2820
		B	542	623	1000	1370	1890	2330	3030	3370