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This is an interactive, self explanatory computer program on real roots of polynomials with real coefficients. This program called POLYROOT has the double feature of tutoring and problem solving. It allows the user to enter on one line any polynomial with real coefficients. Then, the user can find :

1. characteristics of the polynomial, such as degree, number of terms, leading and constant terms.

2. the number of possible positive and negative real roots using Descartes' Rule of signs.

3. all the possible rational roots using the Rational Root Theorem.

4. all the rational roots,and their multiplicities using Synthetic Division.

5. approximations for real roots using the Bisection Method and the Newton's Method.

6. approximations for real roots using the graph of the polynomial over any given interval.

This compiled program is written in BASIC language and is supported by all IBM compatible machines. POLYROOT contains four units which can be run in any order.

This program will support the study of polynomials in Precalculus and Calculus courses. POLYROOT is also useful for Mathematics and Engineering students solving linear differential equations using methods such as finding the zeros of the characteristic equation or forming partial fractions.

REAL ROOTS OF POLYNOMIALS : AN INTERACTIVE COMPUTER APPROACH

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I dedicate this thesis to my mother and father.

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I. INTRODUCTION

1.1 Computer Based Instruction

We have heard much about computers being the technology of the future. Computers, however, are the technology of the present. Only misinformation may create irrational fear of the machinery, and may be responsible for preventing the effective utilization of computers in educational activities. The challenge is clear : to make the educational use of computers maximally beneficial to all concerned.

Because of the decreasing quantity and quality of available human resources, the institutionalization of the computer-based instruction can :

1. offer a uniformly high quality in education available on large scale.
2. reduce reliance on trained instructors.
3. provide rapid update of instructional material.
4. manage instruction by setting objectives, analyzing results, and reporting.

5. provide hands-on,performance-oriented instruction.

6. permit individualization of instruction.

7. augment student involvement and learning.

A computer is not only a machine doing arithmetic rapidly and accurately which is one of its important features, but it can be a valuable tool in teaching mathematics and interacting with the user.

It offers two qualities every student would like to have from his or her teacher :

1. patience : The student sets his or her own pace.

2. persistence : The student can go over the same subject many times until assimilated.

One word of caution: one must be sensitive to potential misuses and abuses of the computer. The ease of access to the machine encourages trial and error methods rather than systematic analysis and may lead to lazy analysis. The computer based learning element should enhance the role of the teacher and should be seen as a complementary component of the learning process offering, not a replacement for the teacher.

1.2 Rationale of POLYROOT

POLYROOT is a program on finding the real roots of polynomials with real coefficients. This subject matter was selected because from high school through higher education, polynomials with their special properties and applications are often studied. One of the principal task is to find roots of polynomial equations. This task can be tedious and time consuming. Finding rational roots involves a considerable number of repetitive, simple, but dull operations, especially if the coefficients of the leading and constant terms are large. Thus the computer is far better suited than humans for performing this job. In Numerical Analysis, another important area of computer applications,POLYROOT not only gives the real roots quickly and accurately to the precision sought, but also provides the user with graphics that make the learning effective, convincing, and pleasant.

1.3 Organization

In chapter II, 'Literature Review', many programs from the present software market dealing with polynomials are listed and discussed. 'Procedure and Documentation of POLYROOT',chapter III, describes how to run the program, the hardware required, the programming language used, and the performance of POLYROOT and its diverse features. In chapter IV, 'Overview of POLYROOT', the functions of the different units of POLYROOT are discussed and some examples of how the mathematics works are demonstrated. Also included are some frames from displays of the computer.

The last chapter deals with the importance of the theory of polynomials and their applications, discusses the rationale for finding the roots of polynomials through the use of computers, and describes places in the curriculum where POLYROOT could be implemented or used. An appendix contains a set of challenging exercises that serve to motivate the interest of the user and to demonstrate the power and versality of POLYROOT.

II. LITERATURE REVIEW

2.1 Educational computer programs :quality, types

Even though possibilities for classroom use of preprogrammed computer software are boundless, our knowledge base for developping good educational computer programs in Mathematics is still in its infancy. We rely primarily on our eager desire to get something started and on our intuitive notions of 'good' teaching. Several criteria enter into determining the quality of a software product. A computer program should be defined by its validity with respect to its stated specifications,its efficiency relative to the complexity of its task, and its clarity of both its structure and its function. A good software package should protect the users from their own errors as well as protect the software from user error. After gaining operational experience with a program, the users often desire additional features. The software must be designed so that new features and enhancements to existing features can be accomplished with a minimum of programming.

In the present market we can find different types of programs. The ones for drill and practice help the student rehearse different elements of thinking. Tutorial programs try to simulate a teacher's live instruction in developing a new skill or concept. Simulation programs model some process or system to make the student gain insights into some real life phenomenon. The most motivating programs for students and teachers alike are the ones involving games. They use audio and visual effects to present the game situation and reward success. They incorporate fantasy elements and can time the player's responses and calculate scores.

2.2 Present market: a summary

To date there is still a shortage of good software and more research is desperately needed. However the production is increasing rapidly because computer hardware is becoming more generally available due to its lower cost. By the end of the decade, the educational software market may be as large as 1.5 million dollars (Karen Pipper Mc Graw,1985). However, among the current software programs already available, very few are interactive and related to the field of polynomials.

MuMath (David Stoutemeyer and Albert Rich,1979) is a powerful symbolic math processing system which performs a host of mathematical operations from simple algebra to complicated differentiation, from high precision arithmetic to taylor series expansions. All processing can be done in symbolic form so that mathematical functions can be manipulated, simplified, integrated, and differentiated algebraically without requiring numerical values. The modular construction of the program allows preservation of adequate working space by only loading the parts of MuMath needed for the operations at hand. In addition to all the built-in functions, MuMath can also define new functions by using the 'muSimp' language that MuMath itself is written in. The documentation is excellent but not for the novice mathematician. No attempt was made to provide any tutorial in the mathematics used. This program is supported by Microsoft corporation and runs on Apple II, II+, IIe, and IBM personnel computers.

An entirely new concept in personal computing software, the 'TK! Solver' (Software Arts, Inc,1983), allows one to write a series of equations which describe the problem much like a mainframe financial modeling package does. Like a spreadsheet, it is capable of solving tabular problems. Unlike a spreadsheet, it is capable of solving equations 'backwards' without restructuring the model. There are eight sheets which are included in the program, each containing different information. The Rule Sheet contains the equations that define the model and the Variable Sheet contains all the variables with their characteristics, such as units and values. The Unit Sheet defines conversions between units of measurements and the Global Sheet sets limits and required constraints for the program, the solvers, and the user's printer. In addition, the lists of values in the model are included in the List Sheet and the functions that have been created are contained in the User Function Sheet. The information that is needed to produce a plot of values is featured in the Plot Sheet while all the information needed to produce a table of values is contained in the Table Sheet. TK! Solver has certain limitations. The mathematical model must be algebraic. one cannot do matrix operations or differential/integral equations. This program runs on Apple IIe, Apple Macintosh, and IBM Personal Computer.

A good graphing program is Arbplot (Harper and Row and Conduit,1982). This program is actually a teaching tool designed for use in colleges and advanced High School mathematics courses. It provides visual representation of geometric processes and concepts in Analytic Geometry, Differential and Integral Calculus. Arbplot is easy to use because it is completely menu driven. The package is broken down into three groups of programs. The first group plots curves from continuous and discontinuous functions defined in several ways. The second illustrates concepts in Calculus such as limits, derivatives, and integration. The third creates and runs 'slide shows', an aid to presenting preplanned sequences of graphic illustrations. Arbplot which runs on Apple II computers also finds the roots of an equation using the Bisection, Newton, and Regula Falsi methods.

Hewlett-Packard Company has a program called Solution to F(X) on an Interval. Given a first guess, this program will search for a real root of the equation F(X)=0, where the user defines the continuous real-valued function F(X) starting at line 5000. Roots are found by marching along at a given step size until a change of sign is encountered. A modified secant method is then used to determine the zero of the function. The user is required to specify the error tolerances for the root and for the function evaluation, as well as the step size and the maximum number of steps and iterations allowed.

Mathematics Series (Spectrum Software, 1982) is a set of programs. One of them is numerical analysis which graphs any two variable equations that the user writes using standard Basic arithmetic functions. It finds roots, maxima and minima, and will plot the integral and derivative of the function. This program includes standard data base management routines for editing, and printer reports of both data and results.

The'Book of IBM Software 1984' did not list any program related to polynomials. However in the 'New 4 th

Edition IBM PC Expansian and Software Guide' (1984) there is a program called 'Roots' which iteratively seeks all the roots simultaneously. The convergence is cubic (and therefore rapid), and stable, requiring no explicit initial guesses from the user. Accuracy is usually within two digits of the precision of the BASIC employed. In addition to displaying the calculated solutions, 'ROOTS' also shows the results of substituting those values into the original polynomial. Any order polynomial may be treated. This program is not an interactive one and has no graphing or tutoring.

Another program calculating the roots of polynomials is written by Hewlett-Packard Company and is called 'Roots of polynomials'. This one uses Barstow's Method. the user must provide the order of the polynomial and its coefficients in order from left to right. The roots of some forms of polynomials cannot be determined by this program. execution time for polynomials of high order may be excessive since many iterations may be required.

For special cases of polynomials, 'Cubic Equation Solver'(Albert E. Hayes,Jr) is a program that solves cubic equations of the form $X^3 + AX^2 + BX + C = 0$
It provides both real and imaginary roots.

The 'Quadratic Equation Solver' written by Hewlett

Packard Co. takes care of all the cases of quadratic equations and provides a graphic representation of the function and its roots.

For the producer of educational programs, the availability of electronic spreadsheets allows an easy implementation of algorithms that are recursive, iterative, or suitable for tabular format.

Data Pro Directory of Microcomputer Software(volume 1 1985) lists many electronic spreadsheets. Among them 'Smart Spreadsheet With Graphics'(Innovative Software,Inc.) combines spreadsheet, graphics and information management capabilities. It supports a matrix of 999 columns by 999 rows and handles up to 80 characters/cell (values and text), and up to 1920 characters/cell(formulas). Multiple windows can be opened on the screen, allowing up to 32 spreadsheets or portions of a larger spreadsheet to be viewed at a time. The Smart Spreadsheet's graphics capabilities include 1, 2, and 3-D bar charts, line, point, scatter graphs, and histograms. A slide show feature for presentations is also provided along with 8087 chip floating-point processor conversion.

The 'Directory of IBM volume 1 number 2' (1985) lists a program called Private Tutor 2.0 which could be a very useful tool to the producer. It is a complete instructional system. It includes four programs on a single diskette.

'Author Program' lets the user write his own lessons on any topic for any age group. 'Preparer Program' takes files from the Author Program and creates code that can be read by the Presenter Program. 'Presenter Program' lets one take courses authored with Private Tutor. The last one, 'Reporter Program', lets the teacher keep student records, list students, and print course completion certificates.

III. PROCEDURE AND DOCUMENTATION OF POLYROOT

3.1 Suporting Hardware And Software

POLYROOT runs on all IBM compatible computers, with monochrome, graphic, or color monitors. It requires 64 KB of memory and one double sided disc drive. However the frame relating to numerical analysis cannot be run on the monochrome display because of the graphics involved. Since the program is written in BASIC and is compiled, one needs to store BASRUN (IBM software) in the same disc of POLYROOT. In case one wants to dump the graphic display into the printer, GRAPHICS, an IBM software, should be run. The IBM personal computer was chosen instead of the APPLE machine, because the former offers not only a better Basic Editor but also the Personnel Editor which allows one to write programs with ease, especially large ones. The Basic Compiler, another plus for this machine, lets one run programs a lot faster. Also, the graphic mode of the IBM PC has a higher resolution (640 X 200) and allows programers to write anywhere on the graphic screen itself. This is very helpful for tutoring.

3.2 Running POLYROOT

This program is self explanatory and needs no guide book or any knowledge of the machine on the part of the user in order to be run. This allows easy access for a large group of people, but one should follow all the directions displayed at the bottom of the screen to run the program properly. Otherwise either information is provided or a sound of a bell is heard to remind the user to be alert. In case of an error, no major disaster happens, as the computer waits for a better move. At this point the user should read what is on the screen and act accordingly (Fig. 2, Pg. 16).

After booting the system, type POLYROOT and press RETURN key. When you see the display of the title press any key to continue.The main menu then appears (Fig. 1,Pg. 15). From this level on, the user is on his own. When entering a polynomial, after typing X (the only variable allowed), the cursor jumps automatically one line up to let you enter your exponent. If the exponent is a single digit, press 'space bar' to go one line down . If the exponent is two digits, as soon as the second digit is typed the cursor will automatically move down. A space of a certain length is reserved for answers. By holding the shift key and pressing 'PrtSc', the screen can be dumped into the printer while the program is running. To get a hard copy of the curves displayed, one should run GRAPHICS (IBM software) before POLYROOT.

M A I N   M E N U

1 GETTING FAMILIAR WITH POLYNOMIALS

2 FINDING RATIONAL ROOTS

3 BISECTION METHOD

4 NEWTON'S METHOD

5 QUIT

SELECT THE TOPIC BY PRESSING THE NUMBER IN FRONT OF YOUR CHOICE

Fig. 1 Main menu

H O M E W O R K

$$P(X) = -234X^{12} + 91X^{11} + 562X^{9} - 45X^{8} + 3074X^{5} - 348X^{4} + 5602X^{3} - 567X^{2}$$

ENTER YOUR POLYNOMIAL WHOSE RATIONAL ROOTS ARE TO BE FOUND BY ANSWERING THE QUESTIONS IN THE BOX BELOW. KEEP IN MIND THAT X^n MEANS X TO THE POWER n. YOU HAVE ONLY ONE LINE TO WRITE YOUR POLYNOMIAL.

TOO LONG.PRESS 1 TO ENTER SHORTER TERM,OR SPACE BAR TO ENTER CONSTANT TERM|

Fig. 2 Prompt following a user's error

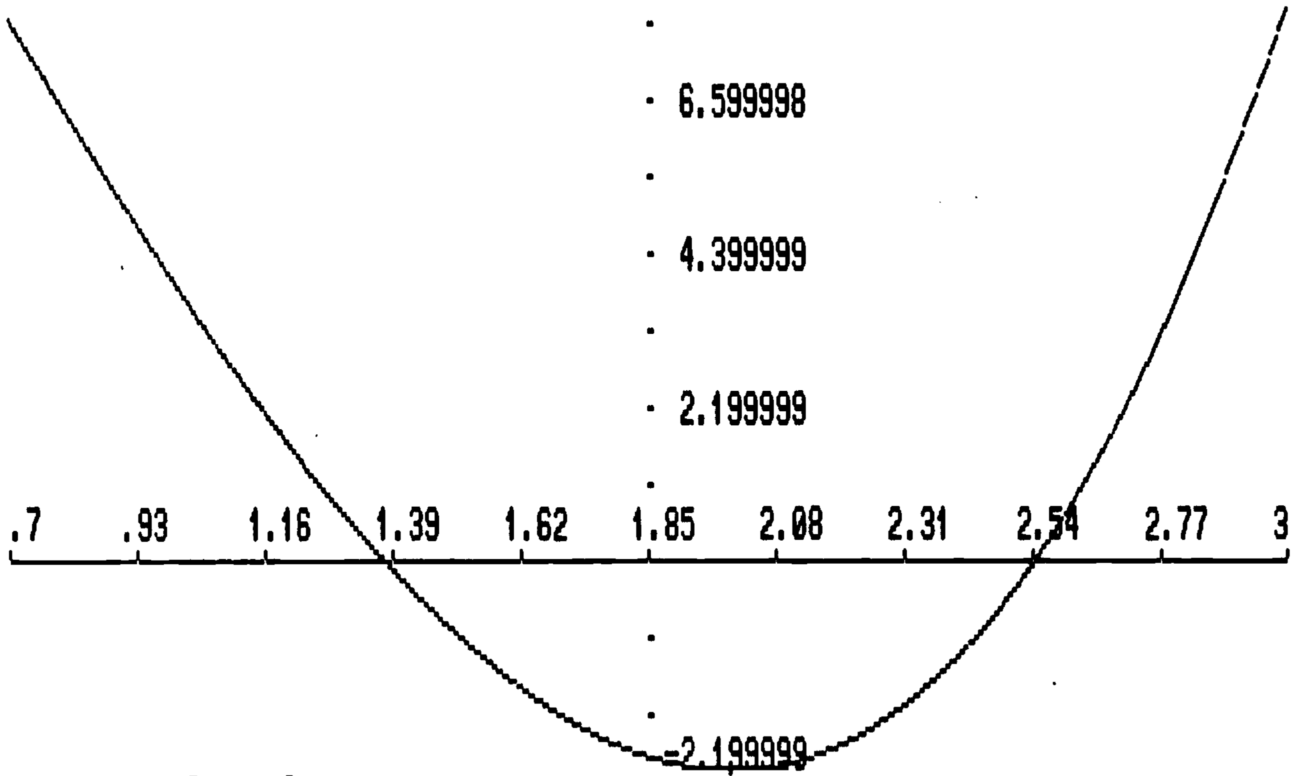
Attempt to enter a term longer than the space allocated. POLYROOT does not accept it and comes up with an appropriate prompt.

3.3 Features : capabilities, limits, and efficiency

This program is the combination of four relatively small units compiled and chained together. This method allows one to load the program one part at a time, and therefore assures a faster run, and maximum use of the memory space of the machine. A special effort was made in this program to give to the user the ability to move from one part of the program to another as quickly as possible without going all the way through a unit. This feature makes the program flexible when following any arbitrarily complicated path through the material, and is especially helpful if the user errs. All the different features of the IBM (highlight, blinking, inversing, sound) were used for pedagogical purpose, and to enhance the learning process. Questions, remarks, and prompts will always appear boxed in the bottom of the screen. The user can thus quickly learn where to look for the next step. For answers, the user is limited by a space of a determined length and by a range of specified inputs depending on the nature of the question. For example if the input is an integer number, then only +, -, and digits from 0 through 9 are allowed and only in the box assigned. In fact the user does not have the control of the cursor. This was decided to prevent the user from destroying the screen, to prevent extravagant inputs, to avoid careless errors, and to exclude data values which will cause the program to fail. During any homework session, all information on a polynomial is recorded and ordered on the screen so that one can dump it into the printer and get a hard copy of the solution.

Even with the screen limitation, polynomials are always displayed as in any mathematical book, with *,^,/ symbols unnecessary. In the first unit which deals with the terminology of polynomials, the user literally types his polynomial exactly as he will write it on a piece of paper. Two lines are reserved for this purpose, one for exponents and the other for coefficients. The program moves the cursor up or down depending on the input and takes care of,at the same time,the spacing and the neatness of the writing prompting directions when necessary. Coefficients are real numbers and allowed five spaces (0 - 99999) without the sign. Exponents which are positive integers may take two spaces (0 - 99). Computations of rational roots are done with double precision.

When graphing polynomials the user is asked to enter a closed domain interval. The program computes the maximum and the minimum function values, and automatically scales the function. Even if the domain interval does not contain zero, the Y axis in the form of a dotted line is still shown in the graph to let the user determine the value of



$$P(X) = +\,2\,X^{3} - 3\,X^{2} - 12\,X + 17$$

CAN YOU GIVE 2 VALUES OF X AT WHICH P(X) HAS OPPOSITE SIGNS, Y OR N ?

Fig. 3 Virtual Y axis

The domain interval does not contain 0 but a virtual Y axis is shown.

the polynomial at different points (Fig.3,Pg. 19). The sketch of the polynomial in any interval is very helpful for the user to find a subinterval for the Bisection Method and a first guess for the Newton Method. The high resolution mode was used for this part instead of the medium resolution even though the former does not offer the possibility of color, but it does give a more precise graphics, and thus is more appropriate for finding the zeros of polynomials graphically.

The graph of any polynomial is computed at 640 points since the whole width of the screen is used, and a high precision of the picture is sought. As one can imagine this involves for each point a very large number of operations, that is, n additions and 2n-1 multiplications for an n degree polynomial if we save the successive powers of X. To speed up the computation and therefore the graphing, an efficient method, the Horner's Method, was chosen. In this method P(X) is written most simply as :

$$P(X) = a_0 + X(a_1 + X(a_2 + \ldots + X(a_{n-1} + a_n X) \ldots)$$

In this form there are n additions and n multiplications, a considerable savings. The use of this method jointly with the compilation of the program makes the graphing quite fast. All computations in this part are done with a single precision.

IV. OVERVIEW OF POLYROOT

The main menu of POLYROOT (Fig. 1,Pg. 15) consists of four independent units that can be run in any order. All units have a tutorial that explains the mathematical concepts, and a homework part where the user enters his own polynomial to find the roots by applying the method studied. The first two units have, in addition, a test part where the user can get a better grasp of the topic in question. At any step of the test or the homework, help can be provided by branching to the coresponding tutorial part without loss of the work achieved at the call.

4.1 Getting familiar with polynomials

This unit has three parts.The first one (Fig.4,Pg.22) presents two pages of definitions and terminology used throughout the program (Figures 5 and 6). The test (Figures 7 and 8) helps the user assimilate what has been learned previously. $P(X) = -4X^5 + 7/3X^2 - 21 + 16X^7 + .51X$ is given and questions such as degree, number of terms, coefficients of leading and constant terms are asked and answered. In the homework part, the user follows special directions (Fig. 9,Pg. 27) to enter any polynomial with real coefficients in any order, then interacts with the

G E T T I N G   F A M I L I A R   W I T H   P O L Y N O M I A L S

1 DEFINITIONS

2 TEST

3 HOMEWORK

4 MAIN MENU

5 QUIT

SELECT THE TOPIC BY PRESSING THE NUMBER IN FRONT OF YOUR CHOICE

Fig. 4 Sub-menu I

Menu of first unit of POLYROOT

D E F I N I T O N S

A POLYNOMIAL FUNCTION IN X IS AN EXPRESSION OF THE FORM :

$$P(X) = a_n X^n + a_{n-1} X^{n-1} + \ldots + a_2 X^2 + a_1 X + a_0$$

a_n IS CALLED THE COEFFICIENT OF X^n . IN THIS PROGRAM WE RESTRICT OUR STUDY TO REAL COEFFICIENTS ONLY .

$a_n X^n$ IS CALLED A TERM . TERMS WILL ALWAYS BE SEPARATED FROM ONE ANOTHER BY + OR − SIGN.

a_0 IS THE CONSTANT TERM

PRESS 'SPACE BAR' TO CONTINUE OR 'M' TO GO BACK TO THE MENU

Fig. 5 Definitions I

First part of definitions

D E F I N I T O N S

THE TERM WHICH CONTAIN THE HIGHEST POWER OF X IS CALLED THE LEADING TERM

THE DEGREE OF THE POLYNOMIAL IS DEFINED AS THE HIGHEST POWER OF X , AND IS AN INTEGER GREATER THAN OR EQUAL TO ZERO .

A ROOT OR ZERO OF A POLYNOMIAL IS ANY COMPLEX NUMBER C (REAL OR NON REAL) SUCH THAT P(C) = 0 .

A POLYNOMIAL OF DEGREE n HAS EXACTLY n ROOTS .

THE n ROOTS ARE NOT NECESSARLY ALL DIFFERENT .

END OF THIS UNIT.PRESS 1 TO SEE IT AGAIN.PRESS M TO GO BACK TO THE MENU.

Fig. 6 Definitions II
Second part of definitions

T E S T

$$P(X) = -\ 4\ X^{5} + 7/3\ X^{2} - 21 + 16\ X^{7} + .51\ X$$

IT HAS DEGREE 7 .

IT HAS 5 TERMS .

+ 16 IS THE COEFFICIENT OF THE LEADING TERM .

WHAT IS THE CONSTANT TERM ?

Fig. 7 Test I in progress

Right answers recorded and last question is asked.

T E S T

$$P(X) = -4X^{5} + 7/3\,X^{2} - 21 + 16X^{7} + .51X$$

IT HAS DEGREE 7 .

IT HAS 5 TERMS .

+ 16 IS THE COEFFICIENT OF THE LEADING TERM .

− 21 IS THE CONSTANT TERM .

END OF THIS UNIT.PRESS 1 TO SEE IT AGAIN.PRESS M TO GO BACK TO THE MENU.

Fig. 8 End of Test I

Records of test and options offered.

H O M E W O R K

USING YOUR OWN POLYNOMIAL , THIS PART HELPS YOU FIND THE ANSWERS TO THE SA-
ME QUESTIONS ASKED IN THE PRECEDENT TEST .

WHILE ENTERING YOUR POLYNOMIAL REMEMBER THE FOLLOWING DIRECTIONS :

* COEFFICIENTS SHOULD BE INTEGERS OR DECIMAL NUMBERS WITH + OR - SIGN .
* COEFFICIENTS CAN OCCUPY AT MOST 5 SPACES AND EXPONENTS AT MOST 2 .
* YOUR POLYNOMIAL CANNOT EXCEED ONE LINE .
* THE COMPUTER WILL LET YOU ENTER YOUR EXPONENTS BY JUMPIMG ONE LINE UP .
* IF YOU WANT TO GO ONE LINE DOWN , PRESS THE SPACE BAR .
* WHEN YOU FINISH ENTERING YOUR POLYNOMIAL , PRESS RETURN .
* WHEN ENTERING YOUR ANSWER , YOU CAN USE 'BACK SPACE' TO CORRECT IT , OR
* 'ESC' TO CANCEL IT ALTOGETHER .

IF YOU FORGET OR MAKE A MISTAKE THE COMPUTER WILL REMIND YOU .

PRESS 'SPACE BAR' TO CONTINUE OR 'M' TO GO BACK TO THE MENU

Fig. 9 Directions

Directions to type in a polynomial.

computer to find the answers to the questions asked in the test (Figures 10 and 11).

4.2 Finding rational roots

This program (Fig. 12,Pg. 31) states Descartes's rule of signs which works on any polynomial P(X) with real coefficients, written in descending powers of X. In the example, the method is applied to

$P(X) = 3X^5 - 7X^4 - 2X^3 - 9X^2 + 21X - 32$ and finds the number of positive and negative zeros (Fig. 13, Pg. 32) All steps of the procedure are shown in a dynamic approach and can be replayed.

In order to find all possible rational roots of any polynomial with integer coefficients POLYROOT applies the rational root theorem, for example, to

$P(X) = 6X^5 + 7X^3 - 12X^2 + X + 3$ (Fig. 14,Pg. 33)

Synthetic division is used in this program to check if a number is a root of a polynomial. This is shown on the polynomial $3X^4 + 2X^3 - 9X^2 + 4$ using 1, -2, 3, and -2/3 as trial numbers (Figures 15,16,17,18 and 19).

In the test part (Figures 20 and 21),

$P(X) = 16X^5 - 88X^4 + 172X^3 - 146X^2 + 56X - 8$ is used to ask for the number of positive and negative real roots, the

H O M E W O R K

$$P(X) = -\ 123\ X^{2} + 342.3\ X^{27} - 12.47\ X^{4} - 91.7\ X^{19} + 1687 - 226\ X^{38} - 23467\ X^{7} - 37\ X$$

IT HAS DEGREE 38 .

IT HAS 8 TERMS .

WHAT IS THE COEFFICIENT OF THE LEADING TERM ?

Fig. 10 Homework I in progress

After typing a polynomial and answering the two first questions, the user is asked a third question.

H O M E W O R K

$$P(X) = -123X^{2} + 342.3X^{27} - 12.47X^{4} - 91.7X^{19} + 1687 - 226X^{38} - 23467X^{7} - 37X$$

IT HAS DEGREE 38 .

IT HAS 8 TERMS .

- 226 IS THE COEFFICIENT OF THE LEADING TERM .

+ 1687 IS THE CONSTANT TERM .

PRESS 1 TO ENTER ANOTHER POLYNOMIAL. PRESS M TO GO BACK TO THE MENU.

Fig. 11 End of Homework I

All four questions were asked and answered.

F I N D I N D   R A T I O N A L   R O O T S

IN THIS PART WE WILL BE USING POLYNOMIALS WITH INTEGER COEFFICIENTS ONLY.

1 DESCARTES'RULE OF SIGNS

2 RATIONAL ROOT THEOREM

3 SYNTHETIC DIVISION

4 TEST

5 HOMEWORK

6 MAIN MENU

7 QUIT

SELECT THE TOPIC BY PRESSING THE NUMBER IN FRONT OF YOUR CHOICE

Fig. 12 Sub-menu II

Menu of the second unit of POLYROOT.

D E S C A R T E S ' R U L E   O F   S I G N S

LET P(X) BE A POLYNOMIAL WITH REAL COEFFICIENTS WRITTEN IN DESCENDING POWERS OF X. COUNT THE NUMBER OF SIGN CHANGES IN THE SIGNS OF THE COEFFICIENTS.

1. THE NUMBER OF POSITIVE REAL ZEROS IS EQUAL TO THE NUMBER OF SIGN CHANGES IN P(X) OR IS EQUAL TO THAT NUMBER DECREASED BY AN EVEN INTEGER.
2. THE NUMBER OF NEGATIVE REAL ZEROS IS EQUAL TO THE NUMBER OF SIGN CHANGES IN P(-X) OR IS EQUAL TO THAT NUMBER DECREASED BY AN EVEN INTEGER.

EXAMPLE:

$$P(X) = 3X^5 - 7X^4 - 2X^3 - 9X^2 + 21X - 32$$

3 SIGN CHANGES. THUS 3 OR 1 POSITIVE REAL ZEROS.

$$P(-X) = -3X^5 - 7X^4 + 2X^3 - 9X^2 - 21X - 32$$

2 SIGN CHANGES. THUS 2 OR 0 NEGATIVE REAL ZEROS.

END OF THIS UNIT.PRESS 1 TO SEE IT AGAIN.PRESS M TO GO BACK TO THE MENU.

Fig. 13 Descartes' Rule of Signs

End of tutorial.

RATIONAL ROOT THEOREM

IF $P(X) = a_n X^n + a_{n-1} X^{n-1} + \ldots + a_1 X + a_0$ HAS INTEGER COEFFICIENTS AND p/q (p/q IS REDUCED) IS A RATIONAL ZERO, THEN

p IS A FACTOR OF a_0 AND q IS A FACTOR OF a_n.

TO USE THIS THEOREM, MAKE A LIST OF ALL FACTORS OF a_0 AND DIVIDE THESE INTEGERS BY THE FACTORS OF a_n.

EXAMPLE: $P(X) = 6X^5 + 7X^3 - 12X^2 + X + 3$

$a_n = 6$, WITH FACTORS ±1, ±2, ±3, ±6 | $a_0 = 3$, WITH FACTORS ±1, ±3

p/q : ±1, $\pm1/2$, $\pm1/3$, $\pm1/6$, ±3, $\pm3/2$ ARE POSSIBLE RATIONAL ROOTS.

END OF THIS UNIT.PRESS 1 TO SEE IT AGAIN.PRESS M TO GO BACK TO THE MENU.

Fig. 14 Rational Root Theorem

End of tutorial

S Y N T H E T I C   D I V I S I O N

SYNTHETIC DIVISION IS A CONDENSED FORM OF THE DIVISION OF A POLYNOMIAL BY X-r (r POSITIVE OR NEGATIVE). IN THIS PROGRAM WE USE IT TO CHECK IF THE VALUE r IS A ROOT.

EXAMPLE: $P(X) = 3X^4 + 2X^3 - 9X^2 + 4$ $\boxed{1,-2,3,-2/3}$

LET US CHECK IF THE NUMBERS IN THE BOX ARE ROOTS OF P(X). WE START WITH 1.

PRESS 'SPACE BAR' TO CONTINUE OR 'M' TO GO BACK TO THE MENU

Fig. 15 Synthetic Division I

Beginning of tutorial.

S Y N T H E T I C   D I V I S I O N

SYNTHETIC DIVISION IS A CONDENSED FORM OF THE DIVISION OF A POLYNOMIAL BY X-r (r POSITIVE OR NEGATIVE). IN THIS PROGRAM WE USE IT TO CHECK IF THE VALUE r IS A ROOT.

EXAMPLE: $P(X) = 3X^4 + 2X^3 - 9X^2 + 4$ $\boxed{1,-2,3,-2/3}$

$\boxed{1}$ 3   2   -9   0   4

WE WRITE DOWN THE COEFFICIENTS OF THE POLYNOMIAL

MAKE SURE TO INCLUDE ZERO COEFFICIENTS FOR MISSING TERMS

PRESS 'SPACE BAR' TO CONTINUE OR 'M' TO GO BACK TO THE MENU

Fig. 16 Synthetic Division II
Rewriting the coefficients of the polynomial.

S Y N T H E T I C   D I V I S I O N

SYNTHETIC DIVISION IS A CONDENSED FORM OF THE DIVISION OF A POLYNOMIAL BY X-r (r POSITIVE OR NEGATIVE). IN THIS PROGRAM WE USE IT TO CHECK IF THE VALUE r IS A ROOT.

EXAMPLE: $F(X) = 3X^4 + 2X^3 - 9X^2 + 4$

$\boxed{1,-2,3,-2/3}$

$\boxed{1}$	3	2	-9	0	4
		3			
	3	5			

MULTIPLY 3 BY 1 TO GET 3
WRITE THE RESULT UNDER 2
ADD THE COLUMN TO GET 5

PRESS 'SPACE BAR' TO CONTINUE OR 'M' TO GO BACK TO THE MENU

Fig. 17 Synthetic Division III
Getting coefficients of the quotient.

S Y N T H E T I C   D I V I S I O N

SYNTHETIC DIVISION IS A CONDENSED FORM OF THE DIVISION OF A POLYNOMIAL BY X-r (r POSITIVE OR NEGATIVE). IN THIS PROGRAM WE USE IT TO CHECK IF THE VALUE r IS A ROOT.

EXAMPLE: P(X) = $3X^4 + 2X^3 - 9X^2 + 4$

$\boxed{1,-2,3,-2/3}$

VALUE r	1	3	2	-9	0	4
			3	5	-4	-4
		3	5	-4	-4	0

COEFFICIENTS OF GIVEN POLYNOMIAL

QUOTIENT

REMAINDER

$3X^3 + 5X^2 - 4X - 4$

DEGREE ONE LESS THAN THE GIVEN POLYNOMIAL

IF REMAINDER EQUALS 0 THEN THE VALUE r WE ARE CHECKING IS A ROOT

PRESS 'SPACE BAR' TO CONTINUE OR 'M' TO GO BACK TO THE MENU

Fig. 18 Synthetic Division IV

Showing 1 as a root of P(X).

S Y N T H E T I C   D I V I S I O N

SYNTHETIC DIVISION IS A CONDENSED FORM OF THE DIVISION OF A POLYNOMIAL BY X-r (r POSITIVE OR NEGATIVE). IN THIS PROGRAM WE USE IT TO CHECK IF THE VALUE r IS A ROOT.

EXAMPLE: P(X) = $3X^4 + 2X^3 - 9X^2 + 4$

1,-2,3,-2/3

	3	2	-9	0	4
1	3	5	-4	-4	0
-2	3	-1	-2	0	
3	3	8	22		
-2/3	3	-3	0		

-2/3 IS A ROOT BECAUSE THE REMAINDER IS 0

END OF THIS UNIT.PRESS 1 TO SEE IT AGAIN.PRESS M TO GO BACK TO THE MENU.

Fig. 19 Synthetic Division V
End of tutorial.

T E S T

$P(X) = +16X^5 - 88X^4 + 172X^3 - 146X^2 + 56X - 8$

#'S OF POSSIBLE POSITIVE REAL ROOT(S): 5,3,1.

#'S OF POSSIBLE NEGATIVE REAL ROOT(S): 0.

DIVISORS OF 8 :±1,±2,±4,±8.

DIVISORS OF 16 :±1,±2,±4,±8,±16.

DIVIDE THE FIRST DIVISOR OF THE CONSTANT TERM BY ALL DIVISORS OF THE LEADING TERM.DO THE SAME THING WITH THE SECOND DIVISOR OF THE CONSTANT TERM AND SO ON DISCARD ALL REPEATING RESULTS. IF A RESULT IS NOT AN INTEGER THEN ENTER IT AS A FRACTION IN THE REDUCED FORM. FOR EXAMPLE 4/6 WILL BE ENTERED AS 2/3. ENTER YOUR ANSWERS EXACTLY IN THE SAME ORDER FOUND AND WITHOUT + OR - SIGN.

WHAT ARE THE POSSIBLE RATIONAL ROOTS ?

Fig. 20 Test II in progress

4 questions were answered. 5th question
and procedure of its solution displayed.

T E S T

$P(X) = +16X^5 - 88X^4 + 172X^3 - 146X^2 + 56X - 8$

THERE ARE 5,3 OR 1 POSSIBLE POSITIVE REAL ROOT(S), AND NO NEGATIVE REAL ROOT.
DIVISORS OF 8 : ±1, ±2, ±4, ±8. DIVISORS OF 16 : ±1, ±2, ±4, ±8, ±16.
POSSIBLE RATIONAL ROOTS : ±1, ±1/2, ±1/4, ±1/8, ±1/16, ±2, ±4, ±8.
RATIONAL ROOTS : 1/2, 1/2, 1/2, 2, 2.

	16	-88	172	-146	56	-8	
1	16	-72	100	-46	10	2	
-1	16	-104	276	-422	478	-486	LOWER BOUND
2	16	-56	60	-26	4	0	
4	16	8	92	342	1372		UPPER BOUND
2	16	-24	12	-2	0		
2	16	8	28	54			
1/2	16	-16	4	0			

THE LAST QUOTIENT IS A QUADRATIC EQUATION AND HAS 1/2 AS DOUBLE ROOT.

END OF THIS UNIT.PRESS 1 TO SEE IT AGAIN.PRESS M TO GO BACK TO THE MENU.

Fig. 21 End of Test II
All results displayed.

divisors of the constant and the leading terms, the possible rational roots, and finally for the rational roots and their multiplicities.

The homework part is exactly the same as the test part but here the user is invited to enter his own polynomial (Figures 22 and 23).

4.3 The Bisection method

This program (Fig. 24, Pg. 44) starts by explaining the method step by step showing graphically how it works (Fig. 25, Pg. 45). The user is then warned about the weakenesses of this method (Fig. 26, Pg. 46). In the homework part, after entering a polynomial, the user is asked to enter a sub-interval containing a zero to start the method. In case he fails, the program helps him find one by graphing the polynomial in any domain of definition (Figures 27 and 28). By pressing space bar, the search continues one more iteration until a zero is found (Fig. 29, Pg. 49).

4.4 Newton's method

Here again the program (Fig. 30, Pg. 50) presents the method (Fig. 31, Pg. 51) and warnings (Fig. 32, Pg. 52). This time, the program asks the user to provide an initial guess of a root, and helps him find one in case of failure

H O M E W O R K

$P(X) = +8X^{4} - 120X^{2} - 80X + 192$

#'S OF POSSIBLE POSITIVE REAL ROOT(S): 2,0.

#'S OF POSSIBLE NEGATIVE REAL ROOT(S): 2,0.

DIVISORS OF 192 :±1,±2,±3,±4,±6,±8,±12,±16,±24,±32,±48,±64,±96,±192.

DIVISORS OF 8 :±1,±2,±4,±8.

POSSIBLE R.ROOTS:±1,±1/2,±1/4,±1/8,±2,±3,±3/2,±3/4,±3/8,±4,±6,±8,±12,±16,±24,
±32,±48,±64,±96,±192

ENTER YOUR ANSWERS IN THE SAME ORDER APPEARED ABOVE.
FRACTIONS SHOULD BE ENTERED WITH SLASH (/)
IF A ROOT IS OF MULTIPLICITY 2 ENTER IT TWICE, AND SO FORTH.

WHAT ARE THE RATIONAL ROOTS ?

Fig. 22 Homework II in progress

4 questions were answered. 5th question
and procedure for its solution displayed.

H O M E W O R K

$P(X) = +8X^4 - 120X^2 - 80X + 192$

#'S OF POSSIBLE POSITIVE REAL ROOT(S): 2,0.

#'S OF POSSIBLE NEGATIVE REAL ROOT(S): 2,0.

DIVISORS OF 192 :±1,±2,±3,±4,±6,±8,±12,±16,±24,±32,±48,±64,±96,±192.

DIVISORS OF 8 :±1,±2,±4,±8.

POSSIBLE R.ROOTS:±1,±1/2,±1/4,±1/8,±2,±3,±3/2,±3/4,±3/8,±4,±6,±8,±12,±16,±24,
±32,±48,±64,±96,±192

RATIONAL ROOTS: 1,-2,-3,4

PRESS 1 TO ENTER ANOTHER POLYNOMIAL. PRESS M TO GO BACK TO THE MENU.

Fig. 23 End of Homework II
All results displayed.

B I S E C T I O N   M E T H O D

1 EXPLANATION

2 HOMEWORK

3 MAIN MENU

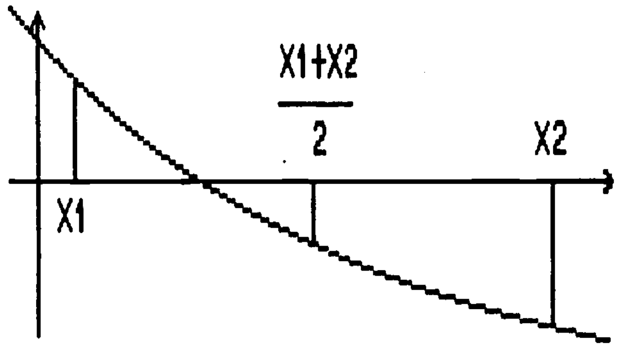
4 QUIT

SELECT THE TOPIC BY PRESSING THE NUMBER IN FRONT OF YOUR CHOICE

Fig. 24 Sub-menu III

Menu of the Bisection Method.

B I S E C T I O N   M E T H O D



THIS METHOD FINDS THE REAL ROOTS OF A CONTINUOUS FUNCTION F(X) ASSUMING THAT AN INTERVAL X1 ≤ X ≤ X2 HAS BEEN FOUND SUCH THAT F(X1)F(X2) < 0.
THE METHOD UNDERTAKES TO DECREASE THE SIZE OF THE INTERVAL. THIS DECREASE IS ACCOMPLISHED BY EVALUATING THE FUNCTION F(X) AT THE MIDPOINT OF THE INTERVAL, THAT IS COMPUTING F((X1+X2)/2).
IF THIS VALUE IS NOT ZERO,THEN THERE IS A SIGN CHANGE IN ONE OF THE TWO HALVES OF THE ORIGINAL INTERVAL AND WE PICK A NEW, DECREASED INTERVAL WITH THE SIGN CHANGE BY :

$$F(X1)F((X1+X2)/2) = \begin{cases} 0 & \text{A ZERO AT } (X1+X2)/2 \\ <0 & \text{NEW INTERVAL } (X1,(X1+X2)/2) \\ >0 & \text{NEW INTERVAL } ((X1+X2)/2,X2) \end{cases}$$

THUS,(X1+X2)/2 REPLACES EITHER X1 OR X2.REPEAT THE PROCESS AGAIN AND AGAIN.
EACH REPETITION HALVES THE LENGTH OF THE INTERVAL.

PRESS 'SPACE BAR' TO CONTINUE OR 'M' TO GO BACK TO THE MENU

Fig. 25 Explanation I

Presentation of the Bisection Method.

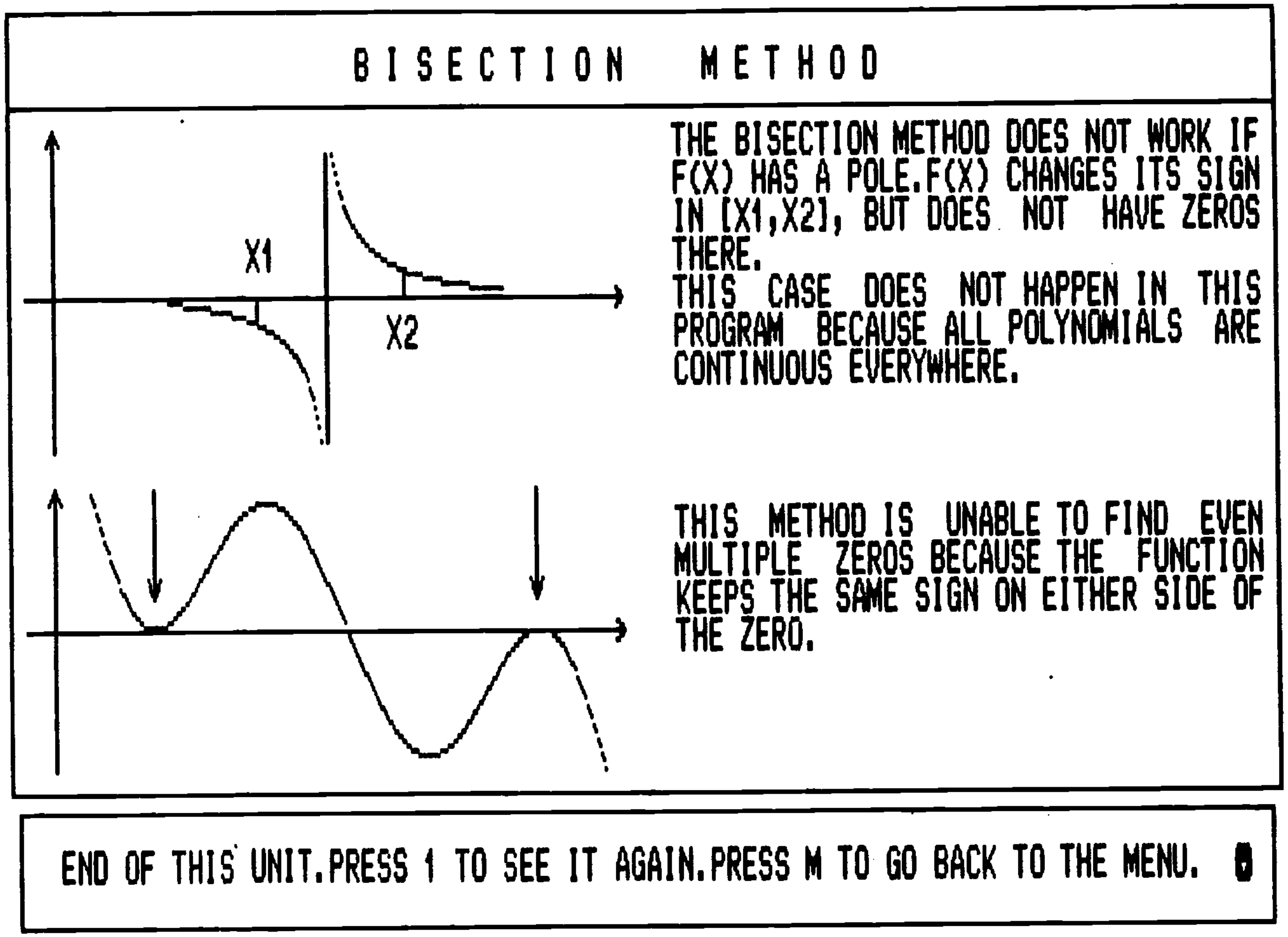


Fig. 26 Explanation II

Warnings on the Bisection Method.

$P(X)= + 3 X^4 - 8 X^3 - 30 X^2 + 72 X + 47$

I WILL GRAPH YOUR POLYNOMIAL IF YOU GIVE ME AN INTERVAL LARGE ENOUGH TO INCLUDE
ALL THE ZEROS.THIS WILL HELP YOU FIND THE SUBINTERVALS FOR THE BISECTION METHOD
WHAT IS THE LOWER BOUND OF YOUR INTERVAL ?

Fig. 27 Search I for a sub-interval

POLYROOT is asking for an interval to graph P(X) after a user failed to provide for a sub-interval.

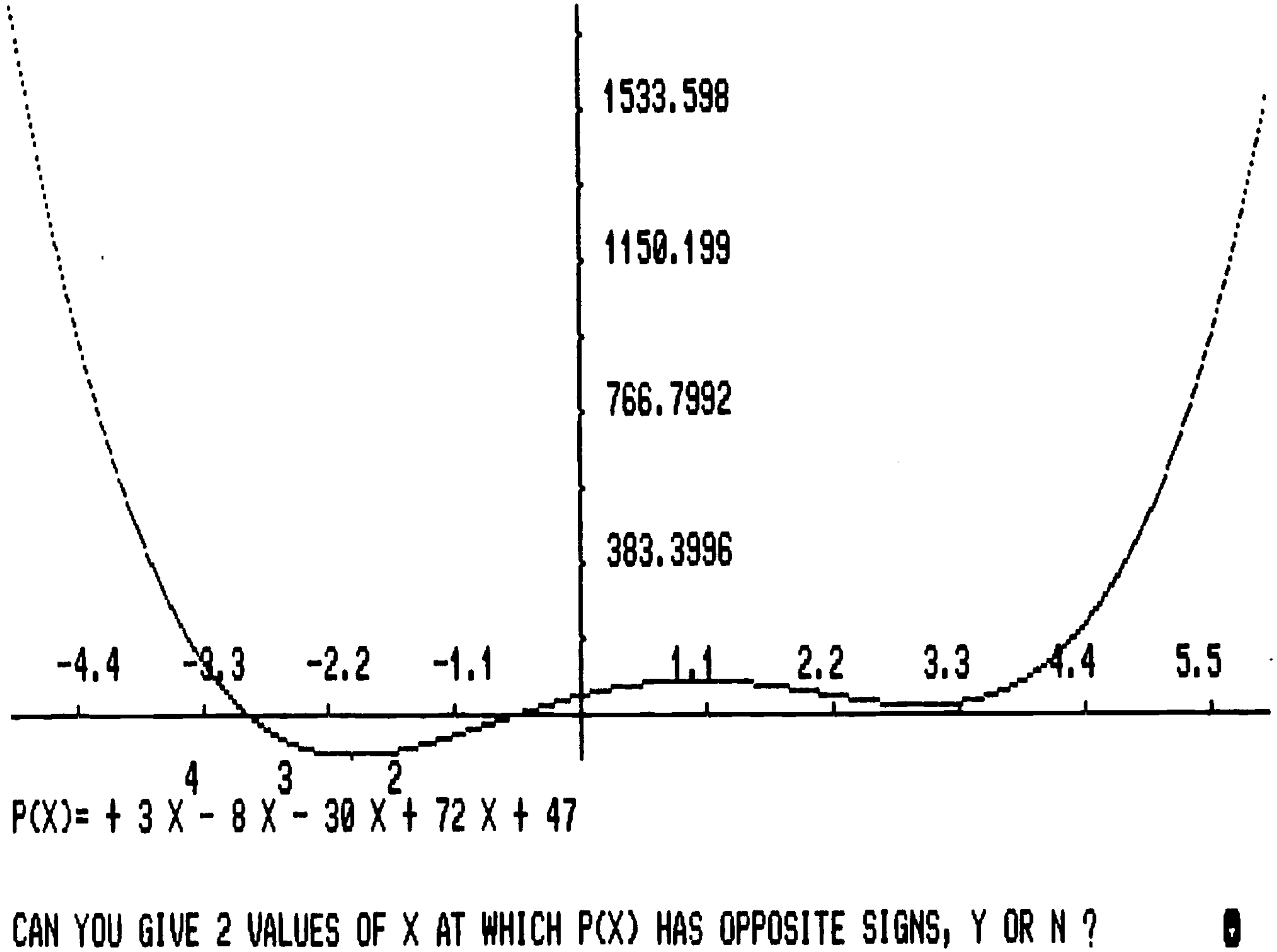
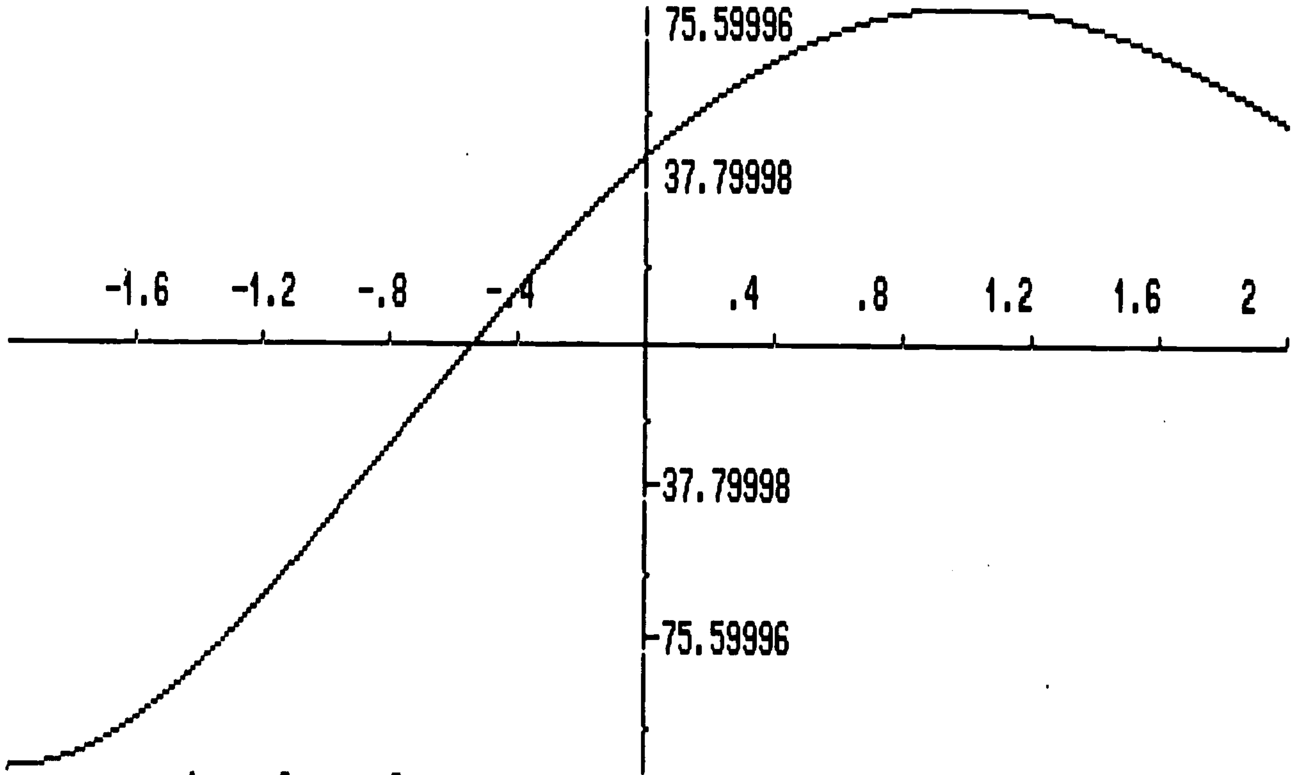


Fig. 28 Search II for a sub-interval

POLYROOT came up with the graph of P(X) to help find a sub-interval.



$P(X) = + 3X^4 - 8X^3 - 30X^2 + 72X + 47$

STEP 19 A ZERO IS IN THE INTERVAL [-.5492783 ,-.5492706]

PRESS SP.BAR FOR ONE MORE STEP,1 TO LOOK FOR 0THER ZEROS,E TO END THIS UNIT

Fig. 29 A step of Homework III

Step 19 of the search of a zero of P(X).
Different options offered.

N E W T O N ' S   M E T H O D

1 EXPLANATION

2 HOMEWORK

3 MAIN MENU

4 QUIT

SELECT THE TOPIC BY PRESSING THE NUMBER IN FRONT OF YOUR CHOICE

Fig. 30 Sub-menu IV
Menu of Newton's Method.

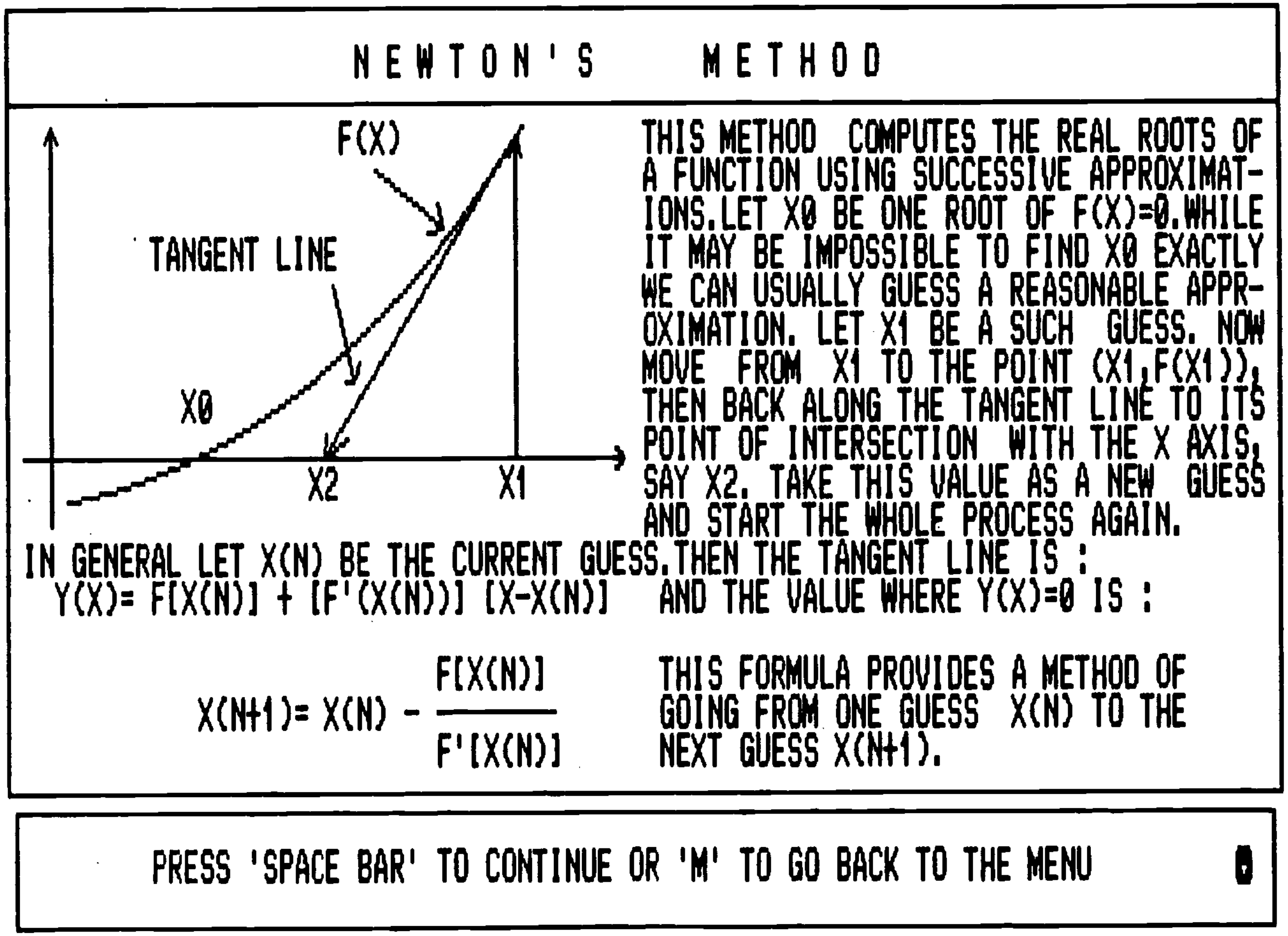
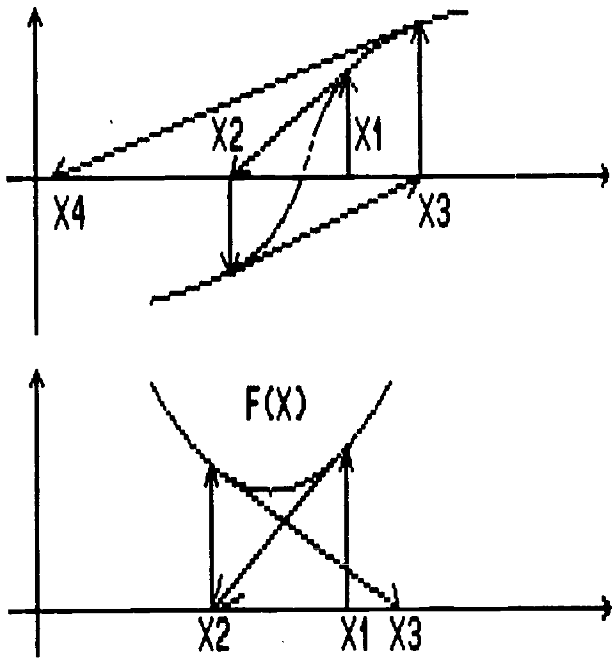


Fig. 31 Explanation III

Presentation of Newton's Method.

N E W T O N ' S   M E T H O D



THE NEWTON'S METHOD WHEN IT WORKS IS FAST BUT THERE ARE RESTRICTIONS :

INFLECTION POINT:

THE METHOD DOES NOT CLOSE IN ON A ZERO.

LOCAL MINIMUM (MAXIMUM IF F(X)<0):

NO CONVERGENCE.

END OF THIS UNIT.PRESS 1 TO SEE IT AGAIN.PRESS M TO GO BACK TO THE MENU.

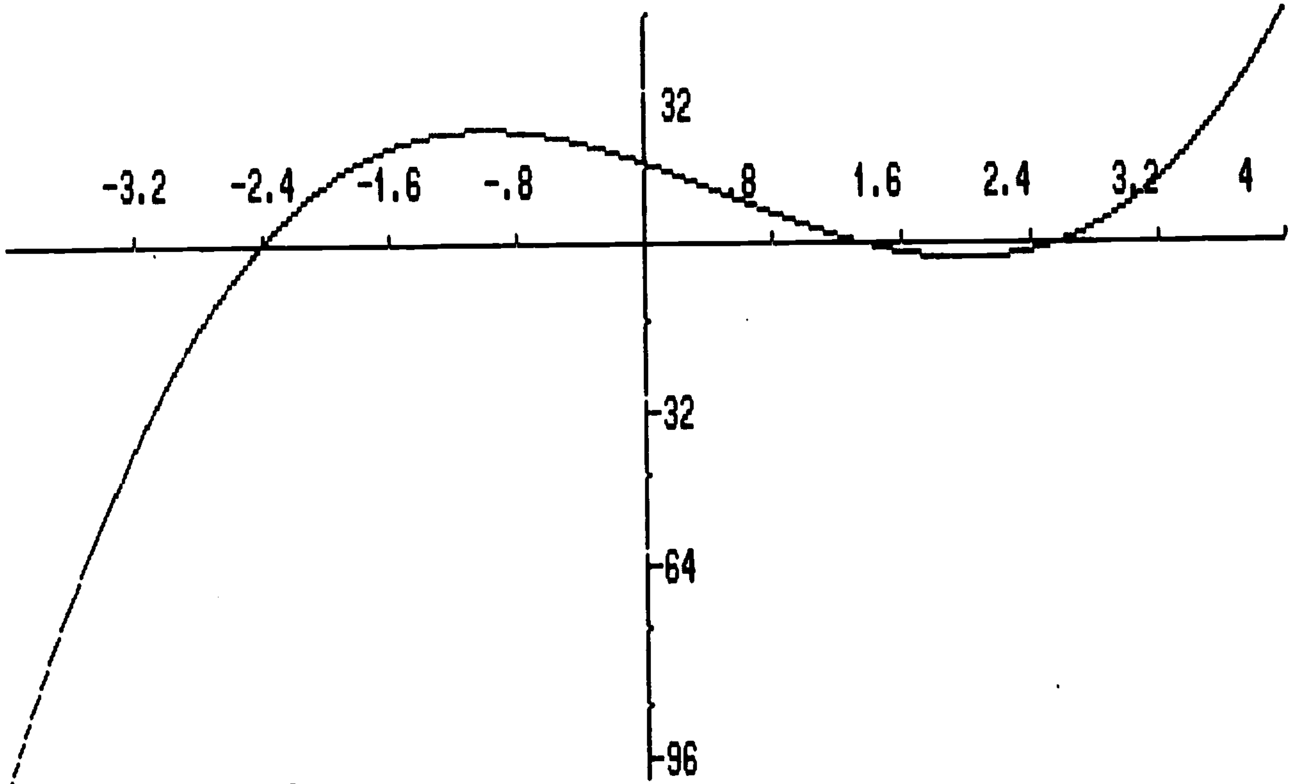
Fig. 32 Explanation IV
Warnings on Newton's Method.

by graphing the polynomial in any interval (Figures 33 and 34). At each iteration, an approximated zero is given and shown on the graph by drawing the tangent to the curve (Fig. 35, Pg. 56).

P(X)= $+ 2 X^3 - 3 X^2 - 12 X + 17$
I WILL GRAPH YOUR POLYNOMIAL IF YOU GIVE ME AN INTERVAL LARGE ENOUGH TO INCLUDE
ALL THE ZEROS.THIS WILL HELP YOU FIND A GUESS OF A ROOT OF P(X).
WHAT IS THE LOWER BOUND OF YOUR INTERVAL ?

Fig. 33 Search I for a guess

POLYROOT is asking for an interval to graph P(X) after a user failed to provide a guess to start the Newton's Method.



$P(X) = +2X^3 - 3X^2 - 12X + 17$

CAN YOU GIVE A VALUE OF X CLOSE ENOUGH TO A ROOT OF P(X), Y OR N ?

Fig. 34 Search II for a guess
POLYROOT came up with a graph of P(X) to help find a guess.

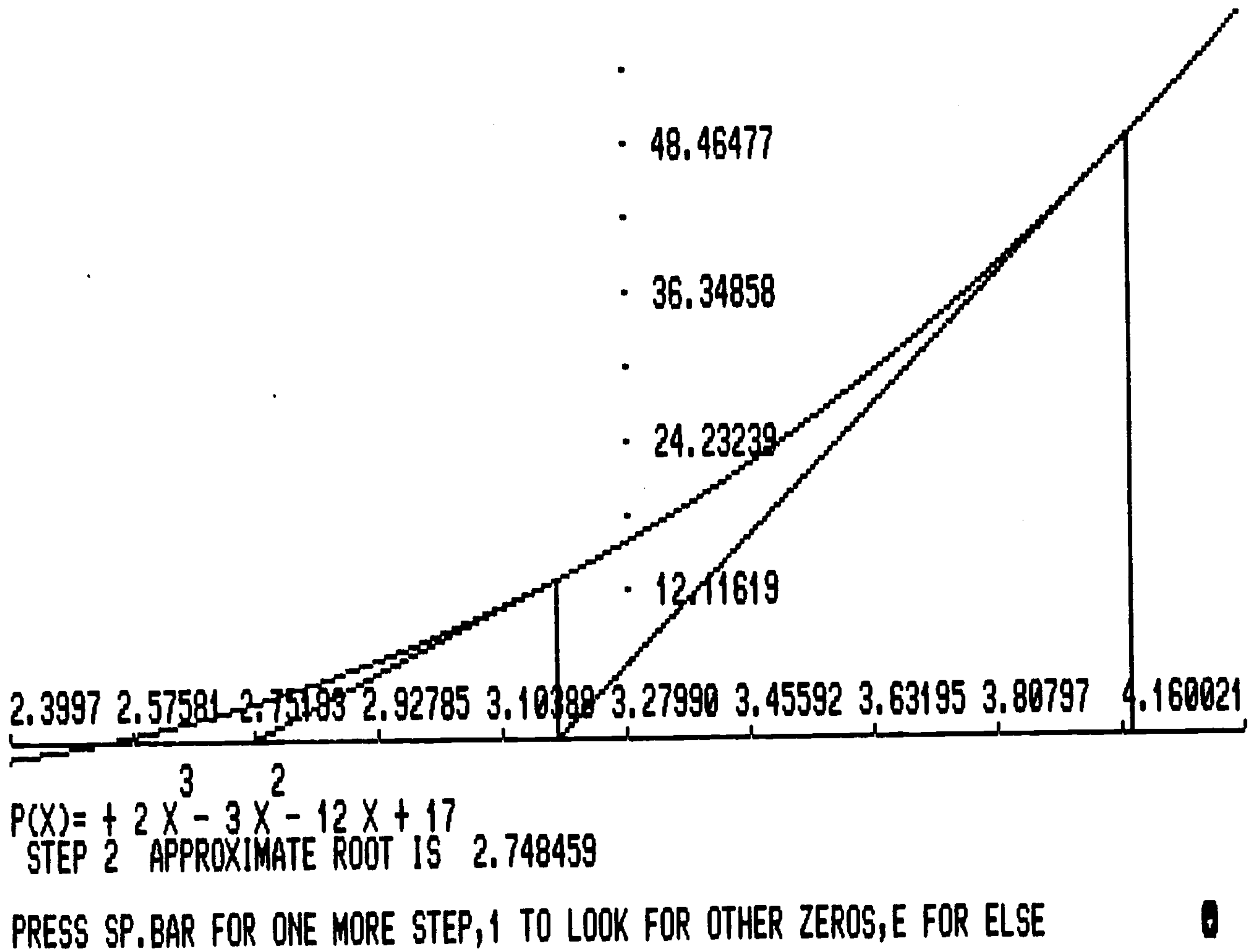


Fig. 35 A step of Homework IV
Approximate root given at step 2. Different options offered.

V. PURPOSE AND IMPORTANCE OF POLYROOT

5.1 Roots of polynomials and their importance

The problem of finding the real zeros of a given continuous function arises frequently in science and engineering. Built from constants and a variable 'X' by means of the fundamental algebraic operations of addition, subtraction, and multiplication, polynomials are a classical subject of Mathematics, and are among the first examples of continuous functions encountered by students. In the study of polynomials the challenge is to find their roots. Finding the zeros of polynomials can be very helpful to the student involved in the solution of mathematical problems in the physical, chemical, engineering, and related sciences. Among their most common use, one can list contributions to the solution of differential equations, using methods such as the characteristic equation, or partial fractions.

5.2 POLYROOT and search for rational roots

Finding the rational roots of a polynomial can be a very time consuming process. Generally many operations are involved in finding all divisors of the coefficients of constant and leading terms, especially if these two numbers are large. Then, depending on luck, the trial of quotients of these divisors will be another more or less lengthy, and always boring task. For example, a polynomial such as

$$20 X^4 - 24 X^3 - 221 X^2 + 360 X - 63 = 0$$

has 72 possible rational roots obtained by dividing each factor of the constant term 63 by each factor of 20, the coefficient of the leading term. Since this task is sequential and repetitive, it is suited for a computer which if well programmed can save time and errors (Figure 36,Pg. 59).

5.3 POLYROOT, graphing, and numerical analysis

This program can graph the same polynomial repeatedly over different intervals in just seconds, task impossible to do in class on the blackboard. For the Bisection Method and the Newton's Method, a subinterval and a first guess of a root are respectively necessary to start the process. Usually one has to evaluate the polynomial at different points and study its sign. No matter how small the chosen step size axis, one always runs the risk of missing some zeros. There may be two zeros (or indeed any positive even number of zeros) in a single subinterval and they may be missed entirely. If there are an odd number of zeros in a subinterval then the bisection process will isolate only

H O M E W O R K

$F(X) = +20X^4 - 24X^3 - 221X^2 + 360X - 63$

#'S OF POSSIBLE POSITIVE REAL ROOT(S): 3,1.

#'S OF POSSIBLE NEGATIVE REAL ROOT(S): 1.

DIVISORS OF 63 :±1,±3,±7,±9,±21,±63.

DIVISORS OF 20 :±1,±2,±4,±5,±10,±20.

POSSIBLE R.ROOTS:±1,±1/2,±1/4,±1/5,±1/10,±1/20,±3,±3/2,±3/4,±3/5,±3/10,±3/20,
±7,±7/2,±7/4,±7/5,±7/10,±7/20,±9,±9/2,±9/4,±9/5,±9/10,±9/20,±21,±21/2,±21/4,
±21/5,±21/10,±21/20,±63,±63/2,±63/4,±63/5,±63/10,±63/20
RATIONAL ROOTS: 1/5,3,3/2,-7/2

PRESS 1 TO ENTER ANOTHER POLYNOMIAL. PRESS M TO GO BACK TO THE MENU.

Fig. 36 POLYROOT and rational roots

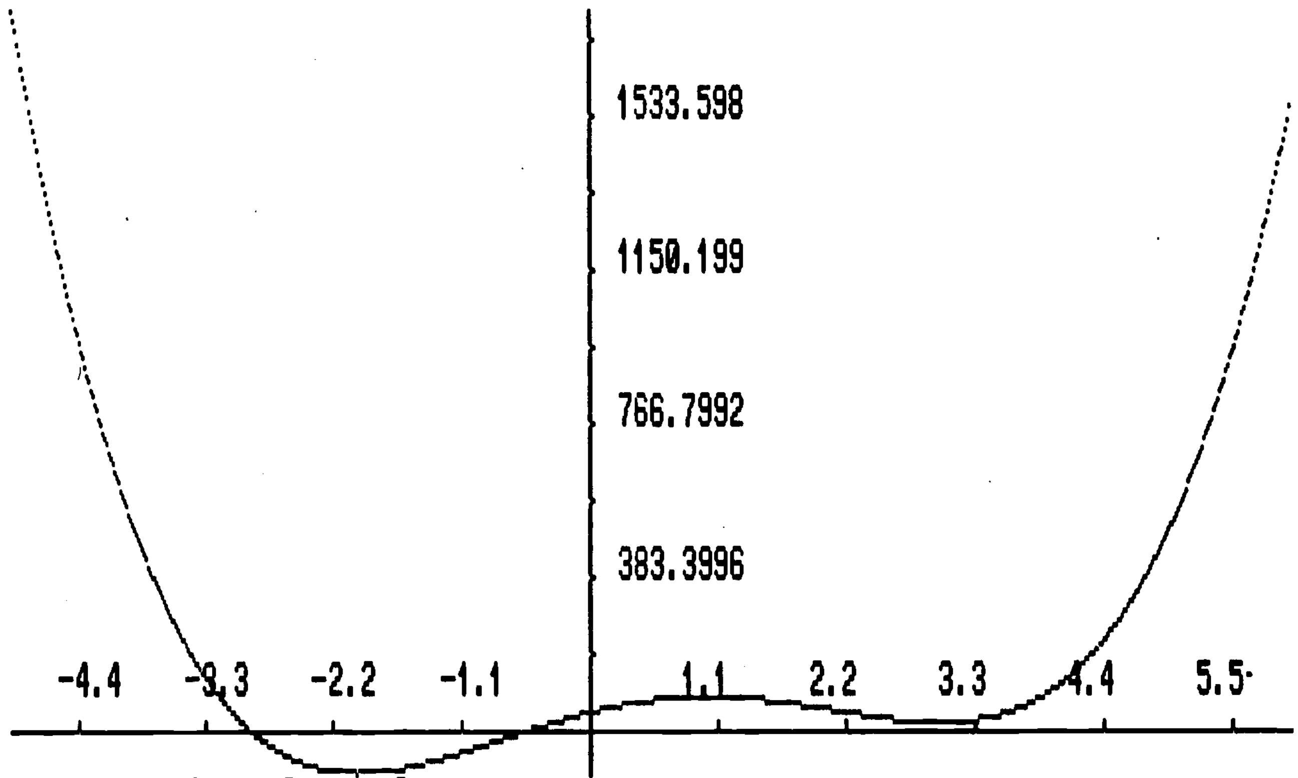
Performance of POLYROOT in finding rational roots.

one of them. Because of the graphing features of this program, one does not have to be especially knowledgeable to avoid these two problems. The user asks the computer to graph his polynomial in any domain of definition and he has the picture before his eyes instantly. By reducing the length of an interval, the user can back off for a global view and find where all the roots lie (Figure 37, Pg. 61), or zoom in on a fine detail and find an approximate root at a desired precision (Figure 38, Pg. 62). In both cases the automatic scaling will always provide the value of the polynomial at all points.

In numerical analysis the power and the ease with wich a computer performs is obvious to everyone. POLYROOT does it even better by showing a visual illustration of the search of real roots, making the whole process a dynamic one. A problem such as # 12, page 71, which needs some insight for its explanation can be easily solved with POLYROOT (Figure 39,40).

5.4 POLYROOT and its place in the curriculum

This program will fit in Precalculus, Calculus, and Engineering courses. The material dealing with rational roots and the search for real zeros by the Bisection Method is within the grasp of anyone with a reasonable knowledge of College Algebra. The part using Newton's

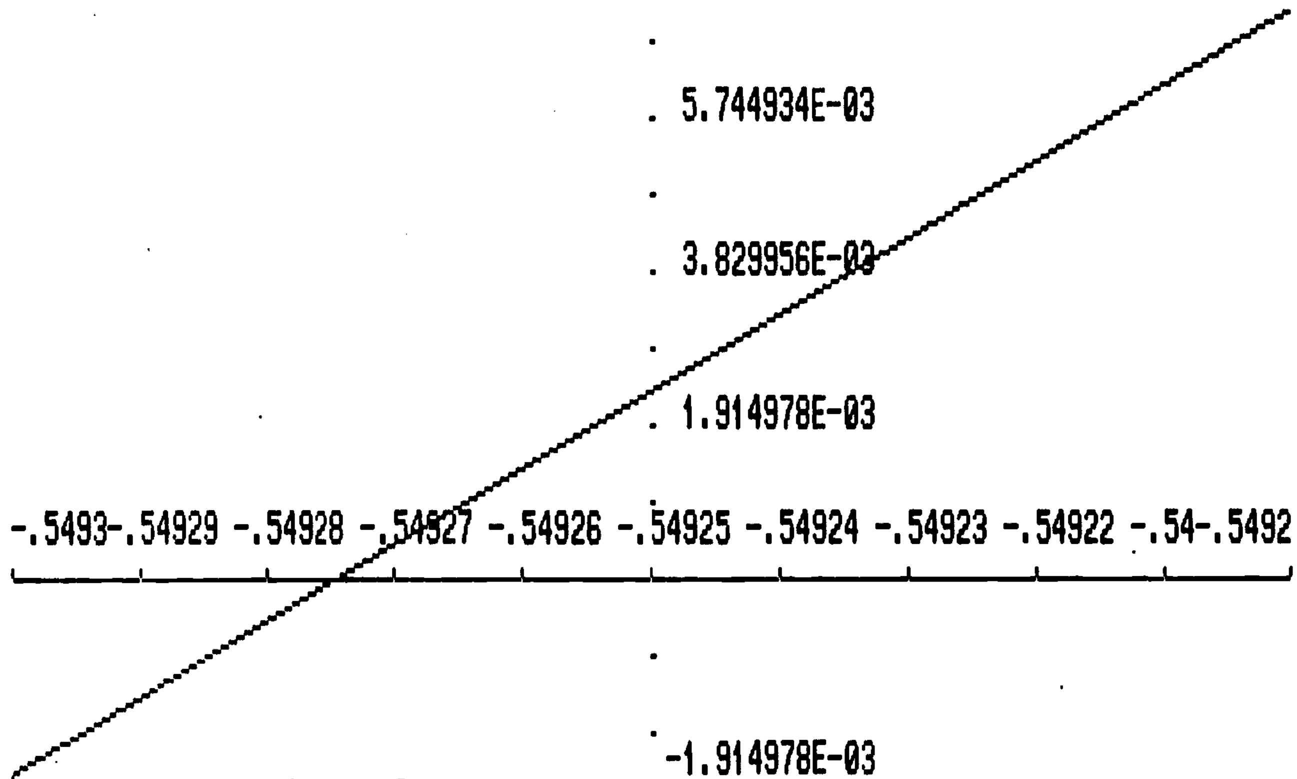


$P(X) = +3X^4 - 8X^3 - 30X^2 + 72X + 47$

CAN YOU GIVE A VALUE OF X CLOSE ENOUGH TO A ROOT OF P(X), Y OR N ?

Fig. 37 POLYROOT and graphing I

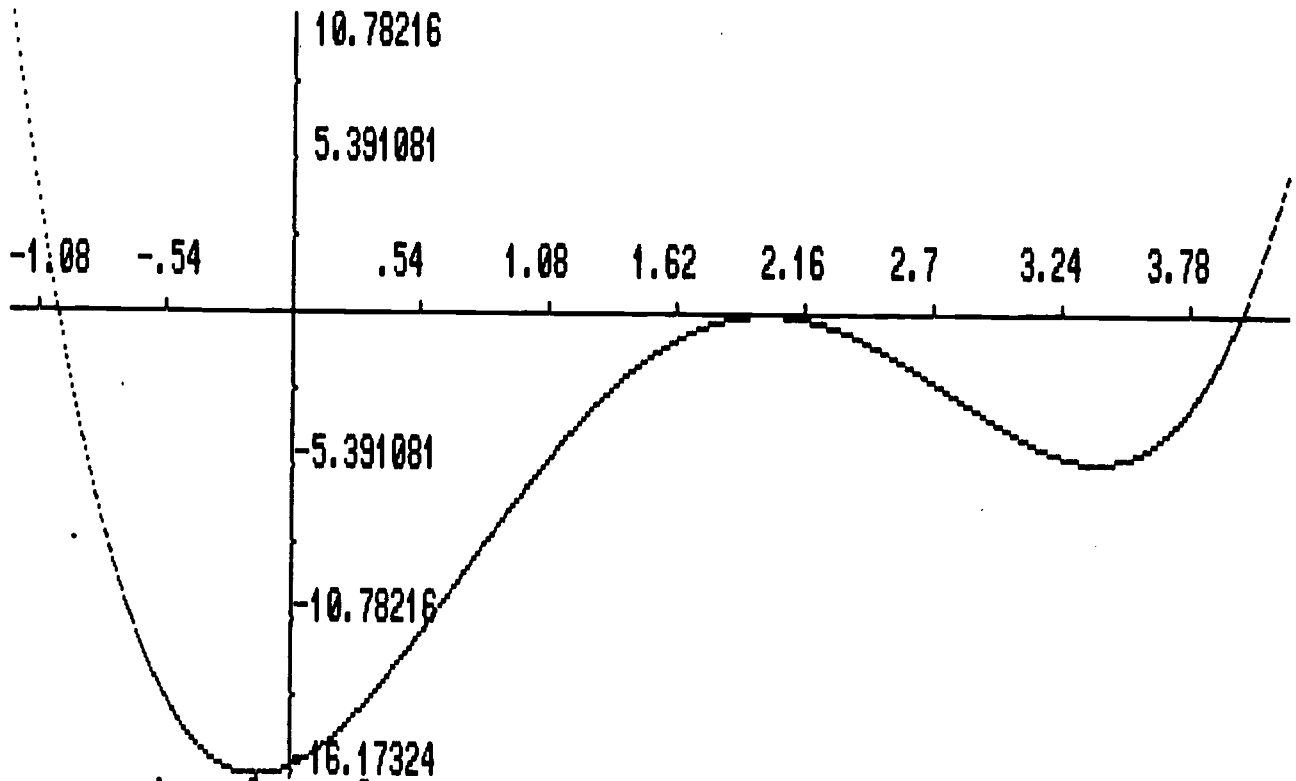
Backing off for a global view of the graph of P(X).



$P(X) = +\ 3\ X^4 - 8\ X^3 - 30\ X^2 + 72\ X + 47$

CAN YOU GIVE A VALUE OF X CLOSE ENOUGH TO A ROOT OF P(X), Y OR N ?

Fig. 38 POLYROOT and graphing II
Zooming in on a root of P(X).

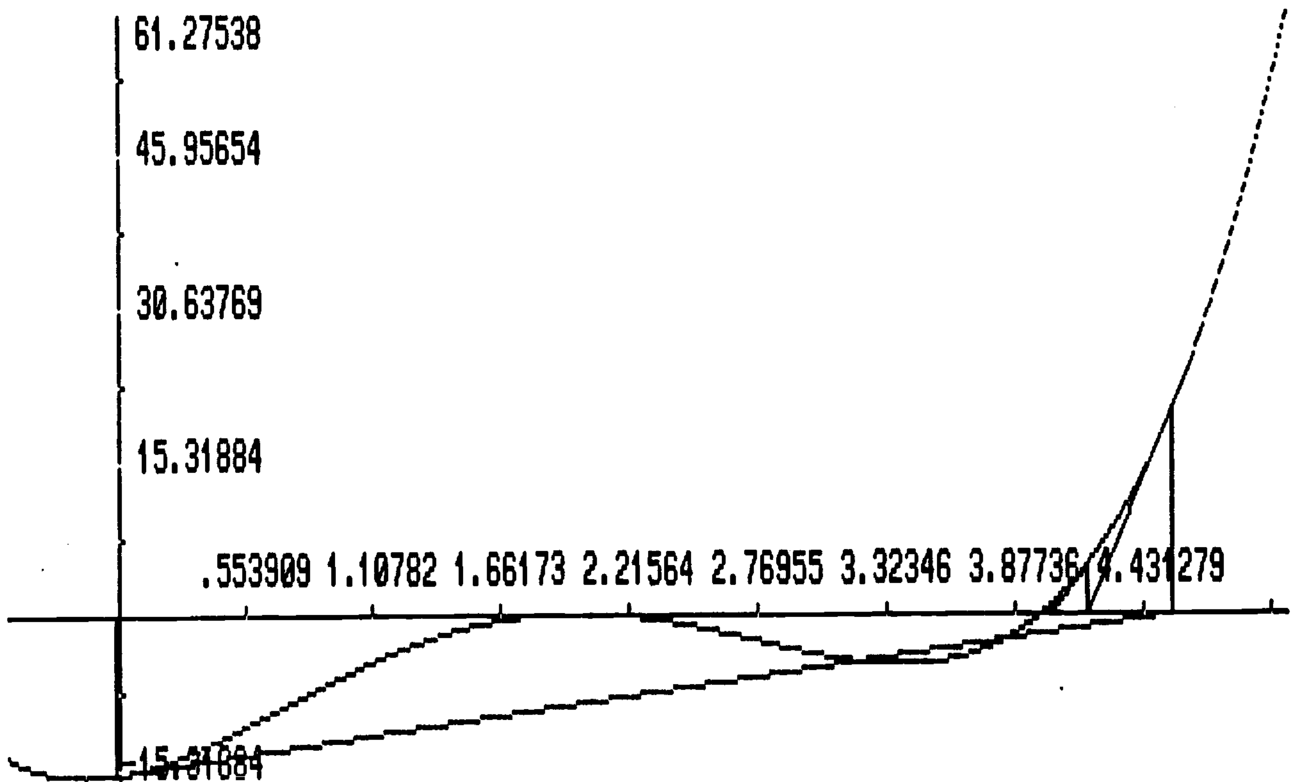


$$P(X) = + X^4 - 7X^3 + 12X^2 + 4X - 16$$

CAN YOU GIVE A VALUE OF X CLOSE ENOUGH TO A ROOT OF P(X), Y OR N ?

Fig. 39 POLYROOT and Newton's Method

From this graph one can see that the tangent to the curve at (-.02,P(-.02)) will cross the X axis on the right of 4.



$P(X) = + X^4 - 7X^3 + 12X^2 + 4X - 16$

STEP 6 APPROXIMATE ROOT IS 4

PRESS SP.BAR FOR ONE MORE STEP,1 TO LOOK FOR OTHER ZEROS,E FOR ELSE

Fig. 40 POLYROOT and Newton's Method II

By taking -0.02 as a first guess POLYROOT finds the extreme right root 4.

Method requires the understanding of the concept of the derivative of a continuous function, and the use of the 'point slope formula' of a line.

VI. BIBLIOGRAPHY

Addison-Wesley Book of Apple Software 1985(Addison-Wesley Publishing Company,Inc.,1985)

Atkinson, Kendall E. An Introduction to Numerical Analysis (John Wiley and Sons,Inc.,1978).52-56, 78-85.

Crothamel, David A. Graphing functions on the Microcomputer Video Display, Computing Teacher 7 (Dec.1979-Jan.1980) 17-20.

Data Pro Directory Of Microcomputer Software,Volume 1 (Data Pro Research Corporation,1984)

Dickson, Leonard E. New First Course In The Theory Of Equations (John Wiley and Sons,Inc.,1939). 6-29,76-80 95-101.

Dobbs, David E. and Hanks, Robert A Modern Course On The Theory Of Equations (Polygonal Publishing House,1980). 113-120, 141-144, 156-165, 171-179

Dorn, William S. Numerical Methods With Fortran 4th Case Studies (John Wiley and Sons,Inc., 1972). 3-16, 25-35.

Hamming, Richard W. Introduction To Numerical Analysis (Mc Graw-Hill,Inc.,1971). 33-45, 49-51, 82-99.

Hansen, Viggo P. Computers In Mathematics Education. National Council Of Teachers Of Mathematics, 1984 Yearbook.

Hastings, Ellen H., and Yates, Daniel S. Microcomputer Unit: Graphing Strait Lines . Mathematics Teacher 76 (Mar.1983):181-186.

Hewlett-Packard. Hewlet-Packard Series 80 Software Catalog. (Reston Publishing Company,Inc.,1982).

Kennedy, Jane B. Graphing Polynomials With Computer Assistance. Mathematics Teacher 74 (Oct.1981).516-519

Lappan, Glenda, and Winter, M.J.A Unit On Slope Functions: Using A Computer In Mathematics Class. Mathematics Teacher 75 (Feb.1982):118-22.

Norris, Donald O. Some Thoughts On Using Microcomputers To Teach Calculus. Journal Of Computers In Mathematics And Science Teaching 2 (Spring 1983): 28-30.

O'Neil, Jr. Harold F. Computer Based-Instruction A State Of The Art Assesment (Academic Press, Inc.,1981).

O'Shea, Tim and Self, John. Learning And Teaching With Computers (Prentice-Hall Inc.,1983). 69-78, 107-112, 152-160, 204-205.

Piper McGraw, Karen. Writing Educational Software: what goes in before the label goes on. In 1985 Programer's Market. (Writer's Digest Books, 1985).

Stanton, Jeffrey. and Wells, Robert P. The Addison Wesley Book Of Apple Software 1985 (Addison-Wesley Publishing Company,Inc.,1985).

Wallace, W.H. What's In Print The Subject Guide To Microcomputer Magazines (Tab Books Inc.,1984).

Wilf, H.S. The Numerical Solution Of Polynomial Equations. In Mathematical Methods For Digital Computers (John Wiley,1961)

APPENDIX

VII. APPENDIX

One of the most useful skills any student of mathematics can acquire is that of constructing his own meaningful problems. This effort will gain him insight into the mathematical structure and promote discovering of unexpected phenomenon. The following is a set of problems one might try to solve, and use to inspire new ideas.

1. Find an integral polynomial whose roots coincide with the roots of the rational polynomial :

$$P(x) = (3/2)\,X^3 + (15/8)\,X^2 - (21/4)\,X + (3/4).$$

2. The 'Weak Version Of Eisenstein's Criterion'states that an integral polynomial :

$$P(X) = a_n X^n + a_{n-1} X^{n-1} + \ldots + a_1 X + a_0$$

of degree n >= 2 has no rational roots if there exists a positive prime integer p with the following three properties :p does not divide the coefficient of the leading term but divides all others, and the square of p does not divide the constant term.

Show that $X^8 + 294\,X^6 + 7\,X^4 - 98\,X + 28$ has no rational roots.

3. Which method(s) should you use to find the cube root of 2, the fourth root of 3?. Find these values.

4. Show that $X^n - 1 = 0$ has just two rational roots when n is even and only one rational root when n is odd.

5. Show that the square root of 2 is not a rational number. And, more generally, show that, if a is an integer, the square root of a is either an integer or it is not rational.

6. A rectangular box is 15 inches long,10 inches wide, and 8 inches high. Each of the three dimensions is to be increased by the same amount X so as to increase the volume of the box by 300 cubic inches. Find X to an accuracy of .00001.

7. What rate of interest is implied in an offer to sell a computer for 2,700 dollars cash, or in annual installments each of 1000 dollars payable 1,2, and 3 years from date?

8. Find an equation whose roots are those of

   $X^4 + X^3 - 3X^2 - X - 4 = 0$ diminished by 2.

9. Find an equation whose roots are those of

   $X^3 + X^2 - 3X + 9 + 0$ increased by 3.

10. Does there exists a real number that exceeds its cube by 1 ?

11. Find three consecutive integers whose product is 720.

12. The equation $X^4 - 7X^3 + 12X^2 + 4X - 16 = 0$ has roots at -1 and +2. Yet, if we apply the Newton's Method with a first guess $X_0 = -0.02$, we reach another root at +4. Explain.

13. Attempt to apply the Newton's Method to the equation:

$X^5 + 8X^4 + 17X^3 - 8X^2 - 14X + 20 = 0$. What happens ?

14. Apply all the methods studied in POLYROOT to the following polynomial:

$X^7 - 28X^6 + 322X^5 - 1960X^4 + 6769X^3 - 13132X^2 + 13068X - 5040$

and compare them.

15. Consider the cubic equation

$$P(X) = X^3 + 9X^2 - X - 105$$

If X_1, X_2, and X_3 are roots of $P(X) = 0$, show that

$$9 = -(X_1 + X_2 + X_3)$$

$$-1 = X_1X_2 + X_1X_3 + X_2X_3$$

$$-105 = -X_1X_2X_3$$

In general, if $P(X) = X^3 + a_2X^2 + a_1X + a_0$ show that

$$a_2 = -(X_1 + X_2 + X_3)$$

$$a_1 = X_1X_2 + X_1X_3 + X_2X_3$$

$$a_0 = -X_1X_2X_3$$

16. The sum of three numbers is 6, their product is -210, the sum of their products two by two is -37. Find the

numbers.

17. The following familiar puzzle problem leads to the solution of a quartic equation.

Two ladders, one 20-ft long and the other 30-ft long, lean against buildings across an alley, as shown below. If the point at which the ladders cross is 8-ft above the ground, how wide is the alley ?

Hint:Gruenberger and Jaffrey, in 'Problems For Computer Solution' (New York: Wiley,1964) show that this problem can be formulated to require solution of the following equation:

$$Y^4 - 16\ Y^3 + 500\ Y^2 - 8000\ Y + 32{,}000 = 0$$

Then $X = \sqrt{(400 - Y^2)}$.