The generation of an alongshore mean flow by the nonlinear interaction of forced barotropic shelf waves over a continental margin is studied using a wind-forced, f-plane model with bottom friction in an attempt to develop a model for poleward eastern boundary undercurrents. Expressions are derived for the alongshore (v) and cross-shelf (u) mean velocities in terms of the lowest order periodic velocities. An expression for the correlation coefficient of $u v_x$, where $x$ is the cross-shelf coordinate, is derived that depends only on the sign of the bottom slope, the magnitude of friction, the local water depth and the forcing frequency.
Empirical orthogonal function (EOF) analysis in the frequency domain has been extended to complex time series giving EOF amplitude and phase for negative and positive frequencies corresponding, respectively, to clockwise and anticlockwise rotation. EOF amplitude functions are calculated directly from the eigenvectors of the rotary cross-spectral matrix and the rotary autospectra. This method is applied to velocity and temperature measurements from the Coastal Upwelling Experiment during July and August 1973 on the continental shelf off Oregon. Rotary EOFs are computed for the diurnal, near-inertial and semidiurnal frequency bands and for frequencies below 0.2 cycles per day.
ON FORCED CURRENTS OVER A CONTINENTAL MARGIN:
THEORY AND ANALYSIS

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Chapter I

INTRODUCTION
ON FORCED CURRENTS OVER A CONTINENTAL MARGIN:
THEORY AND ANALYSIS

1. Introduction

During the summer of 1973 the Coastal Upwelling Experiment (CUE-2) took place on the continental shelf off Oregon. The velocity measurements from this experiment are indicative of a mean poleward undercurrent near the shelf break (Kundu and Allen, 1976). Poleward undercurrents appear to be a ubiquitous feature at eastern ocean boundaries (Wooster and Reid, 1963; Halpern et al., 1978; Hickey, 1979, 1982; Brockmann et al., 1980).

Laboratory experiments have demonstrated that mean flows can be generated by oscillatory forcing (Caldwell and Eide, 1976; Colin de Verdiere, 1979; McEwan et al., 1980). The direction of the mean flow outside the region of forcing is in the direction of long, free shelf wave propagation. This is identical to the poleward flows observed at eastern boundaries.

Analysis of the CUE-2 velocity measurements at the $M_2$ and $K_1$ tidal frequencies, using the response method, was conducted by Torgrimson and Hickey (1979). Kundu (1976) analyzed the CUE-2 data at the inertial frequency band by applying a bandpass filter centered at the inertial frequency and then calculating covariances.
In this thesis a model is developed to investigate the possibility that the mean poleward undercurrent is forced by an oscillatory wind stress. The mean flow in a homogeneous fluid over a continental margin is solved for by assuming the nonlinear terms are small and expanding the variables in powers of the Rossby number. The alongshore wind stress is chosen such that the time mean is zero.

A technique to calculate empirical orthogonal functions from complex time series is developed in order to objectively analyze data at tidal and inertial frequencies in the frequency domain. This technique is applied to a subset of the CUE-2 measurements. The results of the analyses are compared to those from Torgrimson and Hickey (1979) and Kundu (1976).
Chapter II

MEAN FLOW GENERATION ON A CONTINENTAL MARGIN

BY PERIODIC WIND FORCING
1. Introduction

Poleward undercurrents are a common feature at eastern ocean boundaries (Wooster and Reid, 1963). The undercurrents over the continental margin off California, Oregon and Peru are characterized by a core of poleward flowing water evidently of equatorial origin (Halpern et al., 1978; Hickey, 1979, 1982; Brockmann et al., 1980). The core is 20-50 km wide, 200-500 meters in vertical extent and situated near the shelf break. At present, no theory has adequately explained why poleward undercurrents are such a ubiquitous feature at eastern ocean boundaries.

Laboratory experiments by Caldwell and Eide (1976), Colin de Verdière (1979), and McEwan et al. (1980) have demonstrated that mean flows can be generated by oscillatory forcing in homogeneous, rotating fluids with a potential vorticity gradient due to variations in fluid depth. Outside of the forcing region, the mean flow generated is in the direction of long, free shelf wave propagation, i.e. retrograde. Colin de Verdière (1979) and McEwan et al. (1980) demonstrated experimentally that when the forcing travels in a retrograde sense the wave pattern is well ordered and the mean flow relatively strong, whereas, when the forcing travels prograde, the wave pattern is unsteady and the mean flow is weak.

In this paper we investigate the possibility that mean poleward undercurrents at eastern boundaries are forced by an oscillatory wind stress. Recent theoretical studies of mean flow generation are presented in Loder (1980) and Huthnance (1981, 1973), where references to previous studies are given. Huthnance (1973) and Loder (1980) obtained analytical expressions for the tidal forced mean alongshore velocity in a homogeneous fluid over a sandbank by assuming that alongshore gradients are zero. Huthnance (1981) derives an expression for the mean flow generated over a shelf
given the fluctuating velocities, parameterized bottom stress, and vanishingly small friction. These results are not directly applicable to mean flows over continental margins forced by long period (5-12 days) traveling wind-stress since effects of a traveling forcing function, free wave resonances and a coastal boundary condition are not considered.

We solve for the mean flow in a homogeneous fluid over a continental slope with bottom friction by assuming the nonlinear terms are small and expanding the variables in powers of the Rossby number. A traveling wind-stress is chosen such that the time mean over a period is zero. Thus, the first order variables have a zero time mean while the second order variables provide the lowest order contribution to the mean flow. Frictional effects are retained at lowest order. The alongshore length scale is assumed to be greater than the cross-shelf length scale, i.e., a long wave approximation (Allen, 1976), is made. This assumption simplifies the algebra and allows an analytical solution to the second order mean flow in terms of the first order variables. Several important results of the model, however, are found to be independent of the long wave approximation.
2. Formulation

We consider an f-plane model utilizing a straight continental margin, with uniform alongshore topography, adjoining a flat bottomed ocean. The fluid is homogeneous and bottom friction is present. The fluid is assumed to be inviscid away from the surface and bottom boundary layers. Cartesian coordinates \((x', y', z')\) with corresponding velocity components \((u', v', w')\) are used, where \(z'\) is vertical, positive upwards, \(x'\) is cross-shelf, positive onshore, and \(y'\) is alongshore.

The variables \((u', v', w', p')\) and \((x', y', z', t')\) are scaled by \((U, U, UH_0^{-1}, UfL_0)\) and \((L, L, H_0, (\delta f)^{-1})\), respectively, where \(U = \delta \tau_0 (\delta E)^{-1}\) is a characteristic velocity, \(H_0\) the depth of the interior ocean, \(L\) a characteristic alongshore length scale, \(f\) the Coriolis parameter, \(\rho\) the constant fluid density, \(\delta E = [v_T/(fH_0^2)]^{1/2}\) the dimensionless Ekman layer depth, \(\delta\) the ratio of cross-shelf to alongshore length scales, \(\tau_0\) the characteristic wind-stress, and \(v_T\) the constant vertical turbulent eddy viscosity. The characteristic velocity \(U\) is chosen by assuming that the surface and bottom Ekman cross-shelf transports balance at the coast (see Section 5a). The Rossby number, \(\tilde{\varepsilon} = U(fL)^{-1}\), is assumed small such that a perturbation expansion may be made in the limit \(\tilde{\varepsilon} \to 0\). We assume the fluid is hydrostatic and bounded above by a rigid lid. In nondimensional variables, the governing equations are

\[
\begin{align*}
\delta u_t + \tilde{\varepsilon}(u_x + v_y + w_z) - \nu = -p_x + \delta^2 \delta E u_{zz}, \\
\delta v_t + \tilde{\varepsilon}(u_x + v_y + w_z) + u = -p_y + \delta^2 \delta E v_{zz}, \\
\delta w_t + \tilde{\varepsilon}(u_x + v_y + w_z) + w = -p_z, \\
u_x + v_y + w_z = 0,
\end{align*}
\]

* Dimensional variables for which a nondimensional counterpart will be defined are marked with primes.
where the subscripts \((x,y,z,t)\) denote partial differentiation.

An alongshore wind-stress of the following form is assumed:

\[
\tau = \sin(kx+\phi) \cos(2y-\omega t),
\]

where \(k, \ell, \phi,\) and \(\omega\) are respectively the cross-shelf and alongshore wave-numbers, an arbitrary phase, and the forcing frequency.

The model geometry (Figure 1) has a rigid lid at \(z = 0\), a variable depth bottom at \(z = -H(x)\), a coast at \(x = x_0\) \((x_0 \leq 0)\), and a flat bottom extending from \(x = -\delta\) to \(x \to -\infty\). The boundary conditions follow from the assumptions of an imposed alongshore wind-stress at \(z = 0\), a no-slip condition at \(z = -H(x)\), no net mass transport into the coast at \(x = x_0\), and a bounded solution as \(x \to -\infty\). The resulting boundary conditions are:

\[
\begin{align*}
\frac{\partial u}{\partial z} &= w = 0, \quad \text{at } z = 0, \quad (2.3a) \\
u = v = w &= 0, \quad \text{at } z = -H(x), \quad (2.3b) \\
0 & \int_{-H}^{0} u \, dz = 0, \quad \text{at } x = x_0, \quad (2.3c) \\
u, v, w & < \infty, \quad \text{as } x \to -\infty. \quad (2.3d)
\end{align*}
\]

The domain is divided into four regions, inviscid shelf, surface Ekman layer, bottom Ekman layer, and inviscid interior ocean. The variables in the last three of those four regions are denoted by the superscripts \(T, B,\) and \(I,\) respectively. The interior ocean variables are matched to the shelf variables by requiring that pressure and onshore velocity be continuous at \(x = -\delta\).

The momentum equations for the inviscid fluid on the shelf and in the interior ocean are,
\[ \delta u_t + \varepsilon (uu_x + vu_y) - v = -p_x, \]  
\[ \delta v_t + \varepsilon (uv_x + vv_y) + u = -p_y, \]  
\[ u_x + v_y + w_z = 0. \]  
(2.4a, b, c)

The corresponding vorticity equation is,
\[ \delta (v_x - u_y)_t + \varepsilon (uv_x + vv_y)_x - \varepsilon (uu_x + vu_y)_y - w_z = 0. \]  
(2.5)

a. Inviscid shelf

In the shelf region, the long wave approximation, \( \delta \ll 1 \), is made. As a result, the shelf variables are expanded in the following form:
\[ u = u_0 + \varepsilon u_1 + ..., \]  
\[ \v = \v_0 + \v_1 + ... \]  
\[ w = \delta^{-1} (w_0 + \v_1 + ...), \]  
\[ p = p_0 + \v_1 + ... \]  
\[ x = \delta \xi, \]  
(2.6a, b, c, d, e)

where
\[ \v = \v/\delta^2 = (U/\delta)/(f\delta L) \]  
(2.6f)

is the Rossby number formed by the alongshore velocity scale, \( U/\delta \), and the cross-shelf length scale, \( \delta L \). We order the small parameters \( \delta, \delta \xi \), and \( \v \) such that \( \delta \xi/\delta, \delta/\v, \) and \( \delta \xi/\v \) are \( O(1) \) quantities. Substituting (2.6) into (2.4) we obtain at \( O(1) \),
\[ \v_0 = p_0 \xi, \]  
\[ \v_0 t + u_0 = -p_0 y, \]  
\[ u_0 \xi + \v_0 y + w_0 z = 0, \]  
(2.7a, b, c)

and at \( O(\v) \),
\[ \v_1 = p_1 \xi, \]  
(2.8a)
\[ \v_1 t + u_1 = -p_1 y - (u_0 v \xi + v_0 v_0 y), \]  
(2.8b)
The time mean flow is obtained by applying a time average, 
\[ \langle \cdot \rangle = \frac{1}{T} \int_{t_0}^{t_0+T} (\cdot) dt, \]
where \( T = 2\pi/\omega \) and \( t_0 \) is arbitrary, to the \( O(\epsilon) \) equations. The form of the forcing (2.2) implies additional simplifications, i.e., for all \( O(1) \) variables \( \langle \cdot \rangle = 0 \), and for all \( O(\epsilon) \) variables \( \langle \cdot \rangle_y = 0 \).

The depth integrated vorticity equations are formed by utilizing (2.6) with (2.5) and depth integrating to obtain at \( O(1) \),

\[ Hv_0 \varepsilon_t - [w_0(z=0) - w_0(z=-H)] = 0, \]  
\[ (2.9) \]
and at \( O(\epsilon) \),

\[ [\langle w_1(z=0) \rangle - \langle w_1(z=-H) \rangle] = H\langle u_0 \varepsilon_0 \rangle, \]
\[ (2.10) \]
where \( H = H(\xi) \). The vertical velocities \( w_0 \) and \( \langle w_1 \rangle \) at \( z = 0, -H \) are obtained from the Ekman layer solutions (see Appendix A).

The boundary condition (2.3c) at the coast, \( \xi = \xi_0 \), is, for \( O(1) \)

\[ H(\xi_0)u_0 + (\delta_E/\delta) \left( \int_{-\infty}^{0} \bar{u}^Tdn + \int_{0}^{\infty} \bar{u}^B d\zeta \right) = 0, \]
\[ (2.11) \]
and for \( O(\epsilon) \),

\[ H(\xi_0) \langle u_1 \rangle + (\delta_E/\delta) \left( \int_{-\infty}^{0} \bar{u}_1^Tdn + \int_{0}^{\infty} \bar{u}_1^B d\zeta \right) = 0, \]
\[ (2.12) \]
where \( n = z/\delta_E \) and \( \zeta = (z+H)/\delta_E \) are the stretched vertical coordinates and \( u^T = \bar{u}^T/\delta \) and \( u^B = \bar{u}^B/\delta \) are rescaled velocities for the surface and bottom Ekman layers, respectively (see Appendix A).

b. Inviscid interior ocean

Boundary conditions for the shelf variables at the slope boundary, \( \xi = -1 \), are obtained by matching with the appropriate solution for the interior ocean. The interior ocean variables are expanded as follows,
\[ u^I = \delta^{-1}(u_0^I + \varepsilon u_1^I + \ldots), \quad v^I = \delta^{-1}(v_0^I + \varepsilon v_1^I + \ldots), \quad (2.13a,b) \]

\[ w^I = \delta_E \delta^{-1}(w_0^I + \varepsilon w_1^I + \ldots), \quad p^I = \delta^{-1}(p_0^I + \varepsilon p_1^I + \ldots), \quad (2.13c,d) \]

Substituting (2.13) into (2.4) and collecting terms of \( O(1) \), we obtain

\[ v_0^I = p_{0x}^I, \quad u_0^I = -p_{0y}^I, \quad (2.14a,b) \]

\[ u_{0x}^I + v_{0y}^I = 0. \quad (2.14c) \]

The \( O(1) \) depth integrated vorticity equation is formed by utilizing (2.13) with (2.5) and depth integrating to obtain,

\[ (v_0^I - u_0^I)_t - \delta_E/\delta [w_0^I(z=0) - w_0^I(z=-1)] = 0. \quad (2.15) \]

We may write (2.15) in terms of pressure by utilizing (2.14), (A.20), and (2.2) to obtain,

\[ \nu H^2 p_0^I + \gamma \nu H^2 p_0^I = \nu z k \gamma \cos(kx+\phi) \cos(\lambda y-\omega t), \quad (2.16a) \]

where \( \nu H^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 \) and

\[ \gamma = \delta_E/(\sqrt{\delta}). \quad (2.16b) \]

The conditions for matching the variables from the interior ocean and shelf are,

\[ p^I = p, \quad \text{at} \quad x = -\delta, \quad (2.17a) \]

\[ \varepsilon(u^I u_x^I + v^I v_y^I)_t + \varepsilon/\delta (u^I v_x^I + v^I v_y^I) + \delta u_{tt}^I + p_{xt}^I = \]

\[ p_{xt} + \delta u_{tt} + \varepsilon/\delta (uv_x + vv_y) + \varepsilon(uu_x + vu_y)_t, \quad \text{at} \quad x = -\delta, \quad (2.17b) \]

Expanding \( p^I \) in a Taylor series about \( x = 0 \), and utilizing (2.6) and (2.13) we obtain at \( O(1) \),

\[ p_0^I(x=0) = 0, \quad p_{0x}^I(x=0) = p_{0z}(\xi=-1). \quad (2.18a,b) \]
The solution to (2.16) with (2.18a) and (2.3d) is,

\[ p_0^I = - (\cos(kx+\phi) - e^{i\xi x}\cos\phi) G(y,t), \]  \hspace{1cm} (2.19a)

where

\[ G(y,t) = \sqrt{2} k \gamma [(k^2+\xi^2)(\omega^2+\gamma^2)]^{-1} \left[ \gamma \cos(\xi y - \omega t) - \omega \sin(\xi y - \omega t) \right]. \]  \hspace{1cm} (2.19b)

The boundary condition on \( p_0 \) at \( \xi = -1 \), from (2.18b), is,

\[ p_0(\xi=-1) = (k \sin\phi + \xi \cos\phi) G(y,t). \]  \hspace{1cm} (2.20)
3. Analysis

The inviscid shelf motion is determined in three steps. First, the $O(1)$ motion is solved numerically, which allows $H(\xi)$ to be an arbitrary function. Second, the $O(\epsilon)$ Eulerian time mean solution is found analytically in terms of the $O(1)$ solution. Finally, the Lagrangian mean flow is found by adding the Stokes drift to the $O(\epsilon)$ Eulerian mean flow.

a. $O(1)$ solution

We solve the $O(1)$ equations governing the shelf motions by writing (2.9) in terms of pressure. Substituting (A.11) and (A.17) into (2.9), and using (2.7), we obtain,

$$H_p(\xi\xi t) + \gamma_p(\xi\xi) + H(\xi)(p_0 + p_{\xi\xi t}) = 0. \quad (3.1)$$

The boundary condition at the coast is expressed in terms of pressure by substituting (A.10) and (A.16) into (2.11):

$$p_0(\xi) = \sqrt{\xi} \sin\phi \cos(\lambda y - \omega t) - H(\xi_0)\xi^{-1} (p_{0y} + p_{\xi\xi t}), \text{ at } \xi = \xi_0. \quad (3.2)$$

We employ a depth profile for which $H(\xi_0) < \sqrt{\xi} \gamma$ (see Appendix D). Correspondingly, the second term on the right hand side (RHS) of (3.2) is usually small compared to the first term. When $H(\xi_0) < \sqrt{\xi} \gamma$ the surface Ekman transport is primarily compensated at the coast by a bottom Ekman layer transport of equal magnitude. The velocity scaling used in this paper is appropriate for this depth profile.

A forced solution of the form,

$$p_0 = Re[\psi(\xi) \exp[i(\lambda y - \omega t)]], \quad (3.3)$$

is sought where $\psi(\xi)$ is a complex function (see Appendix B).
b. $O(\varepsilon)$ solution

The mean $O(\varepsilon)$ onshore velocity is found by time averaging (2.8b), which gives

$$<u_1> = -<u_0v_{0\xi}>.$$  \hfill (3.4)

The $O(\varepsilon)$ mean vorticity equation, (2.10), may be rewritten, utilizing (A.13), (A.19), and (3.4), as

$$-\gamma <\nabla v_{0\xi} \xi> - \gamma <v_{1\xi}> + (7/20) \gamma <v_{0\xi} v_{0\xi}> \xi = (H<u_0v_{0\xi}>)_\xi.$$  \hfill (3.5)

The boundary condition at $\xi = \xi_0$ is found by substituting (A.12), (A.18), and (3.4) into (2.12):

$$-H<u_0v_{0\xi}> - \gamma <\nabla v_{0\xi} > - \gamma <v_1> + (7/20) \gamma <v_{0\xi} v_{0\xi}> = 0, \text{ at } \xi = \xi_0.$$  \hfill (3.6)

Integrating (3.5) once with respect to $\xi$ and applying (3.6), we obtain

$$<v_1> = -H^{-1} <u_0v_{0\xi}> + (7/20) <v_{0\xi} v_{0\xi}> - \gamma <\nabla v_{0\xi}>.$$  \hfill (3.7)

The $O(\varepsilon)$ Eulerian mean shelf velocities in (3.4) and (3.7) are the lowest order contributions to the mean flow. By comparing (3.5) and (3.6), we note that the mean cross-shelf transport in the interior is balanced by a net cross-shelf transport in the Ekman layers.

Eqs. (3.4) and (3.7) may be expressed in more convenient forms. Multiplying (3.1) by $v_{0\xi}$ and time averaging, we obtain

$$<u_0v_{0\xi}> = \gamma/H_\xi <v_{0\xi}^2>.$$  \hfill (3.8)

Squaring (3.1) and time averaging, we have

$$<u_o^2> = [(H/H_\xi)^2 \omega^2 + \gamma^2/H_\xi^2] <v_{0\xi}^2>.$$  \hfill (3.9)
where the form of the forcing (2.2) has allowed the substitution,

$$\langle v_0^2 \rangle = \omega^2 \langle v_0^2 \rangle. \quad (3.10)$$

Finally, using (3.8), (3.9), and (3.4), we obtain

$$\langle u_1 \rangle = -\gamma H^2 (\gamma^2 + H^2 \omega^2)^{-1} \langle u_0^2 \rangle, \quad (3.11)$$

and using (3.4), (3.7), and (3.11), we find

$$\langle v_1 \rangle = -HH^2 (\gamma^2 + H^2 \omega^2)^{-1} \langle u_0^2 \rangle + (7/20) \langle v_0 v_0 \rangle \langle u_0^2 \rangle$$

$$- \sqrt{2} \langle \tau v_0 \rangle. \quad (3.12)$$

In terms of dimensional variables, (3.11) and (3.12) are

$$\langle u' \rangle = (\delta_{E}^2 / H^2) \chi \{ f \sqrt{2} [(\omega'/f)^2 + \frac{1}{2} (\delta_{E}^2 / H^2)] \}^{-1} \langle u'^2 \rangle, \quad (3.13)$$

and

$$\langle v' \rangle = -H^2 \chi \{ f [\omega'/f)^2 + \frac{1}{2} (\delta_{E}^2 / H^2)] \}^{-1} \langle u'^2 \rangle$$

$$+ (7/20) f^{-1} \langle v'_x \rangle \chi - \sqrt{2} (\delta_{E}^2 f \omega^2)^{-1} \langle \tau v'_x \rangle, \quad (3.14)$$

respectively. The mean cross-shelf velocity (3.11) and the first term on the RHS of (3.12) may be determined from $\delta_{E}$, $H$, $H^2$, $\omega$, $\epsilon$, and the variance of $u_0$. It may be shown that (3.11) is independent of the long-wave approximation. Similarly the first term on the RHS of (3.12) is independent of the long-wave approximation, but the second and third terms on the RHS are not.

Combining (3.8) and (3.9) to form a correlation coefficient, we have

$$R(u_0, v_0) = \langle u_0 v_0 \rangle \langle u_0^2 \rangle^{-\frac{1}{2}} \langle v_0^2 \rangle^{-\frac{1}{2}} = \text{sgn}(H) \omega / \gamma [1 + (H \omega / \gamma)^2]^{-\frac{1}{2}}, \quad (3.15)$$
where \( \text{sgn}(H_\xi) = H_\xi / |H_\xi| \). Writing \( u_0 \) and \( v_{0\xi} \) as,

\[
\begin{align*}
  u_0 &= |u_0| \cos(2y - \omega t + \theta), \\
  v_{0\xi} &= |v_{0\xi}| \cos(2y - \omega t + \phi),
\end{align*}
\]

(3.16a,b)

where \( |u_0| \) and \( |v_{0\xi}| \) are the magnitudes and where \( \theta \) and \( \phi \) the phases of \( u_0 \) and \( v_{0\xi} \), respectively, and substituting (3.16) into (3.15), we obtain,

\[
|\theta - \phi| = \cos^{-1} \left[ 1 + \left( H_\omega \gamma \right)^2 \right]^{-\frac{1}{2}}.
\]

(3.17)

The above expression implies that the phase difference between \( u_0 \) and \( v_{0\xi} \) has a simple relationship that is independent of whether the driving is by wind-stress at the coast or indirectly by interior ocean wind-stress curl, and is independent of the length scales of the forcing and the magnitude of \( H_\xi \).

For a monotonically increasing depth in the offshore direction, i.e., \( H_\xi < 0 \), \( <u_1> \) will always be positive, i.e., onshore. Similarly, the first term on the RHS of (3.12) will be positive, i.e. make a contribution to the alongshore velocity in the direction of long, free wave propagation. The second and third terms, however, can be either positive or negative depending on the profile of \( v_0 \). For the cases investigated in this paper, the boundary condition at the coast (3.2) and the interior ocean-continental margin matching condition (2.20) will tend to produce a profile of \( v_0 \) that is large at the coast and zero at \( \xi = -1 \), i.e. \( v_{0\xi} > 0 \). Thus, the second term will normally make a positive contribution and the third term a negative contribution. The sign of the alongshore velocity (3.12) will therefore depend on the relative magnitudes of the three terms.
**c. Lagrangian mean flow**

The alongshore Lagrangian mean flow (particle drift velocity) is the alongshore Eulerian mean flow plus the alongshore component of the Stokes drift, (Longuet-Higgins, 1959),

\[ \mathbf{v}_L = \langle v \rangle + v_S, \quad (3.18) \]

where the Stokes drift with scaling (2.6) is given approximately by

\[ v_S = \epsilon \langle \int u_0 dv \xi + \frac{t}{t} \int v_0 dv \gamma \rangle + O(\epsilon^2), \quad (3.19) \]

and where \( \int dt \) indicates an indefinite time integral. Thus, the Stokes drift velocity is the same order as the alongshore Eulerian mean flow (3.12).

Following a procedure similar to that used to derive (3.12) we find,

\[ v_{1S} = \frac{H}{\xi}(\gamma^2 + \kappa^2)\langle u_0^2 \rangle + \lambda / \omega \langle v_0^2 \rangle. \quad (3.20) \]

Substituting (3.20) and (3.12) into (3.18) we have,

\[ v_{1L} = \frac{7}{20}\langle v_0 v_0 \rangle - \sqrt{2} \langle \gamma v_0 \rangle + \frac{\lambda}{\omega} \langle v_0^2 \rangle, \quad (3.21) \]

which in dimensional variables is

\[ v_L' = \frac{7}{20}f^{-1}\langle v' v' \rangle - \sqrt{2}(\delta_{\xi \xi} f^2)^{-1}\langle v' v' \rangle + \lambda'/\omega' \langle v'^2 \rangle. \quad (3.22) \]

**d. Topography**

The bottom topography used in the model, \( H(\xi) \), is an alongshore averaged depth profile from the Oregon continental margin. Digitized topography (Peffley and O'Brien, 1976) was smoothed, then averaged over
an alongshore distance of 100 km to produce a smooth profile (Figure 2). An exponential tail was fitted to the profile to extend the depth from 1000 meters, the maximum depth of the digitized data, to 2000 meters, the interior ocean depth. The above procedure produced a depth profile that is smooth, continuous, and monotonically increasing with distance offshore (Figure 3).

e. Forcing

The wind-stress, (2.2), can force the inviscid shelf motion by two separate mechanisms, coastal forcing, where the shelf currents are driven by the boundary conditions at $\xi = \xi_0$ (3.2) ($\phi = \pi/2$, $k = 0$), and wind-stress curl forcing, where the shelf currents are driven by the boundary condition at $\xi = -1$ (2.20) ($\phi = 0$, $k = 2\pi$).

We have found that the wind-stress curl forced response, for reasonable values of $\tau_0$, is small relative to the coastal forced response. For a given $\tau_0$, wind-stress curl forcing generates an $O(1)$ alongshore velocity at $\xi = -1$ whose magnitude varies inversely with frequency (2.20). To help clarify the nature of the response on the continental margin to interior forcing, rather than holding $\tau_0$ constant we vary $\tau_0$ with frequency such that the magnitude of the oceanic velocity at $\xi = -1$ is fixed at $v'(x = -1) = V$ (where $V' = 1$ cm s$^{-1}$).

We have found that when $\omega/k < 0$, i.e., for either the coastal or interior forcing traveling in the direction opposite to that of long, free waves, the response away from a narrow horizontal boundary layer at $\xi = \xi_0$ is approximately $10^{-2}$ times the response for $\omega/k > 0$. Consequently, $\omega$ and $\xi$ are limited to values such that $\omega/k > 0$. 
4. Analytical example

The $O(1)$ depth integrated vorticity equation, (3.1), may be solved analytically for the two-dimensional case where $(\cdots)_y = 0$ for all variables. We may then obtain analytical expressions for the $O(1)$ velocities and the $O(\epsilon)$ mean alongshore velocity for arbitrary $H(\xi)$. The $O(1)$ vorticity equation, (3.1), with the above assumption, may be written as

$$ (Hv_0 + \gamma v_0) = 0. \tag{4.1} $$

Similarly, the boundary condition at the coast, $\xi = \xi_0$, becomes

$$ Hv_0 + \gamma v_0 = \sqrt{2} \gamma \cos(\omega t), \text{ at } \xi = \xi_0, \tag{4.2} $$

where a wind-stress,

$$ \tau = \cos(\omega t), \tag{4.3} $$

consistent with the above assumption is used.

The forced solution to (4.1) with the boundary condition (4.2) is

$$ v_0 = \sqrt{2} \gamma [(\omega H)^2 + \gamma^2]^{-1}[\omega H \sin(\omega t) + \gamma \cos(\omega t)]. \tag{4.4} $$

The boundary condition at $\xi = -1$, (2.20), with the wind-stress (4.3) ($\epsilon \equiv 0$, $\kappa \equiv 0$), is also satisfied by (4.4). The $O(1)$ cross-shelf velocity, from (2.7b) and (4.4), is

$$ u_0 = -\sqrt{2} \gamma \omega[(\omega H)^2 + \gamma^2]^{-1}[\omega H \cos(\omega t) + \gamma \sin(\omega t)]. \tag{4.5} $$

The above expressions for $u_0$ and $v_0$ satisfy (3.15).

The expression for the $O(\epsilon)$ Eulerian alongshore mean velocity, (3.7), may be simplified by utilizing (2.7b), (4.1), and (4.2), to yield
\[ <v_1> = -(13/20) <v_0 v_0>. \]  
(4.6)

Substituting (4.4) into (4.6) we obtain,

\[ <v_1> = (13/20) \gamma^2 \omega^2 HH_z [(\omega H)^2 + \gamma^2]^{-2}, \]  
(4.7a)

which in dimensional variables is,

\[ <v'_1> = \frac{13}{40} \frac{H'_3 f^2 p^2}{H'_x (\omega'/f)^2 + \delta_E^2 (H'/H')^2}. \]  
(4.7b)

Similarly, we find the Lagrangian mean flow by utilizing (4.2), (4.4), and (4.7) with (3.21), to obtain,

\[ v'_{1L} = (33/13) <v_1>. \]  
(4.8)

For vanishingly small friction, i.e., in the limit, \( \delta_E' \rightarrow 0 \), (4.7b) becomes,

\[ \lim_{\delta_E' \rightarrow 0} <v'_1> \approx (13/40) \frac{H'_x^2 (H'_3 f^2 p^2 \omega^2)^{-1}}{\delta_E'}. \]  
(4.9)

Thus, in the limit, \( \delta_E' \rightarrow 0 \), the Eulerian and Lagrangian alongshore mean velocities are nonzero (see Section 5a).

The \( \mathcal{O}(1) \) alongshore velocity, for a forcing period of 5 to 12 days and a wind-stress \( \tau_0 = 1.4 \) dynes.cm\(^{-2}\), has a maximum amplitude of \( \approx 35 \) cm s\(^{-1}\) at \( \xi = \xi_0 \) and decays to \( \leq 1 \) cm s\(^{-1}\) at \( \xi = -1 \). The cross-shelf velocity has an amplitude that is \( \delta_\omega = 0.06 \) times the alongshore velocity.

The mean Eulerian and Lagrangian alongshore velocities, for the depth profile used here, \( (H_\xi < 0) \), are negative everywhere, i.e., opposite to the direction of propagation of long, free shelf waves. The mean Eulerian velocity has a maximum of \(-1.5 \) cm s\(^{-1}\) at \( \xi = \xi_0 \) and decays
quickly offshore. The magnitude at the shelf break, i.e., the 200 meter isobath, is less than \(-0.05 \text{ cm s}^{-1}\). Thus, the flow is restricted to a negative jet at the coast with negligible velocities offshore.

A relationship between the nonlinear interactions that occur in the surface and bottom Ekman layers and the interior may be inferred from the fact that the depth integrated cross-shelf velocity, \(\int_{-H}^{0} u \, dz\), is zero for all \(\zeta\). Consequently, we can write the \(O(\varepsilon)\) depth integrated cross-shelf velocities, utilizing (3.4), (A.8b), and (A.15b), as,

\[
0_{-H}^{0} udz = 0 = -(\delta E / \delta)(\langle v_{1z}^{B}(\zeta=0) \rangle + \int_{0}^{\infty} (\langle u_{0}^{B} v_{0z}^{B} \rangle - \langle v_{0z}^{B} \int_{0}^{\zeta} u_{0z}^{B} d\zeta \rangle) d\zeta) \\
- \langle v_{0z}^{B} [Hu_{0} + (\delta E / \delta) (\int_{0}^{\infty} u_{0z}^{B} d\zeta + \int_{-\infty}^{0} u_{0z}^{T} d\eta)] \rangle
\]

(4.10)

The condition \(\int_{-H}^{0} udz = 0\) also implies that the last term in (4.10) is identically equal to zero, so that

\[
\langle v_{1z}^{B}(\zeta=0) \rangle = - \int_{0}^{\infty} (\langle u_{0}^{B} v_{0z}^{B} \rangle - \langle v_{0z}^{B} \int_{0}^{\zeta} u_{0z}^{B} d\zeta \rangle) d\zeta.
\]

(4.11)

When the bottom Ekman layer results (A.8), (A.10) and (A.12) are utilized (4.11) reduces to (4.6). Thus, the contribution to the alongshore mean flow \(\langle v_{1} \rangle\) from the interior is canceled by the contributions from the surface and bottom Ekman layers and \(\langle v_{1} \rangle\) is forced by the remaining terms from the bottom Ekman layer. This result demonstrates that the mean flows generated by interactions within the surface and bottom boundary layers can make a contribution as important to the mean alongshore flow as those in the interior.
The mean Eulerian velocity for this example is negative everywhere, whereas, the observed mean alongshore velocities at eastern boundaries are positive (Wooster and Reid, 1963). The two dimensional case, however, has the primary deficiency that the dominant response to forcing at the free shelf wave resonances cannot be investigated.
5. Discussion

a. Frictional effects

The existence of friction is necessary to obtain nonzero values of \(<u_0 v_0 >\) the forcing term in the time averaged O(\(\varepsilon\)) momentum equations. Once the solution is obtained, the limiting case \(\delta_E' \to 0\) may be examined.

In the limit, \(\delta_E' \to 0\), (3.13) becomes

\[
\lim_{\delta_E' \to 0} <u'> = 0. 
\]  

(5.1)

Since \(<u_0 v_0 >\) goes to zero as \(\delta_E' \to 0\), (3.8), the above result is expected. However, a nonzero limit is obtained in (3.14):

\[
\lim_{\delta_E' \to 0} <v'> = -H_x' f(H'w'^2)^{-1} <u'^2> + (7/20)f'^{-1} <v_x'v'> - T(x,y,t), 
\]  

(5.2a)

which equivalently is,

\[
\lim_{\delta_E' \to 0} <v'> = -H_x' [f<\int u'dt'>]^t' - (7/20) <v'\int u'dt'> - T(x,y,t), 
\]  

(5.2b)

where (3.1), with \(\delta_E' \to 0\), has been utilized and

\[
T(x,y,t) = \lim_{\delta_E' \to 0} \sqrt{2}(\delta_E' \rho f^2)^{-1} <v_x'>. 
\]  

(5.3)

The term \(T\), (5.3), is finite since the component of \(v_x'\) that is in phase with \(\tau'\) is proportional to \(\delta_E'\) (Brink and Allen, 1978). Similarly, the Lagrangian mean flow, (3.22), may be written as

\[
\lim_{\delta_E' \to 0} v'_L = (7/20)f'^{-1} <v_x'v'> + \ell'/\omega' <v'^2> - T(x,y,t). 
\]  

(5.4)
The nonzero value of $\lim_{\delta E \to 0} <v'>$ may be explained with reference to the boundary condition at $\xi = \xi_0$ (2.10). The $O(\varepsilon)$ interior onshore transport $H<u_1> \approx H\varepsilon E_u u_0^2$, which scales with $\delta E$, and the surface Ekman layer transport $\approx \delta E u_1^T$ are balanced by a transport in the bottom Ekman layer at $\xi = \xi_0$, which also scales with $\delta E$, i.e., $H(\xi_0)u_1 + \delta E u_1^T \approx \delta E u_1^B$. Thus, the velocity in the bottom Ekman layer, $<u_1^B>$, has a component independent of $\delta E$, and since $<v_1> \approx <u_1^B> + \ldots$ (A.12), $<v_1>$ also has a component independent of $\delta E$.

A bottom boundary layer must satisfy two conditions for the limits (5.2) and (5.4) to hold. First, the stress in the layer must be proportional to $v_1$, i.e., $\tau = r(v + \ldots)$, where $r$ is a "resistance coefficient". Second, the transport in the layer must also be proportional to $v$, i.e., $\int_0^{z_0} u_1^B dz = rK(v + \ldots)$, where $z_0$ is the top of the layer and $K$ a constant independent of $r$. These conditions are satisfied when the bottom boundary layer is an Ekman layer. Bulk or slab layers based on Ekman dynamics should also give similar results.

The characteristic velocity used in Section 2 is inappropriate in the limit, $\delta E \to 0$, since $\lim_{\delta E \to 0} U = \infty$. A general characteristic velocity valid in this limit is determined by balancing the three terms in (3.2):

$$U = \tau_0[(\delta E/\delta + H(\xi_0))H_0^2f]^{-1}. \quad (5.5)$$

The appropriate characteristic velocity for $\delta E \to 0$, from (5.5), is

$$U = \tau_0[H(\xi_0)H_0^2f]^{-1}, \quad (5.6)$$

which is the characteristic velocity normally associated with forced inviscid shelf models.
b. Coastal forced results

The results for the coastal forced case were calculated numerically following the procedure outlined in Appendix B with the parameter values given in Table 2. The dimensional magnitude of the alongshore wind-stress, \( \tau' = \frac{\tau_0}{\sqrt{2}} \cos (\lambda y - \omega t) \), is chosen so the maximum value \( \frac{\tau_0}{\sqrt{2}} = \tau_m' = 1 \) dyne cm\(^{-2}\). The results presented here may be extended to other values of \( \tau_m' \), i.e., \( \tau^* \), by multiplying the \( 0(1) \) alongshore velocity \( v_0' \) by \( \tau^*/\tau_m' \) and the Eulerian and Lagrangian mean velocities \( <v'> \) and \( v_L' \) by \( (\tau^*/\tau_m')^2 \).

\( 0(1) \) ALONGSHORE VELOCITY The amplitude and phase of the \( 0(1) \) alongshore velocity, \( v_0' = |v_0'| \cos (\lambda y - \omega t + \phi) \) are plotted as functions of \( \xi \) and the parameter \( \alpha = \omega'/f = \omega \) in Figure 4.

The \( 0(1) \) alongshore velocity amplitude response has two local maxima which correspond to the mode 1 resonant response at \( \alpha = 0.24 \) and the mode 2 resonant response at \( \alpha = 0.06 \). A maximum amplitude of \( \approx 35 \) cm s\(^{-1}\) occurs at \( \xi = \xi_0 \) for \( \alpha = 0.24 \), i.e., the mode 1 resonant frequency. There are two maxima associated with the mode 2 resonant response. A maximum of \( \approx 5 \) cm s\(^{-1}\) at \( \xi = -0.7 \) and \( \approx 25 \) cm s\(^{-1}\) at the coast, \( \xi = \xi_0 \).

The \( 0(1) \) alongshore velocity phase, referenced to the wind-stress, has its largest change with frequency at a resonance. At the mode 1 resonance the phase is nearly constant cross-shelf, equal to \(-6^\circ\) at \( \xi = \xi_0 \) and \( 0^\circ \) at \( \xi = -1 \) so that nearshore fluctuations lead those offshore. This compares with a \( 22^\circ \) phase difference in the same sense for a free wave (Appendix C). At the mode 2 resonant frequency, the phase is \( 0^\circ \) at \( \xi = \xi_0 \), \( -180^\circ \) at \( \xi = -1 \), with 97% of the cross-shelf phase difference occurring between \( \xi = \xi_0 \) and \(-0.5 \), where offshore fluctuations lead those nearshore.

EULERIAN MEAN FLOW The Eulerian and Lagrangian mean alongshore velocities, \( <v'> \) and \( v_L' \), are plotted as functions of \( \xi \) and \( \alpha \) in Figure 5. Eulerian
and Lagrangian mean velocity profiles are plotted as a function of $\xi$ for different values of $a$ in Figure 6.

The Eulerian mean velocity response, in general, has two frequency regimes, a resonant and an off-resonant response (Figure 5). The mode 1 resonant response ($a = 0.24$) has a positive jet of $\approx 1 \text{ cm s}^{-1}$ at $\xi = \xi_0$ and a negative maximum of $\approx -0.5 \text{ cm s}^{-1}$ at $\xi = -0.8$ (Figure 6). At the mode 2 resonant frequency, a maximum of $\approx -0.5 \text{ cm s}^{-1}$ is located at $\xi = -0.85$ and a coastal jet is absent. The off-resonant response consists of a jet of $\approx 1 \text{ cm s}^{-1}$ near $\xi = \xi_0$.

The Eulerian and Lagrangian integrated mass transports,

$$M_E = \int_{-\delta L}^{\delta L} \langle v' \rangle \, dx', \quad \text{and} \quad M_L = \int_{-\delta L}^{\delta L} v'_L \, dx', \quad (5.7)$$

are plotted as functions of $a$ in Figure 7. The magnitude of the alongshore mean transport in the surface and bottom Ekman layers is the order of $\gamma / H(\xi)$ times the shelf contribution, $\langle v_1 \rangle$. This is a small correction, except near $\xi = \xi_0$, and has been ignored.

The large peaks in mass transport occur at the free wave resonances. The offshore velocity maxima in deep water are the principle cause for the mass transport peaks. Although the highest velocities occur away from a resonance, these maxima are near the coast in shallow water and the resultant mass transport is small relative to the mass transport at resonance. A sign change in mass transport occurs near $a = 0.16$; the transport is negative for mode 1 and positive for mode 2. The maximum negative transport of $\approx -0.2 \times 10^{12} \text{ cm}^3 \text{ s}^{-1}$, associated with mode 1, is approximately twice the maximum positive transport of $\approx 0.1 \times 10^{12} \text{ cm}^3 \text{ s}^{-1}$, associated with mode 2.

LAGRANGIAN MEAN FLOW The Lagrangian response has two frequency regimes that are similar to those in the Eulerian response (Figure 5). The
Lagrangian resonant response is also characterized by a positive coastal jet of 2 cm s\(^{-1}\) at the mode 1 and mode 2 resonant frequencies. In general, the Lagrangian response has larger regions of positive flow over the shelf.

The Lagrangian and Eulerian mean alongshore flows, Figure 6, are nearly identical from the interior ocean-continental margin junction to the 400 meter isobath (\(\xi = -0.6\)), i.e., the Stokes drift, \(v_S\), is small in deep water. The Stokes drift is larger than the Eulerian mean flow inshore of the 400 meter isobath.

The Lagrangian alongshore mass transport, Figure 7, is not qualitatively different from the Eulerian mass transport. The principle differences occur at lower frequencies, i.e., \(\alpha \leq 0.05\). The peaks in the mass transport are caused by the resonant responses with the main contribution coming from the alongshore velocities offshore of the 200 meter isobath.

The frequencies at which resonance occurs are dependent on several parameters. These parameters either have a large natural variation, e.g., the shelf width, or are not well known, e.g., the atmospheric length scales of \(\tau\). We can expect that given a forcing function with a reasonably smooth wave-number frequency spectrum the resonant response will dominate.

c. Interior forced results

The results for the interior forced case were calculated numerically by varying \(\tau_0\) with frequency (Section 3e) and applying the procedure described in Appendix B. A dimensional magnitude for the imposed alongshore oceanic velocity, \(V'\), of 1 cm s\(^{-1}\) is used. The results presented may be extended to other values of \(V'\), i.e., \(V^*\), by multiplying the O(1) alongshore velocity \(v_0'\) by \(V^*/V'\) and the Eulerian and Lagrangian mean velocities by \((V^*/V')^2\).
**0(1) ALONGSHORE VELOCITY**  The 0(1) alongshore velocity amplitude response is similar to the coastal forced response (Figure 4). The response is negligible away from the resonant frequencies. A maximum of 30 cm s\(^{-1}\) at \(\xi = -0.1\) is associated with the mode 1 resonance, \(\alpha = 0.24\), and a maximum of 3 cm s\(^{-1}\) occurs at the mode 2 resonant frequency, \(\alpha = 0.06\). The mode 2 resonant response for interior forcing has a single maximum at \(\xi = -0.1\).

The 0(1) alongshore velocity phase, referenced to the imposed oceanic velocity \(V\), rapidly decreases from 0° at \(\xi = -1\) to -120° at \(\xi = -0.8\) and remains nearly constant until \(\xi = -0.2\) where the phase decreases to -135° at \(\xi = \xi_0\), for the mode 1 resonant response. The phase associated with the mode 2 resonant response decreases monotonically, with a nearly constant slope, from 0° at \(\xi = -1\) to +45° at \(\xi = \xi_0\), going through a value of -180° at \(\xi = -0.4\).

**EULERIAN MEAN FLOW**  The interior forced Eulerian and Lagrangian mean alongshore velocities, \(<v'>\) and \(v'_L\), are plotted in Figure 8 as functions of distance offshore, \(\xi\), and frequency, \(\alpha\).

The Eulerian velocity is positive, i.e., in the direction of propagation of long, free shelf waves, everywhere, in contrast to the coastal forced mean flow. The velocity response has two frequency regimes, a resonant and an off-resonant response. The resonant response has a magnitude that is several times larger than the off-resonant response and a maximum that usually occurs near the shelf break (\(\xi = -0.3\), i.e., the 200 meter isobath. A narrow positive jet, situated between \(\xi = \xi_0\) and -0.1, occurs at the mode 1 resonant frequency.

The mode 1 resonant response has two maxima of 0.4 cm s\(^{-1}\) occurring at \(\xi = -0.3\) and -0.7, i.e., the 200 and 600 meter isobaths, respectively.
A break in slope, Figure 3, is associated with each maximum. The response at the mode 2 resonance is negligible relative to the mode 1 response.

The Eulerian alongshore mass transport, Figure 9, has maxima occurring at the free wave resonances. The transport at the mode 1 resonance is \( \approx 0.1 \times 10^{12} \text{ cm}^3 \text{ s}^{-1} \).

**LAGRANGIAN MEAN FLOW** The Lagrangian mean velocity is primarily positive and has its largest values at the free wave resonances. The response associated with the mode 1 resonance is larger than the mode 2 response. In general, the maxima at a resonance are inshore of the shelf break.

The mode 1 resonant response is a single maximum of 1.3 cm s\(^{-1}\) at \( \xi = -0.15 \). The response extends offshore of \( \xi = -0.4 \), where the magnitude has dropped to 0.8 cm s\(^{-1}\). A relatively small response, associated with the mode 2 resonance, of 0.1 cm s\(^{-1}\) occurs at \( \xi = -0.15 \).

The Eulerian and Lagrangian mass transports are nearly identical. The Lagrangian transport is typically 97% of the Eulerian transport.

d. Comparison with other theory

Mean flow generation in a homogeneous fluid over a shelf was studied by Huthnance (1981) (henceforth referred to as H). Small amplitude oscillations and weak friction were assumed. A solution was found by assuming the Rossby number small and expanding in Rossby number. An expression for the mean alongshore velocity \( <v_1> \) given an incident oscillatory current, parameterized bottom stress \( (\tau^B = ry) \), and infinitesimal friction was derived in H (his (5.2)).

We may compare (5.2) in H with (5.2b) here by including in the former the bottom boundary layer momentum flux term neglected in the derivation of (5.2) (that term is in the paragraph containing (2.3) in H).
and a uniform eddy viscosity model, i.e., a bottom Ekman layer. The alongshore bottom stress, for an Ekman layer, is, for $O(1)$,

$$\tau_0^B = r\nu_0,$$  \hspace{1cm} (5.8a)

where $r$ is a 'resistance' coefficient, and for $O(\epsilon)$,

$$\langle \tau_1^B \rangle = r(\langle v_1 \rangle + (3/20) H^{-1}H_\xi^t v_0 \int u_0 dt).$$  \hspace{1cm} (5.8b)

Substituting the above expressions for the bottom stress (5.8a,b) into (3.8) in H we obtain,

$$\langle v_1 \rangle = -H^{-1}H_\xi^t \left[ \langle \int u_0 dt \rangle_2^t - (7/20) \langle v_0 \int u_0 dt \rangle \right],$$  \hspace{1cm} (5.9)

which replaces (5.2) in H.

If we assume that no alongshore gradients exist, then (5.9) may be written as,

$$\langle v_1 \rangle = (27/40) \langle \int u_0 dt \rangle_\xi^t,$$  \hspace{1cm} (5.10)

which is identical to the result given in the paragraph following (5.5) in H, where similar assumptions and bottom stress are used. For no alongshore gradients, (5.2) in H is,

$$\langle v_1 \rangle = \langle \int u_0 dt \rangle_\xi^t.$$  \hspace{1cm} (5.11)

Comparing (5.10) and (5.11) above, we see that, for no alongshore gradients the derived alongshore mean flow is 1.48 times larger if, as in H (5.2), the bottom layer momentum flux term is neglected and a parameterized bottom stress, $\tau^B = r\nu$, is assumed.

A linear model of the wind-forced viscous flow near a continental boundary that retained finite thickness surface and bottom Ekman layers
was developed by Spillane (1980). The model makes use of a homogeneous fluid, alongshore invariant topography, and a spectral form for the wind-stress.

To assess the effects of a finite coastal wall at $\xi = \xi_0$ (3.2), the $O(1)$ results obtained here were compared with those from the Spillane model in which the inclusion of vertical gradients in the analysis allows a coastal wall height, $H(\xi_0)$, of zero. The present model was run with the topography used by Spillane. Excellent agreement between the sea surface elevation from his model ($H(\xi_0) = 0$) and the $O(1)$ pressure here was found for values of $H'(\xi_0)$ from 0 to 34 meters. This insensitivity to the value of wall height was also found by Spillane, i.e., for $H(\xi_0) \leq \sqrt{2} \gamma$ the flow offshore of the boundary was unchanged.

e. Comparison with observations

Evidence for a relatively narrow core of poleward flow off southern Washington was presented by Hickey (1979). The mean flow was observed from current measurements taken from July to September 1972 at 46°N. The core was found to be 20 km wide with a maximum velocity of 16 cm s$^{-1}$. The core occurred at mid-depth in 600 meters of water.

The present model demonstrates that forced barotropic shelf waves with bottom friction can, through non-linear interactions, generate a mean flow with nonnegligible amplitudes. The mean flow generated has a response that is frequency dependent, and a maximum alongshore mass transport at the free wave resonant frequencies. The predominant sign of the mean alongshore velocity depends on the forcing type used.
The mean flow generated by coastal forcing is usually equatorward, i.e., in a direction opposite to the direction of propagation of long, free shelf waves, whereas, the undercurrents at eastern boundaries are observed to be poleward. For this forcing, offshore poleward flows are found only in connection with the mode 2 resonance (see Figure 5). This flow is offshore at the 200 meter isobath with a width of 5-20 km.

For interior forcing, the mean flow generated is poleward, although the magnitude is generally smaller than the coastal forced mean currents. The largest response occurs at the mode 1 resonant frequency. The core of poleward flow is usually centered over the shelf break and has a width of ≈20 km.
6. Summary

The generation of a mean flow over a continental margin by the self interaction of long, forced shelf waves has been studied. In the presence of dissipation provided by bottom friction, the interaction generates a mean flow. The physical reason for the existence of the alongshore mean velocity can be seen from the vertically integrated vorticity balance (2.10). The local mean vorticity change generated by \( \langle u_0 v_0 \rangle \xi \), the transport of potential vorticity, is equal to the stretching caused by the difference in the mean vertical velocity at the surface and bottom boundaries. The above vorticity transport is nonzero when friction is present, i.e., when \( \gamma \neq 0 \).

The expressions for the mean vertical velocities at the surface and bottom boundaries are given by (A.19) and (A.13), respectively. The mean vertical velocity at \( z = 0 \), \( \langle w_1(z=0) \rangle \), is driven by the mean vorticity transport in the surface Ekman layer, \( \langle u_0 v_0 \xi \rangle \). The mean vertical velocity at \( z = -H \), \( \langle w_1(z=-H) \rangle \), is driven by three mechanisms. The first is Ekman pumping which is proportional to the shelf vorticity, \( \langle v_1 \xi \rangle \). The second is \( \langle v_0 v_0 \xi \rangle \xi \) the transport of vorticity in the bottom Ekman layer. The third is the interaction of the mean onshore velocity \( \langle u_1 \rangle \) with the bottom slope. The mean onshore velocity \( \langle u_1 \rangle \) is determined by the gradient of the Reynolds stress \( \langle u_0 v_0 \xi \rangle \), (3.4). Since all terms (except \( \langle v_1 \xi \rangle \)) in the vorticity balance, (2.10), are determined by the transport of vorticity and the gradient of a Reynolds stress, \( \langle v_1 \xi \rangle \) is determined by \( O(1) \) variables.

The mean alongshore velocity \( \langle v_1 \rangle \) may also be explained by mass conservation arguments. The net cross-shelf flow in the interior and Ekman layers driven by the gradients of Reynolds stresses must be balanced by a transport in the bottom Ekman layer. That transport demands the existence of a mean alongshore velocity, (A.12).
Expressions are derived for \( \langle v_1 \rangle \) (3.12), the alongshore mean flow, and \( \langle u_1 \rangle \) (3.11), the cross-shelf mean flow, that depends only on \( O(1) \) variables and model parameter values. An expression for \( R(u_0, v_{0E}) \), the cross-correlation coefficient for \( u_0 \) and \( v_{0E} \), that depends only on the topography, friction, and forcing frequency was derived in (3.15). This form for \( R(u_0, v_{0E}) \) is independent of the boundary conditions at \( \xi = -1 \) and \( \xi_0 \), and the \( O(1) \) variables.

The alongshore mean velocity, \( \langle v' \rangle \), was found to be independent of \( \delta_E \) as \( \delta_E \to 0 \) (5.2). For \( H(\xi_0) \geq \sqrt{2} \gamma \), the presence of infinitesimal friction is sufficient to generate a finite alongshore mean flow, i.e., \( \lim_{\delta_E \to 0} \langle v' \rangle \neq 0 \).

It was shown that the momentum flux and Reynolds stress in the surface and bottom boundary layers contribute to the mean flow. We also determined that for \( H(\xi_0) \approx \sqrt{2} \gamma \) the model results are not sensitive to coastal wall height, \( H(\xi_0) \).

On eastern boundaries the alongshore Eulerian and Lagrangian mean flow, forced by an alongshore wind-stress at the coast, has its largest response associated with the first mode resonance and is directed equatorward. The second largest response is associated with the second mode resonance and directed poleward.

The response forced by an imposed oceanic alongshore velocity is a poleward mean velocity with a maxima near the shelf break. The observed eastern boundary undercurrents are poleward and exhibit an offshore maximum.

We conclude that the model results for coastal forcing are inconsistent with the observations of eastern boundary currents, whereas the results for interior forcing are qualitatively consistent with observations.

Acknowledgments

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## Chapter II

Table 1. Free wave mode characteristics.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\frac{\omega'}{\beta}$</th>
<th>$\frac{\beta'}{f}$</th>
<th>$\frac{2\pi}{\omega'}$ days</th>
<th>$\frac{\beta'}{}$ days</th>
<th>$\frac{\omega'}{\ell'}$ km/day</th>
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<tbody>
<tr>
<td>1</td>
<td>0.246</td>
<td>0.0142</td>
<td>3.0</td>
<td>8.2</td>
<td>333</td>
</tr>
<tr>
<td>2</td>
<td>0.061</td>
<td>0.0197</td>
<td>11.9</td>
<td>5.9</td>
<td>84</td>
</tr>
<tr>
<td>3</td>
<td>0.025</td>
<td>0.0086</td>
<td>29.1</td>
<td>13.5</td>
<td>34</td>
</tr>
</tbody>
</table>
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Table 2. Summary of parameter values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f )</td>
<td>( 10^{-4} ) s(^{-1} )</td>
</tr>
<tr>
<td>( H_0 )</td>
<td>2 km</td>
</tr>
<tr>
<td>( L )</td>
<td>( 10^3 ) km</td>
</tr>
<tr>
<td>( \rho )</td>
<td>1 gm cm(^{-3} )</td>
</tr>
<tr>
<td>( \tau_0 )</td>
<td>1 dyne cm(^{-2} )</td>
</tr>
<tr>
<td>( \delta_E )</td>
<td>4 m</td>
</tr>
<tr>
<td>( \delta_E' )</td>
<td>0.002</td>
</tr>
<tr>
<td>( \ell )</td>
<td>2( \pi )</td>
</tr>
<tr>
<td>( k )</td>
<td>0</td>
</tr>
<tr>
<td>( \phi )</td>
<td>( \frac{1}{4} \pi )</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.094</td>
</tr>
<tr>
<td>( \xi_0 )</td>
<td>-0.05</td>
</tr>
<tr>
<td>( H(\xi_0) )</td>
<td>0.0169</td>
</tr>
<tr>
<td>( H'(\xi_0) )</td>
<td>33.8 m</td>
</tr>
</tbody>
</table>
Chapter II
Fig. 1
Schematic of model geometry.
Chapter II
Fig. 2
Oregon continental margin topography from Peffley and O'Brien (1976). Contour interval is in meters. Nonshaded region denotes extent of alongshore averaging for the depth profile $H(\xi)$. 
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Fig. 3
Depth profile $H(\xi)$ vs. distance offshore.
Chapter II
Fig. 4
Amplitude (left) and phase (right) of coastal forced $O(1)$ alongshore velocity as functions of $\xi$, distance offshore, and $\alpha = \omega'/f$, a nondimensional forcing frequency. The amplitude contours are isotachs in cm s$^{-1}$ (contour interval = 5 cm s$^{-1}$) and phase contours are lines of constant phase in degrees (contour interval = 45 degrees).
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Fig. 5

Coastal forced alongshore mean velocity as a function of $\xi$ and $\alpha = \omega'/f$. The contours are isotachs in cm s$^{-1}$; the double weight contour is the zero cm s$^{-1}$ level. The free wave resonant frequencies for the first two modes are marked. Eulerian $v'$ (contour interval = 0.5 cm s$^{-1}$) and Lagrangian $v_L'$ (contour interval = 1 cm s$^{-1}$) alongshore mean velocities are displayed to the left and right, respectively.
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Fig. 6
The dimensional Eulerian (left) and Lagrangian (right) coastal forced alongshore velocity $v$ vs. $\xi$, distance offshore. The dot-dashed, solid, and dashed lines correspond to mode 1 ($\alpha = \omega'/f = 0.24$) and 2 ($\alpha = 0.6$) resonant frequencies, and a non-resonant ($\alpha = 0.16$) frequency, respectively. The non-resonant frequency is chosen such that the Eulerian mass transport is a minimum.
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Fig. 7
Eulerian (left) and Lagrangian (right) coastal forced mass transport vs. $\alpha = \omega' / f$. Transport is in $10^{12} \text{ cm}^3 \text{ s}^{-1}$. 
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Fig. 8

Interior forced alongshore mean velocity as a function of $\xi$ and $\alpha = \omega'/f$. Notation is the same as for Figure 5 (contour interval is 0.1 cm s$^{-1}$ for Eulerian and 0.2 cm s$^{-1}$ for Lagrangian).
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Fig. 9
Eulerian (left) and Lagrangian (right) interior forced mass transport vs. $\alpha = \omega' / f$. Transport is in $10^{12} \text{ cm}^3 \text{s}^{-1}$. 
Free wave dispersion curves for modes 1-3. Parameter values used are listed in Tables 2 and 3. The dispersion curve plotted as a function of $\omega'/f$ and $\lambda$ and as a function of $\beta'/f$ and $\lambda$ are displayed in the top and bottom rows, respectively.
REFERENCES


Chapter III

ROTARY EMPIRICAL ORTHOGONAL FUNCTION ANALYSIS
OF CURRENTS NEAR THE OREGON COAST
1. Introduction

Empirical orthogonal function (EOF) analysis in the frequency domain was developed for applications in meteorological data analysis by Wallace and Dickinson (1972). It has been applied to oceanographic problems by Wang and Mooers (1977), Hogg (1981) and Halliwell and Mooers (1983). Frequency domain EOF analysis involves computing a band-averaged cross-spectral matrix for several real time series and then calculating the eigenvectors and eigenvalues of that matrix. The complex eigenvectors are used to compute the EOF amplitude and phase.

Rotary spectral analysis is discussed in detail in Gonella (1972) and Mooers (1973). It involves writing a horizontal velocity vector time series as a complex one, where the real and imaginary parts are orthogonal velocity components, and then taking the Fourier transform of the complex time series, which gives Fourier coefficients of positive and negative frequency. Rotary spectra may be interpreted in terms of the hodograph model for two-dimensional velocity vectors (Mooers, 1973). In the hodograph model, the autospectra for positive and negative frequencies are interpreted as resulting from two independent vectors rotating anticlockwise and clockwise, respectively. When these vectors are added together, the resultant vector rotates clockwise or anticlockwise and traces an ellipse whose characteristics are described in Gonella (1972).

In this paper we combine frequency domain EOF analysis and rotary spectral analysis to calculate rotary empirical orthogonal functions.
The rotary EOF analysis is formally similar to single-sided frequency domain EOF analysis. The primary difference is that the time series are complex, rather than real, and the frequency is two-sided, i.e., positive and negative. The extension of single-sided frequency domain EOF analysis with real functions to rotary, two-sided frequency domain EOF analysis with complex functions is analogous to the extension of time domain EOF analysis from real to complex time series described in Kundu and Allen (1976).

The method presented here is applied to velocity and temperature measurements from the Coastal Upwelling Experiment (CUE-II) which took place during the summer of 1973 on the continental shelf off Oregon. Some of the results from the rotary EOF decomposition introduced here may be compared to those obtained from the same data using different techniques. In particular, comparisons are made with results from the response method analysis of the $M_2$ and $K_1$ tides in Torgrimson and Hickey (1979) and with those from the study of near-inertial frequency motions in Kundu (1976).

This investigation consists of two parts. In the first, standard rotary auto and cross-spectral analyses are applied. In the second, rotary EOF decomposition is utilized. The investigation concentrates primarily on the semidiurnal, near-inertial and diurnal frequency bands and on low frequencies of less than 0.25 cycles per day (cpd).
2. Data

The data utilized are a subset of the observations made as part of the Coastal Upwelling Experiment (CUE-II) conducted during the summer of 1973 near 45° 16' N on the continental shelf off Oregon. The observations span nearly two months and include velocity and temperature measurements from five moorings (Fig. 1). The moorings Forsythia, Edelweiss, Carnation, and Aster were deployed by Oregon State University. The data are described in Kundu and Allen (1976). Buoy-B was deployed by the Pacific Marine Environmental Laboratory, and the data are described in Halpern (1976a). The measurements, originally taken at 10 - 15 minute intervals, were filtered with a lowpass filter having a half power point of 12 cycles per day (cpd) and then subsampled to hourly values.

The data are divided into two sets for the purposes of the following analyses. The first is a "vertical" data set which includes 11 horizontal current and 9 temperature measurements from the Buoy-B and Carnation moorings (Table 1). The two moorings were in 100 m of water and were less than 2 km apart. Since the coherence squared between the Buoy-B measurements at 18 m and the Carnation measurements at 20 m for the frequency bands used here are typically greater than 0.7, these two sets of measurements are treated as if they are from a single mooring. A common time period of 54 days beginning July 5, 1973 is used for the analyses. The second set is composed of "cross-shelf"
data which include current measurements at 16 positions from the Forsythia, Edelweiss, Carnation and Aster moorings (Table 2). A common time period of 32 days beginning July 25, 1973 is used for the analyses.

The wind measurements are from the surface mooring at Buoy-B. Hourly wind stress was computed from the hourly wind velocity, extrapolated to the 10 m level with a logarithmic wind profile, using an aerodynamic square law with a constant drag coefficient of 1.3 x $10^{-3}$ (Halpern, 1976b).
3. Rotary empirical orthogonal functions

We consider a set of \( N \) discrete complex time series

\[
w_j(t_m) = u_j(t_m) + iv_j(t_m), \quad m=1, \ldots, M, \tag{3.1}
\]

where \( u_j \) and \( v_j \) are real time series of orthogonal velocity components at position \( j \) (\( j=1, \ldots, N \)), \( t_m = m\Delta t \) is the discrete time, \( \Delta t \) is the sampling interval, \( M \) is the number of data points in the series and \( i = (-1)^{1/2} \). The Fourier coefficients for \( w_j(t_m) \) are

\[
W_j(f_n) = \frac{1}{M} \sum_{m=1}^{M} w_j(t_m) \exp(-2i\pi f_n t_m), \tag{3.2}
\]

\[
n = -M/2, \ldots, M/2-1,
\]

where \( f_n = n/(M\Delta t) \) is the discrete frequency. Positive and negative frequencies correspond to two dimensional, horizontal velocity vectors rotating in anticlockwise and clockwise directions at frequency \( |f_n| \), respectively.

The rotary empirical orthogonal functions (EOFs) are the eigenvectors \( e_j^i \) determined by solving the eigenvalue problem

\[
\sum_{j=1}^{N} c_{kj} e_j^i = \lambda^i e_k^i. \tag{3.3}
\]
where \( e_i^j \) is the \( i \)th \((i=1...N)\) eigenvector with corresponding eigenvalue \( \lambda_i \),

\[
C_{kj} = W_k^* W_j (B_k B_j)^{1/2} = \sum_{m=n-\Delta n}^{n+\Delta n} W_k^* (f_m) W_j (f_m) (B_k B_j)^{1/2}, \tag{3.4}
\]

is the weighted, band-averaged, rotary cross-spectral matrix centered at frequency \( f_n \) with bandwidth \((2\Delta n+1)/(M \Delta t)\). \( \Delta n \) is a positive integer,

\[
\left( \begin{array}{c} \vdots \\
(\quad) \\
\vdots 
\end{array} \right) = \sum_{m=n-\Delta n}^{n+\Delta n} \left( \begin{array}{c} \vdots \\
(\quad) \\
\vdots 
\end{array} \right), \tag{3.5}
\]

is the band-averaging operator, and \(^*\) denotes complex conjugate. The \( B_j \) are real weighting functions and may be used to normalize \( C_{kj} \) or to scale variables with different magnitudes. They are left unspecified at this time. Since \( C_{kj} \) is a Hermitian matrix, the eigenvalues are real and the eigenvectors are orthogonal. We normalize the eigenvectors so that

\[
\sum_{k=1}^{N} e_i^k * e_j^k = \delta_{ij}, \tag{3.6}
\]

where \( \delta_{ij} \) is the Kronecker delta function. Each eigenvector \( e_i^j \) may be multiplied by an arbitrary complex number \( g_i \), where \( g_i^* g_i = 1 \). We choose \( g_i \) such that \( e_i^j \) is real.
The Fourier coefficients may be expanded in terms of the eigenvectors $e_j^i$ such that

$$W_j(f_m) = \sum_{k=1}^{N} e_j^i Z_k^i(f_m) B_j^{-1/2}, \quad m = n - \Delta n, \ldots, n + \Delta n. \quad (3.7)$$

The $Z_k^i(f_m)$ are the rotary EOF amplitude functions and are obtained by multiplying (3.7) by $e_j^i$ and summing over $j$:

$$Z_i^i(f_m) = \sum_{j=1}^{N} e_j^i W_j(f_m) B_j^{1/2}, \quad m = n - \Delta n, \ldots, n + \Delta n. \quad (3.8)$$

The EOF amplitude functions are functions of frequency $f_m$ with a domain of $f_n \pm \Delta n/(M \Delta t)$, i.e., they are defined for the frequencies over which the rotary cross-spectral matrix is band-averaged.

The band-averaged cross-spectra for the rotary EOF amplitude functions are

$$Z_i^j Z_j^i = \sum_{k,n=1}^{N} e^i_k e^j_n = \lambda_i \delta_{ij}. \quad (3.9)$$

The rotary EOF amplitude functions are incoherent over the frequency band $f_n \pm \Delta n/(M \Delta t)$ and have band-averaged autospectra $\lambda_i$.

We point out that frequency domain EOFs and time domain EOFs are formally similar, where frequency and time are the respective independent variables. The band-averaged cross-spectral matrix is analogous to the covariance matrix. The cross-spectra between any
other complex time series and the EOF amplitude function may be computed directly using $Z^i(f_m)$, where the frequency band used for the cross-spectral calculations is the domain of the amplitude function.

The dimensional amplitude and the phase of the EOF are given by

$$A^i_j = |e^i_j| \left( \frac{\lambda^i_j}{B_j} \right)^{1/2}, \quad (3.10)$$

and

$$p^i_j = \tan^{-1} \left[ -\text{Im}(e^i_j) / \text{Re}(e^i_j) \right], \quad (3.11)$$

respectively, for mode $i$ and position $j$. Since $e^i_1$ is real, the EOF phase $p^i_j$ is relative to position $j=1$, i.e., $p^i_1 = 0$. The percent of the variance of the series at position $j$ explained by mode $i$ is given by

$$V^i_j = \left( \frac{A^i_j}{W^i_j W_j} \right) \times 100. \quad (3.12)$$

The percent of the variance explained $V^i_j$ and EOF phase $p^i_j$ may also be determined by computing the coherence squared and phase from the cross-spectra of the mode $i$ amplitude function with series $j$. Utilizing (3.7) and (3.9), we obtain

$$Z^i w_j = \lambda^i_{e^i_j} B_j^{-1/2}. \quad (3.13)$$
The coherence squared and phase from (3.13) are

$$\text{coh}^2_j = \frac{|Z_j^* W_j|^2}{(Z_j^* Z_j^*) W_j W_j^*} = e^{i \phi_j} = A_j^2 / W_j W_j^* = V_j^2 / 100,$$  

(3.14)

and

$$\phi_j = \tan^{-1}[\text{Im}(Z_j^* W_j)/\text{Re}(Z_j^* W_j)]$$

$$= \tan^{-1}[-\text{Im}(e_j^i)/\text{Re}(e_j^i)] = p_j^i, (3.15)$$

respectively. Thus, as with single-sided frequency domain EOFs (Wallace and Dickinson, 1972) the coherence squared between the EOF amplitude function and series j can be interpreted as the percent of the variance of that series explained by the EOF. Also, we see that the phase from the cross-spectra between the EOF amplitude function and series j is the same as the EOF phase.

The trace of the cross-spectral matrix is

$$\sum_{k=1}^{N} C_{kk} = \sum_{i=1}^{N} \chi_i = \sum_{k=1}^{N} W_k^* W_k B_k.$$  

(3.16)

Thus, if the weighting functions $B_k$ are not 1, the trace of $C_{kj}$ is not
equal to \( \sum_{k=1}^{N} W^*_k W_k \), the sum of the band-averaged autospectra of \( w_k(t_m) \). Since \( C_{kj} \) depends on \( B_k \), the total variance explained by mode \( i \), \( \lambda^i \), depends on the weighting.

The rotary statistical quantities presented in Gonella (1972), such as the rotary coefficient, ellipse stability and axis orientation, may be applied to the EOF amplitude functions. We note that the formulas presented in this section are also valid for single-sided EOF analysis if a single-sided Fourier transform is used in place of (3.2). One utility of the method described here is that it allows the direct computation of the cross-spectra between an EOF amplitude function and any other complex series. The same method applies, of course, for single-sided EOFs. This makes the construction of the augmented time series, introduced in Wallace and Dickinson (1972), unnecessary in either case.
4. Rotary autospectra

Rotary spectra are computed for both data sets described in Section 2. Rotary spectra for wind stress measured at Buoy-B and for currents measured at B14 are plotted in Fig. 2. For the wind stress, there are significant peaks in the spectra at the diurnal frequency (±0.98 cpd). For the currents at B14, there are significant peaks in the spectra at the near-inertial (-1.54 cpd) and semidiurnal (±1.91 cpd) frequencies. These peaks, as well as one at the diurnal (+0.98 cpd) frequency, are present in most current spectra. The frequency at which the near-inertial spectral peak occurs (-1.54 cpd) is 1.08 times the local inertial frequency, in agreement with the results of Kundu (1976) from the same data. There are 10 degrees of freedom per frequency band for the spectral and EOF analysis corresponding to bandwidths of 0.093 cpd and 0.16 cpd for the vertical and cross-shelf data sets, respectively.

Rotary autospectral statistics (Gonella, 1972) computed for the vertical and cross-shelf data sets are summarized in Figs. 3 and 4. The rotary coefficient

\[ C_r(f_n) = \frac{(S_+ - S_-)}{(S_+ + S_-)}, \quad \text{(4.1)} \]

where

\[ S_\pm = W^* (\pm |f_n|) W(\pm |f_n|), \quad \text{(4.2)} \]
gives the partition of the total variance between clockwise and anticlockwise rotation. The outer autospectrum

\[ R = W(\{|f_n|\})W(-\{|f_n|\}), \]  

(4.3)

is used to compute the ellipse stability squared

\[ |E|^2(f_n) = R / (S_+S_-)^{1/2}, \]  

(4.4)

and ellipse orientation

\[ \phi_r(f_n) = (1/2) \tan^{-1} \left[ \text{Im}(R) / \text{Re}(R) \right]. \]  

(4.5)

measured anticlockwise from east (Mooers, 1973). In Figs. 3 and 4, ellipses are presented where the major and minor axis lengths (cm s\(^{-1}\)) are the sum and absolute difference, respectively, of the clockwise and anticlockwise autospectra to the one-half power; the sense of the rotation (corresponding to the sign of \(C_r\)) is denoted by an arrow; \(\phi_r\) is represented by the orientation of the ellipse; and a * denotes \(|E|^2\) greater than 0.44, the 90% confidence level. The major axis is given by a line extending from the origin through the ellipse at angle \(\phi_r\), where in the plot vertical (upwards) corresponds to alongshore (northward) and horizontal corresponds to cross-shelf.
a. Vertical data set

The ellipses for the semidiurnal frequency band (1.91 cpd) (Fig. 3) have clockwise rotation and, when the orientation is significant, are aligned with the major axis across the shelf. The magnitude decreases monotonically with depth, with a magnitude of 10 cm s\(^{-1}\) at 3 m and 3 cm s\(^{-1}\) at 96 m. The ellipses for the near-inertial frequency band are nearly circular with clockwise rotation. There is an overall trend of decreasing speeds with depth with a local maximum of 3 cm s\(^{-1}\) at 40 m. The maximum and minimum speeds of \(\sim 5\) cm s\(^{-1}\) and 1 cm s\(^{-1}\) occur at 3 m and 96 m, respectively. In the diurnal frequency band, the sense of rotation for the velocity vectors varies with position. It alternates between clockwise and anticlockwise for each position below 40 m. A large change in major axis length occurs at 8 m, with speeds of 6 cm s\(^{-1}\) at 3 m and 2.5 cm s\(^{-1}\) at 8 m. Below 8 m, the major axis length of the ellipses (\(\sim 1\) cm s\(^{-1}\)) are nearly constant with depth. The ellipses are generally highly eccentric which represents nearly rectilinear motion where the stability is significant. Thus, the motion is rectilinear, aligned across shelf at 8 and 10 m, and aligned alongshore at 40 m.

b. Cross-shelf data set

The rotary ellipses for the semidiurnal frequency band (Fig. 4a), with the exception of E80 and E120, have clockwise rotation. The
ellipse shapes, orientations and magnitudes are similar to those calculated from admittances at the $M_2$ tidal frequency by Torgrimson and Hickey (1979). Thus, the variance in the semidiurnal frequency band appears to be dominated by the $M_2$ baroclinic tide. The rotary spectra for the near-inertial frequency band (Fig. 4b) are characterized by circular ellipses with clockwise rotation. The depth of the subsurface maximum occurs at F80, E80 and C40, and it increases with distance offshore. The ellipse amplitudes at Edelweiss are smaller than the amplitudes at Forsythia or Carnation at the same depth. Rotary spectral results for the diurnal frequency band (Fig. 4c) show clockwise and anticlockwise rotations at different locations, similar to those found in the single mooring data. The rotary spectral ellipse orientation and sense of rotation are somewhat similar to the $K_1$ tidal ellipses calculated from admittances (Torgrimson and Hickey, 1979), however, the spectral ellipse major axis lengths are larger. The ellipses are smallest near the bottom, but no overall pattern in size is obvious.
5. Rotary cross-spectra

Rotary cross-spectral estimates are calculated for the vertical and cross-shelf data sets as a necessary preliminary step to the rotary EOF analysis in Sections 6 and 7. Many of the dynamical implications of the cross-spectra may be seen more clearly from the EOF analysis and a physical interpretation is deferred to those sections.

a. Semidiurnal

Rotary coherence squared and phase for the velocity time series at B3 with each velocity time series in the vertical data set are plotted in Fig. 5 for the frequency band centered at 1.91 cpd. The minimum in coherence at C20 and C40 for clockwise rotation corresponds to a 180° change in phase occurring between those depths. Coherence decreases slowly with depth below 60 m. The phase is constant above 20 m and below 40 m. Rotary cross-spectra for anticlockwise rotation are characterized by monotonically decreasing coherence for increasing depth and small changes in phase over the entire water column.

The rotary cross-spectral results from the cross-shelf data for clockwise rotation are similar to those from the vertical data (Table 3). The coherence is below the 90% confidence level (denoted by **) for most cross-spectral pairs when one velocity time series is from F180, E80 or C40. A phase shift of approximately 180° in the vertical also occurs at these locations. The coherences for anticlockwise rotation
are significant for all pairs except three. Phase differences are typically 35° or less and without the 180° change that exists for clockwise rotation.

b. Near-inertial

The vertical data set results are summarized in Table 4 for clockwise rotation. The time series from adjacent instruments above 20 m are significantly coherent while those from adjacent instruments below 20 m are not coherent. There are, however, nonadjacent instrument pairs whose time series have significant coherence, e.g., C60 and B18. Time series from C80 and C95 are only marginally coherent with those from other instruments. The phase differences are consistent with an upward propagation of phase. For anticlockwise rotation, the pairs above 20 m are coherent while no pairs are significantly coherent below 20 m (Table 5). The phase differences above 20 m indicate a downward propagation of phase.

For the cross-shelf data, the cross-spectral coherences between measurements at different horizontal locations are not significant in the near-inertial frequency band for both clockwise and anticlockwise rotation. The small number of significantly coherent pairs is consistent with white noise, i.e., 10% of the pairs are coherent at the 90% confidence level.
c. Diurnal

For the vertical data set, the coherence for clockwise rotation is generally significant with minimum values occurring when one time series is from depths between 10 and 16 m. The phase is approximately $180^\circ$ for time series from one depth above and one below 15 m. The phase between pairs below 20 m is small. For anticlockwise rotation, coherence is high and the phase is nearly constant throughout the water column.

The coherences from the cross-shelf data for clockwise rotation are low for cross-spectral pairs involving one time series from E20, E120, C60 or A20 but generally significant for other pairs. For anticlockwise rotation, the coherence is high except for pairs involving C20. The phase is fairly uniform across the shelf and phase differences in the vertical are small.
6. Rotary EOFs for the vertical data set

EOF calculations (Section 3) were made using the velocity and temperature observations in the vertical data set. Rotary velocity EOFs were computed where $u_j$ and $v_j$ in (3.1) are the eastward and northward velocity components at location $j$, respectively. Time series of vertical displacement $\bar{\zeta}$ were computed from the temperature time series and the vertical gradient of time averaged temperature $d\bar{T}/dz$ (Fig. 6) where $\bar{\zeta} = \bar{T} / (d\bar{T}/dz)$. Single-sided EOFs for displacement were calculated using the rotary formalism (3.1) with $u_j = \bar{\zeta_j}$ and $v_j = 0$. The rotary spectra for displacement are symmetric about zero frequency. Similarly, rotary EOFs for displacement at $\pm |f_n|$ have amplitudes that are equal and phases with the opposite sign. A benefit of using the rotary formalism with displacement is that the cross-spectral calculations between the displacement EOF amplitude function and rotary spectral coefficients are facilitated.

In calculating the cross-spectral matrix $C_{ij}$ for the vertical data set, the individual time series are weighted in order to minimize the effects of large differences in nearest instrument separations, which range from 2 m to 20 m. Since the total energy for dynamical modes is calculated using space integrals, the weighting is based on the spatial separation of the measurements. The weight $B_j$ for a given measurement location is given by half the distance to the instrument above plus half the distance to the instrument below divided by the total water depth. The individual series from location $j$ are multiplied by $B_j^{1/2}$. 
The position of each instrument and the corresponding weight are given in Table 1. EOFs were computed for both the weighted and unweighted cross spectral matrices. The differences in the EOFs were typically less than 10% for amplitude and 5° for phase. The results for the weighted matrix are presented below.

The selection of EOFs significantly different from those produced from white noise is based on a conservative extrapolation from Table 1 in Overland and Preisendorfer (1982). An EOF mode is assumed to be significant if at least 50% of the total variance is explained by mode 1 or 40% by mode 2. Only significant EOF modes, which here are limited to mode one, are discussed.

a. Semidiurnal

The amplitudes and phases for the rotary first mode velocity EOFs for clockwise and anticlockwise rotation, denoted as mode $1_-$ and mode $1_+$, respectively, are plotted in Fig. 7 as functions of depth for the semidiurnal frequency band centered at $|f| = 1.91$ cpd. The mode $1_-$ EOF, with 85% of the total clockwise variance, has a baroclinic structure with a zero crossing near 30m. The amplitude function is not significantly coherent with the rotary wind stress measured at Buoy-B (Table 6). Phase is nearly constant above and below 30 m with a rapid change of 180° near 30 m. The node in the EOF is near the depth at which low coherence and phase shift of the rotary cross-spectra occur (Fig. 5). The anticlockwise mode $1_+$ EOF amplitude is nearly depth
independent and is smaller than that of the clockwise $l_-$ EOF. The phase is also nearly depth independent, so that the mode $l_+$ structure is similar to that of a barotropic tide. Mode $l_+$ EOF contains 90% of the total anticlockwise variance and its amplitude function is not significantly coherent with the wind stress.

Torgrimson and Hickey (1979) have argued, based on the slope of internal wave characteristics, that the generation of the semidiurnal internal tide in this region is most likely to occur at depths in the ranges 200 to 500 m and 500 to 1000 m. As a result, the baroclinic tide on the shelf in 100 m water depth may be larger in amplitude than the barotropic tide, in agreement with the EOF amplitudes. Gonella (1972) has shown from the following approximate form of the momentum equations: \[ \frac{\partial w}{\partial t} + \text{i} f_0 w = \mathbf{\gamma}, \]
where $w = u + iv$, $\mathbf{\gamma}$ is a periodic forcing function with equal clockwise and anticlockwise components and $f_0$ is the Coriolis parameter, that the response $w$ at the semidiurnal frequency will be primarily clockwise with a theoretical rotary coefficient of -0.95. Consequently, the EOF amplitudes and phases for mode $l_-$ and $l_+$ are consistent with an $M_2$ barotropic tide forcing an $M_2$ baroclinic tide, where the baroclinic response is primarily of clockwise rotation.

Rotary statistics calculated for modes $l_+$ and $l_-$ velocity EOF amplitude functions are presented in Table 6. The rotary coefficient is negative, indicating a clockwise rotating vector. The ellipse is oriented across shelf and has high stability squared.
The displacement EOF has a nearly constant phase with depth and a maximum amplitude at 20 m (Fig. 7). The coherences between the displacement and velocity EOF amplitude functions are significant and are summarized in Table 7. The coherence between the displacement EOF amplitude function and the rotary wind stress is not significant for either clockwise or anticlockwise rotations (Table 8). The baroclinic structure of mode 1, and the location of the maximum in displacement EOF amplitude near the depth of the clockwise velocity EOF node are consistent with a semidiurnal first mode internal wave. The phase between the velocity and displacement amplitude functions, together with the EOF phase for velocity and displacement (Fig. 7), imply that for a positive displacement there is offshore flow above 30 m and onshore flow below 30 m, i.e., there are 180° and 0° phase differences between velocity and displacement above and below 30 m, respectively. This phase relation between the velocity and displacement indicates onshore propagation of the internal wave (Gordon, 1978).

b. Near-inertial

The velocity EOF amplitudes and phases for the near-inertial frequency band centered at \(|f| = 1.54\) cpd are plotted as functions of depth in Fig. 8. There is a greater amount of variance explained by the clockwise EOF than anticlockwise EOF which is consistent with the presence of waves of near-inertial frequency and with the peak in the rotary autospectra for clockwise rotation at -1.54 cpd. The mode 1,
EOF phase has a linear slope above 60 m which corresponds to an upward propagation of phase at a velocity of 0.14 cm s\(^{-1}\). This is in good agreement with the upward propagation of phase at 0.20 cm s\(^{-1}\) found by Kundu (1976) from lagged cross-correlations of band-pass filtered cross-shelf velocity components using the same data. The mode 1\_ EOF amplitude is a maximum at the surface.

The rotary statistics for the EOF amplitude functions (Table 6), \(C_r = 0.9\) and \(|E|^2 = 0.2\), confirm the relatively larger clockwise energy and low coherence between the two rotational components. Neither component is significantly coherent with the wind stress in agreement with previous findings (Gonella, 1972; Davis et. al., 1981). This is possibly due to the response of currents to near-inertial frequency wind stress being governed in part by horizontal variations in the mean flow (Weller, 1982). Loss of coherence would also be caused by the fact that sudden changes in wind stress generate and cancel near-inertial waves in a manner dependent on the relative phase between the wind stress and the existing free waves (Pollard, 1970). The low coherence at the near-inertial frequency between velocities at different cross-shelf locations may also be due to the same processes.

The mode one displacement EOF for the near-inertial frequency band (Fig. 8) has a phase structure similar to the mode 1\_ velocity EOF. Because of the low percent variance explained by this mode for individual displacement time series, however, only one EOF phase value above 40 m is significant. Also, the displacement and velocity EOF amplitude functions are not coherent (Table 7), so the similarity in
phase structure above 40 m may be fortuitous. The displacement EOF amplitude has a local minimum at 40 m depth. Unlike the near-inertial frequency velocity EOF amplitude functions, the mode one displacement EOF amplitude function is coherent with the rotary wind stress (Table 8). The EOF lags the wind stress by 0.30 days for clockwise rotation and leads by 0.08 days for anticlockwise rotation.

c. Diurnal

The clockwise mode 1_ rotary velocity EOF for the diurnal frequency band centered at \(|f| = 0.98\) cpd, has a rapid increase in amplitude above 15 m (Fig. 9). The surface intensification is associated with a 180° phase change near 15 m. The phase and amplitude below 15 m are nearly constant. The mode 1_ EOF accounts for 90% of the clockwise variance. It is also highly coherent with the wind stress (0.90 coherence squared) and lags the wind stress by 0.17 days.

The EOF amplitudes for clockwise and anticlockwise rotation are comparable below 15 m. The mode 1_ EOF phase and amplitude for anticlockwise rotation indicate a depth independent structure below 15 m. The mode 1_ EOF amplitude function is also coherent with the wind stress (0.66 coherence squared). Above 10 m, the clockwise EOF amplitude is larger than the anticlockwise EOF amplitude. This may be related to the fact that at the diurnal frequency the wind stress has more energy for clockwise than anticlockwise rotation (Fig. 2). Also, in time-dependent Ekman layer theory the amplitude of the complex
velocity response to wind forcing with equal magnitudes for both
rotations is greater for clockwise than anticlockwise rotation
(Gonella, 1972).

The rotary ellipse orientation for mode one is across the shelf.
The rotary coefficient indicates clockwise rotation. The EOF
amplitudes and phases for modes 1_+ and 1_-, and the ellipse axis
orientation and stability squared, indicate cross-shelf rectilinear
flow below 15 m and nearly circular clockwise rotation above 15 m.
This structure, together with the mode 1_- surface intensification and
high coherence with the wind stress, indicate a wind forced surface
transport with a compensating cross-shelf transport below. The above
is qualitatively consistent with a two-dimensional mass balance normal
to the coast.

The mode one displacement EOF amplitude function (Fig. 9) is
highly coherent (0.88 coherence squared) with the wind stress for both
rotations. The phase between the displacement EOF amplitude function
and the wind is -57° for clockwise rotation and -107° for anticlockwise
rotation (Table 8). The displacement and velocity EOF amplitude
functions are also coherent. This coherence, the coherence of the wind
stress and the displacement EOF amplitude function, and the surface
intensification in displacement EOF amplitude may correspond to wind
forced advection of horizontal temperature gradients.
d. Low frequency

The first mode EOF amplitude and phase are plotted as functions of depth (m) and frequency (cpd) in Fig. 10. The upper 20 m were contoured separately since the instrument separation is much smaller above 20 m depth.

The EOF amplitude has two maxima. A subsurface maximum of 7 cm s\(^{-1}\) at 60 m and -0.056 cpd, and a surface maximum of 8 cm s\(^{-1}\) at 0.056 cpd. The amplitude is nearly constant with depth at frequencies higher than 0.056 cpd. Below 20 m, the EOFs for clockwise rotation have larger amplitudes. This agrees with the general clockwise rotation found by Kundu and Allen (1976) in hodograph plots of a complex time domain EOF calculated for a similar cross-shelf data set from the CUE-II observations.

The EOF phase is fairly constant in the vertical at each frequency. The phase is most variable in the surface layer. Surface to bottom phase differences are typically 40° or less. EOF phase and amplitude are consistent with current ellipses that are highly eccentric, oriented alongshore and nearly depth independent in amplitude. Since the low frequency velocity fluctuations are nearly rectilinear, rotary EOF analysis may not be the optimum investigative tool for this frequency range.
7. Rotary EOFs for the cross-shelf data set

Rotary EOFs are calculated for the semidiurnal and diurnal frequency bands using the cross-shelf data described in Section 2. EOFs were not computed for the near-inertial frequency band since in that band coherence between currents measured at different across shelf locations was generally not significant (Section 4b).

The weights $B_j$ for the time series are given by the ratio of the area represented by the measurement to the total area for the cross-shelf data set (Fig. 11). Again, individual series are multiplied by $B_j^{1/2}$. The location and weight for each instrument are given in Table 2. The results for the semidiurnal and diurnal frequency bands were not significantly different with or without weighting. The weighted matrix is used to compute the EOFs for the following discussion.

a. Semidiurnal

The mode $l_-$ EOF amplitude and phase are presented using polar coordinates in Fig. 12. The base of the vectors indicates the location of the instrument, the length of the vector represents the amplitude, and the anticlockwise angle from the vertical represents positive phase. The phases are relative to F40. Mode $l_-$, with 79% of the total clockwise variance, has a structure that is consistent with the rotary auto and cross-spectral results in Sections 4 and 5. The phase changes in the vertical through approximately 180° near F180, E80 and C40, and
the EOF amplitude is a minimum at these locations. These features, together with the horizontal differences in EOF phase and the propagation direction inferred in Section 6a, indicate an internal wave with a node in the vertical at approximately 0.4 times the local water depth (F180, E80 and C40) propagating onshore with a cross-shelf wavelength of 40 km. The contours in Fig. 12 of variance explained by mode \( l_- \) for the complex time series show that a small percentage of the variance from instruments near the nodes is explained by mode \( l_- \). This supports the interpretation of a baroclinic mode since the times series at F180, E80 and C40 are not unusually low in variance for this frequency band (Fig. 4a).

The mode \( l_+ \) EOF is nearly depth independent (Fig. 12). Mode \( l_+ \) with 90% of the total anticlockwise variance has an amplitude approximately half the mode \( l_- \) amplitude. All the time series, except that from F80, have a high percentage of their anticlockwise variance explained by mode \( l_+ \). The predominant pattern is nearly depth independent with a small amount of shear present. The cross-shelf EOF phase differences, onshore leading, are in the same sense as those caused by bottom friction in the barotropic tidal model of Battisti and Clarke (1982).

The rotary statistics for the mode one EOF amplitude functions (Table 9) indicate a clockwise rotating vector with the ellipse oriented across shelf. The ellipse stability is significant, indicating the mode \( l_- \) and mode \( l_+ \) EOFs are coherent. The cross-shelf EOF amplitude functions are not coherent with the wind stress.
The separation into baroclinic and barotropic structures by rotation found in these cross-shelf EOFs is also indicated by the EOF results for the vertical data set (Section 6a) and the same explanation for the separation is proposed.

b. Diurnal

The mode 1 EOF amplitude and phase are plotted in Fig. 13. Mode 1 with 57% of the total clockwise variance is fairly depth independent at Forsythia and Carnation. The magnitude of the EOF phase differences are generally less than 90°. At Forsythia the velocity vectors have slight rotation with depth. At Carnation, the phase differences are somewhat larger with the velocity vector at C96 rotated 90° relative to the other Carnation velocities. The four time series at E20, E120, C60 and A20 have only a small percentage of their total variance explained by mode 1. Most of the variance for E20, E120, C60 and A20 is in modes two, four, three and two, respectively.

The amplitude of the mode 1+ EOF, with 84% of the total anticlockwise variance, is nearly independent of depth and offshore distance. The same is generally true for phase with the largest differences occurring at the nearshore locations C20, A20 and A40. The percent of the variance accounted for by the mode 1+ EOF for each time series is high with the smallest value being 59% at C20.

The rotary statistics for the diurnal frequency band are indicative of an anticlockwise rotating velocity vector with alongshore ellipse orientation. The relatively small rotary coefficient (+0.26)
and the appreciable stability squared (0.51) indicate some tendency toward rectilinear flow along 69°, nearly alongshore. The mode one EOF amplitude functions for both rotations are significantly coherent with the wind stress. The mode one EOF leads the wind stress for clockwise and anticlockwise rotation by 0.3 and 0.4 days, respectively. This is the same as the results from the vertical data set and evidently reflects the existence of a wind forced surface layer with compensating cross-shelf transport below.

d. Low frequency

The EOF amplitude and phase are depicted by vectors in Fig. 14 for a frequency band centered at 0.25 cpd. The structure for both $l_-$ and $l_+$ are depth independent with decreasing amplitude offshore. The rotary coefficient $C_r = -0.27$ and stability squared $|E|^2 = 0.77$ indicate slight clockwise rotation consistent with the results from the vertical data set, but the flow is nearly rectilinear. The axis orientation is -77°, nearly alongshore.

From analysis of the same CUE-II data, Hsieh (1982) claims to find that the cross-shelf structure of the horizontal velocity field is dominated by the second barotropic shelf wave mode. The zero crossings of the alongshore velocity in the cross-shelf modes of shelf waves are indicated by changes in the sign of the rotary coefficient $C_r$ (Hsieh, 1982). Rotary EOFs will separate the regions of negative and positive $C_r$ into clockwise and anticlockwise modes, respectively. Thus, the presence of a zero crossing should be accompanied by a change in the
crossshelf direction of the relative amplitudes of the clockwise and anticlockwise EOFs, i.e., $A^1_j(f') > A^1_j(-f')$ should change to $A^1_k(f') < A^1_k(-f')$ across a zero crossing of the alongshore velocity. No indication of a zero crossing is found here in agreement with results from the cross-shelf complex time domain EOFs in Kundu and Allen (1976). This, of course, gives no direct evidence for the presence of cross-shelf coastal trapped wave modes higher than the first.
8. Summary

Current meter data from the CUE-II experiment off Oregon has been analyzed using rotary autospectra, cross-spectra and rotary empirical orthogonal functions (EOFs). The EOF amplitude functions may be calculated from the eigenvectors of the cross-spectral matrix and the autospectra (3.8) and may be used to calculate directly the cross-spectra with other series, e.g., wind stress. The fraction of variance explained by EOF mode $i$ for the series at position $j$ is the coherence squared between EOF amplitude function $i$ and series $j$ (3.14). Likewise, the EOF phase of mode $i$ at position $j$ may be interpreted as the phase between EOF amplitude function $i$ and series $j$ (3.15).

From the spectral analysis, peaks in the rotary autospectra for the currents are found to be ubiquitous for the semidiurnal, near-inertial and diurnal frequency bands. The rotary cross-spectral analyses of the vertical and cross-shelf data sets show good coherence between horizontal velocities over the shelf for semidiurnal and diurnal frequency bands. At the near-inertial frequency, coherences between horizontal velocities from above 20 m depth on mooring Buoy-B are high while those between velocities from different cross-shelf locations are low.

Rotary EOF analysis was performed on the vertical and cross-shelf data sets at the semidiurnal, near-inertial and diurnal frequencies and for frequencies below 0.25 cpd. For the semidiurnal frequency band, the clockwise rotary velocity EOFs have a strong baroclinic structure.
(Fig. 7 and 12) with nodes in the vertical typically at 0.4 times the water depth. The phase relations between displacement and velocity imply onshore propagation with a cross-shelf wavelength of 40 km. The anticlockwise velocity EOF is nearly depth independent and the amplitude function is coherent with the clockwise velocity EOF amplitude function. This is consistent with the $M_2$ barotropic tide forcing a clockwise rotating $M_2$ baroclinic tide.

For the near-inertial frequency band, results from the EOF analysis with the vertical data set (Fig. 8) show that most of the variance is in the clockwise EOF as expected when near-inertial waves are present. The velocity and displacement EOF amplitude functions are not coherent with the wind stress or each other. The depth dependence of the phase for the clockwise velocity EOF is linear with a slope that corresponds to an upward propagation of phase at $0.14 \text{ cm s}^{-1}$. This compares favorably with results of Kundu (1976) from the analysis of the same velocity data using a different method.

For the diurnal frequency band, the velocity EOFs are depth independent below 10 m for clockwise and anticlockwise rotation (Fig. 8). Above 10 m, there is a $180^\circ$ phase shift and a rapid increase in amplitude for clockwise rotation. The velocity EOF amplitude functions are highly coherent with each other and with the wind stress. This evidently corresponds to wind forcing at the diurnal frequency of surface layer motion with a compensating transport in the cross-shelf direction below.
In previous studies of oceanographic data, rotary spectral and rotary cross-spectral calculations have proven useful for the analysis of two-dimensional velocity time series. Likewise, computations of single-sided frequency domain EOFs have been helpful in the cross-spectral analysis of scalar time series. These two techniques are combined here and a formalism for the construction of rotary EOFs for a set of complex time series is developed. Rotary EOF decomposition has the inherent advantages of both rotary spectra and of frequency domain EOFs, i.e., it is insensitive to coordinate orientation and it compactly represents information from the entire cross-spectral matrix. Rotary EOFs naturally separate clockwise and anticlockwise rotating motions. For the near-inertial frequency band, the anticlockwise rotating "noise" is automatically removed from the clockwise rotating "signal". The upward phase propagation of near-inertial internal waves is clearly extracted here by rotary EOF analysis. For the semidiurnal frequency band, we found that baroclinic and barotropic structures are separable by direction of rotation. The calculation of cross-spectra between the EOF amplitude functions and other series is readily accomplished by the method described in Section 3 and can provide additional physical information. For example, in the diurnal frequency band the high coherences of the rotary velocity EOF amplitude functions with the wind stress give strong evidence for wind forcing of velocity structures that are coherent in depth and across the shelf. Based on these positive results, it appears that rotary empirical orthogonal function decomposition should provide another
useful tool for the analysis of oceanographic data.

Acknowledgement

This research was supported for both investigators by the Oceanography Section of the National Science Foundation under Grant OCE-8026131 and also for J. S. Allen by National Science Foundation Grant OCE-8014939. We thank Dr. R. L. Smith and Dr. D. Halpern for providing current, temperature and wind measurements from CUE-II. We also thank Dr. M. Levine and Dr. J. Richman for helpful discussions and comments.
PAGE MISSING
### Chapter III

#### Table 1. Vertical data set.

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Chapter III
Table 2. Cross-shelf data set.

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Table 3. Rotatory cross spectral matrix for cross-shelf data at the clockwise semi-diurnal frequency (1.91 cpd). Values of coherence squared below 0.44, the 90% confidence level, are denoted by *.

| F40 | F80 | F120 | F180 | E20 | E80 | E130 | E180 | E195 | C20 | C40 | C60 | C80 | C95 | A20 |
|-----|-----|------|------|-----|-----|------|------|------|-----|-----|-----|-----|-----|-----|-----|
| F80 | 0.35 |      |      |      |     |      |      |      |     |     |     |     |     |     |     |
| F120| 0.47 | 1.0  | 1.0  | 1.0  |     |      |      |      |     |     |     |     |     |     |     |
| F180| 1.0  | 1.0  | 1.0  | 1.0  |     |      |      |      |     |     |     |     |     |     |     |
| E20 | 0.63 | 0.63 | 0.81 |      |     |      |      |      |     |     |     |     |     |     |     |
| E80 | 0.37 | 0.34 |      |      |     |      |      |      |     |     |     |     |     |     |     |
| E120| 0.46 | 0.79 |      |      |     |      |      |      |     |     |     |     |     |     |     |
| E180| 0.61 | 0.46 |      |      |     |      |      |      |     |     |     |     |     |     |     |
| E195| 0.62 | 0.44 |      |      |     |      |      |      |     |     |     |     |     |     |     |
| C20 | 0.34 | 0.66 |      |      |     |      |      |      |     |     |     |     |     |     |     |
| C40 | 0.65 |      |      |      |     |      |      |      |     |     |     |     |     |     |     |
| C60 | 0.56 | 0.64 |      |      |     |      |      |      |     |     |     |     |     |     |     |
| C80 | 0.72 | 0.68 |      |      |     |      |      |      |     |     |     |     |     |     |     |
| C95 | 0.65 | 0.63 |      |      |     |      |      |      |     |     |     |     |     |     |     |
| A20 | 0.64 | 0.53 |      |      |     |      |      |      |     |     |     |     |     |     |     |
| A40 | 0.69 | 0.37 |      |      |     |      |      |      |     |     |     |     |     |     |     |

Chapter III
### Chapter III

Table 5. Rotary cross spectral matrix for the vertical data set at the anticlockwise inertial frequency (+1.54 cpd). Values of coherence squared below 0.44, the 90% confidence level, are denoted by *.

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Chapter III
Table 4. Rotary cross spectral matrix for the vertical data set at the clockwise inertial frequency (-1.54 cpd). Values of coherence squared below 0.44, the 90% confidence level, are denoted by *. 

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Table 6. First mode rotary velocity EOF for the vertical data set and cross spectra of EOF versus wind stress.

| Frequency (cpd) | Percent Variance | Axis Orientation (deg) | $|E|^2$ | vs Wind Stress | Phase (deg) | EOF Leads (days) |
|-----------------|-------------------|------------------------|-------|---------------|-------------|-----------------|
| -1.91           | 85                | -0.59                  | 2     | 0.62          | 0.49        | -34             | 0.05 |
| -1.54           | 64                | -0.90                  | 56    | 0.02          | 0.16        | -44             | 0.08 |
| -0.98           | 90                | -0.51                  | 13    | 0.67          | 0.90        | 59              | -0.17 |
| -0.24           | 80                | 0.00                   | 24    | 0.33          | 0.72        | 64              | -0.74 |
| -0.15           | 96                | 0.00                   | 81    | 0.71          | 0.48        | -5              | 0.08 |
| -0.06           | 89                | -0.17                  | -29   | 0.26          | 0.66        | 172             | -8.58 |
| 0.06            | 84                |                        |       |               | 0.58        | 67              | 3.33 |
| 0.15            | 90                |                        |       |               | 0.23        | 32              | 0.59 |
| 0.24            | 83                |                        |       |               | 0.73        | 67              | 0.77 |
| 0.98            | 82                |                        |       |               | 0.66        | 113             | 0.32 |
| 1.54            | 58                |                        |       |               | 0.02        | 60              | 0.11 |
| 1.91            | 90                |                        |       |               | 0.47        | -48             | -0.07 |
Table 7. Coherence squared and phase for first mode displacement EOF versus first mode rotary velocity EOF.

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<th>Anticlockwise (+) Velocity EOF</th>
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Chapter III

Table 8. First mode displacement EOF for the vertical data set and cross spectra of EOF versus wind stress.

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<th>vs Clockwise Wind Stress</th>
<th>vs Anticlockwise Wind Stress</th>
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Chapter III

Table 9. First mode rotary velocity EOF for cross-shelf data set and cross spectra of EOF versus wind stress.

| Frequency (cpd) | Percent Variance | Axis Orientation (deg) | $|E|^2$ | vs Wind Stress |
|-----------------|-------------------|------------------------|--------|----------------|
|                 |                   | $C_r$                  | $|E|^2$ | Coh² Phase | EOF Leads |
| -1.97           | 79                | -0.41                  | 6      | 0.04       | -101      | 0.14 |
| -1.03           | 57                | 0.26                   | 69     | 0.84       | -102      | 0.27 |
| -0.25           | 79                | -0.27                  | -77    | 0.39       | -136      | 1.51 |
| 0.25            | 64                |                        |        | 0.25       | 107       | 1.19 |
| 1.03            | 84                |                        |        | 0.78       | 155       | 0.42 |
| 1.97            | 90                |                        |        | 0.25       | 16        | 0.02 |
Chapter III

Fig. 1. Bottom topography and location of the CUE-II current meter moorings used here. Moorings Buoy-B and Carnation were less than 2 km apart.
Chapter III

Fig. 2. Rotary spectra of wind stress measured at Buoy-B (upper left) and of currents measured at B14 (lower left) and rotary cross-spectra between currents at B14 and wind stress (upper right) and between currents at B14 and B10 (lower right) (see Table 1). The bandwidth is 0.093 cpd. Note the spectral peaks for B14 at the semidiurnal (±1.91 cpd), near-inertial (-1.54 cpd) and diurnal (+0.98 cpd) frequencies and for wind stress at the diurnal (±0.98 cpd) frequency. The 90% confidence levels for rotary spectra and coherence squared are indicated by horizontal lines.
Chapter III

Fig. 3. Horizontal current vector ellipses (cm s$^{-1}$) calculated from the rotary autospectra for the vertical data set at the diurnal (0.98 cpd), near-inertial (1.54 cpd) and semidiurnal (1.91 cpd) frequency bands in the the left, center and right columns, respectively. The major and minor axis lengths (cm s$^{-1}$) are the sum and absolute difference, respectively, of the clockwise and anticlockwise autospectra to the one-half power, the sense of the rotation (corresponding to the sign of $C_r$) is denoted by an arrow, $\phi_r$, is represented by the orientation of the ellipse and a * denotes $|E|^2$ greater than 0.44, the 90% confidence level. The major axis is given by a line extending from the origin through the ellipse at angle $\phi_r$.

where in the plot vertical (upwards) corresponds to alongshore (northward) and horizontal (to the right) corresponds to cross-shelf (eastward). The location of the ellipses correspond to the instrument locations.
Chapter III
Fig. 3
Chapter III
Fig. 4. Horizontal current vector ellipses (cm s\(^{-1}\)) calculated from the rotary autospectra for the cross-shelf data set at the (a) diurnal (1.03 cpd), (b) near-inertial (1.50 cpd) and (c) semidiurnal (1.97 cpd) frequencies. The location of the ellipse corresponds to the instrument position. The letters along the top designate the moorings (Table 2).
\[ |f| = 1.97 \text{ cpd} \]

Chapter III
Fig. 4a
b) DISTANCE OFFSHORE (km)

| Depth (m) | 5 cm s⁻¹ | $|f| = 1.50$ cpd |
|-----------|----------|----------------|
| 0         |          |                |
| 50        |          |                |
| 100       |          |                |
| 150       |          |                |
| 200       |          |                |

Chapter III
Fig. 4b
c) $|f| = 1.03$ cpd

Chapter III
Fig. 4c
Chapter III

Fig. 5. Rotary coherence squared (left) and phase (right) for B3 with other currents from the vertical data set at the semidiurnal frequency (1.91 cpd). The clockwise and anticlockwise frequencies are denoted by solid and dashed lines, respectively. The 90% confidence level for coherence squared is denoted by a vertical line at 0.44.
Chapter III  
Fig. 6. The mean vertical temperature gradient $\frac{dT}{dz}$ ($^\circ$C m$^{-1}$) as a function of depth (m) calculated from the mean temperature measured at the Buoy-B and Carnation moorings. The standard deviation is denoted by horizontal lines.
Chapter III

Fig. 7. Rotary first mode velocity and displacement EOF phase (top), amplitude (bottom) and percent variance explained (upper right) as functions of depth (m) for the semidiurnal frequency band (1.91 cpd) and the vertical data set. The velocity and displacement EOF amplitudes are in cm s\(^{-1}\) and meters (m), respectively. Phase for the displacement EOF has been shifted by 180° for display purposes.
Chapter III
Fig. 7
Chapter III

Fig. 8. Rotary first mode velocity and displacement EOF phase (top), amplitude (bottom) and percent variance explained (upper right) as functions of depth (m) for the near-inertial frequency band (1.54 cpd) and the vertical data set. Notation is the same as for Fig. 7. The velocity and displacement EOF amplitudes are in cm s$^{-1}$ and 10$^{-1}$ m, respectively. Phase for the displacement and clockwise velocity EOF has been shifted by 180° for display purposes.
Chapter III
Fig. 8

| \( |f| = 1.54 \text{ cpd} \) |
|-------------------------|
| --- clockwise          |
| --- anticlockwise      |
| --- displacement      |
Chapter III

Fig. 9. Rotary first mode velocity and displacement EOF phase (top), amplitude (bottom) and percent variance explained (upper right) as functions of depth (m) for the diurnal frequency band (0.98 cpd) and the vertical data set. Notation is the same as for Fig. 7. The velocity and displacement EOF amplitudes are in cm s$^{-1}$ and 10$^{-1}$ m, respectively.
Chapter III
Fig. 9

|f| = 0.98 cpd
---
clockwise
-- anticlockwise
--- displacement
Chapter III

Fig. 10. Rotary first mode velocity EOF amplitude (left) and phase (right) as functions of depth (m) and frequency (cpd) for the vertical data set. The EOF amplitude contours are isotachs (1 cm s\(^{-1}\) contour interval) and the EOF phase contours are lines of constant phase (45° contour interval).
Chapter III
Fig. 10
Chapter III

Fig. 11. Schematic of the areas used to calculate the weights $B_j$ for the cross-shelf data set. Instrument locations are denoted by triangles.
Chapter III

Fig. 12. Rotary first mode velocity EOF for clockwise (a) and anticlockwise (b) rotation at the semidiurnal frequency (1.97 cpd) for the cross-shelf data set. The EOF amplitude and phase are presented using polar coordinates. The base of the vector indicates the instrument location, the vector length indicates the amplitude (cm s\(^{-1}\)) and the angle anticlockwise from the vertical indicates positive phase relative to F40. The percent of the variance of the series at position \(j\) explained by this mode is contoured as a function of depth and distance offshore. The contour intervals are 20% and 10% for clockwise and anticlockwise rotation, respectively. The bottom topography is indicated by a solid line of variable depth to the right.
Chapter III
Fig. 12b

b)

DISTANCE OFFSHORE (km)

DEPTH (m)

f = 1.97 (cpd)
Mode 1

(cm s⁻¹)
Chapter III

Fig. 13. Rotary first mode velocity EOF for clockwise (a) and anticlockwise (b) rotation at the diurnal frequency (1.03 cpd) for the cross-shelf data set. Notation is the same as for Fig. 12.
Chapter III
Fig. 13b
Chapter III
Fig. 14. Rotary first mode velocity EOF for clockwise (a) and anticlockwise (b) rotation at 0.25 cpd for the cross-shelf data set. Notation is the same as for Fig. 12.
Chapter III
Fig. 14b
REFERENCES


BIBLIOGRAPHY


APPENDICES
Appendix A

Ekman Layer Solutions

We solve for the surface and bottom Ekman layers to obtain the $O(1)$ and $O(\varepsilon)$ shelf equations in a closed form. The closure is achieved by writing the velocities in the boundary layers as the inviscid interior variables plus a correction and requiring that (2.3a) and (2.3b) are satisfied. We then have for the Ekman layers,

$$u(x,y,z,t) = u^K(x,y,t) + u^E(x,y,z,t),$$

where superscript $K$ denotes interior shelf or ocean variables, superscript $E$ denotes surface $T(\tilde{z} = \eta = z/\delta_E)$ or bottom $B(\tilde{z} = \zeta = (z+H)/\delta_E)$ Ekman layer variables, and $\mathbf{u}$ is the velocity vector.

Substituting (A.1) into (2.1) and subtracting the inviscid interior momentum equation (2.4), we obtain,

$$(A.2a)$$

$$\delta u^E_t + \tilde{c}[(u^K + u^E)u^E_x + u^E u^K + (v^K + v^E)v^E_y + v^E u^K_x + (v^K + v^E)v^E_y + w^E v^E_z] + u^E = \delta^2 u^E_{zz},$$

$$(A.2b)$$

$$\delta v^E_t + \tilde{c}[(u^K + u^E)v^E_x + u^E v^K + (v^K + v^E)v^E_y + v^E v^K_x + (v^K + v^E)v^E_y + w^E v^E_z] + v^E = \delta^2 v^E_{zz},$$

$$(A.2c)$$

$$u^E_x + v^E_y + w^E_z = 0.$$
\( u_z^T = 0, \delta_E v_z^T = \tau, w^T = -w^K(z=0), \) at \( \eta = 0, \) \( \quad \text{(A.3a)} \)
\[ u^T, v^T, w^T \rightarrow 0, \text{ as } \eta \rightarrow -\infty \] \( \quad \text{(A.3b)} \)

and for the bottom Ekman layer are,
\[ u^B = -u^K, v^B = -v^K, w^B = -w^K(z=-H), \text{ at } \zeta = 0, \] \( \quad \text{(A.4a)} \)
\[ u^B, v^B, w^B \rightarrow 0, \text{ as } \zeta \rightarrow \infty. \] \( \quad \text{(A.4b)} \)

a. Shelf

The Ekman layer velocities, in the shelf region, are rescaled and expanded in powers of \( \varepsilon \) to yield,
\[ u^E = \delta^{-1}(u_0^E + \varepsilon u_1^E + \ldots), \quad v^E = \delta^{-1}(v_0^E + \varepsilon v_1^E + \ldots), \] \( \quad \text{(A.5a,b)} \)

and
\[ w^B = \delta^{-1}(w_0^B + \varepsilon w_1^B + \ldots), \] \( \quad \text{(A.5c)} \)
\[ w^T = \delta^{-1}(w_0^T + \varepsilon w_1^T + \ldots). \] \( \quad \text{(A.5d)} \)

Substituting (A.5) and (2.6) into (A.2) we obtain the \( 0(1) \) momentum equations,
\[ -v_0^E = u_0^{E22}, \quad u_0^E = v_0^{E22}. \] \( \quad \text{(A.6a,b)} \)

For the bottom layer, the depth integrated mass conservation equation gives,
\[ w_0(z=-H) = -\delta_E \delta^{-1} \int_0^\infty u_0^B d\zeta - H \xi u_0, \] \( \quad \text{(A.7)} \)

where (A.4) has been utilized. At \( 0(\varepsilon) \) the time averaged momentum equations are,
and the depth integrated time averaged mass conservation equation is

\[
<w_1(z=-H)> = -\delta_E \delta^{-1} \int_0^\infty <u_1 B> d\zeta - \gamma <u_1>. 
\]  

(A.9)

From the standard solutions to (A.6) with boundary conditions obtained from (A.4) it follows that,

\[
\int_0^\infty u_0 B d\zeta = -v_0/\sqrt{2},
\]

(A.10)

and

\[
w_0(z=-H) = \gamma v_0 \xi - H \xi u_0, 
\]

(A.11)

where \( \gamma = \delta_E (\delta \sqrt{2})^{-1} \) (2.16b). The solution to (A.8) with the boundary conditions obtained from (A.4) may be readily obtained and the results corresponding to (A.10) and (A.11) are,

\[
\int_0^\infty <u_1 B> d\zeta = -<v_1>/\sqrt{2} + (7/20)/\sqrt{2} <v_0 v_0 \xi>,
\]

(A.12)

and

\[
<w_1(z=-H)> = -\gamma [-<v_1> \xi + (7/20)<v_0 v_0 \xi>] - H \xi <u_1>. 
\]

(A.13)

For the surface Ekman layer,

\[
w_0^T = 0,
\]

(A.14)
and

\[ \langle v_1^T \rangle = \langle u_1^T \rangle, \quad \langle u_1^T \rangle = \langle v_1^T \rangle = \langle u_0^T v_0 \xi \rangle, \]  
\[ \langle w_1^T \rangle = -\sqrt{2} \gamma \langle v_0^T \xi \rangle. \]  
(A.15a,b,c)

From the solutions to (A.6) and (A.15) with boundary conditions derived from (A.3) it follows that,

\[ \int_{-\infty}^{0} \langle u_0^T \rangle d\eta = \tau, \]  
(A.16)

\[ w_0(z=0) = 0, \]  
(A.17)

and

\[ \int_{-\infty}^{0} \langle u_1^T \rangle d\eta = -\langle \tau v_0 \xi \rangle, \]  
(A.18)

\[ \langle w_1(z=0) \rangle = -\sqrt{2} \gamma \langle \tau v_0 \xi \rangle. \]  
(A.19)

b. Interior ocean

The solutions to the Ekman layers in the interior ocean may be formed in a similar manner. The results for both the bottom and surface Ekman layers are at 0(1),

\[ w_0^I(z=-H) = (v_{0x}^I - u_{0y}^I)/\sqrt{2}, \]  
(A.20a)

\[ w_0^I(z=0) = \tau_x, \]  
(A.20b)

and at 0(\epsilon),

\[ \langle w_1^I(z=-H) \rangle = \langle v_1^I \rangle / \sqrt{2}, \]  
(A.21a)

\[ \langle w_1^I(z=0) \rangle = 0. \]  
(A.21b)
Appendix B

0(1) Numerical Solution

A solution to (3.1) in the form (3.3) is facilitated by writing (3.1) as four coupled first order ordinary differential equations for the four real variables $Y^{(i)}$, $i = 1, 4$,

\[
\begin{align*}
\gamma^{(1)}_{\xi} &= \gamma^{(2)}, \\
\gamma^{(2)}_{\xi} &= -H_{\xi}K[\gamma^{(4)} - kY^{(3)}] + \omega H(\omega Y^{(2)} - kY^{(1)})], \\
\gamma^{(3)}_{\xi} &= \gamma^{(4)}, \\
\gamma^{(4)}_{\xi} &= H_{\xi}K[\gamma^{(2)} - jY^{(1)}] - \omega H(\omega Y^{(4)} - kY^{(3)})],
\end{align*}
\]

where $\psi = Y^{(1)} + iY^{(3)}$ and $K = \left[(\omega H)^2 + \gamma^2\right]^{-1}$.

Utilizing (3.3) with (3.2) and (2.20), we find the boundary conditions,

\[
\begin{align*}
\gamma^{(2)} &= \sqrt{2} \sin \phi - H_{\xi}^{-1}(\omega Y^{(4)} - kY^{(3)}), \text{ at } \xi = 0, \\
\gamma^{(4)} &= H_{\xi}^{-1}(\omega Y^{(2)} - jY^{(1)}), \text{ at } \xi = 0,
\end{align*}
\]

and

\[
\begin{align*}
\gamma^{(2)} &= J, \gamma^{(4)} = \omega J, \text{ at } \xi = -1,
\end{align*}
\]

where $J = \sqrt{2} k \gamma (k \sin \phi + \lambda \cos \phi) \left[((\kappa^2 + \lambda^2)(\omega^2 + \gamma^2))^{-1}\right]$.

We solve (B.1), with the boundary conditions (B.2) and (B.3), using a fifth order Runge-Kutta method. Four independent initial-value problems are created by rewriting the boundary conditions such that only one inhomogeneous condition is specified for each initial-value problem (Acton, 1970). For example, using a subscript on $Y$ to denote the
respective initial-value problem, the boundary condition for these problems, from (B.2) and (B.3), are

\[ Y_1^{(1)} = 1, \quad Y_1^{(2)} = \gamma J, \quad Y_1^{(3)} = 0, \quad Y_1^{(4)} = 0, \text{ at } \xi = -1, \]  
\[ (B.4) \]

\[ Y_2^{(1)} = 0, \quad Y_2^{(2)} = 0, \quad Y_2^{(3)} = 1, \quad Y_2^{(4)} = \omega J, \text{ at } \xi = -1, \]  
\[ (B.5) \]

\[ Y_3^{(1)} = 0, \quad Y_3^{(2)} = \sqrt{2} \sin \phi - H_{Y^{-1}}(\omega - \xi), \quad Y_3^{(3)} = 1, \quad Y_3^{(4)} = 1, \text{ at } \xi = 0, \]  
\[ (B.6) \]

\[ Y_4^{(1)} = 1, \quad Y_4^{(2)} = 1, \quad Y_4^{(3)} = 0, \quad Y_4^{(4)} = H_{Y^{-1}}(\omega - \xi), \text{ at } \xi = 0. \]  
\[ (B.7) \]

The four independent linear solutions are then superimposed to give the solution to the boundary value problem, i.e.,

\[ y(i) = \sum_{j=1}^{4} B_j Y_j^{(i)}, \text{ for } i = 1,4. \]  
\[ (B.8) \]

A system of four linear, homogeneous equations in the four unknowns, \( B_1, \ldots, B_4 \), may be found from the requirement that \( y(i) \) satisfy the boundary conditions (B.2) and (B.3). Solving for \( B_1, \ldots, B_4 \), we then have the solution to the boundary value problem directly without using an iterative technique. The method was tested by comparing the numerical solutions with analytical solutions obtainable in the small slope limit, \( H_\xi << 1 \).
Appendix C

0(1) Free Wave Solution

Equations governing the free wave solution to (3.1), with boundary conditions (2.19) and (3.2), are found by setting the forcing equal to zero, i.e. $\phi = k = 0$. The result is an eigenvalue problem. A wave-like solution of the form,

$$ p_0 = \text{Re}\{\psi(\xi) \exp[i(\xi y - (\alpha - i\beta)t)]\}, \quad (C.1) $$

is used, where $\psi(\xi)$ is the complex eigenfunction and $(\alpha - i\beta)$ is the complex eigenvalue ($\alpha, \beta$ real). We determine $\psi$, $\beta$, and $\alpha$ with a shooting method, that is, for each $\lambda$ we adjust $\gamma$ and $\alpha$ until the boundary conditions are satisfied.

A summary of the results, using the parameter values given in Table 2, are presented in Table 1. The presence of friction in the model causes the long, free waves to be dispersive (Figure 10). The e-folding time varies with frequency and wavenumber (Figure 10). The phase speeds and periods for modes 1 and 2 calculated with the depth profile used here are consistent with those obtained from calculations for Oregon by Cutchin and Smith (1973). The decay time scale generally is longer than the period for the first mode and shorter than the period for higher modes.

Allen and Smith (1981) examined data from the Oregon shelf ($45^\circ 16'N$, see Figure 2) taken during July and August 1973 at a mid-shelf location (100 m depth). Terms in the depth integrated alongshore momentum balance were estimated, including a calculation of the bottom stress using the quadratic drag law on hourly data ($C_D = 1.5 \times 10^{-3}$). They showed that for low pass filtered (40 hour half power point) data the linear
approximation $\tau_B = rv_B$, where subscript $B$ denotes values near the bottom, was reasonably good and by a regression of $\tau_B$ on $v_B$ obtained an estimate of $r = 2 \times 10^{-2}$ cm s$^{-1}$. The equivalent 'resistance' coefficient used here, $fH_0\delta E/\sqrt{2} = 2.8 \times 10^{-2}$ cm s$^{-1}$, is close to that value.

A spindown time of 6.8 days was also estimated by Allen and Smith. This compares with the first and second mode spin down times here of 8.2 and 5.9 days. Peaks in the spectra of depth integrated $v_t$ were found at 0.1 and 0.34 cpd. If these peaks correspond to a resonant response, we note that the corresponding wave lengths for these frequencies for mode 1 are 1100 and 300 km and for mode 2 are 500 and 1300 km, respectively. The wave length used here is 1000 km.
Appendix D
Parameter Values

The parameter values used in this paper (Table 2) are based on the conditions thought to be typical for the Oregon continental margin (see Appendix C).

The model dependence on $\omega$, the forcing frequency, is explored by varying $\omega'$ from 0.02f to 0.32f. Variations in the parameters listed in Table 2 are found to have little qualitative effect on the results. The magnitude of the response is sensitive to the values chosen for $f$, $\delta_E$, and $\tau_0$. 