A Relational Hierarchical Model for Decision-Theoretic Assistance

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Abstract

Building intelligent assistants has been a long-cherished goal of AI and many were built and fine-tuned to specific application domains. In recent work, a domain-independent decision-theoretic model of assistance was proposed, where the task is to infer the user's goal and take actions that minimize the expected cost of the user's policy. In this paper, we extend this work to domains where the user's policies have rich relational and hierarchical structure. Our results indicate that relational hierarchies allow succinct encoding of prior knowledge for the assistant, which in turn enables the assistant to start helping the user after a relatively small amount of experience.

1 Introduction

There has been a growing interest in developing intelligent assistant systems that help users in a variety of tasks ranging from washing hands to travel planning [2, 7, 3]. The emphasis in these systems has been to provide a well-engineered domain-specific solution to the problem of reducing the users' cognitive load in their daily tasks. A decision-theoretic model was proposed recently [10] to formalize the general problem of assistantship as a partially observable Markov decision process (POMDP). In this framework, the assistant and the user interact in the environment to change its state. The goal of the assistant is to take actions that minimize the expected cost of completing the user's task [10]. In most situations, however, the user's task or goal[1] is not directly observable to the assistant, which makes the problem of quickly inferring the user's goals from observed actions critically important. One approach to goal inference is to learn a probabilistic model of the user's policy for achieving various goals and

[1] In this work, we use the words task and goal interchangeably.

then to compute a posterior distribution over goals given the current observation history. However, for this approach to be useful in practice, it is important that the policy be learned as early in the lifetime of the assistant as possible. We call this the problem of "early assistance", which is the main motivation behind this work.

One solution to the early assistance problem, advocated in [10], is to assume that (a) the user's policy is optimal with respect to their goals and actions, the so called "rationality assumption," and that (b) the optimal policy can be computed quickly by knowing the goals, the "tractability assumption." Under these assumptions, the user's policy for each goal can be approximated by an optimal policy, which may be quickly computed. Unfortunately in many real world domains, neither of these assumptions is realistic. Real world domains are too complex to allow tractable optimal solutions. The limited computational power of the user renders the policies to be locally optimal at best.

In this paper, we propose a different solution to the early assistance problem, namely constraining the user's policies using prior domain knowledge in the form of hierarchical and relational constraints. Consider an example of a desktop assistant similar to CALO [30] that helps an academic researcher. The researcher could have some high level tasks like writing a proposal, which may be divided into several subtasks such as preparing the cover page, writing the project description, preparing the budget, completing the biography, etc. with some ordering relationships between them. We expect that an assistant that knows about this high level structure would better help the user. For example, if the budget cannot be prepared before the cover page is done, the assistant would not consider that possibility and can determine the user's task faster. In addition to the hierarchical structure, the tasks, subtasks, and states have a class and relational structure. For example, the urgency of a proposal depends on the closeness of the deadline. The deadline of the proposal is typically mentioned on the web page of the agency to which the proposal is addressed. The collaboration potential of an individual on a proposal depends on their expertise in the areas related to the topic of the proposal. Knowing these relationships and how they influence each other could make the assistant more effective.

This work extends the assistantship model to hierarchical and relational settings, building on the work in hierarchical reinforcement learning[11] and statistical relational learning (SRL).We extend the assistantship framework of [10] by including parameterized task hierarchies and conditional relational influences as prior knowledge of the assistant. We perform inference on the distribution of user's goals given a sequence of their atomic actions by two methods: firstly, we compile this knowledge into an underlying Dynamic Bayesian network and use Bayesian network inference algorithms and secondly, we directly sample the user's tasks given the user's actions. We estimate the parameters for the user's policy and influence relationships by observing the users' actions. Once the user's goal distribution is inferred, we determine an approximately optimal action by estimating the Q-values of different actions using rollouts and picking the action that has the least expected cost. The use of relational hierarchies could potentially be very useful in many real-world assistant systems.

We evaluate our relational hierarchical assistantship model in two different toy domains and compare it to a propositional flat model, propositional hierarchical model, and a relational flat model. Through simulations, we show that when the prior knowledge of the assistant matches the true behavior of the user, the relational hierarchical model provides superior assistance in terms of performing useful actions. The relational flat model and the propositional hierarchical model provide better assistance than the propositional flat model, but fall short of the performance of the relational hierarchical approach.

The rest of the paper[2] is organized as follows: Section 2 summarizes the basic decision-theoretic assistance framework, which is followed by the relational hierarchical extension in Section 3. Section 4 presents the experiments and results, Section 5 outlines some related work and Section 6 concludes the paper.

2 Decision-Theoretic Assistance

In this section, we briefly describe the decision-theoretic model of assistance of [10] which forms the basis of our work. In this setting, there is a user acting in the environment and an assistant that observes the user and attempts to assist him. The environment is modeled as an MDP described by the tuple $\langle W, A, A', T, C, I \rangle$, where W is a finite set of world states, A is a finite set of user actions, A' is a finite set of assistant actions, and $T(w, a, w')$ is a transition function that represents the probability of transitioning to state w' given that action $a \in A \cup A'$ is taken in state w. C is an action-cost function that maps $W \times (A \cup A')$ to real numbers, and I is an initial state distribution over W. An episodic (finite horizon) setting is assumed, where the user chooses a goal and tries to achieve it. The assistant observes the user's actions and the world states but not the goal. After every user's action, the assistant gets a chance to take one or more actions ending with a **noop** action, after which the user gets a turn. The objective is to minimize the sum of the costs of user and assistant actions.

The user is modeled as a stochastic policy $\pi(a|w, g)$ that gives the probability of selecting action $a \in A$ given that the user has goal g and is in state w. The objective is to select an assistant policy π' that minimizes the expected cost given the observed history of the user. The environment is only partially observable to the assistant since it cannot observe the user's goal. It can be modeled as a POMDP, where the user is treated as part of the environment.

In [10], the assistant POMDP is solved approximately, by first estimating the goal of the user given the history of his actions, and then selecting the best assistive action given the posterior goal distribution. One of the key problems in effective assistantship is to learn the task quickly enough to start helping the user as early as possible. In [10], this problem is solved by assuming that the user is rational, i.e., he takes actions to minimize the expected cost. Further, the user MDP is assumed to be tractably solvable for each goal. Hence, their

[2]We extend our ILP'07 paper with the same title by formalizing the problem, including a sampling based method and presenting new experimental results with all the methods

system solves the user MDP for each goal and uses it to initialize the user's policy.

Unfortunately the dual assumptions of tractability MDP and rationality make this approach too restrictive to be useful in real-world domains that are too complicated for any user to approach perfect rationality. We propose a knowledge-based approach to the effective assistantship problem that bypasses the above two assumptions. We provide the assistant with partial knowledge of the user's policy, in the form of a task hierarchy with relational constraints on the subtasks and their parameters. Given this strong prior knowledge, the assistant is able to learn the user's policy quickly by observing his actions and updating the policy parameters. We appropriately adopt the goal estimation and action selection steps of [10] to the new structured policy of the user and show that it performs significantly better than the unstructured approach.

3 A Relational Hierarchical Model of Assistance

In this section, we propose a relational hierarchical representation of the user's policy and show its use for goal estimation and action selection.

3.1 Relational Hierarchical Policies

Users in general, solve difficult problems by decomposing them into a set of smaller ones with some ordering constraints between them. For example, proposal writing might involve writing the project description, preparing the budget, and then getting signatures from proper authorities. Also, the tasks have a natural class-subclass hierarchy, e.g., submitting a paper to ICML and IJCAI might involve similar parameterized subtasks. In the real world, the tasks are chosen based on some attributes of the environment or the user. For instance, the paper the user works on next is influenced by the closeness of the deadline. It is these kinds of relationships that we want to express as prior knowledge so that the assistant can quickly learn the relevant parameters of the policy. We model the user as a stochastic policy $\pi(a|w, T, O)$ that gives the probability of selecting action $a \in A$ given that the user has goal stack T and is in state w. O is the history of the observed states and actions. Learning a flat, propositional representation of the user policy is not practical in many domains. This is due to the fact that in several domains, the state-action space could be prohibitively large. Rather, in this work, we represent the user policy as a *relational task hierarchy* and speed up the learning of the hierarchy parameters via the use of *conditional influence statements* that constrain the space of probabilistic dependencies.

Relational Task Hierarchies. A relational task hierarchy is specified over a set of variables, domain constants, and predicate symbols. There are predicate symbols for representing properties of world states and specifying task names. The task predicates are divided into primitive and abstract tasks. Primitive task predicates will be used to specify ground actions in the MDP that can be

directly executed by the user. Abstract task predicates will be used to specify non-primitive procedures (that involve calling subtasks) for achieving high-level goals. Below we will use the term *task stack* to mean a sequence of ground task names (i.e. task predicates applied to constants).

A relational task hierarchy will be composed of relational task schemas which we now define.

Definition 1 (Relational Task Schema) *A relational task schema is either: 1) A primitive task predicate applied to the appropriate number of variables, or 2) A tuple $\langle N, S, R, G, P \rangle$, where the task name N is an abstract task predicate applied to a set of variables V, S is a set of child relational task schemas (i.e. the subtasks), R is a set of logical rules over state, task, and background predicates that are used to derive a candidate set of ground child tasks in a given situation, G is a set of rules that define the goal conditions for the task, and $P(s|T, w, O)$ is a probability distribution that gives the probability of a ground child task s conditioned on a task stack T, a world state w, and an observation history O.*

Each way of instantiating the variables of a task schema with domain constants yields a ground task. The semantics of a relational task schema specify what it means for the user to "execute to completion" a particular ground task as follows. As the base case, a primitive ground task is executed-to-completion by simply executing the corresponding primitive MDP action until it terminates, resulting in an updated world state.

An abstract ground task, can intuitively be viewed as specifying a stochastic policy over its child subtasks which is executed until its goal condition is satisfied. More precisely, an abstract ground task t is executed-to-completion by repeatedly selecting ground child tasks that are executed-to-completion until the goal condition G is satisfied. At each step given the current state w, observation history O, task stack T, and set of variable bindings B (that include the bindings for t) a child task is selected as follows: 1) Subject to the variable bindings, the rules R are used to derive a set of candidate ground child tasks. 2) From this set we draw a ground task s according to P, properly normalized to only take into account the set of available subtasks. 3) The drawn ground task is then executed-to-completion in the context of variables bindings B' that include the bindings in B along with those in s and a task stack corresponding to pushing t onto T.

Based on the above description, the set of rules R can be viewed as specifying hard constraints on the legal subtasks with P selecting among those tasks that satisfy the constraints. The hard constraints imposed by R can be used restrict the argument of the child task to be of a certain type or may place mutual constraints on variables of the child tasks. For example, we could specify rules that say that the document to be attached in an email should belong to the project that the user is working on. Also, the rules can specify the ordering constraint between the child tasks. For instance, it would be possible to say that to submit a paper the task of writing the paper must be completed first.

We can now define a relational task hierarchy.

Definition 2 (Relational Task Hierarchy) *A relational task hierarchy is rooted acyclic graph whose nodes are relational task schemas that satisfy the following constraints: 1) The root is a special subtask called ROOT. 2) The leaves of the graph are primitive task schemas. 3) There is an arc from node n_1 to node n_2 if and only if the task schema of n_2 is a child of task schema n_1.*

We will use relational task hierarchies to specify the policy of a user. Specifically, the user's actions are assumed to be generated by executing the ROOT task of the hierarchy with an initially empty goal stack and set of variable bindings.

An example of a *Relational Task Hierarchy* is presented in the Figure 1 for a game involving resource gathering and tactical battles. For each task schema we depict some of the variable binding constraints enforced by the R as a logical expression. For clarity we do not depict the ordering constraints imposed by R. From the ROOT task the user has two distinct choices to either gather a resource, *Gather(R)* or attack an enemy, *Attack(E)*. Each of these tasks can be achieved by executing either a primitive action (represented with ovals in the figure) or another subtask. For example, to gather a resource, the user needs to collect the resource (denoted by *Collect(R)*) and deposit the resource at the storage (denoted by *Deposit(R,S)*, which indicates that R is to be deposited in S). Resources are stored in the storages of the same type (for example, gold in a bank, food in a granary etc.), which is expressed as the constraint $R.type = S.type$ in the figure. Once the user chooses to gather a resource (say $gold1$), the value of R in all the nodes that are lower than the node $Gather(R)$ is set to the value $gold1$. R is freed after $Gather$ is completed.

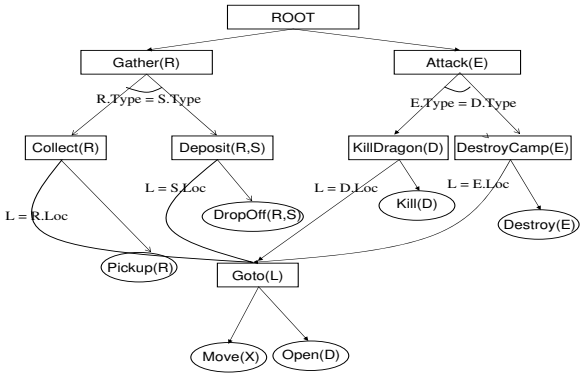

Figure 1: Example of a task hierarchy of the user. The inner nodes indicate subtasks while the leaves are the primitive actions. The tasks are parameterized and the tasks at the higher level will call the tasks at the lower level

Conditional Influences: Often it is relatively easy to hand-code the rule sets R that encode hard-constraints on child tasks. It is more difficult to precisely specify the probability distributions for each task schema. In this work, we take the approach of hand-coding a set of conditional influence statements that

are used to constrain and hence speedup the learning of these probability distributions. The conditional influences describe the objects and their attributes that influence a subtask choice based on some condition, i.e., these statements serve to capture a distribution over the subtasks given some attributes of the environment ($P(subtask \mid worldstate)$). For example, since there could be multiple storage locations for a resource, the choice of a storage may be influenced by its distance to the resource. While this knowledge can be easily expressed in most SRL formalisms such as Probabilistic Relational Language [21] and Bayesian Logic Programs [18], we give an example in First-Order Conditional Influence Language (FOCIL) [22].

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If {Goal(Gather(R)),Completed(Collect(R)),Equal(Type(R),Type(S))} then
Distance(Loc(R), Loc(S))) Qinf subgoal(Deposit(R,S))
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A FOCIL statement of the form

$$If\{Z(\alpha)\} \quad then \quad Y_1(\alpha), \ldots, Y_k(\alpha) \quad Qinf \quad X(\alpha)$$

means that $Y_1(\alpha), \ldots, Y_k(\alpha)$ influence $X(\alpha)$ when $Z(\alpha)$ is true, where α is a set of logical variables. The above statement captures the knowledge that if R is a resource that has been collected, and S is a storage where R can be stored, the choice of the value of S is influenced by the distance between R and S. The probability of choosing a subtask in a given state is determined solely by the attribute values of the objects mentioned in the conditional influence statement, which puts a strong constraint on the user's policy and makes it easier to learn. We outline the relationship of FOCIL to other probabilistic relational languages in the appendix section to reinforce the fact that most of the statistical relational learning formalisms are capable of capturing this knowledge.

3.2 Goal Estimation

In this section, we describe our goal estimation method, given the kind of prior knowledge described in the previous section, and the observations, which consist of the user's primitive actions. Note that the probability of the user's action choice depends in general on not only the pending subgoals, but also on some of the completed subgoals including their variable bindings. Hence, in general, the assistant POMDP must maintain a belief state distribution over the pending and completed subgoals. which we call the "goal structure."

We now define the assistant POMDP. The **state space** is $W \times \mathbf{T}$ where W is the set of world states and $\mathbf{T}$ is the user's goal structure. Correspondingly, the **transition probabilities** are functions between $(w, \mathbf{t})$ and $(w', \mathbf{t})$. Similarly, the **cost** is a function of $\langle state,\ action \rangle$ pairs. The **observation** space now includes the user's actions and their parameters (for example, the resource that is collected, the enemy type that is killed etc).

In this work, we make a simplifying assumption that there is no uncertainty about the completed subtasks. This assumption is justified in our domains, where the completion of each subtask is accompanied with an observation that

identifies the subtask that has just completed. This would simplify the inference process as we do not need to maintain a distribution over the (possibly) completed subtasks. The estimation of the goal stack of the user was performed in two different methods. In the first method a handcoded DBN is used and exact inference is performed on the DBN. In the second method, we directly sample the user's tasks by observing the actions. We present both these methods in this section.

3.2.1 Unrolled DBN

For estimating the user's goal stack, we use a DBN similar to the one used in [19] and present it in Figure 2. T_j^i refers to the task at time-step j and level i in the DAG. O^i refers to the completed subtask at level i. F_j^i is an indicator variable that represents whether T_j^i has been completed and acts as a multiplexer node. If the lower level task is completed and the current task is not completed, the transition function for the current task would reflect choosing an action for the current subtask. If the lower level task is not completed, the current task stays at its current state. If the current task is completed, the value is chosen using a prior distribution over the current task given the higher level tasks.

In the experiments reported in the next section, we compiled the FOCIL statements into a DBN structure by hand. The number of levels of the tasks in the DBN corresponds to the depth of the directed graph in the relational task hierarchy. The values of the different task level nodes will be the instantiated tasks in the hierarchy. For instance, the variable T_j^1 takes values corresponding to all possible instantiations of the second-level tasks. Once the set of possible values for each current task variable in the task is determined, the constraints are used to construct the CPT. For example, the constraint $R.Type = S.Type$ in the Figure 1 implies that a resource of one type can be stored in the storage of the same type. Assume that the user is gathering *gold*. Then in the CPT corresponding to $P(T_j^2 = Store(S, gold) \mid T_j^1 = Gather(gold)$, all the entries except the ones that correspond to a bank are set to 0. The rules R of the task schema determine the non-zero entries of the CPTs, while the FOCIL statements constrain the distributions further. Note that, in general, the subtasks completed at a particular level influence the distribution over the current subtasks at the same level through the hard constraints, which include ordering relationships. In our experiments, however, we have chosen to not explicitly store the completed subtasks at any stage since the ordering of subtasks has a special structure. The subtasks are partitioned into small unordered groups, where the groups are totally ordered. This allows us to maintain a small memory of only the completed subtasks in the current group.

To illustrate the construction of the DBN given the hierarchy and influence statements better, let us consider the example presented in Figure 1. Assume that the user chooses to gather $g1$ (i.e., gold from location 1). Once the episode begins, the variables in the DBN are instantiated to the corresponding values. The task at the highest level T_j^1, would take values from the set $\langle$ $Gather(g1)$, $Gather(g2)$, $Gather(w1)$,$Gather(w2)$, $Destroy(e1)$,$Destroy(e2)$ $\rangle$,

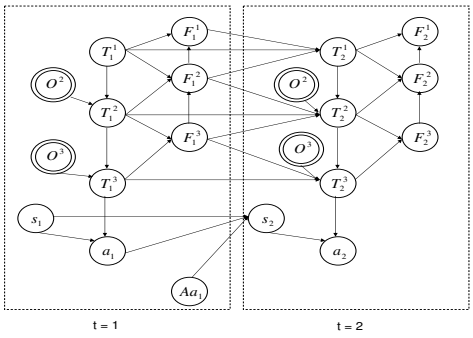

Figure 2: Dynamic Bayesian network that is used to infer the user's goal.

assuming that there are 2 gold and wood locations and 2 enemies. Similarly, the tasks at level n of the DBN would assume values corresponding to the instantiation of the nodes at the n^{th} level of the hierarchy. The conditional influence statements are used to obtain a prior distribution over the goal stack only after every subtask is finished or to minimize uncertainty and retain tractability. Once the prior is obtained, the posterior over the goal stack is updated after every user action. For example, once the user finishes the subtask of $collect(g1)$, the relational structure would restrict the set of subgoals to depositing the resource and the conditional influence statements would provide a prior over the storage locations. Once the highest level task of *Gather* is completed, the DBN parameters are updated using the complete set of observations. Our hypothesis that we verify empirically is that, the relational structure and the conditional influence statements together provide a strong prior over the task stack which enables fast learning.

Given this DBN, we need to infer the value of $P(T_j^{1:d} \mid T_{j-1}^{1:d}, F_{j-1}^{1:d}, a_j, O^{1:d})$, where d is the depth of the DAG i.e, infer the posterior distribution over the user's goal stack given the observations (the user actions in our case) and the completed goal stack. As we have mentioned, we are not considering the completed subgoals due to the fact that most of our constraints are total order and there is no necessity of maintaining them. Since we always estimate the current goal stack given the current action and state, we can approximate the DBN inference as a BN inference for the current time-step. The other issue is the learning of parameters of the DBN. At the end of every episode, the assistant updates the parameters of the DBN based on the observations in that episode using maximum likelihood estimates with Laplace correction. Since the model is inherently relational, we can exploit parameter tying between similar objects and hence accelerate the learning of parameters. The parameter learning in the case of relational models is significantly faster as demonstrated by our experiments.

It should be noted that Fern et al. [10] solved the user MDP and used the

values to initialize the priors for the user's action models. Though it seems justifiable, it is not always possible to solve the user MDP. We show in our experiments that even if we begin with an uniform prior for the action models, the relations and the hierarchical structure would enable the assistant to be useful even in the early episodes.

The high level algorithm that uses a DBN for goal estimation is presented in table 1. The parameters are updated at the end of the episode using MLE estimates. When an episode is completed, the set of completed tasks and the action trajectories are used to update the parameters of the nodes at different levels.

Table 1: Highlevel algorithm for assistance using the DBN

- Iitialize DBNs as in Figure 2 incorporating all hard constraints into the CPTs
- For each episode
  - For each time step
    - Observe any task completed
    - Update the posterior distribution of goal stack based on the observation, the hard constraints, and FOCI statements
    - Observe the next action
    - Update the posterior distribution over the tasks in the task stack
    - Compute the best assistive action
  - Update the DBN parameters

3.2.2 Sampling

The DBN though elegant in its representation of the user's goal stack, is a handcrafted one. Also, as the objects in the domain change, the DBN also needs to be modified accordingly. More importantly, the number of parameters can grow rapidly with the number of objects in the domain. For instance, if there are 10 resources and 10 storage locations, there are 100 possible *Deposit* sub-goals. In this work, we consider a simpler method as an alternative which is to sample the user's goal stack given the observations (user's actions) without converting it to a DBN. Our sampling procedure is inspired by particle filtering where the main idea is to approximate the desired distribution by a set of samples.

The general approach to particle filtering works as follows: Samples are generated according to the prior distribution and propagated according to the transition distribution. The samples are then weighed according to the observed evidence probabilities and new samples are generated according to the weights.

Table 2: Particle filtering algorithm

function PF returns $\{x_t^i\}_{i=1}^N = PF(\{x_{t-1}^i\}_{i=1}^N, y_t)$

1. For each sample i
   - Draw the sample x_t^i using the distribution $x_t^i \sim P(x_t|x_{t-1}^i, y_t)$
   - Compute the weight of the current sample, $w^i \propto Pr(y_t|x_{t-1}^i)$
2. Resampling
   - Resample the current state using, $x_t^i \sim \frac{w^i}{\sum_i w^i}$
   - Set the weights of the current sample, $w^i = 1$

The particle filtering algorithm is presented in Table 2. The state at time t is denoted by x_t and the observation at time step t is denoted by y_t. The distribution $P(x_t|x_{t-1}^i, y_t)$ is given by,

$$P(x_t|x_{t-1}^i, y_t) = \frac{P(y_t|x_t)P(x_t|x_{t-1})}{\sum_{x_t} P(y_t|x_t)P(x_t|x_{t-1})}$$

while,

$$P(y_t|x_{t-1}^i) = \sum_{x_t} P(y_t|x_t)P(x_t|x_{t-1})$$

The key thing to note is that the evidence (y_t) is being used while sampling for the new state. This would ensure that the number of samples required to converge on the true distribution is small. In our model, the hidden state x corresponds to the goal stack $\mathbf{T}$ of the user while the observation y corresponds to the user's action a and the current world state s and is represented as O. Hence the goal of this sampling process is to use the user's actions to generate samples of the goal stack, weigh them according to the user's actions and regenerate the samples. Since, the distribution is represented using the set of samples, there is no necessity of explicitly maintaining the distribution. Note that, the sample counts are the sufficient statistic for the posterior. Thus, we can compute the posterior using the normalized counts i.e.,

$$P(\mathbf{T} = i \mid O) = \frac{N_i}{\sum_{i'} N_{i'}} \tag{1}$$

where N_t is the number of samples whose task stack is equal to i. Similar to the DBN method, the user's policy $P(a \mid s, \mathbf{T})$ is updated after every trajectory. The highlevel algorithm for assistance using sampling is presented in Table 3. The difference with respect to the earlier algorithm is in the goal estimation

step, where we use sampling to obtain a distribution over the user's goal stack. As can be seen, the action selection and the update steps are similar to the earlier case.

Table 3: Highlevel algorithm for assistance using the sampling methods

1. Initialize the parameters of the user's policy using the hard constraints specified in the relational hierarchical model
2. For each episode
   - For each time step
     - Observe any task completed and update the posterior distribution of goal stack based on the observation, the hard constraints, and FOCI statements
     - Observe the next action and the current state
     - Compute the posterior over the user's goals using the particle filter algorithm presented in table 2
     - Compute the best assistive action
   - Update the policy parameters

The advantage of the sampling approach is that it is very general and can be extended across several domains. It is easy to implement and does not require any engineering tailored towards particular application domains. The drawback is that it tends to have a slightly lower accuracy and higher variance in the predicted model when compared to the exact methods. We verify this empiracally in our experiments.

3.3 Action Selection

Given the assistant POMDP M and the distribution over the user's goal stack $P(T^{1:d} \mid O_j)$, where O_j are the observations, we can compute the value of assistive actions. Following the approach of [10], we approximate the assistant POMDP with a series of MDPs $M(t^{1:d})$, for each possible goal stack $t^{1:d}$. Thus, the heuristic value of an action a in a world state w given the observations O_j at time-step j would now correspond to,

$$H(w, a, O_j) = \sum_{t^{1:d}} Q_{t^{1:d}}(w, a) \cdot P(t^{1:d}|O_j)$$

where $Q_{t^{1:d}}(w, a)$ is the value of performing the action a in state w in the MDP $M(t^{1:d})$ and $P(t^{1:d}|O_j)$ is the posterior probability of the goal stack given the observations. Instead of sampling over the goals, we sample over the possible goal stack values. The relations between the different goals would restrict the

number of goal-subgoal combinations. If the hierarchy is designed so that the subgoals are not shared between higher level goals, we can greatly reduce the number of possible combinations and make the sampling process practically feasible. We verify this empirically in our experiments. To compute the value of $Q_{t^{1:d}}(w, a)$, we use the policy rollout technique [6] where the assumption is that the assistant would perform only one action and assumes that the agent takes over from there and estimates the value by rolling out the user policy. Since the assistant has access to the hierarchy, it chooses the actions subjected to the constraints specified by the hierarchy. It should be mentioned that the approach taken here to solve the POMDPs is similar to the QMDP heuristic solution approach taken by Littman et al., [29] and hence suffers from a similar problem in that the assistant cannot take information gathering actions. While these are very useful in many domains, they are not significant in our domains. Hence, we do not consider them explicitly but point out that we can replace the myopic heuristics with POMDP solvers such as sparse sampling [28] when needed.

4 Experiments and Results

In this section, we briefly explain the results of simulation of a user in two domains[3]: a gridworld doorman domain where the assistant has to open the right doors to the user's destination and a kitchen domain where the assistant helps the user in preparing food. We simulate a user in these domains and compare different versions of the decision theoretic model and present the results of the comparison.

The different models that we compare are: the relational hierarchical model that we presented, a hierarchical model where the goal structure is hierarchical, a relational model where there are objects and relations but there is a flat goal structure and a flat model which is a very naive model with a flat goal structure and no notion of objects are relationships. Our hypothesis is that the relational models would benefit from parameter tying and hence can learn the parameters faster and would offer better assistance than their propositional counterparts at earlier episodes. Similarly, the hierarchical model would make it possible to decompose the goal structure thus making it possible to learn faster. We demonstrate through experiments that the combination of relational and hierarchical models would enable the assistant to be more effective than the assistant that uses either of these models.

4.1 Doorman Domain

In this domain, the user is in a gridworld where each grid cell has 4 doors that the user has to open to navigate to the adjacent cell (see Figure 3). The hierarchy presented in Figure 1 was used as the user's goal structure. The goals of the user are to *Gather* a resource or to *Attack* an enemy. To *gather* a

[3]These are modification to the domains presented by Fern et.al[10]

resource, the user has to *collect* the resource and *deposit* it at the corresponding location. Similarly, to *destroy* an enemy, the user has to kill the *dragon* and *destroy* the castle. There are different kinds of resources, namely *food* and *gold*. Each resource can be stored only in a storage of its own type (i.e, *food* is stored in *granary* and *gold* is stored in *bank*). There are 2 locations for each of the resources and its storage. Similarly there are 2 kinds of enemy *red* and *blue*. The user has to kill the *dragon* of a particular kind and *destroy* the castle of the same kind. The episode ends when the user achieves the highest level goal. The actions that the user can perform are to move in 4 directions, open the 4 doors, pick up, put down and attack. The assistant can only open the doors or perform a *noop*. The door closes after one time-step so that at any time only one door is open. The goal of the assistant is to minimize the number of doors that the user needs to open. The user and assistant take actions alternately in this domain.

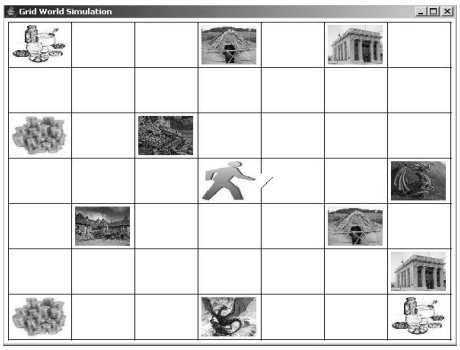

Figure 3: Doorman Domain. Each cell has 4 doors that the user has to open to navigate to the adjacent cell. The goal of the assistant is to minimize the number of doors that the user has to open.

We employed six versions of the assistant that models the user's goal structure. The first two models use a relational hierarchical structure for the user's goal structure. One of them employs the DBN for goal estimation while the other employs sampling for the same. The next two models assume a hierarchical goal structure but no relational structure (i.e., the model does not know that the 2 gold locations are of the same type etc and thus cannot exploit parameter tying), and use the DBN or the sampling method for goal estimation. The fifth model assumes a relational structure of user's goal but assumes flat goals and hence does not know the relationship between collect and deposit of subtasks, while the final model assumes a flat goal structure. A state is a tuple $\langle s, d \rangle$, where s stands for the the agent's cell and d is the door that is open. For the two flat cases, there is a necessity include variables such as *carry* that can take 5 possible values and *kill* that take 3 values to capture the state of the user having collected a resource or killed the dragon before reaching the eventual

destination. Hence the state space of the 2 flat models is 15 times more than that of the hierarchical one.

To compare the different algorithms, we solved the underlying hierarchical MDP and then used the Q-values to simulate the user. For each episode, the higher level goals are chosen at random and the user attempts to achieve the goal. We calculate usefulness of the assistant as the ratio of the correct doors that it opens to the total number of doors that are needed to be opened for the user to reach his goal which is a worst-case measure of the cost savings of the user. We average the usefulness every 10 episodes. The user's policy is hidden from the assistant in all the algorithms and the assistant learns the user policy as and when the user performs his actions. The relational model captures the relationship between the resources and storage and between the dragon's type and the castle's type. The hierarchical model captures the relationship between the different goals and subgoals, for instance, that the user has to collect some resource in order to deposit it, etc. The hierarchical relational model has access to both the kinds of knowledge and also to the knowledge that the distance to the storage location influences the choice of the storage location.

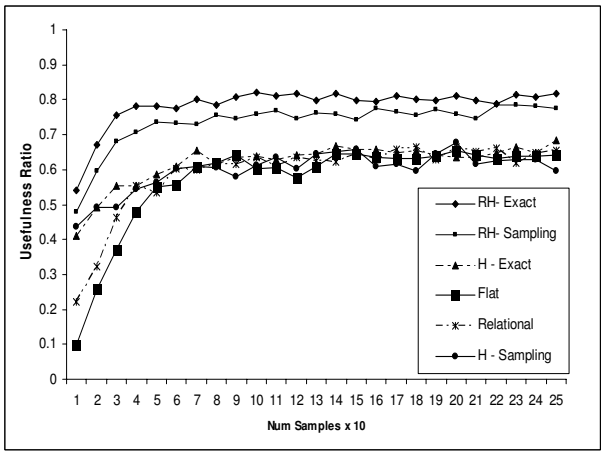

Figure 4: Learning curves for the 4 algorithms in the doorman domain. The y-axis presents the average savings for the user due to the assistant.

The results are presented in Figure 4. The graph presents the average usefulness of the assistant after every 10 episodes. As can be seen from the figure, the assistants that use the relational hierarchical models are more useful than the other models. In particular, they can exploit the prior knowledge effectively as demonstrated by the rapid increase in the usefulness in earlier episodes. It is also observed that the relational hierarchical assistant that employs the handcoded DBN performs better than the sampling method in some situations. This is due to the fact that sampling is an approximate technique for inference. Though in certain cases, the assistant based on sampling does not match the performance

of the hand-coded DBN, sampling is very easy to implement and is a domain-independent inference mechanism that can scale easily to large domains.

The two hierarchical and the relational models also exploit the prior knowledge and hence have a quicker learning rate than the flat model (as can be seen from the first few episodes of the figure). The relational hierarchical models outperform the hierarchical models as they can share parameters and hence have to learn a smaller number of parameters. They outperform the relational model as they can exploit the knowledge of the user's goal structure effectively and can learn quickly at the early stages of an episode. Also, there was no significant difference in the time required for computing the best action of the assistant for all the four algorithms. This clearly demonstrates that the relational hierarchical model can be more effective without increasing the computational cost.

4.2 Kitchen Domain

The other experimental domain is a kitchen domain where the user has to cook some dishes. In this domain, the user has 2 kinds of higher-level goals: one in which he could prepare a recipe which contains a main dish and a side dish and the second in which, he could use some instant food to prepare a main dish and a side dish. There are 2 kinds of main dishes and 2 kinds of side dishes that he could prepare from the recipe. Similarly, there are 2 kinds of main dishes and 2 kinds of side dishes that he could prepare from instant food. The hierarchy is presented in Figure 5. The symbol $\in$ is used to capture the information that the object is part of the plan. For instance, the expression $I \in M.Ing$ means that the parameter to be passed is the ingredient that is used to cook the main dish. The plans are partially ordered. There are 2 shelves with 3 ingredients each. The shelves have doors that must be opened before fetching ingredients and only one door can be open at a time.

The state consists of the contents of the bowl, the ingredient on the table, the mixing state and temperature state of the ingredient (if it is in the bowl) and the door that is open. The user's actions are: open the doors, fetch the ingredients, pour them into the bowl, mix, heat and bake the contents of the bowl, or replace an ingredient back to the shelf. The assistant can perform all user actions except for pouring the ingredients or replacing an ingredient back to the shelf. The cost of all non-pour actions is -1. Unlike in the doorman domain, here it is not necessary for the assistant to wait at every alternative time step. The assistant continues to act until the **noop** becomes the best action according to the heuristic. The episode begins with all the ingredients in the shelf and the doors closed. The episode ends when the user achieves the goal of preparing a main dish and a side dish either with the recipe or using instant food.

The savings is the ratio of the correct non-pour actions that the assistant has performed to the number of actions required for the goal. Similar to the other domain, we compared 6 different types of models of assistance. The first two models are the relational hierarchical models that have the knowledge of the goal-subgoal hierarchy and also has the relationship between the subgoals themselves. They know that the type of the main dish influences the choice of

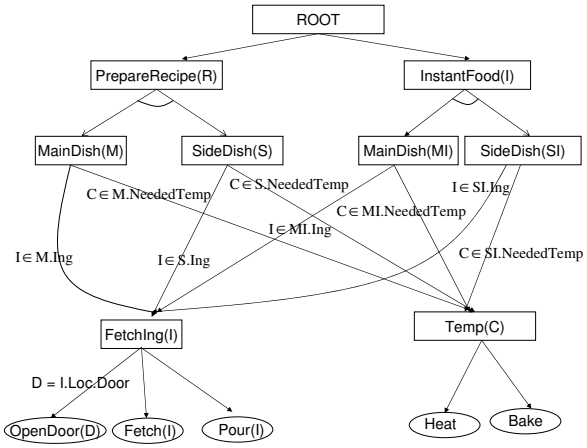

Figure 5: The kitchen domain hierarchy

the side dish. Like the previous domain, the two versions employed the DBN and the sampling methods respectively for goal estimation. The third and fourth models are the exact and sampling versions of the hierarchical model, that have the notions of the goals and subgoals but no knowledge of the relationship between the main dishes and the side dishes and thus have more parameters to learn. The relational model assumes that there are two kinds of food namely the one prepared from recipe and one from instant food and does not possess any knowledge about the hierarchical goal structure. The flat model considers the preparation of each of the 8 dishes as a separate goal and assists the user. Both the flat model and the relational model assume that the user is always going to prepare the dishes in pairs but do not have the notion of main dish and side dishes or the ordering constraints between them.

The results are presented in Figure 6. As can be seen, the hierarchical models greatly dominate the flat ones. Among the models, the relational models have a faster learning rate than their propositional counterparts. They perform better in the earlier few episodes which clearly demonstrates that relational background knowledge accelerates learning. It can be observed that the sampling methods are dominated by the exact models in this domain as well. But, here there is a significant difference between the sampling and the exact methods in the hierarchical model where there is a difference of .9, which corresponds to 9% of non-pour actions. In the relational hierarchical case, this difference is greatly reduced (0.3 to 0.4).

In this domain, the hierarchical knowledge seems to dominate the relational knowledge. This is due to the fact that all the subgoals are similar (i.e, each of them is preparing some kind of food) and the hierarchical knowledge clearly states the ordering of these subgoals. The relational hierarchical models have a better savings rate in the first few episodes as they have fewer parameters to

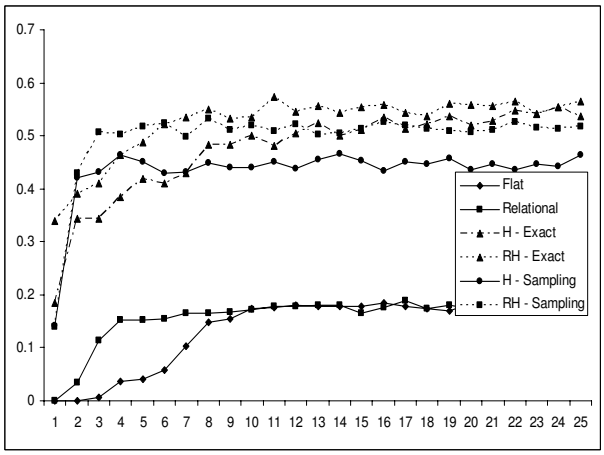

Figure 6: Learning curves of the different algorithms.

learn. Both the flat model and the relational model eventually converged on the same *savings* after 700 episodes. These results demonstrate that though all the models can eventually converge to the same value, the relational hierarchical model converges in early episodes.

There are some differences in the two domains: in the grid world, the agent and the assistant perform alternating actions. In the kitchen domain, the assistant can continue to act till it reaches a *pour* action or it chooses *no-op* as the best action. In grid world, there are no irreversible actions while *pour* is irreversible in kitchen domain. In grid world, the user's goal distribution has a prior that is a function of the number of doors to be opened to the goal while in the kitchen domain it is simply the number of actions in the plan. In grid world, the relations and hierarchy are equally important while in the kitchen domain, the hierarchies seem to be of more importance as the non-relational methods seem to do reasonably well. In spite of these differences, the two domains follow a similarity: the user's goal structure can be modeled as a relational hierarchy and hence can be solved by our proposed model.

5 Related Work

The use of relational hierarchies could potentially be very useful in many real-world assistant systems. For instance, see the work on a real-time desktop assistant [30]. This assistant uses a relational hierarchical model and a particle filter algorithm for tracking the user's goals. There has been several decision-theoretic assistants that have been formulated as POMDPs that are approximately solved offline. For instance, the COACH system helped people suffering from Dementia by giving them appropriate prompts as needed in their daily activities [2].

In this system, there is a single fixed goal of washing hands for the user. In *Electric Elves*, the assistant is used to reschedule a meeting should it appear that the user is likely to miss it [7]. These systems do not have a hierarchical goal structure for the user while in our system, the assistant infers the user's goal combinations and renders assistance.

Several plan recognition algorithms use a hierarchical structure for the user's plan. These systems would typically use a hierarchical HMM [20] or an abstract HMM [1] to track the user's plan. They unroll the HMMs to a DBN and perform inference to infer the user's plan. We follow a similar approach, but the key difference is that in our system, the user's goals are relational. Also, we allow for richer models and do not restrict the user's goal structure to be modeled by a HMM. We use the qualitative influence statements to model the prior over the user's goal stack. We observe that this could be considered as a method to incorporate richer user models inside the plan recognition systems. There has been substantial research in the area of user modeling. Systems that have been used for assistance in spreadsheets [8] and text editing [9] have used handcoded DBNs to infer about the user. Our system provides a natural way to incorporate user models into a decision-theoretic assistant framework.

Hierarchical Task Networks (HTNs)[5] have long been used in planning. HTNs refine plans by applying action decompositions where higher level actions consist of a partially-ordered set of lower level actions. Our work can be understood as using a (relational) HTN for representing the user's goal structure and then performing inference using this network in order to obtain a distribution over user's goals. In recent years, there have been several first-order probabilistic languages developed such as PRMs [17], BLPs [18], RBNs [13], MLNs [27] and many others. One of the main features of these languages is that they allow the domain expert to specify the prior knowledge in a succinct manner. These systems exploit the concept of parameter tying through the use of objects and relations. In this paper, we showed that these systems can be exploited in decision-theoretic setting. We combined the hierarchical models typically used in reinforcement learning with the kinds of influence knowledge typically encoded in relational models to provide a strong bias on the user policies and accelerate learning.

Quite a lot of progress have been made in the design of decision-support sytems. For a detailed review, please see [31]. It would be interesting to understand how the use of relational hierarchical models can help in the decision-making of a complex organization. Also, there has been some work on using hierarchies to organize information collected from huge amounts of texts (such as web documents)[32]. It will be an exciting future direction to use the hierarchical structured learned by this method and track the navigation of the user in the web and provide assistive actions.

6 Conclusions and Future Work

In this work we proposed the incorporation of parameterized task hierarchies to capture the goal structure of a user in a decision-theoretic model of assistance. We used the relational models to specify the prior knowledge as relational hierarchies and as a means to provide informative priors. We evaluated our model against the non-hierarchical and non-relational versions of the model and established that combining both the hierarchies and relational models makes the assistant more useful. We also provided a couple of solutions for estimating the user's goals: an exact method based on DBN and an approximate method based on sampling. The number of parameters can grow rapidly with the number of objects in the case of the DBN. The sampling method on the other hand sacrificies a small amount of accuracy for the ease of design, implementation and scalability. One of our future problems is to investigate the use of Rao-Blackwellization that allows for analytical inference on a part of the network and samples the other part of the network thus increasing the accuracy of the predicted model. Our has been employed in a real desktop assistant [30] but the same ideas can be used in several other assistant domains. For instance, in real-time strategy games where there is a natural hierarchical goal structure an in-built assistant can execute some of the user's goals. In a assistive technology for disabled setting, the user's goals can be modeled hierarchically and the assistant can provide necessary prompts or communication options.

The incorporation of hierarchies would enable the assistant to address several other problems in future. The most important one is the concept of parallel actions. Our current model assumes that the user and the assistant have interleaved actions and cannot act in parallel. Allowing parallel actions can be leveraged if the goal structure is hierarchical as the user can achieve a subgoal while the assistant can try to achieve another one. Yet another problem that could be handled due to the incorporation of hierarchies is the possibility of the user changing his goals midway during an episode. We can also imagine providing assistance to the user in the cases where he forgets to achieve a particular subgoal. Finally, it is important to learn these relational hierarchies using the trajectories of the user. There has been a lot of work on learning rules from uncertain data [33, 34]

APPENDIX

In this section, we represent several first order probabilistic models in FOCIL's syntax to show their commonalities and illustrate the fact that the algorithms are not specific to FOCIL.

Kersting and De Raedt introduced Bayesian Logic Programs [18]. BLPs combine Bayesian Networks with definite clause logic. Bayesian Logic Programs consist of two components: a qualitative component that captures the logical structure of the domain (similar to that of the Bayesian Network structure) and a quantitative component that denotes the probability distributions. An

example of a BLP clause is as follows:

```
bt(X) | father(F,X), bt(F), mother(M,X), bt (M)
```

There is a CPT corresponding to this clause. In this case, the predicates $mother(M, X)$ and $father(F, X)$ would have boolean values. One could then specify the ground facts like $father(John, Tom)$ etc. The function $bt(F)$ represents the blood type of F. The above statement says that a person's blood type is a function of his father's and mother's blood types. The FOCI statement corresponding to the above BLP clause is:

```
If { mother(M,P), father(F,P)} then M.bt, F.bt Qinf P.bt
```

BLPs also use combining rules for combining the distributions due to multiple instantiations of the parent predicates. The main difference between BLPs and FOCI statements is that in the latter, the logical conditions are clearly separated from the influents. BLPs do not make this distinction in the clauses, although they *are* semantically distinguished and implemented by a separate declaration in the model.

Another representation that is closely related to both FOCIL and BLPs is Logical Bayesian Networks [23]. They consist of conditional dependency clauses of the form $X|Y_1, ..., Y_k \longleftarrow Z_1, ..., Z_m$. This can be interpreted as "$Y_1, ..., Y_k$ influence X when $\langle Z_1, ...Z_k \rangle$ are true," where $Y_1, ..., Y_k$ and X are random variables and $\langle Z_1, ..., Z_m \rangle$ are logical literals. The above example of the bloodtype can be represented in LBNs as:

```
bt(X) | bt(M), bt(F) ⟵ Mother(M,X),Father(F,X)
```

More recently Getoor and Grant proposed the formalism of Probabilistic Relational Language (PRL) [21]. The main motivation behind this work is to represent the original work on probabilistic relational models (PRMs) [**?**] in logical notation. While PRMs exclusively use aggregators to combine the influences of multiple parents, both aggregators and combining rules can be used in the PRL framework. The entities and the relationships that are represented as predicates form the logical structure of the domain. The probabilistic structure is composed of non-key attributes that form the random variables in the domain. The general structure of the influence statement is: $DependsOn(X(\alpha), Y_1(\alpha), ...Y_n(\alpha)) \longleftarrow Z(\alpha)$ and can be interpreted as "$\langle Y_1(\alpha)....Y_n(\alpha)$ influence $X(\alpha)$ when $Z(\alpha)$ is true." Consider, our bloodtype example. In PRL, we can represent it as follows:

```
DependsOn(bt(X),bt(M), bt(F)) ⟵ Mother(M,X), Father(F,X)
```

The main difference between the PRLs and LBNs lies in the fact that the PRLs allow for aggregate functions explicitly. The aggregate functions do not pose special problems for parameter learning because often they are deterministic

and are given. However, inference is much more complicated with aggregate functions. In this paper we ignore aggregation and focus on combining rules. Also, in [21] the authors show how to represent several kinds of uncertainties like structure uncertainty, reference uncertainty, and existence uncertainty in PRL. These extensions are out of the scope for the current paper.

Although the different models differ from each other in syntactic details, they all share the same underlying semantics for the core language, and express equivalent pieces of knowledge. All of them also suffer from the multiple-parent problem, which can be addressed through combining rules. Thus, the algorithms discussed in this paper are relevant and applicable to all these formalisms and a few others such as Relational Bayesian Networks (RBNs) [13], Multi-Entity Bayesian Networks (MEBNs) [25], and Probabilistic Logic Programs [26].

Not surprisingly, there are also some statistical relational models that are different compared to FOCIL and it is an interesting research direction to determine how to use these models to specify prior knowledge. For example, PRISM [24] uses a representation that consists of a set of probabilistic atoms called facts, and a set of deterministic non-unit definite clauses called rules. A probability distribution is placed on the interpretations over the facts, and is extended to all literals via the minimal model semantics of definite clause programs. There is no straightforward mapping between the FOCIL representations and PRISM programs. Stochastic Logic Programs (SLPs) are very different from all the above languages since they place distributions on possible proofs of Horn programs rather than on interpretations [15].

Markov Logic Networks [27] and related Conditional Random Fields [16] are based on undirected graphical models and significantly differ from models based on directed graphs. Markov Logic Networks, for example, are more flexible in allowing knowledge to be expressed as weighted first-order formulas, and have correspondingly harder learning and inference problems as functions of the size of the formulas. It is possible to use such a representation for inference but specifying weights seem unnatural for the domains that we consider. Nevertheless, we are currently investigating the problem of using MLNs for specifying prior knowledge.

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