AN ABSTRACT OF THE THESIS OF

ALAN RYAN EK for the Ph. D.
(Name of student) (Degree)

in Forest Management (Mensuration-biometry) presented on April 15, 1969
(Major) (Date)

Title: A COMPARISON OF SOME ESTIMATORS IN FOREST SAMPLING

Abstract approved: ______________________
J. R. Dilworth

The objectives of this study were to ascertain the relative precision and accuracy of certain estimators on several forest populations and to determine if relative performance could be predicted from knowledge of population characteristics. Performance was tested on three populations of trees drawn from stands in northern Ontario. The first population consisted of 479 spruce and fir trees drawn from an uneven aged second growth spruce-fir stand. The second consisted of 309 maple and birch trees from a mature hardwood stand. The third population was composed of 500 red pine drawn from a forty year old plantation. Measurement data obtained for each tree included breast height diameter and total height. For the spruce-fir and hardwood stands, measurements of height and crown area were also obtained from large scale aerial photography.

Estimators for total volume, height and crown area were compared for the test populations. Independent or supplementary

variables employed were diameter, height and crown area plus several transformations and combinations of these variables. Four sample sizes, n = 4, 12, 24, and 40 were employed for each of 25 dependent-independent variable combinations considered. Simple expansion, ratio, unbiased ratio, regression and unequal probability estimators and stratified sampling with the simple expansion estimator were compared using Monte Carlo techniques. Relative performance was evaluated using estimates of sampling variances, biases and mean square errors obtained from repeated sampling of the test populations.

Results indicated linear and parabolic regression and the Horvitz-Thompson pps estimator were usually among the best three estimators for the two largest sample sizes studied. For the smaller sample sizes, linear regression, the Horvitz-Thompson pps and ratio of means estimators were best. For the estimation of volume using diameter-squared as the supplementary variable, linear regression was the best approach. Parabolic regression using diameter and diameter-squared was equally precise for the larger sample sizes.

Major factors affecting the relative performance of estimators were: 1) the form of the dependent-independent variable relationship (linear or curvilinear), 2) the correlation between these variables, 3) the position of the intercept of the population regression line,

4) the variance of the dependent variable given the independent variable and 5) sample size.

A Comparison of Some Estimators in Forest Sampling

by

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A THESIS

submitted to

Oregon State University

in partial fulfillment of
the requirements for the
degree of

Doctor of Philosophy

June 1969

APPROVED:

Head of Department of Forest Management and

in charge of major

Dean of Graduate School

Date thesis is presented April 15, 1969

Typed by Opal Grossnicklaus for Alan Ryan Ek

ACKNOWLEDGMENTS

The author is indebted to his major and minor professors, Dr. John R. Dilworth and Dr. Lyle D. Calvin respectively, for their guidance, suggestions and critical review of the manuscript. He also wishes to thank the rest of his committee: Dr. William K. Ferrell, Professor John F. Bell, Dr. David P. Paine and Dr. Alfred N. Roberts for review of the manuscript and the time and effort they have contributed so freely.

Special thanks are due Dr. Bijan Payandeh, Research Scientist and Dr. Wayne L. Myers, Biometrician, Canada Department of Fisheries and Forestry for their suggestions and constructive criticism during the preparation of the manuscript.

The author is also grateful to his employer, the Canada Department of Fisheries and Forestry, for the support which made this effort possible. Thanks are also due Dr. Leo Sayn-Wittgenstein and Mr. Alan H. Aldred of the Forest Management Institute of the Department for their support and technical assistance in obtaining the basic data.

I would also like to express my gratitude to my wife Carolyn for her patience, understanding and encouragement.

TABLE OF CONTENTS

TABLE OF CONTENTS (CONTINUED)

LIST OF TABLES

LIST OF TABLES (CONTINUED)

A COMPARISON OF SOME ESTIMATORS IN FOREST SAMPLING

I. INTRODUCTION

The estimation of forest characteristics by sampling requires specification of a sampling design. This involves identification of (a) the procedure for drawing sampling units such as trees or plots, (b) methods for measuring or observing the variable of interest on these units and (c) estimation equations or estimators for computing estimates from the information provided by the sample.

In many cases, it is important to consider use of supplementary information that is related to the variable of interest. If this information is relatively inexpensive and/or easy to obtain, it may be of value in improving the precision of estimates. Such information could be used to group units in a population as the basis for stratification or it might be used in the estimator itself. A ratio estimator is an example of the latter usage. The information might also be used to assign selection probabilities to samples or individual sampling units (e.g., probabilities proportional to the size or magnitude of the variable of interest or of variables related to the one of interest). Estimators utilizing this supplementary information are the primary subject of this paper.

The Problem

The overall performance or efficiency of an estimator, considering cost, precision and accuracy, depends largely on population characteristics and the costs associated with obtaining desired information from the sample. Unfortunately, no single estimator is optimally efficient for all forests and all objectives. The selection of an estimator thus depends heavily on prior knowledge of population characteristics plus cost, time, equipment and manpower considerations.

Foresters charged with forest inventory or timber appraisal responsibilities are often acutely aware of forest characteristics and the above mentioned cost, time and resource considerations. Many are also familiar with sampling methods as presented in texts by Cochran (1963) and Yates (1960). Still it is often difficult to choose between estimators. The profusion of new sampling techniques in recent years has further complicated the situation. Possible results from a poor choice are low precision, poor accuracy and unnecessary expenditures of time and money.

As an example, in a double sampling scheme for forest inventory, one might have to choose between use of a regression estimator or stratification with the simple expansion estimator. One might also consider use of a ratio of means versus a mean of ratios estimator (Freese, 1962) for establishing volume/basal area ratios on field plots. In another case, use of a sampling design involving selection with probability proportional to size may have advantages over a design with equal probability selection using a regression estimator or vice versa. In each example estimators differ in one or more respects. In the first, precision might be the major concern or perhaps practical difficulties in stratification. In the second, possible bias may be the most important consideration. In the last case, precision or the complexity of sample selection methods may be the most important factor. Unfortunately, it is not always possible to evaluate factors such as precision and bias to the degree desired. Consequently, it is difficult to integrate these factors with other considerations.

Justification

The difficulty in evaluating estimators and choosing between alternatives is due in part to a lack of comparative knowledge of estimator performance. A discussion and explanation of common forest sampling methods by Freese (1962) underlines this prevalent lack of knowledge. Freese suggested various estimators, e.g., ratio or regression estimators as appropriate for certain problems if basic assumptions are met. Consequences of unfulfilled assumptions were not clearly specified however.

The abundance of literature on this subject in the statistical literature is of little help to foresters. Many papers are difficult to interpret by those with a limited background in mathematics. Other papers consider only cases of limited interest such as that of samples of size 2. Monte Carlo studies in most cases have been limited to comparisons of ratio or pps estimators on computer generated populations. Comparisons utilizing actual populations in most cases have used census or agricultural data. Extrapolation of results to forest populations is thus difficult. The few Monte Carlo comparisons conducted using forestry data (Frauendorfer, 1967; Schreuder, 1966; Ware, 1967) are quite helpful, but it is clear from these that more comparative information is needed.

Scope and Objectives

This study was initiated in order to extend our knowledge of the performance of certain estimators on forest populations. Populations for this study were constructed using data from several forest stands located in northern Ontario. Estimators examined included those currently in use plus several which have potential value for practical applications. These include ratio, unbiased ratio, regression and pps estimators and stratified sampling with the simple expansion estimator. These were applied to different sets of tree measurement data from each poulation.

The objectives were to compare the variance and bias of estimates of total volume, height and crown area. Variance and bias were estimated primarily from results of repeated computer sampling. Several different variables were also used to provide the necessary supplementary information. These included tree diameter, height, crown area, plus transformations and combinations of these. Four sample sizes were also considered.

Specific questions posed were:

A. What is the relative performance, considering variance, bias and mean square error of each estimator under different conditions, i.e., different sample sizes, populations, and dependent-supplementary variable combinations.

B. Based on test results plus theory and results presented in the literature, is it possible to accurately predict estimator performance from knowledge of certain population characteristics, i.e., can results obtained here be generalized? If so, to what extent?

II. LITERATURE REVIEW

Sampling in Forestry

Sampling research in forestry has been concentrated largely on studies of sampling unit size, shape and distribution. Studies concerned with plot size and shape may be traced through ocular cruising, strip samples, circular plots, to Bitterlich point samples. Plot distribution studies have been concerned primarily with random versus systematic sampling. A brief account of these developments is given by Kulow (1966).

Studies of estimator performance have been more limited. Most advances in estimator usage in forestry have relied heavily on theory developed primarily for sampling human populations or agricultural crops. To date, forest sampling efforts have commonly used cluster or stratified sampling schemes, primarily with purposive, systematic or simple random selection methods. Regression estimators have also received considerable attention. Double sampling, either for regression or stratification, has been employed extensively in forest inventory. Examples of the use of most of these methods are given by Schumacher and Chapman (1942), Spurr (1952) and Loetsch and Haller (1964).

The introduction of Bitterlich point sampling (Bitterlich, 1948;

Grosenbaugh, 1952) and more recently "3-P" sampling (Grosenbaugh, 1963, 1965b) stimulated further interest in estimation problems. Grosenbaugh's (1958) paper on point sampling was notable as one of the first forestry papers to stress the concept of probability in Bitterlich point sampling--actually a form of pps sampling. The strong promotion of 3-P sampling for timber sale appraisal has also led to questions of its performance as compared to other methods (Schreuder, 1966; Ware, 1967; Schreuder, Sedransk and Ware, 1968).

Actually, 3-P sampling is a form of pps estimation with a provision for simple sampling. The estimator itself resembles the Horvitz and Thompson (1952) estimator described in the next section. Simple sampling, as defined here, refers to a method of selecting sampling units which does not require a list prior to sampling. The procedure involves determination of an individual sampling unit's inclusion or exclusion from the sample in a random manner as each unit is visited. The advantage of the method is that only one visit to the population is required. Unfortunately the use of this procedure results in a variable sample size.

In the case of 3-P, sampling units (trees) are selected with probabilities proportional to ocular volume or value estimates. In practice, an ocular estimate is made for each tree as it is visited. The estimate is then compared to an appropriate random number. If the estimate is equal or greater than the random number, the tree is included in the sample. This procedure requires only crude estimates of total volume, maximum individual tree volume and population size prior to sampling.

Schreuder (1966) and Ware (1967) considered the basic weakness of the method to be the variability in sample size. They felt this variability may too often result in either insufficient or excessive sample sizes with associated loss in precision or increase in costs. Ware also conducted Monte Carlo sampling trials on several forest populations to compare the performance of 3-P with other methods such as ratio and regression estimation and other pps estimators. General conclusions reached from this work were that 3-P sampling was, in fact, not always superior to other methods. Also, reasons why some estimators performed well on one population and poorly on another were not covered by available theory. Specific results will be discussed in a later review of pertinent simulation studies.

Following this work, Schreuder, Sedransk and Ware (1968) suggested alternatives to 3-P. These included standard ratio, regression and pps estimators plus techniques for simple sampling. Some of these techniques were suitable for use of ratio or regression estimators. Most were designed so as to limit variability in sample size. These authors also pointed out that when two visits to the population are made as suggested by Johnson (1967), there is certainly no need to employ simple sampling. The population list available from the first visit could easily be used to select the sample for the next visit when sample trees are measured. In this way, variability in sample size could be eliminated by use of pps estimators employing a fixed sample size. The list might also be employed with possibly more efficient sampling schemes such as regression or stratification.

To date the Bitterlich and 3-P schemes represent the most significant application of pps estimators in forestry. Grosenbaugh (1965a) has suggested that aerial photo interpretation classes normally used for stratification be used to assign selection probabilities to forest type delineations. This idea has not been tested, however. Despite limited progress, interest in comparative estimator performance stimulated by the introduction of new methods will undoubtedly lead to refinements in estimation methods.

Description of Estimators

Recent advances and trends in sampling theory and practice are well covered by Dalenius (1962) and Murthy (1963). This section will therefore be limited to a description of basic estimators and significant refinements. Estimators discussed here are also given in Appendix A, Table 1. Variance formulae and/or useful approximations for these estimators are given in Appendix A, Table 2.

Let us consider a population of N units, each having a characteristic of interest y_i associated with it. If Y is the population total of this characteristic, then the unbiased simple expansion estimator of Y based on a simple random sample of n units is

$$\hat{Y}_{srs} = \frac{N}{n}\sum^{n} y_i$$

Now let X be the population total of another characteristic x_i which is also associated with each unit. With this added information, we can construct the two most commonly used ratio estimators:

1) ratio of means estimator $\hat{Y}_r = X\sum^{n} y_i/\sum x_i = Xr$

2) mean of ratios estimator $\hat{Y}_{\bar{r}} = X\sum^{n}(y_i/x_i)/n = X\bar{r}$

Both estimators are unbiased if the regression of y on x is linear and through the origin. Under these conditions it can be shown by regression theory that $\hat{Y}_r$ performs the best when the variance of y_i about the regression line is proportional to x_i. If the variance of y_i about the regression line is proportional to x_i^2, $\hat{Y}_{\bar{r}}$ will be the best ratio estimator. Cochran (1963) states that bias in $\hat{Y}_r$ when these conditions are not met is usually unimportant. This follows from Cochran's illustration (p. 160) which shows the bias of this estimator relative to the standard error is of order $1/\sqrt{n}$. Bias of $\hat{Y}_{\bar{r}}$ relative to the standard error was shown by Frauendorfer (1967) to be of order $\sqrt{n}$ and thus may be considerable.

Concern over possible serious bias has led to the development of several unbiased and "nearly" unbiased ratio estimators. Examples of the latter are due to Beale (1962), Durbin (1959), Tin (1965), Nieto de Pascual (1961) and Frauendorfer (1967). Theoretical comparisons of some of these "nearly" unbiased estimators by Tin, and Monte Carlo comparisons by Rao and Beegle (1966), Frauendorfer (1967) and Bowman (1966) indicate that they offer little or no improvement in precision and accuracy over the ratio of means estimator. Due to these negative findings, they will not be considered further.

Two different approaches have been used in the development of unbiased ratio estimators. The first involves the estimation of a correction term which is then added to the estimator. The simplest expression of this type was given by Hartley and Ross (1954). Their estimator of the population total is:

$$\hat{Y}_{u\bar{r}} = X\bar{r} + \frac{n(N-1)}{(n-1)}(\bar{y}-\bar{r}\bar{x})$$

where $\bar{y}$ and $\bar{x}$ are the sample means of y and x. It should be apparent that this estimator, minus the correction term, is identical to the mean of ratios estimator.

An unbiased ratio of means type estimator was developed by Mickey (1959). His method involves dividing the sample at random into g groups of size m, where n = mg. The estimator of the population total is then given as:

$$\hat{Y}_{ur} = X\bar{r}_g + (N-n+m)g\,(\bar{y}-\bar{r}_g\bar{x})$$

where $\bar{r}_g = \sum^{g} r_j/g$ and r_j is the ratio of means estimator computed from the sample after omitting the j^{th} group, i.e., $r_j = (n\bar{y}-m\bar{y}_j)/(n\bar{x}-m\bar{x}_j)$ and $\bar{y}_j$ and $\bar{x}_j$ are the sample means computed from the j^{th} group. This estimator reduces to the Hartley-Ross estimator for the case $n = 2$.

Studies by Rao (1967) indicate that the variance of Mickey's estimator decreases as g increases so that the optimum choice of $g = n$. A Monte Carlo comparison of several ratio estimators by Rao and Beegle (1966) indicates that this estimator performs well under ideal conditions (test conditions will be discussed in more detail in a later section). Under non ideal conditions, they found it considerably more efficient than the Hartley-Ross estimator and also slightly more efficient than the standard ratio of means estimator.

In practice there is normally little need for an unbiased ratio of means estimator since bias of $\hat{Y}_r$ becomes negligible as n increases. The real utility of an unbiased ratio estimator of this type is for small samples or for sampling small populations. A common example would be surveys utilizing many strata and small samples within each. A comparable situation in forestry might be a small timber sale where separate estimates are desired for several species. Here bias may become a serious problem (Goodman and Hartley, 1958). The Hartley-Ross estimator may be of value whenever use of $\hat{Y}_{\bar{r}}$ is indicated but fulfillment of assumptions for unbiasedness is in doubt.

The development of pps estimators represent the other approach to unbiased ratio estimation. Hansen and Hurwitz (1943) are generally recognized as the first to formally introduce selection of sampling units with probabilities proportional to some measure of size. Horvitz and Thompson (1952) then presented a general theory for unequal probability sampling without replacement. Their estimator of Y is:

$$\hat{Y}_{pps} = \sum^{n} y_i/\pi_i$$

where π_i is the inclusion probability, i.e., the probability of including the i^{th} unit in the sample. If π_i is proportional to y_i, $\hat{Y}_{pps}$ will cease to vary and the variance of the estimator will then be zero. This suggests considerable variance reduction may be achieved by making π_i at least approximately proportional to y_i. In practice, π_i, is derived from a supplementary variable x_i, which is positively correlated with y_i. With n fixed and x_i known for each unit in the population, π_i may be set equal to nx_i/X.

This estimator is very similar to the common mean of ratios estimator, however, in some cases it is capable of greater precision and is also unbiased. Assumptions necessary for unbiasedness with normal ratio estimators, such as linearity and regression through the origin, are not required for the pps estimator--though such conditions would aid precision.

Despite the simplicity of this approach, the first procedures for drawing samples without replacement were often complex (Horvitz and Thompson, 1952; Yates and Grundy, 1953; Des Raj, 1956). A relatively simple procedure suggested by Hartley and Rao (1962) appears suitable for most forestry problems. It involves systematic sampling (with a random start) from the cumulated "sizes" or x_i values. In practice, the method involves the selection of a random starting point greater than 0 and less than $K = X/n$ (K must be equal or greater than the maximum x_i); cumulating the x_i as they are observed; and selecting those units whose x_i correspond to every K^{th} interval along the list of cumulated. x_i.

A second pps estimator of interest was proposed by Hájeck (1949); Lahiri (1951) and Midzino (1952). In this scheme, the probability p_s of a particular sample's selection (distinguished from an individual unit's selection probability) is proportional to the sum of sizes $\sum^n x_i$ in that sample, or

$$p_s = \sum^n x_i / \binom{N-1}{n-1} X$$

A method described by Hájeck and Midzino simplifies the sample selection procedure. Here only one unit is drawn proportional to size (x_i/X); the remaining n-1 units are drawn with equal probabilities. The estimator is then

$$\hat{Y}_{pp\Sigma x} = X \sum^{n} y_i / \sum^{n} x_i$$

Similarity with the ratio of means estimator is apparent, however, $\hat{Y}_{pp\Sigma x}$ is unbiased. As with Mickey's estimator, $\hat{Y}_{pp\Sigma x}$ will probably have its greatest value for small samples.

When the relation between y_i and x_i does not go through the origin but is approximately linear, an estimate based on linear regression should perform well. The estimator is

$$\hat{Y}_{lr} = N[\bar{y} + b\ (\bar{X} - \bar{x})]$$

This estimator is used extensively in forestry. It is generally biased but the bias relative to the standard error becomes negligible with increasing sample size. Cochran (1963) states that despite the availability of large sample variance approximations, more information is needed about the behavior of estimates from small samples; especially the value of n required for use of large sample formulae. This author also feels comparisons between regression and ratio estimators are needed since a choice is often made between the two in forestry (Freese, 1962; Johnson, Dahms and Hightree, 1967).

In cases where a curvilinear relationship is suspected, the supplementary variable is sometimes squared to achieve the desired linear relationship. An example is the squaring of tree dbh to achieve linearity with volume. Another approach used in such cases is a parabolic regression, i.e., a 2nd degree polynomial:

$$\hat{Y}_{pr} = N[\bar{y} + b_1 (\bar{X}-\bar{x}) + b_2(\bar{X}' - \bar{x}')]$$

where $\bar{X}'$ and $\bar{x}'$ refer to the population and sample means of $x_i' = x_i^2$. A common use of this estimator is for the "correction" of volume estimates made from aerial photo cruise plots (Paine, 1965), generally in a multiphase sampling scheme. This estimator was studied in comparisons conducted by Cunia and Simard (1967) using samples of size n = 100 drawn from tree volume table data. Their results indicate it was more precise and more accurate than ratio and linear regression estimators which utilized x_i^2 as the supplementary variable. These comparisons were based on large sample variance approximations, however, consequently small sample results remain in doubt.

Stratification is often used in forest inventory to increase precision and also to facilitate separate estimates for particular strata of interest (Bickford, 1952; Macpherson, 1962; Loetsch and Haller, 1964). Despite considerable work on techniques of optimum allocation of sampling units to strata (Cochran, 1963), forestry applications are usually limited by practical difficulties. Examples of these are errors in classifying units for purposes of stratification and lack of knowledge regarding strata sizes and variances prior to sampling.

For timber appraisal problems where complete enumeration of the population is possible, stratification either before or after sample selection may be feasible. As with ratio, regression and pps estimates, the x_i could be ocular estimates. Errors in classifying units into proper strata and their subsequent effect on bias and precision are covered by Cochran (1963) and Dalenius and Ghosh (1967). In the case of post stratification, it is important that the sample and/or strata sizes be large enough to ensure an adequate sample within each stratum. If these conditions are not met, two or more strata may have to be combined with a subsequent loss in precision. With sample sizes in timber sales often exceeding n = 100, i.e., 100 trees, post stratification together with a simple sampling scheme may have some value.

Theoretical Comparisons of Estimators

The intent of this section is to compare theoretically the relative performance of the estimators described earlier. In some cases theoretical results are obtained rather easily, e.g., a comparison of variance formulae by inspection. In other comparisons, certain assumptions are necessary which may not always be valid. This latter situation is complicated by current disagreement as to appropriate theory for finite population sampling (Godambe, 1965, 1966a, 1966b; Hanurav, 1967; Hartley and Rao, 1968a, 1968b).

Most discussions of estimator performance use the simple expansion estimator for simple random sampling as a standard. Considering the ratio of means estimator Cochran (1963), among others, has shown that in large samples the estimator $\hat{Y}_r$ has a smaller variance than $\hat{Y}_{srs}$ if

$$\rho > \frac{CV_x}{2CV_y} = \frac{\text{coefficient of variation of } x_i}{2(\text{coefficient of variation of } y_i)}$$

where ρ is the correlation coefficient between y_i and x_i. If ρ is less than the quantity indicated, the variance of $\hat{Y}_r$, denoted $V(\hat{Y}_r)$, will be greater than $V(\hat{Y}_{srs})$. It follows that when x_i and y_i have approximately equal variability, this ratio estimator will be more precise than simple random sampling if ρ exceeds .5.

Rao (1967) investigated the relative performance of several ratio estimators using two models: 1) when regression of y_i on x_i is linear and x_i normally distributed and 2) when regression of y_i on x_i is linear and x_i has a gamma distribution. Under the assumptions of model 1, Mickey's unbiased ratio estimator with $g = n$ was shown to have a smaller asymptotic variance than $\hat{Y}_r$. For model 2, the exact variance of Mickey's estimator with g = n was smaller than the variance of the Hartley-Ross estimator $\hat{Y}_{u\bar{r}}$. When $n > 8$, the variance of Mickey's estimator was also less than the mean square error of $\hat{Y}_r$.

Using large sample variance approximations, Cochran (1963) showed that the linear regression estimator has a smaller variance than $\hat{Y}_{srs}$ whenever the correlation between x_i and y_i is different from zero. He also pointed out that $V(\hat{Y}_{lr})$ is less than $V(\hat{Y}_r)$ except when the relation between x_i and y_i is a straight line through the origin. In this case these two estimators will have equal variances.

Following Sukhatme (1954) it is apparent that, for large n where the relationship between x_i and y_i is linear with constant residual variance, the variance for stratified sampling with stratification based on the x_i values will always be less than that for the linear regression estimator. This of course requires that the population be divided into an adequate number of strata so as to make the variances within strata small. When the relationship between x_i and y_i is not linear, the use of regression becomes less favorable. Stratified sampling has the advantage that it is unbiased for any type of relationship between x_i and y_i and for any sample size.

With double sampling, for example in forest inventory, Bickford (1966) has indicated that stratified sampling is more precise than linear regression. He also states that use of linear regression is normally attractive only when $\rho > .9$. Des Raj (1964) has shown that when large sample variance approximations are valid, the precision of double sampling using pps, ratio or linear regression estimators at the second phase may be ranked according to the performance of these estimators in single phase sampling. Des Raj (1954) mentioned the difficulty of comparing sampling methods on finite populations when no functional form may be assumed for the distribution of data. To counter this problem he considered a finite population as being drawn from an infinite superpopulation which possessed certain characteristics. Results obtained do not apply to any single population but to the average of all possible finite populations that can be drawn from the superpopulation. Comparisons described below refer to cases when the sampling fraction is negligible and n is large enough for approximate variance formulae to hold.

Several superpopulation models have been considered to date. One used by Des Raj (1958) is

$$y_i = \alpha + \beta x_i + e_i \qquad (1)$$

where α and β are constants, e_i is a random error, the expected value of e_i given x_i equals zero, i.e., $E(e_i|x_i) = 0$, $V(e_i|x_i) = ax_i^{\theta}$, and $\theta > 0$. With $\alpha = 0$, Des Raj has given the following results:

Estimates based on pps sampling with replacement (w/r)[1] (Cochran, 1963) will have a smaller variance than simple random sampling providing $\theta \geq 1$. When $0 \leq \theta \leq 1$ this will be so only if a certain in-inequality is satisfied. Also, this method will be more precise than stratified sampling with proportionate allocation if $\theta > 1$. For $0 < \theta \leq 1$ the latter estimator is more precise. For all θ, stratified sampling with optimum allocation[2] is always more precise than the pps (w/r) estimator. When $\theta = 0$ and α is not restricted to zero, ratio and regression estimators are generally more precise than the pps (w/r) estimator. Cochran (1963) also states that if optimum allocation can be achieved, stratified sampling is never inferior and nearly always better than other methods providing x_i is a constant within strata.

Des Raj (1954) gave a working rule which indicates the pps (w/r) estimator will be more precise than $\hat{Y}_{srs}$ when

$$\rho^2 > [1 + (\alpha^2/\sigma_y^2)][1 + (1/CV_x^2)]$$

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[1]The pps (w/r) estimator has essentially the same form and performance characteristics as $\hat{Y}_{pps}$ when N is large and the sampling fraction is negligible. See Hartley and Rao (1962) for a detailed comparison.

[2]The "optimum allocation" referred to in this paper is that due to Neyman (1934) in which

$$n_h = n \frac{N_h S_h}{\sum^{L} N_h S_h}$$

where L denotes the number of strata and n_h, N_h and S_h^2 refer to the sample size, stratum size and variance, respectively, for the h-th stratum.

This is applicable under the above model 1, assuming the x_i's are fixed and $V(e_i|x_i)$ is independent of x_i. It is apparent from this rule that the precision of pps sampling relative to that of $\hat{Y}_{srs}$ decreases as α increases, i.e., as the y intercept of the regression of y_i on x_i departs from the origin.

Rao (1966) compared the variances of several pps estimators including $\hat{Y}_{pp\Sigma x}$ and $\hat{Y}_{pps}$ using a model similar to (1) where $\alpha = 0$, $a > 0$ and $\theta \geq 0$. He suggested these conditions were quite reasonable for situations where x_i and y_i are highly correlated. Results may be summarized as follows:

$$V(\hat{Y}_{pps}) \lesseqgtr V(\hat{Y}_{pp\Sigma x}) \text{ if } \theta \gtrless 1$$

Since, intuitively, $\hat{Y}_r$ should perform quite like $\hat{Y}_{pp\Sigma x}$ in large samples, it is not surprising that $V(\hat{Y}_r)$ is also less than the variance of the pps (w/r) estimator when $\theta < 1$, as is shown by Cochran (1963). According to Cochran, θ may be expected to lie between 1 and 2 in practice.

It is difficult to establish simple and comprehensive rules from these limited studies, however, some order is apparent. The relative precision of all the estimators discussed here depends in part on one or more of the following factors: ρ, the degree of linearity in the relation between y_i and x_i, the variation of x_i and y_i, departure of the regression line of y_i on x_i from the origin, $(V(e_i|x_i)$ and the distribution form of the x_i. It is probable that knowledge of some of these characteristics would be quite helpful in selecting an appropriate estimator for a given sampling problem.

Numerical Comparisons of Estimators

Relatively few comparative trials of estimator performance have been conducted to date. Fewer still have utilized actual forestry data. Two "Monte Carlo"[3] studies that involved repeated sampling of computer generated populations are described first because they effectively illustrate the influence which population characteristics have on sampling properties of estimators. Results of studies utilizing forestry data will then be summarized.

Rao and Beegle (1966) compared several ratio estimators including $\hat{Y}_r$, $\hat{Y}_{\bar{ur}}$ and $\hat{Y}_{ur}$ by analyzing results of repeated sampling of computer generated data under two different models. For model 1

$$y_i = k(x_i + e_i)$$

where k = 5, x_i has a normal distribution with X = 10, $\sigma_x^2 = 4$ and e_i has a normal distribution independent of x_i with mean 0 and variance 1. Consequently $\rho = .89$ and $CV_x = .2$. For model 2, x_i and y_i

[3]The term "Monte Carlo" is used here to denote the study of sampling distributions of various estimators by means of repeated sampling of test populations. Monte Carlo methods, according to Kendall and Buckland (1967) commonly denote "the solution of mathematical problems arising in a stochastic context by sampling experiments".

have a bivariate normal distribution with $\bar{X} = 5$, $\sigma_x^2 = 45$, $\bar{Y} = 15$, $\sigma_y^2 = 500$, $\rho = .4, .6$ or $.8$ and

$$y_i = \alpha + \beta x_i + e_i$$

where $\alpha = \bar{Y} - \beta \bar{X}$, $\beta = \rho\sigma_y / \sigma_x$ and e_i has normal distribution independent of x_i with mean 0 and variance $= \sigma_y^2 (1-\rho^2)$. For each selected n (4, 6, 10, 20 or 50), 1000 samples of n pairs (x_i, y_i) were used for each model.

As in most of the studies described in this section, test criteria (e.g., variance and bias) were calculated using the Monte Carlo estimates themselves. A major reason for this approach is simply that appropriate formulae for certain estimators and test criteria are often lacking. As an example of methods used, the variance of a particular estimator is calculated using standard variance formulae and assuming each Monte Carlo estimate to be an observation.

Results for model 1 indicate little difference in variances among the estimators tested. It may be noted that all ratio estimators tested are unbiased under this model since the relation between x_i and y_i is linear and through the origin. Conditions for model 2 are not ideal since the relationship between x_i and y_i does not pass through the origin. In this case $\hat{Y}_{ur}$ was considerably more precise than $\hat{Y}_{u\bar{r}}$ and slightly more precise than $\hat{Y}_r$.

Bowman (1966) conducted a study of ratio estimators including $\hat{Y}_r$ and $\hat{Y}_{u\bar{r}}$. He compared the variance, bias, skewness and kurtosis of estimates for n = 6, 10, 14 and 18 and ρ = .5, .7 and .9 using models of the form

$$y_i = \alpha + \beta x_i + e_i$$

Where e_i were normally distributed independent of x_i. Four trials were made, each involving a different distribution for x_i. These distributions were normal, uniform, chi square and exponential with σ_x^2 equal to 10, 10, 20 and 100, respectively. For each n, ρ and x_i distribution, 5000 samples of size n were generated.

Tabulated results showed a decrease in variance and mean square error (MSE) for all estimators as ρ and/or n increased, as expected. Likewise, the distribution of estimates tended toward normality as n increased. Of the five estimators studied, the Hartley-Ross unbiased estimator $\hat{Y}_{u\bar{r}}$ performed the poorest due to its relatively large variance. Most significant, however, was the effect the distribution of x_i had on the sampling distributions of the estimators. For all estimators, as the x_i distribution was changed from uniform to normal, then to chi-square and finally exponential, the bias, variance, skewness and kurtosis of estimates tended to increase.

The precision and accuracy of $\hat{Y}_r$, $\hat{Y}_{\bar{r}}$, $\hat{Y}_{u\bar{r}}$, $\hat{Y}_{ur}$ and eight other ratio type estimators were examined by Frauendorfer (1967) using three sets of actual tree measurement data. The first set, designated population 1, consisted of data on 579 sawtimber trees of mixed species (predominately Douglas-fir, Pseudotsuga menziesii Franco) from the Pacific Northwest. The variables tested were:

y_1 = gross Scribner board foot volume (from form class tables)

$y_2 = y_1$ - field estimated cull deduction

x_1 = dbh by 4 inch classes

x_2 = height in number of 16 foot logs

$x_3 = (dbh)^2$

Three combinations of x and y were examined: (y_1, x_3), (y_2, x_1) and (y_2, x_2). Five hundred Monte Carlo samples (with replacement) were then drawn for samples of size 5, 10, 17 and 28. Results were presented along with coefficients for linear and parabolic regression curves and respective ρ^2 values which served to describe the relationship between x_i and y_i in the population under study.

An analysis of results indicated that $\hat{Y}_{\bar{r}}$ was unsatisfactory in all cases because of its extremely large bias. Bias of $\hat{Y}_r$ was large relative to its standard error (.38 for n = 5 (y_i, x_3)) but decreased slowly as n increased. Ranking of the estimators of interest here considering MSE's would indicate $\hat{Y}_r$ as the best estimator with $\hat{Y}_{ur}$ and $\hat{Y}_{u\bar{r}}$ as second and third respectively.

The second population was a subsample of 31 sugar maple trees

(*Acer saccharum* Marsh.) drawn from a larger population of 373 sawtimber trees of mixed Appalachian hardwoods. The third population consisted of 50 beech trees (*Fagus grandifolia* Ehrh.). Variables for both populations were:

y_1 gross cubic foot volume

y_2 net board foot volume

x_1 diameter at breast height (dbh)

x_2 $(dbh)^2 \cdot$ height

For the second population the combinations (y_1, x_1), (y_2, x_1) and (y_2, x_2) were tested. On population 3, only (y_1, x_1) and (y_1, x_2) were examined. Monte Carlo replications and samples sizes used were similar to those employed for population 1. Results for population 2 were also similar. Results for population 3 appear of little interest since ρ values were all less than .5. This was perhaps due to a limited range of data, however this was not explicitly stated in the text. On the basis of the populations studied, $\hat{Y}_r$ was considered to be the best ratio estimator. Frauendorfer also suggested that for samples of size 4 or larger, the need for unbiased ratio estimators was perhaps overemphasized in the literature.

Ware (1967) described comparisons made on a range of estimators using the same data. He did not subdivide the hardwood data, however, but used it intact as one test population. Estimators compared were $\hat{Y}_{srs}$, $\hat{Y}_r$, $\hat{Y}_{\bar{r}}$, $\hat{Y}_{lr}$, $\hat{Y}_{pps}$, $\hat{Y}_{pps(w/r)}$ and the estimator for 3-P sampling. Variable combinations tested were (y_1, x_3) and (y_2, x_2) for the Pacific Northwest population and (y_1, x_3) for the Appalachian hardwoods. Samples of size 6, 17, 28, 58 and 115 were drawn from each population. The number of samples drawn for each n was 250 for the hardwoods and 100-229 for the conifers.

For both populations Monte Carlo results indicated the MSE for 3-P sampling was much smaller than the variance of simple random sampling, but only moderately smaller than the MSE's for $\hat{Y}_r$ and $\hat{Y}_{\bar{r}}$. The linear regression estimator $\hat{Y}_{lr}$ performed about the same as 3-P however, their MSE's were considerably greater than those for $\hat{Y}_{pps}$ and $\hat{Y}_{pps(w/r)}$. The mean of ratios estimator $\hat{Y}_{\bar{r}}$ also showed a large positive bias (27-30% on the average) for the conifer population and this bias did not decrease with sample size. Ware suggested this might be due to the form of the relationship between x_i and y_i, perhaps the variance structure. For the hardwood population this estimator showed only a 2% positive bias.

Schreuder (1966) compared estimation procedures for double sampling on several populations constructed from data on 640 forest inventory plots in Iowa. Plot cubic foot volume was the primary dependent variable and photo measured height and crown density were used as independent variables along with several transformations. Simple random sampling was used at the first phase and the estimators $\hat{Y}_{pps}$, $\hat{Y}_{pps(w/r)}$, $\hat{Y}_r$, $\hat{Y}_{\bar{r}}$ and $\hat{Y}_{lr}$ were used at the second phase. Stratified sampling with the simple expansion estimator was also used at the second phase. Strata were derived by accumulating ordered x_i so that their sum was just equal or less than X/L where L was the number of strata. This process was then repeated on the remaining x_i to form L strata. Optimum allocation (Neyman, 1934) was used subject to the restriction that at least two units should be sampled within each stratum.

Unfortunately, the number of Monte Carlo replications of the sampling process were limited and thus it was difficult to generalize from the results. Results did indicate, however, that the performance of $\hat{Y}_{\bar{r}}$, $\hat{Y}_{pps}$ and $\hat{Y}_{pps(w/r)}$ was erratic. The behavior of $\hat{Y}_{\bar{r}}$ was especially so, perhaps because of its large apparent bias. The estimators $\hat{Y}_r$, $\hat{Y}_{lr}$ and stratified sampling with the simple expansion estimator performed best.

Cunia and Simard (1967) considered the problem of estimating volume cut from tree length logging operations. Estimators compared included $\hat{Y}_{srs}$, $\hat{Y}_r$, $\hat{Y}_{u\bar{r}}$, $\hat{Y}_{lr}$ and $\hat{Y}_{pr}$. Comparisons were made by applying large sample variance approximations to tree measurement data. The dependent variable y_i was total cubic foot tree volume calculated using four foot sections assuming a parabolic form. Basic independent variables included dbh and stump diameter plus squares and other transformations of these terms. These transformations were actually different volume equations; both constants and function form varied. Variances were calculated from samples of size n = 100 drawn from each of 14 species--watershed groups of data from eastern Canada. For each group, these variances were calculated for numerous sets of x_i, each pertaining to an independent variable or transformation.

Results indicated estimators utilizing supplementary information were usually much more precise than $\hat{Y}_{srs}$. Overall ranking according to precision (smallest variance first) gave $\hat{Y}_{pr}$, $\hat{Y}_{lr}$, $\hat{Y}_r$, $\hat{Y}_{ur}$ and $\hat{Y}_{srs}$. The unbiased ratio estimator $\hat{Y}_{ur}$ - behaved erratically and the authors suggested it be ignored for practical applications. Regarding regression estimators, there was little difference between $\hat{Y}_{lr}$ and $\hat{Y}_{pr}$.

It was concluded that the ratio estimators were greatly affected by the set of x_i used and the tree population under study. The regression estimators were also affected by the tree population but the authors considered them rather insensitive to the set of x_i used.

III. METHODOLOGY

Test Populations

Populations used as the basis for evaluating estimator performance were constructed from tree measurement data obtained from three forest stands located in northern Ontario.[4] Trees comprising these populations were selected to cover the range of size classes present in these stands. The number and species of trees in each test population is indicated below. Measurements obtained for each tree or sampling unit are enclosed in square brackets. Brief descriptions of the parent stands are also given.

Population I. (479 trees)

184 black spruce (Picea mariana (Mill.) BSP.), 40 white spruce (Picea glauca (Moench) Voss) and 255 balsam fir (Abies balsamea (L) Mill.).

[dbh = d, total height = h, photo measured height = hp, and photo measured crown area = ca[5]]

Data were obtained from an overmature and consequently uneven aged, second growth, lowland spruce-fir stand near Searchmont, Ontario.

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[4] Data used here was a subset of that collected for a broad range of sampling studies involving these stands.

[5] Crown area is defined here as the area within a closed two dimensional figure describing the maximum horizontal extent of a tree crown.

Population II. (309 trees) 189 sugar maple (Acer saccharum Marsh.) and 120 yellow birch (Betula alleghaniensis Britt.)

[d, h, hp, ca]

Data were obtained from an overmature uneven aged northern hardwood stand near Searchmont, Ontario.

Population III. 500 red pine (Pinus resinosa Ait.)

[d, h]

Data were obtained from a densely stocked forty year old red pine plantation near Thessalon, Ontario.

This information was obtained by field crews during the summers of 1967 and 1968 and from measurements made on aerial photographs during the intervening winter. Photo measurements were made on large scale (approximately 1:2000) vertical 70 mm photography flown by the author in August, 1967. Kodak Tri-X Aerecon film was used with a Vinten reconnaissance camera and a six inch lens.

Measurements of d were made to the nearest one-tenth inch, h and hp were recorded in feet and ca was noted in square feet. Photo height was determined by conventional parallax measurement methods and ca was estimated by superimposing fine dot grids over stereoscopically viewed tree images. Photo scale was determined using standard radial line plotting techniques.

For each test population, comparisons of estimator performance were made using several different dependent (y_i) and independent (x_i) variable combinations. Some of these were transformations and or combinations of the basic measured variables. The additional variables were d^2, hp· log ca, and vm = merchantable cubic foot tree volume.[6] The variable hp·log ca was used here because it is known to be linearly correlated with tree volume (Sayn--Wittgenstein and Aldred, 1967). Merchantable cubic foot volume was calculated using d^2 and h in tree volume formulae developed by Honer (1967).

Comparisons

Population Characteristics Estimated

As indicated earlier, estimator performance was to be evaluated on the basis of variance and bias of estimates of population totals, specifically total merchantable cubic foot volume VM, total height H and total crown area CA. Total volume was used because it is of considerable interest in forest inventory and appraisal efforts. Total height and crown area were chosen primarily to extend the range of y_i and x_i relationships studied, but they are also of practical interest. The actual (y_i, x_i) combinations used for comparing

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[6] Merchantable volume in this case included that from a one-half foot stump to a minimum top diameter of three inches.

the estimators and the methods and assumptions employed are indicated below.

Description of (y_i, x_i) Combinations

The assessment of estimator performance required the construction of two computer programs. The first was designed to describe the more important characteristics of each of the 25 (y_i, x_i) combinations studied. The x_i used as supplementary variables are shown in Table 1 for each test population and each of the characteristics estimated, VM, H and CA. Descriptions of (y_i, x_i) combinations are given in Appendix B. These descriptions include, for each variable, the mean, variance σ^2, coefficient of variation CV, range and coefficients of skewness γ_1[7] and kurtosis γ_2[8] of its distribution. In addition, the frequency of number of x_i within ten different size classes is shown. These classes are of width [range]/ 10, the first beginning at minimum x_i. These classes are equivalent to strata and may also be used for testing the fit of distribution functions. The variance of y_i within each of these classes is also given. Coefficients are also noted for linear and parabolic regression of y_i

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[7] $\gamma_1 = K_3/\sigma^3$ where K_3 and σ are the third moment and standard deviation of the variable in question, respectively.

[8] $\gamma_2 = K_4/\sigma^4$ where K_4 is the fourth moment of the variable in question.

Table 1. Supplementary variables (x_i) used for prediction of total volume, height and crown area for each test population.

Population characteristic	Supplementary variables (x_i) Test populations		
	I	II	III
Total volume VM	d d^2 h ca hp • log ca	d d^2 h ca hp • log ca	d^2 d_t^2 *
Total height H	ph d ca	ph d ca	d
Total crown area CA	d h	d h	
Total volume VM_t **	d^2 ca		

*d rounded to midpoint of two inch diameter class i.e., 5, 7, 9 etc., prior to squaring.

** $VM_t = \sum^{N} y_{ti}$ where $y_{ti} = y_i + 10$, i.e., a constant was added to individual tree volume in order to alter the intercept of the regression of y_i on x_i.

on x_i plus ρ^2 and ρ_m^2, where ρ_m is the multiple correlation coefficient.

This first program also calculated exact variances for simple random and stratified sampling plus approximate variances for $\hat{Y}_{pps}$, $\hat{Y}_r$, $\hat{Y}_{ur}$, $\hat{Y}_{lr}$ and $\hat{Y}_{pr}$. These values served primarily as checks on variances estimated by the second program, to be described later.

Estimators Compared

All ten of the estimators described earlier were compared. Also, with stratified sampling, 2, 4, 6, 8 and 10 strata were used. Thus, in all, 14 different estimators were examined. These are summarized in Appendix A, Table 1.

In addition, four different sample sizes were used for each (y_i, x_i) combination employed. These were 4, 12, 24 and 40. These particular values were chosen to cover a range of practical situations. Also, they facilitated stratification.

Monte Carlo Procedures and Method of Analysis

Convenient exact formulae or approximations for variance and bias are not available for most of the estimators examined, thus these values were estimated by repeated sampling of test populations. This was accomplished through a second computer program in which a large number of samples were drawn from each population for each sample size and (x_i, y_i) combination of interest. These samples were drawn at random using a computerized algorithm or random number generator (IBM Corporation, 1959, 1967, p. 60).

Estimation of Sampling Variance and Bias

For each estimator, as a sample was drawn, an estimate of Y was calculated. Monte Carlo estimates of true variances $V(\hat{Y})$ were then calculated as the variance of the estimates of Y over all samples drawn. Estimates of bias were calculated by subtracting the average of Monte Carlo estimates of Y from computed true totals. These two Monte Carlo estimates are hereafter designated as V and B, respectively. Other comparative criteria derived from V and B were the estimated mean square error mse = $V + B^2$, estimated bias relative to the standard error $B/\sqrt{V}$ and the estimated bias relative to the true total B/Y.

In the actual performance comparisons, only V, mse, $B/\sqrt{V}$ and B/Y were used. Also, V and mse were converted to a relative basis using the simple expansion estimator for simple random sampling as a standard. These relative values are defined as

$$\text{Relative precision RP} = V_{srs}/V_q$$

$$\text{Relative accuracy RA} = mse_{srs}/mse_q$$

where q refers to the subscript of the estimator in question.

Construction of Estimates

Estimates of Y in the Monte Carlo program for the estimators $\hat{Y}_{srs}$, $\hat{Y}_r$, $\hat{Y}_{\bar{r}}$, $\hat{Y}_{ur}$, $\hat{Y}_{u\bar{r}}$, $\hat{Y}_{lr}$ and $\hat{Y}_{pr}$ were calculated from samples drawn with equal probability without replacement. For a given sample, estimates were calculated for all of these estimators using the same set of $n(y_i, x_i)$ values.

For $\hat{Y}_{pp\Sigma x}$, the selection method used was equivalent to that described by Hajeck (1949) and Midzino (1952). It involved the selection of one sampling unit with probability proportional to size and the remainder with equal probabilities. Results for this estimator are closely tied to those for $\hat{Y}_r$, however, because the (n-1) units selected with equal probabilities were the same ones used for simple random sampling.

With $\hat{Y}_{pps}$, selection was accomplished using the systematic procedure suggested by Hartley and Rao (1962) described earlier.

For stratified sampling $\hat{Y}_{st}$ individual strata were defined by intervals of width [range] /L where L is the number of strata desired. Sampling units were assigned to strata accordingly as their x_i fell into a particular size class. Equal allocation was used as the basis for drawing samples. These procedures (grouping by size classes and equal allocation) seemed realistic in light of current forest sampling practices. Within strata, sampling was with equal probabilities and without replacement. Where n/L was fractional or less than two, consideration of stratified sampling was omitted.

Sample Size for Monte Carlo Trials

Using $\hat{Y}_{srs}$ as a standard, the level of precision specified for each sample size tested (n=4, 12, 24 and 40) was a standard error of the variance estimate SE_V of ± 20%. This was arrived at by considering the cost of computer time and the expected size of differences in estimator variances. The latter were derived from previous work (Ware, 1967) and preliminary trials. Extrapolating from Hansen, Hurwitz and Madow (1953, Vol. II, p. 99) the number of Monte Carlo replications or samples required is

$$N_s = (\gamma_2' - 1)/(SE_V)^2$$

where γ_2' is the coefficient of kurtosis of the estimates $\hat{Y}_{srs}$.

For a normal distribution $\gamma_2' = 3$, but will assume larger values for skewed distributions (Hansen, Hurwitz and Madow, 1953). Studies by Bowman (1966), which were described earlier, gave values of 3-44 for the sampling distributions of several ratio estimators. The largest values were obtained from sampling populations with exponential x_i distributions. Values of γ_2' larger than 10 occurred only with very small samples (n=6), however, and they became progressively smaller as n increased. Only one estimator had values of γ_2'

greater than 16.

Allowing for differences in the distributions of $\hat{Y}_{srs}$ and ratio estimators plus the fact that estimates of γ_2' tend to be skewed,[9] values of 16, 10, 5 and 4 were assumed for samples of size n = 4, 12, 24 and 40, respectively. Substitution of these values in the above equation indicates the required N_s to be 370, 225, 100 and 75 for n = 4, 12, 24 and 40, respectively. For small n where bias may be important, these N_s values were considered to be sufficient to establish the general magnitude of this term.

Analysis

The evaluation of estimators was done by inspection. Using the criteria RP, RA, $B/\sqrt{V}$ and B/Y an attempt was made to group estimators according to their performance under different conditions as indicated by the quantitative descriptions of (y_i, x_i) combinations and different sample sizes. Efforts were then made to assess the most important factors affecting the performance of each estimator and to indicate the specific conditions under which it performs best and also when its use should be avoided. Theory and findings expressed in the literature review were also integrated where possible.

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[9] According to Snedecor and Cochran (1967), the distribution of γ_2' would not approach the normal until the sample size N_s was greater than 1000.

IV. RESULTS

Efficacy of Monte Carlo Procedures

Results of the Monte Carlo comparisons of estimator performance are given in Appendix C. Separate tables are presented for populations I, II, and III. Within each table, values for RP, RA, B/Y and $B/\sqrt{V}$ are listed for each estimator for each (y_i, x_i) combination and sample size utilized. Before examining the performance of estimators, however, several checks were made to ascertain the reliability and validity of the Monte Carlo procedures.

The first check was a comparison of the true variances for $\hat{Y}_{srs}$ with Monte Carlo estimates. Results for the 25 (y_i, x_i) combinations used are shown in Table 2. The percentages given indicate that desired precision (a standard error of the variance estimate of ±20%) was achieved. They also indicate that the γ_2' values used to derive the N_s were appropriate.

Another check involved computation of estimates of γ_2' values for all estimators from the Monte Carlo results. These estimated values were of course different for each estimator, but from insepction it did not appear that they would differ much, on the average, from those obtained for $\hat{Y}_{srs}$. Thus it appeared that adequate precision was obtained for all estimators.

Table 2. Number and percentage of intervals [Monte Carlo variance V ± 20%] which contained $V(\hat{Y}_{srs})$ for three test populations and four sample sizes.

	Number and percentage of intervals containing true variance						
	Population						
Sample size	I Number	I Percent	II Number	II Percent	III Number	III Percent	Overall Percent
4	8	66.7	10	100.0	3	100.0	84.0
12	10	83.3	10	100.0	3	100.0	92.0
24	9	75.0	7	70.0	1	33.3	68.0
40	8	66.7	9	90.0	3	100.0	80.0

For the last check, values for B/Y for all the unbiased estimators were inspected. Considering both the sign and magnitude of these terms (usually less than ± 4%), there was no indication of serious bias in the Monte Carlo procedures.

Grouping of Estimators by Performance

Attempts to group estimators according to similar performance for various (y_i, x_i) combinations proved difficult. All of the descriptive characteristics given in Appendix B appeared to affect performance. The most important were judged to be 1) the form of the (y_i, x_i) relationship (linear or curvilinear), 2) the correlation between y_i and x_i, 3) the deviation of the (y_i, x_i) relationship from the origin i.e., position of y intercept, 4) $V(e_i | x_i)$ and 5) sample size.

All but the regression and stratified sampling estimators were affected by the position of the y intercept. Also the ratio, unbiased ratio, linear regression and unequal probability estimators were quite sensitive to the degree of linearity in the (y_i, x_i) relationship.

Regarding sample size, the relative precision and accuracy of most estimators remained largely constant with increasing sample size. Exceptions were the regression and mean of ratios estimators. The relative precision and accuracy of $\hat{Y}_{lr}$ was sometimes poor for n = 4, but improved and was nearly constant for larger sample sizes. The parabolic regression estimator was imprecise for n = 4 and erratic for n = 12, however, its relative performance improved greatly as sample size increased. As an example, Table 3 in Appendix C shows that the relative precision of $\hat{Y}_{pr}$ increased from less than 1.0 with n = 4 to 131.0 with n = 40 for the (vm, d^2) combination, i.e., y_i = vm and $x_i = d^2$. The relative accuracy of the mean of ratios estimator often decreased with larger sample sizes. This was due to bias which did not appear to decrease as n increased.

Assessment of Estimator Performance

Ratio Estimators

The inequality given earlier for assessing the precision of ratio estimation relative to simple expansion estimates proved appropriate. Of the 25 (y_i, x_i) combinations studied, $\hat{Y}_r$ was more precise and accurate than $\hat{Y}_{srs}$ in 20 cases. Values for the RA of this ratio estimator ranged from 1 to 61 for these combinations. In each such case

$$\rho > \frac{CV_x}{2CV_y} .$$

When

$$\rho < \frac{CV_x}{2CV_y}$$

$\hat{Y}_r$ was usually less precise and accurate than the simple expansion estimate. The only exception was a borderline case where the two sides of the inequality were nearly equal. In that case RP and RA values for $\hat{Y}_r$ were near unity. Ranking $\hat{Y}_r$ and $\hat{Y}_{lr}$ according to precision and accuracy also indicated that the ratio estimator was slightly better than linear regression for n = 4 in 11 of the (y_i, x_i) combinations studied.

Results summarized in Tables 3-5 indicate that $\hat{Y}_r$ was more accurate than $\hat{Y}_{\bar{r}}$ for population I and both more precise and accurate for populations II and III for the important (vm, d^2) combination. This was also true for estimation of volume from d_t^2 values for population III. For the combination (vm, hp$\cdot$log ca), the RP of $\hat{Y}_{\bar{r}}$ was greater than that of $\hat{Y}_r$ for both population I and II, but the RA was greater only for population I. Bias of $\hat{Y}_{\bar{r}}$ for volume estimation using d^2 or hp$\cdot$log ca as supplementary variables averaged more than ten percent. As indicated earlier the bias of $\hat{Y}_{\bar{r}}$ was often this large. It should be noted that $\hat{Y}_r$ had negligible bias in nearly all cases (as compared to the Monte Carlo bias of unbiased estimators). Its bias with n = 4 was less than 3.9 percent for all (vm, d^2) and (vm, hp$\cdot$log ca) combinations.

Unbiased Ratio Estimators

The relative precision of the unbiased ratio estimators $\hat{Y}_{ur}$ and $\hat{Y}_{u\bar{r}}$ was similar for nearly all (y_i, x_i) combinations in which they were more precise than $\hat{Y}_{srs}$. The exception was for highly correlated linear relationships such as (vm, d^2) shown in Tables 3-5. In

Table 3. Performance of estimators for estimation of total volume, height and crown area for (y_i, x_i) combinations of practical interest from population I.[a]

Variables	Performance criteria	Estimators $\hat{Y}_{srs}$	$\hat{Y}_{r}$	$\hat{Y}_{\bar{r}}$	$\hat{Y}_{ur}$	$\hat{Y}_{\bar{ur}}$	$\hat{Y}_{lr}$	$\hat{Y}_{pr}$	$\hat{Y}_{pp\Sigma x}$	$\hat{Y}_{pps}$	$\hat{Y}_{s-2}$	$\hat{Y}_{s-4}$	$\hat{Y}_{s-6}$	$\hat{Y}_{3-8}$	$\hat{Y}_{s-10}$
y_i = vm	RP	1.0	19.6	35.9	11.5	8.7	67.1	60.1	17.5	31.2	1.6				
	RA	1.0	18.5	8.3	11.6	8.7	63.1	60.1	17.6	31.4	1.6				
$x_i = d^2$ [b]	B/Y	.00	.02	.10	.00	.00	.01	.00	.00	.00	.04				
	$B/\sqrt{V}$	.00	.19	2.30	-.02	-.01	.28	.07	-.01	.01	.08				
y_i = vm	RP	1.0	1.8	3.5	1.5	1.5	2.4	2.9	1.7	3.5	2.1	3.5			
	RA	1.0	1.8	2.1	1.5	1.5	2.3	2.8	1.7	3.5	2.2	3.5			
x_i = hp· log ca	B/Y	-.01	.01	.10	.00	.00	.02	.02	.00	.00	.01	.01			
	$B/\sqrt{V}$	-.01	.06	.91	.01	.01	.17	.17	.01	-.06	.06	.03			
y_i = h	RP	1.0	1.1	1.4	1.0	.9	2.6	1.9	1.1	1.5	.5[c]				
	RA	1.0	1.1	.7	1.0	.9	2.5	1.9	1.1	1.5					
x_i = d	BY	.00	.00	-.04	.00	.00	.00	.00	.00	.00					
	$B/\sqrt{V}$	-.02	-.05	-.97	.05	.03	-.12	.14	.01	.04					
y_i = ca	RP	1.0	2.2	2.4	2.2	2.2	2.2	1.7	2.2	2.7	.6[c]				
	RA	1.0	2.2	2.1	2.2	2.2	2.3	1.7	2.3	2.7					
x_i = d	B/Y	.01	.01	.03	.00	.00	.00	.00	.00	.00					
	$B/\sqrt{V}$	.05	.07	.34	.05	.05	.04	.00	.05	.00					

a/ Performance figures for the estimators are based on averages of results over all four sample sizes studied. Figures for $\hat{Y}_{lr}$ obtained by averaging results over sample sizes 12, 24 and 40 only. Figures for $\hat{Y}_{pr}$ obtained by averaging results over sample sizes 24 and 40 only.

b/ For parabolic regression $x_i = d$, $x_i' = d^2$.

c/ RP values based on true variances.

Table 4. Performance of estimators for estimation of total volume, height and crown area for (y_i, x_i) combinations of practical interest from population II.[a/]

Variables	Performance criteria	Estimators $\hat{Y}_{srs}$	$\hat{Y}_{r}$	$\hat{Y}_{\bar{r}}$	$\hat{Y}_{ur}$	$\hat{Y}_{\bar{ur}}$	$\hat{Y}_{lr}$	$\hat{Y}_{pr}$	$\hat{Y}_{pp\Sigma x}$	$\hat{Y}_{pps}$ [c/]	$\hat{Y}_{s-2}$ [d/]	$\hat{Y}_{s-4}$	$\hat{Y}_{s-6}$	$\hat{Y}_{s-8}$	$\hat{Y}_{s-10}$
y_i=vm	RP	1.0	54.3	25.8	49.2	22.8	83.5	84.4	60.0	73.3	.8				
	RA	1.0	53.1	9.1	49.1	22.8	84.0	84.8	60.0						
$x_i=d^2$ [b/]	B/Y	.01	.01	.10	.00	.00	.00	.00	.00						
	$B/\sqrt{V}$	.02	.15	2.36	-.01	.00	.07	-.02	-.04						
y_i=vm	RP	1.0	1.5	1.8	1.5	1.4	2.0	2.3	1.5	1.7	1.4	1.2	1.2	1.3	1.3
	RA	1.0	1.5	1.4	1.5	1.4	2.0	2.2	1.5						
x_i=hp · log ca	B/Y	-.01	.00	.12	-.01	-.01	.01	-.01	-.01						
	$B/\sqrt{V}$	-.03	.02	.73	-.03	-.03	.07	-.06	-.02						
y_i=h	RP	1.0	.4	.3	.4	.3	2.0	2.4	.4	.4	.8	1.0	1.3		
	RA	1.0	.4	.3	.4	.3	1.9	2.4	.4						
x_i=d	B/Y	.00	-.02	-.15	.00	.00	-.01	.00	-.01						
	$B/\sqrt{V}$	.02	-.14	-1.67	-.03	-.03	-.16	.06	-.07						
y_i=ca	RP	1.0	2.5	3.2	2.4	2.4	2.2	1.8	2.5	3.2	1.2	1.9	2.2		
	RA	1.0	2.6	2.8	2.4	2.4	2.2	1.8	2.5						
x_i=d	B/Y	-.01	.00	.03	.00	.00	.00	-.01	.00						
	$B/\sqrt{V}$	-.05	.00	.38	-.02	-.02	-.03	-.12	-.04						

a/ Performance figures for the estimators are based on averages of results over all four sample sizes studied. Figures for $\hat{Y}_{lr}$ obtained by averaging results over sample sizes 12, 24 and 40 only. Figures for $\hat{Y}_{pr}$ obtained by averaging results over sample sizes 24 and 40 only.

b/ For parabolic regression $x_i = d$, $x_i' = d^2$.

c/ RP values based on true variance of $\hat{Y}_{srs}$ and an approximate formula for the variance of $\hat{Y}_{pps}$; see Appendix A, Table 2.

d/ RP values for stratified sampling based on true variances.

Table 5. Performance of estimators for estimation of total volume and height for (y_i, x_i) combinations of practical interest from population III.[a/]

Variables	Performance criteria	Estimators													
		$\hat{Y}_{srs}$	$\hat{Y}_r$	$\hat{Y}_{\bar{r}}$	$\hat{Y}_{ur}$	$\hat{Y}_{\bar{ur}}$	$\hat{Y}_{lr}$	$\hat{Y}_{pr}$	$\hat{Y}_{pp\Sigma x}$	$\hat{Y}_{pps}$	$\hat{Y}_{s-2}$ [b/]	$\hat{Y}_{s-4}$	$\hat{Y}_{s-6}$	$\hat{Y}_{s-8}$	$\hat{Y}_{s-10}$
y_i=vm	RP	1.0	32.0	8.8	32.4	16.3	115.0	107.8	35.6	24.3	1.9	7.1	15.7	23.5	30.8
	RA	1.0	31.3	4.6	32.4	16.3	115.4	107.0	35.6	23.8					
$x_i = d^2$	B/Y	.00	.01	.10	.00	.00	.00	.00	.00	.00					
	$B/\sqrt{V}$	.00	.13	1.95	.00	-.01	.00	-.11	.01	.04					
y_i=vm	RP	1.0	14.5	7.0	14.6	11.4	17.5	16.4	15.6	12.5	1.5	3.7			
	RA	1.0	14.4	3.0	14.6	11.4	17.4	16.3	15.5	12.5					
$x_i = d_t^2$	B/Y	.00	.00	.08	.00	.00	.00	.00	.00	.00					
	$B/\sqrt{V}$	.00	.05	1.26	-.01	-.01	-.05	-.03	.00	-.04					
y_i=h	RP	1.0	.3	.2	.3	.2	2.6	3.9	.3	.3	1.7	2.9	3.7	3.8	3.7
	RA	1.0	.3	.1	.3	.2	2.6	3.9	.3	.3					
x_i = d	B/Y	.00	-.01	-.08	.00	.00	.00	.00	.00	.00					
	$B/\sqrt{V}$	-.02	-.07	-1.32	.02	.02	-.13	-.14	.01	.00					

a/ Performance figures for the estimators are based on averages of results over all four sample sizes studied. Figures for $\hat{Y}_{lr}$ obtained by averaging results over sample sizes 12, 24 and 40 only. Figures for $\hat{Y}_{pr}$ obtained by averaging results over sample sizes 24 and 40 only.

b/ RP values for stratified sampling based on true variances.

such cases, $\hat{Y}_{ur}$ was substantially more precise than $\hat{Y}_{u\bar{r}}$. The mean square error for Mickey's estimator was often equal to and sometimes slightly smaller than that for $\hat{Y}_r$. In all but three cases the RP for $\hat{Y}_{ur}$ was equal to or greater than that for $\hat{Y}_{u\bar{r}}$. Differences for the exceptions were small. The ratio of means estimator was always more accurate than $\hat{Y}_{u\bar{r}}$, however, the latter was often more accurate than $\hat{Y}_{\bar{r}}$.

Unequal Probability Estimators

Little difference in the relative precision and accuracy of $\hat{Y}_r$ and $\hat{Y}_{pp\Sigma x}$ was noted, even for the smallest sample sizes. This was a further indication that bias of $\hat{Y}_r$ was seldom important for $n = 4$.

Generally $\hat{Y}_{pps}$ was of equal or greater precision than $\hat{Y}_r$ and $\hat{Y}_{pp\Sigma x}$. This follows from observed variance relationships assuming Des Raj's (1958) model as appropriate. The model, described earlier, assumed $V(e_i | x_i) = ax_i^{\theta}$. Considering the (vm, d^2) combination; for population I, θ appears to be greater than one. The relative precision of $\hat{Y}_r$ and $\hat{Y}_{\bar{r}}$ substantiate this conclusion. In this case the relative precision of $\hat{Y}_{pps}$ was greater than that of $\hat{Y}_r$ (31.2 and 19.6, respectively). The value of θ for population II appeared only slightly greater than one. Here the difference between the relative precision of $\hat{Y}_r$ and $\hat{Y}_{pps}$ was relatively small (54.3 and 60.0, respectively). For the same combination from population III, θ was apparently less than one, perhaps due to a limited range of data. Except for n = 4, $\hat{Y}_r$ was more precise than $\hat{Y}_{pps}$ (RP values of 32.0 and 24.3, respectively). These results follow from theory given earlier due to Rao (1966) and Cochran (1963). An examination of the variance relationships given in Appendix B indicated values for θ ranging from slightly less than zero to nearly two.

Regarding the importance of the position of the y intercept, trials using (vm_t, d^2) and (vm_t, ca) showed a substantial drop in relative precision for all estimators except linear and parabolic regression and stratified sampling (see Appendix C, Tables 1 and 2). This follows from the rule by Des Raj (1954) given earlier which illustrates the effect α has on the precision of $\hat{Y}_{pps}$.

Regression Estimators

We often choose between use of ratio and linear regression estimators where the (y_i, x_i) relationship is approximately linear and possesses a y intercept near the origin. For all three populations examined, linear relationships such as (vm, d^2), (vm, hp·log ca) and (h, hp) suggested that the linear regression estimator should be used for optimum precision. In some cases, for example (vm, d^2) for all three populations, linear regression was considerably more precise than ratio estimation. Where the y intercept of a linear (y_i, x_i) relationship departed substantially from the origin and/or ρ

was small, $\hat{Y}_{lr}$ was again the best choice (see Appendix C).

For curvilinear relationships $\hat{Y}_{pr}$ performed noticeably better than all other estimators for $n \geq 24$ considering both relative precision and accuracy. For volume estimation Tables 3 and 4 indicate parabolic regression using d as the independent variable and linear regression using d^2 produced similar large gains over $\hat{Y}_{srs}$. Considering use of d^2 only, $\hat{Y}_{lr}$ would have ranked first, $\hat{Y}_{pr}$ second and $\hat{Y}_{pps}$ third according to relative accuracy figures for all three populations (except population III where $\hat{Y}_{pp\Sigma x}$ would have ranked third).

Considering volume estimation from photo measured variables (vm, hp · log ca), ranking of figures given in Tables 3 and 4 from first to third according to accuracy would give $\hat{Y}_{pps}$, $\hat{Y}_{pr}$ and $\hat{Y}_{lr}$ for population I and $\hat{Y}_{lr}$, $\hat{Y}_{pr}$ and $\hat{Y}_{pps}$ for population II. In this last case linear regression was the most accurate for n = 12 and 24 while $\hat{Y}_{pps}$ was the best for n = 4. For the (h, d) combination parabolic regression proved the most accurate with $\hat{Y}_{lr}$ a close second.

Trials on population III using (vm, d^2) are shown in Table 5. These trials showed that rounding the supplementary variable may substantially reduce the precision of all estimators except $\hat{Y}_{srs}$. There was no change in ranks based on relative accuracy, however.

Stratified Sampling

Informative trials of stratified sampling were limited primarily due to skewed x_i distributions which reduced the number of strata into which populations could be divided (see Appendix B). It was possible to form ten strata with only eight (y_i, x_i) combinations. Ranking estimators according to relative accuracy, for population I, stratified sampling with the simple expansion estimator ranked first, second or third best for the combinations (vm, h), (vm, hp· log ca) and (vm_t, ca). Stratified sampling also ranked among the top three for (vm, d) and (h, d) for population II and for (h, d) only on population III (see Appendix C).

Since x_i distributions were skewed to the right and the $V(e_i | x_i)$ often increased in that directions, proportional and optimum allocation would probably be more precise than the equal allocation used here. According to theory, optimum allocation should perform best under such circumstances. It is also probable, judging from results given here, that the optimum number of strata will lie between four and ten for similar populations and (y_i, x_i) combinations.

V. DISCUSSION

Results obtained here lend some support to previous work by Ware (1967), Frauendorfer (1967), Schreuder (1966) and Cunia and Simard (1967). This agreement should broaden the range for inference.

It does appear that for sample sizes equal or greater than four there is no great need for unbiased ratio estimators since the bias of $\hat{Y}_r$ is seldom large. If one is used, it should be Mickey's estimator. Use of the mean of ratios estimator should be avoided because of its bias. The magnitude of $B/\sqrt{V}$ further limits its value. Except for samples of size 4 and 12, bias of regression estimators may be considered negligible. Conversely, since these estimators are normally quite precise, caution should be used in expressing sampling errors for small samples since the bias relative to the standard error may be relatively large.

Unfortunately, the usefulness of much of the available theory depends on quantitative knowledge of population characteristics which is seldom available prior to sampling. Still, rules given by Cochran (1963) and Des Raj (1954) plus results for various linear models such as those stated by Des Raj and Rao (1966) appear useful.

For the more important (y_i, x_i) combinations such as (vm, d) or (vm, d^2), (vm, hp·log ca) and (h, d) most foresters should be able to estimate the nature or degree of the major factors affecting estimator performance, e. g., the form of the (y_i, x_i) relationship. Inserting such information into the rules together with study of empirical research results should reduce the uncertainty in estimator selection. There also appears to be some flexibility present. This follows from the fact that, in some cases, there is little practical difference in the relative precision and accuracy between the best two or three estimators.

Comments on Best Estimators

Linear and parabolic regression and $\hat{Y}_{pps}$ performed consistently well. They ranked as the best three estimators in nearly all cases involving samples of size 24 and 40. Ranking according to mse for samples of size 4 and 12, $\hat{Y}_{lr}$ and $\hat{Y}_{pps}$ were usually first or second and $\hat{Y}_r$ commonly ranked third. For the particularly important (vm, d^2) combination, $\hat{Y}_{lr}$ was best for all three populations. Parabolic regression for volume estimation with $x_1 = d$ and $x_2 = d^2$ was roughly equal to linear regression using d^2 for the larger sample sizes.

Grosenbaugh (1967) reasoned that unequal probability estimators would be best because they, unlike ratio and regression estimators, do not require knowledge of the form of the (y_i, x_i) relationship. He also stated bias may be a problem with ratio or regression estimation. Results obtained here plus those reviewed earlier contradict these statements however. Bias does not appear to be a problem with appropriate use of ratio and regression estimators. Also, the functional form of the (y_i, x_i) relationship does not seem to be a problem for the type of populations examined here. We normally encounter only linear or parabolic relationships when dealing with trees or forest plots in conjunction with inventory or appraisal tasks. Furthermore, foresters should have the background knowledge to guess which form they are dealing with. Even where $\hat{Y}_{1r}$ is used for a parabolic relationship, results indicate this estimator will perform as well as or better than $\hat{Y}_{pps}$. If sample size is large, the parabolic estimator will also work well for linear relationships.

For multiphase sampling Des Raj (1964) has shown that the ranking of estimators by performance will be little different from ranks obtained from comparisons in single phase sampling. With simple sampling, i.e., sampling without a list of the sampling units and knowledge of their x_i values, given appropriate sample selection procedures (Schreuder, Sedransk and Ware, 1968), there is no reason why ratio and regression estimators should not rank nearly as well as or better than Grosenbaugh's (1963) 3-P estimator. For the estimation of merchantable tree volume where the regression of volume on d^2 may not pass through the origin, regression estimation may be preferred.

Choice of Supplementary Variables

Unfortunately, available theory does not help to indicate the degree of differences in estimator performance. In this respect, empirical studies are especially helpful. Results obtained here point to a very real problem in timber appraisal efforts: what should be used as a supplementary variable? Should we use ocular estimates of tree diameter as supplementary variables for regression, pps or 3-P estimation or should we actually measure the diameters? A comparison of results for (vm, d^2) and (vm, d_t^2) for population III indicates the relative precision of the best estimator, linear regression, dropped from 115 to 17.5 due to rounding of d to the nearest two inch class. A similar drop was evident for parabolic regression. The precision of the ratio of means and pps estimators also dropped by approximately 50 percent.

Factors in this drop in precision may be traced in Appendix B, Table 3. Figures given there indicate ρ^2 dropped from .99 for (vm, d^2) to .94 for (vm, d_t^2) and the variance of y_i within strata for the latter combination was roughly three times larger than that for (vm, d^2).

The figures given in Table 5 for the (vm, d_t^2) combination correspond in magnitude to those given by Ware (1967) using four inch diameter classes for similar (y_i, x_i) combinations. One might ask then what precision would Ware have obtained had he used one or two inch diameter classes. Certainly we should consider the cost and precision of measurement versus ocular estimation more carefully. A similar situation arises with the use of variables measured on aerial photographs. Should the supplementary variable photo volume be estimated crudely but cheaply from small scale photographs or precisely from more expensive large scale photography? These are but two of the many similar questions commonly ignored in the literature on forest sampling.

VI. SUMMARY

Simple expansion, ratio, unbiased ratio, regression and unequal probability estimators and stratified sampling with the simple expansion estimator were compared using Monte Carlo techniques. The objective was to ascertain the relative performance of certain estimators for the estimation of some forest characteristics of interest. Performance was tested on three different populations of trees, four sample sizes and 25 dependent-independent variable combinations. A second objective was to ascertain whether or not estimator performance could be accurately predicted from knowledge of certain key population characteristics.

The performance of estimators in terms of relative variance, relative mean square error and bias varied considerably depending on the dependent-independent variable combination employed. Relative mean square errors did not appear to vary significantly over the four sample sizes tested except for the mean of ratios estimator and the linear and parabolic regression estimators. The mean of ratios estimator was also the only estimator which showed serious bias. This bias did not decrease as sample size increased and thus use of this estimator cannot be recommended. The linear regression estimator sometimes performed poorly for sample size four, but was consistently good for larger samples. The performance of the parabolic regression estimator was erratic for samples of size 4 and 12, but improved markedly for larger sample sizes.

The bias of the ratio of means estimator may be considered negligible for sample sizes equal to or greater than four in most cases. Among the unbiased ratio estimators tested, Mickey's estimator was the best. Rarely, however, did this estimator perform better than the ratio of means estimator. Results for the ratio estimators confirmed findings from earlier comparisons discussed in the literature review.

Although it is difficult to name a best estimator, it was apparent that linear and parabolic regression and the Horvitz-Thompson pps estimator performed consistently well. Judging from the size of their respective mean square errors, they were usually the best three estimators for the two largest sample sizes tested. For the smaller sample sizes, the ratio of means, linear regression and unequal probability estimators were best.

For the variable of greatest interest, volume, linear regression using d^2 as the supplementary variable and parabolic regression using d produced essentially the same results for the larger sample sizes tested. Using these supplementary variables, regression was the most precise and accurate approach for volume estimation.

Comparisons involving stratified sampling were limited. However, test results in combination with available theory would indicate it should perform as well as or better than most other estimators. Certainly more extensive empirical comparisons of stratified sampling utilizing proportional and optimum allocation are needed.

Results obtained here indicated the more important factors affecting the relative performance of estimators were: 1) the form of the (y_i, x_i) relationship (linear or curvilinear), 2) the correlation between y_i and x_i, 3) the position of the y intercept of the regression of y_i on x_i, 4) $V(e_i | x_i)$ and 5) sample size.

The author feels that for most populations encountered in forest inventory or appraisal problems, the ranking of estimators,should be comparable regardless of whether or not the y_i and x_i variables refer to tree or plot characteristics. This follows from the fact that we normally encounter only linear or parabolic (y_i, x_i) relationships in such cases. Unfortunately, available theory does little to indicate the degree of differences in performance. Such differences may be more important than ranks would indicate in many cases. Obviously more empirical comparisons are needed.

The tremendous difference in precision between using d^2 and d_t^2 as supplementary variables for volume estimation was one example where ranking inadequately expressed differences in precision. Rounding diameters to two inch classes reduced the relative precision of the best estimator, linear regression, from 115 to 17.5. The precision of other estimators was also substantially reduced. A pertinent question is then: what should be used as the supplementary variable? This question was discussed using current appraisal and inventory methods as examples.

Regarding the extrapolation of results to double or simple sampling problems, the ranking of estimators will probably remain the same. For double sampling this conclusion follows from theory presented by Des Raj (1964). With simple sampling, this conclusion follows from results obtained in this study given appropriate simple sampling procedures such as those suggested by Schreuder, Sedransk and Ware (1968). The degree of differences between estimators will probably be reduced, however, due to the presence of additional terms in variance expressions.

Providing some knowledge is available regarding the major factors influencing performance, it should be possible to narrow down the number of estimators to be considered for a particular estimation problem.

The choice will also depend on the availability of suitable sampling variance estimators--an important consideration for many of the estimators discussed here, especially if sample size is small. Further research should thus be directed towards the derivation and testing of suitable variance estimators, notably for ratio, regression, and pps estimators.

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APPENDIX A

Table 1. Summary of estimators studied.

Sample selection method	Name of estimator	Estimator
Simple random sampling	Simple expansion	$\hat{Y}_{srs} = \frac{N}{n} \sum^{n} y_i$
Simple random sampling	Ratio of means estimator	$\hat{Y}_r = Xr \qquad r = \sum^{n} y_i / \sum^{n} x_i = \bar{y}/\bar{x}$
Simple random sampling	Mean of ratios estimator	$\hat{Y}_{\bar{r}} = X\bar{r} \qquad \bar{r} = \frac{1}{n} \sum^{n} y_i/x_i$
Simple random sampling	Mickey's unbiased ratio estimator; g = n	$\hat{Y}_{ur} = X\bar{r}_g + (N-n+1)g(\bar{y} - \bar{r}_g \bar{x})$ $\bar{r}_g = \sum^{g} r_j / g$ $r_j = (n\bar{y} - y_j)/(n\bar{x} - x_j)$
Simple random sampling	Hartley - Ross unbiased ratio estimator	$\hat{Y}_{u\bar{r}} = X\bar{r} + \frac{n(N-1)}{n-1} (\bar{y} - \bar{r}\bar{x})$

Table 1. Continued.

Sample selection method	Name of estimator	Estimator
Simple random sampling	Linear regression estimator	$\hat{Y}_{lr} = N[\bar{y} + b(\bar{X} - \bar{x})]$
Simple random sampling	Parabolic regression estimator	$\hat{Y}_{pr} = N[\bar{y} + b_i(\bar{X} - \bar{x}) + b_2(\bar{X} - \bar{x})]$
Probability proportional to sample sum of sizes	Midzino's unbiased ratio estimator	$\hat{Y}_{pp\Sigma x} = Xr$
Probability proportional to size	Horvitz - Thompson pps estimator	$\hat{Y}_{pps} = \sum^{n} y_i/\pi_i \quad \pi_i = \frac{nx_i}{X}$
Stratified random sampling	Stratified sampling	$\hat{Y}_{s-L} = \sum_{h=1}^{L} \frac{N_h}{n_h} \Sigma y_{hi}$ L = number of strata h = stratum number

Table 2. Variance formulae for estimators studied.

	Variance formulae	Source
$V(\hat{Y}_{srs})$	$= \frac{N^2}{n} S_y^2(1-f) \qquad S_y^2 = \frac{\sum^{N} (y_i - \bar{Y})}{N-1} \ ; \ f = \frac{n}{N}$	Cochran (1963)
$V(\hat{Y}_r)$	$\doteq N^2 \frac{(1-f)}{n} (S_y^2 + R^2 S_x^2 - 2R\rho S_y S_x)$	Cochran (1963)
$V(\hat{Y}_{u\bar{r}})$	$\doteq \frac{N^2}{n} (S_y^2 + \bar{R}^2 S_x^2 - 2\bar{R} S_{yx}) + \frac{N^2}{n(n-1)} (S_r^2 S_x^2 + S_{rx}^2)$	Cochran (1963)
$V(\hat{Y}_{lr})$	$\doteq \frac{N^2(1-f)}{n} S_y^2 (1-\rho^2)(1 + \frac{1}{n} + \frac{3 + 2\gamma_{2x}^2}{n^2})$	Cochran (1963)
$V(\hat{Y}_{pr})$	$\doteq \frac{N^2}{n} (1-f) S_y^2 (1 - \rho_m^2)$	Cunia and Simard (1967)
$V(\hat{Y}_{pp\Sigma x})$	$= \frac{X}{\binom{N-1}{n-1}} \sum_{s=1}^{\binom{N}{n}} \frac{(\Sigma y_i)_s^2}{n \, (\Sigma x_i)_s} - Y^2$	Rao, (1966)

Table 2. Continued.

	Variance formulae	Source
$V(\hat{Y}_{pps})$	$\doteq \sum^{N} \pi_i \left[1 - \frac{(n-1)}{1}\pi_i\right] \left(\frac{y_i}{\pi_i} - \frac{Y}{n}\right)^2$	Hartley and Rao (1962)
$V(\hat{Y}_{s-L})$	$\doteq \sum_{h=1}^{L} N_h^2 \frac{S_h^2}{n_h} (1 - f_h)$	Cochran (1963)

APPENDIX B

Table 1. Descriptions of (y_i, x_i) combinations from population I.

Variables	Mean	Variance σ^2	Coefficient of variation CV (percent)	Range	Coefficient of skewness γ_1	Coefficient of kurtosis γ_2	For x_i: frequency within each of ten strata[a/] For y_i: variance of y_i within above strata[b/]									
x_i=d	7.59	6.16	32.7	4.5- 25.2	1.92	10.30	194	157	79	34	8	2	3	1	0	1
y_i=vm	7.50	64.73	106.0	1.1-100.4	5.24	47.90	.80	1.68	5.16	4.48						
			Linear regression:	$y_i = -15.576 + 3.041\, x_i$			$\rho^2 = .880$									
			Parabolic regression:	$y_i = .567 + .184\, x_i - .632 x_i^2$			$\rho_m^2 = .991$									
$x_i=d^2$	63.77	2669.57	81.0	20.2-635.0	4.65	40.26	380	76	15	3	2	1	1	0	0	1
y_i=vm	7.50	64.73	106.0	1.1-100.4	5.24	47.90	5.67	7.52	11.38							
			Linear regression:	$y_i = -2.372 + .155 x_i$			$\rho^2 = .988$									
			Parabolic regression:	$y_i = -1.790 + .142 x_i + .000 x_i^2$			$\rho_m^2 = .991$									
x_i=h	48.30	91.38	19.8	24.0- 80.0	.54	3.57	5	28	62	121	104	91	35	18	8	7
y_i=vm	7.50	64.73	106.0	1.1-100.4	5.24	47.90		1.13	2.26	1.95	7.24	10.67	18.82	67.38		
			Linear regression:	$y_i = -21.766 + .606 x_i$			$\rho^2 = .518$									
			Parabolic regression	$y_i = 22.728 - 1.211 x_i + .018 x_i^2$			$\rho_m^2 = .611$									
x_i=ca	81.95	2181.48	57.0	10.0-358.0	1.45	6.63	105	166	112	54	27	9	2	3	0	1
y_i=vm	7.50	64.73	106.0	1.1-100.4	5.24	47.90	6.34	7.29	16.52	57.11	165.98					
			Linear regression:	$y_i = -2.023 + .116 x_i$			$\rho^2 = .455$									
			Parabolic regression:	$y_i = 3.891 - .019 x_i + .001 x_i^2$			$\rho_m^2 = .544$									
x_i=hp· log ca	83.26	680.20	31.3	26.0-209.4	.83	4.25	17	89	151	110	61	33	11	5	1	1
y_i=vm	7.50	64.73	106.0	1.1-100.4	5.24	47.90	1.57	2.01	4.43	7.77	17.89	53.42				
			Linear regression:	$y_i = -12.093 + .235 x_i$			$\rho^2 = .582$									
			Parabolic regression:	$y_i = 12.233 - .333 x_i + .003 x_i^2$			$\rho_m^2 = .742$									

Table 1. Continued.

Variables	Mean	Variance σ^2	Coefficient of variation CV (percent)	Range	Coefficient of skewness γ_1	Coefficient of kurtosis γ_2	For x_i: frequency within each of ten strata[a] For y_i: variance of y_i within above strata[b]									
x_i=hp	44.45	93.04	21.7	15.0- 82.0	.47	3.83	3	12	68	104	149	86	32	18	5	2
y_i=h	48.33	91.38	19.8	24.0- 80.0	.54	3.56			37.70	27.61	22.47	24.70	38.25	89.58		
	Linear regression:	$y_i = 12.100 + .815x_i$					$\rho^2 - .676$									
	Parabolic regression:	$x_i = 11.322 + .850x_i - .000x_i^2$					$\rho_m^2 = .676$									
x_i=d	7.59	6.16	32.7	4.5- 25.2	1.92	10.30	194	157	79	34	8	2	3	1	0	1
y_i=h	48.33	91.38	19.8	24.0- 80.0	.54	3.56	39.25	33.57	51.91	57.76						
	Linear regression:	$y_i = 25.227 + 3.044x_i$					$\rho^2 = .625$									
	Parabolic regression:	$y_i = 13.821 + 5.638x_i - .129x_i^2$					$\rho_m^2 = .664$									
x_i=ca	81.95	2181.48	57.0	10.0-358.0	1.45	6.63	105	166	112	54	27	9	2	3	0	1
y_i=h	48.33	91.38	19.8	24.0- 80.0	.54	3.56	47.70	66.38	61.35	84.09	90.81					
	Linear regression:	$y_i = 39.259 + .111x_i$					$\rho^2 = .292$									
	Parabolic regression:	$y_i = 37.948 + .141x_i - .000x_i^2$					$\rho_m^2 = .295$									
x_i=d	7.59	6.16	32.7	4.5- 25.2	1.92	10.30	194	157	79	34	8	2	3	1	0	1
y_i=ca	81.95	2181.48	57.0	10.0-358.0	1.45	6.63	551.89	1010.55	1784.92	2206.15						
	Linear regression:	$x_i = -22.973 + 13.824x_i$					$\rho^2 = .540$									
	Parabolic regression:	$y_i = -28.488 + 15.079x_i - .063x_i^2$					$\rho_m^2 = .540$									
x_i=h	48.33	91.38	19.8	24.0- 80.0	.54	3.56	5	28	62	121	104	91	35	18	8	7
y_i=ca	81.95	2181.48	57.0	10.0-358.0	1.45	6.63		692.89	960.88	827.48	1331.55	2190.90	2413.47	3107.47		
	Linear regression:	$y_i = -45.723 + 2.642x_i$					$\rho^2 = .292$									
	Parabolic regression:	$y_i = 31.391 + .507x_i + .031x_i^2$					$\rho_m^2 = .295$									

Table 1. Continued.

Variables	Mean	Variance σ^2	Coefficient of variation CV (percent)	Range	Coefficient of skewness γ_1	Coefficient of kurtosis γ_2	For x_i: frequency within each of ten strata[a/] For y_i: variance of y_i within above strata[b/]									
$x_i=d^2$	63.77	2669.57	81.0	20.2-635.0	4.65	40.26	380	76	15	3	2	1	1	0	0	1
$y_i=vm_t$[c/]	17.50	64.73	46.0	11.1-110.4	5.24	47.90	5.67	7.52	11.38							
			Linear regression:	$y_i = 7.628 + .155x_i$				$\rho^2 = .988$								
			Parabolic regression:	$y_i = 8.210 + .142x_i + .000x_i^2$				$\rho_m^2 = .991$								
$x_i=ca$	81.95	2181.48	57.0	10.0-358.0	1.45	6.63	105	166	112	54	27	9	2	3	0	1
$y_i=vm_t$	17.50	64.73	46.0	11.1-110.4	5.24	47.90	6.34	7.29	16.52	57.11	165.98					
			Linear regression:	$y_i = 7.977 + .116x_i$				$\rho^2 = .455$								
			Parabolic regression:	$y_i = 13.891 - .019x_i + .001x_i^2$				$\rho_m^2 = .544$								

a/ Strata ranked from smallest to largest x_i values.

b/ Variances omitted where frequency within strata less than 15.

c/ $vm_t = vm + 10$, i.e., a constant was added to individual tree volumes in order to alter the intercept of the regression of y_i on x_i.

Table 2. Descriptions of (y_i, x_i) combinations from population II.

Variables	Mean	Variance σ^2	Coefficient of variation CV (percent)	Range	Coefficient of skewness γ_1	Coefficient of kurtosis γ_2	For x_i: frequency within each of ten strata[a/] For y_i: variance of y_i within above strata[b/]									
x_i=d	15.05	47.74	45.9	4.5- 39.6	.57	2.94	57	49	61	61	23	33	16	7	0	2
y_i=vm	44.38	1839.13	96.6	1.2-263.7	1.71	6.93	5.44	12.64	20.10	51.74	73.15	146.45	194.38			
			Linear regression:	$y_i = 45,222 + 5.953x_i$			$\rho^2 = .920$									
			Parabolic regression:	$y_i = -.610 - .424x_i + .187x_i^2$			$\rho_m^2 = .990$									
$x_i=d^2$	274.33	59019.57	88.6	20.2-1568.2	1.64	6.74	132	93	26	31	16	6	3	0	0	2
y_i=vm	44.38	1839.13	96.6	1.2- 263.7	1.71	6.93	65.35	79.89	80.92	143.27	184.21					
			Linear regression:	$y_i = -3.795 + .175x_i$			$\rho^2 = .990$									
			Parabolic regression:	$y_i = -3.192 + .171x_i + .000x_i^2$			$\rho_m^2 = .990$									
x_i=h	69.70	227.24	21.6	25.0-101.0	-.82	3.33	8	12	12	21	27	53	82	60	27	7
y_i=vm	44.38	1839.13	96.6	1.2-263.7	1.71	6.93				22.36	134.02	463.95	955.85	1883.48	2144.09	
			Linear regression:	$y_i = -83.615 + .184x_i$			$\rho^2 = .417$									
			Parabolic regression:	$y_i = 59.551 - .300x_i + .038x_i^2$			$\rho_m^2 = .483$									
x_i=ca	423.33	82062.28	67.7	67.0-1475.0	1.12	3.87	83	73	47	44	20	19	12	6	2	3
y_i=vm	44.38	1839.13	96.6	1.2- 263.7	1.71	6.93	244.92	743.71	446.75	942.93	872.73	2311.16				
			Linear regression:	$y_i = -3.012 + .112x_i$			$\rho^2 = .559$									
			Parabolic regression:	$y_i = -8.780 + .141x_i - .000x_i^2$			$\rho_m^2 = .563$									
x_i=hp· log ca	168.39	2398.17	29.1	45.7-289.9	-.24	2.45	10	14	32	36	46	66	47	45	8	5
y_i=vm	44.38	1839.13	96.6	1.2-263.7	1.71	6.93			24.50	379.38	248.67	880.12	1202.10	2636.92		
			Linear regression:	$y_i = -59.708 + .618x_i$			$\rho^2 = .498$									
			Parabolic regression:	$y_i = 4.757 - .257x_i + .003x_i^2$			$\rho^2 = .530$									

Table 2. Continued.

Variables	Mean	Variance σ^2	Coefficient of variation CV (percent)	Range	Coefficient of skewness γ_1	Coefficient of kurtosis γ_2	For x_i: frequency within each of ten strata[a/] For y_i: variance of y_i within above strata[b/]									
x_i=hp	65.50	178.30	20.4	25.0- 93.0	-.74	3.26	5	12	9	24	28	53	76	62	34	6
y_i=h	69.70	227.24	21.6	25.0-101.0	-.82	3.33				77.39	183.53	92.21	114.53	96.30	91.24	
	Linear regression: $y_i = 16.696 + .809x_i$						$\rho^2 = .514$									
	Parabolic regression: $y_i = -25.101 + .227x_i - .012x_i^2$						$\rho_m^2 = .549$									
x_i=d	15.05	47.74	45.9	4.5- 39.6	.57	2.94	57	49	61	61	23	33	16	7	0	2
y_i=h	69.70	227.24	21.6	25.0-101.0	-.82	3.33	123.96	89.39	47.84	95.20	43.60	69.21	60.94			
	Linear regression: $y_i = 45.793 + 1.588x_i$						$\rho^2 = .530$									
	Parabolic regression: $y_i = 24.710 + 4.602x_i - .088x_i^2$						$\rho_m^2 = .657$									
x_i=ca	423.33	8206.28	67.7	67.0-1475.0	1.12	3.87	83	73	47	44	20	19	12	6	2	3
y_i=h	69.70	227.24	21.6	25.0- 101.0	-.82	3.33	232.07	170.69	120.12	66.61	44.69	56.76				
	Linear regression: $y_i = 56.853 + .030x_i$						$\rho^2 = .332$									
	Parabolic regression: $y_i = 49.375 + .068x_i - .000x_i^2$						$\rho_m^2 = .382$									
x_i=d	15.05	47.74	45.9	4.5- 39.6	.57	2.94	57	49	61	61	23	33	16	7	0	2
y_i=ca	423.33	8206.28	67.7	67.0-1475.0	1.12	3.87	5011.32	9271.80	11647.48	33472.92	75704.26	56228.07	97931.63			
	Linear regression: $y_i = -64.578 + 32.413x_i$						$\rho^2 = .611$									
	Parabolic regression: $y_i = -114.160 + 39.501x_i - .208x_i^2$						$\rho_m^2 = .613$									
x_i=h	69.70	227.24	21.6	25.0- 101.0	-.82	3.33	8	12	12	21	27	53	82	60	27	7
y_i=ca	423.33	8206.28	67.7	67.0-1475.0	1.12	3.87				7227.61	19816.66	32595.13	65306.98	78686.45	65500.69	
	Linear regression: $y_i = -340.239 + 10.956x_i$						$\rho^2 = .332$									
	Parabolic regression: $y_i = 428.546 - 14.990x_i + .204x_i^2$						$\rho_m^2 = .382$									

a/ Strata ranked from smallest to largest x_i values.

b/ Variances omitted where frequency within strata less than 15.

Table 3. Descriptions of (y_i, x_i) combinations from population III.

Variables	Mean	Variance σ^2	Coefficient of variation CV (percent)	Range	Coefficient of skewness γ_1	Coefficient of kurtosis γ_2	For x_i: frequency within each of ten strata[a/] For y_i: variance of y_i within above strata[b/]									
$x_i = d^2$	62.33	1297.79	58.8	5.3-174.2	.50	2.57	82	77	72	92	59	52	39	13	10	4
$y_i = vm$	7.81	27.59	67.3	6.2- 21.7	.47	2.42	.39	.61	.48	.78	.83	.71	.81			
		Linear regression:		$y_i = -1.247 + .145x_i$				$\rho^2 = .993$								
		Parabolic regression:		$y_i = -1.411 + .151x_i - .000x_i^2$				$\rho_m^2 = .993$								
$x_i = d_t^2$	64.36	1439.08	58.9	9.0-169.0	.54	2.66	139	0	128	0	142	0	80	0	0	11
$y_i = vm$	7.81	27.59	67.3	6.2- 21.7	.47	2.42	1.13		1.22		2.55		2.88			
		Linear regression:		$y_i = -.820 + .134x_i$				$\rho^2 = .938$								
		Parabolic regression:		$y_i = -1.229 + .149x_i - .000x_i^2$				$\rho_m^2 = .939$								
$x_i = d$	7.53	5.64	31.5	2.3-13.2	-.45	2.15	14	51	60	55	78	84	67	63	20	8
$y_i = h$	51.32	39.39	12.23	23.0-63.0	-1.19	4.65		16.80	7.38	8.16	6.14	4.89	11.05	8.30	4.19	
		Linear regression:		$y_i = 34.972 + 2.172x_i$				$\rho^2 = .676$								
		Parabolic regression:		$y_i = 17.781 + 7.289x_i - .342x_i^2$				$\rho_m^2 = .785$								

a/ Strata ranked from smallest to largest x_i values.

b/ Variances omitted where frequency within strata less than 15.

APPENDIX C

Table 1. Estimator performance for (y_i, x_i) combinations from population I.

Variables	Sample size	Performance criteria	$\hat{Y}_{srs}$	$\hat{Y}_r$	$\hat{\bar{Y}}_r$	$\hat{Y}_{ur}$	$\hat{\bar{Y}}_{ur}$	$\hat{Y}_{lr}$	$\hat{Y}_{pr}$	$\hat{Y}_{pp\,\Sigma x}$	$\hat{Y}_{pps}$	$\hat{Y}_{s-2}$	$\hat{Y}_{s-4}$	$\hat{Y}_{s-6}$	$\hat{Y}_{s-8}$	$\hat{Y}_{s-10}$
			(3689861)													
y_i = vm	4	RP	2779097[b/]	2.76	5.28	1.73	1.65	20.55	.06	1.63	2.89	1.05				
x_i = d		RA	1.00	2.61	3.22	1.74	1.66	12.66	.06	1.65	2.92	1.06				
		B/Y	.050	.074	.164	.033	.035	.082	.141	.007	-.008	.015				
		B/√V	.108	.267	..812	.093	.097	.801	.073	.019	-.028	.034				
			(1209238)													
	12	RP	1062206	2.27	4.67	1.99	1.82	15.79	27.82	2.24	4.97	1.07				
		RA	1.00	2.20	2.00	1.98	1.82	10.73	27.95	2.23	5.02	1.08				
		B/Y	.029	.039	.155	.025	.025	.050	.004	.023	.002	-.008				
		B/√V	.100	.205	1.164	.122	.117	.697	.075	.121	.015	-.030				
			(589083)													
	24	RP	527612	2.10	5.37	1.91	1.76	9.28	60.20	1.81	5.54					
		RA	1.00	2.11	1.61	1.91	1.76	8.43	60.52	1.81	5.60					
		B/Y	-.020	-.005	.134	-.014	-.014	..022	.002	-.016	.001					
		B/√V	-.099	-.036	1.538	-.095	-.093	.333	.067	-.108	.016					
			(341021)													
	40	RP	407923	1.97	6.18	1.85	1.69	9.40	59.96	2.18	11.11					
		RA	1.00	1.97	1.19	1.85	1.69	8.90	59.66	2.18	11.01					
		B/Y	-.002	-.002	.147	-.004	-.003	..014	.002	-.001	.005					
		B/√V	.099	.036	1.538	.095	.093	.333	.072	.108	.016					
			(3689861)													
y_i = vm	4	RP	4983464	32.09	49.71	9.45	8.29	123.74	.10	27.99	40.84	1.57				
x_i = d^2		RA	1.00	28.56	20.66	9.45	8.29	105.13	.10	27.95	40.87	1.56				
		B/Y	-.017	.039	.105	-.006	-.005	.024	-.086	.005	.000	.040				
		B/√V	.028	.353	1.187	-.030	-.022	.422	-.045	.046	.005	.081				

Continued on next page

Table 1 Continued.

Variables	Sample size	Performance criteria	Estimators $\hat{Y}_{srs}$	$\hat{Y}_{r}$	$\hat{Y}_{\bar{r}}$	$\hat{Y}_{ur}$	$\hat{Y}_{\bar{ur}}$	$\hat{Y}_{lr}$	$\hat{Y}_{pr}$	$\hat{Y}_{pp\,\Sigma x}$	$\hat{Y}_{pps}$	$\hat{Y}_{s-2}$	$\hat{Y}_{s-4}$	$\hat{Y}_{s-6}$	$\hat{Y}_{s-8}$	$\hat{Y}_{s-10}$
			(1209238)													
	12	RP	963893	15.25	26.37	11.12	8.69	46.75	6.37	12.05	24.11					
		RA	1.00	14.33	5.64	11.20	8.72	44.89	6.47	12.22	24.34					
		B/Y	.034	.020	.103	.008	.010	.010	-.001	.004	.004					
		$B/\sqrt{V}$	.126	.286	1.937	.093	.110	.240	-.011	.046	.072					
			(589083)													
	24	RP	564643	14.35	25.89	11.73	8.62	66.50	16.18	14.36	29.90					
		RA	1.00	14.00	3.87	11.73	8.61	60.29	16.04	14.35	29.89					
		B/Y	.002	.009	.098	.002	.002	.008	-.005	-.001	-.001					
		$B/\sqrt{V}$	.010	.160	2.385	.028	.035	.321	-.092	-.026	-.022					
			(341021)													
	40	RP	391864	16.75	41.52	13.81	9.11	88.15	45.19	15.74	29.81					
		RA	1.00	17.17	2.92	13.85	9.15	84.23	46.02	15.93	30.57					
		B/Y	-.028	-.001	.100	-.007	-.008	.005	-.002	-.005	-.001					
		$B/\sqrt{V}$	-.162	-.032	3.686	-.151	-.146	.272	-.087	-.119	-.028					
			(3689861)													
y_i = vm x_i = h	4	RP	3106738	1.56	2.24	1.33	1.29	3.29	.01	1.55	1.64	1.59				
		RA	1.00	1.53	1.99	1.32	1.29	2.75	.01	1.54	1.64	1.59				
		B/Y	.035	.060	.119	.038	.038	.122	-.296	.050	.028	.019				
		$B/\sqrt{V}$	.070	.152	.363	.089	.087	.453	-.057	.127	.072	.049				
			(1209238)													
	12	RP	1341596	1.41	2.37	1.32	1.28	3.64	3.23	1.40	2.17	2.61	3.85	4.31		
		RA	1.00	1.41	1.91	1.32	1.29	3.46	3.23	1.40	2.16	2.61	3.85	4.31		
		B/Y	.007	.013	.103	.004	.004	.039	-.007	.008	-.015	-.004	.005	.002		
		$B/\sqrt{V}$	.023	.047	.491	.013	.013	.230	-.039	.029	-.070	-.019	.032	.010		
			(589083)													
	24	RP	706161	1.34	2.24	1.30	1.27	2.45	3.62	1.34	2.07	2.08	3.85	4.18	4.40	
		RA	1.00	1.35	1.94	1.31	1.28	2.53	3.74	1.35	2.13	2.15	3.98	4.32	4.55	
		B/Y	-.044	-.032	.070	-.037	-.038	.004	-.004	-.035	-.016	-.008	.005	-.004	.004	
		$B/\sqrt{V}$	-.189	-.160	.449	-.182	-.182	.030	-.033	-.173	-.098	-.048	.039	-.039	.033	

Continued on next page

Table 1 Continued.

Variables	Sample size	Performance criteria	Estimators $\hat{Y}_{srs}$	$\hat{Y}_{r}$	$\hat{Y}_{\bar{r}}$	$\hat{Y}_{ur}$	$\hat{Y}_{\bar{ur}}$	$\hat{Y}_{lr}$	$\hat{Y}_{pr}$	$\hat{Y}_{pp\,\Sigma x}$	$\hat{Y}_{pps}$	$\hat{Y}_{s-2}$	$\hat{Y}_{s-4}$	$\hat{Y}_{s-6}$	$\hat{Y}_{s-8}$	$\hat{Y}_{s-10}$
			(341021)													
	40	RP	210266	1.24	2.17	1.22	1.20	1.71	1.97	1.25	1.45	1.36	2.30		3.31	4.03
		RA	1.00	1.21	.81	1.20	1.19	1.59	2.01	1.24	1.54	1.41	2.43		3.32	4.24
		B/Y	.033	.035	.118	.032	.033	.037	.019	.031	-.006	.016	-.009		-.018	-.008
		B/√V	.260	.303	1.363	.281	.281	.379	.219	.274	-.055	.144	-.109		-.253	-.130
			(3689861)													
y_i = vm, x_i = ca	4	RP	3440716	2.46	2.42	1.83	1.77	1.80	.01	2.41	2.40	.81				
		RA	1.00	2.44	2.43	1.82	1.76	1.71	.01	2.42	2.38	.81				
		B/Y	.016	.032	-.001	.024	.027	.088	.198	.001	-.033	.023				
		B/√V	.032	.096	-.003	.063	.070	.230	.029	.004	-.098	.040				
			(1209238)													
	12	RP	1487380	2.16	3.40	1.83	1.83	2.36	1.07	1.95	5.66	1.14	1.70			
		RA	1.00	2.17	3.39	1.84	1.83	2.39	1.06	1.97	5.72	1.16	1.72			
		B/Y	-.040	-.019	-.024	-.027	-.027	.008	-.045	-.020	-.009	.015	-.004			
		B/√V	-.119	-.082	-.132	-.108	-.110	.036	-.137	-.083	-.060	.047	-.014			
			(589083)													
	24	RP	399052	1.53	1.99	1.38	1.47	1.54	1.68	1.53	3.32	.69				
		RA	1.00	1.60	2.13	1.45	1.55	1.58	1.75	1.61	3.54	.73				
		B/Y	.048	.024	-.001	.021	.022	.030	.022	.021	.007	-.023				
		B/√V	.272	.171	-.006	.141	.154	.215	.162	.151	.075	-.106				
			(341021)													
	40	RP	244301	1.71	2.13	1.64	1.68	1.59	1.25	1.69	1.78					
		RA	1.00	1.69	2.11	1.63	1.68	1.57	1.24	1.66	1.76					
		B/Y	.001	.009	-.008	.007	.008	.011	.007	.014	.011					
		B/√V	.004	.083	-.085	.067	.075	.105	.060	.137	.104					
			(3689861)													
y_i = vm, x_i = hp' Log ca	4	RP	4743911	2.35	3.78	1.76	1.65	3.87	.07	2.23	3.70	2.87				
		RA	1.00	2.34	3.41	1.77	1.65	3.29	.07	2.23	3.70	2.87				
		B/Y	-.027	.027	.103	-.008	-.011	.130	-.062	.005	.007	.010				
		B/√V	-.044	.068	.331	-.017	-.022	.422	-.027	.012	.022	.029				

Continued on next page

Table 1 Continued.

Variables	Sample size	Performance criteria	Estimators $\hat{Y}_{srs}$	$\hat{Y}_{r}$	$\hat{Y}_{\bar{r}}$	$\hat{Y}_{ur}$	$\hat{Y}_{u\bar{r}}$	$\hat{Y}_{lr}$	$\hat{Y}_{pr}$	$\hat{Y}_{pp \Sigma x}$	$\hat{Y}_{pps}$	$\hat{Y}_{s-2}$	$\hat{Y}_{s-4}$	$\hat{Y}_{s-6}$	$\hat{Y}_{s-8}$	$\hat{Y}_{s-10}$
			(1209238)													
	12	RP	1202943	1.63	3.77	1.42	1.39	2.98	3.60	1.55	3.34	2.11	3.47			
		RA	1.00	1.63	2.58	1.41	1.38	2.92	3.60	1.54	3.35	2.12	3.49			
		B/Y	-.020	-.007	.108	-.021	-.020	.027	.012	-.020	-.004	.010	.006			
		$B/\sqrt{V}$	-.065	-.029	.684	-.083	-.078	.153	.077	-.082	-.025	.049	.034			
			(589083)													
	24	RP	421449	1.67	3.33	1.59	1.54	2.16	2.54	1.62	2.40	1.75				
		RA	1.00	1.63	1.24	1.58	1.54	2.07	2.48	1.63	2.48	1.78				
		B/Y	.034	.033	.132	.027	.028	.034	.028	.022	-.003	.016				
		$B/\sqrt{V}$	.186	.233	1.331	.190	.194	.277	.245	.155	-.028	.117				
			(341021)													
	40	RP	304155	1.46	3.24	1.41	1.38	1.92	3.18	1.53	4.76	1.86				
		RA	1.00	1.46	1.21	1.41	1.38	1.91	3.16	1.54	4.60	1.87				
		B/Y	-.009	-.003	.111	-.006	-.006	.010	.008	-.003	-.014	.004				
		$B/\sqrt{V}$	-.057	-.021	1.301	-.050	-.048	.088	.095	-.026	-.199	.033				
			(5208745)													
$y_i = h$	4	RP	5471271	2.63	2.16	2.64	2.60	2.55	.03	2.31	3.25	1.45				
$x_i = hp$		RA	1.00	2.63	2.12	2.63	2.60	2.55	.03	2.31	3.25	1.44				
		B/Y	.004	.000	-.010	.003	.003	.001	-.003	.001	.003	.006				
		$B/\sqrt{V}$	.037	-.002	-.140	.045	.043	.009	-.006	.008	.045	.069				
			(1707006)													
	12	RP	1951474	2.57	2.07	2.57	2.55	2.41	1.49	2.58	2.68	1.95	1.32	1.70		
		RA	1.00	2.57	1.85	2.57	2.55	2.41	1.50	2.58	2.68	1.95	1.31	1.70		
		B/Y	.002	-.001	-.014	-.000	-.000	.002	.001	.000	-.001	.002	.002	.000		
		$B/\sqrt{V}$	.025	-.037	-.343	-.010	-.012	.060	.020	-.003	-.020	.049	.044	.006		
			(831572)													
	24	RP	1026189	2.64	1.95	2.65	2.58	3.85	2.79	2.91	2.45	1.78	1.61	1.55	2.46	
		RA	1.00	2.62	1.56	2.64	2.57	3.84	2.78	2.88	2.45	1.79	1.62	1.56	2.41	
		B/Y	-.003	-.003	-.016	-.003	-.003	-.002	-.003	-.003	-.002	.000	.000	-.001	.004	
		$B/\sqrt{V}$	-.080	-.128	-.513	-.111	-.111	-.095	-.097	-.131	-.080	-.007	-.001	-.028	.161	

Continued on next page

Table 1 Continued.

Variables	Sample size	Performance criteria	Estimators													
			$\hat{Y}_{srs}$	$\hat{Y}_{r}$	$\hat{Y}_{\bar{r}}$	$\hat{Y}_{ur}$	$\hat{Y}_{\bar{ur}}$	$\hat{Y}_{lr}$	$\hat{Y}_{pr}$	$\hat{Y}_{pp \Sigma x}$	$\hat{Y}_{pps}$	$\hat{Y}_{s-2}$	$\hat{Y}_{s-4}$	$\hat{Y}_{s-6}$	$\hat{Y}_{s-8}$	$\hat{Y}_{s-10}$
			(481398)													
	40	RP	547330	3.50	2.51	3.51	3.47	3.88	2.81	3.80	2.27	2.05	1.35			
		RA	1.00	3.45	2.06	3.45	3.41	3.82	2.75	3.71	2.17	2.05	1.26			
		B/Y	.000	.002	-.009	.002	.002	.002	.003	.002	.005	.000	.007			
		B/√V	.000	.124	-.471	.136	.140	.117	.147	.152	.218	.008	.268			
			(5208745)													
$y_i = h$	4	RP	5061512	.98	1.20	.69	.67	2.23	.01	.90	1.39	.57[a/]				
		RA	1.00	.98	1.05	.68	.66	2.20	.01	.89	1.39					
$x_i = d$		B/Y	-.003	-.002	-.035	.014	.014	-.007	-.073	.007	.000					
		B/√V	-.028	-.025	-.391	.116	.112	-.113	-.062	.069	.003					
			(1707006)													
	12	RP	1936227	1.21	1.49	1.14	1.06	2.36	.82	1.18	1.76	.52[a/]				
		RA	1.00	1.15	.80	1.12	1.04	2.30	.80	1.16	1.78					
		B/Y	.005	-.013	-.046	-.009	-.010	-.007	.013	-.009	.000					
		B/√V	..090	-.239	-.937	-.167	-.166	-.185	.194	-.163	-.006					
			(831572)													
	24	RP	1008437	1.24	1.43	1.20	1.11	2.65	1.43	1.26	1.90					
		RA	1.00	1.23	.60	1.20	1.11	2.59	1.41	1.26	1.88					
		B/Y	-.002	-.004	-.042	-.002	-.002	-.004	.005	-.002	.003					
		B/√V	-.038	-.100	-1.168	.045	-.048	-.148	.133	-.063	.101					
			(481398)													
	40	RP	521478	1.05	1.32	1.02	.92	2.68	2.34	1.06	.99					
		RA	1.00	1.03	.46	.98	.89	2.70	2.31	1.03	.99					
		B/Y	-.003	.005	-.037	.007	.007	.000	.003	.006	.002					
		B/√V	-.096	.177	-1.379	.221	.219	-.012	.144	.209	.068					
			(5208745)													
$y_i = h$	4	RP	5378763	.16	.07	.12	.08	.62	.00	.17	.16	.65				
$x_i = ca$		RA	1.00	.15	.04	.12	.08	.62	.00	.17	.16	.65				
		B/Y	.005	-.071	-.327	.001	.004	.002	-.008	.003	.011	-.004				
		B/√V	.05	-.284	-.851	.006	.012	.020	-.003	.012	.045	-.034				

Continued on next page

Table 1 Continued.

Variables	Sample size	Performance criteria	$\hat{Y}_{srs}$	$\hat{Y}_{r}$	$\hat{Y}_{\bar{r}}$	$\hat{Y}_{ur}$	$\hat{Y}_{\bar{ur}}$	$\hat{Y}_{lr}$	$\hat{Y}_{pr}$	$\hat{Y}_{pp\ \Sigma x}$	$\hat{Y}_{pps}$	$\hat{Y}_{s-2}$ [a]	$\hat{Y}_{s-4}$	$\hat{Y}_{s-6}$	$\hat{Y}_{s-8}$	$\hat{Y}_{s-10}$
			Estimators													
			(1707006)													
	12	RP	2033340	.18	.07	.17	.09	1.52	.73	.21	.22	.71	.67			
		RA	1.00	.18	.03	.17	.09	1.52	.73	.21	.22	.71	.67			
		B/Y	.001	-.010	-.302	.013	.015	.001	.000	.006	.003	.001	.001			
		$B/\sqrt{V}$	.020	-.068	-1.334	.084	.077	.023	.003	.045	.022	.008	.014			
			(831572)													
	24	RP	839647	.16	.07	.16	.09	1.33	.92	.17	.17	.50				
		RA	1.00	.16	.01	.16	.09	1.33	.93	.18	.16	.50				
		B/Y	-.005	-.006	-.304	.004	.007	-.004	-.004	.001	-.018	.003				
		$B/\sqrt{V}$	-.130	-.065	-1.993	.039	.050	-.106	-.100	.013	-.181	.051				
			(481398)													
	40	RP	275831	.10	.04	.10	.06	1.01	.90	.10	.11					
		RA	1.00	.10	.00	.10	.06	1.00	.88	.10	.10					
		B/Y	.001	-.018	-.323	-.012	-.016	-.003	-.004	-.014	-.018					
		$B/\sqrt{V}$	.027	-.251	-2.794	-.169	-.168	-.132	-.152	-.199	-.272					
			(124345200)													
y_i = ca	4	RP	104785300	1.92	2.04	1.81	1.83	.82	.00	1.86	2.13	.59[a]				
x_i = d		RA	1.00	1.93	2.01	1.84	1.85	.84	.00	1.89	2.17					
		B/Y	.035	.017	.032	.011	.012	-.006	.020	.004	-.002					
		$B/\sqrt{V}$	.133	.089	.177	.057	.062	-.019	.001	.020	-.009					
			(40750330)													
	12	RP	43735580	2.23	2.43	2.19	2.17	2.15	.62	2.30	3.10	.58[a]				
		RA	1.00	2.23	2.40	2.18	2.17	2.15	.63	2.29	3.12					
		B/Y	-.012	-.008	.015	-.010	-.010	-.006	.003	-.010	.002					
		$B/\sqrt{V}$	-.069	-.070	.135	-.091	-.090	-.056	.015	-.089	.018					
			(19851600)													
	24	RP	21109430	2.41	2.45	2.40	2.36	2.36	1.77	2.53	2.85					
		RA	1.00	2.39	2.12	2.39	2.35	2.34	1.78	2.52	2.82					
		B/Y	-.008	.008	.030	.007	.007	.009	.003	.008	.008					
		$B/\sqrt{V}$	-.070	.108	.405	.091	.087	.120	.031	.102	.109					

Continued on next page

Table 1 Continued.

Variables	Sample size	Performance criteria	Estimators $\hat{Y}_{srs}$	$\hat{Y}_{r}$	$\hat{Y}_{\bar{r}}$	$\hat{Y}_{ur}$	$\hat{Y}_{\bar{ur}}$	$\hat{Y}_{lr}$	$\hat{Y}_{pr}$	$\hat{Y}_{pp \Sigma x}$	$\hat{Y}_{pps}$	$\hat{Y}_{s-2}$	$\hat{Y}_{s-4}$	$\hat{Y}_{s-6}$	$\hat{Y}_{s-8}$	$\hat{Y}_{s-10}$
			(11492110)													
	40	RP	10561780	2.34	2.49	2.33	2.31	2.22	1.54	2.24	2.72					
		RA	1.00	2.41	1.88	2.41	2.38	2.32	1.60	2.30	2.80					
		B/Y	.018	.008	.033	.007	.007	.003	-.001	.008	-.007					
		B/$\sqrt{V}$	.222	.142	.625	.128	.133	.059	-.022	.148	-.130					
			(124345200)													
$y_i = ca$	4	RP	120745900	1.39	1.53	1.32	1.32	.81	.00	1.30	1.15	1.21				
$x_i = h$		RA	1.00	1.39	1.53	1.31	1.31	.81	.00	1.29	1.14	1.21				
		B/Y	-.012	-.011	.003	-.017	-.016	.005	.664	-.025	-.222	-.010				
		B/$\sqrt{V}$	-.042	-.047	.014	-.069	-.067	.016	.032	-.104	-.086	-.038				
			(40750330)													
	12	RP	42658910	1.45	1.53	1.44	1.43	1.35	.86	1.35	1.96	1.34	.88	.97		
		RA	1.00	1.46	1.52	1.44	1.43	1.35	.86	1.35	1.91	1.35	.88	.97		
		B/Y	-.010	-.004	.014	-.006	-.006	.011	-.004	-.008	.021	-.002	-.007	-.003		
		B/$\sqrt{V}$	-.060	-.027	.101	-.040	-.041	.079	-.022	-.054	.173	-.012	-.042	-.020		
			(19851600)													
	24	RP	17959380	1.36	1.50	1.35	1.35	1.29	1.11	1.38	1.63	1.11	.96	.82	.82	
		RA	1.00	1.36	1.40	1.35	1.29	1.18	1.11	1.57	1.11	.96	.81	.80	1.17	
		B/Y	.006	.007	.024	.006	.006	.008	-.001	.007	-.017	.009	-.002	-.018	-.019	
		B/$\sqrt{V}$	.053	.078	.277	.069	.070	.079	-.011	.075	-.205	.084	-.016	-.151	-.160	
			(11492110)													
	40	RP	11171360	1.53	1.68	1.52	1.52	1.67	1.67	1.64	1.23	1.11	.84		.75	1.14
		RA	1.00	1.51	1.69	1.50	1.49	1.65	1.65	1.62	1.24	1.12	.85		.76	1.11
		B/Y	-.011	-.012	.007	-.012	-.012	-.010	-.006	-.012	-.008	-.003	-.003		.000	.017
		B/$\sqrt{V}$	-.130	-.171	.106	-.179	-.177	-.155	-.088	-.182	-.099	-.032	-.030		-.003	.216
			(3689861)													
$y_i = vm_t$ g/	4	RP	4984393	2.36	2.26	1.02	.78	120.85	.10	2.58	3.39	1.57				
$x_i = d^2$		RA	1.00	2.13	1.06	1.02	.78	103.05	.10	2.57	3.39	1.56				
		B/Y	-.007	-.057	-.189	.010	.008	.010	-.037	-.009	.004	.017				
		B/$\sqrt{V}$	-.028	-.331	-1.068	.027	.027	.417	-.045	-.056	.031	.081				

Continued on next page

Table 1 Continued.

Variables	Sample size	Performance criteria	Estimators													
			$\hat{Y}_{srs}$	$\hat{Y}_{r}$	$\hat{Y}_{\bar{r}}$	$\hat{Y}_{ur}$	$\hat{Y}_{\bar{ur}}$	$\hat{Y}_{lr}$	$\hat{Y}_{pr}$	$\hat{Y}_{pp\Sigma x}$	$\hat{Y}_{pps}$	$\hat{Y}_{s-2}$	$\hat{Y}_{s-4}$	$\hat{Y}_{s-6}$	$\hat{Y}_{s-8}$	$\hat{Y}_{s-10}$
			(1209238)													
	12	RP	964413	1.39	1.30	1.16	.70	45.53	6.36	1.25	1.71					
		RA	1.00	1.27	.28	1.16	.70	43.79	6.46	1.25	1.71					
		B/Y	.015	-.034	-.197	-.015	-.019	.004	-.000	-.012	-.011					
		B/√V	.126	-.338	-1.924	-.135	-.140	.237	-.012	-.116	-.122					
			(589083)													
	24	RP	564868	1.56	1.31	1.38	.79	64.42	16.08	1.67	2.28					
		RA	1.00	1.52	.20	1.38	.79	58.60	15.95	1.66	2.27					
		B/Y	.001	-.011	-.184	-.001	-.003	.004	-.002	.002	.004					
		B/√V	.010	-.155	-2.343	-.012	-.026	.315	-.093	.031	.070					
			(341021)													
	40	RP	391977	1.78	1.69	1.58	.83	83.84	44.14	1.76	2.87					
		RA	1.00	1.82	1.56	1.58	.83	80.42	44.96	1.76	2.93					
		B/Y	-.012	.003	-.183	.010	.013	.002	-.001	.009	.003					
		B/√V	-.162	.046	-3.181	.167	.161	.264	-.087	.167	.068					
			(3689861)													
$y_i = vm_t$ c/	4	RP	3441693	1.11	.52	.96	.65	1.80	.01	1.38	1.03	.81				
$x_i = ca$		RA	1.00	1.09	.35	.96	.65	1.71	.01	1.38	1.03	.81				
		B/Y	.007	-.033	-.217	.012	.015	.038	.085	.004	-.006	.010				
		B/√V	.032	-.159	-.708	.053	.055	.230	.029	.022	-.026	.040				
			(1209238)													
	12	RP	1487926	1.37	.58	1.32	.94	2.36	1.07	1.45	1.70	1.14	1.70			
		RA	1.00	1.36	.25	1.33	.95	2.39	1.07	1.47	1.73	1.16	1.73			
		B/Y	-.017	-.018	-.221	-.006	-.006	.003	-.019	-.005	-.002	.006	-.002			
		B/√V	-.119	-.142	-1.158	-.046	-.040	.036	-.137	-.040	-.021	.046	-.014			
			(589083)													
	24	RP	399273	1.03	.42	1.00	.75	1.54	1.68	.96	.83	.69				
		RA	1.00	1.09	.09	1.07	.80	1.58	1.75	1.03	.89	.72				
		B/Y	.020	-.009	-.240	-.003	-.007	.013	.009	-.003	-.001	-.010				
		B/√V	.272	-.126	-2.075	-.044	-.085	.215	.162	-.034	-.008	-.106				

Continued on next page

Table 1 Continued.

Variables	Sample size	Performance criteria	Estimators $\hat{Y}_{srs}$	$\hat{Y}_{r}$	$\hat{Y}_{\bar{r}}$	$\hat{Y}_{ur}$	$\hat{Y}_{\bar{ur}}$	$\hat{Y}_{lr}$	$\hat{Y}_{pr}$	$\hat{Y}_{pp \Sigma x}$	$\hat{Y}_{pps}$	$\hat{Y}_{s-2}$	$\hat{Y}_{s-4}$	$\hat{Y}_{s-6}$	$\hat{Y}_{s-8}$	$\hat{Y}_{s-10}$
			(341021)													
	40	RP	244468	.96	.33	.95	.65	1.59	1.25	.94	.78					
		RA	1.00	.95	.59	.93	.64	1.57	1.24	.91	.77					
		B/Y	.000	.004	-.219	.008	.010	.005	.003	.010	.003					
		$B/\sqrt{V}$	.004	.072	-2.125	.131	.139	.104	.059	.171	.047					

[a] RP values for stratified sampling based on true variances.

[b] Monte Carlo estimate of variance given in place of RP value of 1.00. Figure in parentheses is true variance.

[c] $vm_t = vm + 10$, i.e., a constant was added to individual tree volume in order to alter the intercept of the regression of y_i on x_i.

Table 2. Estimator performance for (y_i, x_i) combinations from population II.

Variables	Sample size	Performance criteria	Estimators													
			$\hat{Y}_{srs}$	$\hat{Y}_{r}$	$\hat{Y}_{\bar{r}}$	$\hat{Y}_{ur}$	$\hat{Y}_{u\bar{r}}$	$\hat{Y}_{lr}$	$\hat{Y}_{pr}$	$\hat{Y}_{pp\,\Sigma x}$	$\hat{Y}_{pps}$	$\hat{Y}_{s-2}$	$\hat{Y}_{s-4}$	$\hat{Y}_{s-6}$	$\hat{Y}_{s-8}$	$\hat{Y}_{s-10}$
			(43472810)													
y_i = vm	4	RP	41666140[b]	3.31	4.55	2.53	2.16	9.34	.17	3.37	4.30	2.90[a]				
x_i = d		RA	1.0	3.17	2.41	2.54	2.16	7.00	.17	3.36	4.26					
		B/Y	,007	.054	.208	.000	.002	.089	.033	.010	-.024					
		B/$\sqrt{V}$	.016	.211	.942	.001	.006	.577	.030	.039	-.104					
			(14110840)													
	12	RP	12652180	3.17	4.57	2.96	2.34	9.89	45.34	3.30	5.01	2.90	7.76	15.74		
		RA	1.0	3.15	1.21	2.92	2.34	9.17	45.16	3.30	4.96					
		B/Y	-.004	.013	.203	-.005	-.005	.023	-.003	-.006	-.012					
		B/$\sqrt{V}$	-.016	.087	1.671	-.033	-.030	.281	-.066	-.045	-.105					
			(6770356)													
	24	RP	9355418	3.45	4.50	3.36	2.52	14.71	95.06	3.90	9.52	2.89				
		RA	1.0	3.48	1.00	3.37	2.52	13.92	95.64	3.93	9.55					
		B/Y	-.018	.001	.198	-.008	-.009	.015	.001	-.004	.004					
		B/$\sqrt{V}$	-.083	.006	1.884	-.064	-.066	.252	.027	-.033	.060					
			(3834159)													
	40	RP	3354848	3.20	4.44	3.15	2.40	9.85	73.74	2.85	6.66	2.88				
		RA	1.0	3.14	.38	3.14	2.40	9.54	74.06	2.83	6.72					
		B/Y	.012	.012	.209	.008	.009	.009	-.001	.010	.000					
		B/$\sqrt{V}$	.090	.163	3.292	.101	.100	.202	-.062	.123	.005					
			(43472810)													
y_i = vm	4	RP	38785880	38.80	21.70	26.46	15.53	46.02	.06	52.53	72.53[c]	.83				
x_i = d^2		RA	1.0	34.97	9.43	26.53	15.56	45.16	.06	52.73						
		B/Y	.036	.025	.112	.005	.007	.011	.109	.004						
		B/$\sqrt{V}$	.079	.341	1.147	.060	.064	.159	.060	.056						
			(14110840)													
	12	RP	13239010	61.76	28.80	56.14	28.43	71.62	19.66	65.10	72.8[illegible]	.82				
		RA	1.0	61.55	6.10	55.87	28.31	71.80	19.62	64.80						
		B/Y	-.014	.003	.095	-.003	-.004	.001	-.004	-.003						
		B/$\sqrt{V}$	-.052	.079	1.931	-.086	-.085	.162	-.067	-.086						

Continued on next page

Table 2 Continued.

Variables	Sample size	Performance criteria	Estimators $\hat{Y}_{srs}$	$\hat{Y}_{r}$	$\hat{Y}_{\bar{r}}$	$\hat{Y}_{ur}$	$\hat{Y}_{\bar{ur}}$	$\hat{Y}_{lr}$	$\hat{Y}_{pr}$	$\hat{Y}_{pp \Sigma x}$	$\hat{Y}_{pps}$	$\hat{Y}_{s-2}$	$\hat{Y}_{s-4}$	$\hat{Y}_{s-6}$	$\hat{Y}_{s-8}$	$\hat{Y}_{s-10}$
			(6770356)													
	24	RP	6931281	60.08	25.88	58.55	24.66	98.84	68.11	66.30	73.43					
		RA	1.0	60.40	3.53	58.13	24.58	99.21	66.32	64.25						
		B/Y	-.014	-.000	.095	-.003	-.004	-.001	-.004	-.005						
		$B/\sqrt{V}$	-.073	-.001	2.524	-.112	-.093	-.041	-.180	-.193						
			(3834159)													
	40	RP	3992344	56.47	26.75	55.51	22.54	80.14	55.77	55.94	74.27					
		RA	1.0	55.45	17.38	55.75	22.56	80.00	56.33	56.51						
		B/Y	.018	.004	.108	.002	.003	.001	-.001	.001						
		$B/\sqrt{V}$	.124	.184	3.824	.104	.119	.077	-.071	.071						
			(43472810)													
y_i = vm	4	RP	43350670	1.34	1.52	1.27	1.25	.84	.00	1.23	1.44	.89				
x_i = h		RA	1.0	1.33	1.41	1.27	1.25	.80	.00	1.23						
		B/Y	.014	.035	.113	.013	.013	.121	-18.436	-.003						
		$B/\sqrt{V}$	.029	.084	.290	.030	.031	.232	-.049	-.006						
			(14110840)													
	12	RP	14427800	1.35	1.49	1.33	1.30	1.41	.68	1.36	1.45	.89	1.04	.99		
		RA	1.0	1.34	1.23	1.33	1.30	1.39	.68	1.36						
		B/Y	.005	.010	.103	.003	.004	.028	-.015	.009						
		$B/\sqrt{V}$	.019	.041	.454	.012	.016	.121	-.046	.038						
			(6770356)													
	24	RP	5332816	1.29	1.51	1.28	1.26	1.40	1.40	1.27	1.45	.87	1.02	.98	.94	
		RA	1.0	1.29	.91	1.29	1.26	1.39	1.38	1.27						
		B/Y	.011	.012	.112	.009	.009	.016	-.003	.013						
		$B/\sqrt{V}$	.068	.082	.820	.058	.060	.109	-.022	.087						
			(3834159)													
	40	RP	3888602	1.27	1.60	1.27	1.25	1.37	1.92	1.30	1.46	.86	1.00		.92	1.00
		RA	1.0	1.28	.99	1.27	1.25	1.39	1.94	1.31						
		B/Y	-.017	-.013	.090	-.015	-.015	-.001	-.001	-.010						
		$B/\sqrt{V}$	-.119	-.099	.796	-.114	-.113	-.006	-.067	-.082						

Continued on next page

Table 2 Continued.

Variables	Sample size	Performance criteria	$\hat{Y}_{srs}$	$\hat{Y}_r$	$\hat{Y}_{\bar{r}}$	$\hat{Y}_{ur}$	$\hat{Y}_{\bar{ur}}$	$\hat{Y}_{lr}$	$\hat{Y}_{pr}$	$\hat{Y}_{pp\,\Sigma x}$	$\hat{Y}_{pps}$	$\hat{Y}_{s-2}$	$\hat{Y}_{s-4}$	$\hat{Y}_{s-6}$	$\hat{Y}_{s-8}$	$\hat{Y}_{s-10}$
			Estimators													
			(43472810)													
y_i = vm	4	RP	39162200	1.77	1.41	1.79	1.72	.48	.01	2.40	2.27	1.16				
x_i = ca		RA	1.0	1.76	1.37	1.79	1.72	.48	.01	2.38						
		B/Y	.013	.025	.062	.016	.018	-.009	-.150	.029						
		$B/\sqrt{V}$	.030	.072	.160	.048	.053	-.014	-.027	.098						
			(14110840)													
	12	RP	15655800	1.99	1.47	1.96	1.89	1.77	.36	2.11	2.27	1.15	1.45	1.53		
		RA	1.0	1.99	1.44	1.96	1.89	1.77	.36	2.11						
		B/Y	-.012	-.006	.038	-.008	-.008	-.013	-.042	-.008						
		$B/\sqrt{V}$	-.043	-.029	.162	-.040	-.040	-.062	-.086	-.041						
			(6770356)													
	24	RP	6467656	1.95	1.28	1.93	1.92	1.86	1.29	1.98	2.27	1.14	1.43	1.51	1.56	
		RA	1.0	1.97	1.08	1.95	1.94	1.87	1.29	2.00						
		B/Y	-.016	.005	.071	.003	.001	.005	-.010	.001						
		$B/\sqrt{V}$	-.085	.039	.435	.025	.009	.037	-.063	.011						
			(3834159)													
	40	RP	3256104	2.00	1.26	2.01	2.05	1.84	1.51	2.08	2.27	1.12				
		RA	1.0	2.03	1.07	2.03	2.08	1.86	1.51	2.10						
		B/Y	.015	.000	.052	-.001	.002	-.006	-.013	.003						
		$B/\sqrt{V}$	.118	.001	.440	-.007	.020	-.062	-.119	.029						
			(43472810)													
y_i = vm	4	RP	47024240	1.49	1.72	1.40	1.36	.98	.00	1.35	1.68	1.41				
x_i = hp· Log ca		RA	1.0	1.49	1.60	1.39	1.36	.98	.00	1.34						
		B/Y	-.030	-.001	.106	-.033	-.033	.039	-1.035	-.043						
		$B/\sqrt{V}$	-.061	-.003	.277	-.078	-.077	.077	-.071	-.102						
			(14110840)													
	12	RP	11851470	1.48	1.81	1.46	1.39	1.62	1.33	1.51	1.69	1.43	1.20	1.24		
		RA	1.0	1.47	1.12	1.46	1.39	1.60	1.33	1.51						
		B/Y	.018	.023	.147	.014	.014	.026	-.005	.018						
		$B/\sqrt{V}$	.073	.113	.787	.067	.066	.134	-.021	.087						

Continued on next page

Table 2 Continued.

Variables	Sample size	Performance criteria	Estimators $\hat{Y}_{srs}$	$\hat{Y}_{r}$	$\hat{Y}_{\bar{r}}$	$\hat{Y}_{ur}$	$\hat{Y}_{u\bar{r}}$	$\hat{Y}_{lr}$	$\hat{Y}_{pr}$	$\hat{Y}_{pp \Sigma x}$	$\hat{Y}_{pps}$	$\hat{Y}_{s-2}$	$\hat{Y}_{s-4}$	$\hat{Y}_{s-6}$	$\hat{Y}_{s-8}$	$\hat{Y}_{s-10}$
			(6770356)													
	24	RP	8264023	1.57	1.82	1.56	1.48	2.28	2.18	1.54	1.70	1.40	1.19	1.23	1.33	
		RA	1.0	1.57	1.16	1.56	1.48	2.29	2.16	1.55						
		B/Y	-.014	-.006	.118	-.010	-.011	.002	-.018	-.010						
		$B/\sqrt{V}$	-.069	-.035	.761	-.061	-.062	.013	-.126	-.060						
			(3834159)													
	40	RP	4424832	1.57	1.81	1.57	1.48	2.24	2.34	1.68	1.71	1.40	1.16		1.30	1.27
		RA	1.0	1.58	1.82	1.57	1.48	2.24	2.34	1.69						
		B/Y	-.008	-.002	.126	-.005	-.005	.005	.001	.000						
		$B/\sqrt{V}$	-.055	-.016	1.104	-.037	-.039	.052	.006	.001						
			(5371461)													
$y_1 = h$	4	RP	5712341	1.86	1.75	1.87	1.85	1.12	.01	1.86	1.85	1.52				
$x_1 =$ hp		RA	1.00	1.89	1.74	1.90	1.88	1.13	.01	1.88						
		B/Y	.012	-.002	-.011	.001	.001	-.010	.126	.002						
		$B/\sqrt{V}$	.112	-.019	-.126	.014	.014	-.096	.106	.022						
			(1743523)													
	12	RP	1525033	1.72	1.61	1.72	1.72	1.57	.58	1.82	1.85	1.51	1.39	1.35		
		RA	1.00	1.75	1.54	1.75	1.74	1.54	.59	1.86						
		B/Y	-.008	-.003	-.012	-.002	-.002	-.009	-.004	-.000						
		$B/\sqrt{V}$	-.136	-.066	-.262	-.049	-.047	-.197	-.058	-.010						
			(836539)													
	24	RP	1028714	1.77	1.77	1.77	1.75	1.91	1.87	1.85	1.85	1.51	1.38	1.32	1.29	
		RA	1.00	1.78	1.58	1.79	1.77	1.88	1.90	1.86						
		B/Y	-.006	-.003	-.013	-.003	-.003	-.006	-.002	-.003						
		$B/\sqrt{V}$	-.128	-.087	-.372	-.077	-.074	-.177	-.049	-.086						
			(473745)													
	40	RP	433270	1.66	1.71	1.66	1.65	1.77	1.96	1.68	1.85	1.50	1.35		1.26	1.26
		RA	1.00	1.74	1.59	1.74	1.73	1.85	2.05	1.76						
		B/Y	.007	.002	-.009	.002	.002	.003	.003	.002						
		$B/\sqrt{V}$	.240	.083	-.373	.090	.090	.111	.119	.088						

Continued on next page

Table 2 Continued.

Variables	Sample size	Performance criteria	$\hat{Y}_{srs}$	$\hat{Y}_{r}$	$\hat{Y}_{\bar{r}}$	$\hat{Y}_{ur}$	$\hat{Y}_{\bar{ur}}$	$\hat{Y}_{1r}$	$\hat{Y}_{pr}$	$\hat{Y}_{pp\ \Sigma x}$	$\hat{Y}_{pps}$	$\hat{Y}_{s-2}$	$\hat{Y}_{s-4}$	$\hat{Y}_{s-6}$	$\hat{Y}_{s-8}$	$\hat{Y}_{s-10}$
			Estimators													
			(5371461)													
$y_i = h$	4	RP	5116565	.33	.31	.30	.24	1.06	.01	.40	.42	.77				
$x_i = d$		RA	1.00	.32	.19	.30	.25	.98	.01	.40						
		B/Y	.006	-.037	-.149	-.003	-.001	-.029	.117	-.016						
		$B/\sqrt{V}$	.054	-.201	-.795	-.016	-.004	-.286	.094	-.099						
			(1743523)													
	12	RP	1587761	.40	.32	.39	.28	1.93	1.28	.35	.42	.76	1.04	1.34		
		RA	1.00	.40	.11	.39	.28	1.85	1.29	.35						
		B/Y	-.005	-.006	-.148	.006	.007	-.009	.003	.003						
		$B/\sqrt{V}$	-.080	-.064	-1.440	.060	.065	-.225	.055	.028						
			(836539)													
	24	RP	881281	.48	.34	.48	.35	2.22	2.25	.50	.42	.76				
		RA	1.00	.45	.60	.47	.34	2.05	2.25	.48						
		B/Y	-.001	-.017	-.161	-.012	-.014	-.009	-.001	-.013						
		$B/\sqrt{V}$	-.027	-.274	-2.149	-.190	-.185	-.293	-.050	-.207						
			(473745)													
	40	RP	437129	.27	.22	.27	.19	1.80	2.48	.29	.42	.74				
		RA	1.00	.28	.34	.28	.20	1.82	2.45	.30						
		B/Y	.004	-.002	-.153	.001	.001	.001	.003	.000						
		$B/\sqrt{V}$	.126	-.003	-2.310	.018	.011	.039	.173	.008						
			(5371461)													
$y_i = h$	4	RP	5164088	.09	.05	.07	.04	.53	.00	.12	.11	.69				
$x_i = ca$		RA	1.00	.08	.02	.07	.04	.50	.00	.12						
		B/Y	.001	-.115	-.490	-.019	-.015	-.032	-.218	-.016						
		$B/\sqrt{V}$	.009	-.319	-1.010	-.048	-.030	-.220	-.040	-.054						
			(1,743.523)													
	12	RP	1825959	.12	.05	.11	.06	1.34	.55	.13	.1.	.68	.59	.70		
		RA	1.00	.11	.01	.11	.06	1.32	.54	.13						
		B/Y	.003	-.034	-.464	-.002	-.006	-.006	.012	-.001						
		$B/\sqrt{V}$	.042	-.188	-1.735	-.011	-.022	-.105	.145	-.004						

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Table 2 Continued.

Variables	Sample size	Performance criteria	Estimators $\hat{Y}_{srs}$	$\hat{Y}_{r}$	$\hat{Y}_{\bar{r}}$	$\hat{Y}_{ur}$	$\hat{Y}_{u\bar{r}}$	$\hat{Y}_{lr}$	$\hat{Y}_{pr}$	$\hat{Y}_{pp\Sigma x}$	$\hat{Y}_{pps}$	$\hat{Y}_{s-2}$	$\hat{Y}_{s-4}$	$\hat{Y}_{s-6}$	$\hat{Y}_{s-8}$	$\hat{Y}_{s-10}$
			(836539)													
	24	RP	819285	.11	.05	.11	.05	1.58	1.30	.12	.11	.67	.58	.69	.66	
		RA	1.00	.11	.01	.11	.05	1.54	1.30	.11						
		B/Y	-.002	-.019	-.468	-.004	-.003	-.005	.001	-.008						
		B/√V	-.046	-.148	-2.569	-.034	-.018	-.163	.040	-.062						
			(473745)													
	40	RP	430027	.11	.04	.11	.05	1.16	1.18	.12	.11	.65				
		RA	1.00	.11	.04	.11	.05	1.16	1.16	.12						
		B/Y	.001	-.002	-.462	.006	.008	.001	.003	.004						
		B/√V	.017	.019	-2.996	.070	.061	.026	.124	.049						
			(1939767000)													
y_i = ca	4	RP	2122254000	3.11	3.53	2.82	2.77	1.70	.03	2.88	3.11	1.22				
x_i = d		RA	1.00	3.15	3.48	2.58	2.80	1.72	.03	2.91						
		B/Y	-.038	.006	.031	-.002	-.002	-.002	.046	-.011						
		B/√V	-.107	.028	.168	-.009	-.011	-.009	.022	-.054						
			(629629500)													
	12	RP	586077500	2.51	3.23	2.41	2.42	2.30	.82	2.46	3.11	1.22	1.89	2.21		
		RA	1.00	2.52	2.76	2.42	2.44	2.31	.82	2.47						
		B/Y	.018	.009	.043	.006	.007	.005	-.022	.009						
		B/√V	.095	.080	.422	.052	.058	.044	-.107	.076						
			(302094900)													
	24	RP	385001200	2.58	3.50	2.55	2.50	2.52	2.07	2.66	3.16	1.20				
		RA	1.00	2.59	3.02	2.55	2.50	2.52	2.05	2.66						
		B/Y	-.004	.001	.032	.000	.000	-.002	-.010	-.004						
		B/√V	-.029	.013	.398	.000	.000	-.022	-.093	-.046						
			(171081100)													
	40	RP	160366600	1.92	2.50	1.90	1.89	1.88	1.62	1.95	3.22	1.19				
		RA	1.00	1.94	2.01	1.91	1.90	1.89	1.61	1.95						
		B/Y	-.014	-.008	.032	-.008	-.009	-.008	-.012	-.010						
		B/√V	-.141	-.110	.523	-.122	-.125	-.118	-.156	-.142						

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Table 2 Continued.

Variables	Sample size	Performance criteria	Estimators													
			$\hat{Y}_{srs}$	$\hat{Y}_{r}$	$\hat{Y}_{\bar{F}}$	$\hat{Y}_{ur}$	$\hat{Y}_{\bar{ur}}$	$\hat{Y}_{lr}$	$\hat{Y}_{pr}$	$\hat{Y}_{pp \Sigma x}$	$\hat{Y}_{pps}$	$\hat{Y}_{s-2}$	$\hat{Y}_{s-4}$	$\hat{Y}_{s-6}$	$\hat{Y}_{s-8}$	$\hat{Y}_{s-10}$
			(1939767000)													
y_i = ca	4	RP	2039245000	1.32	1.51	1.26	1.27	.79	.00	1.39	1.49	.88				
x_i = h		RA	1.00	1.32	1.48	1.26	1.27	.78	.00	1.39						
		B/Y	.008	.012	.042	.003	.004	.047	.687	.007						
		B/$\sqrt{V}$	.023	.041	.150	.010	.013	.121	.057	.025						
			(629629500)													
	12	RP	540443700	1.35	1.51	1.33	1.32	1.29	.81	1.26	1.49	.87	.90	.84		
		RA	1.00	1.34	1.35	1.33	1.32	1.28	.81	1.26						
		B/Y	.012	.013	.051	.010	.010	.020	.019	.011						
		B/$\sqrt{V}$	.070	.087	.351	.066	.067	.129	.095	.069						
			(302094900)													
	24	RP	365675500	1.37	1.56	1.36	1.35	1.58	1.41	1.51	1.49	.86	.89	.83	.84	
		RA	1.00	1.38	1.48	1.37	1.36	1.59	1.43	1.52						
		B/Y	-.018	-.010	.031	-.012	-.012	.007	-.004	-.010						
		B/$\sqrt{V}$	-.120	-.082	.261	-.094	-.096	.064	-.034	-.085						
			(171081100)													
	40	RP	127784600	1.34	1.40	1.33	1.33	1.33	1.30	1.32	1.51	.85	.87		.82	.81
		RA	1.00	1.34	1.09	1.34	1.33	1.33	1.30	1.32						
		B/Y	.004	.000	.039	.000	.000	.001	-.001	-.004						
		B/$\sqrt{V}$	.042	.007	.540	-.004	-.002	.015	-.016	-.047						

[a] RP values for stratified sampling based on true variances.

[b] Monte Carlo estimate of variance given in place of RP value of 1.0. Figure in parentheses is true variance.

[c] RP values for remaining (y_i, x_i) combinations in this column based on true variance of Y_{srs} and an approximate formula for the variance of Y_{pps}; see Appendix A, Table 2.

Table 3. Estimator performance for (y_i, x_i) combinations from population III.

Variables	Sample size	Performance criteria	$\hat{Y}_{srs}$	$\hat{Y}_r$	$\hat{Y}_{\bar{r}}$	$\hat{Y}_{ur}$	$\hat{Y}_{\bar{ur}}$	$\hat{Y}_{lr}$	$\hat{Y}_{pr}$	$\hat{Y}_{pp \Sigma x}$	$\hat{Y}_{pps}$	$\hat{Y}_{s-2}$	$\hat{Y}_{s-4}$	$\hat{Y}_{s-6}$	$\hat{Y}_{s-8}$	$\hat{Y}_{s-10}$
			Estimators													
			(1713771)													
y_i = vm $x_i = d^2$	4	RP	1934682 b/	25.73	9.33	25.46	13.69	100.82	.10	32.81	31.14	1.96 a/				
		RA	1.00	23.93	5.30	25.40	13.67	100.85	.10	32.64	30.22					
		B/Y	.007	.019	.101	.003	.004	.000	-.080	.005	-.011					
		B/√V	.020	.275	.872	.053	.049	.008	-.071	.074	-.175					
			(562043)													
	12	RP	490306	30.37	8.28	31.14	15.33	104.35	59.87	34.93	21.49	1.96	7.14	15.75		
		RA	1.0	29.68	2.37	31.22	15.37	104.66	59.81	35.02	21.54					
		B/Y	.010	.005	.099	.001	.001	.000	-.001	-.001	.001					
		B/√V	.055	.162	1.582	.023	.026	.001	-.062	-.026	.020					
			(274111)													
	24	RP	216097	36.74	8.24	37.48	19.06	110.10	84.59	35.83	16.02	1.94	7.09	15.65	23.56	
		RA	1.0	36.47	1.27	37.45	19.05	110.09	83.51	35.78	15.12					
		B/Y	-.001	.001	.097	-.001	-.001	-.000	-.001	.001	.007					
		B/√V	-.005	.085	2.346	-.028	-.026	-.013	-.114	.039	..243					
			(158938)													
	40	RP	157499	35.07	9.26	35.52	16.93	130.70	131.00	38.66	28.44	1.92	7.02		23.36	30.82
		RA	1.0	35.26	9.32	35.59	16.92	131.42	130.47	38.76	28.50					
		B/Y	-.001	.000	.100	-.001	-.002	-.000	-.001	-.009	.001					
		B/√V	-.074	.016	2.999	-.061	-.076	-.001	-.097	-.052	.059					
			(1713771)													
y_i = vm $x_i = d_t^2$ c/	4	RP	1772902	11.32	6.26	11.20	9.35	7.40	.27	13.56	12.82	1.47				
		RA	1.0	11.16	4.69	11.23	9.37	7.41	.27	13.60	12.85					
		B/Y	.018	.014	.079	.001	.004	-.003	.003	.001	.000					
		B/√V	.052	.134	.582	.014	.040	-.025	.004	.013	.002					
			(562043)													
	12	RP	558633	15.05	6.57	15.42	11.66	16.61	10.97	15.23	14.42	1.46	3.74			
		RA	1.0	14.96	3.20	15.42	11.66	16.60	10.97	15.23	14.42					
		B/Y	-.001	.004	.077	.001	.000	-.001	-.001	.000	.000					
		B/√V	-.008	.079	1.027	.016	.003	-.022	-.020	.010	.001					

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Table 3 Continued.

Variables	Sample size	Performance criteria	$\hat{Y}_{srs}$	$\hat{Y}_r$	$\hat{Y}_{\bar{r}}$	$\hat{Y}_{ur}$	$\hat{Y}_{\bar{ur}}$	$\hat{Y}_{lr}$	$\hat{Y}_{pr}$	$\hat{Y}_{pp\ \Sigma x}$	$\hat{Y}_{pps}$	$\hat{Y}_{s-2}$	$\hat{Y}_{s-4}$	$\hat{Y}_{s-6}$	$\hat{Y}_{s-8}$	$\hat{Y}_{s-10}$
			Estimators													
			(274111)													
	24	RP	312943	16.42	7.72	16.69	13.04	17.85	15.65	16.81	14.97	1.45[a]	3.70			
		RA	1.0	16.42	2.74	16.52	12.89	17.57	15.47	16.71	14.79					
		B/Y	-.014	-.004	.070	-.005	-.006	-.005	.-005	-.004	-.005					
		B/√V	-.100	-.101	1.359	-.143	-.147	-.162	-.147	-.125	-.148					
			(158938)													
	40	RP	170597	15.07	7.63	15.15	11.66	17.91	17.11	16.60	7.98	1.43	3.64			
		RA	1.0	14.99	1.44	15.16	11.66	17.98	17.05	16.47	8.02					
		B/Y	.007	.003	.080	.002	.002	.001	.002	.003	.000					
		B/√V	.069	.099	2.080	.065	.070	.028	.092	.110	.003					
			(2446973)													
$y_i = h$	4	RP	2611601	.28	.21	.28	.24	2.16	.00	.28	.27	1.68				
$x_i = d$		RA	1.0	.27	.15	.28	.24	1.91	.00	.28	.27					
		B/Y	-.000	-.018	-.082	.000	-.000	-.015	.120	-.002	.006					
		B/√V	-.005	-.147	-.593	.001	-.001	-.358	.050	-.017	.050					
			(802502)													
	12	RP	910100	.30	.22	.31	.26	2.61	2.46	.31	.42	1.68	2.93	3.74		
		RA	1.0	.30	.11	.31	.26	2.57	2.47	.31	.42					
		B/Y	.003	-.009	-.083	-.004	-.004	-.003	.001	-.002	-.003					
		B/√V	.074	-.135	-1.059	-.054	-.055	-.138	.033	-.027	-.044					
			(391384)													
	24	RP	300399)	.30	.21	.30	.25	2.13	2.91	.28	.24	1.67	2.93	3.74	3.80	
		RA	1.0	.30	.05	.30	.24	2.11	2.84	.28	.23					
		B/Y	-.000	-.003	-.085	-.000	.000	-.001	-.002	-.002	-.006					
		B/√V	-.023	-.072	-1.836	-.001	.005	-.095	-.156	-.042	-.139					
			(226937)													
	40	RP	257132	.30	.21	.31	.25	3.07	4.89	.31	.25	1.67	2.93		3.78	3.74
		RA	1.0	.31	.05	.31	.25	3.07	4.90	..30	.25					
		B/Y	-.003	.003	-.077	.005	.005	-.002	-.001	.005	.006					
		B/√V	-.137	.084	-1.773	.127	.130	-.145	-.130	.141	.142					

[a] RP values for stratified sampling estimators based on true variances.

[b] Monte Carlo estimate of variance given in place of RP value of 1.0. Figure in parentheses is true variance.

[c] d rounded to midpoint of two inch diameter class i.e., 5, 7, 9 etc., prior to squaring.