RESONANCE CAPTURE OF NEUTRONS BY URANIUM CYLINDERS

by

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RESONANCE CAPTURE OF NEUTRONS BY URANIUM CYLINDERS

INTRODUCTION

The resonance capture of neutrons by natural uranium is of sufficient importance in reactors that only by careful considerations of geometry to allow a minimum of resonance capture can a reactor be made to operate with natural uranium for fuel. The operating reactors fueled with natural uranium in the manner associated with this discussion are termed heterogeneous reactors, in that the uranium is in discrete lumps in the moderator. The Hanford Test Pile (HTP), the reactor used in these experiments, is composed of natural uranium rods imbedded in a matrix of graphite moderator and is a low power reactor. Consisting of only two regions (the fuel and the moderator), the reactor is amenable to a mathematical analysis. The HTP is an uncooled reactor, roughly cubical, with edges 18 feet long and is surrounded by five feet of concrete shielding. There is no provision to replace the fuel, since the normal flux is sufficiently low to eliminate the problem of the burnout of the fissionable isotopes. The uranium to moderator volume ratio is on the order of 0.025; therefore, it is seen that the bulk of the volume of the reactor is composed of the moderator.

An operating reactor is a multiplying system in which

the neutron density at any time is a function of the density of the previous generation where a generation of neutrons is a cycle in which a neutron, formed from the fission of the nucleus, causes a fission and is replaced by another neutron. As is shown in Figure 1, there are four distinct processes competing for the neutrons between the liberation of the neutrons at fission energies and the final capture of the neutrons. The first is the capture of neutrons by the isotope of uranium with a mass of 238 atomic mass units (amu). This capture, at energies above the fission threshold of this isotope, will lead to fission and to the enhancement of the neutron population. Neutrons which escape capture at energies above the fast fission threshold are moderated to energies corresponding to capture resonances in the uranium. These neutrons have a probability of being captured in either U-238 or U-235; however, those that are captured in the U-235 will not be discussed in this paper. The neutrons that interact with the U-238 nuclei in the resonance region lead to radiative captures and remove the neutrons from the system.

The third process is parasitic capture in the reactor components other than the uranium. The capture in the uranium, a fraction of which leads to fission and a reproduction of the neutron cycle, is the fourth process. The ratio of the density of neutrons in any two successive generations is termed the multiplication factor and is

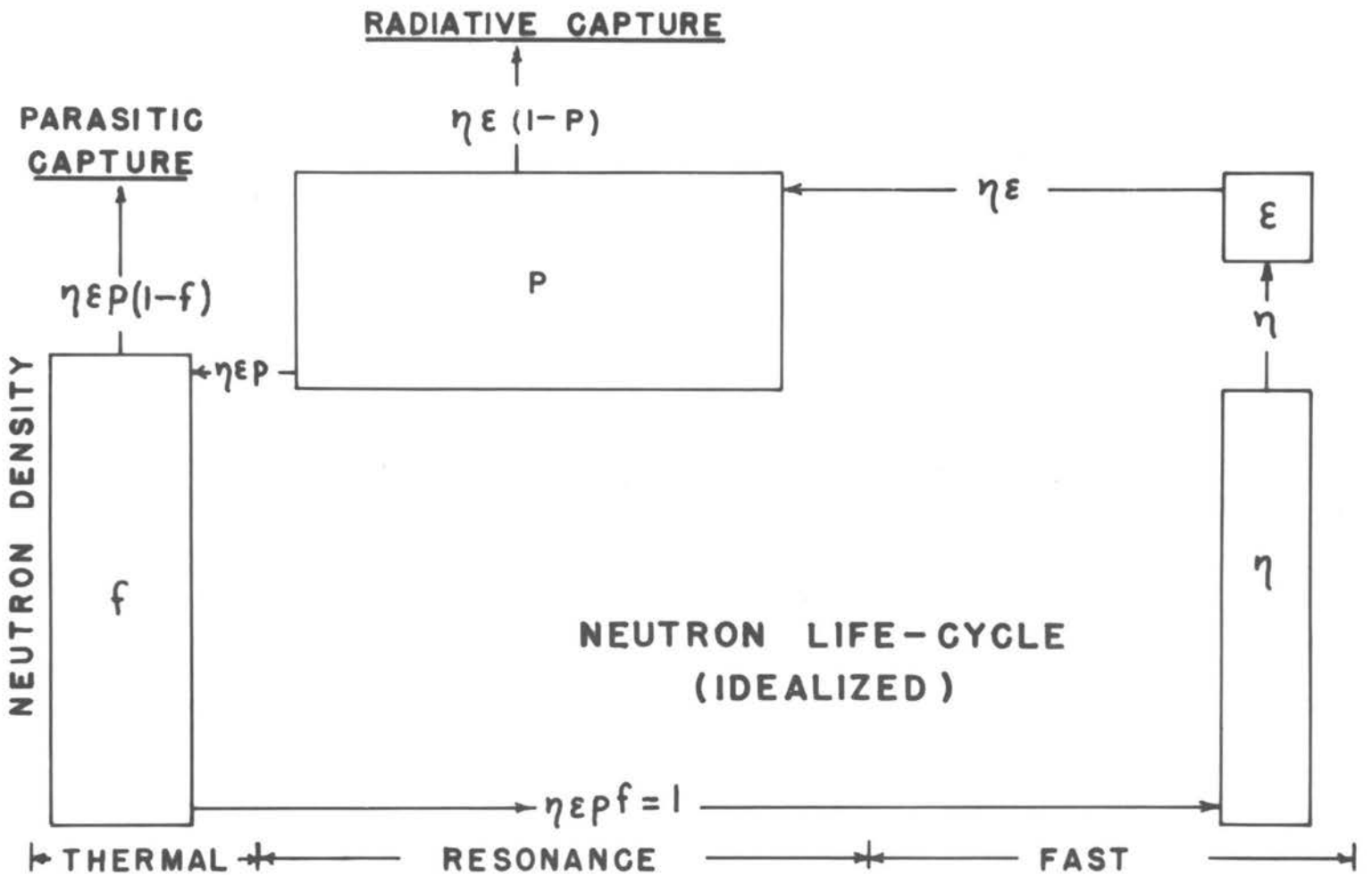

RADIATIVE CAPTURE
PARASITIC CAPTURE
$\eta\varepsilon(1-P)$
$\eta\varepsilon$
ε
P
$\eta\varepsilon P(1-f)$
$\eta\varepsilon P$
η
NEUTRON DENSITY
f
η
NEUTRON LIFE-CYCLE
(IDEALIZED)
$\eta\varepsilon pf = 1$
THERMAL
RESONANCE
FAST
NEUTRON ENERGY

FIGURE 1

denoted by k. The above four processes may be expressed more succinctly as follows:

$$k = \varepsilon p f \eta \qquad (1)$$

where the terms on the right correspond to the four processes described above. Appropriately, this is termed the four-factor-formula.

The terms on the right in equation (1) have the following definitions. The fast effect, ε , is the total number of neutrons reaching the resonance energy region of the absorber per neutron produced by primary fission. Here, primary fission is that resulting from the capture of neutrons that are in thermal equilibrium with the moderator. The resonance escape probability, p, equals the fraction of the neutrons that escape capture in the energy region corresponding to the resonances of the uranium. The thermal utilization, f, is the fraction of the neutrons in thermal equilibrium with the moderator that is captured in the uranium. And η is the number of neutrons formed per thermal neutron captured in the uranium. These last two events take place well below the energy level where resonance capture occurs.

The cycle of neutron behavior and the interactions of the neutrons are best discussed as a combination of classical and quantum effects. The neutrons, released with fission energies which may range as high as several million electron volts (9, p. 60), will make elastic

collisions with the moderator until they are in thermal equilibrium with the moderator nuclei. This slowing down process is a mechanical process which may be described in the following fashion:

In the energy range between the fission energies ($\sim 10^6$ electron volts) and thermal energies ($\sim$ 0.10 electron volts), the methods of classical mechanics adequately treat the situation in which there is only elastic scattering. There is a negligible amount of inelastic scattering in graphite during slowing down (13, pp. 76-78), and since coherent scattering in graphite occurs below thermal energy these two effects will be ignored. Another effect, which is of no consequence in this development, is any anisotropy of the neutron scattering from the carbon nucleus.

As a result of the number of collisions to reduce the energy of a neutron from nascent to thermal, and, further, since the mean free path of a neutron for scattering events is on the order of 2.5 cm, a neutron may migrate several meters before coming to thermal equilibrium with the moderator. This migration tends to minimize small physical discontinuities in the system. The fission neutrons that escape capture in the U-238 and leave the uranium rod will migrate throughout the reactor until they leave the system or are captured. The migration will be a process of moderation of the neutron energy until the neutrons are in thermal equilibrium with the moderator.

Neutrons in thermal equilibrium with the moderator have a statistical distribution of kinetic energy analogous to that of molecules of a gas in thermal equilibrium with the surroundings. This is a point particle problem that has the following Maxwell-Boltzmann distribution of energies:

$$\frac{n(E)}{n} = \frac{2}{\sqrt{\pi}} (k_o T)^{-\frac{3}{2}} E^{\frac{1}{2}} e^{-\frac{E}{kT}}$$

Here, n is the neutron density, k_o Boltzmann's constant, E the neutron energy, and T the temperature of the moderator in degrees Kelvin.

The Maxwell-Boltzmann energy distribution occurs since the mean life of the thermalized neutron is long compared with the moderation time. (6, p. 184) There are, however, two reasons why this picture lacks completeness. These are: first, that neutrons are continually falling into this distribution from high energies; and, secondly, since absorption cross sections are approximately proportional to the reciprocal of the neutron speed, the lower energy neutrons are constantly being removed from the system. This tends to give a skewed distribution with an average value somewhat higher than that expressed by the above equation. The exact equivalent temperature of the neutrons in a heterogeneous reactor is not accurately known because of uncertainties associated with cross section data and current measuring techniques. There is also a spatial

dependence of the neutron temperature resulting from the proximate, preferential neutron absorbers.

Between fission and thermal energies the collision density in the graphite is related to the average logarithmic energy loss per collision $\xi = \overline{\ln E_1 - \ln E_2}$ where E_1 and E_2 are the energies of the neutron before and after collision with a graphite nucleus.

If a collision density per unit logarithmic energy interval is identified as K(E), it has been shown by Placzek (11) that asymptotically K(E) approaches the value

$$K(E) \rightarrow \frac{Q}{\xi E}$$

Here, Q is the number of source neutrons entering the system per unit time per unit volume, and ξ is the logarithmic energy decrement given by $\xi = 1 + \alpha(1-\alpha)^{-1}\ln\alpha$ where, for collisions with a nucleus of mass A, $\alpha = [(A-1)/(A+1)]^2$ (6, p. 143). Further, the product of the neutron density and the neutron velocity is written as $n(E)v = \varphi(E)$, the neutron flux. The flux is related to the collision density and the macroscopic scattering cross section Σ_s by

$$\varphi(E) = \frac{K(E)}{\Sigma_s} = \frac{Q}{\Sigma_s \xi E}$$

In the slowing down region the flux, $\varphi(E)$, is proportional to 1/E, where the slowing down region extends from approximately $\alpha^3(E_o)$ to thermal energies. The collision density initially exhibits transient behavior as

a result of the lower limit to which the neutron energy may be degraded on the first and higher collisions. Figure 2 reproduces the results from Placzek for the case of graphite. Here, the coordinates have been changed somewhat and assume E_o to be two mev. From the energy spectrum for fission neutrons (9, p. 60), it is assumed that $\alpha^3 E_o \geq \alpha^3(0.25 \text{ mev})$ or that the upper limit of the region in which the flux is proportional to 1/E is 0.367 x 0.25 mev = 0.10 mev. Experiments to determine the lower limit of this region, i.e., the point at which the 1/E flux meets the Maxwellian distribution, indicate that the two are in equilibrium at approximately 0.17 electron volts. (9, p. 131) The resonance region, between 5 and 1,000 electron volts, is in the region well removed from the end points of the 1/E flux and justifies the assumption of a constant collision density.

For the spectrum of the neutron energies experienced in a nuclear reactor and for interactions with the nuclei in the reactor, the pertinent cross sections are: neutron scattering cross section, slowly varying capture cross section, and resonance capture cross sections. Scattering cross sections are discussed in several standard works (2 and 12), and from the method of partial waves the total scattering cross section is given as

$$\sigma = 4\pi\left(\frac{\lambda}{2\pi}\right)^2 \sum_l (2l+1) \sin^2 \delta_l$$

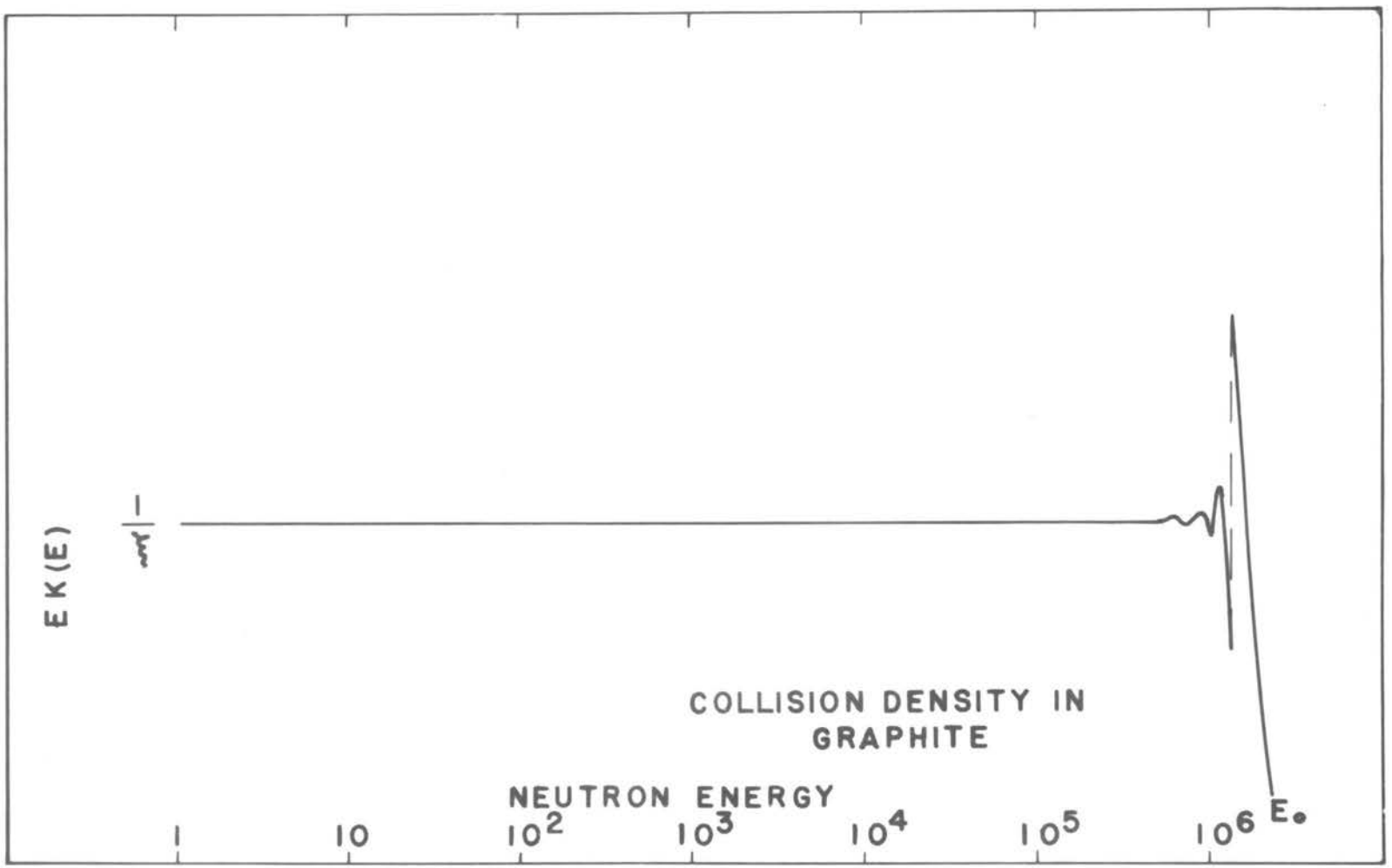

E K(E)
COLLISION DENSITY IN
GRAPHITE
NEUTRON ENERGY
1
10
10^2
10^3
10^4
10^5
10^6
E•

FIGURE 2

where λ is the de Broglie wave length, ℓ is the angular-momentun quantum number, and δ_ℓ is the angle of phase shift. This method of solution does not explain resonances, and it was not until the Breit-Wigner discussion of the compound nucleus that these were amenable to quantitative analysis.

N. Bohr (3, p. 345) suggested a model of the nucleus in which the mode of decay is independent of the mode of formation. The mode of decay of this compound nucleus is dependent on the quantum level to which the nucleus is raised. The compound nucleus is excited into an energy level equal to the binding energy of the extra nucleon plus the kinetic energy of the bombarding particle. If this excitation energy corresponds to one of the virtual levels of the compound nucleus, there will be a resonance and the cross section for neutron interactions will be a varying function of neutron energy as the excitation energy of the neutron approaches a virtual state. Breit and Wigner (4) suggested a method by which these resonance phenomena may be discussed. The width of these states may be estimated from the uncertainty principle,

$$\Gamma \tau_\ell \sim \hbar$$

where τ_ℓ is the mean life of the state, Γ is the width of the level, and $\hbar$ is Planck's constant divided by 2π.

The observed lives of these states are greater than

10^{-14} seconds; consequently, Γ becomes

$$\Gamma \leq \frac{6.6\,(10)^{-14}}{\tau_a} \text{ ev}$$

which states that the level width is no greater than 6.6 electron volts.

For the probability of interaction of slow neutrons incident on even-even nuclei, the Breit-Wigner formula is written (4, p. 529)

$$\sigma = 4\lambda^2 \frac{\Gamma_n \Gamma_\gamma}{(E-E_r)^2+(\Gamma/2)^2} = \frac{\sigma_0}{1+\left(\frac{E-E_r}{\Gamma/2}\right)^2} \qquad (2)$$

where Γ_n/Γ and Γ_γ/Γ are relative probabilities of neutron and gamma emission respectively and E_r is the energy of the resonance. In the low energy region (region with no resonances) the terms in the above formula will be nearly constant except Γ_n. Γ_n is proportional to the neutron velocity, so the capture cross section becomes $\sigma_a =$ constant $(v)^{-1}$ where v is the neutron speed. The 1/v dependence of the cross section for neutron capture in regions removed from the resonances is in agreement with observed data. The result of measurements of total cross section of the U-238 nucleus is shown in Figure 3. Here, the cross section in units of barns is plotted as a function of the neutron energy - (1 barn = 10^{-24} cm^2). The values of the parameters associated with the resonances appear in Table 1 (8, p. 30).

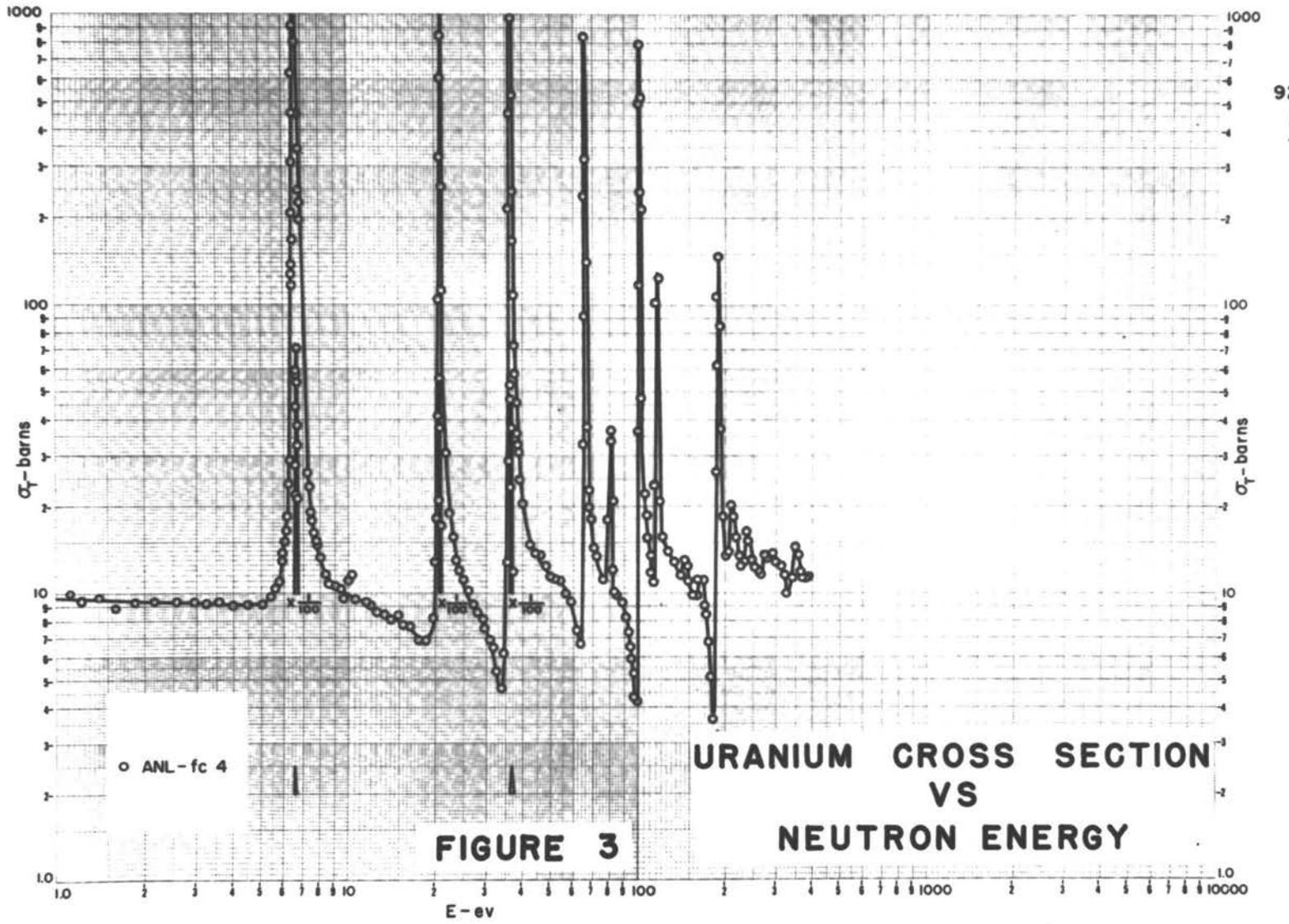


FIGURE 3

URANIUM CROSS SECTION
VS
NEUTRON ENERGY

TABLE 1

RESONANCE PARAMETERS OF U-238

E_r * (ev)	$\alpha^3 E_r$ (ev)	σ_o (k barns)	Γ_n (mv)	Γ_γ (mv)	$\frac{\Gamma_n}{\Gamma_n + \Gamma_\gamma}$
6.70	2.46	22.50	1.52	24	0.06
20.9	7.67	7.46	5.0	25	0.17
37.0	13.58	14.97	3.9	25	0.13
66.5	24.40	3.23	4.3	25	0.15
81.6	29.95	0.43	11.5	25	0.32
90	33.0	0.01	2.9	25	0.10
104	38.2	3.23	6.7	25	0.21
118	43.3	0.43	3.0	25	0.11
146	53.6	0.03	5.8	25	0.19
166	60.9	0.03	6.7	25	0.21
192	70.5	0.43	2.9	25	0.10
212	77.8	0.02	3.5	25	0.12
242	88.8	0.02	3.3	25	0.12
258	94.7	0.02	6	25	0.19
278	102	0.02	12	25	0.32
297	109	0.02	13	25	0.34
368	135	0.02	22	25	0.47
418	153	0.02	41	25	0.62

* Legend: 1 k barn = 10^3 barns
1 mv = 10^{-3} electron volts

Note: mv adapted from Harvey, et al (8).

RESONANCE CAPTURE OF NEUTRONS

Resonance Escape Probability

In an operating reactor, the probability of an incident neutron interacting with a uranium nucleus is a function of the neutron energy. In a region where the neutron density is n(E) with velocity v, the differential probability of interaction can be written

$$dI(E) = n(E)\, v\, \Sigma(E)\, dE$$

where Σ (E) is the cross section per unit volume at energy E.

The probability, I, of interaction leading to capture then becomes

$$I = \int_E \varphi(E')\, \Sigma_a(E')\, dE'$$

where φ(E), the flux, is written for n(E)v; and Σ_a(E), the absorption cross section, replaces the total cross section Σ (E).

This is not a function that can be treated analytically as is seen from the curve of cross section as a function of neutron energy shown in Figure 3. Therefore, approximate methods and/or experimental measurements have been utilized in order to arrive at the result (8, p. 23)

$$\frac{\int_E \sigma_a(E')\,\varphi(E')\,dE'}{\int_E \varphi(E')\,dE'} = 276(10)^{-24}\ \text{cm}^2$$

The value of the above integral, termed the resonance integral, is applicable to dilute mixture of uranium in moderator, i.e., the flux at any point will conform to the 1/E distribution of neutron energy.

As is shown in Figure 2, the neutron must lose energy to approximately $\alpha^3 E_o$ before the oscillations of the collision density can be ignored. This is also the situation that exists in the resonance region. The resonance bands appear as negative sources and as such will introduce oscillations into the collision density. In order to have no oscillations of the collision density at a resonance then, it is necessary that the resonances be separated in energy by at least $\alpha^3 E_r$, where E_r is the neutron energy at the resonance.

Referring to Table 1, it is seen that the resonances in U-238 do not conform to this criterion, and that perturbations by resonances will result, preventing a high order analysis of a dilute mixture of uranium in graphite moderator. The heterogeneous lattice, however, is treated in a different manner and quite readily yields results which may be evaluated and checked experimentally. A straightforward derivation that has physical acceptance assumes contigual, narrow resonances. The development

of the effect on a reactor resulting from neutron captures in the resonances can be more expeditiously treated as the effect summed over all resonances rather than the sum of the effects attributed to each resonance.

The effect of the captures in the resonance region can be related to the resonance escape probability, p, in the following fashion suggested by Wigner. (15, p. 2) For resonances so separated in energy that there can be no interaction between resonances, EK(E) will regain the value $1/\bar{\xi}$ between resonances. The lower three resonances in U-238 approximate this spacing and contribute over 90 per cent of the resonance integral, so the resonances in U-238 will be treated as independent.

Referring to Table 1, it is seen that $\Delta E_i \ll \bar{\xi} E_i$, where ΔE is the width assigned to a line resonance and $\bar{\xi}$ is the average value of ξ for the system.

The number of neutrons scattered into ΔE_i is the same with or without capture; consequently, a balance of neutrons at E_1 is

$$\Sigma_s \left(\frac{Q}{\Sigma_s \xi E_i} \right) \Delta E_i = (\Sigma_a + \Sigma_s)\, \varphi(E_i) \Delta E_i$$

Here, the right hand side of the equation gives the number leaving and the left side of the equation shows the number of neutrons scattered into ΔE_i.

The probability of finding a neutron in ΔE_i is, then, per unit source strength,

$$\frac{1}{Q}\left(\frac{Q}{\xi E_i}\right)\Delta E_i = \frac{\Delta E_i}{\xi E_i} .$$

The probability of absorption in ΔE_i is

$$\frac{\Sigma_a}{\Sigma_a+\Sigma_s}\ \frac{\Delta E_i}{\xi E_i}$$

so the probability that a neutron will escape absorption in ΔE_i is

$$p = 1 - \frac{\Sigma_a}{\Sigma_a+\Sigma_s}\ \frac{\Delta E_i}{\xi E_i} .$$

For the case of an infinite density of independent resonances, the probability of escaping capture becomes (6, p. 167)

$$p(E) = \exp\left[-\int_E^{E_0} \frac{\Sigma_a}{\Sigma_a+\Sigma_s}\ \frac{dE'}{E'}\right].$$

This is the manner in which the resonance escape probability is usually expressed.

For a constant scattering cross section and writing $N\sigma = \Sigma$ the above becomes

$$p(E) = \exp\left[-\frac{N_0}{N_q\sigma_{s_q}\xi}\int_E^{E_0} \frac{\sigma_a}{1+\frac{N_0(\sigma_s+\sigma_a)_0}{N_q\sigma_{s_q}}}\ \frac{dE'}{E'}\right]. \qquad (4)$$

The integral in the last equation can be identified with the resonance integral presented in equation (3).

The above elementary derivation shows only the form of the resonance escape probability for the heterogeneous assembly of uranium in graphite. In the situation that

exists with lumped fuel assemblies, the results are modified by effectively altering the resonance cross section of the uranium nuclei and by including the spatial dependence of the neutron flux.

Geometry Dependence of Resonance Capture

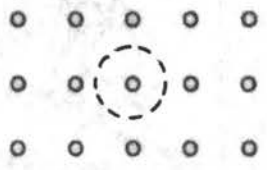
Qualitatively, the lumping of the uranium will reduce the resonance capture in the lump by allowing only a small fraction of the neutrons with energies corresponding to resonances to penetrate through a thin shell of the metal. This physical picture prompted Wigner (16, p. 264) to suggest that the resonance capture in a lump could be treated as two separate effects. The incident neutrons with energies matching the resonance energies will be preferentially captured in the surface creating a surface effect, and the capture of the neutrons of all other energies in the resonance region will be distributed throughout the uranium lump. Obviously, the resonance integral of equation (4) is no longer complete. The resonance integral will be replaced by an effective resonance integral

$$\int_{E}^{E_o} \frac{\sigma_a}{1 + \frac{N_o(\sigma_a+\sigma_s)_o}{N_q \sigma_{sq}}} \frac{dE'}{E'} \longrightarrow \int_{E}^{E_o} (\sigma_{a_o})_{eff} \frac{dE'}{E'}.$$

Into the effective resonance absorption cross section are then collected all variations from the theoretic 1/E neutron

flux of the dilute assembly.

In the heterogeneous lattice there exists still another difference; there is now a well-defined region in which moderation occurs but little absorption takes place. There is also the uranium in which there is strong resonance absorption but little moderation; consequently, the energy dependent neutron flux will vary in density through the lattice cell where the cell in the square array is taken in the same sense as the Wigner-Seitz (17, p. 805) approximation to a crystal lattice cell. Here, the rectangular cell of the reactor is replaced with an equivalent cylindrical cell which contains an equal volume of moderator as shown in Figure 4.



EQUIVALENT CELL IN A SQUARE ARRAY

FIGURE 4

The spatial dependence of the flux can be expressed functionally for the three conditions existing in the moderator and uranium. In the graphite moderator,

$$\bar{\varphi}_g = \frac{1}{V_g} \int_{V_g} \varphi(r)\, dV$$

where V_q is the volume of moderator per unit length of cell. Since the uranium columns are very long compared to the cell radius, there is no angular dependence of the flux nor any end effects.

The average flux incident on the surface of the uranium cylinders is

$$\bar{\varphi}_s = \frac{1}{S} \int_s \varphi(r)\, dS.$$

Here, S is the uranium surface per unit length of cell, and as before the flux has no angular dependence. Throughout the volume of the uranium the average flux can be expressed by

$$\bar{\varphi}_o = \frac{1}{V_o} \int_{V_o} \varphi(r)\, dV$$

where V_o is the uranium volume per unit length of cell and $\varphi(r)$ has only radial spatial dependence.

Rewriting the effective resonance cross section in the form suggested by Wigner (16)

$$(\sigma_{a_o})_{eff} = a(E) + b(E)\frac{S}{M}$$

with a(E) an energy dependent function proportional to the number of U-238 nuclei per unit length of cell and b(E)S/M a description of the surface absorption per unit mass of uranium per unit length of cell.

The effective resonance integral may be written (7, p. 260)

$$\int_E (\sigma_{a_o})_{eff} \frac{dE'}{E'} = \int_E a(E') \frac{dE'}{E'} + \frac{S}{M} \int_E b(E') \frac{dE'}{E'}.$$

The first integral on the right now expresses the effective cross section of the U-238 nuclei which are protected by the outer layers of U-238 nuclei. The second integral is the effective cross section of the nuclei in the surface section of the uranium in which absorptions at resonance occur.

Substituting the spatial dependence and effective resonance integral into equation (4), the resonance escape probability for a heterogeneous reactor now becomes (6, p. 262)

$$p(E) = \exp\left\{ -\frac{N_o V_o}{V_g f_g \Sigma_{s_g}} \left[\int_E^{E_o} \frac{\bar{\varphi}_o(E')}{\bar{\varphi}_g(E')} a(E') \frac{dE'}{E'} + \frac{S}{M} \int_E^{E_o} \frac{\bar{\varphi}_s(E')}{\bar{\varphi}_g(E')} b(E') \frac{dE'}{E'} \right] \right\}.$$

This may be rewritten as

$$p(E) \approx \exp\left\{ -\frac{N_o V_o}{V_g f_g \Sigma_{sg}} \left(\frac{\bar{\varphi}_o}{\bar{\varphi}_g} \right) \int_E (\sigma_{a_o})_{eff} \frac{dE'}{E'} \right\} \qquad (5)$$

since $\bar{\varphi}_o / \bar{\varphi}_g$ is approximately energy independent.

Temperature Dependence of Resonance Capture

For a constant energy spectrum of the incident

neutrons, the effective resonance integral is also dependent upon the temperature of the absorber atoms. This temperature dependence arises from a broadening of the resonances that is in turn a result of the thermal motion of the uranium nuclei. The energy E appearing in equation (2) is the relative energy of the neutron and the target nucleus. If the target nucleus is in thermal motion, the relative energy will be (1, p. 140)

$$E = \frac{m}{2}(\vec{v}_n - \vec{v}_A)^2$$

Here, v_n is the neutron velocity, m is the neutron mass, and v_A is that component of the nuclear velocity that is parallel to v_n. Expanding this expression and taking the neutron mass as unity on the atomic scale,

$$E = \tfrac{1}{2} v_n^2 - v_n v_A + \tfrac{1}{2} v_A^2.$$

The velocity of the uranium nucleus is small compared to the neutron velocity; consequently,

$$E \approx E_n \pm (2E_n)^{\frac{1}{2}} v_A$$

where the total kinetic energy has an energy range expressed by the second term on the right.

Substitution of the last expression for the relative energy of the neutron into equation (2) results in an associated range of values for the cross section at neutron energy E_n. This concomitant change in cross

section when the energy of the nucleus is changed is referred to as a "Doppler Effect." The discussion of the Doppler Effect parallels that of Bethe (1, p. 140). The distribution of the components of nuclear velocity parallel to that of the incident neutron can be expressed by

$$W(V_A)\,dV_A = \left(\frac{A}{2\pi k_0 T}\right)^{\frac{1}{2}} \exp\left[-\frac{AV_A^2}{2k_0T}\right] dV_A$$

where A is the mass of the nucleus.

The probability of finding a neutron of energy E_n with a relative energy between E and E + dE is then

$$W(E)\,dE = \pi^{-\frac{1}{2}} \exp\left[-\frac{(E-E_n)^2}{\Delta^2}\right] \frac{dE}{\Delta}$$

where the "Doppler Width" is $\Delta = 2(Ek_0T/A)^{\frac{1}{2}}$. The Doppler width is a function not only of the neutron energy but also of the temperature of the uranium.

The effective cross section which includes the Doppler broadening results from combining the above with equation (2).

$$\sigma = \int_E \sigma(E')\,W(E')\,dE' = \pi^{-\frac{1}{2}} \int_{-\infty}^{\infty} \frac{\sigma_{0_r}}{1+\left(\frac{E-E_r}{\Gamma/2}\right)^2} \frac{1}{\Delta} e^{-\frac{(E-E_n)^2}{\Delta^2}} dE.$$

Rewriting this equation after making the substitutions $x = (E_n - E_r)/(\Gamma/2)$ and $y = (E - E_r)/(\Gamma/2)$,

$$\sigma = \frac{\sigma_o \Gamma}{2\sqrt{\pi}\Delta} \int_{-\infty}^{\infty} \frac{e^{-\left(\frac{\Gamma}{2\Delta}\right)^2 (x^2 - y^2)}}{1+y^2} \, dy.$$

Integration of the above over all available neutron energies leads to $\sigma = \pi \sigma_o \Gamma / 2$, an expression which is independent of the temperature of the uranium.

For the well-defined resonances of U-238, the dilute resonance integral will be temperature independent since the energy is practically constant over each resonance; however, the effective resonance integral for uranium cylinders is temperature dependent. The temperature dependence of the resonance capture by uranium cylinders results from changing the effective cross section in the extremes of the resonances. From the last equation, the cross section averaged over the energy of the resonance is constant; consequently, $\sigma_o \Gamma$ must be constant with temperature. At higher temperatures and smaller values of σ_o , the value of Γ is correspondingly greater, accounting for the term "temperature broadening." This broadening raises the value of the cross sections in the extremes of the resonance as the peak cross section, σ_o , is reduced. The value of σ_o at these temperatures is still of sufficient magnitude to insure that neutrons with corresponding energies will be selectively removed near the surface, so raising the cross section between the resonances will then have the effect of raising the volume

absorption. This changes the effective resonance integral and gives to the resonance escape probability a temperature dependence. With negligible loss in precision, the resonance escape probability may be expressed as the first two terms of the expansion of the exponent of equation (5) where it is to be remembered that the resonance integral is dependent upon the uranium temperature.

$$p \approx 1 - \frac{N_o V_o \bar{\varphi}_o}{N_g V_g \sigma_{sg} \xi_g \bar{\varphi}_g} \int (\sigma_{a_o})_{eff} \frac{dE'}{E'}.$$

The resonance capture, 1-p, then becomes

$$1 - p = \frac{N_o V_o \bar{\varphi}_o}{N_g V_g \sigma_{sg} \xi_g \bar{\varphi}_g} \int (\sigma_{a_o})_{eff} \frac{dE'}{E'}. \qquad (6)$$

The resonance capture relates the loss of resonance neutrons from the system to the resonance integral as is shown in equation (6).

THEORY OF REACTIVITY MEASUREMENTS

In order to determine the effects of temperature and geometry on the resonance capture of neutrons by the uranium, it is necessary to relate the capture of the neutrons to experimentally measureable quantities. This is done by relating the time rate of change in the neutron density in the reactor to the multiplication factor k_{eff}.

The effective multiplication factor of the reactor is the multiplication factor of the infinite lattice, the four-factor-formula, corrected for the leakage of neutrons from the system (6, p. 197).

$$k_{eff} = k \frac{e^{-B^2\tau}}{1+L^2B^2}. \qquad (7)$$

The term $\exp(-B^2\tau)$ accounts for the loss of neutrons from the system during the process of moderation from fission to thermal energies. The geometrical buckling B^2 is a function of the geometry of the reactor, and τ is the moderation area for thermal neutrons in the graphite. The denominator of equation (7) accounts for the leakage of thermal neutrons from the reactor with L^2 the square of the distance that the thermalized neutron will difuse in graphite before being captured. For small changes in the multiplication factor from the just critical assembly, the leakage terms are essentially constant; so,

$$\left(\frac{dk}{k}\right)_{eff} = \frac{dk}{k}.$$

The time dependence of the neutron flux for the near critical assembly is given by

$$\varphi(t) = \varphi(0) \exp(\omega t)$$

where ω is the reciprocal of the period of the reactor and is termed reactivity. Here the period is defined as the time necessary for the neutron density to change by a factor of e, the base of the natural logarithms.

For reactor periods longer than approximately one hour, there is a linear relationship between the reactivity and the multiplication factor k.

$$\omega = \text{const } \frac{dk}{k}.$$

The value of the constant term is 385 inhours per per cent of dk/k (9, p. 52) with the inhour defined as that amount of excess reactivity necessary to give a period of one hour.

Denoting the multiplicative constant in the above equation by A', a change in the critical reactor leading to a non-critical situation can then be expressed by

$$\omega = A' \frac{dk}{k} = A' \left[\frac{d\varepsilon}{\varepsilon} + \frac{dp}{p} + \frac{df}{f} + \frac{d\eta}{\eta}\right].$$

Here, the measured reactivity relates the change in the

flux density per unit time to a change in the terms comprising the multiplication constant with an initial reactivity of zero. For a more general situation in which the reactivity changes are between two non-critical configurations,

$$\delta\omega = A'\left[\frac{d\varepsilon}{\varepsilon} + \frac{dp}{p} + \frac{df}{f} + \frac{d\eta}{\eta}\right]. \tag{8}$$

This equation permits assessing the changes in the four-factor-formula when the measured reactivities (described in Appendix II) between two configurations of the entire reactor are known. Altering only a fraction of the system rather than the entire reactor will still cause a change in measured reactivity; consequently, there must be assigned a statistical weight to the measurement in order to analyze the perturbed fraction of the reactor.

The statistical weight (6, pp. 372-375) of a region U that is a central part of a large reactor is shown to be

$$W(u) = \frac{\int_u \varphi^2(v)\,dV}{\int_v \varphi^2(v)\,dV}$$

where the volume integrals are over the region U and over the total volume of the reactor V.

For the reflected reactor, the statistical weighting is written

$$W(u) = \frac{\int_u \varphi^2(v)dV \Big/ \int_v \varphi^2(v)dV}{\int_L \varphi^2(v)dV \Big/ \int_v \varphi^2(v)dV} = \frac{\int_u \varphi^2(v)dV}{\int_L \varphi^2(v)dV}$$

where L is the loaded volume of the reactor. The statistical weighting for the HTP is derived in Appendix I.

The effect of a perturbation introduced into a region will be averaged over the reactor and will appear as a smaller perturbation per unit volume over the entire reactor, or

W(U) x (perturbation of U) = measured perturbation.

From the measured values can be derived the changes in k introduced into a region U. In this discussion the region, U, will be a unit cell where the unit cell has the radial volume associated with the lattice and a length defined by the experiment. The general relation is then

$$\frac{dk}{k} = \frac{1}{(W(u))(3.85\times10^4)}\,\delta\omega. \qquad (9)$$

CALCULATION OF THE RESONANCE ESCAPE PROBABILITY FOR THE HTP

The evaluation of the expression for the resonance escape probability is not made directly for this study. The analytic form of the integrals is not known sufficiently to permit a solution either analytically or by the use of numerical techniques. An appropriate method of evaluation is indirect and depends on the other parameters of the operating reactor.

The equation expressing the critical condition of a finite reactor is given as

$$\frac{k e^{-B^2\tau}}{1+L^2B^2} = 1.$$

Expanding this expression and neglecting terms in higher order of B than the second,

$$k = 1 + B^2(L^2+\tau).$$

Substituting values from Table 2 into the above expression gives a value of $k = 1.071$. From equation (1) it is seen that k is also equal to $\varepsilon p f \eta$, or that

$$p = \frac{1.071}{\varepsilon f \eta}.$$

Again, substituting from Table 2, it is found that the resonance escape probability for the HTP is 0.872.

TABLE 2

LATTICE CONSTANTS FOR HTP

Constants	Values
Square of Lattice Diffusion Length	$L^2 = (250\ cm^2)$
Geometrical Buckling	$B^2 = 114(10^{-6})cm^{-2}$
Moderation Area to Thermal Energies	$\tau = 370\ cm^2$
Fast Effect	$\varepsilon = 1.03$
Neutrons per Thermal Neutron Captured in Uranium	$\eta = 1.32$
Thermal Utilization	$f = 0.904$
Resonance Escape Probability	$P = 0.872$
$N_o/(N_g \xi_g \sigma_{sg})$	$\beta = 0.796$
$\alpha^3 = [(A-1)^2/(A+1)^2]^3$	$\alpha^3 = 0.367$
Normal Moderator to Uranium Volume Ratio	$V_g/V_o = 42.6$

EXPERIMENTAL PROCEDURE AND RESULTS

Variation of Uranium Geometry

The determinations of the geometry dependence of the resonance integral are all measurements related to a change in reactivity between two predetermined configurations of the reactor loading. The separation of the resonance neutrons from those having other energies is accomplished by: (a) covering the uranium with a cladding which removes the thermal neutrons, thereby reducing f and η to zero by definition; and (b) surrounding this assembly with sufficient moderator to insure that the fast effect can be ignored. Since the fast fission threshold for U-238 is at about one mev (13, p. 304) and the nearest source of fast fission neutrons is approximately eight mean free scattering paths away, the fast effect can be assumed to be unity if the resonance fissions in U-235 are ignored. The fast effect for the normal lattice is 1.03, the situation that exists when neutrons of all energies are incident on the uranium. Removing all of the thermal neutrons removes approximately 95 per cent of the total number of fast neutrons resulting from fissions occurring in the U-235. This tends to change the value of $\epsilon - 1$ by approximately the same amount, thereby reducing the value of the fast effect in this

region to less than 1.0015 and making any changes in ϵ outside the limit of measure of the HTP.

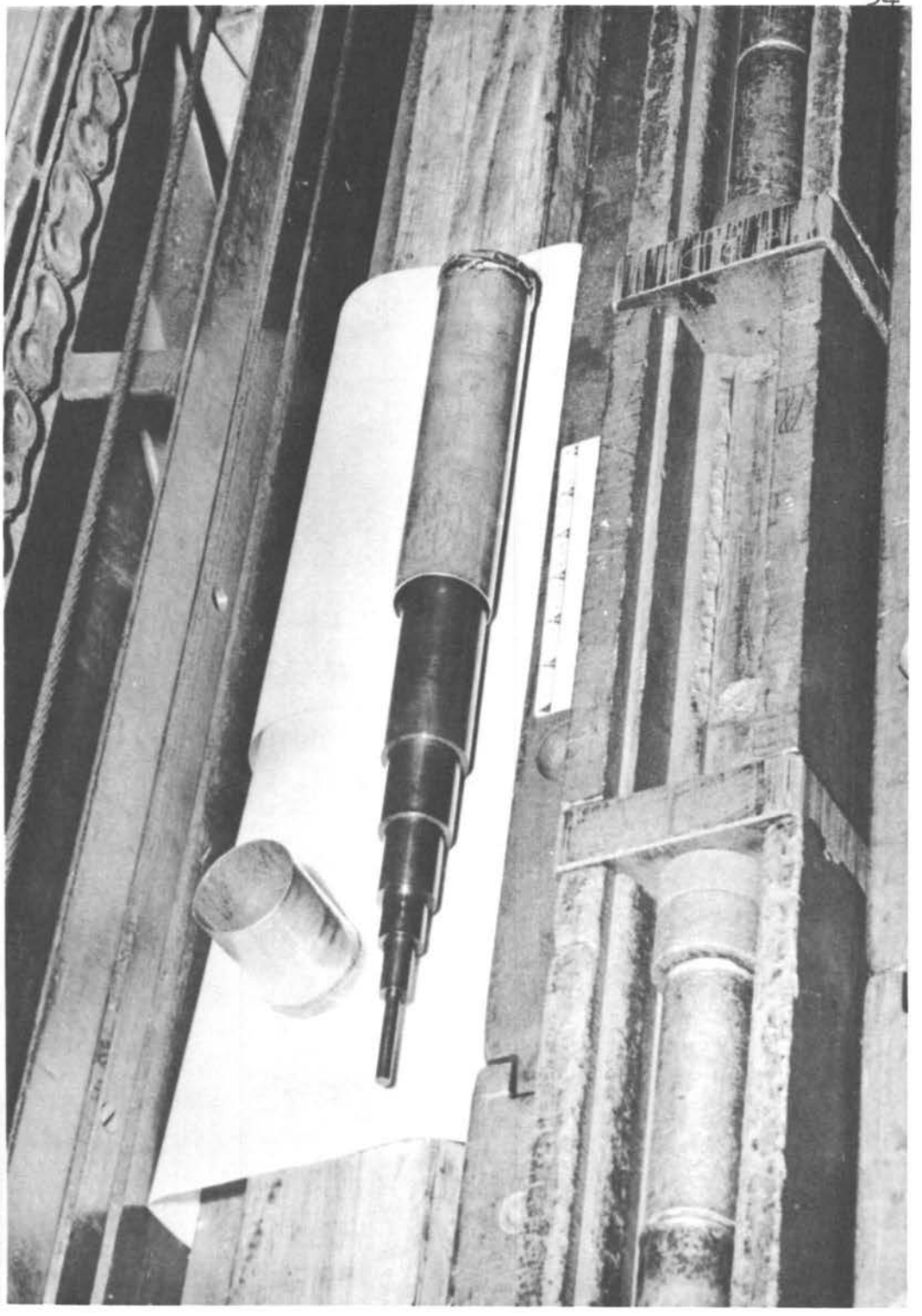
The thermal neutrons are removed by covering the cylinders with cadmium having a thickness of 0.51 mm (0.020 inches). This thickness of cadmium will remove all incident neutrons with energies above 0.47 electron volts (9, p. 132). The sharp resonance and high thermal cross section (13, p. 186) make cadmium a particularly useful filter, and in experimental reactor physics the lower limit of the resonance region is customarily taken at cadmium cut-off (9, p. 129).

The cadmium covered portion of test cell shown in Figure 5 is a system sensitive only to the incident neutrons with resonance energies. Any change in the measured reactivity resulting from altering the configuration inside the cadmium cover can be attributed to a change in the resonance capture, or

$$\frac{dk}{k} = \frac{dp}{p}.$$

With the exception of the resonance escape probability, the terms in the multiplication factor have been eliminated by the cadmium shield and the graphite moderator. This now allows any changes in the reactivity to be related to the resonance escape probability.

The empty cadmium cover is placed in the test position at the center of the reactor and the reactivity measured



URANIUM CYLINDERS AND HTP TEST CELL

FIGURE 5

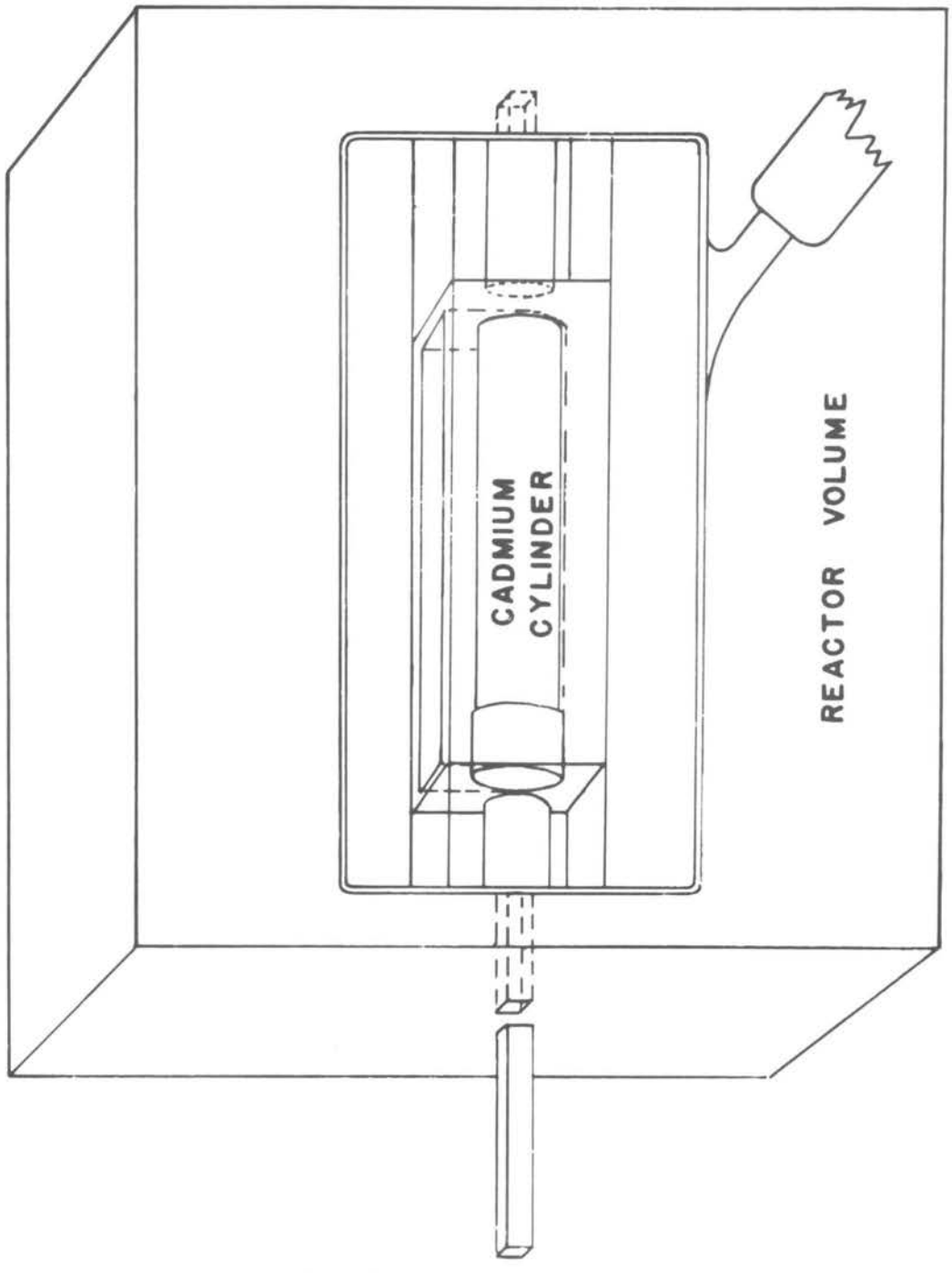
in the manner described in Appendix III. After a value is found for this configuration (referred to as the standard) the removable column shown in Figure 6 is withdrawn and the uranium cylinder to be measured is placed inside the cadmium cover. The difference between the measured reactivity of the reactor with the cadmium covered uranium cylinder at the standard position and the reactivity measured with the emply cadmium cover at the standard position is proportional to the capture of neutrons of resonance energies, or $\delta\omega \sim$ 1-p. Between the reactivity values measured for the different uranium cylinders (all with the same length), there then exists the following relationship:

$$\frac{\delta\omega_i}{\delta\omega_j} = \frac{1-P_i}{1-P_j} \simeq \frac{\left[(V_o \bar{\Phi}_o / \bar{\Phi}_g) \int (\sigma_{ao}) \frac{dE'}{E'}\right]_i}{\left[(V_o \bar{\Phi}_o / \bar{\Phi}_g) \int (\sigma_{ao}) \frac{dE'}{E'}\right]_j}.$$

All the assemblies have the same statistical weight; therefore, the A' is cancelled from the above ratio.

The relationship between the measured reactivities and the effective resonance integral is then

$$\frac{\delta\omega_i}{\delta\omega_j} = \frac{\left[V_o \bar{\Phi}_o \int (\sigma_{ao})_{eff} \frac{dE'}{E'}\right]_i}{\left[V_o \bar{\Phi}_o \int (\sigma_{ao})_{eff} \frac{dE'}{E'}\right]_j}$$



TEST POSITION OF CADMIUM CYLINDER

FIGURE 6

where $\bar{\varphi}_{oi} / \bar{\varphi}_{oj}$ is the ratio of the average fluxes (a quantity that essentially remains unity). This equation relates the effective resonance integrals to measurable quantities and, in terms of the j'th component, the effective resonance integral of the i'th assembly is

$$\int (\sigma_{ao})_{eff} \frac{dE'}{E'}\Big|_i = \left(\frac{V_o}{\delta\omega}\right)_j \left(\frac{\delta\omega}{V_o}\right)_i \int (\sigma_{ao})_{eff} \frac{dE'}{E'}\Big|_j .$$

For these relative measurements it is assumed that the j'th component has the standard dimensions of the uranium (r = 1.84 cm) discussed in this paper, and the last equation becomes

$$\int (\sigma_{ao})_{eff} \frac{dE'}{E'}\Big|_i = \text{const} \ \frac{\delta\omega_i}{V_{oi}} . \qquad (10)$$

Here, the effective resonance integral for the absorber is related to the experimentally determined quantities, reactivity, and uranium volume. The value of the effective resonance integral for the standard configuration is included in the constant in equation (10) and is determined from the accepted form of the integral (6, p. 260) to be 10.68 barns.

The values, as measured, contain a contribution from the fission of U-235 in the energy region above the cadmium cut-off. The correction necessary to account for this effect is empirical and is made as follows. Three cylinders

of uranium of equal volume and density, but possessing different amounts of U-235, were tested in the above manner with the results as shown in Figure 7. The correction term particular to the standard geometry (in this case a radius of 1.84 cm) was found by extrapolating the results to correspond to zero U-235. The U-235 content in natural uranium is of sufficient dilution (0.0071 of the uranium is U-235) that the correction is proportional to the mass of the uranium, or

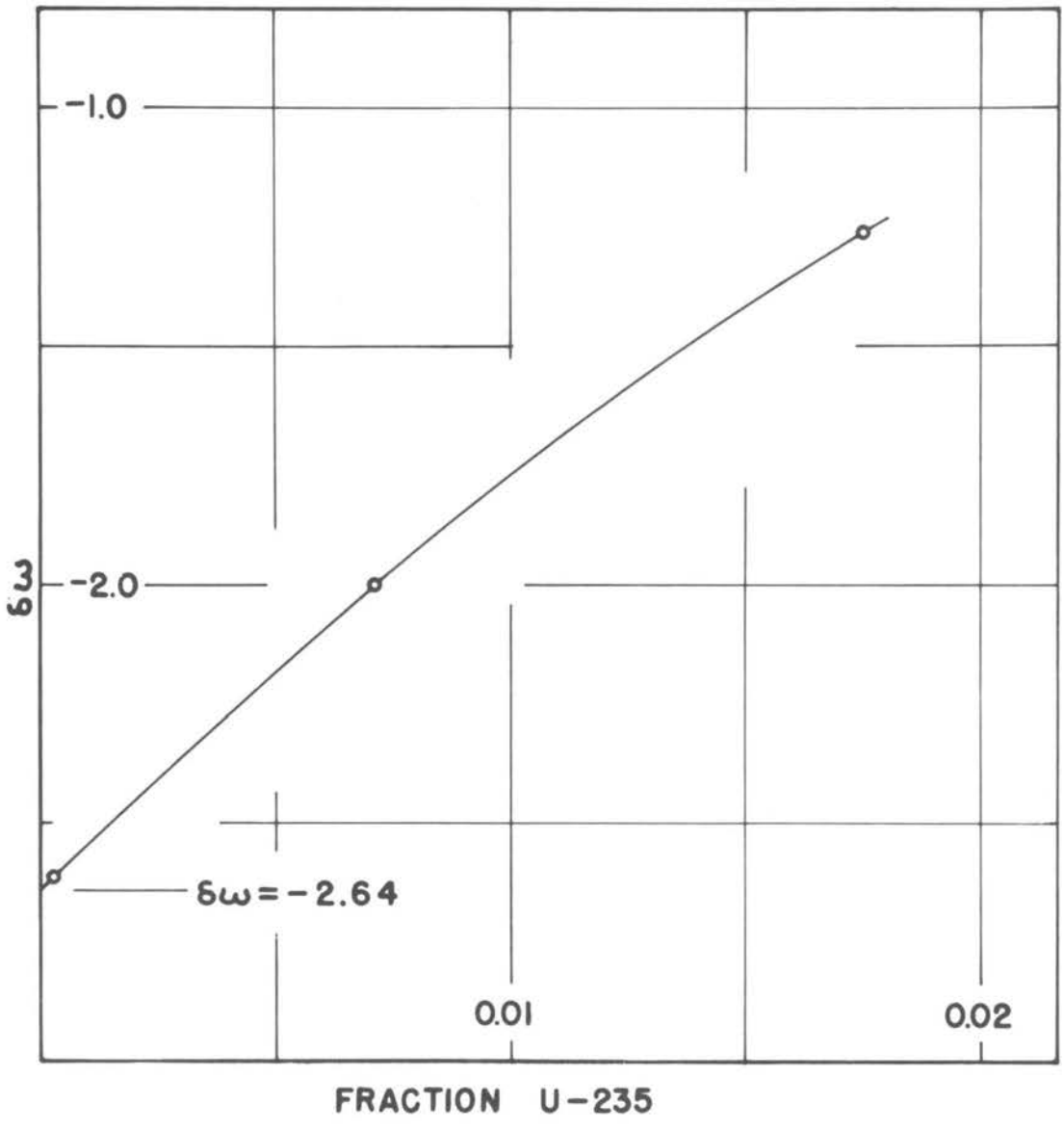
$$\delta\omega(235)_i = \delta\omega(235)_{std} \frac{M_i}{M_{std}} .$$

The corrected value to assign to the cylinders of U-238 is then

$$\delta\omega(238)_i = \delta\omega(\text{measured})_i - \delta\omega(235)_i .$$

This is the result that is tabulated in Table 3 as $\delta\omega_{28}$.

The measured and corrected results of the effect of varying the uranium geometry are given with other pertinent data in Table 3 and are displayed in Figures 8 and 9. In Figure 8, the corrected data are presented as functions of the uranium cylinder wall thickness. All results for a constant outside radius are seen to lie on a smooth curve with the different curves forming a consistent family of results. The standard deviation associated with each



REACTIVITY OF CADMIUM COVERED URANIUM
AS A FUNCTION OF U-235 CONTENT

FIGURE 7

TABLE 3

DATA ON URANIUM CYLINDERS

Outside Radius (cm)	Inside Radius (cm)					
	0	0.318	0.635	0.952	1.270	1.588
1.905	2.09	2.06	1.98	1.85	1.60	1.18
	2.78	2.73	2.59	2.37	1.98	1.39
	4363	4244	3883	3279	2424	1337
	0.058	0.060	0.066	0.078	0.105	0.191
	10.60	10.70	11.09	12.02	13.58	17.29
1.588	1.62	1.60	1.50	1.32	0.97	
	2.10	2.06	1.90	1.63	1.14	
	3026	2907	2546	1942	1087	
	0.070	0.073	0.083	0.109	0.194	
	11.54	11.78	12.41	13.96	17.44	
1.270	1.15	1.13	1.01	0.78		
	1.46	1.42	1.24	0.92		
	1939	1820	1459	855		
	0.086	0.092	0.115	0.196		
	12.52	12.98	14.13	17.89		
0.952	0.76	0.71	0.57			
	0.93	0.86	0.67			
	1084	965	604			
	0.115	0.129	0.206			
	14.27	14.81	18.45			
0.635	0.41	0.36				
	0.49	0.42				
	480	361				
	0.172	0.229				
	16.98	19.35				
0.318	0.14					
	0.16					
	119					
	0.344					
	22.36					

Legend: Standard Uranium

(-1) $\delta\omega_U$ (ih)	2.00
(-1) $\delta\omega_{28}$ (ih)	2.64
Mass (gm)	4040
S/M (cm^2/gm)	0.058
$\int(\sigma_{ad})_{eff} dE/E$ (barns)	10.68

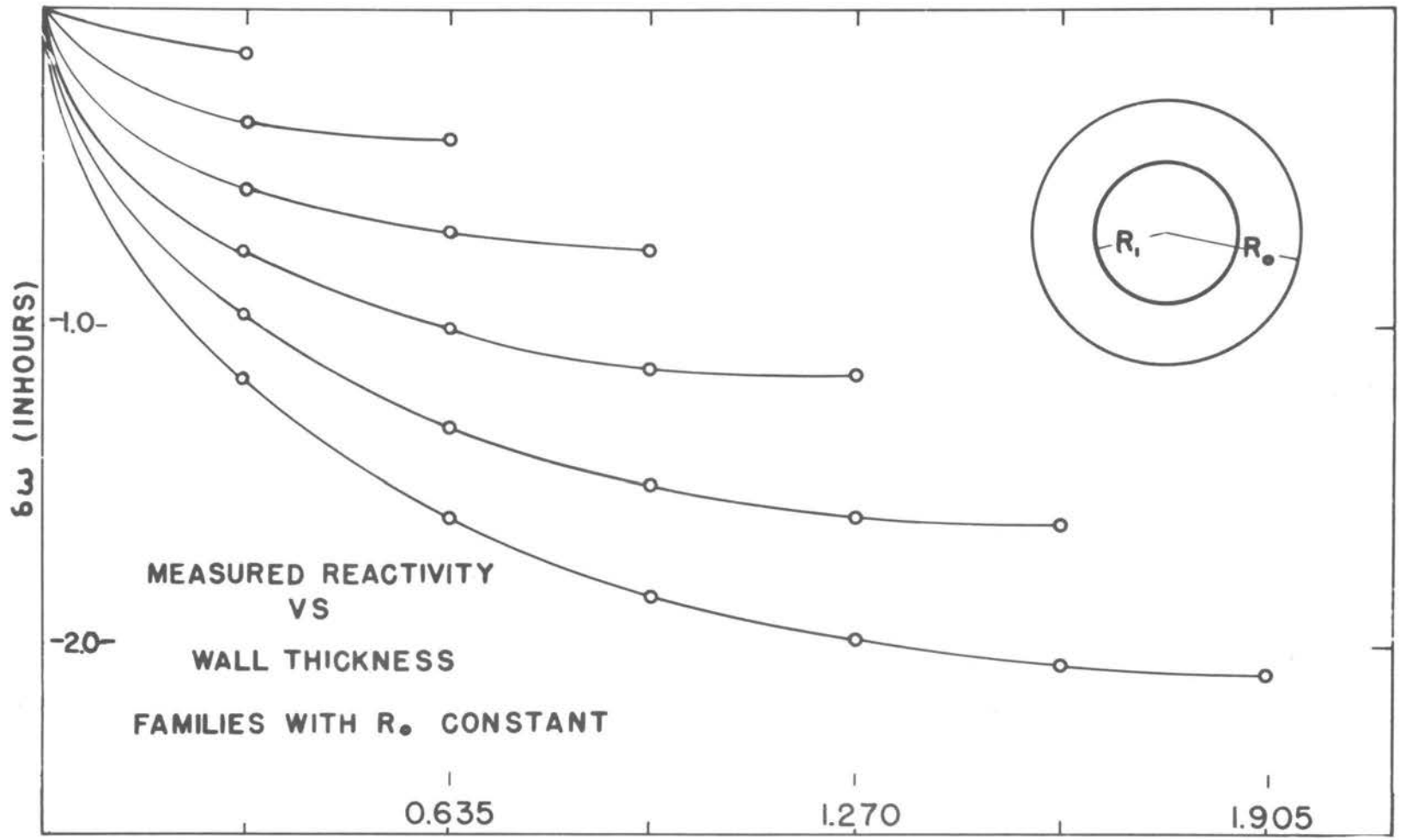

δω (INHOURS)
-1.0-
-2.0-
MEASURED REACTIVITY
VS
WALL THICKNESS
FAMILIES WITH R. CONSTANT
R,
R.
0.635
1.270
1.905
URANIUM WALL THICKNESS (R.-R,) (CM)

FIGURE 8

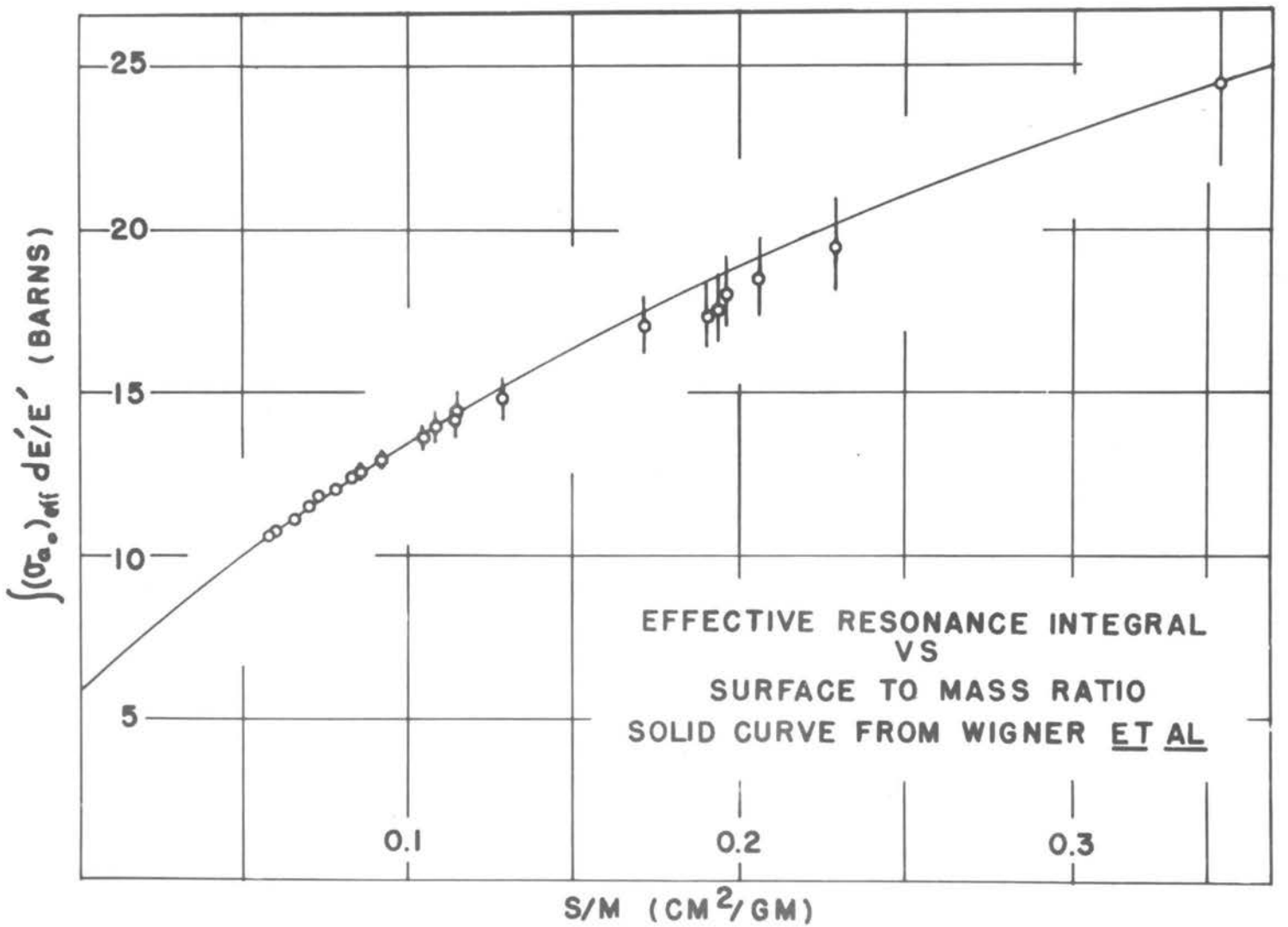

25
20
15
10
5
0.1
0.2
0.3
$\int(\sigma_{a_0})_{eff}\, dE'/E'$ (BARNS)
S/M (CM^2/GM)
EFFECTIVE RESONANCE INTEGRAL
VS
SURFACE TO MASS RATIO
SOLID CURVE FROM WIGNER ET AL

FIGURE 9

of the points is discussed in Appendix IV and is within the circle chosen to represent the data points.

The data presented in Figure 9 are the result of evaluating equation (10) for each of the values depicted in Figure 8. The error associated with each of the positions results from the precision of the HTP and the magnitude of the measured perturbation. The largest values attributed to the effective resonance integral are those corresponding to the largest surface to mass ratio but the smallest measured reactivity changes. It is seen that it is not feasible to attempt to assign a linear fit to these data; however, a numerical fit to the results is approximated by the curve

$$\int (\sigma_{a0})_{eff} \frac{dE}{E} = 6.0 \left[1 + 15.6 \frac{S}{M} \left(1 - 2.48 \frac{S}{M} + 3.68 \left(\frac{S}{M}\right)^2\right)\right].$$

This form of the curve was chosen because it constitutes only a correction term to the currently accepted formula for the effective resonance integral.

The solid line drawn in Figure 9 is the theoretical curve from Wigner, et al (15, p. 270). The agreement between the experimental data and the theoretical curve shows that for strong resonance absorbers the capture of neutrons may be thought of as being two separate processes: (1) the surface effect, where the neutrons with energies corresponding to the resonances are removed; and (2) the volume effect, where the rest of the absorptions occur.

Variation of Uranium Temperature

For incident neutrons with a constant energy spectrum, the change in the resonance capture of neutrons by cylinders of U-238 can be found from equation (6). From this it is seen that

$$\frac{1}{P}\frac{\partial P}{\partial T} = -\beta \frac{V_0 \bar{\varphi}_0}{V_9 \bar{\varphi}_9} \int (\sigma_{a0})_{eff} \frac{dE'}{E'}.$$

There are two other effects associated with changing the temperature of the uranium cylinders. The first is a change in the volume of the uranium due to thermal expansion. This results in a different surface to mass ratio and the concomitant change in the neutron capture. The second effect is change in the thermal utilization of the neutrons resulting from a change in the uranium volume and from reducing the amount of parasitic neutron absorber in the region of the heated assembly. These systematic effects are discussed quantitatively in Appendix III. The change in the multiplication factor is then written

$$\frac{1}{k}\frac{dk}{dT} = \frac{1}{P}\frac{\partial P}{\partial T} + \zeta(T) + \rho(T).$$

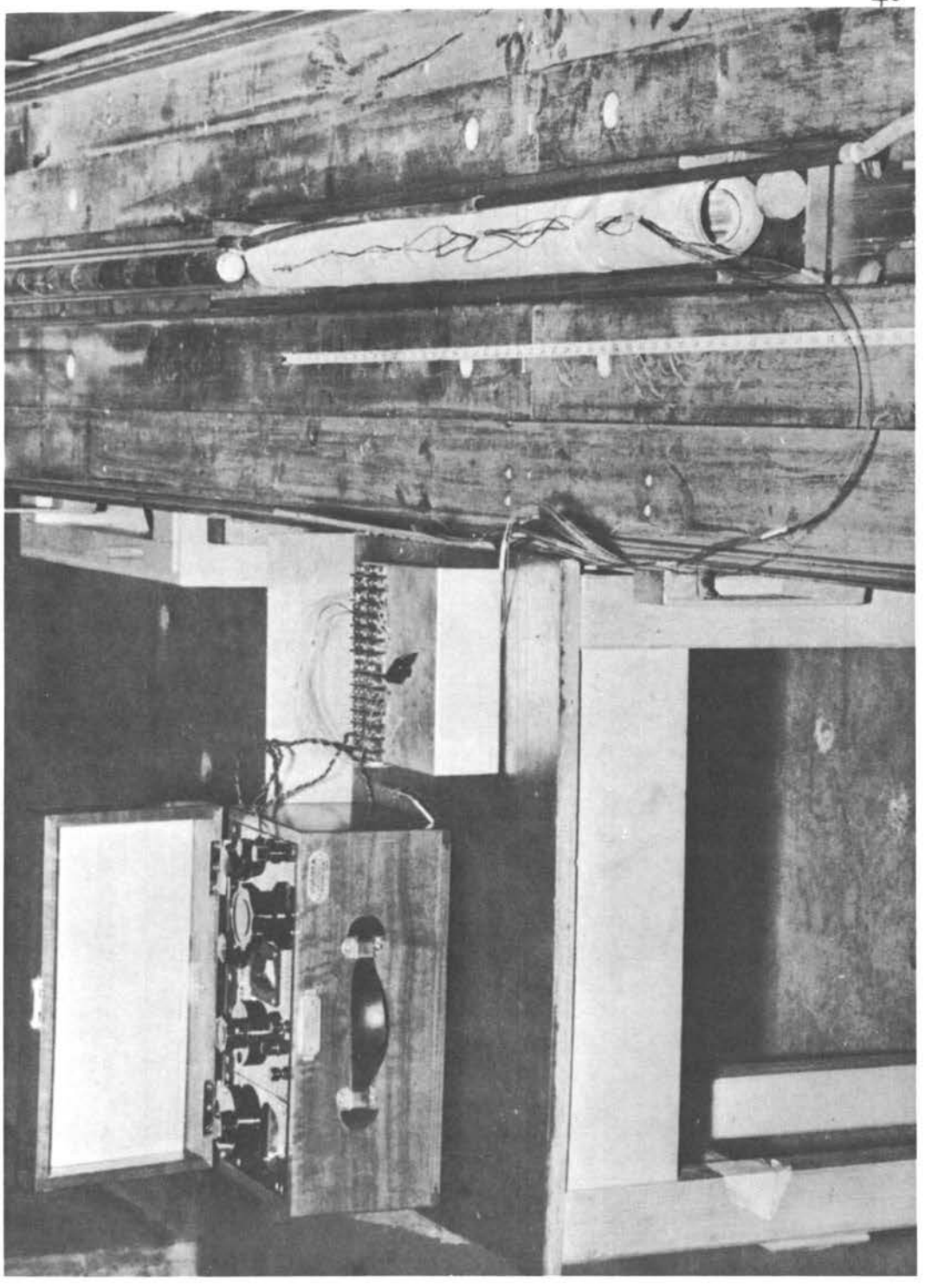
Here, $\zeta(T)$ is the calculated temperature coefficient associated with the thermal utilization and $\rho(T)$ is the change in the resonance capture resulting from the change

in the volume of the uranium cylinder. From equation (8) the relation between temperature dependence of the multiplication factor and the temperature dependence of the measured reactivity is

$$\frac{\delta \omega}{\delta T} = A'\left(\frac{1}{k}\frac{dk}{dT}\right).$$

Since the measurements to determine the temperature effects are not related to a standard configuration as was done in the previous measurement, and since only a small fraction of the HTP was perturbed in this experiment, it is necessary to apply the statistical weighting to the evaluation of the above equation. The statistical weighting of the region is discussed in Appendix I with numerical results applicable to this experiment.

The experiment is conducted in a manner similar to that suggested by Borst (5). An aluminum-clad column of uranium 92 cm in length and with a 1.70 cm radius is heated in an electric furnace to a temperature slightly below the melting point of the cladding ($\sim 660^{\circ}C$). After an equilibrium is reached with the furnace temperature ($\sim 600^{\circ}C$), the uranium is placed in the central tube of the insulated cartridge shown in Figure 10. This aluminum assembly is double-walled aluminum covered with an outer layer of asbestos paper insulation. The construction is



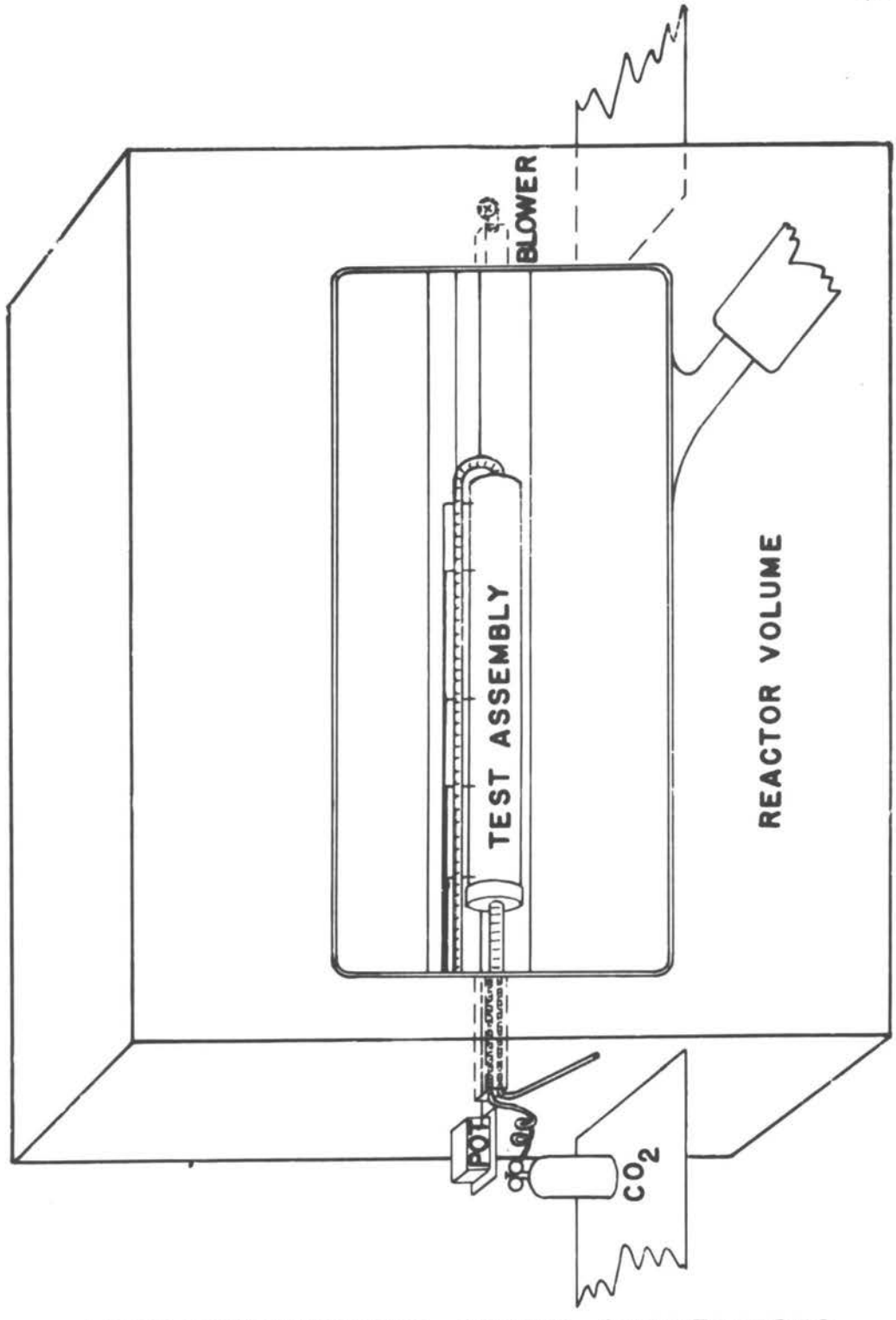
TEST CELL FOR TEMPERATURE-REACTIVITY MEASUREMENTS

FIGURE 10

such that the center tube is suspended in the larger tube by point contacts. Thermocouple wells are provided in order that temperature monitors may be placed in contact with the surface of the heated cylinders.

After the uranium is placed in the central assembly and the thermocouples inserted, the cap is placed over the outer tube and the complete assembly then placed at the center of the reactor as shown in Figure 11. The equipment is so designed that a cooling gas may be used to replace the static air in the assembly. Restricting the gas to the assembly prevents the transfer of heat to the moderator and allows the air to be replaced with different coolants. The flow of the coolant may also be controlled by regulating the flow of the gas through the tubing.

After the temperature of the uranium and the power level of the reactor have reached a state of dynamic equilibrium, simultaneous measurements are made of the temperature in the assembly and the reactivity of the HTP. These measurements are performed at the rate of 12 measurements per hour for the period of time necessary for the uranium to cool to approximately 30°C. The range of temperature then covered by the experiment is seen from Table 4 to be from 250° to 300° C. The range of temperatures is limited by the rapid cooling at the time when the power level of the reactor is in a transient condition



TEST CONFIGURATION DURING SIMULTANEOUS TEMPERATURE-REACTIVITY MEASUREMENTS
FIGURE II

TABLE 4

REACTIVITY-TEMPERATURE DATA
COOLING GAS - AIR

Run 1		Run 2		Run 3		Run 4		Run 5	
$\omega - \omega_o$	$T_o - T$	$\omega - \omega_o$	$T_o - T$	$\omega - \omega_o$	$T_o - T$	$\omega - \omega_o$	$T_o - T$	$\omega - \omega_o$	$T_o - T$
0	0	0	0	0	0	0	0	0	0
0.05	42	0.07	43	0.08	45	0.07	49	0.06	35
0.11	79	0.14	80	0.14	83	0.13	92	0.10	70
0.17	113	0.18	114	0.18	113	0.19	127	0.16	93
0.22	144	0.25	142	0.23	139	0.24	157	0.18	118
0.25	169	0.28	166	0.25	159	0.28	181	0.21	139
0.28	190	0.32	187	0.29	179	0.30	203	0.25	157
0.33	210	0.35	206	0.31	195	0.34	222	0.27	173
0.36	227	0.36	223	0.35	210	0.37	237	0.29	188
0.38	241	0.40	238	0.37	223	0.39	248	0.32	202
0.39	254	0.41	250	0.37	230	0.41	260	0.34	215
0.38	266	0.42	261	0.39	240	0.42	268	0.36	225
0.42	276	0.43	270	0.40	248	0.43	275	0.38	235
0.45	284	0.44	278			0.45	280	0.39	241
0.46	292	0.44	285					0.40	253
0.49	298	0.47	290						
0.48	303								
0.49	307								

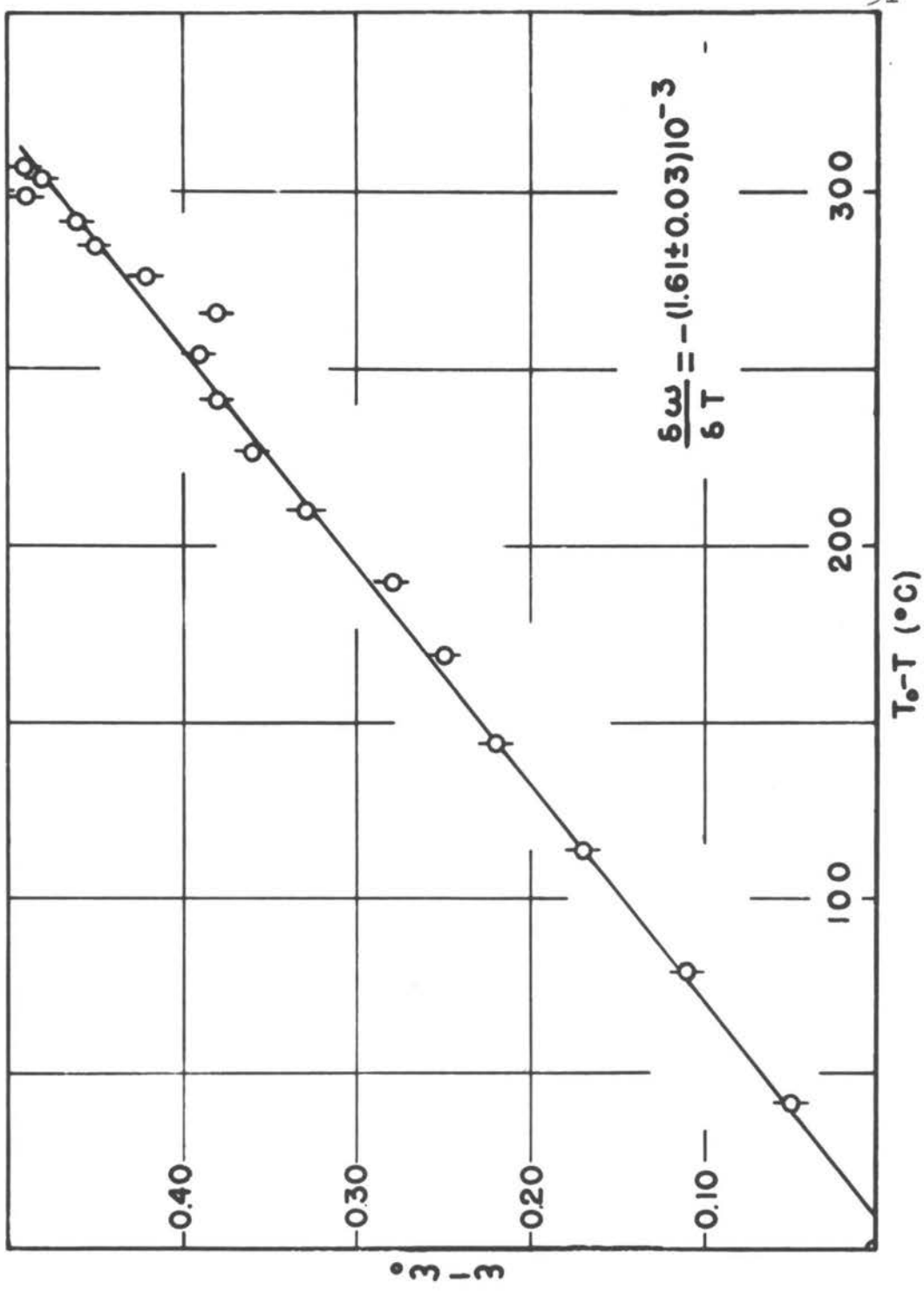
and reactivity measurements may not be conducted. The upper limit of the temperature is set by the melting point of the uranium cladding. Bare uranium may not be heated while exposed to the atmosphere because the rate of oxidation is so high that the area would be contaminated with radioactive material. Lowering the temperature to that of liquid nitrogen is not found to be feasible due to the condensation of moisture from the surrounding moderator. The local concentration of the water changes the moderation in the vicinity of the assembly and introduces perturbations into the measured values of the excess reactivity of the HTP.

The results of the experiments using air as a coolant are tabulated in Tables 4 and 5 and displayed in Figure 12. The precision of the measurements is represented by the size of the circles on the graph. The error in the reactivity measurements is discussed in Appendix IV but the error in the temperature, determined from the experiment, is less than 5° C.

TABLE 5

LEAST SQUARES ANALYSIS OF TEMPERATURE-REACTIVITY DATA

Data Run	$\frac{\delta \omega}{\delta T}$ $(10)^3$ $\frac{ih}{^oC}$
1	- 1.61 ± 0.03
2	- 1.58 ± 0.12
3	- 1.61 ± 0.06
4	- 1.60 ± 0.05
5	- 1.60 ± 0.05



TEMPERATURE COEFFICIENT
RUN I AIR COOLANT

FIGURE 12

The temperature is measured with 30-gauge iron-constantan thermocouples electrically welded at the junctions. The reference junction is exposed to the ambient temperature and the emf measured with a Rubicon Potentiometer.[1] The ambient temperature may vary a fraction of a degree during an experimental run, but this is small compared to the uncertainty in assuming the average temperature.

The average temperature is determined from measurements of the surface and center temperatures at six different positions in the assembly, these positions being sufficient to determine both the longitudinal and radial gradients of the temperature distribution. With the exception of the line contacts on which the uranium is suspended, the heat is transferred by conduction and radiation through the air space to the inner liner. The outer surface of this liner is cooled by a stream of air or of carbon dioxide gas that enters the system at approximately ambient temperature. The flow of gas is adjusted to limit the cooling time of the uranium between the limits of approximately 500° C and 30° C to periods of approximately two hours. For cooling periods of this magnitude, the conduction through the uranium is much more rapid than

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[1] Rubicon Potentiometer, cat. number 2732, manufactured by Rubicon Co., Ridge Avenue at 35th Street, Philadelphia 32, Pa.

the transfer of heat from the uranium cladding to the surface; consequently, the temperatures throughout the uranium are reasonably uniform. It was found that the maximum variation between any two positions in the metal was approximately 20 degrees. The temperature is averaged over the volume with the approximate averaging procedure over the radius, R, assumed to be that resulting from using a linear gradient, i.e.,

$$\bar{T} = \frac{\int_0^R T(r)\,r\,dr}{\int_0^R r\,dr} = T(R) + \tfrac{1}{3}\left(T(0) - T(R)\right).$$

These average temperatures are the values assigned to the data. The temperature changes per unit time, however, are practically constant over the entire column of uranium, and the differences measured in the temperature changes at any two data points vary by less than a degree. Since it is the temperature change that is of interest and not the absolute value, it is seen that the uncertainty assigned the temperature data is pessimistic.

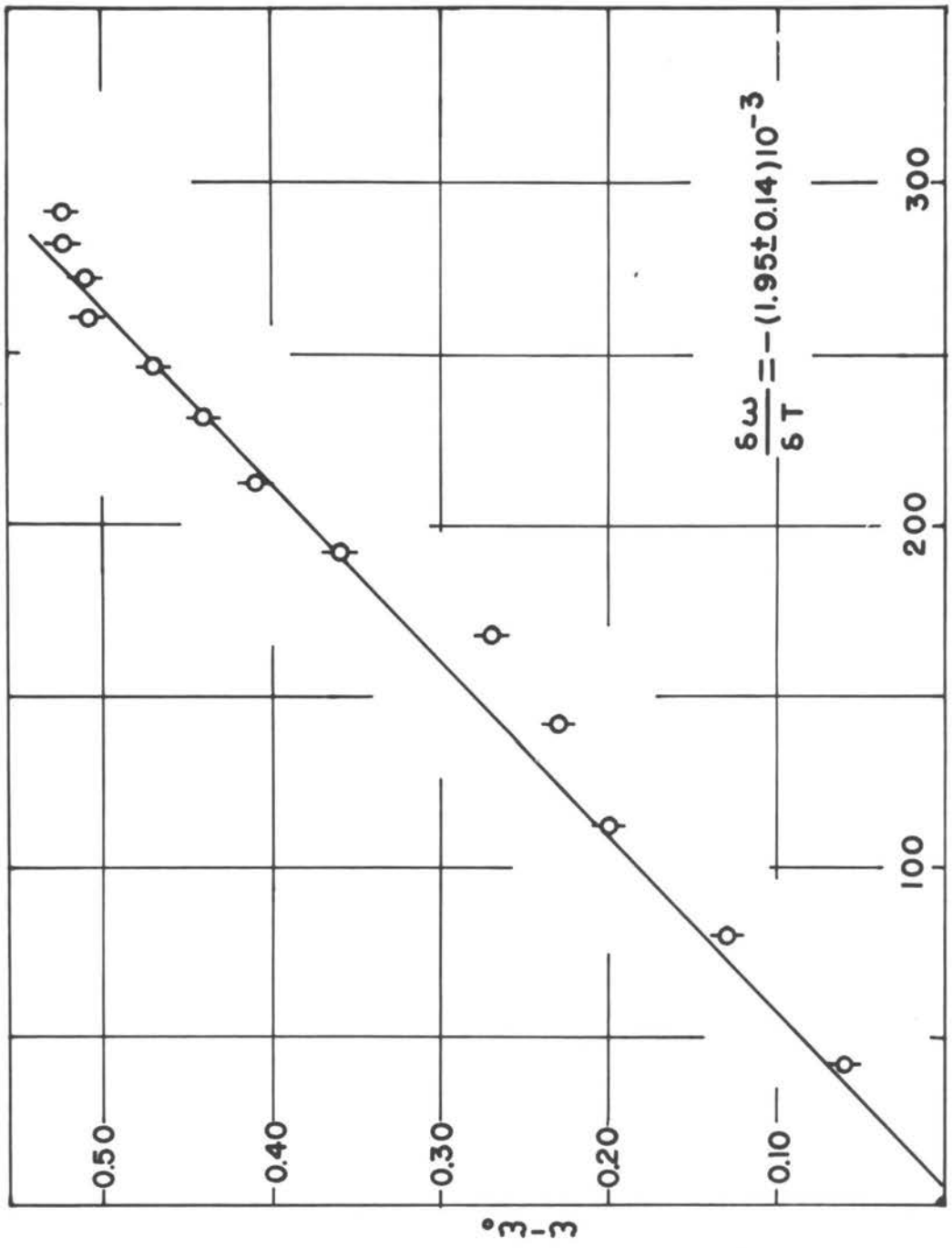
In order to determine the amount of heat transferred to the atmosphere of the reactor during the experiment, thermocouples are arranged to monitor the temperature near the surface of the assembly. The complete assembly is cooled by a blower that is arranged to project a stream of air through the reactor as is shown in Figure 11. The resultant change in the moderator temperature was less

than 20° C at the peak value and when averaged over the cell was less than 2° C.

The results from these measurements then yield a final value of the reactivity change of the reactor as a function of the temperature change of the fuel as is shown in Figure 12, where the coolant is air. The test of the accuracy of the calculation presented in Appendix III is accomplished by replacing the air in the assembly with carbon dioxide. The carbon dioxide has a cross section for the capture of thermal neutrons approximately three orders of magnitude less than that of air. Consequently, any change in this absorption as a result of a change in the density of the gas is beyond the limit of sensitivity of the HTP. A sample of the results using carbon dioxide coolant is presented in Table 6 and in Figure 13.

The solid lines drawn through the data in Figures 12 and 13 are the results of a least squares analysis of the data with the quoted error being the standard deviation. The methods of approach to the evaluation of the data are discussed in Appendix IV. The interpretation of these results necessitates an analysis of the lattice values of this perturbed cell and of the effective fraction of the reactor represented by the volume of the test cell.

The only perturbations in lattice parameters affecting this measurement are the change in the resonance escape probability accompanying the removal of the



TEMPERATURE COEFFICIENT
CARBON DIOXIDE COOLING

FIGURE 13

TABLE 6

REACTIVITY-TEMPERATURE DATA
COOLING GAS - CARBON DIOXIDE

$\omega - \omega_o$	$T_o - T$
0	0
0.06	42
0.13	80
0.20	112
0.23	142
0.27	168
0.36	192
0.41	212
0.44	231
0.47	247
0.51	260
0.51	272
0.53	282
0.53	291

moderator around the test assembly and the change in thermal utilization resulting from the removal of the graphite and the addition of the aluminum assemblies. The change in the resonance escape probability is measured by the method similar to that discussed earlier. The effective resonance integral is measured in the normal lattice configuration and in the perturbed configuration to establish the value of the constant appearing in equation (10).

$$\frac{\delta \omega_1}{\delta \omega_2} = \frac{(\text{const})_1 \int (\sigma_{ao})_{eff} \frac{dE'}{E'}}{(\text{const})_2 \int (\sigma_{ao})_{eff} \frac{dE'}{E'}}.$$

In both the normal and perturbed lattice, the effective resonance integral has the same value, so the ratio of the reactivities is the ratio of the multiplicative constants. The ratio of the reactivities for the two configurations of graphite, but with a constant amount of uranium, is

$$\frac{\delta\omega_2}{\delta\omega_1} = \frac{-1.98 \pm 0.03}{-1.85 \pm 0.04} = 1.07 \pm 0.04.$$

The value of $N_o V_o \bar{\Phi}_o / N_g V_g \sigma_{sg} \xi_g \bar{\varphi}_g$ in the perturbed lattice is, then, 1.07 ± 0.04 the value assigned to the same terms for normal lattice, or

$$(\text{constant})_2 = (1.07 \pm 0.04)\left(\frac{0.137}{10.68}\right)$$

$$= 0.0137 \pm 0.0005.$$

The numerical results of the temperature measurements are then as follows: the value found for the reactivity coefficient from the data is

$$\frac{\delta\omega}{\delta T} = -(1.606 \pm 0.021)10^{-3}\ ih/°C.$$

Applying the weighting theorem from Appendix I leads at once to the result for the change in the multiplication factor of the entire reactor

$$\frac{1}{k}\frac{dk}{dT} = -(1.815 \pm 0.024)10^{-5}\ /°C.$$

Correcting this value for the systematic errors $P(T)$ and $\zeta(T)$,

$$\frac{1}{P}\frac{\partial P}{\partial T} = -2.23\,(10)^{-5}\ /°C.$$

The change in the volume term of the effective resonance integral is

$$\frac{\partial}{\partial T}\int(\sigma_{ao})_{eff}\frac{dE'}{E'} = 1.65\,(10)^{-3}\ \text{barns}\ /°C.$$

No precision of measure has been assigned the last two expressions. This is a result of calculating the corrections for the systematic errors with no obvious limit of precision. The correction for the expansion of the air is found by experiment as is shown in Table 6 and Figure 13. Here, the difference between air coolant and CO_2 coolant as measured is seen to be

$$\left(\frac{\delta\omega}{\delta T}\right)_{air} - \left(\frac{\delta\omega}{\delta T}\right)_{CO_2} = 0.34\,(10)^{-3}\ ih/°C.$$

The measured results have an associated uncertainty of

$$\pm\ 0.14\,(10)^{-3}\ ih/°C.$$

The effect of the air is then $(0.34 \pm 0.14)10^{-3}$ ih/°C. This may be compared to the calculated value of $0.36(10)^{-3}$ ih/°C. From this an uncertainty of ± 50 per cent is assigned to the correction, a large uncertainty in view of the customary accuracy of the theory. This leads to the following results:

$$\frac{1}{P}\frac{\partial P}{\partial T} = -(2.23 \pm 0.19)\,10^{-5}\ /°C$$

and

$$\frac{\partial}{\partial T}\int(\sigma_{ao})_{eff}\frac{dE'}{E'} = (1.65 \pm 0.14)10^{-3} \text{ barns }/^{\circ}C.$$

The logarithmic form expressing the relationship between the volume term of the effective resonance integral and the uranium temperature can be written as

$$\frac{1}{A}\frac{\partial A}{\partial T} = \frac{-1}{\ln \frac{1}{p}}\left(\frac{1}{P}\frac{\partial P}{\partial T}\right).$$

The temperature coefficient of the volume absorption of effective resonance integral is then seen to be

$$\frac{1}{A}\frac{\partial A}{\partial T} = (1.47 \pm 0.13)10^{-4}/^{\circ}C.$$

The results from other methods to determine the temperature coefficient of the volume absorption for uranium metal are presented in the literature (14, p. 601). The reported values range from $1.1(10)^{-4}/^{\circ}C$ to $3(10)^{-4}/^{\circ}C$, a range that includes the value which was determined by this experiment. From this it is seen that there is agreement between reactivity measurements and other methods used to determine the temperature coefficient of uranium cylinders.

CONCLUSIONS

The results of this study are of primary importance in the field of reactor physics and engineering where it is necessary to evaluate the resonance capture both for production and for safety considerations.

The agreement between the theory and the experimental results implies that, for a strong resonance absorber like uranium, approximate methods are sufficient to calculate resonance capture.

The most significant aspect of this study is: differential experiments using sample amounts of uranium can be performed in a low power reactor and lead to results which are independent of the lattice structure for a heterogeneous reactor that is fueled with natural uranium.

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APPENDICES

APPENDIX I

STATISTICAL WEIGHTING OF THE HTP

The loaded volume of the HTP is an ovate cylinder containing 292 channels of uranium metal. Since the solutions of the Helmholtz equation are not tabulated for this geometry, the loading is approximated by an equivalent circular cylinder where the equivalence is approximated by measuring the neutron density as a function of position and fitting these values to the following development of the statistical weighting.

Assuming elliptic geometry as an approximation,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

but $x = R \cos \theta$ and $y = R \sin \theta$

so $$R^2 = \frac{a^2 b^2}{b^2 \cos^2 \theta + a^2 \sin^2 \theta} .$$

The equivalent radius is found from

$$\overline{R^{-2}} = \frac{\int_0^{\frac{\pi}{2}} R^{-2}\, d\theta}{\int_0^{\frac{\pi}{2}} d\theta} = \frac{2}{\pi a^2 b^2}\left[b^2 \int_0^{\frac{\pi}{2}} \cos^2\theta d\theta + a^2 \int_0^{\frac{\pi}{2}} \sin^2\theta d\theta\right]$$

$$= \frac{1}{2a^2} + \frac{1}{2b^2} .$$

As a result of more moderation in the x direction, it is found that the augmented values of a and b do not have the same ratio as the loaded dimensions. The measured values of the augmented dimensions are

$$a = 244.5 \text{ cm}$$

$$b = 282.5 \text{ cm}$$

$$\frac{1}{R^2} = \frac{1}{(262)^2}$$

or the radius of the equivalent cylinder is 262 cm. The equivalent radius of the loaded volume is

$$\frac{1}{R^2_L} = \frac{1}{2(183.5)^2} + \frac{1}{2(237.5)^2} = \frac{1}{205^2}$$

The loaded length of the lattice is 470 cm with an augmented length of 579 cm.

The neutron flux distribution throughout the volume of the reactor is then

$$\varphi(V) = \varphi(0) J_o\left(\frac{\alpha r}{262}\right) \cos\frac{\pi z}{579}$$

where α is the first root of the Bessel's function of zero order and $\varphi(0)$ is the neutron flux at the origin of the coordinate system, the center of the reactor.

The statistical weight of a region (7, pp. 378-381) is shown to be proportional to the square of the neutron flux, so it is seen that an average flux can be found as follows:

$$\frac{\bar{\varphi}^2 \int_L dV}{\int_V \varphi^2(V)\,dV} = \frac{\int_L \varphi^2(V)\,dV}{\int_V \varphi^2(V)\,dV}$$

where L is the loaded volume of the reactor and V is the augmented volume. From this, it is seen that

$$\bar{\varphi}^2 = \varphi^2(0) \quad \frac{\int_L J_o^2 \frac{\alpha r}{262} \cos \frac{\pi z}{579}\, dV}{\int_L dV}$$

$$= \varphi^2(0) \left(\frac{2}{470(205)^2}\right) \int_0^{235} \cos^2 \frac{\pi z}{579}\, dz \int_0^{205} J_0^2 \frac{\alpha r}{262} r\, dr$$

$$= 0.257\, \varphi^2(0)$$

The effective average flux squared is then 0.257, the flux at the test position of the assemblies.

The loading of the reactor is 292 columns of uranium each 470 cm in length. In terms of fuel elements, each 8 inches or 20.32 cm in length, there are $292 \times 470/20.32 = 6754$ fuel elements in the reactor. An equivalent reactor with a uniform reactivity is found from

$$\bar{\varphi}^2\, V = \varphi^2(0)\, V'$$

where V' is the volume of the reactor with a constant flux $\varphi(0)$

$$V' = 0.257V$$

so the equivalent reactor would have

$$0.257\ (6754) = 1736 \text{ fuel elements.}$$

The fraction of the reactor entering into these experiments is, weighted by reactivity,

$$\frac{4}{1736} = 0.0023.$$

This implies that if all of the uranium in the loaded reactor experienced the same temperature change as the test sample, there would be a reactivity change 434 times as large as that measured in the experiments. With 3.85×10^4 ih/ $\frac{dk}{k}$, the value of $\frac{1}{k}\frac{dk}{dT}$ becomes

$$\left.\frac{1}{k}\frac{dk}{dT}\right|_{\text{Test}} = 1.13 \times 10^{-2} \left.\frac{\delta\omega}{\delta T}\right|_{\text{Measured.}}$$

APPENDIX II

REACTIVITY MONITORING

The excess reactivity of the Hanford Test Pile is determined from measured changes in the current from the ion chambers shown in Figure 14.

If the current through the level galvanometer G_L is i, then the current through the differential galvanometer G_D is $i' = ai + b$, where a and b are constants.

The current from the ion chambers is expressed by

$$i = i_o \exp(t/T)$$

where t is time and T is the period of the reactor measured in seconds.

The ratio of galvanometer sensitivities is measured and found to be K, and, further, the deflection of the level galvanometer is proportional to the current from the ion chambers

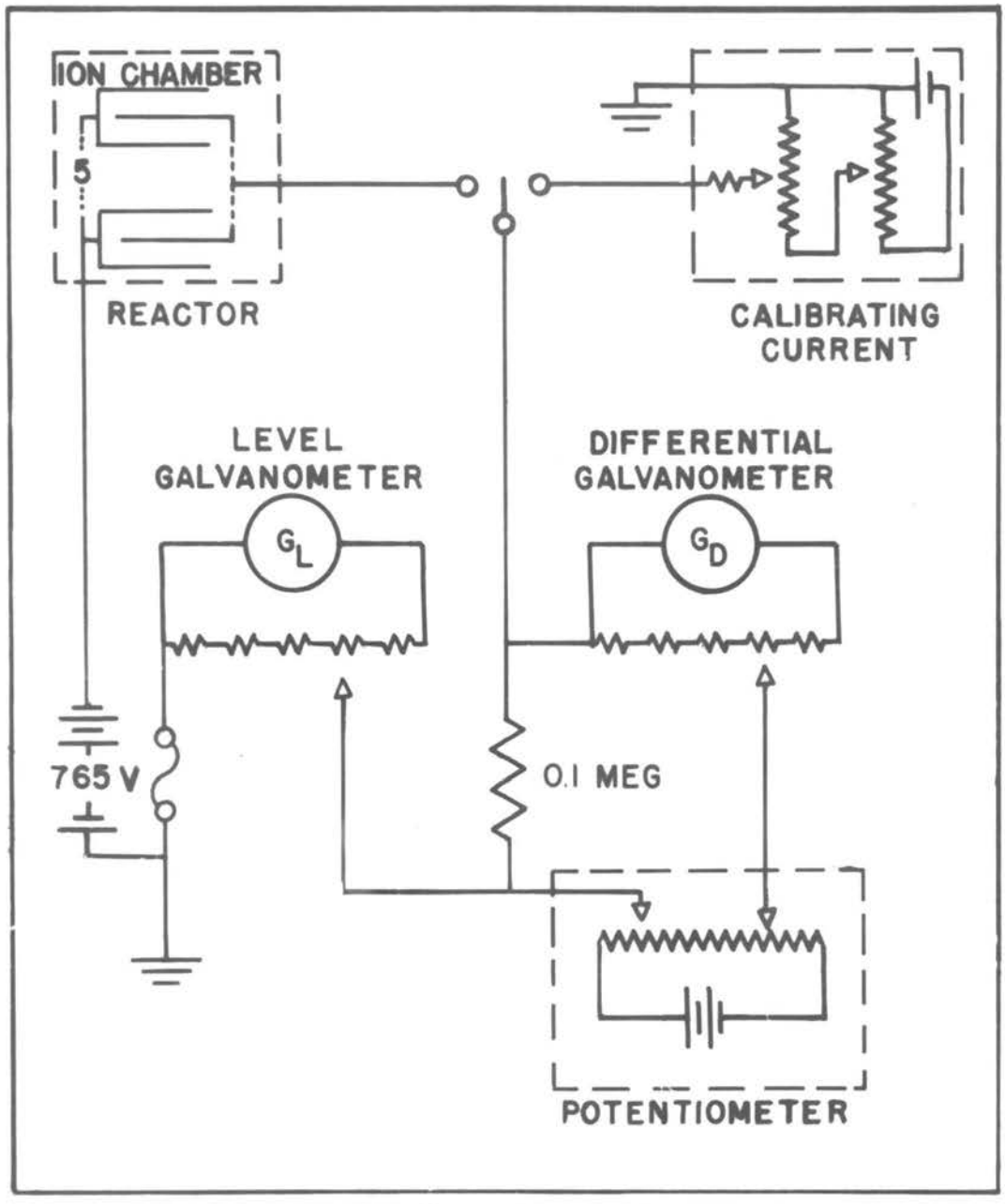
$$d_L = \text{const } i_o \exp(t/T)$$

$$d_L = d_{Lo} \frac{\Delta t}{T}$$

$$\frac{\Delta d_L}{\Delta d_D} = K$$

$$\Delta d_D = \frac{d_{Lo}}{K} \frac{\Delta t}{T} . \qquad (a)$$

For periods of the reactor that are long ($\geq$ 3600 sec),



SCHEMATIC DIAGRAM
POWER MONITORING SYSTEM

FIGURE 14

the reactivity and the period have the following relation:

$$\omega(\text{ih}) = 3600/T$$

or, substituting from (a),

$$\omega(\text{ih}) = \frac{3600K}{d_{Lo}} \frac{\Delta d_D}{\Delta t} .$$

For a constant power level about which the reactor power drifts and for a predetermined length of time t, the reactivity of the unit has a constant relation to the differential galvanometer.

The mechanics of making a reactivity measurement are as follows. With the reactor in the standard configuration, the control rods are removed from the effective region of the reactor. This results in a multiplication factor greater than unity and a concomitant rise in the neutron density. The change in the neutron density is manifested by a change in the current through the level galvanometer, G_L, shown in Figure 14. After the neutron density, or reactor power level, has reached a predetermined value, the calibrated control rod is adjusted until the deflection of the level galvanometer is a constant. With the value of the reactivity associated with the control rod position plus that found from any slight period of the reactor, the reactivity of the "standard" is determined.

Replacing the standard configuration with the test cell and repeating the above leads to a value of the reactivity assigned to the test configuration. The

difference between the standard and the test configuration is termed $\delta\omega$ and is thus used in this work.

APPENDIX III

RELATION BETWEEN URANIUM TEMPERATURE AND SYSTEMATIC ERRORS

For the perturbed region of the Hanford Test Pile, the fraction of the thermal neutrons captured in the fuel can be expressed by

$$f = \frac{\Sigma_{ao} V_o \bar{\varphi}_o}{\Sigma_{ao} V_o \bar{\varphi}_o + \Sigma_{ag} V_g \bar{\varphi}_g + \Sigma_{al} V_{al} \bar{\varphi}_{al}}$$

where Σ_a is the macroscopic absorption cross section

V is the volume per unit cell length

$\bar{\varphi}$ is the average flux in V

and the subscripts, o and g, correspond respectively to uranium and graphite, and al designates the aluminum.

At a constant neutron energy, there is no change in the cross sections. This makes $\Sigma_a V$ a constant with temperature; so, if only the uranium temperature is altered

$$\frac{1}{f} = 1 + \frac{\text{constant}}{\bar{\varphi}_o}$$

and

$$\frac{1}{f}\frac{df}{dT} = (1-f)\ \frac{1}{\bar{\varphi}}\frac{d\bar{\varphi}}{dT}$$

The diffusion-theoretic expression for the neutron flux in the uranium is

$$\varphi(r) = \varphi(0)\ I_o\ (\mu r)$$

where $\varphi(0)$ is the neutron flux at the origin and $I_o(\mu r)$ is a modified Bessel's function of the first kind and zero order. The term μ is the reciprocal of the diffusion length in the uranium and is defined by (7, p. 116)

$$\mu^2 = 3\,\Sigma_a\Sigma\,(1-\overline{\cos\theta})\left(1-\frac{4}{5}\frac{\Sigma_a}{\Sigma}+\frac{\Sigma_a}{\Sigma}\frac{\overline{\cos\theta}}{1-\overline{\cos\theta}}\right)$$

$$\mu^2 \approx (\text{constant})\,N_o^2.$$

Here, N_o is the number of nuclei per unit volume. All the terms unaffected by heating the uranium have been lumped into one constant term.

The volume average of the flux is found to be

$$\bar{\varphi} = \frac{\int_o^R \varphi(r)rdr}{\int_o^R rdr} = \frac{2\,I_1(\mu R)}{\mu R}.$$

The product of μV is constant for a constant amount of uranium, or

$$\mu R \sim \frac{1}{R^2}.$$

The change in $\bar{\varphi}$ then becomes

$$\frac{1}{\bar{\varphi}}\frac{d\bar{\varphi}}{dT} = -4\left[\frac{(\mu R)I_o(\mu R)}{2I_1(\mu R)} - 1\right]\frac{1}{R}\frac{dR}{dT}$$

but dR/RdT is the linear coefficient of expansion α so

$$\frac{1}{\bar{\varphi}}\frac{d\bar{\varphi}}{dT} = -4\left[\frac{(\mu R)I_o(\mu R)}{2I_1(\mu R)} - 1\right]\alpha.$$

The first term in the brackets on the right is seen to be the surface to average flux in the uranium. The value of the first term for uranium with a radius of 1.70 cm may be found assuming the following cross sections (13, p. 6)

$$\sigma_s = 8.20 \text{ barns}$$

$$\sigma_a = 7.75 \text{ barns}$$

$$\sigma = \sigma_a + \sigma_s$$

The number of nuclei per unit volume at a uranium density of 19.05 gm/cm^3 is (8, p. 623) $N_o = 0.048 \times 10^{-24}/\text{cm}^3$. The value of the macroscopic cross sections becomes

$$\Sigma_a = 0.375 \text{ cm}^{-1} \qquad \Sigma = 0.767 \text{ cm}^{-1}$$

with $(1-\overline{\cos\theta}) = 0.9972$. Combining these terms, (6, p. 116) it is found that the inverse diffusion length is $\mu^2 = 0.526 \text{ cm}^{-2}$ or $\mu = 0.726 \text{ cm}^{-1}$; then, $I_o(\mu R) = I_o(1.24) = 1.42$ and $I_1(1.24) = 0.747$.

Using a value of the linear coefficient of thermal expansion averaged over the different crystal axes (8, p. 250) $\alpha = 2.0 \times 10^{-5}/^{\circ}\text{C}$

$$\frac{1}{\bar{\varphi}} \frac{d\bar{\varphi}}{dT} = -1.4 \times 10^{-5}/^{\circ}\text{C}.$$

This solution normalizes all values to a value $\varphi(o)$ at the origin, a value that is undefined and yields a resultant magnitude analysis only. The normalizing condition is to a constant incident flux. This results in raising, rather than reducing, the average flux through

the cylinder of uranium for a positive change in the uranium temperature or,

$$\frac{1}{\Phi}\frac{d\Phi}{dT} = 1.4 \times 10^{-5}/^{o}C.$$

The thermal utilization of this perturbed cell as is determined by diffusion theory is as follows: the addition of the aluminum to the cell perturbs the thermal utilization which is written as

$$\frac{1}{f_1} = 1 + \frac{\Sigma_{ag} V_g \bar{\Phi}_g}{\Sigma_{ao} V_o \bar{\Phi}_o} + \frac{\Sigma_{a_{al}} V_{al} \bar{\Phi}_{al}}{\Sigma_{ao} V_o \bar{\Phi}_o}$$

The reciprocal of the thermal utilization of the lattice is represented by the first two terms and is equal to $(0.904)^{-1}$ or 1.106. The ratio of $\Sigma_{a_{al}} V_{al} / \Sigma_{ao} V_o$, as measured, is found to be 0.094

$$\frac{1}{f_1} = 1.106 + 0.094 \frac{I_o(\varkappa R)}{2I_1(\varkappa R)/\varkappa R}$$

$$= 1.215$$

or $$f_1 = 0.823$$

From this, then

$$\frac{1}{f_1}\frac{df_1}{dT} = (1-f_1)\frac{1}{\Phi}\frac{d\Phi}{dT} = \zeta_1(T)$$

$$= 0.25 \times 10^{-5}/^{o}C.$$

The remaining effect on the thermal utilization results from the change in the density of the nitrogen molecules in the vicinity of the heated uranium. Within the outer jacket of the assembly there is a void of 1300 cm^3. At STP the air has an absorption cross section

$\Sigma_a = 7.66 \times 10^{-5}$ cm^{-1} so within the void there is 0.10 cm^2 of thermal neutron absorber. From the gas law $NT = N_oT_o$

$$\frac{\Delta N}{\Delta T} = - \frac{N}{T}.$$

The cross section is proportional to the number of molecules, so the area of absorber, A, at an average temperature $T = 293^oK$

$$\frac{\Delta A}{\Delta T} = - \frac{0.10}{293} = - 3.4 \times 10^{-4} \frac{cm^2}{^oC}$$

In the test position, the introduction of 1.50 cm^2 of absorber will cause a reactivity change of -1 inhour, or

$$\frac{\Delta A}{\Delta \omega} = - 1.50 \frac{cm^2}{ih}.$$

From the above relationships

$$\frac{\delta \omega}{\delta T} = \frac{3.4 \times 10^{-4}}{1.50} = 0.23 \times 10^{-3} \frac{ih}{^oC}.$$

Or, using the weighting theorem

$$\frac{1}{f_2} \frac{df_2}{dT} = \frac{0.23 \times 434}{3.85} 10^{-7}$$

$$= 0.26(10)^{-5}/^oC = \gamma_2(T).$$

From this,

$$\gamma(T) = \gamma_1(T) + \gamma_2(T) = 0.51(10)^{-5}/^oC.$$

The change in resonance escape probability associated with the thermal expansion of the uranium is as follows:

$$P = \exp \left(- \beta A (1 + \mu \frac{S}{M}) \right)$$

where β is now constant with slight changes in the uranium volume

$$\frac{1}{p}\frac{dp}{dT} = -\beta \frac{\partial}{\partial T} A\left(1+\mu \frac{S}{M}\right) = \rho(T)$$

$$(T) = -\beta \frac{A\mu S}{M}\left(\frac{1}{S}\frac{dS}{dT}\right)$$

$$\frac{1}{S}\frac{dS}{dT} = \frac{1}{r}\frac{dr}{dT}$$

where $\frac{1}{r}\frac{dr}{dT}$ is the linear coefficient of thermal expansion

$$\beta = 0.013 \quad \frac{A\mu S}{M} = 3.84 \quad \frac{1}{r}\frac{dr}{dT} = 2 \times 10^{-5}/^{\circ}C$$

$$\rho(T) \approx -0.10\ (10)^{-5}/^{\circ}C.$$

From these results, then

$$\zeta(T) + \rho(T) = 0.41\ (10)^{-5}/^{\circ}C.$$

Interpreting this value in terms of $\frac{\delta\omega}{\delta T}$ for the experiment

$$\frac{\delta\omega}{\delta T} = 0.36(10)^{-3}\ ih\ /^{\circ}C.$$

APPENDIX IV

STATISTICAL ANALYSIS

Test Pile Precision

The precision of the measurements made with the HTP is the result of periodic determination of the change in reactivity resulting from replacing the primary uranium standard fuel elements with the secondary standard fuel elements. The standard deviation of this measured reactivity is then found from

$$\sigma^2 = \frac{N \sum_1^N \delta\omega_i^2 - (\sum_1^N \delta\omega_i)^2}{N(N-1)}$$

Here, N is the number of observation and $\delta\omega$ the measured reactivity change. The value for σ is

$$\sigma = \pm \ 0.01 \text{ ih/measurement.}$$

Curve Fitting for $\delta\omega/\delta T$

A least squares fit to a curve of the form $\omega = a + bT$ in order to establish the coefficient, b, is as follows. To make the sum of the square of the errors a minimum,

$$\frac{\partial}{\partial a} \sum_i (a + bT_i - \omega_i)^2 = 0 = \sum_i (a + bT_i - \omega_i)$$

$$\frac{\partial}{\partial b} \sum_i (a + bT_i - \omega_i)^2 = 0 = \sum_i (aT_i + bT_i^2 - \omega_i T_i)$$

$$b = \frac{N \sum_1^N \omega_i T_i - \sum_1^N \omega_i \sum_1^N T_i}{N \sum_1^N T_1^2 - (\sum_1^N T_i)^2}$$

The standard deviation of the $\delta\omega / \delta T$ is found from the following formula

$$\sigma_b^2 = \frac{\sum_i \omega_i^2 - \frac{(\sum_i \omega_i)^2}{N} - \left[\frac{\sum_i \omega_i T_i - \frac{\sum_i \omega_i \sum_i T_i}{N}}{\sum_i T_i^2 - \frac{(\sum_i T_i)^2}{N}}\right]^2}{(N-2)\left[\sum_i T_i^2 - \frac{(\sum_i T_i)^2}{N}\right]}$$

The standard deviation is multiplied by 2.65, the numerical factor necessary to insure 99 per cent confidence limits at 13 degrees of freedom, the minimum number for any of the data runs.

The average value from the five measurements is found from

$$\bar{b} \pm \overline{\sigma_b}^2 = \frac{\sum_i \frac{b_i}{\sigma_i^2}}{\sum_i \frac{1}{\sigma_i^2}} \pm \frac{1}{\sum_i \frac{1}{\sigma_i^2}}$$

APPENDIX V

DEFINITION OF SYMBOLS

A	Atomic mass number or volume absorption coefficient
A'	Constant
B^2	Geometrical Buckling of a reactor
E	Kinetic energy
H	Heat capacity
K(E)	Collision density per unit logarithmic energy
L^2	Square of the interval distance a thermal neutron can diffuse in the reactor
N	Number
N_{sub}	Number of particles per unit volume
Q	Source strength of neutrons per unit volume
T	Temperature Kelvin scale
C	Temperature Centigrade scale
U	A region or volume of the reactor
V	Volume
W	Maxwell-Boltzmann distribution
W(U)	Statistical weight of a region U
a(E) b(E)	Volume and surface terms relating energy dependence of cross sections to effective uranium cross section in a lump
f	Fraction of the thermal neutrons captured in the uranium (thermal utilization)

g	Subscript denoting graphite
$\hbar$	Planck's constant divided by 2π
i, j	Indices to denote separate conditions
k	Multiplication factor of the lattice
k_{eff}	Multiplication factor of the reactor
ℓ	Angular momentum quantum number
n	Density of neutrons
o	Subscript denoting uranium
p	Resonance escape probability
r	Radius in cm
v	Velocity
Γ_n	Probability of neutron emission
Γ_γ	Probability of gamma emission
Δ	Doppler width of resonance
Σ_a	Macroscopic absorption cross section
Σ_s	Macroscopic scattering cross section
Σ	Macroscopic total cross section
α	$[(a-1)/(A+1)]^2$
β	Constant
γ	Gamma ray or photon
δ_ℓ	Phase shift to determine anisotropy of scattering in the center of mass coordinate system
ε	Fast fission factor or augmentation of multiplication factor by fissions in U-238 at energies above approximately one million electron volts
ξ	Logarithmic energy decrement

$\zeta(T)$	Temperature dependence of the thermal utilization
η	Fast neutrons formed per thermal neutron captured in the uranium
ϑ	Scattering angle of neutrons in the center of mass coordinate system
ρ	Density (gm/cm^3)
σ_a	Microscopic absorption cross section
σ_s	Microscopic scattering cross section
σ	Microscopic total cross section
σ_o	Microscopic peak resonance cross section
τ	Moderation area or Fermi "age"
τ_ℓ	Mean life of a level
λ	DeBroglie wave length divided by 2
φ	Neutron flux product of neutron density and neutron velocity
ω	Eigen state corresponding to stable period of the reactor